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A Generalized Order-Up-To Policy and Altruistic Behavior in a Three-level Supply Chain

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Abstract: Assuming a stochastic external market demand, this research studies the benefit of the order coordination in a serially linked three-level supply chain. Each player's cost is represented by the infinite horizon standard deviation of the end of period net stock levels. To represent the activity of a player in a supply chain, the generalized order-up-to policy proposed by Hosoda and Disney (2006a) is exploited. It is shown that to minimize the total supply chain cost, the attitude of the first level player to cost increases is essential. This type of order coordination is called "altruistic behavior" herein and can produce a large cost reduction (more than 20%) to the overall supply chain. A coordination model which may be more applicable in practical settings is also introduced with this benefit.

Keywords: order-up-to policy, altruistic behavior, order coordination

1 Introduction

An order coordination policy based on the Order-Up-To (OUT) policy that minimizes the *total* inventory costs for a three-level supply chain will be examined. For a single level of a supply chain, Vassian (1955) introduced an ordering policy with a Work In Progress (WIP) feedback loop and showed that this ordering policy minimizes the variance of the end of period net stock levels. In addition, Vassian showed that the minimized variance of the end-period net inventory level is identical to the variance of the error in the forecast of demand over the lead-time plus review period. In this research, Vassian's ordering policy is called as *the traditional OUT policy* [1]. From Vassian's seminal contribution, it is obvious that in a single level supply chain case, the traditional OUT policy is an optimal policy for minimizing the variance of the end of period net stock levels over time. In a multi-level supply chain scenario, however, it might be reasonable to assume that a sequence of traditional OUT policies may not be optimal anymore as there is no guarantee that a succession of local minimizations will result in a global optimum, as shown in Hosoda and Disney (2006a). Since the traditional OUT policy does not provide

[1] It should be noted that several researchers adopt an alternative expression for the OUT policy that exploits a time varying OUT target (see, Lee et al. 2000, for example), however, the dynamics given by these two expositions is identical, as shown in Hosoda and Disney (2006b).

much freedom to manipulate the dynamics of the ordering process, Hosoda and Disney (2006a) have investigated a two-level supply chain using the traditional OUT policy modified to include a proportional controller. This brings more flexibility to alter the dynamics of the ordering process, and shows that a sequence of traditional OUT policies is no longer optimal. They also show that to enjoy the cost saving, the attitude of the first level player to cost increases is an essential factor. They call this attitude "altruistic behavior". In this chapter the model shown in Hosoda and Disney (2006a) will be extended to a three-level supply chain model, and the benefit of the altruistic behavior and roles of the first and the second level players in a three-level supply chain will be analyzed[2]. In addition, as a benchmark for performance comparisons, a sequence of three traditional OUT policies supply chain model shown in Hosoda and Disney (2006b) will be used.

2 Literature review

As a type of supply chain coordination, information sharing has been studied by many researchers. However, counter-intuitively, not all results support the benefit of information sharing.

Graves (1999) studies a two-level supply chain with the OUT policy (termed "adaptive base-stock policy" in his paper), a non-stationary demand process, IMA(1,1) process, with Minimum Mean Square Error (MMSE) forecasting[3]. Graves finds that sharing demand information brings no benefit to the upstream player, if the upstream players know the coefficients of the customer demand process. Kim and Ryan (2003) analyse the value of demand information sharing using the model with an unknown demand process and an exponential smoothing forecast. They conclude that sharing demand data can significantly reduce the costs in upper-stream players of the supply chain. However, the benefit is limited when the upper-stream player has a large amount of historical order data, as by exploiting this data, the upper-stream player can improve its forecast accuracy. Assuming a known demand process and the MMSE forecast, Raghunathan (2001) reports similar results in that the set of order history data contains all the necessary information to allow the upper-stream player to reduce his costs. Assuming an AR(1) demand process, Lee et al. (2000) develop a two-level supply chain model and investigate the benefit of demand information sharing. Under their assumption that the manufacturer uses only the latest observed demand information in its forecast, they conclude that the manufacturer can obtain inventory and costs reductions with information sharing. Hosoda et al.

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[2] Readers are encouraged to visit our web site
http://www.bullwhip.co.uk/bwExplorer.htm
to see how the altruistic behavior brings the benefits to a supply chain.
[3] For the details of an MMSE forecasting, see Box et al. (1994).

(2008) investigate the benefit of sharing the market demand information using a set of data obtained from a real retail supply chain. It is shown that there is a benefit of information sharing, and a source of such benefit is the error terms, which are originally hidden in the market demand process and difficult to extract without shared market demand information. In addition to information sharing, some researchers have analysed operational coordination of supply chains, such as Vendor Managed Inventory (VMI). This field of research has attracted abundant attention since the late 1990s. Disney and Towill (2003a) develop a two-level VMI supply chain model and compare the measured bullwhip with a traditional serially linked supply chain. They report that the VMI scheme can substantially reduce the bullwhip. In their VMI scheme, information about the first-level stock level, the goods in transit, the second-level stock level, and the reorder point is used to determine the target inventory level. Using their VMI model, Disney and Towill (2003b) investigate each of the potential sources of the bullwhip proposed by Lee et al. (1997). They show that two of the four causes; the rationing game and order batching, can be completely eliminated by the adoption of VMI scheme in a supply chain and other two causes also can be reduced significantly.

Aviv and Federgruen (1998) study the benefits of a VMI scheme using a two-echelon supply chain model consisting of a single supplier and J retailers. They study three scenarios: 1) a traditional decentralised system, 2) a VMI system, and 3) a system with full information sharing between players. Under the VMI program, the timing and magnitude of the replenishment shipments to the retailers are decided by the supplier on the basis of the full information given by all retailers. A comparison was made and they conclude that the VMI program (where information on inventory levels is also shared) has much more potential and can reduce costs by 4.7% on average. The benefits of VMI against the full information sharing scenario become larger when capacity is tight, since VMI scheme enables the supplier to increase its utilization rate. Using a serially linked two-level supply chain with an AR(1) market demand, Hosoda and Disney (2006a) investigate the impact of altruistic behavior on the overall supply chain cost. To realize altruistic behavior at the first level, they introduced a traditional OUT policy with a single proportional controller in the system feedback loop. This proportional controller enables us to manipulate the order placed by the retailer to achieve lower total supply chain cost. The sum of the standard deviations of net stock levels at each level was used as an objective function to be minimised. It is suggested that altruistic behavior by the first level player mitigates the bullwhip effect, and this lower bullwhip is the source of the benefit at the second level. Also, the cost benefit at the second level is large enough to compensate the loss at the first level. It is shown that on average more than 10% cost reduction can be achieved.

Some researchers assume that the second level player can modify the first level player's order pattern by offering incentives and find that the first level player should be altruistic to achieve lower total costs. In his two-level supply chain model, Gavirneni (2006) assumes that the supplier can alter the pattern of orders placed by the retailer, by offering fluctuating prices. As the result of this incentive, the retailer's ordering pattern is not optimum for itself anymore and thus the retailer's cost will increase. However, the benefit at the supplier is sufficient enough to compensate the increase at the retailer. The overall supply chain performance can be improved by 5% on average with the aid of information sharing. Luo (2007) considers a coordination scheme in a two-level supply chain consisting of a vendor and a buyer. The vendor asks the buyer to change its order quantity to achieve lower set up, ordering and inventory holding costs of the vendor. To convince the buyer, a credit period incentive is offered by the vendor. It is shown that the benefit to the vendor is always greater than the loss of the buyer so that this cooperation scheme can bring the benefits to overall supply chain. From these two papers, it might be reasonable to conclude that the type of incentives for the first level player affects the total amount of saving costs. Other incentives to encourage the first level player to incur cost increase include quantity flexibility (Tsay 1999), quantity discounts (Weng 1995), and revenue sharing (Giannoccaro and Pontrandolfo 2004).

The literature review suggests some useful insights to our problem. First, sharing market demand information may bring benefits to a supply chain, but the amount of such benefit is not clear. In our model, therefore, to negate the benefit coming from sharing the market demand information, it is assumed that up-to-date market demand information is shared and common knowledge in the supply chain. This assumption enables us to focus on the benefits only from the altruistic behavior. Secondly, it might be better to assume a centralized supply chain model to quantify the benefit of the altruistic behavior. In the case of a decentralized supply chain, incentives and/or a way of redistribution of the generated benefits may significantly affect the behavior of each player in a supply chain. The centralized supply chain assumption allows us to ignore such issues. Therefore, we will assume that in the supply chain there are no incentive conflicts, all necessary information is shared, and all players will cooperate to minimize the total cost.

