Kinematic redatuming by two source interferometry

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Abstract

Interferometry is a method of redatuming physical sources to receiver locations. Under idealized assumptions stacking the cross correlogram of the two common receiver gathers yields a bandlimited Green's function between the receivers. Geometrically this process amounts to isolating a physical source, which generates a path containing both receivers, and canceling the common part. In this paper, we show that in order to recover the travel time between two receivers, one could creatively use rays from more than one physical source. With this approach redatuming is possible even in situations where the conventional interferometry fails.

1 Introduction

Interferometry is a method for redatuming physical shots to the location of the physical receivers [1, 2, 3, 4, 5]. One of the physical receivers becomes a virtual source (VS), and the rest are virtual receivers (VR). The goal is to obtain an approximation to the gather we would observe had a physical source been placed in lieu of the virtual one. Mathematically, the redatuming is done by cross-correlating common receiver gathers at the virtual source and the virtual receiver locations, and then stacking the resulting correlogram

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[6, 7]. For a high-frequency source wavelet, this process isolates a physical source that emits a ray, which passes through the virtual source and is recorded by the virtual receiver. The common path from the physical source to both receivers is cancelled to yield the sought ray between the virtual source and the virtual receiver [7]. The contributions to the correlogram by other sources are cancelled by the staking operator. We will henceforth refer to this approach as *single source interferometry* to emphasize that in principle only one (*a priori* unknown) physical source is needed to obtain one virtual trace.[1]

Interferometric formula [8, 5, 9] relies on assumptions that are hard to meet in reality. Its blind application will therefore result in an erroneous virtual gather. The gather may contain events corresponding to no real physical structure or miss other events that should have been registered. When the medium is not completely surrounded by physical sources as dictated by the theory, certain structures inside are not properly illuminated. As interferometry does not create new information, the virtual gather will not contain reflections from those objects. This may complicate the use of the gather for imaging purposes or even the interpretation of some available events, e.g., multiples.

In a medium with rich structure, multiple scattering can make up for insufficient source coverage. Randomly positioned diffractors, for example, will act as secondary sources that provide illumination from all angles and create a field satisfying the equipartition condition. Under this condition, interferometry works successfully as before [10].

In this paper, we present a method of recovering missing reflections in the virtual gather due to poor illumination. We consider a simple model with a single reflector and thus no multiple scattering. Single source interferometry then fails to produce a virtual gather with events matching the true gather, as no source gives rise to a ray with correct geometry. We show how to use two reciprocal sources to generate rays that contain the virtual reflection path. Similarly to the conventional approach, other parts of the rays are cancelled by a suitable correlation.

[1]In the case of a bandlimited pulse, all source locations within one Fresnel zone contribute constructively to the virtual gather. Using literally one source in this case may lead to loss of important phase information. Yet all of those sources are situated in the neighborhood of a *single* stationary phase location, and the same ray may be associated to all of them. We therefore continue to use the term 'single source interferometry' in this geometric sense to contrast this approach to what will follow.

The proposed approach is capable of recovering the kinematics. Further processing of the gather could be used to obtain good fits for the phase and amplitudes. Any such analysis would be external to the algorithm proposed here, and in the interest of concise exposition we do not spend time discussing that.

2 Two source interferometry

The setup for our problem is as follows. Consider a 2D acoustic model (Figure 1). The medium contains two flat layers separated by a reflector at depth $z_\ell = 2000$. 100 sources and collocated receivers $\boldsymbol{x}_\mathrm{s} = (x_\mathrm{s}, z_\mathrm{s})$ are located on the surface $z_\mathrm{s} = 0$ at offsets $-2500 \leq x_\mathrm{s} \leq 2500$. 12 additional receivers are put in a horizontal well at depth $z_\mathrm{r} = 2500$ below the reflector. The single receiver at $(-300, 2500)$ will be a virtual source, and 11 remaining receivers at the same depth with offsets from 300 to 1000 will become virtual receivers.

Recall that interferometry aims at reconstructing the virtual trace for the virtual source and the virtual receivers as if the source were physical (Figure 2).

The gather corresponding to this fictitious model is shown in Figure 3. It contains the direct wave as well as the reflection off the boundary between the layers from below. In what follows, we will show how to reconstruct that reflection from the available data.

For each physical shot, the surface receivers record the direct arrival and the reflections from the boundary between the layers. We mute the direct wave and show an example of the reflections in Figure 4. The receivers down the well record only the transmitted wave (Figures 5, 6). As the sources on the surface generate only a down-going wave, neither the direct arrival from the VS to the VR, nor the reflection off the interface from below can be recovered by the single source interferometry.

