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Abstract. Entanglement is arguably the key quantum-mechanical resource for improving the performance of communication, precision measurement and computing systems beyond their classical-physics limits. Yet entanglement is fragile, being very susceptible to destruction by the decoherence arising from loss and noise. Surprisingly, Lloyd (2008 *Science* **321** 1463) recently proved that a very large performance gain accrues from use of entanglement in single-photon target detection within an entanglement-destroying lossy, noisy environment when compared to what can be achieved with unentangled single-photon states. We extend Lloyd's analysis to the full multiphoton input Hilbert space. We show that the performance of Lloyd's single-photon 'quantum illumination' system is, at best, equal to that of a coherent-state transmitter of the same average photon number, and may be substantially worse. We demonstrate that the coherent-state system derives its advantage from the coherence between a sequence of weak—single photon on average—transmissions, a possibility that was not allowed for in Lloyd's work. Nevertheless, as shown by Tan *et al* (2008 *Phys. Rev. Lett.* **101** 253601), quantum illumination may offer a significant, although more modest, performance gain when operation is *not* limited to the single-photon regime.

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Entanglement is arguably the key quantum-mechanical resource for improving the performance of communication systems [3], precision-measurement sensors [4] and computer algorithms [5] beyond their classical limits. Yet entanglement is fragile, being very susceptible to destruction by the decoherence arising from loss and noise. Thus, the recent paper by Lloyd [1] is especially exciting. Lloyd showed that a very large performance gain accrues from use of entanglement in single-photon target detection within an entanglement-destroying lossy, noisy environment when compared to what can be achieved with unentangled single-photon states. His work teaches us *not* to dismiss the value of entanglement in all scenarios wherein decoherence destroys that nonclassical behavior prior to quantum measurement.

The present work extends Lloyd's analysis to include the full multiphoton input Hilbert space that allows coherences between different photon-number states. In particular, we compare Lloyd's strictly single-photon 'quantum illumination' transmitter with a weak coherent-state transmitter operating at unity average photon number per pulse. We get a somewhat surprising result: Lloyd's quantum-illumination system yields performance that, at best, equals that of the coherent-state system, and may be substantially worse. Since [1] appeared, Tan *et al* [2] reported a full Gaussian-state treatment of quantum-illumination target detection—outside the single-photon regime—revealing that entanglement *does* provide a performance gain, albeit more modest, compared to coherent-state systems of the same average photon number. Thus we shall also address the cause for Lloyd's single-photon quantum illumination losing its luster when compared with its weak coherent-state counterpart. Here, the answer turns out be the coherence between a sequence of single photon on average coherent-state transmissions.

Consider a quantum-illumination radar system in which, on each transmission, one photon from a maximally entangled pair interrogates a region of space in which a target might be located, whereas the other photon is retained at the receiver for use in a joint measurement with what is received from that region. Lloyd [1] showed that such a transmitter affords dramatically improved photodetection sensitivity in an entanglement-breaking scenario in comparison to what is achieved with unentangled single-photon states. His quantum Chernoff bound on the error probability—when the optimum quantum measurement over N-repeated entangled-state transmissions is employed—can be stated as follows:

$$\Pr(e)_{\mathrm{QI}} \leqslant \begin{cases} \mathrm{e}^{-N\kappa}/2, & \text{for } \kappa \ll 1,\, MN_{\mathrm{B}} \ll 1 \text{and } \kappa \gg N_{\mathrm{B}}/M; \\ & \text{the 'good' regime} \\ \mathrm{e}^{-N\kappa^2 M/8N_{\mathrm{B}}}/2, & \text{for } \kappa \ll N_{\mathrm{B}}/M \ll MN_{\mathrm{B}} \ll 1; \\ & \text{the 'bad' regime.} \end{cases} \tag{1}$$

Here, κ is the transmitter-to-receiver coupling when the target is present, N_{B} is the average number of received background photons per mode, and $M \gg 1$ is the number of temporal modes over which the transmitter state is entangled. The distinction between 'good' and 'bad' regimes is that performance in the former is independent of the background noise, whereas the performance in the latter is dominated by that noise.

