

Singularity analysis: a tool for extracting lithologic and stratigraphic content from seismic data

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Abstract

In this work, we test an amplitude-independent method of seismic data analysis designed to extract lithologic information about stratigraphic horizons. We apply the method of singularity characterization in an attempt to determine the sharpness of lithologic boundaries. We infer the sharpness of the boundary based upon a fractional integration of the seismic trace. The order of fractional integration is taken to represent the abruptness of the lithologic transition responsible for a given reflector. We find that the method output behaves in a geologically reasonable manner which suggests that our method is responding to lithologic variations along boundaries responsible for prominent reflectors in the data.

1 Introduction

Seismic data is a valuable tool in that it provides the geologist with information on the geometrical and temporal relationships between subsurface lithologic boundaries. However, this form of data is lacking in direct information about the exact lithologies associated with a given seismic reflector. Generally, spatial information from seismic interpretations is used to construct conceptual geologic models to explain the origins of stratigraphic features. It is from such models that we infer lithologic content.

Lithologic transitions are an example of geologic features that do not lend themselves readily to quantification. As a result, and for simplicity, we often treat such transitions as sharp and well-defined. This simplification is effective in the case of seismic data sets where reflectors appear as discrete events. In actuality, however, these seismic features are often generated by complex lithologic changes composed of sub-seismic scale layering or gradational lithologic transitions, not the presumed step functions. By treating transitions between rock types as jump discontinuities, we lose an aspect of geologic content that is vital for an accurate understanding of the geologic system.

In this study, we test a method designed to identify and quantify lithologic boundaries. Specifically, we attempt to extract information about the boundaries causing seismic reflectors by characterizing local maxima within a seismic data set. We test this method by focusing our analysis on two subsets of a seismic line within the the Jeanne d'Arc Basin: an area containing an unconformity that is lithologically well-constrained by core data but poorly imaged, and an area where the geometry of the surrounding units is well-constrained but lacks definite lithologic information. We pose two general questions: 1) does singularity characterization represent stratigraphy that we cannot see in the seismic data but is known to exist; and 2) does the method have the potential to provide us with additional information that would be useful in the development of geologic models and/or interpretations? If, in fact, this method allows us to extract detailed lithologic information, not only will we have a tool that provides increased insight into subsurface geology but we will have a method for quantifying a previously descriptive geologic feature. Quantifying lithologic boundaries, in turn, will give us a means of directly comparing numerical geologic model outputs to real geology and, therefore, a basis for assessing the strength of those numerical models.

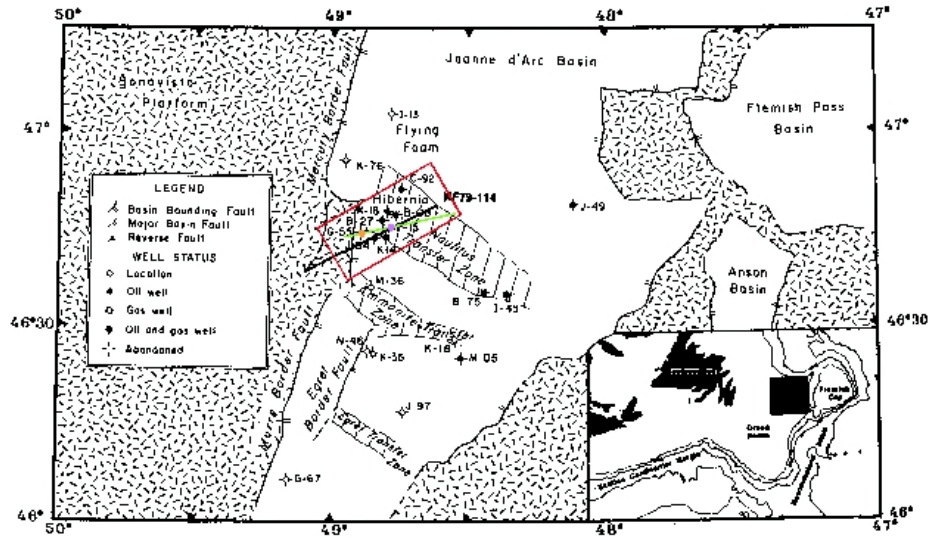


Figure 1: Map of the southern Jeanne d'Arc Basin including study area (red box), well locations, and major border faults. Inset provides regional context. The seismic line used in this study (green) intersected the G-55 well (orange) and the P-15 well (purple). From [Driscoll and Hogg \(1995\)](#).

2 The Jeanne d'Arc Basin

We chose the Jeanne d'Arc Basin (JdA) in the Grand Banks region off the coast of Newfoundland (Figure 1) to test the method of singularity characterization. Hydrocarbon exploration and production of the Grand Banks began in the late 60's/early 70's and the Hibernia field was discovered in the JdA in 1979. Hibernia proved to be a large field with up to 1 billion bbl of recoverable reserves ([Meneley, 1986](#)). By the late 1980's over 300,000 km of reflection seismic lines had been collected in the Grand Banks area with more than 6000 km of those being three-dimensional seismic data. In addition, by mid-1987, 56 deep exploration wells were drilled in the JdA alone ([Tankard and Welsink, 1987](#)).

The Jeanne d'Arc Basin is part of a larger rift system that developed during the breakup of the Pangean super-continent (Figure 1). The Grand Banks resulted from late Triassic/early Cretaceous brittle extension and the consequent development of an extensive system of normal faults. The JdA is bounded on the west by two major, eastward dipping listric normal faults: the Mercury Border Fault and the Murre Border Fault (Figure 1). Faulting was controlled, in part, by the preexisting fabric of the Avalon and Meguma Terranes ([Haworth and Keen, 1979](#); [Tankard and Welsink, 1987](#); [Enachescu, 1988](#)). Later stages of basin development (from the end of the early Cretaceous to the present) are due primarily to post-rift thermal subsidence ([Tankard and Welsink, 1987](#); [Enachescu, 1988](#)). For this work, we extracted a line from a larger, 3-D data set and focused our analysis on two specific portions of that line (Figures 1 and 2).

