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Optimal Taxation in a Two Sector Economy with Heterogeneous Agents

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Abstract:

In this paper we show that in a two sector economy with heterogeneous agents and competitive markets, in a steady state the optimal capital income tax rate is in general different from zero. The optimal tax policy in this setting depends on the relative price difference. In a two sector economy capital and labour margins are interdependent, which is why a difference between investment good's price and consumption good's price allows the government to tax capital income in one sector and undo the tax distortion by differential labour income taxation. This policy serves efficiency purpose as it restores production efficiency. For instance, if investment goods are more expensive than consumption goods, it is optimal to tax capital income in consumption sector, and set zero capital income tax and lower labour income tax in investment sector. This policy discourages work and investment in consumption sector, and encourages agents to shift capital and working time to investment sector. This increases production in investment sector and restores production efficiency. In a model with two classes of agents, we show that this policy can also serve redistributive purpose.

Keywords: Optimal taxation, Ramsey problem, Two Sector model.

***JEL* Codes:** C61, E13, E62, H21.

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1. Introduction.

In this paper, we show that in a standard two sector economy with heterogeneous agents, in a steady state the optimal capital income tax rate is in general different from zero. In an economy where consumption goods and investment goods are two final goods, capital and labour margins are interdependent. If there is a difference between the equilibrium price of investment goods and equilibrium price of consumption goods, the interdependence of capital and labour margins allows the government to choose an optimal policy that taxes/subsidizes capital income from one sector and set different rates of labour income taxes across sectors. This policy is optimal since the distortions of the capital income tax can be undone by differential labour income taxation, and the tax mix restores the production efficiency condition. We consider a special form of the general model with two classes of agents: workers and capitalists. We show that the optimal policy that taxes/subsidizes capital income due to relative price difference can also serve the redistributive purpose.

Our model is a competitive equilibrium version of the standard two sector neoclassical growth model with a government that finances an exogenous stream of government purchases, and where agents are of different types. We consider two sectors that produce consumption goods (consumption sector, hereafter) and investment goods (investment sector, hereafter), using raw labour and capital on which government levies distorting flat-rate taxes. The problem is to determine the optimal settings over time for two labour income tax rates and two capital income tax rates for the two sectors. We characterize this problem as the standard Ramsey (1927) problem. We extend the important works of Judd (1985), Chamley (1986), Jones, Manuelli and Rossi (1993), Jones et al. (1997), Judd (1999) and Atkeson, Chari and Kehoe (1999), all of which discuss the optimality of zero capital income tax.

In a one sector neoclassical growth model where one final good is used for consumption and investment, Chamley (1986) and Judd (1985) show that if an equilibrium has an asymptotic steady state, then the optimal policy is eventually to set the tax rate on capital income to zero. Their key argument is that capital income taxation serves neither efficiency not redistributive purpose in the long run. The optimality of the zero capital tax extends also to a one sector economy with heterogeneous agents: an idea that was mentioned by Chamley (1986) and an analysis that was explored in depth by Judd (1985). Judd (1985) shows that in a one sector economy with heterogeneous agents, unanticipated redistributive capital taxation has severely limited effectiveness since it depresses wages. His paper argues that if the government only values the welfare of workers, since taxing capital in the long run is not optimal, there will not be any redistribution in the limit and government expenditures will be financed solely by levying wage taxes on workers. Thus in Judd's (1985) analysis, capital owners who are assumed not to work will be exempt from taxation in the steady state.

Jones et al. (1993) and Jones et al. (1997) show that the optimality of zero capital income tax extends to a model with endogenous growth through human capital accumulation. One of their main arguments was that since the distortion created by physical capital income tax and human capital income tax is compounding in nature, it is a bad idea to tax these income. Judd (1999) explores the intuition by showing that in an economy with competitive markets, a long run policy that involves a capital income tax creates exponentially growing distortions in intertemporal allocations --- something which is inconsistent with the commodity tax principle. Atkeson, Chari and Kehoe (1999) summarizes these findings in a paper showing that optimality of zero capital income tax is analytically strong even if one relaxes some of Chamley's (1986) assumptions[2]. They show that this result holds in an economy with heterogeneous agents, or in an economy with endogenous growth, or in an open economy, or in an economy where the agents live in overlapping generations.

We present a two sector neoclassical model which is possibly one of the simplest extensions of Chamley's (1986) original model. We show that if the equilibrium of a standard two sector neoclassical growth model with heterogeneous agents has an asymptotic steady state, the optimal capital income tax rate in investment sector is zero, but the optimal capital income tax rate in consumption sector is in general different from zero. We argue that in a two sector economy where investment and consumption are produced as two final goods, capital and labour margins are interdependent, and so is the optimal policy of taxing income from these factors. Due to this interdependence, capital income taxation in our model can serve both efficiency and redistributive purposes. We show that in a steady state it is optimal to set zero tax on capital income from both sectors if and only if price of investment goods and price of consumption goods are equal. The difference in relative price of investment and consumption creates a difference in social marginal value of capital in the two sectors, and a tax/subsidy on capital income in one sector, leaving the other capital income tax at zero rate can undo this difference which in turns serves the efficiency purpose. The distortion created by this capital income tax can be undone by setting different rates of labour income taxes across sectors.

For instance, in a steady state if investment goods are cheaper than consumption goods, there will be over accumulation of capital which in turns make return to capital very low. We argue

[2] Chari and Kehoe (1999), Ljungqvist and Sargent (2000, ch. 12), and Erosa and Gervais (2001) also present comprehensive coverage of these models and the discussion on the optimality of zero capital income tax.

that in this case the optimal policy should subsidize capital income from consumption sector, and set higher labour income tax and zero capital income tax in investment sector. This policy is optimal because a capital subsidy and a lower labour income tax in consumption sector encourages more capital and working time in consumption sector and higher production of consumption goods, which in turns undoes the relative price difference. On the contrary, in a steady state if consumption goods are cheaper than investment goods, the optimal policy should encourage production of investment goods, for which the optimal policy should tax capital income in consumption sector, and set lower labour income tax and zero capital income tax in investment sector.

