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**TIME DEPENDENT ESTIMATES OF DELAYS
AND DELAY COSTS AT MAJOR AIRPORTS**

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Time Dependent Estimates of Delays
and Delay Costs at Major Airports

by

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Abstract

Two queuing models appropriate for estimating time dependent delays and delay costs at major airports are reviewed. The models use the demand and capacity profiles at any given airport as well as the number of runways there to compute bounds on queuing statistics. The bounds are obtained through the iterative solution of systems of equations describing the two models. This computational procedure is highly efficient and inexpensive. The assumptions and limitations of the models are discussed.

Common characteristics and properties of delay profiles at major airports are illustrated through a detailed example. Potential applications to the exploration of the effect of air traffic control innovations on congestion and to the estimation of marginal delay costs are also described.

I. INTRODUCTION

The problem of air traffic congestion at major airports has been the subject of numerous studies in the past. Since these airports are generally acknowledged to be the principal bottlenecks on the airside of the air transportation system, this attention is certainly well deserved.

Understanding of all facets of the airport congestion phenomena is becoming increasingly essential for a number of reasons. Airports and runways, first, are enormously expensive facilities and it is to the best interest of society to use these facilities as efficiently as possible. Second, the primary future hurdle to further growth of the air transportation system will most likely be the cost or unavailability of fuel. Circling in the air over an airport or waiting for long periods next to a runway to take-off are notoriously poor ways of utilizing expensive fuel resources, especially in cases where a trip is over a short or medium distance. A third fact is that there is currently throughout the world a tendency to adopt a wait-and-see attitude on planning major airport-related construction programs. In view of the current questions concerning the future growth of air transportation, claims regarding the need for new airport facilities are viewed with doubt and scepticism. The major determinants of whether a new facility is indeed needed are the costs, nature, and causes of airport delays.

The major deficiency of most work on airport queues has been that, due to lack of analytical tools, the time-varying nature of

airport congestion phenomena has not been explicitly considered and accounted for.

In 1969, CARLIN AND PARK [1] took a highly practical approach to the problem of congestion in New York City's airports, considering the time-dependence of delay costs due to the demand profile. They estimated, among other things, total delay costs at the airports during peak and off-peak hours.

More recently, KOOPMAN [6] pointed out that delay estimates are relatively insensitive to the precise queuing model used, as long as the probabilistic nature of the queuing process was explicitly recognized. Through a computer-aided analytic solution of sets of transition-probability equations, he obtained upper and lower bounds on the actual time-dependent delay statistics and demonstrated that, for the parameter values prevalent at major airports, these bounds are very close to each other.

This paper applies KOOPMAN's approach to multiple server queuing systems. By using a set of computer programs carefully written to account for some of the numerical intricacies of the queuing models, it provides a detailed example of congestion analysis at a specific airport and attempts to cast light on several important practical problems: It reviews the sensitivity of waiting times to changes in airport capacity and airport demand; it computes the total daily costs of delays and places a price-tag on the non-uniformity of demand through the day; finally it illustrates the concept of marginal delay costs, by estimating the costs of adding new flights at different times of the day.

The results illustrate the potential of this approach to future work on runway pricing and on evaluating the need for new facilities.

In the following sections, we first review the queuing models, their assumptions, and limitations in an informal theoretical section (part 2). In part 3, we present some results from the detailed case study that was mentioned earlier. Part 4 discusses the results and the approach to a number of important airport-related problems. A set of notes, that supplement the text, provide mostly background information on the subjects discussed.

2. THE MODELS

The theoretical model presented here is based on the earlier work of KOOPMAN [6] and is a quite straight-forward extension of that work to the case of multiple servers (i.e., multiple runway airports). For this reason we shall only describe the bare essentials of the theoretical foundations here and, instead, concentrate on providing an intuitive explanation of the basic rationale, of the assumptions used, and of the limitations of the models. For a rigorous treatment of the theoretical questions, the reader is referred to [6].

The model considers an airport as a set of independent, parallel servers (the runways). A schematic representation of this system is shown in figure 1.

It is assumed that the total demand at the airport - that is, the sum of the demands for landings and for take-offs - is a Poisson process with a time-dependent average demand rate, given by $\lambda(t)$. The Poisson assumption for airport demand is consistent with actual observations at several major airports and has been used extensively in the literature [4], [8], [10] (see Note 1).

By contrast, the form of the probability law describing the duration of a service at the runways is still a matter for speculation [4], [8], [10]. The duration of the period during which a runway is busy with an aircraft depends on such diverse factors as type of operation being conducted, weather, aircraft mix, runway configuration in use, runway surface conditions, location of runway exits, air traffic control equipment, requirements for minimum separations

