Will we observe black holes at LHC?

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The generalized uncertainty principle, motivated by string theory and non-commutative quantum mechanics, suggests significant modifications to the Hawking temperature and evaporation process of black holes. For extra-dimensional gravity with Planck scale O(TeV), this leads to important changes in the formation and detection of black holes at the the Large Hadron Collider. The number of particles produced in Hawking evaporation decreases substantially. The evaporation ends when the black hole mass is Planck scale, leaving a remnant and a consequent missing energy of order TeV. Furthermore, the minimum energy for black hole formation in collisions is increased, and could even be increased to such an extent that no black holes are formed at LHC energies.

The possible existence of large extra dimensions (LEDs) [1] has opened up new and exciting directions of research in quantum gravity. In this scenario, standard-model particles are confined to the observable 4-dimensional "brane" universe, whereas gravitons can access the whole d-dimensional "bulk" spacetime, being localized at the brane at low energies. The effects of LEDs show up in new particle states at energies below the fundamental gravitational scale [2] and in nonperturbative effects in the trans-Planckian energy regime (for a review see [3].) The fundamental Planck scale in LED models may be as small as a TeV, within reach of near-future experiments. This leads to many interesting possibilities in "quantum gravity phenomenology". One of the most exciting quantum gravity signatures would be production and decay of black holes (BHs) and other extended objects in cosmic ray air-showers and at particle colliders such as the Large Hadron Collider (LHC) [4, 5, 6].

Hawking evaporation provides the observable signature of BH formation at the TeV energy scale. After formation, BHs are expected to lose the hair associated with multipole and angular momenta, decay via Hawking radiation, and eventually either disappear completely or leave a Planck-sized remnant. The final state of BH decay is presently not understood, being in the realm of quantum gravity theory. Here we investigate the implications for BH decay of the generalized uncertainty principle (GUP).

It is commonly believed that quantum gravity implies the existence of a minimum length [7]. This in turn leads to a modification of the quantum mechanical uncertainty principle:

$$\Delta x_i \gtrsim \frac{\hbar}{\Delta p_i} \left[1 + \left(\alpha' \ell_{\rm Pl} \frac{\Delta p_i}{\hbar} \right)^2 \right] , \qquad (1)$$

where $\ell_{\rm Pl}$ is the Planck length and α' is a dimensionless constant of order one which depends on the details of the quantum gravity theory. Equation (1) can be derived in the context of non-commutative quantum mechanics [8], string theory [9] or from minimum length considerations [10]. In the classical limit, $\Delta x_i \gg \ell_{\rm Pl}$, we recover the standard Heisenberg uncertainty principle $\Delta x \Delta p \gtrsim \hbar$. The correction term in Eq. (1) becomes effective when momentum and length scales are near the Planck scale. Equation (1) implies a minimum length scale:

$$\Delta x_i \gtrsim \Delta x_{\rm min} \equiv 2\alpha' \ell_{\rm Pl} . \qquad (2)$$

The Hawking thermodynamical quantities can be derived heuristically by applying the uncertainty principle to 4-dimensional BHs, and the GUP leads to modified thermodynamical quantities [11]. Here we generalize this to d-dimensional BHs, which are relevant for BH production at TeV scales.

A d-dimensional Schwarzschild BH has gravitational potential $\Phi \propto (r_s/r)^{d-3}$, where the horizon radius is given by

$$r_s = \omega_d \ell_{\rm Pl} m^{1/(d-3)} , \; m = \frac{M}{M_{\rm Pl}} . \qquad (3)$$

Here M is the mass and the dimensionless area factor is $\omega_d = \{16\pi/[(d-2)\Omega_{d-2}]\}^{1/(d-3)}$, where Ω_{d-2} is the area of S^{d-2}. Since Hawking radiation is a quantum process, the "emitted" quanta must satisfy the uncertainty principle. By modelling a black hole as a $(d-1)$-dimensional cube of size $2r_s$, the uncertainty in the coordinate of a massless Hawking particle at emission is estimated as

