Hover flight control of helicopter using optimal control theory

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Abstract: *This paper represents the optimal control theory and its application to the full scale helicopters. Generally the control of a helicopter is a hard task, because its system is very nonlinear, coupled and sensitive to the control inputs and external disturbances which might destabilize the system. As a result of these instabilities, it is essential to use a control process that helps to improve the systems performance, confirming stability and robustness. The main objective of this part is to develop a control system design technique using Linear Quadratic Regulator (LQR) to stabilize the helicopter near hover flight. In order to achieve this objective, firstly, the nonlinear model of the helicopter is linearized using small disturbance theory. The linear optimal control theory is applied to the linearized state space model of the helicopter to design the LQR controller. To clarify robustness of the controller, the effects of external wind gusts and mass change are taken into concern. Wind gusts are taken as disturbances in all directions which are simulated as a sine wave. Many simulations were made to validate and verify the response of the linear controller of the helicopter. The results show that the use of an optimal control process as LQR is a good solution for MIMO helicopter system, achieving a good stabilization and refining the final behavior of the helicopter and handling the external wind gusts disturbances as shown in the different simulations.*

Key Words: *helicopter, state space model, full state feedback, linear quadratic regulator, wind gust, robust control.*

1. INTRODUCTION

The helicopter motion has six degrees of freedom: longitudinal motion (up-down, fore-aft), lateral motion (left-right), and angular motion (pitching, rolling, and yawing). These motions are accomplished by collectively varying the pitch of all MR blades, therefore the rotor thrust increased; this control is called (collective pitch).

The blades pitch can be cyclically varied as a sinusoidal function of azimuth which inclines the tip-path-plane to any direction and changes the thrust vector. This control is called (cyclic pitch). Changing pitch of the tail rotor blades collectively lead to change of the tail rotor thrust and hence of the yaw moment. This control is called (pedal or TR collective).

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A helicopter pilot must simultaneously control three forces and moments, hence, the control of a helicopter, is a difficult task indeed. A helicopter pilot typically has at his disposal a cyclic stick to control both fore/aft motions (pitch control) and left/right motion (roll control), a collective lever to control up and down motions (vertical control), and pedals to control left and right yawing motions (yaw control) [1] and [2].

State feedback control, also known as pole placement, involves the relocation of the system poles to the left half s plane in achieving stability in the closed-loop, with the assumption that the system is controllable or observable [3].

Methods based on the linear quadratic regulator (LQR) have been proven to be very efficient and are relatively simple ways used even for a highly nonlinear system like a helicopter**,** also linear quadratic optimal control is that LQR designs are robust with respect to fairly large plant variations [5]. A lot of researchers exploited the LQR theory for helicopter control, see references [6], [7], [8], [9], and [10].

This paper describes the helicopter dynamics model and the stability of the unstable helicopter is discussed. The paper is organized as follows: Section-2 deals with helicopter nonlinear dynamic model; section-3 describes the linear model of helicopter; section-4 is devoted to the controller structure using LQR controller with full state feedback for helicopter stabilization; section-5 describes the helicopter linear model for mass change; section-6 describes the helicopter linear model in presence of gust disturbances; section-7 presents the simulation results of the helicopter response to test the LQR controller performance for mass change and gust disturbances. Main conclusions can be found in section-8.

2. HELICOPTER NONLINEAR DYNAMIC MODEL

Helicopter can be modeled as the combination of many interacting subsystems. By combining aerodynamics of each subsystem, we can get the integrated helicopter dynamic equations**.** The rotor interacts with the air and induces the so called "inflow". The existence of inflow can affect the rotor aerodynamics heavily, so that the inflow effect must be considered; also the rotor flapping dynamic will be considered [11]. The helicopter model used in this paper has a four blades main rotor and a three blade tail rotor with conventional mechanical controls. A model of helicopter normally consists of a total of four inputs: θ_o, θ_{1s}, θ_{1c}, θ_{oT} and nine states**:** u, v, w, φ, θ, ψ, p, q, r. The six degree-of-freedom rigid body motion of helicopter can be described as following: [11].

- **Force equations:**

$$X = m.\left(\dot{U} - R.V + Q.W + g.\sin(\Theta)\right) \tag{1}$$

$$Y = m.\left(\dot{V} - P.W + R.U - g.\sin(\Phi)\cos(\Theta)\right) \tag{2}$$

$$Z = m.\left(\dot{W} - Q.U + P.V - g\,.\cos(\Phi)\cos(\Theta)\right) \tag{3}$$

- **Moment equations:**

$$L = \dot{P}.I_{xx} + Q.R.\left(I_{zz} - I_{yy}\right) - \left(\dot{R}. + P.Q\right).I_{xz} \tag{4}$$

$$M = \dot{Q}.I_{yy} - P.R.(I_{zz} - I_{xx}) + (P^2 - R^2).I_{xz} \tag{5}$$

$$N = \dot{R}.I_{zz} + P.Q.\left(I_{yy} - I_{xx}\right) + \left(Q.R - \dot{P}\right).I_{xz} \tag{6}$$

The three kinematic equations are obtained by relating the three body axes system rates P, Q, and R with the three Euler rates $\dot{\Phi}$, $\dot{\Theta}$, $\dot{\Psi}$ in the earth axes system.

