An integrated strategy for vehicle active suspension and anti-lock braking systems

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A B S T R A C T

In this paper, a decentralized integrated control structure is developed based on a quarter car vehicle model including longitudinal and vertical dynamics. In this structure, the anti-lock braking system (ABS) is designed to decrease the stopping distance by regulating the longitudinal slip for improved safety during hard braking while the active suspension system (ASS) decreases the sprung mass acceleration to improve the ride comfort on irregular roads. During hard braking, it is preferred for conventional ASS to control the variations of tire deflection to improve the braking performance. However, in a new strategy, it is shown that if the ABS controller follows the optimal longitudinal slip varied with the vehicle speed and tire normal force instead of a constant value, the dependency of ASS and ABS is decreased. In this way, the ABS performance has high quality performance even in the presence of passive suspension. Application of ASS causes more reduction in the body vibration to provide more ride comfort during braking. As a conclusion, when the ASS is integrated with the proposed strategy of ABS, the overall ride and safety performances are simultaneously improved during hard braking on a good road spectrum.

1. Introduction

In the past, vehicle chassis control (VCC) systems have been developed dramatically to improve the overall vehicle behavior in relation to safety, ride comfort and handling. Among the conventional VCC systems, the active suspension system (ASS) and the anti-lock braking system (ABS) are employed to attain the ride comfort and safety of the vehicle during braking respectively [1, 2]. The anti-lock braking system controls the longitudinal slip of the wheels to

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generate the maximum braking and prevents the wheels from becoming locked [2]. As a result, the minimum stopping distance is achieved and the safety of the vehicle during hard braking is improved. In recent years, many studies have been conducted on the design of the conventional ABS. In these studies, different control methods have been employed such as sliding mode [3, 4], linear predictive control [5], non-linear optimal control [6, 7] and fuzzy control [8]. In these studies, only longitudinal dynamics is controlled for straight-line braking maneuverers on symmetric flat roads. However, braking on irregular roads causes some variations in the tire normal force which affects the stopping distance [9, 10]. In this situation, the control of normal force via active suspension can improve the braking performance.

In normal driving conditions on irregular roads, the main goal of ASS is to isolate the vehicle and its occupants from the road roughness. This objective which is to comply with the ride comfort criterion is achieved by decreasing the sprung mass acceleration [1, 11]. However, during accelerating or braking, the ASS must obtain the best possible contact between the tires and the road to meet the safety criterion. This aim of ASS can assist the other vehicle control systems like ABS.

It is considered from the above discussion that the control strategy for any VCC system is dependent on the driving conditions and other control systems. This can be a good motivation for studying the integrated vehicle dynamic control (IVDC) systems. Generally, the motivation of IVDC is to combine and manage all control subsystems which affect vehicle responses. Different structures are usually proposed for integrated vehicle dynamic control systems in the literature. In the decentralized control structure, each system is designed separately but coordinated with the others for a certain purpose. In contrast, in the centralized control structure, an integrated multi-input multi-output (MIMO) controller is developed to control all subsystems [12]. Although the centralized structure reduces the necessary sensors, it is not used for most dynamic systems to avoid extra complexity in the controller formulation.

In what follows, some strategies presented for vehicle integrated control using suspension and braking/traction systems are reviewed. Lin and Ting [13] succeeded to simulate the ABS and ASS in the case of quarter car model. They applied two back-stepping controllers in a decentralized manner for ASS and ABS. This integrated control greatly improved the stopping distance by controlling the tire deflection in the squeezed situation and increased the normal force. However, the performance of the suspension system is not discussed in the paper unfortunately. Wang et al. [14] integrated ABS and ASS by a T–S fuzzy-neural controller. Because conventional multi-input multi-output fuzzy neural networks cannot cope with a large number of variables, they used a hierarchical fuzzy neural network with learning structure. In other work, semi-active MR vehicle suspension system was integrated with the braking and steering control system using fuzzy and sliding mode control [15]. In this work, it was shown that the designed switching multi-layer control strategies achieve good vehicle performance in different situations. In another study, gain-scheduled suspension and brake control was used based on vehicle status to control the bounce and yaw motions of a light vehicle [16]. Therefore, both vehicle yaw stability and attitude are improved using the gain-scheduled robust methodology.

There is a unanimous agreement among researchers that the proper functioning of suspension is very effective during hard braking to meet the safety criterion. A positive interaction of suspension and braking systems requires the best possible contact between the tire and the road

[17-19]. With this view angle, all studies reviewed above tried to improve the ABS performance with the assistant of ASS. However, there are not enough studies to design the ABS controller considering the suspension system characteristics. In fact, when the tire/road interaction is considered in the design process of ABS controller, the dependency of ASS and ABS can be decreased. In this way, the ASS is designed individually to meet the ride comfort criterion as its main aim while the ABS performance has still high quality. Also, the extra complexity in the integrated controller formulations is avoided. This point can contribute to design a novel control strategy for integrated ASS and ABS.