We now describe the algorithm for redatuming to the virtual source the surface reflection off the boundary between the layers. In an effort to simplify the exposition, we will assume straight rays. We stress, however, that both theory and numerics will work for bending rays as well, although geometry and formulas become more cumbersome.

Fix two receiver locations in the well: the virtual source $\boldsymbol{x}_\mathrm{r}^1$ and the virtual receiver $\boldsymbol{x}_\mathrm{r}^2$ (Figure 7). The point on the boundary $\widetilde{\boldsymbol{x}_\ell} = (\widetilde{x_\ell}, z_\ell)$ where the ray going from $\boldsymbol{x}_\mathrm{r}^1$ to $\boldsymbol{x}_\mathrm{r}^2$ reflects off the interface is determined by

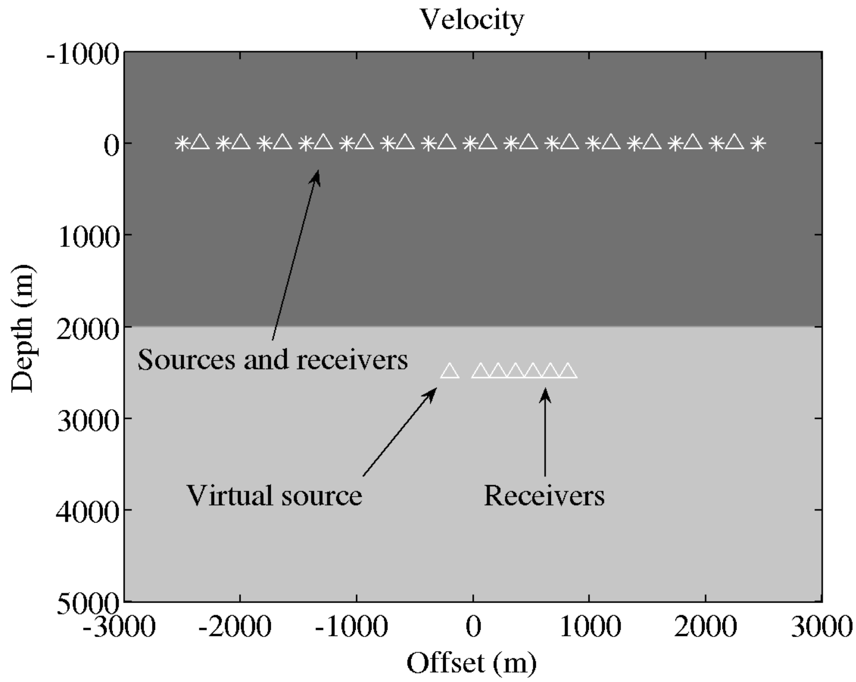

Figure 1: Complete model. Sources are on the surface. Receivers are on the surface as well as inside the medium. The single receiver at $(-300, 2500)$ will act as a virtual source, while the rest of the buried receivers will become virtual receivers.

Snell's law. In the case of a flat reflector, its offset $\widetilde{x_\ell} = \dfrac{x_{\mathrm{r}}^1 + x_{\mathrm{r}}^2}{2}$ is right in the middle between the receivers. Sources on the surface $\widetilde{\boldsymbol{x}_{\mathrm{s}}^1} = \left(\widetilde{x_{\mathrm{s}}^1}, 0\right)$ and $\widetilde{\boldsymbol{x}_{\mathrm{s}}^2} = \left(\widetilde{x_{\mathrm{s}}^2}, 0\right)$, where

$$\widetilde{x_{\mathrm{s}}^1} = \frac{x_{\mathrm{r}}^1 z_\ell}{z_\ell - z_{\mathrm{r}}}, \quad \widetilde{x_{\mathrm{s}}^2} = \frac{x_{\mathrm{r}}^2 z_\ell}{z_\ell - z_{\mathrm{r}}}, \tag{1}$$

generate rays that pass through the reflection point $\widetilde{\boldsymbol{x}_\ell}$ and are received at $\boldsymbol{x}_{\mathrm{r}}^1$ and $\boldsymbol{x}_{\mathrm{r}}^2$ respectively (Figure 7). The time of the virtual reflection can then

