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A delayed demand supply chain: Incentives for upstream players

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Abstract

We study a decentralized supply chain where only delayed market demand information is available for making replenishment decisions. The impact of this delay is quantified in a serially linked two-level supply chain where each player exploits the order-up-to replenishment policy. The market demand is assumed to be a first-order autoregressive process. It is shown that the first level of the supply chain benefits from shorter time delays; however, the benefit for the second level is quite minor at best and can sometimes even be (counter-intuitively) detrimental. We conclude that the second level does not have a strong incentive to reduce the time delays in the shared market demand information.

Keywords: Information delay, inventory, manufacturing, order-up-to policy, stochastic process, supply chain management, time series, RFID

1. Introduction

We consider a situation where the available market demand information used to make replenishment decisions is accurate but tardy. Delayed market demand information is not unusual in the real world. The recent popularity of Radio Frequency IDentification (RFID) technology suggests that delays

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and errors in information flows may be quite common as it is often advocated that RFID technology allows the sharing of accurate real-time supply chain information. Here we quantify the impact of the time delay on a serially linked two-level supply chain performance under a market demand information sharing scheme[1]. The supply chain consists of a distributor at the first level and a manufacturer at the second level and is operated in a decentralised manner. It is assumed that the delayed demand information is common knowledge in the supply chain thanks to an information sharing scheme. This enables us to isolate the impact of the time lag from the impact of sharing market demand information. The manufacturer exploits the shared market information to improve its forecast accuracy. To represent the market demand we use the first-order autoregressive (AR(1)) process as it has been shown that many actual market demand processes can be modeled by this structure, [1] and [2]. To quantify the impact of the delay in the market demand information, we measure both the distributor's and the manufacturer's inventory cost as well as the manufacturer's production variability costs.

To the best of our knowledge, [3] and [4] may be the first research on the delayed information problem. They focus on the optimum ordering policy for a single supply chain echelon when inventory information is delayed and show that when a constant/stochastic time delay in inventory information exists, an Order-Up-To (OUT) type policy is the optimum replenishment policy. The objective of our research is slightly different from [3] and [4]. We quantify the economic consequences of shorter time delays in the market demand information for each player in a two-level supply chain. If the economic benefit for the manufacturer is large, he would voluntarily cooperate with the distributor (e.g. by affixing RFID tags to each item) to reduce the time delay in the market demand information. Otherwise, there is no reason for the manufacturer to take this action. This aspect of our research is a key difference from [3] and [4]. A similar but different issue, inaccuracy of information in a supply chain, has attracted much research recently; [5], [6], [7], [8] and [9]. However, here we will focus solely on the issue of the time delayed demand information in the two-level supply chain.

Using a single level supply chain model with the OUT policy, [10] con-

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[1]In this paper, 'delay' refers to the backward shift of the information in the time dimension.

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sider the case where the most recent demand information is available to the replenishment decision maker but is not available to the forecaster. Thus, order quantities are based on inventory information that includes the most recent demand information but the forecasts that are affected by the information delay. It is shown that when such forecasts are used in the OUT policy, the bullwhip effect can be reduced. Assuming that the most recent demand information is not available, [11] propose a replenishment policy which can overcome the disadvantage of the demand information delay in a single level supply chain. It is shown that without eliminating the delay in the demand information, a manufacturer can reduce its production cost by exploiting the new policy that can be easily implemented and requires little to no running cost.

Recently, there is a stream of literature discussing the impact of better information / knowledge on supply chain performance. In a setting where the market demand process is mis-specified by a retailer and this erroneous information is shared with a manufacturer, [9] observe that this is not always detrimental for the supply chain as a whole. Using a two-level decentralized supply chain model, [12] show that the sharing of the more accurate demand information can decrease the expected profit of the supply chain. [13] consider the situation where the manufacturer is selecting a retail partner. They show that the manufacturer's expected profit depends on the retailer's ability to generate accurate demand forecasts. [14] consider the case where the actual replenishment lead-time is not known for supply chain participants and propose an algorithm to identify the unknown delay in the lead-time.

First we consider the forecasting method used by the distributor. It is shown that the conditional expectation of the demand over the replenishment lead-time plus the review period does not provide the distributor with the minimum inventory cost any more (we call this forecast a "sub-optimum forecast", as the forecast is sub-optimum for the distributor), contrary to the findings of [15] and [16]. Instead, the conditional expectation of the demand over the sum of the replenishment lead-time, the time delay and the review period yields the optimum forecast. This yields the minimum inventory cost for the distributor (called "optimum forecast" herein, as the forecast is optimum only for the distributor). This finding is, we believe, one of the contributions of this research. Thus it might be reasonable to assume that the distributor does not know the optimum forecast under the information delay setting and believes (wrongly) that the sub-optimum forecast is the optimum forecast for him. We will quantify the impact of the sub-optimum

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forecast on the distributor and on the supply chain as a whole.

Additionally, it is shown that only when the optimum forecast is used does the impact of the replenishment lead-time and the time delay on the distributor's inventory cost become identical. Therefore, the impact of reducing the time delay in information may not be the same as that of the lead-time delay if an optimum forecast is not made. We show that irrespective of the forecasting method used, the distributor's inventory cost always becomes smaller when the time delay in the demand information is reduced.

In terms of the impact on the manufacturer, when the distributor exploits the optimum forecast, there is an economic benefit for the manufacturer as well, but the magnitude of this benefit is quite minor. Furthermore, we will show a counter-intuitive result: the manufacturer's cost could increase when the information time delay is shortened or eliminated. This can happen when the distributor exploits the sub-optimum forecast. Therefore, the manufacturer may not have a strong incentive to eliminate the time delay in the demand information if the distributor's forecasting performance is poor. This result might explain why the implementation of RFID technologies is almost always initiated by downstream supply chain players and sometimes ends without any clear benefits (see [17], for example).

This paper is organized as follows: the model will be described with its properties in the next section. Then to illustrate those properties, some results of numerical analysis will be shown in Section 3. We conclude in Section 4.

2. Model

This section presents details of the serially linked two-level supply chain model. The model is based on the previous works of [1], [11] [16], and [18]. To describe the model, let us use an example where products are sold to the end customer on a consignment basis. Fig. 1 represents a schematic of the model. Sales representatives from the distributor pick products from the on-hand inventory in the distributor's warehouse, and put them into the trunk (or, boot) of their vehicles for delivery. This "trunk inventory" will become the consignment inventory when the products are delivered to the customer's stocking point. Customer demand, D_t, is satisfied from the consignment inventory. The distributor cannot observe the value of D_t directly. Only by observing the left-over consignment inventory level does the sales
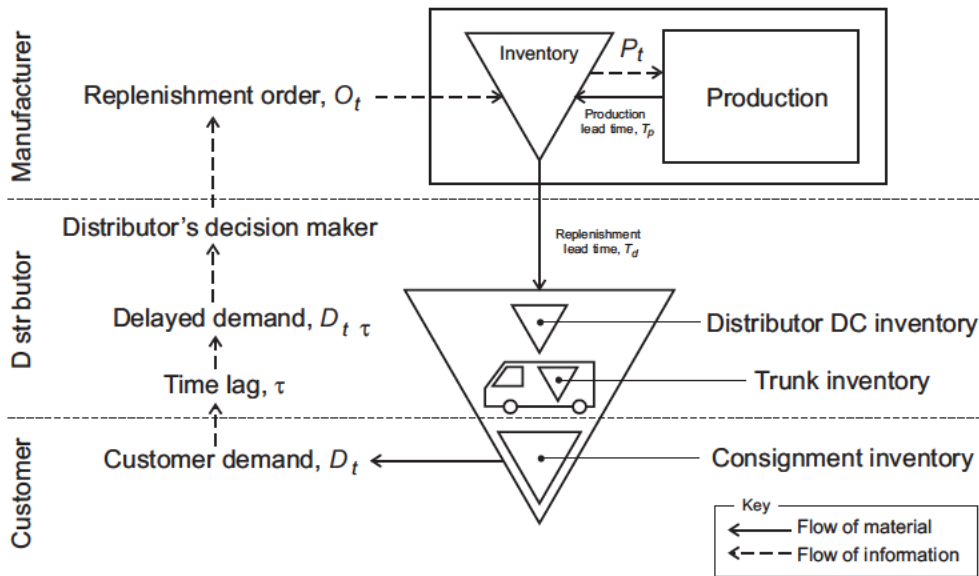
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Figure 1: Schematic of the model

representatives become aware of D_t at the end of period t, and this information will be used in the replenishment decision at the distributor after $\tau(=0, 1, 2, \dots)$ time periods, where τ is a constant value. Thus, at time period t, the distributor knows only the values of $\{D_{t-\tau}, D_{t-\tau-1}, \dots\}$. It is assumed that if the consignment inventory cannot meet all of the customer's demand, then the trunk inventory in the field is used immediately. If the trunk inventory is still not large enough, then the on-hand inventory at the warehouse can be used to satisfy the demand immediately. If there is still unmet demand, such demand will be backlogged. Since we assume that sales representatives have easy access to all types of inventory in the field, we can consider consignment inventory, trunk inventory and warehouse inventory as a single pile of inventory. From now, the net stock level of this combined pile of inventory at t is represented by NS_t. We also assume that the number of products placed in the trunk of their vehicles by the sales representatives is independent from the customer demand. Therefore, the distributor cannot obtain any knowledge about the customer demand by simply observing left-over inventory in its warehouse.