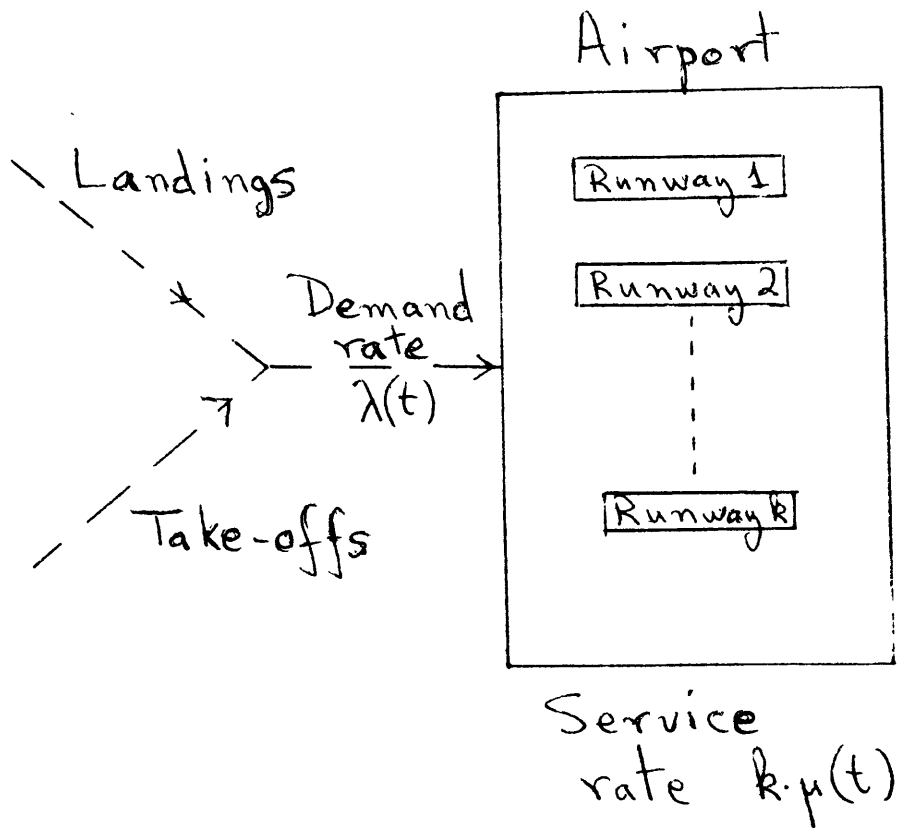


Figure 1: Schematic representation of the model.

between aircraft, pilot and air traffic controller performance, etc. Following the example of [6], we shall sidestep this issue by making this intuitively reasonable observation: the duration of the service times must be "less random" than the perfect randomness described by the negative exponential probability density function and "less regular" than the perfect regularity described by deterministic service times.

This last point is a crucial one as it drives our whole approach to the problem: we shall seek to obtain upper and lower bounds on congestion-related statistics by noting that a worst case is provided by the negative exponential service assumption and a best case by the deterministic service assumption. The rationale, of course, is that, if - for the set of parameter values prevalent in the systems under consideration, i.e. the major commercial airports - the upper and lower bounds turn out to be reasonably close to each other, then either bound (or any reasonably weighted combination of the two) can be used as a good approximation of the actual statistics desired. As will be seen in what follows, the bounds do indeed turn out to be close for all practical purposes, and under widely varying sets of conditions.

Here then is the strategy to be followed: Given an airport with k independent runways each of which has a time-dependent average service rate $\mu(t)$, we shall solve iteratively and for the desired period of time two systems of equations, one describing an $M/M/k$ queuing system and the other an $M/D/k$ queuing system. The actual values of interest will then be bounded from above and below by

the values obtained from these two queuing models. This whole approach is dictated by the fact that the integro-differential equations that describe an M/G/k queuing system - a more realistic model for the case of interest - are unwieldy even for the purpose of obtaining numerical solutions.

Assumptions in the Model

To complete the description of our queuing models, we now list some assumptions that were made, mostly for reasons of computational feasibility. The most important of these, from a practical viewpoint, is the assumption of the existence of a single queue of aircraft awaiting use of the runways on a strictly first-come, first-served basis. Thus, we make no distinction between landing and departing aircraft but are instead interested only in overall measures of congestion. While, in practice, the average service times (and the probability distributions) for landings and take-offs are different (see Note 2), we use here what is in effect a single weighted average service time for both kinds of operations (see Note 3).

Another assumption is that all active runways (or, all the parallel servers in figure 1) operate independently and are identical. In practice, runways often can not be operated independently, since operations at one may affect those on another, due to airport geometry. Again, from the practical viewpoint, this assumption is not too restrictive since dependencies among the servers, if they exist, can be accounted for by adjusting the service rates accordingly. As an example, consider an airport with a single runway which can handle,

say, 50 aircraft movements per hour, i.e. the average service time is 72 seconds. Suppose now that operations are begun at a second runway which intersects the first one. Then, the overall airport capacity might increase to, say, 80 operations per hour; and not to 100 as it would if the two runways were independent. To account for this in our model, we would then assume the existence of a single independent server, with an average service time of 45 seconds, for an overall airport capacity of 80 movements per hour.

Obviously, the number of state-transition equations, describing the queuing models and being iteratively solved by the computer, must be finite. Since the number of such equations is equal to the number of states in the queuing model, a further condition must be that the capacity of the airport queue is finite. Thus, it is assumed that the queuing system of figure 1, can accommodate up to a maximum of m aircraft (including the ones in service at the k servers). In practice, this is entirely inconsequential since m can be selected large enough to make it highly unlikely that the number of aircraft in the terminal area at any given instant will be equal to m . This is further discussed later in this paper.

Finally, it is assumed that successive service times are statistically independent. This is substantially true in reality, as little attempt is made, under today's air traffic control regime, to sequence operations in anything but a first-come, first-served way. Successive service times are, therefore, randomly mixed according to the mix of aircraft with little or no inter-dependence among them.

The M/M/k System Equations

We now list the equations that describe the two queuing systems under consideration here. First, for the M/M/k model, we have Poisson arrivals at a time-dependent average rate of $\lambda(t)$. These arrivals are served by k parallel servers, each operating at an average service rate, $\mu(t)$. It is assumed, that individual service times are distributed as negative exponential random variables with exponent equal to the value of $\mu(t)$ at the instant t when service is initiated. The queue capacity is equal to m .

