NATURAL RESOURCES IN A COMPETITIVE ECONOMY

by

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ABSTRACT

Both a naive logistic model and a sophisticated age class model are used to investigate competitive equilibria for a renewable natural resource. A rational expectations model is presented and tested for Douglas Fir stumpage supply. Forest Service policy is discussed: It is concluded that explicit valuation of standing timber stock for other than lumber purposes leads to a coherent forest policy consistent with the Multiple-Use Sustained Yield Act of 1960.

Public ownership and government regulation are prevalent in the renewable natural resource field. Fish, lobsters and other valuable marine life are protected by a maze of catch limitations. Public ownership of timber lands is not uncommon in many countries. In the U.S. about one-third of all timber land is publicly owned. Government ownership and regulation makes the government responsible for deciding how much of these resources are used in the present and how much will be available in the future. Resources have two sources of value: the end products they produce are consumer (fish cakes, wood frame houses, lobster dinners) and the stock of the resource provides externalities by its very existence (forests provide recreation, fish provide food for other fish, etc.). The price consumers are willing to pay for a resource is an adequate measure of the resource's private value. The public value is admittedly much harder to measure. (What would the last Dodo bird or carrier pigeon have been worth?)

The United States Forest Service appears to make its harvest decisions without placing any weight on prices. Below it is shown that a policy very like the one the Forest Service actually follows can be arrived at by maximizing the present discounted value of the timber stock for public (recreation of wildlife) purposes. Further, the analysis of present discounted value gives a proper criterion for judging forest improvement projects while the present analytical framework (Maximum Sustainable Yield) does not. The difficulty with present discounted value is that the future prices are unknown. Much of this thesis is devoted to building a rational expectations model to predict future prices. An outline of the sections of this paper follow.

Section 1: Discusses the difference in growth functions between fish and trees. Develops the formula for an optimal policy for a present discounted value maximizing producer facing constant prices.

Section 2: Uses a simple logistic model to discuss a renewable resource in a simple competitive world. The inclusion of a demand curve in the model causes there to be a smooth flow of the resource. If the initial stock of the resource is large compared to the eventual steady state stock, then the stock is slowly reduced to the steady state stock and, at the same time, prices are rising. The model is extended to include a nonrenewable resource; it is seen that the inclusion of the nonrenewable resource, in the case cited, can cause the extinction of the renewable one.

Section 3: Develops a rational expectations model for trees. It is seen that finding the expected equilibrium is equivalent to maximizing a consumer surplus expression.

Section 4: The model of section three is estimated.

Section 5: Forest Service policy is discussed in light of the model above. It is found that a small value to standing stock will justify the difference between competitive action and current government policy.

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Section 1

The Effect of Growth Specification on Optimal Policy with Constant Price

Natural resources are usually partitioned by their ability to reproduce. Those resources that have no sex life are called non-renewable, while those that reproduce themselves (or are somehow regenerated) are called renewable. The economic analysis of non-renewable resources is particularly simple. Under simple assumptions about extraction cost, a necessary condition for the resource to be made available in every time period is that MR - MC increases at the producer's rate of time discount. Given that prices must rise exponentially, demand equations, and the amount of the resource involved, it is not difficult to find the quantity sold in every period.

The foremost example of a renewable resource is fish. Here a growth law is postulated ($\dot{x} = f(x) - c$) where x is the stock of fish and c is the catch. (Noitce that a non-renewable resource is just one for which $f(x) \equiv 0$.) The simplest case is where x is one-dimensional and f is concave. It is easy to show that faced with an exponentially growing price, a present value maximizing fishery owner will adopt a bang-bang control -- he will reduce (increase) his stock to some optimal level and maintain it there.[1] It is an implication of a logistic type growth law that only part of a homogenous population will be harvested.

Section 1.1

Age Class:

A simple model of a natural resource, $\dot{x} = f(x) - c$, says that growth is a function of the biomass (or possibly number of individuals) of the population. This model will best describe populations where the age distributions of the individuals is irrelevant for population growth. Bacteria, locusts or short-lived fish are examples of populations well-described by a simple biomass equation. Additionally, any population harvested in such a way as to keep the age distribution constant will be well described by a simple biomass equation. Since trees, and for that matter long-lived fish, grow at different rates at different ages, it is necessary to keep track of each age class.[2] Colin Clark works out the optimal harvest policy for fish and finds the present discounted value maximizing policy is likely to involve the use of a fishing net with a mesh that catches large fish but not small fish. When there are high fixed costs to fishery, Clark finds the optimal policy is to catch "all" the fish in one year and not return until the fishery has rebuilt its stock. It would seem that Clark's age class model could be carried over to trees.

This is not so.

The crucial difference between fish and trees (besides flavor) is their sex lives, a stylized version of which follows. Fish are recruited (which means survive until some critical age) in constant numbers per year regardless of conditions. Mortality and slowed growth of individuals account for the familiar logistic shape of the function "f" in the biomass equation $\dot{x} = f(x) - c$.

That is, more fish mean less food (or oxygen or hiding places) per capita. In turn, less food means less growth. The primary restraint on growth is the crowding effect.

Trees have a different history: new trees sprout when old ones are removed. When Douglas firs are young, they are packed tightly together. As they age, the hardier and faster growing trees shade out the less hardy trees. This is the same crowding effect fish exhibit. The difference between fish and trees is that the harvest of fish alleviates the crowding while the harvest of trees may or may not alleviate their crowding. Trees are subject to two kinds of harvest: thinning and cutting. Thinning -- and other methods of timber stand improvement -- is the removal of only some of the trees on a given plot. The object of thinning is to give the remaining trees more light and hasten their growth. Sometimes the removed timber is commercially valuable and sometimes it is not. Cutting a stand of trees does nothing for the remaining trees because there are no remaining trees. Cutting a stand replaces old growth with seedlings. Thus cutting and thinning have different effects. Thinning is akin to the harvest of fish while we take cutting as our model of the harvest of trees. Thinning is to be considered as a suboptimization problem that has already been carried out and is embodied in the relevant growth functions. This assumption makes attention to age class in trees of paramount importance. Since trees of different age classes grow at different rates, a simple biomass equation will not be able to distinguish between a young and old stand of equal volume. Another unpleasant effect of the biomass formulation is that it leads to a policy of cutting only some trees of an even-aged stand on the false assumption the rest will grow faster. In short, one must consider the age class problem for trees.

While it is an externality that complicates the optimal policy of a fishery, it is the simple economic concept of rent that makes it difficult to find the optimal policy for forest management. The proceeds of the sale of a natural resource are attributable as a rent to the ownership of the land or sea that supports the resource. Since no one owns the sea, fishermen do not worry about the "rent" a fish should be paying. This is why fish are called a common property resource.

Much is said about the common property nature of fish. Spence's study of blue whales, Reddy's model of lobsters, and Vernon Smith's[4] theoretical article are all recent examples of work that shows lack of private ownership makes fishing inefficient.

Although the rent to growing trees is appropriated by the owners of the land, rent is still an important consideration in the growing of trees. Old trees occupy space that would otherwise be devoted to young trees (or ranch houses, shopping centers, etc.). The best alternative use of a land parcel determines the rent of the parcel. Should an old tree no longer be able to pay the competitive rent for the land on which it stands, then a profit maximizing business would cut the tree down. Rent is one of the determinants of optimal rotation age. The effects of rent are most clearly seen in the sections on forestry in a constant price world, and in the linear forest model.

Footnotes

1 Colin Clark, "Mathematical Problems in Biological Conservation," American Math. Monthly (forthcoming). Clark's article contains an extensive bibliography.

2 Colin Clark, Gordon Edwards, and Michael Friedlaender, "Beverton-Holt Model of a Commercial Fishery: Optimal Dynamics," Journal Fisheries Research Board of Canada 30, No. 11 1973.

3 Bernard J. Reddy, "The New England Lobster Pot Fishery: An Empirical Study," Massachusetts Institute of Technology, 1975 (unpublished).

4 Vernon Smith, "General Equilibrum with a Replenishable Natural Resource," Review of Economic Studies, 1974 Symposium, p. 105.

Section 1.2

Forestry In A World With Constant Price

Although the profit maximizing policy for managing a forest in a regime of constant price expectations has been known since 1849 and is a straightforward exercise in dynamic programming, many famous economists have published incorrect solutions.[1] The source of many of their errors is the concept of ground rent. The model that follows shows the role of ground rent in determining when to cut trees. The model also presents the technology of a forest economy in its simplest form. Later we join this technology with a market mechanism.

Before continuing any further it is necessary to explain the problem and the conventions used (assumptions made). By a forest I mean a plot of ground that will support the growth of trees, and any trees on that ground. When trees are cut from this forest they become immediately and costlessly available to consumers. All the costs of maintaining this forest are assumed to come at cutting time. In reality this is not so -- taxes, thinning expenditures, fire protection, nursing young trees -- are all significant expenses occurring throughout the forest's life. The assumption is made purely for simplicity and is not defensible in any other way.[2] When a price is quoted it means price net of cutting and reforesting costs. These costs are assumed constant per unit output. Replanting is assumed to happen immediately on cutting. The act of cutting a tree is also the act of planting a tree. The forester is assumed to maximize present discounted value with rate of interest r. Under these conditions we show that there is an optimal rotation for trees -- an age younger than which trees are allowed to grow and older than which they are cut. Let X(a) be the stock of trees of an even age class stand. "a" is the age of these trees $a = T_{n+1} - T_n$ where the T's are the n^{th} and $n{+}1^{st}$ time the forest is cut. Let $V(T_n, X(T_n - T_{n-1}))$ be the value of the timber at time T_n. The present value (PV) of the stand is thus

$$PV(0) = e^{-rT_1} V(T_1, X(T_1 - 0)) + e^{-rT_2} V(T_2, X(T_2 - T_1)) + PV(T_2) e^{-rT_2}$$

The forester's problem is to choose a set $(T_1, \ldots, T_n)$ of times to cut the forest. The first order conditions for a maximum are $\partial(PV)/\partial T_i = 0$. Thus:

$$\left[\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x}\frac{dX}{dt}\right]\bigg|_{T_1} - e^{-r(T_2-T_1)} \frac{\partial V}{\partial x}\frac{dX}{dt}\bigg|_{T_2} = rV(T_1)$$

Value is equal to price times quantity: $V = q \cdot X$. (A more general formulation of value would include a cost function: $V = q \cdot X - C(X)$, where C(X) is the cost of removal and replanting. If C(X) is proportional to X then the simple formulation gives the right answer.)

Making the substitution:

$$[\dot{P}(T_1)X(T_1) + P(T_1)\dot{X}(T_1)] - [e^{-r(T_2-T_1)} \dot{P}(T_2)\dot{X}(T_2-T_1)]$$

$$= rP(T_1)X(T_1)$$

The above expression explains the experiment of lengthening the first period a little while shortening the second period the same amount. The first term is the gain from holding the stock a little longer. There are two parts to the gain. First the price changes. Second the quantity changes. The second term is the discounted value of the timber that is lost from the second period harvest. Thus the lefthand side is the change in timber value from altering the plan. The righthand side is the loss attributable to the discount rate from postponing the cut. Their equality is obvious. Before finding the optimal policy for the case of constant or exponentially changing price, one more variational experiment should be undertaken.

$$\partial PV(0)/\partial T_2=0 \Rightarrow [\dot{P}(T_2)X(T_2) + P(T_2)\dot{X}(T_2)]\exp(-rT_2) +$$

$$\partial PV/\partial t|_{T_2} \exp(-rT_2) = r[P(T_2)X(T_2) + PV(2)]\exp(-rT_2)$$

Comparing this formulation to the one previous, one sees that the loss from lengthening a period by epsilon is the same as the loss from putting off the entire plan by epsilon. Perhaps this makes the importance of ground rent (which is the present value) a little clearer. When deciding when to cut a forest one must remember that only on cutting is the land freed for another rotation. This matter is neglected in many forestry and economics texts.

Now we are ready to use the first formula to derive the optimal rotation age in the event that $\dot{p} = bP$ (b constant). Since the problem facing the forester is the same (up to a multiplication constant) at the beginning of period n+1 as it was at the beginning of period n, his action must be the same. This implies a uniform tree life of L. That is $T_n - T_{n-1} = L$, all n. Using $\dot{p} = bp$ and dividing the first form of the variational equation by p:

$$bX(L) + \dot{X}(L) - e^{(b-r)L}\dot{X}(L) = r\dot{X}(L)$$

or

$$\frac{\dot{X}(L)}{X(L)} = \frac{(r - b)}{1 - e^{(b-r)L}} = \frac{(r - b)e^{(r-b)L}}{(e^{(b-r)L} - 1)}$$

It is easier to interpret the above equation for L if we multiply again by p and use a familiar series expansion

$$\frac{P\dot{X}}{PX} = \frac{r - b}{1 - e^{(b-r)L}} = (r - b) \sum_{1=0}^{\infty} e^{(b-r)L_i}$$

We have proved Theorem 1.1:

Theorem 1.1

If the assumptions of this section hold and price is increasing at the exponential rate b and the interest rate is r, then the present value maximizing rotation age for trees is implicitly defined by:

$$\frac{\dot{X}(L)}{X(L)} = (r - b)/(1 - \exp(b - r)L)) .$$

This says that the percentage change in value from holding a stock a little longer equals the interest rate times a correction factor that accounts not only for putting off the plan a little this cut ($e^{-rL \cdot 0} = 1$) but also every future period i, (e^{-rL_i}). Clearly, large r and L make the correction factor irrelevant. (For Douglas fir $L \approx 90$, $r \approx 10\%$, $1 - e^{-9}$ can be taken to equal 1 for our purposes. For southern pines L is approximately 18 and the issue has some relevance.) Samuelson[fn] discusses the relation between L and the rotation ages proposed by other authors. One of these ages, M, the maximum sustainable yield, is what the U. S. Forest Service appears to have as its goal and, perhaps, policy. M corresponds to an interest rate of zero and constant price (or a constant present value price -- b = r). M is the rotation length that maximizes the volume of timber removed and M > L. A later part of this paper will be directed to Forest Service policy.