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$\Delta x \sim 2r_s$. The true value is $\Delta x = 2Kr_s$, where K is a correction factor of order one that can in principle be calculated for the spherical geometry of the horizon. The Hawking temperature T can be identified up to normalization with the energy uncertainty of the emitted particles, $T \sim \Delta E \sim c\Delta p$ (the Boltzmann constant is set to one). This leads to the formula for the BH temperature [11]. If one uses the standard uncertainty relation, one deduces the standard formula [11], and this is readily generalized to recover the d-dimensional Hawking temperature [12]

$$T_0 = \left(\frac{d-3}{4\pi\omega_d}\right) M_{\mathrm{Pl}}c^2\, m^{1/(3-d)}\,. \tag{4}$$

For the GUP case, $\Delta p = (2\hbar/\Delta x)[1 + \sqrt{1 - 4\ell_{\mathrm{Pl}}^2\alpha'^2/\Delta x^2}]^{-1}$ and $\Delta x = 2Kr_s$. This leads to

$$T = 2T_0 \left(1 + \sqrt{1 - \frac{\alpha^2}{\omega_d^2 m^{2/(d-3)}}}\right)^{-1}\,, \tag{5}$$

where $\alpha = \alpha'/K$. Equation (5) generalizes the 4-dimensional result of Ref. [11] to d dimensions, and also to incorporate the GUP parameter α' and the geometrical correction factor K. In Ref. [11], $\alpha = 1$.

From Eq. (5) it is evident that GUP-quantum gravity effects increase the characteristic temperature of the BH. Therefore, we expect quantum BHs to be *hotter, shorter-lived and with a smaller entropy* than semiclassical BHs of the same mass. The BH temperature is undefined for $M < M_{\mathrm{min}}$, where

$$M_{\mathrm{min}} = \frac{d-2}{8\Gamma\left(\frac{d-1}{2}\right)} \left(\alpha\sqrt{\pi}\right)^{d-3} M_{\mathrm{Pl}}\,. \tag{6}$$

BHs with mass less than M_{min} do not exist, since their horizon radius would fall below the minimum allowed length. Hence Hawking evaporation must stop once the BH mass reaches M_{min}. This can be shown by calculating the mass loss during evaporation. The energy radiated per unit time is governed by the Stefan-Boltzmann law. Following Ref. [13], we assume that the radiation takes place mainly along the 4-dimensional brane. (We neglect the energy losses due to emission of gravitons into the bulk.) Since the surface gravity is constant over the horizon, the Hawking temperature of the higher-dimensional BH and that of the induced BH on the brane are identical. Thus Eq. (5) can be used to calculate the emission rate. The mass loss for a BH emitting on an n-dimensional brane is given by [14]

$$\frac{dm}{dt} = -\frac{1}{cM_{\mathrm{Pl}}}\bar{\sigma}_n A(n)T^n\,, \tag{7}$$

where $A(n)$ is the area of the induced BH on the brane and $\bar{\sigma}_n$ is the effective n-dimensional Stefan-Boltzmann constant,

$$\bar{\sigma}_n = \frac{\Omega_{n-3}\Gamma(n)\zeta(n)}{(2\pi\hbar c)^{n-1}(n-2)} \sum_i c_i(n)\Gamma_{s_i}(n)f_i(n)\,. \tag{8}$$

Here the sum is over all particle flavours, c_i are the n-dimensional degrees of freedom of the individual species, the Γ_{s_i}'s are the n-dimensional greybody factors [15], and $f_i(n) = 1$ or $1 - 2^{1-n}$ for bosons or fermions. The area of the induced BH on the brane is taken as the optical area in $n-2$ dimensions and is given by

$$A(n) = \Omega_{n-2}r_c^{n-2}\,, \tag{9}$$

where $r_c = [(d-1)/2]^{1/(d-3)}[(d-1)/(d-3)]^{1/2}\,r_s$ is the optical radius [13]. Using Eq. (5) the BH rate of mass loss on a 4-dimensional brane can be written as

$$\frac{dm}{dt} = 16\left(\frac{dm}{dt}\right)_0 \left(1 + \sqrt{1 - \frac{\alpha^2}{\omega_d^2 m^{2/(d-3)}}}\right)^{-4}\,, \tag{10}$$

where the standard rate is

$$\left(\frac{dm}{dt}\right)_0 = -\frac{\mu}{t_{\mathrm{Pl}}}m^{-2/(d-3)}\,, \tag{11}$$

with

$$\mu = \frac{\pi^2}{120}\frac{(d-1)^{(d-1)/(d-3)}}{2^{2/(d-3)}\omega_d^2}\left(\frac{d-3}{4\pi}\right)^3 \sum_i c_i(4)\Gamma_{s_i}(4)f_i(4)\,. \tag{12}$$

