# Incorporating Cycle Time Uncertainty to Improve Railcar Fleet Sizing 

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#### Abstract

This thesis involves railcar fleet sizing strategies with a specific company in the chemical industry. We note that the identity of the company in this report has been disguised, and some portions of the fleets have been omitted to mask their actual sizes. However, all analysis in this thesis was conducted on actual data. In our research, we evaluate the appropriateness of both deterministic and stochastic fleet sizing models for this company. In addition, we propose an economic model that is adapted from a basic inventory management policy that can be applied to fleet sizing in order to arrive at a cost-driven solution. Through our research, we demonstrate that the fleet sizing strategy of this company can be improved by incorporating transit time variability into the fleet sizing model. Additionally, we show that fleet sizes can be reduced by accurately characterizing the distributions of the underlying transit and customer holding time data. Finally, we show the potential value of considering economic factors to arrive at a fleet sizing decision that balances the cost of over-capacity with the cost of an insufficient supply of railcars.


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## I. Introduction

## I.a. Motivation

Determining the appropriate fleet size for a private shipper that requires a fleet is a challenging task. In selecting the size of the fleet, fleet managers must balance the high customer service expectations of their channel partners, the variability of product demand and transit operating times, and the desire to achieve high asset utilization performance from the capital invested in their fleet. As we will document in greater detail later in this thesis, the challenges faced by fleet managers are magnified in the chemical industry, where additional constraints such as specialized vehicle requirements add complexity to a task that is already difficult to manage.

In this research, we address the issue of developing a method for a private fleet manager to determine the appropriate number of railcars in a fleet. Specifically, we focus on the incorporating the variability of railcar cycle time (the time it takes a railcar to make a complete trip from origin to destination and back to the origin) into the fleet-sizing decision. In addition, we recommend a process that enables fleet managers to use simulation to understand the expected requirements of their fleet capacity. Finally, we suggest an alternative approach to interpreting the results of the simulation in making the fleet sizing decision.

The intent of this research is to improve the process by which managers perform the fleet-sizing analysis and to develop a method that provides greater insight into the effect of cycle time variability on fleet size requirements. By considering the effect of cycle time variability on requirements of the fleet, managers can ensure that their fleet size selection is in line with the risk tolerance appropriate to balance the needs of customer service with asset utilization of the fleet. In addition, by understanding the sources of variability in cycle time performance,
managers can begin to identify strategies that mitigate the impact of this variability. Finally, we seek to investigate the potential of economic considerations in the fleet sizing decision.

The remainder of the report is outlined below. We begin with a review of published literature on railcar fleet topics. The majority of this literature is focused on the challenges and issues specific to the petrochemical and chemical industries.

In the next section of the report, we describe the data that was analyzed in this research. The data provided for analysis consisted of historical railcar cycle time data for several of the previously mentioned fleets. We describe the nature of this data, how it was cleaned, and the approach taken in the absence of sufficient data. In addition, we describe the statistical techniques used to perform distribution fitting of theoretical probabilistic distributions to historical cycle time data. These distributions are used in a simulation technique described later in the document.

Following our description of the data, we describe three different fleet sizing methods, and provide a general explanation of the benefits and limitations of each method. The first method described is the process that is currently implemented by KSCC at the time of this research. The second method is a deterministic approach in which basic descriptive statistics of railcar cycle time data are used to arrive at recommended fleet sizes. The final method explained is a stochastic method in which Monte Carlo simulation is used to create histograms of expected fleet size requirements.

In the next section, we discuss an economic model adapted from inventory management and describe how it could be applied to a fleet sizing decision process. In this model, we describe a cost driven approach that could enable a fleet manager to arrive at a recommended fleet size from a distribution of possible requirements based on a ratio that compares the cost of
excess capacity to the cost of insufficient capacity. After describing the model, we demonstrate its potential to be used as a means to reduce the fleet size by lowering the cost of insufficient capacity.

In the next section, we review and analyze the results of the three different fleet sizing models. We discuss the differences between the model outputs and the implications of the differences on fleet sizing decisions. We then proceed to describe the advantages of the insight gained by the simulation analysis. We conclude with a summary of our findings in the analysis section.

In the final section of this report, we describe future research opportunities that could be conducted to further develop concepts identified during our research. We describe the framework for a more advanced simulation model that could be developed with access to additional information about order arrival data and promised delivery time commitments. We describe how such a model could be used to test the results of the fleet sizing models described in this report.

## I.b. Characteristics of the Sponsor Company Fleet

In this section of the report we describe the characteristics of the railcar fleets for which data was provided for analysis. The scope of this project includes the analysis of three different railcar sub-fleets of Kendall Square Chemical Company (Fleet X, Fleet Y, and Fleet Z). As shown in Figure 1, each sub-fleet is dedicated to a product group. Within each product group, there can one or more specific chemicals. The chemicals transported by these fleets are primarily destined for industrial use by customers. These chemicals are raw materials for products such as packaging materials, sunscreens, lubricants, detergents, plastics, and resins. These products are generally shipped to large industrial customers, including several competitors.

The railcars used for transporting these products are either owned or leased; in rare instances, spot cars are used as well. Spot cars are usually hard to find, and therefore cars within a product group are interchangeable if the railcar is purged before using it for another product. For instance, a railcar carrying product gamma must be purged before carrying product zeta. But a railcar carrying Fleet X or Fleet Y product are assigned solely for the product and cannot be used to load any other products. Purging is done in a cleaning railcar spot specifically assigned for this purpose. Kendall Square Chemical Company estimates that five to six cars can be cleaned every day. In addition, Kendall Square Chemical Company's railcar staging, loading, and maintenance practices result in approximately ten cars being required for loading, in the process of loading, or out of service for preventative or corrective maintenance.


Figure 1: Kendall Square Chemical Company Fleet Structure

## II. Literature Review

In this literature review, we provide a summary of existing research related to our thesis. This review consists of four sections topics. In the first section, we summarize work that describes some of the challenges of operating efficient rail fleets in the chemical industry. In the second section, we describe literature that explains the process of a railcar cycle in order to introduce basic terminology and discuss the essential operations of the process. In the third section, we describe the sources of variability that exist in each stage of the railcar cycle in order to emphasize the importance of understanding cycle time variability. In the final section, we describe existing literature on various existing fleet sizing strategies. We conclude with a discussion of how our research applies some of the existing research to data of an actual railcar fleet. In addition, we describe how part of our research is an extension of the concepts introduced by several authors.

## II.a. Challenges to High Supply Chain Asset Utilization in the Chemical Industry

Closs, Mollenkopf, and Keller (2005) discuss the increasing attention placed on the performance of supply chain assets in the chemical industry, and they suggest a rationale for the recent increase in focus. Reasons include:

- A recent decline in operating margins due to increased competition, forcing companies in the industry to investigate the costs associated with their operations.
- The need to avoid capital investments in operating equipment and inventory in order to preserve cash of other business ventures.

They also state that an area of improvement for the supply chains of chemical companies is in the asset utilization of their rail fleets. A combination of high customer service level targets and the necessity for specialized railcars has led many companies in the industry to invest in private fleets in order to ensure continuous railcar availability. As a result of this business decision, significant investments have been made in railcar fleets. Their research suggests that efforts to improve the performance of these fleets would result in reduced requirements for working capital investments and also provide tangible benefits such as a reduction in product inventory levels by reducing the required levels of safety stock.

Poor utilization performance of the chemical fleets can be attributed to several characteristics specific to the products that are shipped by these companies (Young, Swan, \& Burn, 2002). The characteristics of the products being transported by chemical companies play a significant role in the need to invest in specialized fleets. These unique product characteristics result in chemical companies having to maintain private, specialized fleets. This customization reduces the flexibility of the car, and thus, reduces the utilization performance of the fleet. Because railcar specialization can reduce fleet utilization, many railroads have not made investments in chemical railcar fleets, thus forcing these chemical companies to maintain private fleets.

In the following sections we introduce common terminology and explanations of the stages of a railcar cycle. After an explanation of terms, we will then investigate the sources of uncertainty that result in variable cycle times

## II.b. An Explanation of the Railcar Cycle

For the purposes of this thesis, a railcar cycle is considered to include the process by which a railcar is loaded with product at a shipper's point of origin, shipped to the customer site, unloaded and held at a the customer site, transported back to the shipper origin, and then held empty at the shipper origin until it is re-loaded with product. As shown in Figure 2, Closs, Mollenkopf, and Keller (2003) describe this process and divide it into four stages. In Figure 2, we have modified the descriptions from Closs, Mollenkopf, and Keller stages to reflect those commonly used within the sample data we are working with.


Figure 2: Stages of a Railcar Cycle (Modified from Closs, Mollenkopf, and Keller, 2003)

The total time required for a complete cycle is considered to be the sum of the four components listed in Figure 2.

## II.c. Sources of Variability in Railcar Cycle Times

In each of the stages described above, sources of uncertainty and variability exist which can extend the total length of time of a railcar cycle. Reasons for this are described below: Stage 1: Empty at shipper origin. In this stage, delay is caused by multiple factors including cleaning time, maintenance repairs, loading position spotting, and awaiting customer demand (Young et al., 2002).

Stage 2: Loaded transit time. In this stage, delay is caused by variable shipping distances based on customer location and railroad switching time (Closs, Keller, \& Mollenkopf, 2003). In addition, lane traffic volumes and car priority have also been shown to affect in-transit loaded (Young et al., 2002). High value or perishable goods may receive higher priority.

Stage 3: Time at customer. Delay in this stage of the cycle is common and can be attributed to several factors. Because of the extent of problem, specific research on the sources of this delay is summarized in articles on Excessive Customer Holding Time. We have summarized some of this research in the following section.

Stage 4: Empty transit times. In-transit empty railcar times suffer from the same sources of variability as Stage 2 transit. However, as Closs, Mollenkopf, and Keller (2003) discuss, Stage 4 times can often be longer because of the lower priority assigned to empty cars.

## II.c.i) Excessive Customer Holding Time

Excessive customer holding time (ECHT) is a problem which plagues the owners of private railcar fleets, particularly in the chemical industry. ECHT occurs when the customers of shippers hold railcars longer than the actual time needed to unload the car. Young, Swan, and

Burn cite many causes of this phenomenon. These causes include contractual allowances, poor channel coordination and a lack of adequate demand planning, the unwillingness of customers to invest in storage facilities large enough to accept full car loads, and ineffective demurrage policies. As a result of ECHT, railcar fleets often run at low utilization levels, and are larger in size than would be needed if cars were returned in a timely manner. Young, Swan, and Burn describe several policies that fleet owners have enacted in an attempt to reduce ECHT. Among these policies, implementing a demurrage process is the most common.

## II.d. Existing Fleet Sizing Strategies and Models

The need to develop an appropriate fleet size for a pool of vehicles is a decision that is found in many industries across a variety of transportation modes. In each case, managers must consider the factors driving the demand of cars, the cycle time performance of the fleet, the costs associated with maintaining the fleet, and the expectations of customer service. In this section, we describe some of the existing work on fleet sizing strategies.

Tyworth (1977) describes a study of the private railcar fleets of forest product firms in the northwestern United States. In this study, statistical regression analysis was performed to identify correlation between fleet policies and operational fleet performance metrics. The regression performed demonstrated that the distance shipped was the highest correlated factor. However, it also indicated that whether or not a shipper monitored customer holding time influenced fleet performance.

Beaujon and Turnquist (1990) provide examples of several different industries and applications where the common goal is to balance adequate car availability with efficient use of the fleet. They equate this type of analysis to that of inventory management, in which managers
must weigh the tradeoff of holding costs against that of a stockout. In addition, many fleet operators attempt to operate with constant fleet sizes and minimize cost through routing decisions. Beaujon and Turnquist suggest that by not adjusting fleet sizes, managers are failing to recognize the poor investment performance of the fleet asset. They highlight the important benefits of measuring the asset performance of the investment, and not solely focusing on the operational costs of the fleet.