$$\dot{\Phi} = \mathrm{P} + \mathrm{Q}.\sin(\Phi).\tan(\Theta) + \mathrm{R}.\cos(\Phi).\tan(\Theta) \tag{7}$$

$$\dot{\Theta} = \mathrm{Q}.\cos(\Phi) - \mathrm{R}.\sin(\Phi) \tag{8}$$

$$\dot{\Psi} = \mathrm{Q}.\sin(\Phi).\sec(\Theta) - \mathrm{R}.\cos(\Phi).\sec(\Theta) \tag{9}$$

Notice that the equations uses uppercase characters (U, V, W, P, Q, R, Φ, Θ, Ψ) in the nonlinear equations and lowercase characters (u, v, w, p, q, r, φ, θ, ψ) in the linearized equations. The rotor forces and moments are functions of these states and inputs besides the rotor induced velocity v_i and flapping angles β_o, β_{1s}, β_{1c} [11, 14].

Thrust vector in body frame is defined as a function the tilt of tip path plane (β_{1s}, β_{1c}).

By controlling the TPP inclination, the pilot indirectly controls the direction of the propulsion forces [15].

3. LINEAR MODEL OF HELICOPTER

Assume the model to have only small perturbations and applying the small-disturbance theory, we assume that the motion of the helicopter consists of small deviations about a steady flight condition.

The linearized equations of motion for the full 6 DoFs, describing perturbed motion about a general trim condition, can then be written as:

$$\dot{\mathrm{x}} = \mathrm{Ax} + \mathrm{Bu} \tag{10}$$

Equation (10) represents the state space model of the linear model, where A is the stability matrix which represents the characteristic of the system and B is the control matrix [11]. The state space model consists of a 9×9 state matrix A and a 9×4 input matrix B as shown in equations (11)

$$[\dot{\mathrm{x}}]^{9\times1} = \begin{bmatrix} \mathrm{A}^{4\times4}_{\text{longitudinal}} & \mathrm{A}^{4\times5}_{\text{cross coupling}} \\ \mathrm{A}^{5\times4}_{\text{cross coupling}} & \mathrm{A}^{5\times5}_{\text{lateral}} \end{bmatrix}^{9\times9} .\,[\mathrm{x}]^{9\times1} + \begin{bmatrix} \mathrm{B}^{4\times4}_{\text{longitudinal}} \\ \mathrm{B}^{5\times4}_{\text{lateral}} \end{bmatrix}^{9\times4} .\,[\mathrm{u}]^{4\times1} \tag{11}$$

The output matrix is:

$$[\mathrm{y}]^{9\times1} = [\mathrm{I}]^{9\times9}.[\mathrm{x}]^{9\times1} + [0]^{9\times4}.[\mathrm{u}]^{4\times1} \tag{12}$$

Where x and u are defined as:

$$[\mathrm{x}] = [\mathrm{u\ w\ q\ \theta\ v\ p\ \emptyset\ r\ \psi}]^{T} \tag{13}$$

$$[\mathrm{u}] = [\theta_{o}\ \theta_{1s}\ \theta_{1c}\ \theta_{oT}\ ]^{\mathrm{T}} \tag{14}$$

Finally, there are 36 stability derivatives and 24 control derivatives in the standard 6 DoFs set. These derivatives are derived and listed in [11] and [14].

Stability and control derivatives of the example helicopter are calculated near hover according to [14].

The derivatives are inserted in matrices A and B and G. The eigenvalues of stability matrix, A, are shown in Table.1.

From this table, it is shown that the system has complex poles with positive real parts which indicate that the system is unstable.

Table 1. Uncontrolled Helicopter Poles

Poles
0
-5.56
-0.962
0.0721+0.362i
0.0721−0.362i
-0.439
-0.24
-0.00896+0.0591i
-0.00896+0.0591i

4. LQR CONTROLLER DESIGN

The Linear Quadratic Regulator (LQR) was implemented to control the helicopter. It is one of the most commonly used methods to design full state feedback control for linear systems which seeks a solution for the linear full state feedback problem [19]. Consider the linear time-invariant system:

$$\dot{x} = Ax + Bu,\ y = Cx + Du \tag{15}$$

The state feedback control law has the form

$$u(t) = -Kx(t) \tag{16}$$

As all of the states are measured, the resulting feedback system is called a full state feedback system [16]. An equation for the closed loop system is derived as follow:

$$\dot{x}(t) = (A - BK)x(t) \tag{17}$$

This system is stable if and only if the system matrix, A-BK, has all its eigenvalues in the left half plane. The closed loop system with full state feedback is shown in Figure 1.

