Self-Similarity in Abrasiveness of Hard Carbon-Containing Coatings

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The abrasiveness of hard carbon-containing thin films such as diamond-like carbon (DLC) and boron carbide (nominally B_4C) towards steel is considered here. First, a remarkably simple experimentally observed power-law relationship between the abrasion rate of the coatings and the number of cycles is described. This relationship remains valid over at least 4 orders of magnitude of the number of cycles, with very little experimental scatter. Then possible models of wear are discussed. It is assumed that the dominant mechanism of steel wear is its mechanical abrasion by nano-scale asperities on the coating that have relatively large attack angles, i.e. by the so-called sharp asperities. Wear of coating is assumed to be mainly due to physical/chemical processes. Finally, models of the abrasion process for two basic cases are presented, namely a coated ball on a flat steel disk and a steel ball on a coated flat disk. The nominal contact region can be considered as constant in the former case, while in the latter case, the size of the region may be enlarged due to wear of the steel. These models of the abrasion process are based on the assumption of self-similar changes of the distribution function characterizing the statistical properties of patterns of scattered surface sharp asperities. It is shown that the power-law relationship for abrasion rate follows from the models.
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1 Introduction

Carbon-containing thin films such as diamond-like carbon (DLC) and boron carbide (nominally B_4C) have the ability to enhance the fatigue resistance of heavily loaded steel components such as gears, bearings, and cams [1]. It has been proposed [2] that this ability stems from the fact that these coatings can polish away asperities on the counterparts, reducing the magnitude and the number of the high intensity stresses that they cause. It is known, for example, that the lifetime of gears can be extended by polishing the contacting faces [3]. Recently we have begun studying the kinetics of abrasive polishing by these coatings, measuring the decline in the rate at which they abrade steel and the changes in their morphology that are associated with that decline.

The wear equation developed by Archard [4] (see also [5,6]), sometimes used to describe abrasive wear, is

$$M = c\,\frac{d}{3}\,\frac{P}{H}$$

where M is the amount of material worn away, d is the sliding distance, P is the normal load, H is the indentation hardness value of the softer of the wearing pair, and c is a constant. Harris and Weiner [7] showed, however, that for a fixed P and H, d is insufficient to determine M, indicating that Archard's formula cannot be directly applied to describe their pin-on-disk experiments in which a steel ball is worn by a nominally flat DLC-coated surface. From studies of metal containing DLC (Me:DLC) coatings, it was found that (i) The abrasiveness is strongly dependent on the coating hardness [8]; (ii) The abrasion rate does not depend on surface roughness features with horizontal length scales on the order of micrometers or larger, but it is strongly correlated to roughness features with horizontal length scales on the order of nanometers [9]; and (iii) The abrasion rate drops significantly with every pass of the ball, coinciding with the loss of relatively sharp nanometer-scale DLC asperities, even as the micrometer scale structure is unchanged [7,9].

Finally, Harris and his co-workers [7,9,10] discovered a remarkably simple relationship, with very little experimental scatter that predicts $\overline{A(n)}$, the average abrasion rate (volume removed per meter) of a steel ball by a coating during the first n cycles,

$$\overline{A(n)} = A_1 n^{\beta} \tag{1.1}$$

where A_1 is the abrasion rate on the first cycle, and β is an experimentally observed exponent. This relationship has been shown to be valid for both Me:DLC and B_4C, and it remains valid over at least 4 orders of magnitude of n. Such a simple, accurate, and widely applicable relationship is rarely observed in tribology. Because its origin is not understood, further theoretical study is necessary to describe observed experimental results.

In this paper, we propose a theoretical explanation for the relationship in Eq. (1.1) and connect this experimentally observed phenomenon with some nano-characteristics of the coated surfaces. In our models we employ the experimental observation that relatively large attack angles of nano-scale asperities are necessary to create microchips of the steel. This observation which is well known for cutting and other abrasive conditions (see, e.g., [11]), was recently confirmed for the abrasion of steel by DLC coatings (see [7,9]). Our models are based on the concept of self-similarity of the spatial pattern of nano-scale sharp asperities of the coatings.

We first need to define the term "similarity" in a quantitative fashion. In applied mathematics the term has been used with two rather different meanings: (i) two individuals (e.g., objects or phenomena) can be transformed one to another by some transformations; and (ii) two individuals look alike. The former meaning of similarity is used in various branches of science, in particular in dimensional analysis and group analysis of differential equations. Classical geometrical similarity is an example of this meaning of the term. The latter meaning is used in cluster analyses and recognition of images. According to this meaning, two individuals are similar if they belong to the same cluster, i.e. the measure of difference between the individuals is sufficiently small. There are various mathematical methods for clustering individuals (see, e.g. [12]).

