

Improved Green's functions from seismic interferometry

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Abstract

Under certain theoretical assumptions, the theory of seismic interferometry allows the construction of artificial (or virtual) sources and receivers at the locations of receivers in a physical experiment. This is done by redatuming the physical sources to be at the locations of the physical receivers. Each redatumed trace is formed by stacking the cross-correlations of appropriate recorded traces from each physical shot. For the resulting stacked traces to be a valid approximation certain requirements, like an adequate number of surface sources with a small enough spacing in the acquisition geometry, must be met. If these requirements are not met, the resulting virtual shot gather will contain artifacts. In this paper, we analyze both the sets of correlated traces (correlograms) and their stack. We observe that it is possible to reduce certain artifacts in the stacked traces by novel filtering operations. These filtering operations may have broad utility in all of seismic interferometric applications.

1 Introduction

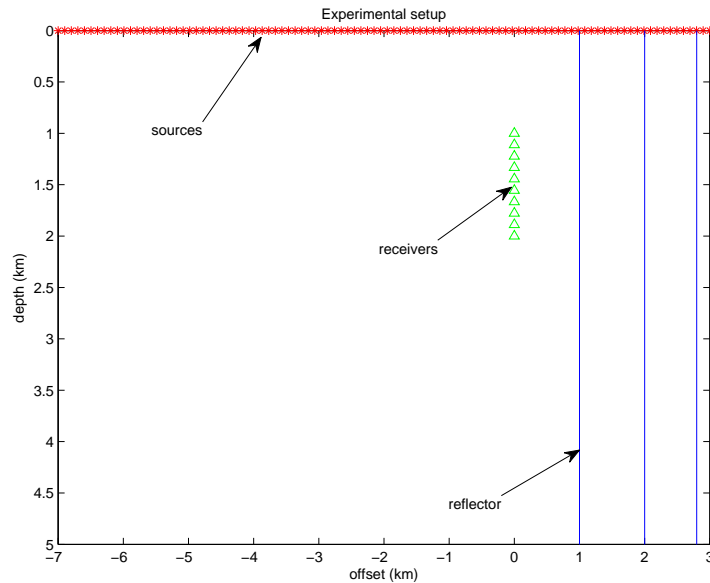


Figure 1: Experimental setup

We begin by assuming that we have a physical source and a collection of physical receivers lying in an acoustic medium. A VSP geometry is shown in Figure 1, with many surface sources and ten receivers located in a well. The source emits a pulse, which then propagates into the medium and is recorded at the receivers to form a shot-gather. It is typically desirable in imaging applications to place the source into the medium so as to maximize the illumination of an area of interest. This unfortunately often proves to be technologically impossible or prohibitively costly. Interferometry is an approach to processing seismic data that allows the physical source to be moved, or redatumed to a receiver location to produce there a virtual source (*Rickett and Claerbout, 1996; Derode et al., 2003;*

Bakulin and Calvert, 2004; Schuster et al., 2004; Wapenaar et al., 2005). If properly constructed, the virtual source could be used for imaging as if it were physical.

Assume we have a pair of receivers and we would like to construct a seismic trace that would be obtained at the second receiver had a physical source been positioned at the location of the first one. The construction is accomplished by stacking of pairwise cross-correlations of received signals at the two locations over all available physical sources. In conditions with sufficient source coverage, the stack will contain the (bandlimited) Green's function from one receiver to another. The first receiver is then called a virtual source and the second one is called a virtual receiver. Figure 2 shows the geometry for a virtual source simulated at the location of most shallow receiver from the surface VSP sources shown in Figure 1.

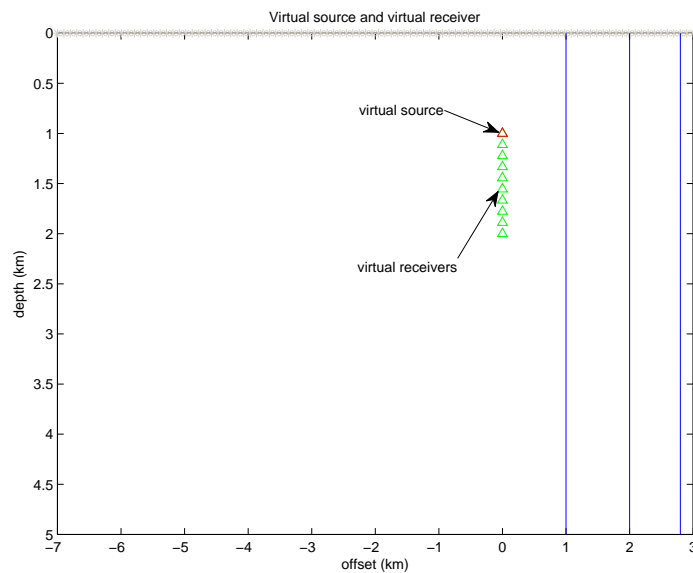


Figure 2: Virtual source and virtual receivers

The interferometric redatuming process does not create new seismic energy. Rather, it is a filter that attempts to only pass events with ray paths starting at one of the surface sources, passing through the virtual source location, bouncing off a reflector, and then traveling to the virtual receiver. Events with this type of ray path form stationary phases on the set of correlograms (*Snieder, 2004*). The process of stacking of correlograms removes from them all correlated events except for stationary phase contributions. The latter are produced by physical sources whose rays that pass through the virtual source and are received at the virtual receiver (*Lu et al., 2008*). Other parts of the correlograms are generally regarded as noise and they are intended to be filtered out as a result of stacking. A stationary phase point in a correlogram has two components: the offset of the physical source that generated the correct ray (which is source location of the correlogram trace with the stationary phase) and the ray’s travel time from the virtual source to the virtual receiver (which is the lag at which the stationary phase occurs). Stacking explicitly ignores the former and only preserves the latter. Under idealized assumptions, this is justified as the stack can be shown to contain the bandlimited virtual trace exactly. When those assumptions are violated stacking leads to undesirable artifacts and as a consequence to incorrect estimation of the Green’s function (*Mehta et al., 2008a,b*).

Conceptually, if we knew which sources generated stationary phase contributions we could throw away contributions from all other sources to the virtual trace. This paper explores the concept of extracting stationary phase point contributions directly from correlograms and using those to construct a virtual trace. The direct benefit of this approach lies in an improved quality

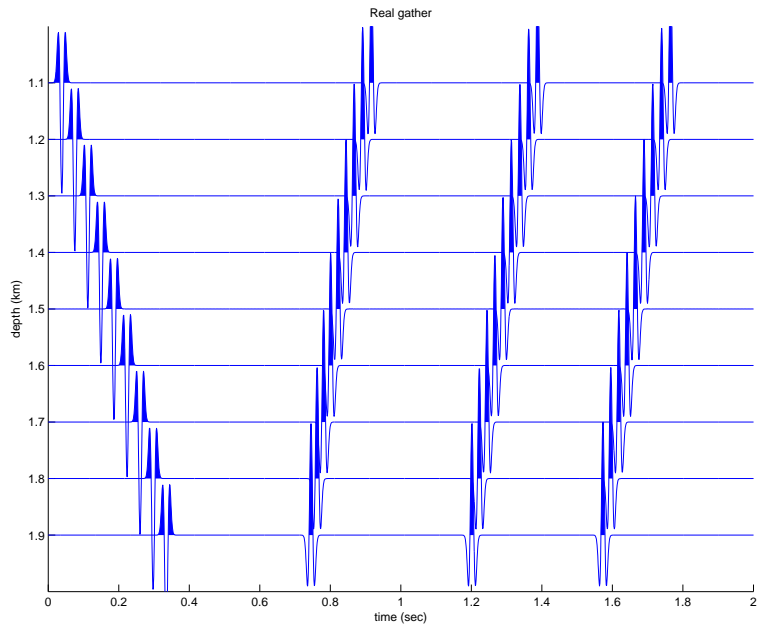


Figure 3: Real Green's function at the virtual source

of the virtual trace. More specifically this technique potentially allows for:

- removal of artifacts caused by limited physical source aperture;
- removal of artifacts caused by uneven angular source coverage;
- removal of the “ringing” (aliasing) due to spatially sparse acquisition;
- selective enhancement of desired reflections;
- compensation for missing source coverage by means of an interpolation or extrapolation.

2 Problem setup

Throughout the discussion below we will assume for specificity a two-dimensional acoustic VSP scenario (see Fig. 1). More specifically, we have a vertical borehole and physical sources positioned on the surface with offsets ranging from -7 km to 3 km. We will look at three different cases of source spacing that correspond to geometries routinely encountered in practice: 25 m, 50 m and 100 m. 10 equidistant receivers are positioned into the borehole at depths varying from 1 km to 2 km. The background velocity $c = 3$ km/sec is assumed constant and we will also assume single scattering. These assumptions are not crucial for what follows, and the proposed methodology is applicable to more general cases.

Immersed inside the medium described above are 3 vertical reflectors, whose offsets are $1, 2$ and 2.8 km correspondingly. If a physical source could be positioned inside the borehole at the location of the first receiver (see Fig. 2), the other receivers would record a shot-gather illustrated in Fig. 3.

We note four clearly visible events: a direct arrival and three primary reflections, as well as the absence of any artifacts. Our goal in this paper is to propose a novel interferometry-based approach that will allow to reconstruct this theoretical shot-gather from actual physical data with the best possible quality.

3 Correlogram space and interferometry

Recall that a wave propagation between any two points is defined by the Green's function. If a source is located at one point then a wavefield re-

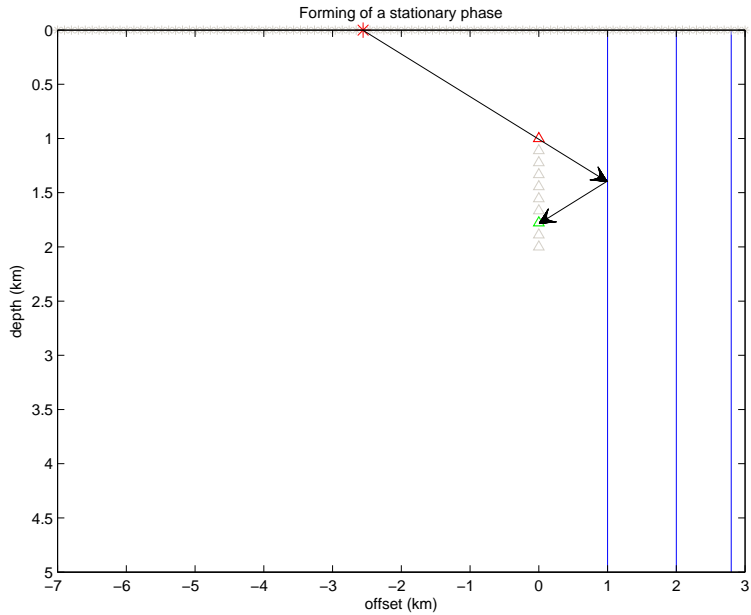


Figure 4: Correct ray for a pair of a virtual source and a virtual receiver

ceived at another point is a convolution of the Green's function between the two points with the source wavelet. Let us first compare an actual physical Green's function and its virtual reconstruction by interferometry. The physical Green's function does not depend on the experimental setup as it is solely a property of the acoustic medium. Any reconstruction obtained from recorded data contains an imprint of the geometry of the experiment. For an arbitrary reflector to be registered in the reconstructed Green's function, the direction from the virtual source to the reflector must be illuminated by a physical source (see Fig. 4).

