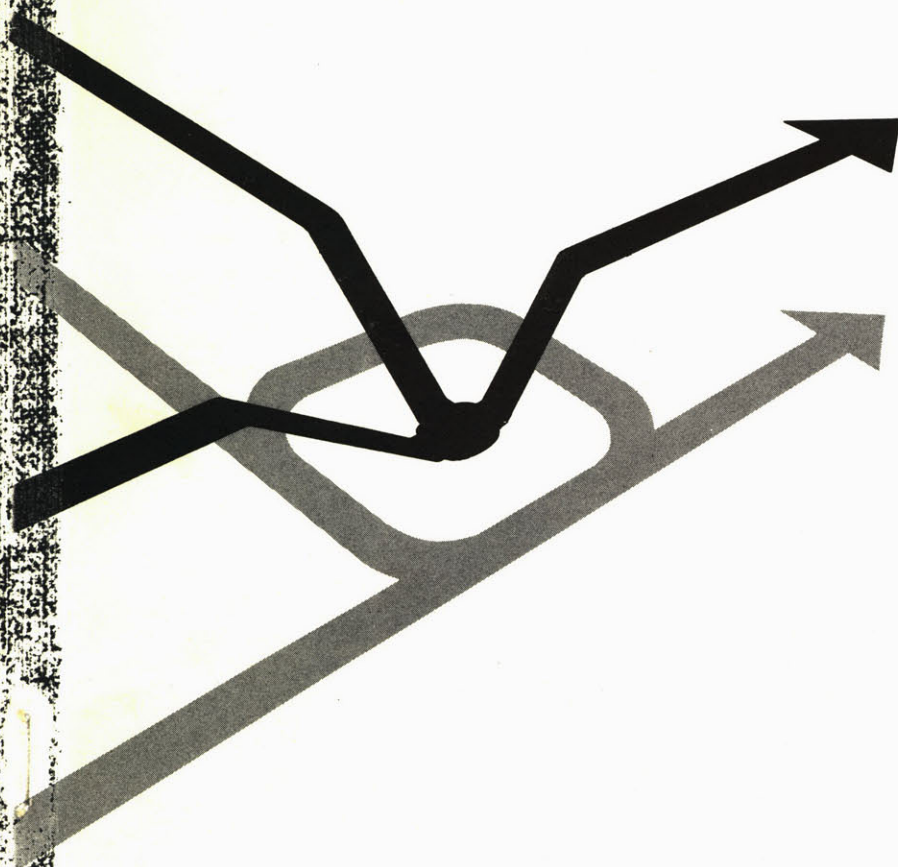


A METHOD FOR DETERMINATION OF OPTIMUM VEHICLE SIZE AND FREQUENCY OF SERVICE FOR A SHORT HAUL V/STOL AIR TRANSPORT SYSTEM

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Abstract

To compete successfully in short haul markets under 200 miles, an air transport system must offer a high daily frequency of service, N , as well as short air travel times. In a given market, N can be increased by using vehicles of smaller seat capacity, C , which are more expensive per seat to operate. A method of determining optimal values of N and C for assumed market behavior in terms of fare and time elasticities is presented. By defining total trip time to include the average wait for service, and using a demand model developed for the Northeast Corridor, the air share of total demand in any market can be calculated as a function of N and the competing fares. Plotting daily passengers versus N , and relating this to the maximum and breakeven load factors for a family of vehicles of different seating capacities, determines the values of N and C which maximize return to the operator.

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1. INTRODUCTION

In competition with other modes of intercity passenger transportation for trips between 10 and 500 miles, frequency of service for a V/STOL short haul air system will have a strong effect on generating new passengers and attracting passengers away from other modes, i.e. the total revenue in a given market is a function of the frequency of service N , and this argues for a high frequency of service using vehicles of small capacity C .

But the cost of providing highly frequent service using small vehicles will be higher, since unit costs in terms of cents/seat-mile are higher for smaller capacity aircraft. The question then is: What are the size of vehicle and frequency of service which, in a given market competition, defined by fares and trip times, maximize net income for the air system?

The following discussions outline an approach to structuring this problem and obtaining a solution. The first step is to show from results of optimal dispatching investigations that the level of service to the passenger as measured by average wait for service, \bar{D} , is strongly

related to N . Then, by defining total travel time T_t as the sum of en route plus wait time, and using it in a travel prediction model, the market share or revenue can be found as a function of N for a given modal market competition in terms of fares and travel times. By making an appropriate correction for vehicle capacity, maximum net contribution to overhead can then be obtained for a family of $V/STOL$ vehicles of varying capacity. Optimum values of N and C , an optimal dispatch pattern and average wait for service are all then specified for the given market competition.

The work described here assumes that in these short haul markets, passengers may not be required to make a seat reservation, and indeed that there may not be knowledge on their part of the operations timetable. In other words, the system may be operated quite differently from present airline service. It is also assumed that only one size of vehicle will be assigned to a given route. The method is part of a larger network scheduling process.

2. OPTIMAL DISPATCHING

In a given transportation market, the distribution of departures during the day will be an optimal dispatch pattern if it maximizes net income for the system operator or maximizes service to the passengers.

But a dispatch pattern is the combination of two elements:

- (1) The number of flights operated daily or frequency of service, N
- (2) The pattern of the N flights at various times during the day.

The optimal frequency of service N will be calculated, in part 4 of this report, in order to maximize net contribution to overhead for the operator.

The pattern of N flights during the day should then be such as to maximize the service to passengers. Since in terms of scheduling that service may be measured by the average wait for service, or delay per passenger \bar{D} , the problem of dispatch optimization will be that of minimizing, for a given N , the delay imposed on passengers. We now have good methods for solving this problem (Reference

1), and we wish to draw some conclusions from recent applications of these methods.

Constant Arrival Rate

Given a constant arrival rate of p passengers per hour between times t_0 and t_N ($T_d = t_N - t_0$), we want to dispatch N vehicles of fixed capacity C at times t_1, t_2, \dots, t_N in order to minimize the total delay D

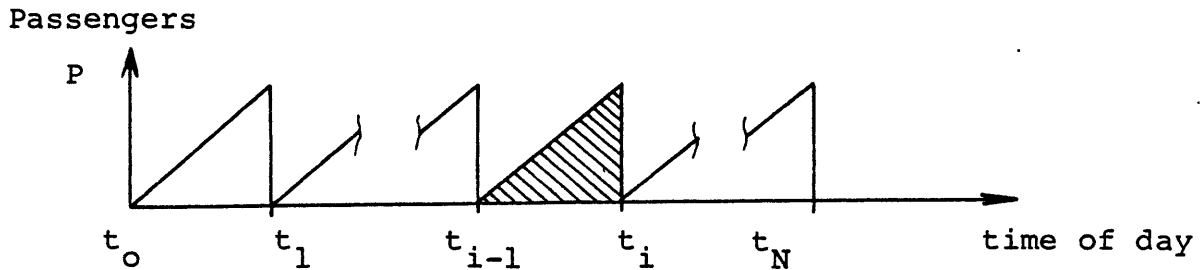


Figure A

For passengers arriving between times t_{i-1} and t_i and waiting until departure at t_i , the delay, represented by the hatched area in Figure A, is:

$$\int_{t_{i-1}}^{t_i} P(t) \cdot dt = \frac{p}{2} (t_i - t_{i-1})^2$$

If there are N such departures, the total delay over the day will be

$$D = \frac{p}{2} \cdot \sum_{i=1}^N (t_i - t_{i-1})^2$$

Calculus of variations shows that the minimum value of D is obtained for departures distributed at equal intervals of time during the day :

$$t_i - t_{i-1} = \frac{T_d}{N}, \quad \forall i$$

$$\text{Then } D = \frac{p}{2} \cdot \sum_{i=1}^N \left[\frac{T_d}{N} \right]^2 = \frac{p}{2} \cdot \frac{T_d^2}{N}$$

Since the total number of passengers is

$$\bar{P} = p \cdot T_d$$

the average delay per passenger will be :

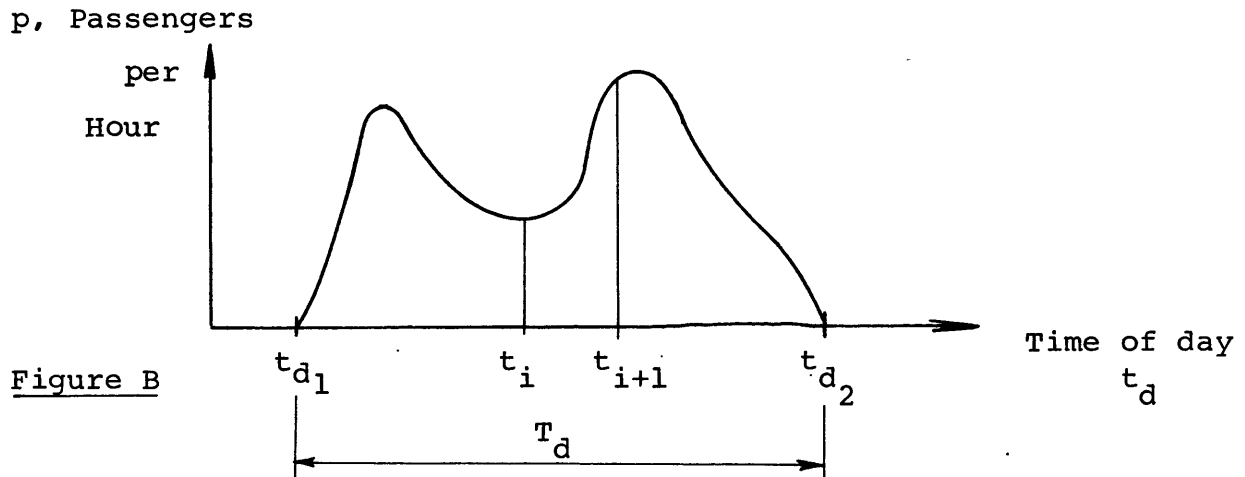
$$\boxed{\bar{D} = \frac{T_d}{2N}} \quad 2.1$$

This shows that in the case of optimal, least delay dispatching for constant arrival rate, the average delay is only dependent on the frequency of service, N , and equals one half the headway time, or interval between departures.

General Case

We now are concerned with the problem of dispatching independent vehicles of fixed capacity C to satisfy a

periodic, deterministic but non-uniform demand which is non-zero from t_{d_1} to t_{d_2} , as shown in Figure B.



In this case, we minimize a weighted average of delay and dispatch costs; the optimization can then be stated as:

$$\text{Min } \left[Z = \lambda \cdot D + N \cdot TC \right]$$

where TC is the cost of dispatching a vehicle, or trip cost

N is the frequency of service

D is the total passenger delay over the day

λ represents the unit cost (or loss of revenue)

per minute of passenger delay for the operator

Z is the weighted cost of dispatching

The parameter λ can be considered as a Lagrange multiplier which will determine the value of N in the optimization. As suggested by D. E. Ward (Ref. 3) this problem can be solved using a discrete dynamic program. Figure 1 shows a typical demand and resulting dispatch patterns.

