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**Time-Selective Signaling and Reception for  
Communication over Multipath Fading Channels**

by

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# Time-Selective Signaling and Reception for Communication over Multipath Fading Channels

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## Abstract

The mobile wireless channel offers inherent diversity by virtue of multipath and Doppler shifts. Multipath diversity is exploited via spread-spectrum signaling employed in code division multiple access (CDMA) systems. However, the RAKE receiver commonly used in CDMA systems exploits only multipath diversity, and consequently suffers from significant performance degradation under fast fading. We develop new signaling and reception schemes in the context of CDMA systems that fully exploit the channel via joint multipath-Doppler diversity. The signaling waveforms are spread in time and frequency. Receiver structures are developed to deal with the inter-symbol interference (ISI) introduced by overlapping successive symbols. Analytical and simulated performance results indicate that the effects of ISI are negligible due to the excellent correlation properties of the spreading codes. Moreover, even the small Doppler spreads encountered in practice can yield significant performance gains. Additionally, the time-selective signaling scheme allows for substantially higher level of diversity and thereby brings the fading channel closer to an additive white Gaussian noise channel. This facilitates the use of error control codes developed for the Gaussian channel.

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# Chapter 1

## Introduction

The demand for mobile wireless communication systems is growing rapidly. This demand, in combination with the limited availability of the radio frequency spectrum, therefore, calls for communication networks that can operate with high bandwidth efficiency. The design of efficient signal modulation and demodulation techniques for physical layer transmission is of utmost importance in this regard.

The design of efficient transmission and reception techniques depends heavily on the nature of the communication channel (physical layer). The problem of reliable communication over an additive white Gaussian noise (AWGN) channel is well understood. However, the several characteristics of the wireless channel make it much more difficult to achieve reliable communication over wireless channels. The most important characteristic is that the wireless channel is a time-varying channel. The transmitted signal arrives at the receiver from different paths with randomly time-varying nature. The mobility of the user also contributes significantly to the time-varying nature of the channel. This results in amplitude variations in the received signal, usually referred to as fading. Multipath fading is a significant limiting factor in the performance of existing mobile communication systems.

The performance of traditional modulation schemes like Phase Shift Keying (PSK) suffer significant degradation on fading channels as compared to their performance on additive white Gaussian noise (AWGN) channels. The probability of error decreases only inversely with signal-to-noise ratio (SNR) in contrast to the exponential decrease



in an AWGN channel [12]. However, acceptable performance can be obtained by the use of redundant signaling.

There have been many approaches to introducing redundancy. One approach has been to develop coding and coded-modulation schemes [8, 20] for fading channels taking note that the design criteria for fading channels are different from those for the AWGN channel. Another method tries to use modulation and demodulation techniques that would combat fading and transform the channel into an AWGN channel [31]. Traditional coding schemes designed for AWGN channels are used in this approach. Since the coding schemes for AWGN channels are well understood, we will focus on the second approach, which tries to design the physical layer communication system to transform fading channels into Gaussian channels.

Diversity signaling is a powerful technique to combat fading and transform the channel into an AWGN channel. This technique provides several independently faded replicas of the same information signal at the receiver. It significantly reduces the probability that all the signal replicas suffer from amplitude fading at the same time. Frequency, time and space diversity are commonly used forms of diversity [12]. The performance of diversity signaling schemes and their relationship with coding need to be explored to distinguish between these approaches.

The mobile wireless channel offers inherent diversity, by virtue of multipath and Doppler shifts [16, 17, 1]. Multipath shifts, that arise due to the different delays of the propagation paths, provide time-shifted copies of the information signal. Doppler shifts arise due to the time variations and provide frequency-shifted copies of the information signal. This inherent diversity can be completely exploited by the appropriate choice of signaling and reception schemes.

Multipath diversity is exploited by spread-spectrum signaling and multipath RAKE reception employed in existing Code Division Multiple Access (CDMA) communica-

tion systems [22]. In spread-spectrum signaling, the signals used for transmission have a bandwidth much greater than the information rate. The RAKE receiver then is able to use this information to distinguish between different time-shifted paths and combine them constructively. However, this multipath RAKE receiver does not exploit Doppler diversity and consequently suffers significant performance degradation under fast fading conditions. We develop signaling and reception schemes that take advantage of multipath and Doppler diversity jointly.

There are other existing methods that exploit channel variations. However, the mechanism by which improved performance is achieved - Doppler diversity - is not clearly identified and is therefore not completely exploited. One commonly used technique to exploit the time-variations in the channel is interleaving [12, 28]. Interleaving is a procedure by which the coded symbols are reordered so that successively transmitted symbols will suffer almost independent fading. This allows us to exploit time diversity in the presence of an error control code. The efficiency of various interleaving schemes are studied in [4, 14].

Another approach that has been recently proposed in [30, 31] uses symbols that are spread in time to transform an arbitrary Rayleigh fading channel into a non-fading marginally Gaussian noise channel. Diversity is implicitly exploited by the process of time-spreading and equalization. Since transmitted symbols overlap in time, the spreading codes are chosen to achieve low inter-symbol interference (ISI) levels. Thus, ISI need not be dealt with at the receiver. As the spreading codes are obtained by optimizing over the real-valued space to minimize ISI, the resulting codes are also real valued. Real-valued codes may not be desirable in practice due to complexity of implementation.

There has also been an attempt to design codes appropriate for systems with multiple transmit antennas (space diversity) [21]. This method also exploits time-variations in the channel.

It has also been shown in [25] that independent and identically distributed diversity channels bring the fading channel towards an AWGN channel asymptotically as the number of diversity channels increases, and that coded-modulation techniques developed for the AWGN channel perform well on such a diversity channel.

We concentrate on completely exploiting inherent channel diversity and studying its effect on the choice of the coding scheme.

## 1.1 Contribution

We propose and evaluate time-selective signaling and reception schemes that jointly exploit multipath and Doppler diversity. The use of these signaling and reception techniques result in

- significant performance gain compared to existing wireless communication systems that exploit only multipath diversity
- asymptotic convergence of the fading channel to an AWGN channel as the number of diversity components increases

We later study the relationship between diversity and error control coding using simulations. The simulation results indicate that

- error control codes designed for an AWGN channel can be effectively used for a fading channel with diversity
- the performance of the code can be enhanced by additional diversity

We also show using simulations that the proposed signaling scheme performs better than a block interleaving scheme under a delay constraint. Since our focus is on the effects of the channel, we restrict ourselves to the single-user case.

Our scheme is based on a recently developed [17] representation for the multipath fading channel. It decomposes the channel into a series of independent fading channels corresponding to different multipath-Doppler-shifted signal components. Under our scheme, the signaling waveforms are spread both in time and frequency. The spreading codes are chosen to be binary valued. The time-spreading of the signal aids the receiver in constructively combining the various Doppler shifted signals received. Although the time duration of the signaling waveform is increased, the data rate is maintained by allowing symbols to overlap with each other. This introduces intersymbol interference (ISI). The time-frequency channel representation and the signaling scheme are described in Chapter 2.

A time-frequency RAKE receiver [17] is used for combined multipath-Doppler diversity. An optimal maximum likelihood sequence detector is developed to deal with ISI. Due to its computational complexity, a sub-optimal linear decorrelating detector (zero-forcing equalizer) with reduced computational complexity is also derived. Analytical and simulated performance results indicate that the effect of ISI is relatively negligible compared to the diversity gain achieved in realistic situations. Also, the mechanism by which improved performance is achieved, Doppler diversity, is clearly identified and exploited. The various reception techniques and their performance are discussed in Chapter 3.

We derive the constraints on the channel for the asymptotic convergence to an AWGN channel as the number of diversity components increases. While a previous study [25], proves the asymptotic convergence of a diversity channel to an AWGN channel for independent and identically distributed diversity channels, it is not di-

rectly applicable to the time-selective signaling scheme since the diversity channels (multipath and doppler shifted signals) do not necessarily have identical strengths. Based on this convergence result, it can be expected that coding schemes developed for the Gaussian channel can be effectively used.

We study the effect of diversity on the coding scheme employed using the example of a simple convolutional code under realistic channel and mobility assumptions. The simulation results confirm the fact that diversity brings the fading channel closer to an AWGN channel. Besides the diversity gain achieved due to the additional diversity exploited, the coding gain is also improved in certain cases due to time-spreading.

Unlike interleaving, the time-selective signaling scheme provides performance improvements even in an uncoded system. Also, we show by simulations that the time-selective signaling and reception scheme performs better than a block interleaving scheme under a delay constraint in a coded system. Often, such a delay constraint is imposed by the nature of data being supported by the communication system. The asymptotic analysis and the relationship between diversity and coding are discussed in Chapter 4.

In Chapter 5, we discuss related work and finally, in Chapter 6, we present our conclusions and directions for future research.