Let us define by $P_i(t)$, $i = 0, 1, 2, \dots, m$, the probability that at time t there are i aircraft in the terminal area. Then, for any t , we can write the well-known set of Chapman-Kolmogorov equations for the derivatives $P'_i(t)$ of the state probabilities. Suppressing, for reasons of conciseness, the time-dependence of the arrival and service rates, i.e. writing $\lambda = \lambda(t)$ and $\mu = \mu(t)$, we have:

$$P'_0(t) = -\lambda P_0(t) + \mu P_1(t) \quad (1.1)$$

$$P'_i(t) = \lambda P_{i-1}(t) - (\lambda + i\mu)P_i(t) + (i+1)\mu P_{i+1}(t) \quad \text{for } 1 \leq i \leq k-1 \quad (1.2)$$

$$P'_i(t) = \lambda P_{i-1}(t) - (\lambda + k\mu)P_i(t) + k\mu P_{i+1}(t) \quad \text{for } k \leq i \leq m-1 \quad (1.3)$$

$$P'_m(t) = \lambda P_{m-1}(t) - k\mu P_m(t) \quad (1.4)$$

The above $m + 1$ equations can be solved iteratively for any desired period of time T , using the approximation $P_i(t+\Delta t) = P_i(t) + P'_i(t) \cdot \Delta t$, where Δt is a time interval chosen sufficiently small to be consistent

with the Poisson assumptions regarding the arrival and service processes. A boundary set of values $P_i(0)$, $i = 0, 1, 2, \dots, m$, and the functions $\lambda(t)$ and $\mu(t)$ for $0 \leq t \leq T$ must be provided.

The M/D/k System Equations

Turning to the corresponding system of equations for the model in which service is assumed to be deterministic, we define the increment of time as equal to the duration of a single service time. We assume further that all k parallel servers begin and end service simultaneously (see Note 4). It is then possible to write equations relating the sets of state probabilities $P_i(t)$ and $P_i(t+1)$ - remember that t is now being increased at discrete intervals equal to the average service time. (Since time intervals are normalized to $1/\mu$, the demand rate must also be normalized to $\rho = \lambda/\mu$, the demand per unit of service.) These equations are based on the fact that the probability that exactly n aircraft will attempt to join the system between t and $t+1$ is equal to $\rho^n \cdot \exp(-\rho)/n!$ due to the Poisson law for the demand pattern.

We then have:

$$P_0(t+1) = \exp(-\rho)q_k(t) \quad (2.1)$$

$$P_i(t+1) = \exp(-\rho) \left[q_k(t) \frac{\rho^i}{i!} + P_{k+1}(t) \frac{\rho^{j-1}}{(i-1)!} + P_{k+i}(t) \right] \quad (2.2)$$

for $1 \leq i \leq m-k$

$$P_i(t+1) = \exp(-\rho) \left[q_k(t) \frac{\rho^i}{i!} + P_{k+1}(t) \frac{\rho^{i-1}}{(i-1)!} + \dots \dots \dots \right. \\ \left. \dots + P_m(t) \frac{\rho^{i+k-m}}{(i+k-m)!} \right] \quad \text{for } m-k+1 \leq i \leq m-1 \quad (2.3)$$

$$P_m(t+1) = q_k(t)b_m + P_{k+1}(t) \cdot b_{m-1} + \dots + P_m(t) \cdot b_k \quad (2.4)$$

where $q_k(t) = \sum_{i=0}^k P_i(t)$ and $b_j = \exp(-\rho) \sum_{i=j}^{\infty} \frac{\rho^i}{i!}$.

Strictly speaking, (2) assumes that the new arrivals during a unit of time join the queue at the end of the service unit at which time the capacity limit, m , applies.

Again, beginning with a set of initial conditions $P_i(0)$, $i = 0, 1, 2, \dots, m$, the above set of equations can be solved iteratively to obtain numerical answers for demand and service rate profiles, $\lambda(t)$ and $\mu(t)$ (we have, for conciseness, suppressed the time variable in the equations).

Related Quantities

KOOPMAN [6] has shown that for "relatively slow varying" $\lambda(t)$ and $\mu(t)$ the sets of equations for the M/M/k and M/D/k systems possess unique periodic solutions with period T whenever the demand and service rates are both periodic with period T . In the case of airports, demand and service rates can indeed be considered to be periodic quantities with period $T=24$ hours. It remains, therefore, to solve the two sets of equations numerically to obtain estimates of the state probabilities, $P_i(t)$, for all $0 \leq t \leq T$. The state probabilities, in turn, can be used to compute other quantities of interest. Of those, we shall specifically refer to:

i) The probability that all runways are busy and, therefore, that a newly-arriving aircraft will experience positive delay,

$$B(t) = 1 - \sum_{i=0}^k P_i(t) \quad (3)$$

ii) The expected number of aircraft in the queue at time t ,

$$Q(t) = \sum_{i=k+1}^m (i-k)P_i(t) \quad (4)$$

iii) The average waiting time in the queue for aircraft that arrive at time t (see Note 5)

$$W(t) = \frac{1}{k \cdot \mu(t)} \sum_{i=k}^m (i-k+1)P_i(t) \quad (5)$$

This last quantity is only an approximation in the case when $\mu(t)$ is a function of time. The reason is that the rate of service, $\mu(t)$, may change in the future if the waiting time is long (see Note 6).

In all cases, two estimates of these parameters of interest are obtained, one based on the M/M/k and the other based on the M/D/k model.

The Computer Programs

Computer programs were written [5] to compute numerical solutions for the two queuing models. The inputs to the programs are: the hourly demand levels; the hourly service rates; and the number of independent servers at the airport of interest. The piecewise linear functions that result from connecting the half-hour points of the demand and service rates are then taken to represent $\lambda(t)$ and $\mu(t)$, respectively (for instance, figure 4 shows the function $\lambda(t)$ that results from the demand depicted in figure 2). The outputs of the programs are the state probabilities, $P_i(t)$, as computed in (1) and (2), as well as other desired quantities such as those obtained from (3) through (7).

Particular attention needs to be given to numerical control problems due to the magnitudes of some of the coefficients in the equations and to the propagation and build-up of truncation errors in the iterative solution. Double precision arithmetic is used throughout as well as the procedures outlined below.

The iterative solution of the set of differential equations, (1), is accomplished with the aid of a standard Runge-Kutta subroutine. The time increment between successive iterations, Δt , is varied internally during the period of interest, T , according to the magnitudes of the parameters $\lambda(t)$ and $\mu(t)$. Specifically, Δt can be doubled or halved on successive iterations depending on the magnitude of the total truncation error which is not allowed to exceed a prespecified level. At the same time, Δt is not allowed to exceed a preset maximum interval which is consistent with the Poisson assumptions.

