Discussion

Discussion: "Normal Indentation of Elastic Half-Space With a Rigid Frictionless Axisymmetric Punch" (Fu, G., and Chandra, A., 2002, ASME J. Appl. Mech., 69, pp. 142–147)

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The authors presented an interesting consideration of an axisymmetric frictionless contact problem with the aid of mathematical software MATHEMATICA. Evidently, the use of modern analytical software gives a possibility to obtain new results, check known solutions, and correct possible misprints. However, some papers in the field should be added to their reference list.

In 1939 an analytical solution for a punch described by a monomial function of r of a positive even degree α was obtained by Shtaerman [1]. It is worth mentioning that after A. E. H. Love had obtained his solution, the problem for conical punch was also solved by Lur'e [2] in 1941. The problem for a punch described by a monomial function of r of an arbitrary real degree α was solved by Galin (see Chap. 2, paragraph 5 in Ref. [3]). Then this problem was also analyzed by Sneddon [4]. In 1957 the problem was analyzed by Segedin [5] for a punch whose shape is represented by a series (a polynomial function of r) with integer degrees α. For a punch described by a fractional power series of r, the problem was analyzed in Ref. [6]. The analysis in Ref. [6] was based on the Galin's solution ([3]). It was shown that the solution can be also used in the case when the punch is a transversally isotropic solid and the half space has homogeneous initial stresses. In particular, a formula similar to formula (17) was obtained.

References

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Closure to "Discussion of 'Normal Indentation of Elastic Half-Space With a Rigid Frictionless Axisymmetric Punch' " (2003, ASME J. Appl. Mech., 70, pp. 783)

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Our work ([1]) was based on Green's solution ([2]). We thank the discussors for pointing out a different approach taken by Borodich [3] following the work of Galin [4]. At the time of publication, we were unaware of the work by Borodich. The usage of our derived solution is straightforward. With modern mathematical software, hypergeometric function will be like a regular elementary function and the final result is easy to be obtain. It can also be used to check analytical expressions for possible misprints.

As it is pointed out in the paper, the power of the "polynomial" can be any non-negative number, such as 0, 2, 1/12, e, π. With this solution, one can use multiple terms to define the punch shape instead of a monomial function of the punch radius.

We appreciate the fact that there exist numerous contributions to this field in the Russian literature, and our understanding of this work is mainly based on the books by Gladwell [5] and Sneddon [6]. Johnson [7] also mentioned the solutions by Shtaerman and Galin in his book.

References

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