Sayarshad and Goseiri (2008) describe a fleet sizing methodology in which cycle time performance and product demand are deterministic in nature. They use a Simulated Annealing optimization approach to develop a method for fleet sizing decisions. In their work, they describe the relationship between fleet size and the asset utilization of the fleet as a critical consideration for fleet sizing decisions.

Turnquist and Jordan (2008) describe a fleet sizing problem in which parts are manufactured at a single source of origin and distributed to number of assembly facilities. This network design is described as a one-to-many network. In their model, the supply of parts is assumed to be deterministic while, because of several sources of uncertainty, transportation times are treated as stochastic. In this model, the central goal is to provide a solution for recommended fleet size based on the probability of having insufficient capacity available to load based on the uncertainty of container cycle time. In their work, they suggest that an alternative to this approach would be to base the fleet sizing decision on a shortage cost.

In our research, we seek to apply the concepts of previous work by testing the implications of the assumptions of railcar cycle times treated as deterministic or stochastic. We test these assumptions on the historical data of a private fleet operator in the chemical industry to understand the benefits and limitations of each. Additionally, we build on the work of Turnquist
and Jordan, and Beaujon and Turnquist, by developing an economic model that allows a fleet sizing decision to be made by comparing the cost of over-capacity of railcars with the cost of having an insufficient fleet to satisfy demand.

## III. Data Description

In the following section, we describe the data that were provided for analysis in this project. After the describing the nature of the data, we will proceed to discuss how these data were used in our analysis.

For this project, railcar cycle time data from Kendall Square Chemical Company's railcar tracking system were provided and analyzed. For each complete railcar cycle, the time required to complete each of the four stages of the cycle is captured and transferred to spreadsheet form. Therefore, for each railcar trip between an origin-destination pairing, the dataset included the length of time spent in the following stages: Empty at origin, loaded transit time, dwell time at destination, and empty transit time back to origin. Furthermore, these data were provided to us categorized by product family categories, as there are differences between the railcar requirements of each family. An example of these data are provided in Table 1. In this table, we have provided a sub-set of data from a single sub-fleet to illustrate the nature of the data and demonstrate ways in which we addressed certain anomalies of the data. Note that the data provided in Table 1 is not actual data from the fleet, but rather a set of data randomly generated for illustrative purposes. The actual data analyzed in this thesis was real cycle time data from the tracking system of Kendall Square Chemical Company. This table was randomly generated in order to conceal propriety data.

Table 1: Illustrative Railcar Transit Data

| Row | Fleet | Origin | Destination | Shipment ID | Time at Origin (days) | Loaded <br> Transit Time (days) | Time at Customer Site (days) | Empty Transit <br> Time Back to Origin (days) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | A | Origin 1 | City A | 1001 | 5.8 | 8.6 | 22.8 | 16.2 |
| 2 | A | Origin 1 | City D | 1002 | 19.6 | 3.4 | 8.9 | 16.6 |
| 3 | A | Origin 1 | City E | 1003 | 4.0 | 20.5 | 23.6 | 21.3 |
| 4 | A | Origin 1 | City A | 1004 | 10.5 | 7.5 | 26.3 | 6.1 |
| 5 | A | Origin 1 | City F | 1005 |  |  |  |  |
| 6 | A | Origin 1 | City F | 1006 | 7.0 | 2.5 | 19.9 | 19.9 |
| 7 | A | Origin 1 | City C | 1007 | 18.2 | 0.0 | 19.2 | 11.3 |
| 8 | A | Origin 1 | City D | 1008 | 15.8 | 22.0 | 1.1 | 15.6 |
| 9 | A | Origin 1 | City A | 1009 | 18.3 | 4.5 | 16.5 | 4.8 |
| 10 | A | Origin 1 | City A | 1010 | 19.3 | 5.9 | 9.3 | 0.9 |
| 11 | A | Origin 1 | City H | 1011 | 10.7 | 6.8 | 17.9 | 7.0 |
| 12 | A | Origin 1 | City C | 1012 | 5.0 | 14.2 | 13.7 | 3.1 |
| 13 | A | Origin 1 | City B | 1013 | 15.9 | 21.6 | 21.2 | 3.7 |
| 14 | A | Origin 1 | City B | 1014 | 18.3 | 18.5 | 15.5 | 10.3 |

As can be seen in row 5 of Table 1, in some cases, data for all four legs of the railcar cycle were blank. In these cases, the entire railcar cycle was omitted from our analysis. In other instances, such as in row 7, column Loaded Transit Time, 0.0 days was registered as the length of time spent in one or more stages of the trip, while the remaining stages registered intervals of time within the expected range. In these instances, after discussions with individuals familiar with the data collection process, we omitted the zero values from our analysis and left the remaining values as part of the analysis. The integrity of the data remaining in the other stages of the cycle where a zero value was omitted was validated through discussions with Kendall Square Chemical Company stakeholders who were familiar with the data collection process of the railcar tracking system.

While we were provided with data on four stages of the railcar cycle, our analysis treats the Time at Origin data as a fixed value, which was determined through discussions with Kendall Square Chemical Company stakeholders. The reason is that the time a car spends empty at the origin is a function of the fleet size and not of the rail transit process. The time spent empty at origin is held constant at five days in our analysis. This number was derived through discussions with Kendall Square Chemical Company about the amount of time required for a railcar to be received in the rail yard, cleaned (if required), staged for loading, loaded with product, and staged to be picked up for transport. This five day time period represents an operational constraint in KSCC's process, and is not a form of buffering.

The remaining three stages of the railcar cycle were assumed to be independent random variables for the purposes of this analysis. By treating these stages as independent variables, our analysis was conducted under the assumption that the performance of one stage of the railcar cycle does not influence the performance of another stage. Additionally, we assumed independence between the individual origin-destination pairings. For example, we assumed that the cycle time performance of City A has no impact on the cycle time performance of City B. After discussions with KSCC stakeholders, we determined that this was a fair assumption.

After the data cleaning, we began our analysis. For the purposes of this project, we were concerned with three product group sub-fleets:

1. Fleet X
2. Fleet $Y$
3. Fleet $Z$

For each origin-destination pair, we looked at all railcar cycles that occurred during Q1 and Q2 of 2010, which represented the most recent data provided to us. In the cases where this
provided enough data points for statistical analysis, we proceeded. We used only Q1 and Q2 data when possible in order to reflect the most recent patterns in customer holding time behavior and transit time performance. Through conversations with KSCC we determined that demand is relatively constant throughout the year, and that minimal seasonality exists. In instances where the data from Q1 and Q2 of 2010 did not provide adequate data, we added more historical data to the set until it was sufficient for a statistical analysis. For the purposes of this analysis, we considered 25 data points as sufficient for statistical analysis.

After ensuring that sufficient data existed, we proceeded with our analysis. In some cases, sufficient data was not available for a valid statistical analysis. In these cases, we omitted these origin-destination pairs from our analysis. Therefore, the fleet sizing recommendations contained in this report do not reflect actual fleet sizes at the company. In order to estimate the railcar requirements in such situations, Kendall Square Chemical Company planners currently use a process that involves estimating the cycle time performance of these pairs by considering factors such as the transit time performance of customers with destinations that are in close geographic proximity, the holding time behavior of the customer at other destinations, and knowledge of the contractual agreements with customers concerning the allowance of railcar holding time. The planner combines this information with intuition from his or her experience to determine an estimate on the expected cycle time performance. After verifying the origindestination pairings where sufficient data were available, and ensuring that the data were cleaned of zeros and blank rows, we began to analyze individual cycle stages of each origin destination pairing separately.

## IV. Data Analysis

In the Methods section of this report, we will describe two different categories of fleet sizing models; deterministic models and stochastic models. For each type of model, parameters are required for the model to calculate a recommended fleet size or a range of possible fleet sizes. In the following section of this report, we describe how the inputs to these models were derived from the cycle time data previously described.

## IV.a. Deterministic Analysis

In the deterministic models, the inputs are cycle time mean and cycle time standard deviation for each origin-destination pairing. Because of the assumption of independence between cycle stages, the descriptive statistics of each stage of the cycle within an origindestination paring had to be combined to compute the total origin-destination values. In order to determine cycle time for each origin-destination pairing, the mean time for the loaded transit time stage, the time at customer stage, and the empty transit stage of the cycle were calculated, and then these means were summed. In addition, five days was added to these three means to capture the effect of the time spent empty at the origin. To determine the standard deviation of cycle times for each pairing, a similar process was used. This process is shown in Equation (1):

$$
\begin{equation*}
\sigma_{O D}^{2}=\sqrt{\left(\sigma_{L T}^{2}+\sigma_{C H}^{2}+\sigma_{E T}^{2}\right)} \tag{1}
\end{equation*}
$$

where:
$\sigma_{O D}^{2}=$ standard deviation of the origin-destination pairing cycle time
$\sigma_{L T}^{2}=$ standard deviation of the loaded transit stage
$\sigma_{C H}^{2}=$ standard deviation of the customer holding stage
$\sigma_{E T}^{2}=$ standard deviation of the empty transit stage

First, the standard deviation of each stage of the cycle was calculated. Next, the sum of the squares of these standard values was computed. Finally, the square root of the sum of the squares was found to get a total cycle time standard deviation of each origin-destination pairing. The time at origin component of the railcar cycle was not included in this calculation as it is assumed to be a static value with no variability.

## IV.b. Stochastic Analysis

In the methods section of this report, we will describe a fleet sizing model that treats the three variable components of the railcar cycle as stochastic random variables. In order to develop these inputs, we followed a process that fits probabilistic distributions to historical cycle time data. These distributions are then used in place of the historical data as the inputs to a simulation model. In this section of the report, we will describe the process by which best-fit distributions were selected.

## IV.b.i) Distribution Fitting

The process of distribution fitting begins by matching historical railcar cycle data with the "best-fit" distribution. This is a standard statistical method, which can be completed using a variety of spreadsheet analysis tools. Choosing theoretical distributions is preferable to using empirical distributions for the following reasons

- Theoretical distributions "smooth out" data and may provide insight into the underlying distribution.
- Empirical distributions may not generate values outside of the range of sample data.
- Empirical distributions are difficult to use if the data set is large.

For this project, we used the @RISK software package to conduct this analysis. However, the methods used and the statistical analysis conducted can be replicated using other means. We will describe both the process used in @RISK and general statistical concepts employed in the following sections.

For each origin-destination pairing, we analyzed the distribution profile of each stage of the railcar cycle. A chi-squared goodness-of-fit test was performed on the data in order to identify probability distributions that most closely match the characteristics of the data. The chisquare test performs a comparison of the actual data provided by the railcar tracking system to a number of theoretical distributions. The @RISK software computes a chi-square value for each comparison and then ranks the various types of distributions in terms of the their "fit" with the actual data set. For a normal distribution, the equation to determine the test statistic ( $\chi^{2}$-value) for a chi-square goodness-of-fit is shown by Equation (2) (Albright, Winston, Zappe, \& Broadie, 2011):

$$
\begin{equation*}
\chi^{2} \text { value }=\sum_{i=1}^{C} \frac{\left(O_{i}-E_{i}\right)^{2}}{E_{i}} \tag{2}
\end{equation*}
$$

where $\mathrm{O}_{\mathrm{i}}$ is the observed data point and $\mathrm{E}_{\mathrm{i}}$ is the expected value of that data point based on the theoretical distribution. The @RISK software computes the $\chi^{2}$-value for each distribution provided by the software package. In each case, we reviewed this value for abnormalities, and then selected the distribution with the best fit.

We offer the following illustrative example using loaded transit time data for the City E destination within the Fleet X sub-fleet. In Figure 3, a histogram is shown which represents
historical data from the railcar cycle time data sheet. In addition, a best-fit distribution line is shown. This distribution was chosen based on it having the lowest chi-squared test value.


