Secret

CORNER REFLECTORS

by

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Introduction

A device which returns a microwave beam directly towards the transmitter would, if available, find many radar applications. It could be used to increase the visibility of lifeboats and balloons. Towed in a raft it could be used as a practice target for the training of gun laying operators.

This is accomplished by the cut glass reflectors of highway signs when they return a headlight beam to a motorist regardless of his car's position. Such "corner" reflectors have been used occasionally in radar work, but few of their properties were known. It was hoped that with more specific knowledge corner reflectors could be designed to fill some of these needs.

This report contains an analysis of corner reflector theory and a discussion of experimental tests followed by some reccomendations for future research and for reflector design.

Corner Reflector Theory

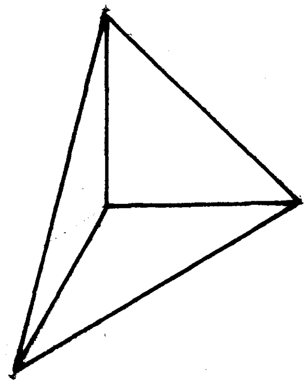
A trihedral corner reflector consists of three mutually perpendicular plane metallic reflectors (Figure 1). A plane electromagnetic wave incident on this configuration may be reflected from each of the three mirrors in succession. Irrespective of the original direction of incidence, this triple reflection reverses the direction of the wave heading it back towards the source.

A dihedral corner reflector consists of two plane reflectors forming a dihedral angle of 90°. This type of corner returns the beam towards the source only if the direction of the incident beam is perpendicular to the line of intersection of the planes (Figure 2). These remarks will be justified in detail by the following analysis.

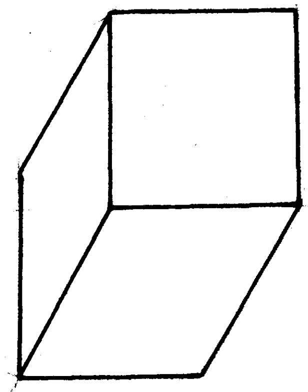
Reflection from plane mirrors.

It is known from the theory of images that the field of an oscillating dipole and a reflector can be considered as coming from the original dipole and an image dipole arranged as shown in Figure 3.[1] A dipole of any

[1] See for example, J. C. Slater, <u>Microwave Transmission,</u> p. 270, McGraw-Hill Book Co, New York, 1942

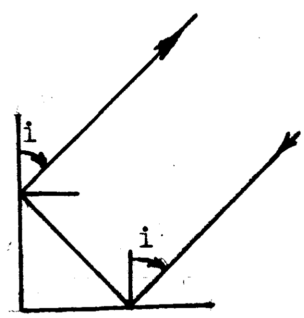


Triangular Corner



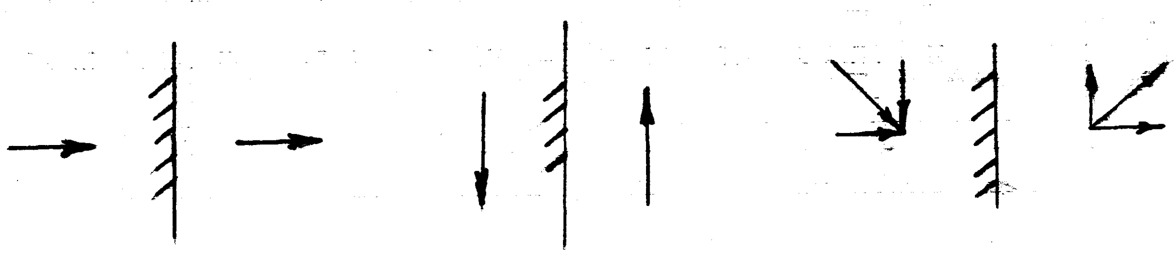
Square Corner

Figure 1



Dihedral Corner

Figure 2



Dipole Images

Figure 3

orientation can be resolved into components perpendicular and parallel to the reflecting plane. The components are treated independently and then combined, as illustrated. Use will be made of this principle in the following discussion.

Reflection from dihedral corners.

Consider a dihedral reflector formed from plane mirrors in the xz and yz planes and a radiating dipole at x,y,z with components of dipole moment E_x, E_y, and E_z. There will be two singly reflected images:

1. At x, -y, z with components $-E_x$, E_y, $-E_z$
2. At -x, y, z with components E_x, $-E_y$, $-E_z$

There will be one doubly reflected image at -x, -y, z with components $-E_x$, $-E_y$, and E_z.