The sequence of events at the distributor is as follows: at the beginning of the time period t, the distributor receives a shipment from the manufacturer and this shipment is stored in the distributor's warehouse. Then the distribu-

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tor's sales representatives pick products for their trunk inventory. At the end of the period, the distributor places an order with the knowledge of the delayed customer's demand information ($D_{t-\tau}$). The manufacturer receives the demand from the distributor without delay and dispatches the products from its on-hand inventory. The constant replenishment lead-time for the distributor is $T_d(= 0, 1, 2, \dots)$. If the manufacturer's does not have enough on-hand inventory to meet all the demand from the distributor, unmet demand is filled from an external source by using an expediting strategy. The expediting strategy assumption is widely used in multi-level supply chain research (see [1][18][19][20][21], for example) and as it allows an analytical model capable of producing managerial insights to be created. Detailed discussion on the expediting assumption can be seen in [1]. Finally the manufacturer makes a production request, P_t, at the end of the period. After a constant production lead-time, $T_p(= 0, 1, 2, \dots)$, P_t will be completed. It is assumed that the manufacturer has infinite raw material.

For an objective function, the inventory costs at both the distributor and the manufacturer and the production cost at the manufacturer are considered. A unit holding cost for on-hand inventory (h_d) and a unit backlog cost for unmet demand (b_d) are incurred by the distributor at the end of each period. The manufacturer incurs a holding cost per unit of on-hand inventory (h_m) and an expediting cost per unit of unmet demand (b_m), charged at the end of each period. When P_t is greater than the standard capacity of the production line G, the manufacturer is charged an overwork cost (w) per period for each product over the capacity G whilst incurring an opportunity cost (u) per period for each unit of lost production below the capacity G.

We note that the setting where time delay exists in the inventory information is quite similar to the setting where the demand information is delayed. This is especially true considering the distributor's inventory balance equation, $NS_t = NS_{t-1} + O_{t-T_d-1} - D_t$, where O_{t-T_d-1} is the replenishment order rate placed by the distributor at $t - T_d - 1$. If the distributor knows NS_t and NS_{t-1}, then the distributor can determine D_t using this inventory balance equation since O_{t-T_d-1} is locally available information, even though the distributor does not observe D_t directly. Therefore, when $\tau \geq 1$ we also must also assume that the distributor does not know $\{NS_t, NS_{t-1}, \dots, NS_{t-\tau+1}\}$ either.

2.1. Market demand model and information sharing

We assume that the customer's demand follows an AR(1) process. An AR(1) process is a well-accepted model to represent supply chain demand processes. It is given by

$$D_t = \mu + \rho(D_{t-1} - \mu) + \varepsilon_t, \tag{1}$$

where D_t is the non-negative market demand realized at time t, μ is a mean of the demand, ρ is an autoregressive parameter constrained to $|\rho| < 1$ and ε_t is a normally distributed white noise element in time t with a mean of zero and a standard deviation of σ_ε. Thus when $\rho = 0$, D_t is a white noise process. The variance of D_t is $\sigma_\varepsilon^2/(1 - \rho^2)$. We consider the case when $0 \leq \rho < 1$ as we assume a negative value of ρ is rather rare in reality. Evidence of non-negative values of ρ have been provided in [1] and [2]. It is assumed that the distributor has complete knowledge of the market demand process (i.e. Eq. 1 is known to the distributor), and this knowledge is shared with the manufacturer. The manufacturer exploits this shared knowledge for its production request decision making.

It should be noted that in [1] and [18], a different form of the AR(1) demand process is used,

$$D_t = d + \rho D_{t-1} + \varepsilon_t,$$

where d is a constant value. The value of the mean of this AR(1) process becomes $d/(1 - \rho)$, which is dependent upon the value of ρ. To avoid unnecessary complexity from an unnecessary initial transient response, we use Eq. 1.

2.2. Distributor's ordering policy and costs

A traditional periodic review OUT policy without time delays (i.e. $\tau = 0$) can be described by the following set of formulae (see, [1] for example),

$$\begin{cases} O_t &= D_t + (S_t - S_{t-1}), \\ S_t &= \hat{D}_t + z_d \sigma_d, \end{cases} \tag{2}$$

where O_t is the order rate at t, S_t is the OUT level determined at t and $\hat{D}_t$ represents the conditional expectation of the total demand during the period $(t, t + T_d + 1]$, where $\hat{D}_t = E\left[\sum_{i=1}^{T_d+1} D_{t+i} | D_t\right]$. σ_d is the standard deviation

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of the sum of the forecast errors over the lead-time and review period, where $\sigma_d^2 = Var\left[\hat{D}_t - \sum_{i=1}^{T_d+1} D_{t+i}\right]$. z_d is set to $z_d = \Phi^{-1}[b_d/(b_d+h_d)]$ to minimize the expected holding and backlog cost in the period $t + T_d + 1$. $\Phi^{-1}[\cdot]$ is the inverse of the cumulative distribution function for the standard normal distribution. $z_d\sigma_d$ represents the target left-over net stock level at the end of each period - the "*safety stock*". $\hat{D}_t$ and σ_d^2 can be shown to be equivalent to

$$
\begin{aligned}
\hat{D}_t &= (T_d+1)\mu + \rho\,\frac{1-\rho^{T_d+1}}{1-\rho}(D_t-\mu), \\
\sigma_d^2 &= \frac{\left((T_d+1)(1-\rho^2)+\rho(1-\rho^{T_d+1})(\rho^{T_d+2}-\rho-2)\right)}{(1-\rho)^2(1-\rho^2)}\sigma_\varepsilon^2.
\end{aligned}
\tag{3}
$$

Note that as shown in Appendix 1, σ_d is identical to the standard deviation of the net stock levels at the end of each period. Therefore, for a given σ_d, the minimized distributor's expected inventory cost, C_d, can be obtained by the classic newsvendor approach. It is $C_d = (h_d + b_d)\phi[z_d]\sigma_d$, where $\phi[\cdot]$ is the probability density function of the standard normal distribution. Thus, the distributor's concern is to minimise σ_d as C_d is proportional to σ_d when h_d and b_d are given and the safety stock is to $z_d\sigma_d$.

In what follows, we will consider the case that the demand information is delayed. The sequence of events is the same as those assumed in the traditional OUT policy. When the most up-to-date demand information is not available, only $D_{t-\tau}$ is available, a replenishment ordering decision maker who is familiar with Eq. 2 might exploit the following ordering policy, instead of Eq. 2.

$$
\begin{cases}
O'_t &= D_{t-\tau} + (S'_t - S'_{t-1}), \\
S'_t &= \hat{D}_{t-\tau} + z_d\acute{\sigma}_d,
\end{cases}
\tag{4}
$$

where O'_t and S'_t are the order rate and the OUT level determined at t respectively when the demand information is delayed. $\hat{D}_{t-\tau}$ represents the conditional expectation of the total demand during the period $(t, t + T_d + 1]$ given $D_{t-\tau}$, where $\hat{D}_{t-\tau} = E\left[\sum_{i=1}^{T_d+1} D_{t+i}|D_{t-\tau}\right]$. $\acute{\sigma}_d$ is the standard deviation of the forecast error subject to $\hat{D}_{t-\tau}$, where $\acute{\sigma}_d^2 = Var\left[\hat{D}_{t-\tau} - \sum_{i=1}^{T_d+1} D_{t+i}\right]$.

$\hat{D}_{t-\tau}$ and $\acute{\sigma}_d^2$ can be shown to be:

$$\hat{D}_{t-\tau} = (T_d + 1)\mu + \rho^{\tau+1}\frac{1 - \rho^{T_d+1}}{1 - \rho}(D_{t-\tau} - \mu), \tag{5}$$

$$\acute{\sigma}_d^2 = \frac{\sigma_\varepsilon^2}{1 - \rho^2}\left(\rho^{\tau+1}\frac{1 - \rho^{T_d+1}}{1 - \rho} - \rho\frac{1 - \rho^{\tau+T_d+1}}{1 - \rho}\right)^2 +$$

$$\frac{\left(\begin{array}{l}(\tau + T_d + 1)(1 - \rho^2)+ \\ \rho(1 - \rho^{\tau+T_d+1})(\rho^{\tau+T_d+2} - \rho - 2)\end{array}\right)}{(1 - \rho)^2(1 - \rho^2)}\sigma_\varepsilon^2. \tag{6}$$

Details are shown in Appendix 2. Appendix 2 also shows that $\acute{\sigma}_d^2$ still represents the variance of the net stock levels, even though the demand information is delayed. However, as shown in the following property, $\hat{D}_{t-\tau}$ does not minimize the variance (or the standard deviation) of the net stock levels at the distributor.

