Reduced Order Modeling of delayed PEEC circuits

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Abstract—We propose a novel model order reduction technique that is able to accurately reduce electrically large systems with delay elements, which can be described by means of neutral delayed differential equations. It is based on an adaptive multipoint expansion and model order reduction of equivalent first order systems. The neutral delayed differential formulation is preserved in the reduced model. Pertinent numerical results validate the proposed model order reduction approach.

Index Terms—Delayed Partial Element Equivalent Circuit method, reduced order systems, neutral delayed differential equations.

I. INTRODUCTION

With the increase of operating frequencies, the accurate modeling of high-speed components and modern integrated circuits requires electromagnetic (EM) 3-D methods [1], [2] as necessary analysis and design tools. Large systems of equations are usually generated by the use of these methods and model order reduction (MOR) techniques are crucial to reduce the complexity of EM models and the computational cost of the simulations, while retaining the important physical features of the original system [3], [4]. Among all EM methods, the Partial Element Equivalent Circuit (PEEC) method [2] uses a circuit interpretation of the Electric Field Integral Equation (EFIE) [5], therefore it is especially suitable to problems involving both electromagnetic fields and circuits [2], [6]. If we consider a quasi-static PEEC formulation [2], which approximates the full-wave PEEC approach [7], an equivalent RLC circuit is obtained, since the time delays between the elements in the full-wave PEEC formulation are neglected. Systems of ordinary differential equations (ODE) are obtained. Standard MOR techniques for ODE systems can be used to reduce the size of a quasi-static PEEC model [3], [4]. When signal waveform rise times decrease and the corresponding frequency content increases or the geometric dimensions become electrically large, time delays must be taken into account and, therefore, included in the modeling.

A PEEC formulation which include delay elements, called τPEEC [8], becomes necessary and leads to systems of neutral delayed differential equations (NDDE) [7]. While several successful MOR methods for large ODE systems have been proposed over the years, the reduction of large NDDE systems is still a very challenging research topic, since standard MOR techniques for ODE systems cannot be directly applied to NDDE systems. Especially, the reduction of electrically large structures where delays among coupled elements cannot be

neglected or easily approximated by rational basis functions needs to be investigated and addressed.

Some techniques for the reduction of NDDE systems have been proposed [9]–[12]. In [9] an equivalent first order system is computed by means of a Taylor expansion, and then MOR Krylov subspace methods [3], [4] are applied. The NDDE formulation is not preserved in the reduction process. Multiple expansion points and the construction of rational local approximations of τPEEC systems are used in [10]. Then, each rational local approximation is reduced by standard MOR methods and portions of these reduced rational models are combined to obtain a global reduced model for the frequency range of interest. As in [9], the NDDE formulation is not preserved in the reduction process. The approach in [11] extracts exponential terms (primary phase factors) and then the smoother remainders are expanded into a linear form and then projected to obtain the reduced model. Hence, the extraction of primary phase factors and the segregation of the system into multiple remainder phase matrices, each corresponding to a primary phase factor, are needed. In [12], an equivalent first order system is computed by means of a single point Taylor expansion [9] and a corresponding orthogonal projection matrix is computed by means of a block Arnoldi algorithm [3]. Then, an orthogonal projection matrix for the original NDDE system is extracted and a reduced NDDE system is obtained.

This paper presents a novel MOR method for τPEEC models that is able to accurately reduce electrically large structures where delays among coupled elements cannot be neglected or easily approximated by rational basis functions. It is based on an adaptive multipoint expansion and MOR of equivalent first order systems [12]. The NDDE formulation is preserved in the reduced model.

The paper is organized as follows. Section II describes the modified nodal analysis (MNA) equations of the τPEEC method. Section III describes the proposed MOR method for NDDE systems. Finally, a pertinent numerical example based on the τPEEC method validates the proposed technique in Section IV.