Our main result (optimality of nonzero capital income tax) was primarily hinted by Atkeson et al. (1999) in an analysis of optimal taxation in a one sector economy with heterogeneous agents. Atkeson et al. (1999) impose additional restrictions on the optimal taxation problem in order to restrict tax rates on capital income and tax rates on labour income to be same across all types of agents. These restrictions are (1) in the Ramsey equilibrium, intertemporal marginal rate of substitution of consumption across all agent types must be equal, and (2) the ratio of intratemporal marginal rate of substitution between consumption and labour across types of agents is equal to the ratio of marginal product of labour across types of agents. Their paper argues that in a competitive economy with heterogeneous agents, zero capital income taxation in the steady state is optimal if these extra constraints (in particular, 2) do not depend on the capital stock. According to their analysis, if capital and labour income taxes are same for all types of agents, in a steady state zero rate of capital income tax is optimal if the production function is separable between capital and labour. The intuition behind our main result stems from the interdependence of the capital and labour margins, which in turns presumes that the production functions are not separable in capital and labour.

In a model with workers and capitalists, we show that the optimal policy with nonzero capital income tax in consumption sector and differential labour income taxation can serve the redistributive purpose. We show that in such an economy where the workers are exogenously constrained to not hold any assets and the capitalists are exogenously constrained not to work, even if the government cares only about the welfare of workers, there may be redistribution in the limit, and capitalists may have to bear part of the burden of the tax. This result extends Judd's (1985) finding that a capital income tax in the long run does not serve redistributive purpose.

2. A Decentralized Economy with Heterogeneous Agents.

Time is discrete and runs forever. The economy has two production sectors indexed by $j \in \{C, X\}$, where C and X denote the consumption sector and investment sector, respectively. All markets are perfectly competitive. There is a finite (integer) number of different classes of agents, N, and each class is of same size. The consumption, labour supply and capital stock of the representative agent in class $i \in \mathrm{N}$ are denoted by c_t^i, n_{jt}^i and k_{jt}^i, respectively. Class i's utility function is $u^i(c_t^i, n_{ct}^i, n_{xt}^i)$, but the discount factor $b \in (0,1)$ is identical across all agents. The utility function is strictly increasing in consumption and decreasing in labour supply, separable in consumption and labour, linear in labour and satisfies standard regularity conditions[3]. The agents purchase new investment goods and rents capital to the firms for one period. Capital decays at the fixed rate $d \in (0,1)$. Firms return the rented capital stock next period net of depreciation, and pay unit cost of capital employed, equal to r_j. Firms also pay wages, denoted by w_j. Agents of type i are each endowed with one unit of time at each period and $k_0^i > 0$ units of capital at period 0. The consumption sector's technology is:

$$c_t + g_t = f^c(k_{ct}, n_{ct}) \tag{1.1}$$

where g_t is exogenously determined government consumption expenditure, and c_t is the level of aggregate private consumption. The investment sector's technology is:

$$x_{ct} + x_{xt} = f^x(k_{xt}, n_{xt}) \tag{1.2}$$

where x_j denotes the level of new investment goods. The technology $f^j(.)$ satisfies standard regularity conditions (including linear homogeneity) and exhibits constant returns to

[3] We assume that utility is separable in consumption and labour and linear in labour in order to simply our algebra. We will assume the general form of utility function where marginal disutility of work may be different across sectors, i.e. working in consumption sector and working in investment sector may have different disliking. We do this since it leads us to the more general results. As will be clear from the analysis to follow, our main results are not sensitive to these assumptions; i.e. our analysis with this general specification shows that our main results hold for a much broader class of utility functions. Later, in a sample case of heterogeneity in our general model, we will impose restrictions on the preferences such that the results extend to models with commonly used preferences, e.g. utility functions of the form $v(c_t, 1 - n_{ct} - n_{xt})$.

scale. We will denote the marginal product of capital, say, in sector j evaluated at period t by $f_k^j(t)$. Capital's law of motion is:

$$x_{jt} = k_{jt+1} - (1-d)k_{jt}; \qquad j \in \{C, X\} \tag{1.3}$$

The government finances the exogenous stream of purchases $\{g_t\}_{t=0}^{\infty}$ solely by linearly taxing income from capital and labour employed in both sectors. We will assume that the government has access to some commitment device, or a *commitment technology* that allows the government to commit itself once and for all to the sequence of tax rates announced at period 0. The government taxes labour income and capital income at rates t_t^j per unit and q_t^j per unit, respectively. The government runs a balanced budget each period. The government makes non-negative class-specific lump sum transfer $TR_t^i \geq 0$ (but there are no lump sum taxes). We also assume that the government has a social welfare function that is simply a non-negatively weighted average of individual utilities, with the weight $a^i \geq 0$ on class i, with $\sum_{i=1}^{N} a^i = 1$. The government's budget constraints are:

$$g_t + TR_t = t_t^c w_{ct} n_{ct} + t_t^x w_{xt} n_{xt} + q_t^c r_{ct} k_{ct} + q_t^x r_{xt} k_{xt} \tag{2}$$