Property 1. *When the demand information is delayed, the forecast given by,*

$$\hat{D}_{t-\tau}^* = (\tau + T_d + 1)\mu + \rho\,\frac{1 - \rho^{\tau+T_d+1}}{1 - \rho}(D_{t-\tau} - \mu), \tag{7}$$

minimizes the variance of the net stock levels. The minimum variance of the net stock for the distributor is

$$\acute{\sigma}_d^{*2} = \frac{\left(\begin{array}{l}(\tau + T_d + 1)(1 - \rho^2)+ \\ \rho(1 - \rho^{\tau+T_d+1})(\rho^{\tau+T_d+2} - \rho - 2)\end{array}\right)}{(1 - \rho)^2(1 - \rho^2)}\sigma_\varepsilon^2,$$

$$\acute{\sigma}_d^{*2} \leq \acute{\sigma}_d^2.$$

Proof 1. *Details are shown in Appendix 2.*

Note that from Eq. 3, Eq. 5 and Eq. 7, it is easy to check that when $\tau = 0$, we have $\hat{D}_t = \hat{D}_{t-\tau} = \hat{D}_{t-\tau}^*$.

From property 1, we may obtain the ordering policy which minimises the distributor's expected inventory cost,

$$\begin{cases} O_t'^* = D_{t-\tau} + (S_t'^* - S_{t-1}'^*), \\ S_t'^* = \hat{D}_{t-\tau}^* + z_d \cdot \acute{\sigma}_d^*, \end{cases} \tag{8}$$

where $O_t'^*$ and $S_t'^*$ are the order rate and the OUT level at time period t respectively, subject to $\hat{D}_{t-\tau}^*$. This ordering policy has the following property.

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Property 2. *When $\hat{D}^*_{t-\tau}$ is exploited within the OUT policy, the impact of the time delay, τ, on the variance (and the standard deviation) of the net stock levels, $\acute{\sigma}^{*2}_d$ (or, $\acute{\sigma}^*_d$), becomes exactly identical to that of the replenishment lead-time, T_d.*

Proof 2. *Appendix 2 provides the necessary steps to obtain this property.*

In other words, when a non-optimum forecast (e.g. $\hat{D}_{t-\tau}$) is used, the impact of τ and T_d on the variance (and the standard deviation) of the net stock levels for the distributor are not identical: it depends on the values τ and T_d used in a sub-optimum forecast. Indeed when $\hat{D}_{t-\tau}$ is exploited, the variance of the forecast errors is given by Eq. 6 which suggests that the impact of τ on $\acute{\sigma}^2_d$ is not the same as that of T_d. Under the special case of $\rho = 0$, however, we have the following property.

Property 3. *When the market demand process follows a white noise process (i.e. $\rho = 0$), even if $\hat{D}_{t-\tau}$ is exploited,*

- *the variance of the net stock levels are minimized and we have $\acute{\sigma}^2_d = \acute{\sigma}^{*2}_d$, and*

- *the impact of τ on the variance (or, standard deviation) of the net stock levels becomes identical to that of T_d.*

Proof 3. *It is obvious since when $\rho = 0$, $\acute{\sigma}^2_d = \acute{\sigma}^{*2}_d = (\tau + T_d + 1)\sigma^2_\varepsilon$.*

Property 3 shows us that when the market demand is a white noise process, both $\hat{D}_{t-\tau}$ and $\hat{D}^*_{t-\tau}$ yield the minimum inventory cost, C_d, for the distributor since C_d is proportional to the distributor's standard deviation of the net stock levels.

Property 4. *When $0 < \rho < 1$, both $\acute{\sigma}_d$ and $\acute{\sigma}^*_d$ are increasing in τ.*

Proof 4. *Details are shown in Appendix 3.*

Since the distributor incurs only the inventory cost (C_d) which is a linear function of the standard deviation of the net stock levels, property 4 suggests that reducing the value of τ is always beneficial for the distributor, irrespective of its forecasting method.

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In the rest of the paper, we will exploit not only $\hat{D}^*_{t-\tau}$ but also $\hat{D}_{t-\tau}$ when we quantify the impact of τ on a supply chain cost. This is reasonable as the distributor might not have any knowledge of property 1 and uses the ordering policy given by Eq. 4 with $\hat{D}_{t-\tau}$, believing it yields the optimum inventory cost for the distributor even though the demand information is delayed. The other valid reason is that $\hat{D}_{t-\tau}$ has the following interesting property.

Property 5. *When $0 < \rho < 1$ and $0 < \tau$,*

- *the variance of the distributor's orders when the suboptimal forecast is used O'_t, $Var[O']$, is always less than the distributors orders when the optimal forecast is used O'^*_t, $Var[O'^*]$,*

- *$Var[O']$ is decreasing in τ, whilst $Var[O'^*]$ is increasing in τ.*

Proof 5. *The proof is shown in Appendix 4.*

Property 5 means that the combined use of the delayed demand information and the non-optimum forecast (i.e. $\hat{D}_{t-\tau}$) can mitigate the well-known Bullwhip effect (see, [22], for example), when the market demand is positively correlated over the time. Furthermore, surprisingly, the level of Bullwhip reduction will increase as the time delay becomes longer. A simple explanation of this is that when the demand information delay is longer, the sub-optimum forecast becomes less responsive to demand fluctuations since $\rho^{\tau+1}$ in $\hat{D}_{t-\tau}$ (see, Eq. 5) becomes smaller as τ increases. Note that it is well recognized that in a multi-level supply chain, lower Bullwhip can bring benefit to upper levels of a supply chain (see, [23], for example). Therefore, property 5 raises a question about whether there is actually some economic benefit to the manufacturer from time delays in the demand information. To answer this question, first let us develop a model for the manufacturer.

2.3. Manufacturer's ordering policy and costs

It is assumed that the review period used by the manufacturer is the same as that by the distributor. The manufacturer receives an order from the distributor without delay. The market demand information, $D_{t-\tau}$, is also shared by the distributor and is known at time period t for the manufacturer. Therefore, the ordering policy exploited by the manufacturer can be described as

$$\begin{cases} P_t &= \tilde{O}_t + (M_t - M_{t-1}), \\ M_t &= \hat{O}_t + z_m \sigma_m, \end{cases}$$

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where $\tilde{O}_t$ is the distributor's demand at time t and can be $O_t^{'*}$ or O_t', depending on the distributor's forecasting method. M_t is the OUT level determined at t and $\hat{O}_t$ is the conditional expectation of the total demand from the distributor during the period $(t, t + T_m + 1]$ given $\tilde{O}_t$ and $D_{t-\tau}$. That is $\hat{O}_t = E\left[\sum_{i=1}^{T_m+1} \tilde{O}_{t+i} | \tilde{O}_t, D_{t-\tau}\right]$. $z_m = \Phi^{-1}[b_m/(b_m + h_m)]$ and σ_m is the standard deviation of the forecast error, where $\sigma_m^2 = Var\left[\hat{O}_t - \sum_{i=1}^{T_m+1} \tilde{O}_{t+i}\right]$. The manufacturer's minimized inventory cost is given by $C_m = (h_m + b_m)\phi[z_m]\sigma_m$.

The manufacturer also incurs a production cost. The expected production cost period is $uE\left[(G - P_t)^+\right] + wE\left[(P_t - G)^+\right]$, where $(x)^+ = \max[x, 0]$. By applying newsvendor logic we may obtain C_P, the minimized value of the capacity cost, $C_P = (u + w)\phi[z_p]\sigma_P$, where $z_p = \Phi^{-1}[w/(w + u)]$ and σ_P is the standard deviation of P_t. The optimal capacity is given by $G^* = \mu + \sigma_P z_p$.

General expressions of $\hat{O}_t$, σ_m^2 and σ_P^2 are

$$\hat{O}_t = (T_m + 1)\mu + (\rho + \rho K - K)\frac{1 - \rho^{T_m+1}}{1 - \rho}(D_{t-\tau} - \mu),$$

$$\sigma_m^2 = \frac{\begin{pmatrix} (T_m + 1)(\rho^2 - 1) + K^2(\rho - 1)^2(\rho^{2(T_m+1)} - 1) + \\ 2K(\rho - 1)(\rho^{T_m+1} - 1)(\rho^{T_m+2} - 1) + \\ \rho(\rho^{T_m+1} - 1)(\rho(\rho^{T_m+1} - 1) - 2) \end{pmatrix}}{(\rho - 1)^3(1 + \rho)}\sigma_\varepsilon^2,$$

$$\sigma_P^2 = \frac{\left(\rho^{T_m+1}(\rho + \rho K - K)\right)^2}{1 - \rho^2}\sigma_\varepsilon^2 + \left(\frac{1 - \rho^{T_m+1}(\rho + \rho K - K)}{1 - \rho}\right)^2\sigma_\varepsilon^2,$$

where $K = \rho(1 - \rho^{\tau + T_d + 1})/(1 - \rho)$ for the case when the distributor uses the optimal forecast and $K = \rho^{\tau+1}(1 - \rho^{T_d+1})/(1 - \rho)$ for the case when the distributor uses the sub-optimal forecast. Details are shown in Appendix 5.