Ideally sources should cover the medium from all possible angles or else the Green's function risks containing errors or omissions. In a realistic seismic surveying experiment sources can only be positioned on the surface and even

there a continuous coverage is impractical. It is therefore to be expected that the reconstructed or virtual Green’s function will contain a number of artifacts or errors relative to its physical counterpart.

A conventional approach to interferometric reconstruction of the virtual shot-gather is as follows (*Korneev and Bakulin, 2006; Lu et al., 2008*). A series of explosions is conducted on the surface, each resulting in a shot-gather recorded at each receiver location in the borehole (see Figs 5, 6, 7).

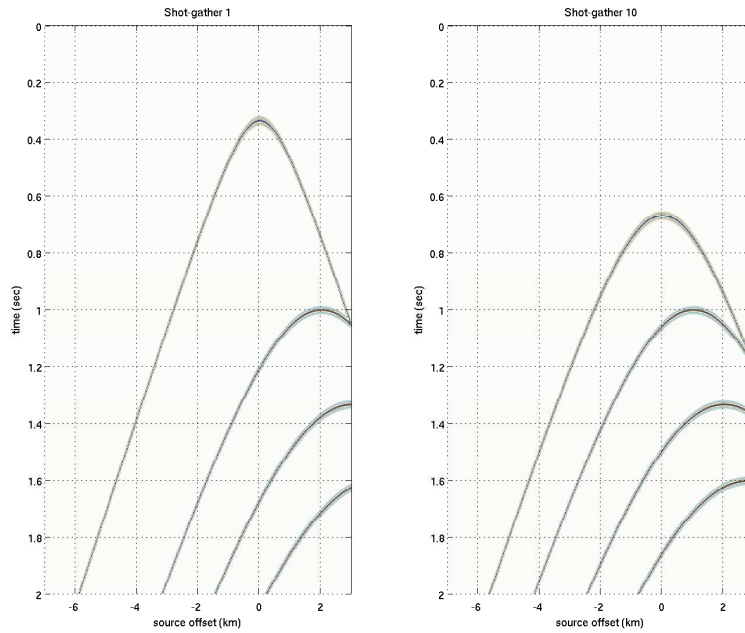


Figure 5: Shot-gather (source spacing 25m)

A virtual source will be at the location of one of the receivers, say the first receiver, and the other nine receivers become virtual receivers. We construct cross-correlograms by cross-correlating traces corresponding to the same physical source from the first receiver and every other receiver (see Figs

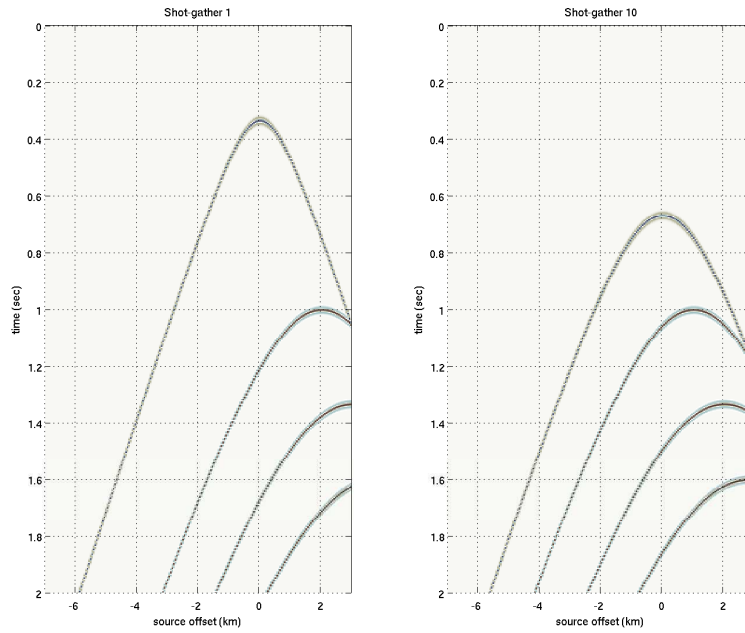


Figure 6: Shot-gather (source spacing 50m)

8, 9, 10).

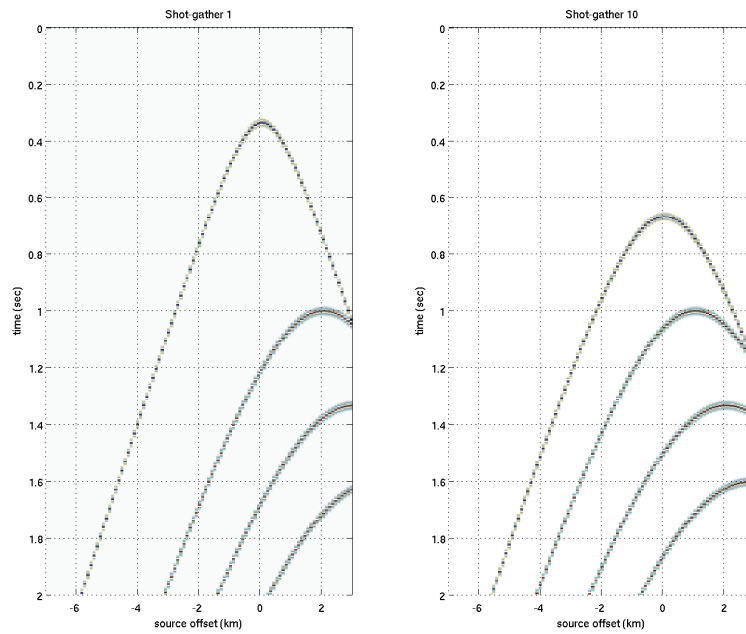


Figure 7: Shot-gather (source spacing 100m)

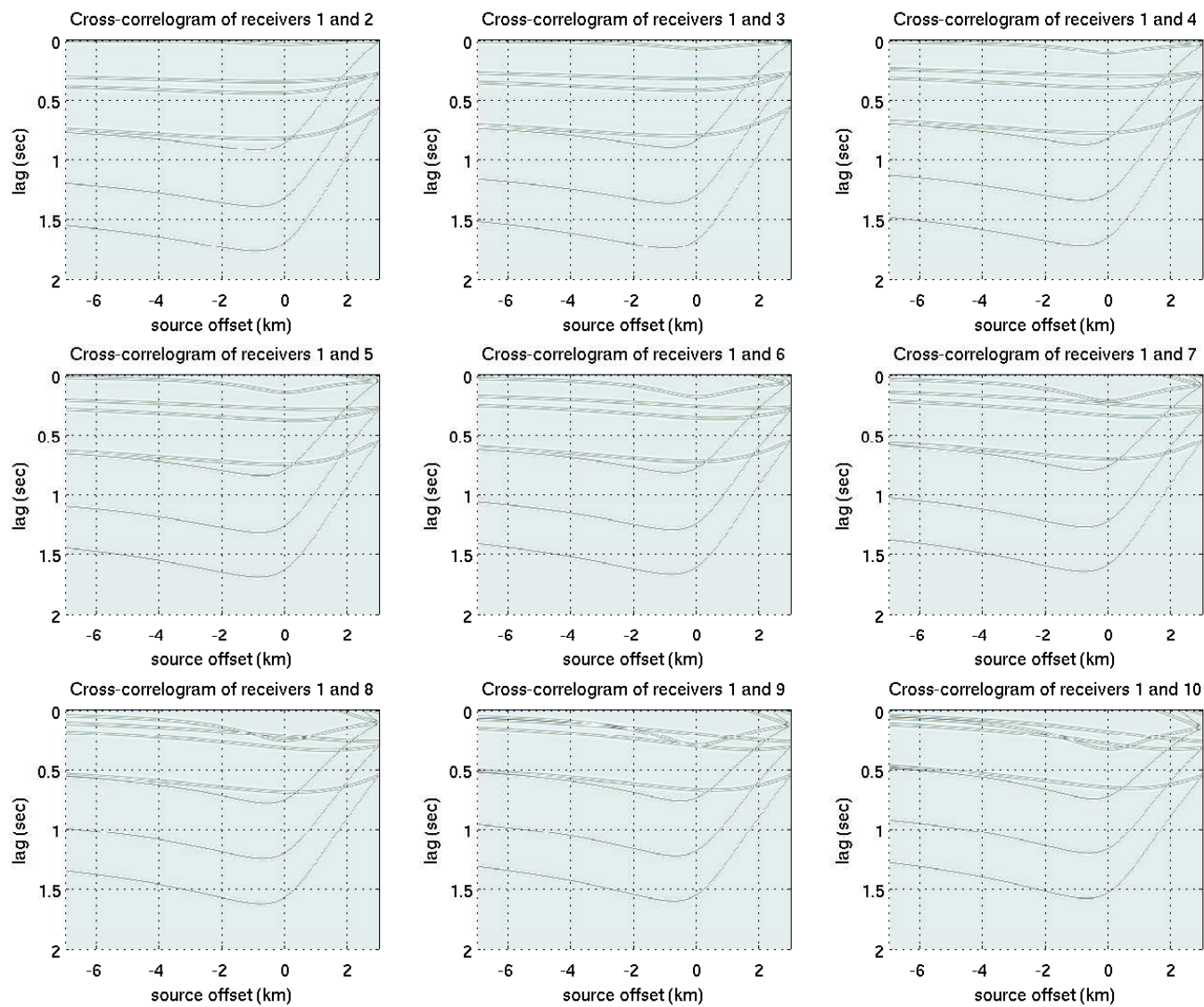


Figure 8: Correlogram space (source spacing 25m)