As expected, the GUP-corrected mass loss is larger than the standard Hawking mass loss. Equation (10) can be integrated to give the decay time,

$$\tau = \left(\frac{d-3}{16\mu}\right)\left(\frac{\alpha}{\omega_d}\right)^{d-1} I(4, d-6, \omega_d m^{1/(d-3)}/\alpha)\,t_{\mathrm{Pl}}\,, \tag{13}$$

where

$$I(m, n, x) = \int_1^x dz\, z^n \left(z + \sqrt{z^2 - 1}\right)^m\,. \tag{14}$$

(This integral can be solved analytically.) Setting $\alpha = 0$ we obtain the standard Hawking decay time in d dimensions,

$$\tau_0 = \frac{1}{\mu}\left(\frac{d-3}{d-1}\right)m^{(d-1)/(d-3)}\,t_{\mathrm{Pl}}\,. \tag{15}$$

The BH entropy is

$$S = 2\pi\omega_d\left(\frac{\alpha}{\omega_d}\right)^{d-2} I(1, d-4, \omega_d m^{1/(d-3)}/\alpha)\,, \tag{16}$$

and is always smaller than the standard ($\alpha = 0$) Hawking value,

$$S_0 = \left(\frac{4\pi\omega_d}{d-2}\right)m^{(d-2)/(d-3)}\,. \tag{17}$$

From Eq. (11) and Eq. (15), it is evident that the final stage of standard Hawking evaporation is catastrophic. The BH reaches in a finite time a stage with zero mass,

infinite radiation rate and infinite temperature. By contrast, in the GUP scenario the existence of a minimum length prevents the mass becoming smaller than $M_{\min}$, Eq. (6). At this point the emission rate, Eq. (10), becomes imaginary. Since the emission rate is finite at the end, it could be argued that the BH decays nonthermally by emitting a hard Planck-mass quantum in a finite time $\mathcal{O}(t_{\mathrm{Pl}})$, once the final stage of evaporation has been reached. However, the specific heat, $\mathcal{C} = T\partial S/\partial T$, is

$$\mathcal{C} = -2\pi\omega_d m^{(d-2)/(d-3)}\sqrt{1-\frac{\alpha^2}{\omega_d^2 m^{2/(d-3)}}} \times$$
$$\times \left(1+\sqrt{1-\frac{\alpha^2}{\omega_d^2 m^{2/(d-3)}}}\right), \qquad (18)$$

and vanishes at the endpoint, so that the BH cannot exchange heat with the surrounding space. Thus the endpoint of Hawking evaporation in the GUP scenario is characterized by a *Planck-sized remnant* with maximum temperature,

$$T_{\max} = 2T_0\Big|_{M=M_{\min}}. \qquad (19)$$

The way that the GUP prevents BHs from evaporating completely is similar to the way that the standard uncertainty principle prevents the hydrogen atom from collapsing. The existence of a remnant as a consequence of the GUP was pointed in Ref. [11], in the context of primordial BHs in cosmology. Primordial BH remnants are possible candidates for dark matter. Remnants have also been predicted in certain models of quantum black holes [16].

The multiplicity of a particle species i produced in BH decay is [14]

$$N_i = N\frac{c_i\Gamma_{s_i}f_i(3)}{\sum_j c_j\Gamma_{s_j}f_j(3)}, \qquad (20)$$

where the total multiplicity N is

$$N = \frac{30\zeta(3)}{\pi^4}S\frac{\sum_i c_i\Gamma_{s_i}f_i(3)}{\sum_j c_j\Gamma_{s_j}f_j(4)}. \qquad (21)$$