Property 6. *When $0 < \rho < 1$ and the optimal forecast $\hat{D}_{t-\tau}^*$ is exploited, both σ_m and σ_P are increasing in τ. However, when the sub-optimal forecast $\hat{D}_{t-\tau}$ is exploited, both σ_m and σ_P are decreasing in τ, if $0 < \rho < 1$.*

Proof 6. *The proof is given in Appendix 6.*

Property 6 suggests that when the distributor exploits $\hat{D}_{t-\tau}^*$, reducing the delay in the market demand information decreases the inventory and the production costs of the manufacturer. Therefore, the manufacturer might

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have an incentive to work together with the distributor to reduce the information delay. On the other hand, in the case of $\hat{D}_{t-\tau}$, τ has an opposite effect on the two costs: reducing τ results in higher inventory and production costs of the manufacturer, as we conjectured from property 5. Therefore, in this case the manufacturer does not have any incentive to cooperate with the distributor to improve the market demand information availability.

Property 7. *When $\rho = 0$, both σ_m^2 and σ_P^2 are independent of τ and the distributor's forecasting method.*

Proof 7. *Setting $\rho = 0$, we have $\sigma_m^2 = (T_m + 1)\sigma_\varepsilon^2$ and $\sigma_P^2 = \sigma_\varepsilon^2$.*

Property 7 means that when the market demand follows a white noise process, the manufacturer may not be interested in cooperating with the distributor to reduce τ, since the manufacturer's costs are proportional to σ_m and σ_P, which are independent of τ.

To illustrate the properties found, in the next section we will now conduct a numerical analysis.

3. Numerical analysis

In this section, the following values will be used $\tau = 0, 1, 2, 3$, $T_d = 4$ and $T_m = 4$. Unless otherwise stated, the cost parameters are assumed to be; $h_d = 2$, $b_d = 50$, $h_m = 1$, $b_m = 25$, $u = 2$, $w = 50$. In terms of the market demand, $0.0 \le \rho \le 0.9$ and $\sigma_\varepsilon = 10$ is assumed. We measure total supply chain cost with the sum of the inventory cost at each level, C_d, C_m, and the manufacturers production cost, C_P. Since the closed form expressions to obtain C_d, C_m and C_P have been given, interested readers may efficiently conduct their own numerical analysis with different parameter settings and will find similar findings to those shown in this section. In addition to the costs, we use the following measure to quantify the impact of eliminating the information delay:

$$\% \text{ reduction} = \frac{(\text{total cost when } \tau = k - 1) - (\text{total cost when } \tau = k)}{\text{total cost when } \tau = k} \times 100,$$

where $k = 1, 2$, or 3.

Table 1 shows calculated values of % reduction of total supply chain cost. Fig. 2 shows the calculated values of the distributor's cost, C_d, and the manufacturer's total cost, $C_m + C_P$, for each forecasting method. Recall that

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Table 1: Percentage reduction in costs

|	τ	Distributor		Manufacturer	
		Optimum	Sub-optimum	Optimum	Sub-optimum
$\rho = 0.9$	$3 \to 2$	-14.3%	-17.2%	-4.1%	3.7%
	$2 \to 1$	-16.5%	-20.3%	-4.8%	4.0%
	$1 \to 0$	-19.5%	-22.1%	-5.6%	4.2%
	Average	-16.8%	-19.9%	-4.8%	4.0%
$\rho = 0.7$	$3 \to 2$	-10.3%	-11.2%	-0.6%	3.6%
	$2 \to 1$	-12.2%	-13.8%	-0.9%	5.1%
	$1 \to 0$	-14.9%	-16.1%	-1.3%	7.1%
	Average	-12.4%	-13.7%	-1.0%	5.2%
$\rho = 0.5$	$3 \to 2$	-8.2%	-8.4%	0.0%	1.2%
	$2 \to 1$	-9.7%	-10.4%	-0.1%	2.6%
	$1 \to 0$	-11.8%	-12.6%	-0.2%	5.4%
	Average	-9.9%	-10.5%	-0.1%	3.1%
$\rho = 0.3$	$3 \to 2$	-7.2%	-7.2%	0.0%	0.3%
	$2 \to 1$	-8.4%	-8.5%	0.0%	0.9%
	$1 \to 0$	-10.0%	-10.4%	0.0%	3.1%
	Average	-8.5%	-8.7%	0.0%	1.4%
$\rho = 0.0$	$3 \to 2$	-6.5%	-6.5%	0.0%	0.0%
	$2 \to 1$	-7.4%	-7.4%	0.0%	0.0%
	$1 \to 0$	-8.7%	-8.7%	0.0%	0.0%
	Average	-7.5%	-7.5%	0.0%	0.0%

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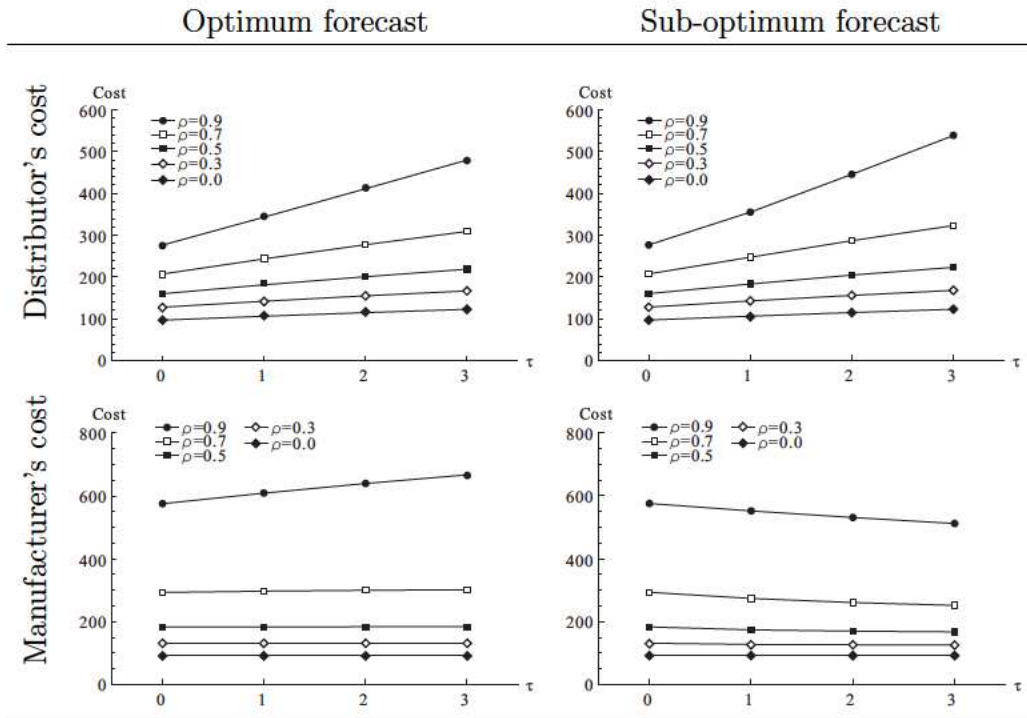
Figure 2: Impact of forecast on each player's costs

when $\tau = 0$, we have $\hat{D}_{t-\tau} = \hat{D}^*_{t-\tau}$ (see Eq. 5 and Eq. 7). Thus when $\tau = 0$ the cost given by the sub-optimum forecast becomes equal to the corresponding cost given by the optimum forecast as illustrated in Fig. 2. Based on Table 1 and Fig. 2, we can observe: For the distributor, there is always a cost benefit ranging from -6.5% to -22.1%. This benefit arises from property 4. Therefore, the distributor always has a strong incentive to reduce τ. However, for the manufacturer, most of the time the cost reduction is quite small and may even be detrimental (a positive % reduction indicates that the manufacturer's cost has increased). This troubling situation may exist when a sub-optimum forecast ($\hat{D}_{t-\tau}$) is exploited by the distributor, as reducing τ will actually *increase* the manufacturer's cost (recall, property 6). Therefore, in the case of a non-MMSE forecast, the manufacturer has no incentive for (or, indeed may even be against) eliminating the time delays. On the other hand, when the distributor uses the optimum forecast ($\hat{D}^*_{t-\tau}$), the manufacturer can enjoy some benefits by reducing τ. However, the amount of the
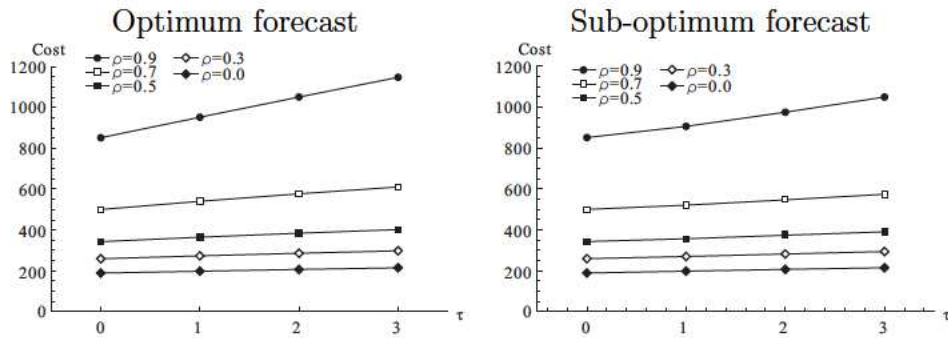
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Figure 3: Impact of forecast on total supply chain cost