Figure 3: Comparison of Empirical Data Histogram to Best Fit Theoretical Distribution
Figure 4 shows the screen shot of the chi-square value for all distributions that were tested against this data set:

| Fit Ranking |  |  | $\boldsymbol{\nabla} \mid$ |
| :--- | ---: | :---: | :---: |
| Fit | Chi-Sq |  |  |
| Expon | 52.9643 |  |  |
| Loglogistic | 59.7143 |  |  |
| Pearson5 | 59.7143 |  |  |
| InvGauss | 60.0357 |  |  |
| Lognorm | 60.0357 |  |  |
| Extvalue | 64.8571 |  |  |
| Logistic | 66.4643 |  |  |
| Pareto | 74.8214 |  |  |
| Triang | 82.8571 |  |  |
| Norrmal | 104.7143 |  |  |
| Uniform | 162.5714 |  |  |
| SetaGeneral | $\mathrm{N} / \mathrm{A}$ |  |  |
| Gamma | $\mathrm{N} / \mathrm{A}$ |  |  |
| Pearson6 | $\mathrm{N} / \mathrm{A}$ |  |  |
| Weibul/ | $\mathrm{N} / \mathrm{A}$ |  |  |

## Figure 4: Screenshot from@RISK of Chi-Squared Results

For this data set, the Exponential distribution was selected as the best fit.

## IV.b.ii) Parameter Estimation using Maximum Likelihood Estimators

@RISK uses Maximum Likelihood Estimators (MLEs) to determine the parameters of a theoretical distribution and once the distribution is fit, it determines the goodness of fit using Chi-Square distributions. For example, in Figure 5, @RISK returns the Pearson5 as one of the recommended distributions for the loaded transit time from the plant to customer A.


Figure 5: Theoretical Probability Distribution: Pearson 5 Distribution

Using the MLE functions, the parameters of the distribution are calculated. In this instance, alpha is 2.5576 and beta is 26.512 . The mean of the Pearson5 distribution is given by Equation (3) and the variance is calculated using Equation (4).

$$
\begin{gather*}
\text { Mean }=\frac{\beta}{\alpha-1}  \tag{3}\\
\text { Variance }=\frac{\beta^{2}}{(\alpha-1)^{2}(\alpha-2)} \tag{4}
\end{gather*}
$$

Substituting alpha and beta values and adjusting for shifts, the mean is calculated as 14.59 and the standard deviation is 22.79 . The mean of the sample data is 16.9 whereas the fitted theoretical distribution has a slightly lower mean value. We discuss the cause of this difference and its effect on fleet size requirements in the Section VII of this report.

This distribution fitting process was repeated for all origin-destination pairs within each of the three product groups for the following three stages of the railcar cycle: loaded transit time, days at customer, and empty transit time. As previously stated, empty days at origin was not assigned a probabilistic distribution. The input value of the empty days at origin stage, along with the probabilistic distributions of the other three stages served as the inputs for our simulation model.

Table 7 and Table 8 of the appendix provide all of the probability distributions that were created during our analysis. For each railcar cycle stage of each origin-destination pair, we list the name of the distribution chosen and provide a graph of the distribution. In addition, Table 7 and Table 8 contain a comparison of the distribution mean and standard deviation to the historical sample data mean and standard deviation. All graphs in these tables were created using @RISK software.

## V. Methods of Fleet Sizing Models

In this section of the document, we describe three different fleet sizing strategies that are applied to the Kendall Square Chemical Company fleet. These strategies fall into one of two types of models: deterministic or stochastic. In the deterministic models, the input variables (three stages of the railcar cycle) are treated as deterministic, and the two primary descriptive statistics (mean and standard deviation) of the empirical data are used to develop recommended fleet strategies. In the stochastic model, the input variables (three stages of the railcar cycle) are treated as random variables and their characteristics are used in a simulation to construct histograms of potential fleet size requirements. In this section, we will describe the methods used to construct these models. In the Analysis section of this report, we will examine the output of the models and discuss their implications. We begin by describing the deterministic methods.

## V.a. Fleet Sizing Model \#1: The MEAN BUFFERING Method

As previously discussed, the current method employed by Kendall Square Chemical Company to determine an appropriate fleet size begins by determining the mean cycle for each origin-destination pair within a fleet. The calculation of this mean cycle time was previously described in the Data section of this report. After determining the mean cycle time (T), annual demand forecasts (D) and the capacity of one railcar $\left(C_{R}\right)$ are used to determine the number of cycles required in one year to satisfy demand. Finally, the number of cycles required per year and the mean cycle time are used to compute the number of railcars required to service each origin-destination pairing. This calculation is shown in Equation (5):

$$
\begin{equation*}
\text { Mean Req. Railcars }=\frac{\frac{D}{C_{R}}}{\frac{363}{T}} \tag{5}
\end{equation*}
$$

where:
$\mathrm{D}=$ Annual demand forecast for the destination (Tons)
$\mathrm{C}_{\mathrm{R}}=$ Capacity of one railcar (Tons/Car)
$T=$ Mean cycle time for the OD pairing (Days)

This process is then repeated for each origin-destination pairing in the sub-fleet. Once the requirements are determined for each pairing, Kendall Square Chemical Company buffers the total fleet size by increasing these mean requirements by fifteen percent. This buffering is intended to protect against uncertainty and variability in the system. Kendall Square Chemical Company has determined that the cost of not having railcars available when required is high enough to require extra cars to be held to guarantee that demand is satisfied at the loading station. Kendall Square Chemical Company has indicated that this fifteen percent value has historically provided adequate protection against uncertainty and variability.

While this process of buffering the mean has provided adequate levels of service for Kendall Square Chemical Company in the past, there are several limitations of this method. The most significant limitation of this model is that it does not use the variability of railcar cycle time as input to help determine an appropriate fleet size. By failing to use this property of the cycletime data, the current model is forced to use a percentage buffer against the mean. As we will explain later in this document this action prevents the fleet managers from being able to fully understand what level of risk they should take on based on the extent of variability in the fleet. Additionally, it prevents the fleet manager from being able to gain a more complete understanding of exactly which customers and cycle time stages have the most variability, and
therefore significantly impact on fleet size. These limitations will be discussed in greater detail in the analysis section of this report.

## V.b. Fleet Sizing Model \#2: The NORMAL DISTRIBUTION Method

In this fleet sizing model, we improve upon Kendall Square Chemical Company's current method of fleet sizing by incorporating cycle time variability in the model. This eliminates the need to buffer with a percentage of the mean fleet size because it allows the modeler to use the inherent variability of cycle times to determine an appropriate fleet size. The two descriptive statistics that are used in this method are cycle-time mean and standard deviation. The sample mean and standard deviation of cycle time for each origin-destination pair are computed using the method described in Section IV of this report.

With these two descriptive statistics of each origin-destination pairing, the fleet modeler can then use the properties of the normal distribution to arrive at an appropriate fleet size. By assuming that the distribution of total cycle time for each pairing is normally distributed and knowing the mean and standard deviation of the distribution, the fleet size can be determined by selecting a point on the cumulative distribution function of this distribution. In this method, we select the $65^{\text {th }}$ percentile value on the cumulative distribution function in order to offer an alternative to the current method of buffering the mean by $15 \%$. The results of this $65^{\text {th }}$ percentile values for each pairing are then summed to find a total recommended fleet size at each of the two points on the cumulative function.

This method is an improvement over the current process because it recognizes that in order to protect against the variability of cycle times, a model must incorporate a measure of this uncertainty for each origin-destination pairing. This model accomplishes this goal by factoring
in the standard deviation of cycle time components. With this included in the model, a fleet manager can make more informed decisions about the size of the fleet relative to the variability in cycle times.

Although this method offers an improvement over the current process, it is not without its limitations. The most significant limitation of this model is that it assumes that the railcar cycle time follows a normal distribution. We will discuss later in this document why this assumption is, in some cases, invalid and describe the implications of this assumption. An additional limitation of this model is that it offers no quantitative method for selecting how much risk should be taken with the fleet size. This model contains no method to determine which percentile of fleet size provides the most appropriate risk management. These limitations will be discussed in greater detail in the Section VII of this report.

## V.c. Fleet Sizing Model \#3: The SIMULATION Method

Our third fleet sizing model uses a Monte Carlo technique to simulate railcar cycle times in order to create a distribution of required fleet sizes. As described in the Section IV of this report, the first step in this approach is to use historical cycle time data to fit probabilistic distributions for each leg of the railcar cycle for each origin-destination pairing, and then use these distributions in place of the historical data as inputs to the model. In the Monte Carlo simulation, railcar cycle times are replicated using a random number generator and the probabilistic distributions of cycle times. Finally, these simulated cycle times are converted to railcar requirements and form a histogram of required fleet sizes. We begin the description of this process with an explanation of Monte Carlo simulation.

## V.c.i) Simulation Technique

KSCC's method for performing fleet sizing is deterministic, where averages are used as estimates for transit and dwell components of cycle time. For instance, the unloading time at a customer City B in Fleet X product group varied from 3 days to 29 days with an average of 14 days. To analyze this type of uncertainty in a deterministic fleet sizing method, individual variables are modified to study the impact on fleet size. In the case of KSCC, they use a contingency factor to pad the fleet size to manage uncertainty. Contingency factors may help manage risk, but they do not capture the underlying uncertainty. One can imagine that with hundreds of customers and multiple product groups, there will be thousands of uncertain variables, and thereby it is infeasible to perform this type of deterministic analysis.

Monte Carlo simulation creates distributions for variables and therefore maps all possible realizations and gives them a probability of occurrence based on historical data. The primary output is a histogram that maps the range of possible outcomes for fleet sizes according to various instances of the random variables. Using this methodology will help Kendall Square Chemical Company choose the appropriate fleet size by combining uncertainty with relevant costs and risks. Further, commercial packages available to perform Monte Carlo Simulations can provide additional outputs that help study issues such as; Which variables have a significant impact on total fleet size? This would help management focus their efforts on reducing uncertainty in these variables or managing the contracts with the customer involved. The @RISK simulation model computes regression coefficients for the transit time random variables. These regression coefficients can provide a measure of how much the output will change if the input is changed by one standard deviation by only assuming that such a change could be implemented independently.

## V.c.ii) Simulation Model Description

In our simulation, we used a Monte Carlo simulation technique to develop a histogram of required fleet sizes. The details of the mechanics of this simulation are provided in this section. For each iteration of the simulation, a random number is generated for each probability distribution. The @RISK software package uses a portable random number generator based on a subtractive method (Guide to Using @RISK, 2010). This random number is an output of the given probability distribution, and this technique is known as Monte Carlo sampling (Albright et al., 2011). In the long run, the outcomes are expected to occur with the frequencies specified by the probabilities in the distribution (Albright et al., 2011). A distinct number is generated for each individual stage which represents the length of time required for that stage, in that specific iteration. For each iteration, the lengths of time for each stage are summed to determine the expected total cycle time for that origin-destination pair, in that specific iteration.

Once the expected cycle time for an origin-destination pair is determined, that information is used to determine the expected number of railcars required to service the customer. The calculation to determine the number of railcars for each customer uses the expected railcar cycle time output and the forecasted demand by the customer. The forecasted demand by customer is used to determine the total number of cycles required in a year based on the forecasted demand and the capacity of a railcar. This operation is shown in Equation ).

$$
\begin{equation*}
\text { Total Required Cycles Per Year }=\frac{\text { ForecastedAnnualDemand }}{\text { RailcarCapacity }} \tag{6}
\end{equation*}
$$

Then, as shown in Equation (7), the number of cycles that one railcar is able to make in a year is determined by dividing the available days in a year (363) by the cycle time outcome for that iteration.

$$
\begin{equation*}
\text { Cycles Per Railcar Per Year }=\frac{363}{\text { Expected Cycle Time }} \tag{7}
\end{equation*}
$$

Once this value is determined, the model divides the total number of "cycles required" in a year to satisfy demand by the number of cycles that one railcar can complete in a year. This value is the recommended fleet size for one iteration, for one OD pairing. This operation is shown in Equation (8).

$$
\begin{equation*}
\text { Required Railcars Per Year }=\frac{\text { Total Required Cycles Per Year }}{\text { Cycles Per Railcar Per Year }} \tag{8}
\end{equation*}
$$

In the simulation, this process is repeated for 10,000 iterations in order to build a histogram of recommended fleet sizes for each customer, within each product group. These distributions are then combined to develop a histogram of required fleet sizes for the entire product group. KSCC can then use this histogram to select a fleet size based on their desired level of risk tolerance.