Footnotes

1 Paul A. Samuelson, "Forestry in an Evolving Society," _Economic Inquiry_ (forthcoming).

2 Mason Gaffney, "Taxes on Yield," British Columbia Institute for Economic Policy Analysis, 1975 (unpublished).

3 Samuelson, op cit.

Section 2

The Effect on Optimal Policy of a Market Mechanism in a World with Simply Specified Growth

The market mechanism makes it undesirable to cut large sections of a forest or harvest a large percentage of all fish in any given time period. This section explicates the relation between harvest, stock, price and the rate of price increase. Crudely, it is seen that a large initial stock calls forth a policy of gradual reduction in population and increase in price.

Section 2.1

A Simple Renewable Resource with a Market Partial Equilibrium

It is unfortunate that the detail of the technology of growing trees obscures the price dynamics of a simple equilibrium model. To obtain the clearest picture of the price mechanism over time, I adopt a simple technology: $\dot{x} = f(x) - c$, and a simple description of the rest of the world.

The world has one producer who is a price taker. (One could have many identical producers -- but it only adds constants to the calculations.) At every instant in time he faces a (twice continuously differentiable) downward sloping demand for his product. He knows this demand and uses it to predict correctly the future path of prices. That is to say, he has rational expectations and those expectations are fulfilled.

As before, the producer is assumed to maximize the present discounted value of resource landings with discount rate r. The resource pool, of which he is the sole owner, has a reproduction law of the form $\dot{x} = f(x) - c$, where x is the stock and c is the harvest at time t, the time subscript being suppressed. "f" is twice continuously differentiable and is assumed to have the usual shape:

$$f(0) = 0 \quad f'' < 0 \quad f(k) = 0 \quad f(x) > 0 \quad x \varepsilon [0,k]$$

An example of such a function is the logistic, $xg(1 - x/k)$. For a further explanation, see Colin Clark's American Math. Monthly article.

The producer's problem is:

$$\max \int_0^\infty e^{-rt} \, q \, c \, dt$$

$$\text{s.t. } \dot{x} = f(x) - c$$

$$x \geq 0 \qquad c \varepsilon [0,\infty]$$

where q(t) is the expected price at time t. Consumers are represented by a downward sloping demand curve D(q) which is continuously differentiable and for convenience, is assumed to be the same in every period. If we had information on the relative prices of the resource and other goods in the future,we would include it in the demand curve.

The market clearing equation states D = c or catch equals demand at every instant.

The first step in solving this problem is to apply the maximum principle of Pontragin et al. to the producer's problem. Let

$$H = e^{-rt}q \cdot c + \lambda(f(x) - c)$$

be the Hamiltonian. Necessary conditions for an optimum are:

$$\dot{x} = \partial H/\partial\lambda = f(x) - c$$

$$\dot{\lambda} = -\partial H/\partial x = -f'(x)\lambda ,$$

$$\text{transversality condition} \quad \lim_{t\to\infty} \lambda x = 0$$

and the maximum principle: choose c to max H at every time t. A quick examination of H shows that either c is one of zero and infinity or $\lambda = e^{-rt}q$. We now impose an additional restriction on the demand curve. Zero is demanded only at infinite price and an infinite amount is demanded at zero price. This is sufficient to rule out any possibility that $\lambda \neq e^{-rt}q$. In the more usual constant price case the information that c takes an interior value is manipulated to find the steady state solution.[1] We use this information to reduce our system to two equations. Substitute $qe^{-rt} = \lambda$ into $\dot{\lambda} = f'\lambda$ to get $\dot{q} = (r - f')q$.

Substitute $c = D(q)$ to get

$$\dot{x} = f(x) - D(q) .$$

So far it has been shown that the first order conditions for a maximum are

$$\dot{q} = (r - f')q$$

$$\dot{x} = f - D(q)$$

$$x_0 = x(0)$$

$$\lim_{t\to\infty} q \exp(-rt)\, x = 0$$

(A remark: Another way to formulate the expected equilibrium problem is the following Hamiltonian:

$$H = \exp(-rt)\ (q\ c + \int_q^\infty D(z)dz) + \lambda(f(x) - c)$$

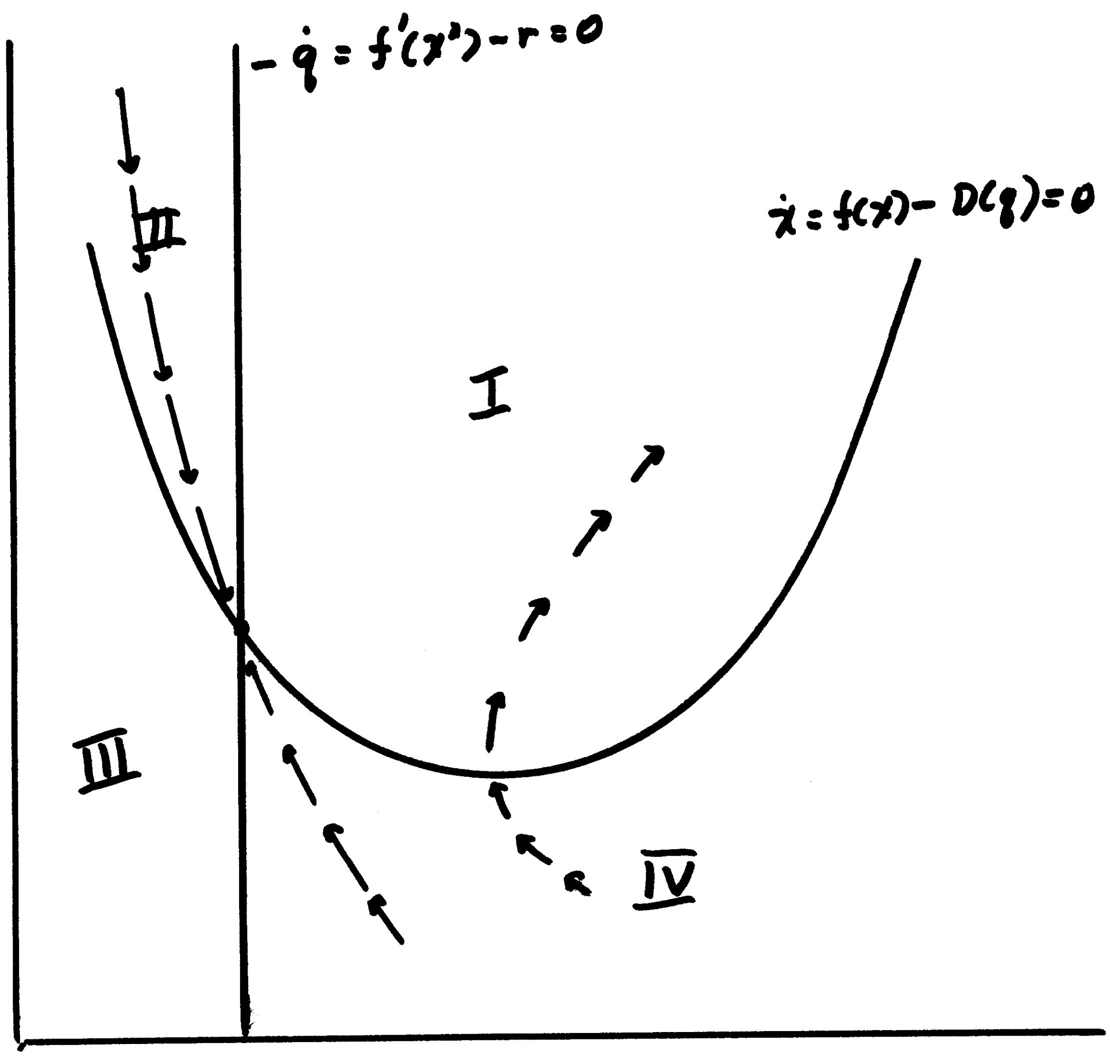
The q and the c are regarded as state variables: the first order conditions will be as above. The equation has the interpretation as minimizing on q the sum of producer and consumer surplus -- with consumer surplus defined from Marshallian demand curves -- while maximizing surplus on c. This is a two-point boundary value problem. It is complicated by the nature of the end time or transversality condition. The way to find the nature of the solution is by way of a phase diagram. Consider the vector field $V(q,x) = (\dot{q},\dot{x})$. First we show that V is singular at only one point which we call a steady state and denote (x^*, c^*). Then we show that only solutions that converge to the steady state meet the transversality condition.

The system has an obvious steady state x^*, c^* defined by $f'(x^*) = r$ and $f(x^*) = c^*$. The steady state is unique. The solution to $f' = r$ is unique under the assumption that $f'' < 0$. Could there be another steady state? Let x_s be such a quantity, $r - f' \neq 0$ by assertion. Moreover, f' is constant. Thus $q = a \exp((r - f')t)$. The single valued property of downward sloping demand finishes off the possibility that x_s is a steady state.

The table below shows the direction of a solution passing through an arbitrary point of each of the quadrants in (x,q) space.

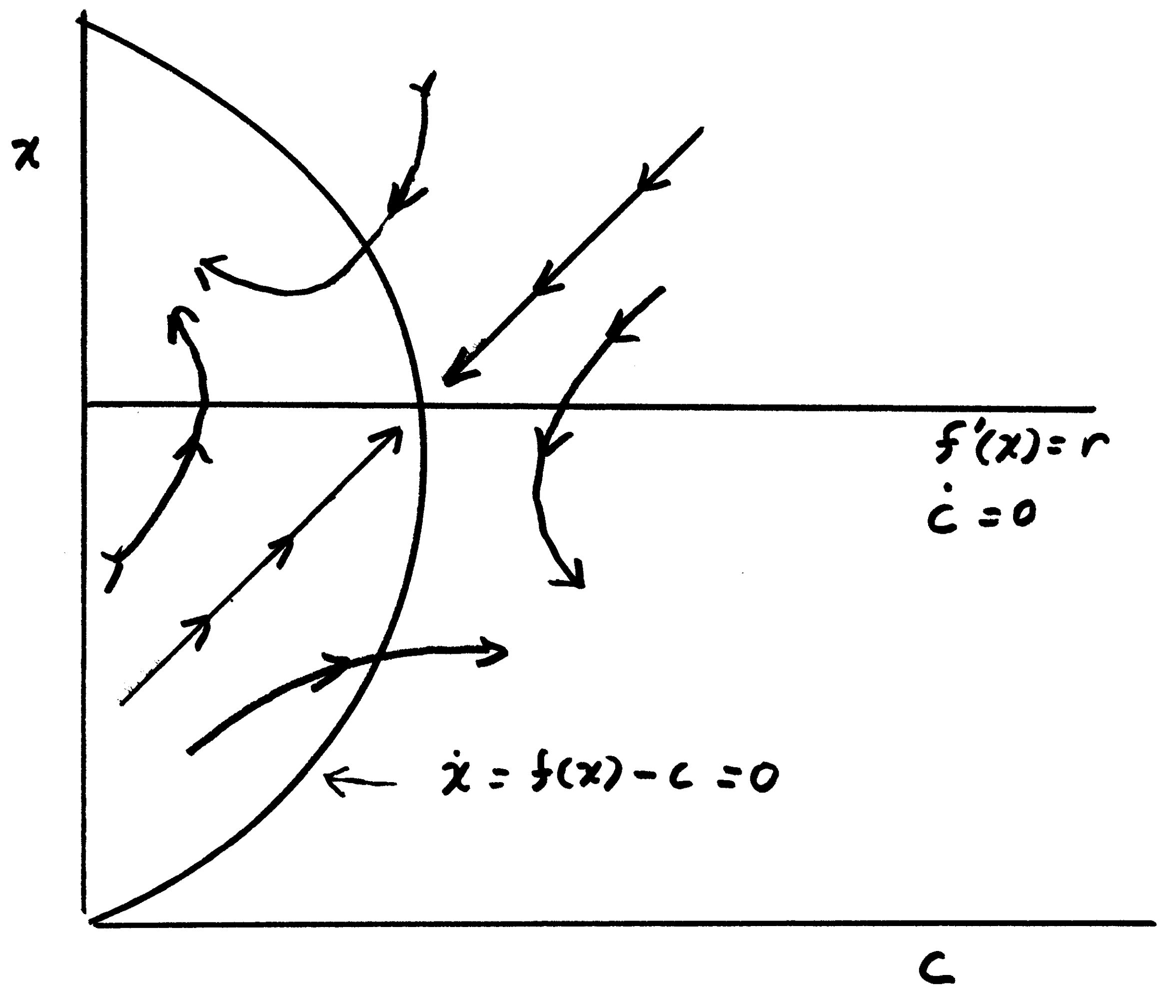
Quadrant	$(\dot{x},\dot{q})$ or V
I	(+,+)
II	(+,-)
III	(-,-)
IV	(-,+)

Suppose that $(x,q)_0$ is in quadrant I. Then both x and q increase without bound. The assumption that there is a maximum size to the population assures that f' is zero for large enough x. Thus q increases at the rate of interest in the long run and $q \exp(-rt)x$ increases without bound. This is a violation of the transversality condition. Similarly an initial point in quadrant three can be viewed as violating the transversality condition by going to negative infinity. If one insists on the variables



$-\dot{q} = f'(x^*) - r = 0$
$\dot{x} = f(x) - D(q) = 0$
I
II
III
IV

Phase Diagram in x,c Space



being positive, then it is ruled out by the assumption of infinite demand at zero price and zero demand at infinite price. Either way one views it, initial values of q that place $(x,q)_0$ in quadrant III are impossible.

Before proceeding to even-numbered quadrants, we need to examine the boundaries of the quadrants: that is the $\dot{x} = 0$ and $\dot{q} = 0$ curves. Except for the singular point at (x^*,q^*), any solution that begins on one of these curves will immediately cross into quadrants I or III and thus be inadmissable.

In quadrants II and IV, x is either steadily decreasing or steadily increasing. Therefore, any characterisitc passing through these quadrants must intersect their boundaries and pass into quadrants I or III, or the characteristic must terminate at (x^*,q^*).

Because we have made strong regularity assumptions on f and D, there exists exactly one characteristic that passes through the point (x^*,c^*).[2]

We have shown that only paths that end at the steady state meet the transversality condition. This allows us to identify one characteristic along which all solutions must lie and thus relate the starting (or any other time) price to the initial quantity.