It is observed that natural processes in a surprisingly large number of cases are, broadly speaking, self-similar during their intermediate stage of development when the behavior of the processes

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has ceased to depend on the details of the boundary or initial conditions [13–15]. This idea is undergoing an upsurge of interest due to the introduction of the concept of fractals (see, e.g., [16]), although the concept of self-similarity is broader than that of fractals. The models presented in this paper rely on the assumption that the spatial patterns of sharp asperities within the nominal contact region during an intermediate stage are self-similar. Although the meaning of statistical self-similarity is somewhat between the above two meanings of the term, it is based on the concept of transformations, namely the transformation of coordinate dilations. We will consider two types of pin-on-disk experiments. For the sake of clarity, we will consider first the problem of wear of a flat steel surface (disk) caused by a coated slider (ball). In this process each of the sharp asperities of the coated surface is continuously within the nominal contact region and abrades the steel surface. This process has some mathematical features related to a self-similar model of development of multiple contact points between two layers in multilayer steel stacks under increasing pressure [17] and to self-similar models of damage accumulation [13]. We will then consider the problem of wear of a steel ball caused by a nominally flat coated surface. In this process each of the sharp asperities on the coating can abrade the steel surface only during a short interval of the cycle when it is within the nominal contact region between the steel ball and the coated surface. Both schemes (a ball coated with a hard film and a steel ball against a coated disk) have been studied experimentally (see, e.g., [9,18,19]).

When considering the contact of rough surfaces, one should distinguish the nominal A_n and real A_r areas of contact between a ball and a surface. If a perfectly smooth ball is pressed into a perfectly smooth flat surface then the initial nominal region of contact can be obtained by solving the Hertz contact problem. The real contact area is made up from discrete regions defined by points of mutual interaction between asperities on the surface and the ball. Usually, A_r is a small fraction of A_n. As the asperities on the surfaces wear, A_r increases while A_n may remain the same if the slider is more resistant to wear than the flat surface. Note that in the second experimental scheme (a steel ball on a coated disk), the radius of the nominal contact region after few cycles will substantially exceed the initial nominal (Hertzian) radius r_H due to wear of the ball surface. The nominal contact area will remain the same $A_n = \pi r_H^2$ in the first experimental scheme (coated ball on a steel disk). The paper is organized as follows. First we review experimental results concerning the abrasiveness of carbon-containing thin films. Then we discuss some possible geometrical models of sharp asperities and show that modeling asperities as cones has some advantages. In particular, the predictions of this model for an asperity are independent of the material hardening exponent. We next discuss the abrasiveness of coated surfaces and show that the problem is reduced to calculating statistical properties of the spatial pattern of sharp asperities. Finally, by assuming that the distribution function of the pattern changes in a self-similar manner, we derive formulas for the abrasion rate which are asymptotically equivalent to the power-law relationship Eq. (1.1) observed experimentally.

2 Experimental Background

The experiments to be modeled have been described in detail previously [7]. They were performed using a ball-on-disk apparatus on which a 3.2 mm diameter 52100 steel ball with a nominal initial R_a of 25 nm was held in dry sliding contact against a steel coupon that had been coated with a sputtered metal-containing diamondlike carbon (Me:DLC) or boron carbide (nominal formula, B_4C). Both W- and Ti-containing DLC coatings were examined; their chemical compositions are given in [19]. The wear tests were conducted in air at room temperature with a humidity of 35 percent ± 15 percent. The coupon was polished to $R_a = 10$ nm before coating. After coating, the surface R_a roughness was about 100 to 400 nm. Thus, all of the asperities discussed in this work are intrinsic to the coating and have nothing to do with the original surface roughness of the coupon. The load on the balls was varied between 0.05 and 11 Newtons, corresponding to initial nominal maximum contact stresses between 0.36 and 2.2 GPa. However, the nominal contact stresses dropped significantly as the size of the wear scar on the ball increased. Of course, since steel was removed during the experiments, local contact stresses must have reached 6–7 GPa, the nominal hardness of the steel balls. In order to determine the volume of steel removed from the ball we measured the surface profile of the wear scar on the ball and used that profile to calculate the missing volume by numerical integration [20].

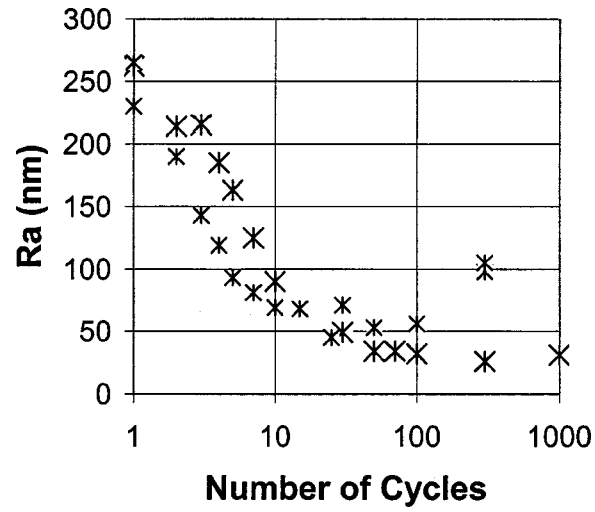


Fig. 1 The time history of the surface roughness R_a of a 52100 steel ball after sliding against a sputtered boron carbide coating

Friction coefficients are not reported here because they were highly variable and do not correlate with our abrasion measurements. An example of this variability is presented in Fig. 2 of [19], which shoes that the friction coefficient sometimes rises and sometimes falls initially. In the boron carbide experiments we did not detect any boron on the ball with Auger spectroscopy, indicating that little or no boron carbide was transferred during our wear tests. We did detect carbon on the balls, when run against either boron carbide or DLC, but we could not be certain whether this carbon came as a transfer layer, was from the steel, or was adventitious.

