Development of a model for predicting dynamic response of a sphere at viscoelastic interface: A dynamic Hertz model

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Abstract. A model for predicting the dynamic response of a sphere at viscoelastic interface is presented. The model is based on Hertz contact model and the model for a sphere in a medium. In addition to the elastic properties of medium and the size of sphere, the model considers the density of sphere, the density and viscosity of medium, and damping of oscillations of sphere due to radiation of shear waves. The model can predict not only the effects of the mechanical properties of medium, the physical properties of sphere, and the amplitude of excitation force on sphere displacement, but also the effects of these parameters on shift of resonance frequency. The proposed model can be used to identify the elastic and damping properties of materials, and to understand the dynamic responses of spherical objects at viscoelastic interfaces in practical applications.

1. Introduction

Spherical objects, such as bubbles and spheres, embedded in mediums and at viscoelastic interfaces are encountered in many applications [1-3]. Spherical objects embedded in a material have been used to determine material properties, such as the elasticity modulus [1,4]. However, in practice, for example, in therapeutic and diagnostic ultrasound applications that use microbubbles [5] and in atomic force microscopy or indentation tests that use spheres [6-8], the spherical objects are at viscoelastic interfaces.

More recently, the use of microbubbles in a fluid to push against tissue under ultrasound exposure was proposed to improve the contrast and spatial resolution of elasticity imaging [9]. In addition, mathematical models for the displacements of a bubble located at medium interfaces in response to external loads were proposed [10,11] and these models were evaluated experimentally [12]. The Hertz model, or modified Hertz models, are widely used in various applications, including atomic force microscopy and nanoindentation, to predict the responses of spheres at medium interfaces under external loads [6,8,13]. However, the models in the literature for material identification are based on some fictive arrangements of springs, do not take into account the effect of the sphere mass for dynamic loading and radiation damping, or require the use of a variety of parameters including relaxation times under constant load and deformation [14,15]. Different models have been proposed to investigate the nonlinear vibrations of Hertzian contact, however, they are based on a mass-spring-damper system [3,16]. There are some models for studying dynamic-contact stiffness at the interface between a vibrating rigid sphere and a semi-infinite viscoelastic solid [17]. Overall, most studies in the literature focus on the steady-state deformation of the sphere or relaxation [18,19]. There are some studies on the dynamic response analysis of a spherical subject impacting a metal plate [20]. It should be noted that different approaches are used to handle the stiffness of the system consisting a sphere in contact with an elastic body [21]. There are some studies in which the oscillatory response is investigated, however, the sphere is attached to an auxiliary system such as a pendulum in these studies [22,23]. A practical model to analyse the dynamic response of a sphere at viscoelastic interface, considering wave propagation using a continuum linear-viscoelastic medium and the Hertz contact model, will be invaluable, especially for material identification purposes.

In this paper, a model for the dynamic response of a sphere at viscoelastic interface exposed to an external dynamic load is presented. The model is based on the Hertz contact model [13] and the model for the sphere embedded in a medium [24-25]. In addition to the elastic properties of medium and the size of sphere, the model takes into account the density and viscosity of medium and the mass of sphere. In the model, the damping of the oscillations of the sphere due to the radiation of shear waves is taken into account. The proposed model was assessed using the model of the sphere embedded in a medium. The results show that the model can predict not only the effects of the mechanical properties of medium, the physical properties of sphere and the amplitude of the excitation force on the displacement of the sphere, but also the effects of these parameters on the shift of the resonance frequency. The model proposed in this study can be used to identify the elastic and damping properties of materials, and to understand the dynamic responses of spherical objects at viscoelastic interfaces in practical applications

2. Mathematical model

The spheres embedded in a viscoelastic medium and at a viscoelastic interface are shown in Figure 1. The model for the dynamic response of the sphere at the viscoelastic interface here was developed based on the Hertz contact model [13] and the model for a sphere embedded in a medium [24,25]. It is assumed that the sphere is always in contact with the medium.