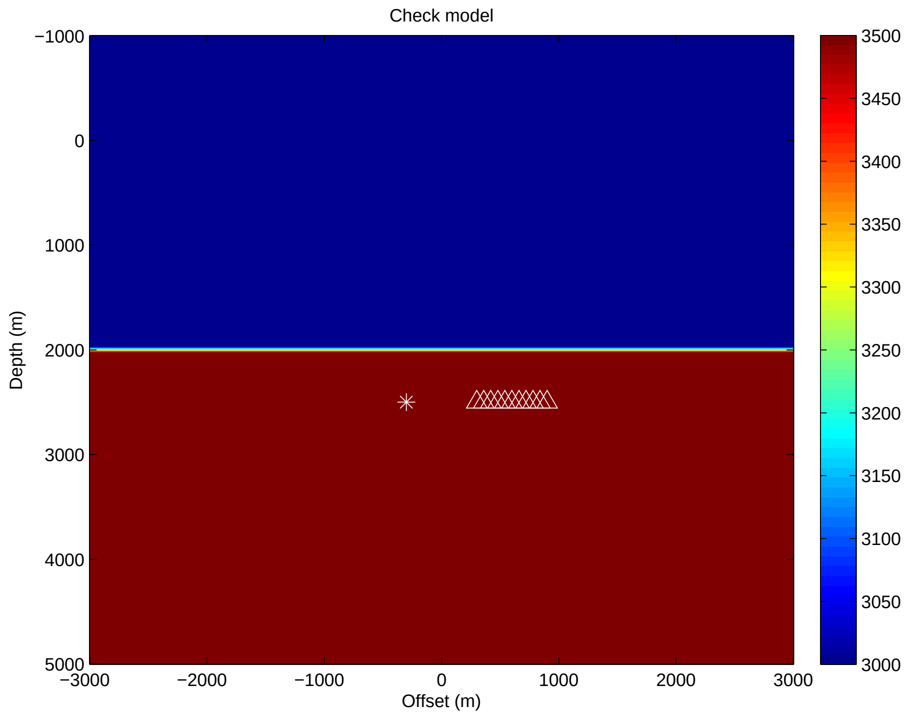

Figure 2: A fictitious setup with a physical source underneath, which we try to emulate using interferometry.The direct wave is of less interest to us. The focus is on recovering the reflection from the boundary at $z = 2000$.

be calculated using the following trivial identity:

$$\begin{aligned}\underbrace{\tau\left(\boldsymbol{x}_{\mathrm{r}}^{1}, \boldsymbol{x}_{\ell}\right)+\tau\left(\boldsymbol{x}_{\ell}, \boldsymbol{x}_{\mathrm{r}}^{2}\right)}_{\text{VS-VR reflection time}} = & \underbrace{\tau\left(\widetilde{\boldsymbol{x}_{\mathrm{s}}^{1}}, \boldsymbol{x}_{\mathrm{r}}^{1}\right)}_{\text{first direct time}}+\underbrace{\tau\left(\widetilde{\boldsymbol{x}_{\mathrm{s}}^{2}}, \boldsymbol{x}_{\mathrm{r}}^{2}\right)}_{\text{second direct time}} \\ & -\underbrace{\left(\tau\left(\widetilde{\boldsymbol{x}_{\mathrm{s}}^{1}}, \boldsymbol{x}_{\ell}\right)+\tau\left(\boldsymbol{x}_{\ell}, \widetilde{\boldsymbol{x}_{\mathrm{s}}^{2}}\right)\right)}_{\text{surface reflection time}},\end{aligned} \tag{2}$$

where τ always denotes the travel time between two points. The source locations $\widetilde{\boldsymbol{x}_{\mathrm{s}}^{1}}$ and $\widetilde{\boldsymbol{x}_{\mathrm{s}}^{2}}$ are analogs of the stationary phase source in the single source interferometry. They correspond to the only sources that produce the correct ray-paths and hence true time delays. As the exact depth of the reflector is not known *a priori*, neither are the locations of these sources. Much