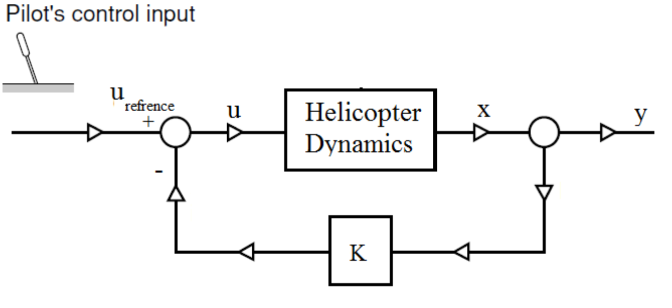


Fig. 1: Full state feedback of helicopter [11]

Where K is the feedback gain matrix which minimize the following performance index,

$$J = \frac{1}{2}\int_0^\infty (x^T Q x + u^T R u)\ dt \tag{18}$$

Q and R are the weighting matrices for states and inputs, respectively. Q must be positive semi definite and R positive definite. Q and R matrices contain all zeroes except along the main diagonal. Considering this assumption that the system is controllable and observable, then the optimal Feedback matrix K that minimize the performance index (J) is given by equation (19). [17] and [18]

$$K = R^{-1}BP \tag{19}$$

Where the matrix P is the solution of the Riccati equation and after solving Equation (20), matrix P will be used to find feedback gain matrix K.

$$A^TP + PA - PBR^{-1}B^TP + Q = 0 \tag{20}$$

Finally the optimal performance index will be calculated using P.

$$J^* = \frac{1}{2}X_0^TPX_0 \tag{21}$$

In [8], the theory of LQR controller design has been investigated and a different approach based on Bryson's rule [19] has been adopted to select the weighting matrices. Bryson's rule will be used in this paper, because it depends on the response and actuators constraints in the selection of Q and R weighting parameters.

R matrix is shown in equation (22).

$$R = \begin{bmatrix} U_1^{-2} & 0 & 0 & 0 \\ 0 & U_2^{-2} & 0 & 0 \\ 0 & 0 & U_3^{-2} & 0 \\ 0 & 0 & 0 & U_4^{-2} \end{bmatrix} \tag{22}$$

According to Bryson's rule, U is the maximum acceptable input actuator.

The states matrix Q can be found using Bryson's rule as shown in equation (23).

$$Q = \begin{bmatrix} Q_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Q_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Q_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Q_{77} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q_{88} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q_{99} \end{bmatrix} \tag{23}$$

According to Bryson's rule:

$$Q_{ii} = \frac{1}{|x_i|^2} \tag{24}$$

Where x_i is the maximum acceptable value for each state.

The maximum accepted control inputs and maximum states for the helicopter in this paper are shown in Table 2. and Table 3.

Table 2. Helicopter Actuators Range

Actuator Input	Range (degree)
θ_0	19
θ_{1s}	± 20
θ_{1c}	± 10.5
θ_{0T}	-15 to + 27

Table 3. LQR Weighting States

State	Maximum
q	12 deg/sec (0.209 rad/sec)
Θ, ψ	± 30 degree
p	20 deg/sec (0.349 rad/sec)
Φ	± 60 degree
r	10 deg/sec (0.1745 rad/sec)

The weighting of states values are selected according to the aeronautical design standard (ADS-33 Level 1) (military aircraft). [11]. The weighting parameters of linear velocities are selected to give a resonable results near hover flight at constant altitude. For different initial longitudinal and lateral velocities conditions, the time responses of the states and control inputs are used to demonstrate the controller performance and closed-loop system behavior that are achieved. The initial conditions are considered to be up to ±10 ft/sec for both longitudinal and lateral velocities. The weighting matrices Q and R are selected to be for the initial speed range of ±10 ft/sec as below.

$$Q=\mathrm{diag}(0.0156,\ \mathbf{10^6},\ 22.8,\ 3.6481,\ 0.0091,\ 8.2082,\ 0.912,\ 32.8329,\ 3.6481)$$

$$R=\mathrm{diag}(9.0950,\ 8.2082,\ 29.7804,\ 4.5038)$$

The resulted feedback gains from LQR controller could obtain the stable poles as shown in Table 4.

Table 4. Controlled Helicopter Poles

Pole
-780
-30.4
-11.3
-4.78
-0.837
-0.525– 0.456i
-0.525– 0.456i
-0.317
-0.075

5. TEST OF LQR CONTROLLER FOR MASS CHANGE

One of the assumptions made for modelling the helicopter was to treat the gross weight (mass) of the helicopter as constant disregarding fuel consumption. Besides fuel consumption the helicopter might also have to offload a payload in hover in midair. This could be in a military configuration offloading troopers or weapons or it could be in fire-fighter configuration offloading water or chemicals. Helicopter in this paper has a gross weight of 20000 lb, fuel weight of 3000 lb which it represents 15% of the gross weight. The minimum operating empty weight is 10700 lb which it represents about 53.5 of the gross weight.

It has been chosen here to focus on the change in mass and inertia and to disregard the change in CG. The estimation of helicopter inertia is a complex problem, so an approximate relation is suggested here; this relation depends on the radius of gyration for each axis. The radius of gyration ρ_g can be calculated through the relations in equation (25)

$$\rho_{g_{xx}} = \sqrt{\frac{I_{xx}}{m}} \quad , \rho_{g_{yy}} = \sqrt{\frac{I_{yy}}{m}} \quad \text{and} \quad \rho_{g_{zz}} = \sqrt{\frac{I_{zz}}{m}} \tag{25}$$

The radius of gyration is calculated for each axis and assumed to be constant, and then the new moment of inertia for any change in mass can be calculated.

