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# New method of estimating regional stress orientations: application to focal mechanism data of recent British earthquakes

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## SUMMARY

It is demonstrated how a recently devised graphical method for estimating palaeostresses from exposed striated fault planes can be adapted for use with earthquake focal mechanism data. Provided the data set includes one or more earthquakes for which the fault plane can be identified, the principal stress directions can be determined more precisely than by using the alternative Right Dihedra method. Using the new so-called Right Trihedra method stress directions are found which are compatible with recent British earthquakes. The method can also be used, in the case of some focal mechanisms, to resolve the ambiguity regarding which focal plane corresponds to the fault plane.

**Key words:** earthquake mechanism, inversion, stereographic method, stress axes.



## INTRODUCTION

Seismologists and structural geologists share the common aim of deducing the stresses responsible for the observed motions on fault planes. During the last two decades both groups of scientists have devised methods which estimate stress orientations from groups of faults in a region.

Seismologists have made frequent use of the $P$–$T$ axis method for this purpose. This involves using the centres of the quadrants on the focal sphere defined by dilatational and compressional first arrivals of $P$-waves [the so-called pressure ($P$) and tension ($T$) axes] to estimate the orientation of axes of greatest and least principal stress ($\sigma_1$ and $\sigma_3$). This simple method, which allows the stress directions to be estimated from a single earthquake, has the added advantage of not requiring the identification of the fault plane from the two focal planes. Its main disadvantage however arises from the severe assumption regarding the angular relationship of the fault plane to the principal stresses, namely that fault plane is inclined at 45° to the $\sigma_1$ and $\sigma_3$ axes. For newly generated fault surfaces this assumption is clearly at odds with most rock failure criteria whereas faults undergoing reactivation are likely to have less-constrained orientations relative to the imposed stresses. In the latter case, McKenzie (1969) pointed out that $\sigma_1$ and $\sigma_3$ orientations responsible for a given reactivated fault could lie anywhere in the dilatational and compressional quadrants, respectively. On this basis the quadrantal data from a single earthquake imposes only the broadest of limits on the orientation of the principal stresses.

Structural geologists concerned with determining palaeostresses from ancient faults, and probably for this reason more concerned by the problems of reactivation, have sought alternatives to the $P$–$T$ axis method. Angelier & Mechler (1977) realized that the imprecise stress estimates yielded by the quadrantal data from a single fault can be greatly improved if the compressional and dilatational quadrants of several faults are superimposed. This widely-used method, referred to as the Right Dihedra method, is performed graphically with use of the stereographic projection.

In addition to this graphical method, numerical methods have been developed which are based on Bott's (1959) theoretical demonstration that the rake (pitch) of the slip direction on a fault plane of specified dip and strike is a function of the stress ratio [$R = (\sigma_1 - \sigma_2)/(\sigma_2 - \sigma_3)$] as well as of the orientations of the principal stress axes. Carey & Brunier (1974), Vasseur, Etchecopar & Philip (1983), Angelier (1979), Gephart & Forsyth (1984) and Michael (1984) have all proposed procedures for inverting data consisting of orientations of fault planes and associated slip directions to estimate stress ratios and axial orientations.

In spite of the availability of above-mentioned inversion methods, the need continues for graphical forms of analysis especially as a preliminary to computer processing to ascertain visually the degree of coherence of the data set and to identify rogue faults. This paper describes a graphical method of finding the directions of principal stresses which represents an improvement on the Right Dihedra method and shows, for the first time, how the new method can be applied to earthquake data.

## THE METHOD

The focal sphere of any earthquake can be divided into four quadrants by a right dihedron formed by two perpendicular planes; the fault plane itself and the auxiliary plane. On the basis of the polarity of first arrivals of *P*-waves, dilatational and compressional quadrants are distinguished. Assuming that slip on the fault plane occurs in a direction of maximum resolved shear stress in the plane, it has been shown that maximum and minimum compressive stress axes ($\sigma_1$ and $\sigma_3$) must lie in the dilatational and compressional quadrants, respectively (McKenzie 1969).