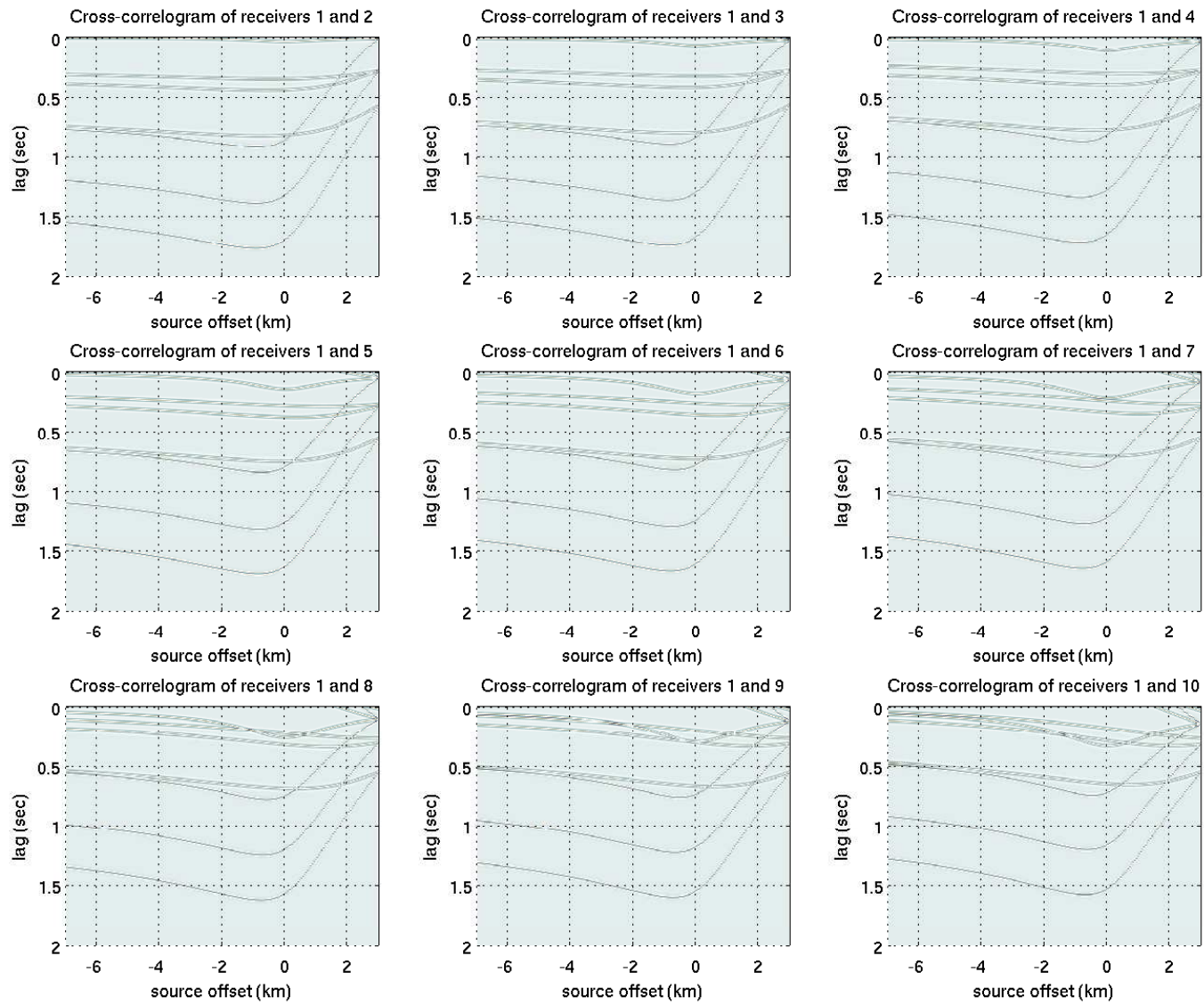


Figure 9: Correlogram space (source spacing 50m)

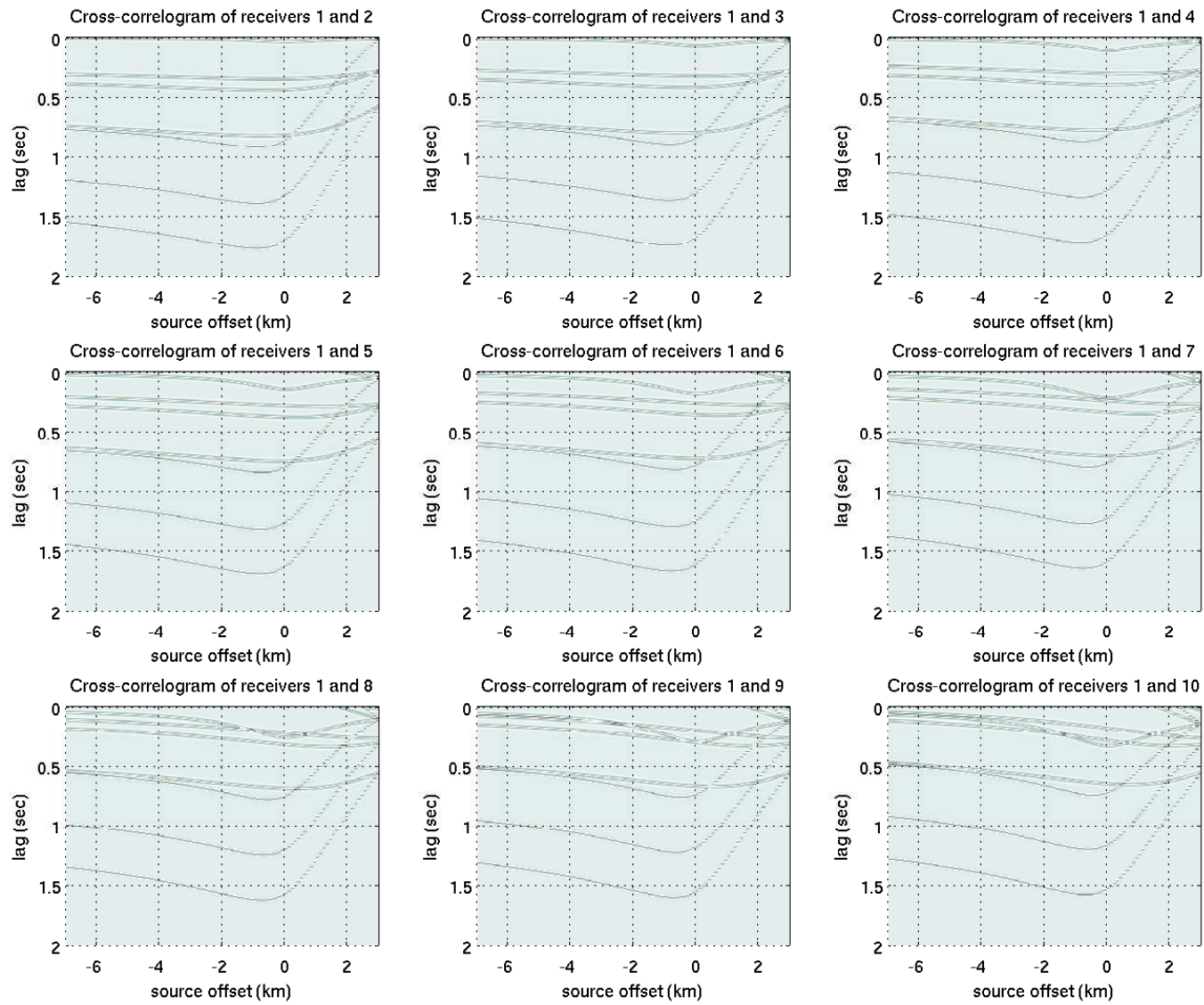


Figure 10: Correlogram space (source spacing 100m)

These cross-correlograms are finally stacked in the horizontal direction to form a virtual gather (Figs 11, 12, 13).

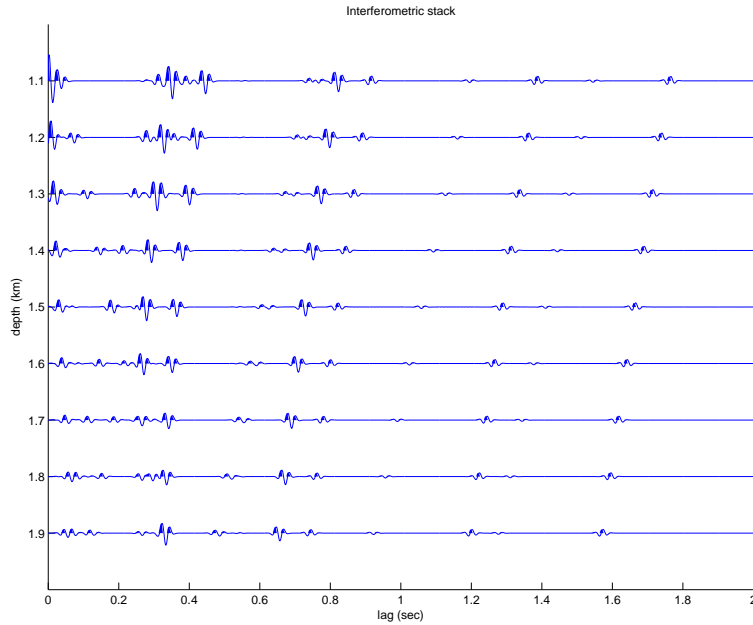


Figure 11: Interferometric stack (source spacing 25m)

A couple of observations are immediately in order about those gathers.

- It can be checked by visual inspection that all three gathers contain events from the real gather plotted in Fig. 3.
- In addition to those events, a large number of artifacts are present, and those obscure the desired physical events, most strongly when the source spacing is large.

The process of reconstructing the Green's function at the virtual source is based on Rayleigh's reciprocity theorem (*Wapenaar, 2004; Wapenaar et al.,*

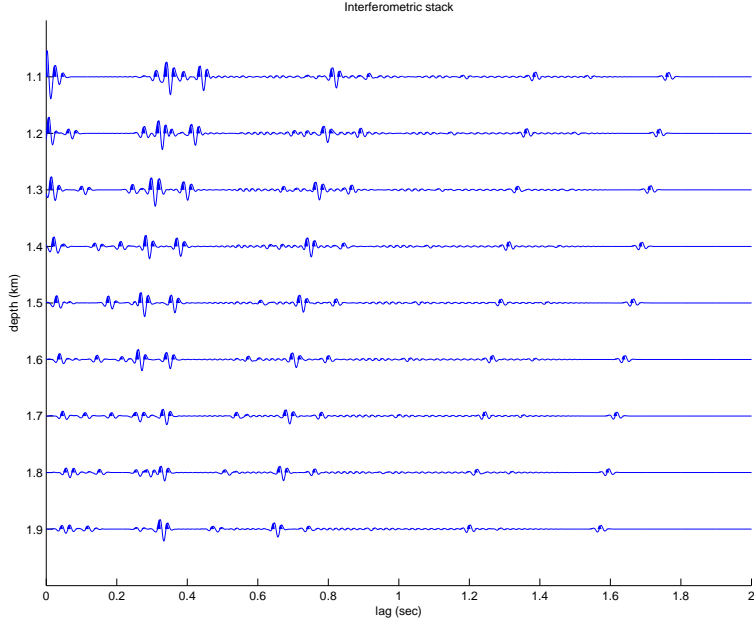


Figure 12: Interferometric stack (source spacing 50m)