## V.c.iii) Model Outputs

The SIMULATION Method described above develops a histogram of recommended fleet sizes based on the uncertainty of cycle times in three of the stages of the railcar cycle. Figure 6 shows an example of this output for one customer in the Fleet X Fleet.


Figure 6: Example of Distribution of Railcar Fleet Requirements

After constructing a distribution for each origin-destination pairing, we combined these distributions to create a total sub-fleet distribution for the Fleet X fleet. Figure 7 shows the cumulative distribution of this distribution in graphical and chart form.


| Percentile | Required <br> Fleet Size |
| ---: | ---: |
| $5 \%$ | 80.4 |
| $\mathbf{1 0 \%}$ | 85.8 |
| $\mathbf{1 5 \%}$ | 89.9 |
| $20 \%$ | 93.3 |
| $\mathbf{2 5 \%}$ | 96.2 |
| $\mathbf{3 0 \%}$ | 99.2 |
| $35 \%$ | 102.1 |
| $40 \%$ | 104.9 |
| $45 \%$ | 107.8 |
| $50 \%$ | 110.9 |
| $55 \%$ | 114.0 |
| $60 \%$ | 117.4 |
| $65 \%$ | 121.1 |
| $70 \%$ | 125.3 |
| $75 \%$ | 130.1 |
| $80 \%$ | 135.8 |
| $85 \%$ | 143.0 |
| $90 \%$ | 153.1 |
| $95 \%$ | 171.0 |

## Figure 7: Cumulative Distribution Graph and Chart for Fleet X

After using the simulation technique to construct a distribution of fleet sizes required for each sub-fleet, the fleet manager must then decide the point on this distribution he or she should select in order to have a fleet size that is large enough such that it provides adequate railcar availability at the origin site, but also small enough so that the cars are in service often enough to achieve suitable levels of asset utilization. There are multiple methods that could be used in order to select a point on these distributions to select the appropriate fleet size. In our Economic Models section, we will describe a method that is designed to enable a fleet manager to select a size which balances the cost of over capacity against the cost of railcar shortages.

## V.c.iv) Limitations of the Simulation Model

While we feel that there are several significant advantages of this model over Kendall Square Chemical Company's current process, it is important to understand its limitations. We see two important limitations of this model that should be noted.

1. The model assumes uniform customer ordering throughout the year. By making this assumption, the model does not account for any variability in customer ordering patterns. It assumes that orders are placed in a uniform manner, and that customer ordering patterns will not impact the required fleet size. If this is not the case, and order patterns are not uniform, a more robust model would need to be constructed to incorporate this. Possible adaptions to this model to account for non-uniform ordering include the use of a queuing model or the use of a discrete event simulation to estimate the effect of order patterns. Each of these options would require us to have access to historical customer order data, which was not available for this study.
2. The model relies on historical data to develop distributions for the simulation, and therefore is not immediately applicable to new destinations or destinations where limited historical data is present. This limitation is also found in Kendall Square Chemical Company's current fleet sizing model, and is overcome by Kendall Square Chemical Company's planners using a process that involves estimating these expected fleet requirements using intuition to estimate the expected cycle time performance based on analysis of destinations in similar regions, the customers holding time behavior at other destinations, and several other factors. Our proposed model relies on similar analysis to be done by Kendall Square Chemical Company's planners to account for a lack of historical data.

## VI. Economic Model

In this method, we propose that Kendall Square Chemical Company use an approach that allows the percentile that is selected to be calculated based on a comparison of the cost of not having an adequate number of cars to service customers with the cost of having excess capacity. We describe this method in the following section.

## VI.a. Cost Driven Fleet Sizing Model: The COST PERCENTILE Method

In selecting an appropriate fleet size from the distribution, Kendall Square Chemical Company should seek to determine a fleet size that balances the cost of not having a car available when needed with the cost of having an excess car that is not being utilized. While the cost of an excess car is relatively easy to determine, the cost of a not having a car can be difficult to calculate. There are many factors that should be included in cost of not having an available car, including:

- Contractual penalties from late shipments to customers
- Under-utilization of loading labor and assets
- The costs of switching to a different transportation mode to expedite the shipment (truck shipment)
- Additional plant changeovers forced by a schedule change as a result of not having the correct railcar
- The production of alternative products (not on the schedule) in order to keep the plant running, resulting in inventory carrying costs and margin loss.
- Unmet demand due to railcar shortages may be lost to competitors.

The above list is not comprehensive but should serve as the basis for determining the cost of not having a railcar. Through our conversations with Kendall Square Chemical Company employees, we have identified the costs associated with parts of the above list, and have arrived at estimates of the total cost. Gaining more accuracy in this number will be important for Kendall Square Chemical Company to be able to determine what risk level is appropriate for the business in terms of fleet sizing.

Our recommendation for Kendall Square Chemical Company as to the appropriate fleet size for their fleet involves adapting a theory from inventory management known as the Newsvendor model. In the traditional inventory context, a Newsvendor analysis is applied to a situation in which a one-time decision must be made to select a purchase quantity of goods to satisfy a percentage of expected demand over a finite time period (Silver, Silver, Pyke, \& Peterson, 1998). In order to determine the targeted percentage of expected demand to be satisfied, a ratio of the cost of underage compared to the cost of overage is calculated, resulting in a number between zero and one. This number is then used as the percentile to be selected from the cumulative distribution of possible results. This process will be discussed in more detail later on in this section. Before describing the process further, we first discuss the applicability of this inventory model to Kendall Square Chemical Company's fleet sizing scenario.

A Newsvendor analysis is typically applied to inventory management in situations that exhibit some or all of the following features (Silver et al., 1998):

- A finite and well-defined selling season exists.
- Purchasers are forced to commit to stocking quantities before the beginning of the selling season.
- There may or may not be opportunities to purchase additional quantities during the selling season.
- Demand during the selling season is uncertain.
- There is a shortage cost, known as $\mathrm{C}_{\mathrm{u}}$ (cost of underage), associated with not fulfilling each unmet unit of demand.
- There is an overage cost, known as $\mathrm{C}_{0}$ (cost of overage), associated with each unit of inventory that is carried beyond what is required of demand.

While the applicability of this approach to that of railcar fleet sizes may not be inherently obvious by reading the list of features provided above, we argue that it provides intuition in order to select a fleet size from the cumulative distribution of our simulation results. The basis for this argument is that KSCC's fleet sizing problem has the following characteristics:

- KSCC must determine its fleet size prior to the beginning of a quarter and generally maintains that size throughout the quarter. KSCC may have opportunities to acquire additional cars on the spot market, but given the specialized requirements of many cars in their fleet, spot cars cannot always be acquired.
- There is a measurable cost of having extra railcars - either a leasing expense or the capital cost of owning a car.
- There is a measurable cost of not having a car available when needed. Through interviews with Kendall Square Chemical Company employees we have determined three types of shortage costs. This will be discussed in more detail later.
- Railcar cycle times are uncertain, therefore creating an effect comparable to that of uncertain product demand in the traditional newsvendor application.

Because of the similarities of between Kendall Square Chemical Company's fleet sizing characteristics and a Newsvendor inventory model, and the previously cited work by Beaujon and Turnquist (1991) suggesting that the fleet sizes be calculated using a shortage cost, we have used an adaptation of this technique to recommend Kendall Square Chemical Company's fleet size.

The most significant argument against the applicability of this ratio technique is that of the effect due to the dynamic nature of railcars being loaded, sent out for shipped, and then returned for reuse within the same time period of analysis. Therefore, a "shortage" could be experienced one day, and then cars could be returned from their cycles and thus result in an "overage" the next day. This phenomenon does not exist in the Newsvendor inventory model. Once product is sold out, it is presumed that all future orders with be unfulfilled. While this argument limits the model from being directly applied to fleet sizing, there are enough similarities that the concept of using the ratio of the cost of an extra car versus the cost penalties of not having a car when needed is a useful method and should help guide the fleet modeler in providing a recommended fleet size. For the remainder of this document, we will refer to this method as the COST PERCENTILE model.

In order to develop a COST PERCENTILE method, we first had to determine the cost overage. The primary costs to consider when determining the cost of overage are the ownership cost of the car and expected maintenance costs associated to maintain the car for the given time period. Through a combination of interviews with Kendall Square Chemical Company and general industry research, we estimate the total ownership cost of one railcar to be $\$ 9,500$ annually. Because the minimum ownership period of a railcar is equal to a complete railcar
cycle, we determined the cost of overage to be equal to the lease cost over the average cycle time of 41 days. Therefore, the cost of overage is given by:

$$
\begin{equation*}
\$ 9,500 \times 41 / 365=\$ 1,067 \tag{9}
\end{equation*}
$$

After determining the cost of overage, we then had to determine the cost of underage. As stated previously, there are numerous costs, which contribute to the cost of underage. Some of these costs are obvious and relatively easy to measure, while others are less obvious and thus harder to quantify. In addition, a "shortage" event can result in one of several different outcomes, with each outcome having a different associated cost. Through a combination of research and interviews with an Kendall Square Chemical Company stakeholder, we have identified three different types of shortage events. Those types of shortage events include:

- Type 1 Shortage: A car is not available, but the order can be delayed until a car arrives. The only costs incurred are those of under-utilized labor and equipment. The cost of a Type 1 Shortage is estimated to be $\$ 150$.
- Type 2 Shortage: A car is not available and the order must be shipped via truckload immediately. The primary costs associated with a Type 2 Shortage are the cost penalty of shipping via truck instead of rail and the increased labor requirements of loading several trucks instead of one railcar. The cost of a Type 2 Shortage is estimated to $\$ 2,000$.
- Type 3 Shortage: A car is not available and the product cannot be transported via truck because of safety regulations or lack of availability. As a result the order is canceled, possible contractual fines result, and Kendall Square Chemical Company loses out on the opportunity to sell this load as well as future sales. The cost of a Type Three Shortage is estimated to be $\$ 30,000$.

Having established three different shortage event possibilities, it becomes necessary to combine these possibilities into a single cost for the COST PERCENTILE formula. In order to combine these costs, we assigned probabilities of occurrence to each shortage type, and then found the weighted average cost of not having a railcar. Through interviews with an Kendall Square Chemical Company stakeholder, we have assigned the following probabilities of occurrence to the shortage types when a shortage even occurs:

Probability that the shortage is a Type 1 Shortage: $45 \%$
Probability that the shortage is a Type 2 Shortage: $50 \%$
Probability that the shortage is a Type 3 Shortage: 5\%
By using these probabilities as the coefficients for our weighted average cost of a shortage, we are able to compute the cost of underage to be:

$$
\begin{equation*}
(\$ 150 \times 0.45)+(\$ 2,000 \times 0.50)+(\$ 30,000 \times .05)=\$ 2,475 \tag{10}
\end{equation*}
$$

Having established both the cost of overage and the cost of underage, we can proceed with our calculation of the COST PERCENTILE. Returning to the methodology used in the Newsvendor analysis, the point on the cumulative distribution which balances the cost of excess capacity with the cost of a stockout can be found using the relationship in Equation (11). We have applied this ratio in order to determine the COST PERCENTILE in our analysis.

$$
\begin{equation*}
\text { COST PERCENTILE }=\frac{\text { Cost of Underage }}{\text { Cost of Underage }+ \text { Cost of Overage }} \tag{11}
\end{equation*}
$$

COST PERCENTILE $=\$ 2,475 /(\$ 2,475+\$ 1,067)=0.70$

While this COST PERCENTILE method can be a useful exercise for fleet managers to use to determine an appropriate size fleet, it is not without its limitations. In a later section of this report, we discuss the benefits, potential uses, and limitations of this methodology.