Using the dynamic program for various market densities (as determined by the ratio \bar{T}/C of the total daily traffic in passengers, \bar{T} , to the vehicle capacity C) and different values of the parameter λ to give variations in N , computations have shown the following results:

The relation for average delay per passenger, $\bar{D} = \frac{T_d}{2N}$, shown in the case of constant arrival rate, seems to hold for optimal dispatching in a non-uniform demand situation, at least as long as no flight is dispatched at full capacity.

Figure 2 shows how close the optimal average delay \bar{D} is to the function $\bar{D} = \frac{T_d}{2N}$. The agreement is extremely good, and computational experience with several non-uniform demand distributions shows similar results. This computational result indicates that optimal average delay is only dependent on N .

Experience with other dispatching policies which result in regularly distributed patterns (such as equal load

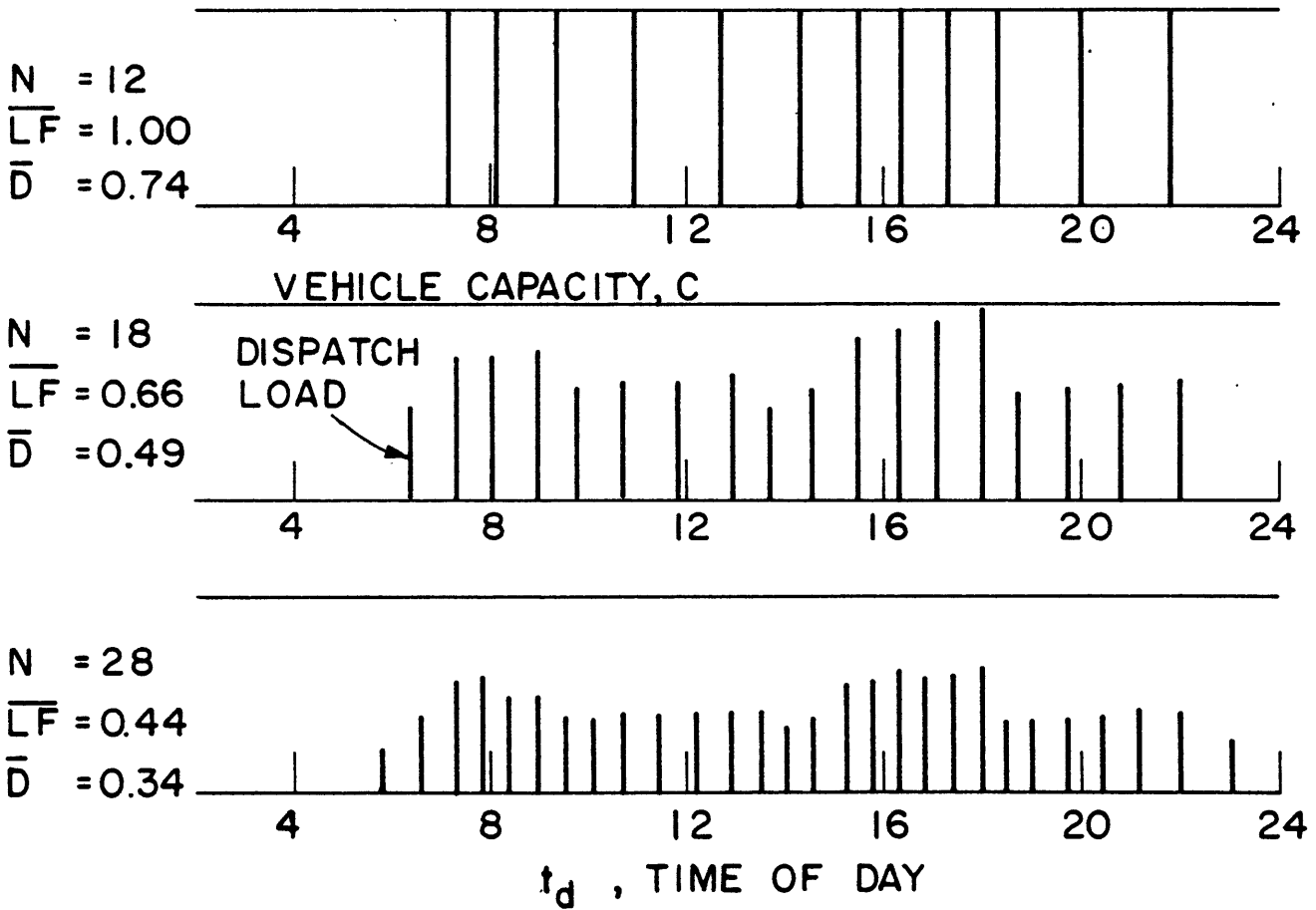
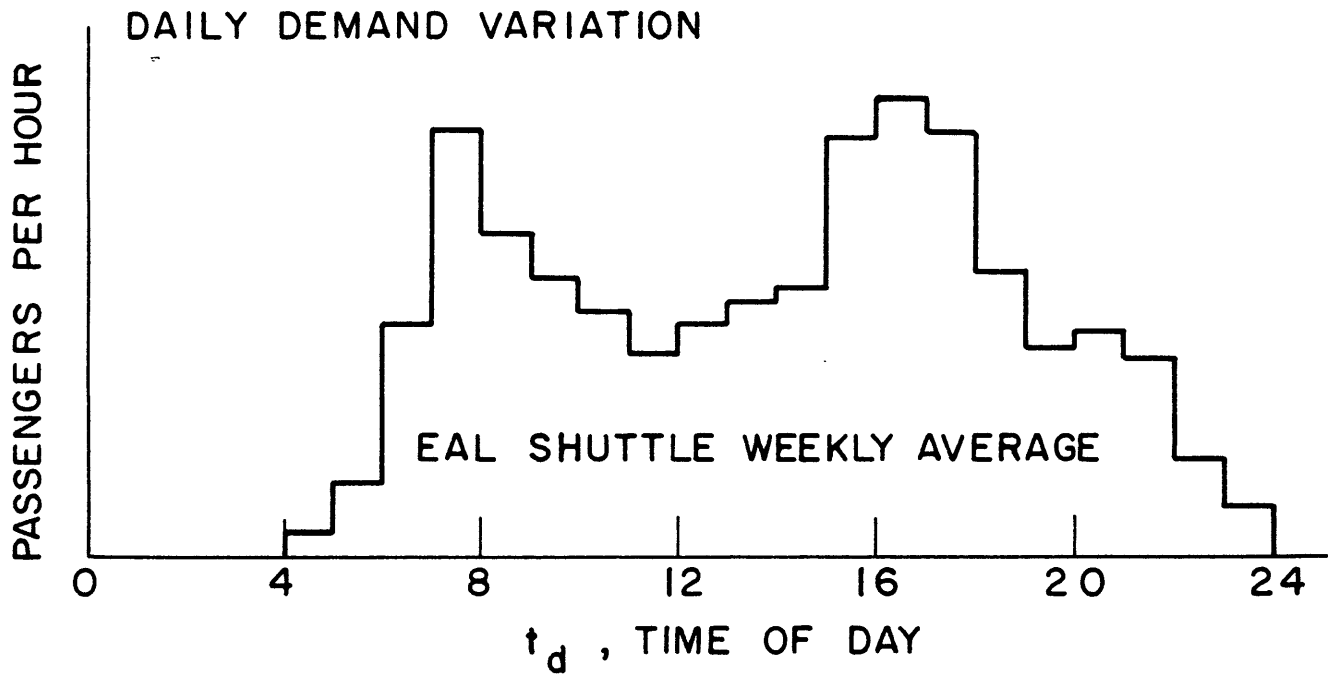


FIG. 1 OPTIMAL DISPATCHING PATTERNS FOR $\overline{T}/C = 12.5$

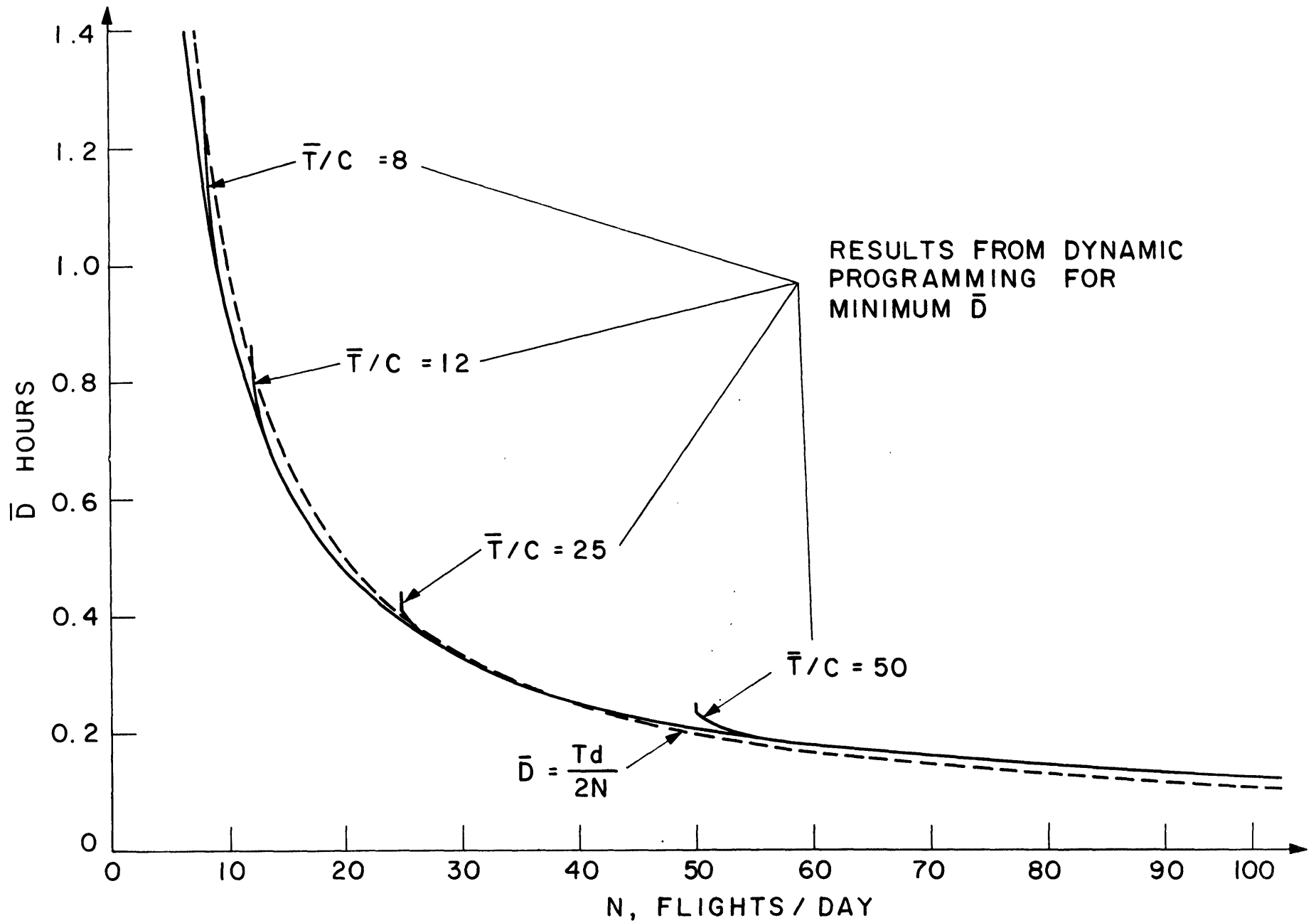


FIG. 2 AVERAGE DELAY PER PASSENGER, \bar{D} VS DAILY FREQUENCY, N

dispatching) has indicated that the average delay is strongly and predominantly determined by N, and that the relationship 2.1 is a good measure of average wait for service, \bar{D} .