The first area of interest for this study is a portion of the Avalon Sand identified by [Driscoll and Hogg \(1995\)](#) as the G-55 wedge (Figure 3). The Avalon is one of the producing reservoirs in the JdA basin and it posed an interesting problem. The P-15 well was found to produce oil, however, the stratigraphically up-dip G-55 well encountered only wet sand (Figure 4; [Tankard and Welsink \(1987\)](#)). Based upon seismic data and well logs, Tankard and Welsink posited that deposition of the Avalon sequence was dominated by shallow marine shelf or basinal processes and that the Eastern Shoals/Avalon Sand packages represented a transgressive-regressive sequence ([Tankard and Welsink, 1987](#)). To explain the up-dip presence of the water in the Avalon reservoir, they appealed to a structural trap - a synthetic normal fault isolating the down-dip oil reservoir from the up-dip aquifer (Figure 4).

While Tankard and Welsink based their analysis of the Avalon stratigraphy solely on seismic data and well logs, [Driscoll and Hogg \(1995\)](#) included cores in their assessment (Figures 3 and 5). From this additional

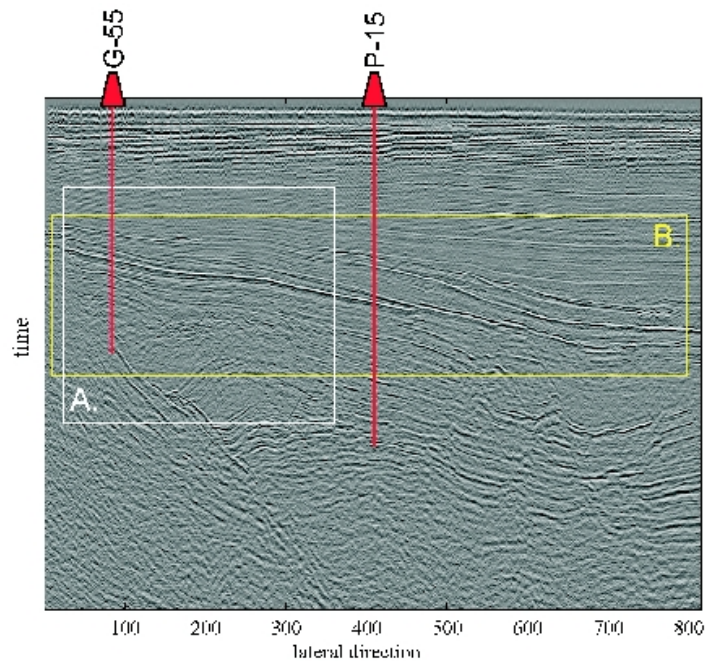


Figure 2: Seismic line extracted for analysis in this study. Two particular areas of interest are noted: the G-55 wedge (A.) and the prograding clinoforms (B.) (see also Figure 3). The two wells used by Driscoll and Hogg (1995) to constrain the lithology of the G-55 wedge are indicated in red.

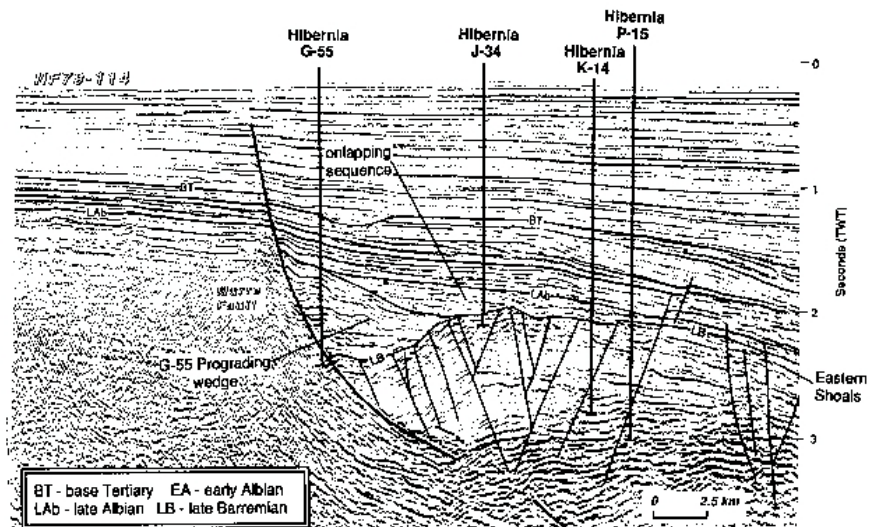


Figure 3: Interpreted seismic line (NF79-114) from Driscoll and Hogg, 1995. Two regions of this line are of interest in the present study: 1) the Avalon Formation which includes the two packages identified above as the “G-55 Prograding wedge” and the “onlapping sequence” and 2) the prograding clinoforms found immediately below the base Tertiary unconformity (“BT”) to the right side of this figure.