The Right Dihedra method (Pegaroro 1972; Angelier & Mechler 1977) delimits the range of possible orientations of $\sigma_1$ and $\sigma_3$ axes from a composite stereogram constructed from the $\sigma_1$ and $\sigma_3$ quadrants of several earthquakes which are considered to have occurred in response to the same stress field (Fig. 1). The extent to which this procedure reduces the size on the sphere of the fields of potential $\sigma_1$ and $\sigma_3$ directions depends on the variability of the orientations of the fault planes.

The new method used here considerably improves the efficiency of the search procedure for the stress directions involved in the Right Dihedra method. The improved search strategy is made possible by the discovery by Lisle (1988) that a given fault and associated slip direction places closer geometrical constraints on the principal stress directions than those originally demonstrated by McKenzie (1969). This additional restriction can be expressed as follows.

To the fault plane and auxiliary plane which define the right dihedron for a given earthquake a third perpendicular plane is added. These three mutually-orthogonal planes together form a right trihedron (Fig. 2). The dihedron formed of the third plane in conjunction with the auxiliary plane divides the sphere into two pairs of quadrants (labelled *A* and *B* pairs in Fig. 2a). *The principal stress axes $\sigma_1$ and $\sigma_3$ may not be oriented so that they both lie in the same pair of quadrants.* For example, if $\sigma_1$ is known to lie in the *A* pair of quadrants then $\sigma_3$ is obliged to lie in the *B* pair and vice versa.

The proof of the above statement is given in the Appendix.

In the proposed method this additional restriction on principal stress orientation leads to the improvement of the Right Dihedra (RD) method. The new method, when applied to earthquake focal mechanism data, requires the identification of the auxiliary plane.

First a group of earthquake data are analysed using the original RD method to produce fields on the stereogram containing potential $\sigma_1$ and $\sigma_3$ directions. Then, for each earthquake in turn, the *A* and *B* quadrants are superimposed stereographically on these $\sigma_1$ and $\sigma_3$ fields, as illustrated in Fig. 2(b). If either set of quadrants (lets say the *A* pair) are found to enclose completely the field of one stress axis (e.g. $\sigma_1$) it can be concluded, from the newly-established property, that the real orientation of the other stress axis ($\sigma$ in this case) must be within the other set of quadrants (the *B* quadrants in this present example). Therefore any portion of the orientation field of the latter stress axis (here $\sigma_3$) which lies within the 'wrong' quadrant (i.e. the *A* quadrant pair) can be eliminated from the orientation field. The final outcome of this procedure, after the *A* and *B* quadrants of all earthquakes have been considered in this way, is usually a significant reduction in the size of the $\sigma_1$ and $\sigma_3$ orientation fields.

The new method, named the Right Trihedra method, is more efficient than the RD method since, from the same data, it produces more precise estimates of the principal stress directions. For small data sets, the graphical procedure using the stereographic projection is feasible; when many earthquakes are to be analysed the procedure is more conveniently performed with a computer program (Lisle 1988).

## APPLICATION TO RECENT BRITISH EARTHQUAKES

Marrow & Walker (1988) estimated intraplate stresses from available mechanisms for the whole UK using the RD method. The same published data, i.e. the focal mechanisms of the earthquakes at Kintail (Assumpçao 1981), Dunoon