The quantum-illumination performance in (1) is substantially better than the quantum Chernoff bound Lloyd found for N-repeated transmissions of a single-photon pure state,

$$\Pr(e)_{\mathrm{SP}} \leqslant \begin{cases} \mathrm{e}^{-N\kappa}/2, & \text{for } \kappa \ll 1 \text{ and } \kappa \gg N_{\mathrm{B}} \ll MN_{\mathrm{B}} \ll 1; \\ & \text{the 'good' regime} \\ \mathrm{e}^{-N\kappa^2/8N_{\mathrm{B}}}/2, & \text{for } \kappa \ll N_{\mathrm{B}} \ll MN_{\mathrm{B}} \ll 1; \\ & \text{the 'bad' regime,} \end{cases} \tag{2}$$

where M-mode photodetection has been assumed, as in the entangled case. Comparing (1) and (2) shows that when both systems are in their 'good' regimes, they achieve identical performance. However, the 'good' regime for quantum illumination extends to M-times higher background levels than does that for unentangled single-photon transmission. Moreover, when both systems are in their 'bad' regimes, quantum illumination achieves an error-probability exponent that is M times higher than that for unentangled single-photon transmission. Note that M may be a very large number. For example, when the entangled state is obtained from a continuous-wave spontaneous parametric downconverter source operating in the biphoton limit, a T-sec-long pulse from a source whose phase-matching bandwidth is W Hz comprises $M = WT$ modes. Taking a reasonable $W = 1$ THz value for the phase-matching bandwidth, we find that $M = 10^3$ for a $T = 1$ ns pulse.

Lloyd's results were predicated upon the assumption of single-photon operation. For any given transmission he assumed that at most one photon was received, be it from target return or background light. He also indicated that a full Gaussian-state analysis of quantum illumination in the $\kappa \ll 1$ regime would lift the preceding restriction. That analysis has now been performed by Tan *et al* [2]. Their work, as we will now show, demonstrates a significant difference from what Lloyd found for the single-photon limit. Before turning to that demonstration, it is important to note that Chernoff bounds are exponentially tight. Specifically, if

$$\Pr(e) \leqslant \mathrm{e}^{-N\mathcal{E}}/2 \tag{3}$$

is the Chernoff bound on the error probability achieved with N repeated uses of a particular transmitter state, then

$$\lim_{N \to \infty} [\ln(\Pr(e))/N] = -\mathcal{E}, \tag{4}$$

so that system performance is well characterized by the Chernoff bound error-probability exponent $\mathcal{E}$.

For a transmitter that emits a coherent state with N photons on average—e.g. by N-repeated transmissions of a coherent state with unity average photon number—the quantum Chernoff bound from Tan *et al* is

$$\Pr(e)_{\mathrm{CS}} \leqslant \mathrm{e}^{-N\kappa(\sqrt{N_{\mathrm{B}}+1}-\sqrt{N_{\mathrm{B}}})^2}/2 \approx \mathrm{e}^{-N\kappa}/2, \quad \text{for } N_{\mathrm{B}} \ll 1. \tag{5}$$

This performance equals that of Lloyd's quantum-illumination transmitter in that system's 'good' regime, and is *superior* to that transmitter's performance in its 'bad' regime. Indeed, when (1) is compared with (5) we see that the latter has no κ restriction on its validity, i.e. $N_{\mathrm{B}} \ll 1$ is enough to ensure that repeated transmission of coherent states with unity average photon number leads to minimum error-probability target detection performance that is *not* background-noise limited. Thus, whereas Lloyd's quantum-illumination system will fall prey to background noise—within its assumption of single-photon operation—when $\kappa \ll N_{\mathrm{B}}/M \ll M N_{\mathrm{B}} \ll 1$, no such thing happens for the coherent-state system.