In this paper, two optimal control laws for ASS and ABS in a decentralized structure are designed based on a non-linear 4DOF model. The minimized performance indexes related to braking and suspension control are defined separately and the control laws are individually derived using a prediction approach. In the derived optimal control laws, the suspension controller is adjusted to meet both ride comfort and road holding. On the other hand, the ABS controller is designed to follow the reference wheel slip in such a way that the maximum braking force can be achieved at each time. Two reference models for wheel slip are employed to be followed by the ABS controller. In the first strategy conducted by most previous works, a constant slip value between 0.1-0.2 namely 0.15 is selected. In the second case, the controller follows a wheel slip reference model compatible with the tire/road conditions and the vehicle speed. This reference model considers the effect of tire normal load and road friction on the maximum value of braking force. The performances of ABS in two strategies are compared with each other on irregular roads in the presence of both passive and active suspensions. Also, the effect of ASS on the ABS performance with two strategies are investigated and the obtained results are discussed. The simulation studies are conducted using the 4DOF non-linear vehicle model excited by the standard good road profile according to ISO-8608.

2. Modeling

According to Fig. 1, a 4DOF non-linear quarter car model including longitudinal and vertical dynamics are employed to design the integrated control system.
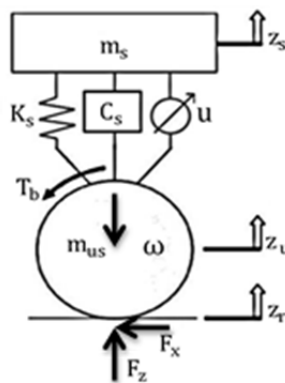

Fig. 1. 4DOF non-linear vehicle model

For the vehicle model, two degrees of freedom are related to the vertical motion of the sprung mass m_s and unsprung mass m_{us}. The other two degrees of freedom involving brake parts

include the wheel angular speed ω and the vehicle velocity V. The dynamic equations of the wheel are as follow:

$$\begin{bmatrix} \dot{V} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} -\dfrac{1}{M_t}(F_x) \\ -\dfrac{1}{V}\left[\dfrac{F_x}{M_t}(1-\lambda) + \dfrac{R^2 F_x}{I_t}\right] + (\dfrac{R}{VI_t})T_b \end{bmatrix} \tag{1}$$

where R is the wheel radius, I_t is the total moment of inertia of the wheel, M_t is the total mass of quarter vehicle, V is the vehicle speed, λ is the wheel longitudinal slip, F_x is the wheel longitudinal force and T_b is the wheel braking torque.

The longitudinal slip during the braking, $V > R\omega$, is calculated as:

$$\lambda = \frac{V - R\omega}{V} \tag{2}$$

For vertical dynamics, the state space form of governing equations is written as follow [2, 13]:

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} z_2 - z_4 \\ -\dfrac{1}{m_s}(f_s + f_d) - g + \dfrac{u}{m_s} \\ z_4 - \dot{z}_r \\ \dfrac{1}{m_{us}}(f_s + f_d - f_{st} - f_{dt}) - g - \dfrac{u}{m_{us}} \end{bmatrix} \tag{3}$$

where $z_1 = z_s - z_u$ is the suspension deflection or wheel travel, $z_2 = \dot{z}_s$ is the sprung mass vertical velocity, $z_3 = z_u - z_r$ is the tire deflection, $z_4 = \dot{z}_u$ is the tire vertical velocity, z_r is the road input, g is the gravitational acceleration and u is the active suspension force determined by the control law. Also, $f_s = K_s z_1$ and $f_d = C_s \dot{z}_1$ are the suspension spring and damper forces respectively by denoting K_s and C_s as the suspension stiffness and damping coefficients. Finally, $f_{st} = K_t z_3$ and $f_{dt} = C_t \dot{z}_3$ are the tire spring and damper forces in which K_t and C_t denote the tire stiffness and damping coefficients respectively.

In this study, the nonlinear Dugoff's tire model is used to describe the behavior of the tire because of its simplicity and its good fitness to experimental data [20]. In this model, the relation for longitudinal force of the tire is as follows:

$$F_x = \frac{C_i \lambda}{1 - \lambda} f \tag{4}$$

where

$$f(s) = \begin{cases} S(2 - S) & if \ \ S < 1 \\ 1 & if \ \ S \geq 1 \end{cases} \tag{5}$$

and

$$s = \frac{\mu F_z \left(1 - \ _r V\sqrt{\lambda^2 + tan^2\alpha}\right)(1 - \lambda)}{\sqrt{C_i^2 \lambda^2 + C_\alpha^2 tan^2\alpha}} \tag{6}$$

Here, μ is the road coefficient of friction and $_r$ is the reduction factor of road adhesion. Also, C_i and C_α are the longitudinal and cornering stiffness of the tire respectively. The Dugoff's model is based on the friction ellipse idea considering the saturation property of tire force.

It should be noted that the tire normal force F_z is the intersection point of the suspension and braking dynamics. The longitudinal braking force is directly dependent on the tire normal force according to Eqs. (4) to (6). Also, F_z is the sum of f_{st} and f_{dt} forces in Eq. (3). This means that the suspension control system is able to have influence on the tire longitudinal dynamics by controlling the tire normal force.

3. Controller design

In this section, two integrated controllers are individually designed for ABS and ASS based on the nonlinear dynamic model described in the previous section. After that, the coordination strategies for the two systems are discussed. A prediction method is employed for the designed controllers [1, 2]. In this method, concisely, the system response is first predicted for the next interval by Taylor series expansion and then the current control signal is calculated by minimizing the predicted error.

