A Short Note on Modeling Wave Propagation in Media with Multiple Sets of Fractures

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Abstract

Wave propagation and scattering in fractured formations have been modeled with finite-difference programs and the use of equivalent anisotropic media description of discrete fractures. This type of fracture description allows a decomposition of the compliance matrix into two parts: one accounts for the background medium and another accounts for the fractures. The compliance for the fractures themselves can be a sum of compliances of various fracture sets with arbitrary orientations. Non-orthorgonality of the fractures, however, complicates the compliance matrix. At the moment, we can model an orthorhombic medium (9 independent elastic constants) with the two orthogonal fracture sets. However, if the fractures are non-orthogonal, this results in more general anisotropy (monoclinic) for which we need to specify 11 independent parameters.. Theoretical formulation shows that the finite difference program can be extended to simulate wave propagation in monoclinic media with little additional computational and storage cost.

1. Introduction

Fractures control fluid flow in hydrocarbon reservoirs. Knowledge of the direction, separation and dimensions of fractures is very important in developing reservoir production and stimulation programs. For this reason, characterization of fractures has gained increasing interest over the last decade or so. Fractures are often studied using AVO/AVA analysis. In these studies it is assumed that closely spaced, aligned fractures result in effective anisotropy.

More recently, one begins to consider the effect of discrete fractures on recorded signals. In particular, one group at ERL focuses on developing methods that estimate fracture parameters from waves scattered off the fractures (Willis et al., 2004). To understand the response from discrete fractures, Pearce et al. (2003) used an elastic finite difference scheme developed by Lawrence Berkeley National Laboratory (Nihei et al., 2002). This code implements the Coates-Schoenberg equivalent anisotropic medium approach for fractures (Coates and Schoenberg, 1995). We also developed a similar finite difference program (ERLSMP) that has more functions and a user-friendly interface to simulate the wave propagation in fractured media.

The Coates-Schoenberg formulation is quite general as it allows for modeling

multiple intersecting sets of fractures with arbitrary orientations. However, in order to fully benefit from this flexibility, the finite-difference code should allow for fairly arbitrary anisotropy (at least monoclinic).

In this note, we review briefly the method developed by Coates and Schoenberg (1995) and Hood (1991). Then we discuss how to compute the equivalent stiffness matrices of fractured media in various scenarios, and detail the formulation of modeling wave propagation in media with non-orthogonal fracture sets.

2. Modeling Discrete Fractures

For a system of aligned cracks, or fractures, an effective anisotropic medium can be derived when the dominant wavelength is long compared to the typical scales of the fractures (like width and spacing). This long-wavelength equivalent medium theory was used by Schoenberg and Muir (1989) to derive the effective properties of a finely layered medium.

To accurately model the response of seismic waves through boundaries in an elastic solid, Muir et al. (1992) used the idea of Schoenberg and Muir (1989) to represent stiffness (or compliance) coefficients in a grid cell traversed by a fracture that is in some way the 'average' of a stack of parallel layers. Later yet, Coates and Schoenberg (1995) used the same idea together with a linear slip model for the boundary to implement discrete or single fracture(s) in a finite difference scheme. The Coates-Schoenberg method has been used since by several researchers to understand scattering off discrete fractures (see Coates and Schoenberg, 1995, Nihei et al., 2002 and Vlastos et al., 2003, for example).

Pearce et al. (2003) used the same formulation to model the response of a set of parallel vertical fractures as a simplified model of a fractured reservoir. The fracture is modeled as an interface across which the traction is continuous, but the displacement jumps. This can be expressed by the slip interface condition (Coates and Schoenberg, 1995).

As pointed out by Coates and Schoenberg (1995), this approach can be implemented in a fairly straightforward manner when the fracture is aligned with the finite difference grid. However, this is not so simple for fractures making an angle with respect to the FD grid. Additional difficulty occurs when one fracture is cut by another fracture. Nichols et al. (1989) described the problem of modeling rocks with multiple sets of fractures based on the theory outlined by Schoenberg and Muir (1989). They also showed explicitly how to obtain the resultant compliance tensor for an orthogonal fracture set embedded in an isotropic medium. They showed that such a fracture set renders the medium orthorhombic.

Now, in the formulation of Coates and Schoenberg (1995), the fractures are not required to be parallel to the finite difference grid. Even with two sets of parallel fractures, the grid can be rotated, such that the stiffness tensor corresponds to one of an orthorhombic medium. However, if the fractures are not orthogonal, it is impossible to rotate the grid such that the medium becomes orthorhombic. One always has a monoclinic medium, which is characterized by 13 elastic parameters. On account of constraints imposed by the vertical fractures, this number can be reduced to 11 (Schoenberg, 1998).