A plane wave approaching a double corner from a direction <u>S</u> and plane polarized in a direction <u>E</u> will be reflected and return along <u>S</u>. The returning wave is pictured below the incident wave in figure 4. <u>E</u> can be resolved into E_z and E_{xy} both perpendicular to <u>S</u>. E_{xy} can be resolved into E_x and E_y. The incident plane wave can be thought of as originating from a dipole located at infinity in the <u>S</u> direction and having moment components proportional to E_x, E_y, and E_z. The image dipole at infinity along $-\underline{S}$ will have components $-E_x$, $-E_y$, and E_z. The reflected wave will be polarized in the direction of the image dipole $\underline{E}'$. $\underline{E}'_{xy} = -\underline{E}_{xy}$ but is still perpendicular to $\underline{S}$. In general $\underline{E}'$ is different from $\underline{E}$. Thus for a double reflection there is a rotation of the plane of polarization equal to 2β where β is the angle $\underline{E}$ makes with $\underline{E}_z$.

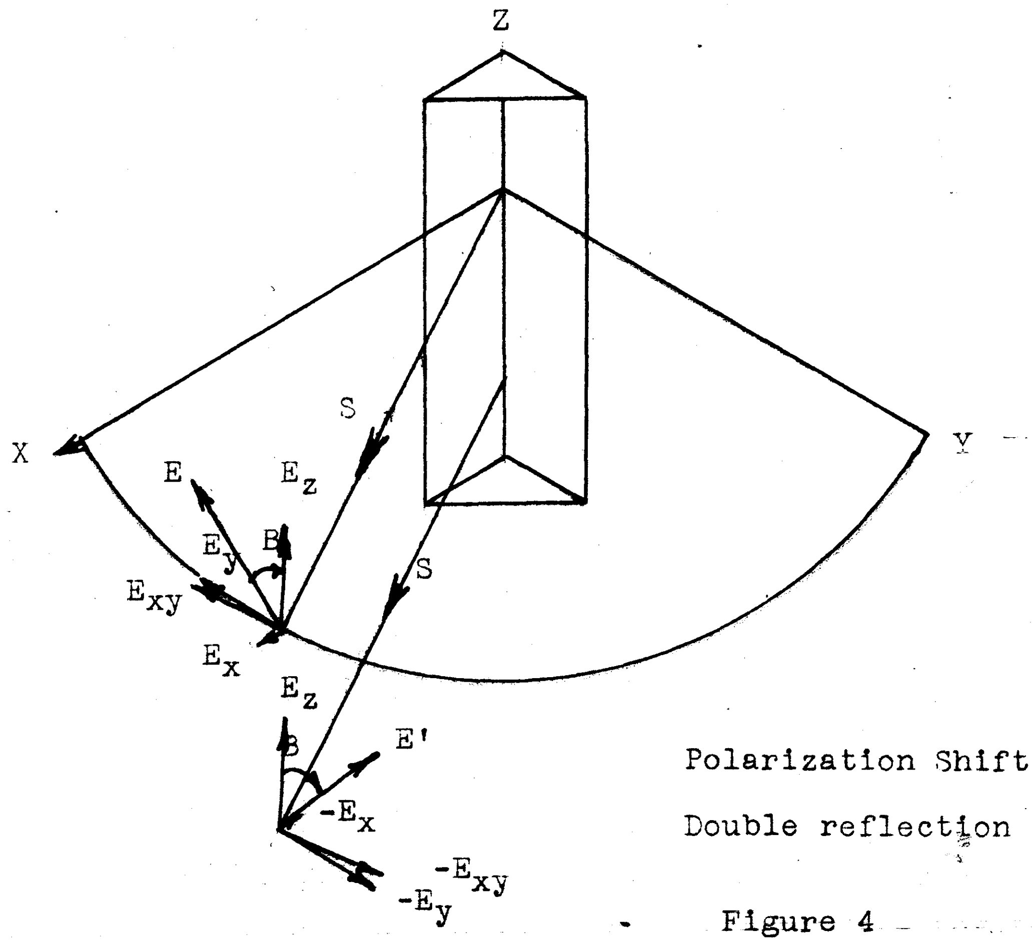
The geometrical aperture of a dihedral reflector with rectangular sides is calculated in Appendix I. It is shown that the aperture is greatest when $\underline{S}$ makes an angle of 45° with the x axis (Figure 4) and falls to zero on either side. Notice, however, that when this angle is near zero, a strong single reflection is returned.

Reflection from a trihedral corner.