3.1. ABS controller design

In the anti-lock braking system, the wheel slip $\lambda(t)$ is controlled to track its desired response $\lambda_d(t)$. This control variable is considered as the output of the system $y_1 = \lambda$. Therefore, a performance index that penalizes the next instant tracking errors and the current control input is considered in the following form:

$$J_1 = \frac{1}{2}\rho_1 e^2(t + h_1) + \frac{1}{2}\rho_2 T_b^2 \tag{7}$$

where $e = \lambda - \lambda_d$ is the tracking error of the wheel slip, $\rho_1 > 0$ and $\rho_2 \geq 0$ are the weighting factors indicating the relative importance of the corresponding terms and h_1 is the predictive period.

The predicted response for the output in the next interval $t + h_1$ is approximated by a kth-order Taylor series expansion at t. The expansion order k is selected to be equal with the relative degree of the corresponding output in the non-linear system [1, 2, 19]. This selection leads to a small control effort and prevents the complexity of control law by eliminating the derivatives of the control input in the prediction. In this way, the control inputs will be constant in the prediction interval.

According to Eq. (1), the relative degree of λ is $r_1 = 1$ which is a well-defined number. This number is determined as the lowest order of the derivative of the system output in which the control input $T_b(t)$ first appears explicitly [21]. Therefore, the first-order Taylor series is sufficient for λ,

$$\lambda(t + h_1) = \lambda(t) + h_1\left[f_1(\lambda, V) + \frac{R}{VI_t}T_b\right] \tag{8}$$

where

$$f_1(\lambda, V) = -\frac{1}{V}\left[\frac{F_x}{M_t}(1 - \lambda) + \frac{R^2 F_x}{I_t}\right] \tag{9}$$

The control input T_b is derived by applying the optimality condition as follows.

$$\frac{\partial J_1}{\partial T_b} = 0 \tag{10}$$

This equation leads to,

$$T_b(t) = a[\rho_1 b\{e + h(f_2 - \dot{\lambda}_d)]] \tag{11}$$

where

$$a = -\frac{1}{\rho_1 b^2 + \rho_2}, \quad b = \frac{hR}{VI_t} \tag{12}$$

In the conventional ABS, the desired value for the longitudinal wheel slip λ_d is selected such that the maximum braking forces are achieved during braking. In most of the previous studies, a constant value, e.g. 0.15, has been used for the reference wheel slip [2, 5]. However, during the hard braking, to achieve the maximum braking force at each time, the optimum value of longitudinal slip is proposed to be tracked by the ABS controller. In this study, the optimum value of wheel slip $\lambda_{d\,opt}$ can be instantaneously calculated by differentiating the longitudinal force with respect to the wheel slip. Therefore, by using the Dugoff's tire model described by Equations (4) to (6), the optimum wheel slip can be obtained by online solving of the following algebraic equation:

$$\frac{\partial F_x}{\partial \lambda}\Big|_{\lambda=\lambda_d} = 0 \quad \Rightarrow \quad (2-S)(1-\varepsilon_r V\lambda) - (2-2S)(1-\varepsilon_r V\lambda^2) = 0 \tag{13}$$

Note that a simplified Dugoff's model is used in Eq. (13) with no slip angle. This is for the reason that the ABS controller will be active in the limit of $S < 1$ which describes the nonlinear behavior of the tire force. As an important feature of Eq. (13), the effect of vehicle speed, tire normal load and road friction coefficient is considered in calculating the optimum slip.

In the simulation studies of the present paper, both a constant slip value ($\lambda_{d\,cte} = 0.15$) and the online optimum slip value $\lambda_{d\,opt}$ calculated by Eq. (13) which is dependent on the tire vertical load and vehicle speed are employed to be tracked by the ABS controller. The online optimum wheel slip calculated by Eq. (13) for a wide range of speeds and normal forces are presented in Fig. 2. The effect of tracking the optimum longitudinal slip on the dependency of ASS and ABS will be investigated in simulation studies.
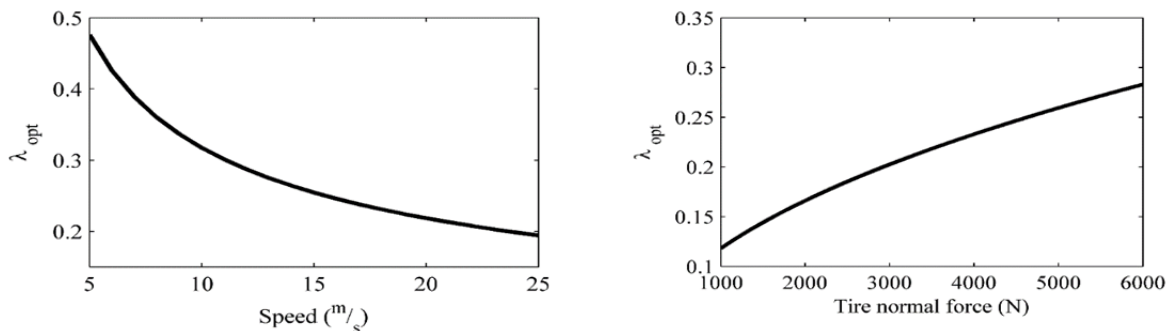
Fig. 2. Optimum slip value vs the speed and normal force (right: at V=25 m/s , left: at Fz=3500 N)

3.2. ASS controller design

The main aim of the ASS is to control the actual suspension responses such as sprung mass acceleration $\dot{z}_2$, suspension deflection z_1 and tire deflection z_3 close to their rest situation by using a minimum external control force u. To this aim, the prediction-based method is applied again for calculating the external control force. Note that no real active suspension can vanish all suspension responses simultaneously. However, by applying a tunable control law, an appropriate trade-off between opposite responses can be obtained.