Given this result, one can then relate the level of service measured by \bar{D} to the market load factor \bar{LF} and the market density.

The market load factor over the day is :

$$\bar{LF} = \frac{\bar{T}}{N \cdot C} = \frac{1}{N} \cdot \left[\frac{\bar{T}}{C} \right]$$

and substituting 2.1, we obtain the relationship

$$\bar{D} = \frac{T_d}{2 \cdot \left(\bar{T}/C \right)} \cdot (\bar{LF}) \quad 2.2$$

which states that, for a given \bar{T}/C , \bar{D} is proportional to \bar{LF} . The linearity of this result is indicated by Figure 3 which compares 2.2 with optimal dispatching computations. This figure shows that for low density markets in terms of \bar{T}/C , it is difficult to obtain good economic load factors

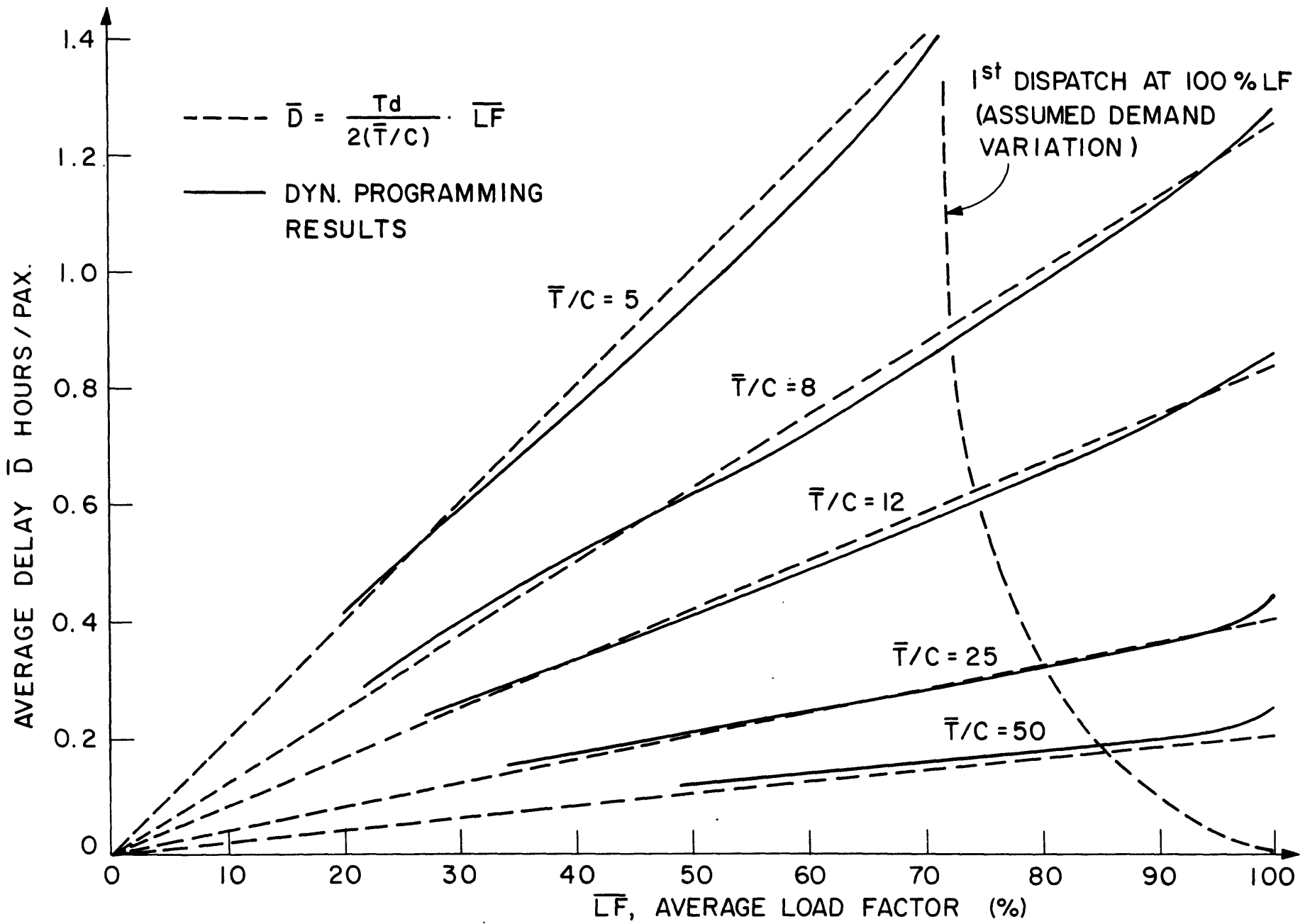


FIG. 3 RELATIONSHIP OF AVERAGE DELAY, \bar{D} AND AVERAGE LOAD FACTOR, \bar{LF}

without incurring high values of \bar{D} . Conversely in high density markets, it is possible to operate above 70% load factor and still have low values of passenger delay.

Total Travel Time

If total travel time, T_t , is taken as time from true origin to true destination, it becomes (for any public transportation system) the sum of:

1. Access time to the system, T_a
2. Wait time for next service, \bar{D}
3. System en route time, T_b (block time)
4. Egress time from system, T_e

If the system en route time is known, and some estimate can be made for average access and egress times, then the relation 2.1 for \bar{D} may be used in computing T_t . Since \bar{D} is a function of N , then T_t becomes a function of N . Typical variations in total travel time against travel distance are shown in Figure 5. It shows that despite the much greater speed of a V/STOL aircraft, the total travel time for auto or bus can be less. The crossover distance

where $V/STOL$ becomes the fastest means of transportation is a function of N . Figure 5 assumes that, given N dispatches, an optimal dispatch pattern, or at least a regularly distributed pattern of N dispatches will be used.

Thus, using the expression 2.1, total transit time offered by a transportation system becomes an explicit function of N , the daily frequency of service.

3. TRAVEL PREDICTION MODEL

To structure the problem we need some model of market behavior given competing transportation systems which offer the passenger a set of alternatives described in terms of fares and travel times. There are a variety of such models at present, and the one used here is shown in Figure 4. It describes the traffic for a given mode in terms of the best values of fare and frequency in the market, and the modal fares relative to these best values. The values of $\alpha, \beta, \gamma, \delta$ have the economic interpretation of elasticities of travel demand with respect to fares and travel times, and must be accurately known before any selection of optimal values of N and C can be made with confidence.

The inputs to the model for a given set of competing modes such as air, rail, bus, auto, and V/STOL are total travel time as indicated in Figure 5, and total travel cost as given by Figure 6. Travel cost is defined as sum of

1. Access cost to the system (e.g. taxi fare)
2. System fare
3. Egress cost from the system

FIG. 4 TRAVEL PREDICTION MODEL - ESTIMATING THE ROLES OF FARE AND TOTAL TRAVEL TIME

$$\bar{P}_{ij_k} = K_{ij} \cdot (F_{ij}^m)^{\alpha} \cdot (F_{ij_k}^r)^{\beta} \cdot (T_{ij}^m)^{\gamma} \cdot (T_{ij_k}^r)^{\delta}$$

where :

\bar{P}_{ij_k} = Volume of travel by mode k from i to j

K_{ij} = Constant for a given city pair. Function of the population at terminal cities and economic factors such as per capita income