II. DELAYED PEEC FORMULATION

The PEEC method [2] stems from the integral equation form of Maxwell's equations. With respect to other EM methods, it is worth pointing out its capability to provide a circuit interpretation of the EFIE equation, thus allowing to handle

complex problems involving both circuits and electromagnetic fields.

In the standard approach, volumes and surfaces are discretized into elementary regions, hexahedra and patches respectively [8] over which the current and charge densities are expanded into a series of basis functions.

Following the standard Galerkin's testing procedure, topological elements, namely nodes and branches are generated and electrical lumped elements are identified modeling both the magnetic and electric field coupling.

Conductors are modeled by their ohmic resistance, while dielectrics require modeling the polarization charge due to the dielectric polarization [13]. Magnetic and electric field coupling are modeled by partial inductances and coefficients of potential, respectively.

An example of τPEEC circuit for a conductor elementary cell is illustrated, in the Laplace domain, in Fig. 1 where the current controlled voltage sources $sL_{p,ij}I_j$ and the charge controlled current sources sQ_i model the magnetic and electric field couplings, respectively.
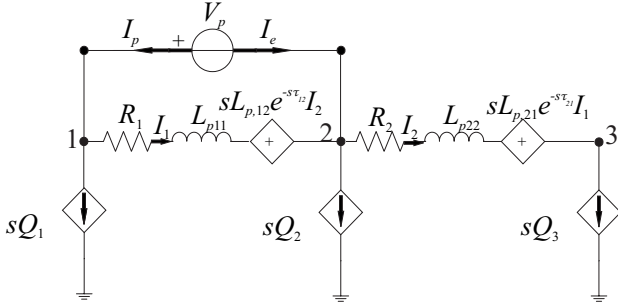

Fig. 1. Illustration of τPEEC circuit electrical quantities for a conductor elementary cell.

Let us assume that the meshing process of conductors and dielectrics has generated n_i volume cells where currents flow and n_n surface cells where charge is located; the resultant number of elementary cells of conductors and dielectrics is n_c and n_d, respectively and that of electrical nodes is n_n. Furthermore, let us assume to be interested in generating an admittance representation $\mathbf{Y}(s)$ having n_p output currents $\mathbf{i}_p(t)$ under voltage excitation $\mathbf{v}_p(t)$. If the MNA approach [14] is used, the global number of unknowns is $n_u = n_i + n_d + n_n + n_p$ and an admittance representation of the PEEC circuit is obtained

$$
\begin{aligned}
\mathbf{C}\left(\boldsymbol{\tau}\right)\frac{d\mathbf{x}(t)}{dt} &= -\mathbf{G}\left(\boldsymbol{\tau}\right)\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \qquad (1a)\\
\mathbf{i}_p(t) &= \mathbf{L}^T\mathbf{x}(t) \qquad (1b)
\end{aligned}
$$

where $\mathbf{x}(t) \in \Re^{n_u \times 1}$ and $\boldsymbol{\tau} \in \Re^{n_\tau \times 1}$ contains all delays $\boldsymbol{\tau}_L, \boldsymbol{\tau}_C$, which denote the center-to-center delay matrices for the magnetic and electric field coupling. Since this is an n_p-port formulation, whereby the only sources are the voltage sources at the n_p-port nodes, $\mathbf{B} = \mathbf{L}$ where $\mathbf{B} \in \Re^{n_u \times n_p}$. Each delayed

entry of matrices $\mathbf{C}\left(\boldsymbol{\tau}\right)$ and $\mathbf{G}\left(\boldsymbol{\tau}\right)$ act as a delay operator for the corresponding entry of vector $\mathbf{x}(t)$. Hence, (1) can be re-written in the Laplace domain as:

$$
\begin{aligned}
s\mathbf{C}(s)\mathbf{X}(s) &= -\mathbf{G}(s)\mathbf{X}(s) + \mathbf{B}\mathbf{V}_p(s) \qquad (2)\\
\mathbf{I}_p(s) &= \mathbf{B}^T\mathbf{X}(s) \qquad (3)\\
\mathbf{C}(s) &= \mathbf{C}_0 + \sum_{k=1}^{n_\tau}\mathbf{C}_k e^{-s\tau_k} \qquad (4)\\
\mathbf{G}(s) &= \mathbf{G}_0 + \sum_{k=1}^{n_\tau}\mathbf{G}_k e^{-s\tau_k} \qquad (5)
\end{aligned}
$$