For the M/D/k model, the terms $\exp(-\rho) \cdot \rho^i / i!$ are computed at the beginning of each iteration (note that ρ is a function of time). All terms with value greater than a prespecified number (we have used 10^{-9}) are included. This provides the coefficients in (2), including the b_j .

A useful feature of the computer programs is an option under which the capacity, m , of the queuing system is adjusted internally so that the probability of system saturation, $P_m(t)$, is maintained arbitrarily small. When this option is in use $P_m(t)$, for the current value, m , of the system capacity, is the first state probability to be calculated on each iteration. If $P_m(t)$ turns

out to be greater than a prespecified tolerance level of saturation, the queue capacity is increased in steps of 1 unit, until the probability of a saturated queue is below the required level. The system of equations is then solved for the iteration in question using the new value of m . Conversely, if at the beginning of a new iteration the value of $P_m(t)$ is less than a required level, the queue capacity is decreased in steps of 1 unit. Since the number of operations per iteration in each algorithm is proportional to m^2 , this leads to improved efficiency. In addition, by not allowing the queue to saturate, the full potential extent of congestion can be explored.

On the other hand, if it is believed that an airport and terminal area do indeed have only a specified number of waiting slots for aircraft, then m can be maintained fixed.

The programs are being used at present to obtain delay estimates for various demand profiles at major airports in the United States in a project sponsored by the Federal Aviation Administration. The programs are written in FORTRAN H language. Typical execution times for a 24-hour case, such as the one described in the next section, run to a total of about 25 seconds of CPU time for the two queuing models on an IBM 370/168 computer.

3. A DETAILED CASE STUDY

The example chosen for detailed study was Logan International Airport in Boston. The average demand profile (landings and take-offs) over the weekdays of a two-week period (16-29 September 1970) for this airport [3] is shown in figure 2 (see Note 6).

For initialization purposes, 4 a.m. was chosen as the beginning of the 24-hour period. Due to very low traffic activity at that time it can be assumed that the initial conditions on the state probabilities are as follows:

$$P_0(0) = 1$$

$$P_i(0) = 0 \quad \text{for } i = 1, 2, \dots, n$$

The theoretical capacity of Logan International Airport depends on weather conditions. When visibility is good (Visual Flight Rules weather) average airport capacity is considered to be 80 operations/hour. In poor visibility (Instrument Flight Rules weather) the average capacity is reduced to 70 operations/hour (see Note 7). As two runways are active most of the time at Logan Airport, k was chosen equal to 2 for all computer runs. Thus, each runway is considered to have a capacity of 40 and 35 operations/hour in VFR and IFR weather, respectively. As explained earlier, no distinction is made between landings and take-offs.

Average Queue Lengths and Waiting Times

For the demand profile shown in figure 2, the computer programs

AVERAGE HOURLY OPERATIONS - BOSTON (WEEKDAYS, 16-29 SEPT. 1970)