Figure 1 shows the time history of the surface roughness R_a of a 52100 steel ball after sliding against a sputtered boron carbide film coated on a steel disk. (Boron carbide is similar to DLC, in the sense that it is a hard, amorphous, carbon-containing film, and DLC coatings often contains large amounts of other elements such as Si, N, B, Ti, Cr, and W.) The steel surface becomes highly polished, with roughness decreasing by an order of magnitude to a mirror finish of 25 to 50 nm by 100 cycles. Except for instances where debris in the contact temporarily increases R_a, the roughness remains approximately constant beyond this point. This polishing appears to be primarily mechanical in nature. The SEM images in Figs. 2(*a*) and 2(*b*) show that the surface of the coating also changes rapidly and drastically during the polishing process, even though it is much harder than the steel. The abrasiveness of the film shown in Fig. 2(*b*) (after 500 cycles) is about 3 orders of magnitude lower than that of the film shown in Fig. 2(*a*) (unworn). (The terraces shown in Fig. 2(*b*) were examined with an AFM and were found to be almost atomically flat [21]). Analogous AFM images for DLC are shown in Figs. 2(*c*–*e*). Since the coatings are much harder than the steel, we do not expect that they could have been plastically deformed directly by the balls. XPS analysis of a boron-carbide coated coupon after 500 cycles

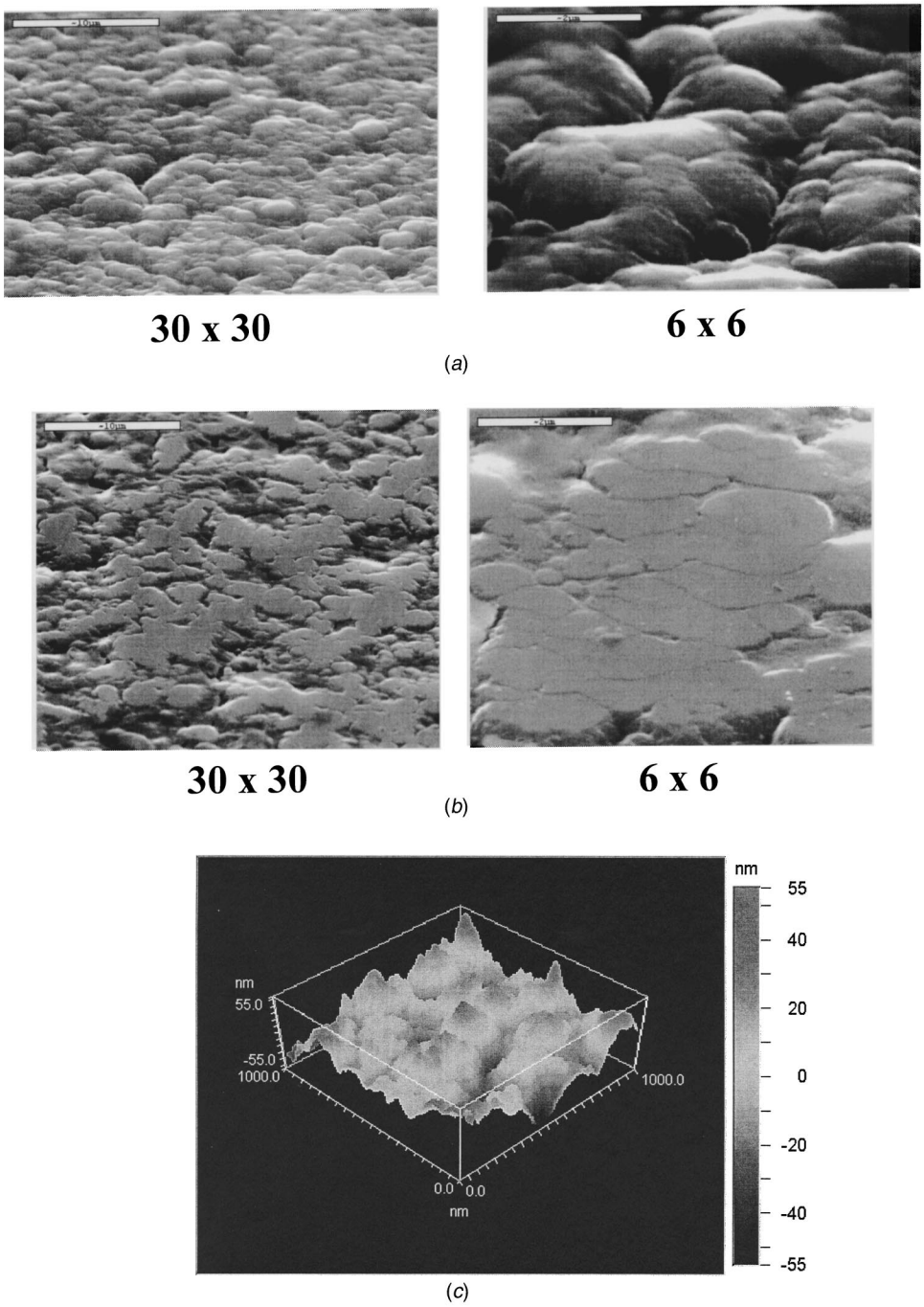
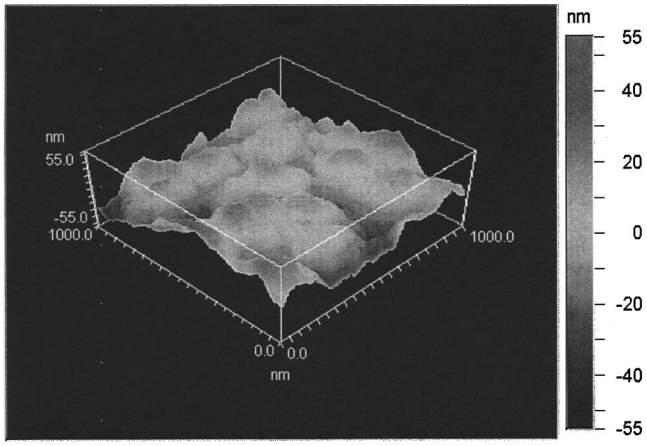