benefit is quite minor, especially when the value of ρ is small. Only when the market demand is highly correlated across the time (e.g. $\rho = 0.9$) and the optimum forecast is used, does the manufacturer may have an incentive to cooperate with the distributor to reduce the demand information delays in the supply chain. As we expected from property 7, when the market demand follows a white noise process (i.e. $\rho = 0$), the manufacturer's costs are constant irrespective of τ and the distributor's forecasting method. Therefore, the manufacturer may not be at all interested in reducing the delay in the demand information in this case. Together with property 3, we may conclude that the white noise demand process assumption yields only quite special results.

In our model setting, we do not assume that any coordination schemes exist in the supply chain. Only the demand information sharing scheme is assumed. However, it might be interesting to consider the impact of τ on the total supply chain cost, $C_d + C_m + C_P$, as shown in Fig. 3. Here the total supply chain cost in monotonically increasing in τ with both forecasting methods. As property 4 and 6 suggest, when the optimum forecast is exploited, shorter delays result in reduced total supply chain cost. However, this result is not always applicable for the case of the sub-optimum forecast as suggested by property 6. Property 6 suggests that in the case of the sub-optimum forecast, if the manufacturer's cost is dominant, the total supply chain cost could increase as the result of eliminating the demand information delay. Fig. 4 shows the results of different cost parameter settings where $h_d = 2$, $b_d = 50$, $h_m = 1$, $b_m = 25$, $u = 30$ and $w = 200$ (left), and $h_d = 2$, $b_d = 50$, $h_m = 5$, $b_m = 100$, $u = 2$ and $w = 50$ (right), respectively. Under
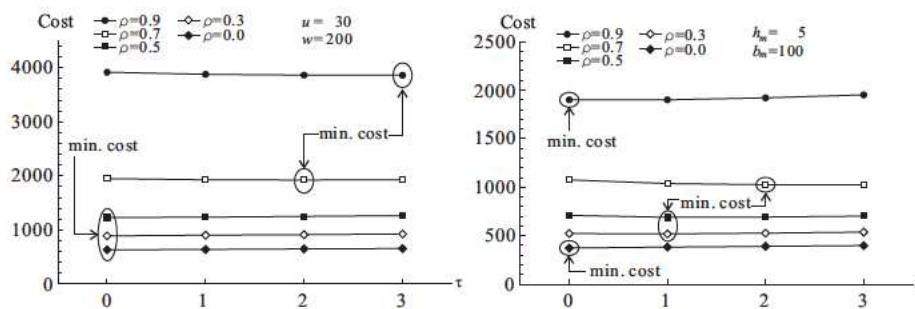
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Figure 4: Total supply chain cost: Sub-optimum forecast with different cost parameters

these settings, almost all lines are quite flat suggesting eliminating the information delay does not have a significant impact on the total supply chain cost. Interestingly, in some cases, the total cost is minimized with the value of $\tau = 0$, 1, 2 or 3.

4. Conclusions

We have investigated the impact of delays in the availability of market demand information in a two-level decentralized supply chain with information sharing. As a performance indicator, the distributor's inventory cost and the manufacturer's inventory and production costs are considered.

It is shown that the impact of the delay in the market demand for the distributor is quite straightforward. Shorter demand information delays lower the distributor's inventory cost (property 4). However, the impact of reducing time delays for the manufacturer is not so simple and is largely dependent on the distributor's forecasting method. When the distributor exploits the optimum forecast, both the manufacturer's inventory and production costs decrease as the time delay is shortened, but the cost reduction is quite minor. When a sub-optimum forecast is used, the manufacturer's inventory and production costs become higher as the delay is reduced. This counter-intuitive finding results from the fact that when there is delayed demand information the Bullwhip effect is reduced by the sub-optimal forecast (property 5). Furthermore longer time delays yield smaller Bullwhip. This Bullwhip reduction leads to lower inventory and production costs at the manufacturer (property 6). It is also shown that when there is no correlation in the market demand across the time (i.e. demand is white noise), reducing delays in the market demand information does not influence the manufacturer's costs.

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Together with the numerical analysis, we may conclude that whichever forecasting method the distributor uses, the manufacturer may not want to take the initiative in eliminating market demand information delays, since reducing time delays in the demand information is not hugely effective at reducing its own local costs. It is also shown that the total supply chain cost is always reduced as the delay is shortened or reduced, if and only if the optimum forecasting method is used. However, in the case of the sub-optimum forecast, it is shown that the total supply chain cost can increase as the result of the cost increase of the manufacturer.

Finally, we mention the limitations of our research and point towards potential future research directions. The contributions of this research are largely based on the AR(1) market demand process assumption. As a further study, a different demand model such as an ARIMA process could be considered to examine information delays in a supply chain. The setting of a serially linked supply chain could be another limitation of this research. Considering other types of supply chain structure, such as a divergent supply chains, could also be an interesting future research direction.

Acknowledgement

Appendix 1: The link between the variance of the sum of the forecast errors and the net stock variance

Let us use another form of the OUT policy that is dynamically equivalent, [16],

$$
\begin{aligned}
O_t &= S_t - (NS_t + WIP_t) \\
&= S_t - IP_t^-,
\end{aligned}
$$

where NS_t is the end of period net stock level at t. WIP_t is the Work-In-Progress (or, on-order inventory) at t, where $WIP_t = \sum_{i=1}^{T_d} O_{t+i}$. $IP_t^-(= NS_t + WIP_t)$ is the inventory position just before an order O_t is placed.

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Therefore, the inventory position just after the order O_t is placed, $IP_t^+(= IP_t^- + O_t)$, is always equal to S_t, if the OUT policy is exploited. At this moment, the on-orders are $O_{t-T_d}, O_{t-T_d+1}, \ldots, O_t$, and all those orders will be delivered by the time $t + T_d + 1$. Orders placed during $(t, t + T_d + 1]$ will be delivered after $t + T_d + 1$. Therefore NS_{t+T_d+1} can be described as

$$
\begin{aligned}
NS_{t+T_d+1} &= IP_t^+ - \sum_{i=1}^{T_d+1} D_{t+i} = S_t - \sum_{i=1}^{T_d+1} D_{t+i} \\
&= \hat{D}_t + safety\ stock - \sum_{i=1}^{T_d+1} D_{t+i},
\end{aligned}
\tag{9}
$$

where $\hat{D}_t$ is the forecast of the demand made at t. Note that at this moment, we do not specify any type of forecasting method for $\hat{D}_t$. Since the *safety stock* is a constant value over the time, we can ignore that value when we consider the variance of the net stock levels. From Eq. 9, the variance of NS_{t+T_d+1} can be written as

$$
Var[NS_{t+T_d+1}] = Var\left[\hat{D}_t - \sum_{i=1}^{T_d+1} D_{t+i}\right].
\tag{10}
$$

Clearly, Eq. 10 is time-independent. Thus we can simplify to

$$
Var[NS] = Var\left[\hat{D} - \sum_{i=1}^{T_d+1} D_i\right]
$$

which indicates that the variance of the end of period net stock levels is identical to the variance of forecast errors over $T_d + 1$ periods. And also the variance of the net stock levels depends on the value of $\hat{D}$. Similar results are shown by [15] using discrete variable servo theory.