## Chapter 2

### Time-Selective Signaling

The determination of optimal modulation and demodulation techniques depends heavily on the accurate characterization of the communication channel. In this chapter, we will discuss the characterization of the mobile wireless channel. The final representation of the channel output signal will be obtained in the context of code division multiple access (CDMA) schemes. Therefore, spread-spectrum signaling and code division multiple access communications will be introduced before we obtain the final channel and signal representation. We will then use this representation to motivate the choice of the signaling scheme.

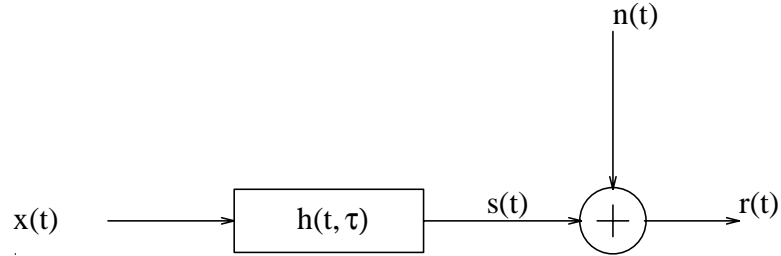
#### 2.1 Channel Characterization

The received signal at the output of a mobile wireless channel is a superposition of multiple signals arriving from different paths. Each of these paths generally have different delays and attenuation. If the delays and attenuation are constant over time, i. e., a static multipath channel, the channel impulse response can be characterized by a time-invariant function of the delay. However, the characteristics of the different paths vary with time in a realistic channel. Thus, the impulse response of the channel has to be a function of time.

The input-output relation for a linear time-varying channel with additive white Gaussian noise (AWGN), shown in figure 2.1, can be written in the form [1, 12]

$$r(t) = s(t) + n(t) = \int_0^{\infty} h(t, \tau)x(t - \tau)d\tau + n(t) \quad (2.1)$$

where  $h(t, \tau)$  is the time-varying impulse response that describes the channel,  $x(t)$  is the transmitted signal and  $n(t)$  is the AWGN. The variable  $\tau$  represents the multipath delays. The *spreading function* of the channel,  $H(\theta, \tau)$ , is defined as



**Figure 2.1** Mobile wireless channel : Linear time-varying channel

$$H(\theta, \tau) = \int h(t, \tau)e^{-j2\pi\theta t} dt \quad (2.2)$$

where  $\theta$  corresponds to the Doppler shifts (frequency shifts) introduced by the channel. The corresponding representation of  $s(t)$  is given by

$$s(t) = \int \int H(\theta, \tau)x(t - \tau)e^{-j2\pi\theta t} d\theta d\tau. \quad (2.3)$$

The spreading function  $H(\theta, \tau)$  quantifies the time-frequency spreading introduced by the channel.  $x(t - \tau)e^{-j2\pi\theta t}$  represents a time-shifted (by  $\tau$ ) and Doppler shifted (by  $\theta$ ) version of the transmitted signal. Thus, an arbitrary time-varying linear system admits such a representation in terms of time and frequency shifts [1].

The mobile wireless channel is a randomly time-varying channel. Therefore, the time-variant channel impulse response  $h(t, \tau)$  is best modeled by a stochastic process.

The wide-sense stationary uncorrelated scatterer (WSSUS) model [1, 12] with  $h(t, \tau)$  represented as a stationary complex Gaussian process is a realistic model. In this model, the channel responses at different delays (different paths) are assumed to be uncorrelated. Thus, the channel is characterized by second-order statistics given by (assuming a zero-mean process, which corresponds to Rayleigh fading)

$$E\{H(\theta_1, \tau_1)H^*(\theta_2, \tau_2)\} = \Psi(\theta_1, \tau_1)\delta(\theta_1 - \theta_2)\delta(\tau_1 - \tau_2) \quad (2.4)$$

where

$$\Psi(\theta, \tau) \triangleq E\{|H(\theta, \tau)|^2\} \quad (2.5)$$

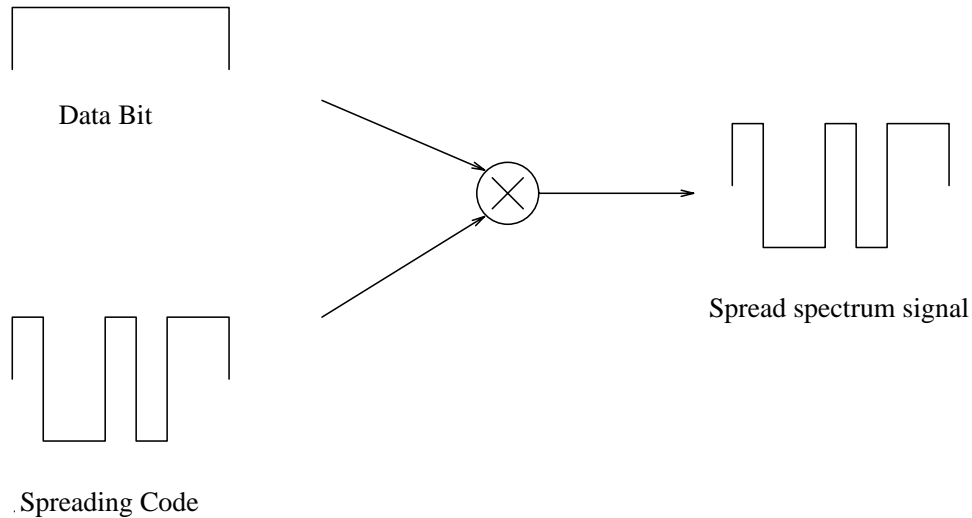
The maximum ranges of  $\tau$  and  $\theta$  over which  $\Psi(\theta, \tau)$  is essentially non-zero are called the *multipath spread* (denoted by  $T_m$ ) and *Doppler spread* (denoted by  $B_d$ ) respectively.

## 2.2 Spread-Spectrum Signaling and CDMA Systems

Spread-spectrum signals use a transmission bandwidth much greater than the information rate. This inherent redundancy is used to overcome severe interference from other users of the same channel. Also, pseudo randomness is introduced to make the signals appear like random noise to other receivers.

Direct sequence (DS) and frequency-hopped (FH) spread spectrum signals are two common types of spread-spectrum signals. They differ in way the data bit is modulated into a spread spectrum signal. In frequency hopping, a pseudo-random sequence is used to select the frequency of the transmitted signal. In direct sequence spread spectrum signaling, the data bit is directly multiplied by a pseudo-random sequence called the *spreading* sequence (or spreading code) to spread its spectrum as shown in figure 2.2. A detailed discussion of the properties of spread-spectrum signals can be found in [12].





**Figure 2.2** Direct sequence spread-spectrum signaling

The multiple access scheme based on spread-spectrum signaling, where each user is identified by a pseudo-random code, is the *code division multiple access* (CDMA) scheme. Each user is allowed to use the entire available bandwidth at all times. The signals of the various users are separated at the receiver using the spreading code assigned to each user. Ideally, we would like the spreading codes to be orthogonal so that the interference can be completely mitigated. However, this is impossible because of the asynchronous nature of the transmissions of different users. Therefore, spreading codes that possess reasonably low cross-correlation properties for all relative delays are used. Low auto-correlation values are also desired to distinguish between the multiple paths received.

Other multiple access schemes like *time division multiple access* (TDMA) and *frequency division multiple access* (FDMA) are also used in existing cellular communication systems. These schemes partition the available time duration or bandwidth among the different users to distinguish between them. One of the main advantages of CDMA schemes over these multiple access techniques is its ability to reuse the same

spectrum in all cells, since users are distinguished based on their spreading code. In TDMA and FDMA schemes, adjacent cells cannot use the same time duration or bandwidth. Another important characteristic of existing CDMA systems is that they can distinguish between the multiple paths received and improve performance. Time-selective signaling and reception schemes, to be developed in the following sections, add to this list of advantages by enabling the receiver to distinguish between frequency shifted copies of the signal as well. All these techniques provide ways to increase the bandwidth efficiency of the system and thereby help support more users.

Henceforth, we will restrict our discussion to DS spread-spectrum systems.