F_{ij}^m = Minimum cost (over all modes) of traveling from i to j

$F_{ij_k}^r$ = Relative cost of mode k from j

T_{ij}^m = Minimum value (over all modes) of total travel time from i to j

$T_{ij_k}^r$ = Relative time for mode k from i to j

α = Elasticity of traffic with respect to minimum fare

β = Elasticity of traffic with respect to relative fare

γ = Elasticity of traffic with respect to minimum travel time

δ = Elasticity of traffic with respect to relative travel time

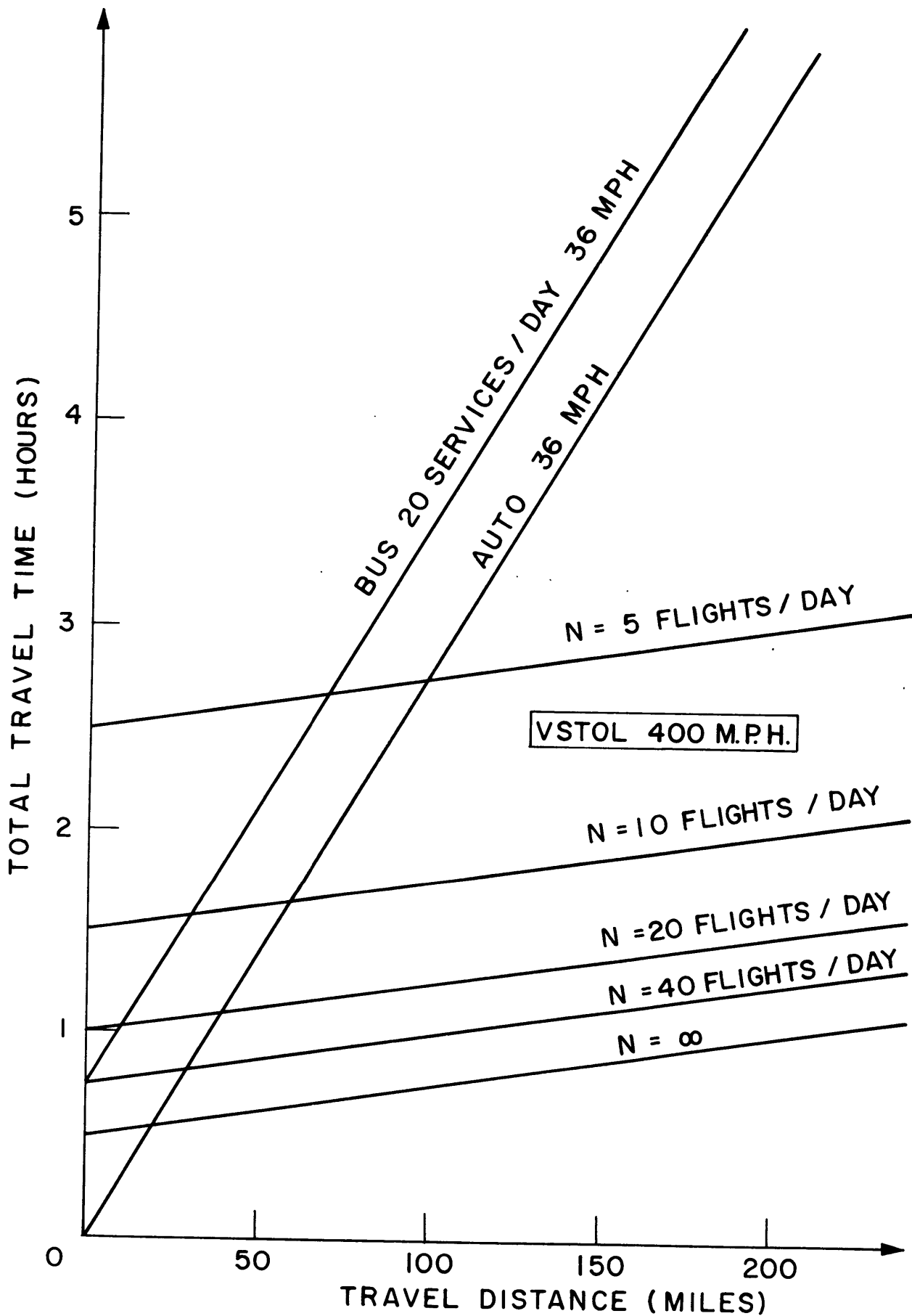


FIG. 5 ASSUMED VARIATION OF SYSTEM TRAVEL TIME WITH DAILY FREQUENCY, N

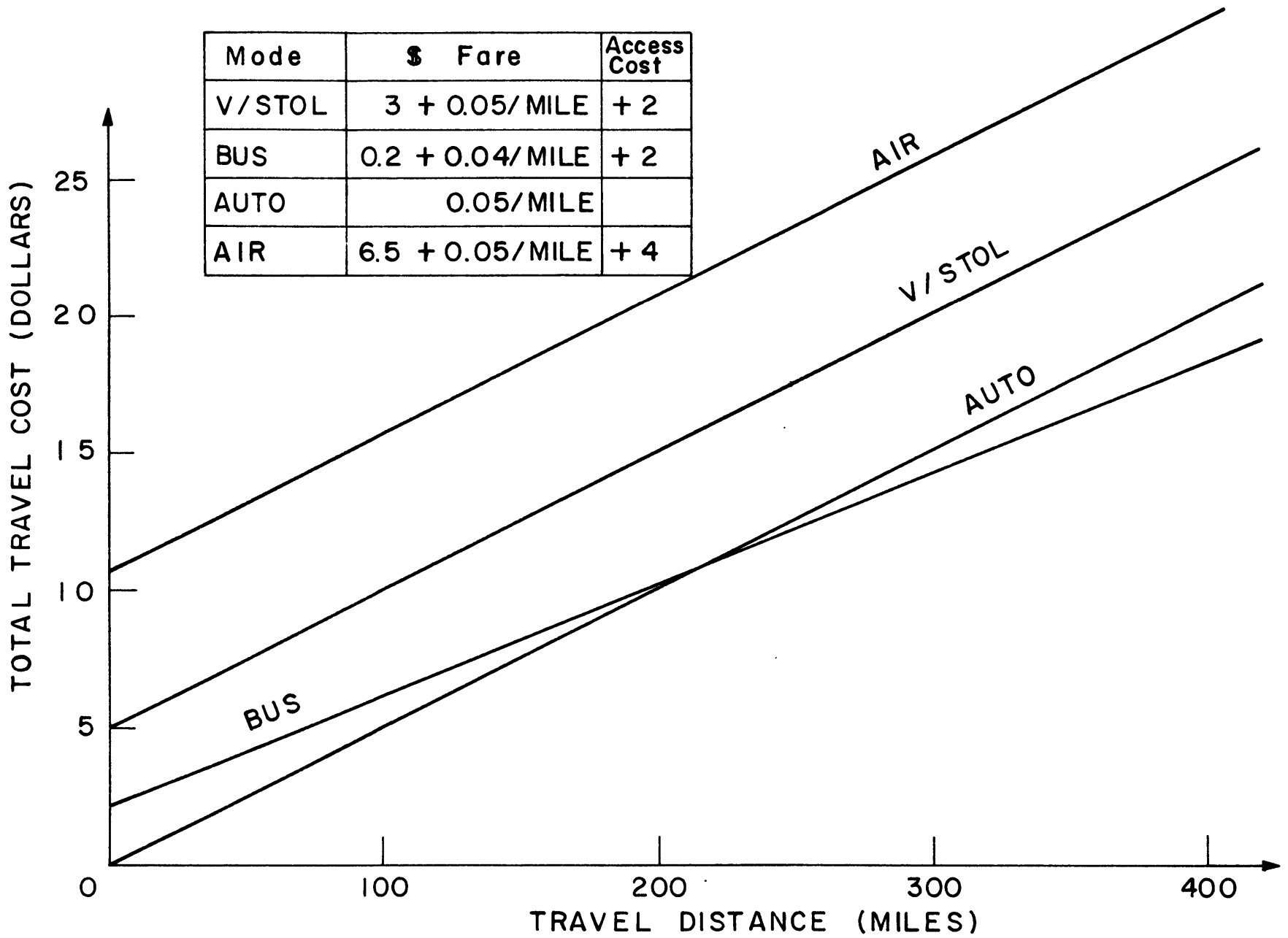


FIG. 6 ASSUMED VARIATION OF SYSTEM TRAVEL COSTS WITH DISTANCE

Figure 6 gives typical, out of pocket costs for present competing modes. The VTOL fare structure has been arbitrarily assumed to be roughly proportional to system costs as a function of terminal costs, and travel distance. The fare structure assumption will affect the optimal values of N and C , and one may raise the question of determining an optimal fare structure for the mode, particularly if the vehicle size C is known.

For the given inputs, typical market model results are shown in Figures 7, 8 and 9. For a market presently served by auto and bus, Figure 7 shows the market share for an assumed V/STOL system at $N=5$, and $N=30$ flights/day as a function of market distance. It shows that as distance increases, the time savings due to the high speed of a V/STOL system attracts an increasing percentage of the market. Conversely, at distances below 50 miles, the time savings may be negligible (unless a higher frequency of V/STOL service is offered) and the market is dominated by the automobile.

By crossplotting such results, the market share may be plotted as a function of N for a given distance as shown in Figure 8. Here at 800 miles the air mode shows

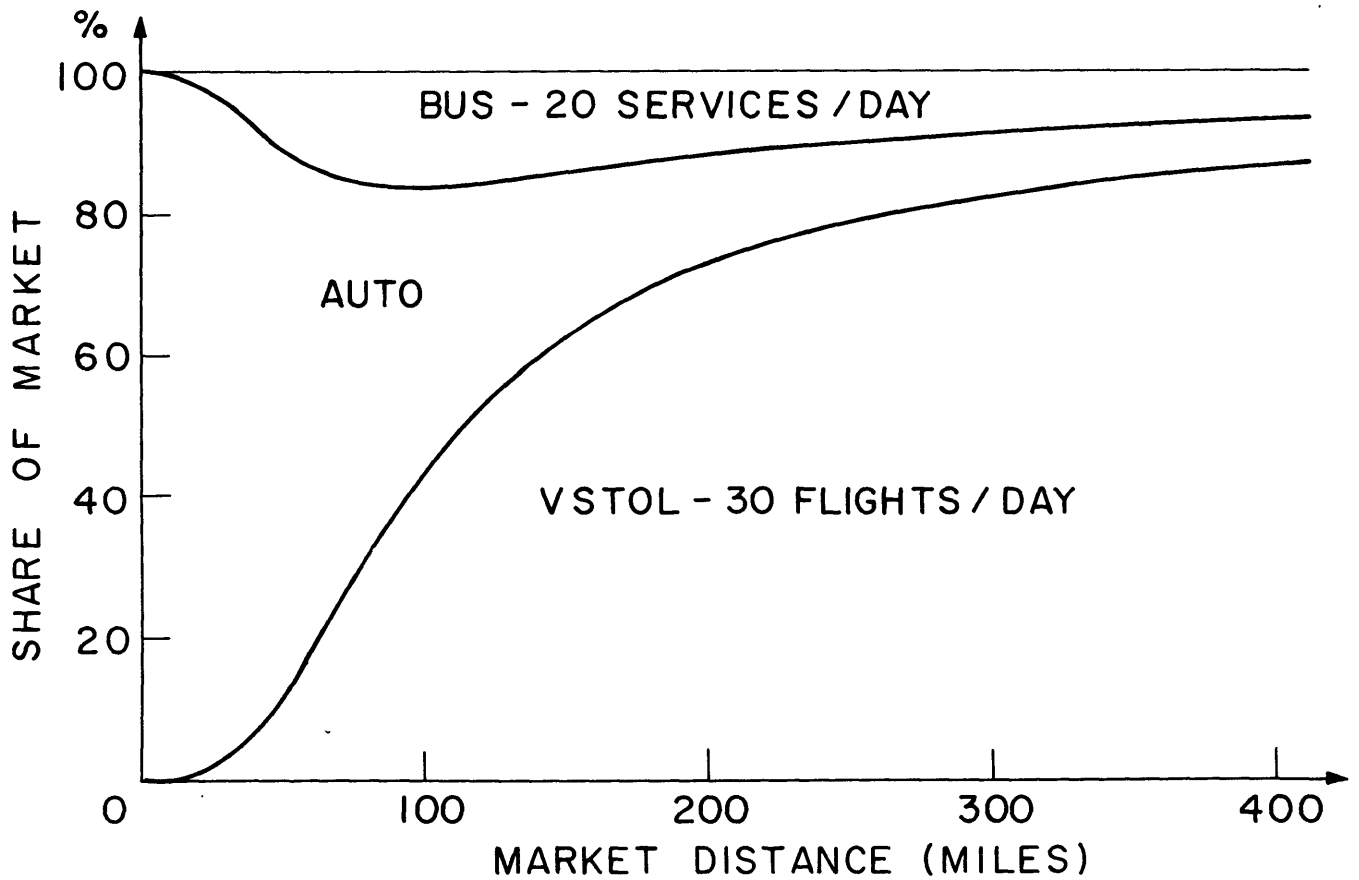
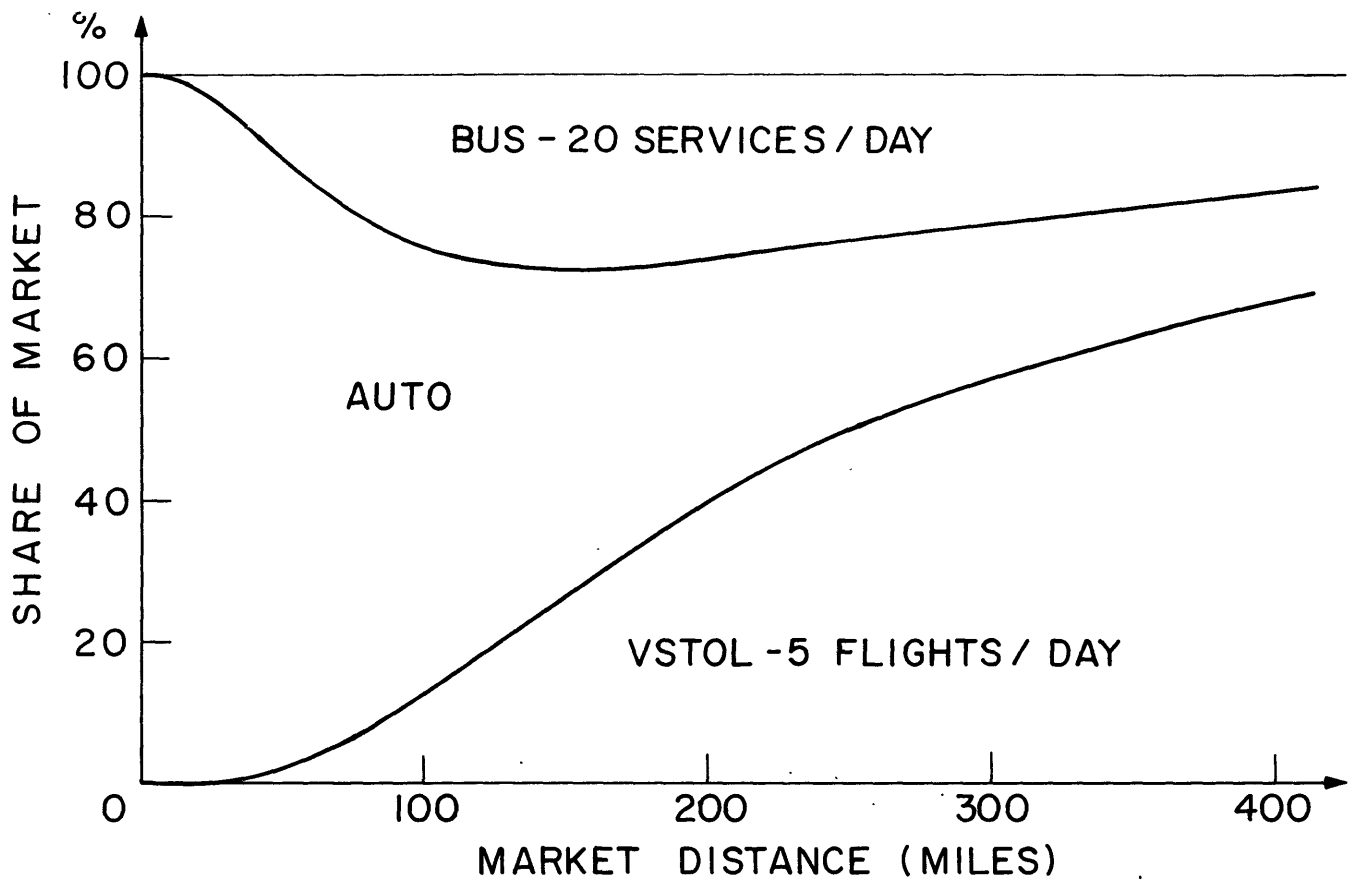