For robust control the state space model is extended with two uncertainty matrices $\bar{A}$ and $\bar{B}$ as seen in equation (26)

$$\dot{x} = (A + \varepsilon\bar{A})x + (B + \varepsilon\bar{B})u \tag{26}$$

Where ε is the uncertainty parameter for the mass change and it can be defined as equation (27)

$$\varepsilon = \frac{\text{Gross weight}}{\text{New weight}} - 1 \tag{27}$$

The uncertainty matrix $\bar{A}$ will be written as shown in equation (28)

$$\bar{A} = \begin{bmatrix} \frac{X_u}{m} & \frac{X_w}{m} & \frac{X_q}{m} & 0 & \frac{X_v}{m} & \frac{X_p}{m} & 0 & \frac{X_r}{m} & 0 \\ \frac{Z_u}{m} & \frac{Z_w}{m} & \frac{Z_q}{m} & 0 & \frac{Z_v}{m} & \frac{Z_p}{m} & 0 & \frac{Z_r}{m} & 0 \\ \frac{M_u}{I_{yy}} & \frac{M_w}{I_{yy}} & \frac{M_q}{I_{yy}} & 0 & \frac{M_v}{I_{yy}} & \frac{M_P}{I_{yy}} & 0 & \frac{M_r}{I_{yy}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{Y_u}{m} & \frac{Y_w}{m} & \frac{Y_q}{m} & 0 & \frac{Y_v}{m} & \frac{Y_p}{m} & 0 & \frac{Y_r}{m} & 0 \\ L'_u & L'_w & L'_q & 0 & L'_v & L'_p & 0 & L'_r & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ N'_u & N'_w & N'_q & 0 & N'_v & N'_p & 0 & N'_r & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{28}$$

6. TEST OF LQR FOR ATMOSPHERIC DISTURBANCE

The equations of motion modified to account for atmospheric disturbances. The aerodynamic forces and moments acting on the helicopter depend on the relative motion of the helicopter to the atmosphere [13]. For example; the modification of X-force equation will be as shown in equation (29)

$$\begin{aligned} \frac{X_{GM}}{m} = \frac{X_u}{m}(u - u_g) + \frac{X_w}{m}(w - w_g) + \frac{X_q}{m} q + \frac{X_v}{m}(v - v_g) + \frac{X_p}{m} p + \frac{X_r}{m} r + \frac{X_{\theta_o}}{m}\theta_o + \frac{X_{\theta_{1s}}}{m}\theta_{1s} \\ + \frac{X_{\theta_{1c}}}{m}\theta_{1c} + \frac{X_{\theta_{oT}}}{m}\theta_{oT} \end{aligned} \tag{29}$$

The new model can be written in state space model as equation (30) or (31).

$$\dot{x} = Ax + Bu + G\xi \tag{30}$$

The state space model can be written as:

$$\dot{x} = Ax + \hat{B}\begin{bmatrix} u \\ \xi \end{bmatrix} \tag{31}$$

Where $\hat{B}$ is the new input matrix including the control and gust matrix together as shown in equation (32).

$$\hat{B} = [B \quad G] \tag{32}$$

The full input matrix, $\hat{B}$, and gust input vector, ξ , are shown in equations (33) and (34)

$$\hat{B} = \begin{bmatrix} \frac{X_{\theta_o}}{m} & \frac{X_{\theta_{1s}}}{m} & \frac{X_{\theta_{1c}}}{m} & \frac{X_{\theta_{oT}}}{m} & -\frac{X_u}{m} & -\frac{X_w}{m} & -\frac{X_v}{m} \\ \frac{Z_{\theta_o}}{m} & \frac{Z_{\theta_{1s}}}{m} & \frac{Z_{\theta_{1c}}}{m} & \frac{Z_{\theta_{oT}}}{m} & -\frac{Z_u}{m} & -\frac{Z_w}{m} & -\frac{Z_v}{m} \\ \frac{M_{\theta_o}}{I_{yy}} & \frac{M_{\theta_{1s}}}{I_{yy}} & \frac{M_{\theta_{1c}}}{I_{yy}} & \frac{M_{\theta_{oT}}}{I_{yy}} & -\frac{M_u}{I_{yy}} & -\frac{M_w}{I_{yy}} & -\frac{M_v}{I_{yy}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{Y_{\theta_o}}{m} & \frac{Y_{\theta_{1s}}}{m} & \frac{Y_{\theta_{1c}}}{m} & \frac{Y_{\theta_{oT}}}{m} & -\frac{Y_u}{m} & -\frac{Y_w}{m} & -\frac{Y_v}{m} \\ L'_{\theta_o} & L'_{\theta_{1s}} & L'_{\theta_{1c}} & L'_{\theta_{oT}} & -L'_u & -L'_w & -L'_v \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ N'_{\theta_o} & N'_{\theta_{1s}} & N'_{\theta_{1c}} & N'_{\theta_{oT}} & -N'_u & -N'_w & -N'_v \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{33}$$