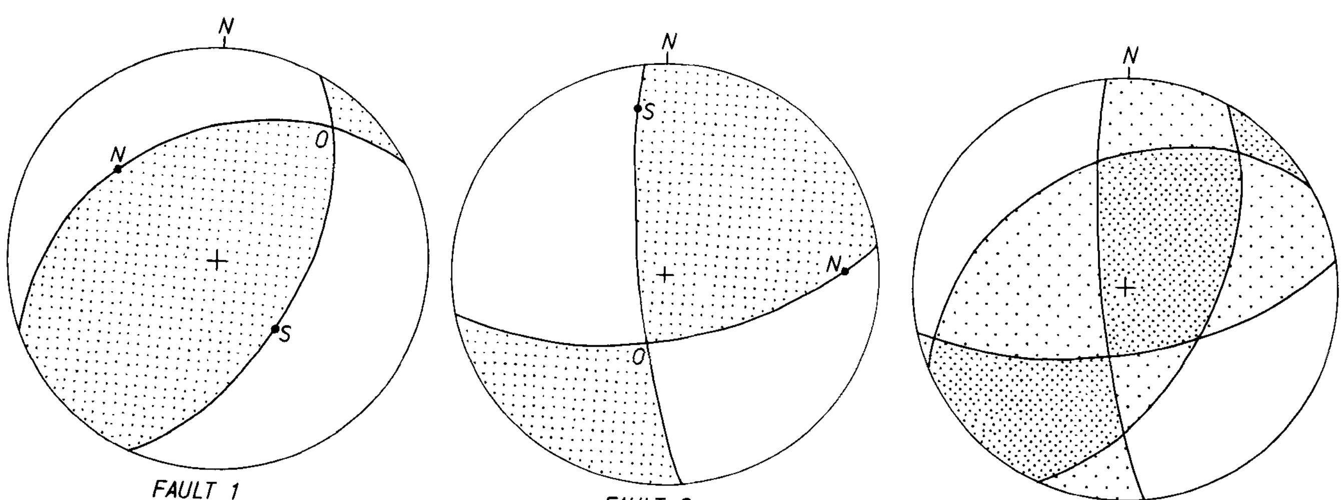


**Figure 1.** The principle of the Right Dihedra method (Angelier & Mechler 1977). The superimposition of $P/T$ quadrantal pattern of a number of earthquakes leads to a reduction in the size of the fields of potential $\sigma_1$ and $\sigma_3$ orientations.