2005; *Schuster and Zhou, 2006*), which under further simplifying assumptions yields:

$$G(\mathbf{x}_r^1, \mathbf{x}_r^j, -t) + G(\mathbf{x}_r^1, \mathbf{x}_r^j, t) \propto \oint_{\mathcal{C}_s} G(\mathbf{x}_s, \mathbf{x}_r^1, -t) \star G(\mathbf{x}_s, \mathbf{x}_r^j, t) d\mathbf{x}_s. \quad (1)$$

Here

- $\mathbf{x}_r^j, j = 1, \dots, 10$ are receiver locations;
- t is time;
- G is the Green's function of the medium;
- \mathbf{x}_s denotes a source location on a continuous closed curve \mathcal{C}_s surrounding our medium; and

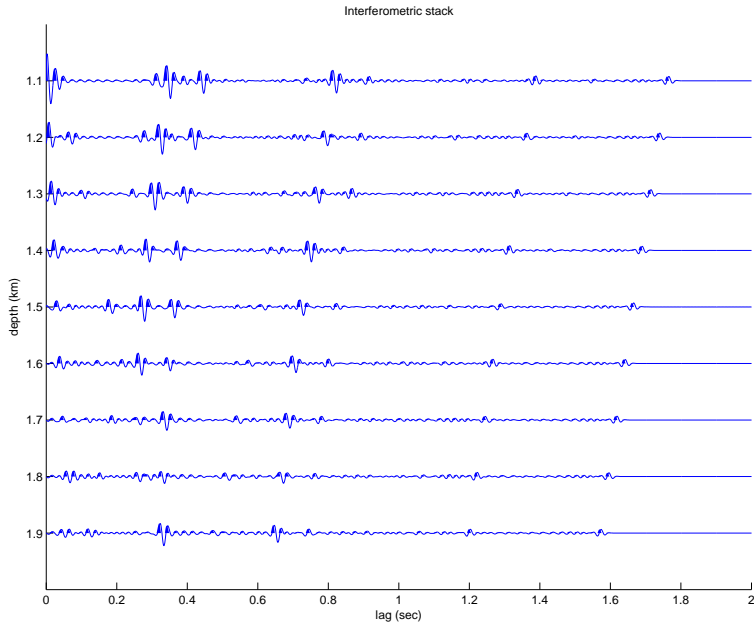


Figure 13: Interferometric stack (source spacing 100m)

- \star denotes convolution in time.

It is clear that the assumptions of this theorem are violated in all three of our setups in that the sources do not enclose the entire medium but instead illuminate it only from the surface, and furthermore the source coverage is not continuous but instead it is restricted to a finite and progressively sparser grid (on the three decimated data sets). This disconnect between theory and practice results in poor performance of the interferometric algorithm, and requires that extra steps be taken in order to alleviate observed problems.

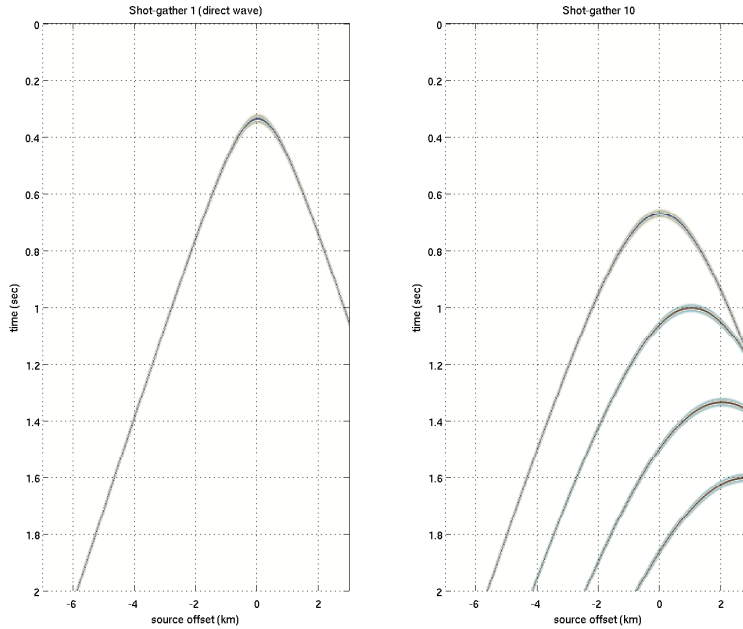


Figure 14: Muted shot-gather (source spacing 25m)

4 Cross-terms and a virtual source from a direct wave

We proceed towards constructing a better virtual Green's function by coming back to the set of raw correlograms and examining them more closely. These correlograms are comprised of pairwise cross-correlations of events from initial shot-gathers (Figs 5, 6, 7). Of physical significance are cross-correlations of the direct wave at the first gather with primary reflections on the rest, as those correspond to a physical process of wave propagation from the virtual source to the virtual receivers. Cross-correlations between various reflections themselves are non-physical, and we would like to filter those out. (Note: in the case with multiple scattering, it is possible that certain of these cross

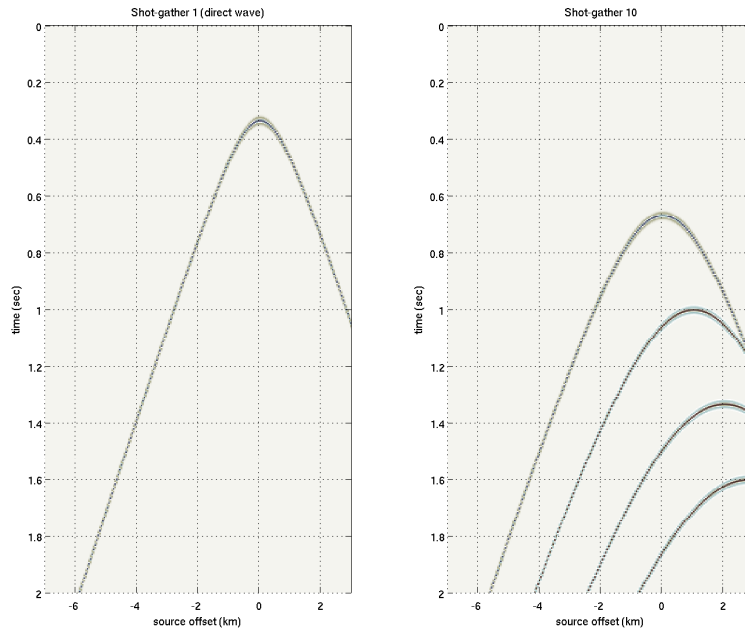


Figure 15: Muted shot-gather (source spacing 50m)

correlations may add constructively as additional smaller amounts of illumination energy. For now, however, we will ignore this effect and address it in a future paper.)

This is accomplished by muting off the reflections on the traces at the virtual source and using only the direct wave to compute cross-correlations (*Bakulin and Calvert, 2004*). The modified shot-gathers are depicted in Figs 14, 15, 16, and the resulting cross-correlograms are given in Figs. 17, 18, 19.

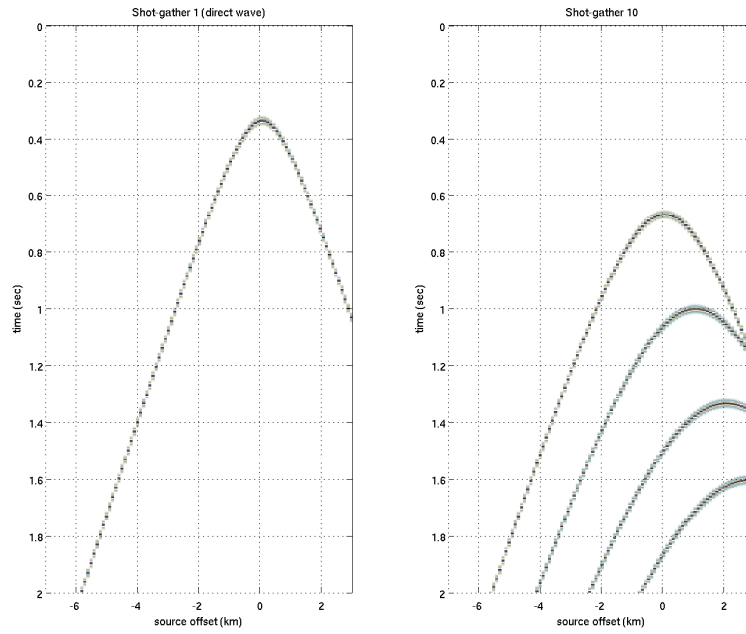


Figure 16: Muted shot-gather (source spacing 100m)

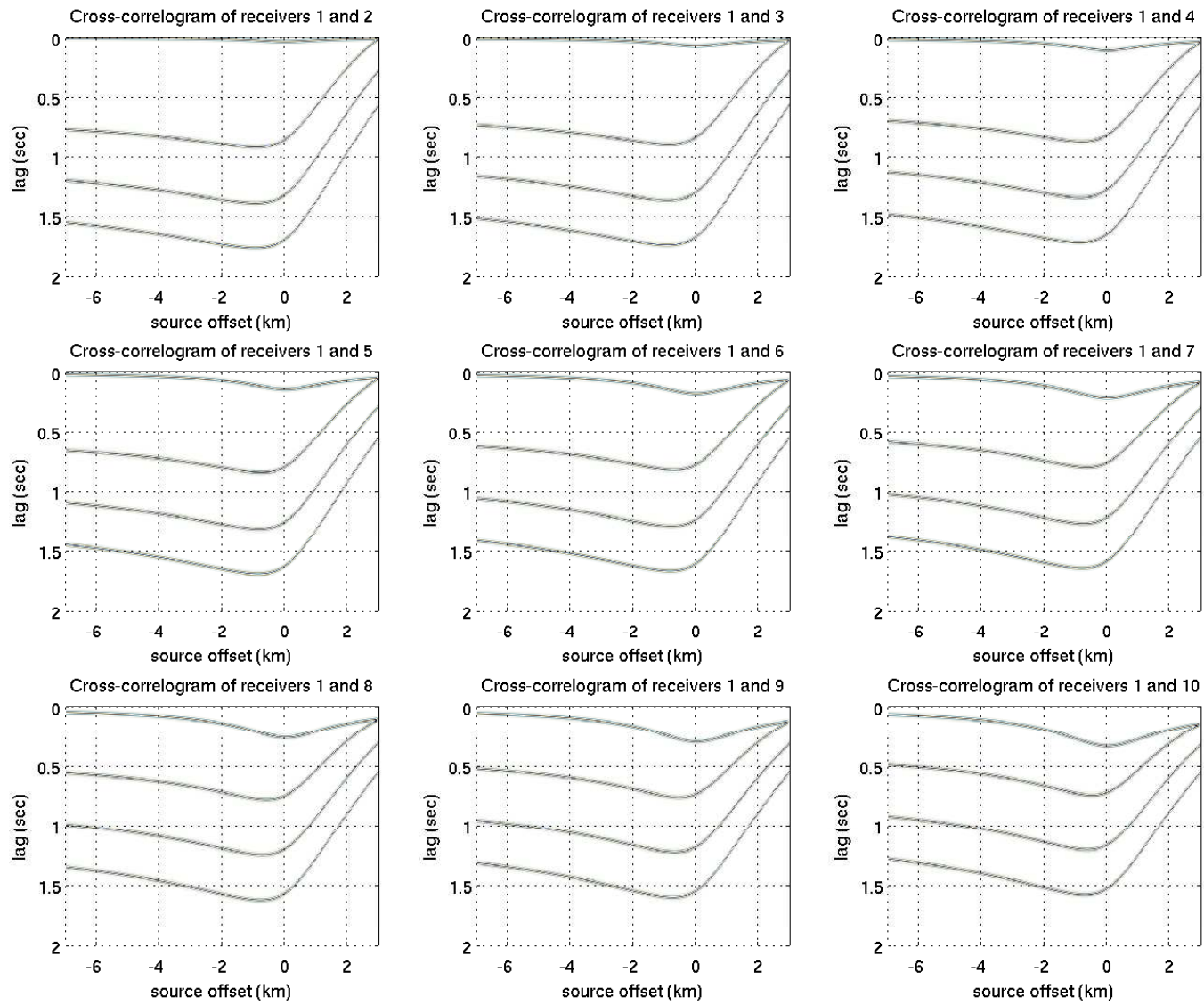


Figure 17: Muted correlogram space (source spacing 25m)