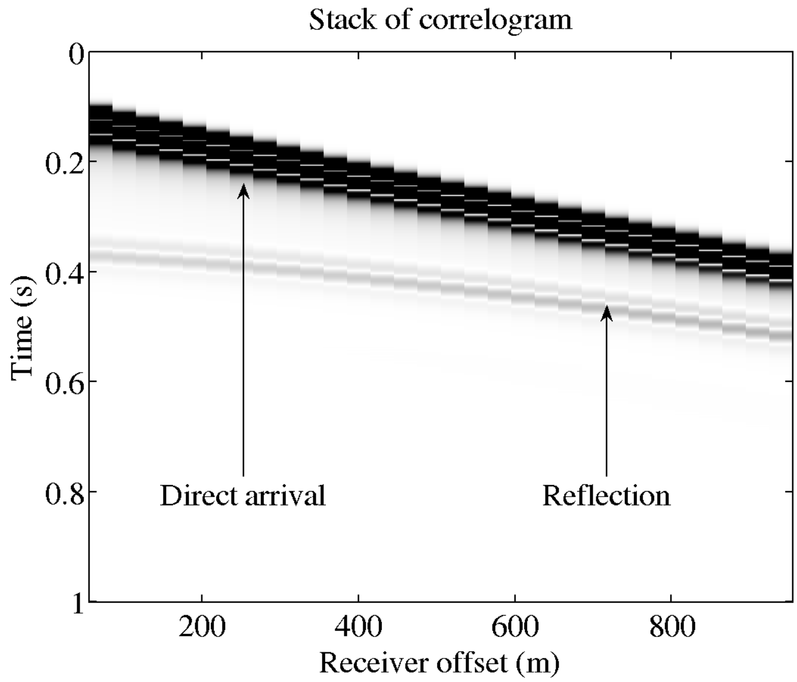

Figure 3: A shot gather that corresponds to a fictitious setup presented in Figure 2. Our goal is to automatically recover the reflection from the boundary between the two layers.

like in the single source interferometry we have to obtain the delay through an interferometric integral, which would automatically pick the contribution from these sources while canceling the input from all other source pairs.

Consider two general source locations $\boldsymbol{x}_\mathrm{s}^1$ and $\boldsymbol{x}_\mathrm{s}^2$ (Figure 7). Recalling that convolution of two traces adds travel times and correlation subtracts them, we proceed as follows:

1. We first compute the convolution of the traces recorded at $\boldsymbol{x}_\mathrm{r}^1$ and $\boldsymbol{x}_\mathrm{r}^2$:

$$D^2\big(\boldsymbol{x}_\mathrm{s}^1, \boldsymbol{x}_\mathrm{s}^2; \boldsymbol{x}_\mathrm{r}^1, \boldsymbol{x}_\mathrm{r}^2; t\big) = G\big(\boldsymbol{x}_\mathrm{s}^1, \boldsymbol{x}_\mathrm{r}^1; t\big) \star_t G\big(\boldsymbol{x}_\mathrm{s}^2, \boldsymbol{x}_\mathrm{r}^2; t\big) \,. \tag{3}$$

2. Then we correlate the result with the surface reflection $R(\boldsymbol{x}_\mathrm{s}^1, \boldsymbol{x}_\mathrm{s}^2; t) = G(\boldsymbol{x}_\mathrm{s}^1, \boldsymbol{x}_\ell; t) \star_t G(\boldsymbol{x}_\ell, \boldsymbol{x}_\mathrm{s}^2; t)$:

$$C\big(\boldsymbol{x}_\mathrm{s}^1, \boldsymbol{x}_\mathrm{s}^2; \boldsymbol{x}_\mathrm{r}^1, \boldsymbol{x}_\mathrm{r}^2; t\big) = D^2\big(\boldsymbol{x}_\mathrm{s}^1, \boldsymbol{x}_\mathrm{s}^2; \boldsymbol{x}_\mathrm{r}^1, \boldsymbol{x}_\mathrm{r}^2; t\big) \star_t R\big(\boldsymbol{x}_\mathrm{s}^1, \boldsymbol{x}_\mathrm{s}^2; -t\big) \,. \tag{4}$$