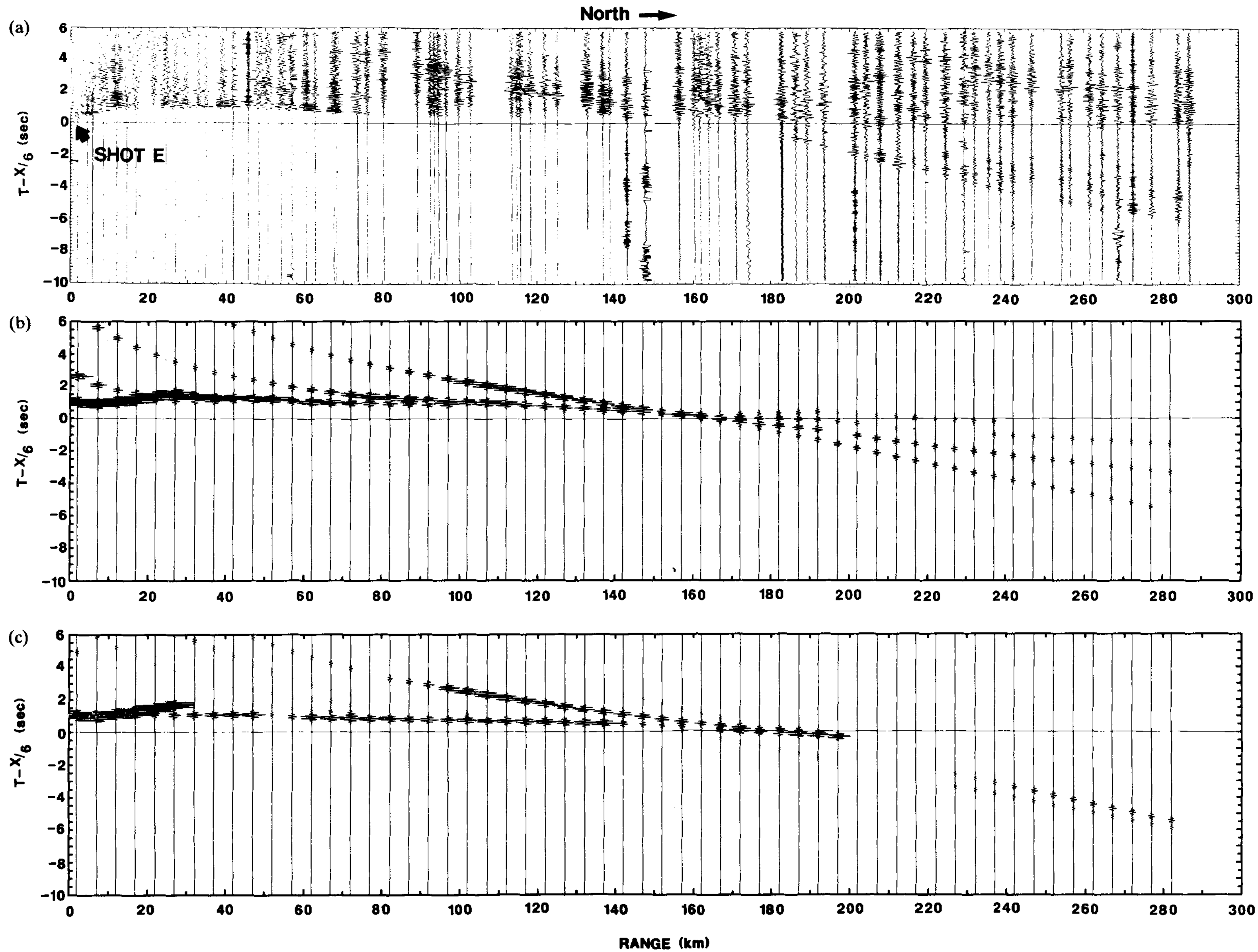


**Figure 5.** (a) LISPB record section for shot E (north), reproduced from Bamford *et al.* (1978). (b) Synthetic seismograms for shot E (north) generated from the 1978 model (Fig. 2b). (c) Synthetic seismograms for shot E (north) generated from the new LISPB model (Fig. 7a). (d) LISPB record section for shot E (south), reproduced from Bamford *et al.* (1978). (e) Synthetic seismograms for shot E (south) generated from the 1978 model (Fig. 2b). (f) Synthetic seismograms for shot E (south) generated from the new LISPB model (Fig. 7a). All plotting parameters as in Fig. 3.

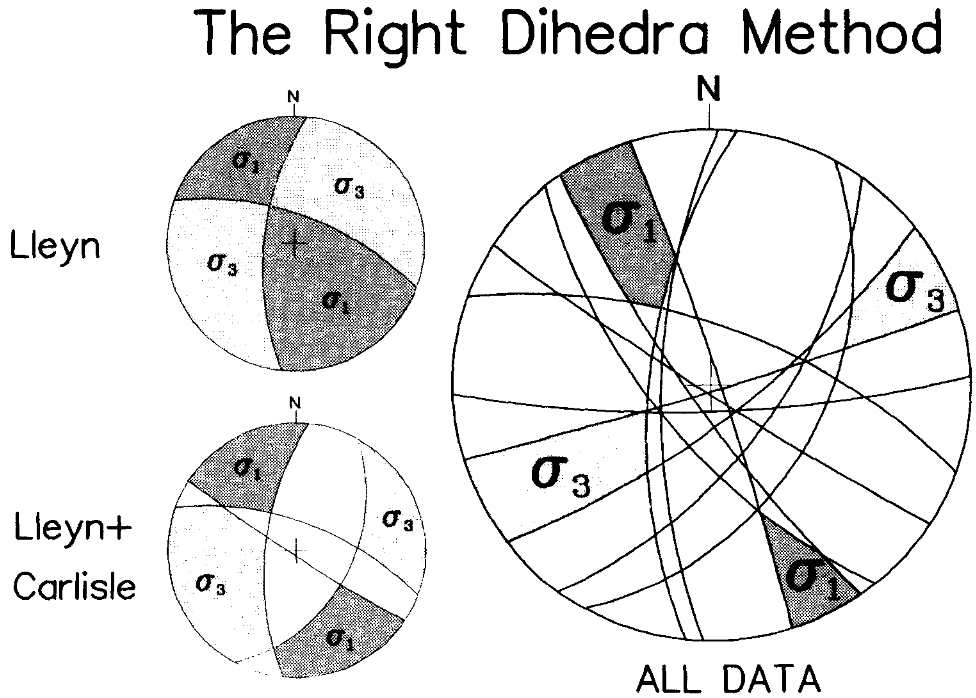


**Figure 4.** The Right Dihedra method applied to the data from the Lleyn and Carlisle events to illustrate the principle (left), and to all events (right).