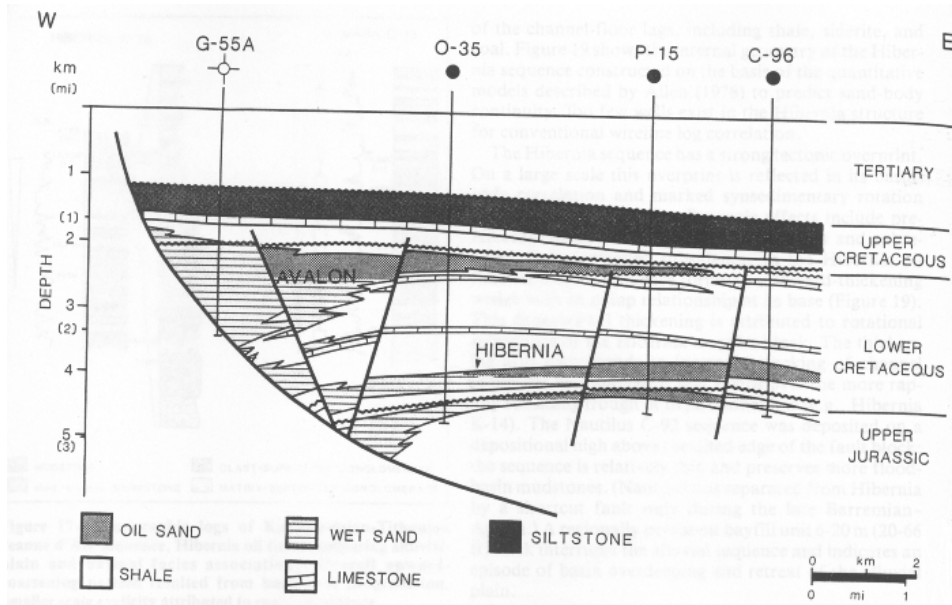


Figure 4: Stratigraphic model of Tankard and Welsink (1987) explaining the presence of water up-dip of the Avalon sand. Here, an appeal is made to a synthetic normal fault as a structural trap for the Avalon oil. From Tankard and Welsink (1987).

information, it was apparent that the Tankard and Welsink model required modification. The cores revealed that the muds underlying the Avalon sand were shallow water facies, not deep water as previously thought (Tankard and Welsink, 1987; Driscoll and Hogg, 1995), and that the Eastern Shoals Formation represented diminished marine influence up-section. This led to a regressive-transgressive model – the inverse of that constructed by Tankard and Welsink. This revised model allowed for a new mechanism for partitioning the Avalon sand: a stratigraphic trap. Driscoll and Hogg argued that the late Barremian hiatus was actually a rift onset unconformity and that the western extent of the Avalon sand represented prograding fluvial/fan delta deposits. They based this interpretation, in part, on the presence of diverging reflectors in the region proximal to the Murre fault (Figure 3). Analysis of the G-55 and P-15 cores were consistent with this interpretation; the sands overlying the southwestern extent of the unconformity (Well G-55) contained high-angle cross-bedding and pebble lags and lacked bioturbation while the sands to the northeast (Well P-15) were either massive or contained low-angle cross-bedding and were often highly bioturbated. It was, they argued, this lateral lithologic change from fluvial/fan delta deposits to on-lapping basinal sands that caused the partitioning of the Avalon reservoir (Figure 5).

Driscoll and Hogg’s interpretation of the presence of the fluvial/fan delta deposits is based upon core analysis and seismic stratigraphy, but the late Barremian unconformity is poorly imaged in the seismic data used for this study (Figures 2 and 6). An identification of the G-55 wedge would be difficult from the seismic data alone and a tool for emphasizing the stratigraphy in this section of the data would be extremely helpful in constructing an accurate geologic model. In the case of the G-55 wedge, singularity analysis would be potentially useful in two ways: 1) it could highlight stratigraphy that is “lost” in migration artifacts and noise and/or 2) it could allow us to locate the late Barremian unconformity more accurately than is possible from the seismic data alone, by emphasizing angular relationships of bedding.

While the first area of our study, the Avalon Sands, are faulted syn-rift deposits, the second area analyzed in this study is located within the younger post-rift sediments (Figures 2, 5 and 7). These sediments, which sit above the Petrel Limestone and below the base Tertiary unconformity, represent an overall period of clinof orm progradation and sea level fall. There is a general sedimentation pattern that consists of progradation (and slight aggradation) followed by a basin-ward shift in deposition. At least three cycles of clinof orm development/depocenter shift occur between the Petrel and the base Tertiary unconformity

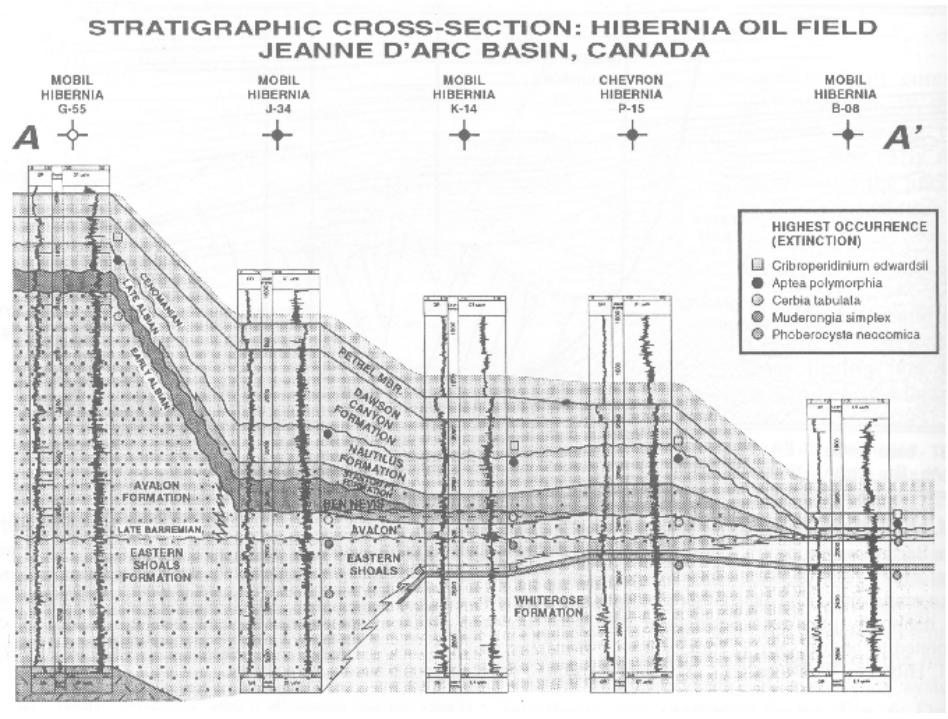


Figure 5: Stratigraphic correlation between wells in the Jeanne d’Arc Basin illustrating the presence of a stratigraphic trap in the Avalon Formation (Driscoll and Hogg, 1995). The sediments of the Avalon near the G-55 well contain pebble-size clasts and so Driscoll and Hogg interpreted these deposits to be a prograding wedge (the G-55 wedge). The remainder of the Avalon sediments are basal sand deposits. From Driscoll and Hogg (1995).