3 The model

A serially linked three-level supply chain system is analyzed. All three players exploit a periodic review system, and the replenishment lead-time is constant and known. The ordering policy used herein is the OUT policy. The OUT level is adjusted each time period according to the latest updated demand forecast and the shared information. The knowledge about the market demand process captured by the first level player is shared with all other players without delay. It is assumed that the true market demand process is correctly captured. The cost parameters and the ordering policies in the supply chain are common knowledge.

3.1 Sequence of events and costs

The sequence of events in any period at any level is as follows: the order placed earlier is received, and the demand is fulfilled at the beginning of the period, the net stock level is reviewed and ordering decision is made at the end of the period. We will now describe the three-level supply chain model where each level uses the OUT policy with the MMSE forecasting scheme. We assume a periodic review policy but do not assume a specific length of the review period. All of the results herein are consistent whatever review period is adopted (day, week, month, etc.). We will use the subscript n ($= 1, 2, 3$) to represent the level of the supply chain. It is assumed that the costs in the supply chain are directly proportional to the standard deviation of the net stock level at each level as in Hosoda and Disney (2006a). Therefore, the objective function used in this research can be written as

$$J = \sum_{n=1}^{3} \sqrt{V[NS_n]} = \sqrt{V[NS_1]} + \sqrt{V[NS_2]} + \sqrt{V[NS_3]}, \tag{3.1}$$

where $V[NS_n]$ represents the stable variance of the net stock level at n th level of the supply chain.

3.2 Market demand

Let us assume the demand pattern faced by the retailer is an AR(1) process. The AR(1) demand process assumption is common when autocorrelation exists among the demand process. Many researchers employ this assumption (see, Hosoda et al. 2008, for example). The formulation of AR(1) process is given by

$$D_t = d + \rho D_{t-1} + \varepsilon_t \,,$$

where D_t is the observed market demand at time period t, d is the constant term, ρ is the autoregressive coefficient, $|\rho| < 1$, and ε_t is an i.i.d. white noise process with a mean of zero and a variance of σ_ε^2. The stable variance of D_t, $V[D_t]$, is $\sigma_\varepsilon^2/(1-\rho^2)$. Detailed discussions about an AR(1) model can be seen in Box et al. (1997).

3.3 Ordering policy

The traditional OUT policy for the player at level n in the supply chain can be described as follows (Vassian 1955)

$$O_{t,n} = S_{t,n} - (WIP_{t,n} + NS_{t,n}),$$
$$S_{t,n} = \hat{O}_{t,n-1}^{L_n} + safety\ stock,$$

where $O_{t,n}$ is the order rate at time t, S_t is the OUT level at time t and $WIP_{t,n}$ is the sum of orders that are already placed but not yet received at time t and can be expressed as $WIP_{t,n} = \sum_{i=1}^{L_n-1} O_{t-i,n}$. $NS_{t,n}$ is the end of period net stock level at time t, and $\hat{O}_{t,n-1}^{L_n}$ is the conditional estimate of the total demand from the $n-1$ level player over L_n time periods, which is the lead-time plus review period. For $n=1$, $\hat{O}_{t,n-1}^{L_n}$ is denoted as $\hat{D}_t^{L_1}$. To realize our generalized OUT policy, let us begin by modifying the traditional OUT policy.

$$\begin{aligned}
O_{t,n} &= S_{t,n} - (WIP_{t,n} + NS_{t,n}) \\
&= \hat{O}_{t,n-1}^{L_n} - (WIP_{t,n} + NS_{t,n}) + safety\ stock \\
&= \tilde{O}_{t,n-1}^{L_n} + \hat{O}_{t,n-1}^{L_n-1} - (WIP_{t,n} + NS_{t,n}) + safety\ stock \\
&= \tilde{O}_{t,n-1}^{L_n} + (\hat{O}_{t,n-1}^{L_n-1} - (WIP_{t,n} + NS_{t,n})) + safety\ stock \\
&= \tilde{O}_{t,n-1}^{L_n} + (DIP_{t,n} - (WIP_{t,n} + NS_{t,n})) + safety\ stock,
\end{aligned} \tag{3.2}$$

where $\tilde{O}_{t,n-1}^{L_n}$ is $E[O_{t+L_n,n-1} \mid O_{t,n-1}]$, the conditional estimate of the demand in time period $t+L_n$ made at t. Therefore, $\tilde{O}_{t,n-1}^{L_n} + \hat{O}_{t,n-1}^{L_n-1} = \hat{O}_{t,n-1}^{L_n}$. When $n = 1$, $\tilde{O}_{t,n-1}^{L_n}$ is $E[D_{t+L_1} \mid D_t]$ and denoted as $\tilde{D}_t^{L_1}$. $DIP_{t,n}$ is a Desired Inventory Position at time t and $DIP_{t,n} = \hat{O}_{t,n-1}^{L_n-1} = E[\sum_{i=1}^{L_n-1} O_{t,n-1} \mid O_{t,n-1}]$. Note that $DIP_{t,n} = 0$, if $L_n = 1$. Incorporating a proportional controller, F_n, into Eq. 3.2 yields the ordering policy, the generalized OUT policy.

$$O_{t,n} = \tilde{O}_{t,n-1}^{L_n} + F_n(DIP_{t,n} - (WIP_{t,n} + NS_{t,n})) + safety\ stock\ ,$$

where $0 < F_n < 2$ as shown in Hosoda and Disney (2006a). Obviously, if $F_n = 1$, the policy is identical to the traditional OUT policy. In what follows, for simplicity, we will set $d = 0$ and $safety\ stock = 0$ without loss of generality, since these values are time invariant values and do not affect the value of J.

4 Scenarios

Three different scenarios will be considered herein. Scenario 1 is the three-level traditional OUT policy supply chain that was investigated in Hosoda and Disney (2006b). Scenario 1 will form the baseline scenario for the other scenarios to be compared against. Scenario 2 is the generalized OUT policy supply chain case where the first and the second level players exploit the generalized OUT policy and the third level player adopts the traditional OUT policy. Scenario 3 is a special case of Scenario □; here not only the third level player, but also the first level player, adopts the traditional OUT policy to minimize its own variance of the net stock. Only the middle level player is concerned with minimizing the objective function by tuning its proportional controller, F_2. Scenario 3 is expected to bring enough benefit so that this scenario might be a more acceptable strategy for a retail supply chain where usually the unit cost of the net stock at the first level (retail store, for example) is the most expensive. If Scenario 3 is successful, the variance of net stock level at the first level player, namely the retail store, is minimized due to the traditional OUT policy, and at the same time, the complete supply chain can also enjoy a cost reduction generated by the altruistic behavior of the second level player.

4.1 Scenario 1: The traditional OUT policy supply chain

In what follows $V[\overline{NS}_n]$ will be used to show the variance of net stock level in a traditional OUT policy supply chain. As shown in Hosoda and Disney (2006b), the expressions of the variances of net stock level at each level in a serially linked three-level supply chain model are expressed as;

$$\begin{aligned}
V[\overline{NS}_1] &= \frac{\left(L_1(1-\rho^2)+\rho(1-\rho^{L_1})(\rho^{L_1+1}-\rho-2)\right)}{(1-\rho)^2(1-\rho^2)}\sigma_\varepsilon^2, \\
V[\overline{NS}_2] &= \frac{\left(L_2(1-\rho^2)+\rho^{L_1+1}(1-\rho^{L_2})(\rho^{L_1+1}+\rho^{L_1+L_2+1}-2\rho-2)\right)}{(1-\rho)^2(1-\rho^2)}\sigma_\varepsilon^2, \\
V[\overline{NS}_3] &= \frac{\left(L_3(1-\rho^2)+\rho^{L_1+L_2+1}(1-\rho^{L_3})(\rho^{L_1+L_2+1}+\rho^{L_1+L_2+L_3+1}-2\rho-2)\right)}{(1-\rho)^2(1-\rho^2)}\sigma_\varepsilon^2.
\end{aligned} \tag{4.1}$$

Therefore the objective function for Scenario 1, J_{S1}, becomes

$$
\begin{aligned}
J_{S1} &= \sqrt{V[\overline{NS}_1]} + \sqrt{V[\overline{NS}_2]} + \sqrt{V[\overline{NS}_3]} \\
&= \sqrt{\frac{\left(L_1(1-\rho^2)+\rho(1-\rho^{L_1})(\rho^{L_1+1}-\rho-2)\right)}{(1-\rho)^2(1-\rho^2)}\sigma_\varepsilon^2} + \\
&\quad \sqrt{\frac{\left(L_2(1-\rho^2)+\rho^{L_1+1}(1-\rho^{L_2})(\rho^{L_1+1}+\rho^{L_1+L_2+1}-2\rho-2)\right)}{(1-\rho)^2(1-\rho^2)}\sigma_\varepsilon^2} + \\
&\quad \sqrt{\frac{\left(L_3(1-\rho^2)+\rho^{L_1+L_2+1}(1-\rho^{L_3})(\rho^{L_1+L_2+1}+\rho^{L_1+L_2+L_3+1}-2\rho-2)\right)}{(1-\rho)^2(1-\rho^2)}\sigma_\varepsilon^2}.
\end{aligned}
$$

4.2 Scenario 2: The generalized OUT policy supply chain

Scenario 2 assumes that the generalized OUT policy is used in the three-level supply chain. To minimize the objective function (Eq. 3.1), from the *Principle of Optimality* (Bellman 1957), the highest level player must use the policy which minimizes his own variance of the net stock level, as shown in Hosoda and Disney (2006a). Thus, the third level player should use the traditional OUT policy. As the result, only the first two players in the supply chain employ the generalized OUT policy.