(Redmayne & Musson 1987), Carlisle (Marrow & Roberts 1985), Lleyn (Trodd, Warburton & Pooley 1985), induced events in Cornwall (Walker 1984) together with the more recent Bishop's Castle earthquake (Ritchie, Musson & Woodcock 1990) are used here to examine the performance of the Right Trihedra method. The data are displayed in Fig. 3.

As a first stage, all six mechanisms are analysed by the RD method. The results are shown in lower hemisphere stereographic projection in Fig. 4. In common with the results of Marrow & Walker (1988) based on five earthquakes, they indicate a NW–SE trending direction of greatest compression ($\sigma_1$) and a NE–SW directed $\sigma_3$ axis.

This solution can now be refined by applying the RT

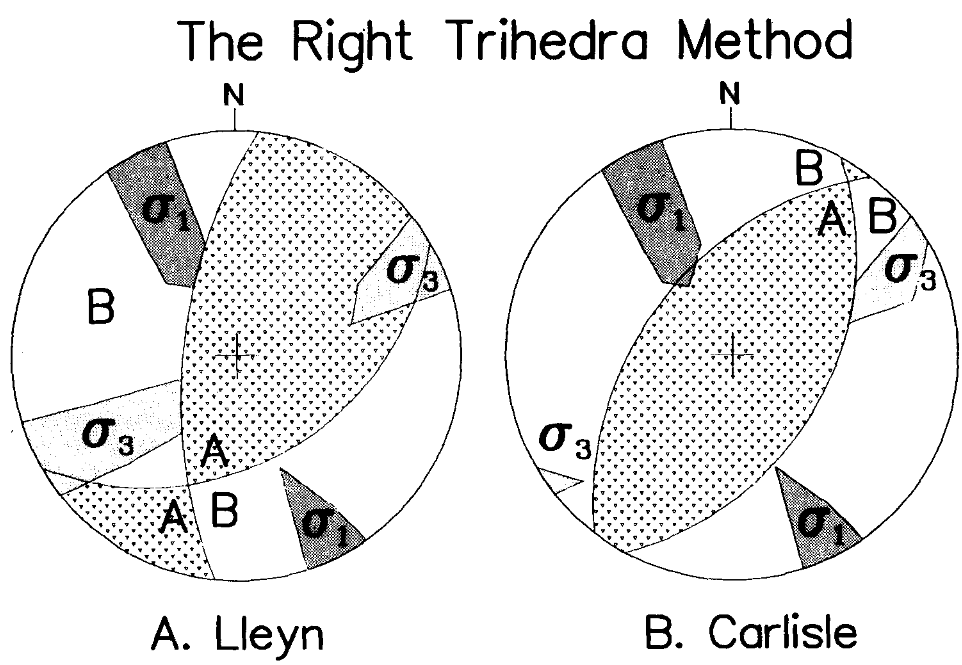


**Figure 5.** The Right Trihedra method used to refine the estimates produced by the Right Dihedra method in Fig. 4. Consideration of the *A* and *B* quadrants of each event in turn leads to a progressive reduction of the size of the orientation fields (see text for explanation).