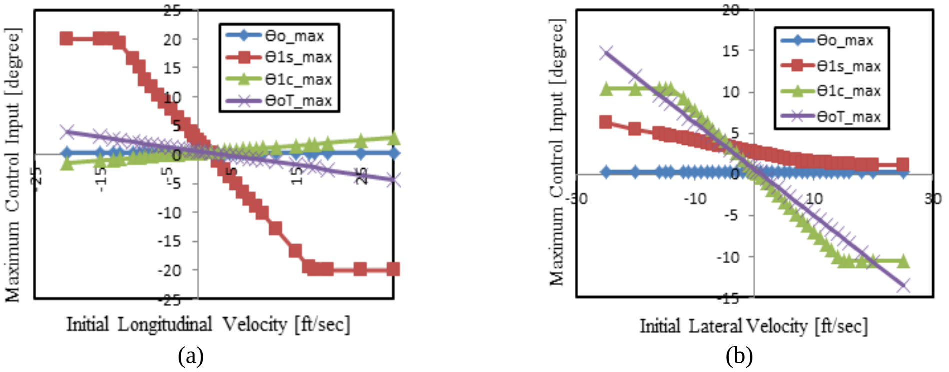
$$\xi = \begin{bmatrix} u_g \\ w_g \\ v_g \end{bmatrix} \tag{34}$$

For robust control the state space model including the change in mass and gust effect is extended with two uncertainty matrices $\bar{A}$ and $\grave{B}$ as seen in equation (35).

$$\dot{x} = (A + \varepsilon\bar{A})x + (\hat{B} + \varepsilon\grave{B})\begin{bmatrix} u \\ \xi \end{bmatrix} \tag{35}$$

7. SIMULATION RESULTS

In this section, the simulation results are presented to investigate the performance of the LQR controller for helicopter response in a gusty environment. Control inputs are mechanically and aerodynamically limited to produce always possible required lift forces and limited to not exceed the stall limits for rotor discs. So control law can be limited with a saturation function to illustrate the boundary of the control mechanism. Firstly, the variation of maximum control inputs needed at each initial condition is obtained in Fig. 2.



(a) (b)

Fig. 2 Variation of control inputs maximum values with the initial velocities

Fig. 2 shows that the actuators inputs are more affected by the initial side velocities except the main rotor collective, θ_o which remains constant to maintain the helicopter altitude in hover. Finally, it can be said that longitudinal stabilization requires less control effort than lateral one.

A simulation study is performed for the helicopter in this paper to test the effect of sinusoidal gusts with different frequencies and different amplitudes from x, y and z directions. It was found that the low gust frequencies have a significant influence on the steady state amplitudes of the lateral states response (v, p, ∅, r, ψ).

All results are accepted for near hover flight for all gusts and the vertical gust has the significant influence on steady state amplitudes in case of low frequencies. In fact, vertical wind gusts can be neglected compared with horizontal gusts since the main factor influencing thrust in hover comes from the horizontal gusts [20].

Fig. 3 through Fig. 5 give a comparison between the effect of three different gust frequencies ($\omega = 0.3, 0.7$ and 1.5) rad/sec from x, y and z direction on the roll angle (∅).