Consider a trihedral reflector located at the origin of xyz coordinates with the mirrors in the xy, xz, and yz planes and with a radiating dipole at x, y, z with moments E_x, E_y, and E_z. The images are as follows:

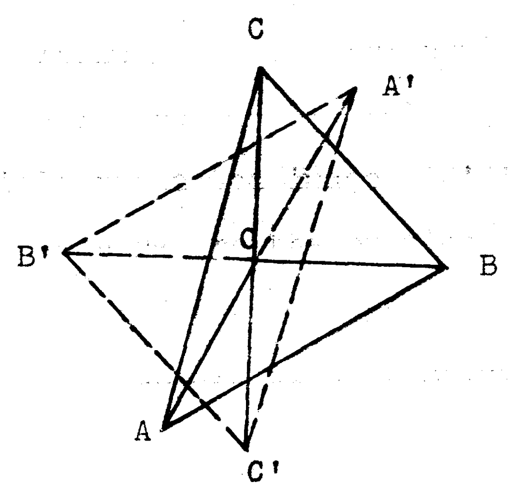
Three single reflections

1. At x, y, -z with components $-E_x$, $-E_y$, E_z
2. At x, -y, z with components $-E_x$, E_y, $-E_z$
3. -x, y, z with components E_x, $-E_y$, $-E_z$



Polarization Shift for Double reflection

Figure 4



Aperture Stops for Trihedral Corner

Figure 5

Three double reflections

1. At x, -y, -z with components E_x, $-E_y$, $-E_z$
2. At -x, y, -z with components $-E_x$, E_y, $-E_z$
3. At -x, -y, z with components $-E_x$, $-E_y$, E_z

One triple reflection at -x, -y, -z with components E_x, E_y, and E_z.

Using the same reasoning as for the dihedral reflector we see that there is no change in the plane of polarization for triple reflections. The trihedral corner can thus be replaced by a plane mirror placed at the origin and oriented so as to be normal to the incident beam - except that there is no phase shift on reflection.

The calculation of aperture size for trihedral corners is considerably more difficult than for dihedral reflectors. Appendix II contains a consideration of the optical stops of the system. It is shown there for the cases of square and triangular corners that there are two apertures which simultaneously limit the radiation from the image dipole. For the triangular corner of figure 5, one aperture is the equilateral triangle ABC, and the other is its triply reflected image A'B'C'. Similarly the stops for square corners are the outside boundary of the reflector and its triply reflected image, but in this case the apertures are both six sided figures neither of which is a plane figure.

The calculation of the aperture of a triangular corner is outlined in Appendix III using a method due to R. C. Spencer. P. D. Crout has calculated the apertures for both triangular and square corners using graphical projection methods.

These calculations show that the aperture is greatest when the incident beam is advancing along the axis of symmetry of the corner. The aperture decreases as the incident direction deviates from the symmetry axis. When this direction lies in one of the edges of the corner or in one of its faces, the aperture is zero. However, in these positions the corner returns a single or a double reflection.

Of course, the geometrical picture presented above is incomplete. The real solution of the problem is the old story of Maxwell's equations with the specified boundary conditions. In practice, two corrections to the geometrical theory need consideration.

1. The energy intercepted from the incident beam by the corner is not returned as a plane wave of limited cross section. Diffraction causes the beam to spread. If A, the aperture of the corner is large, the gain of the corner would be $\frac{4\pi A}{\lambda^2}$ (the gain of a uniformly illuminated area A).

2. For small reflectors an edge effect may become important. A boundary of the reflector disturbs the surface current distribution near the boundary, and

hence the radiation field from the reflector is slightly distorted.

For a large corner the reflecting cross section would be the geometrical cross section. If P is the power transmitted by a radar parabola of area A_p, the power p received from a corner of area A at a distance r is [1]

$$p = P \frac{1}{4\pi r^2} \frac{4\pi A_p}{\lambda^2} A \frac{1}{4\pi r^2} \frac{4\pi A}{\lambda^2} A_p$$

assuming the illumination of the parabola is uniform. In practice both the A_p's should be multiplied by about .67 to correct for the nonuniform illumination.

$$\frac{p}{P} = \frac{A_p^2 \quad A^2}{(r^2\lambda^2)^2}$$

Thus the power received goes up as the <u>fourth</u> power of a linear dimension of the corner.

