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TECHNICAL NOTE

Calculation of the daylight envelope for plane failure of rock slopes

R. J. LISLE*

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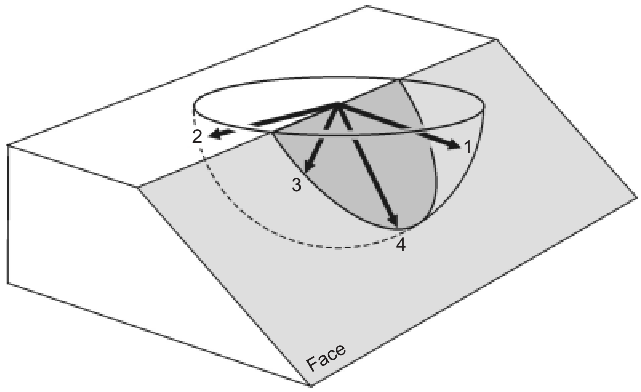
INTRODUCTION

Plane failure—the downdip sliding on a single set of discontinuities—constitutes an important mode of failure of rock slopes. In assessing the potential for such failure, the orientation of the discontinuities relative to that of the face of the slope is a crucial consideration. Discontinuities that dip in towards the face cannot act as sliding surfaces because movement is prevented by the lack of space for the displaced mass (Fig. 1(a)). On the other hand, discontinuities that dip generally in the direction of the rock face are kinematically viable: such planes of weakness are said to *daylight*.

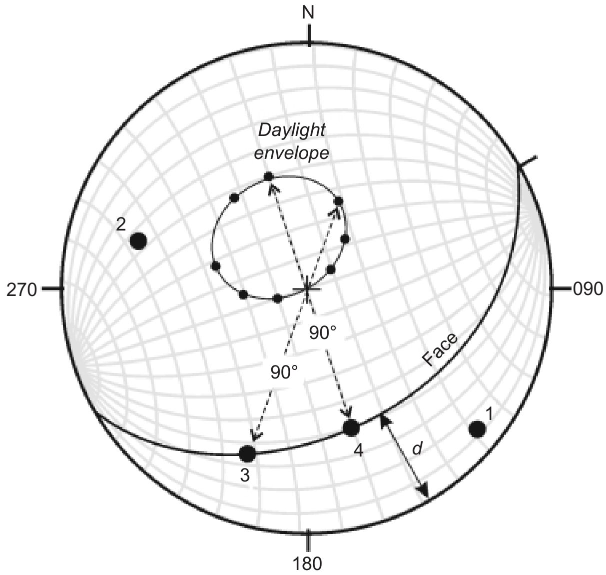
It is common practice to use hemispherical projection techniques to establish the range of orientations of daylighting discontinuities from a consideration of the attitude of the rock face (Hoek & Bray, 1981; Bell, 1987; Lisle & Leyshon, 2004). On the projection, the normals of daylighting planes occupy an area bounded by a curve known as the *daylight envelope*. The envelope is defined by the poles of planes that are marginal in terms of the daylighting condition: these are planes whose downdip lines lie within the plane of the rock face (e.g. lines 3 and 4 in Fig. 1(a)). To construct the daylight envelope by hand therefore involves the following steps (Fig. 1(b)):

(*a*) The plane of the face is drawn as a great circle using a Lambert equal-area net.
(*b*) Any point is selected on the great circle (e.g. point 3 in Fig. 1(b)). This represents the downdip line of a discontinuity whose orientation is transitional in terms of daylighting.
(*c*) From the point in (*b*) a vertical 90° angle is set out radially to locate the pole of the discontinuity in (*b*) (Fig. 1(b)).
(*d*) Stages (*b*) and (*c*) are repeated for other points on the face's great circle, such as point 4 (Fig. 1(b)). The resulting array of poles defines the daylight envelope.

This manual procedure is tedious because more than 20 points are required to draw the envelope with reasonable accuracy. It is therefore worthwhile to incorporate the construction of the daylight in computer programs used to plot projections. This note provides the necessary equations for the direct computation of the daylight envelope.



(a)



(b)

Fig. 1. Manual construction of the daylight envelope. Downdip directions of discontinuities (1, 2, 3 and 4) in relation to a rock face. Slip direction 1 is pointing out of the face and is associated with a daylighting discontinuity, whereas slip direction 2 plunges into the rock mass and therefore does not daylight. Slip directions 3 and 4 are marginal cases: the normals to planes associated with these slip directions lie on the daylight envelope. To construct the daylight envelope draw the great circle of the rock face and, from an arbitrary point 3 on the great circle, set out a vertical angle of 90°. This locates a pole lying on the daylight envelope. The procedure is repeated for other points such as 4 until the shape of the envelope can be drawn

DERIVATION OF EQUATIONS

Figure 2 shows the angular relations of the problem on a Lambert equal-area projection. A rock face, sloping eastwards at an angle d, is shown as a great circle. On this great circle, an arbitrary downdip line of a discontinuity that is parallel to the plane of the face plots as a point, D. This line makes an angle p with the horizontal in a vertical plane (i.e. it plunges at angle p), and makes an angle r with the

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* School of Earth, Ocean and Planetary Sciences, Cardiff University, UK.