## VI.b. Using the COST PERCENTILE Method to Select a Fleet Size

In a previous section of this report, we explained the general approach of the critical ratio method and showed how a value of 0.70 was calculated for the Kendall Square Chemical Company fleet. In this section, we demonstrate how this method can be used to select an appropriate fleet size. Then, we proceed with a discussion of the sensitivity of this process to changes in the assumptions about the cost of overage and underage. Finally, we discuss how this approach can be an effective tool for managers to use in order to reduce the overall requirements of their fleets.

## VI.b.i) Selecting a Fleet Size

In order to select an appropriate fleet size using the critical ratio, we first developed a histogram of potential required fleet sizes based on the output of our Monte Carlo simulation. As described in the Section V of this report, this distribution is the result of the simulation procedure in which the fleet size requirements of every origin-destination are summed after each iteration of the simulation. Therefore, running 10,000 iterations of the simulation results in having 10,000 fleet size requirements. These potential requirements are then used to create the cumulative distribution function, which is shown in Figure 8.

| Percentile | Required Fleet Size |
| ---: | ---: |
| $\mathbf{5 \%}$ | 80.4 |
| $\mathbf{1 0 \%}$ | 85.8 |
| $\mathbf{1 5 \%}$ | 89.9 |
| $\mathbf{2 0 \%}$ | 93.3 |
| $\mathbf{2 5 \%}$ | 96.2 |
| $\mathbf{3 0 \%}$ | 99.2 |
| $\mathbf{3 5 \%}$ | 102.1 |
| $\mathbf{4 0 \%}$ | 104.9 |
| $\mathbf{4 5 \%}$ | 107.8 |
| $\mathbf{5 0 \%}$ | 110.9 |
| $\mathbf{5 5 \%}$ | 114.0 |
| $\mathbf{6 0 \%}$ | 117.4 |
| $\mathbf{6 5 \%}$ | 121.1 |
| $\mathbf{7 0 \%}$ | 125.3 |
| $\mathbf{7 5 \%}$ | 130.1 |
| $\mathbf{8 0 \%}$ | 135.8 |
| $\mathbf{8 5 \%}$ | 143.0 |
| $\mathbf{9 0 \%}$ | 153.1 |
| $\mathbf{9 5 \%}$ | 171.0 |

Figure 8: Chart of Cumulative Distribution of Fleet Size Requirements

After developing this cumulative distribution function, we plot the function to show a graphical representation, as shown in Figure 9.


Figure 9: Cumulative Distribution Graph of Fleet $X$ with Cost Percentile

Once the cumulative distribution function is created, we select the point on the distribution that corresponds to the COST PERCENTILE value of 0.70 . This point is at the intersection of the two arrows in Figure 9, and occurs at a value of 125 railcars. Therefore, the recommendation for the Fleet X fleet using the COST PERCENTILE method with the stated overage and underage assumptions is to select a fleet size of 125 railcars.

In order to evaluate this method, we first performed sensitivity analysis of the recommended fleet size relative to changes in the two parameters of the COST PERCENTILE formula. To accomplish this, we tested the sensitivity of the recommended fleet size to changes in the estimation of the cost of underage and the cost of overage. In Table 2 and Table 3, we show the results of this sensitivity analysis on each input.

Table 2: Sensitivity Analysis for Changes in the Cost of Overage

| \% Change in Cost of Overage | Cost of Overage |  | COST <br> PERCENTILE | Fleet <br> Size | \% Change in Fleet Size |
| :---: | :---: | :---: | :---: | :---: | :---: |
| -50\% | \$ | 533.56 | 0.82 | 138 | 10.4\% |
| -40\% | \$ | 640.27 | 0.79 | 134 | 7.2\% |
| -30\% | \$ | 746.99 | 0.77 | 132 | 5.6\% |
| -20\% | \$ | 853.70 | 0.74 | 129 | 3.2\% |
| -10\% | \$ | 960.41 | 0.72 | 126 | 0.8\% |
| 0\% | \$ | 1,067.12 | 0.70 | 125 | 0.0\% |
| 10\% | \$ | 1,173.84 | 0.68 | 123 | -1.6\% |
| 20\% | \$ | 1,280.55 | 0.66 | 122 | -2.4\% |
| 30\% | \$ | 1,387.26 | 0.64 | 120 | -4.0\% |
| 40\% | \$ | 1,493.97 | 0.62 | 119 | -4.8\% |
| 50\% | \$ | 1,600.68 | 0.61 | 118 | -5.6\% |

Table 3: Sensitivity Analysis for Changes in the Cost of Underage

| \% Change in Cost <br> of Underage | Cost of <br> Underage | COST <br> PERCENTILE | Fleet <br> Size | \% Change in <br> Fleet Size |  |
| ---: | :--- | :--- | :--- | ---: | ---: |
| $-50 \%$ | $\$$ | $1,237.50$ | 0.54 | 113 | $-9.6 \%$ |
| $-40 \%$ | $\$$ | $1,485.00$ | 0.58 | 115 | $-8.0 \%$ |
| $-30 \%$ | $\$$ | $1,732.50$ | 0.62 | 119 | $-4.8 \%$ |
| $-20 \%$ | $\$$ | $1,980.00$ | 0.65 | 121 | $-3.2 \%$ |
| $-10 \%$ | $\$$ | $2,227.50$ | 0.68 | 123 | $-1.6 \%$ |
| $0 \%$ | $\$$ | $2,475.00$ | 0.70 | 125 | $0.0 \%$ |
| $10 \%$ | $\$$ | $2,722.50$ | 0.72 | 126 | $0.8 \%$ |
| $20 \%$ | $\$$ | $2,970.00$ | 0.74 | 129 | $3.2 \%$ |
| $30 \%$ | $\$$ | $3,217.50$ | 0.75 | 130 | $4.0 \%$ |
| $40 \%$ | $\$$ | $3,465.00$ | 0.76 | 131 | $4.8 \%$ |
| $50 \%$ | $\$$ | $3,712.50$ | 0.78 | 133 | $6.4 \%$ |

The analysis above demonstrates that a $50 \%$ change in either the cost of overage or the cost of underage results in less than a $10.5 \%$ change in fleet size. While a $10.5 \%$ change a the fleet size may seem insignificant, it must be noted that if an incorrect estimation in the cost of overage
results in a fleet manager selecting a fleet which is $10.5 \%$ greater than required, this result would have a substantial impact on the asset utilization performance of the fleet. There will be excess railcars on site, and problems such a lack of storage space will add further financial burdens on top of the low asset utilization. Similarly, underestimating of the cost of underage will result in a fleet size manager selecting a fleet with is too small to adequately service demand at the loading station and will result in significant expediting costs, poor customer service levels, and increased transportation costs. Because this method is sensitive to the underlying assumptions of the costs of underage and overage, and those costs are difficult to accurately estimate, we recommend that additional research be conduct before this approach is implemented fully.

## VI.c. Using the COST PERCENTILE Fleet Sizing Approach to Reduce Fleet

## Size Requirements

The method of using the COST PERCENTILE to select fleet sizes requires additional research before the model should be implemented into commercial use. However, the methodology shows particular promise when viewed from the perspective of a fleet manager attempting to reduce the size of his or her fleet while maintaining the same level of customer service at the loading station.

As we identified in citing previous research performed in the chemical industry, variability in railcar transit times force fleet managers to maintain excess capacity of railcar stock in order to satisfy customer demand. In this context, if a fleet manager wishes to reduce the size of the fleet, he or she must seek to reduce the variability in cycle times or reduce the mean transit and customer holding times. While this may seem like a reasonable undertaking, many of the causes of variable and excessive cycle times are outside of the direct control of most fleet
managers (Closs et al., 2005). In most cases, fleet managers will have limited influence over the rail operators who control the transportation of their cars. In addition, customers are often given an excessive allowance for railcar holding times that is contractual committed (Young, et al. 2003). Although these contracts may limit KSCC from reducing customer holding time length in some cases, it is likely that KSCC will have a better chance of impacting holding time behavior than of affecting railcar transit time performance. Therefore, it may appear that a fleet manager's only option in reducing fleet size is to accept a greater risk of not having available cars when needed. In most cases, this option is unacceptable.

However, we argue that the COST PERCENTILE approach to fleet sizing offers an alternative to this dilemma. By taking measures to reduce the cost of underage, a fleet manager can lower the COST PERCENTILE of his or her fleet, and thus reduce the required fleet size needed. If the actions taken to reduce the cost of underage add flexibility to the fleet's shipping options, then a manager can reduce the fleet size while maintaining the ability to ship product. An example of this is provided below.

## VI.c.i) Managing the Cost of Underage

In our original COST PERCENTILE calculation, we explained three types of possible underage events and the probabilities associated with their occurrence. In the worst case scenario, a Type 3 shortage, a $\$ 30,000$ penalty is occurred because there is no option to transport the load, and there is a five percent chance of this occurring when a shortage event takes place. If investments are made to the loading capabilities of the origin destination and partnerships are formed with regional trucking operations, it should be possible to reduce the possibility of this occurrence. If this were the case, Type Three shortages would become Type Two, and thus result in a penalty of $\$ 2,000$. If reduced to zero the COST PERCENTILE yields the following:

$$
\begin{equation*}
\text { Cost of Underage }=(\$ 150 \times 0.45)+(\$ 2,000 \times 0.55)=\$ 1,075 \tag{13}
\end{equation*}
$$

The cost of overage remains unchanged from the previous analysis at $\$ 1,067$. With the newly calculated cost of underage, the COST PERCENTILE becomes:

$$
\begin{equation*}
\text { COST PERCENTILE }=\frac{\text { Cost of Underage }}{\text { Cost of Underage }+ \text { Cost of Overage }} \tag{14}
\end{equation*}
$$

$$
\begin{equation*}
\operatorname{COST} \operatorname{PERCENTILE}=\$ 1,075 /(\$ 1,075+\$ 1,067)=0.51 \tag{15}
\end{equation*}
$$

At this COST PERCENTILE, the new recommendation for the Fleet X fleet size is 110 cars. This is a 15-car reduction from the fleet size calculated by reducing the probability of a Type Three shortage. Assuming all cars would have been leased and incur maintenance fees, this reduction would result in a savings of \$9,500 per car, for a total cost savings of \$142,500 annually.

The above example does not factor in the upfront costs associated with upgrading loading equipment and other expenses that would be incurred to allow for additional loading of trucks. However, the cost savings generated by the reduction in fleet could be combined with the upfront cost of the equipment improvements in an economic cash flow analysis.

## VII. Analysis

In the Analysis section of this report we begin by presenting the fleet sizing requirements for the Fleet X for each of the three models that were discussed in the previous section of the report. We then proceed with a discussion of these results and the differences between the model outputs. We follow this with an explanation of what accounts for these differences, and why they are significant in understanding the nature of the fleet sizing problem.

## VII.a.Model Output Results

Table 4 shows the results of the three different fleet sizing models. As discussed in Section I, KSCC states that ten railcars are required for spotting or are out of service for maintenance repairs. The impact of this is captured in the "Cars Staged" line of the requirements table. This is an operational constraint, not an additional form of buffering.

As can be seen from the results, the railcars required at the mean cycle time for both deterministic methods (118 cars) and the output of the simulation (116.8) are very similar. However, when generating values above the mean using each of the three methods, differences in the recommended fleet size begin to surface. In the following sections, we discuss the reasons for the differences in these results.