### 2.3 Time-Selective Signaling

Considering a spread-spectrum signaling system with symbols spread in time, the transmitted signal  $x(t)$  can be represented as

$$x(t) = \sqrt{E_s} \sum_i b^i q(t - iT) \quad (2.6)$$

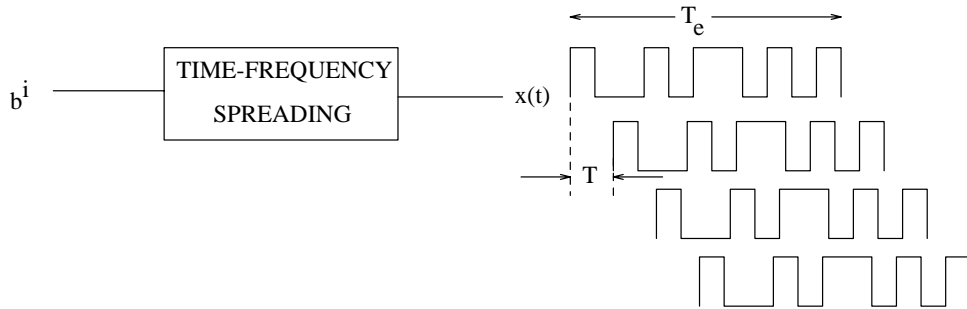
where  $b^i$  are the transmitted bits ( $\pm 1$ ),  $E_s$  is the signal energy per bit,  $q(t)$  is the spreading code,  $1/T$  is the data rate and  $q(t)$  has time support  $T_e > T$ . Using equation (2.3), a time-frequency sampled representation can be obtained [17]. This sampled representation is possible since the transmitted symbol is limited in time over the interval  $[0, T_e]$  and approximately band-limited in frequency. Therefore, the received signal can be expressed as

$$r(t) \approx \sqrt{E_s} \frac{T_c}{T_e} \sum_i b^i \sum_{n=0}^{N-1} \sum_{k=-P}^P \hat{H}^i\left(\frac{k}{T_e}, nT_c\right) u_{n,k}^i(t) + n(t) \quad (2.7)$$

where

$$u_{n,k}^i(t) = q(t - iT - nT_c) e^{j\frac{2\pi kt}{T_e}}, \quad N = \left\lceil \frac{T_m}{T_c} \right\rceil + 1, \quad K \triangleq 2P + 1 = \lceil B_d T_e \rceil + 1$$

$\hat{H}^i(\frac{k}{T_e}, nT_c)$  are samples of the spreading function of the band-limited version of the channel,  $T_m$  is the multipath spread,  $B_d$  is the Doppler spread and  $T_c$  is the chip duration. The received signal is, therefore, a randomly weighted linear combination of time and frequency shifted versions of the transmitted signal or, equivalently, a diversity of  $NK$  is available. Equation (2.7) clearly indicates that making the channel time-selective by spreading the symbol in time (increasing  $T_e$ ) increases the Doppler diversity ( $K$ ) achievable. Therefore, we choose  $T_e$  large enough to be able to exploit the Doppler diversity. We can always make the channel vary within the symbol duration by increasing the symbol duration, as long as the channel is not a static channel. The time-signaling signaling scheme is illustrated in figure 2.3. Each of the symbols shown is spread in spectrum using a spreading code and is spread in time to a duration  $T_e$ . However, since symbols overlap because of spreading, we need to



**Figure 2.3** Time-selective signaling

analyze the effect of ISI on the performance. This is done in Chapter 3.

## Chapter 3

### Reception Techniques and Performance

In this chapter, we will develop optimal and sub-optimal reception techniques for the signaling scheme proposed in Chapter 2 and discuss their performance. We have seen in Chapter 2 that, to exploit Doppler diversity, the symbol waveform has to be spread in time. However, we need to overlap symbols to maintain the data rate. Therefore, ISI is introduced. Receivers that can combine multipath and Doppler diversity need to be developed. Also, due to ISI, all symbols need to be detected jointly for optimal detection since at the very least each symbol overlaps with the preceding and succeeding symbol. Thus, there are two functions of the receiver prior to detection, diversity combining and ISI cancellation.

#### 3.1 Reception Techniques

##### 3.1.1 Optimal Detector

The optimal detector is the Maximum Likelihood Sequence Detector. This detector can be developed as follows [23, 6, 5]. Conditioned on the channel coefficients and the bit sequence, the log likelihood function corresponds to that of a complex Gaussian process and can be written as

$$\Lambda(r(t)/\{h_{n,k}^i\}, \{b^i\}) = - \int_{-\infty}^{\infty} \left| r(t) - \sum_i \sum_{n,k} b^i h_{n,k}^i u_{n,k}^i(t) \right|^2 dt \quad (3.1)$$

where for simplicity of expression  $\frac{T_c}{T_e} \hat{H}(\frac{k}{T_e}, nT_c) \triangleq h_{n,k}^i$ .

Neglecting terms independent of the bit sequence, we can reduce this expression to obtain an iterative metric given by

$$\sum_i \left\{ \sum_{n,k} 2 \operatorname{Re} (b^i z_{n,k}^i h_{n,k}^{i*}) - \sum_{j=i-B+1}^{i-1} b^i b^j \sum_{n',k',n,k} h_{n',k'}^{i*} h_{n,k}^j R_{n',k',n,k}^{i,j} \right\} \quad (3.2)$$

where

$$z_{n,k}^i = \int r(t) u_{n,k}^{i*}(t) dt \quad R_{n',k',n,k}^{i,j} = \int u_{n',k'}^{i*}(t) u_{n,k}^j(t) dt$$

It is clear that the first term in the metric corresponds to maximal ratio combining [3] of the multipath-Doppler diversity components. The other term corresponds to the ISI. When the correlations between the different time and frequency shifted spreading codes ( $R_{n',k',n,k}^{i,j}$ ) are zero, there is no ISI. Therefore, the diversity components would be uncorrelated and maximal ratio combining would be optimal.

Since this metric can be expressed as a summation over  $i$  where each term of the summation can be computed at time interval  $i$  and is dependent only on a finite number of bits ( $B$ ), we can implement this detection scheme using the Viterbi algorithm.  $B$  is determined by the number of bits that overlap because of the spreading in time. A trellis can be built with  $2^{B-1}$  states and sequence detection performed. Although this detection technique is optimal, we note that the number of states to be considered is exponentially complex in the number of bits that overlap. Therefore, sub-optimal linear detectors that are less complex computationally but still perform reasonably well will be developed.

### 3.1.2 Sub-optimal Linear Detectors

We know from equations (2.7) and (3.2) that given the channel coefficients,  $\{h_{n,k}^i\}$ , the sufficient statistics for detecting the block of symbols corresponds to the outputs of the matched filters,  $z_{n,k}^i$ , corresponding to each of the time and frequency shifted

codes  $u_{n,k}^i(t)$ . To develop the sub-optimal receiver, we will rewrite the output of the matched filters conveniently in matrix notation using the following definitions. The statistics  $\mathbf{z}$  is defined as the  $(2I+1)NK \times 1$  vector of the matched filter outputs,  $\mathbf{b}$  is the  $(2I+1) \times 1$  vector of bits,  $\mathbf{h}^i$  is the  $NK \times 1$  vector of channel coefficients for the  $i^{\text{th}}$  symbol and  $\mathbf{n}$  is the  $(2I+1)NK \times 1$  vector of noise components at the outputs of each of the matched filters. The channel coefficients are written in matrix form,  $\mathbf{H} \in \mathbb{C}^{(2I+1)NK \times (2I+1)}$ , as

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}^{-I} & 0 & \cdots & 0 \\ 0 & \mathbf{h}^{-I+1} & 0 & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \mathbf{h}^I \end{bmatrix} \quad (3.3)$$

Therefore, we can write,

$$\mathbf{z} = \mathbf{R}\mathbf{H}\mathbf{b} + \mathbf{n} \quad (3.4)$$

where  $\mathbf{R}$  is a  $(2I+1)NK \times (2I+1)NK$  matrix defined as

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_{-I,-I} & \mathbf{R}_{-I,-I+1} & \cdots & \mathbf{R}_{-I,I} \\ \mathbf{R}_{-I+1,-I} & \mathbf{R}_{-I+1,-I+1} & \cdots & \mathbf{R}_{-I+1,I} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{R}_{I,-I} & \mathbf{R}_{I,-I+1} & \cdots & \mathbf{R}_{I,I} \end{bmatrix} \quad (3.5)$$

and

$$\mathbf{R}_{i,j} = \int_{-IT}^{IT+T_e} \mathbf{u}^{i*}(t)\mathbf{u}^j(t)^H dt \quad (3.6)$$

Thus, a number of detectors similar to those developed in [9, 12, 19, 24, 32] can be applied here. One sub-optimal linear detector that we will consider is the decorrelating detector. The detector can be represented as

$$\hat{\mathbf{b}} = \text{sign}\{\text{Re}(\mathbf{H}^H \mathbf{D} \mathbf{R}^{-1} \mathbf{z})\} \quad (3.7)$$

In the case of the decorrelating detector (zero-forcing equalizer),  $\mathbf{R}^{-1}$  is the stage that removes the ISI. But, because of this, the noise at the output of the zero-forcing stage is enhanced and is correlated across the various diversity branches. Therefore, a noise whitening stage is applied to the output before maximal ratio combining. As will be seen from the performance analysis and the simulation results, the effect of ISI is not significant compared to the gains achieved due to increase in Doppler diversity. Thus, a *one-shot* detector ignoring the ISI given by

$$\hat{\mathbf{b}} = \text{sign}\{\text{Re}(\mathbf{H}^H \mathbf{z})\} \quad (3.8)$$

can be used. This detector performs only the linear combining of the various multipath and Doppler diversity components. Therefore, significant diversity gain can be achieved compared to systems that exploit only multipath diversity at the low cost in complexity of just increasing the duration of the spreading code and using the corresponding matched filters at the receiver for the various diversity components.