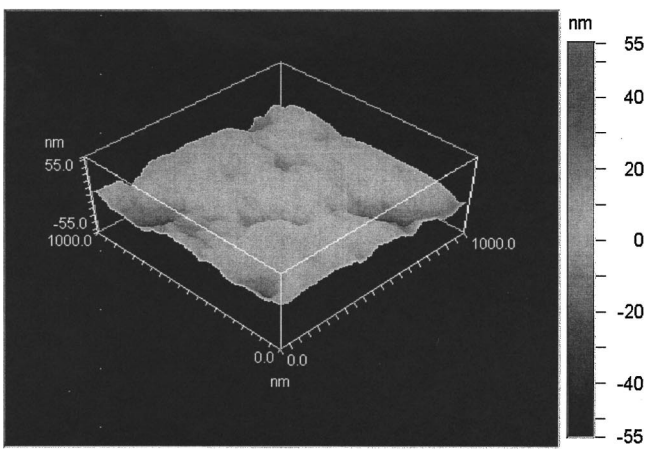
Fig. 2 SEM images of the surface of a sputtered boron carbide coating under different magnifications (*a*) unworn surface; (*b*) the surface after 500 cycles. AFM images of a metal-containing amorphous hydrogenated DLC coating: (*c*) unworn surface; (*d*) the surface after 10 cycles; (*e*) the same surface after 1000 cycles.

showed an oxide of boron in the wear track. We take this partial oxidation of the boron carbide as evidence that the counter-polishing of the boron carbide by the steel ball is chemical in nature [10,22]. The fact that the abrasion rates are independent of sliding speed variations of up to an order of magnitude strongly suggests that this process has little or no temperature dependence. However, we have not identified specific chemical pathways or rate-controlling steps that would explain this remarkable smoothing of very hard films by steel.

Considering that the abrasion rate of the steel is controlled by details of the morphology of the surfaces together with the local pressures, then the enormous changes in morphology exhibited in Figs. 1 and 2 would lead one to expect a very complex time-history for the abrasion rate. In particular, we consider it to be remarkable that the abrasion rate caused by the surfaces shown in Figs. 2(*b*) and 2(*e*) could be related to or predicted by the abrasion rate caused by the surfaces shown in Figs. 2(*a*) and 2(*c*) especially since the steel surface has also undergone dramatic changes. Nevertheless, the variation of the abrasion rate with the number of cycles follows an extraordinarily simple time history, as shown in Fig. 3 for loads of 1, 5, and 11 Newtons. In this graph the ordinate, the average abrasion rate, is defined as



(*d*)



(*e*)

Fig. 2 (***continued***)

$$\overline{A(n)} = \frac{M}{d} = \frac{M}{2\pi R n} \tag{2.1}$$

where M is the total amount of steel removed after n cycles, and d is the total distance traveled by the steel ball with a pin-on-disk radius R. The linear relationships mean, for example, that for a given load the abrasion rate after 1000 or 10,000 cycles can be predicted knowing only the abrasion rate on the first half-dozen cycles. (We note that while the surface morphology of the coatings changes dramatically during the experiments, the changes amount, in effect, to shaving off the top 0.1 to 0.2 μm of the coating. Since the coating is approximately 2 μm thick, the coating thickness is only slightly reduced in our experiments, and it is not completely worn off anywhere. In fact, a profilometer scan through the wear track using a low horizontal resolution does not detect the presence of the wear track.) Since the slopes of these lines are nearly identical, we can accurately predict the abrasion rate for any load on the 1000th cycle or the 10,000th cycle knowing only the abrasion rate on the first ten (or even fewer) cycles, even though the abrasion rate changes strongly and continuously. The straight lines observed in Figs. 3 and 4 imply the power-law relationship Eq. (1.1), where β, the slope, must lie between 0 and -1 [7], and is typically -0.8. The relationship between the instantaneous abrasion rate on the ith cycle A_i and the average abrasion rate after n cycles is