Table 4: Comparison of Method Results for Fleet X

| Destination City | Cars <br> Required @ <br> Deterministic <br> Mean | Method 1: <br> Cars Required to Buffer Mean 15\% | Method 2: Cars Required @ $65 \%$ of Normal Distribution | Cars Required at Simulation Mean Cycle Time | Method 3: Cars Required @ $65 \%$ Simulation Distribution |
| :---: | :---: | :---: | :---: | :---: | :---: |
| City A | 7.8 | 9.0 | 8.7 | 7.8 | 8.2 |
| City B | 30.5 | 35.1 | 38.7 | 28.7 | 29.4 |
| City C | 9.0 | 10.3 | 10.0 | 9.1 | 9.9 |
| City D | 2.0 | 2.3 | 2.1 | 2.0 | 2.1 |
| City E | 11.3 | 13.0 | 13.3 | 11.4 | 11.8 |
| City F | 24.0 | 27.7 | 25.2 | 24.4 | 25.6 |
| City G | 10.2 | 11.7 | 11.5 | 10.1 | 10.6 |
| City H | 13.6 | 15.6 | 15.1 | 13.3 | 13.7 |
| Cars Staged | 10 | 11.5 | 10 | 10 | 10 |
| Total Fleet Size | 118.4 | 136.2 | 134.6 | 116.8 | 121.1 |

## VII.a.i) Comparison of Methods \#1 and \#2

In Method \#1, the mean fleet size is calculated, and then buffered by adding an additional fifteen percent of the mean. In Method \#2, the same sample data is used to determine mean cycle time and standard deviation. To calculate a fleet size requirement, the sample cycle time data is assumed to follow a normal distribution, and then a percentile is selected from the cumulative distribution function of the distribution. As we will discuss in the following section, for certain cycle stages of some origin-destination pairings, the assumption of normal distributions is appropriate, while in many others, it is not. This limitation of Method \#2 will be addressed in a later section.

We begin our comparison of the results of these two methods by evaluating the aggregate fleet requirement of Fleet X. As can be seen in Table 4, Method \#1 recommends a fleet size of 136.2 railcars after accounting for buffering, while Method \#2 recommends 134.6 at the $65^{\text {th }}$ percentile. At the aggregate level, these two methods produce a result that appears comparable.

However, upon further inspection of the railcar requirements at the origin-destination pairing level, the differences in the outputs of the two methods can be seen.

Table 5 shows a comparison of Method \#1 and Method \#2 at the origin-destination level. At this level, it becomes obvious that the methods produce differing results. For example, Table 5 shows that Method \#1 produces a railcar requirement of 35.1 for City B, while Method \#2 results in a 38.7 car requirement, thus requiring 3.6 railcars more. In the case of City F , the opposite is true. Method \#1 produces a requirement of 27.7 railcars, while Method \#2 requires only 25.2 railcars. The underlying cause for this difference can be explained by understanding the effect of the standard distribution of sample data on Method \#2. As can be seen in Table 5, City B has a high standard deviation of cycle time relative to the mean, resulting in a coefficient of variation of 0.69 . Thus, it is intuitive that a model, which accounts for variability in cycle time data, such as Method \#2, will produce a larger fleet size requirement in this instance. Likewise, City F has a relatively low coefficient of variation at 0.13 , and thus Method \#2 produces a lower fleet size relative to Method \#1. This benefit can attributed to the impact of risk pooling. As we will describe in the following section, this property of Method \#2 provides advantages over the traditional fleet sizing model.

## Table 5: Comparison of Model 1 and Model 2

| Destination City | Deterministic <br> Mean Cycle <br> Time | Deterministic <br> Cycle Time <br> Standard <br> Deviation | Deterministic Coefficient of Variation | Cars <br> Required @ <br> Deterministic <br> Mean | Method 1: <br> Cars Required to Buffer Mean 15\% | Method 2: Cars Required @ 65\% of Normal Distribution |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| City A | 27 | 8 | 0.28 | 7.8 | 9.0 | 8.7 |
| City B | 49 | 34 | 0.69 | 30.5 | 35.1 | 38.7 |
| City C | 38 | 12 | 0.32 | 9.0 | 10.3 | 10.0 |
| City D | 36 | 4 | 0.11 | 2.0 | 2.3 | 2.1 |
| City E | 31 | 14 | 0.46 | 11.3 | 13.0 | 13.3 |
| City F | 65 | 8 | 0.13 | 24.0 | 27.7 | 25.2 |
| City G | 46 | 16 | 0.35 | 10.2 | 11.7 | 11.5 |
| City H | 37 | 10 | 0.28 | 13.6 | 15.6 | 15.1 |
| Cars Staged |  |  |  | 10 | 11.5 | 10 |
| Total Fleet Size |  |  |  | 118.4 | 136.2 | 134.6 |

Fleet Managers will select a railcar fleet size above that required by mean cycle times in order to protect against the uncertainty of several factors, one of the primary of which is uncertain cycle time performance. Therefore, measuring this variability and including it in the fleet sizing process provides the benefit exemplified in the examples of City B and City F above; in cases of high variability, larger buffers are required, while in cases of low variability, smaller buffers are sufficient. Method \#1, by using a fifteen percent of mean buffer, is unable to account for these differences. In addition, Method \#2 would help a fleet manager to easily identify the components of cycle time for destinations that account for the most variability. He or she can then focus on managing the variability in these components in an effort to reduce overall fleet size requirements.

Nevertheless, Method \#2 does not account for the true underlying distribution of the data, and therefore is only a modest improvement over the existing model. Obtaining a fleet size using a certain percentile over mean, still does not account for the actual risks of running out of rail cars or leasing/owning extra cars. Without a clear understanding of these costs and way to use
these costs in identifying risk, the model would return fleet sizes in excess/short of ideal requirements. It is intuitive to conclude that Kendall Square Chemical Company may have fleet sizes in excess of need, based on analysis of 2010 loading time distributions, in which the loading times were in excess of the purported five days required for loading; therefore, the 15 percent buffer may be in excess of requirements. To bridge the gap in this method, we attempt in the next section to identify the true underlying distributions behind the transit data and compare the results to this method.

## VII.a.ii) Comparison of Method \#2 to Method \#3

In this section, we evaluate the assumption of normality in Method \#2 against Method \#3, which fits the transit data to its best-fit theoretical distributions. In this section, we seek to study the impact of this method of fleet sizes requirements. As we described in the Methods section of this report, Method \#3 uses a stochastic process which fits historical data to a theoretical distribution, replaces this data with the distribution, and then uses the distribution as an input to a Monte Carlo simulation.

Table 7 shows a comparison of Method \#2 to Method \#3 by origin-destination pairing and by aggregate fleet size. We note that at the mean cycle time value, the deterministic and stochastic methods return comparable fleet size recommendations, with 118.4 and 116.8 , respectively. However, when comparing the cumulative fleet size requirements of the two methods at the $65^{\text {th }}$ percentile of their distributions, the fleet size requirement of Method \#2 is 134.6 railcars, which is significantly larger than the 121.1 railcar requirement of Method \#3. This result was initially surprising, especially considering the similar results of the standard deviation of railcar car cycle times between the empirical data used in Method \#2 and the
simulation results of Method \#3. With nearly identical means and similar coefficients of variation, we expected a similar fleet sizing result at the $65^{\text {th }}$ percentile of each distribution.

Table 6: Comparison of Model 2 and Model 3 Results

| Destination City | Deterministic <br> Mean Cycle <br> Time | Deterministic Cycle Time Standard Deviation | Deterministic <br> Coefficient of <br> Variation | Cars <br> Required @ <br> Deterministic <br> Mean | Method 2: Cars <br> Required @ <br> $65 \%$ of Normal <br> Distribution | Simulation <br> Mean <br> Cyde Time | Simulation Cyde Time Standard Deviation | Simulation Coefficient of Variation | Cars Required at Simulation Mean Cycle Time | Method 3: Cars <br> Required @ <br> $65 \%$ Simulation <br> Distribution |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| City A | 27 | 8 | 0.28 | 7.8 | 8.7 | 26.6 | 7.4 | 0.28 | 7.8 | 8.2 |
| City B | 49 | 34 | 0.69 | 30.5 | 38.7 | 45.4 | 25.4 | 0.56 | 28.7 | 29.4 |
| City C | 38 | 12 | 0.32 | 9.0 | 10.0 | 37.8 | 10.2 | 0.27 | 9.1 | 9.9 |
| City D | 36 | 4 | 0.11 | 2.0 | 2.1 | 36.2 | 4.0 | 0.11 | 2.0 | 2.1 |
| City E | 31 | 14 | 0.46 | 11.3 | 13.3 | 31.5 | 12.6 | 0.40 | 11.4 | 11.8 |
| City F | 65 | 8 | 0.13 | 24.0 | 25.2 | 66.2 | 7.9 | 0.12 | 24.4 | 25.6 |
| City G | 46 | 16 | 0.35 | 10.2 | 11.5 | 45.9 | 17.0 | 0.37 | 10.1 | 10.6 |
| City H | 37 | 10 | 0.28 | 13.6 | 15.1 | 37.7 | 12.4 | 0.33 | 13.3 | 13.7 |
| Cars Staged |  |  |  | 10 | 10 |  |  |  | 10 | 10 |
| Total Fleet Size |  |  |  | 118.4 | 134.6 |  |  |  | 116.8 | 121.1 |

In order to understand the factors which cause this significant difference in the fleet size requirements of Method \#2 and Method \#3, we again investigated differences in railcar requirements at the origin-destination pairing level. Using the output displayed in Table 6, we observed that some individual origin-destination pairings were nearly identical when comparing the two methods at the $65^{\text {th }}$ percentile, while others exhibited significant differences. For example, the railcar requirement at the $65^{\text {th }}$ percentile for City $C$ under Method $\# 2$ is 10.0 railcars, while using Method \#3 the requirement is 9.9 . However, in City B, Method \#2 requires 38.7 railcars while Method \#3 requires only 29.4 railcars. Furthermore, in the case of railcar requirements using Method \#3 for City B, we observed that the $65^{\text {th }}$ percentile was only slightly larger than the requirements at the mean ( 28.7 cars at the mean versus 29.4 at the $65^{\text {th }}$ percentile). Again, this was a surprising result, especially considering the relatively high coefficient of variation, 0.56 , of the City B simulation result. A similar comparison of the Method \#2 and Method \#3 results for all origin-destination revealed that, in all the cases where there was greater than a two percent difference in fleet size between the two methods, the requirements of Method \#3 were less than Method \#2. In summary, it appeared that in some cases the normal distribution
and the simulation distribution produced the same results, but when differences occurred, the normal distribution was over-estimating the fleet size requirements when compared the simulation results of Method \#3. In order to understand the reasons for this difference, investigated the effect of the underlying data distributions being used un the models.

## VII.b.The Impact of Skewness of Input Distributions

In this section, we explain our findings of how the distributions of certain cycle time stages impacts the overall fleet size requirements. We describe distributions modeled during our analysis and relate the properties of these distributions to statistics theory to explain their impact on our fleet sizing Methods.

Method \#2 relies on the assumption of a normal distribution for all origin-destination fleet size requirement distributions. But, frequently, the underlying data for these components follow a theoretical distribution like in Figure 10; the distribution is not symmetrical around the mean.


Figure 10: Cycle Time Distribution with Positive Skewness

Symmetrical distribution of data implies that there are an equal number of observations on both sides of the mean. The mean and the median are equal in symmetrical distributions (Hardy \& Bryman, 2004). In distributions like the one pictured in Figure 10, where there are unequal observations on both sides of the mean, the Coefficient of Skewness helps describe the distribution of the underlying data better. The Coefficient of Skewness describes the relationship between the mean and the median of a distribution and is a signed number indicating which tail of the distribution contains the fewest number of values. In Figure 10, extreme positive values drag the mean to the right tail of the distribution; therefore, the mean is greater than the median.