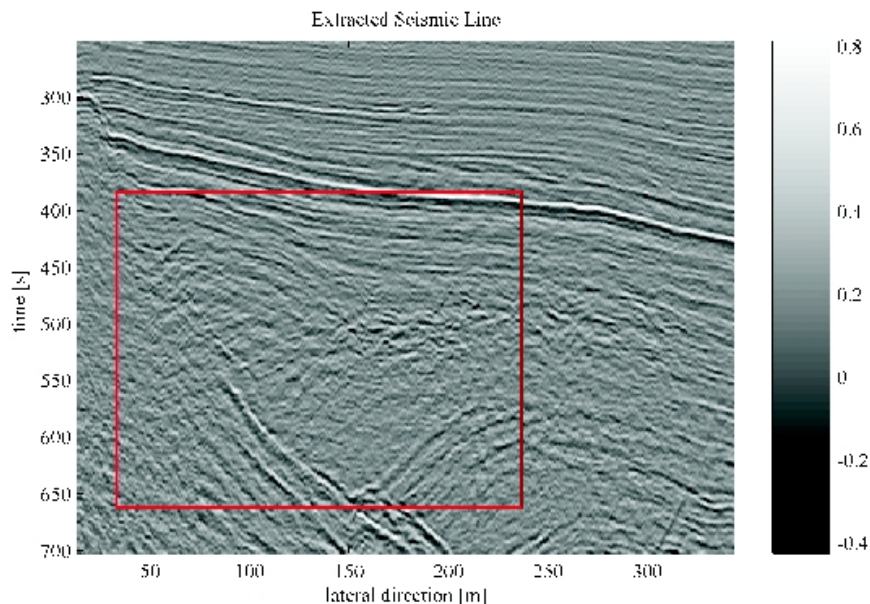


Figure 6: Seismic data from the G-55 wedge (Region A. in Figure 2). Red box indicates the area of the wedge. Imaging of the stratigraphy in this region is not clear; determining true dip of the beds in the wedge, as well as the exact position of the late Barremian unconformity, is difficult.

(Figure 7, bottom). These cycles are of interest in the search for hydrocarbons because the basinward shift in sedimentation results in the deposition of material in the form of low-stand or basin-floor fans. However, these basin-floor deposits are not always sandy; on occasion they prove to be mud rich and, therefore, poor reservoirs (Kolla and Macurda, 1988; Posmentier and Vail, 1988). Unfortunately, determination of lithology cannot be made based upon geometry of the deposits alone (Kolla and Macurda, 1988; Posmentier and Vail, 1988). Singularity characterization would be valuable in these circumstances if it could provide us with information on the lithology of the fan deposits and allow us to distinguish between sand-rich fans versus mud-rich fans. In this example, we use singularity analysis, not to constrain stratigraphic geometry as with the G-55 wedge but, to provide information about the abruptness of the bounding surfaces between deposits.

3 Data

The data set used in this study is a 3-D seismic cube collected for Petro Canada. The volume covers an area of about 20 km x 25 km and extends down to 4.459 s two-way travel time (Figure 1). The two wells described above (G-55 and P-15) were used for lithologic constraint of the late Barremian surface. We relied upon Driscoll and Hogg (1995) for that lithologic analysis as well as for stratigraphic interpretation. The G-55 and P-15 wells are positioned at $N46^{\circ}44'17.01''/W48^{\circ}53'10.75''$ and $N46^{\circ}44'58.98''/W48^{\circ}46'51.18''$, respectively (Figures 1 and 2).

The seismic data were visualized and interpreted using Schlumberger's GeoQuest software. A line intersecting both wells was extracted and converted from Schlumberger's SEG Y format to standard ASCII via Seismic Unix. This was required to provide portability between software packages. The code for characterizing the singularities was written in Matlab by Herrmann.