In (1.1), (1.2), (1.3) and (2), for $z = c, n_c, n_x, k_c, k_x, x_c, x_x, TR$, let $z_t \equiv \sum_{i=1}^{N} z_t^i$. Competitive pricing ensures that factors are paid their marginal revenue product. Optimality in the production sectors requires that marginal products are equated with the rental prices of the production factors. Defining the relative price of new investment goods as p_t, the conditions that characterize optimality in the production sectors are $r_{ct} = f_k^c(t), w_{ct} = f_n^c(t), r_{xt} = p_t f_k^x(t), w_{xt} = p_t f_n^x(t)$. The representative agent in class $i \in \mathrm{N}$ chooses allocations $\{c_t^i, n_{ct}^i, n_{xt}^i, k_{ct+1}^i, k_{xt+1}^i\}_{t=0}^{\infty}$ in order to maximize discounted lifetime utility subject to the following budget constraints:

$$c_t^i + p_t[k_{ct+1}^i + k_{xt+1}^i] = (1-t_t^c)w_{ct}n_{ct}^i + (1-t_t^x)w_{xt}n_{xt}^i + p_t[k_{ct}^i R_t^c + k_{xt}^i R_t^x] + TR_t^i \tag{3}$$

where $R_t^j \equiv [p_t^{-1}(1-q_t^j)r_{jt} + (1-d)]$. Optimality conditions for the agent i's problem include transversality conditions, (3), and:

$$u_{nc}^i(t) = -u_c^i(t)(1-t_t^c)w_{ct} \tag{4a}$$

$$u_{nx}^i(t) = -u_c^i(t)(1-t_t^x)w_{xt} \tag{4b}$$

$$u_c^i(t) = \frac{b}{p_t}u_c^i(t+1)\{(1-q_{t+1}^c)r_{ct+1} + p_{t+1}(1-d)\} \tag{4c}$$

$$u_c^i(t) = \frac{b}{p_t}u_c^i(t+1)\{(1-q_{t+1}^x)r_{xt+1} + p_{t+1}(1-d)\} \tag{4d}$$

Given the current model, a feasible allocation is a sequence $\{k_{ct}, k_{xt}, c_t, n_{ct}, n_{xt}, x_{ct}, x_{xt}, g_t\}_{t=0}^{\infty}$ that satisfies equations (1.1), (1.2) and (1.3); a price system is a 5-tuple of nonnegative bounded sequences $\{w_{ct}, w_{xt}, r_{ct}, r_{xt}, p_t\}_{t=0}^{\infty}$; a government policy is a 6-tuple of sequences $\{t_t^c, t_t^x, q_t^c, q_t^x, g_t, TR_t\}_{t=0}^{\infty}$. A competitive equilibrium in this economy is a feasible allocation, a price system, and a government policy such that (a) given the price system and the government policy, the allocation solves both sets of the firms' problems and the agents' problems, and (b) given the allocation and the price system, the government policy satisfies the sequence of government budget constraints (2). The competitive equilibrium dynamics in this environment can be characterized by a system of equations that include the transversality conditions, optimality conditions in the production sectors, (1.1), (1.2), (1.3), (2), and for each agent i, (3), (4a), (4b), (4c) and (4d).

Notice the interdependence of the capital and labour margins in this two sector model. From (4a) and (4b), it is straightforward to show that if the competitive equilibrium has a steady state,

$$p = \frac{(1-t^c)f_n^c u_{nc}^i}{(1-t^x)f_n^x u_{nx}^i} \tag{4e}$$

Furthermore, (4c) and (4d) together imply that in a steady state,

$$p = \frac{(1-q^c)f_k^c}{(1-q^x)f_k^x} \tag{4f}$$

(4e) and (4d) together imply that competitive equilibrium allocations must satisfy:

$$\frac{(1-t^{c})(1-q^{x})}{(1-t^{x})(1-q^{c})} = \frac{f_k^c f_n^x}{f_k^x f_n^c}\left(\frac{u_{nx}^i}{u_{nc}^i}\right) \qquad (4g)$$

(4g) implies that due to the interdependence of labour and capital margins, the optimal policy for capital income taxation will depend on the optimal policy for labour income taxation.

3. The Ramsey Problem.

We use Chamley's (1986) approach to Ramsey (1927) problem, and derive the conditions that characterize the Ramsey allocation. Then we look for the taxes that can implement the second-best wedges. Taxes that implement the second best wedges are optimal taxes. In line with Chamley (1986), we assume that the government chooses after tax returns to maximize welfare such that the chosen after tax returns generate allocations that are implementable in a competitive equilibrium. We define after tax returns as $\tilde{r}_{jt} \equiv (1-q_t^j) r_{jt}, \tilde{w}_{jt} \equiv (1-t_t^j) w_{jt}; j \in \{C, X\}$. We consider (2) and invoke linear homogeneity property of the production functions. With the linear homogeneity property, the government budget constraint can be expressed only in terms of allocations and after tax returns, such as:

$$g_t + TR_t = f^c(k_{ct}, n_{ct}) + p_t f^x(k_{xt}, n_{xt}) - \tilde{r}_{ct} k_{ct} - \tilde{r}_{xt} k_{xt} - \tilde{w}_{ct} n_{ct} - \tilde{w}_{xt} n_{xt} \qquad (5)$$

In a model with only one class of agents, given the preset revenue target, the Ramsey problem is therefore the government's problem of choosing the after tax returns that maximizes welfare and generates allocations and prices that satisfy the i invariant equations (5), (4), (1.1), (1.2) and (1.3). Since there are many classes of agents, one needs to incorporate the agents' budget constraints as a competitive equilibrium condition that the optimal taxes must satisfy. Put differently, the optimal taxes must generate allocations and prices that satisfy equilibrium conditions for each class of agents.

Notice that we assume tax rates on capital income and tax rates on labour income do not differ across classes of agents. Atkeson et al. (1999) solve a similar problem for a one sector economy using the primal approach. The primal approach to optimal taxation, due primarily to Atkinson & Stiglitz (1980, ch. 12), characterizes the set of allocations that can be implemented as a competitive equilibrium with distorting taxes by two simple conditions: a set of resource constraints and a set of implementability constraints. If one solves the current problem using the primal approach and restricts the capital income tax rates and labour income tax rates not to vary across classes of agents, in addition to these two constraints, the Ramsey problem must include additional constraints: *(a)* $\frac{u_c^i(t)}{u_c^i(t+1)} = \frac{u_c^i(t)}{u_c^i(t+1)}$; *(b)* $\frac{u_{nc}^i(t)}{u_c^i(t) f_{ni}^c(t)} = \frac{u_{nc}^i(t)}{u_c^i(t) f_{ni}^c(t)}$; and *(c)* $\frac{u_{nx}^i(t)}{u_c^i(t) p_t f_{ni}^x(t)} = \frac{u_{nx}^i(t)}{u_c^i(t) p_t f_{ni}^x(t)}$, $i \neq i$. We recast the Ramsey problem using Chamley's (1986) approach where these constraints are incorporated with the detailed equilibrium conditions for all classes of agents[4]. More precisely, we recast the government's problem as one in which the government chooses allocations to maximize welfare subject to (5), (4), (3), (1.1), (1.2) and (1.3) for all class i. The problem in Lagrangian form is:

$$\hat{\mathbf{L}} = \sum_{t=0}^{\infty} b^t \left\{ \begin{aligned} & \sum_{i=1}^{N} a^i u^i(c_t^i, n_{ct}^i, n_{xt}^i) \\ & + y_t [f^c(k_{ct}, n_{ct}) + p_t f^x(k_{xt}, n_{xt}) - \tilde{r}_{ct} k_{ct} - \tilde{r}_{xt} k_{xt} - \tilde{w}_{ct} n_{ct} - \tilde{w}_{xt} n_{xt} - g_t - TR_t] \\ & + f_{1t} [f^c(k_{ct}, n_{ct}) - c_t - g_t] \\ & + f_{2t} [f^x(k_{xt}, n_{xt}) + (1-d)(k_{ct} + k_{xt}) - k_{ct+1} - k_{xt+1}] \\ & + \sum_{i=1}^{N} m_{1t}^i [u_{nc}^i(t) + u_c^i(t) \tilde{w}_{ct}] + \sum_{i=1}^{N} m_{2t}^i [u_{nx}^i(t) + u_c^i(t) \tilde{w}_{xt}] \\ & + \sum_{i=1}^{N} m_{3t}^i [u_c^i(t) - \frac{b}{p_t} u_c^i(t+1) \{\tilde{r}_{ct+1} + p_{t+1}(1-d)\}] \\ & + \sum_{i=1}^{N} m_{4t}^i [u_c^i(t) - \frac{b}{p_t} u_c^i(t+1) \{\tilde{r}_{xt+1} + p_{t+1}(1-d)\}] \\ & + \sum_{i=1}^{N} e_t^i [p_t R_{ct} k_{ct}^i + p_t R_{xt} k_{xt}^i + \tilde{w}_{ct} n_{ct}^i + \tilde{w}_{xt} n_{xt}^i - c_t^i - p_t k_{ct+1}^i - p_t k_{xt+1}^i + TR_t^i] \end{aligned} \right\} \tag{6}$$

[4] In the approach we follow to characterize the optimal taxation problem, we do not require any additional restriction on production functions in order to derive our main result. As we will show later, our analysis can recover the Chamley-Judd result; but for that we do not require any restrictions on production functions. We can work with the general form of neoclassical production function and show that the optimal capital income tax in a steady state depends on the relative price difference. The Chamley-Judd result holds only if the relative prices of investment and consumption are same. Since our key intuition is based on the interdependence of capital and labour margins, we find the hint by Atkeson et al. (1999) relevant.

where $y_t \equiv \sum_{i=1}^{N} y_t^i$, for $y = c, n_c, n_x, k_c, k_x, TR$, and $y_t, f_{1t}, f_{2t}, m_{1t}^i, m_{2t}^i, m_{3t}^i, m_{4t}^i, e_t^i$ are Lagrange multipliers for (5), (1.1), (1.2&1.3), (4a), (4b), (4c), (4d) and (3), respectively. Notice that since in the Ramsey problem the budget constraint and first order conditions of each class of agents are included, the social marginal value of an increment in the capital stock depends now on whose capital stock is augmented. If in equilibrium all classes behave in the same manner, their unilateral actions determine the social marginal value of capital. In this section we derive the equilibrium in the general case for all class i.

The Ramsey problem's first order condition with respect to k_{ct+1}^i and k_{xt+1}^i are:

$$f_{2t} + e_t^i p_t = b\{y_{t+1}[f_k^c(t+1) - \tilde{r}_{ct+1}] + f_{1t+1} f_k^c(t+1) + f_{2t+1}(1-d) + e_{t+1}^i[\tilde{r}_{ct+1} + p_{t+1}(1-d)]\} \quad (7.1)$$

$$f_{2t} + e_t^i p_t = b\{y_{t+1}[p_{t+1} f_k^x(t+1) - \tilde{r}_{xt+1}] + f_{2t+1}[f_k^x(t+1) + (1-d)] + e_{t+1}^i[\tilde{r}_{xt+1} + p_{t+1}(1-d)]\} \quad (7.2)$$

and with respect to n_{ct}^i and n_{xt}^i are:

$$a^i u_{nc}^i(t) + (y_t + f_{1t}) f_n^c(t) = \tilde{w}_{ct}(y_t - e_t^i) \quad (7.3)$$

$$a^i u_{nx}^i(t) + (y_t p_t + f_{2t}) f_n^x(t) = \tilde{w}_{xt}(y_t - e_t^i) \quad (7.4)$$

Proposition 1: In a two sector economy with heterogeneous agents, the steady state tax rate on capital income from investment sector is zero.