We have demonstrated the following two theorems:

<u>Theorem 2.1</u>

<u>The partial equilibrium model with autonomous demand has a unique steady state and converges to the steady state from any initial allocation. The steady state values are implicitly defined by $f'(x^*) = r$ and $f(x^*) = c^*$ and $D(q^*) = c^*$.</u>

Footnotes

1 Colin Clark, American Math. Monthly, op cit.

2 Witold Hurewicz, Lectures on Ordinary Differential Equations. Cambridge: MIT Press, 1958.

Theorem 2.2

If x_0 is greater than x^*, then q_0 is less than q^* and q increases monotonically at the rate $r - f'(x)$ while x falls monotonically. In particular, $\dot{q}/q$ is greater than r when x exceeds x_{max} is equal to r when x equals x_{max} and is positive but less than r when x is greater than x^* but less than x_{max}.

Section 2.2

Linearization of the Simple Model

What is left to figure out is the initial price, q_0. To this end we know the "end time" price q^*: $D(q^*) = f(x^*) = c^*$. But $q_0 = q^*/(\exp \int_0^\infty r - f'(x(t))dt)$ and depends on the time path of $x(t)$. One can approximate q_0 by linearizing the differential equations that make up this system. Let Δq and Δx be the state variables expressed as a deviation from the steady state values. That is $\Delta q = q - q^*$ and $\Delta x = x - x^*$. To a first order approximation the problem of a renewable resource is:

$$\begin{matrix} \dot{\Delta q} \\ \dot{\Delta x} \end{matrix} = \begin{pmatrix} 0 & -f''(x^*)q^* \\ -D'(q^*) & f'(x^*) \end{pmatrix} \begin{matrix} \Delta q \\ \Delta x \end{matrix}$$

The characteristic polynomial is $b^2 - bf' - D'f''q^*$ and the characteristic roots are

$$b_1 = (f' + \sqrt{f'^2 + 4D'f''q^*})/2$$

$$b_2 = (f' - \sqrt{f'^2 + 4D'f''q^*})/2$$

which are real and of opposite sign, b_2 being negative. The eigenvectors are:

$$b_i = \begin{pmatrix} 1 \\ -b_i/f''q^* \end{pmatrix} \qquad i = 1,2$$

Thus solutions following the laws of motion of this system -- intertemporal myopic efficiency -- are of the form

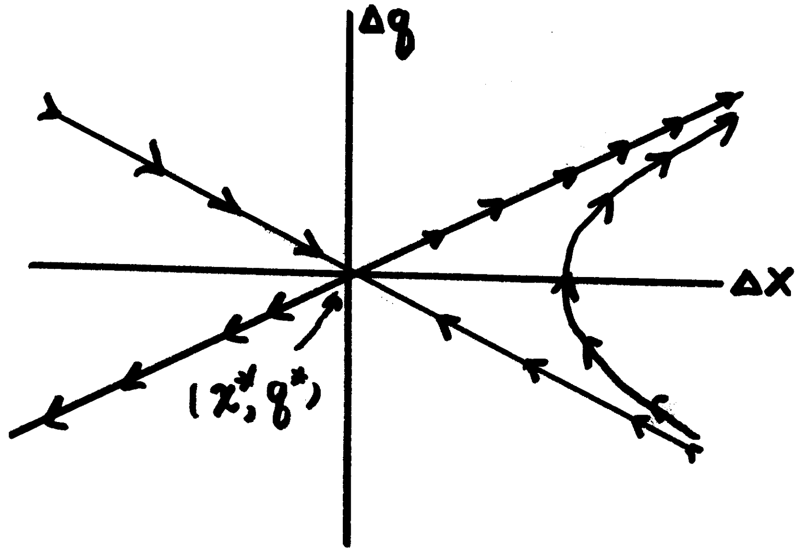
$$z(t) = a_1h_1 \exp (b_1t) + a_2h_2 \exp (b_2t)$$

But the transformed end time conditions are that $\lim_{t\to\infty} z = 0$. This implies $a_1 = 0$. Thus the initial price q_0 can be found from the following:

$$\Delta q = \frac{2\Delta x(-f''q^*)}{f' - \sqrt{f'^2 + 4D'f''q^*}}$$

If $\Delta x(0)$ is the initial stock, then $\Delta q(0)$ is found from the above and $q(0) = \Delta q(0) + q^*$.

The above formula shows that Δq has the opposite sign of Δx Thus if the stock is greater than the steady state stock, price will start below the equilibrium price and rise to it. One can draw a phase diagram of this system:



Any path that does not start on b_2 will end on b_1 and call for infinite quantity and price. One can separate the economic problem into two parts: intertemporal myopic efficiency -- being on some trajectory defined by the model -- and end time conditions -- ending at a steady state rather than an infeasible solution.

If anything is surprising in this model it should be the statements on price. Why should the price rise along an optimal path when $x > x^*$? If price didn't go up the producer would dump all his surplus resource on the market now. The price system must create a capital gain for the producer to induce him to hold his resource. This is particularly important in the case of trees. There we will find price must rise on virgin timber fast enough to offset both the interest rate and the opportunities for new growth.

<u>Section 2.3</u>

<u>Changes in Demand</u>

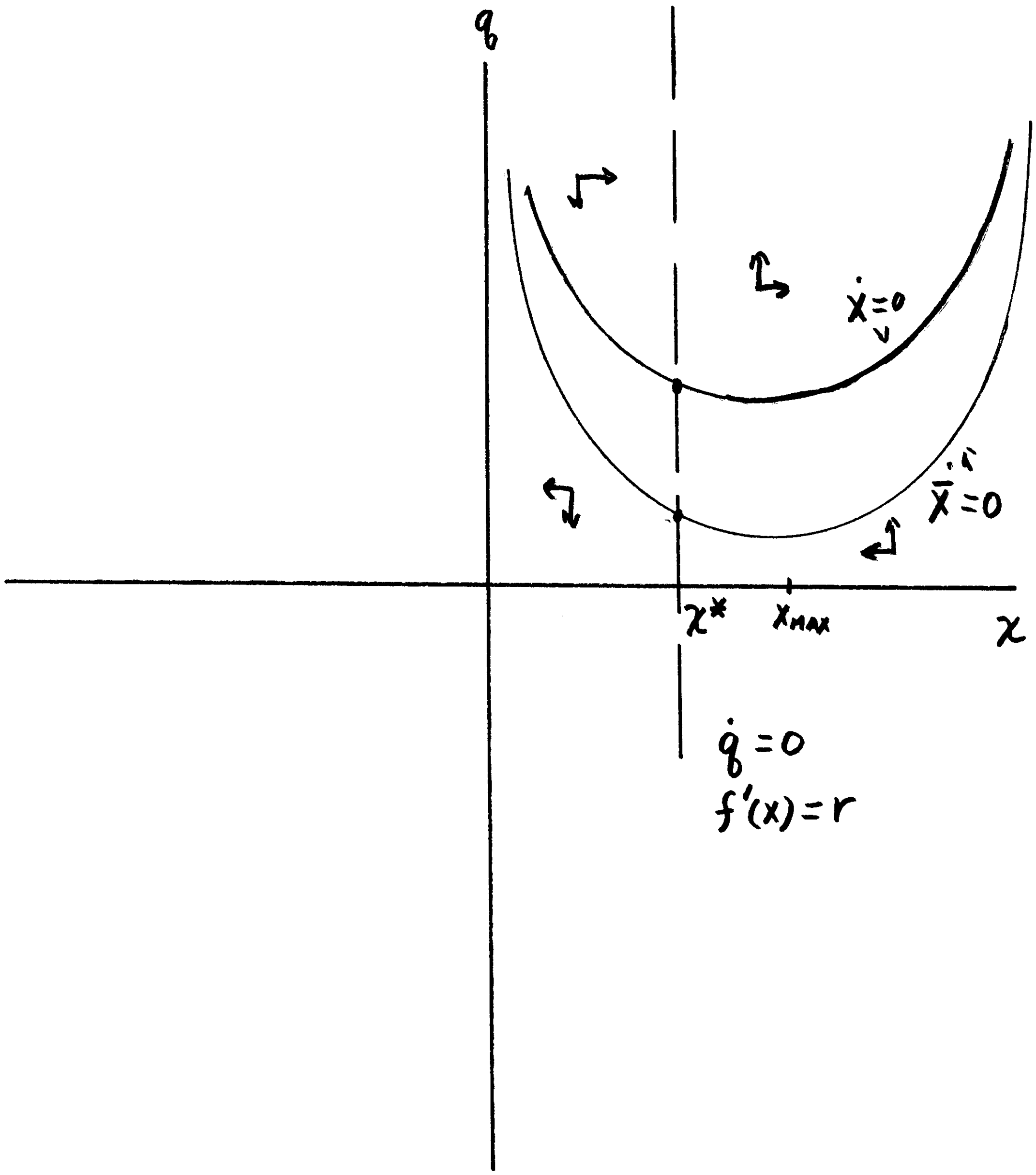
One can inquire how shifts in demand change the allocation of a nonrenewable resource over time. The untranslated phase space provides the easiest exposition of the basic result on shifts in demand. If $D(q)$ is a demand curve then $\bar{D}(q) = D(q + a)$ is an inward shift of the demand curve D. Let all the variables of the shifted system be represented with overbars.

We have:

<u>Theorem 2.3</u>

<u>If "a" is a positive number, the shifted system will converge to a steady state $(\bar{x}^*, \bar{q}^*)$ and $\bar{x}^* = x^*$ while $\bar{q}^* = q^* - a$. Moreover, if $z(t) = (x(t), q(t))$ is the optimal path of the initial system and $\bar{z}(t) = (\bar{x}(t), \bar{q}(t))$ is the optimal path of the transformed system, then the trace of $\bar{z}$ will lie below the trace of z, assuming that x_0 is greater than zero.</u>

The first part of the theorem is a consequence of Theorem 2.1 and elementary algebra. For the second part, consider the phase space (x,q).


q
$\dot{x}=0$
$\dot{\bar{x}}=0$
x^*
x_{MAX}
x
$\dot{q}=0$
$f'(x)=r$

The $\dot{x} = 0$ locus is described by the equation $\dot{x} = f(x) - D(q)$. Thus, changing D to $\overline{D}$ increases the RHS of the $\dot{x}$ equation by decreasing demand. This requires an adjustment of x, a decrease if x is less than x_{max} and an increase otherwise. The result, as can be seen on the diagram, is a lowering of the $\dot{x} = 0$ curve. What remains to be shown is that z and $\overline{z}$ don't corss. Suppose z and $\overline{z}$ did cross at the point (q,x). $\dot{q} = \dot{\overline{q}}$ since the price equation is the same for both. $\dot{\overline{x}}$ is greater than $\dot{x}$, from the equation for $\dot{x}$. Thus the bar system will move above the original system and can never be below it. But the bar system has its equilibrium below the original system. This is a contradiction and the theorem stands.

The above theorem shows that for autonomous demand, a shift in the demand curve does not change the longrun stock or flow of the resource; the shift merely changes price.

Actually, a demand shift may have further effect, it may change the slope of the converging arm z(t), and the speed at which the system moves up the arm. The slope of the arm describes how much prices and (through autonomous demand) how much consumption changes over time. Thus a system with a flat arm describes a system with equality across time (generations) while one with a highly sloped arm has a great difference in the resource flows across generations.

The experiment I have in mind is a differential shift in the demand curve. $\overline{D} = D(q + a)$ and vary "a" a small amount, w. We know that $d\overline{q}^*/dw = -1$ and can find

Theorem 2.4

$db_2/dw = f''(D' + D''q^*)/\sqrt{r^2 + 4D'f''q^*}$

The sign of $db_2/dw = -\text{sgn}(D' + D''q^*)$

or

$db_2/dw > 0$ if $|D'| > D''q^*$ or $1 + D''q^*/D' > 0$

< 0 if $|D'| < D''q($ or $1 + D''q^*/D' < 0$

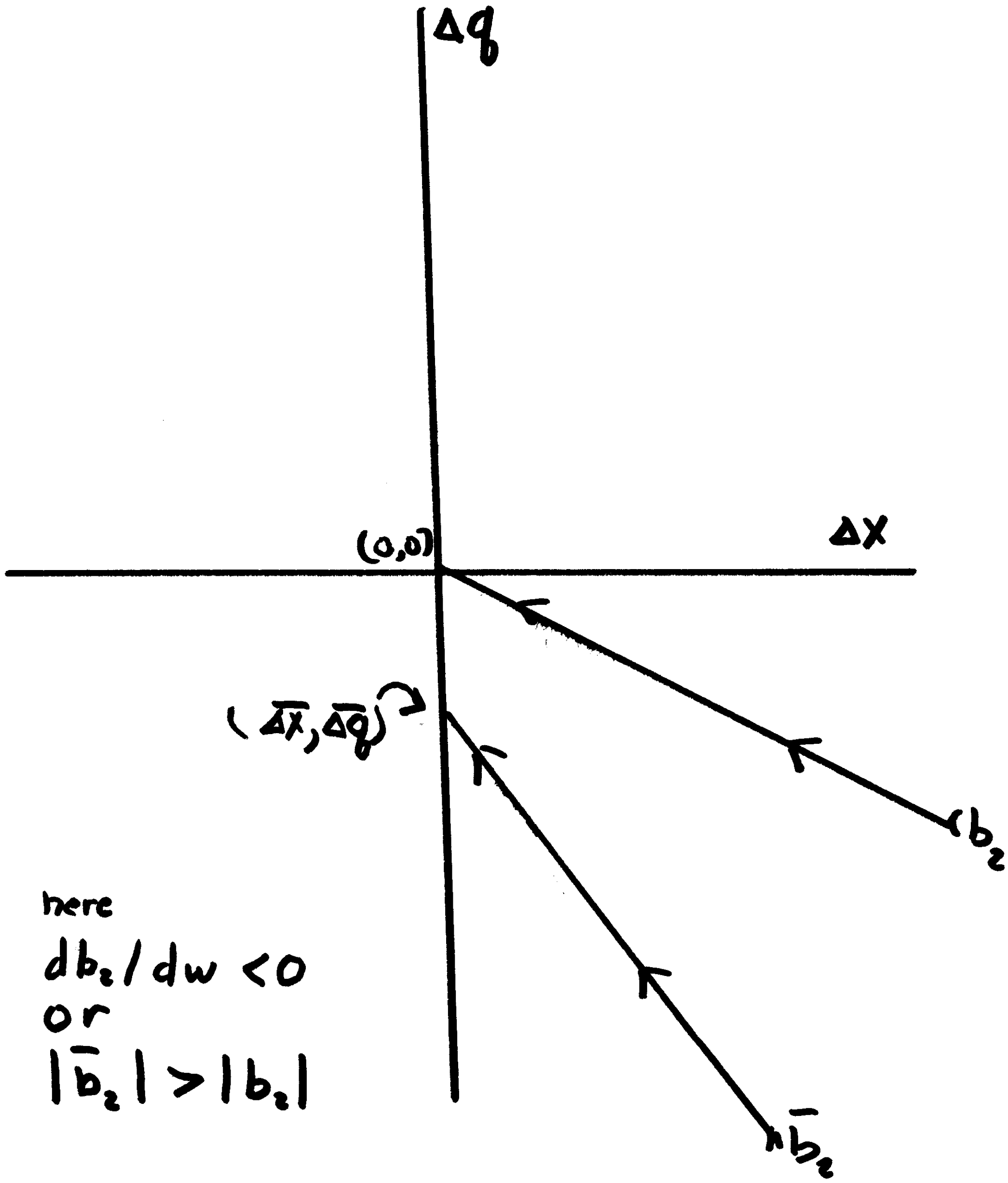
where $D''q^*/D'$ is the elasticity of the first derivative of demand with respect to price.