4.2.1 The ordering process and MMSE forecasts

To obtain an MMSE forecast, knowledge of the structure of the order process is required. In the case of Scenario 2, the process of $O_{t,1}$ and $O_{t,2}$, the volume of orders placed by the first and the second players respectively, can be described as

$$O_{t+1,1} = (1-F_1)O_{t,1} + \rho^{L_1}(\rho+F_1-1)D_t + (\rho^{L_1}+F_1\cdot\Lambda_{L_1})\varepsilon_{t+1}, \tag{4.2}$$

$$
\begin{aligned}
O_{t+1,2} &= (1-F_2)O_{t,2} + (1-F_1)^{L_2}(F_2-F_1)O_{t,1} + \\
&\quad \left(\rho^{L_1}(1-F_1)^{L_2}(F_1-F_2)+\rho^{L_1+L_2}(\rho+F_2-1)\right)D_t + \xi\cdot\varepsilon_{t+1},
\end{aligned} \tag{4.3}
$$

where

$$\Lambda_{L_1} = \frac{1-\rho^{L_1}}{1-\rho}, \text{ and } \xi = \frac{(1-F_1)^{L_2}(F_2-F_1)(1-\rho^{L_1})+\rho^{L_1+L_2}(\rho+F_2-1)-F_2}{\rho-1}.$$

Hosoda (2005) provides details. Eq. 4.2 and Eq. 4.3 yield expressions for the MMSE forecasts of $O_{t,n}$ over L_{n+1} time periods, $\hat{O}_{t,n}^{L_{n+1}}$ where $n = 1$ and 2.

$$\hat{O}_{t,1}^{L_2} = E\left[\sum_{i=1}^{L_2} O_{t+i,1} \mid D_t, O_{t,1}, \rho, F_1, L_1\right]$$
$$= \frac{(1-F_1)(1-(1-F_1)^{L_2})}{F_1} O_{t,1} + \frac{\rho^{L_1}\begin{pmatrix}((1-F_1)^{L_2+1}-1)(\rho-1)+ \\ F_1(\rho^{L_2+1}-1)\end{pmatrix}}{F_1(\rho-1)} D_t,$$

$$\hat{O}_{t,2}^{L_3} = E\left[\sum_{i=1}^{L_3} O_{t+i,2} \mid D_t, O_{t,1}, O_{t,2}, \rho, F_1, F_2, L_1, L_2\right]$$
$$= \frac{(1-F_1)^{L_2}((1-F_1)^{L_3}-1)(F_1-1)}{F_1} O_{t,1} - \frac{(1-F_1)^{L_2}((1-F_2)^{L_3}-1)(F_2-1)}{F_2} O_{t,1} +$$
$$\frac{((1-F_2)^{L_3}-1)(F_2-1)}{F_2} O_{t,2} + \frac{((1-F_1)^{L_3}-1)(1-F_1)^{L_2+1}\rho^{L_1}}{F_1} D_t +$$
$$\frac{((1-F_2)^{L_3}-1)(1-F_1)^{L_2}(F_2-1)\rho^{L_1}}{F_2} D_t - \frac{((1-F_2)^{L_3}-1)(F_2-1)\rho^{L_1+L_2}}{F_2} D_t +$$
$$\frac{(\rho^{L_3}-1)\rho^{L_1+L_2+1}}{\rho-1} D_t.$$

Hosoda (2005) provides details. Note that if a sequence of the traditional OUT policies are used in the supply chain (that is $F_1 = F_2 = 1$), then $\hat{O}_{t,1}^{L_2} = \frac{\rho^{L_1+1}(\rho^{L_2}-1)}{\rho-1} D_t$ and $\hat{O}_{t,2}^{L_3} = \frac{(\rho^{L_3}-1)\rho^{L_1+L_2+1}}{\rho-1} D_t$.

4.2.2 The objective function

From here, the expression $V[\overline{\overline{NS}}_n]$ will be used for the variance of net stock levels of the generalized OUT policy at the n th level. As shown in the appendix and Hosoda (2005), the net stock levels at the first and the second levels follow ARMA(1, L_n – 1) processes, where n = 1 and 2 respectively. By exploiting this property, we can have the following:

$$V[\overline{\overline{NS}}_1] = V[\overline{NS}_1] + \frac{\Omega^2\Psi^2}{(\rho-1)^2(1-\Psi^2)}\sigma_\varepsilon^2, \tag{4.4}$$

$$V[\overline{\overline{NS}}_2] = V[\overline{NS}_2] + \frac{\sigma_\varepsilon^2}{(\rho-1)^2}\left(\frac{(\Psi^{2L_2}-1)\Omega^2\Psi^2}{\Psi^2-1} + 2\Omega\Psi\left(\frac{1-\Psi^{L_2}}{1-\Psi} - \frac{\rho^{L_1+1}((\rho\Psi)^{L_2}-1)}{\rho\Psi-1}\right)+\right. \tag{4.5}$$

$$\left.\frac{\left((1-(1-F_1)^{L_2})-\rho^{L_1}(\rho^{L_2}-(1-F_1)^{L_2})\right)^2(F_2-1)^2}{(2-F_2)F_2}\right),$$

where $\Psi = 1-F_1$, $\Omega = \rho^{L_1}-1$. Detailed steps to obtain Eq. 4.4 and Eq. 4.5 are shown in the appendix. Since the third level player adopts the traditional OUT policy to contribute to the minimization of the objective function, the forecast error over the lead-time plus review period can be used as an alternative.

$$\begin{aligned}V[\overline{\overline{NS}}_3] &= E\left[\left(\hat{O}_{t,2}^{L_3}-\textstyle\sum_{i=1}^{L_3}O_{t+i,2}\right)^2\right]= \\ &\xi^2\cdot\sigma_\varepsilon^2+ \\ &\textstyle\sum_{r=2}^{L_3}\left(\left(\sum_{i=2}^{r}\left(\frac{-1}{(F_1-1)(\rho-1)\rho}\right.\right.\right. \\ &\left((1-F_2)^{i-1}\left((F_1-1)(F_2-1)(\rho-1)\rho^{L_1+L_2}\left(\rho/(1-F_2)\right)^i-\right.\right. \\ &\rho F_1(1-F_1)^{L_2}\left((F_1-1)/(F_2-1)\right)^i(F_2-1)(\rho^{L_1}-1)+ \\ &\left.\left.\left.\left.\left.\rho F_2(F_1-1)\left(1-(1-F_1)^{L_2}+\rho^{L_1}((1-F_1)^{L_2}-\rho^{L_2})\right)\right)\right)\right)+\xi\right)^2\right)\sigma_\varepsilon^2.\end{aligned} \tag{4.6}$$

Details of the derivation of Eq. 4.6 are shown in Hosoda (2005). It should be noted that Eq. 4.6 cannot be used when $F_1=1$, $F_2=1$, and/or when $\rho=0$ because of a singularity in the denominator. However, solutions do exist at the singularity, and they are also shown Hosoda (2005). In this section, Eq. 4.6 will be used in the analysis.