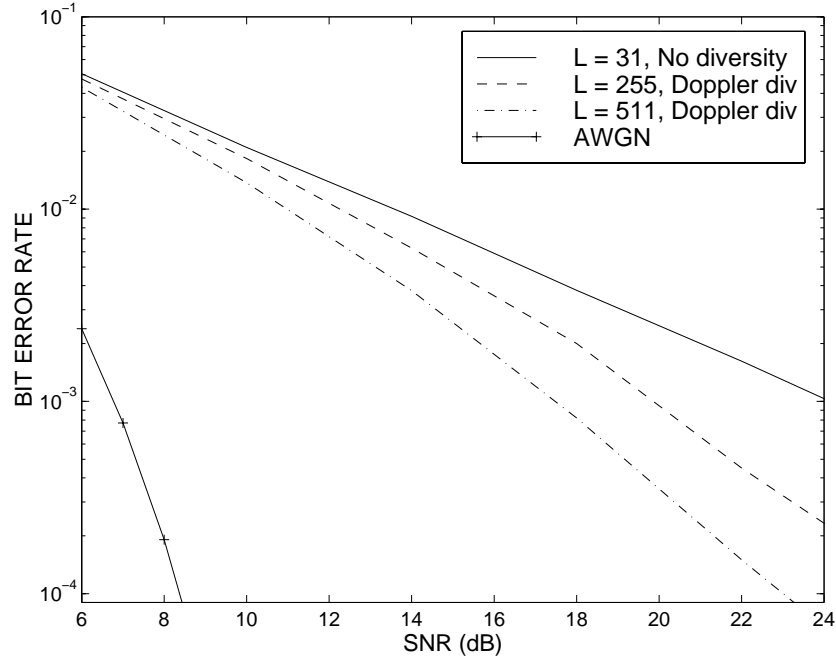
## 3.2 Performance Analysis and Numerical Results

### 3.2.1 Diversity Gain

In this section, we discuss the simulation results and their implications and analyze the performance of the techniques for time-selective signaling and reception proposed earlier.

In our simulations, a spread-spectrum signaling system using a spreading code of length 31 (or 15) with non-overlapping symbols and employing the multipath RAKE receiver, whenever multipath diversity is achievable, is used for comparison. Time-selective signaling systems with overlapping codes of length 255 and 511 (in chips) with a new symbol transmitted every 31 (or 15) chips and using time-frequency RAKE

receivers proposed are compared with this system. The Jakes fading channel model with a carrier frequency of 1.8 GHz and mobile speed of 50 miles/hr (which gives a Doppler spread of about 130 Hz) is used at a data transmission rate of 10 kHz.

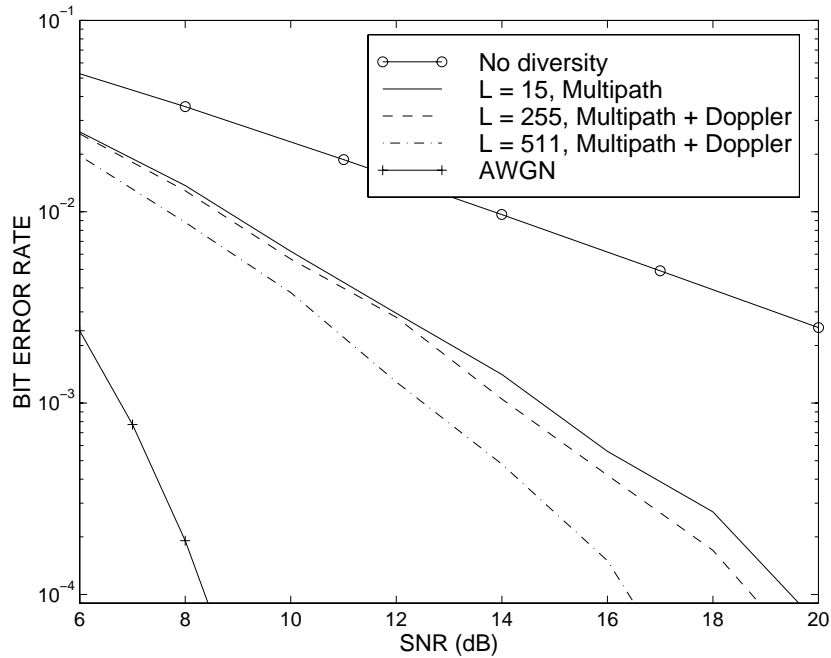


**Figure 3.1** Doppler diversity gain: Single path case - Data Rate = 10 kHz,  $L$  = Spreading code length in chips,  $T/T_c = 31$ , Mobile Speed = 50 miles/hr.

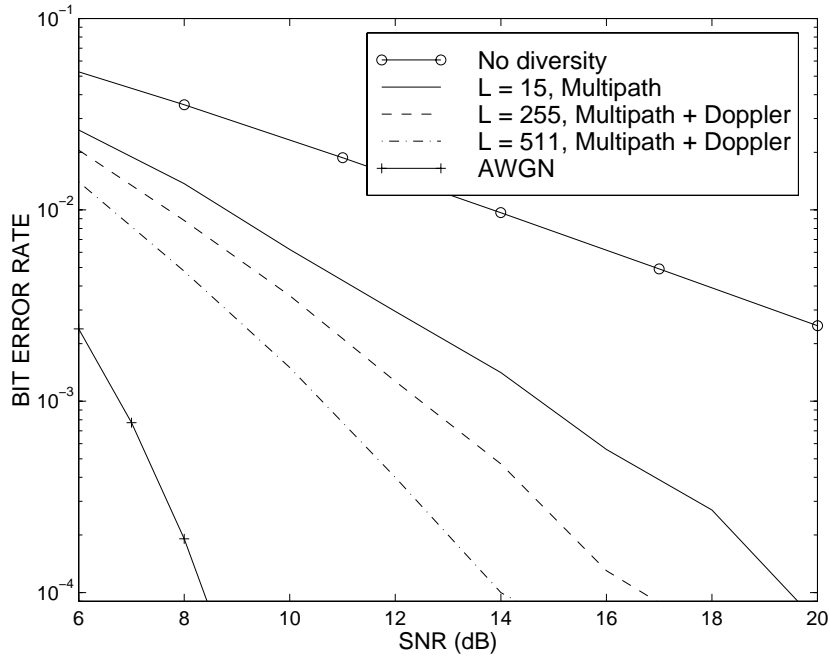
Figures 3.1, 3.2 and 3.3 summarize the simulated performance of the decorrelating receiver for different lengths of temporally-spread symbols, and different number of multipath components. Figure 3.1 corresponds to a single path, whereas figures 3.2 and 3.3 show results for a frequency-selective fading channel with two paths, with the mobile speed being 50 and 100 miles per hour respectively. The performance of a fading channel with no diversity and an AWGN channel are also shown in all the three figures for comparison. As evident from the figures, time-selective signaling can deliver significant gains, especially for longer code lengths. For example, in figure 3.1,



the system with a spreading code length of 511 chips is about 6 dB better than the conventional system with a spreading code length of 31 chips at  $P_e \approx 10^{-3}$ . It should be also noted that the performance for  $L = 255$  at 100 miles per hour in figure 3.3 is almost the same as the performance for  $L = 511$  and a mobile speed of 50 miles per hour in figure 3.2. Therefore, the same performance gain obtained at 100 miles per hour can be obtained at 50 miles per hour by increasing the symbol duration by a factor of 2. This demonstrates the ability of the signaling scheme to take advantage of channel characteristics appropriately by the choice of signal duration.



**Figure 3.2** Doppler diversity gain : Two path case - Data Rate = 10 kHz,  
 $L$  = Spreading code length in chips,  $T/T_c = 15$ , Mobile Speed = 50 miles/hr,  
 Ratio of average energy of the two paths = 2:1.



**Figure 3.3** Doppler diversity gain : Two path case - Data Rate = 10 kHz,  $L$  = Spreading code length in chips,  $T/T_c = 15$ , Mobile speed = 100 miles/hr, Ratio of average energy of the two paths = 2:1.