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1. $P \frac{1}{4\pi r^2}$, the intensity at the reflector for an isotropic transmitter, times $\frac{4\pi A_p}{\lambda^2}$ (gain) gives the actual intensity at the corner. This times A gives the power intercepted by the reflector. If the reflector were an isotropic scatterer, the intensity of the scattered radiation at the receiver would be the above quantity times $\frac{1}{4\pi r^2}$. The actual intensity is $\frac{4\pi A}{\lambda^2}$ (gain of corner) times this. The received power is the intensity times A_p, the absorbing cross section of the parabola.

Experimental Results

Measurements of the reflections from corner reflectors have been made in collaboration with R.D. O'Neal. The measurements were made at ten centimeters using trihedral corners of sizes from six inches to four feet (measured along an edge formed by the intersection of two of the mirrors.) Separate parabolas were used for transmission and reception. The corners were supported on a portable wooden tower which permitted rotating the corner about three different axes. Transmitter monitor and receiver both consisted of bolometers and small battery operated receivers of rather uncertain characteristics. Only relative power measurements could be made, and the fraction of the total power which was received is not known.

Several conditions must be satisfied by the apparatus if these measurements are to be relied upon. The corner must be far enough away from the transmitter for the wave front to be considered plane across the aperture of the corner. The receiving parabola must be close enough to the transmitting parabola for the received energy to be essentially the same as that returned to the transmitter. This requirement is more stringent for large corners which give a sharper beam.

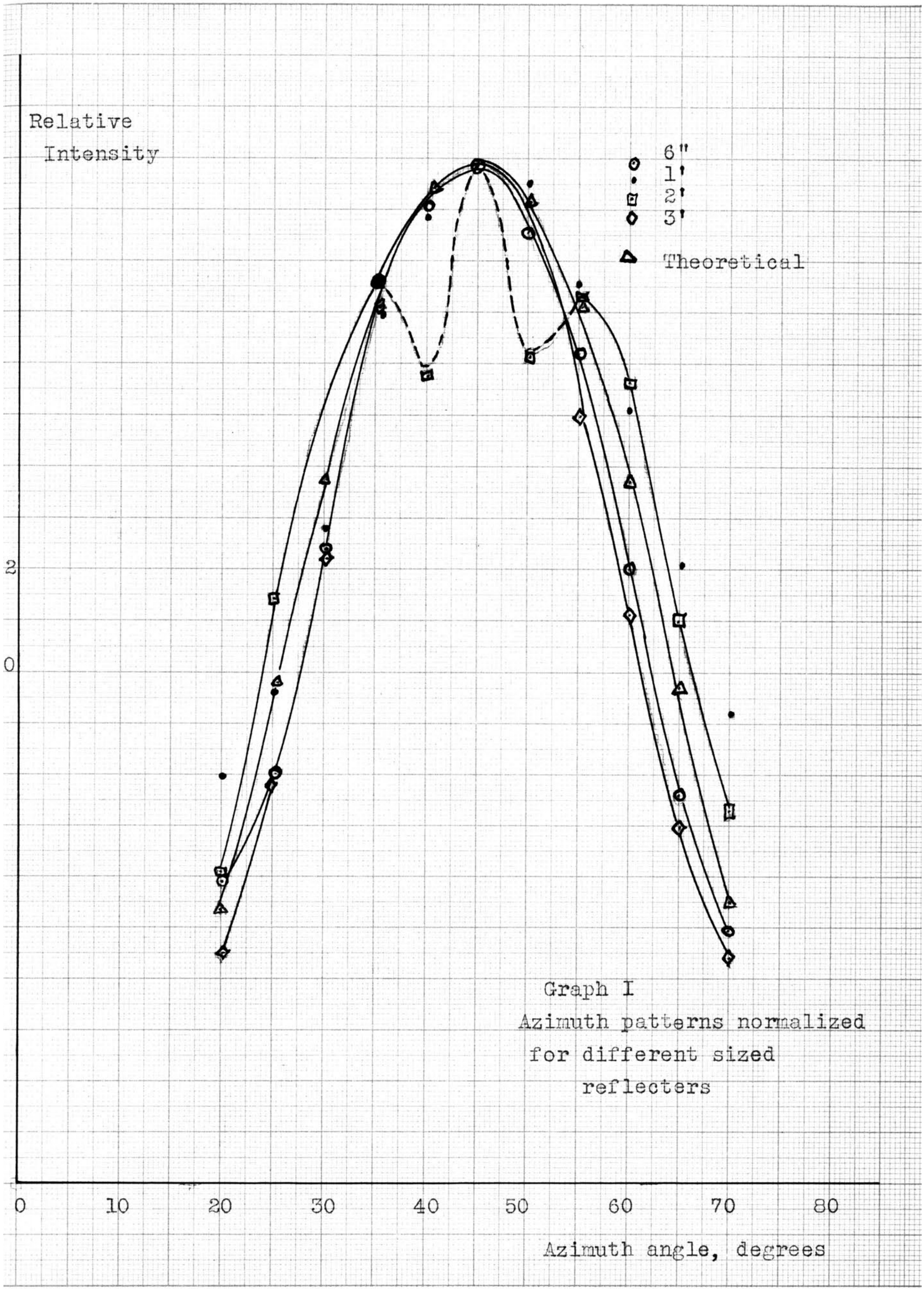
Typical curves of received power versus azimuth angle for triangular trihedral corners are shown in graph I. Curves for different size corners are normalized to the same peak intensity. Graph II shows the agreement between the experimental points and Crout's theoretical curves.