In the case that db_2/dw is greater than zero, the overbar system will converge more slowly to equilibrium since $\bar{b}_2$ will be less negative than b_2 and $z = a_2h_2 \exp(\bar{b}_2t)$. Moreover, $\bar{h}_2$ will be less steeply sloped than h_2 since

$$\bar{h}_2 = (-b_2/f''q^*)^{1} \quad \text{and} \quad |\bar{b}_2| < |b_2|$$

and f'' is less than zero.

Thus the effects of slope and speed work together. Greater slope and higher convergence speed go together (at least locally), and provide for faster changes in quantity consumed over a given time interval.



Δq
ΔX
(0,0)
($\overline{\Delta X}$, $\overline{\Delta q}$)
b_2
$\bar{b}_2$
here
$db_2/dw < 0$
or
$|\bar{b}_2| > |b_2|$

Section 2.4

Simple General Equilibrium Models

A genral equilibrium setting is more satisfactory than a partial equilibrium one since the general equilibrium setting seems less artificial, accounts for income effects, and permits rigorous welfare statements. It is less satisfactory for estimation purposes, especially if the income effects are small. The simplest of the general equilibrium models is the one in which there is only the renewable resource. It turns out to be identical to the partial equilibrium model. The simple general equilibrium model is just a Ramsey type problem and its solution is well known. The special characteristics of a renewable resource -- a limit to the population size -- never become important. Another case we consider, and this case also amounts to a Ramsey type model, is the case of a renewable resource and a perishable resource. In both cases the important feature is the autonomy of the derived demand function. The following general equilibrium model is mostly useful for its extension to the case where the resource provides an externality and for the comparison to the nonautonomous case.

A Perishable Resource

Let m be labor or some other resource that cannot be stored from period to period and is provided at a constant rate. If there is one consumer with utility function $U(c,m) \exp(-rt)$ where c, as usual, is the harvest of a renewable resource, and $\dot{x} = f(x) - c$, then the consumer's problem of maximizing his happiness subject to the constraints of the population he preys on results in the Hamiltonian:

$$H = \exp(-rt)\, U(c,m) + \lambda(f(x) - c)$$

and first order conditions,

$$MUc = \exp(rt)\lambda$$

$$\dot{\lambda} = -f\lambda$$

Making the usual substitution of $q = \exp(rt)$ and inverting the marginal utility condition we get

$$\dot{q} = (r - f')q$$

$$c = MUc^{-1}(q)$$

This is exactly the system described in the previous section (if MU has the reasonable property of being everywhere positive). For comparison with a succeeding section we solve this for the case $U = c^{\alpha}m^{\beta}$ and $\alpha + \beta$cl. Clearly,

$$c = \frac{m^{\beta/1-\alpha}}{\alpha^{1/1-\alpha}}\, q^{1/\alpha - 1}$$

is the demand curve and $f'(x^*) = r$, $f(x^*) = c^*$ define the steady state stock and quantity. Inverting the demand curve gives the steady state price, q^*.

The model has further meaning if one assumes that labor and the resource are used to produce one good while leisure constitutes another good. Suppose U(c,m) is of the form U(w,L) where w is a produced good w = w(h,c), w neoclassical, and L + h = m where L is leisure and h is hours worked. Will the man work more or less as time progresses?

If x_0 is greater than x* we know that q rises and x and c fall along an optimal path. For any given time we know the consumer solves $\max_h U(w(h,c), m - h)$ by setting its derivative equal to zero:

$$U_1 w_1 - U_2 = 0 \quad .$$

Since we know c decreases over time, we totally differentiate the first order conditions to find dh/dc, multiply it by a negative number (the change in c) and find whether or not the man works more.

First translate the F.O.C. to their equivalent form $MP_L - MRS = 0$, where MP_L is the marginal product of labor and MRS is the marginal rate of substitution or MU_h/MU_c. We get the comparative statics result:

$$-\frac{dh^*}{dc^*} = \frac{\frac{\partial}{\partial c}(MP_L - MRS)}{\frac{\partial}{\partial h}(MP_L - MRS)}$$

Although the derivative of the marginal product of labor can be signed by regularity conditions on the production function, the derivatives of the MRS can take either sign and depend, ultimately, on the magnitude and sign of the income effect. Moreover, even knowing the sign of the change in the MRS does not tell what its magnitude is. For instance, take the case where both functions are Cobb-Douglas. The MRS = bc/h b dependent on which Cobb-Douglas utility function is chosen (where b is a constant). Let $w = c^{\alpha} h^{\beta}$, $\alpha+\beta<1$.

$$\text{F.O.C.} \qquad MP_L - MRS = \frac{\beta w}{H} - \frac{bc}{H}$$

$$\frac{\partial \text{F.O.C.}}{\partial c} = \frac{1}{H}\left(\frac{\beta\alpha w}{c} - b\right)$$

$$\frac{\partial \text{F.O.C.}}{\partial H} = \frac{c}{H^2}\left(\frac{\beta(\beta - 1)w}{c} + b\right)$$

Thus

$$\frac{\partial \text{F.O.C.}}{\partial c} > 0 \text{ iff } \frac{w}{c} > b/\alpha\beta$$

$$\frac{\partial \text{F.O.C.}}{\partial H} < 0 \text{ iff } \frac{w}{c} > b/\beta(1 - \beta)$$

Assume that labor's share is greater than the resource share, $\alpha > \beta$. Thus, $\beta(1 - \beta) \geq \alpha\beta$ and we distinguish three cases:

$$dH/dc > 0 \text{ when } w/c > b/\beta(1 - \beta) > b/\alpha\beta$$

$$dH/dc < 0 \text{ when } b/\beta(1 - \beta) > w/c > b/\alpha\beta$$

$$dH/dc > 0 \text{ when } b/\beta(1 - \beta) > b/\alpha\beta > w/c$$

In the normal first and third cases more work is done and less leisure spent as the resource flow diminishes. Notice that only with less than constant returns to scale can the amount of leisure spent go up as the resource flow is diminished.

Section 2.5

General Equilibrium with the Stock Valued

The introductory chapter suggested one might value the stock of a natural resource for its positive externalities. Forests have the public good attribute of recreation. (The model that follows applies equally well to management in a regime of taxes or the management of a pest population. Simply make the externality negative.)

Again assume one consumer; this time let his utility function be $U(c,x) \exp(-rt)$; as usual the natural resource is modelled $\dot{x} = f(x) - c$. Now the Hamiltonian is

$$H = U(c,x) \exp(-rt) + \lambda(f(x) - c)$$

and first order conditions after the usual transformation

$$\lambda = \exp(-rt)\, q$$

are

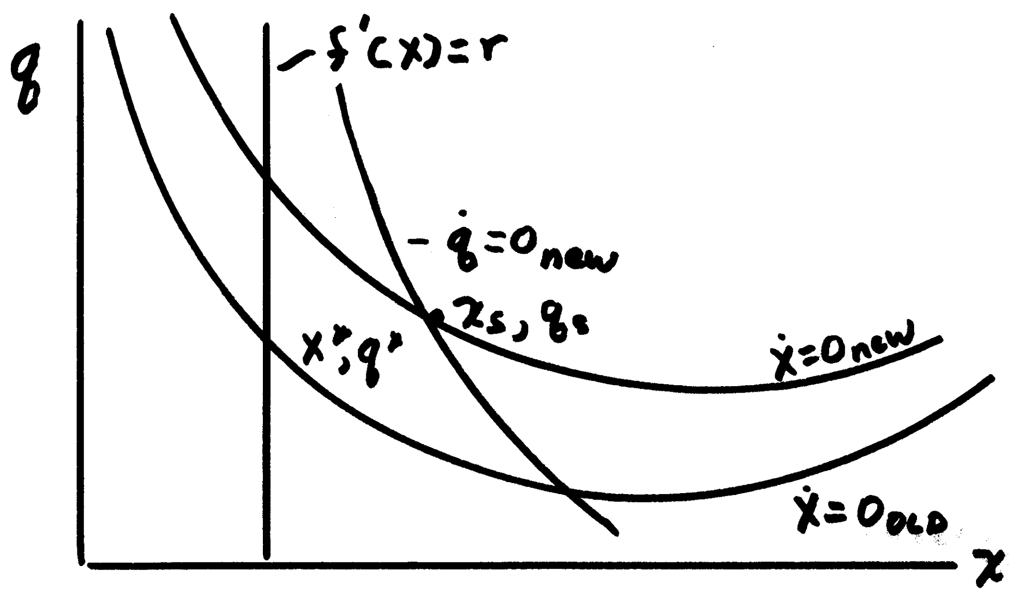
$$MUc = q \text{ or } c = MUc^{-1}(q,x) \quad \text{where } MU_{c_1} < 0 \text{ and } MU_{c_2} > 0$$

$$\dot{q} = (r - f')q - MUx$$

$$\dot{x} = f(x) - MUc^{-1}(q,x)$$

and $$\lim_{t \to \infty} \lambda x = 0$$

Note that the demand equation is still autonomous. x and q are related by the transversality condition. Thus the results of the section on autonomous demand curves hold here too. The $\dot{q} = 0$ locus is found by solving $(r - f')q - MUx(f(x),x) = 0$. We assume the last term decreases in x over the relevant range. Thus $\dot{q} = 0$ lies to the right of the line $f'(x) = r$. $\dot{x} = 0 = f(x) - MUc^{-1}(q,x)$ is raised by the inclusion of x in the demand so the phase diagram is:



and x_s, the new steady state, is higher than the old steady state and may even exceed x_{max}. The steady state resource flow in $f(x_s)$ and may have any relation to $f(x^*)$ the old steady state resource flow. The steady state price, q_s, will be higher if x_s is greater than x_{max}, otherwise it is indeterminate. We have shown

Theorem 2.5

Valuing the stock of a resource will cause the steady state stock to be higher than it would otherwise have been. The stock increase may raise or lower the steady state resource flow.

Consider the example of Cobb-Douglas utility and $f(x) = x^{\gamma}$ for a growth function, $1 > \gamma > 0$. f behaves badly -- it has no maximum -- but the analysis is easy.

$$u = c^{\alpha} x^{\beta}$$

$$MU_c = \alpha c^{\alpha-1} x^{\beta} = q; \quad MU_x = \beta c^{\alpha} x^{\beta-1}$$

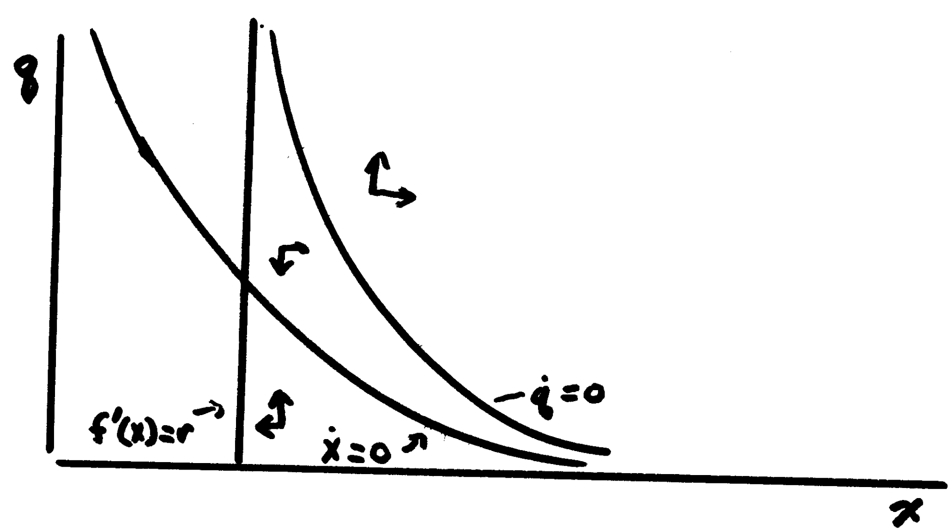
$$\dot{q} = (r - \gamma x^{\gamma-1})q - \beta x^{\beta+\gamma\alpha-1}$$

$$\dot{x} = x^{\gamma} - (qx^{1-\beta}/\beta)^{1/\alpha}$$

$$\dot{x} = 0 \text{ implies } q = \beta x^{\gamma\alpha+\beta-1}$$

$$\dot{q} = 0 \text{ implies } q = \beta x^{\gamma\alpha+\beta-1}/(r - \gamma x^{\gamma-1})$$

It is immediate that x* is infinite and q* is zero. The phase diagram is:



and the result is that conservation dictates an ever-increasing resource stock.

This sort of example can be used to impute a value of stock to an organization observed to be following a plan that does not lead to an expected market equilibrium.