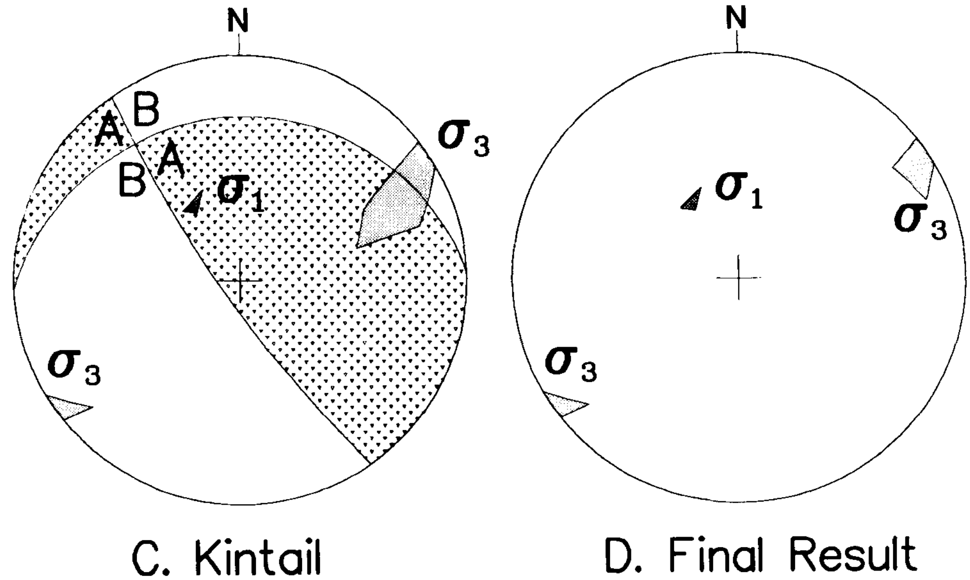


**Figure 5.** *(continued)*

method. For four of the earthquakes there is published evidence suggesting which of the two focal planes corresponds to the fault plane and therefore for these cases focal trihedra can be constructed. Fig. 5 shows how the consideration of the $A$ and $B$ quadrants of each earthquake in turn usually leads to a progressive reduction in the size of the $\sigma_1$ and $\sigma_3$ orientation fields.

In this instance, only the Bishop's Castle earthquake did not contribute to this elimination process, i.e. although it fits the obtained solution it does not help constrain that solution. During the application of the method to other data sets it is sometimes found that one or both of the stress orientation fields is completely lost by the elimination process. Such a result would imply that no single stress tensor is compatible with all the earthquakes and would therefore suggest temporal/spatial heterogeneity of the stress. However in this case, small $\sigma_1$ and $\sigma_3$ search regions remain intact after the exercise. Although this outcome could be fortuitous in the sense that it is favoured by the fact that the number of faults analysed is small, we still have no reason to reject the notion of a homogeneous current UK stress field. The indicated $\sigma_1$ axis plunges at an angle of 48° towards 328° and the axis of least compression is almost horizontal (plunges 5°) in a direction 064°.

Besides providing an efficient search procedure for principal stress axes, the application of the Right Trihedra method is capable of yielding additional geometrical information on the mechanism of individual earthquakes. In particular, if independent estimates of the regional stress directions are available, it can assist in the selection of the fault plane from the two focal planes. To illustrate this consider the mechanism of the Dunoon earthquake (Fig. 6) for which uncertainty exists regarding whether the NE or NW trending focal planes matches the fault plane (Redmayne & Musson 1987). On the assumption that the deduced stress directions in Fig. 5 are also responsible for the Dunoon event, the $\sigma_1$ and $\sigma_3$ directions must fall in different quadrants of the dihedron formed from the auxiliary and third (trihedral) planes. In the Dunoon example, the separation of $\sigma_1$ and $\sigma_3$ into separate quadrants is only produced when the NW-striking focal plane is used as the auxiliary plane. Thus the other focal plane, the NE-trending one, is the more likely fault plane.