$$\overline{A(n)} = \frac{1}{n}\sum_{i=1}^{n} A_i\,. \tag{2.2}$$

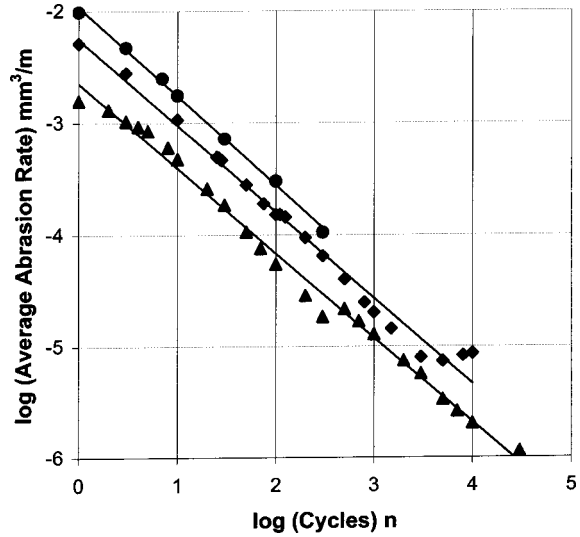


Fig. 3 The variation of the abrasion rate for boron carbide coatings with the number of cycles for loads of 1 (triangles), 5 (diamonds), and 11 (circles) Newtons, respectively

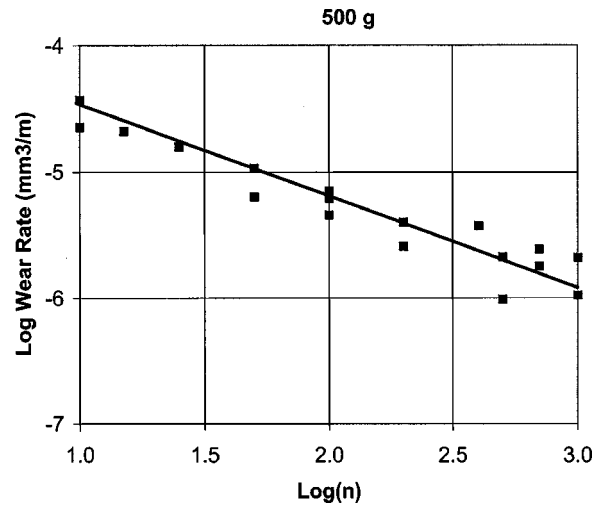


Fig. 4 Log wear rate for Me:DLC vs. Log number of cycles at 500 g load (after [32])

Combining Eq. (1.1) and Eq. (2.2) we have

$$A_n = A_1[n^{1+\beta} - (n-1)^{1+\beta}] = A_1 n^{1+\beta}[1-(1-1/n)^{1+\beta}]$$
$$= A_1 n^{1+\beta}[(1+\beta)(1/n) - (1+\beta)(\beta/2!)(1/n)^2 + \cdots]$$

When $n \geqslant 6$ and β is -0.8, we can write with error less than 1 percent

$$A_n = (1+\beta)A_1 n^{\beta} \tag{2.3}$$

3 Modeling Sharp Asperities at the Nanoscale

The description of surface roughness at the level of individual asperities is a complex problem. It was shown (see, e.g., [23]) that results obtained by modeling the surface roughness as collections of spheres [24,25] are not scale independent, so various other approaches, including the fractal approach, were developed (see, e.g., a recent review by Borodich and Onishchenko, [16]). Since fractal properties of surfaces are usually observed within some interval of scales whose lower cutoff is greater than the nanoscale, it is assumed that nanoscale asperities are smooth or piecewise smooth.

Let us put the origin of a Cartesian coordinate system *Oxyz* at the peak of an asperity. We direct the axis z into the depth of the asperity and the axes x and y along the plane of the surface. So, the asperity can be described as some function $f: z=f(x,y)$. If f is a power law function, then an asperity can be approximated as

$$z=B(\theta)r^{\gamma}, \quad \text{where } 1\leqslant\gamma\leqslant 2. \tag{3.1}$$

Evidently, the size of an abrasion groove in steel made by a hard DLC asperity depends on the depth of its penetration which in turn depends on the external load. If an asperity is described with Eq. (3.1) then the effective attack angle $\arctan f'$ increases with the depth of penetration for $\gamma>1$. So, an asperity can be "dull" for small loads and "sharp" for larger loads.

What happens when the external load on an asperity changes? To answer this question we can take advantage of the self-similarity of Hertz-type contact problems (not to be confused with the statistical self-similarity that will be employed later in the paper). This means that from a solution for one value of external load the solution for any other load can be obtained by re-scaling. Hertz problems are self-similar if the constitutive relationships are homogeneous with respect to the strains or the stresses and if the indenter's shape is described by a homogeneous function whose degree is greater or equal to unity [26]. So, if the stress-strain relation of the coating is $\sigma\propto\varepsilon^{\kappa}$ where κ is the work-hardening exponent of the constitutive relationship, then the Hertz type contact problem for an asperity described by Eq. (3.1) is self-similar. In particular, the size of the contact region r_H for this asperity and the depth of penetration h depend on the external load L_a as [27,28]

$$r_H(L_a)=\left(\frac{L_a}{L_1}\right)^{1/2+\kappa(\gamma-1)} r_H(L_1), \quad h(L_a)=\left(\frac{L_a}{L_1}\right)^{\gamma/2+\kappa(\gamma-1)} h(L_1) \tag{3.2}$$

where L_1 is some initial value of the external load. If an asperity is described by Eq. (3.1), and the depth is h then the cross sectional area of a groove that it plows is

$$A_a=2\left(hx_*-\int_0^{x_*}Bx^{\gamma}dx\right)=2h\left(\frac{h}{B}\right)^{1/\gamma}\left(\frac{\gamma}{\gamma+1}\right), \quad x_*=\left(\frac{h}{B}\right)^{1/\gamma}. \tag{3.3}$$