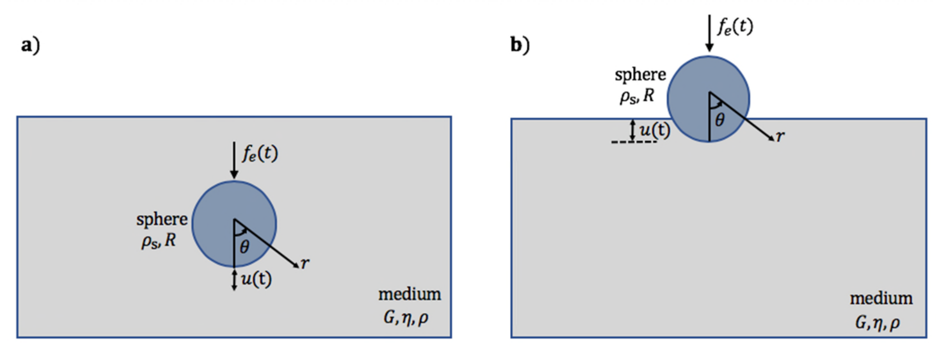

Figure 1. The spheres embedded in a viscoelastic medium (a) and at a viscoelastic interface (b).

The frequency-domain equation that couples the displacement of the sphere embedded in an elastic medium for a dynamic external force is as follows [24]:

$$F_e = -m_s\omega^2 U + 6\pi GRU\left(1 - jkR - \frac{1}{9}k^2R^2\right) \quad (1)$$

where m_s and R are the mass and radius of sphere, G is the shear modulus of medium, $k = \frac{\omega}{\sqrt{G/\rho}}$ is the wave number of the shear wave with the frequency ω, ρ is the density of medium, $j = \sqrt{-1}$ and F_e and U are the Fourier transforms of the external force $f_e(t)$ and displacement $u(t)$, respectively. The mass of a sphere is given by $m_s = \frac{4}{3}\pi R^3 \rho_s$ where ρ_s is the density of the sphere. Aglyamov et al. [25]

considered the external force as a rectangular pulse with the amplitude of f_0 and a duration of τ (i.e., the constant force f_0 is applied for a short time τ and then removed). It should be noted that the response to the rectangular pulse simulates the impulse response for small τ values and the step response for large τ values with $0 \leq t \leq \tau$. It can be calculated that the Fourier transform of the rectangular pulse is $F_e = -\frac{jf_0}{\omega}\left(e^{j\omega\tau} - 1\right)$. Aglyamov et al. [25] replaced the shear modulus G with $(G - j\omega\eta)$ in equation (1), and hence they obtained the equation that couples the displacement of the sphere in a viscoelastic medium for a dynamic external force as follows:

$$U = \frac{(-jf_0/\omega)(e^{j\omega\tau}-1)}{6\pi(G-j\omega\eta)R\left[1-jkR-\frac{1}{9}(1+2\beta)k^2R^2\right]} \tag{2}$$

where $\beta = \rho_s/\rho$. By using the inverse Fourier transform, $u(t) = \mathcal{F}^{-1}\{U\}$, the time-domain response of the sphere in the viscoelastic medium can be found as [25]:

$$u(t) = -\frac{jf_0}{12\pi^2 R}\int_{-\infty}^{\infty}\frac{(e^{j\omega\tau}-1)e^{-j\omega t}}{\omega(G-j\omega\eta)\left(1-jkR-\frac{1}{9}(1+2\beta)k^2R^2\right)}\mathrm{d}\omega \tag{3}$$

Using $k^2 = \frac{\omega^2}{G/\rho}$, the last term in equation (1) becomes $-6\pi GRU\frac{1}{9}k^2R^2 = -\frac{2}{3}\pi\rho R^3\omega^2 U$. Hence, equation (1) can be written as follows:

$$F_e = \frac{4}{3}\pi R^3\rho_s(-\omega^2 U) + \frac{2}{3}\pi R^3\rho(-\omega^2 U) + 6\pi GRU - 6\pi GRUj\omega\sqrt{\frac{\rho}{G}}R \tag{4}$$