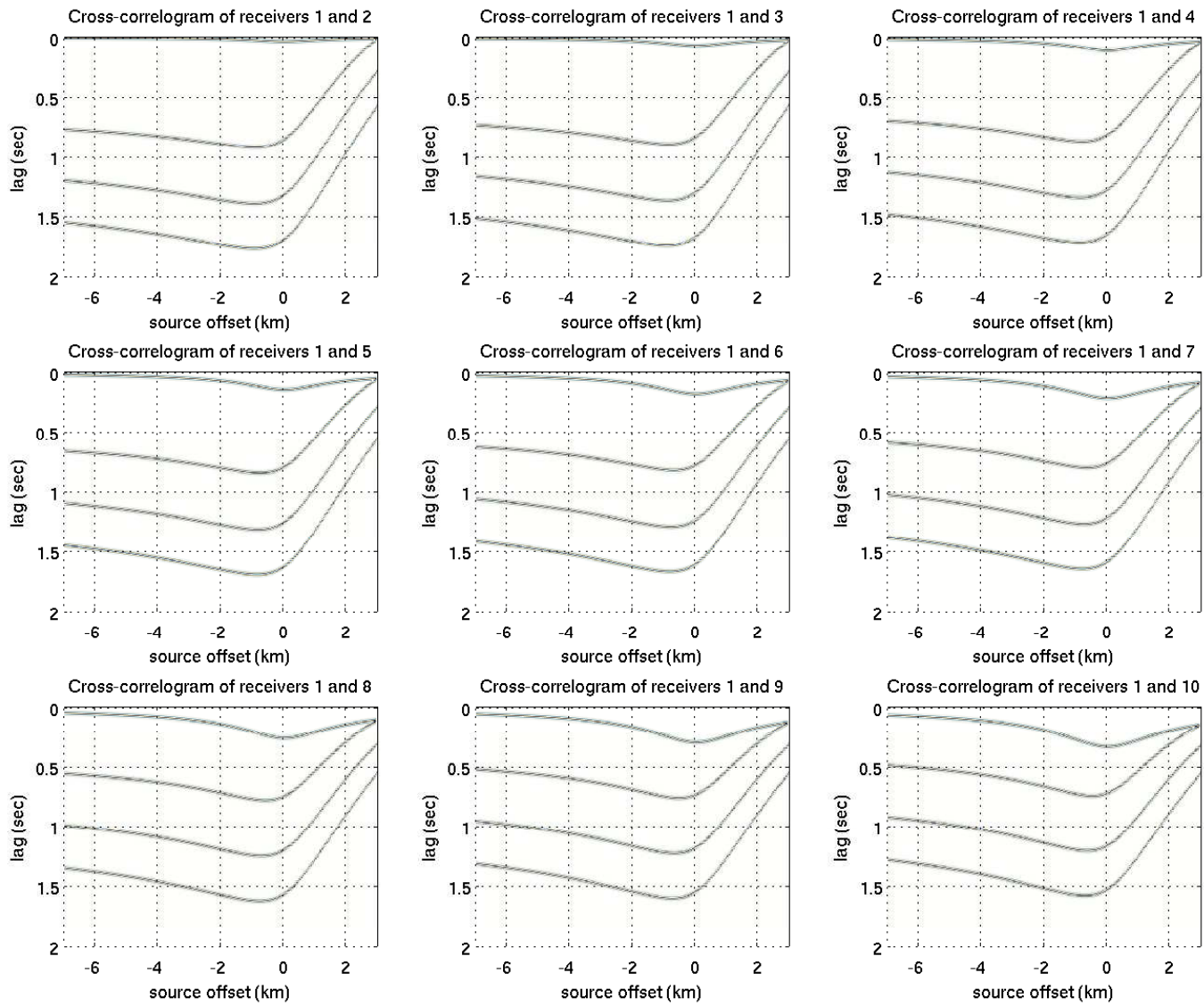


Figure 18: Muted correlogram space (source spacing 50m)

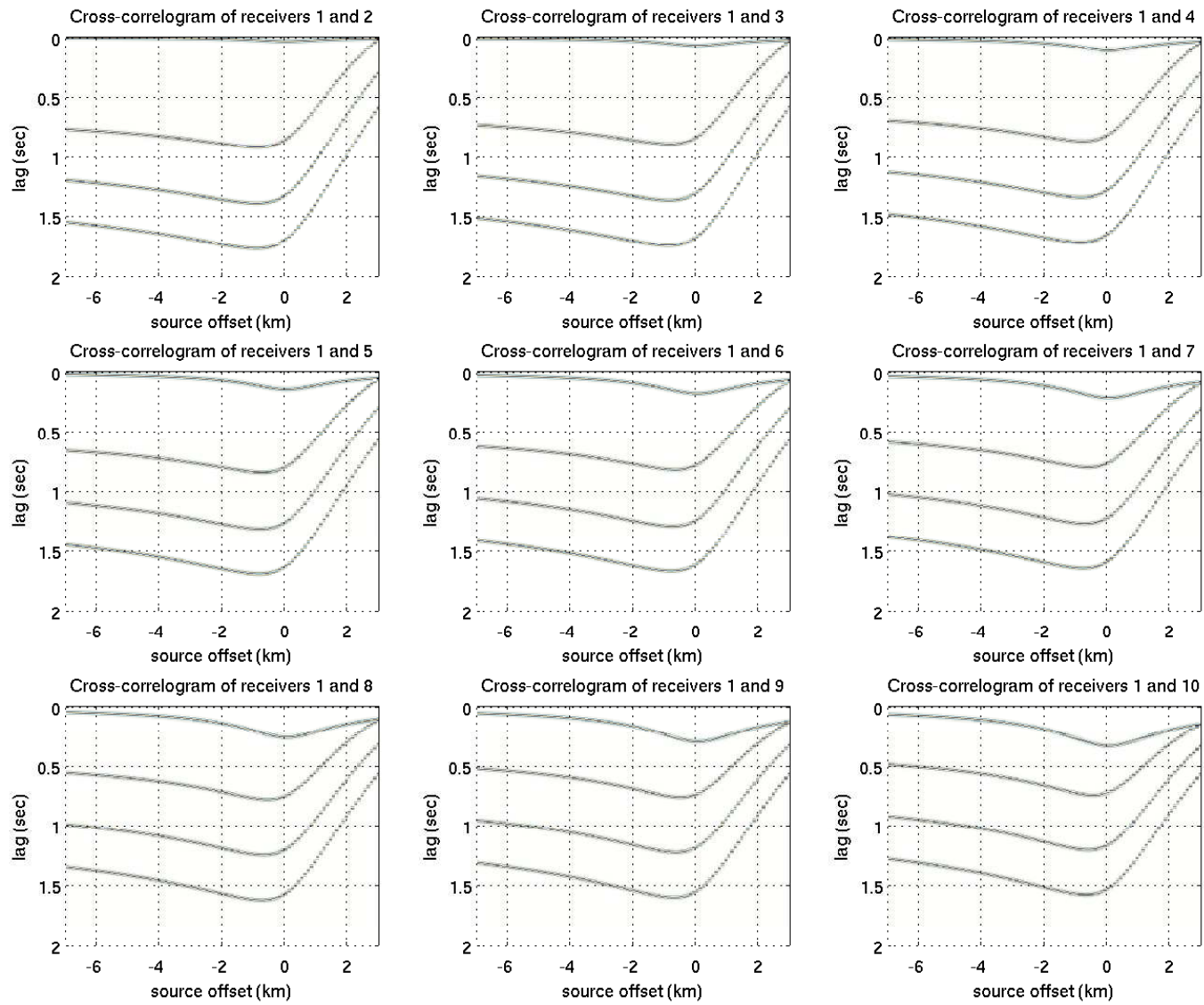


Figure 19: Muted correlogram space (source spacing 100m)

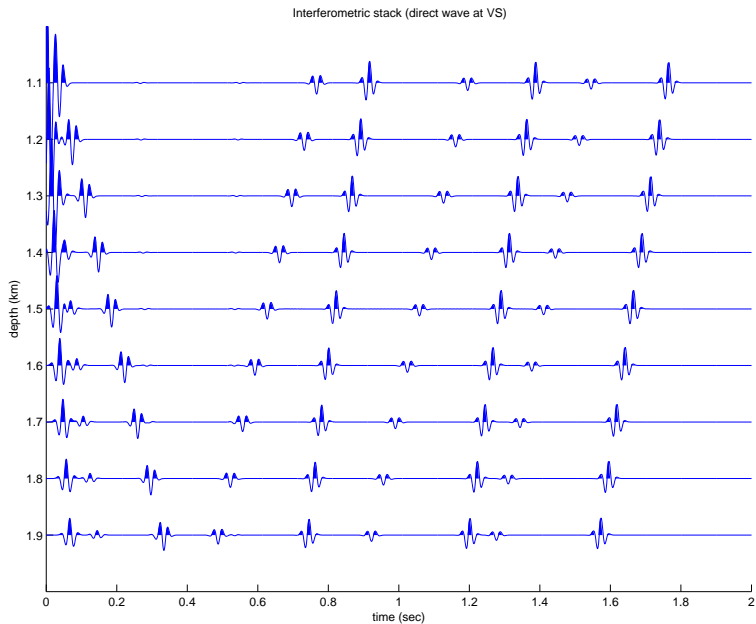


Figure 20: Muted interferometric stack (source spacing 25m)

Stacks obtained from cross-correlograms constructed using only the direct waves at the virtual source are noticeably better in quality (compare Figs 13 and 22). However, they still suffer from remaining artifacts. Their nature is two-fold: troublesome coherent cross-term events are due to the finiteness of the real acquisition surface, and incoherent ringing is due to spatial aliasing caused by sparsely located sources.