A relatively few large values on the right hand side can increase the mean to the right side of the distribution although the distribution may be positively skewed; in other words, the
mean is greater than the median, and there are more values to the left of the mean than the right. This leads to an important conclusion about distributions such as in Figure 10: The more skewed the distribution, the more it becomes necessary to use more than one measure of central tendency to describe it (Hardy \& Bryman, 2004). The medians ( $50^{\text {th }}$ percentile) in positively skewed distributions such as in Figure 1 are less than the mean, and using medians to calculate fleet sizes would yield values less than using means. Therefore, since distributions of the underlying data of KSCC's transit times are frequently not symmetrical, the assumption of normal distributions in fleet size requirements may not appropriately forecast fleet sizes. As the distributions are mostly positively skewed (reference Figure 13 and Figure 14 in the Appendix), using the normal distribution would lead to estimating higher than required fleet sizes than if using relative measures of central tendency such as the median ( $50^{\text {th }}$ percentile).

To illustrate, consider the histograms of fleet sizes of the following two destination locations: City H and City F. As shown in Figure 11, in the case of City F, the distribution resembles a normal distribution, and the mean and the median are the same. The mean fleet size from the empirical sample data and the simulation result require approximately the same fleet size; 24.4 and 24.0 , respectively. Furthermore, the fleet size requirement at the $65^{\text {th }}$ percentile for Method \#2 is 25.2 railcars, and for Method \#3 it is 25.6. Thus, it is intuitive to understand that the distribution of required fleet sizes for City F appears to follow a Normal distribution. In the case of City F, the assumption of normality in Method \#2 is appropriate.


Figure 11: City F Fleet Size Histogram; Symmetrical Distribution Example

Now, we will examine the distribution of required fleet sizes for City H. As shown in Figure 12, City H has a distribution that is positively skewed (Mean, 13.3 is higher than the median of 12.3). Again, the railcar requirement at the mean cycle from the empirical data is comparable to that of the simulation result. However, there is a significant difference at the $65^{\text {th }}$ percentile of Method \#2 and Method \#3. In Method \#2, the requirement is 15.1 , while in Method \#3 it is only 13.7. Thus, it becomes clear that the simulation output of City H cannot be characterized by the normal distribution.


Figure 12: City H Fleet Size Histogram; Positively Skewed Distribution Example

To understand why the normal distribution can be an appropriate assumption in some cases and in others is not, we must look at the characteristics of the underlying data input into the model. Figure 13 shows the three input distribution for the railcar cycle time stages at City F. While viewing the graphical illustration of the inputs to City F, it becomes obvious that the resulting fleet size distribution will resemble a normal distribution.


Figure 13: Cycle Stage Input Distributions for City F

However, the cycle time input distributions for City H are substantially different. From Figure 14, it is intuitive to understand that with these inputs to the simulation, the distribution of the resulting fleet size requirement will be substantially skewed. The underlying distributions of cycle time data for City H account for the skewness in the distribution of total fleet size requirements for City H seen in Figure 12.


Figure 14: Cycle Stage Input Distribution for City H

## VII.c. Summary of Analysis

The distributions of transit time stages have a significant impact on railcar fleet sizing requirements for Kendall Square Chemical Company. Buffering the mean by a static percentile and the assumption of normal distribution each fail to recognize these distributions, and thus produce results that do not account for the differences in transit time characteristics. Modeling the underlying data using theoretical distributions and running Monte Carlo simulations adjusts for the flawed assumptions in the above methodologies. This methodology accounts for the
positive skewness that is evident in the underlying data and models risk more accurately. We recommend this methodology for Kendall Square Chemical Company.

## VIII. Conclusion

This section has been prepared in order to summarize the findings of our research. In this conclusion, our intent is to provide an explanation of the research findings in a context that can be applied to the management of a railcar fleet.

The implications of the above findings for a fleet manager as follows:

1. Incorporating cycle time variability into a fleet sizing model is necessary in order to gain an accurate understanding of the underlying sources of volatility in the fleet. By measuring variability and using it to drive fleet sizing decisions, managers can begin to identify lanes that are forcing excessive fleet requirements and focus efforts to reduce variability.
2. In the case of Kendall Square Chemical Company, this over-estimation of railcar requirements can be prevented by recognizing the distributions of cycle time data. In many cases, transit and customer holding time data exhibit positive skewing. This skewing results in the mean providing an inaccurate depiction of the central tendency of the data. The use of distribution fitting and Monte Carlo simulation can help a fleet manager gain a more accurate understanding of fleet size requirements.
3. There is potential to factor in economic considerations when selecting a fleet size from a distribution of size requirements. By comparing the cost of excess capacity against the cost of under-capacity, a fleet manager can determine how much risk he or she should take in selecting a fleet size.

## IX. Recommendations for Future Research

In this section of the report we discuss future research that could be conducted to build upon the findings of our project.

## IX.a. Improvements to the Existing Simulation Model

While the simulation model described in this report offers advantages over the current method in that it recognizes the importance not only of incorporating variability in railcar cycle time, but also of characterizing the distributions of the cycle stages, it assumes a uniform order arrival rate and constant, known demand. While discussions with Kendall Square Chemical Company employees led to the conclusion that these assumptions are appropriate for this project, in reality, there will be some degree of uncertainty associated with demand and order arrival. By constructing a model that incorporates these uncertainties, a model can be developed that allows a fleet manager to more accurately replicate all sources of uncertainty inherent in the fleet sizing process.

In addition, a more comprehensive model would be able to incorporate order delivery promise dates. By simulating order arrival, railcar cycles, and performance against promised arrival date at given fleet sizes, a modeler would be able to use an iterative process in which railcar fleet sizes are determined by an economic model, and then those sizes are tested in the simulation to determine its impact on customer service performance.

## IX.b. Testing Assumptions of Independence

As previously discussed Section III, we have made two assumptions of independence in our analysis. The first assumption is that the stages of the railcar cycle within an origindestination pairing are independent of each other. The second assumption is that the cycle
performance of individual origin-destination pairings is independent. Both of these assumptions should be tested through additional research.

## IX.c. Testing the "Shortage" Event Limitation in the COST PERCENTILE Model

As we discussed in Section VI.a, our adoption of the Newsvendor analysis is limited by the difference between a shortage event in inventory management and a shortage event in the case of a railcar fleet. Additional research is required to impact of this difference on the application of the COST PERCENTILE calculation. This research would be valuable to fully develop a cost driven fleet sizing model.

## X. Appendix

Table 7: Fleet X Input Distributions for Simulation Model

| Destination | Cycle Stage | Distribution Graph | Distribution Type | Distribution Mean | Distribution Std Dev | Sample Mean | Sample <br> Std Dev |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CITY A | Loaded Transit |  | Log Logistic | 4.7 | 1.1 | 4.7 | 2.1 |
| CITYA | Unloading Time |  | Weibull | 12.9 | 7.3 | 12.9 | 7.1 |
| CITY A | Empty Transit |  | Gamma | 4.0 | 1.3 | 4.0 | 1.6 |
| СГІү ${ }^{\text {B }}$ | Loaded Transit |  | Log Logistic | 14.1 | 24.2 | 16.9 | 33.2 |
| СГТץ ${ }^{\text {B }}$ | Unloading Time |  | Log Logistic | 13.5 | 5.8 | 13.5 | 5.8 |
| СпTY B | Empty Transit |  | Logistic | 13.5 | 5.3 | 13.5 | 5.8 |
| cITY | Loaded Transit |  | Logistic | 7.5 | 4.2 | 7.3 | 6.6 |
| CITY | Unloading Time |  | Triangular | 14.3 | 7.3 | 11.3 | 8.6 |
| CITY | Empty Transit |  | Gamma | 11.3 | 4.7 | 13.9 | 4.7 |
| CITY ${ }^{\text {D }}$ | Loaded Transit |  | Logistic | 11.3 | 2.3 | 11.2 | 2.3 |
| CITY ${ }^{\text {D }}$ | Unloading Time |  | Weibull | 11.0 | 2.8 | 11.0 | 2.8 |
| CITY ${ }^{\text {D }}$ | Empty Transit |  | Exponential | 8.8 | 1.9 | 8.9 | 1.4 |



Table 8: Fleet Y Input Distributions for Simulation Model

| Destination | Cycle Stage | Distribution Graph | Distribution Type | Distribution Mean | Distribution Std Dev | Sample <br> Mean | Sample <br> Std Dev |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CITY AA | Loaded Transit |  | Logistic | 11.8 | 5.9 | 11.2 | 6.5 |
| CITY AA | Unloading Time |  | Exponential | 4.7 | 4.0 | 4.9 | 3.9 |
| CITY AA | Empty Transit |  | Logistic | 7.9 | 3.7 | 7.6 | 3.9 |
| CITY BB | Loaded Transit |  | Beta General | 11.9 | 10.2 | 12.0 | 9.8 |
| CITY BB | Unloading Time |  | Log Logistic | 12.7 | 7.3 | 12.2 | 4.7 |
| CITY BB | Empty Transit |  | Inverse Gaussian | 14.3 | 11.4 | 14.3 | 7.0 |
| CITY CC | Loaded Transit |  | Log Logistic | 9.4 | 10.6 | 9.6 | 10.6 |
| CITY CC | Unloading Time |  | Exponential | 9.1 | 4.0 | 9.2 | 5.8 |
| CITY CC | Empty Transit |  | Exponential | 14.8 | 11.1 | 15.3 | 10.1 |
| CITY DD | Loaded Transit |  | Inverse Gaussian | 8.0 | 10.5 | 8.0 | 10.1 |
| CITY DD | Unloading Time |  | Beta General | 8.5 | 3.5 | 8.5 | 3.6 |
| CITY DD | Empty Transit |  | Inverse Gaussian | 8.7 | 3.8 | 8.7 | 3.6 |


| CITY | Loaded Transit |  | Log Normal | 11.9 | 9.0 | 11.9 | 9.1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CITY 区 | Unloading Time |  | Logistic | 9.4 | 3.9 | 9.8 | 4.7 |
| CITY ${ }^{\text {Ex }}$ | Empty Transit |  | Log Normal | 10.4 | 4.0 | 10.5 | 4.2 |
| CITYFF | Loaded Transit |  | Log Normal | 8.5 | 10.5 | 8.9 | 13.6 |
| CITY FF | Unloading Time |  | Log Normal | 11.5 | 7.8 | 11.5 | 7.4 |
| CITYFF | Empty Transit |  | Exponential | 10.6 | 4.8 | 10.8 | 6.1 |