According to the requirements of suspension control system, three control variables z_1, z_2 and z_3 are considered as the outputs of the system $y_2 = [z_1 \quad z_2 \quad z_3]$. Therefore, a performance index minimizing the next tracking errors and current control effort is defined as follows:

$$J_2 = \frac{1}{2}\sum_{i=2}^{3} \eta_i z_i^2(t + h_2) + \frac{1}{2}\eta_4 u^2(t) \tag{14}$$

where η_i $(i = 1,2,3) > 0$ and $\eta_4 \geq 0$ are weighting factors and h_2 is the predictive period. Again, the nonlinear response of each output is predicted by Taylor series expansion to obtain the performance index (14) as a function of current control input u. The expansion order is adopted to be equal with the relative degree of the corresponding output as mentioned before. According to Eq. (3), the system has the well-defined relative degree $r_2 = 1$ for z_2 and $r_3 = 2$ for both z_1 and z_3. Therefore, the second-order Taylor series for z_1 and the first-order Taylor series for z_2 and and z_3 will be sufficient. That is,

$$z_1(t + h_2) = z_1(t) + h_2(z_2 - z_4) + \frac{h_2^2}{2!}\left[g_1 - g_2 + u\left(\frac{1}{m_s} + \frac{1}{m_{us}}\right)\right] \tag{15}$$

$$z_2(t + h_2) = z_2(t) + h_2\left(g_1 + \frac{u}{m_s}\right) \tag{16}$$

$$z_3(t + h_2) = z_3(t) + h_2(z_4 - \dot{z}_r) + \frac{h_2^2}{2!}\left(g_2 - \frac{u}{m_{us}} - \ddot{z}_r\right) \tag{17}$$

where

$$g_1 = -\frac{1}{m_s}(f_s + f_d) - g, \qquad g_2 = \frac{1}{m_{us}}(f_s + f_d - f_{st} - f_{dt}) - g \tag{18}$$

The ASS control input u is derived by applying the optimality condition as follows:

$$\frac{\partial J_2}{\partial u} = 0 \tag{19}$$

which leads to:

$$u(t) = c\{\eta_1 d_1\left[z_1(t) + h_2(z_2 - z_4) + \frac{h_2^2}{2!}(g_1 - g_2)\right] + \eta_2 d_2[z_2(t) + h_2 g_1] + \eta_3 d_3[z_3(t) \tag{20}$$
$$+ h_2(z_4 - \dot{z}_r) + \frac{h_2^2}{2!}(g_2 - \ddot{z}_r)]$$

where

$$c = -\frac{1}{\eta_1 {d_1}^2 + \eta_2 {d_2}^2 + \eta_3 {d_3}^2 + \eta_4} \tag{21}$$

$$d_1 = \frac{h^2}{2!}\left(\frac{1}{m_s} + \frac{1}{m_{us}}\right), d_2 = \frac{h}{m_s}, d_3 = -\frac{h^2}{2m_{us}} \tag{22}$$

4. Simulation results

In this section, at first, the 4DOF non-linear quarter car model including longitudinal and vertical dynamics is verified with the results of references [13] and [19]. For validating the braking dynamics, the initial velocity of the vehicle is considered 30 m/s at the start of hard braking on flat road with the friction coefficient 0.85. In this condition, the stopping distance for the model in the case of without braking control is calculated as 85 m which is consistent with the result of [13] for the same parameters. On the other hand, the suspension dynamics is validated with the results of reference [19] for the road excited by two bumps. Fig. 3 shows the comparative body acceleration results for the same conditions.
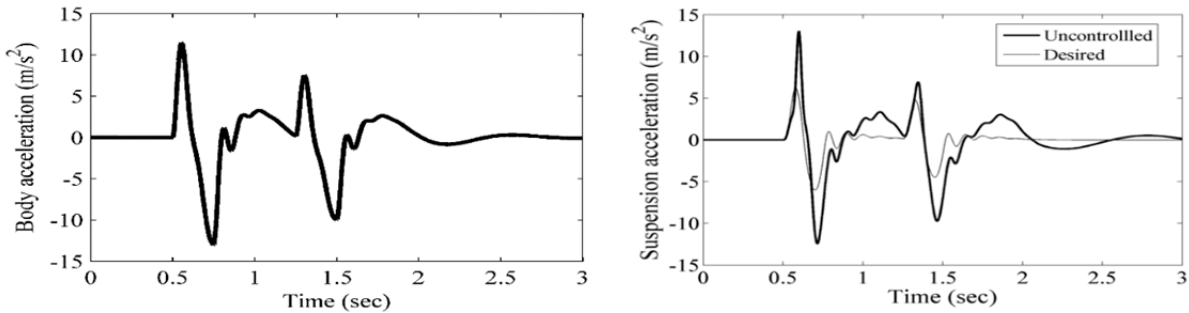