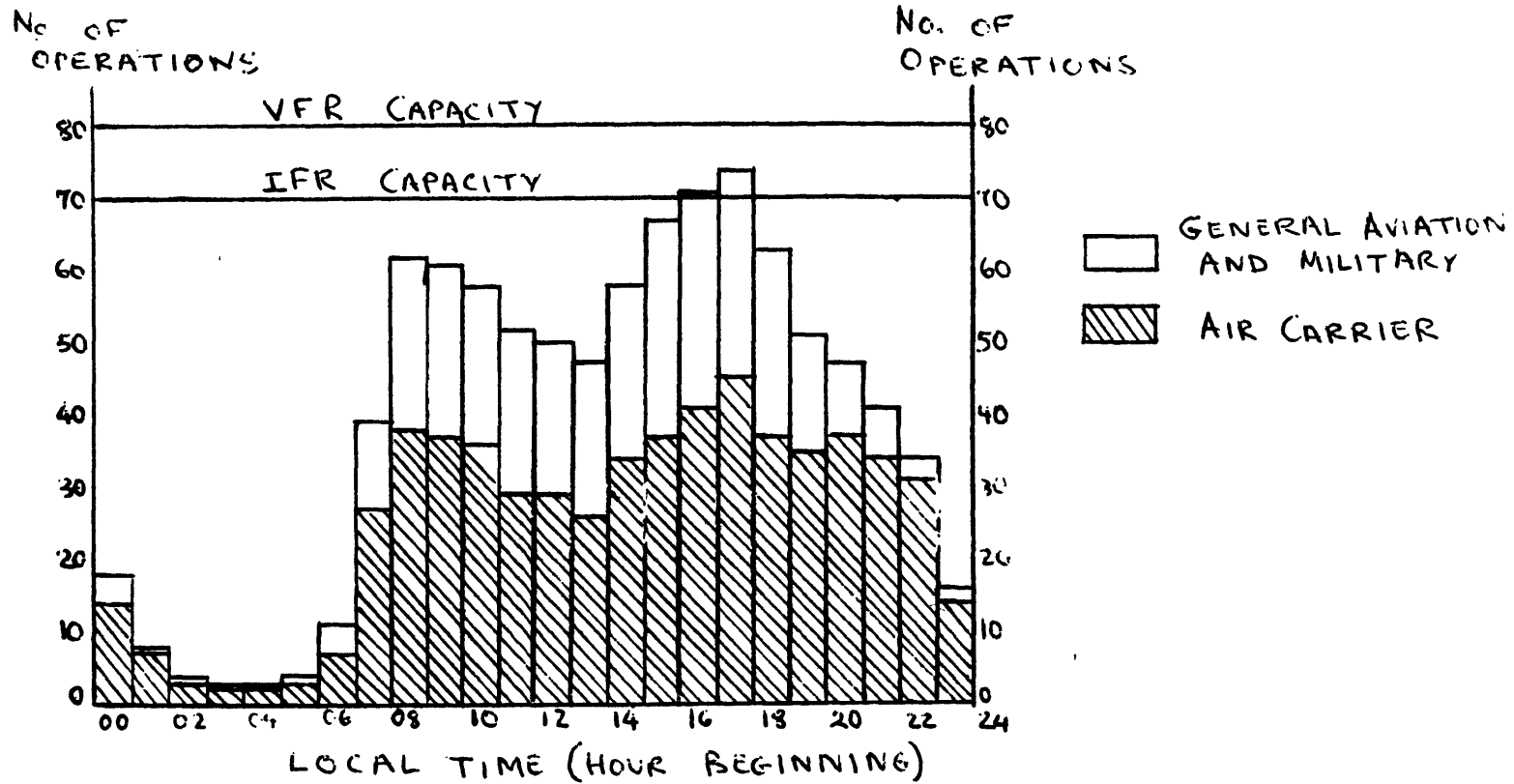


Figure 2: Demand profile at Logan International Airport

were run twice for 24-hour periods, once with an overall airport capacity of 80 operations/hour and the second time with a capacity of 70 operations/hour throughout the day.

A set of results for these two cases is shown in figures 3 and 4. Figure 3 compares the average queue lengths $Q(t)$, for the two cases (or, rather, the bounds on the average queue lengths). It should be noted that an increase in capacity by 10 operations per hour (from 70 to 80) reduces average queue length by roughly a factor of 3 at the peak hour (6p.m.).

Figure 4 concentrates on waiting times, $W(t)$, for the case in which the capacity is 70 operations/hour in order to focus attention on the dynamic properties of this queuing system. One point to note is the strongly non-linear nature of the relationship between demand and average waiting time. A peak demand of about 62 operations in the morning results in relatively modest delays. By contrast, an increase of the demand to a maximum of 74 operations in late afternoon implies very severe waiting times averaging to 10 or more minutes per aircraft. A second observation is that there exists a time phase between demand changes and the attendant congestion effects. This time phase is especially evident during the morning and evening peak hours. It is not determined by any simple relationship, depending on the whole past history of the demand rate.