The overall experimental error of the measurements is difficult to estimate. It includes errors in aligning the tower with the transmitter, errors in monitoring the transmitted power, errors in received signal measurements, and errors in the construction of the corners. When an unusual curve was obtained, it was difficult to tell if the apparatus was out of alignment or if the corner angles were wrong.

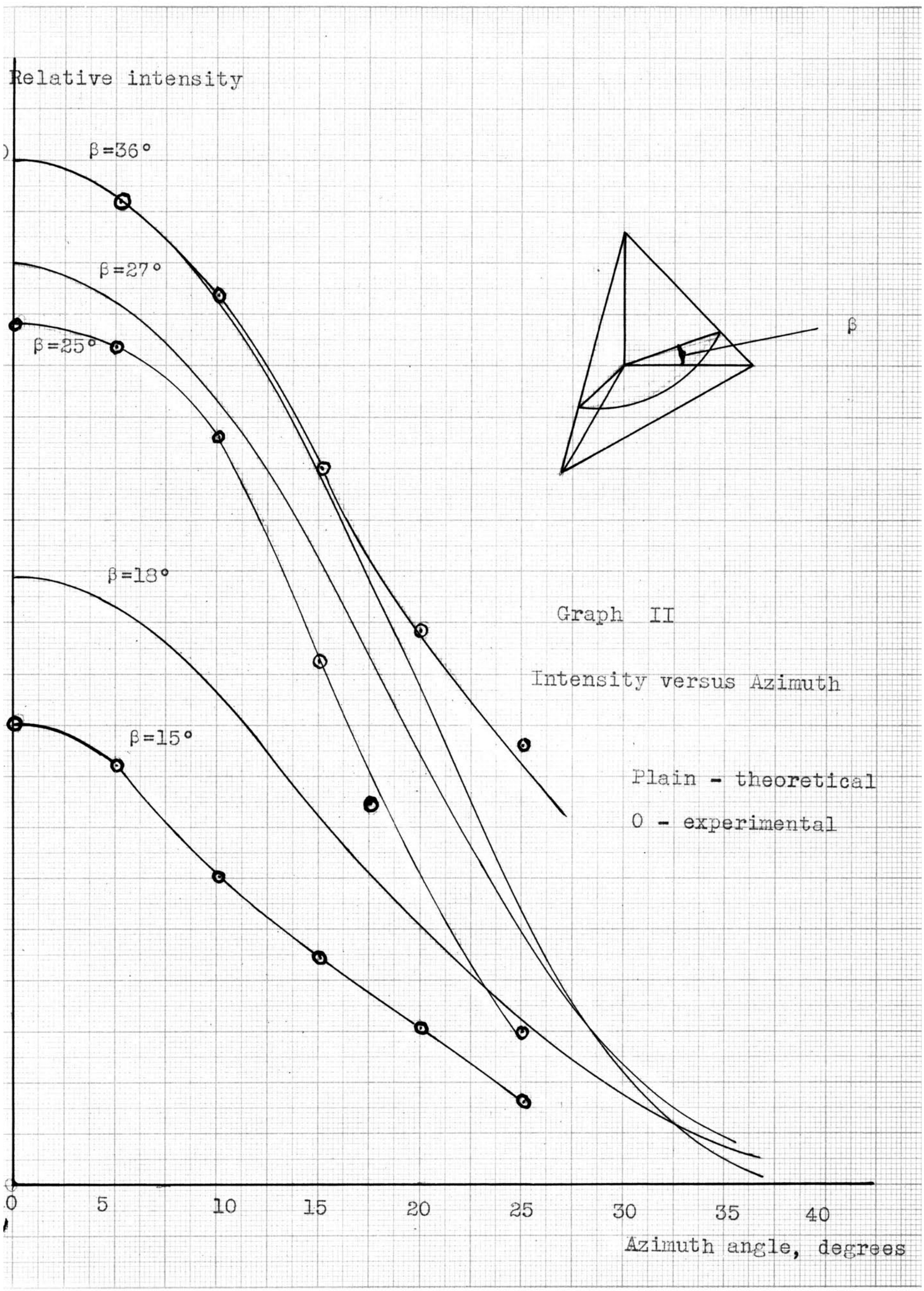
It was verified that the plane of polarization is shifted for a double reflection and is not for a triple reflection.