Fig. 3. Comparison between body or suspension acceleration response of passive suspension for two bumps excitation (Left: present study, Right: reference [19])

Now, simulation results of the non-linear 4DOF vehicle model are presented to show the effectiveness of the proposed integrated control of ABS and ASS in different strategies. For the simulation study, the initial velocity of the vehicle is considered 20 m/s at the start of hard braking. Since a flat road is not able to excite the vertical dynamics of quarter car model, an irregular ISO 8608 standard C quality road [22] as shown in Fig. 4 is proposed to simulate the vertical dynamics of the vehicle. Different strategies for integrating braking and suspension systems in the case of with and without control are discussed. For each case, the vehicle stopping distance together with ride comfort indexes are reported and compared.
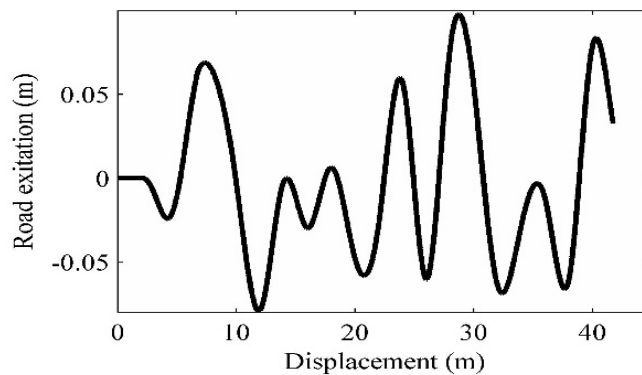

Fig. 4. Road excitation spectrum

The first simulation results refer to manual braking with passive suspension system. This simulation serves as the base result to be compared with other strategies. Fig. 5 shows that the vehicle is stopped in 48.25 m by manual hard braking within 3.75 seconds. Fig. 6 indicates the

locking of the wheel for the case without ABS. In this case, the wheel angular speed becomes zero within a short period of time while the linear speed of the tire center V is not zero. This means that the tire is locked and the longitudinal slip is reached to its maximum value (100%).
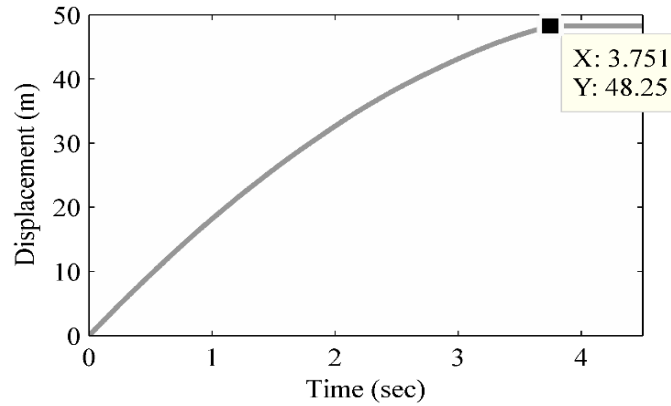
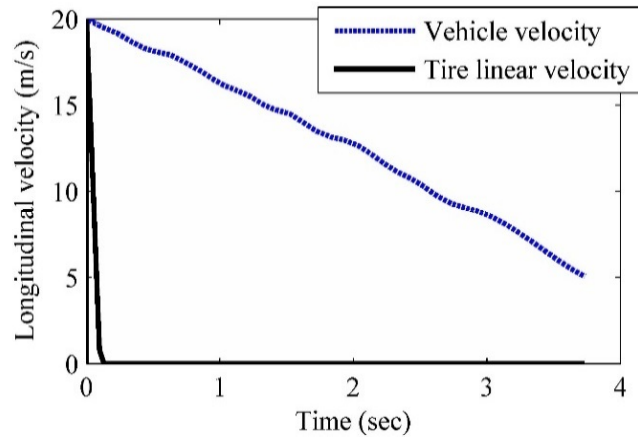**Fig. 5.** Stopping distance in manual braking