Delay Costs

In working with economic quantities a weighted average of

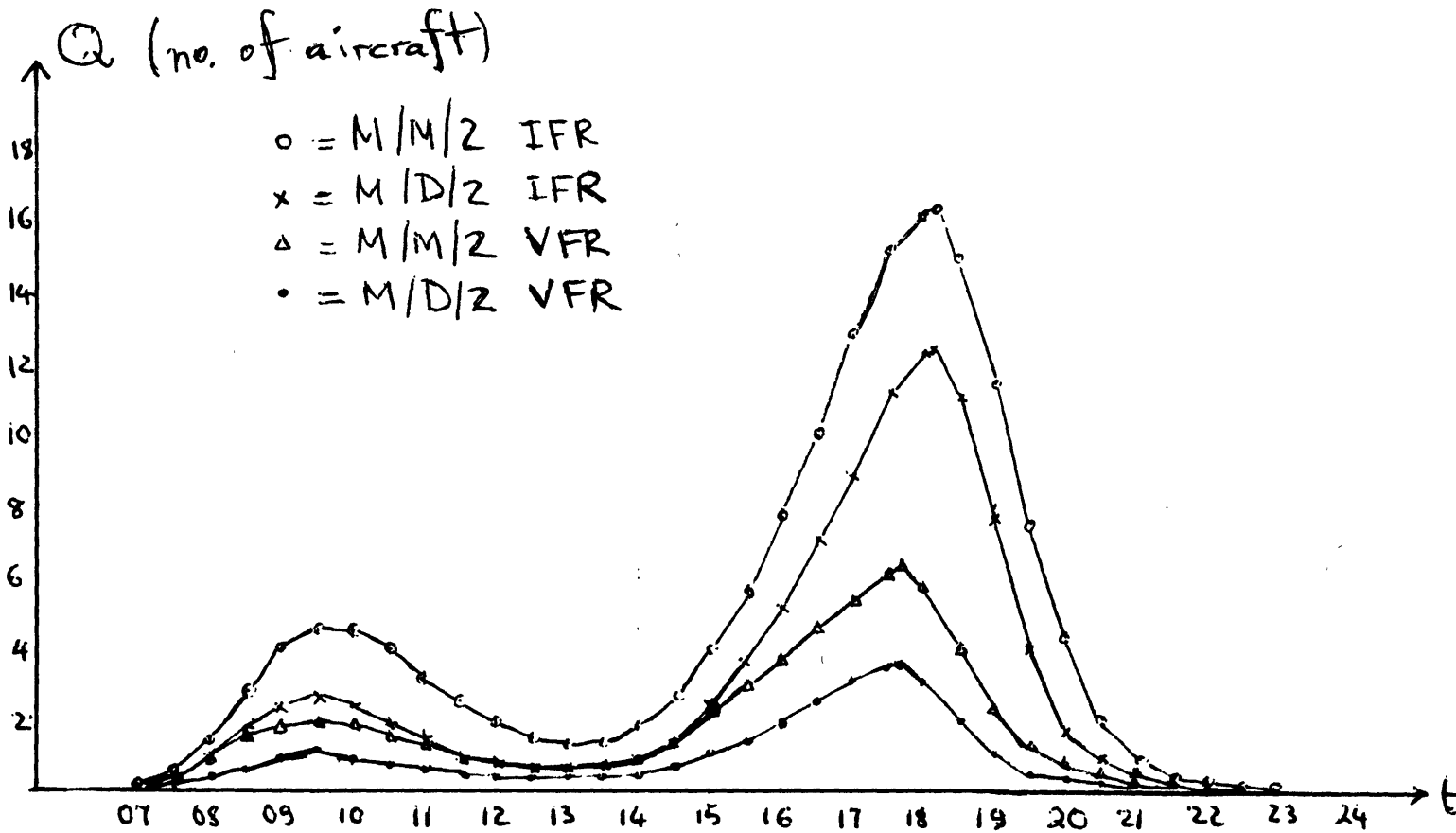


Figure 3: The average queue length

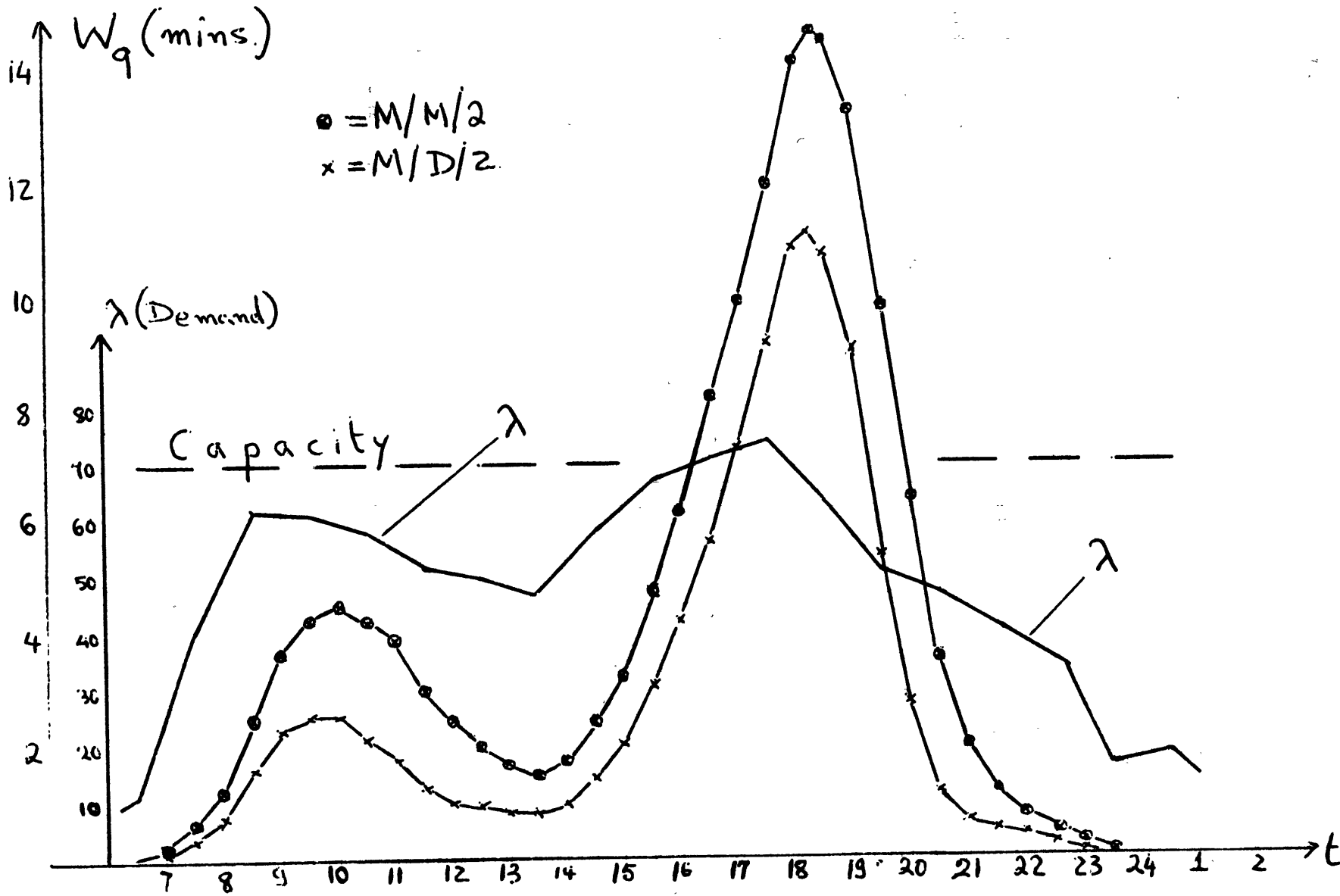


Figure 4: Demand profile and average waiting time

the two bounds was used as an estimator of the actual queuing statistics. Specifically, the estimator of average delay that was employed is:

$$W_e(t) = 1/3 W(t)_{M/M/2} + 2/3 \cdot W(t)_{M/D/2}$$

The reason for this particular weighting is that a deterministic service time distribution is a better approximation to the actual service time distribution than a negative exponential distribution. So, it was felt that the low bound should be weighed more heavily.

Uniformly Distributed Demand

In order to obtain an estimate of the congestion costs due to the non-uniformity of demand, a hypothetical case in which traffic demand was maintained constant at about 53 operations per hour for 18 hours (6 a.m. to midnight) was compared to the status quo as represented by figure 2. The same number of operations is performed in both cases.

The results for the two cases are compared in figure 1. Uniform demand reduces delay costs by 45% and 62% in the VFR and IFR cases, respectively.

From table I, it is also possible to obtain estimates of the current annual delay costs at the airport. Weather at Logan Airport is of VFR type about 85% of the time, and IFR 15% of the time. For the annual delay costs, we thus compute $365 \cdot [(.15)(17,611) + (.85)(7,288)] = \$2,915,000$ (see Note 10).

	CURRENT DEMAND		UNIFORM DEMAND	
	VFR	IFR	VFR	IFR
Cumulative Delay Costs for 24-hour Period (943 operations)	\$6,288	\$17,611	\$3,480	\$6,746
Delay Costs Per Operation	\$6.67	\$18.67	\$3.69	\$7.16

Table I: Delay Costs for Two Demand Distributions

Adding New Flights

Finally, it is possible to quantify the impact that additional users at different times have on the queuing statistics and on delay costs.

To illustrate this, the existing operations pattern as shown on figure 2, was taken as the status quo . Assuming a capacity of 80 operations per hour at the airport, we compare four cases each of which involves demand for 8 additional operations per hour, i.e. an increase equal to 10% of the hourly airport capacity.