### 3.2.2 Performance of the Decorrelating Detector

Now, we will study the performance of the decorrelating detector analytically. At the output of the decorrelating stage, there is no interference between the different overlapping symbols, and the noise is distributed as  $\text{Gaussian}(\mathbf{0}, \mathbf{R}^{-H})$ . Thus, the output block (of multipath-Doppler components) corresponding to a symbol of interest (say  $i^{\text{th}}$  bit) is corrupted by a noise vector that is  $\text{Gaussian}(\mathbf{0}, (\mathbf{R}^{-H})_i)$  where  $(\mathbf{R}^{-H})_i$  is the block of  $\mathbf{R}^{-H}$  corresponding to the  $i^{\text{th}}$  bit. The noise whitening filter is given as  $\mathbf{D} = \mathbf{C}^{-1}$  where  $\mathbf{C}$  is obtained by the Cholesky factorization of  $(\mathbf{R}^{-H})_i$ . The probability of bit error after combining these diversity branches can be obtained

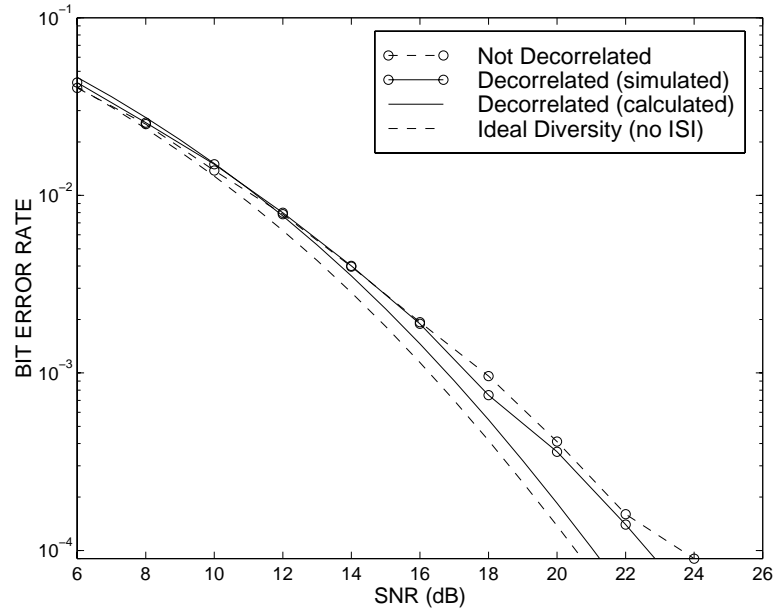
as [10, 19, 32]

$$P_e = \frac{1}{2} \sum_{k=1}^P \pi_k \left[ 1 - \sqrt{\frac{\gamma_k}{1 + \gamma_k}} \right] \quad (3.9)$$

where  $\pi_k$  is defined as

$$\pi_k = \prod_{i=1, i \neq k}^P \frac{\lambda_k}{\lambda_k - \lambda_i} \quad (3.10)$$

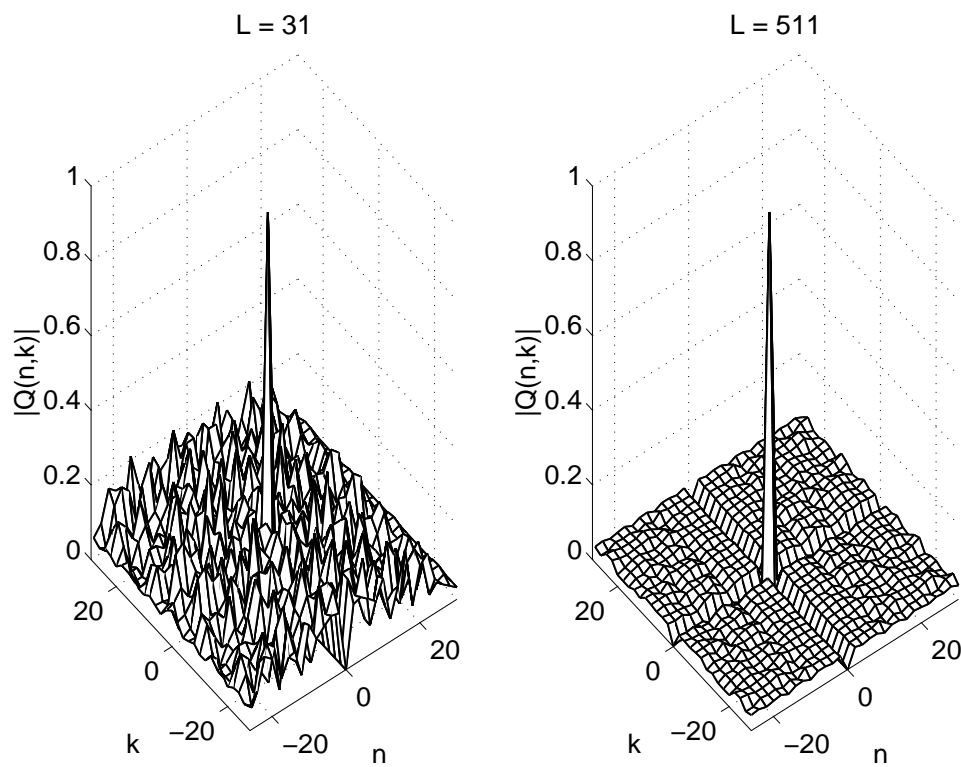
$\gamma_k = \lambda_k E_s / \mathcal{N}_0$  is the effective average SNR for the  $k^{\text{th}}$  diversity branch where  $\lambda_k$  is the  $k^{\text{th}}$  distinct eigenvalue of  $\mathbf{\Phi}_i(\mathbf{R}^{-H})_i^{-1}$ ,  $\mathbf{\Phi}_i$  is the covariance matrix of the channel coefficients corresponding to the diversity components of the  $i^{\text{th}}$  bit and the total number of diversity components  $P = NK$ .



**Figure 3.4** Comparison of the decorrelating detector with the one-shot detector - Data Rate = 10 kHz,  $L$  = Spreading code length in chips = 511,  $T/T_c = 31$ , Mobile speed = 50 miles/hr.

This analytical result is now compared with the simulation results for performance with and without the decorrelating stage and also with the analytical performance

of an ideal diversity system with no ISI. The comparison is shown in figure 3.4 and clearly shows the negligible effect of overlapping symbols on performance. First of all, we observe from figure 3.4 that the simulated performance of a decorrelating receiver (solid line with circles for simulation points) agrees well with the analytical result (solid line) in equation (3.9). The performance without the decorrelating stage (dashed line with circles) is very close to the performance of the decorrelating detector. Also, all of these systems perform almost as well as an ideal diversity system with no ISI (dashed line). Thus, we can conclude that ISI is not significant.



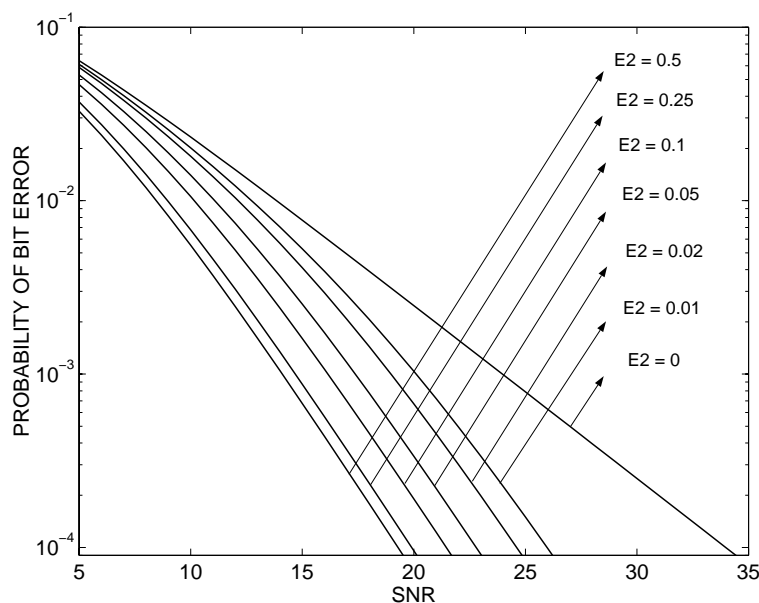
**Figure 3.5** Correlation function of spreading code (m-sequence) for time and frequency shifts,  $L =$  Length of the spreading code

The lack of degradation with larger overlap can be attributed to the correlation properties of m-sequences. As the length of spreading code increases, the number of

symbols that overlap increases. However, the orthogonality properties of the code also improve, resulting in reduced ISI contribution due to each overlapping symbol. The net effect is virtually negligible ISI. The improvement in the time-frequency correlation properties of the spreading codes is illustrated in figure 3.5 which plots the magnitude of the time-frequency correlation function

$$Q(n, k) = \int q(t)q(t - nT_c)e^{-\frac{j2\pi kt}{T_e}} dt \quad (3.11)$$

for two different lengths. The correlation values are much smaller for  $L = 511$  than for  $L = 31$ .



**Figure 3.6** Bit error probability versus SNR for a two branch diversity system

We will now illustrate that the gains shown in previous examples are possible with small Doppler spreads, i.e.,  $B_d T_e \approx 0.1, 0.2$  for  $L = 255, 511$  respectively. For such

Doppler spreads, the strength of the signal captured by the Doppler matched-filter is about 1-2 % of the total signal strength, corresponding to the projection of the Doppler shifted-signal on to matched-filter waveform. To illustrate this significant gain for such small signal strengths in diversity component, we show the performance of a system with 2 diversity branches under varying proportions of energies in them in figure 3.6. These are calculated analytically [12]. We see that most of the gain in performance is achieved with 1 – 2% of the energy in the additional diversity component ( $E_2 = 0.01, 0.02$ ).