Appendix 2: Derivation of the forecasts and the net stock variances

The ordering policy given by Eq. 4 can be rewritten as follows:

$$
\begin{cases}
A_{t-\tau} &= D_{t-\tau} + (S_{t-\tau} - S_{t-\tau-1}), \\
O_t' &= A_{t-\tau},
\end{cases}
$$

and the sequence of the order decision making can be restated like this: The order, $A_{t-\tau}$ is determined at $t - \tau$ by using the OUT policy. $A_{t-\tau}$ is held

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until t and is received by the manufacturer as an order (O'_t) at the end of t. Let us use $IP^+_{t-\tau}$, which is the inventory position at $t - \tau$ right after $A_{t-\tau}$ is determined and is the sum of the net stock level at $t - \tau$ and the total of on-orders, $\{A_{t-\tau}, A_{t-\tau-1}, \ldots, A_{t-\tau-T_d}\}$. All those on-orders will become the manufacturer's on-hand inventory during $(t - \tau, t + T_d + 1]$. Therefore, whatever ordering policy is used, we must have

$$NS_{t+T_d+1} = IP^+_{t-\tau} - \sum_{i=1}^{\tau+T_d+1} D_{t-\tau+i}.$$

Using the same way of reasoning as used in Appendix 1, we can have

$$
\begin{aligned}
Var[NS_{t+T_d+1}] &= Var\left[\hat{D} - \sum_{i=1}^{\tau+T_d+1} D_{t-\tau+i}\right] \\
&= E\left[\left(\hat{D} - \sum_{i=1}^{\tau+T_d+1} D_{t-\tau+i}\right)^2\right]. \quad (11)
\end{aligned}
$$

This suggests that the forecast $(\hat{D})$ should cover the demand over $\tau + T_d + 1$ periods to minimize $Var[NS_{t+T_d+1}]$. Let $\hat{D}^*_{t-\tau}$ be the conditional expectation of the demand over $\tau + T_d + 1$ periods given $D_{t-\tau}$, where $\hat{D}^*_{t-\tau} = E\left[\sum_{i=1}^{\tau+T_d+1} D_{t-\tau+i}|D_{t-\tau}\right]$. $\hat{D}^*_{t-\tau}$ minimizes Eq. 11 since $\hat{D}^*_{t-\tau} - \sum_{i=1}^{\tau+T_d+1} D_{t-\tau+i}$ yields a sequence of independent unknown error terms. A closed form of $\hat{D}^*_{t-\tau}$ is

$$
\begin{aligned}
\hat{D}^*_{t-\tau} &= E\left[\sum_{i=1}^{\tau+T_d+1} D_{t-\tau+i} \,\bigg|\, D_{t-\tau}\right] \\
&= (\tau + T_d + 1)\mu + \rho(1 + \rho + \cdots + \rho^{\tau+T_d})(D_{t-\tau} - \mu) \\
&= (\tau + T_d + 1)\mu + \rho\frac{1 - \rho^{\tau+T_d+1}}{1 - \rho}(D_{t-\tau} - \mu),
\end{aligned}
$$

and the minimized value of $Var[NS_{t+T_d+1}]$, $\acute{\sigma}^{*2}_d$, can be obtained using the results shown in a previous research (e.g. [16]) and is

$$
\begin{aligned}
\min\left[Var[NS_{t+T_d+1}]\right] &= \acute{\sigma}^{*2}_d \\
&= \frac{\begin{pmatrix}(\tau + T_d + 1)(1 - \rho^2) + \\ \rho(1 - \rho^{\tau+T_d+1})(\rho^{\tau+T_d+2} - \rho - 2)\end{pmatrix}}{(1 - \rho)^2(1 - \rho^2)}\sigma^2_\varepsilon. \quad (12)
\end{aligned}
$$

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Eq. 12 tells us that $\acute{\sigma}_d^{*2}$ is a function of the sum of the two delays, $(\tau + T_d)$, when ρ and σ_ε^2 are given. Therefore, the impact of τ on $\acute{\sigma}_d^{*2}$ is identical to that of T_d, when the optimum forecast, $\hat{D}_{t-\tau}^*$, is exploited. Using $\hat{D}_{t-\tau}^*$, we can modify Eq. 11,

$$
\begin{aligned}
Var[NS_{t+T_d+1}] &= E\left[\left(\hat{D} - \sum_{i=1}^{\tau+T_d+1} D_{t-\tau+i}\right)^2\right] \\
&= E\left[\left(\left(\hat{D} - \hat{D}_{t-\tau}^*\right) + \left(\hat{D}_{t-\tau}^* - \sum_{i=1}^{\tau+T_d+1} D_{t-\tau+i}\right)\right)^2\right].
\end{aligned}
$$
(13)

Since $E\left[\hat{D}_{t-\tau}^* - \sum_{i=1}^{\tau+T_d+1} D_{t-\tau+i} | D_{t-\tau}\right] = 0$, Eq. 13 can be simplified to

$$
\begin{aligned}
Var[NS_{t+T_d+1}] &= E\left[\left(\hat{D} - \hat{D}_{t-\tau}^*\right)^2\right] + E\left[\left(\hat{D}_{t-\tau}^* - \sum_{i=1}^{\tau+T_d+1} D_{t-\tau+i}\right)^2\right] \\
&= E\left[\left(\hat{D} - \hat{D}_{t-\tau}^*\right)^2\right] + \acute{\sigma}_d^{*2}.
\end{aligned}
$$
(14)

Using the recursive characteristic of an AR(1) model, future values of the demand (i.e. $D_{t+1}, D_{t+2}, \dots$) given $D_{t-\tau}$ can be described as

$$
D_{t+1} = \mu + \rho^{\tau+1}(D_{t-\tau} - \mu) + \sum_{i=0}^{\tau} \varepsilon_{t+1-i},
$$

$$
D_{t+2} = \mu + \rho^{\tau+2}(D_{t-\tau} - \mu) + \sum_{i=0}^{\tau+1} \varepsilon_{t+2-i},
$$

$$
\vdots
$$

$$
D_{t+T_d+1} = \mu + \rho^{\tau+T_d+1}(D_{t-\tau} - \mu) + \sum_{i=0}^{\tau+T_d} \varepsilon_{t+T_d+1-i}.
$$

Therefore, $\hat{D}_{t-\tau}$ is

$$
\begin{aligned}
\hat{D}_{t-\tau} &= E[D_{t+1} + D_{t+2} + \dots + D_{t+T_d+1} | D_{t-\tau}] \\
&= (T_d+1)\mu + \rho^{\tau+1}\left(1 + \rho + \dots + \rho^{T_d}\right)(D_{t-\tau} - \mu) \\
&= (T_d+1)\mu + \rho^{\tau+1}\frac{1 - \rho^{T_d+1}}{1 - \rho}(D_{t-\tau} - \mu).
\end{aligned}
$$

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If we substitute $\hat{D}_{t-\tau}$ into $\hat{D}$ in Eq. 14, we obtain $\acute{\sigma}_d^2$ Eq. 6. As we expect, we can see $\acute{\sigma}_d^{*2} \leq \acute{\sigma}_d^2$, since the RHS of Eq. 6 is the sum of a non-negative value and $\acute{\sigma}_d^{*2}$. Only when $\tau = 0$ and/or $\rho = 0$ do we have $\acute{\sigma}_d^{*2} = \acute{\sigma}_d^2$. $\square$

Appendix 3: Analysis of the distributor's net stock variance

First, we will show that $\acute{\sigma}_d^*$ is increasing in $\tau(= 0, 1, 2, \dots)$. For convenience, we will use a new notation $f(\tau)$, where

$$f(\tau) = \acute{\sigma}_d^* = \frac{\left(\begin{array}{c} (\tau + T_d + 1)(1 - \rho^2) + \\ \rho(1 - \rho^{\tau + T_d + 1})(\rho^{\tau + T_d + 2} - \rho - 2) \end{array} \right)}{(1 - \rho)^2 (1 - \rho^2)} \sigma_\varepsilon^2.$$

To prove property 4, it is enough to show that $f(\tau + 1) - f(\tau) > 0$. Actually it is as:

$$f(\tau + 1) - f(\tau) \quad = \quad \frac{(\rho^{\tau + T_d + 2} - 1)^2}{(\rho - 1)^2} \sigma_\varepsilon^2 > 0.$$

Next, we will show that $\acute{\sigma}_d$ is increasing in τ. From Eq. 6, $\acute{\sigma}_d^2$ can be rewritten using $\acute{\sigma}_d^{*2}$ as

$$\acute{\sigma}_d^2 = \frac{\sigma_\varepsilon^2}{1 - \rho^2} \left(\rho^{\tau + 1} \frac{1 - \rho^{T_d + 1}}{1 - \rho} - \rho \frac{1 - \rho^{\tau + T_d + 1}}{1 - \rho} \right)^2 + \acute{\sigma}_d^{*2}. \tag{15}$$

Since we already know that $\acute{\sigma}_d^*$ is increasing in τ, we can focus only on the first term of the RHS of Eq. 15. Let us set

$$f(\tau) = \frac{\sigma_\varepsilon^2}{1 - \rho^2} \left(\rho^{\tau + 1} \frac{1 - \rho^{T_d + 1}}{1 - \rho} - \rho \frac{1 - \rho^{\tau + T_d + 1}}{1 - \rho} \right)^2.$$