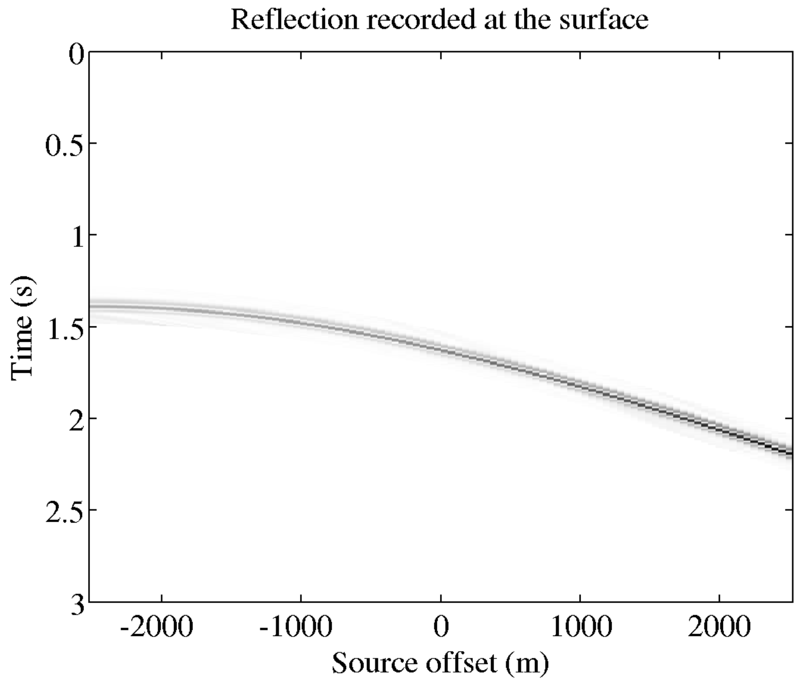

Figure 4: Reflections recorded at the surface. The direct arrivals are filtered out.

3. Finally the two source interferometric stack is obtained by integrating the correlogram over source locations:

$$\widetilde{G}\big(\boldsymbol{x}_{\mathrm{r}}^1, \boldsymbol{x}_{\mathrm{r}}^2; t\big) = \iint\limits_{\left\{x_{\mathrm{s}}^1 \le x_{\mathrm{s}}^2\right\}} C\big(\boldsymbol{x}_{\mathrm{s}}^1, \boldsymbol{x}_{\mathrm{s}}^2; \boldsymbol{x}_{\mathrm{r}}^1, \boldsymbol{x}_{\mathrm{r}}^2; t\big)\ dx_{\mathrm{s}}^1\, dx_{\mathrm{s}}^2. \tag{5}$$

We emphasize that the two-dimensional integral in Equation (5) is over *two distinct* source locations, and not over the two coordinates of the same point. The stack (5) will contain only the contributions from the two reciprocal source locations $\boldsymbol{x}_{\mathrm{s}}^1 = \widetilde{\boldsymbol{x}_{\mathrm{s}}^1}$ and $\boldsymbol{x}_{\mathrm{s}}^2 = \widetilde{\boldsymbol{x}_{\mathrm{s}}^2}$. Correlations produced by other sources will be filtered out by the summation. The justification for this based on the stationary phase argument is given in Appendix A. We note that a similar approach is taken when identifying multiples from the surface seismic data. Ours, however, is a different problem for a different setup.

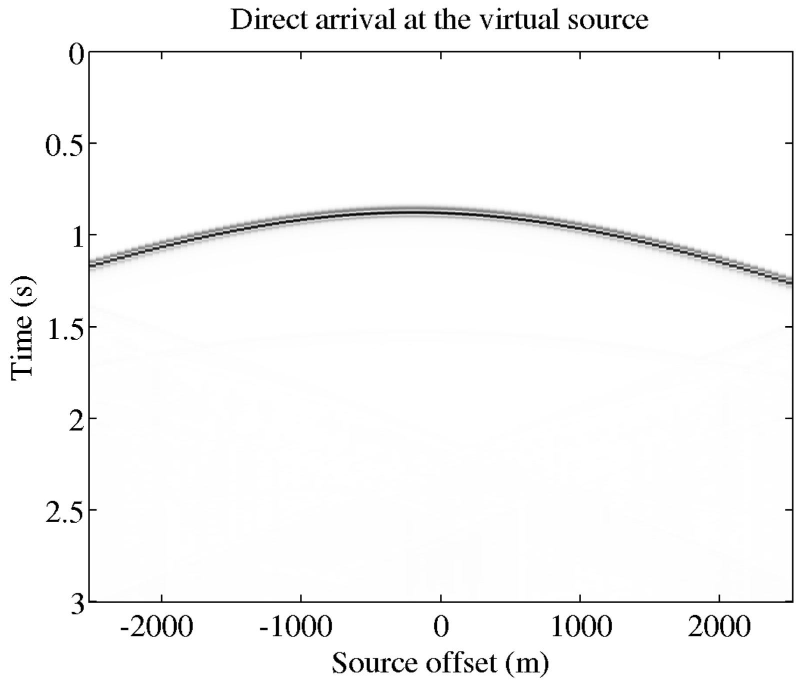

Figure 5: Gather recorded at the virtual source inside the medium (see Figure 1). Only the direct arrival is visible. The existence of the reflector is not manifested in any way.

We now show the numerical simulations confirming the theoretical conclusions presented above. The interface is the result of a velocity jump, so ray bending will occur. As was noted above, this is an impediment neither for the theory nor for the numerics.