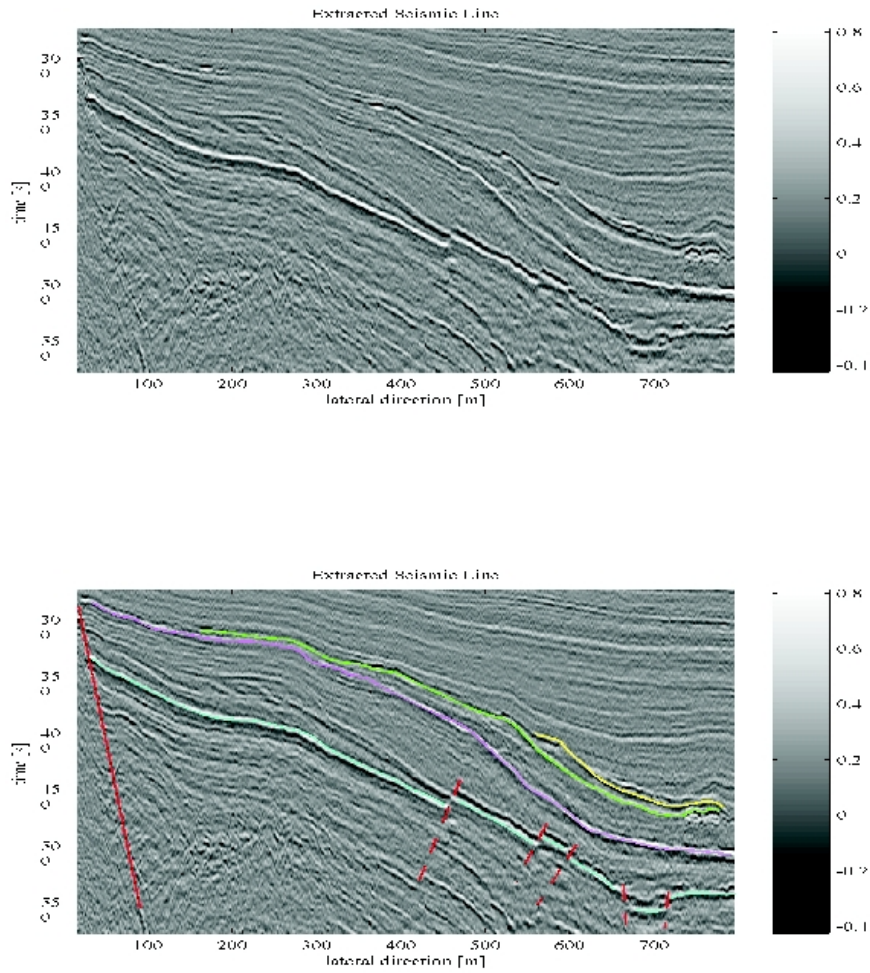


Figure 7: Uninterpreted (top) and interpreted (bottom) data from area B in Figure 2, highlighting the bounding surfaces between clinoforms (note: vertically distorted). Four boundaries are mapped. The Petrel Limestone is indicated in blue while three clinoform surfaces are mapped in purple, green and yellow. The Murre Border Fault is indicated in red and smaller faults that offset the Petrel are indicated in dashed red.

4 Method

Singularities are points in a data series at which the derivative is undefined. From a purely mathematical perspective, the derivative at such a point is a Dirac or delta function. Lithologic transitions can be thought of as singular transitions in grain-size. In seismic data, such “lithologic singularities” are manifested not by the ideal delta function, but rather as a local maxima in the seismic trace. The method of singularity analysis applied in this study identifies a local maximum in the data set and then estimates the order of the corresponding singularity by fractionally integrating until that maximum is removed. The value of the fractional integration that removes the local maximum is the order of the singularity. We describe the method below. For a rigorous description of the mathematical concepts behind singularity analysis and the details of characterization, the reader is referred to [F. \(1997\)](#); [Herrmann and Stark \(2000\)](#); [Verhelst \(2000\)](#) and Herrmann (these proceedings).

This analysis is based upon the fact that the reflectivity response of a “real” (i.e. gradational) lithologic transition is not the idealized delta function that would result from a true step function transition (Figure 8). [Herrmann and Stark \(2000\)](#); [Verhelst \(2000\)](#) argue that a given lithologic boundary has a unique reflectivity response, and that the reflected wavelet is actually a convolution of the source wavelet with that response function (Figure 8, top). The goal of the method applied in this study is to identify the position and order of the local maxima that results from a lithologic transition, and to then extract information about the transition based upon the fractional integration required to eliminate the local maxima (Figure 8, bottom). Essentially, the singularity algorithm identifies a local maximum in the derivative of the original data set and then searches, within a window, for the corresponding maximum in successive fractionally integrated versions of that data set. This can be written as follows:

$$f \mapsto \sigma^\beta \frac{d^\beta}{dt^\beta} (f * \phi_\sigma) \quad (1)$$

where f is the original (seismic) data, ϕ is a smoothing function of scale σ designed to remove high frequency events, and β is a fractional order of integration/differentiation. For β , the following applies: $\beta < 0$ represents fractional integration while $\beta > 0$ represents fractional differentiation.

The order of the singularity (β in Equation 1) is determined to be the largest fractional integration in which the local maximum is identifiable. This value of integration is then mapped to the appropriate location in the original data set. In a seismic data set, this process is repeated for successive vertical traces. The more negative the value of β , the “sharper” or more persistent the singularity. In terms of geologic boundaries, we would expect that a sand-on-sand interface would generate a less-sharp singularity in the data and a larger (less negative) value for β than would a sand-on-mud contact.

One final detail of the singularity analysis algorithm bears mentioning. The method is directional in the sense that we can decide to characterize the singularities associated with positive or negative local maxima. In seismic data, this translates into whether we analyze black (positive) seismic events or white (negative) events. Further testing is required to determine which “direction” is more effective, but it is likely that the direction selected will depend upon the features that we hope to identify. We processed the JdA data looking for both positive and negative events and found no appreciable difference. All figures presented in this paper are from positive runs.

5 Discussion

The results of the singularity algorithm are presented in Figures 9 and 10. Subsets of Figures 9 and 10 were extracted for the analyses presented below. From Figure 10, it is apparent that, to first order, the method of singularity characterization presented here captures stratigraphy in the seismic data.

5.1 The G-55 Wedge

In the JdA seismic data set, the late Barremian surface and the overlying and underlying stratigraphy is poorly imaged; this is particularly true in the vicinity of the G-55 well (Figures 2 and 7). This ambiguity may have contributed to Tankard and Welsink’s appeal to a structural trap to explain the wet G-55 well.