The objective function for the three-level generalized supply chain model is

$$\begin{aligned}J_{S2} &= \sqrt{V[NS_1]}+\sqrt{V[NS_2]}+\sqrt{V[NS_3]} \\ &= \sqrt{V[\overline{\overline{NS}}_1]}+\sqrt{V[\overline{\overline{NS}}_2]}+\sqrt{V[\overline{\overline{NS}}_3]} \\ &= \sqrt{\left(\frac{\left(L_1(1-\rho^2)+\rho(1-\rho^{L_1})(\rho^{L_1+1}-\rho-2)\right)}{(1-\rho)^2(1-\rho^2)}\sigma_\varepsilon^2+\frac{(\rho^{L_1}-1)^2(F_1-1)^2}{(\rho-1)^2(2-F_1)F_1}\sigma_\varepsilon^2\right)}+\end{aligned}$$

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$$
\sqrt{\left(\begin{array}{l}
\frac{\sigma_\varepsilon^2}{(\rho-1)^2}\left(\begin{array}{l}
L_2+\frac{((1-F_1)^{2L_2}-1)(\rho^{L_1}-1)^2(1-F_1)^2}{(1-F_1)^2-1}+\\
\frac{\rho^{L_1+1}(\rho^{L_2}-1)(\rho^{L_1+L_2+1}+\rho^{L_1+1}-2\rho-2)}{\rho^2-1}+\\
2(\rho^{L_1}-1)\times\\
(1-F_1)\left(\frac{1-(1-F_1)^{L_2}}{F_1}-\frac{\rho^{L_1+1}\left((\rho(1-F_1))^{L_2}-1\right)}{\rho(1-F_1)-1}\right)
\end{array}\right)+\\
\frac{((1-(1-F_1)^{L_2})-\rho^{L_1}(\rho^{L_2}-(1-F_1)^{L_2}))^2(F_2-1)^2}{(\rho-1)^2(2-F_2)F_2}\sigma_\varepsilon^2
\end{array}\right)}+ \qquad (4.7)
$$

$$
\sqrt{\left(\begin{array}{l}
\xi^2\sigma_\varepsilon^2+\\
\sum_{r=2}^{L_3}\left(\left(\sum_{i=2}^{r}\left(\frac{-1}{(F_1-1)(\rho-1)\rho}\right.\right.\right.\\
\left((1-F_2)^{i-1}\left((F_1-1)(F_2-1)(\rho-1)\rho^{L_1+L_2}\left(\rho/(1-F_2)\right)^i-\right.\right.\\
\rho F_1(1-F_1)^{L_2}\left((F_1-1)/(F_2-1)\right)^i(F_2-1)(\rho^{L_1}-1)+\\
\left.\left.\left.\left.\left.\rho F_2(F_1-1)\left(1-(1-F_1)^{L_2}+\rho^{L_1}((1-F_1)^{L_2}-\rho^{L_2})\right)\right)\right)\right)+\xi\right)^2\right)\sigma_\varepsilon^2
\end{array}\right)}.
$$

4.3 Scenario 3: The generalized OUT policy supply chain when $F_1 = 1$

In Scenario 3, a special case of scenario 2, where $F_1 = 1$, will be considered. In this scenario, only the second player employs the generalized OUT policy in the supply chain in order to manipulate the dynamics of the supply chain.

In Scenario 3, from Eq. 4.3, the ordering process can be expressed as

$$
O_{t+1,2} = (1-F_2)O_{t,2} + \rho^{L_1+L_2}(\rho+F_2-1)D_t + \frac{F_2-\rho^{L_1+L_2}(\rho+F_2-1)}{1-\rho}\varepsilon_{t+1}.
$$

From Eq. 4.5, by setting $F_1 = 1$, the variance of the net stock level at the second in Scenario 3, $V[\overline{\overline{NS}}_2 \mid F_1 = 1]$ can be expressed as

$$V[\overline{\overline{NS}}_2 \mid F_1 = 1] = \frac{\left(L_2(1-\rho^2)+\rho^{L_1+1}(1-\rho^{L_2})(\rho^{L_1+1}+\rho^{L_1+L_2+1}-2\rho-2)\right)}{(1-\rho)^2(1-\rho^2)}\sigma_\varepsilon^2 + \frac{(1-\rho^{L_1+L_2})^2(F_2-1)^2}{(\rho-1)^2(2-F_2)F_2}\sigma_\varepsilon^2. \tag{4.8}$$

The variance of the net stock level at the third level in Scenario 3, $V[\overline{\overline{NS}}_3 \mid F_1 = 1]$, can be written as

$$\begin{aligned} &V[\overline{\overline{NS}}_3 \mid F_1 = 1] \\ &= \sum_{r=1}^{L_3}\left(\left(\frac{(1-F_2)^r+\rho^{L_1+L_2}(\rho^r-(1-F_2)^r)-1}{\rho-1}\right)^2\right)\sigma_\varepsilon^2 \\ &= \frac{\sigma_\varepsilon^2}{(\rho-1)^2}\left(L_3 - 1 + \frac{2((1-F_2)^{L_3}-1)(F_2-1)(\rho^{L_1+L_2}-1)}{F_2} + \right. \\ &\quad \frac{((1-F_2)^{2(L_3+1)}-1)+((1-F_2)^{2L_3}-1)(F_2-1)^2\rho^{L_1+L_2}(\rho^{L_1+L_2}-2)}{(F_2-2)F_2} + \\ &\quad \frac{\rho^{L_1+L_2+1}(\rho^{L_3}-1)(\rho(\rho^{L_1+L_2}+\rho^{L_1+L_2+L_3}-2)-2)}{(\rho^2-1)} + \\ &\quad \left. \frac{2\rho^{L_1+L_2+1}(F_2-1)((\rho-F_2\rho)^{L_3}-1)(1-\rho^{L_1+L_2})}{1+(F_2-1)\rho}\right). \end{aligned} \tag{4.9}$$

Details are shown in Hosoda (2005). By using Eq. 4.1, Eq. 4.8, and Eq. 4.9, J_{S3}, the objective function for Scenario 3 can be described as

$$\begin{aligned} J_{S3} &= \sqrt{V[NS_1]} + \sqrt{V[NS_2]} + \sqrt{V[NS_3]} \\ &= \sqrt{V[\overline{NS}_1]} + \sqrt{V[\overline{\overline{NS}}_2 \mid F_1 = 1]} + \sqrt{V[\overline{\overline{NS}}_3 \mid F_1 = 1]} \end{aligned}$$

$$
\begin{aligned}
&= \sqrt{\frac{\left(L_1(1-\rho^2)+\rho(1-\rho^{L_1})(\rho^{L_1+1}-\rho-2)\right)}{(1-\rho)^2(1-\rho^2)}\sigma_\varepsilon^2} + \\
&\sqrt{\left(\frac{\left(L_2(1-\rho^2)+\rho^{L_1+1}(1-\rho^{L_2})(\rho^{L_1+1}+\rho^{L_1+L_2+1}-2\rho-2)\right)}{(1-\rho)^2(1-\rho^2)}\sigma_\varepsilon^2 + \frac{(1-\rho^{L_1+L_2})^2(F_2-1)^2}{(\rho-1)^2(2-F_2)F_2}\sigma_\varepsilon^2\right)} + \\
&\sqrt{\left(\frac{\sigma_\varepsilon^2}{(\rho-1)^2}\left(L_3-1+\frac{2((1-F_2)^{L_3}-1)(F_2-1)(\rho^{L_1+L_2}-1)}{F_2} + \frac{((1-F_2)^{2(L_3+1)}-1)+((1-F_2)^{2L_3}-1)(F_2-1)^2\rho^{L_1+L_2}(\rho^{L_1+L_2}-2)}{(F_2-2)F_2} + \frac{\rho^{L_1+L_2+1}(\rho^{L_3}-1)(\rho(\rho^{L_1+L_2}+\rho^{L_1+L_2+L_3}-2)-2)}{(\rho^2-1)} + \frac{2\rho^{L_1+L_2+1}(F_2-1)((\rho-F_2\rho)^{L_3}-1)(1-\rho^{L_1+L_2})}{1+(F_2-1)\rho}\right)\right)}.
\end{aligned}
\qquad (4.10)
$$

Due to the rather unwieldy expressions of the objective functions, further analytical investigations are difficult to present. Thus, numerical investigations will be exploited.

5 Numerical investigations

In this section, the three scenarios with two lead-time settings $L_1 = 2, L_2 = 2$, $L_3 = 3$ and $L_1 = 1$, $L_2 = 3$, $L_3 = 3$ will be investigated numerically. $\sigma_\varepsilon^2 = 1$ is assumed. By using Eq. 4.7 and Eq. 4.10, the values of J_{S2} have been plotted in Fig. 5.1 and the values of J_{S3} in Fig. 5.2 - 5.3 with the restriction that $0 < F_1 < 2$ and $0 < F_2 < 2$, when $\rho =$ -0.7, 0.0 and 0.7 for both lead-time settings. From these figures, it can be seen that J_{S2} and J_{S3} have an unique minimum value for the given values of ρ, L_1, L_2 and L_3. The optimum values of the proportional controllers, F_1^*, and F_2^*, to minimize the objective functions, J_{S2} and J_{S3} respectively, are obtained by using the cylindrical algebraic decomposition algorithm (Collins et al. 2002). J_{S2}^* and J_{S3}^* will be used to represent the minimized values of J_{S2} and J_{S3}, respectively.

5.1 Benefit of Scenario 2

Tables 5.1 – 5.2 show the results of Scenario 1 and Tables 5.3 – 5.4 highlight the results for Scenario 2. From Tables 5.1 – 5.4, the following insights can be obtained.