Then we have

$$f(\tau + 1) - f(\tau) \quad = \quad \frac{\rho^{\tau + 2} \left(2 - \rho^\tau (1 + \rho) \right)}{(\rho - 1)^2 (1 + \rho)} \sigma_\varepsilon^2 > 0,$$

when ρ is positive. $\square$

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Appendix 4: Analysis of the distributors order variance

For convenience, we will use $K = \rho(1 - \rho^{\tau+T_d+1})/(1 - \rho)$ and $\hat{D}^*_{t-\tau} = (\tau + T_d + 1)\mu + KD_{t-\tau}$. Now, O'^*_t in Eq. 8 can be rewritten as

$$
\begin{aligned}
O'^*_t &= D_{t-\tau} + (S'^*_t - S'^*_{t-1}) \\
&= D_{t-\tau} + (\hat{D}^*_{t-\tau} - \hat{D}^*_{t-\tau-1}) \\
&= D_{t-\tau} + K(D_{t-\tau} - D_{t-\tau-1}) \\
&= \mu + \rho(D_{t-\tau-1} - \mu) + \varepsilon_{t-\tau} + K(\mu + \rho(D_{t-\tau-1} - \mu) + \varepsilon_{t-\tau} - D_{t-\tau-1}) \\
&= \mu + (\rho + \rho K - K)(D_{t-\tau-1} - \mu) + (1 + K)\varepsilon_{t-\tau}.
\end{aligned}
$$

Then using knowledge that a constant value does not affect the value of the variance, a general expression of the variance of O'^*_t can be obtained as

$$
\begin{aligned}
Var[O'^*] &= Var\left[(\rho + \rho K - K)D_{t-\tau-1} + (1 + K)\varepsilon_{t-\tau}\right] \\
&= (\rho + \rho K - K)^2\, Var[D] + (1 + K)^2\sigma_\varepsilon^2 \\
&= \frac{(\rho + \rho K - K)^2}{1 - \rho^2}\sigma_\varepsilon^2 + (1 + K)^2\,\sigma_\varepsilon^2.
\end{aligned}
$$

Substitute $\rho(1 - \rho^{\tau+T_d+1})/(1 - \rho)$ into K, and simplification will yield

$$
Var[O'^*] = \frac{1 + \rho + 2\rho^{2(\tau+T_d+2)} - 2\rho^{\tau+T_d+2}(1 + \rho)}{(\rho - 1)^2(1 + \rho)}\sigma_\varepsilon^2.
$$

Clearly $Var[O'^*]$ is the function of $(\tau + T_d)$ when ρ and σ_ε^2 are given. Therefore, the impact of τ on $Var[O'^*]$ is identical to that of T_d. By following the same steps, we may obtain the variance of O'_t,

$$
Var[O'] = \frac{\left(1 + \rho\left(2\rho^\tau\left(\rho^{T_d+1} - 1\right)\left(\rho - \rho^{1+\tau} + \rho^{2+T_d+\tau} - 1\right) - 1\right)\right)}{(\rho - 1)^2(1 + \rho)}\sigma_\varepsilon^2.
$$

As $\rho^{\tau+1}(2\rho^{T_d+1} - 1) - 1 < 2\rho^{T_d+1} - 2 < 0$ when $0 \leq \rho < 1$, we can show the following relationship.

$$
Var[O'^*] - Var[O'] = \frac{2\rho(\rho^\tau - 1)(\rho^{\tau+1}(2\rho^{T_d+1} - 1) - 1)}{(\rho - 1)^2(1 + \rho)}\sigma_\varepsilon^2 \geq 0.
$$

Therefore, $Var[O'^*] \geq Var[O']$, when $0 < \rho < 1$ and $\tau > 0$. Only if $\rho = 0$ and/or $\tau = 0$ do these two variances have the same value which is σ_ε^2.

The second property in property 5 can be proved by showing $(\partial Var[O'])/(\partial \tau) < 0$ and $(\partial Var[O^{*'}])/(\partial \tau) > 0$. This gradient is given by

$$\frac{\partial Var[O']}{\partial \tau} = \frac{2\rho^{\tau+1}(\rho^{T_{d1}+1} - 1)(2\rho^{\tau+T_{d1}+2} - 2\rho^{\tau+1} + \rho - 1)\log\rho}{(\rho - 1)^2(1 + \rho)}\sigma_\varepsilon^2.$$

Under the condition $0 < \rho < 1$, we have;

$$2\rho^{\tau+1}(\rho^{T_{d1}+1} - 1) \; < \; 0,$$
$$2\rho^{\tau+T_{d1}+2} - 2\rho^{\tau+1} + \rho - 1 < 2\rho^{\tau+1} - 2\rho^{\tau+1} + \rho - 1 = \rho - 1 \; < \; 0,$$
$$\log\rho \; < \; 0,$$
$$(\rho - 1)^2(1 + \rho) \; > \; 0,$$
$$\sigma_\varepsilon^2 \; > \; 0.$$

Therefore, $(\partial Var[O'])/(\partial \tau) < 0$ must be true.

Finally, we will show $(\partial Var[O^{*'}])/(\partial \tau) > 0$, which can be written as

$$\frac{\partial Var[O^{*'}]}{\partial \tau} = \frac{2\rho^{\tau+T_{d1}+2}(2\rho^{\tau+T_{d1}+2} - \rho - 1)\log\rho}{(\rho - 1)^2(1 + \rho)}\sigma_\varepsilon^2.$$

When $0 < \rho < 1$, it is easy to check;

$$2\rho^{\tau+T_{d1}+2} \; > \; 0,$$
$$2\rho^{\tau+T_{d1}+2} - \rho - 1 \leq 2\rho - \rho - 1 = \rho - 1 \; < \; 0,$$
$$\log\rho \; < \; 0,$$
$$(\rho - 1)^2(1 + \rho) \; > \; 0.$$

Therefore we have $(\partial Var[O^{*'}])/(\partial \tau) > 0$. $\square$

Appendix 5: Derivation of the manufacturer's replenishment policies and their variances

For convenience, let us begin with the case of $O_t'^*$. $\hat{O}_t$ can be given as

$$
\begin{aligned}
\hat{O}_t &= E\left[O_{t+1}'^* + O_{t+2}'^* + \cdots + O_{t+T_m+1}'^* | O_t'^*\right] \\
&= E\left[O_{t+1}'^* + O_{t+2}'^* + \cdots + O_{t+T_m+1}'^* | D_{t-\tau}, K\right] \\
&= (T_m + 1)\mu + (\rho + \rho K - K)(1 + \rho + \ldots + \rho^{L_2-1})(D_{t-\tau} - \mu) \\
&= (T_m + 1)\mu + (\rho + \rho K - K)\frac{1 - \rho^{T_m+1}}{1 - \rho}(D_{t-\tau} - \mu), \qquad (16)
\end{aligned}
$$

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where $K = \rho(1 - \rho^{\tau+T_d+1})/(1-\rho)$. For the case of $O_t = O_t'$, by following the same steps as in Appendix 4, we find that $K = \rho^{\tau+1}(1 - \rho^{T_d+1})/(1-\rho)$. A general expression for the forecast error over the lead-time plus the review period is

$$
\begin{aligned}
\hat{O}_t - \sum_{i=1}^{T_m+1} O_{t+i} &= (1 + K + (\rho + \rho K - K)(1 + \rho + \ldots + \rho^{L_2-2}))\varepsilon_{t-\tau+1} \\
&\quad + (1 + K + (\rho + \rho K - K)(1 + \rho + \ldots + \rho^{L_2-3}))\varepsilon_{t-\tau+2} \\
&\quad + \cdots \\
&\quad + (1 + K + (\rho + \rho K - K))\varepsilon_{t-\tau+T_m} \\
&\quad + (1 + K)\varepsilon_{t-\tau+T_m+1},
\end{aligned}
$$

and σ_m^2 is

$$
\begin{aligned}
\sigma_m^2 &= E\left[\left(\hat{O}_t - \sum_{i=1}^{T_m+1} O_{t+i}\right)^2\right] \\
&= \sum_{i=1}^{T_m+1}\left((1+K) + (\rho + \rho K - K)\sum_{j=2}^{i}\rho^{j-2}\right)^2 \sigma_\varepsilon^2 \\
&= \frac{\left(\begin{array}{l}(T_m+1)(\rho^2-1) + K^2(\rho-1)^2(\rho^{2(T_m+1)}-1) + \\ 2K(\rho-1)(\rho^{T_m+1}-1)(\rho^{T_m+2}-1) + \\ \rho(\rho^{T_m+1}-1)(\rho(\rho^{T_m+1}-1)-2)\end{array}\right)}{(\rho-1)^3(1+\rho)}\sigma_\varepsilon^2,
\end{aligned}
$$

where $K = \rho(1 - \rho^{\tau+T_d+1})/(1-\rho)$ for the case of $O_t = O_t'^*$ and $K = \rho^{\tau+1}(1 - \rho^{T_d+1})/(1-\rho)$ for the case of $O_t = O_t'$. With knowledge of Eq. 16 and Eq. 1, P_t can be rewritten as

$$
\begin{aligned}
P_t &= O_t + (M_t - M_{t-1}) \\
&= O_t + (\hat{O}_t - \hat{O}_{t-1}) \\
&= O_t + (\rho + \rho K - K)\frac{1 - \rho^{T_m}+1}{1-\rho}(D_{t-\tau} - D_{t-\tau-1}) \\
&= \mu + (\rho + \rho K - K)(D_{t-\tau-1} - \mu) + (1+K)\varepsilon_{t-\tau} + \\
&\quad (\rho + \rho K - K)\frac{1 - \rho^{T_m}+1}{1-\rho}(\mu + \rho(D_{t-\tau-1} - \mu) + \varepsilon_{t-\tau} - D_{t-\tau-1}) \\
&= \rho^{T_m+1}(\rho + \rho K - K)(D_{t-\tau-1} - \mu) + \frac{1 - \rho^{T_m+1}(\rho + \rho K - K)}{1-\rho}\varepsilon_{t-\tau}.
\end{aligned}
$$