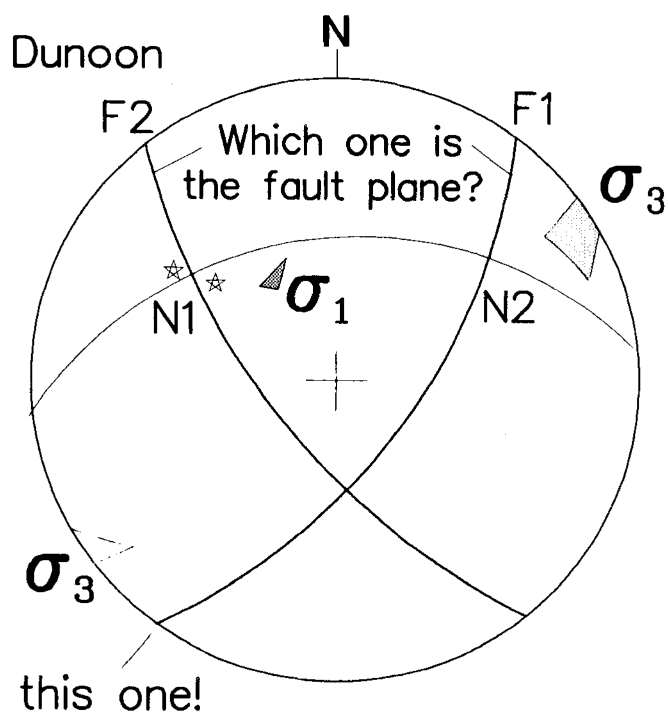


**Figure 6.** By assuming the stress orientations deduced in Fig. 5(d) were those responsible for the Dunoon event, the fault plane for the latter can be identified by using the $A$ and $B$ quadrant property explained in Fig. 2. Of the two potential fault planes (labelled F1 and F2), only the focal plane with a NE–SW strike (F1) separates the $\sigma_1$ and $\sigma_3$ axes by its $A$ and $B$ quadrants.

## CONCLUSIONS

(1) For the purposes of calculating regional stress orientations, the $P/T$ quadrant superimposition method of Angelier & Mechler (1977) provides a conservative, easily understood method of analysis.

(2) However, when the focal data includes the knowledge of the fault plane orientation or one or more earthquakes, the Right Trihedra method offers improved definition of principal stress orientations.

(3) Once regional stress directions are estimated, the use of the geometrical property of the stress tensor described by Lisle (1988) makes it possible to identify the fault plane orientation associated with some events.

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## APPENDIX

### Proof that $\sigma_1$ and $\sigma_3$ directions lie in opposite quadrants bounded by planes *SN* and *ON* (Fig. A1)

Fig. A1 is a stereographic projection showing the fault plane normal $N$, slip direction $S$ and direction $O$ perpendicular to $N$ and $S$. It is assumed that $S$ is the direction within the fault plane of maximum resolved shear stress.

The aim is to prove that if $\sigma_1$ and $\sigma_3$ axes lie on the same side of plane $SN$ they will lie on the opposite sides of plane $ON$, and vice versa.

The stress vector $T$ which acts on plane $SO$ has components given by Cauchy's formula (Means 1976, p. 103):

$$l\sigma_1, m\sigma_2, n\sigma_3,$$

where $l$, $m$ and $n$ are the direction cosines of $N$. The direction of $S$ is given by the line of intersection of planes $NT$ and $SO$ whilst $O$, which is the direction in the fault at right angles to $S$, is the normal to plane $NTS$.

The equation of a plane can be found from the

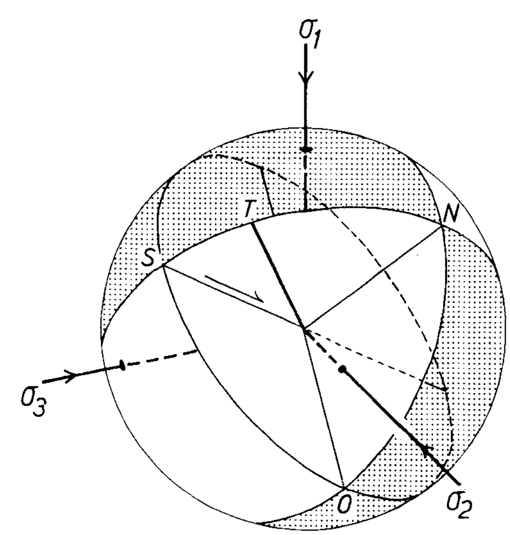


**Figure A1.** Plane $SO$ is a fault plane with direction of maximum resolved shear stress $S$ which is inclined to stress axes $\sigma_1$, $\sigma_2$ and $\sigma_3$. Line $O$ is within the fault plane and is perpendicular to $S$. $N$ is the normal to the fault plane. $T$ is the stress vector acting on the fault plane so that $S$, $N$ and $T$ lie in the same plane.