For example, consider the United States Forest Service. The Multiple-Use Sustained Yield Act of 1960 instructs the Forest Service to manage the nation's forests for lumber, recreation, and wildlife. The forest service achieves these goals through a system of regulation that provides for equal timber harvests in every year and timber harvests at less than the rate one would expect from a competitive firm in early years. The long run stock is x_{max}, and the resource harvest is $f(x_{max})$. $f(x_{max})$ is greater than f(x) for any other x. The preceeding description of the Forest Service's policy is a little crude; a more complete discussion of this matter will be found in the next chapter. For the purpose of what follows it is sufficient to say that the Forest Service has ill-defined goals that include using forest land for timber production and for other uses. Although the Forest Service does not reduce its valuation of standing timber to a single number, an economist trying to understand what the Forest Service does could treat the Forest Service as if it were trying to maximize an instantaneous utility function of the following variety:

$$U(c,x) = q\, c + kx.$$

Below the possibility that the Forest Service takes maximim yield as its goals is also discussed.

Let consumers be represented by U(c) exp (-rt) while the resource holder (the Forest Service) decides to attach a value, $k = k_0 \exp(-rt)$ to standing resource. Thus the demand equation is $c = MUc^{-1}(q)$, but the producer has elected to $\max_c \int_0^\infty \exp(-rt)(q(t)\, c + kx)dt$ s.t. $\dot{x} = f(x) - c$. The Hamiltonian and F.O.C. for this problem are:

$$H = \exp(-rt)(q\, c + kx) + \lambda(f(x) - c)$$

$$\exp(-rt)\, q = \lambda \quad \text{if} \quad c \neq 0$$

$$\dot{q} = (r - f')q - k$$

$$\dot{x} = f(x) - MUc^{-1}(q)$$

Letting the population grow to x_{max} ($f'(x_{max}) = 0$) is often called maximum sustained yield. It is of particular interest that maximum sustained yield implies a steady state with $f'(x_s) = 0$ or $x_s = x_{max}$ so that

$$k = rq = rMUc\,(f(x_{max})).$$

Although the concept of maximum sustainable yield (MSY) implies a steady state valuation of the resource stock, it is without implication for the dynamics. Does the conservation agency use the discounted steady state price $k^0 \exp(-rt)$ to make its intertemporal decisions? Do they have a utility function that settles down to $MUx(f(x),x) = k_0$ at the maximum yield point? Neither question is amenable to theoretical solution.

We summarize this discussion on MSY in

Theorem 2.6

Maximum sustainable yield implies the marginal utility of holding the resource at the maximum yield point is a constant k equal to rq or $MUc^{-1}(f(x_{max}))$. MSY has no implication for the time path to reach the point x_{max}.

Section 2.6

A Renewable Resource in a General Equilibrium Setting with a Nonrenewable Resource.

In this model there is both a renewable and a nonrenewable resource. The stock of the nonrenewable resource is y(t) and its rate of extraction is g. This leads to the following planner's problem, the answer to which is also the perfect foresight competitive equilibrium for a one-consumer (or Hicksian in the sense of Arrow and Hahn) world.

$$\max \int_0^\infty U(c,g)e^{-rt}dt$$

$$\text{s.t.} \quad \dot{y} = -g$$

$$\dot{x} = f(x) - c$$

$$x(0) = x_0 \quad \text{and} \quad y(0) = y_0$$

$$x,y,c,g \geq 0$$

This is similar to Dasgupta and Heal.[1] They have "capital" instead of a renewable resource; they require the nonrenewable resource to reproduce capital; and they have only one consumption good, $\bar{c}$. Their model is:

$$\max \int_0 U(\bar{c})e^{-rt}\, dt$$

$$\text{s.t.} \qquad \dot{x} = h(x,g) - \bar{c}$$

$$\dot{y} = -g$$

$$x(0) = x_0 \quad \text{and} \quad y(0) = y_0$$

$$x, y, \bar{c},\ g \geq 0$$

How the economy fares in the long run is shown to be a function of the elasticity of substitution in the function h. Dasgupta and Heal's model differs from mine in two major ways: 1. In my model there is a limit to how much of the renewable resource (or capital good) may be stored. 2. In my model the nonrenewable resource cannot be converted into another capital good for storage and later use. These differences assure, in the long run of my model, that the flow of goods will be diminished. It is the utility function that determines how much lesser amounts of the nonrenewable resource will hurt. (The utility function could be regarded as being of a single argument, z, U(z) and z being a production function of c and g, the resources: $z = z(c,g)$ and z not storable. Perhaps this makes the relation to Dasgupta and Heal clearer.)

My model is solved by forming the Hamiltonian and finding the first order conditions.

$$H \;=\; e^{-rt}U(c,g) + \quad (f(x) - c) + \gamma(-g)$$

Conditions are placed on U to assure that g and c do not vanish. (Marginal Utility (MU) at zero is unbounded.) The first order conditions are:

$$e^{-rt}MUc \;=\; \lambda$$

$$e^{-rt}MUg \;=\; \gamma$$

$$\dot{\lambda} \;=\; -\lambda f'$$

$$-\dot{\gamma} \;=\; 0$$

Introduce the current price variables s and v and get:

$$MUc \;=\; v \;=\; \lambda e^{rt}$$

$$MUg \;=\; s \;=\; \gamma e^{rt}$$

Plainly $s = s_0 \exp(rt)$ (since g is greater than zero everywhere). The two conditions involving marginal utility can be solved for one resource as a function of the other and own price.

$$c \;=\; MUc^{-1}(v,g) \quad \text{where} \quad MUc^{-1} < 0 \qquad MUc^{-1} > 0$$

$$g \;=\; MUg^{-1}(s,c) \qquad\qquad MUg^{-1} < 0 \qquad MUg^{-1} > 0$$

Solving the above one gets

$$c = MUc^{-1}(v, MU_g^{-1}(s_0, e^{rt}, c))$$

as the nonautonomous implicit derived demand for c as a function of own price, v, and s_0 the price of the nonrenewable resource. The rest of the system is :

$$\dot{v} = (r - f')v$$

$$\dot{x} = f(x) - c$$

Unlike the previous systems, the steady state is no longer clear. In the event the MU_g^{-1} increases at an exponential rate it may be possible to find a "steady state" in which v decreases exponentially. For example, if the utility function is Cobb-Douglas then a "steady state" can be found in which g goes to zero but c and x are constant.

$$u = c^{\alpha}g^{\beta} \quad \text{and} \quad \alpha + \beta < 1$$

$$g = MUg^{-1}(s,c) = (s_0e^{rt}/c^{\alpha}\beta)^{1/(\beta - 1)}$$

$$c = MUc^{-1}(v,c) = (v/g^{\beta}\alpha)^{1/\alpha - 1}$$

which can be solved for c to reveal that

$$v = v_0e^{(\beta/\beta - 1)rt}$$

is consistent with an unchanging value of c , call it c'.

$$c' = (v_0/\alpha)^{(\beta - 1)/(1 - \alpha - \beta)} (s_0/\beta)^{-\beta/(1 - \alpha - \beta)}$$

c' is also determined from the equations $\dot{x} = f(x) - c' = 0$ and $\dot{v} = (r - f'(x))v$. Since $r - f' = (\beta/\beta - 1)r$, fixes x' and thus c' and the product $s_0 v_0$. But s_0 is chosen so that $MUg^{-1}(s_0 \exp (rt), c')dt = y_0$. This equation provides a solution for the steady state. (It is easy to show that the system always converges to the unique steady state.)

Because f" is less than zero, the steady state with a non-renewable resource as well as a renewable one has a higher renewable resource rate of interest and a lower steady state stock than a similar economy with just a renewable resource. For instance, consider the economy with a constant flow of some resource M -- say labor -- and the usual renewable resource. Such an economy will generate the usual steady state ($f'(x) = r$)). The economy with the exhaustible resource will (if it has a steady state) generate a steady state with a lower renewable resource stock and a smaller flow of renewable resource. This occurs because the (second) cross partials of the utility function are positive. That is, higher consumption generates higher marginal utility.

All of these models have the possibility of exhaustion and extinction. For instance, suppose

$$f'(x) = r \quad \text{or} \quad f'(x) = r(1/1 - \beta)$$

has no solution. Then it would appear that the corner solution $x = 0$ is the long run equilibrium. Thus if β is large (the nonrenewable resource is much more important than the renewable one), it will be right to drive the renewable resource to extinction. Because the marginal utility of zero is unbounded, extinction will only happen asymptotically.

This discussion leads to

Theorem 2.7

In the context of the model of this section, if U_{12} is positive, then a steady state, if one exists, will involve a resource stock lower than that implicitly defined by $f'(x) = r$.

Theorem 2.8

If the utility indicator is Cobb-Douglas, then the steady state with a nonrenewable resource will be defined by $f'(x) = 1/(1 - \beta)r$; price of the renewable resource, v, will be decreasing exponentially at the rate $r - f'(x_s)$; and the stock of the nonrenewable resource will be run to zero. By comparison, if the second resource were perishable instead of nonrenewable, then the steady state would be defined by $f''(x) = r$, price constant.

This completes our discussion of the two perculiarities our model is designed to wed: demand equations for a renewable resource and the reproductive manners of trees. In the next section we present our full supply model.

Footnotes

1 P. Dasgupta and G. M. Heal, "The Optimal Depletion of Exhaustible Resources," Review of Economic Studies, Symposium 1975, p. 3.

Section 3 A Model for Estimation

Section 3.1

Continuous Time

What would an equilibrium model look like in continuous time? Let X(t,a) be the number of acres of timber at time t of age a. This is just like our optimal rotation age model except we now allow there to be forests of different ages at the same time. We insist that at any two times the amount of land in the forest is the same. If A is the set of possible ages, $\int_{z \in A} X(t,z)dz$ = a constant for all t. The growth equation for this system is just an aging equation. Define C(t,a) as the removal from class a in year t.

$$X(t,a) = X(t-z,a-z) - \int_0^z C(t-g,a-g)dg$$

for any z > 0. This equation says that what there is today is what there was last week or year less what was cut in the interim. The t-z,a-z occurs because z days ago the current class a was z days younger. What remains to be accounted for is what is cut.

$$X(t,0) = \int_{a \in A} C(t,a)da$$

To define the objective function we need to know one more thing: how much wood (or value) is there to an acre of age class a? The function M(a) answers that question. It is the growth function. The producers' problem is to max PV subject to the above equations.

$$PV = \int_0^\infty Qe^{-rt}[\int_{a \varepsilon A} C(t,)M(a)da]dt$$

Additionally, we require for each t that Q and (the term above in brackers) supply be compatible with some demand equation. It is no wonder that the mind boggles at solving this.

Before continuing to the discrete time model -- on which there is some information -- it is time to justify the use of a finite time horizon to approximate the infinite horizon. For any bounded set of prices, P and positive discount rate, r, and number E > 0 there is a time T so far away that the loss from neglecting times greater than T is less than E. Since there is a limited amount of land and prices are bounded, only a limited amount of value can be lost in any time period. Call that amount V. Clearly $\sum_{t=T}^{\infty} e^{-rt}V$ can be made as small as desired by appropriate choice of T.

The simplest model of a forest economy is one with perfectly anticipated prices, a linear technology, and a linear objective function.

The most common objective function for a resource holder is present discounted value. Given prices, present value is linear in quantity harvested. Although this linearity is a great computational advantage, it has some unpleasant implications for the evaluation of risk. Agents with linear objective functions are not risk averse. (They are risk neutral.) In a world of perfectly anticipated prices (no risk) this is not a serious drawback. The second problem with present value is that an agent using the present value criterion feels no need to spread his income over time. He may well execute a plan calling for no income for the plan's first one hundred years. The assumption of ability to borrow or lend at a fixed rate of interest makes consumption plans other than 100 years of starvation feasible. Present value is not an ideal objective function, but computational ease justifies its use.

Section 3.2

Linear Forest Model

The technology for a forest is easily described. Land with trees gets one year older every year until it reaches maturity and then it stays the same age. (A better description of the technology would include mortality. Some fraction of land bearing mature trees is returned to the zero age class because of the death of the trees.) To each age there corresponds a volume of marketable product: timber. (This formulation is close to that of Jungenfelt.)[1] Trees of zero age are bare land with tree seeds already planted. Harvesting is the act of converting old land (with old trees) to young land (with tree seeds). The land's age at harvest determines the volume of timber. Harvest can be thought of as producing a joint product: timber and seeded land. (There is forestry literature on the optimal amount of effort to replant and nurse trees but it always assumes constant prices.) Formally, let $X_{t,i}$ be the quantity of land at time t occupied by trees of age i. In the absence of cutting, $(0 < i < n) X_{t+1,i+1} = X_{t,i}$. (Unless i = n -- where n is the hypothesized maximum age for trees -- in which case $X_{t+1,n} = X_{t,n-1} + X_{t,n}$.) Let $C_{t,i}$ be the quantity of land in age class i cut in year t. Then $X_{t+1,i+1} + C_{t,i} = X_{t,i}$ if $0 < i < n$. The interpretation is X and C are an instantaneous division of land into two classes, one to cut and one to mature further. This allocation is made at the beginning of each time period. The recursion equation for X (bare land at time t) depends on the cut in all age classes. All of the above can be put into matrix notation:

Let

$$A = \begin{matrix} 0 & \cdots\cdots & 0 \\ \vdots & I & \vdots \\ 0 & \cdots\cdots & 0 \end{matrix}$$

$$B = \begin{matrix} 0, & 1,\ 1\ldots & 0 \\ \vdots & -I & \vdots \\ 0 & \cdots\cdots & \vdots \end{matrix}$$

then $X_t - BC_t = AX_{t-1})$

$$X_t = (X_{t,0}, \ldots, X_{t,n-1}$$

$$C_t = (C_{t,0}, \ldots, C_{t,n-1})$$

The minus sign before B is because of the definition of B. Its diagonal elements are non-positive.