Both square and triangular corners were measured. However, no reliable figure for the ratio of the power received from square and triangular corners of the same edge length was obtained. The square corners have the disadvantage of being more difficult to construct rigidly.

Considerable effort was made to construct a beacon from corner reflectors. Such a device would return a beam to the transmitter over an azimutl range of 360°. The desired range in elevation angle for the beacon may vary from a few to 180°. To obtain a uniform return over all angles, it is necessary to have the beams from two



Graph I
Azimuth patterns normalized
for different sized
reflecters



Graph II

Intensity versus Azimuth

or more corners to overlap. Measurements were made on several beacon arrangements, but none of them proved satisfactory. Interference of the beams in the region of overlap caused the pattern to be a series of sharp maxima and minima rather than a smooth curve. The interference situation was furthur complicated by the presence of the double reflections in the overlap region. Double reflection beams have still different phases and polarizations.

The measurement program should be continued. The following points are of particular importance:

1. Measurement of the edge effect for small reflectors. The dimensions at which this become important are not known.

2. Measurement of the spatial distribution of the energy scattered by a corner. This would verify the theoretical diffraction pattern due to the finite aperture and enable a gain calculation to be made.

3. Measurement of the ratio of received to transmitted power. In particular more accurate checks on the ratio of the returned signal from square and triangular corners of the same edge length are needed.

4. Determination of the necessary manufacturing tolerances of the dihedral angles between the mirror planes. When these angles are not exactly 90°, multiple images are formed, and the performance changes.

5. The beacon problem will require more work. In this connection some of the new corners described below should be tested.

Suggestions for the design of beacons and frequency selective corners.

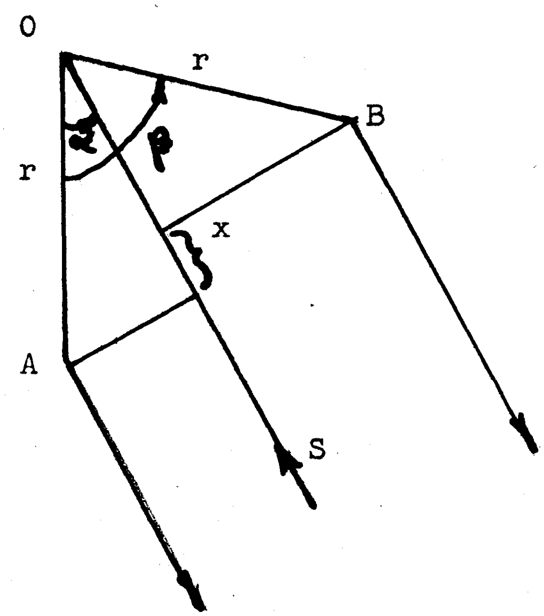
In figure six, A and B are the vertices of two trihedral reflectors both located at a distance r from the center O of the beacon. Consider a plane wave advancing in the direction S. The triple reflections will differ in phase by $\frac{2\pi x}{\lambda}$ radians (see figure for definition of x).

$$x = r[\cos\alpha - \cos(\beta - \alpha)]$$
$$= r[\cos\alpha - \cos\alpha\cos\beta - \sin\alpha\sin\beta]$$
$$= r[\cos\alpha(1 - \cos\beta) - \sin\alpha\sin\beta]$$

To have constructive interference, x should be $n\lambda$ independent of α. This can be accomplished only by making $r = 0$.

If r is zero, however, the packing of the corners in a beacon is seriously restricted. Moreover, the double reflections have not been considered, and these might have to be reduced.