For a fixed virtual source and each of the 11 virtual receivers, we compute the two source interferometric stack. The resulting virtual gather is shown in Figure 8.

Comparison with the check gather (Figure 3) shows that the kinematics of the reflection is recovered exactly. A better phase match could be obtained by deconvolving the source from the transmitted waves before the convolution. The correct amplitude of this virtual reflection cannot be recovered directly because the reflector is never physically excited from below. Surface data could be used to constrain the amplitude. We intentionally choose to forego all this post-processing of the correlogram and/or of the stack as secondary

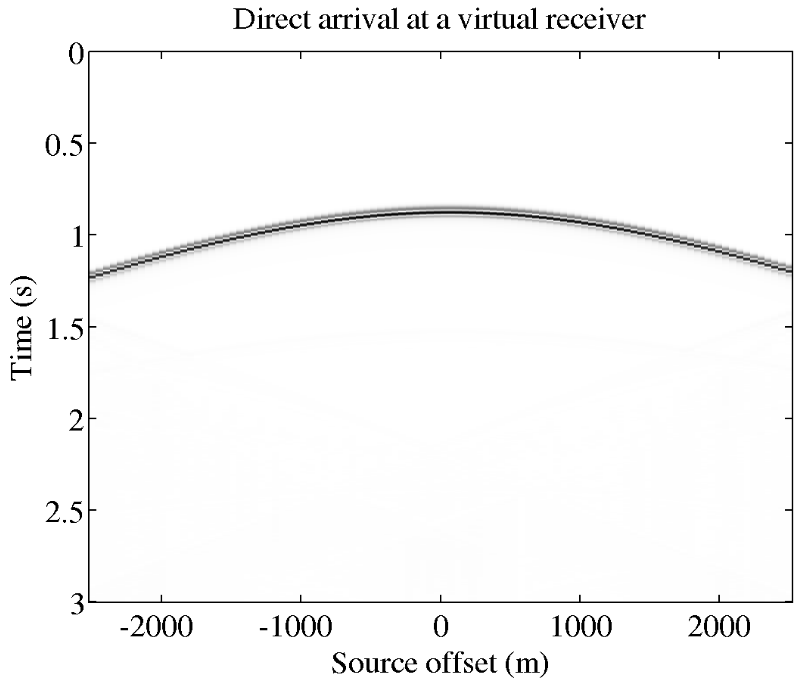

Figure 6: Gather recorded at one of the virtual receivers inside the medium (see Figure 1). Only the direct arrival is visible. The existence of the reflector is not manifested in any way.

to the main point of this note.

3 Conclusions

Interferometry is a rapidly developing field. Its goal is to estimate the Green's function between two receivers from recorded data. Many creative and ingenious methods are used depending on the setup geometry, properties of the medium, etc. The underlying idea behind many of those techniques is to find a physical source that would produce a wavefield phase, whose path contains the ray from the virtual source to the virtual receiver. Contributions by all other sources are cancelled by the interferometric integral.

In this paper, we presented a simple setup where no single source provides illumination adequate for the purposes of interferometry. The down-going