The observable effects of the GUP at particle colliders are related to the existence of a minimum mass and to the multiplicity of the decay. We assume that the parton-parton center-of-mass (CM) energy E_{cm} is larger than $M_{\min}$. Since the GUP-corrected entropy is smaller than the standard one, we expect a significantly smaller multiplicity, i.e., a *smaller number of emitted particles* in the evaporation process, and a *larger average energy* of the produced quanta. Moreover, the BH stops evaporating when it reaches the minimal mass. This should lead to the detection of a missing energy of the order of the minimal mass plus the missing energy due to invisible decay products.
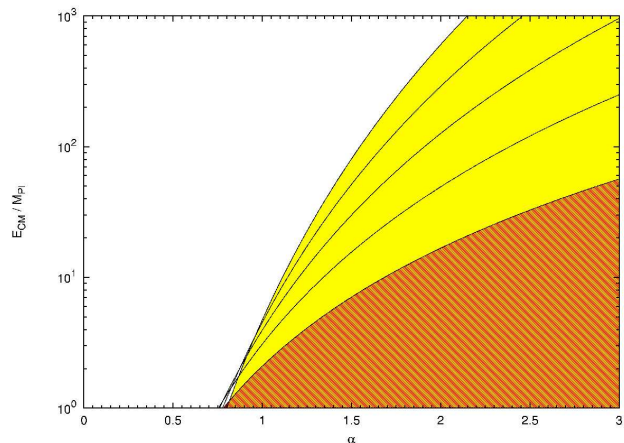

FIG. 1: Minimal mass for BH formation vs the parameter α, for $d = 6,\ldots,10$ dimensions (from below). The yellow and red regions are the exclusion zones for $d = 10$ and $d = 6$ respectively.

What happens if $E_{\mathrm{cm}} < M_{\min}$? In this case, the collision will produce no BH. Either a different nonperturbative gravitational object (perhaps a brane [5] or string ball [6]) forms in the collision, or the scattering is dominated by nongravitational effects. In the former case, depending on the details of the quantum gravity theory, we could have formation of a gravitational object that is either stable or does not produce any quanta during the decay phase. Therefore, the formed object could be totally invisible, with the only observable effect being a missing energy of about the minimal mass in the final state of the collision.

The minimal BH mass depends sensitively on the (unknown) $\mathcal{O}(1)$ parameter α and on the spacetime dimension d. Figure 1 gives the parton-parton E_{cm} in Planck units vs the parameter α, for $d = 6,\ldots,10$ spacetime dimensions. A BH in d dimensions at fixed α can form only if $E_{\mathrm{cm}}/M_{\mathrm{Pl}}$ is above the curve corresponding to the spacetime dimension. For instance, if $\alpha = 1$ and $d = 10$, BHs with mass smaller than $\sim 4.72 M_{\mathrm{Pl}}$ do not form. If $\alpha = 2$, the minimum BH mass will be $\sim 600 M_{\mathrm{Pl}}$! In this case, LHC will not see any BH. The situation is somewhat better in atmospheric ultra high-energy cosmic ray events, because the collisions are expected to have a larger E_{cm} than in particle colliders, and BHs are expected to form with a higher average mass. However, the bulk of the BHs which are formed in cosmic ray collisions have mass of the order of few M_{Pl}, so that the rate of BH formation in the GUP scenario could be dramatically suppressed. In the worst case of large parameter α, no BHs will be observed, either at particle colliders or in cosmic ray airshowers.

The region of BH detection at LHC for $d = 6$ (yellow) and $d = 10$ (red) dimensions is shown in Fig. 2. The upper line correspond to the CM energy of LHC ($M_{\mathrm{Pl}} = 14$ TeV). The two lower lines correspond to the current ex-

TABLE I: Thermodynamical quantities for two typical 10-dimensional BHs produced at LHC (masses $M = 9$ and 12 TeV), assuming $M_{\mathrm{Pl}} = 1$ TeV. The values in brackets give the percentage deviation from standard Hawking quantities.

$M = 9$ TeV

	Min. mass (TeV)	Temperature (TeV)	Decay time (TeV^{-1})	Entropy	Multiplicity
$\alpha = 0$	–	.508	.689	15.5	5
$\alpha = 0.5$	.037	.538 (+6%)	.511 (-26%)	13.3 (-8%)	5 (0%)
$\alpha = 1.0$	4.72	.721 (+42%)	.071 (-90%)	5.24 (-66%)	2 (-60%)