If the two mirrors of a dihedral reflector do not meet (Figure 7), the range of angles over which the double reflection aperture exists is sharply reduced. Similarly the working region of a trihedral corner could



Interference of beams

from two corners

Figure 6

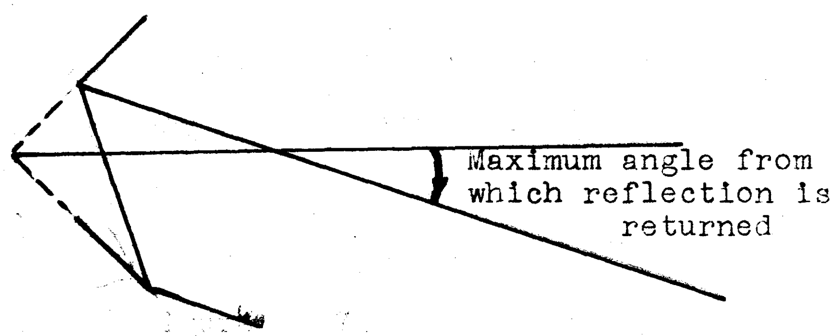
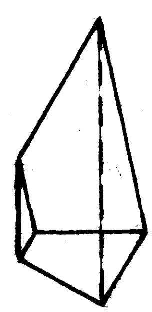


Figure 7

Open Dihedral

Reflector



Truncated Trihedral

Reflector

Figure 8

be sharply reduced by truncating the corner pyramid as in figure 8. Note that the double reflections from a trihedral corner can be limited by the preceeding method.

If the aperture cut off can be made sharp enough, the region of beam overlap necessary in beacons would be reduced. This would greatly simplify the interference problem.

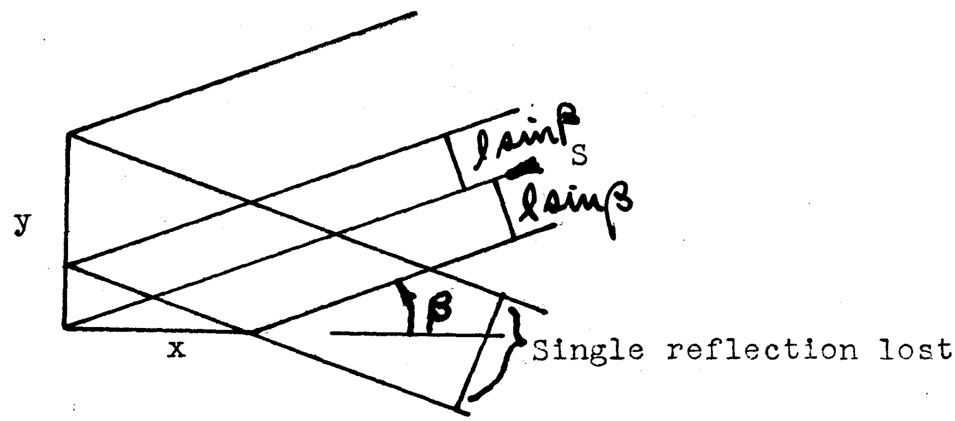
If the opening left by truncating the double or triple corner is covered by a metal sheet, additional energy can be reflected in the region of normal incidence. Moreover, by adjusting the distance of this new reflector from the vertex of the corner the radiation from the single and triple parts can be made to interfere either constructively or destructively. By making the effective apertures of the two reflecting portions equal, nearly complete cancellation could be obtained for some frequencies, but a strong reflection for others. The sharpness of the frequency response curve would be greater for greater vertex to single reflector distances, because this would increase the distance between the two image dipoles.

A frequency selective corner could be made to work for radar of one frequency but not for another. It might be possible to conduct IFF (identification, friend or foe) work on this basis. Friendly objects equipped with frequency selective corners large enough to return a major portion of the total power returned by the object could be seen with radar of one wavelength, but its signal in a radar set of different wavelength would be greatly reduced. Foreign objects not equipped with the corners could be seen with equal ease by both radar sets.

Appendix I

Calculation of Aperture of Rectangular Dihedral Reflector

Consider a reflector made from two mirrors each ℓ by a with common edge ℓ. Let $\underline{S}$ be the direction of the incident beam (see figure).



Assume β to be less than 45°. A beam of cross sectional area $\ell a \sin\beta$ strikes mirror x and is completely reflected by mirror y. All of the beam striking y first is not reflected again by x, but a beam of area $\ell a \sin\beta$ is returned. The total aperture area is thus $2\ell a \sin\beta$.

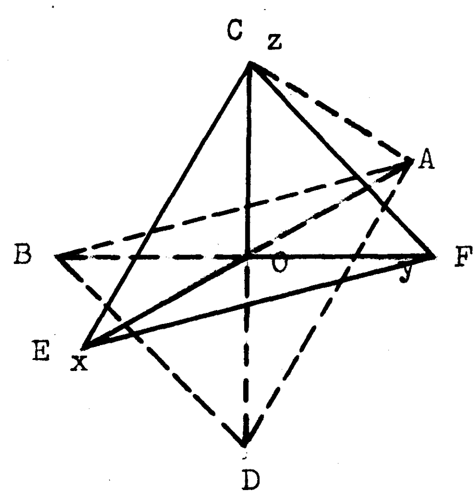
Similarly, if β is greater than 45°, the aperture area is $2\ell a \cos\beta$.