Performance gains, implicitly due to Doppler diversity, have also been reported in other non-CDMA systems [6] for  $B_d T_e \approx 0.1$ . However, we note that non-spread spectrum systems cannot exploit both multipath and Doppler diversity simultaneously due to a small time-bandwidth product (TBP) of nearly 1. CDMA systems have a unique ability to exploit joint multipath-Doppler diversity by achieving arbitrarily large TBP. Time-selective signaling corresponds to increasing the TBP of the system by increasing the time-spread of the signaling waveform.

## Chapter 4

### Diversity and Coding

We have seen in Chapter 3 that Doppler diversity can significantly improve the performance of communication systems over multipath fading channels. In this chapter, we will evaluate the asymptotic behavior of the time-selective signaling and reception scheme developed in Chapters 2 and 3 as the number of diversity components increases. Also, in almost all communication systems error control codes have to be used in addition to other physical layer performance enhancement techniques (like diversity) to achieve acceptable performance. Coding introduces redundancy over time and exploits the time diversity in the channel. Therefore, it is important to investigate the effect of diversity, exploited in the receivers developed in Section 3, on the performance of the coding scheme. The extent to which coding is needed for these systems also need to be investigated. We study the effect of diversity on coding using the example of a rate  $1/2$ , constraint length 3 convolutional code. Using the same convolutional code, the proposed scheme is also compared with a  $N \times N$  block interleaving scheme under a delay constraint.

#### 4.1 Asymptotic Behavior of the Signaling Scheme

Now, we will discuss the asymptotic behavior of the proposed system when the number of Doppler diversity components ( $K$ ) is increased by increasing the time spread ( $T_e$ )

of the signal. The received signal is represented as

$$r(t) = \sqrt{E_s} \sum_i b^i \sum_{n=0}^{N-1} \sum_{k=0}^{K-1} h_{n,k}^i u_{n,k}^i(t) + n(t) \quad (4.1)$$

where we have relabeled the index  $k$  to run from 0 to  $K - 1$  for convenience. The coherently combined output of the matched filters, corresponding to the various diversity components, can be written as

$$Z^i = S^i + N^i = \sqrt{E_s} \sum_{n=0}^{N-1} \sum_{k=0}^{K-1} |h_{n,k}^i|^2 b^i + \sum_{n=0}^{N-1} \sum_{k=0}^{K-1} h_{n,k}^{i*} n_{n,k}^i \quad (4.2)$$

under the following orthogonality assumption

$$\langle u_{n,k}^i(t), u_{n',k'}^j(t) \rangle = \int u_{n,k}^i(t) u_{n',k'}^{j*}(t) dt = \begin{cases} 1 & \text{when } (i, n, k) = (j, n', k'), \\ 0 & \text{else.} \end{cases} \quad (4.3)$$

**Lemma 4.1** If  $\{h_{n,k}^i\}$  are zero-mean independent complex Gaussian random variables and conditions

1.  $\sum_{n=0}^{N-1} \sum_{k=0}^{K-1} E[|h_{n,k}^i|^2] = C$  (constant) (4.4)

2. The Lindeberg condition (see Appendix A, [2]) for Central Limit Theorem (CLT) for the set of random variables  $\{h_{n,k}^{i*} n_{n,k}^i\}$

3.  $\lim_{K \rightarrow \infty} \sum_{n=0}^{N-1} \sum_{k=0}^{K-1} E^2[|h_{n,k}^i|^2] = 0$  (4.5)

are satisfied, then

$$Z^i \rightarrow \text{Gaussian}(C\sqrt{E_s}, CN_0) \text{ as } K \rightarrow \infty \quad (4.6)$$

where  $\mathcal{N}_0 = E[|n(t)|^2]$ .



**Proof** We know that  $n(t)$  is a zero mean complex Gaussian process with  $E[|n(t)|^2] = \mathcal{N}_0$ . Also,  $n(t)$  is independent of  $\{h_{n,k}^i\}$ . Therefore,

$$E[N^i] = 0 \quad \text{and} \quad \text{Var}[N^i] = \sum_{n=0}^{N-1} \sum_{k=0}^{K-1} E[|h_{n,k}^i|^2] \mathcal{N}_0 \quad (4.7)$$

Using CLT on  $N^i$  [2] and equation (4.4), we can see that

$$N^i \rightarrow N(0, C\mathcal{N}_0) \quad (4.8)$$

The Lindeberg condition is a sufficient condition for convergence of  $N^i$ . Let  $b^i = 1$ .

Now, we consider  $S^i$ .

$$E[S^i] = \sqrt{E_s} \sum_{n=0}^{N-1} \sum_{k=0}^{K-1} E[|h_{n,k}^i|^2] \quad (4.9)$$

and

$$\text{Var}[S^i] = E_s \sum_{n=0}^{N-1} \sum_{k=0}^{K-1} \left\{ E[|h_{n,k}^i|^4] - E^2[|h_{n,k}^i|^2] \right\} \quad (4.10)$$

Since  $h_{n,k}^i$  are zero-mean complex Gaussian random variables, we have [15]

$$E[|h_{n,k}^i|^4] = 2E^2[|h_{n,k}^i|^2] \quad (4.11)$$

Therefore,

$$\text{Var}[S^i] = E_s \sum_{n=0}^{N-1} \sum_{k=0}^{K-1} E^2[|h_{n,k}^i|^2] \quad (4.12)$$

Also, since the additive noise is independent of the fading coefficients,  $S^i$  and  $N^i$  are uncorrelated. Now,

$$S^i \rightarrow E[S^i] = C\sqrt{E_s} \quad (4.13)$$

if equation (4.5) holds. Combining equations 4.13 and 4.8, we get

$$Z^i \rightarrow N(C\sqrt{E_s}, C\mathcal{N}_0) \quad (4.14)$$

□

The three conditions impose constraints on the channel to ensure convergence to an additive Gaussian noise channel with Signal to Noise Ratio (SNR) given as  $CE_s/\mathcal{N}_0$ . Equation (4.4) ensures that the sum of the energies of the channel coefficients corresponding to the various diversity branches is finite. The Lindeberg condition is a sufficient condition for CLT. In addition to convergence it also imposes the constraint that [2]

$$\max_k \frac{E[|h_{n,k}^i|^2]}{\sum_{n=0}^{N-1} \sum_{k=0}^{K-1} E[|h_{n,k}^i|^2]} \rightarrow 0 \quad (4.15)$$

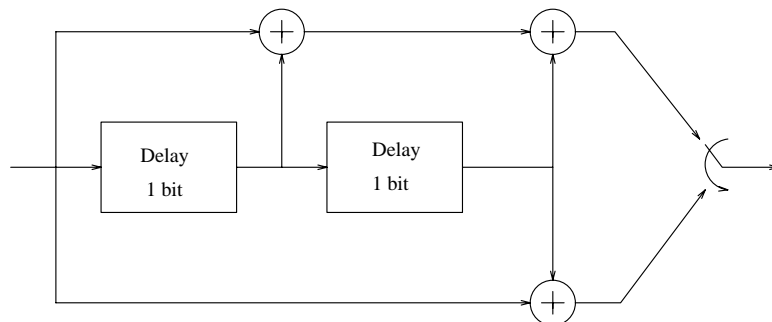
This condition means that asymptotically no diversity component has a finite fraction of the total received signal energy. It can be easily seen that independent and identically distributed  $\{h_{n,k}^i\}$  satisfy all the conditions. However, the channel coefficients need not necessarily be identically distributed. Similar convergence results have been recently shown in [25, 31]. In [25], the convergence is shown for the case when the various diversity components are identically distributed. The approach in [31] also assumes a similar orthogonality property by using the fact that the precoding filter is lossless.

## 4.2 Effect of Diversity on Coding Gain

As seen in Section 4.1, diversity brings the fading channel closer to an AWGN channel. The asymptotic results suggest that codes developed for an AWGN channel could be used effectively on a diversity channel. We study the relationship between diversity and coding by simulating the effect of varying levels of diversity (achieved by time-selective signaling and reception) on the performance of a convolutional code.

A rate( $r$ ) 1/2, constraint length( $l$ ) 3 convolutional code is chosen to study this relationship between diversity and coding gain. This code is chosen since it is compu-

tationally simple and is developed for an AWGN channel. The convolutional encoder is shown in figure 4.1.



**Figure 4.1** Convolutional encoder (Rate = 1/2, Constraint length = 3)

Soft-decision Viterbi decoding is used. The soft-decision is obtained by coherently combining the various multipath-Doppler diversity components. Based on the results in Chapter 3, the ISI is assumed to be negligible. The coding gain that can be achieved on an AWGN channel by this code can be bounded using the following equation [12, 29].

$$\text{Coding Gain} \leq 10 \log_{10}(r d_{free}) \quad (4.16)$$

where  $r$  is the rate of the code and  $d_{free}$  is its minimum free distance. In this case,  $r = 1/2$  and  $d_{free} = 5$ . Therefore, the upper bound is 3.98 dB. However, the actual coding gain on real channels surpasses this bound as discussed in [7].