3. Case studies for Fracture Representation

3.1. A vertical fracture embedded in an isotropic medium with fracture strike parallel to the finite difference grid

For a transverse isotropic medium with a horizontal symmetry axis (HTI), the stiffness matrix can be written as:

$$
C = \begin{bmatrix}
c_{11} & c_{12} & c_{13} & & & \\
c_{12} & c_{11} & c_{13} & & & \\
c_{13} & c_{13} & c_{33} & & & \\
& & & c_{44} & & \\
& & & & c_{44} & \\
& & & & & c_{66}
\end{bmatrix}, \tag{1}
$$

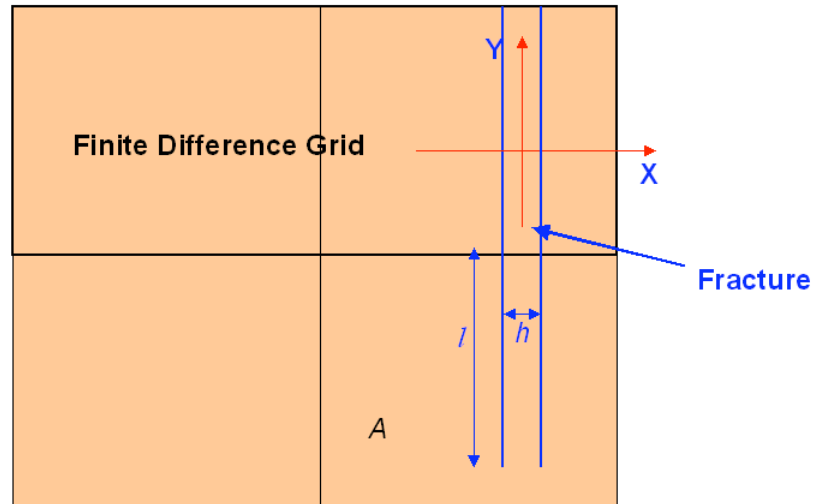where $c_{12} = c_{11} - 2c_{66}$ and only five elastic constants are independent.

Figure 1. A vertical fracture embeds in a homogenous background formation. The normal and strike directions of the fracture are parallel to the finite difference grid.

For vertical discrete fractures embedded in a homogenous background formation (Figure 1), Coates and Schoenberg (1995) showed that the equivalent medium in the fracture coordinate system (x-y system) possesses the property as an HTI medium. Axes x and y are normal and parallel to the fracture strike. Figure 1 shows that in a 2-D finite difference cell with area A, the fracture length is l and the thickness of the fracture is h. In 3-D, A is replaced by V, the volume of the cell, and l is replaced by a, the area of the fault or fracture lying within the 3-D cell volume.
Define

$L = \dfrac{l}{A}$ for 2-D and $L = \dfrac{a}{V}$ for 3-D.

The explicit expression of the equivalent medium stiffness matrix can be written as

$$
C = \begin{bmatrix}
(\lambda+2\mu)(1-r^2\delta_N) & \lambda(1-r\delta_N) & \lambda(1-\delta_N) & & & \\
\lambda(1-r\delta_N) & (\lambda+2\mu)(1-r^2\delta_N) & \lambda(1-\delta_N) & & & \\
\lambda(1-\delta_N) & \lambda(1-\delta_N) & (\lambda+2\mu)(1-\delta_N) & & & \\
& & & \mu(1-\delta_T) & & \\
& & & & \mu(1-\delta_T) & \\
& & & & & \mu\delta_N
\end{bmatrix}
$$

$$(2)$$

where

$$r = \frac{\lambda}{\lambda+2\mu}, \quad \delta_T = \frac{Z_T\mu}{L+Z_T\mu}, \quad \delta_N = \frac{Z_N(\lambda+2\mu)}{L+Z_N(\lambda+2\mu)},$$ and Z_T and Z_N are the tangential

and normal compliances of the fracture.

Comparing to equations (1) and (2), we can see that when the finite difference grids are parallel to the fracture coordinate system, the effect of the fracture on wave propagation can be simulated using an equivalent HTI medium.