Fig. 6. Vehicle and tire linear velocity in manual braking

On the other hand, Fig. 7 shows the responses of passive suspension system on the irregular road profile. As shown in Fig. 7(a), the body acceleration magnitude reaches to 7 m/s^2 which should be reduced for ride comfort. The root mean square of body acceleration is calculated as 3.41 m/s^2 for the passive suspension system. The variation of wheel travel is shown in Fig. 7(b). Also, the tire deflection variation is presented in Fig. 7(c). Fig. 7(d) shows that the tire normal force which has a direct impact on tire longitudinal force has changed severely but always has positive value which guarantees the contact between the tire and the road until the vehicle is stopped.
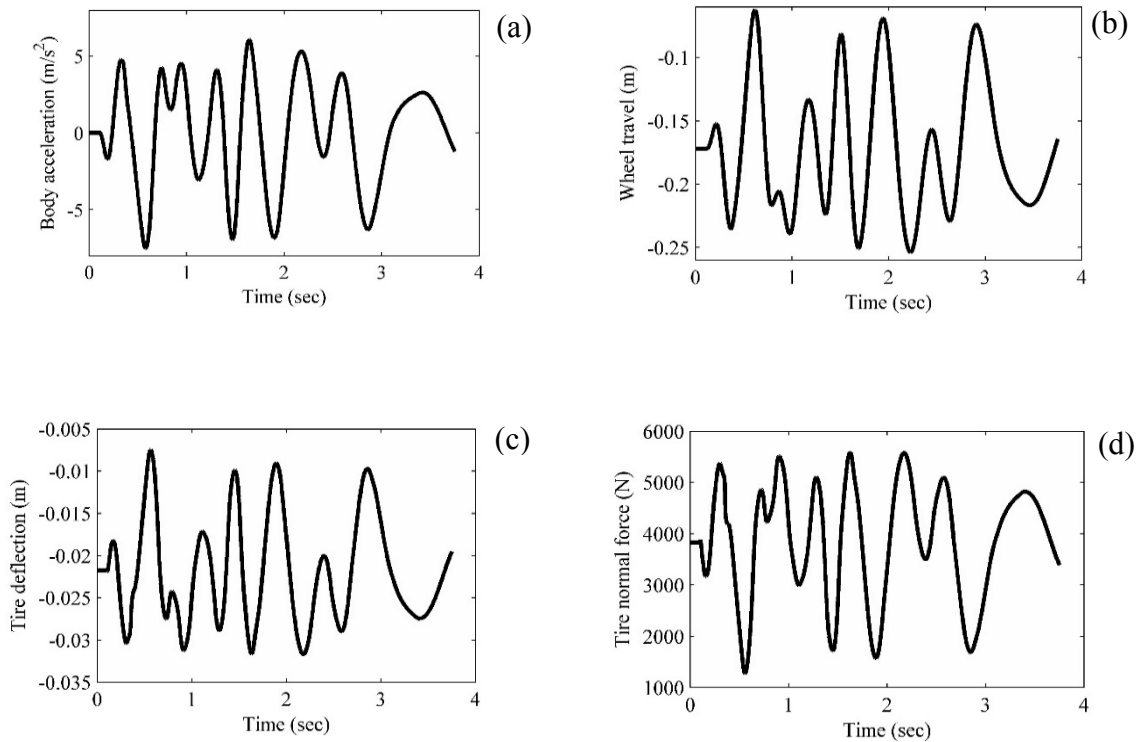
Fig. 7. Responses of passive suspension system, (a) Body acceleration, (b) Wheel travel,
(c) Tire deflection and (d) Tire normal force

For the next simulation, it is supposed that the stand-alone braking control is activated by ABS in the presence of passive suspension system. In this case, two strategies of ABS are employed. In the first strategy, a constant value of wheel slip namely $\lambda_{d\ cte} = 0.15$ is selected as a reference value to be tracked by the ABS controller. However, the controller of second strategy tracks the optimum wheel slip varied by road condition. Fig. 8(a) illustrates the variation of the optimum slip derived by Eq. (13) in the presence of varying normal load. Tracking this variable reference model leads to the maximum brake force applied to the wheel. This fact is illustrated by comparing the responses of the manual braking and the ABS with two strategies (Fig. 8). According to the results, the stopping distance for the manual braking is calculated as 48.25 m while for the ABS is 42.21 m and 41.71 m respectively by tracking the constant and varied wheel slip as the reference model. The above results clearly show that the stopping distance has been decreased by a greater value if the ABS tracks the optimum wheel slip varied with road conditions. As a result, the ABS performance is improved by generating the maximum braking force. The braking torques are also shown in Fig. 8(c). Normal force variations cause the variations of braking torques.
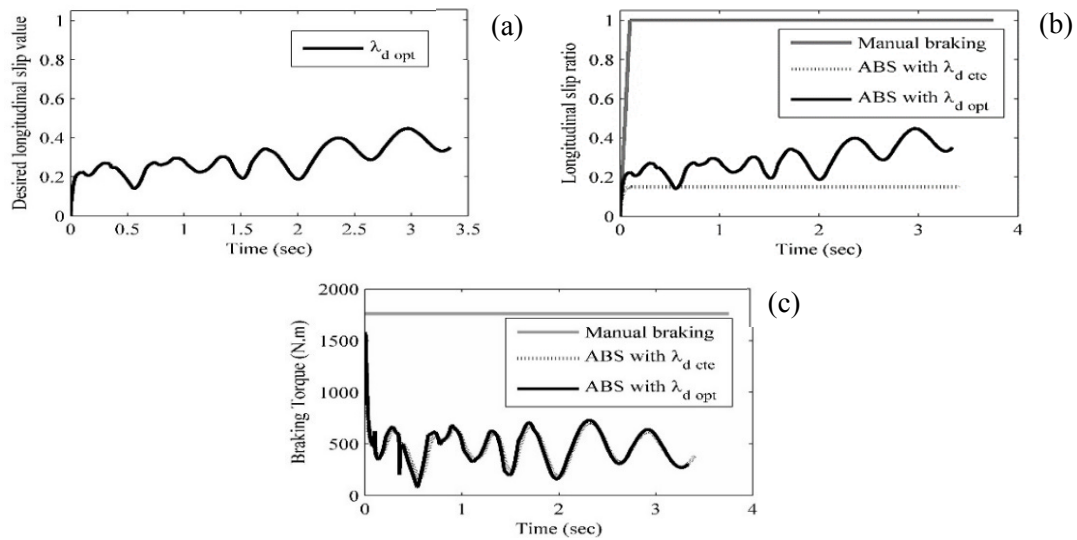
Fig. 8. Comparison of the ABS controller performance in tracking the two different reference models with passive suspension: (a) variable optimum wheel slip (b) longitudinal slip (c) braking torque.