- $J^*_{S2} < J_{S1}$ for all values of ρ and all lead-time settings. This means that the generalized OUT policy supply chain always outperforms the traditional OUT policy supply chain.
- Both F_1^* and F_2^* never have unit value.
- The value of F_n^* $(n = 1, 2)$ is affected by both the value of ρ and the lead-time settings.
- $J^*_{S2} < J_{S1}$ is achieved by altruistic behavior in the first level player, by accepting a greater level of net stock to achieve a predetermined customer service level. That is accepting $\sqrt{V[\overline{\overline{NS}}_1]} > \sqrt{V[\overline{NS}_1]}$.
- In almost all parameter settings, the second level player enjoys the benefit, that is $\sqrt{V[\overline{\overline{NS}}_2]} < \sqrt{V[\overline{NS}_2]}$. The only exception in the points of the solution space we have chosen is the case when $L_1 = 1$, $L_2 = 3$, $L_3 = 3$ and $\rho = 0.9$.

Hosoda, T. and Disney, S.M., (2008), “A generalized order-up-to policy and altruistic behavior in a three-level supply chain”, in “Supply Chain Management and Knowledge Management: Integrating Critical Perspectives in Theory and Practice”, Edited by Dwivedi, A. and Butcher, T., Palgrave Macmillan, pp190–216, ISBN 978-0-230-57343-7.

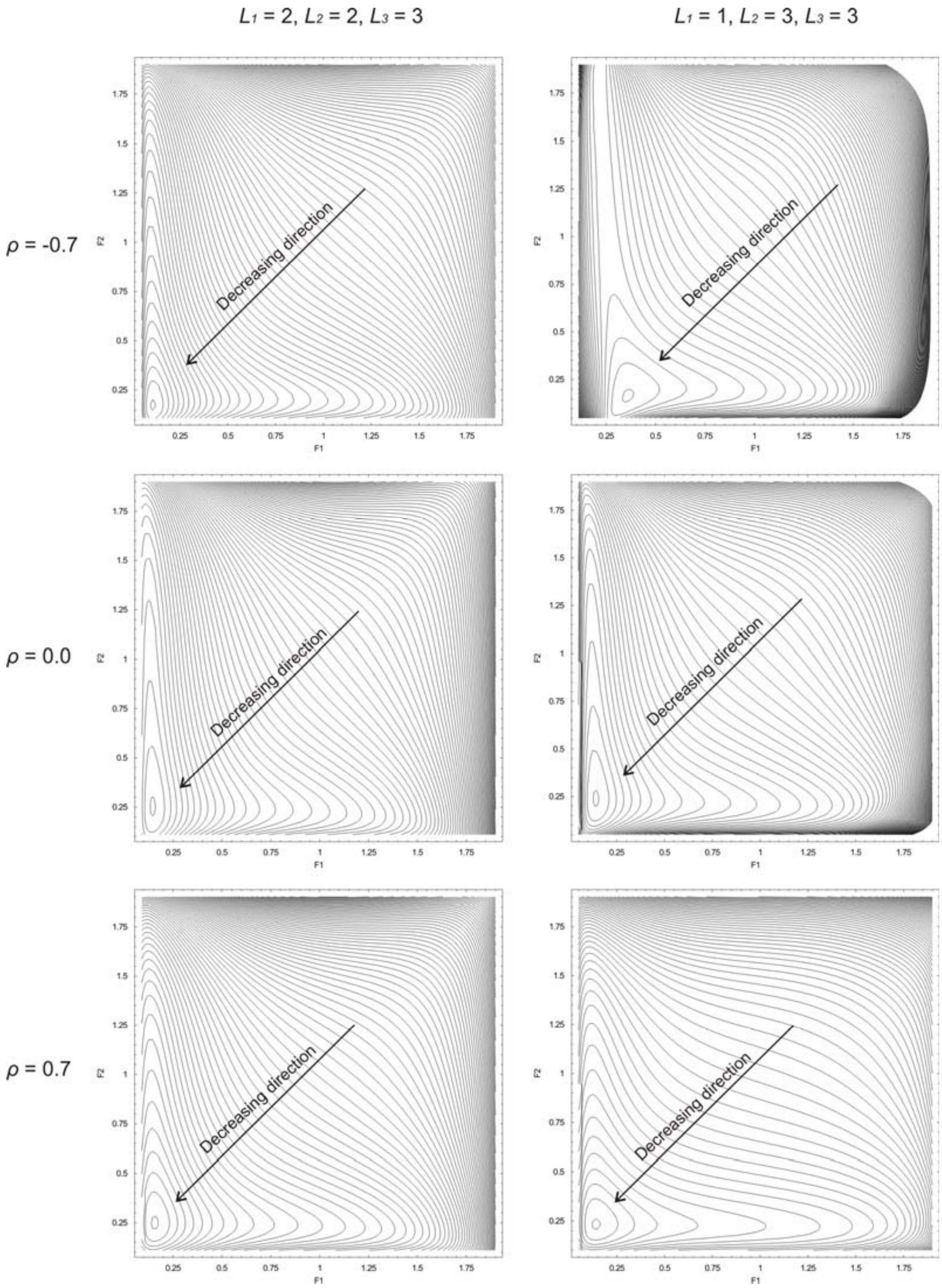

Figure 5.1: The values of J_{S2}

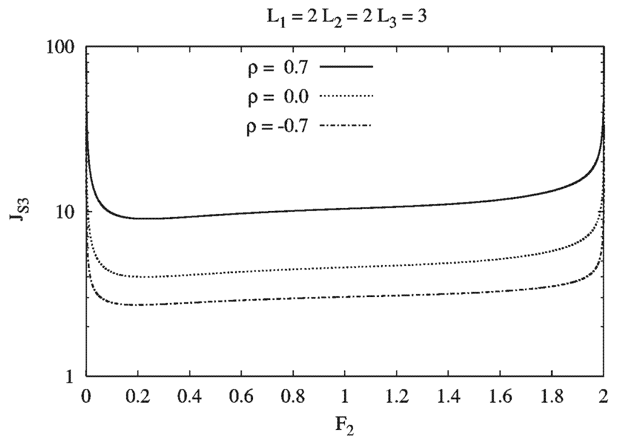

Figure 5.2: The values of J_{S3} when $L_1 = 2$, $L_2 = 2$, $L_3 = 3$

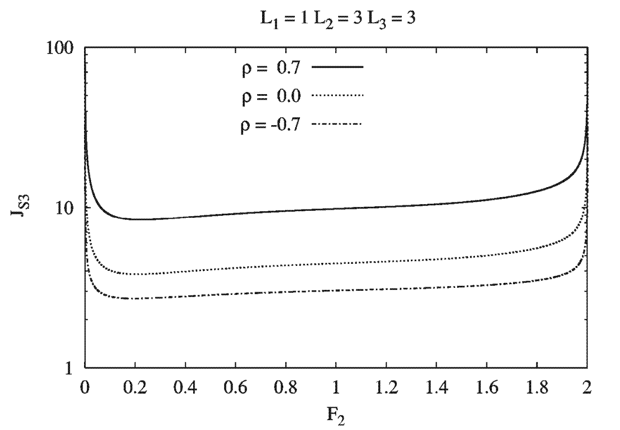

Figure 5.3: The values of J_{S3} when $L_1 = 1$, $L_2 = 3$, $L_3 = 3$

Hosoda, T. and Disney, S.M., (2008), "A generalized order-up-to policy and altruistic behavior in a three-level supply chain", in "Supply Chain Management and Knowledge Management: Integrating Critical Perspectives in Theory and Practice", Edited by Dwivedi, A. and Butcher, T., Palgrave Macmillan, pp190–216, ISBN 978-0-230-57343-7.