It is easy to understand the origin of the performance advantage afforded by N-repeated transmissions of a unity average photon-number coherent state. All of these transmissions have been assumed to be phase coherent. Hence they are equivalent to transmitting the coherent state $|\sqrt{N}\rangle$, with average photon number N, in a single mode that is the coherent superposition of the modes excited by each individual transmission. Unity quantum efficiency photon counting (direct detection) on this super-mode will achieve error probability $\mathrm{e}^{-N\kappa}/2$, when $N\kappa \gg N_{\mathrm{B}} \ll 1$. The optimum quantum receiver for the coherent-state system is not photon counting, but, as shown in (5), it is always in the 'good' regime when $N_{\mathrm{B}} \ll 1$.

Another way to exhibit the role played by coherence in achieving the performance given in (5) is to consider a receiver that eschews this coherence. Suppose that on each of the N coherent-state transmissions we employ a minimum error-probability receiver to decide—based on the return from that transmission alone—whether or not the target is present. When $\kappa \gg N_B \ll 1$, the error probability for this single-transmission receiver is given by

$$\Pr(e)_{CS_1} \approx \frac{1 - \sqrt{1 - e^{-\kappa}}}{2} \approx \frac{1 - \sqrt{\kappa}}{2}, \quad \text{for } \kappa \ll 1, \tag{6}$$

where we have used the background-free error probability of the optimum receiver in the first approximation [6]. Now suppose that the *decisions* made on each of the N individual transmissions are combined, by majority vote, to determine a final decision as to target absence or presence. This majority-vote receiver ignores the coherence between the different coherent-state transmissions. The Chernoff bound on its error probability is

$$\Pr(e)_{MV} \leqslant \frac{[2\sqrt{p(1-p)}]^N}{2}, \quad \text{where } p \equiv \Pr(e)_{CS_1}. \tag{7}$$

Making use of the $\kappa \ll 1$ approximation for p we can reduce this Chernoff bound to

$$\Pr(e)_{MV} \leqslant e^{-N\kappa/2}/2, \tag{8}$$

which has an error-probability exponent that is a factor of two worse than what is obtained with coherent processing of the N coherent-state transmissions.

The single-shot receiver needed to achieve $\Pr(e)_{CS_1}$ is a complicated photodetection feedback system [7]. Thus it is of interest to exhibit an alternative, for use with the coherent-state transmitter, whose performance may exceed that of Lloyd's quantum-illumination system. Suppose that a coherent-state transmitter is used in conjunction with unity quantum-efficiency homodyne detection. The Chernoff bound for this system is

$$\Pr(e)_{hom} \leqslant e^{-N\kappa/(4N_B+2)}/2 \approx e^{-N\kappa/2}/2, \quad \text{for } N_B \ll 1, \tag{9}$$

so that the performance of this conventional (coherent-state transmitter, homodyne-detection receiver) laser radar is a factor of two worse in error-probability exponent than Lloyd's quantum-illumination system when that system is in its 'good' regime. The conventional system, however, performs far better than Lloyd's quantum-illumination system when the latter is in its 'bad' regime.

In conclusion, the full Gaussian-state analysis suggests that Lloyd's quantum illumination is unlikely to substantially improve radar performance in the low-noise regime wherein $N_B \ll 1$. Tan *et al* [2], however, showed that a factor-of-four error-probability exponent improvement, over coherent-state transmission, *is* achieved with a spontaneous parametric downconverter entangled-photon source and the optimum quantum measurement in the lossy ($\kappa \ll 1$), noisy ($N_B \gg 1$), low-brightness ($N/M \ll 1$) scenario. Thus, the ultimate conclusion remains the same, i.e. we should be careful *not* to dismiss the potential benefits of entanglement in scenarios wherein decoherence destroys that entanglement prior to quantum measurement.

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