For an asperity that penetrates rather deeply into the metal surface, a cone $z=B(\theta)r$, i.e., $\gamma=1$, is a very effective approximation. It follows from Eq. (3.2) that neither the contact region size nor the depth of penetration h depend on the work-hardening exponent κ.

For a circular cone $B(\theta)$ is a constant and the cross-section is a wedge, and it follows from Eq. (3.2) that $A_a=h^2/B$ (see, also [29]). The amount of steel $m(L_a)$ which is plowed by a conical asperity loaded by the force L_a during a unity displacement is $m=k_1A_a=k_1L_ah^2(1)/B$ where k_1 is a constant and $h(1)$ the depth of indentation of the cone for $L_a=1$. It is possible to show that if there is a system of independently acting conical asperities $z=\mathrm{Br}$ loaded by the total load L_Σ then the abrasiveness of the system depends neither on the particular load distribution in the system nor on the number of asperities. This conclusion is valid for an arbitrary depth of asperity penetration. However, the depth usually cannot exceed some critical depth h_c which depends on the roughness parameters of the coating. So, if we suppose that during the self-similar stage, all sharp asperities have the critical depth h_c, and if the average load L_a acting on a sharp asperity does not change, then the amount of steel m which is abraded by each conical asperity during a unity displacement along the surface is

$$m=k_1A_a=k_1h_c^2/B$$

Hence, the abrasiveness of a system of N conical asperities penetrated on the critical depth h_c is just the sum of the abrasiveness of the individual asperities, i.e., Nm.

4 Wear of a Steel Surface by a Coated Ball

Let us discuss the first scheme of pin-on-disk experiments, i.e., a coated ball sliding under an external load L against a steel surface. As we have mentioned, in this process each of the sharp asperities abrades the steel surface continuously.

4.1 Assumptions of the Model. The main assumptions of our model are:

1. The nominal contact region G_H with an area A_n is constant and its value can be obtained from the Hertz solution.
2. After some initial stage, the wear process becomes statistically self-similar. During the self-similar stage, all sharp asperities are described as similar conical asperities penetrating to the critical depth h_c, and the current abrasiveness of a coating is determined by the number of sharp asperities within the nominal region of contact G_H. The average amount of steel removed by each of the sharp asperities is proportional to the distance D it traveled through the surface.

4.2 Statistical Self-Similarity of Sharp Asperity Patterns. Let us consider a set of random points within the contact region $G(t)$ at time t comprising the peaks of sharp asperities. We assume that the points of the pattern are generated by some underlying random wear mechanism. We are mainly interested in the number of the sharp asperities $N(t)$ within the contact region at the moment t, because the average amount of steel removed is the same for each of the asperities. It is assumed that the point pattern transforms with process time in a statistically self-similar way. The self-similarity means that the distribution of the point of sharp asperity peaks within the contact region, which is normalized by the average distance between sharp asperities, is independent of time t; i.e., only the mean of the probability distribution changes its value while all other dimensionless central moments remain unaltered. Hence, the images of the pattern at t_1 and t_2 cannot be distinguished statistically if the average distance between the points is not known. If the process is at steady-state, we can write

$$\frac{l(t_1)}{l(t_2)}=f\left(\frac{t_1}{t_2}\right). \tag{4.1}$$

Similarly, we obtain

$$\frac{l(t_1)}{l(t_3)}=f\left(\frac{t_1}{t_3}\right)$$

and

$$\frac{l(t_2)}{l(t_3)}=f\left(\frac{t_2}{t_3}\right).$$

If we denote $x=t_1/t_3$ and $y=t_1/t_2$ then we have

$$\frac{f(x)}{f(y)}=f\left(\frac{x}{y}\right). \tag{4.2}$$

Evidently, $f(1)=1$. However, it is known that if the above equation is valid for a differentiable function, then $f(x)$ is a power-law function. Indeed, after differentiation with respect to x, we have

$$\frac{1}{f(y)}\frac{df(x)}{dx}=\frac{1}{y}f'\left(\frac{x}{y}\right),$$

and setting $x=y$, we obtain

$$f(x)=Cx^{f'(1)}.$$

Because $f(1)=1$, we have $C=1$ and

$$f(x)=x^{\alpha}, \quad \alpha=f'(1). \tag{4.3}$$

Thus, we have obtained a power-law relationship for the average distance between sharp asperities,

$$l(t)=l(t_1)\left(\frac{t}{t_1}\right)^{\alpha}, \quad \alpha>0.$$

4.3 The Number of Sharp Asperities. The distance between asperities at time t is related to the contact area $|G(t)|$ and the number of asperities $N(t)$ by

$$l(t)=\sqrt{|G(t)|/N(t)}. \tag{4.4}$$

Therefore, we can calculate the number at any time t by

$$N(t)=N(t_1)\left(\frac{t}{t_1}\right)^{-2\alpha}, \quad \alpha>0.$$

It is obvious from inspection of Figs. 2(*a*) and 2(*b*) that these surfaces do not transform into each other upon dilation. Nevertheless, our analysis shows that if the asperities remain randomly distributed in the sense discussed above, then the distribution function for the distance between the asperities is self-similar.