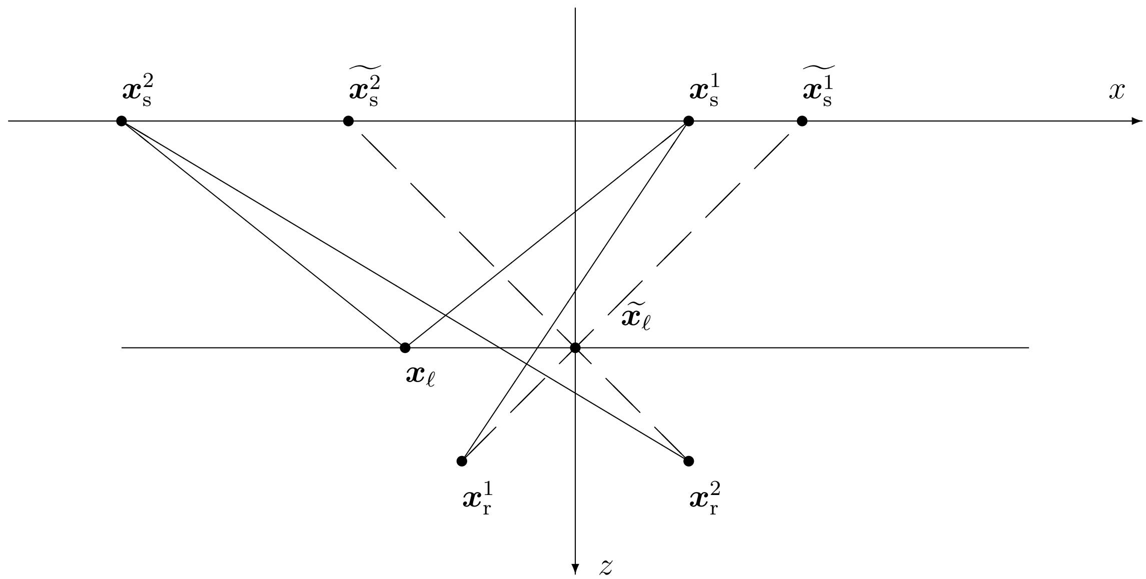

Figure 7: The delay time of the reflection off a horizontal layer $z = z_\ell$ from the virtual source $\boldsymbol{x}_\mathrm{r}^1$ to the virtual receiver $\boldsymbol{x}_\mathrm{r}^2$ can be computed from the direct arrivals and the surface reflection if the physical sources are at stationary phase locations: $\boldsymbol{x}_\mathrm{s}^i = \widetilde{\boldsymbol{x}_\mathrm{s}^i}$, $i = 1, 2$. Solid lines represent rays generated by arbitrary sources. Dashed lines are the rays generated by stationary phase sources.

waves are transmitted through the interface with no visible imprint. No part of the Green's function can then be correctly recovered from these records. It is natural in such a circumstance to try to use more than one ray in order to construct the desired path using basic geometric considerations. We use acoustic reciprocity to combine direct arrivals at two receivers in the well with the surface reflections to produce kinematically correct virtual reflections.

The proposed method is not in any way a substitution for the existing techniques. Because of its natural physical interpretation, single source interferometry certainly remains the preferred approach when at all feasible. However, physical limitations may prevent it from working successfully in some cases. Other techniques such as the one presented in this note could then be tried.

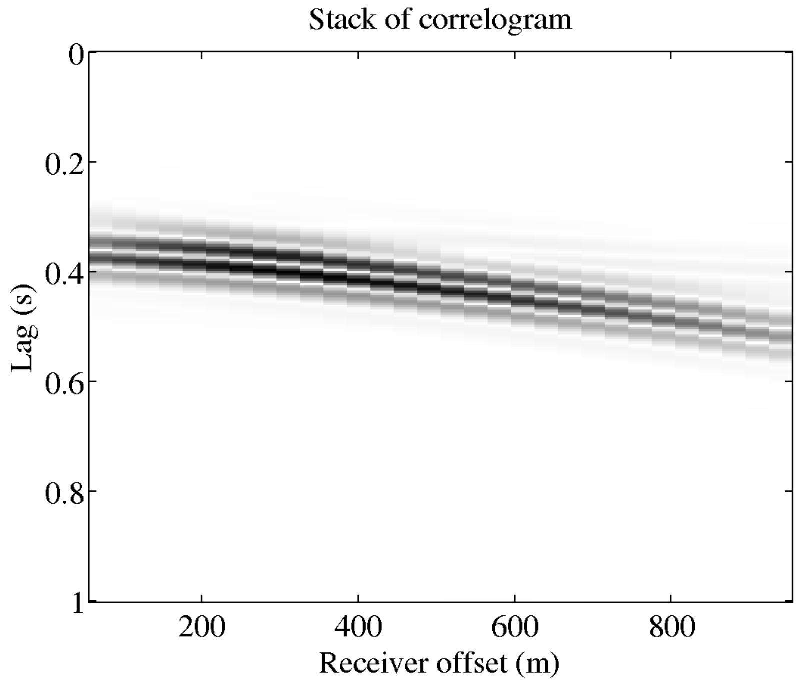

Figure 8: Virtual gather obtained by redatuming surface reflections to all virtual receivers inside the medium shown in Figure 1. The kinematics of the reflection is recovered exactly (compare with Figure 3). Better phase match could be obtained by deconvolving the source.

4 Acknowledgements

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Appendices

A Stationary phase analysis

As changing of scale and shift of spatial variable amounts to simply choosing convenient physical units, we can choose the coordinate system, in which

$$z_{\mathrm{s}} = 0, \quad x_{\mathrm{r}}^1 = -x_{\mathrm{r}}^2, \quad x_{\mathrm{r}}^1 < x_{\mathrm{r}}^2. \tag{6}$$

Assume there is a reflector above the receivers at some unknown depth $z_\ell < z_{\mathrm{r}}$. To simplify the presentation assume also no ray bending. Then we can suppose that the velocity $v = 1$ throughout the medium.