3.2. A vertical fracture embedded in an isotropic medium with fracture strike making an angle with the finite difference grid
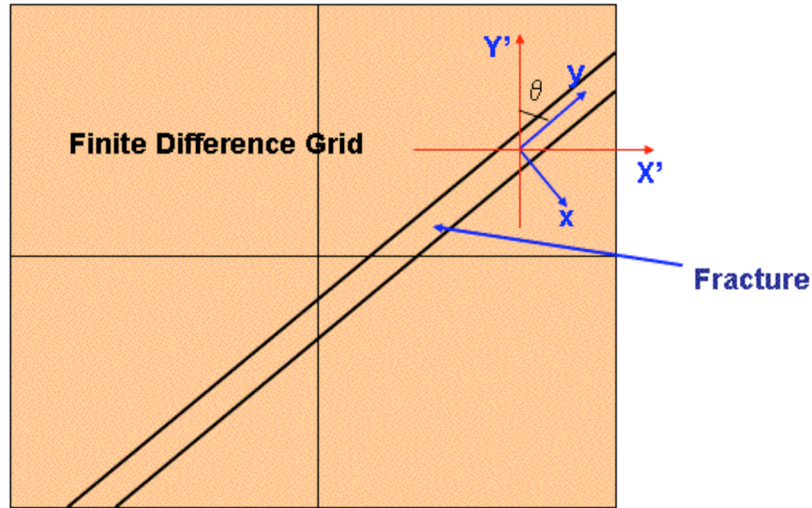

Figure 2. A fracture embeds in a homogeneous background medium. The y axis of the fracture coordinates makes an angle with the y' axis of the finite difference grid coordinates

If the fracture strike makes an angle an angle θ to the grid coordinate system (Figure 2), we need to rotate the stiffness matrix for the HTI medium using the Bond transformation (Auld, 1990). The stiffness matrix C has 13 non-zero elements and shows monoclinic symmetry:

$$C = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & c_{16} \\ c_{12} & c_{22} & c_{23} & 0 & 0 & c_{26} \\ c_{13} & c_{23} & c_{33} & 0 & 0 & c_{36} \\ 0 & 0 & 0 & c_{44} & c_{45} & 0 \\ 0 & 0 & 0 & c_{45} & c_{55} & 0 \\ c_{16} & c_{26} & c_{36} & 0 & 0 & c_{66} \end{bmatrix}. \tag{3}$$

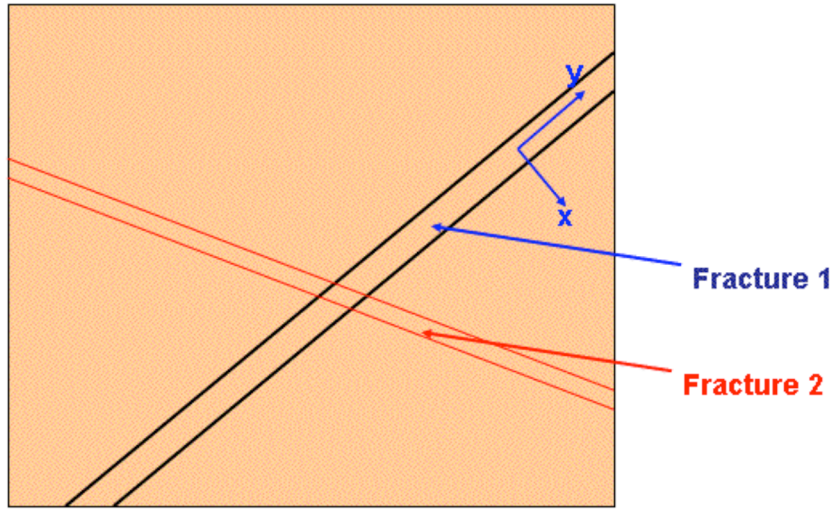## 3.3. Multiple sets of non-orthogonal vertical fractures

Figure 3. Non-orthogonal fractures embed in a homogeneous background medium.