Proof: The time invariant version of (7.2), after substituting for equilibrium factor prices, is:

$$f_2 + pe^i[1 - b(p^{-1}\tilde{r}_x + 1 - d)] = b[y(r_x - \tilde{r}_x) + f_2(p^{-1} r_x + 1 - d)] \quad (8.1)$$

The time invariant version of (4d) for all class i is

$$1 = b[p^{-1}\tilde{r}_x + 1 - d] \quad (8.2)$$

Since optimal taxes generate allocations that satisfy both (8.1) and (8.2), in (8.1) $1-b(p^{-1}\tilde{r}_x+1-d)=0$, and thus:

$$f_2[1-b(p^{-1}r_x+1-d)]=by(r_x-\tilde{r}_x) \qquad (8.3)$$

(8.2) and (8.3) together imply:

$$b(\tilde{r}_x-r_x)(p^{-1}f_2+y)=0 \qquad (8.4)$$

Since $(p^{-1}f_2+y)\neq 0, b\neq 0$, it must be that Ramsey taxes satisfy $(\tilde{r}_x-r_x)=0, i.e. q^x=0$. •

In the current environment, the zero capital income tax policy is optimal for income from capital in the investment sector. Notice that Ramsey optimality condition in a steady state, (8.3), has a straightforward interpretation. Rewrite (8.3) as:

$$f_2=b[y(r_x-\tilde{r}_x)+f_2(f_k^x+1-d)] \qquad (8.5)$$

(8.5) states that a marginal increment of capital in investment sector increases the quantity of available capital goods by the amount $[f_k^x+1-d]$, which has social marginal value f_2. In addition, there is an increase in tax revenues equal to $[r_x-\tilde{r}_x]$, which enables the government to reduce other taxes by the same amount. Since y is the shadow price of government's resources, the reduction of this excess burden equals $y[r_x-\tilde{r}_x]$. The sum of these two effects is discounted by discount factor b, and is equal to the social marginal value of capital in investment sector, given by f_2. Since the optimal policy is to set $q^x=0$, investment in investment sector is consistent with the condition $1=b(f_k^x+1-d)$, which characterizes the socially optimal allocation of capital.

Proposition 2: In a two sector economy with heterogeneous agents, the steady state tax rate on capital income from consumption sector is given by

$$1 - q^c = \frac{(y + f_1)(1 - t^c) f_n^c u_{nx}^i}{f_2 (1 - t^x) f_n^x u_{nc}^i + y (1 - t^c) f_n^c u_{nx}^i}$$

Proof: The time invariant version of (7.1), after substituting for equilibrium factor prices, is:

$$f_2 + pe^i [1 - b(p^{-1}\tilde{r}_c + 1 - d)] = b[y(r_c - \tilde{r}_c) + f_1 r_c + f_2 (1 - d)] \quad (9.1)$$

The time invariant version of (4c) for all class i is

$$1 = b[p^{-1}\tilde{r}_c + 1 - d] \quad (9.2)$$

Since optimal taxes generate allocations that satisfy both (9.1) and (9.2), in (9.1) $1 - b(p^{-1}\tilde{r}_c + 1 - d) = 0$, and thus:

$$f_2 [1 - b(1 - d)] = b[y(r_c - \tilde{r}_c) + f_1 r_c] \quad (9.3)$$

(9.2) and (9.3) together imply:

$$(1 - q^c)(p^{-1} f_2 + y) = y + f_1 \quad (9.4)$$

Since (4a) and (4b) hold for all class i, and since the optimal tax policy generates implementable allocations, the optimal tax policy must be consistent with equilibrium price of investment goods, which is given by (4e). Invoking the equilibrium price in (9.4) gives

$$1 - q^c = \frac{(y + f_1)(1 - t^c) f_n^c u_{nx}^i}{f_2 (1 - t^x) f_n^x u_{nc}^i + y (1 - t^c) f_n^c u_{nx}^i} \quad \bullet$$

Proposition 2 says that in a steady state the optimal capital income tax in the consumption sector is in general different from zero. This result is one of the main contributions of the current paper. Notice that the intuition behind this result can be drawn from the interdependence of capital and labour margins in this multi-sector economy. Unlike a one sector neoclassical model where the final good is either consumed or invested to augment capital stock, in the current economy capital is a good produced in a different sector. This is why equilibrium capital and labour margins are interdependent. Since capital and labour margins are interdependent, equilibrium price of investment goods depend on the optimal policy of taxing labour income and equilibrium labour margins. Together with the steady state versions of the Euler equations (4c) and (4d), it is straightforward to verify that the optimal policy of taxing income from capital and labour are also interdependent.

Let us characterize the optimal capital income tax rate in consumption sector which will explain why it is nonzero in general, and zero only conditionally. More precisely, we will show that the optimal capital income tax policy depends on the relative price of investment goods, which in turns depend on the optimal policy for taxing labour income. Due to this interdependence, there exists only one equilibrium price of investment goods for which it is optimal to tax capital income tax in consumption sector at zero rate. The zero capital income tax policy is therefore one of many implementable optimal policies. To simplify the derivations, we impose the restriction $u^i_{nc} = u^i_{nx}$ on preferences, i.e. we assume that the marginal disutility from working in two sectors is same. This is common with utility functions of the type $u^i = u(c^i_t, 1 - n^i_{ct} - n^i_{xt})$. The equilibrium price of investment goods simplifies to:

$$p = \frac{(1 - t^c) f^c_n}{(1 - t^x) f^x_n} \tag{9.5}$$

With (9.5), in a steady state the optimal capital income tax rate in consumption sector is given by:

$$1 - q^c = \frac{(y + f_1)(1 - t^c) f^c_n}{f_2 (1 - t^x) f^x_n + y(1 - t^c) f^c_n} \tag{9.6}$$

Proposition 3: In a two sector economy with heterogeneous agents, the steady state tax rate on capital income from consumption sector is *zero* if and only if in equilibrium price of investment goods and price of consumption goods are *equal*. This policy is supported by a labour income tax policy that prescribes *equal* labour income tax rates across sectors.

Proof: Using (9.5), rewrite (9.6) as:

$$1-q^{c} = \frac{(y + f_1)}{p^{-1} f_2 + y} \qquad (9.7)$$

which is same as in (9.4). In this proof we will first show that in a steady state of the Ramsey equilibrium, a zero capital income tax policy generates a set of allocations and prices in which price of investment goods and price of consumption goods are equal (i.e. $q^{c} = 0 \Rightarrow p = 1$). Then we show that if in equilibrium price of investment goods and price of consumption goods are equal, the optimal policy is to set zero tax on capital income from consumption sector (i.e. $p = 1 \Rightarrow q^{c} = 0$).