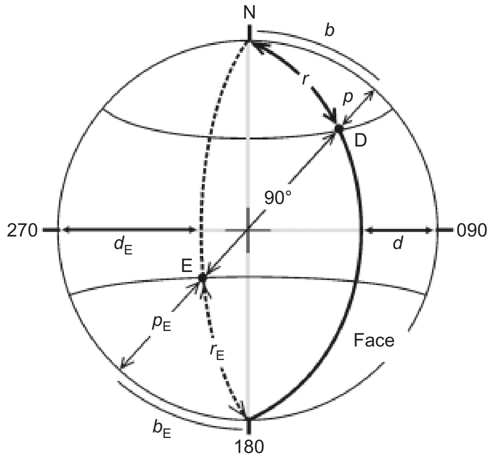

Fig. 2. Definition of angles used in the derivation of the formulae for the coordinates of points on the daylight envelope

horizontal in the plane of the face. The angle between the direction of the vertical plane through D and the strike of the face equals b (Fig. 2). The angles d, p, r and b are related by (Ragan, 1984)

$$\cos r = \cos b \cos r \tag{1}$$

$$\tan d = \frac{\tan p}{\sin b} \tag{2}$$

$$\sin r = \frac{\sin p}{\sin d} \tag{3}$$

The coordinates of D on the projection are given by (Weisstein, 2003, p. 1682)

$$x = k \sin r \cos d \tag{4}$$

$$y = k \cos r \tag{5}$$

where

$$k = \sqrt{2/(1 + \sin r \sin d)}$$

By setting d and varying r, equations (1) and (2) can be used to draw points lying on a great circle of slope angle d. Similarly, small circles can be drawn by fixing r and varying d.

The point E, a point lying on the daylight envelope, has coordinates

$$x_E = k_E \sin r_E \cos d_E \tag{6}$$

$$y_E = k_E \cos r_E \tag{7}$$

where

$$k_E = \sqrt{2/(1 + \sin r_E \sin d_E)}.$$

As point E makes an angle of 90° from D in a vertical plane (Fig. 2), $p_E = 90 - p$ and $b_E = b$. From equations (1), (2) and (3) respectively we get

$$\cos r_E = \cos r \tan p \tag{8}$$

$$\tan d_E = \frac{\tan d}{\tan^2 p} \tag{9}$$

$$\sin r_E = \frac{\cos p}{\sin d_E} \tag{10}$$

After substitution into equations (6) and (7) and some manipulation we obtain

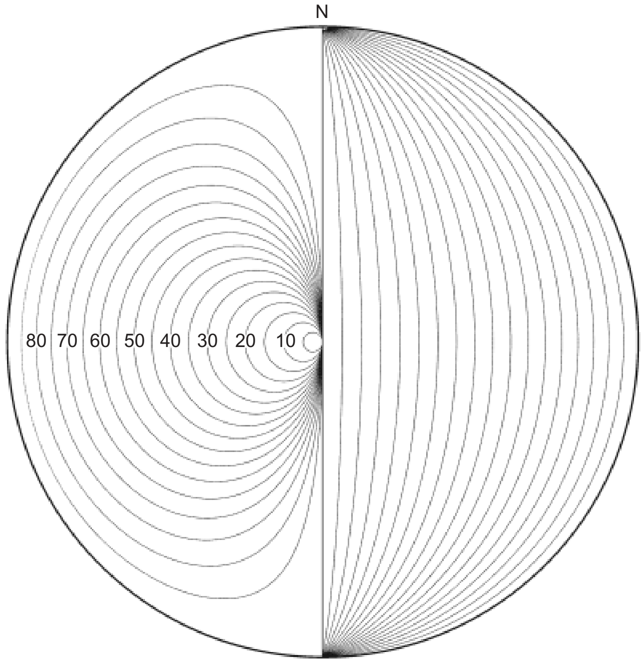

Fig. 3. Examples of computer-drawn daylight envelopes for a series of dipping rock faces with angles of dip ranging from 5° to 85°. This figure could be used as a reference equal-area net for tracing off the appropriate daylight curve

$$x_E = K \cos d \sin r \tag{11}$$

$$y_E = K \cos r \tag{12}$$

where

$$K = \frac{\sin d \sin r}{\sqrt{1 - \sin^2 d \sin^2 r}} \sqrt{\frac{2}{1 + \sqrt{1 - \sin^2 d \sin^2 r}}}$$

Where the face does not slope eastwards, the coordinates given by equations (11) and (12) need to be transformed to produce the required rotation through an anticlockwise angle θ:

$$x' = \cos \theta x_E + \sin \theta y_E \tag{13}$$

$$y' = -\sin \theta x_E + \cos \theta y_E \tag{14}$$

RESULTS

The equations derived above form the basis of computer drafting of daylight envelopes. Fig. 3 shows examples of computed daylight envelopes for a family of east-sloping rock faces with slopes in 5° increments. As an alternative to computer drafting, Fig. 3 could be used as a set of reference curves: that is, the appropriate curves could be traced to obtain the daylight envelope required for slope stability analysis.

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