- Case 1: 8 additional flights between 1 p.m. and 2 p.m., a time with relatively low air traffic activity.
- Case 2: 8 additional flights for each of the three hours between 11 a.m. and 2 p.m.; during this whole period the airport is only moderately utilized.
- Case 3: 8 additional flights between 5 p.m. and 6 p.m., a time when the airport, as it is, experiences the maximum demand rate of the day.
- Case 4: 8 additional flights for each of the three hours between 3 p.m. and 6 p.m.; these three hours are already associated with the highest sustained demand rate for the whole day.

The computer results for the delay costs are summarized in table II. Two observations can be made from these results. First, the after-effects of cases 3 and 4 are much more pronounced than those of cases 1 and 2. The disturbance introduced by the addi-

TIME	STATUS QUO	CASE 1	CASE 2	CASE 3	CASE 4
4 A.M.	\$.03	\$ -	\$ -	\$ -	\$ -
5	.05	-	-	-	-
6	.44	-	-	-	-
7	19.08	-	-	-	-
8	167.94	-	-	-	-
9	418.49	-	-	-	-
10	390.57	-	-	-	-
11	274.56	-	352.54 (28)	-	-
12	183.41	-	362.52 (98)	-	-
1 P.M.	147.25	196.24 (33)	298.79 (103)	-	-
2	192.82	273.88 (42)	279.43 (45)	-	-
3	425.34	431.96 (2)	432.26 (2)	-	555.72 (31)
4	782.14	782.99 (.1)	783.06 (.1)	-	1494.18 (91)
5	1167.21	-	-	1411.48 (21)	2577.95 (121)
6	1236.91	-	-	2086.45 (69)	2993.79 (142)
7	523.13	-	-	800.41 (53)	1170.60 (123)
8	172.64	-	-	194.76 (13)	236.87 (37)
9	102.36	-	-	103.03 (.2)	104.36 (2)
10	59.04	-	-	-	-
11	18.46	-	-	-	-
12	4.35	-	-	-	-
1 A.M.	2.22	-	-	-	-
2	.21	-	-	-	-
3	.05	-	-	-	-
TOTAL COSTS	6,288.61	6,426.70	6,792.34	7,682.50	11,012.36
COST PER OPERATION	6.67	6.76	7.02	8.08	11.39

Table II: Hourly total delay costs in \$. Figures in parenthesis indicate % increase over status quo. A "-" indicates no appreciable change from status quo.

	ADDITIONAL TOTAL DELAY COSTS FOR 24-HOUR PERIOD (\$)	PERCENTAGE INCREMENT OVER STATUS QUO (%)	MARGINAL DELAY COSTS FOR EACH ADDITIONAL OPERATION (\$)
CASE 1	138.09	+2	17.26
CASE 2	503.72	+8	20.99
CASE 3	1393.89	+22	173.46
CASE 4	4723.75	+75	196.82

Table III: Summary of results of Table 2. The last column is obtained by simply dividing additional total costs by the appropriate number of operations. For example $138.09/8 = 17.26$.

tional demand on the system virtually disappears about an hour later for cases 1 and 2. The after-effects of cases 3 and 4, on the other hand, last for three and four hours. The second observation relates to the strongly non-linear behavior of congestion phenomena. The results of table II, in this respect, are summarized by table III. The effects of cases 1 and 2 vary from those of cases 3 and 4, respectively by factors of 10, i.e., a new operation conducted during a peak traffic hour introduces marginal delay costs an order of magnitude larger than those caused by a new operation added at a relatively off-peak hour.

Even in absolute terms the figures for cases 3 and 4 are quite impressive. It is rather remarkable, for instance, that an addition of a total of 24 operations (case 4) to the already existing total of 943 operations, i.e., a 2.5% increase, implies an increase of 75% in total delay costs for the day!

4. DISCUSSION

The approach outlined and illustrated in the earlier sections should prove useful in clarifying a number of issues related to air traffic congestion at airports as well as in exploring several new questions in the future. Congestion dynamics similar to those that we have already observed in our numerical example can be reasonably expected to apply to most major airports, since Logan International is a rather typical example of these transportation centers.

Several points have been illustrated, all with important practical implications for airport congestion. For instance, it has been shown that relatively small improvements in the service rate or a limited reduction in demand can have a significant effect on delays. (The reverse, of course, is also true.) The Federal Aviation Administration (FAA) in the United States and similar agencies in Japan, the Soviet Union, and several West European countries are currently in the process of introducing major innovations in terminal area air traffic control equipment (see Note 11). While the primary purpose of these innovations, at this stage, is to alleviate the workload of air traffic controllers, they can also be expected to make possible marginal increases in airport capacity. From the above, it is clear that even such marginal improvements can provide substantial delay-saving benefits.

Another, perhaps less obvious way of decreasing delay costs is through a modification of the demand pattern during the course of a typical day. The aim here is to "smoothen" the demand profile, to the extent possible. Our example clearly demonstrated the effects of time-variations in demand and placed a price tag on the costs of these variations. A rather crude way to even out the time-distribution of demand is through imposition of upper limits (or "quota") on the number of operations that can be conducted at a given airport during certain periods of a day. This has actually been done beginning in 1968 when the FAA imposed hourly quota on operations at several major airports (see Note 12).

Unfortunately, the quota method is economically inefficient since it simply propagates the status quo instead of actually auctioning off the available time-slots to those flights for which an operation during a peak hour is most valuable. An apparently simple way of implementing a market-like environment is through the use of a time-varying schedule of runway usage fees. (However, one can not emphasize too strongly that these schedules must also be cognizant of other public policy objectives, in addition to that of economic efficiency, with regard to runway use. These objectives are described particularly well by LITTLE AND McLEOD [7]). Although practical experience with such time-varying fees is very limited (see Note 13) due to the reluctance of airport operators to use them, several economists [2] [3] [7] [13] have argued cogently and persuasively in favor of this pricing mechanism in recent years (see Note 14). A major gap, that has severely hampered

the application of the aforementioned body of work, has been the inability to compute congestion costs in an accurate way that reflects the actual time-varying nature of demand instead of being based on the traditional steady-state queuing models. Through the method described here, such items as average delay costs as a function of time and, more importantly, marginal delay costs imposed on other airport users by new flights at different times of the day can be computed. It is expected that future studies of this issue will take advantage of this capability.