For the first proof, notice that in (9.7), $f_1 = p^{-1} f_2 \Leftrightarrow q^{c} = 0$. This implies that a zero capital income tax in consumption sector is optimal if and only if $p = (f_1)^{-1} f_2$. Thus a zero capital income tax in consumption sector is optimal if and only if in equilibrium price of investment goods is equal to the ratio of social marginal value of investment goods to social marginal value of consumption goods. Rewrite (9.3) as:

$$f_2 = b[y(r_c - \tilde{r}_c) + f_1 f_k^{c} + f_2(1-d)] \qquad (9.8)$$

If in a steady state of the Ramsey equilibrium, the government taxes capital income from consumption sector at zero rate, (9.8) together with the condition $p = (f_1)^{-1} f_2$ implies:

$$1 = b[(f_2)^{-1} f_1 f_k^{c} + 1 - d] \qquad (9.9)$$

The zero tax policy is optimal only if the resulting allocations replicate the socially optimal allocation of capital in consumption sector, for which $1 = b(f_k^{c} + 1 - d)$ must hold. Together with (9.9), this implies that in a steady state of the Ramsey equilibrium if the government sets $q^{c} = 0$, it should generate allocations that are consistent with $(f_2)^{-1} f_1 = 1$, i.e. allocations consistent with $p = 1$.

We now show the converse. Say in equilibrium price of investment goods and price of consumption goods are equal, i.e. $p = 1$. From (9.7),

$$1-q^{c} = \frac{(y + f_1)}{f_2 + y} \tag{9.10}$$

which now defines the optimal capital income tax policy in a steady state. This policy generates allocations which are consistent with steady state of both the decentralized equilibrium and the Ramsey equilibrium, i.e. this policy must satisfy:

$$1 = b\left[\frac{y + f_1}{y + f_2} f_k^c + 1 - d\right] \tag{9.11}$$

$$f_2 = b\left[y f_k^c\left(1 - \frac{y + f_1}{y + f_2}\right) + f_1 f_k^c + f_2(1-d)\right] \tag{9.12}$$

(9.11) and (9.12) together imply that the optimal policy implements socially optimal level of capital allocation if the optimal policy is consistent with the condition $\frac{y + f_1}{y + f_2} = 1$. The only optimal policy that satisfies this condition is to set $q^c = 0$.

If $p = 1$, consider (7.3) and (7.4) in a steady state, and impose the preference restrictions. This gives:

$$1 - t^c = \frac{a^i u_n^i + (f_1 + y) f_n^c}{w_c (y - e^i)} \tag{9.13}$$

$$1 - t^x = \frac{a^i u_n^i + (f_2 + y) f_n^x}{w_x (y - e^i)} \tag{9.14}$$

From (9.5), (9.13), (9.14), it is straightforward to show that if in equilibrium price of investment goods and price of consumption goods are equal, $f_n^c = f_n^x$, and thus $q^c = 0 \Leftrightarrow t^c = t^x$. •

Following proposition 3, if price of investment goods and price of consumption goods are not equal in equilibrium, the government can implement an optimal policy that taxes/subsidizes capital income in consumption sector and taxes labour income from two sectors at different rates in order to undo the capital income tax distortion. Notice that with $u_{nc}^i = u_{nx}^i$, the competitive equilibrium condition (4g) is consistent with production efficiency condition if

$(1-q^c)(1-t^x)=(1-q^x)(1-t^c)$. With proposition 1, if there is no difference in relative prices (of consumption and investment), the policy that satisfies production efficiency must involve $q^c = 0, t^c = t^x$ (proposition 3). This policy is one of many implementable Ramsey policies, which is optimal *only if* there is no difference in relative prices. This analysis recovers the Chamley-Judd result in the current setting. If that is not the case, the optimal policy that serves efficiency purpose includes $q^c = 1-\frac{(1-t^c)}{(1-t^x)}$, with $t^c \neq t^x$. Let us consider an example. Say the economy is in a steady state with an inefficiently large production of consumption goods and low production of investment goods, such that investment goods are more expensive than consumption goods. The (long run) optimal policy should encourage production of investment goods by setting a tax on capital income and a higher tax on labour income from consumption sector. This policy, supported by a zero capital income tax and a lower labour income tax in investment sector, encourages agents to shift more capital and working hours to the investment sector, which in turns increases investment goods production and minimizes the relative price difference.

The Ramsey equilibrium conditions explain how the distortions of a capital income tax can be undone. Consider (9.8), which states that a marginal increment of capital in consumption sector increases the quantity of available consumption goods by the amount f_k^c, which has social marginal value f_1. This increment is adjusted by capital depreciation in investment sector, which has social marginal value f_2. Thus the aggregate increment in the quantity of available consumption goods net of depreciation in social marginal value terms is equal to $[f_1 f_k^c + f_2(1-d)]$. The first term is due to an increase in capital in consumption sector, while the second terms stands for an indirect increase in production of consumption good through increase in depreciated capital in investment sector. This is obvious since in a steady state of the Ramsey optimum, the capital income tax in investment sector is zero, and it is optimal to keep depreciated capital in investment sector. The increased tax revenue, equal to $[r_c - \tilde{r}_c]$, enables the government to reduce other taxes by the same amount, and the reduction of this excess burden equals $y\,[r_c - \tilde{r}_c]$. The sum of these two effects is discounted, and is equal to the social marginal value of the available capital.

It is optimal to set zero tax on capital income from consumption sector when social marginal value of investment and consumption goods are same, implying in turns that their relative prices are same. Any difference in social marginal value of these two is reflected in a relative price difference, which can be undone by the optimal policy that taxes/subsidizes capital income in consumption sector and sets differential labour income tax rates. It cannot be optimal to set nonzero tax on capital income from investment sector as it removes the shifting option. In that sector, a zero capital income tax allows agents to shift depreciated capital to that sector along the transition, and avoid compounding capital tax liabilities in consumption sector. The optimal capital income tax in consumption sector is thus not zero, in general, and the Chamley-Judd result is a special case in this setting.