III. DELAYED MODEL ORDER REDUCTION

In [12], an equivalent first order system is computed by means of a single point Taylor expansion [9] and a corresponding orthogonal projection matrix is computed by means of a block Arnoldi algorithm [3]. Then, an orthogonal projection matrix for the original NDDE system is extracted and a reduced NDDE system is obtained. The NDDE formulation is preserved in the reduction process. The proposed novel MOR algorithm is based on an adaptive multipoint expansion and MOR of equivalent first order systems. As in [12], the NDDE formulation is preserved. The multipoint feature allows to reduce electrically large structures with large delays ($2\pi freq_{max}\tau_{max} > 10$) [11] that cannot be neglected or easily approximated by a single point expansion and rational functions. The equivalent first order system obtained after the single point Taylor expansion of exponential terms has an order equal to qn_u, where q is the order of the Taylor expansion and n_u the order of the original NDDE system [12]. Since exponential terms with large delays need many terms in the Taylor expansion to be accurately approximated, the reduction of equivalent first order systems becomes computationally expensive and sometimes not feasible. The multipoint expansion [15] addresses this issue and is able to accurately reduce NDDE systems with large delays, since a small expansion Taylor order can be used for each expansion point and the accuracy of the reduced model is increased by adding new expansion points. An adaptive algorithm is used to choose the expansion points, assuming that the order of the Taylor expansion is fixed for each expansion point. It is not described here due to the lack of space. At each expansion point, the MOR algorithm described in [12] is applied and the corresponding projection matrix $\mathbf{Q}_i$, $i = 1, \ldots, n_{points}$ is computed. The final projection matrix $\mathbf{Q}$ is based on the orthogonalization of the stack column collection of all single expansion point projection matrices. The computation of $\mathbf{Q}$ in the case of multipoint expansion is explained in the flowchart in Fig. 2. The MOR algorithm described in [12] is called *One point DMOR* in Fig. 2, where $n_r = qn_p$ represents the reduced order for each expansion point.

Once $\mathbf{Q}$ is computed, it is applied to the original NDDE system (2)-(3) and a reduced NDDE system

Algorithm 1: MULTIPOINT($NDDE_{orig}, \{s_k\}_{k=1}^{n_{points}}, n_r, q$)

comment: Multipoint computation of basis $\mathbf{Q}$

$$
\begin{cases}
\mathbf{Q}_1 = One\ point\ DMOR(NDDE_{orig}, s_1, n_r, q); \\
\mathbf{Q} = \mathbf{Q}_1; \\
\textbf{for}\ k \leftarrow 2\ \textbf{to}\ n_{points} \\
\quad \textbf{do}\ \begin{cases} \mathbf{Q}_k = One\ point\ DMOR(NDDE_{orig}, s_k, n_r, q); \\ \mathbf{Q} = ([\mathbf{Q}, \mathbf{Q}_k]); \end{cases} \\
\mathbf{Q} = orth(\mathbf{Q});
\end{cases}
$$

Fig. 2. Pseudocode for computing the orthogonal basis $\mathbf{Q}$ in the multipoint expansion case.