FIG. 7 TYPICAL MARKET SHARE RESULTS FOR VSTOL VS. AUTO, BUS

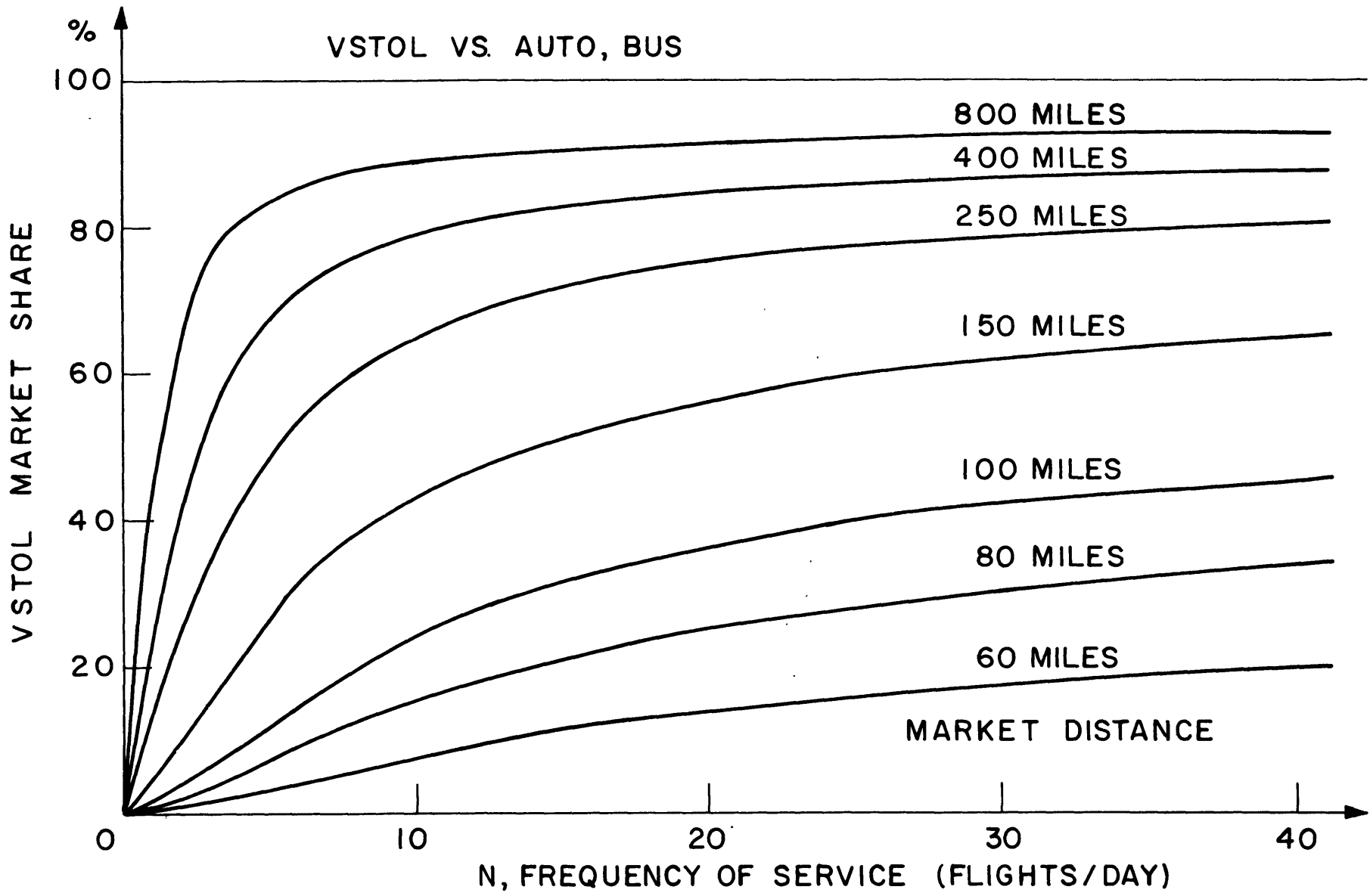


FIG. 8 VARIATION OF VSTOL MARKET SHARE WITH DAILY FREQUENCY, N

a market share variation which increases rapidly to above 80% of the market as is typically expected for airline systems. However, at shorter distances the market share is obtained much more slowly as N is increased, and even for very high values of N, the market share may only be 30-40% of the total market. As had been hypothesized, at distances below 150 miles, these market share curves have a double curvature which means that there exists a region where increasing the frequency of service (and therefore total seats/day) will lead to increased load factors since the V/STOL traffic increases faster than the added capacity. This effect has been observed in some present short haul helicopter markets where a minimal daily frequency exists before the market can be developed.

These market share curves have to be corrected for total market size since total travel in terms of passengers/day is much larger for the shorter distances. An indication of this is given by Figure 9, which shows the typical hyperbolic increase in total passengers/day as distance decreases. The market shares for auto, bus, V/STOL, and fixed wing airline are shown for typical values of travel costs and times. The air markets show their typical peaking in the vicinity.