Table 5.1: Values of J_{S1}: $L_1 = 2,\ L_2 = 2,\ L_3 = 3$

ρ	$\sqrt{V[\overline{NS}_1]}$	$\sqrt{V[\overline{NS}_2]}$	$\sqrt{V[\overline{NS}_3]}$	J_{S1}
−0.9	1.005	0.928	1.169	3.102
−0.8	1.020	0.902	1.079	3.000
−0.7	1.044	0.908	1.071	3.023
−0.6	1.077	0.935	1.105	3.117
−0.5	1.118	0.976	1.164	3.258
−0.4	1.166	1.031	1.240	3.437
−0.3	1.221	1.098	1.333	3.652
−0.2	1.281	1.182	1.444	3.906
−0.1	1.345	1.286	1.575	4.206
0.0	1.414	1.414	1.732	4.560
0.1	1.487	1.570	1.924	4.982
0.2	1.562	1.759	2.165	5.486
0.3	1.640	1.985	2.472	6.097
0.4	1.720	2.252	2.871	6.844
0.5	1.803	2.565	3.401	7.769
0.6	1.887	2.929	4.111	8.926
0.7	1.972	3.348	5.069	10.390
0.8	2.059	3.830	6.366	12.255
0.9	2.147	4.378	8.120	14.646

Table 5.2: Values of J_{S1}: $L_1 = 1,\ L_2 = 3,\ L_3 = 3$

ρ	$\sqrt{V[\overline{NS}_1]}$	$\sqrt{V[\overline{NS}_2]}$	$\sqrt{V[\overline{NS}_3]}$	J_{S1}
−0.9	1.000	0.933	1.169	3.102
−0.8	1.000	0.924	1.079	3.002
−0.7	1.000	0.956	1.071	3.027
−0.6	1.000	1.017	1.105	3.122
−0.5	1.000	1.097	1.164	3.261
−0.4	1.000	1.192	1.240	3.433
−0.3	1.000	1.302	1.333	3.636
−0.2	1.000	1.428	1.444	3.871
−0.1	1.000	1.570	1.575	4.144
0.0	1.000	1.732	1.732	4.464
0.1	1.000	1.917	1.924	4.842
0.2	1.000	2.130	2.165	5.294
0.3	1.000	2.373	2.472	5.844
0.4	1.000	2.652	2.871	6.523
0.5	1.000	2.971	3.401	7.372
0.6	1.000	3.337	4.111	8.448
0.7	1.000	3.755	5.069	9.825
0.8	1.000	4.232	6.366	11.598
0.9	1.000	4.773	8.120	13.893

Hosoda, T. and Disney, S.M., (2008), “A generalized order-up-to policy and altruistic behavior in a three-level supply chain”, in “Supply Chain Management and Knowledge Management: Integrating Critical Perspectives in Theory and Practice”, Edited by Dwivedi, A. and Butcher, T., Palgrave Macmillan, pp190–216, ISBN 978-0-230-57343-7.

Table 5.3: Values of J_{S2}^*: $L_1 = 2,\ L_2 = 2,\ L_3 = 3$

ρ	F_1^*	F_2^*	$\sqrt{V[\overline{\overline{NS_1}}]}$	$\sqrt{V[\overline{\overline{NS_2}}]}$	$\sqrt{V[\overline{\overline{NS_3}}]}$	J_{S2}^*
−0.9	0.07346	0.10481	1.035	0.845	0.934	2.814
−0.8	0.09560	0.14682	1.104	0.731	0.654	2.489
−0.7	0.10825	0.17411	1.200	0.640	0.499	2.339
−0.6	0.11694	0.19493	1.314	0.564	0.424	2.302
−0.5	0.12391	0.21274	1.441	0.500	0.398	2.339
−0.4	0.12979	0.22753	1.576	0.449	0.403	2.427
−0.3	0.13482	0.23843	1.717	0.412	0.425	2.554
−0.2	0.13911	0.24527	1.863	0.395	0.459	2.717
−0.1	0.14270	0.24865	2.014	0.400	0.505	2.918
0.0	0.14562	0.24952	2.169	0.434	0.561	3.164
0.1	0.14796	0.24890	2.327	0.504	0.632	3.463
0.2	0.14988	0.24776	2.489	0.618	0.725	3.832
0.3	0.15146	0.24678	2.653	0.785	0.853	4.291
0.4	0.15277	0.24629	2.819	1.013	1.037	4.869
0.5	0.15388	0.24631	2.987	1.310	1.310	5.608
0.6	0.15482	0.24678	3.156	1.684	1.721	6.562
0.7	0.15564	0.24762	3.327	2.143	2.336	7.806
0.8	0.15638	0.24874	3.498	2.694	3.246	9.439
0.9	0.15708	0.25011	3.670	3.347	4.570	11.587

Table 5.4: Values of J_{S2}^*: $L_1 = 1,\ L_2 = 3,\ L_3 = 3$

ρ	F_1^*	F_2^*	$\sqrt{V[\overline{\overline{NS_1}}]}$	$\sqrt{V[\overline{\overline{NS_2}}]}$	$\sqrt{V[\overline{\overline{NS_3}}]}$	J_{S2}^*
−0.9	0.76356	0.13470	1.029	0.927	0.968	2.924
−0.8	0.52431	0.16327	1.137	0.784	0.767	2.688
−0.7	0.35779	0.16386	1.305	0.608	0.651	2.564
−0.6	0.27910	0.17316	1.443	0.501	0.562	2.506
−0.5	0.22998	0.18930	1.567	0.425	0.505	2.497
−0.4	0.19440	0.21008	1.688	0.367	0.473	2.527
−0.3	0.16710	0.23270	1.807	0.332	0.461	2.600
−0.2	0.14897	0.24737	1.904	0.351	0.472	2.727
−0.1	0.14191	0.24474	1.947	0.459	0.509	2.915
0.0	0.14054	0.23656	1.956	0.638	0.566	3.161
0.1	0.14049	0.23072	1.956	0.866	0.641	3.464
0.2	0.14058	0.22772	1.956	1.140	0.738	3.834
0.3	0.14060	0.22679	1.956	1.463	0.870	4.289
0.4	0.14055	0.22724	1.956	1.843	1.060	4.858
0.5	0.14048	0.22856	1.957	2.287	1.339	5.583
0.6	0.14043	0.23043	1.957	2.804	1.757	6.518
0.7	0.14043	0.23264	1.957	3.402	2.379	7.738
0.8	0.14048	0.23505	1.957	4.089	3.297	9.343
0.9	0.14060	0.23762	1.956	4.873	4.630	11.459

Fig. 5.4 shows ΔJ_{S2}, a measure of the benefit of altruistic behavior, described as (J_{S1} – J^*_{S2}) / J_{S1}. The average values of the ΔJ_{S2} are 26.1% and 22.7% for the lead-time settings $L_1 = 2, L_2 = 2,$ $L_3 = 3$ and $L_1 = 1,$ $L_2 = 3,$ $L_3 = 3,$ respectively. If it is assumed that ρ is positive as in Lee et al. (2000), then the average values become as high as 26.9% and 23.7%, respectively.

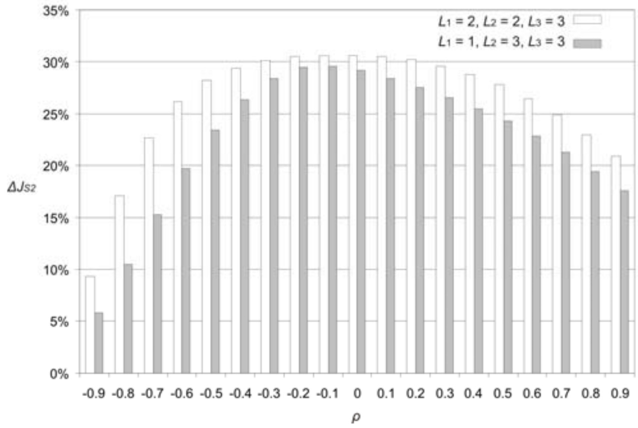

Figure 5.4: ΔJ_{S2} Objective function reduction (%)

5.2 Benefit of Scenario 3

Tables 5.5 – 5.6 provide the results of the numerical investigation. In Scenario 3, since the value of F_1 is constant ($F_1 = 1$), only the optimum values of F^*_2 are shown in these tables. In this scenario, the first level player's standard deviation of the net stock level is identical to $\sqrt{V[\overline{NS}_1]}$ because of the unit value of F_1.

From Tables 5.5 – 5.6, the following insights may be obtained.

- $J^*_{S3} < J_{S1}$ for all values of ρ and lead-time settings. This means that the generalized OUT policy supply chain always outperforms the traditional OUT policy supply chain.
- F^*_2 never has unit value.
- The value of F^*_2 is affected by both the value of ρ and the lead-time settings.
- $J^*_{S3} < J_{S1}$ is achieved by altruistic behavior of the second level player. That is by accepting $\sqrt{V[\overline{\overline{NS}}_2]} > \sqrt{V[\overline{NS}_2]}$.