The results obtained over multipath fading channels with varying diversity are shown in table 4.1. Each row in the table corresponds to a different channel condition or data rate specified by the mobile speed and the data rate columns. Four different schemes of varying diversity levels are compared for each of these channels. Of these four systems, we consider two single path systems and two multipath systems (with 2 paths). Under each of these two categories, we choose one time-selective signaling

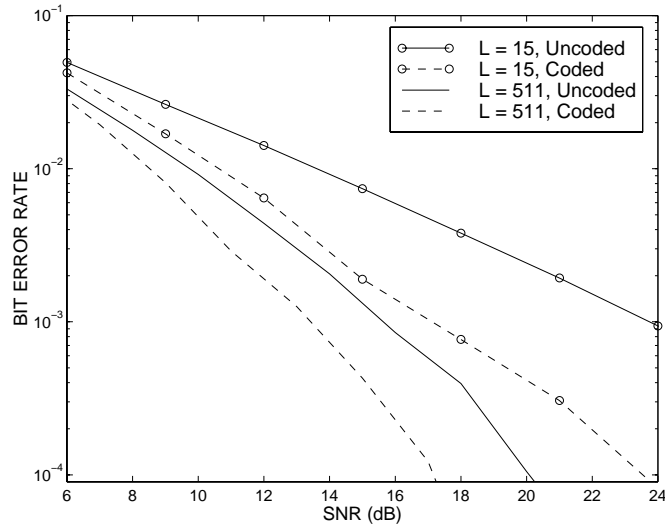
scheme that exploits joint multi-Doppler diversity ( $L = 511$ ) and a traditional system that exploits only multipath diversity ( $L = 15$ ).

Coding Gains (dB) at $P_e = 10^{-2}$					
Data Rate (kbps)	Mobile Speed (miles/hr)	1 Path		2 Paths	
		$L = 15$	$L = 511$	$L = 15$	$L = 511$
10	50	1.28	0.64	0.8	1.28
5	50	2.80	1.27	1.76	1.76
5	75	3.19	1.64	1.84	1.84
Coding Gains (dB) at $P_e = 10^{-3}$					
Data Rate (kbps)	Mobile Speed (miles/hr)	1 Path		2 Paths	
		$L = 15$	$L = 511$	$L = 15$	$L = 511$
10	50	3.85	1.76	2.08	2.88
5	50	6.71	2.26	2.64	3.12
5	75	7.77	2.90	3.28	3.76

**Table 4.1** Coding gains for the Rate = 1/2,  
Constraint length = 3 convolutional code

The following conclusions can be drawn from the simulation results.

1. The time-selective signaling system with joint multipath-Doppler diversity outperforms a traditional system with multipath diversity (seen earlier for an uncoded system) in the presence of coding. Figure 4.2 is an example.
2. The system with no diversity, i. e., the single path system with  $L = 15$ , has the highest coding gain (see table 4.1). However, the combination of diversity gain and coding gain for the systems with diversity is always greater than the coding gain in the absence of diversity leading to overall performance improvement.
3. Some of the coding gain that is lost by the addition of the first diversity component is regained by adding more diversity components. We see that, in the two-path case, which is more realistic, that besides the diversity gain from time-spreading, there is an increase in the coding gain also compared to the coding



**Figure 4.2** Performance comparison of coded system with and without diversity - Uncoded bit rate = 5 kbps,  $T/T_c = 15$ , Single path, 50 miles/hr,  $L$  = length of symbol in chips

gain for the system with just multipath diversity. For example, the coding gain for a multipath diversity system ( $L = 15$ ) at a data rate of 5 kbps and a mobile speed of 50 miles per hour at  $P_e = 10^{-3}$  is 2.64 dB. However, the coding gain for the time-selective signaling system with joint multipath-Doppler diversity ( $L = 511$ ) under the same channel conditions is 3.12 dB. Ultimately, as the number of diversity components tends to infinity, we expect the coding gain to be similar to that of the coding gain for an AWGN channel.

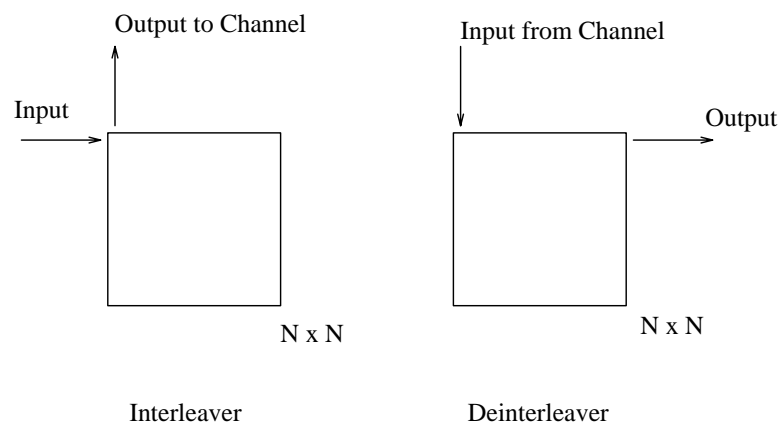
4. The coding gain increases as the speed of channel variations increases for the same signaling and reception scheme. This can be attributed to the fact that the code is able to see more states of the channel within its constraint length and is therefore able to exploit the time diversity. This result is similar in essence to our ability to achieve more diversity with more time-variability.

These results emphasize the importance of exploiting diversity to bring the fading channel closer to an AWGN channel and how it enhances the performance of simple codes. The use of simple codes helps us reduce the decoding complexity at the receiver.

### 4.3 Comparison with a Block Interleaving Scheme

A commonly used technique to exploit the time-variations in the channel is interleaving [28]. In interleaving, coded input bits are reordered before transmission and put back in the original order before decoding. This is done to achieve time-variability of the fading process within the constraint length of the code and thereby achieve diversity. Unlike interleaving, the time-selective signaling scheme provides performance improvements even in an uncoded system.

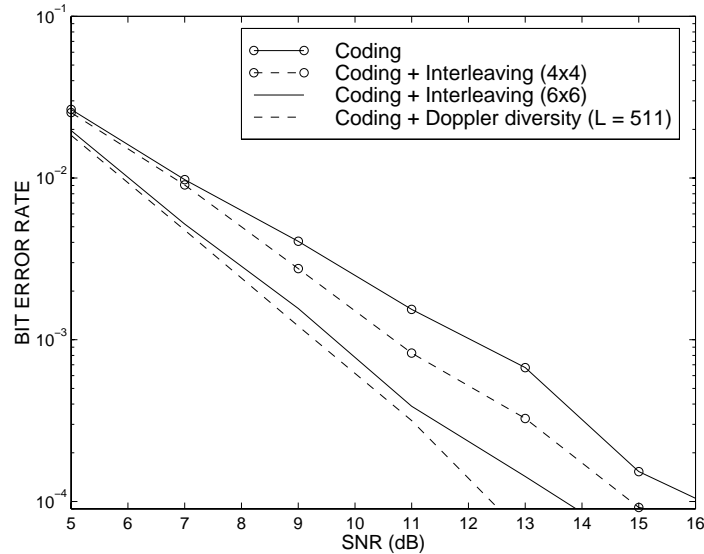
In this section, the proposed time-selective signaling scheme is compared with a block interleaving scheme using simulations. The block interleaving scheme is shown in figure 4.3.



**Figure 4.3**  $N \times N$  block interleaver

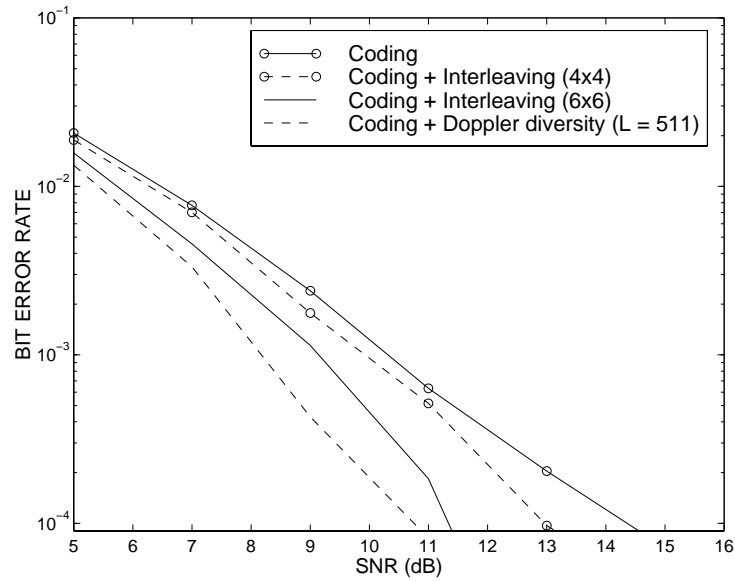
In the block interleaver, the input bits are written into a matrix in rows and transmitted along the columns. The deinterleaver works in a similar manner.

Both time-selective signaling and block interleaving incur a delay to achieve time diversity. Therefore, in order to compare these two schemes the delay suffered is fixed. The delay in bits for a time-selective signaling scheme is equal to the ratio  $T_e/T$ , for one-shot detection neglecting ISI given in equation (3.8). The delay involved in a  $N \times N$  block interleaver-deinterleaver combination is  $2N^2$ .