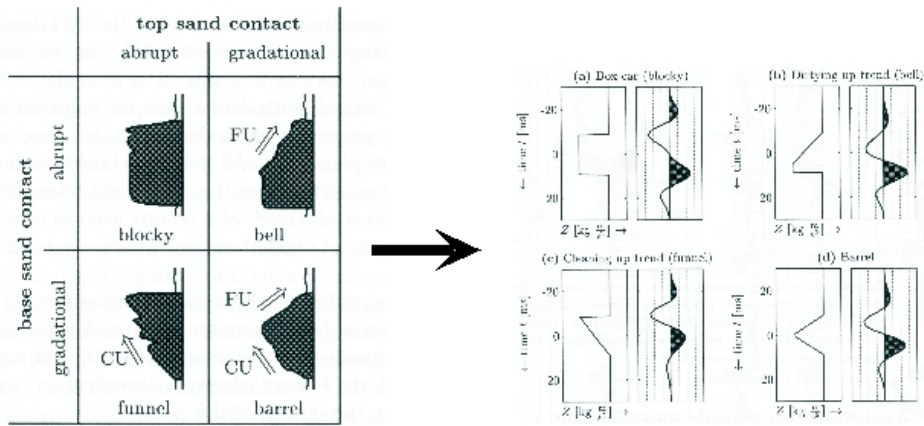
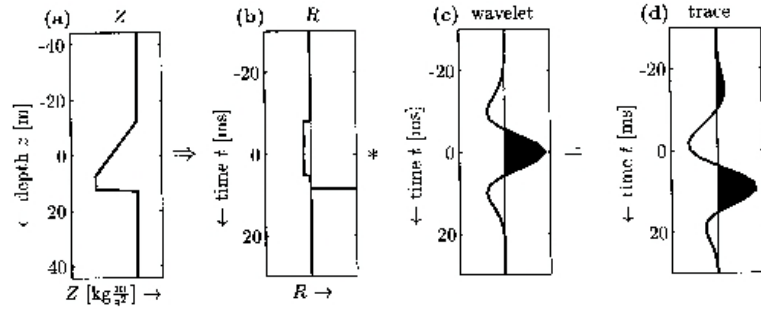


Figure 8: (top) The basic assumption behind singularity analysis is that subsurface lithologies can be translated into specific impedance structures (a) that result in unique reflectivity response functions (b). When those response functions are convolved with a source wavelet (c), in this case a zero-phase Ricker wavelet, unique reflected wavelets (d) result. (bottom) Therefore, different lithologic transitions should result in diagnostic wavelets in the seismic trace. Fractionally integrating these diagnostic wavelets can therefore yield information about the corresponding lithologic transition. From [Verhelst \(2000\)](#).

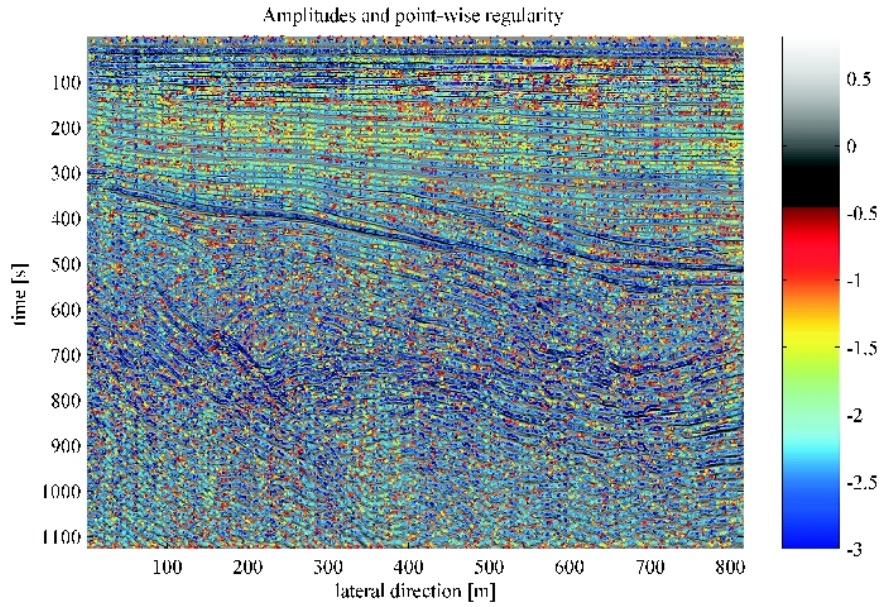


Figure 9: Full seismic line with singularity order mapped to data.

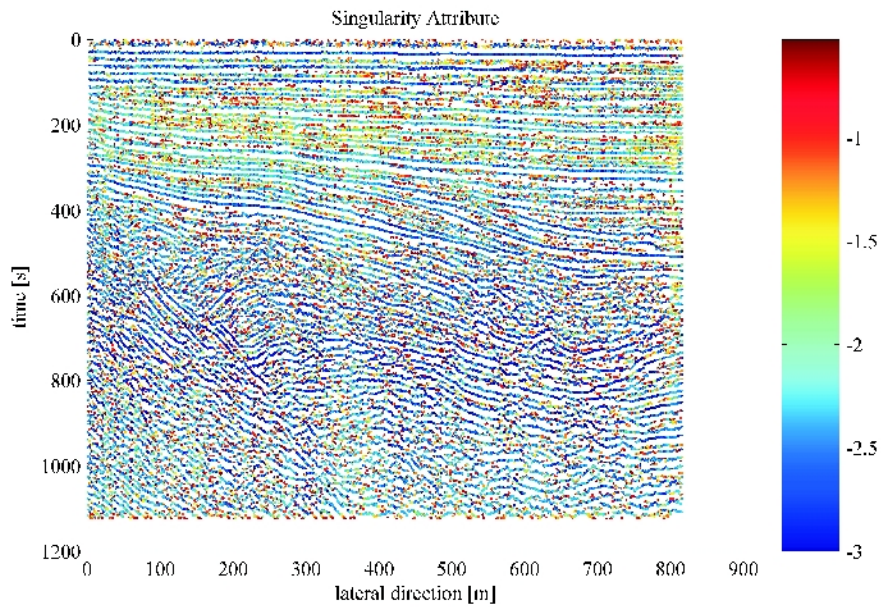


Figure 10: Map of singularity order without original seismic data. In this form, it is apparent that the singularity algorithm is mapping to stratigraphy.

This poor data quality appears to be, in part, a function of the migration process with the end result being that the true stratigraphic relationships are not clearly apparent. To aid in data interpretation, we would like a method that more clearly identifies the late Barremian unconformity and highlights the surrounding stratigraphic relationships (e.g. on-lap and diverging reflectors).