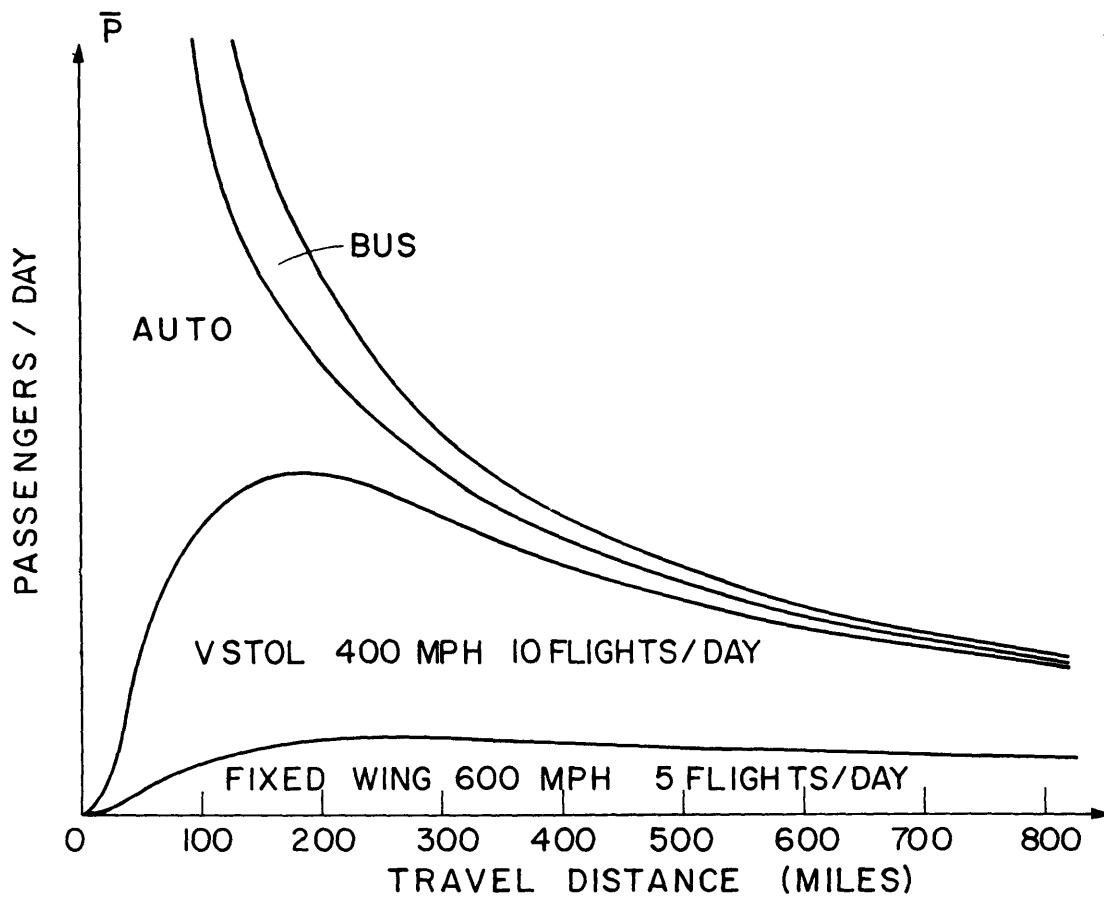
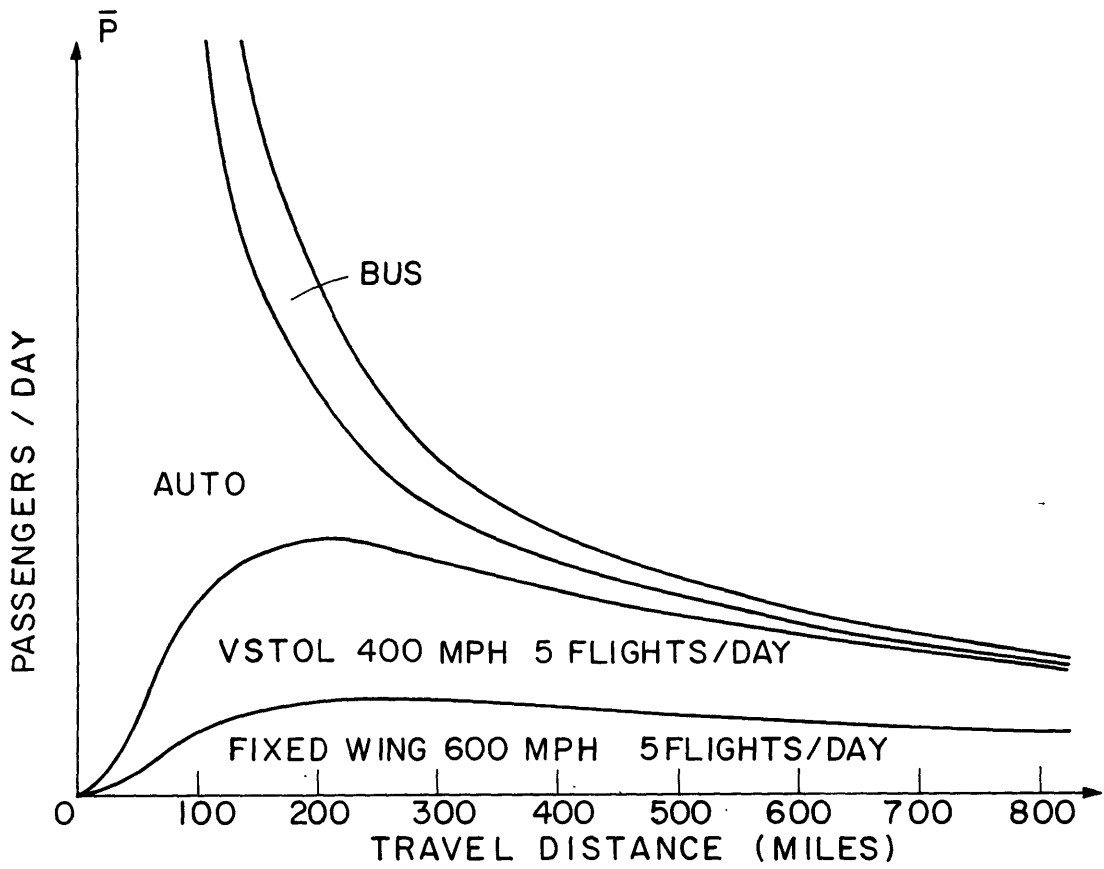


FIG. 9 TYPICAL TRAVEL MODEL RESULTS, FOR VSTOL VS AUTO, BUS, AND AIR

of 200 miles, with automobiles serving a vast market at lower distances. For these assumptions, the potential market area is indicated by Figure 9 to lie between 50 and 500 miles with a supersingly large share of the longer haul traffic. This potential market area can be placed at lower ranges by increasing the V/STOL fare structure in terms of cents/seat mile, or reducing V/STOL cruising speeds. Similarly, higher V/STOL daily frequencies cause a higher penetration of the very short range automobile travel market. One can establish the roles which the various modes might play by setting fares, cruising speeds, and frequency of service.

4. DETERMINING N AND C TO MAXIMIZE CONTRIBUTION
TO OVERHEAD

In a given market competition, the travel prediction model now allows us to predict the daily passenger demand \bar{P} as a function of the frequency of service N, all other parameters such as time and fare structures being fixed for a certain market and vehicle.

But the demand \bar{P} , as given by the model, is an expected or average demand. The expected number of passengers or traffic \bar{T} can only be derived by combining the random variability of demand with constraints due to vehicle capacity.

It has been assumed that air travel demand is normally distributed about an average demand \bar{P} and that the standard deviation of demand is a linear function of the average demand:

$$\sigma_p = K \cdot \bar{P}$$

A value of $K = 0.22$ has been used in our analyses.

The amount of traffic is obviously limited by the number of seats $N \cdot C$. If, for a frequency of service N_0 , the average passenger demand is P_0 and the seat capacity $N_0 \cdot C$, Figure 10 shows that a certain percentage of flights will be dispatched at full capacity while the passenger load on other flights will be distributed according to the demand distribution. The expected traffic T_0 is the mean of the passenger load distribution, and is less than P_0 since passengers will be lost to other modes on over-capacity days.

Figure 10 shows a typical $\bar{P}(N)$ curve for a given market, i.e. the average number of potential passengers for a V/STOL system given N frequencies. For a given vehicle size C , the straight line $N \cdot C$ represents 100% load factor, and obviously when $\bar{P}(N)$ is above this straight line, the system cannot physically carry its potential daily demand. The traffic curves, calculated assuming a normal distribution of $P(N)$ are therefore a function of vehicle capacity, $\bar{T}(N, C)$. Note that average load factor is given by the ratio of $\bar{T}(N, C) / N \cdot C$ at any given N , and that constant load factor curves are straight lines from the origin on this plot.

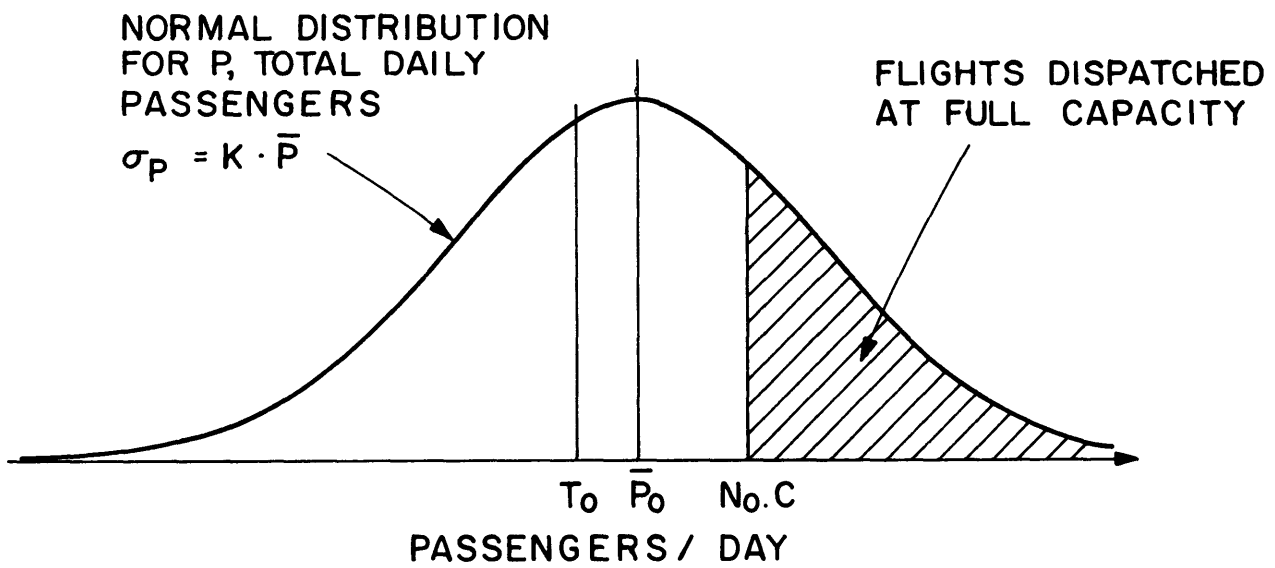
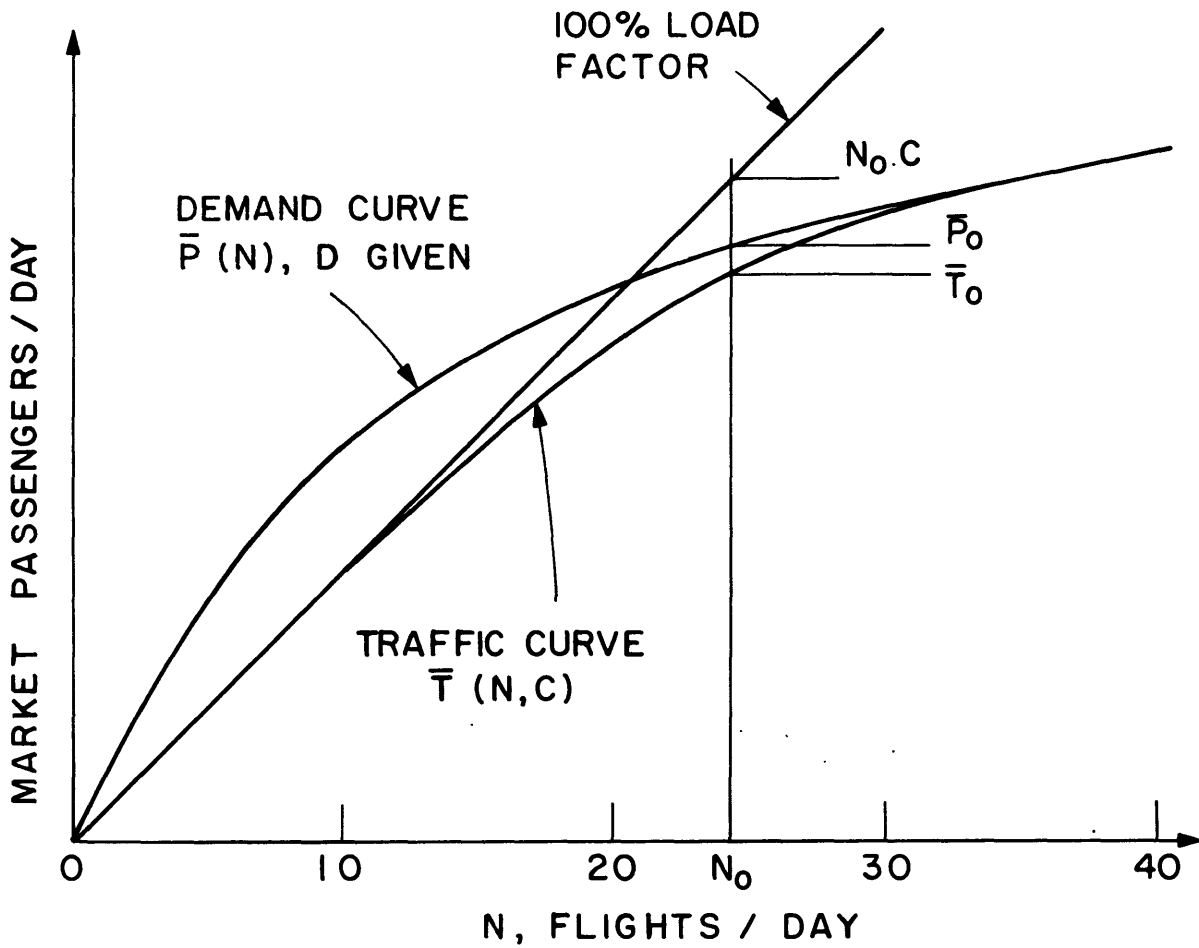