For all previous results, the ASS was inactive. By activation of this control system, the effect of suspension control on the ABS performance and overall vehicle behavior is investigated. At first, the responses of vehicle suspension system in the case of with and without control are compared in Fig. 9. As it is seen, the ASS controller reduces the normal force and the tire deflection variations significantly. Also, the ASS control is able to reduce the body acceleration which improves the vehicle ride performance. The reduction of tire deflection variation can have positive effects on ABS performance. To show these important results and in order to have a comprehensive comparison of different strategies in the cases of stand-alone and integrated control systems, the root mean square (RMS) of suspension system outputs together with vehicle stopping distance are reported in Table 1.
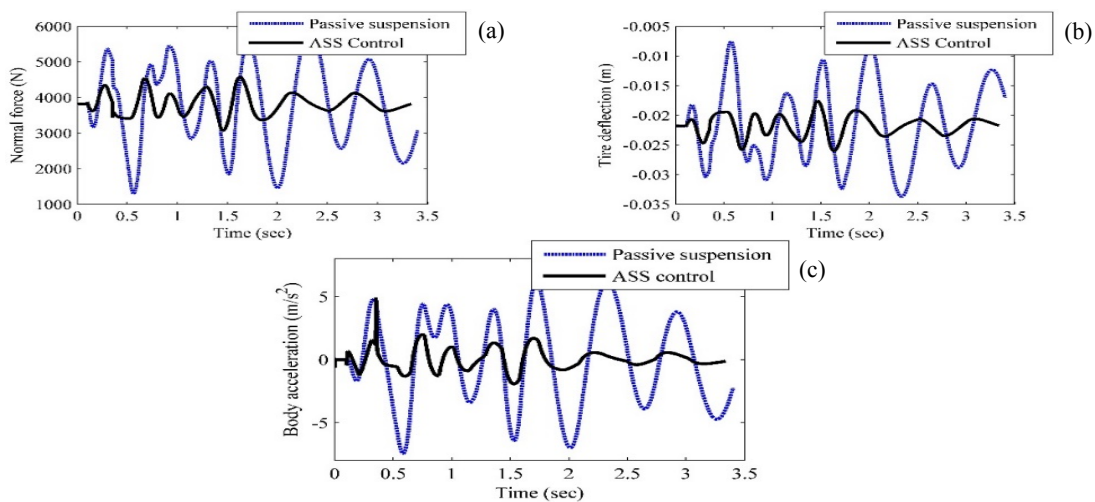

Fig. 9. Comparison of the responses of active and passive suspension systems: (a) normal force (b) tire deflection (c) body acceleration

Table 1. Stopping distance and RMS of suspension outputs in different strategies

Braking mode	Reference slip value	ASS mode	RMS of wheel travel (mm)	RMS of Tire deflection (mm)	RMS of body acceleration (m/s^2)	Stopping distance (m)	Stopping distance decrement (m)
manual	-	Passive	180	6.30	3.41	48.25	0
ABS	$\lambda_{d\,cte}$	Passive	178	6.82	3.67	42.21	6.04
ABS	$\lambda_{d\,cte}$	Active	182	1.72	0.57	41.81	6.44
ABS	$\lambda_{d\,opt}$	Passive	180	6.84	3.69	41.71	6.54
ABS	$\lambda_{d\,opt}$	Active	182	1.72	0.57	41.58	6.67

The results of Table 1 show that the ASS controller decreases the body acceleration and tire deflection variations about 84% and 75%, respectively. Also, it is shown in the results that the ASS performance is not affected by different ABS strategies. In fact, the braking strategies in tracking two reference models cannot have influence on the tire deflection, wheel travel and normal force variations. In contrast, the ASS activation improves the ABS performance and reduces the stopping distance.

From the results of Table 1, the performance of ABS in achievement of a short stopping distance is improved in two ways. One way is to track the optimal longitudinal slip varied with tire normal force and vehicle speed instead of a constant value and the second way is to integrate the ABS with ASS as an assistant control system. It is considered that the first way is most effective on the safety performance so that the stopping distance in this way is more decreased rather than the activation of ASS. Therefore, when the safety would be the only criterion for vehicle control, tracking the optimal reference slip by ABS can be enough to achieve a short stopping distance even in the presence of passive suspension. This strategy decreases the stopping distance about 50 cm which is obtained by comparing second and fourth rows of Table 1. When the ABS tracks the optimal reference slip, application of ASS decreases the stopping distance about 13 cm by comparing the two last rows of Table 1. This reduction may not be remarkable for vehicle safety but when the ride comfort as a second priority of vehicle control is considered, application of ASS can be very useful. The results show that the body acceleration is greatly reduced by ASS which indicates the ride comfort improvement. As a conclusion, when the ASS is integrated with the second strategy of ABS which tracks the optimal slip varied with tire normal force and vehicle speed, overall ride and safety performances are greatly improved during hard braking on a good road spectrum.