4.4 Abrasiveness of a Coated Ball. We assume that all sharp asperities that are in contact with the steel are equally abrasive, but the abrasion process eventually reduces the sharpness of any given asperity below a critical value, at which point the abrasiveness of that asperity goes to zero, thus increasing l. If the sliding velocity v is constant then we can calculate the average amount of steel removed during the self-similar stage of the wear process by the time t

$$M(t)=\nu\int_1^t mN(\tau)d\tau=\nu\int_1^t mN(t_1)\left(\frac{\tau}{t_1}\right)^{-2\alpha}d\tau$$

or

$$M(t)=\nu mN(t_1)\left(\frac{1}{t_1}\right)^{-2\alpha}\left(\frac{1}{1-2\alpha}\right)(t^{1-2\alpha}-t_1^{1-2\alpha})$$

where m is the average amount of steel removed by an asperity during a cycle. The abrasion rate averaged over the time $t-t_1$ of the self-similar regime is

$$\overline{A(t)}=\frac{M(t)}{\nu(t-t_1)}=\frac{t_1}{t-t_1}mN(t_1)\left(\frac{1}{1-2\alpha}\right)\left[\left(\frac{t}{t_1}\right)^{1-2\alpha}-1\right].$$

If the self-similar stage starts quickly, then the period $T\gg t_1$ and

$$\overline{A(t_1+T)}=\overline{A_1}=\frac{t_1}{T}mN(t_1)\left(\frac{1}{1-2\alpha}\right)\left[\left(1+\frac{T}{t_1}\right)^{1-2\alpha}-1\right]$$
$$\approx\frac{mN(t_1)}{1-2\alpha}\left(\frac{T}{t_1}\right)^{-2\alpha}.$$

Similarly, we have

$$\overline{A(t_1+nT)}=\frac{t_1}{nT}\left(\frac{mN(t_1)}{1-2\alpha}\right)\left[\left(1+\frac{nT}{t_1}\right)^{1-2\alpha}-1\right]$$
$$\approx\frac{mN(t_1)}{1-2\alpha}\left(\frac{nT}{t_1}\right)^{-2\alpha}.$$

Asymptotically, the abrasion rate of a coated ball is governed by the following power-law equation

$$\overline{A(t_1+nT)}\approx\overline{A_1}n^{-2\alpha}.$$

5 Wear of a Steel Ball by a Coated Surface

We now discuss the second scheme of pin-on-disk experiments, i.e., a steel ball sliding under an external load L against a coated surface. In this process each of the sharp asperities on the coating abrades the steel only during a rather short part of the cycle when it is within the nominal region of contact between the moving ball and the surface. Nominally during a cycle, a ball contacts the disk surface in a ring $C_R: R-r_w<r<R+r_w$ with area $A_R=4\pi Rr_w$, where R is the radius of the circle of the pin motion (the wear track radius) and r_w is the current radius of the (circular) contact region between the worn ball and the surface. Initially, $r_w=r_H$. Since the contact radius of the slider increases, fresh sharp asperities come into the play within the periphery of the ring. We will take the period T as the unit of time. The average time of real contact of an asperity $\langle t\rangle$ can be estimated using the ratio of the area of the ball contact region to the area of the contact ring

$$\langle t\rangle\approx\frac{\pi r_w^2}{A_R}T=\frac{\pi r_w^2}{4\pi Rr_w}T=\frac{r_w}{4R}T.$$

5.1 Assumptions of the Model. The main assumptions of our model are:

1. The nominal contact region increases due to wear of the steel by the coated surface
2. There are no changes in the DLC surface at the level of microns or larger; all changes are at the nano-scale, where sharp asperities may become blunt. For example, Fig. 2(*b*) shows that the B_4C surface becomes considerably smoother, perhaps because of tribo-chemical wear, but some sharp asperities remain that cause wear of the steel. (In previous work using an AFM [9] we found that the RMS asperity angle on a DLC surface dropped from 18 ± 7 deg initially, when abrasiveness was high, to 6 ± 2 deg after 1000 cycles, when abrasiveness was low. These values give some sense of the difference between a "sharp" and a "blunt" asperity.)
3. After some initial period, the pattern of sharp asperities within the normalized contact region can be described in statistically self-similar way. During this stage all sharp asperities are described as similar conical asperities that penetrate to the critical depth h_c, and the current abrasiveness of a coating is determined by the number of the sharp asperities within the normalized nominal region of contact $\tilde{G}_n$. The average amount of steel removed by each of the sharp asperities is proportional to the distance d it traveled through the steel surface.