For multiple sets of non-orthogonal vertical fractures (Figure 3), Nichols et al. (1989) show that the compliance matrix for the equivalent medium is

$$S = S_b + \sum_{i=1}^{m} \Delta S_i \tag{4}$$

where m is the number of fracture sets, S_b and ΔS_i are the compliance of background medium and contribution from each fracture set i. It is obvious that the order in which the fractures are included does not affect the final compliance. Assuming each fracture strike forms an angle θ_i to the finite difference grid direction, the Bond transformation matrix can be written as

$$B = \begin{bmatrix} \dfrac{1+\cos 2\theta_i}{2} & \dfrac{1-\cos 2\theta_i}{2} & 0 & 0 & 0 & \sin 2\theta_i \\ -\dfrac{\sin 2\theta_i}{2} & \dfrac{\sin 2\theta_i}{2} & 0 & 0 & 0 & \cos 2\theta_i \\ 0 & 0 & 0 & -\sin\theta_i & -\cos\theta_i & 0 \end{bmatrix}, \tag{5}$$

and

$$\Delta S_i = B^T Z_i B, \tag{6}$$

where in fracture coordinate system, the compliance of each fracture set can be written as

$$Z_i = \begin{bmatrix} Z_{Ni} & & \\ & Z_{Vi} & \\ & & Z_{Hi} \end{bmatrix}, \tag{7}$$

where Z_{Ni}, Z_{Vi}, and Z_{Hi} represent the normal, vertical and horizontal compliance of the fracture, respectively.

Inversion of the compliance matrix gives the stiffness matrix. Such fractured media show monoclinic symmetry. The constitutive equation can be written as:

$$\begin{bmatrix} \tau_{11} \\ \tau_{22} \\ \tau_{33} \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & c_{16} \\ c_{12} & c_{22} & c_{23} & 0 & 0 & c_{26} \\ c_{13} & c_{23} & c_{33} & 0 & 0 & c_{36} \\ 0 & 0 & 0 & c_{44} & c_{45} & 0 \\ 0 & 0 & 0 & c_{45} & c_{55} & 0 \\ c_{16} & c_{26} & c_{36} & 0 & 0 & c_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{bmatrix}. \tag{8}$$

The stiffness matrix has 13 elastic constants, but 11 of them are independent due to the constraints of vertical fractures.

3.4. Two orthogonal sets of fractures embedded in an isotropic background medium

As a special case, an isotropic background medium embedded with two orthogonal sets of fractures can be described by an orthorhombic elastic stiffness matrix. This conclusion can be deduced from the more general case 3 by choosing θ_i be 0 and 90 degrees. The orthorhombic elastic stiffness matrix can be written as follows:

$$C = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \tag{9}$$
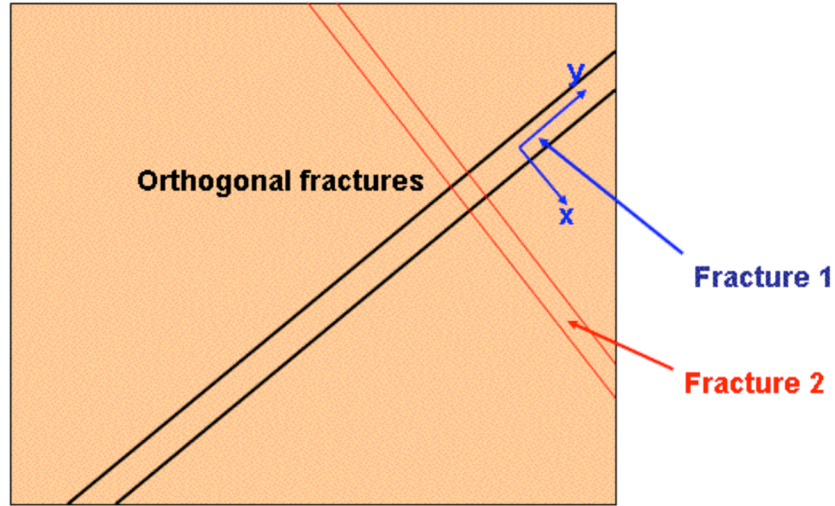
Figure 4. Two orthogonal sets of fractures embedded in an isotropic background medium is equivalent to an orthorhombic medium

3.5. A vertical fracture imbedded in a TI medium

In fact, the background medium can be arbitrary anisotropic for describing the fractures using equivalent media approaches (Nicols et al, 1989, Hood, 1991). A special scenario of interests is a set of vertical fracture imbedded in a layered TI medium, which is equivalent to an orthorhombic medium by Schoenberg and Helbig (1997). We represent a layered medium (transversely isotropic media with a vertical symmetric axis) as:

$$
C = \begin{bmatrix}
c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\
c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\
c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & c_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & c_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & c_{66}
\end{bmatrix}. \tag{10}
$$