The first term, $\frac{4}{3}\pi R^3\rho_s(-\omega^2 U)$, in equation (4) or the corresponding term $\frac{4}{3}\pi R^3\rho_s\ddot{u}$ in the time domain shows the inertia force due to the mass of the sphere. It should be noted that the inertia force due to the mass of the sphere at the viscoelastic medium will be the same as the one for the sphere in a viscoelastic medium, i.e., $F_{\mathrm{i,sphere}} = \frac{4}{3}\pi R^3\rho_s(-\omega^2 U)$. The second term, $\frac{2}{3}\pi R^3\rho(-\omega^2 U)$, in equation (4) or the corresponding term $\frac{2}{3}\pi R^3\rho\ddot{u}$ in the time domain is the inertia force due to the mass of the elastic medium involved in motion ($m_a = \frac{2}{3}\pi R^3\rho$ is the induced mass). It is noted that, depending on some other parameters including the submergence depth, the added mass can be much less than the mass of the sphere [26]. Therefore, based on the results in the literature [27,28], it is assumed that the inertia force due to the mass of the elastic medium involved in motion for the sphere at the medium interface is half of that of the sphere embedded in a medium, i.e., $F_{\mathrm{i,medium}} = \frac{1}{3}\pi R^3\rho(-\omega^2 U)$. Meanwhile, it can be shown that the effect of the induced mass is low and the mass of the sphere mainly dominates inertia forces, even for a sphere embedded in a medium. The m_s/m_a ratio is more than 15 for practical applications (i.e., for the steel sphere with $\rho_s = 7800$ kg/m^3 embedded in the gel with $\rho = 1000$ kg/m^3).

The third term, $6\pi GRU$, in equation (4) is the equivalent force related to the stiffness of the system. The fourth term in equation (4), being the product of the equivalent force related to the stiffness of the system and the term $-j\omega\sqrt{\frac{\rho}{G}}R$, denotes the damping of the oscillations of the sphere due to the radiation of shear waves. As can be seen, the damping due to shear wave propagation for the sphere embedded in a medium depends on the stiffness of the system, the displacement induced, the frequency (or wave number), and the sphere radius. The same parameters should also affect the damping term due to shear wave propagation for the sphere at a medium interface. Therefore, firstly, the correct stiffness term should be determined for the model of the sphere at a medium interface.

The Hertz theory describes the contact mechanics between the elastic sphere and an elastic half-space. In the Hertz model, the displacement (y) induced by the elastic sphere is correlated with the external force (f) with $f=\frac{4E^*\sqrt{R}}{3}y^{3/2}$ where E^* is the reduced Young's modulus, computed as $1/E^*=\left(1-\nu_{\text{sphere}}^2\right)/E_{\text{sphere}}+(1-\nu^2)/E$ and R is the relative radius [13]. It is known that the Hertz model accurately predicts the displacement of the sphere at an elastic half-space under static loading. Therefore, the displacement-force relationship for the sphere at a medium interface can be taken from Hertz theory. As opposed to the constant stiffness component for the sphere embedded in a medium, the stiffness component for the Hertz model is a function of applied force or induced displacement. Using the Hertz model, we can determine the stiffness as a function of sphere displacement as $k=\frac{df}{dy}=1.5\left(\frac{4E^*\sqrt{R}}{3}\right)y^{1/2}$. As it is well known, the stiffness changes from zero (at the beginning of the deformation) to the maximum value $k=1.5\left(\frac{4E^*\sqrt{R}}{3}\right)u^{1/2}$ for a displacement of u. The use of this stiffness and the aforementioned inertia force provides that the frequency of oscillation of the sphere at the viscoelastic interface is kept in accordance with Hertz theory which produces accurate estimations. In this study, such a model for the sphere at the viscoelastic interface is developed so that, not only the frequency of oscillations, but also the steady-state displacement of the sphere is the same as that of the Hertz-model; note that the accuracy of the Hertz model has already been validated. For this purpose, the effective stiffness of the sphere at an interface is defined as $k_1=k+\alpha$ where α is a constant stiffness component. Hence, for the force of amplitude f producing a steady-state displacement of u, we should have $f=k_1u=(k+\alpha)u=\left[1.5\left(\frac{4E^*\sqrt{R}}{3}\right)u^{1/2}+\alpha\right]u$. From this, we find $\alpha=-0.5\frac{f}{u}$, hence, we have $k_1=1.5\left(\frac{4E^*\sqrt{R}}{3}\right)u^{1/2}-0.5\frac{f}{u}$. As we can write $f^{1/3}=\left(\frac{4E^*\sqrt{R}}{3}\right)^{1/3}u^{1/2}$, it becomes $k_1=1.5\left(\frac{4E^*\sqrt{R}}{3}\right)^{2/3}f^{1/3}-0.5\frac{f}{u}$. Now, we can write the equation of motion for the sphere at an interface in time domain as:

$$f=\left(\frac{1}{3}\pi R^3\rho+\frac{4}{3}\pi R^3\rho_{\text{s}}\right)\ddot{u}+\left[1.5\left(\frac{4E^*\sqrt{R}}{3}\right)^{2/3}f^{1/3}-0.5\frac{f}{u}\right]u+f_{\text{r}} \quad (5)$$

where f_r is the force component related to the damping of the oscillations of the sphere due to the radiation of shear waves for the sphere at the viscoelastic medium interface. Equation (5) can be arranged as:

$$1.5f=\frac{1}{3}\pi R^3(4\rho_{\text{s}}+\rho)\ddot{u}+1.5\left(\frac{4E^*\sqrt{R}}{3}\right)^{2/3}f^{1/3}u+f_{\text{r}} \quad (6)$$

Equation (6) in frequency domain can be written as:

$$1.5(-jf_0/\omega)\left(e^{j\omega\tau}-1\right)=\frac{1}{3}\pi R^3(4\rho_{\text{s}}+\rho)(-\omega^2U)+1.5f_0^{1/3}\left(\frac{4E^*\sqrt{R}}{3}\right)^{2/3}U+F_{\text{r}} \quad (7)$$

It should be noted that, for the sphere at a viscoelastic interface, as the upper half-space does not contribute to the radiation of shear waves, the damping force for this case will be half of that of the sphere embedded in a medium. Hence, the force component related to the damping of the oscillations of the sphere due to the radiation of shear waves for the sphere at the viscoelastic medium interface can be obtained as $F_{\text{r}}=\frac{1}{2}\left(-j\omega\sqrt{\frac{\rho}{G}}R\right)1.5f_0^{1/3}\left(\frac{4E^*\sqrt{R}}{3}\right)^{2/3}U$. Now, the equation for the sphere at an elastic interface in frequency domain is obtained as follows:

$$1.5(-jf_0/\omega)\left(e^{j\omega\tau}-1\right)=\frac{1}{3}\pi R^3(4\rho_{\text{s}}+\rho)(-\omega^2U)+1.5f_0^{1/3}\left(\frac{4E^*\sqrt{R}}{3}\right)^{2/3}\left(1-\frac{1}{2}j\omega\sqrt{\frac{\rho}{G}}R\right)U \quad (8)$$

It should be noted that, the reduced Young's modulus becomes $E^*=E/(1-\nu^2)$ when the sphere is not deformable. The elasticity modulus is related to the shear modulus by $E=2G(1+\nu)$ for homogeneous isotropic materials. Hence, we obtain $E^*=2G(1+\nu)/(1-\nu^2)$ for a homogeneous isotropic material and a rigid sphere. Considering the effect of the viscosity of the medium (η), that is, by replacing G with $(G - j\omega\eta)$, as similar to other studies [24,25], equation (8) can be rewritten as follows:

$$(-jf_0/\omega)\left(e^{j\omega\tau}-1\right) = \frac{2}{9}\pi R^3(4\rho_s+\rho)(-\omega^2 U) + f_0^{1/3}\left[\frac{8(G-j\eta\omega)(1+\nu)\sqrt{R}}{3(1-\nu^2)}\right]^{2/3}\left(1-\frac{1}{2}j\omega\sqrt{\frac{\rho}{G-j\eta\omega}}R\right)U \quad (9)$$

Hence, we obtained a very useful expression that contains the correct displacement-force relationship for the sphere at a medium interface for dynamic loading as well as the damping term for the oscillations of the sphere at an interface due to the radiation of shear waves. Overall, the dynamic response of the sphere at the viscoelastic interface is obtained as follows:

$$U = \frac{(-jf_0/\omega)\left(e^{j\omega\tau}-1\right)}{-\frac{2}{9}\pi R^3(4\rho_s+\rho)\omega^2 + f_0^{1/3}\left[\frac{8(G-j\eta\omega)(1+\nu)\sqrt{R}}{3(1-\nu^2)}\right]^{2/3}\left(1-\frac{1}{2}j\omega\sqrt{\frac{\rho}{G-j\eta\omega}}R\right)} \quad (10)$$