FIG. 10 CALCULATION OF TRAFFIC FOR A GIVEN MARKET AND VEHICLE CAPACITY

If F is the fare charged for the actual flight service, the total revenue per day is simply: $TR = F \cdot \bar{T}$, hence the variation of total revenue with N and C in a given market can be represented on a similar curve.

We now want to determine on these traffic or total revenue curves the optimal point in terms of profitability to the operator. Because of the short-term aspect of our optimization in construction a schedule over a network of routes, we want to select the conditions which maximize contribution to overhead, as this will lead to a maximization of profit on a network basis. For other planning purposes, maximization of net profit can be a suitable objective.

For the short term schedule planning, the usual airline direct and indirect operating costs both have fixed and variable terms, and we will optimize using the variable costs only. The marginal revenue is taken as total revenue which is a function of N and C . The daily contribution to overhead (or marginal income) is then defined as

$$\begin{aligned} \text{CON} &= \text{TR} - \text{variable costs} \\ &= F \cdot \bar{T}(N, C) - \text{VDC}(C) \cdot N - \text{VIC} \cdot \bar{T}(N, C) \end{aligned}$$

where VDC = variable direct costs per trip

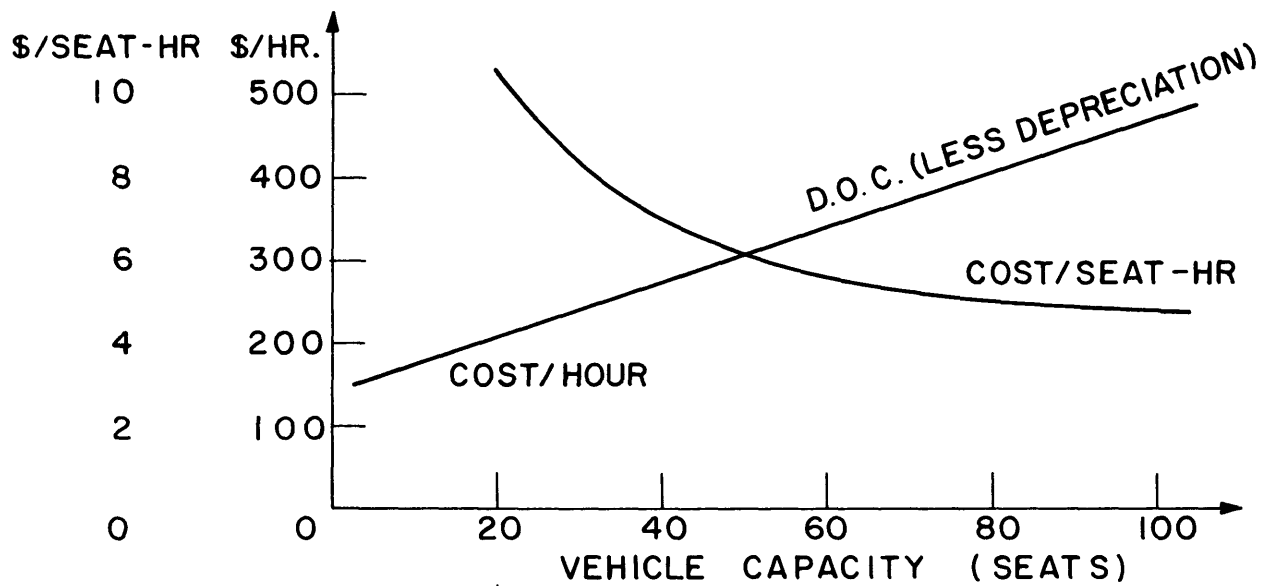
VIC = variable indirect costs per passenger

In our analyses of typical short-haul markets, we have been concerned with a family of vehicles, the characteristics of which have been determined by a computer program developed at the Flight Transportation Laboratory of M.I.T. (Ref. 2) . The basic vehicle used as an example here is a V/STOL aircraft of the tilt wing type with 4 engines, 2 propellers, a cruise speed of 400 MPH, a design range of 400 miles and technologically feasible in 1980. Figure 11 shows how the operational costs vary with the vehicle capacity.

The larger size vehicles have lower unit costs in terms of cost/seat·hour or cost/seat·mile , and would seem to allow lower fares. However, the smaller size vehicles have lower trip costs (or cost/hour) which allow lesser dispatch costs for a given frequency of service. The costs shown in Figure 11 are direct operating costs for the vehicles less the fixed depreciation costs. As well, one must estimate the variable indirect costs associated with boarding, processing of passengers, etc. in the ground operations of the V/STOL system. These have been estimated at \$2 per passenger for an efficient ground operation in future automated terminals.

Given these cost characteristics, the number of passengers to break even can be determined for any size of

VARIABLE OPERATING COST



VARIABLE COST BREAKEVEN LOAD FACTOR

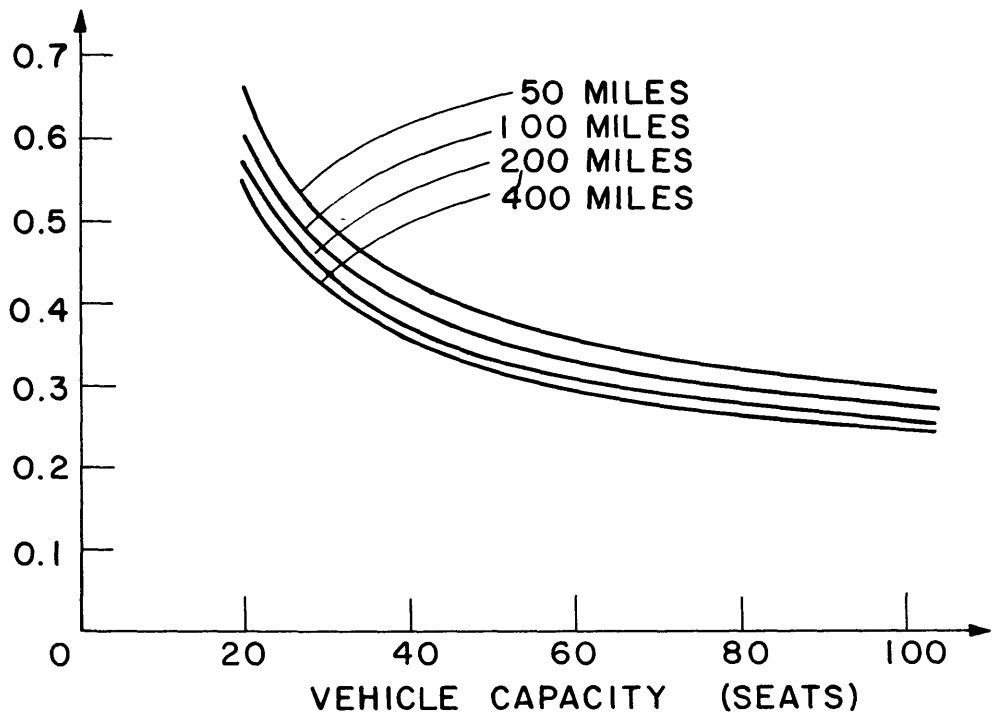


FIG. II TYPICAL ECONOMIC CHARACTERISTICS FOR A FAMILY OF VSTOL AIRCRAFT (1980 TILT WING, 4 ENGINES, 2 PROPELLERS) 400 MPH CRUISE 400 MILES DESIGN RANGE

vehicle. This breakeven load factor can now be plotted on the traffic-frequency plot such as Figure 12. The difference between the traffic curve \bar{T} and the breakeven line is proportional to the contribution to overhead expressed as passengers/day or net revenue.

Figure 12 shows plots of traffic curves for a market of 100 miles distance, and a potential demand of 1000 passengers/day at $N = \infty$, and for vehicle capacities of 30, 60, and 100 seats. The $\bar{T}(N,C)$ curves are different because of the capacity saturation effect, and the load factor lines are different because of capacity and cost effects. As aircraft size increases, the optimum daily frequency N_0^* decreases, and also the traffic carried in the market. In this case, the maximum net contribution to overhead occurs for the smaller 30 passenger vehicle at the highest frequency per day, and traffic size.