The result of the singularity analysis on the G-55 wedge can be seen in Figure 11. The analysis does not emphasize the Barremian unconformity itself; the exact position of the surface is not identifiable by a coherent singularity “event”. However, the method does emphasize the stratigraphy of the G-55 wedge and the Eastern Shoals formation and therefore allows us to constrain the position of the unconformity more closely than is possible simply from the seismic data. If we look at the angular relationships between the singularities mapped in the Eastern Shoals Formation, and those in the G-55 wedge, we can locate the late Barremian unconformity where those angles change (Figure 11, bottom).

Singularity characterization enhances our knowledge of the stratigraphy around the G-55 well because it “filters out” migration-related artifacts that overshadow the actual stratigraphy in the data. If we view the attribute map alone without the original seismic data (Figure 11), the stratigraphic layering becomes more apparent and we begin to see the southwestward dipping Eastern Shoals Formation and the diverging reflectors of the G-55 fan. This additional information aids our interpretation in two ways: 1) we can more accurately position the late Barremian unconformity by mapping the on-lap relationship of the G-55 fan onto the Eastern Shoals, and 2) we can better identify the diverging reflectors in the G-55 fan which argues for a sun-rift depositional package (Driscoll and Hogg, 1995) and against a post-deposition synthetic fault (Tankard and Welsink, 1987).

5.2 The Progradational Clinofolds

In the second area of interest for this study (area B, Figure 2), we are not concerned with identifying stratigraphy (the prograding delta forms are quite distinct in the seismic data) but rather with extracting more information from the data about the lithologic characteristics of stratigraphy that we know to exist. Here, we would like to see if the singularity orders change in a consistent way along a bounding surface. We focus our attention on three clear horizons between sets of basinward-stepping delta packages (Figure 7). From our stratigraphic model of regressive sequences, we would expect to see a basin-ward shift in sand deposition. Up-dip, this shift would generally place coarse sediment on coarse. Down-dip, however, such a shift in depocenters should result in coarser sediments being deposited on fine (Posmentier and Vail, 1988). If singularity analysis responds to variations in lithologic composition, then the up-dip sand-on-sand relationship should generate a different singularity order than the down-dip sand-on-mud case.

We see that, in fact, the singularity characterization does change down-dip and does so consistently along more than one bounding surface (Figure 12). In contrast, the singularity order does not change along the similarly-dipping event representing the Petrel Limestone. At this point in the development of singularity analysis, the actual value of singularity cannot be interpreted – we have yet to calibrate values with specific lithologic combinations. However consistent behavior along bounding surfaces of similar genesis suggests that the method is responding to characteristics common to those surfaces. It is important to note that relative values of singularity order are relevant and a careful look at the clinofold bounding surfaces yields an interesting result.

As stated above, singularity order should theoretically provide information about the sharpness of the lithologic transition at a stratigraphic bounding surface; the sharper the transition, the more negative the order of singularity. If we follow along a surface between sets of prograding clinofolds, we see that the singularity order becomes more negative as we proceed down-dip (Figure 12). In other words, the lithologic boundary becomes sharper as we go from proximal to distal. This is in agreement with what we know about prograding clinofolds. The proximal ends of the clinofolds are generally sand-on-sand, which generates a smooth transition between lithologies because there is relatively little difference between the two grain sizes. However, down-dip we are consistently placing coarser sediments on finer and generating a sand-on-mud transition. This latter transition is expected to produce a more negative singularity corresponding to a more persistent local maximum and is, in fact, what we see. The output of the singularity method is in agreement with our geologic interpretation. If, as appears to be the case, the singularity analysis is responding to the lithologic make-up of the clinofold sediments, then there is reason to believe that this method could be used