5. Conclusion

In this paper, a decentralized integrated controller is developed to control both ASS and ABS with different strategies. In this way, two optimal control laws for ASS and ABS are individually designed based on a 4DOF non-linear quarter-car model. The results show that the ASS not only improves the ride comfort remarkably by decreasing the body acceleration, but it also assists the ABS in reducing the stopping distance. In a new strategy, the ABS performance is improved by

tracking the optimal slip compatible with tire/road conditions and vehicle speed. In this way, the dependency of ABS to ASS is decreased. However, as an important result, when the ASS is integrated with the proposed strategy of ABS which tracks the optimal slip varied with tire normal force and vehicle speed, overall ride and safety performances are improved simultaneously during hard braking on a good road spectrum.

References

[1] A. Malekshahi, M. Mirzaei, S. Aghasizade, Non-linear predictive control of multi-input multi-output vehicle suspension system, Journal of Low Frequency Noise, Vibration and Active Control, 34 (2015) 87-105.

[2] H. Mirzaeinejad, M. Mirzaei, A novel method for non-linear control of wheel slip in anti-lock braking systems, Control Engineering Practice, 18 (2010) 918-926.

[3] R. Verma, D. Ginoya, P.D. Shendge, S.B. Phadke, Slip regulation for anti-lock braking systems using multiple surface sliding controller combined with inertial delay control, Vehicle System Dynamics, 53 (2015) 1150-1171.

[4] A. Okyay, E. Cigeroglu, S.Ç. Başlamışlı, A new sliding-mode controller design methodology with derivative switching function for anti-lock brake system, Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, 227 (2013) 2487-2503.

[5] S. Anwar, B. Ashrafi, A predictive control algorithm for an anti-lock braking system, in, SAE Technical Paper, 2002.

[6] H. Mirzaeinejad, M. Mirzaei, A new approach for modelling and control of two-wheel anti-lock brake systems, Proceedings of the Institution of Mechanical Engineers, Part K: Journal of Multi-body Dynamics, 225 (2011) 179-192.

[7] M. Mirzaei, H. Mirzaeinejad, Optimal design of a non-linear controller for anti-lock braking system, Transportation Research Part C: Emerging Technologies, 24 (2012) 19-35.

[8] G.F. Mauer, A fuzzy logic controller for an ABS braking system, IEEE Transactions on Fuzzy Systems, 3 (1995) 381-388.

[9] R. Safvat, M. Mirzaei, S. Aghasizade, H. Mirzaeinejad, Optimal control of nonlinear vehicle suspension system for improvement of the ABS performance, in: 12th International Symposium on Advanced Vehicle Control, Tokyo, Japan, 2014.

[10] C. Zhengke, The research of vehicle's ride comfort in the nonlinear suspension system, in: 5th International Conference on Education, Management, Information and Medicine (EMIM), Shenyang, China, 2015.

[11] E. Guglielmino, T. Sireteanu, C.W. Stammers, G. Ghita, M. Giuclea, Semi-active suspension control: improved vehicle ride and road friendliness, Springer Science & Business Media, 2008.

[12] P. Gáspár, Design of integrated control for road vehicles, in: Robust Control and Linear Parameter Varying Approaches, Springer Berlin Heidelberg, 2013, pp. 213-235.

[13] J.S. Lin, W.E. Ting, Nonlinear control design of anti-lock braking systems with assistance of active suspension, IET Control Theory & Applications, 1 (2007) 343-348.

[14] W.Y. Wang, M.C. Chen, S.F. Su, Hierarchical T–S fuzzy-neural control of anti-lock braking system and active suspension in a vehicle, Automatica, 48 (2012) 1698-1706.

[15] S.B. Lu, Y.N. Li, S.B. Choi, L. Zheng, M.S. Seong, Integrated control on MR vehicle suspension system associated with braking and steering control, Vehicle System Dynamics, 49 (2011) 361-380.

[16] C. Poussot-Vassal, O. Sename, L. Dugard, P. Gaspar, Z. Szabo, J. Bokor, Attitude and handling improvements through gain-scheduled suspensions and brakes control, Control Engineering Practice, 19 (2011) 252-263.

[17] M. Valasek, O. Vaculin, J. Kejval, Global chassis control: integration synergy of brake and suspension control for active safety, in: 7th International Symposium on Advanced Vehicle Control (AVEC), HAN University, Arnhem, Netherlands, 2004, pp. 495-500.

[18] O. Vaculín, J. Svoboda, M. Valášek, P. Steinbauer, Influence of deteriorated suspension components on ABS braking, Vehicle System Dynamics, 46 (2008) 969-979.

[19] A. Malekshahi, M. Mirzaei, Designing a non-linear tracking controller for vehicle active suspension systems using an optimization process, International Journal of Automotive Technology, 13 (2012) 263-271.

[20] D.E. Smith, J.M. Starkey, Effects of model complexity on the performance of automated vehicle steering controllers: Model development, validation and comparison, Vehicle System Dynamics: International Journal of Vehicle Mechanics And Mobility, 24 (1995) 163-181.

[21] J.J.E. Slotine, W. Li, Applied nonlinear control, Prentice-Hall Englewood Cliffs, NJ, 1991.

[22] M. Agostinacchio, D. Ciampa, S. Olita, The vibrations induced by surface irregularities in road pavements–a Matlab® approach, European Transport Research Review, 6 (2014) 267-275.