The above formulation completely describes the dynamics of this model. It remains to describe the objective function. At the beginning of any period t, $(C_{t,0}, \ldots, C_{t,n})$ acres of land (bearing timber) are harvested. Assume M_i units (board feet) of timber are on each unit (acre) of land in age class i. Thus $C_{t,i}M_i$ board feet of timber are removed from land of class i in age t. Total removals are $\sum_{i=0}^{n} C_{t,i}M_i$ or $\langle C_t, M \rangle$. (inner product) at time t. If the prices to the resource holder net of cutting and planting costs are Q(t) the objective function is

$$PV = \sum_{t=1}^{T} e^{-rt} Q(t) \langle C_t, M \rangle$$

set $P(t) = e^{-rt}Q(t)$

The problem facing a resource holder is to maximize present value subject to initial conditions and a dynamic constraint. Thus

$$\max \sum_{t=1}^{T} P(t) \langle C_t, M \rangle$$

$$\text{s.t.} \quad \begin{pmatrix} I & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 \\ -A & I & 0 & \cdots & 0 & i & -B & 0 & \cdots & 0 \\ 0 & -A & I & \cdots & 0 & 1 & 0 & -B & \cdots & 0 \\ \vdots & \vdots & \vdots & & & 1 & \vdots & \vdots & & \vdots \\ & & & & & 1 & & & & \\ 0 & 0 & 0 & \cdots & -A & 1 & 0 & 0 & \cdots & -B \end{pmatrix} \begin{pmatrix} X(0) \\ \vdots \\ X(T) \\ C(1) \\ \vdots \\ C(T) \end{pmatrix} \leq \begin{pmatrix} \bar{X}_0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

This is a linear programming problem. Notice that the boundary conditions $X(0) = \bar{X}_0$ are incorporated in the first line of the matrix. There are only T time periods. No choices are made in period zero. Period zero just determines the stock available for division in period one. If the above matrix is abbreviated (A/B), the dual problem can be written

$$\min \langle \lambda(0), X(0) \rangle$$

$$\text{s.t.} \quad \lambda(A/B) \geq (0,\ldots,0,M\,P(1),\ldots,M\,P(T))$$

This instructs one to find the minimum value for land with trees in period or

$$\varepsilon(P_T\ M'(i^t) + M(i^*)P') \leqq P_T M(\ )$$

or rearranging and taking $\lim\ \varepsilon \to 0$

$$P_T\ M'(i^*) + M(i^*) + P_T' \leq P_T\ M'(0)$$

which is easy to interpret. The terms on the left are the change in the value of the standing timber. The right hand term is the value of the lost new growth. In a two-period model, with the periods very close together, it is both necessary and sufficient for cutting. Now one can inquire about i's other than i^*, say $i^* + z$. A necessary condition is:

$$P_T\ M'' + M'P' \qquad 0$$

or

$$\frac{M''}{M'} + \frac{P'}{P} \leq 0 \qquad \forall\ i > i^*$$

Notice that $M'' < 0$, $M' > 0$, $p < 0$ and $p' > 0$. Thus perversity depends on the curvature of the growth function compared to the percentage growth in price. When price doesn't rise there is no perversity. The cut-no-cut sequence just requires a reversal of all the inequality signs.

zero subject to the constraint that the minimum value be greater than or equal to the value of the anticipated harvests from time zero to time T. The value of land with trees (or without) in period zero is nothing other than the competitive selling price of the land. The dual problem is to find this competitive price of land: it is instructive to expand the dual constraint:

$$\lambda_t I - \lambda_{t+1} A \geq 0$$

$$-\lambda_t B \geq P_t M$$

or $$\lambda_{t,j} \geq P_t M_j + \lambda_{t,0}$$

$$\lambda_{t,j} - \lambda_{t+1,\min(j+1,n)} \geq 0$$

The equations say the shadow price of land in period t may not be less than either the value of the timber plus the shadow price of bare land or the shadow price of land of one year older age class in the next periid. To put this another way, in each period the entrepreneur chooses either to cut or to save trees of given age class. He chooses the option that maximizes his profits. This recursive formulation makes it very easy to solve the dual problem.

Remember that we are searching for a min of $<\lambda_0 - \lambda_0>$ where $X_0 > 0$. In the terminal period T, $\lambda_{t,j} \geq P_T M; + \lambda_{T,0}$ and $\lambda_{T,0} = 0$ ($M_0 = 0$ -- seeded land at the end of time is worthless). Why shouldn't $\lambda_{T,j}$ exceed $P_T M_j$? Because $\lambda_{T-1,j-1} \geq \lambda_{T,j}$ and one is trying to minimize $\lambda_{0,j}$. To find $\lambda_{T-1,j}$ one first needs to find $\lambda_{T-1,0}$. $\lambda_{T-1,0} - \lambda_{T,1}$. We conclude $\lambda_{T-1,0} = \lambda_{T,i} = P_T M_1$. Now we can see that

$$\lambda_{T-1,k} \geq P_{T-1} M_j + \lambda_{T-1,0} = P_{T-1} M_j + P_T M_1$$

$$\lambda_{T-1,j} \geq \lambda_{T,j+1} = P_T M_{j+1} \quad \text{or} \quad \lambda_{T-1,i} = \text{MAX} <P_T M_{j+1}; P_{T-1} M_j + P_T M_1>$$

Obviously this can be solved for any finite number of periods. For instance, one might find

$$\lambda_{T-3,j-3} = \lambda_{T-2,j-2} = \lambda_{T-1,0} + P_{T-1,}M_{j-1} = P_{t-1}M_{j-1} + P_T M_1$$

In general $\lambda_{0,j}$ will be the sum of a sequence of $P_t M_j$'s determined by the rule

$$\lambda_{t-1,j} = \text{Max} <\lambda_{t,j+1}; P_{t-1}M_j + \lambda_{t-1,0}>$$

Whenever

$$\lambda_{t-1,j} = P_{t-1}M_j + \lambda_{t-1,0}$$

we shall say

$$j \varepsilon J^*(t) = <j/\lambda_{t,j} = P_t M_j + \lambda_{t,0}>$$

It will not be hard to show that $C_{t,j} \neq 0$ iff $j \varepsilon J^*(t)$, that timber is cut iff the shadow price of timber is the market value of the timber plus the shadow price of bare land.

Another way to state this problem is to find a saddle point of

$$L(X,C,\lambda) = \sum_{t=1}^{T} <C_t, M> P(t) - \sum_{t=1}^{T} \lambda_t (X_t - BC_t - AX_{t-1})$$

The F. O. C.are:

1. $\partial L/\partial C_t = m\,P(t) + \lambda_t B \leq 0$

2. $\partial L/\partial X_t = -\lambda_t + \lambda_{t+1} A \leq 0$

3. $\partial L/\partial \lambda_t = X_t - B\,C_t - A\,X_{t-1} = 0$

The Kuhn-Tucker theorem further asserts that $C_{t,j} \frac{\partial L}{\partial C_{t,j}} = 0$ so $M_j(P(t) + (\partial_t B)_j < 0$ means $C_{t,j} = 0$. Similarly

$$\partial L/\partial X_{t,j} = 0$$

so $-\lambda_{t,j} + (\lambda_{t+1}A)_j < 0$ implies $X_{t,j} = 0$ since X_t - BC must equal a constant. Setting equation 1 equal to zero implies the other should be set equal to the appropriate component of AX_{t-1}. (It may happen that both $\partial L/\partial C_t$ and $\partial L/\partial X_t = 0$ -- this is an "edge" equilibrium -- any feasible mixture of C_t and X_t produces the same value of the objective function.) Thus we see that (for the case of present value) the dual problem can be solved recursively with great ease. The solution to the primal then follows from the complementary slackness conditions. The value of the solution to the dual problem is $\langle\lambda(0), X_0\rangle$ where $X_{0,j}$ is the value of bare land. Here "value" means present discounted value and it is also the price of land.

In short:

Theorem 3.1

Timber of class i is cut at time t if $\lambda_{t,0} + P_t M_i > \lambda_{t+1,i+1}$. Timber may be cut or saved if $\lambda_{t,0} + P_t M_i = \lambda_{t+1,i+1}$. Otherwise timber is saved.

Footnote

1 Carl Jungenfelt, personal discussion based on a study of his on Swedish forests. See: Mål och medel i skogspolitiken (Ends and Means in Forestry Policy), Sweden, 1973.

Section 3.3

The Linear Forest: Perversity

The usual description of forestry includes the policy "cut the oldest trees first." It is easy to show that this is not always an optimal policy. For example, consider a two-period forest with prices $P_{T-1} = 1$ and $P_T = 2$ and a growth function described by $M = (0, 8, 13, 14, 14.6, 15.19, 15.67)$. Note that $M_{i+1} - M_i$ decreases with i and M_{i+1}/M_i decreases uniformly towards one. These regularity conditions assure the growth function is concave. Now $\lambda_{t,i} = P_T M_j$ and $\lambda_{t-1,j} = \text{Max}\{\lambda_{T,j+1}; P_{T-1}M_j + P_T M_1\} = \text{Max}\{ P_T M_{j+1}; PM_j; +P_T M_1\}$

For our example:

j	T,j+1	$P_{T-1}M_j+P_TM_1$	delta	T-1,j
0	16	16+0 = 16	0	16
1	26	16+8 = 24	2	26
2	28	16+13 = 29	-1	29
3	29.2	16+14 = 30	- .8	30
4	30.38	16+14.6 = 30.6	- .22	30.6
5	31.34	16+15.19 = 31.19	+ .15	31.34

In period T-1, age classes 2, 3 and 4 should be cut while classes 1 and 5 should be allowed to grow. Thus the set $J^*(t)$ introduced earlier is not connected. It is not enough to specify a division point in the age distribution of the forest; all of the calculations must be done.

By our example we have proved

Theorem 3.2

In an optimal policy, old timber may be cut after younger timber.

The result cited above -- cut younger trees first -- flies in the face of intuition. Were there no rent, the condition for indifference between cutting and not would be the percent change in present value of growing stock equals the rate of interest.

$$\frac{\dot{PV}}{PV} = \frac{\dot{X}}{X} + \frac{\dot{P}}{P} = r$$

In this sort of world faster growing things always stand longer than slower growing ones. But in the world described in the preceeding models does have rent. Present value (of growing stock) is price times quantity plus the land value.

$$PV = P \cdot M + \text{rent}$$

$$\dot{PV} = \dot{P}_L M + P_L \dot{M}$$

$$\frac{\dot{PV}}{PV} = \frac{\dot{P}}{P + \text{rent}/M} + \frac{\dot{M}}{M + \text{rent}/P}$$

where, as usual, M is the growth function and P is price. The second term of the RHS behaves properly, it decreases in the age of trees (M), but the first term of the RHS behaves paradoxically: It increases in age. Why does the capital gain term from holding growing stock increase with age (or volume)? It is because it is a percent change in value which the timber but not the rent undergoes and rent is a larger proportion of the value of young stands than it is of old age stands. Since the price of bare land is constant while the timber volume may be great or small, the percent of the present value made up by the timber volume varies as the timber volume. The owner of the plot of land owns both the land and the timber. While the timber grows the land does not. But to keep from switching to the best alternative use, the owner needs the return on his whole investment to rise at the rate of interest. Should timber be only a small fraction of the value of his investment, then the gain (in percentage terms) from a price increase will be less than the gain would be if timber were a large fraction of the investment, because only timber and not land are subject to the increase.

Let us further explore the phenomenon of cutting younger trees first. Let ε be the length of the time periods. There is some i^* at which one is indifferent between cutting and not.

$$P_{T-\varepsilon}M(i^*) + P_T M(\varepsilon) \quad = \quad P_T M(i^* + \varepsilon)$$

Moreover, assume for small z, $i^* + z$ age trees are worth cutting.

$$P_{T-\varepsilon}M'(i^*) \;>\; P_T M'(i^* + \varepsilon)$$

One would like to know if a no-cut regime will occur for any age older than i^*. That is, does

$$P_{T-\varepsilon}M(i^* + s) + P_T M(\varepsilon) \;<\; P_T M(i^* + \varepsilon + s)$$

This depends on

$$P_{T-\varepsilon}\int_0^s M'(i + z)dz \;<\; P_T \int_0^s M'(i + \varepsilon + z)dz$$

for some s. Since M is concave $(d^2_M/d_i^{\,2} < 0)$ one knows that the integral on the right is smaller than the one on the left for any value of s. But this is not enough to rule out perversity. One does not know how much larger the left hand integral is; the difference in $P_{T-\varepsilon}$ and P_T could outweigh the good effects of the concavity. However, this does clarify one case: if $P_T - \varepsilon \;\geq\; P_T$ and a no-cut-cut transfer occurs, no-cut will not recur.

This can also be stated for small -- in differential form. Again $P_T M(i + \varepsilon) \;\leq\; P_T M(\varepsilon) + P_T - \varepsilon M(i^*)$ which is the condition for cutting at i^*. Equivalently:

$$P_T M(i^*) + P_T \varepsilon M'(i^*) \;\leq\; P_T M(\varepsilon) + M(i^*)P_T - \varepsilon \frac{dP}{dT} M(i^*)$$

So far, the discussion has been couched only in terms of a two-period model. In a multiperiod model, the failure of price to rise is again sufficient to assure no perversity. Suppose class i is to be cut at time t in an optimal program.

$$P_T M(i) + \lambda_{t,0} \geq \lambda_{t+1,i+1}$$

and

$$\lambda_{t+1,i+1} = \text{Max}\{P_{t+n} M(i + n) + \lambda_{t+n,0} | n \epsilon z,\ t+n < T,\ n > 0\}$$

Thus we have T-t inequalities of the form

$$P_T M(i) + \lambda_{t,0} > P_{t+n} M(i + n) + \lambda_{t+n,0}$$

and the same analysis we used above guarantees us that all ages older than i will be cut.

Theorem 3.3

Older timber will not be cut after younger timber if

$$M''/M' + P'/P \leq 0$$

and this implies that P/P negative is sufficient to rule out the perversity.

Section 3.4

Partial Equilibrium and the Linear Forest

A partial equilibrium world is interesting because it corresponds to an approximation of a rational expectations world. Solving for the partial equilibrium gives a set of prices and actions that are mutually compatible. That is, if the producers believed the prices from the partial equilibrium model and acted as if these prices would obtain in the future, then the producers would take actions that would make those prices come true.[1] The partial equilbrium is an approximation because it ignores income effects. No account is taken of the money producers get from selling lumber. In particular, it is assumed that an addition to stumpage prices which changes the income of firms and therefore of consumers, does not change the consumer's demand for stumpage. Two justifications are offered. First, the numerical size of the income effect is so small it can be ignored. Second, a change in instantaneous price does not effect permanent income and thus does not cause an income effect.[2]

Consider a partial equilbrium world where the linear model defines the supply set and correspondence for lumber and a set of functions, $D_t(P_t)$, $t = 1,\dots,T$, defines demand. The model becomes producer:

$$\max \sum_{t=1}^{T} I_t$$

$$\text{s.t.} \quad \begin{aligned} X_3 - BC_3 &= AX_2 \\ X_2 - BC_2 &= AX_1 \\ X_1 - BC_1 &= AX_0 \quad , \qquad \text{etc.} \end{aligned}$$

$I_t = P_t \langle M,C \rangle$

Consumer: $D_t(P_t)$

where demand is a function of own period price only

Prices: $P = (P_1,\ldots,P_T)$

Balance: $D_t(P_t) = \langle M,C \rangle$

To make the competitive assumption more plausible, one could imagine n identical producers and j identical consumers and divide all quantities by j or n. Of course, this wouldn't change anything.