4. The Two Sector Two Class Agents' Economy.

We now illustrate the case of 2 classes of agents in a two sector model following Judd's (1985) version of heterogeneity. This enables us to examine the redistributive properties of the optimal policy when agent classes have distinct exogenous restrictions on investment and work. Assume there exist two classes of agents in the model economy, such that $i \in \{1, 2\}$, and each class is of measure one. Agent type 1 are workers who only work and do not save, and agent type 2 are capitalists who only save and do not work. We will continue with the same set up in production sectors and for the government. We will continue with the notation $\tilde{r}_{jt} \equiv (1-q_t^j) r_{jt}, \tilde{w}_{jt} \equiv (1-t_t^j) w_{jt}; j \in \{C, X\}$. In addition, in order to simply the algebra, we will assume that workers' utility function is separable in consumption and leisure, linear in labour services, and that marginal disutility from working in the two sectors are same. In particular, we assume that workers have preferences described by the utility function over infinite horizon

$$U^1 \equiv \sum_{t=0}^{\infty} b^t u^1(c_t^1, 1 - n_{ct}^1 - n_{xt}^1) \tag{10}$$

The workers' budget constraints for all time t are:

$$c_t^1 = \tilde{w}_{ct} n_{ct}^1 + \tilde{w}_{xt} n_{xt}^1 + TR_t^1 \tag{11}$$

The representative worker is endowed with one unit of time at each period, and chooses consumption and labour supply to maximize (10) subject to (11). The consolidated first order conditions, assuming $u_{nc}^1(t) = u_{nx}^1(t) = u_n^1(t)$, include (11) and:

$$u_n^1(t) = -u_c^1(t)\tilde{w}_{jt}; \qquad j \in \{C, X\} \tag{12}$$

The capitalists do not work and only invest by purchasing investment goods and renting capital to firms. Each capitalist is endowed with $k_0^2 > 0$ units of capital at period 0. They have preferences described by the utility function over infinite horizon

$$U^2 \equiv \sum_{t=0}^{\infty} b^t u^2(c_t^2) \tag{13}$$

Budget constraint for capitalists for all time t is:

$$c_t^2 + p_t(k_{ct+1}^2 + k_{xt+1}^2) = \tilde{r}_{ct}k_{ct}^2 + \tilde{r}_{xt}k_{xt}^2 + (1-d)(k_{ct}^2 + k_{xt}^2)p_t + TR_t^2 \tag{14}$$

The representative capitalist chooses $\{c_t^2, k_{ct+1}^2, k_{xt+1}^2\}_{t=0}^{\infty}$ in order to maximize (13) subject to (14). The consolidated first order conditions that characterize their equilibrium behaviour include the transversality conditions, (14), and the Euler equation:

$$u_c^2(t) = bu_c^2(t+1)\frac{p_{t+1}}{p_t}\left(\frac{\tilde{r}_{jt+1}}{p_{t+1}} + 1 - d\right) \qquad j \in \{C, X\} \tag{15}$$

Since firms' problems are unchanged, the competitive equilibrium factor prices in this model are $r_{ct} = f_k^c(t), w_{ct} = f_n^c(t), r_{xt} = p_t f_k^x(t), w_{xt} = p_t f_n^x(t)$. The resource constraints are:

$$f^c(k_{ct}^2, n_{ct}^1) - c_t^1 - c_t^2 - g_t = 0 \tag{16a}$$

$$f^x(k_{xt}^2, n_{xt}^1) + (1-d)(k_{ct}^2 + k_{xt}^2) - (k_{ct+1}^2 + k_{xt+1}^2) = 0 \tag{16b}$$

We continue with the assumption that government expenditure is exogenous, and government can commit to a plan of tax rates. The government budget constraint for all time t is:

$$g_t + TR_t^1 + TR_t^2 = f^c(k_{ct}^2, n_{ct}^1) + p_t f^x(k_{xt}^2, n_{xt}^1) - \tilde{r}_{ct}k_{ct}^2 - \tilde{r}_{xt}k_{xt}^2 - \tilde{w}_{ct}n_{ct}^1 - \tilde{w}_{xt}n_{xt}^1 \tag{17}$$

Given the current model, a feasible allocation is a sequence $\{k_{ct}^2, k_{xt}^2, c_t^1, c_t^2, n_{ct}^1, n_{xt}^1, g_t\}_{t=0}^{\infty}$ that satisfies equations (16a) and (16b); a price system is a 5-tuple of nonnegative bounded sequences $\{w_{ct}, w_{xt}, r_{ct}, r_{xt}, p_t\}_{t=0}^{\infty}$; a government policy is a 7-tuple of sequences $\{t_t^c, t_t^x, q_t^c, q_t^x, g_t, TR_t^1, TR_t^2\}_{t=0}^{\infty}$. A competitive equilibrium in this economy is a feasible allocation, a price system, and a government policy such that (a) given the price system and the government policy, the allocation solves both sets of the firms' problems and the agents' problems, and (b) given the allocation and the price system, the government policy satisfies the sequence of government budget constraints (17).