Appendix II

Optical Stops of Trihedral Reflectors

A trihedral reflector consists of six optical systems in parallel. Consider a reflector with its vertex at the origin of xyz coordinates and its mirrors in the coördinate planes. By the x plane is meant the plane perpendicular to the x axis, etc. Then the light can be reflected from the planes in the following order: xyz, xzy, yxz, yzx, zxy, zyx.

Consider the optical system xyz. To find the stops of the system form the images of all the elements of the system in object space. See figure.



The image of the x plane is the x plane itself.
The image of the y plane is the OAC plane (Y).
The image of the z plane is the OAB plane (Z).

Any straight line intersecting the Z and x triangles intersects the Y triangle. Therefore the Y triangle never acts as a stop for the system. Similarly it can be shown that the triangles y and Z are the stops of the yxz system.

Proceeding in the same manner for the other parallel optical paths, the complete stop system is found to consist of the reflector planes themselves and the three triangles OAB, BOD, and DOA.

Inspection of the figure will show that this system of stops is equivalent to the two triangles ABD and CEF.

Exactly the same argument will hold for a square corner; but for corners whose three mirrors are not the same or are reentrant figures it would be possible for the second stop to limit the beam for some angles of incidence. The analysis for such a case would be much more complicated.

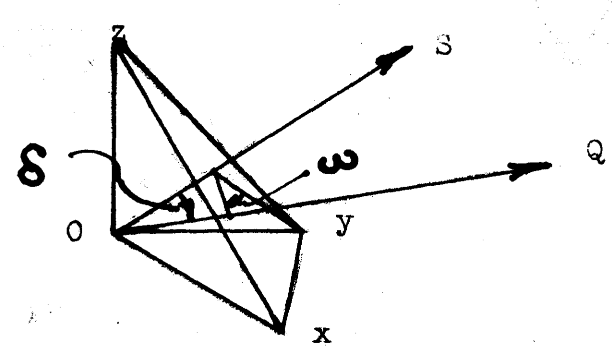
Appendix III

The Aperture Size of a Triangular Corner

Given a triangular corner reflector whose edges are of length ℓ (Figure). The limiting apertures are two equilateral triangles of sides $\ell\sqrt{2}$ located in two parallel planes a distance $2\ell/\sqrt{3}$ apart (see appendixII). The problem is to find the projected area common to these two triangles on a plane of arbitrary orientation.

Now the projection of a triangle on a plane is the same as the projection of a congruent triangle located in a plane parallel to the plane of the original triangle. Furthurmore, the projections coincide if the congruent triangles have their sides respectively parallel, and the second triangle is located so that its center projects to the same point as the center of the original triangle.

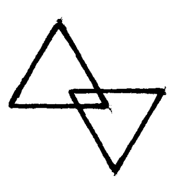
Consider the corner illustrated.

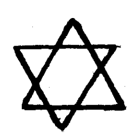


S is the axis of symmetry of the corner. Q is the direction of the incident plane wave. Q will be specified by the following two angles: δ, the angle between Q and S, and ω, the dihedral angle between the plane QOS and the plane SO and the y axis. Now the projection of the rear aperture triangle in the direction Q is the same as the projection of some triangle C in the plane of the front triangle. This new triangle will be located according to the preceding argument. As the direction Q changes, triangle C will slide over the front aperture triangle.

The problem is now reduced to that of finding the area of overlap of the two triangles as they slide over each other. The addition of a cos δ factor will then give the projected area in the direction Q.

Two cases are to be distinguished depending on the shape of the figure produced by the overlapping. This figure may have four or six sides as illustrated.





With a little plane geometry and considerable work the answer is found to be:

Case I: $A=\frac{\sqrt{3}}{6}\cos\delta\left[\,4s^2-4sp\tan\delta\cos\omega+p^2\tan^2\delta(1-4\sin^2\omega)\right]$

Case II: $A=(s^2-p^2\tan^2\delta)\frac{\sqrt{3}}{2}\cos\delta$ where $s=\ell\sqrt{\frac{2}{3}}$ and $p=\frac{2\ell}{\sqrt{3}}$.

Note A is independent of ω in case II.

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