Table 9: Fleet Z Input Distributions for Simulation Model

| Destination | Cycle Stage | Distribution Graph | Distribution Type | Distribution Mean | Distribution Std Dev | Sample <br> Mean | Sample <br> Std Dev |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| City AAA | Loaded Transit |  | Inverse Gaussian | 9.1 | 1.5 | 9.1 | 1.5 |
| City AAA | Unloading Time |  | Exponential | 10.4 | 7.4 | 10.6 | 6.7 |
| City AAA | Empty Transit |  | Log Logistic | 9.3 | 1.5 | 9.3 | 1.4 |
| City BBB | Loaded Transit |  | Log Logistic | 14.2 | 4.0 | 14.3 | 4.2 |
| City BBB | Unloading Time |  | Logistic | 25.8 | 8.0 | 25.6 | 8.4 |
| City BBB | Empty Transit |  | Log Normal | 17.9 | 7.5 | 17.9 | 7.1 |
| City CCC | Loaded Transit |  | Log Normal | 13.4 | 2.7 | 13.4 | 2.7 |
| City CCC | Unloading Time |  | Weibull | 21.3 | 12.1 | 21.4 | 11.8 |
| City CCC | Empty Transit |  | Exponential | 12.2 | 3.2 | 12.3 | 3.5 |
| City DDD | Loaded Transit |  | Exponential | 6.6 | 3.4 | 6.8 | 4.6 |
| City DDD | Unloading Time |  | Extreme Value | 5.5 | 3.3 | 5.6 | 3.7 |
| City DDD | Empty Transit |  | Inverse Gaussian | 5.6 | 6.1 | 5.6 | 5.5 |
| City ${ }^{\text {mex }}$ | Loaded Transit |  | Log Normal | 5.3 | 2.3 | 5.3 | 2.5 |
| City ExE $^{\text {I }}$ | Unloading Time |  | Exponential | 6.6 | 5.0 | 6.6 | 4.4 |
| City ${ }_{\text {ExE }}$ | Empty Transit |  | Log Logistic | 5.1 | 6.8 | 4.9 | 2.2 |


| City FFF | Loaded Transit | $8$ | Gamma | 14.0 | 3.1 | 14.0 | 3.2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| City FFF | Unloading Time | $-5$ | Log Logistic | 16.3 | 7.5 | 16.0 | 7.4 |
| City FFF | Empty Transit | $\stackrel{8}{8}$ | Gamma | 12.6 | 2.5 | 12.5 | 2.9 |
| City GGG | Loaded Transit | $\stackrel{4}{7}$ | Beta General | 7.3 | 0.9 | 7.3 | 0.9 |
| City GGG | Unloading Time | $\stackrel{-5}{\nabla}$ | Inverse Gaussian | 10.3 | 6.6 | 10.3 | 6.2 |
| City GGG | Empty Transit |  | Exponential | 6.2 | 2.0 | 6.2 | 3.3 |
| City HHH | Loaded Transit |  | Garma | 17.9 | 5.8 | 17.9 | 6.8 |
| City HHH | Unloading Time | ${ }^{-5}$ | Extreme Value | 17.6 | 9.3 | 17.7 | 9.3 |
| City HHH | Empty Transit |  | Weibull | 14.9 | 2.0 | 14.9 | 2.1 |
| City III | Loaded Transit |  | Logistic | 10.9 | 2.7 | 11.2 | 3.3 |
| City III | Unloading Time | $8 \quad 10$ | Exponential | 2.1 | 2.0 | 2.2 | 2.6 |
| City III | Empty Transit |  | Log Logistic | 27.1 | 20.4 | 26.2 | 17.3 |
| City JJJ | Loaded Transit | $8 /{ }^{8}$ | Triangular | 13.5 | 2.5 | 13.6 | 2.5 |
| City JJJ | Unloading Time |  | Normal | 30.8 | 14.1 | 29.9 | 15.0 |
| City JJ | Empty Transit | $19$ $\stackrel{28}{\nabla}$ | Inverse Gaussian | 14.6 | 3.8 | 14.6 | 3.9 |
| City KKK | Loaded Transit |  | Log Logistic | 5.8 | 0.8 | 5.9 | 0.9 |
| City KKK | Unloading Time | 素 | Exponential | 14.9 | 8.6 | 15.1 | 7.9 |
| City KKK | Empty Transit |  | Beta General | 7.2 | 1.0 | 7.2 | 1.0 |


| City ய | Loaded Transit | $\% \quad 22$ | Gamme | 14.8 | 2.3 | 14.9 | 2.4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| City [1 | Unloading Time |  | Log Logistic | 11.2 | 23.4 | 10.0 | 6.9 |
| City ש | Empty Transit |  | Weibull | 14.3 | 2.5 | 14.3 | 2.5 |
| City MMM | Loaded Transit |  | Garma | 14.2 | 2.6 | 14.2 | 2.7 |
| City MMM | Unloading Time |  | Exponential | 14.3 | 13.3 | 14.7 | 11.1 |
| City MMM | Empty Transit |  | Logistic | 13.3 | 2.4 | 13.0 | 3.2 |
| City NNN | Loaded Transit |  | Gamma | 9.4 | 1.4 | 9.4 | 1.4 |
| City NNN | Unloading Time |  | Exponential | 14.5 | 9.4 | 14.7 | 8.1 |
| City NNN | Empty Transit |  | Gamme | 10.3 | 2.7 | 10.3 | 2.9 |
| Cily 000 | Loaded Transit |  | Exponential | 10.3 | 2.1 | 10.4 | 1.8 |
| City 000 | Unloading Time |  | Inverse Gaussian | 5.7 | 4.0 | 5.7 | 3.9 |
| City 000 | Empty Transit |  | Log Normal | 10.6 | 3.4 | 10.7 | 3.9 |
| City PPP | Loaded Transit |  | Weibull | 16.7 | 4.6 | 16.7 | 4.7 |
| City PPP | Unloading Time |  | Exponential | 2.3 | 1.8 | 2.3 | 1.7 |
| City PPP | Empty Transit |  | Normal | 11.4 | 2.2 | 11.4 | 2.2 |
| City QQQ | Loaded Transit |  | Weibull | 11.8 | 1.8 | 11.8 | 1.8 |
| City QQQ | Unloading Tme |  | Logistic | 12.5 | 3.6 | 12.2 | 3.5 |
| City QQQ | Empty Transit |  | Log Logistic | 12.7 | 2.4 | 12.6 | 2.1 |


| City RRR | Loaded Transit |  | Weibull | 9.4 | 1.6 | 9.4 | 1.7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| City RRR | Unloading Time |  | Extreme Value | 6.5 | 2.8 | 6.6 | 3.3 |
| City RRR | Empty Transit |  | Gamma | 11.1 | 2.2 | 11.1 | 2.2 |
| City SSS | Loaded Transit | $\begin{array}{\|l\|r\|} \hline 8 & 28 \\ \hline \end{array}$ | Log Logistic | 16.1 | 3.8 | 16.0 | 3.4 |
| City SSS | Unloading Time | $0$ | Weibull | 3.8 | 1.8 | 3.9 | 1.9 |
| City SSS | Empty Transit | $\begin{array}{\|lr\|} \hline 12 & 30 \\ \nabla & 7 \end{array}$ | Inverse Gaussian | 17.2 | 3.6 | 17.2 | 3.2 |
| City TIT | Loaded Transit | $9$ | Inverse Gaussian | 12.1 | 1.4 | 12.1 | 1.3 |
| City TIT | Unioading Time |  | Log Normal | 9.4 | 7.8 | 8.9 | 4.4 |
| City TTT | Empty Transit |  | Log Logistic | 11.5 | 2.2 | 11.4 | 1.8 |
| City UUU | Loaded Transit |  | Weibull | 6.1 | 1.0 | 6.1 | 1.1 |
| City UUU | Unloading Time |  | Exponential | 7.2 | 5.9 | 7.4 | 6.1 |
| City UUU | Empty Transit | $3 / 10$ | Log Logistic | 5.2 | 1.6 | 5.2 | 1.6 |
| City WV | Loaded Transit | $\stackrel{1}{*}$ $\begin{gathered} 12 \\ \nabla \end{gathered}$ | Extreme Value | 6.5 | 1.5 | 6.5 | 1.4 |
| City WV | Unloading Time | $\%$ | Inverse Gaussian | 4.7 | 3.9 | 4.7 | 3.1 |
| City W | Empty Transit |  | Log Logistic | 5.8 | 1.2 | 6.0 | 2.2 |
| City www | Loaded Transit | $\mid 5$ $35$ | Log Logistic | 18.5 | 5.0 | 18.4 | 4.2 |
| City www | Unloading Time | $01$ | Exponential | 2.5 | 1.5 | 2.5 | 1.6 |
| City WWW | Empty Transit |  | Beta General | 14.2 | 2.1 | 14.2 | 2.1 |



Table 10: Fleet Sizes Results of All Methods for Fleet Y

|  | Cars Required <br> @ Deterministic <br> Mean | Method 1: Cars <br> Required to <br> Buffer Mean 15\% | Method 2: Cars <br> Required @ 65\% of <br> Normal Distribution | Cars Required at <br> Simulation Mean <br> Cycle Time | Method 3: Cars <br> Required @ 65\% <br> Simulation Distribution |
| :--- | ---: | :--- | :--- | :--- | :--- |
| City AA | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 |
| City BB | 18.0 | 20.7 | 20.0 | 18.1 | 18.9 |
| City CC | 17.3 | 19.9 | 20.0 | 17.0 | 17.9 |
| City DD | 23.7 | 27.3 | 27.2 | 23.8 | 24.1 |
| City EE | 33.7 | 38.8 | 37.6 | 33.3 | 35.4 |
| City FF | 16.1 | 18.5 | 18.9 | 15.9 | 16.4 |
| Cars Staged | 10 | 11.5 | 10 | 10.0 | 10.0 |
| Total Fleet Size | $\underline{119.1}$ | $\underline{137.0}$ | $\mathbf{1 3 4 . 0}$ | $\underline{118.4}$ | $\mathbf{1 2 3 . 0}$ |

Table 11: Fleet Sizes Results of All Methods for Fleet Z

| Destination City | Cars Required @ Deterministic Mean | Method 1: Cars Required to <br> Buffer Mean 15\% | Method 2: Cars Required @ 65\% of Normal Distribution | Cars Required at Simulation Mean Cycle Time | Method 3: Cars Required <br> @ 65\% Simulation <br> Distribution |
| :---: | :---: | :---: | :---: | :---: | :---: |
| City AAA | 9.6 | 11.0 | 10.4 | 9.5 | 9.7 |
| City BBB | 63.5 | 73.0 | 68.1 | 63.6 | 66.6 |
| City CCC | 41.5 | 47.7 | 45.3 | 41.4 | 43.6 |
| City DDD | 1.4 | 1.6 | 1.6 | 1.4 | 1.4 |
| City EEE | 51.1 | 58.7 | 56.0 | 51.6 | 53.5 |
| City FFF | 37.5 | 43.1 | 40.1 | 37.8 | 39.7 |
| City GGG | 12.9 | 14.8 | 14.1 | 12.9 | 13.4 |
| City HHH | 9.2 | 10.6 | 10.0 | 9.2 | 9.7 |
| City III | 9.6 | 11.1 | 11.1 | 9.7 | 10.1 |
| City JJJ | 12.6 | 14.5 | 13.8 | 12.7 | 13.8 |
| City KKK | 11.8 | 13.6 | 12.9 | 11.7 | 11.9 |
| City LLL | 16.9 | 19.4 | 18.0 | 17.3 | 17.3 |
| City MMM | 3.9 | 4.5 | 4.3 | 3.9 | 4.0 |
| City NNN | 9.8 | 11.3 | 10.6 | 9.7 | 10.0 |
| City 000 | 5.3 | 6.1 | 5.6 | 5.3 | 5.4 |
| City PPP | 24.7 | 28.4 | 26.2 | 26.4 | 26.2 |
| City QQQ | 12.4 | 14.3 | 13.0 | 12.5 | 13.0 |
| City RRR | 4.3 | 4.9 | 4.5 | 4.3 | 4.4 |
| City SSS | 14.0 | 16.1 | 14.6 | 14.0 | 14.3 |
| City TTT | 3.7 | 4.3 | 3.9 | 3.9 | 3.7 |
| City UUU | 3.6 | 4.2 | 4.0 | 3.6 | 3.6 |
| City VVV | 12.5 | 14.4 | 13.4 | 12.4 | 12.7 |
| City WWW | 23.3 | 26.8 | 24.4 | 23.4 | 24.0 |
| City XXX | 22.2 | 25.6 | 23.5 | 22.4 | 22.8 |
| City YYY | 9.7 | 11.2 | 10.2 | 10.2 | 10.6 |
| City Z 72 | 14.3 | 16.5 | 15.3 | 14.3 | 15.0 |
| Cars Staged | 10.0 | 11.5 | 10.0 | 10.0 | 10.0 |
| Total Fleet Size | 451.1 | 518.8 | 484.8 | 455.0 | 470.4 |

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