As explained before, the time-domain response of the sphere at the viscoelastic interface can be found by the inverse Fourier transform as:

$$u(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{(-jf_0/\omega)\left(e^{j\omega\tau}-1\right)e^{-j\omega t}}{-\frac{2}{9}\pi R^3(4\rho_s+\rho)\omega^2 + f_0^{1/3}\left[\frac{8(G-j\omega\eta)(1+\nu)\sqrt{R}}{3(1-\nu^2)}\right]^{2/3}\left(1-\frac{1}{2}j\omega\sqrt{\frac{\rho}{G-j\omega\eta}}R\right)}\mathrm{d}\omega \quad (11)$$

3. Results and discussion

The excitation duration τ was divided into *N* (e.g., 1000) points and the calculations were repeated over the entire time period of interest using the Matlab software (Mathworks, Natick, MA). The displacements of the sphere at the viscoelastic medium interface for different force levels and the excitation time of $\tau = 0.67$ ms are shown in Figure 2a. Here, the properties of medium are $G = 1300$ Pa, $\rho = 1000$ kg/m^3, $\eta = 0.1$ Pa s and $\nu = 0.45$ and the radius and density of sphere are $R = 0.5$ mm and ρ_s = 3980 kg/m^3. In addition, the displacements of the same sphere embedded in the same viscoelastic medium are shown in Figure 2b. In addition to the time-domain data, the spectrums for both cases are shown in Figures 2c and d. As expected, the sphere displacement increases as the force level increases for both models. It is clearly seen that the frequency (or period) of oscillations depends on the amplitude of the excitation force for the model of the sphere at the viscoelastic medium interface; the frequencies of oscillations are 120, 170 and 240 Hz for $f_0 = 0.023$, 0.23 and 2.3 mN, respectively. These results show that the proposed model has the ability to estimate the stiffness change due to the amplitude of the excitation force. As the amplitude of the excitation force increases, the stiffness of the system increases, therefore the frequency of oscillations increases. On the other hand, the frequency of oscillations does not change with force level for the model of the sphere in a medium, since the system's stiffness parameters do not change with the force level for this model. It is seen that there is a peak at 190 Hz for the model of the sphere in a medium for all force levels. Energy concentrates around natural frequencies for all force levels and, as the force level increases, the energy concentration moves to higher frequencies for the interface model. On the other hand, although the energy level increases with force amplitude, its distribution as a function of frequency is the same for all force levels for the embedded model.

The displacements of the sphere for the interface and embedded models and their spectrums for different densities of sphere are shown in Figure 3. It is seen that the frequency of oscillations decreases with the increase of the sphere density for the interface model; the frequencies of oscillations are 310, 240 and 180 Hz for $\rho_s = 1990, 3980$ and 7960 kg/m^3, respectively. The amplitude of the resonance peak decreases and the peak becomes wider as the sphere density decreases for the interface model. It should be noted that, for a viscously damped system, the damping ratio is expressed by $\zeta = \frac{c}{c_{\rm cr}} = \frac{c}{2\sqrt{km}}$. Therefore, the system's damping ratio increases as the density of the sphere (or the effective mass of the system) decreases and this results in a decrease in the displacement amplitude as seen in Figure 3, though the medium viscosity is the same for all three curves in Figure 3.