Using the just mentioned method, we write the stiffness matrix of the equivalent medium as:

$$C = \begin{bmatrix} c_{11}(1-\delta_N) & c_{12}(1-\delta_N) & c_{13}(1-\delta_N) & 0 & 0 & 0 \\ c_{12}(1-\delta_N) & c_{11}\left(1-\delta_N\dfrac{c_{12}^2}{c_{11}^2}\right) & c_{13}\left(1-\delta_N\dfrac{c_{12}}{c_{11}}\right) & 0 & 0 & 0 \\ c_{13}(1-\delta_N) & c_{13}\left(1-\delta_N\dfrac{c_{12}}{c_{11}}\right) & c_{33}\left(1-\delta_N\dfrac{c_{13}^2}{c_{11}c_{13}}\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44}(1-\delta_V) & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66}(1-\delta_H) \end{bmatrix}$$

$$(11)$$

where

$$\delta_N = \frac{Z_N\rho c_{11}}{1+Z_N\rho c_{11}}, \quad \delta_V = \frac{Z_V\rho c_{44}}{1+Z_V\rho c_{44}}, \quad \text{and} \quad \delta_H = \frac{Z_H\rho c_{66}}{1+Z_H\rho c_{66}}.$$

We assume the x axis of the TI media is normal to the fracture.

4. Finite Difference Implementation

We use the constitutive equation for orthorhombic media for our finite difference program. In implementation, we apply time differentiation to both sides the of constitutive equation, written out explicitly as:

$$\frac{\partial \tau_{xx}}{\partial t} = c_{11}\frac{\partial v_x}{\partial x} + c_{12}\frac{\partial v_y}{\partial y} + c_{13}\frac{\partial v_z}{\partial z},$$

$$\frac{\partial \tau_{yy}}{\partial t} = c_{12}\frac{\partial v_x}{\partial x} + c_{22}\frac{\partial v_y}{\partial y} + c_{23}\frac{\partial v_z}{\partial z},$$

$$\frac{\partial \tau_{zz}}{\partial t} = c_{13}\frac{\partial v_x}{\partial x} + c_{23}\frac{\partial v_y}{\partial y} + c_{33}\frac{\partial v_z}{\partial z}, \qquad (12)$$

$$\frac{\partial \tau_{yz}}{\partial t} = c_{44}\left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right),$$

$$\frac{\partial \tau_{xz}}{\partial t} = c_{55}\left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right),$$

$$\frac{\partial \tau_{xy}}{\partial t} = c_{66}\left(\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right),$$

where τ_{ij} and v_i are elements of the stress tensor and velocity, respectively, and $i, j = x$, y, z.

To model non-orthogonal sets of fractures (monoclinic equivalent media), we need to extend equation (12) to the following:

$$\frac{\partial \tau_{xx}}{\partial t} = c_{11}\frac{\partial v_x}{\partial x} + c_{12}\frac{\partial v_y}{\partial y} + c_{13}\frac{\partial v_z}{\partial z} + c_{16}\left(\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right),$$

$$\frac{\partial \tau_{yy}}{\partial t} = c_{12}\frac{\partial v_x}{\partial x} + c_{22}\frac{\partial v_y}{\partial y} + c_{23}\frac{\partial v_z}{\partial z} + c_{26}\left(\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right),$$

$$\frac{\partial \tau_{zz}}{\partial t} = c_{13}\frac{\partial v_x}{\partial x} + c_{23}\frac{\partial v_y}{\partial y} + c_{33}\frac{\partial v_z}{\partial z} + c_{36}\left(\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right),$$

$$\frac{\partial \tau_{yz}}{\partial t} = c_{44}\left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right) + c_{45}\left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right),$$

$$\frac{\partial \tau_{xz}}{\partial t} = c_{55}\left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right) + c_{45}\left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right),$$

$$\frac{\partial \tau_{xy}}{\partial t} = c_{66}\left(\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right) + c_{16}\frac{\partial v_x}{\partial x} + c_{26}\frac{\partial v_y}{\partial y} + c_{36}\frac{\partial v_z}{\partial z}.$$

(13)

Comparing equation (12) and (13), we find that we can model wave propagation non-orthogonal fracture systems using the finite difference method without technical difficulty. We only need to take into account the additional terms in equation (13). This will only increase the memory storage of the extra elastic constants, but no additional differentiation computation is needed.

5. Conclusion

We summarize and clarify the approaches to represent discrete fractures embedded in background media. Our current finite difference code can model orthogonal sets of fractures, and the algorithm can be efficiently extended to study non-orthogonal sets of fractures. This will result in a small increase in the memory storage and computation.

6. Acknowledgements

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