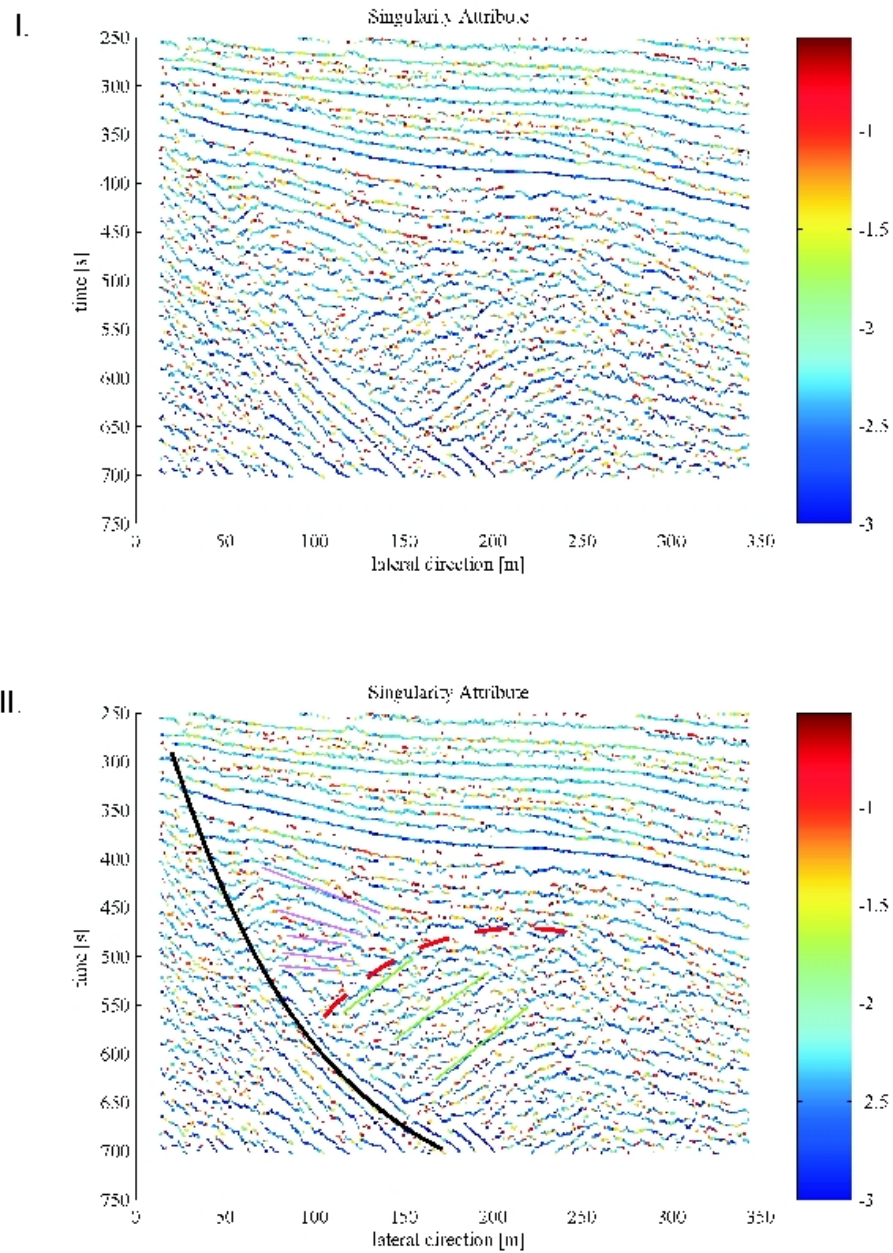


Figure 11: Uninterpreted (top) and interpreted (bottom) maps of singularity analysis from the region of the G-55 wedge (area A, Figure 2). The angular relationship between the divergent reflectors in the wedge (purple) and the west-ward dipping, subparallel reflectors of the underlying Eastern Shoals Formation (green) is apparent and facilitates the mapping of the late Barremian unconformity (dashed red). The Murre Border fault is indicated in black.

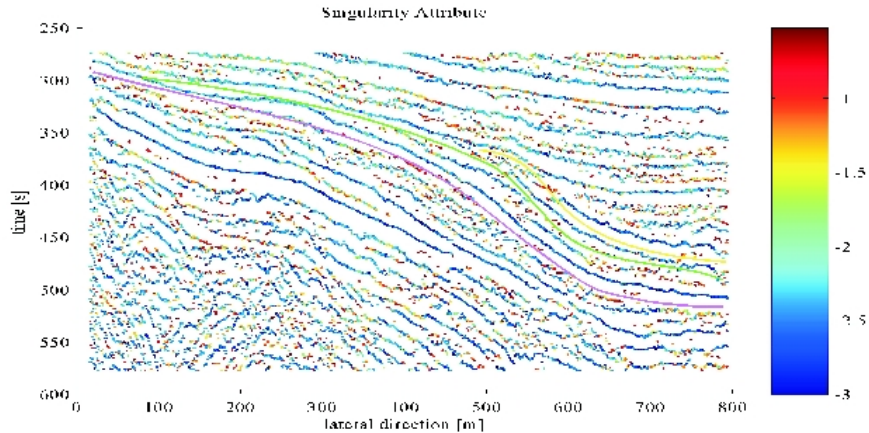


Figure 12: Singularity attributes for clinoforms (area B, Figure 2). Note consistent down-dip decrease in singularity order along clinoform boundaries. The three boundaries mapped in Figure 7 are indicated here with the appropriate color underlining the boundary.

to characterize the composition of packages like basin-floor fans; a muddy fan overlying deep-water deposits should generate a lower order (less negative) singularity than the more abrupt transition created by a sandy fan overlying fine, deep-water deposits.

6 Conclusion

From applying singularity characterization to a known data set and, more specifically, to seismic events with known or confidently predicted lithologic properties, we make the following observations:

1. The method of singularity analysis produces results that are geologically intuitive (Figures 10, 11 and 12). That is, if we identify the positions of singularities in the data set, they follow stratigraphic architecture that we know to exist based upon the amplitude structure of the data set.
2. Singularity analysis can emphasize stratigraphy (Figure 11). In the poorly imaged G-55 wedge, the singularity analysis seems to filter out artifacts and highlight stratigraphic relationships. In the data set used for this study, it is difficult to position the late Barremian unconformity based solely on the seismic data. However, the angular relationships between the bedding of the G-55 wedge and the Eastern Shoals Formation are more apparent in the map of singularity characteristics and those relationships aid in locating the late Barremian surface.
3. Singularity analysis order changes consistently along stratigraphic surfaces of similar genesis (Figure 12). In the particular case of the bounding surfaces between packages of delta foreset deposits, the singularity order changes as we move down-dip on the surface. This is consistent with the fact that we anticipate changes in the lithologic characteristics along this boundary and suggests that the singularity analysis is responding to lithologic characteristics of the interface.
4. In contrast to 3. above, singularity order is consistent along surfaces like the Petrel Limestone that are expected to be relatively homogeneous. Because of motion on the bounding faults and subsidence, the Petrel has a profile similar to that of the clinoform surfaces but does not show similar trends in singularity order.
5. Not only does singularity order change consistently along boundaries, as in the case of the clinoform packages, it also changes in a way that is consistent with a reasonable lithologic model (Figure 12). As we move from proximal to distal along the clinoform boundary, singularity order becomes more

negative indicating a more abrupt lithologic transition. This corresponds to an increased sharpness to the lithologic boundary which is what we would predict in a progradational system.

Based upon the results of this work, we conclude that singularity analysis is a potentially useful tool in the development of stratigraphic models. We say this for two reasons: 1) maps of singularity characteristics mirror known stratigraphy and, in some cases, highlight stratigraphic structure and relationships, and 2) singularity orders change consistently and predictably along surfaces with well-constrained lithologic characteristics. This method is not yet able to provide us with detailed information about lithology, but the results are encouraging and suggest that this method could be used to constrain subsurface lithology and to extract significant geologic content from seismic data.

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