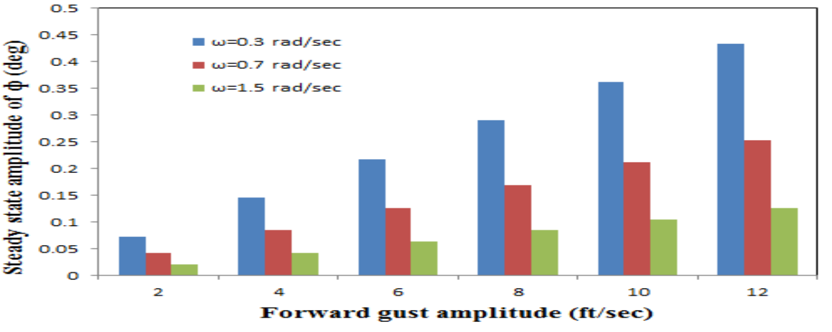


Fig. 3 Effect of forward gust on steady state amplitude of roll angle

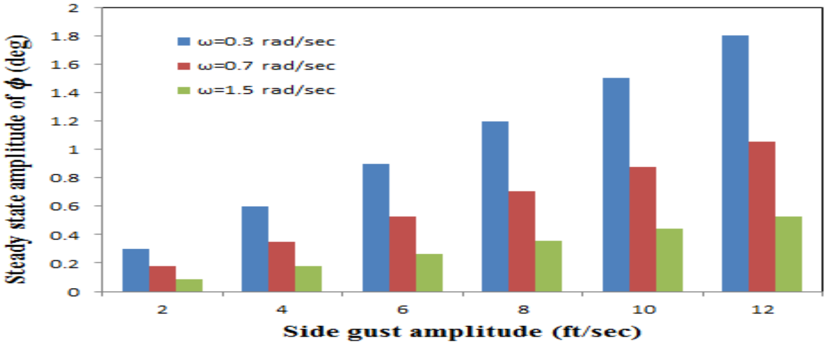


Fig. 4 Effect of side gust on steady state amplitude of roll angle

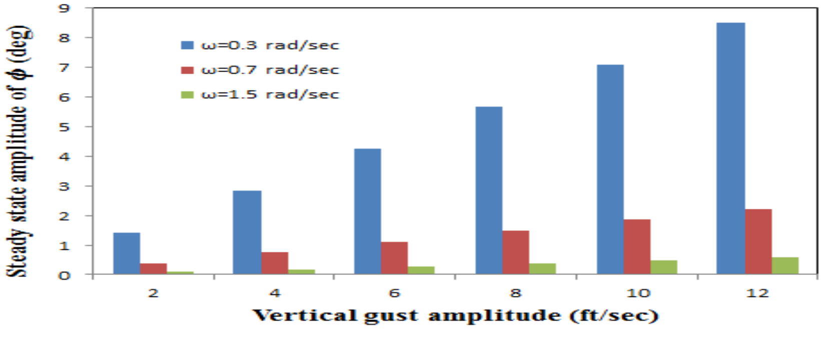


Fig. 5 Effect of vertical gust on steady state amplitude of roll angle

From Fig. 3, Fig. 4 and Fig. 5, it is clear that the vertical gust has the largest influence on the steady state amplitude of roll angle response in case of low frequencies. Also the steady state amplitude increases with gust amplitude in all gusts.

Fig. 5 through Fig. 8 give a comparison between the effect of three different gust frequencies ($\omega = 0.3, 0.7$ and 1.5) rad/sec from x, y and z direction on side velocity (v).

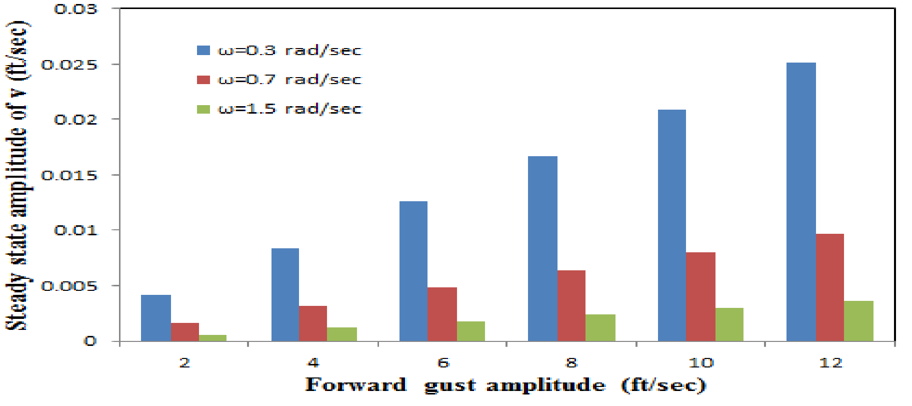


Fig. 6 Effect of forward gust on steady state amplitude of side velocity

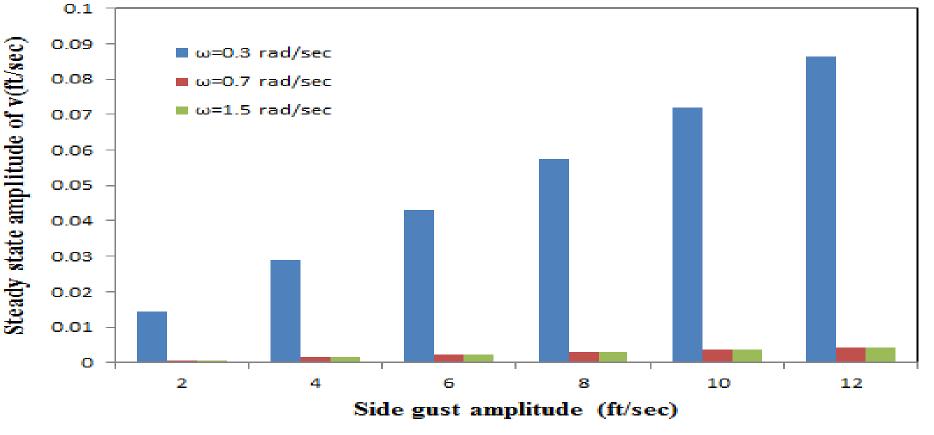


Fig. 7 Effect of side gust on steady state amplitude of side velocity

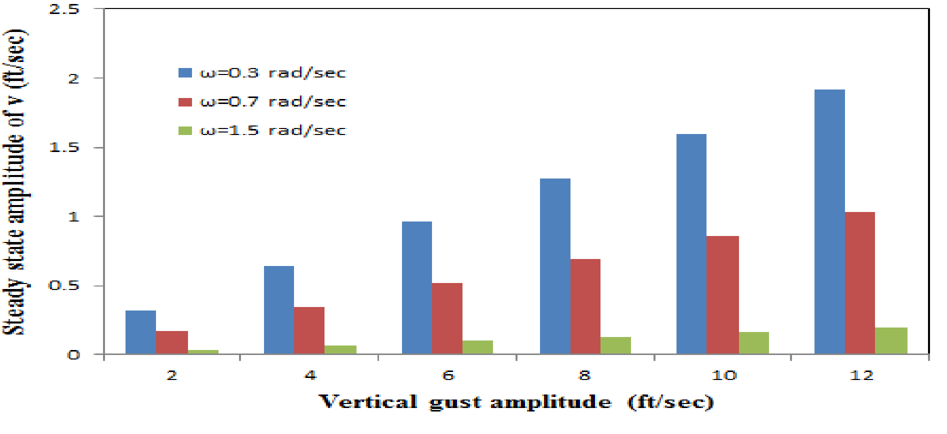


Fig. 8 Effect of vertical gust on steady state amplitude of side velocity

From Fig. 6, Fig. 7 and Fig. 8, it is clear that the vertical gust has the largest influence on the steady state amplitude of side velocity response in case of low frequencies. Also the steady state amplitude increases with gust amplitude in all gusts.