We prove:

Theorem 3.4

There is a unique multimarket partial equilibrium.

The existence of a partial equilibrum is easy to demonstrate. Let $S(P_1,\ldots,P_t)$ be the supply correspondence from R^T to R^T. Because the technology is neoclassical (closed, convex, contains 0, no free producetion), S is upper semi-continuous. Let p^* be the inverse of the demand function. If p^* is continuous, then $S \times p^*$ is upper-semi-continuous from the space of prices crossed with the space of quantities to itself ($R^T \times R^T$). If one could find a (nonempty) compact, convex restriction of $R^T \times R^T$, call it A, with the property that $S \times p^*(A)$ A one could apply the Kakutani fixed point theorem. Since we take the quantity of land is fixed, the set of possible productions, R, is easily seen to be compact and convex. If p^* is continuous, then the set of possible prices $p^*(R)$ is also compact and its closed convex hull $\bar{p}^*(R)$ has all the desired properties. Thus $A = R \text{ v } \bar{p}^*(R)$ is the sought after restriction and there is a fixed point in A. The fixed point is the multi-market partial equilibrium.

Does this provide a unique solution? Suppose p^*, c^* is a partial equilibrium. Could p', c' be another partial equilibrium? Let $p = p^* - p'$ and $c = c^* - c'$. $\Delta p \Delta c \geq 0$ because the technology is neoclassical. But the assumption that demand curves slope down means $\Delta p \Delta c < 0$. Obviously, the two condiitons cannot both be met. There can be only one partial equilibrium.[3]

We now know there is a unique multimarket partial equilibrium. Unfortunately that does not tell us how to find it. In theory, Scarf's algorithm will find any fixed point. A more practical method takes advantage of the problem's structure. It is possible to break the problem down into a sequence of subproblems, one for each period. The assumptions on demand (negative -quasi -semi -definite jacobian) make it possible to view the problem as one of maximizing a concave function. There are well-known algorithms whose convergence are guaranteed, that will do this. But first we turn to Scarf's algorithm.

In theory, one could use Scarf's algorithm. In practice there are a few drawbacks. Scarf suggests it is computationally unfeasible for a model with many markets. Here the contemplated dimension is many hundred years -- thus many hundred markets. The second objection to Scarf's algorithm is that it provides only a good approximation to the true location of the equilibrium price-quantity. That is, it guarantees to terminate with quantities so chosen to that supply-demand is small, but not necessarily quantities near (in the Euclidean metric sense) the true equilibrium.[4] To break the problem down to a period by period problem, first look at the producer's first order conditions for the last period:

$$P_T M + \lambda_T B \leq 0$$

$$-\lambda_T \leq 0$$

but λ_T as small as possible

and $P_T = P(<C_t, M>)$ imply λ is a function of AX_{T-1}

$X_T - BC_T = AX_{T-;}$ and $<$ implies $C_{T,J} = 0$

$$P_{T-1}M_j + (\lambda_{T-1}, B)_j \leq 0$$

$$\lambda_T A \leq \lambda_{T-1}$$

$$X_{T-1} - BC_{t-1} = AX_{T-2}$$

$$P_{T-1} = P(<C_t, M>)$$

$$-\lambda_{T-1,j} + (\lambda_T A)_j < 0 \text{ implies } X_{T,j} = 0$$

yields $\lambda_{T-1}(AX_{T-2})$

The implication here is that $\lambda_{T-1}(AX_{T-Z})$ can be found by reference to $\lambda_T(AX_{T-1})$ and reference to the appropriate functions in period T-1. Moreover, the solution must exist and it is unique. Thus, the partial equilibrium solution for every period, as a function of its endowment, can be found by reference to the next period's shadow prices and the own period production and demand functions. This would be a very nice property if there was any hope of an analytic solution for the shadow prices. Even though an analytic solution is beyond reach, the recursive nature of the model allows the easy use of numerical methods. $\lambda_T(AX_{T-1})$ can be tabulated for a couple of hundred values and extrapolated for all values inbetween. At each step in the procedure, only the approximations used in the previous step need be kept, if all that is wanted is the initial shadow values and plan. This greatly reduces storage requirements.

This leaves us to solve the repeated problem: given AX_{T-1}, $\lambda_{T+1}(AX_T)$, $P_T = P(C_T, M)$, find C_T

s.t. $P_T = P(\langle C_T, M\rangle)$ -- supply equals demand

$X_T + BC_T = AX_{T-1}$ -- production function

$\lambda_{T,i} = \max\langle P_T M_i + \lambda_{T,0}(AX_T); \lambda_{T+1,i+1}(AX_T)A\rangle$

$M_i P_T + (\lambda_T B)_i < 0 \Rightarrow (C_{T,i} = 0$

$-\lambda_{T,i} + (\lambda_{T+1}A)_i < 0 \Rightarrow X_{T,i} = 0$

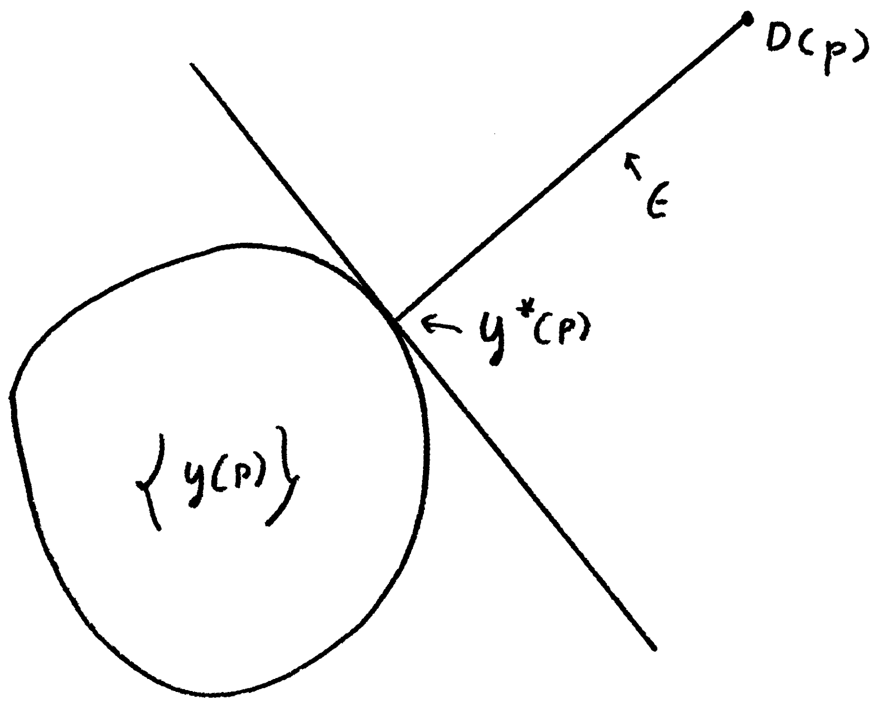
Suppose we were to choose a trial solution C_T'. By the first three conditions above we would have λ_T', X_T', and P_T'. On examination of conditions four and five, we would likely find there is an i for which $M_i P_T + (\lambda_T B)_i < 0$ and $C_{T,i}' \neq 0$ and some for which $-\lambda_{T,j} + (\lambda_{T+1}A)_j < 0$ and $X_{T,j}' \neq 0$. Consider what happens if in the latter case $C_{T,j}$ is increased some. One gets a new P_T lower than P_T'. $X_{T,o} > X_{T,0}'$, $X_{T,j} < X_{T,j}'$. Since there is less timber in period T+1, $\forall_i\ \lambda_{T+1,i} \geq \lambda_{T+1,i}'$, what happens to λ_T? Well, $P_T M$ does fall, but $\lambda_{T,0}$ rises. Thus, we would not appear to know what happens to $P_T M + \lambda_{T,0}$, though we would expect it to decrease. In short, although we don't know for sure, we expect the shadow prices for good j to move in the right direction to redress $-\lambda_{T,j} + (\lambda_{T+1}A)_j < 0$. Unfortunately, the shadow prices for all the other age classes behave this way, and many of those classes already satisfied the complementary slackness conditions. Thus, a naive movement in what appears to be the right direction might do much more harm than good.

The way to find the equilibrium is

Theorem 3.5

A plan that minimizes the sum of consumer surplus (with Marshallian not Hicksian demands) and producer surplus is also the market equilibrium.

Figure 3.1



Footnotes

1 John F. Muth, "Rational Expectations and the Theory of Price Movements," Econometrica 29, No. 3, July 1961.

2 Truman Bewley and Hal Varian, Consumer Surplus and the Permanent Income Hypothesis, Harvard and M.I.T., 1975 (unpublished); and Robert D. Willig, "Consumer Surplus Without Apology," AER (forthcoming).

3 Gerard Debreu, The Theory of Value. New York: Wiley, 1959.

4 Herbert Scarf with collaboration of Terje Hansen, The Computation of Economic Equilibrium. New Haven: Yale University Press, 1973.

Section 3.5

Consumer Surplus -- A Solution

This section exhibits a constructive algorithm to find the multi-market partial equilibrium. Let $\pi(p)$ be the restricted profit function at prices p and land allotment X_0. $\pi(p) = \lambda_o X_o = \sum_t P_t Y_t$ where $Y_t = \{C_t, M\}$ and is the amount of timber cut in period t. $\{Y(p)\}$ is the supply correspondence $Y(p)\varepsilon\{Y(0)\}$ is some point in the supply correspondence $\pi(p) = p \cdot Y(p)$.

Theorem 3.6

With these preliminaries, define $V(p) = \sum_{t=1}^{N} (\int_{P_t}^{\infty} D_t(z)dz) + \pi(p)$ V will be shown to have a unique minimum and at that minimum $D(p) - \{Y(p)\} = 0$. Minimizing $V(p)$ is equivalent to finding the equilibrium.

Claim: If $D(p) + \{Y(p)\} \neq 0$, then there is a direction of decrease for V at p.

Let $Y^*(p)\varepsilon\{Y(p)\}$ minimize the euclidean distance between D(p) and the set $\{Y(p)\}$. The direction of decrease will be

$$\varepsilon = \frac{-D(p) + Y^*(p)}{||D(P) - Y^*(p)}$$

First it must be shown that $\{Y^*(p)\}\varepsilon\overline{\{Y(P+t)\}|t = Y_2, Y_1, 1/8, \text{etc.}}\}$ where the ——— denotes closure. Y^* must be a limit point because, if not, setting ε = excess supply at point Y^* would produce a new supply point that could be anywhere in the supply set. The assertion on Y^* is true because of the law of supply: $\Delta p \Delta Y \geq 0$. Let $\Delta p = t_2$ and choose Y^* as the starting point. $Y = Y^* - Y_{new}$. The definition of ε assures us that ε is orthogonal to a supporting hyperplane for the supply set at Y^*. All candidates for Y_{new} must either be on that hyperplane or not in the supply set. Thus Y^* is certainly a member of $\langle \lim_{t \to 0^+} y(p + t\varepsilon) \rangle$. Now we are ready to examine V.

$$D_\varepsilon V = D_\varepsilon \Sigma \int D(z) + \lim_{t \to 0} \frac{\pi(p + \varepsilon t) - \pi(p)}{t}$$

and is the directional derivative. $D_t \varepsilon \int D(z) = \langle \varepsilon \cdot D(p) \rangle$.

$$\lim_{t \to 0^+} \frac{\pi(p + \varepsilon t) - \pi(p)}{t} = \lim_{t \to 0^+} \frac{p\{y(p + t\varepsilon)\} - p \cdot \{y(p)\}}{t}$$

$$+ \lim_{t \to 0^+} \varepsilon \cdot y(p + t\varepsilon)$$

The first term vanishes. The second term $\to \varepsilon\, y^*$ because of the lemma above. Thus $D_\varepsilon V = \varepsilon \cdot (-D(p) + Y^*(p)) < 0$ and p not an equilibrium prices implies there is a direction of decrease for V.

Since one can choose prices so high that all goods are in excess supply and so low that all goods are in excess demand, one can limit p to a compact subspace, and assure that V does not take its maximum value on the boundary of that subspace. This assures that V has a finite max (because it is continuous). The sequence of V's created by always moving in the direction ε is both monotonically decreasing and bounded. It converges to the max of V and the equilibrium of the multi-market system.

Remark: At the bottom of this exposition is the negative quasi-definiteness of the demand function's jacobian and the Poincaré-Hopf lemma. [1]

Although it is now possible to find the equilibrium, it is only possible to tell a little about it. If one starts with a virgin forest, discounted producer price P must rise fast enough to induce the resource holders not to cut. After inventories have been liquidated, price goes into a "limit cycle" (because it is bounded). One would like to know more about the long run behavior, but all that can be said is that the limit cycles are gentle. Choose λ_{t+2}, then $\lambda_{t+1,j} = \max(P_{t+1}M_i + \lambda_{t+1,2}, \lambda_{t+2,t+2})$ let class i be some class cut in t + 1

$$\lambda_{t+1,i} = P_{t+1}M_i + \lambda_{t+1,1}$$

Let j be cut in t. What would happen if we let it grow?

$$\lambda_{t,j} = P_t M_j + \lambda_{t+1,1} \geq P_{t+1}M_{j+1} + \lambda_{t+2,1}$$

Now $\lambda_{t+1,1} \geq \lambda_{t+2,2}$ and $\geq P_{t+1}M_1 + \lambda_{t+2,1}$. Thus

$$-P_t M_j + P_{t+1}M_{j+1} \leq \lambda_{t+1,1} - \lambda_{t+2,1}$$

since $\lambda_{t+2,3}$ can only underestimate the truth:

$$\lambda_{t+2,2} - \lambda_{t+1,1} \geq P_{t+1}M_{j+1} - P_t M_j$$

and by use of the lemma on the following page

$$\lambda_{t+2,2}\left(1 - \frac{m(1)}{m(2)}\right) \geq P_{t+1}M_{j+1} - P_t M_j$$

which limits the change in prices.