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Thus, σ_P^2 is

$$
\begin{aligned}
\sigma_P^2 &= \left(\rho^{T_m+1}(\rho + \rho K - K)\right)^2 Var[D] + \left(\frac{1 - \rho^{T_m+1}(\rho + \rho K - K)}{1 - \rho}\right)^2 \sigma_\varepsilon^2 \\
&= \frac{\left(\rho^{T_m+1}(\rho + \rho K - K)\right)^2}{1 - \rho^2} \sigma_\varepsilon^2 + \left(\frac{1 - \rho^{T_m+1}(\rho + \rho K - K)}{1 - \rho}\right)^2 \sigma_\varepsilon^2,
\end{aligned}
$$

where $K = \rho(1 - \rho^{\tau + T_d+1})/(1 - \rho)$ for the case of $O_t = O_t'^*$ and $K = \rho^{\tau+1}(1 - \rho^{T_d+1})/(1 - \rho)$ for the case of $O_t = O_t'$. $\square$

Appendix 6: The influence of the delay on the manufacturer

The fact that σ_m and σ_P is increasing in τ when the optimal forecast is present (the first property in property 6) is proved by showing $(\partial \sigma_m^2)/(\partial \tau) > 0$ and $(\partial \sigma_P^2)/(\partial \tau) > 0$, when $\hat{D}_{t-\tau}^*$ is exploited. In such case, $(\partial \sigma_m^2)/(\partial \tau)$ is given as

$$
\frac{\partial \sigma_m^2}{\partial \tau} = \frac{\left(\begin{array}{c} 2\rho^{\tau+T_d+2}(\rho^{T_m+1} - 1) \times \\ (\rho^{\tau+T_d+2} + \rho^{\tau+T_d+T_m+3} - \rho - 1)\log\rho \end{array}\right)}{(\rho-1)^3(1+\rho)} \sigma_\varepsilon^2.
$$

For $0 < \rho < 1$, it is easy to check;

$$
\begin{aligned}
2\rho^{\tau+T_d+2} &> 0, \\
\rho^{T_m+1} - 1 &< 0, \\
\rho^{\tau+T_d+2} + \rho^{\tau+T_d+T_m+3} - \rho - 1 \le 2\rho - \rho - 1 = \rho - 1 &< 0, \\
\log\rho &< 0, \\
(\rho-1)^3(1+\rho) &< 0, \\
\sigma_\varepsilon^2 &> 0.
\end{aligned}
$$

Therefore, $(\partial \sigma_m^2)/(\partial \tau) > 0$.

In the case that $\hat{D}_{t-\tau}^*$ is used by the distributor, $(\partial \sigma_P^2)/(\partial \tau)$ is

$$
\frac{\partial \sigma_P^2}{\partial \tau} = \frac{\left(\begin{array}{c} 2\rho^{\tau+T_d+T_m+3} \times \\ (2\rho^{\tau+T_d+T_m+3} - \rho - 1)\log\rho \end{array}\right)}{(\rho-1)^2(1+\rho)} \sigma_\varepsilon^2.
$$

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For $0 < \rho < 1$, it is necessary to check;

$$
\begin{aligned}
2\rho^{\tau+T_d+T_m+3} &> 0, \\
2\rho^{\tau+T_d+T_m+3} - \rho - 1 \leq 2\rho - \rho - 1 = \rho - 1 &< 0, \\
\log \rho &< 0, \\
(\rho-1)^2(1+\rho) &> 0, \\
\sigma_\varepsilon^2 &> 0.
\end{aligned}
$$

Therefore, we have $(\partial \sigma_P^2)/(\partial \tau) > 0$.

Now, we will now show that σ_m and σ_P are decreasing in τ when the non-optimal forecast is present by showing $(\partial \sigma_m^2)/(\partial \tau) < 0$ and $(\partial \sigma_P^2)/(\partial \tau) < 0$. In this case, $(\partial \sigma_m^2)/(\partial \tau)$ is given by

$$
\frac{\partial \sigma_m^2}{\partial \tau} = \frac{\left(\begin{array}{c} 2\rho^{\tau+1}(\rho^{T_d+1}-1)(\rho^{T_m+1}-1)\times \\ (\rho^{T_m+2} - \rho^{\tau+1} + \rho^{\tau+T_d+2} - \rho^{\tau+T_m+2} + \rho^{\tau+T_d+T_m+3} - 1)\log \rho \end{array}\right)}{(\rho-1)^3(1+\rho)}\sigma_\varepsilon^2.
$$

Since $0 < \rho < 1$, we have

$$
\begin{aligned}
2\rho^{\tau+1}(\rho^{T_d+1}-1)(\rho^{T_m+1}-1) &> 0, \\
\log \rho &< 0, \\
(\rho-1)^3(1+\rho) &< 0, \\
\sigma_\varepsilon^2 &> 0.
\end{aligned}
$$

For convenience, let us set $f_m(\tau) = (\rho^{T_m+2} - \rho^{\tau+1} + \rho^{\tau+T_d+2} - \rho^{\tau+T_m+2} + \rho^{\tau+T_d+T_m+3} - 1)$ to show $f_m(\tau)$ is negative. $(\partial f_m(\tau))/(\partial \tau)$ is

$$
\frac{\partial f_m(\tau)}{\partial \tau} = \rho^{\tau+1}(\rho^{T_d+1}-1)(\rho^{T_m+1}+1)\log \rho > 0.
$$

Therefore, the maximum value of $f_m(\tau)$ is achieved when $\tau \to \infty$:

$$
\max f_m(\tau) = \lim_{\tau \to \infty} f(\tau) = \rho^{T_m+2} - 1 < 0.
$$

This shows $f_m(\tau) < 0$, and we conclude that $(\partial \sigma_m^2)/(\partial \tau) < 0$. By following the same steps above, we will show $(\partial \sigma_P^2)/(\partial \tau) < 0$. $(\partial \sigma_P^2)/(\partial \tau)$ is given as

$$
\frac{\partial \sigma_P^2}{\partial \tau} = \frac{\left(\begin{array}{c} 2\rho^{\tau+T_m+2}(\rho^{T_d+1}-1)\times \\ (\rho(2\rho^{T_m+1}(1 - \rho^\tau + \rho^{\tau+T_d+1}) - 1) - 1)\log \rho \end{array}\right)}{(\rho-1)^2(1+\rho)}\sigma_\varepsilon^2.
$$

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With the knowledge that $0 < \rho < 1$, we have

$$
\begin{aligned}
2\rho^{\tau+T_m+2} &> 0, \\
\rho^{T_d+1} - 1 &< 0, \\
\log \rho &< 0, \\
(\rho-1)^2(1+\rho) &> 0, \\
\sigma_\varepsilon^2 &> 0.
\end{aligned}
$$

For convenience, we set $f_P(\tau) = \rho(2\rho^{T_m+1}(1-\rho^\tau+\rho^{\tau+T_d+1})-1)-1$ and will show $f_P(\tau)$ is negative by using $(\partial f_P(\tau))/(\partial \tau)$. Since $(\partial f_P(\tau))/(\partial \tau)$ is

$$
\frac{\partial f_P(\tau)}{\partial \tau} = 2\rho^{\tau+T_m+2}(\rho^{T_d+1}-1)\log \rho \geq 0,
$$

$f_P(\tau)$ yields the maximum value when $\tau \to \infty$, which is

$$
\begin{aligned}
\max f_P(\tau) &= \lim_{\tau \to \infty} f_P(\tau) = 2\rho^{T_m+2} - \rho - 1 \\
&< 2\rho - \rho - 1 \\
&= \rho - 1 < 0.
\end{aligned}
$$

Therefore, $f_P(\tau) < 0$ and we can conclude that $(\partial \sigma_P^2)/(\partial \tau) < 0$. $\square$

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