$$s\mathbf{C}_r(s)\boldsymbol{\chi}(s) = -\mathbf{G}_r(s)\boldsymbol{\chi}(s) + \mathbf{B}_r\mathbf{V}_p(s) \tag{6}$$

$$\mathbf{I}_p(s) = \mathbf{B}_r^T\boldsymbol{\chi}(s) \tag{7}$$

$$\mathbf{C}_r(s) = \mathbf{C}_{r,0} + \sum_{k=1}^{n_\tau} \mathbf{C}_{r,k}e^{-s\tau_k} \tag{8}$$

$$\mathbf{G}_r(s) = \mathbf{G}_{r,0} + \sum_{k=1}^{n_\tau} \mathbf{G}_{r,k}e^{-s\tau_k} \tag{9}$$

is obtained, where the following congruence transformations are used

$$\mathbf{C}_{r,i} = \mathbf{Q}^T\mathbf{C}_i\mathbf{Q},\ i = 0, \ldots, n_\tau \tag{10a}$$

$$\mathbf{G}_{r,i} = \mathbf{Q}^T\mathbf{G}_i\mathbf{Q},\ i = 0, \ldots, n_\tau \tag{10b}$$

$$\mathbf{B}_r = \mathbf{Q}^T\mathbf{B} \tag{10c}$$

$$\mathbf{L}_r = \mathbf{Q}^T\mathbf{L} \tag{10d}$$

IV. NUMERICAL RESULTS

Concerning the error criterion, let us define the weighted RMS-error as

$$
\begin{aligned}
Err &= \\
&= \sqrt{\frac{\sum_{i=1}^{(n_p)^2} \sum_{k=1}^{K_s} \left| w_{Y_i}(s_k)\left(Y_{r,i}(s_k) - Y_i(s_k)\right)\right|^2}{(n_p)^2 K_s}}
\end{aligned} \tag{11}
$$

with

$$w_{Y_i}(s) = |(Y_i(s)^{-1}| \tag{12}$$

It is used to assess the accuracy of reduced order models over the bandwidth of interest, which is sampled over K_s samples.

A. Multiconductor system

A multiconductor system composed of four coplanar conductors with a length $\ell = 10$ cm, a width $W = 1$ mm, a thickness $t = 1$ mm and an horizontal spacing $S_x = 3$ mm has been modeled in this example. Fig. 3 shows its cross section. The system is analyzed on the frequency range $[0.0001, 10]$ GHz.
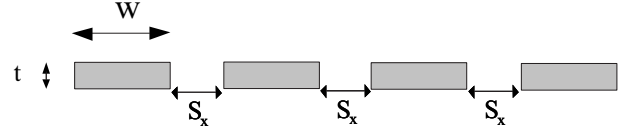

Fig. 3. Cross section of the multiconductor system.

The six ports of the system are defined between successive conductors.

The order of the original τPEEC model is equal to $n_u = 2390$, the number of delays is equal to 632 and the largest delay gives $2\pi freq_{max}\tau_{max} = 21$. Table I shows some parameters of the proposed MOR method.

TABLE I
MOR PARAMETERS.

Parameter	Adaptive method
Taylor terms	2
Expansion points	7
Reduced order	84
Weighted RMS-error reduced model	0.025

Figs. 4-7 compare the magnitude and phase of the original and reduced NDDE model of $\mathbf{Y}_{33}(s)$ and $\mathbf{Y}_{36}(s)$. As clearly seen, the presented MOR method can reduce a large NDDE system with large delay terms, while accurately preserving the behavior of the original system.
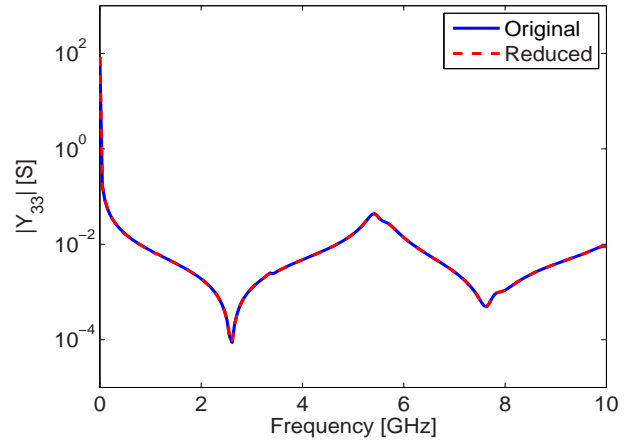

Fig. 4. Magnitude of $\mathbf{Y}_{33}$.