**Figure 4.4** Comparison of time-selective signaling scheme with block interleaving at a mobile speed of 50 miles/hr,  $L =$  length of symbol in chips,  $T/T_c = 15$

Figures 4.4 and 4.5 show the simulation results for the comparison for two different mobile speeds, 50 miles/hr and 75 miles/hr respectively. The performance of the time-selective signaling scheme is plotted for a spreading code length of 511 chips, corresponding to a bit delay of approximately 34 bits (since  $T_e = 15$  chips). The performance of the block interleaving scheme is shown for  $N = 4$  and 6, corresponding



**Figure 4.5** Comparison of time-selective signaling scheme with block interleaving at a mobile speed of 75 miles/hr,  $L$  = length of symbol in chips,  $T/T_c = 15$

to delays of 32 and 72 bits respectively. It can be clearly seen that the time-selective scheme is able to exploit the time diversity better than a simple block interleaving scheme under a delay constraint.

It should also be noted that asymptotically, as we increase the delay, the time-selective signaling scheme approaches the performance achieved on an AWGN channel. However, bit interleaving schemes will not be able to completely exploit the diversity. With perfect bit interleaving, a good approximation of the performance is the performance of a diversity system with number of diversity components equal to the minimum free distance of the convolutional code [12].



## Chapter 5

### Related Work

#### 5.1 Exploiting Channel Variations

The most common approach to combat fading has been interleaving. Interleaving involves reordering coded symbols to achieve time variability of the fading process between adjacent symbols. Various realizations of interleavers with minimum possible delay and storage capacity have been discussed in [14]. The performance of interleavers is discussed in [4] for the classic bursty channel. These properties have been used to exploit time diversity over a fading channel.

Recently, other signaling and reception techniques have been proposed that directly exploit the channel variations for improved performance [30, 31, 6]. Techniques to spread the transmitted symbols in time have been proposed by G. W. Wornell in [31]. The performance of these techniques for a single-user system and multi-user system have been analyzed in [31] and [30] respectively. These techniques involve real-valued spreading codes for transmission. The inherent channel diversity is exploited implicitly using an equalizer. However, we propose signaling and reception schemes that directly exploit the inherent channel diversity. Another approach that attempts to exploit joint multipath and Doppler diversity in the channel is proposed by B. D. Hart and D. P. Taylor in [6]. In this approach, time-frequency diversity receivers are used to exploit diversity. However, the time-frequency spread of the transmitted symbols is not chosen based on the characteristics of the channel. This limits the

extent to which these schemes can exploit diversity. We appropriately choose both signaling and reception techniques to completely exploit inherent channel diversity.

## 5.2 Time-Frequency Channel Representation

Time-selective signaling and reception schemes are based on a fundamental time-frequency representation of the channel developed and used in [17, 16, 19, 18]. The fundamental channel representation and its ability to characterize the inherent channel diversity are developed by A. M. Sayeed and B. Aazhang in [17]. This representation has been used in synchronization and detection techniques in [16, 18, 19]. In [18], multiuser timing acquisition schemes have been developed. Multiuser detection schemes have been developed for fast fading channels in [19].

## 5.3 Reception Techniques

Optimal and sub-optimal detection techniques for the ISI channel are well known. The optimal maximum likelihood sequence detection technique for the ISI channel was developed by G. D. Forney in [5]. A complete overview of the various sub-optimal adaptive equalization techniques for ISI channels has been provided in [13] by S. U. H. Qureshi.

Verdu proposed and analyzed the optimal multiuser detector in [26]. However, because of its computational complexity, several sub-optimal multiuser detector solutions which are more feasible to implement have been proposed over the last decade [9, 10, 24]. The linear decorrelating detector has been extensively analyzed in [9, 10]. Additional references on multiuser detectors can be found in [11, 27].

## 5.4 Coding

An overview of coded-modulation techniques for fading channels can be found in [8, 20]. Also, coded-modulation has been regarded as a way of introducing time diversity. The performance of coded-modulation techniques designed for an AWGN channel and a fading channel have been analyzed and compared for a diversity system in [25]. Space-time codes designed specifically for communication systems with transmit diversity have been developed in [21].

## Chapter 6

### Conclusions

Time-selective signaling and reception techniques developed in this thesis can significantly improve the performance of CDMA systems in multipath fading channels. While the RAKE receiver in existing CDMA systems exploits only multipath diversity, our approach is based on a time-frequency generalization of the RAKE receiver that maximally exploits the channel via joint multipath-Doppler diversity. This is achieved by spreading the symbol in time as well as in frequency. Analytical and simulated results show compelling gains due to time-selective signaling and reception for realistic fading scenarios. For instance, there is a gain of about 6 dB at  $P_e \approx 10^{-3}$  in the single path case, for a mobile speed of 50 miles/hr at a data rate of 10 kbps.

Our results also show that the inter-symbol interference (ISI) introduced by the overlapping temporally-spread codes is relatively negligible in the single-user case. For instance, the performance of the decorrelating detector was not significantly different from the *one-shot* receiver that ignored ISI. This can be attributed to the excellent correlation properties of m-sequences that improve with increasing code length.

The asymptotic analysis indicates that the higher level of diversity afforded by time-selective signaling and reception makes the fading channel behave like an additive white Gaussian noise (AWGN) channel thereby suggesting the use of existing powerful coding techniques.

The simulation results for a coded system show the significant performance gain achieved by combining diversity with coding. It was observed in the two-path case

(which is more realistic) that, due to the Doppler diversity exploited, the coding gain is also larger compared to the coding gain for the system with just multipath diversity. This confirms the earlier result on the convergence of the diversity channel to an AWGN channel. The convolutional code is able to perform better as the channel gets closer to an AWGN channel. Simulation results also indicate that the proposed scheme performs better than a  $N \times N$  block interleaving scheme under a delay constraint.

## 6.1 Future Work

Since our study focussed on the effects of the channel, we restricted ourselves to the single-user case. However, the results can be easily extended to multiuser scenarios. Multiuser detectors can be derived based on the detectors developed in [19]. A simple extension of the decorrelating detector to a multiuser scenario would involve increasing the dimensions of the correlation matrix to incorporate the correlations between the various users as well. In this case, the structure of the detector would still remain the same although the complexity is increased because of the increase in dimensionality.

One important practical consideration that needs to be studied is the effect of improper channel estimates on the performance. The degradation in performance due to noisy channel estimation may not be significant in the presence of a pilot channel for channel estimation. However, it becomes important when the resources for an additional channel for pilot estimation are not available. Also, the speed of channel tracking required for rapidly fading channels may lead to poor channel estimates. Therefore, it is important to investigate clearly, the effect of improper channel estimation on performance. Since we assume perfect channel estimation, maximal ratio combining has been used in all our detectors. Equal gain combining [3], which does not require channel estimation, would be an useful alternative. A

comparison of these two linear diversity combining techniques [3] in the context of time-selective signaling and reception schemes would be helpful. It must also be noted that although the error in the channel estimates will get larger for fast fading channels, the potential diversity gain achievable also increases for fast fading channels.

A comparison of the time-selective signaling scheme with other interleaving schemes that are more efficient than block interleaving [4, 14] would be interesting.

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## Appendix A

### Lindeberg Condition

The Lindeberg condition is a sufficient condition for central limit theorem (CLT) and not necessary. Consider, for each  $n$ , a set of  $r_n$  independent random variables

$$X_{n1}, X_{n2}, \dots, X_{nr_n}$$

Suppose that

$$E[X_{nk}] = 0, \quad \sigma_{nk}^2 = E[X_{nk}^2], \quad s_n^2 = \sum_{k=1}^{r_n} \sigma_{nk}^2 \quad (\text{A.1})$$

The Lindeberg condition is

$$\lim_{n \rightarrow \infty} \sum_{k=1}^{r_n} \frac{1}{s_n^2} \int_{|X_{nk}| \geq \epsilon s_n} X_{nk}^2 dP = 0 \quad (\text{A.2})$$

for  $\epsilon > 0$ . It can be shown that if Lindeberg condition is satisfied, then [2]

$$\frac{S_n}{s_n} \rightarrow \text{Gaussian}(0, 1) \quad \text{as } n \rightarrow \infty \quad (\text{A.3})$$

where  $S_n = X_{n1} + X_{n2} + \dots + X_{nr_n}$ . Although, the Lindeberg condition is only a sufficient condition for CLT, it becomes necessary and sufficient when a extra condition is imposed on the random variables. The additional condition is

$$\max_{1 \leq k \leq r_n} \frac{\sigma_{nk}^2}{s_n^2} \rightarrow 0 \quad (\text{A.4})$$

Detailed analysis of convergence and the Lindeberg condition can be found in [2].

In lemma 4.1, the Lindeberg condition is required for the set of random variables

$\{h_{n,k}^{i*} n_{n,k}^i\}$ . Therefore,

$$S_n = N^i, \quad s_n^2 = CN_0. \quad (\text{A.5})$$