5 Edge effects and tapering

Stacking of a set of cross-correlograms is intended to preserve stationary phase points. They can be identified by the horizontal (or zero) slope of events. Integrating (horizontally) over all sources removes in theory slanted (dipping) branches as negative parts of cross-correlograms cancel positive

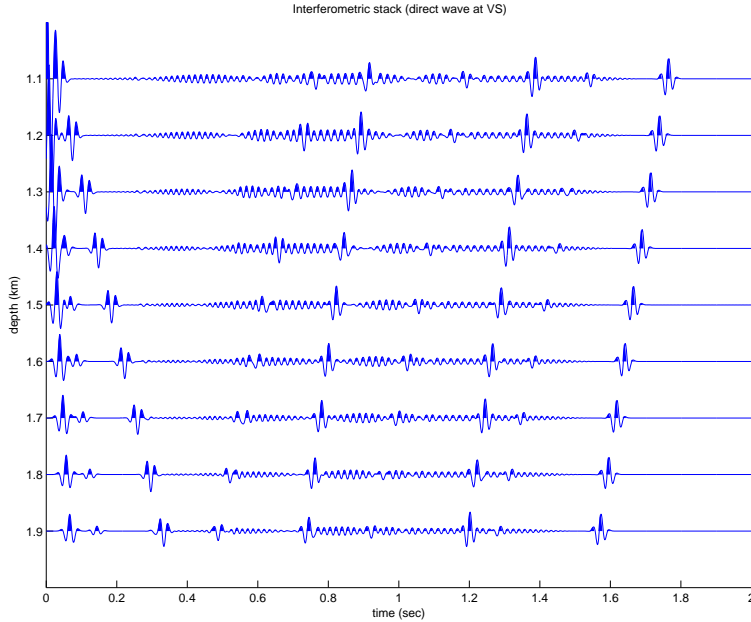


Figure 21: Muted interferometric stack (source spacing 50m)

ones. A non-trivial contribution is generated only where the integration line touches an event at one point. That is the stationary phase point.

When an acquisition surface is finite, additional uncompensated contributions arise along edges of cross-correlograms. In order to prevent them from appearing in the final stack as artificial events, we apply a smooth taper. Figure 23 shows an example taper function which applies a single scalar multiplier to each trace in the set of correlograms. The point of this taper is to reduce the far source offset contributions which abruptly stop and are no longer canceled out by additional sources. Multiplying the cross-correlation inside the integral (1) by the function plotted in Fig.23 smoothes out uncompensated events along the edges on the cross-correlograms and results in far better stacks (Figs 24, 25, 26). We note that in the case of the finest source

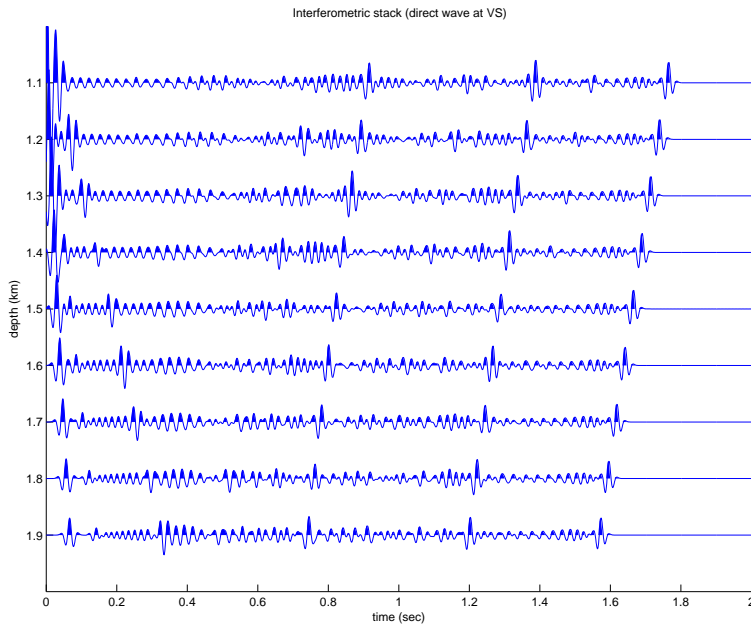


Figure 22: Muted interferometric stack (source spacing 100m)

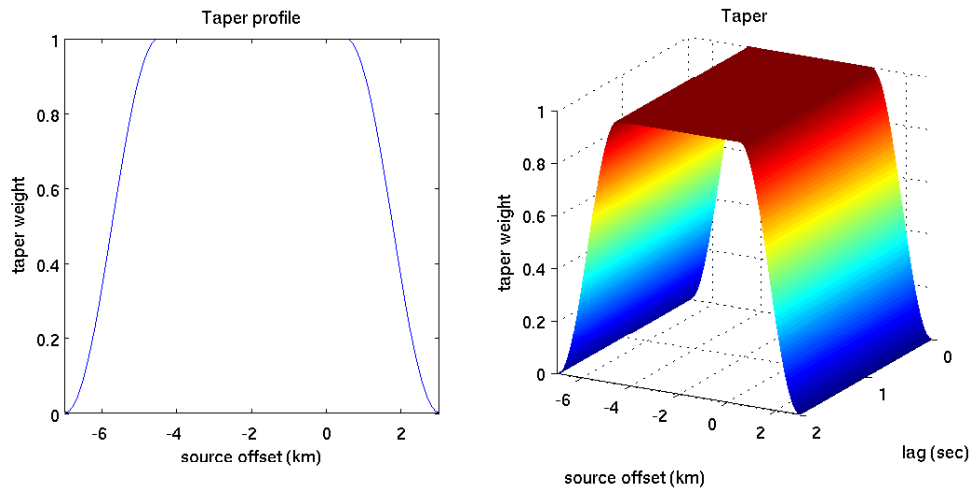


Figure 23: Taper

spacing, tapering of the edges results in a virtually perfect reconstruction of

the Green’s function. As the spacing increases, we see the remaining “ringing” due to spatial aliasing. Dealing with this problem is a subject of the following section.

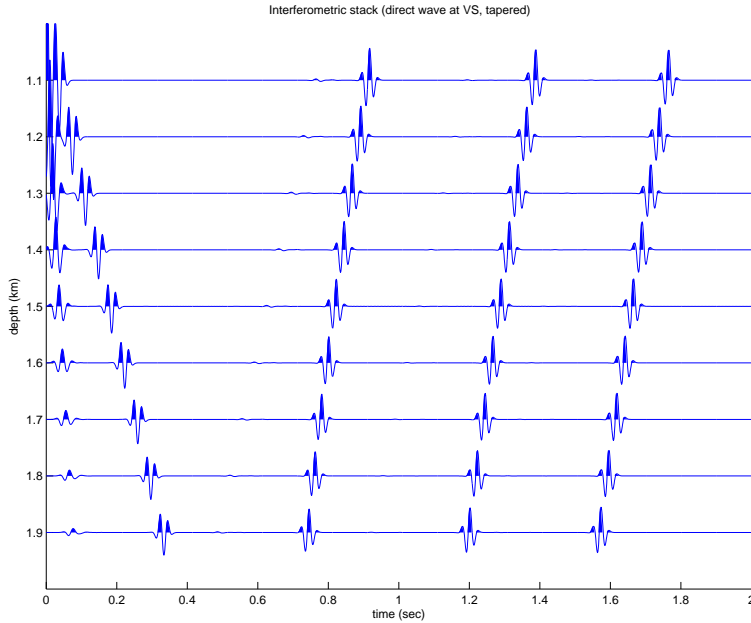


Figure 24: Tapered interferometric stack (source spacing 25m)

6 Spatial aliasing

6.1 Direct identification of stationary phase points

The simple but key observation is that stacking is a tool for identifying stationary points, not the end goal. As was elaborated above, in an idealized scenario stacking picks all contributions from stationary phase points while at the same time filtering out everything else. When those ideal conditions are not met, stacking should not be employed without a prior and proper

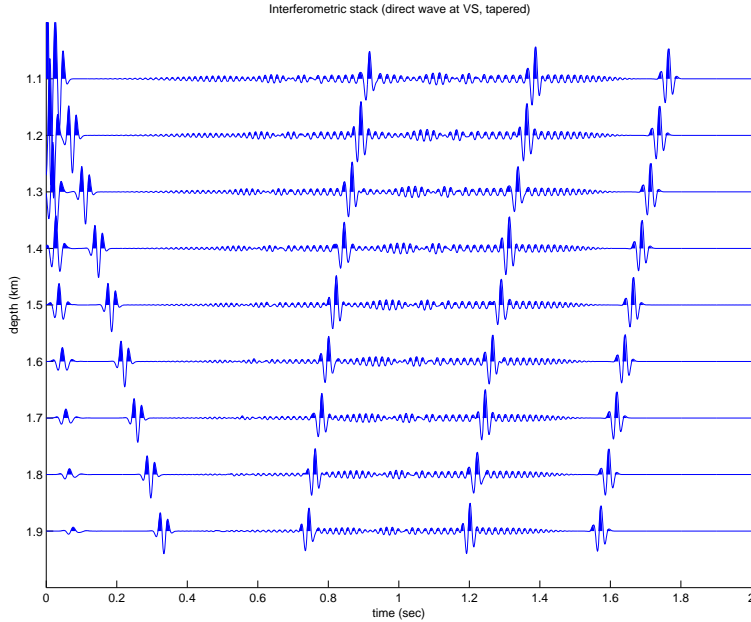


Figure 25: Tapered interferometric stack (source spacing 50m)

processing of the correlograms.

Spatial aliasing is a phenomenon that leads to significant noise in the stack due to incomplete destructive interference of the steeply dipping branches within a correlogram. This is because of too large time shifts between the traces in the event branches, particularly where the slope is especially steep. It is extremely problematic to interpolate between the samples of the cross-correlogram without a loss in resolution unless additional information about reflectors is available.

We instead propose an algorithm that enables us to directly identify stationary phase points in the correlogram space and use those to produce an enhanced stack void of “ringing” induced by spatial aliasing.