Table 5.5: Values of J_{S3}^{*}: $L_1 = 2,\ L_2 = 2,\ L_3 = 3$

ρ	F_2^*	$\sqrt{V[\overline{NS_1}]}$	$\sqrt{V[\overline{\overline{NS_2}}]}$	$\sqrt{V[\overline{\overline{NS_3}}]}$	J_{S3}^*
−0.9	0.13673	1.005	0.978	0.956	2.939
−0.8	0.17795	1.020	1.019	0.703	2.741
−0.7	0.19758	1.044	1.089	0.571	2.704
−0.6	0.20904	1.077	1.170	0.514	2.760
−0.5	0.21693	1.118	1.254	0.502	2.874
−0.4	0.22213	1.166	1.343	0.517	3.026
−0.3	0.22508	1.221	1.443	0.548	3.211
−0.2	0.22644	1.281	1.559	0.591	3.430
−0.1	0.22689	1.345	1.698	0.645	3.688
0.0	0.22694	1.414	1.867	0.709	3.990
0.1	0.22698	1.487	2.073	0.788	4.348
0.2	0.22724	1.562	2.325	0.889	4.776
0.3	0.22783	1.640	2.628	1.025	5.293
0.4	0.22877	1.720	2.990	1.218	5.929
0.5	0.23005	1.803	3.420	1.501	6.724
0.6	0.23165	1.887	3.924	1.921	7.732
0.7	0.23351	1.972	4.511	2.546	9.029
0.8	0.23560	2.059	5.188	3.466	10.713
0.9	0.23788	2.147	5.963	4.801	12.911

Table 5.6: Values of J_{S3}^{*}: $L_1 = 1,\ L_2 = 3,\ L_3 = 3$

ρ	F_2^*	$\sqrt{V[\overline{NS_1}]}$	$\sqrt{V[\overline{\overline{NS_2}}]}$	$\sqrt{V[\overline{\overline{NS_3}}]}$	J_{S3}^*
−0.9	0.13632	1.000	0.983	0.956	2.939
−0.8	0.17592	1.000	1.040	0.701	2.741
−0.7	0.19295	1.000	1.135	0.566	2.700
−0.6	0.20129	1.000	1.247	0.503	2.750
−0.5	0.20594	1.000	1.367	0.484	2.851
−0.4	0.20817	1.000	1.495	0.491	2.987
−0.3	0.20868	1.000	1.634	0.515	3.150
−0.2	0.20827	1.000	1.789	0.552	3.341
−0.1	0.20768	1.000	1.964	0.599	3.563
0.0	0.20740	1.000	2.166	0.658	3.824
0.1	0.20774	1.000	2.399	0.733	4.132
0.2	0.20880	1.000	2.672	0.829	4.502
0.3	0.21055	1.000	2.992	0.962	4.954
0.4	0.21289	1.000	3.365	1.151	5.517
0.5	0.21570	1.000	3.801	1.431	6.232
0.6	0.21884	1.000	4.307	1.850	7.156
0.7	0.22219	1.000	4.892	2.473	8.364
0.8	0.22567	1.000	5.564	3.391	9.955
0.9	0.22921	1.000	6.332	4.726	12.058

By using a measure of benefit of $\Delta J_{S3} = (J_{S1} - J^*_{S3}) / J_{S1}$, the benefit of Scenario 3 has been plotted in Fig. 5.5. The average benefit in Scenario 3 is 11.9% and 13.2% for the lead-time settings $L_1 = 2, L_2 = 2,\ L_3 = 3$ and $L_1 = 1,\ L_2 = 3,\ L_3 = 3$, respectively. With the assumption of positive values of ρ, these average benefits will increase to 13.0% and 14.8%, respectively.

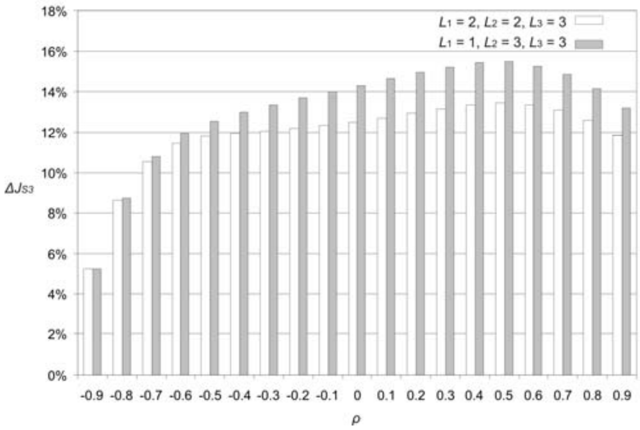

Figure 5.5: ΔJ_{S3} Objective function reduction (%)

6 Conclusion

By using a three-level supply chain model, three different scenarios have been investigated and some interesting insights have been obtained. To obtain analytical expressions of the variances of the end-period net stock levels at each level in the generalized OUT policy supply chain, a newly developed method is exploited. The traditional OUT policy supply chain has been used as a benchmark for performance in Scenario 1.

In Scenario 2, two proportional controllers were incorporated, one at the first level, and the other at the second level. By adjusting the values of the proportional controllers properly, a significant amount of benefit can be obtained. Neither of these two controllers takes unit values; however, $\sqrt{V[\overline{\overline{NS}}_2]}$ is less than $\sqrt{V[\overline{NS}_2]}$, and only altruistic behavior of the first level is required to enjoy such a benefit, in almost all parameter settings. The quantified benefits are quite large, and it is shown that such *benefits come from each player in the supply chain doing what is the best for itself and the supply chain, rather than doing what is the best for its own selfish interests*. In other words, a sequence of optimum policies does not provide a global minimum cost of a supply chain.

Scenario 2 has shown the lowest cost function in the model settings; however, to enjoy the benefit, the altruistic behavior at the first level must be accepted. But this is usually where the most expensive inventory holding costs are incurred. In addition, the redistribution of the inventory costs among players might be a barrier to implementation of Scenario 2, as we discussed in Literature review. Some additional incentives for the first level player may be necessary, since the overall benefit completely depends on the degree of altruistic behavior given by the first level player.

To overcome incentive conflict issues in a supply chain, Scenario 3 is considered. Scenario 3 may be a case of a three-level supply chain that is governed by two organizations: the first level inventory is managed by a retailer, and both second and third level inventories are managed by a supplier, for example. The retailer's concern is to minimize its own inventory related costs. The supplier's interest is to minimize the sum of the inventory related cost at both second and third levels. The retailer can help the supplier by providing up-to-date market demand information. To achieve the goal independently, the retailer may use the traditional OUT policy, which minimizes its own standard deviation of net stock level, and the supplier incorporates F_2 into the OUT policy at the second level and employs the traditional OUT policy at the third level to minimize its total inventory related costs. Having worked in the real business world, the two organization three-level supply chain in Scenario 3 might become realistic. Since the supplier behaves altruistically and it is the supplier who enjoys the benefit from Scenario 3, it may be more acceptable to a real business world than Scenario 2. Therefore, Scenario 3 could bring a "win-win" situation in a supply chain easily with less difficulty in implementation and operation.

There might be some challenges to enjoy the benefit. The results shown herein depend on a crucial assumption that all necessary information is shared without delay and exploited in a proper manner to obtain optimum values of F_n. To share the information without delay, the use of information technologies such as Internet and/or EDI might be essential. For the latter point, since the value of the objective function is not so sensitive to the value of F_n (see, Figure 5.1 – 5.3, for example), even if the values of proportional controllers actually used in a supply chain are slightly different from the optimum values, the supply chain still can reduce its total costs by exploiting the generalized OUT policy.

Appendix

To optimize supply chain costs, analytical expressions of a cost function are essential. In this research, analytical expressions of variances are exploited. These expressions are able to be obtained through the following steps (Hosoda 2005).

Step 1: Express the end-period net stock level process as an ARMA(1,*q*) process, where *q* is a non-negative integer.

Step 2: Obtain the analytical expression of the variance of the ARMA(1,*q*) process.

The most significant advantage of this method is that it is not necessary to specify the value of the lead-times in a supply chain to gain analytical expressions.

From now, the details about how to obtain $\sqrt{V[\overline{\overline{NS}}_1]}$ will be shown. By following the same steps, $\sqrt{V[\overline{\overline{NS}}_2]}$ is also obtainable.

In our model, the order placed by the first level player is expressed as

$$O_{t,1} = \widetilde{D}_t^{L_1} + F_1(DIP_{t,1} - (WIP_{t,1} + NS_{t,1})). \tag{A.1}$$

It is assumed herein that $NS_{t,1}$ can be described as

$$NS_{t,1} = NS_{t-1,1} + O_{t-L_1,1} - D_t.$$

From above equation,

$$O_{t,1} = NS_{t+L_1,1} - NS_{t+L_1-1,1} + D_{t+L_1}, \tag{A.2}$$

can be obtained. $WIP_{t,1}$ can be expressed as

$$WIP_{t,1} = \sum_{i=1}^{L_1-1} O_{t-i,1}. \tag{A.3}$$

Consider first the case when L_1 is greater than one. Substituting Eq. A.2 into Eq. A.3, another expression of $WIP_{t,1}$ is

$$WIP_{t,1} = NS_{t+L_1-1,1} - NS_{t,1} + \sum_{i=1}^{L_1-1} D_{t+i}. \tag{A.4}$$

After incorporating Eq. A.2 and Eq. A.4 into the LHS and the RHS of Eq. A.1, respectively, some algebraic simplification yields

$$NS_{t+1,1} = (1-F_1)NS_{t,1} + (\widetilde{D}_{t+1-L_1}^{L_1} - D_{t+1}) + F_1(DIP_{t+1-L_1,1} - \sum_{i=1}^{L_1-1} D_{t+1-L_1+i}). \tag{A.5}$$