coordinates of three non-colinear points $(x_1, y_1, z_1)$ $(x_2, y_2, z_2)$ and $(x_3, y_3, z_3)$ which lie within it (Spiegel 1968, p. 47). If the point $(x_1, y_1, z_1)$ is the origin, the equation of this plane can be written:

$$\begin{vmatrix} y_2 & z_2 \\ y_3 & z_3 \end{vmatrix} x + \begin{vmatrix} z_2 & x_2 \\ z_3 & x_3 \end{vmatrix} y + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} z = 0. \tag{A1}$$

Plane $NS$ passes through the point $(x_2 = l, y_2 = m, z_2 = n)$ on line $N$ and point $(x_3 = l\sigma_1, y_3 = m\sigma_2, z_3 = n\sigma_3)$ on $T$ and therefore from equation (A1) has the equation:

$$mn(\sigma_3 - \sigma_2)x + ln(\sigma_1 - \sigma_3)y + lm(\sigma_2 - \sigma_1)z = 0. \tag{A2}$$

Line $O$ is perpendicular to this plane and therefore has direction ratios $mn(\sigma_3 - \sigma_2)$, $ln(\sigma_1 - \sigma_3)$, $lm(\sigma_2 - \sigma_1)$, so that there must exist along $O$ a point with coordinates $[mn(\sigma_3 - \sigma_2), ln(\sigma_1 - \sigma_3), lm(\sigma_2 - \sigma_1)]$. Using this latter point together with the point $(l, m, n)$ located on fault plane normal $N$, we can again use equation (1) to arrive at the equation of plane $ON$. Line $S$ is perpendicular to plane $ON$ and therefore has direction ratios: $l[m^2(\sigma_2 - \sigma_1) - n^2(\sigma_1 - \sigma_3)]$, $m[n^2(\sigma_3 - \sigma_2) - l^2(\sigma_3 - \sigma_2)]$, $n[l^2(\sigma_1 - \sigma_3) - m^2(\sigma_3 - \sigma_2)]$.

The sign of the product $\cos(O^\wedge\sigma_1) \cdot \cos(O^\wedge\sigma_3)$ will be positive if $\sigma_1$ and $\sigma_3$ both lie on the same side of plane $SN$, but will be negative if these axes lie on opposite sides. Similarly $\cos(S^\wedge\sigma_1) \cdot \cos(S^\wedge\sigma_3)$ will be positive or negative depending on whether $\sigma_1$ and $\sigma_3$ are on the same or opposite sides respectively of plane $ON$.

It follows that $\sigma_1$ and $\sigma_3$ will lie in opposite quadrants (shaded in Fig. A1) when

$$\frac{\cos(S^\wedge\sigma_1) \cdot \cos(S^\wedge\sigma_3)}{\cos(O^\wedge\sigma_1) \cdot \cos(O^\wedge\sigma_3)} < 0.$$

This condition therefore depends on the signs of the direction cosines of lines $S$ and $N$. Since the direction ratios will share the same signs as the direction cosines the above condition can be written, after some simplification as:

$$\frac{[m^2(\sigma_2 - \sigma_1) - n^2(\sigma_1 - \sigma_3)][l^2(\sigma_1 - \sigma_3) - m^2(\sigma_3 - \sigma_2)]}{m^2(\sigma_3 - \sigma_2)(\sigma_2 - \sigma_1)} < 0.$$

For triaxial stress states $(\sigma_1 > \sigma_s > \sigma_3)$ it can be readily shown that the numerator of the above expression will always be negative whilst the denominator will invariably be positive. Therefore the value of the expression is always negative and hence our original proposition is proven.