V. CONCLUSIONS

We have presented a new MOR technique for large NDDE systems, which is applicable to τPEEC models. It is able to accurately reduce electrically large structures where delays among coupled elements cannot be neglected or easily approximated by rational basis functions. It is based on an adaptive multipoint expansion and MOR of equivalent first order systems. The NDDE formulation is preserved in the
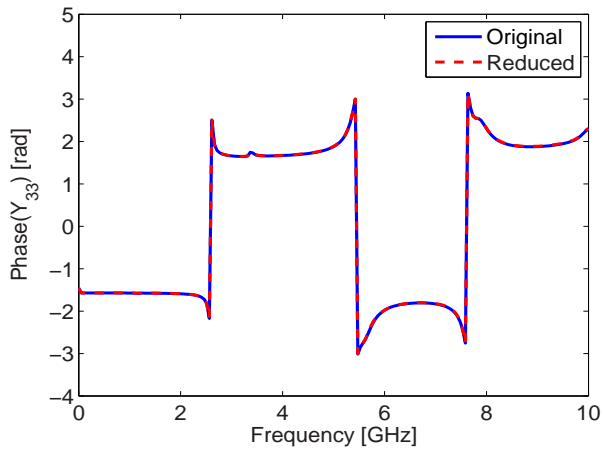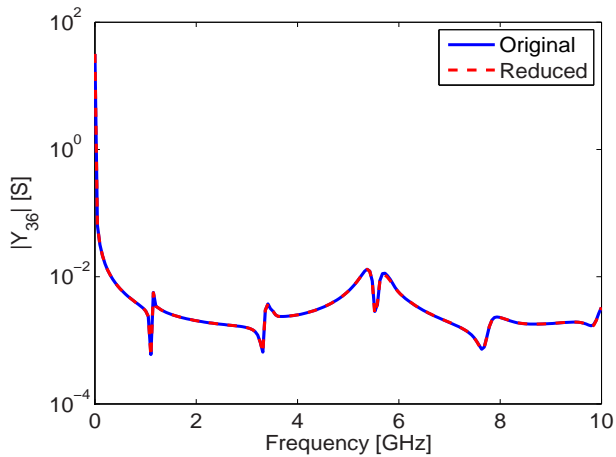
Fig. 5.   Phase of $\mathbf{Y}_{33}$.

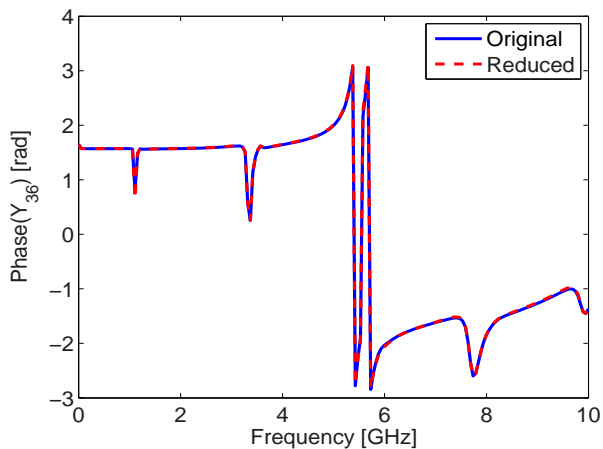Fig. 6.   Magnitude of $\mathbf{Y}_{36}$.

Fig. 7.   Phase of $\mathbf{Y}_{36}$.

reduced model. Numerical results based on the τPEEC method

have validated the proposed MOR approach.

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