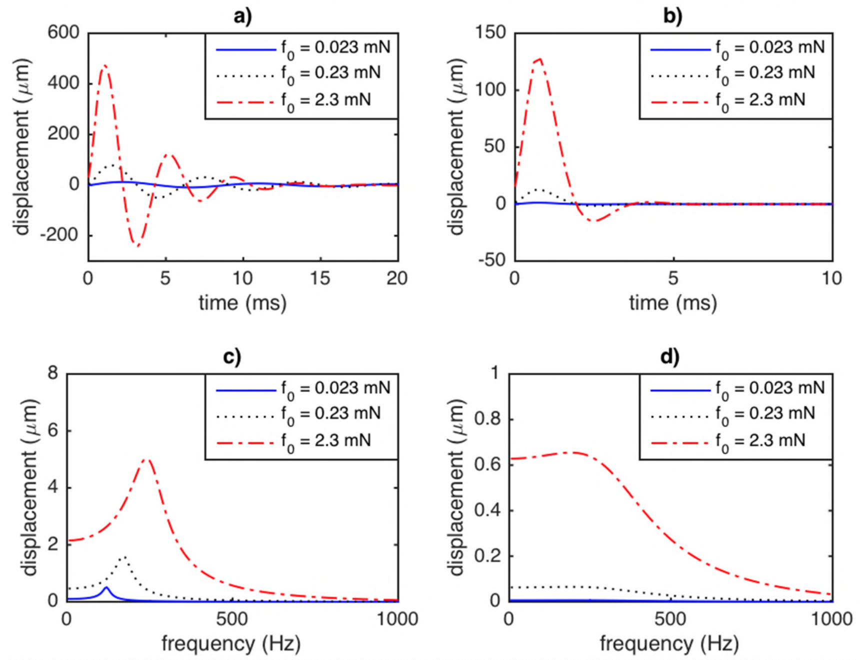

Figure 2. Sphere displacements predicted using the interface (a) and embedded (b) models for different force levels and the excitation time of $\tau = 0.67$ ms and the spectrums obtained using the interface (c) and embedded (d) models ($G = 1300$ Pa, $\rho = 1000$ kg/m^3, $\eta = 0.1$ Pa s, $\upsilon = 0.45$, $R = 0.5$ mm, $\rho_s = 3980$ kg/m^3).

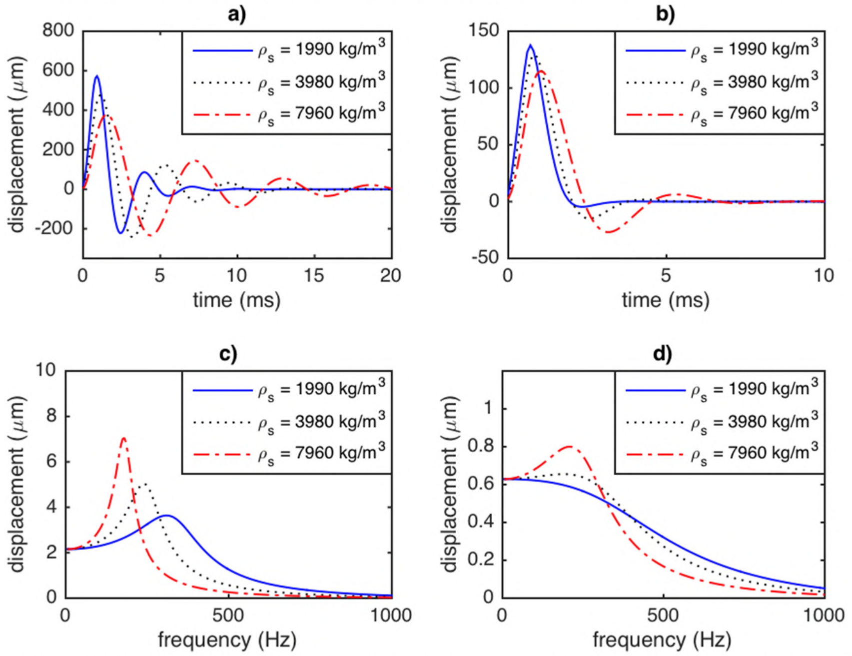

Figure 3. Sphere displacements predicted using the interface (a) and embedded (b) models for the different densities of the sphere and the spectrums obtained using the interface (c) and embedded (d) models ($G = 1300$ Pa, $\rho = 1000$ kg/m^3, $\eta = 0.1$ Pa s, $v = 0.45$, $R = 0.5$ mm, $f_0 = 2.3$ mN, $\tau = 0.67$ ms).

The proposed model can be used in various applications. When the displacements of a sphere with known physical properties at a viscoelastic medium interface to a given excitation force are measured, the model can be used to determine the shear modulus and viscosity of the medium. The model can be used to identify the physical properties of spherical inhomogeneities at viscoelastic interfaces with known mechanical properties by using the measured displacements of the inhomogeneities to a given excitation force. It should be noted that there are some studies for determining the effects of surface elasticity and surface stress by measuring the shifts of resonant frequencies of a beam [29]. The model in this study can provide a new experimental approach for determining the surface elasticity based on the shift of the resonance frequency of the sphere at viscoelastic interface or by using the measured displacements of the sphere with known physical properties at the viscoelastic medium interface to a given excitation force. In some applications, it is difficult, or not possible, to determine the force, such as the acoustic radiation force on spherical inhomogeneities in ultrasound applications. The model proposed in this study can be used to identify the unknown force by measuring the displacements of the sphere with known physical properties at the viscoelastic medium interface with known mechanical properties or the resonance shifts for different force levels.

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