7.1 Influence of mass change on helicopter response

The purpose of this section is to view how the LQR controller handles the influence of the mass changes on the helicopter. The test was carried out using four uncertainties as shown in Table 5.

Table 5. Different uncertainties for helicopter weight

Type	Helicopter weight (lb)	ε
Gross weight	20000	0
50 % drop in fuel	18500	0.081
Minimum operating weight	10700	0.869
Extra weight of 1000 lb	21000	−0.0476

Two similar gust disturbances were applied on the four systems to view their behaviors. The test was performed with a sine wave disturbance in y and z directions.

Fig. 9 through Fig. 18 show the helicopter response in the four tests using 3 ft/sec initial forward velocity condition and gust disturbance of 1 rad/sec frequency and 5 ft/sec amplitude.

The figures arranged such that compare the same states to show the effect of gust type on the four uncertainties.

- **Influence of mass change on linear velocities:**

The influence of mass change on helicpter linear velocities is shown in Fig. 9 through Fig. 12.

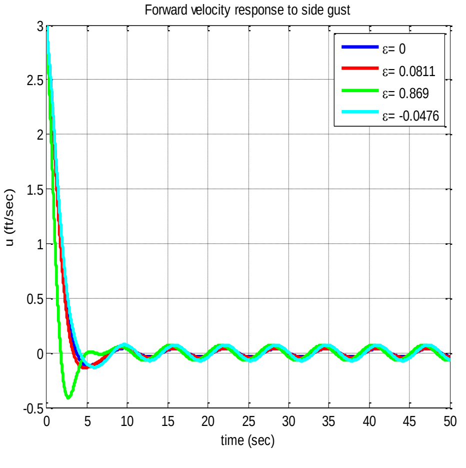


Fig. 9 Influence of mass change on forward velocity in case of side gust

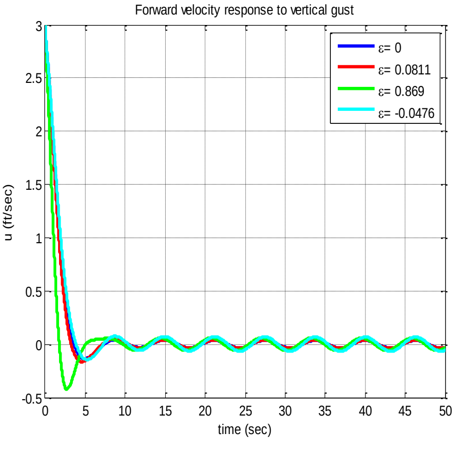


Fig. 10 Influence of mass change on forward velocity in case of vertical gust

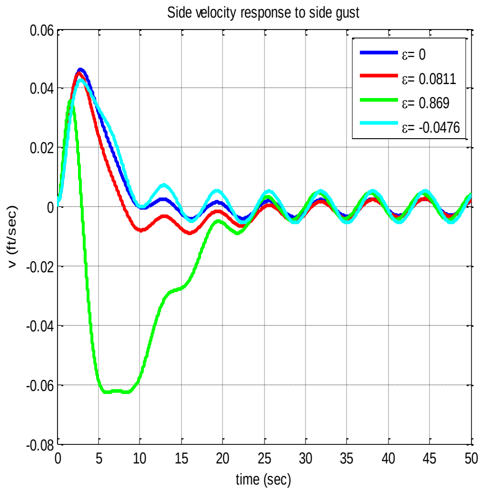


Fig. 11 Influence of mass change on side velocity in case of side gust

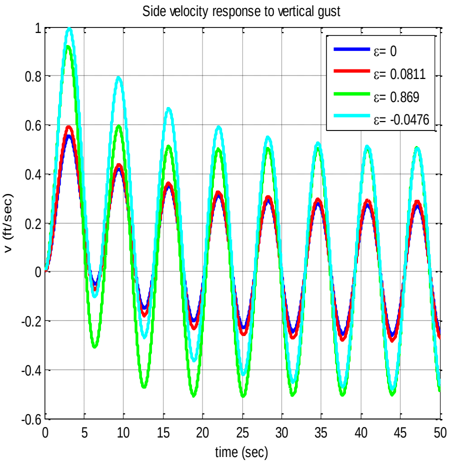


Fig. 12 Influence of mass change on side velocity in case of vertical gust

From Fig. 9 through Fig. 12, it is clear that the LQR controller gives accepted results for the two case of gust.

The most significant influence is on the side velocity in case of minimum operating weight and extra weight in the presence of vertical gust.

In case of side gust, the side velocity response has a largest impact in transient region in case of minimum operating weight.

- **Influence of mass change on Euler angles:**

The influence of mass change on Euler angles is shown in Fig. 13 through Fig. 18.