There are many competitive equilibria indexed by different government policies, and this multiplicity motivates the Ramsey problem. Assume that the government has a social welfare function that is a positively weighted average of individual utilities, with the weight $a^i \geq 0; i \in \{1,2\}$, $\sum_{i=1}^{2} a^i = 1$. The Ramsey problem's Lagrangian is:

$$
\tilde{L} = \sum_{t=0}^{\infty} b^t \left\{
\begin{array}{l}
a^1 u^1(c_t^1, 1-n_{ct}^1 - n_{xt}^1) + a^2 u^2(c_t^2) \\
+ y_t [f^c(k_{ct}^2, n_{ct}^1) + p_t f^x(k_{xt}^2, n_{xt}^1) - \tilde{r}_{ct} k_{ct}^2 - \tilde{r}_{xt} k_{xt}^2 - \tilde{w}_{ct} n_{ct}^1 - \tilde{w}_{xt} n_{xt}^1 - g_t - TR_t^1 - TR_t^2] \\
+ f_{1t}[f^c(k_{ct}^2, n_{ct}^1) - c_t^1 - c_t^2 - g_t] \\
+ f_{2t}[f^x(k_{xt}^2, n_{xt}^1) + (1-d)(k_{ct}^2 + k_{xt}^2) - (k_{ct+1}^2 + k_{xt+1}^2)] \\
+ m_{1t}^1 [u_n^1(t) + u_c^1(t)\tilde{w}_{ct}] + m_{2t}^1 [u_n^1(t) + u_c^1(t)\tilde{w}_{xt}] \\
+ m_{3t}^2 [u_c^2(t) - \dfrac{b}{p_t} u_c^2(t+1)\{\tilde{r}_{ct+1} + p_{t+1}(1-d)\}] \\
+ m_{4t}^2 [u_c^2(t) - \dfrac{b}{p_t} u_c^2(t+1)\{\tilde{r}_{xt+1} + p_{t+1}(1-d)\}] \\
+ e_t^1 [\tilde{w}_{ct} n_{ct}^1 + \tilde{w}_{xt} n_{xt}^1 + TR_t^1 - c_t^1] \\
+ e_t^2 [\tilde{r}_{ct} k_{ct}^2 + \tilde{r}_{xt} k_{xt}^2 + (1-d)(k_{ct}^2 + k_{xt}^2) p_t + TR_t^2 - c_t^2 - p_t (k_{ct+1}^2 + k_{xt+1}^2)]
\end{array}
\right\}
\tag{18}
$$

The first order condition with respect to k_{ct+1}^2 is:

$$
f_{2t} + e_t^2 p_t = b\{y_{t+1}[f_k^c(t+1) - \tilde{r}_{ct+1}] + f_{1t+1} f_k^c(t+1) + f_{2t+1}(1-d) + e_{t+1}^2 [\tilde{r}_{ct+1} + p_{t+1}(1-d)]\}
\tag{19.1}
$$

and with respect to k_{xt+1}^2 is:

$$f_{2t} + e_t^2 p_t = b\{y_{t+1}[p_{t+1} f_k^x(t+1) - \tilde{r}_{xt+1}] + f_{2t+1}[f_k^x(t+1) + (1-d)] + e_{t+1}^2[\tilde{r}_{xt+1} + p_{t+1}(1-d)]\} \quad (19.2)$$

and with respect to n_{ct}^1 and n_{xt}^1 are:

$$a^1 u_n^1(t) + (y_t + f_{1t}) f_n^c(t) = \tilde{w}_{ct}(y_t - e_t^1) \quad (19.3)$$

$$a^1 u_n^1(t) + (y_t p_t + f_{2t}) f_n^x(t) = \tilde{w}_{xt}(y_t - e_t^1) \quad (19.4)$$

The conditions (19.1), (19.2), (19.3) and (19.4) are symmetric to conditions (7.1), (7.2), (7.3) and (7.4), respectively. Furthermore, symmetry of (4a), (4b) from the general model (given the preference restriction), and (12) from the current model, and symmetry of (4c), (4d) from the general model and (15) from the current model imply that the current set up is also covered by the preceding analysis of optimal taxation in a steady state. In this set up, therefore, proposition 1, 2, and 3 hold; i.e. if there is a steady state, optimal capital income tax rate in investment sector is zero, and optimal capital income tax rate in consumption sector is in general different from zero.

We now discuss the redistributive properties of this optimal policy under the current setting. We will only focus on redistribution in a limiting steady state, and will not discuss about how much redistribution is accomplished in the transition period. We first consider the special case that extends Chamley-Judd result in our setting. In deciding the optimal policy, a^2 plays no role, and thus we can conduct this analysis from the point of view where government cares only about workers' welfare, i.e. $a^1 > a^2 = 0$. If investment good's price and consumption good's price are same in equilibrium, optimal capital income tax rate is zero. With this policy, the government collects all revenue to finance its purchases by levying labour income taxes. Even if the government cares only about the workers' welfare, there will not be any redistribution in the limit. This result extends one of Judd's (1985) main results to the current setting.

If investment goods are more expensive than consumption goods, say, it is optimal to tax capital income and set higher labour income tax in consumption sector. With this policy, the government collects revenue from three tax instruments, and both workers and capitalists bear the burden of taxes. This happens even if the government cares only about the workers' welfare. With this policy, there is redistribution in the limit. We therefore show that a capital income tax can serve efficiency as well as redistributive purposes. If investment goods are cheaper than consumption goods, say, it is optimal to subsidize capital income in consumption sector, which is accomplished by levying taxes on labour income. The revenue collected from labour income taxation will be used for both government purchases and capital subsidy. Since we assume there is no lump sum tax, the optimal policy involves some redistribution in the limit in the form of capital subsidy.

5. Concluding Remarks.

We examine optimal income taxation in a two sector economy with heterogeneous agents. We contribute by showing that in such an economy, if the competitive equilibrium has a steady state, the optimal capital income tax rate in consumption sector is in general different from zero. This result can be extended to a more general result if one replaces the sector-specific income taxes with one capital income tax rate and one labour income tax rate which are applied to capital income and labour income from both sectors (the average effective tax rates on capital income and labour income). This simplification assists in smoother algebra and the optimal policy, given such a tax code, is similar to what we have found. This tax code, however does not explain the characteristics of a tax mix. Our analysis shows that if there is a difference between equilibrium prices of investment and consumption, the optimal policy is to set zero tax on capital income from investment sector, a tax/subsidy on capital income from consumption sector, and different rates of labour income taxes across sectors. Our tax code thus assists in explaining how the shifting occurs. In the model with workers and capitalists, we show that our tax code explains the redistributive property of the optimal policy.

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