Travel times of waves transmitted to the well are given by

$$\tau(\boldsymbol{x}_{\mathrm{s}}^i, \boldsymbol{x}_{\mathrm{r}}^i) = \frac{1}{v}\|\boldsymbol{x}_{\mathrm{s}}^i - \boldsymbol{x}_{\mathrm{r}}^i\| = \sqrt{(x_{\mathrm{s}}^i)^2 + 2x_{\mathrm{s}}^i\, x_{\mathrm{r}}^i + (x_{\mathrm{r}}^i)^2 + z_{\mathrm{r}}^2}, \; i = 1, 2. \tag{7}$$

At the same time, the surface reflection delays are

$$\begin{aligned} \tau(\boldsymbol{x}_{\mathrm{s}}^1, \boldsymbol{x}_{\mathrm{s}}^2) &= \frac{1}{c}\left[\|\boldsymbol{x}_{\mathrm{s}}^1 - \boldsymbol{x}_\ell\| + \|\boldsymbol{x}_\ell - \boldsymbol{x}_{\mathrm{s}}^2\|\right] \\ &= \sqrt{(x_{\mathrm{s}}^1)^2 + 2x_{\mathrm{s}}^1\, x_{\mathrm{r}}^2 + (x_{\mathrm{s}}^2)^2 + 4}. \end{aligned} \tag{8}$$

The moveout curve of the reflected event in the correlogram given by Equation (4) is

$$\begin{aligned} T(x_{\mathrm{s}}^1, x_{\mathrm{s}}^2) &= \frac{1}{c}\left[\|\boldsymbol{x}_{\mathrm{s}}^1 - \boldsymbol{x}_{\mathrm{r}}^1\| + \|\boldsymbol{x}_{\mathrm{s}}^2 - \boldsymbol{x}_{\mathrm{r}}^2\| - \|\boldsymbol{x}_{\mathrm{s}}^1 - \boldsymbol{x}_\ell\| - \|\boldsymbol{x}_\ell - \boldsymbol{x}_{\mathrm{s}}^2\|\right] \\ &= \sqrt{(x_{\mathrm{s}}^1)^2 + 2x_{\mathrm{s}}^1\, x_{\mathrm{r}}^1 + (x_{\mathrm{r}}^i)^2 + z_{\mathrm{r}}^2} \\ &\quad + \sqrt{(x_{\mathrm{s}}^1)^2 + 2x_{\mathrm{s}}^1\, x_{\mathrm{r}}^1 + (x_{\mathrm{r}}^i)^2 + z_{\mathrm{r}}^2} \\ &\quad - \sqrt{(x_{\mathrm{s}}^1)^2 + 2x_{\mathrm{s}}^1\, x_{\mathrm{r}}^2 + (x_{\mathrm{s}}^2)^2 + 4}. \end{aligned} \tag{9}$$

We now write the stationary phase condition as follows:

$$\begin{cases} \dfrac{\partial T(x_{\mathrm{s}}^1, x_{\mathrm{s}}^2)}{\partial x_{\mathrm{s}}^1} = 0, \\ \dfrac{\partial T(x_{\mathrm{s}}^1, x_{\mathrm{s}}^2)}{\partial x_{\mathrm{s}}^2} = 0, \\ x_{\mathrm{s}}^2 < x_{\mathrm{s}}^1 \end{cases} \tag{10}$$

After a bit of algebra the solution to the system (10) takes the form:

$$\begin{aligned} x_{\mathrm{s}}^1 &= \frac{x_{\mathrm{r}}^1 z_\ell}{z_\ell - z_{\mathrm{r}}} = \widetilde{x_{\mathrm{s}}^1} \\ x_{\mathrm{s}}^2 &= \frac{x_{\mathrm{r}}^2 z_\ell}{z_\ell - z_{\mathrm{r}}} = \widetilde{x_{\mathrm{s}}^2}. \end{aligned} \tag{11}$$

Without the inequality $x_{\mathrm{s}}^2 < x_{\mathrm{s}}^1$, the system (10) would produce multiple solutions, only one of which is geometrically admissible. The system in its stated form has the unique solution (11). Comparing this solution to (1), we conclude that the sources satisfying the stationary phase condition are the ones that produce geometrically correct rays. The integral in Equation (5) will then pick only the correct delay with the rest of the integrand stacking out, and the entire algirithm is therefore justified.

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