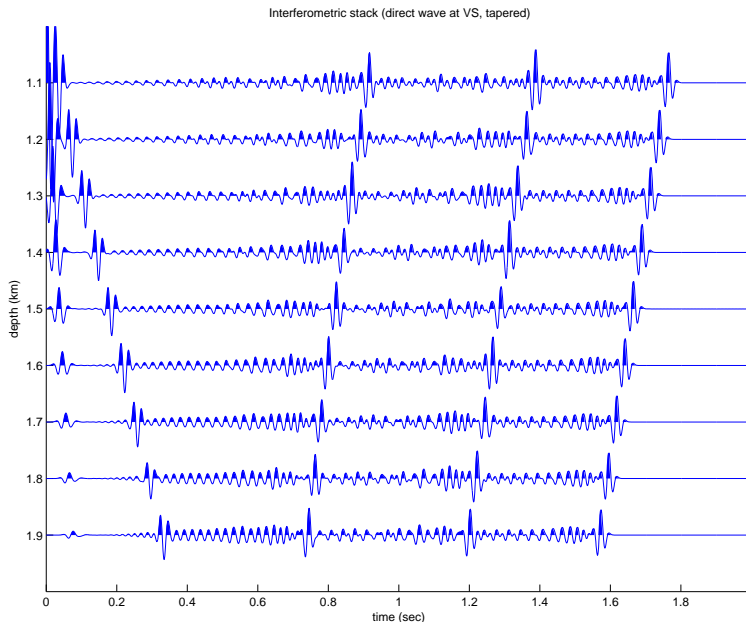


Figure 26: Tapered interferometric stack (source spacing 100m)

6.2 Correlation weight functions and enhanced stacking

Consider a cross-correlogram $C(\mathbf{x}_r^j, \mathbf{x}_s, \tau)$ and a standard (tapered but aliased) interferometric stack $s(\mathbf{x}_r^j, \tau)$ introduced above. Each coherent contribution in the stack is introduced by a stationary point in the correlogram. We set up a sliding window of the size Δt comparable to the temporal wavelength of the source.

For any fixed recording time t_0 we compute a local zero-lag correlation of the stack with each individual cross-correlogram:

$$Z(\mathbf{x}_r^j, t_0, \mathbf{x}_s) = \int_{t_0 - \frac{\Delta t}{2}}^{t_0 + \frac{\Delta t}{2}} s(\mathbf{x}_r^j, \tau) C(\mathbf{x}_r^j, \mathbf{x}_s, \tau) d\tau. \quad (2)$$

The maximum contribution to the stack at lag t_0 of a particular physical source \mathbf{x}_s will result in a maximum value of the function $Z(t_0, \cdot)$. Let

$$\mathbf{x}_{st_0}(\mathbf{x}_r^j) = \arg \max_{\mathbf{x}_s \in \mathcal{C}_s} Z(\mathbf{x}_r^j, t_0, \mathbf{x}_s), \quad (3)$$

and

$$z_{\mathbf{x}_r^j, t_0} = Z(\mathbf{x}_r^j, t_0, \mathbf{x}_{st_0}(\mathbf{x}_r^j)). \quad (4)$$

Our assertion is that if the stack contains an event at time t_0 , then \mathbf{x}_{st_0} is the offset of the physical source that produced that contribution. The significance of the contribution is measured by the weight map $Z(\mathbf{x}_r^j, t_0, \mathbf{x}_s)$. For our three setups their corresponding weight maps are plotted in Figs 27, 28, 29.

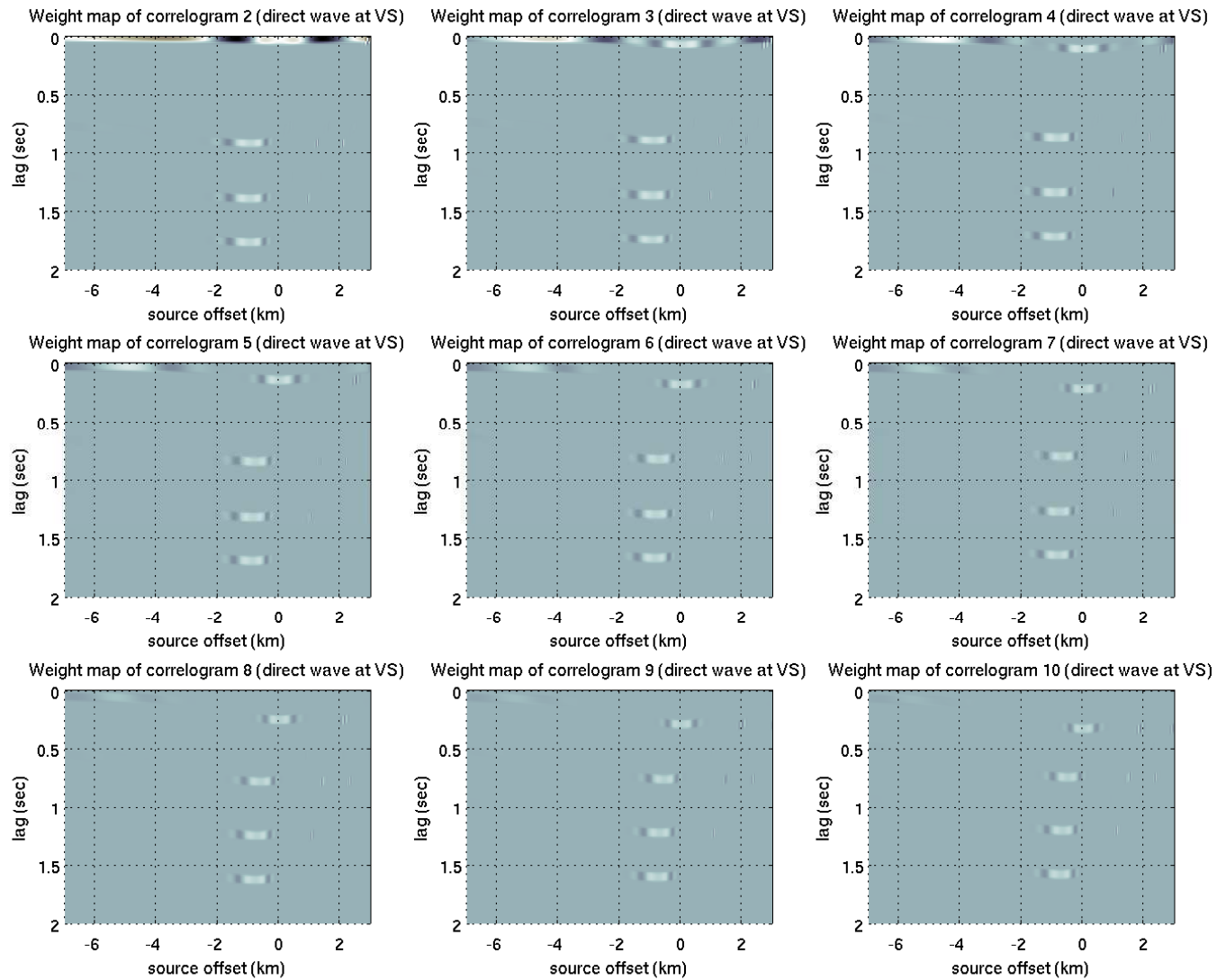


Figure 27: Weight maps (source spacing 25m)

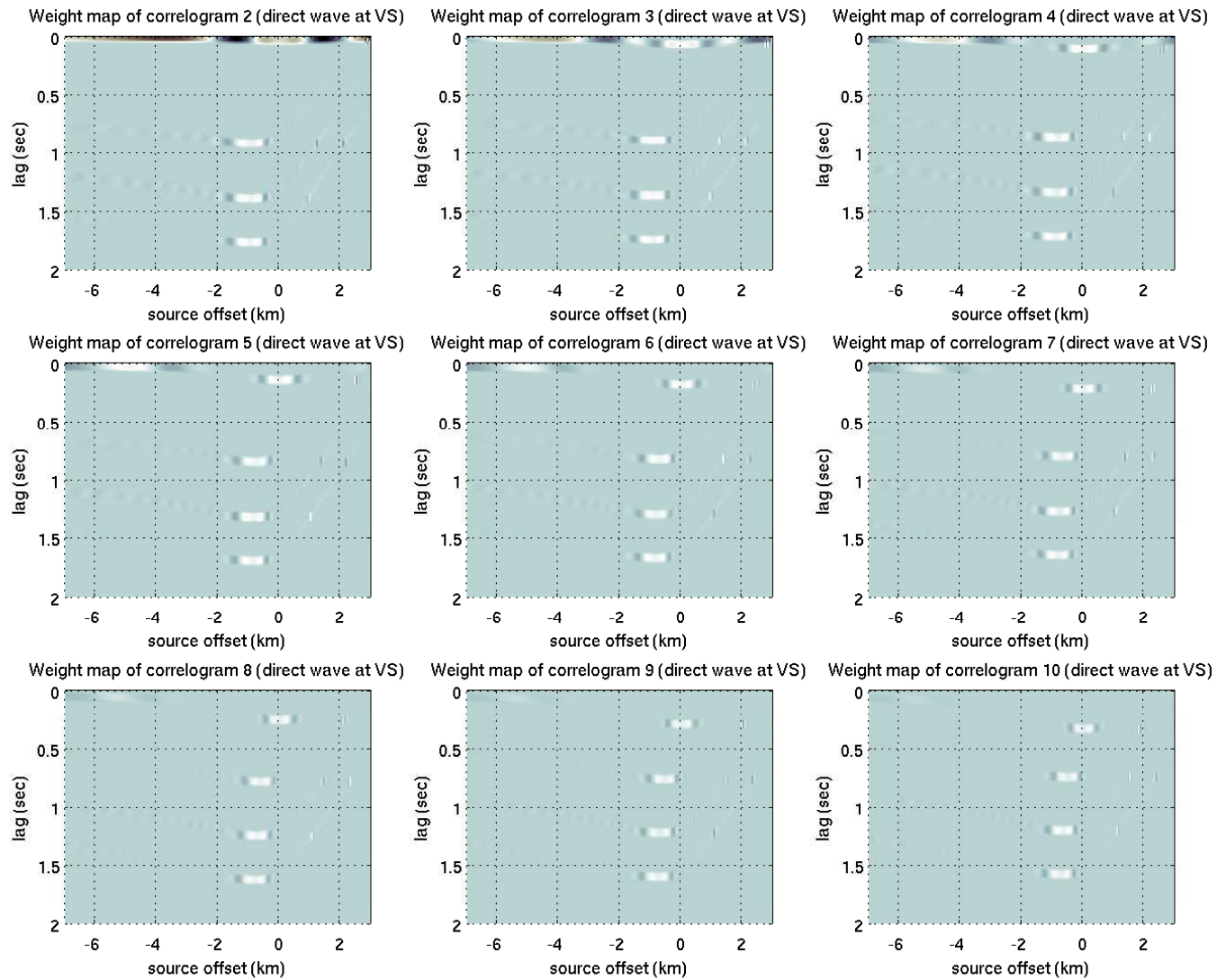


Figure 28: Weight maps space (source spacing 50m)