Since the nominal region of contact varies during the process, we cannot use Eq. (4.4) in a direct way. Indeed, the average distance between sharp asperities of the larger contact region will be greater than the distance between them in the smaller region, even if the number of sharp asperities is the same for both regions. To compare patterns in varying regions, we will adopt a typical pattern analysis technique for size normalization (see, e.g., [30]) and consider the normalized nominal region of contact. The normalization may be done by linear mapping of the nominal contact region to a fixed standard region. The region at the end of the first cycle of the self-similar stage $G_n(t=T)$ can be taken as the standard region. Hence, if the ratio of the contact radii at t and at kt is λ then the radius $\tilde{r}_w$ of the normalized nominal region of contact $\tilde{G}(t=kT)$ is constant, namely $\tilde{r}_w=r_w(kT)/\lambda_k=r_w(T)$, and Eq. (4.4) transforms into the following

$$\tilde{l}(t)=\sqrt{|\tilde{G}(t)|/N(t)}. \tag{5.1}$$

Assuming self-similarity on the steady-state stage of the process, we can write

$$\frac{\tilde{l}(t_1)}{\tilde{l}(t_2)}=f\left(\frac{t_1}{t_2}\right), \quad t_1=T, \quad t_2=kT. \tag{5.2}$$

Thus, we obtain the same Eq. (4.2) as in the case of a coated ball $f(x)/f(y)=f(x/y)$, which leads to the power law relation Eq. (4.3) and the power-law of average distances among sharp asperities

$$\tilde{l}(kT)=\tilde{l}(T)(k)^{\alpha_1}, \quad \alpha_1>0. \tag{5.3}$$

Using Eqs. (5.1) and (5.3), we can calculate the number of sharp asperities at any time $t=kT$

$$N(t=kT)=N(T)(k)^{-2\alpha_1}, \quad \alpha_1>0. \tag{5.4}$$

where $N(T)$ is the number of the asperities within the contact region $|\tilde{G}(T)|$ at $t_1=T$. Since the ball speed is $\nu=2\pi R/T$, the distance a sharp asperity travels through the counterpart during a cycle is

$$D=\nu\langle t\rangle=k_1\pi r_w/2.$$

Therefore, the total amount of steel M removed from the slider during n cycles of the self-similar regime is

$$M=\sum_{i=1}^{n} M_i=M_1\sum_{i=1}^{n} i^{-2\alpha_1}, \quad M_1=mN(1).$$

The abrasion rate averaged over the first n cycles of the self-similar regime is

$$\overline{A(n)}\approx\frac{1}{2\pi Rn}\sum_{i=1}^{n} M_i.$$

We can approximate the sum by an integral. Then we have

$$\overline{A(1)}\approx\frac{1}{2\pi R}\int_0^1 M(x)dx=\frac{mN(1)}{2\pi R}\int_0^1 x^{-2\alpha_1}dx=\frac{mN(1)}{1-2\alpha_1}.$$

Similarly, we obtain for the abrasion rate averaged over n cycles of the self-similar regime

$$\overline{A(n)}\approx\frac{1}{2\pi Rn}\int_0^n M(x)dx=\frac{mN(1)}{2\pi Rn}\int_0^n x^{-2\alpha_1}dx=\overline{A(1)}\frac{n^{1-2\alpha_1}}{n}.$$

Finally, for $\beta=-2\alpha$ we obtain the Harris abrasion law obtained earlier empirically. Note that the value of the exponent α_1 may differ from the value α obtained in the first model for a coated ball.

6 Discussion and Conclusions

It was shown that the abrasiveness of hard carbon-containing thin films such as diamond-like carbon (DLC) and boron carbide (nominally B_4C) towards steel follows Eq. (1.1). It is plausible [31] that this mechanism by which the shape of asperities changes, is the same for lubricated wear of a DLC coating against another DLC coating. The self-similar relationship between l and n specified in Eqs. (4.1) and (5.2) is clearly not the only relationship that could be imagined. For example, if the probability that an asperity loses its abrasiveness in any time interval were constant, then we would have obtained an exponential (rather than a power-law) relationship between A_n and n. However, the experimental data cannot be fit with an exponential function. Eqs. (4.1) and (5.2) imply instead that the probability of losing abrasiveness in any given time interval declines with n. This result may be due to the fact that the first asperities to lose their abrasiveness stand out from the surface, while the remaining asperities are protected by being partially ''hidden'' in valleys (see Fig. 2(b)). (We note that because the coating is so much harder than the steel, elastic deformation of the coating is minimal. Thus, we expect that asperities that appear hidden in our micrographs would remain hidden even under load.) The hypothesis of statistical self-similarity leads to Eqs. (5.4) and (1.1). The fact that Eq. (1.1) predicts the experimentally observed data so well over so many orders of magnitude strongly constrains the mechanism by which the morphology of the surface changes. These constraints will be explored in future work. Finally, we note that there could be other models that also describe the experimental data shown in Figs. 3 and 4. However, the remarkable simplicity of the graphs shown in Figs. 3 and 4 argues strongly that some very simple underlying principle, such as that proposed here, controls the abrasion kinetics of these carbon-containing films.

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