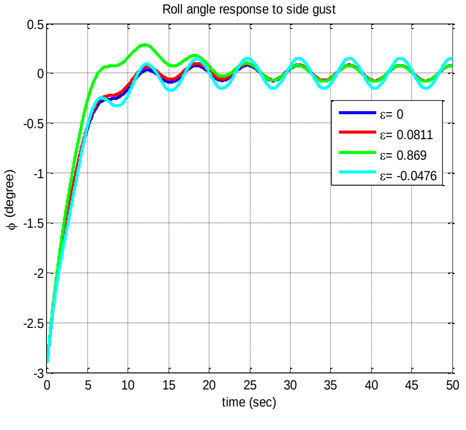


Fig. 13 Influence of mass change on roll angle in case of side gust

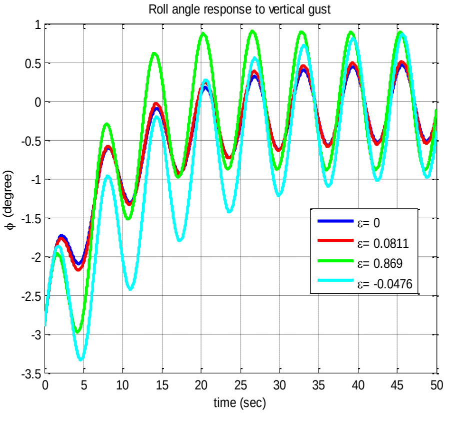


Fig. 14 Influence of mass change on roll angle in case of vertical gust

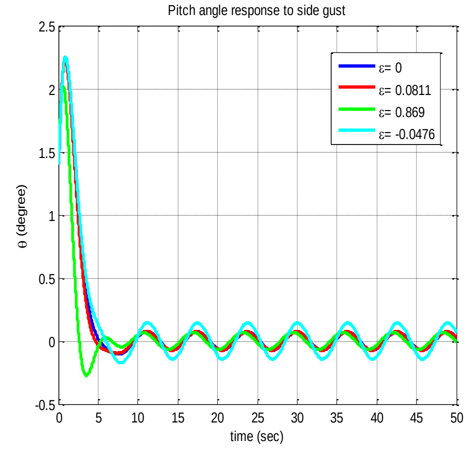


Fig. 15 Influence of mass change on pitch angle in case of side gust

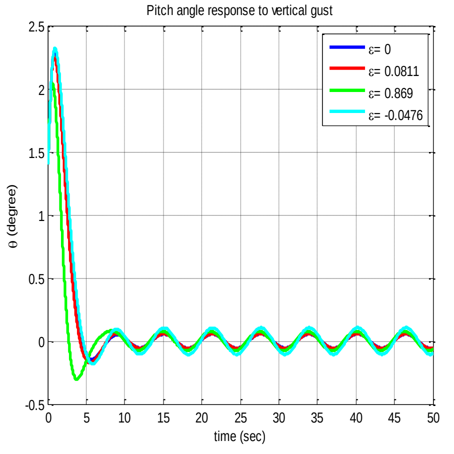


Fig. 16 Influence of mass change on pitch angle in case of vertical gust

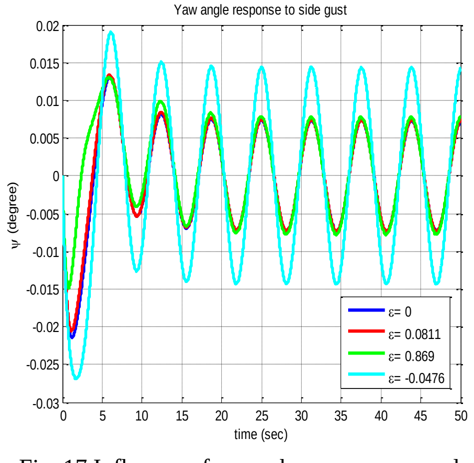


Fig. 17 Influence of mass change on yaw angle in case of side gust

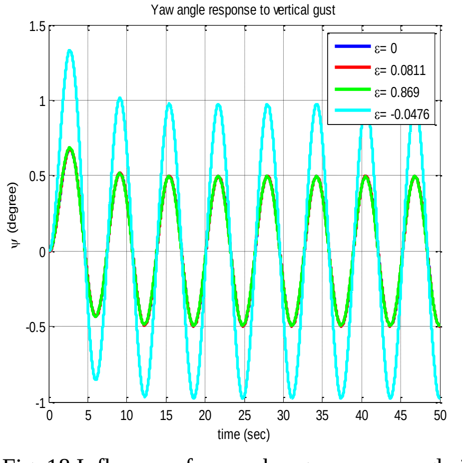


Fig. 18 Influence of mass change on yaw angle in case of vertical gust

From Fig. 13 through Fig. 18, it is clear that the LQR controller gives accepted results for the two cases of gust. The most significant influence is on roll and yaw angles in case of minimum operating weight and extra weight in the presence of vertical gust.

8. CONCLUSIONS

In this paper, the optimal control theory has been applied to control the dynamics of the helicopter in the presence of external sinusoidal wind gusts. The LQR controller is designed based on the linear model of helicopter. The simulation results presented clearly show that the LQR controller gives accepted results for the helicopter dynamics in high frequencies gusts. In Low frequencies the LQR controller performs good for all dynamics except in case of vertical gust which has more influence to the helicopter lateral dynamics. The robustness of the controller was performed for mass change and the controller gives accepted results for all mass changes. Finally the use of LQR is a good solution for MIMO systems. It gives a good stabilization and improving the behavior of the helicopter as it was shown in the different simulations.

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