On a more general level, the numerical results vividly illustrate two properties of time-dependent queues often alluded to in the literature [9]:

- i) As in the well-known cases of constant demand, so too in the case when demand is time-dependent, there exists a strongly non-linear relationship between the demand rate and the average queue length (and average waiting time). The exact nature of this relationship depends on the time-history of the demand pattern.
- ii) A non-constant time phase exists between the demand pattern and the attendant congestion phenomena.

Finally, it may be pointed out that, while the computer-aided approach to time-dependent queues which was outlined here can, naturally, be used in contexts other than airport congestion, the answer to whether or not the upper and lower bounds are "sufficiently" close will depend on the parameters and requirements of the particular problem at hand.

5. NOTES

1. STEUART[12] has recently cast some doubt on the Poisson assumption by discovering evidence of short-term periodicity ("banks") within hours due to airline schedules. However, STEUART's data come from airport gate occupancies -- and a single specific group of gates at that -- and, therefore, require further exploration.

2. The average service time for take-offs at major airports, i.e., the average time gap between the completion of successive departures from the same runway when there is a departure queue, is of the order of 80 to 100 seconds (35 to 45 take-offs per hour). The corresponding range for landings is more like 90 to 120 seconds.

3. It is simple theoretically, to extend the approach here in such a way that separate queues are maintained for arrivals and for departures with distinct service times for each type of operation and a set of priority rules to determine the order of service. One of the co-authors (ODONI) is presently working on such a problem. However, in practice, severe penalties in terms of program complexity and computational effort have to be paid for differentiating between landings and take-offs.

4. This assumption introduces an additional error in the computation of waiting times for the M/D/k queue. The reason is that those aircraft which arrive at a time when one of the servers is idle will have to wait until the beginning of the next service

period to enter service. This delay, however, is equal to half a service time on the average and thus of the order of a half minute. It applies only to those aircraft finding the system in states $P_0(t)$, $P_1(t)$, ... $P_{k-1}(t)$. Since the delays of interest in practice are those that exceed the 4 or 5 minute level, the error involved is small for practical purposes.

5. For those aircraft that join the queue, the perceived service rate of the k servers at time t is $k \cdot \mu(t)$. For the aircraft that enter the queue when there are $i \geq k$ other aircraft in the system, the number of aircraft before them in the queue is equal to $i - k$. These preceding aircraft must enter service before the last one to arrive (hence the term $(i - k)/(k\mu(t))$). In addition, there is a waiting time until the next service of those already being served is completed. This introduces an additional $1/(k \cdot \mu(t))$ delay -- exactly for the M/M/k queue, an overestimation by $1/(2k \cdot \mu(t))$ for the M/D/k queue. Note that, for the time-dependent queuing system, $W(t)$ is not equal to $Q(t)/\lambda(t)$.

6. For all the runs in the Logan International Airport example that follows, we have used a constant service rate μ throughout the 24-hour period. In any case, for relatively slow-varying $\mu(t)$, expression (5) should be quite accurate.

7. In terms of number of operations, air traffic volume in Boston has remained substantially constant over the period from 1970 to 1974 (the time when this is written). With the energy cri-

sis and the increasing use of large aircraft, it can also be expected that air traffic volume will not change significantly for several more years.

8. Clearly these numbers provide only average estimates. For instance, it is known that the airport has on occasion been able to accommodate up to almost 100 operations/hour. Conversely, in very poor weather conditions, capacity may be reduced all the way to zero (when the airport closes down).

9. We did not attempt to include other costs, such as the costs of lost passenger time in the estimates of delay costs.

10. An accurate estimate of P requires exact knowledge of the traffic mix at an airport, as well as the marginal delay costs in the air (waiting to land) and on the ground (waiting to depart) for each type of aircraft in the mix. No such detailed calculation was performed by the authors. The \$5 figure was selected after a quick review of: i) marginal direct operating costs for various classes of aircraft; and ii) the mix of aircraft using Logan International Airport.

11. There are about 10 commercial airports in the United States which are believed to operate at a congestion level similar to that of Boston. Four other airports (New York's JFK International and La Guardia, Chicago's O'Hare International, and Washington's National Airport) operate with congestion problems which are definitely more severe.

12. Most notable among these innovations is the ARTS III System (Automatic Radar Tracking System) in the United States and similar systems elsewhere. These systems automate to some extent the air traffic control operations near an airport by performing several time-consuming functions that formerly had to be performed manually.

13. For a variety of reasons [11] , congestion problems are much less severe now than in 1968. Only four airports, those mentioned under Note 10, are still operating with a quota system on hourly scheduling.

14. The authors are aware of only two cases in which a time-varying schedule of landing fees has been implemented. By far the most important of the two is the schedule of charges used by the British Airports Authority at Heathrow Airport. This schedule was initiated in April 1972 and, among other things, imposed a surcharge of about \$50 for landings or take-offs between 9 a.m. and 1 p.m. on weekdays during the peak season. This schedule of charges has been revised recently (beginning on April 1, 1974) and the surcharge may now amount up to \$250 for a 747 jet on an intercontinental flight.

The second pertinent case is the imposition in 1969 and thereafter of a \$25 fee on general aviation aircraft using New York's JFK International Airport during selected time periods. (The runway fee at other times is \$5.) Despite the apparent modesty of these amounts, the effects on the distribution of general aviation demand at that airport were dramatic.

15. Current landing fees are computed primarily on the basis of maximum gross take-off weight of the aircraft. There is, however, considerable variation on the exact formula used from place to place. The range of landing fees varies widely, ranging from about \$150 in some U.S. airports to about \$1,500 in most European airports and up to \$4000 in Sydney, Australia -- all for the same aircraft (B747).

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