Now, D_{t+1} can be expressed by using D_{t+1-L_1},

$$\begin{aligned} D_{t+1} &= \rho D_t + \varepsilon_{t+1} \\ &= \rho(\rho D_{t-1} + \varepsilon_t) + \varepsilon_{t+1} \\ &= \cdots \\ &= \rho^{L_1} D_{t+1-L_1} + \rho^{L_1-1}\varepsilon_{t+2-L_1} + \rho^{L_1-2}\varepsilon_{t+3-L_1} + \cdots + \rho\varepsilon_t + \varepsilon_{t+1}. \end{aligned}$$

Thus, since $\tilde{D}^{L_1}_{t+1-L_1} = E[D_{t+1} \mid D_{t+1-L_1}]$, $\tilde{D}^{L_1}_{t+1-L_1}$ is given by

$$\tilde{D}^{L_1}_{t+1-L_1} = \rho^{L_1} D_{t+1-L_1}.$$

Therefore, $\tilde{D}^{L_1}_{t+1-L_1} - D_{t+1}$ can be written as

$$\tilde{D}^{L_1}_{t+1-L_1} - D_{t+1} = -\sum_{i=0}^{L_1-1}(\rho^i \varepsilon_{t+1-i}). \tag{A.6}$$

$\sum_{i=1}^{L_1-1} D_{t+1-L_1+i}$ can be described as

$$\begin{aligned} \sum_{i=1}^{L_1-1} D_{t+1-L_1+i} &= D_{t+2-L_1} + D_{t+3-L_1} + \cdots + D_t \\ &= \rho D_{t+1-L_1} + \varepsilon_{t+2-L_1} + \rho(\rho D_{t+1-L_1} + \varepsilon_{t+2-L_1}) + \varepsilon_{t+3-L_1} + \cdots \\ &\quad + \rho^{L_1} D_{t+1-L_1} + \rho^{L_1-2}\varepsilon_{t+2-L_1} + \rho^{L_1-3}\varepsilon_{t+3-L_1} + \cdots + \varepsilon_t \\ &= \frac{\rho(\rho^{L_1-1}-1)}{\rho-1} D_{t+1-L_1} + \sum_{i=0}^{L_1-2}\sum_{j=0}^{L_1-2-i}(\rho^j \varepsilon_{t+2-L_1+i}). \end{aligned}$$

And $DIP_{t+1-L_1,1}$ is

$$DIP_{t+1-L_1,1} = E\left[\sum_{i=1}^{L_1-1} D_{t+1-L_1+i} \mid D_{t+1-L_1}\right] = \frac{\rho(\rho^{L_1-1}-1)}{\rho-1} D_{t+1-L_1}.$$

Thus, an expression for $F_1(DIP_{t+1-L_1,1} - \sum_{i=1}^{L_1-1} D_{t+1-L_1+i})$ becomes

$$F_1\left(DIP_{t+1-L_1,1} - \sum_{i=1}^{L_1-1} D_{t+1-L_1+i}\right) = -F_1\left(\sum_{i=0}^{L_1-2}\sum_{j=0}^{L_1-2-i}(\rho^j \varepsilon_{t+2-L_1+i})\right). \tag{A.7}$$

By substituting Eq. A.6 and Eq. A.7 into Eq. A.5, the final expression of the end-period net stock level process at the first level can be expressed as

$$\begin{aligned} NS_{t+1,1} &= \eta_1 NS_{t,1} - \lambda_{0,1}\varepsilon_{t+1} - \lambda_{1,1}\varepsilon_t - \cdots - \lambda_{L_1-1,1}\varepsilon_{t+2-L_1} \\ &= \eta_1 NS_{t,1} - \sum_{i=0}^{L_1-1}\lambda_{i,1}\varepsilon_{t+1-i}, \end{aligned} \tag{A.8}$$

where $\eta_1 = 1 - F_1$ and $\lambda_{i,1} = \rho^i + \dfrac{F_1(\rho^i - 1)}{\rho - 1}$.

From Eq. A.8, it can be seen that $NS_{t,1}$ follows ARMA($1, L_1 - 1$) process with AutoreRressive (AR) coefficient η_1 and Moving Average (MA) coefficients $\lambda_{0,1}$,…, $\lambda_{L_1-1,1}$. It should be noted that in the case of the traditional OUT policy where $F_1 = 1$, the AR coefficient η_1 becomes zero and $NS_{t,1}$ follows ARMA(0, L_1 – 1) process. This result coincides with Gilbert (2005). Generally, the variance of ARMA(1,*q*) process at level n can be expressed as

$$\frac{\sigma_\varepsilon^2}{1-\eta_n^2}\left(\sum_{i=0}^{q}\lambda_{i,n}^2 + 2\eta_n\sum_{i=1}^{q}\left(\lambda_{i,n}\sum_{j=0}^{i-1}(\eta_n^{i-1-j}\lambda_{j,n})\right)\right). \tag{A.9}$$

After substituting the AR and the MA coefficients into Eq. A.9, some algebraic simplification yields Eq. 4.4. The same conclusions can be obtained for the case of $L_1 = 1$, where $WIP_{t,1} = 0$, by following the same steps described herein.

References

Y. Aviv and A. Federgruen, ‘The operational benefits of information sharing and vendor managed inventory (VMI) programs’, Working paper, Washington University, St. Louis, MO (1998).

R. Bellman, *Dynamic Programming*, (NJ: Princeton University Press, 1957), p. 83.

G. E. P. Box, G. M. Jenkins and G. C. Reinsel, *Time Series Analysis: Forecasting and Control*, 3rd edn (NJ: Prentice Hall, Englewood Cliffs, 1994).

G. E. Collins, J. R. Johnson and W. Krandick, ‘Interval arithmetic in cylindrical algebraic decomposition’, *Journal of Symbolic Computation*, 34 (2002) 145–157.

S. M. Disney and D. R. Towill, ‘The effect of vendor managed inventory (VMI) dynamics on the Bullwhip Effect in supply chains’, *International Journal of Production Economics*, 85 (2003a) 199–215.

S. M. Disney and D. R. Towill, 'Vendor-managed inventory and bullwhip reduction in a two-level supply chain', International *Journal of Operations & Production Management*, 23 (2003b) 625–651.

S. Gavirneni, 'Price fluctuations, information sharing, and supply chain performance', *European Journal of Operational Research*, 174 (2006) 1651-1663.

I. Giannoccaro and P. Pontrandolfo, 'Supply chain coordination by revenue sharing contracts' *International Journal of Production Economics*, 89 (2004) 131-139.

K. Gilbert, 'An ARIMA supply chain model', *Management Science*, 51 (2005) 305–310.

S. C. Graves, 'A single-item inventory model for a nonstationary demand process', *Manufacturing & Service Operations Management*, 1 (1999) 50–61.

T. Hosoda, 'The principles governing the dynamics of supply chains', PhD dissertation, Cardiff University, UK (2005).

T. Hosoda and S. M. Disney, 'The governing dynamics of supply chains: The impact of altruistic behavior', *Automatica*, 42 (2006a) 1301-1309.

T. Hosoda and S. M. Disney, 'On variance amplification in a three-echelon supply chain with minimum mean square error forecasting', *OMEGA: The International Journal of Management Science*, 34 (2006b) 344-358.

T. Hosoda, M. M. Naim, S. M. Disney and A. Potter, 'Is there a benefit to sharing market sales information? Linking theory and practice', *Computers & Industrial Engineering,* 54 (2008) 315-326.

H. Kim and J. K. Ryan, 'The cost impact of using simple forecasting techniques in a supply chain', *Naval Research Logistics*, 50 (2003) 388–411.

H. L. Lee, V. Padmanabhan, and S. Whang, 'Information distortion in a supply chain: The bullwhip effect', *Management Science*, 43 (1997) 546–558.

H. L. Lee, K. C. So and C. S. Tang, 'The value of information sharing in a two-level supply chain', *Management Science*, 46 (2000) 626–643.

J. Luo, ‘Buyer-vendor inventory coordination with credit period incentives’, *International Journal of Production Economics*, 108 (2007) 143-152.

S. Raghunathan, ‘Information sharing in a supply chain: A note on its value when demand is nonstationary’, *Management Science*, 47 (2001) 605–610.

A. A. Tsay, ‘The quantity flexibility contract and supplier-customer incentives’, *Management Science,* 45 (1999) 1339-1358.

H. J. Vassian, ‘Application of discrete variable servo theory to inventory control’, *Journal of the Operations Research Society of America*, 3 (1955) 272–282.

Z. K. Weng, ‘Channel coordination and quantity discounts’, *Management Science* 41 (1995) 1509-1522.