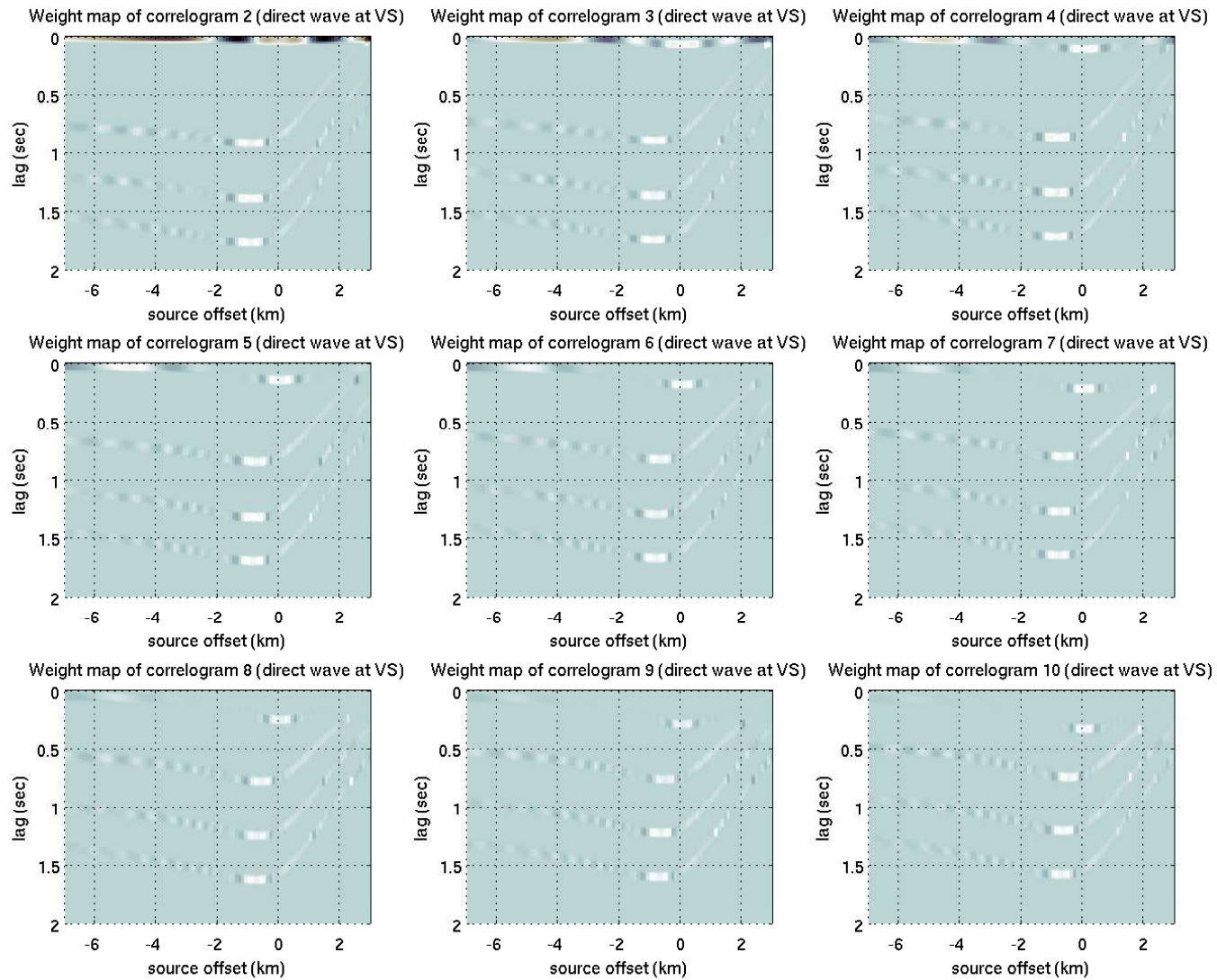


Figure 29: Weight maps (source spacing 100m)

Define a small threshold parameter ζ , set to zero any contribution that is below the threshold, i.e.

$$Z^\zeta(\mathbf{x}_r^j, t, \mathbf{x}_s) = \begin{cases} Z(\mathbf{x}_r^j, t, \mathbf{x}_s), & |Z(\mathbf{x}_r^j, t, \mathbf{x}_s)| > \zeta \\ 0, & \text{otherwise} \end{cases}, \quad (5)$$

and construct an enhanced stack $s_{\text{enh}}(t)$ as follows:

- set $s_{\text{enh}}(\mathbf{x}_r^j, t) \equiv 0$.
- for each window $[t_0 - \frac{\Delta t}{2}, t_0 + \frac{\Delta t}{2}]$

$$s_{\text{enh}}(\mathbf{x}_r^j, t) = s_{\text{enh}}(\mathbf{x}_r^j, t) + C(\mathbf{x}_r^j, t, \mathbf{x}_{s_t}) \cdot Z^\zeta(\mathbf{x}_r^j, t, \mathbf{x}_{s_t}),$$

$$t \in \left[t_0 - \frac{\Delta t}{2}, t_0 + \frac{\Delta t}{2} \right]$$

- end for each

In words, the enhanced stacks contain local pieces of cross-correlograms that correlate most prominently with the standard stack. As the “ringing” does not correlate as well, its contribution is suppressed through the weight map Z . Figs 30, 31, 32 demonstrate that this technique allows for a nearly perfect reconstruction of the virtual gather even in the case of significant spatial aliasing.

7 Conclusions

Interferometry is a relatively new area with a significant potential. One’s ability to correctly redatum physical sources to the physical receiver locations may have tremendous applications in seismic imaging. At the same

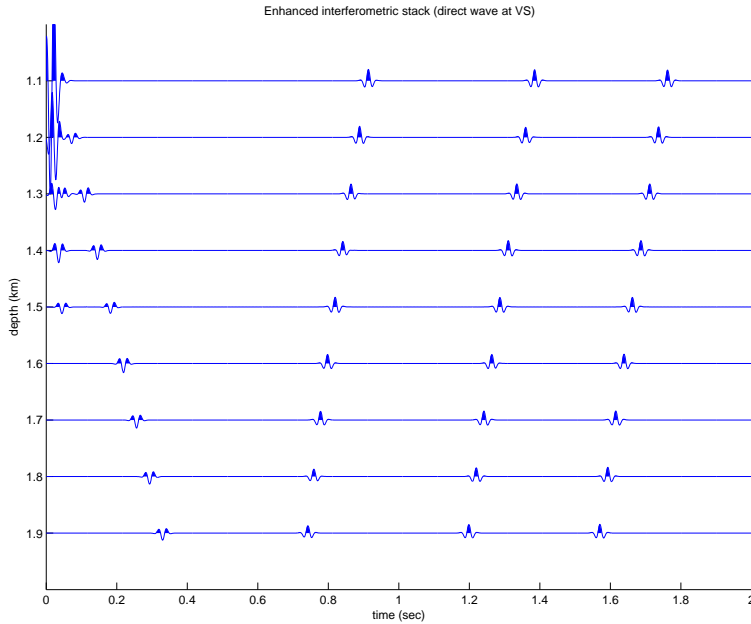


Figure 30: Enhanced interferometric stack (source spacing 25m)

time classical interferometric techniques work flawlessly only in idealized situations. Ignoring practical limitations and/or failure to correct for them may result in a poor reconstruction of a virtual gather and as a result in poor images.

Conventional interferometry obtains a virtual gather by stacking cross-correlograms in order to extract stationary phase points. This process may lead to introducing additional artifacts into the stack. In this paper, we have proposed to extract stationary phase points directly from correlograms. By so doing, we explicitly identify physical events in the correlogram space and avoid having to rely on the stacking procedure to filter out noise. We have demonstrated that the proposed approach may allow mitigating problems caused by edge effects and spatial aliasing. We are at the beginning of this

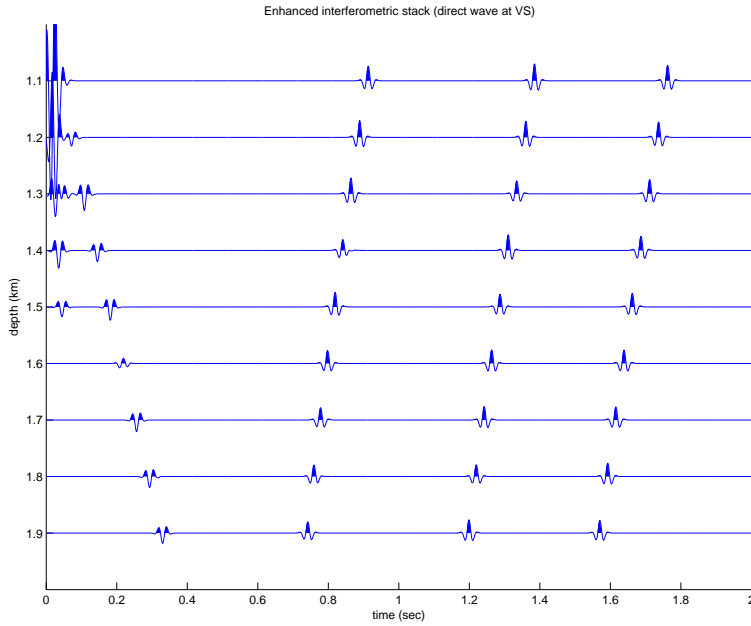


Figure 31: Enhanced interferometric stack (source spacing 50m)

analysis and have not addressed the full amplitude aspects of this methodology. Additional effort will be made to more fully take into consideration these concerns.

8 Acknowledgement

This work was supported by the Earth Resources Laboratory Founding Member Consortium.

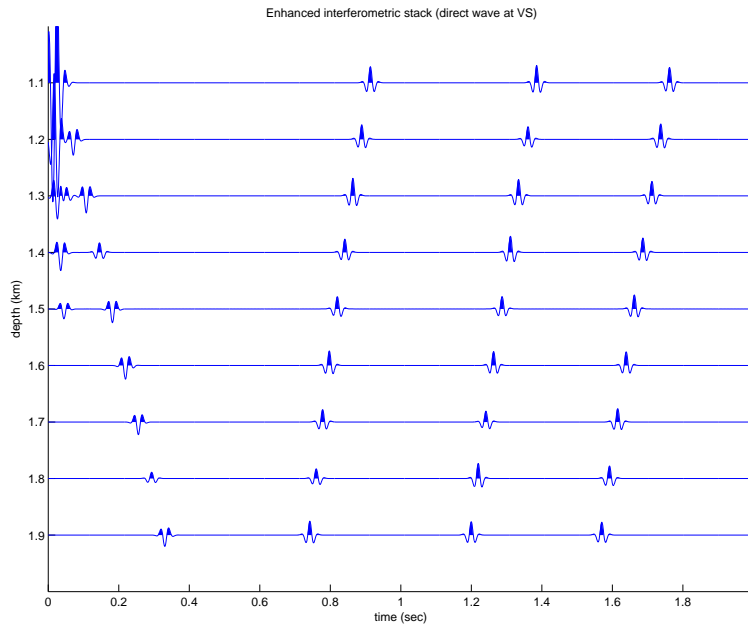


Figure 32: Enhanced interferometric stack (source spacing 100m)

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