If we examine different size markets over different distances, we can show the area where aircraft of varying sizes would be optimal for the market assumptions of the example used in this report. Figure 13 shows such a result, where market potential (P for air if $N = \infty$)

TRAVEL DISTANCE = 100 MILES, $P(N = \infty) = 1000$ PASSENGERS/DAY

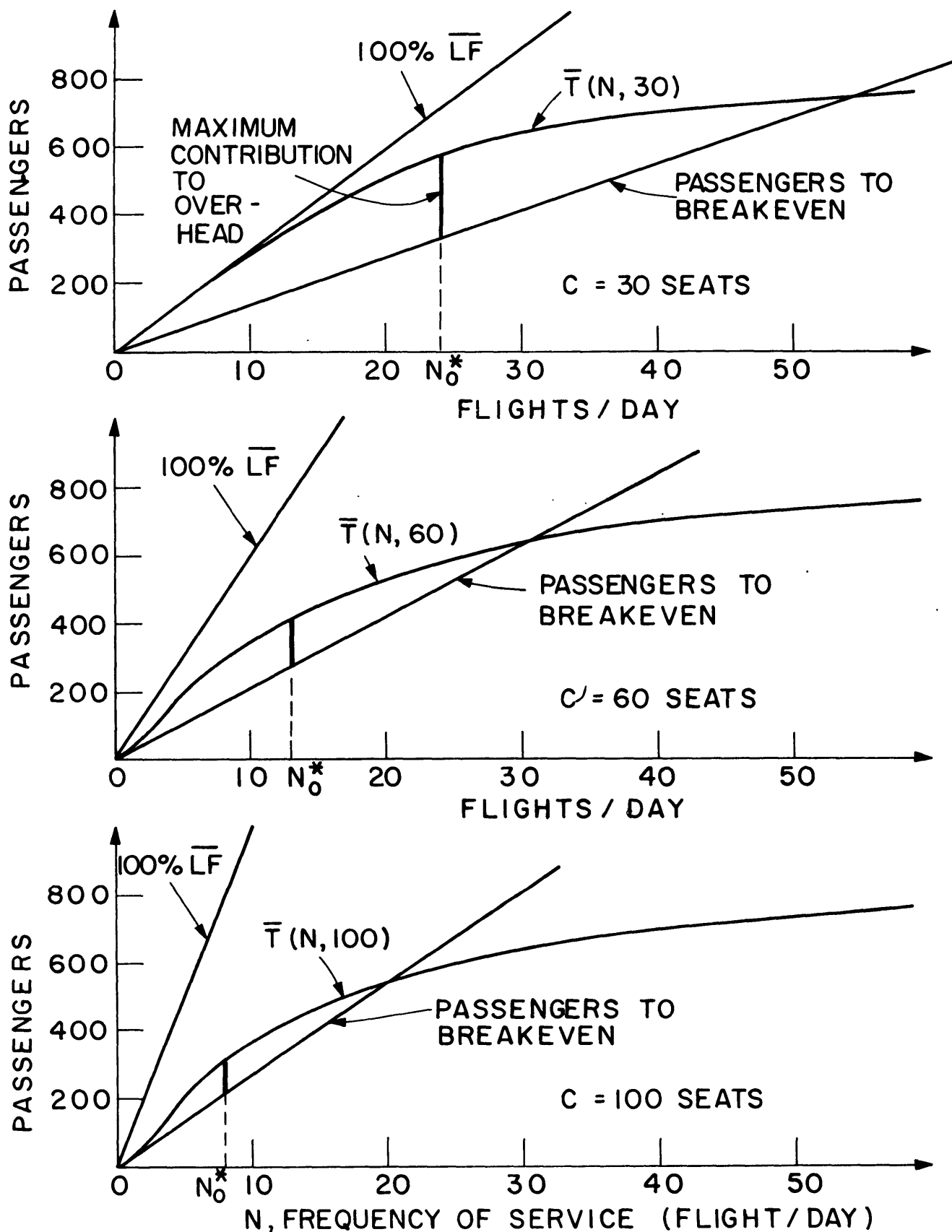


FIG. 12 MAXIMIZING CONTRIBUTION OVER VEHICLE SIZE AND FREQUENCY OF SERVICE

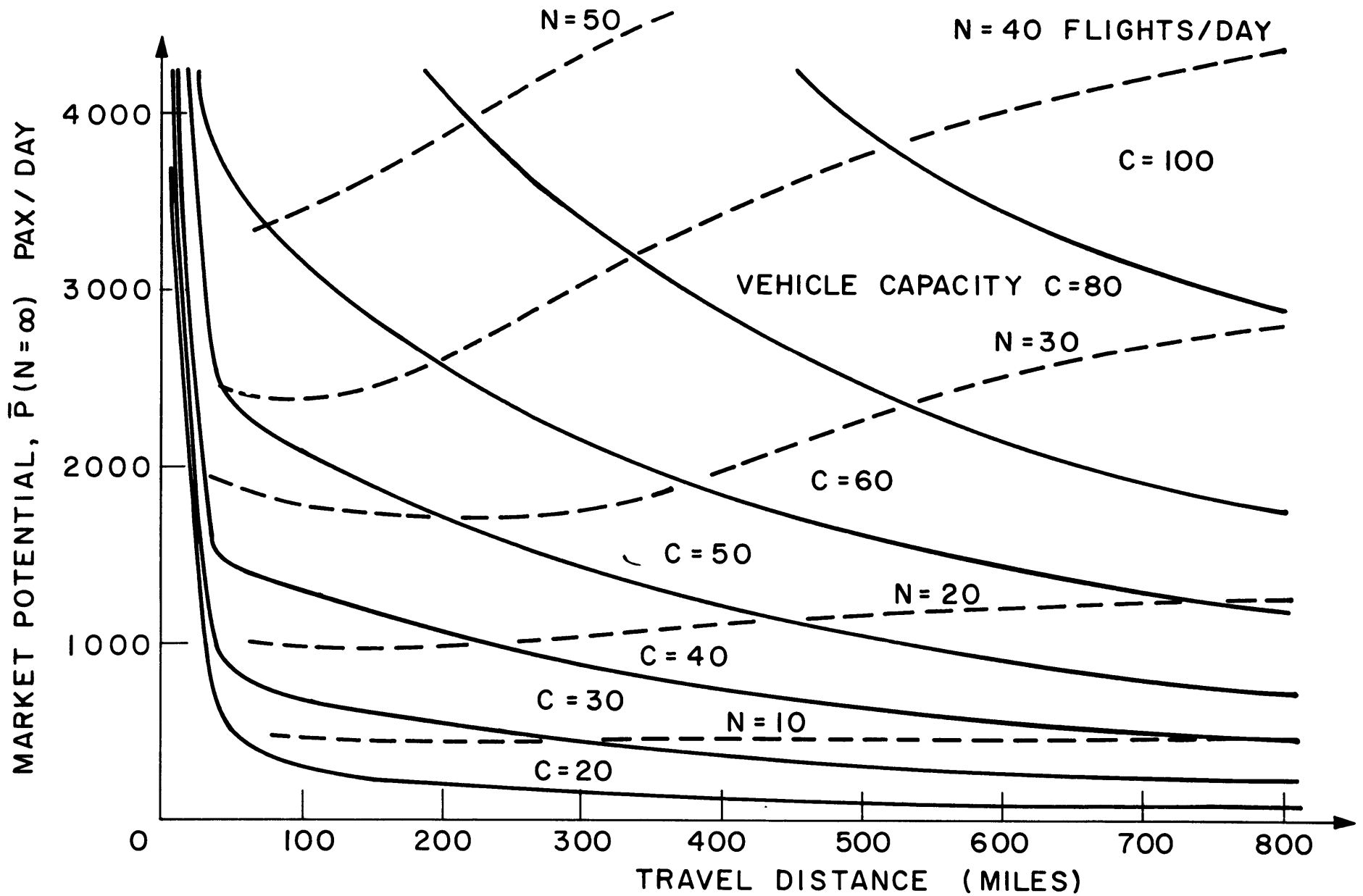


FIG. 13 OPTIMAL AIRCRAFT SIZE ALLOCATION IN A TYPICAL MARKET COMPETITION

is used to give market size in passengers per day. Given the market potential and distance, one can select the optimal vehicle size, and solve for the optimal frequency, N_0^* , and the actual market size for that frequency. For example, if the market distance were 200 miles, and the potential V/STOL traffic at $N = \infty$ were 2000 passengers/day, the optimal V/STOL tilt wing would have approximately 50 seats, and the optimal frequency of service is approximately 33 flights/day. Using a 50 seat vehicle, one can then solve for the exact optimal frequency and market size.

5. ESTIMATING λ AND THE DISPATCH PATTERN

In Figure 12 the optimal frequency for maximizing net contribution occurs when the slope of the \bar{T} (N,C) curve is parallel to the breakeven load line.

$$\text{i.e.} \quad \left[\frac{\partial \bar{T}}{\partial N} \right]_{N_0^*} = \overline{LF}_{BE} \cdot C \quad 5.1$$

where \overline{LF}_{BE} is the variable cost breakeven load factor shown in Figure 11.

For the double curvature \bar{T} (N,C) curves such as shown for the 100 seat vehicle in Figure 12, another operating point which could be selected is the frequency which gives maximum load factor. This occurs at a lesser load and frequency than the net contribution point, and maximizes net contribution per passenger.

The value of λ required to determine an optimal dispatch pattern of N_0^* flights can now be calculated for use in the dynamic program of part 2. This value represents the loss of daily revenue caused by a unit of passenger delay.

$$\text{i.e.} \quad \lambda = -\frac{\partial R}{\partial D}$$

Using relation 5.1

$$\left[\frac{\delta R}{\delta D} \right]_{N^*_0} = F \cdot \left[\frac{\delta N}{\delta D} \right]_{N^*_0} \cdot \overline{LF}_{BE} \cdot C \quad 5.2$$

From 2.1 for optimal dispatching,

$$D = \bar{T} \cdot \left[\frac{T_d}{2N} \right], \text{ or } N = \frac{\bar{T} \cdot T_d}{2D}$$

Therefore,

$$\frac{\delta N}{\delta D} = \left[\frac{-T_d \cdot \bar{T}}{2} \right] \cdot \frac{1}{D^2} = \frac{-N}{D} = \frac{-2N^2}{\bar{T} \cdot T_d}$$

Substituting in 5.2

$$\left[\frac{\delta R}{\delta D} \right]_{N^*_0} = \frac{-F \cdot 2N^*_0{}^2 \cdot \overline{LF}_{BE}}{T_d (\bar{T}/C)}$$

Therefore,

$$\lambda^* = \frac{F \cdot \overline{LF}_{BE}}{T_d} \cdot \frac{2N^*_0{}^2}{(\bar{T}/C)} \quad 5.3$$

Relation 5.3 gives a value of λ which may be used in the dynamic programs of part 2 to produce an optimal dispatch pattern of N^*_0 flights. It is not a precise determination of λ^* since N is a discrete variable, but in practice, it either produces N^*_0 flights, or gives a

value close to that required.

It is interesting to note that λ^* is a function of vehicle and market variables such as fare, breakeven load factor, market density and level of service as measured by delay or frequency. As indicated in part 2 of this report, λ may be regarded as a Lagrange multiplier, and given the interpretation of the cost per minute of passenger delay to the operator. The optimal value of λ will vary depending on the market competition and vehicle used.

6. SUMMARY

For a given model of travel market behavior, a method of determining an optimal vehicle size and frequency of service for a short haul V/STOL air transport system has been determined. For each individual city pair market, a predicted travel volume, and net income can be calculated for any vehicle size, and its optimal frequency of service. The problem now turns to selecting in some fashion, a small number of different aircraft sizes to be routed over the collection of city pair markets on a given network of routes. On the network, we may desire to mix the aircraft types in order to improve utilization or decrease the total number of aircraft required. The method described in this report is one component of a larger problem of schedule generation for such a transportation system.

The model results have tended to show that frequency of service is important to short haul air systems which are in competition with the automobile, and that smaller, more costly vehicles may be more economic in the sense that they generate higher revenues through higher schedule frequency.

The travel behavior model used here has a number of points of agreement with expected traffic behavior, but before it can be used with any confidence, further experimental verification and statistical testing seems necessary.

All of the method described in this report has been coded for an IBM 360 Model 65 at MIT such that a link by link evaluation of a number of vehicle sizes can be carried out for individual link market competitions from a given network.

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