$M = 12$ TeV

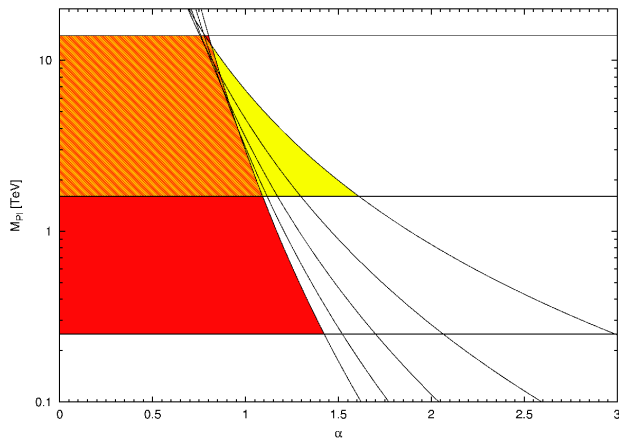|	Min. mass (TeV)	Temperature (TeV)	Decay time (TeV^{-1})	Entropy	Multiplicity
$\alpha = 0$	–	.488	.997	21.5	7
$\alpha = 0.5$	.037	.513 (+5%)	.760 (-24%)	20.0 (-7%)	7 (0%)
$\alpha = 1.0$	4.72	.657 (+35%)	.157 (-84%)	9.62 (-55%)	3 (-57%)

FIG. 2: $(\alpha, M_{\mathrm{Pl}})$ moduli space for BH observation at LHC. The yellow (red) region is for $d = 6$ ($d = 10$) spacetime dimensions.

perimental limits on M_{Pl} for $d = 6$ ($M_{\mathrm{Pl}} = 1.6$ TeV, from submillimeter tests of the gravitational inverse-square law [17]) and $d = 10$ ($M_{\mathrm{Pl}} = 0.25$ TeV, from collider experiments [18]). The maximum value of the parameter α that allows observation of BH formation at LHC is $\alpha \sim 1.6$ ($d = 6$) and $\alpha \sim 1.4$ ($d = 10$).

If the parameter α is sufficiently small, BHs can still be observed at LHC. If this is the case, what are the observable signatures of the GUP? We have mentioned that the multiplicity of the decay process is significantly smaller than in the standard case. Table 1 shows the parameters for two typical 10−dimensional BHs produced at LHC with masses $M = 9$ and 12 TeV, and parameters $\alpha = 0$, 0.5, 1, assuming $M_{\mathrm{Pl}} = 1$ TeV. The most significant difference between the standard BH and the GUP-corrected BH is given by the multiplicity of the decay products. As α increases, the total number of particles produced during the evaporation phase becomes smaller. The sup-

pression in the multiplicity is most severe for $\alpha = 1$ and $M = 9$ TeV. Indeed, as α increases and M decreases, the minimum BH mass M_{min} becomes a significant fraction of the BH mass M. Therefore, a smaller amount of energy can be converted into Hawking thermal radiation.

The decrease in multiplicity can lead to important observational signatures. The hadron to lepton ratio of the decay products is roughly expected to be 5:1. In the case $M = 12$ TeV, $\alpha = 0$, for instance, the BH evaporates into five quarks, one charged lepton, and one gluon, with the charged leptons accounting for about 15% of the emission. As the parameter α increases the leptonic component becomes negligible. When $\alpha = 1$, for example, the charged leptonic component is reduced to $\sim 6\%$ of the emission.

What happens after Hawking evaporation has stopped? The BH remnant becomes invisible to the detector. This leads to a large missing energy. (For $\alpha = 1$ the missing energy of the 9 TeV BH is more than 50% of the total CM energy.) Therefore, an event with characteristics of a standard BH event (large visible transverse energy and high sphericity), but with smaller multiplicity and large missing energy, would be the GUP smoking gun.

In summary, the generalized uncertainty principle of string theory or non-commutative quantum mechanics brings important qualitative and quantitative changes to BH thermodynamics. The BH is hotter, evaporates more rapidly, and has smaller entropy, relative to the standard case. BH evaporation involves a smaller number of particles with higher average energy. And the endpoint of evaporation is a Planck-scale remnant with zero heat capacity. We have shown, in the context of TeV-scale higher-dimensional gravity, that these changes could have significant implications for the possible formation and detection of BHs at LHC (and also in cosmic ray showers). In particular, the minimum energy for formation is increased, and could be pushed beyond the reach of LHC even if $M_{\mathrm{Pl}} \sim 1$ TeV.

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