Imagine the managers of land of class j follow the policy for land of class i instead of following the policy optimal for land of class j. Call the rent λW. The first time the land is cut, the manager earns $P_{t+z}M(i+z)$ while the managers of class j land earn $P_{t+z}M(j+z)$ or $\frac{M(i+z)}{M(j+z)}$ as much as the j managers. Since uniform

$$\lim_{s\to\infty} \frac{M(i+s)}{M(j+s)} \to 1$$

we know that $\lambda_{t,i} \geq \lambda W_{t,i} \geq \frac{M_i}{M_j} \lambda_{t,i}$

Thus

$$\lambda_{t,i} - \lambda_{t,j} \leq \lambda_{t,j}(1 - \frac{M_i}{M_j})$$

The above discussion shows that:

<u>Theorem 3.7</u>

<u>In the long run p is restricted to a closed interval and its rate of change is bounded.</u>

This theorem refers to a model in which the time horizon is infinite. It is true because prices are bounded and it is a very weak statement. The theorem one would like to prove is that the price path would converge to some exponential path in the long run. With periods separated by finite time intervals long run exponential prices won't necessarily happen. (I have created cyclic examples for the infinite time case.) I conjecture that the multimarket equilibrium model will settle to a stable equilibrium with even age distribution if it is specified in continuous time.

A model of interest to the forestry profession is that of a forest with an initial condition of equal amounts of land at all ages less than a certain rotation age and no land bearing trees older than the rotation age. This is a perfectly regulated forest. If there is the same demand curve every period, and the rotation age is determined from the Faustmann formula of Chapter One, $\dot{X}(L)/X(L) = r/(1 - \exp(-rL))$, then it is obvious that the policy of cutting all trees at the rotation age and having the price be the price that clears the market will be a steady state and a partial equilibrium. This is Theorem 3.8.

Theorem 3.8.

If 1. Demand is constant over time.

2. The rotation age L is defined by the Faustmann formula.

3. Land is divided evenly among all classes of age less than L.

Then, a policy of cutting trees when they reach age L will result in a constant price for stumpage and constant supply of stumpage. The state of the economy is a multi-market partial equilibrium. It is also a steady state.

There is an easy extension of this model to a world with uncertain demand. Say $P_i = P_i(\langle M,C\rangle)\cdot\varepsilon_i$ where ε is a random variable of known distribution $F(\varepsilon)$, the ε_i's are i.i.d. For the sake of comparibility, choose ε so that $\bar{\varepsilon} = 1$; ε_t revealed in its own period and $\varepsilon \geq 0$. Now our problem is:

$$\text{producer:} \quad \max \Sigma E(P_t(\langle C_t,M\rangle))$$

$$\begin{aligned}
\text{s.t.} \quad & X_3 - BC_3 = AX_2 \\
& X_2 - BC_2 = AX_1 \\
& X_1 - BC_1 = AX_0, \quad \text{etc.}
\end{aligned}$$

$$\text{consumer} \quad P_t = P(D_t)\varepsilon_t^{\max}$$

$$\begin{aligned}
\text{balance} \quad & D_t = \langle C_t,M\rangle \\
& I_t \equiv D_t
\end{aligned}$$

"E" denotes the expected value.

Look at the last period first. Given X_2 we could find (if we were lucky) $E(I_t) = \int I_t(\varepsilon)dF = P(\langle C_t,M\rangle)\int \varepsilon dF = P(\langle C_t,M\rangle)\bar{\varepsilon}$ and $\bar{\varepsilon} = 1$. Thus, $\lambda_3(AX_2)$ can be found just as before. Now consider period 2. The choice the producer makes in period 2 is dependent on the realization of ε in period 2. For instance, if it is zero, the optimal policy is to cut nothing at all. For any given ε one easily finds the correct solution for all the variables and finds $\lambda_2(AX_1)|_{e_2}$. Duality assures us that $\lambda_2(AX_1)/e_2\cdot AX_1 = \max_{t=2} \sum^{3} E(I_t)$ subject to the constraints and given e_2. We take the expected value of both sides of this equation to get $\lambda_2(AX_2)$. One proceeds to solve this in the same way for each earlier period.

One remark on comparing this to the certainty model: Imagine that an entrepreneur made plans for the certainty case and executed them in the uncertain demand world just described. The expected value of his plan in the uncertain world would be equal to the value of his plan in the world with certainty. Therefore, an optimal plan in the uncertain world has a greater expected value than the certain plan does. The following (by now familiar) rule should make this clear:

$$\lambda_{t,i}\,|_{\varepsilon} = \max< P_t \varepsilon M_i + \lambda_{t+1,1};\ \lambda_{t+1,i+i}>$$

Thus $\lambda_{t,i}$ is bounded from below by $\lambda_{t+1,i+1}$. So $E\lambda_{t,i}$ exceeds both $P_t\bar{e}M_i + \lambda_{t+1}$ and $\lambda_{t+1,i+1}$ because $P_t\varepsilon M_i + \lambda_{t+1,0}$ is chosen when ε is large, and $\lambda_{t+1,i+1}$ is chosen when ε is small.

One should notice that the introduction of ε poses only a small additional computation burden, that of solving each period's problem enough times to estimate $E\lambda_t(AX_{t-1})$.

Footnote

1 This section benefitted greatly from a discussion with Hal Varian.

Section 4

Section 4.1

Brief Description of Model:

There are two types of agents, consumers and producers. Consumers are represented by their demand curves. Their demand in each period is taken to be the translog factor share approximation discussed in the [1] succeeding section with the constraint that the log own price term be zero. (That the long run elasticity of demand is one.) For periods in the sample time (1950 to 1972) the demand equation is the translog estimate with the actual values of the independent variables. For the periods after the sample time, the independent variables are estimated by ordinary least squares regression on a constant and the log of a trend term. These ordinary least squares estimates are then projected into the future. See Table 4.3. The major problems with this procedure are two-fold. The estimate of the demand curve has a good deal more price elasticity than seems warranted from the data, even in the long run. This problem will be discussed again later. The demand cures are supposed to represent the producer's (rational) expectations of demand. Can the producers perceive shifts in the value of construction fast enough to be able to smooth the year to year price differences? Although by using the actual value of construction for the years of the sample period it is assumed the producers are able to perceive the year to year changes in demand, it is not clear that this is so. Using the trended value of output even for the sample period would correspond to the assumption that producers can not respond in time to smooth year to year fluctuations. In retrospect this seems like the better assumption.

Producers are assumed to maximize the present value of their profits from the sale of timber, subject to biological constraints on the growing of timber and their expectations of price. Producers expect prices to be such that supply and demand are equated at every time t. This is rational expectations. The most severe problem here is that there are at least two important classes of producers -- private and public. Although the profit maximizing assumption makes sense for the private producers, it is not clear that it makes sinse for the Forest Service. The Forest Service loudly announces that it makes its decisions based on noneconomic criteria. The position taken here is that although the Forest Service cuts less than the competitive producers in the early part of the period, they are subject to heavy industry pressure and the effect of that pressure has been to assure management practices (although not rhetoric) very close to that of the prive producers in the late part of the period. (If, indeed, the Forest Service cuts less than a competitive firm would there are at least three justifications: 1. the Forest Service is charged with providing recreation and range from the lands it administers, private industry is not. 2. The Forest Service tends to own land of lower productive capacity than the private sector. There is less incentive to cut such land for its future growth. 3. If it is right for the private sector to hold even one of a given age class, then, because of constant returns to scale, it is right for the Forest Service to hold <u>any</u> amount of trees in that age class.)

Producers view the world as having seven periods and the technology for tree growing described in the sections called Linear Supply Model (Section 3) and Supply Considerations (Section 4.3). Briefly, the supply model says that, unless land with trees is cut, it gets one year older every year. Land with older trees has more lumber than land with younger trees. Land that is cut becomes land of zero age. Given the technology described by the linear supply model and the functional values chosen in the section called Supply Considerations, the producers are viewed as profit maximizing competitors. For any set of prices p and behavioral parameter, r, each producer chooses an action (harvest) c to maximize

$$\pi = \sum_{t=1}^{7} < C(t), M > e^{-rt} \; p(t) \qquad \text{s, t. Biological Constraints}$$

where M is a vector of the lumber contents of one acre of forest at various ages. The interest rate parameter is the producers' rate of nominal time discount. It includes a constant expected rate of inflation. (Because all the pieces of the model are linear homogenous of degree zero in prices and income, a changed unexpected rate of inflation will not affect the policy decisions of the producers.)

It may be that many policies, C, will maximize profits for given p. For the purpose of the next few paragraphs, assume that C is unique. The producers' profit maximization problem is sold for C(p), the supply function, and $\pi(p) = \langle p, \langle C(p), M \rangle\rangle$ the profit function.

The question of what prices to use remains. Rational expectations are posited. That is, $< C(p), M > = D(p)$ is to be sold for price. In fact, this is done by the computer by maximizing $\sum_{t=1,7} \int_b^P D_i(p_i) - \pi(p)$, where b is any small number. The first order conditions for a max are $D_t = < C(p), M >_t$, which are nothing more than the conditions for an expected equilibrium. (The second order conditions are obviously satisfied because $\frac{\partial^2 V}{\partial p^2}$ is negative definite.)

Footnotes

1 Ernst R. Berndt and David Wood, "Technology, Prices, and the Derived Demand for Energy," Review of Economics and Statistics LVII, No. 3 (August 1975), p. 259.

Section 4.2

Supply Model Details

The description above is too heuristic for those who might wish to reproduce the results below. Following the notation of Section 3.1, define A, B, and E as in Table 4.2.1 and Bioconstraints as in Table 4.2.2.

The producers' problem, given prices, is to choose C(t), t = 1,7 to maximize

$$\pi(p) = \sum_{t=1,2} < C(t), M > P(t) + \sum_{t=3,7} e^{-30rt} < C(t), M > P(t)$$

subject to the bioconstraints.

Notice that the bioconstraints and the profit function both imply that land gets no older between periods one and two. This is because periods one and two are one year apart while the other periods are thirty years apart. The reasons for this are described below.

The computer maximizes $V = \sum_{i=1,7} {}_b\int^P D(p)\, dp - \pi(p)$ on "p" by the following gradient algorithm.

1. Choose $P_o \varepsilon R^7$, set n = 0.
2. Find $D(P_n)$, C at P_n -- call it C_n and $V(P_n)$.
3. If C has many values, i.e., if π takes its maximum value on a set $\{C_n\}$, choose $C_n \varepsilon \{C_n\}$ to minimize $||D(P_n) - C_n, M||$.
4. $G_n = D(P_n) - \langle C_n, M \rangle$.
5. Find a step size, E, small enough so $V(P_n + EG) > V(P_n) + G^2$. Section 3 shows that this can be done.
6. $P_{n+1} = P_n + EG$, n = n + 1, go to 2.

The algorithm will converge to p the expected prices and C the anticipated actions. As described below, $\langle C, M \rangle$ is the estimate of supply, given the producers' rate of discount, r. The discount rate, r, is then chosen to make the harvest predicted by the model and the actual harvest as close as possible.

Table 4.2.1

A =

0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0
0	0	1	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	1	0	0	0
0	0	0	0	0	1	0	0
0	0	0	0	0	0	1	1

B =

0	1	1	1	1	1	1	1
0	-1	0	0	0	0	0	0
0	0	-1	0	0	0	0	0
0	0	0	-1	0	0	0	0
0	0	0	0	-1	0	0	0
0	0	0	0	0	-1	0	0
0	0	0	0	0	0	-1	0
0	0	0	0	0	0	0	-1

E =

0	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Table 4.2.2

Bioconstraints

$$X(0) = X_0$$

$$X(1) = X_0 + B\ C(1) - E\ C(1)$$

$$X(2) = X(1) + B\ C(2) + E\ C(1)$$

$$X(3) = X(2) + B\ C(3)$$

$$X(n) = X(n-1) + B\ C(n) \qquad n = 4,7.$$

Section 4.3

Supply Considerations: Data and Growth

The data to build a supply model come from two souces. The Forest surveys of 1970 and 1963 (called Outlook and Timber Trends, respectively)[2] provide observation of the forest inventory and removals in those years. The Department of the Census, Current Industrial Reports series, provides data on lumber and plywood production on a yearly basis.[2] If the 1970 and 1963 data were comparable, one could construct a growth function from this information. In fact, they are noncomparable. What I have done is use a normal yield table for the shape of the growth function and the data in the Forest Surveys for the scale of the function. Using the forest service estimates of growth (5206M bd. ft. in 1970, 4582M bd. ft. in 1963) and the census removals estimate, there is a huge discrepancy. The 1970 survey "discovered" 17,000M bd. ft.

In 1930 Richard McArdle, then a Forest Service Silviculturist, summarized forest measurements in a series of normal yield tables for second growth Douglas Fir.[3] Their tables were updated and revised by Walter Meyer and later Donald Bruce (1949) and were revised again in 1961. The yield tables list the following data: Average dbh, normal number of trees per acre, volume per tree, and diameter growth by stand age and initial diameter. "Dbh" is an abbreviation for "diameter at breast height." Normal number of trees per acre is the number of trees per acre if there were no natural catastrophes and if the acre were completely reseeded to begin with. Normality is the ratio of actual trees per acre to the normal number. In practice these conditions are rarely met.

These tables were meant to be used to predict the growth or yield of a homogeneous stand of trees. A forester could count the trees per acre to estimate normality and core the trees to estimate age. Age is important because trees grow much faster under good conditions than bad conditions. For a given diameter at breast height, a younger tree is expected to increase diameter faster than an older tree. The younger tree is said to grow on higher site class land. To use the normal yield tables for an aggregative growth function (a purpose they were not intended for), one must know the diameter-age relationship (how good a site is) and normality. The survey data in Outlook gives the distribution of growing stock by diameter classes, acres occupied, and annual growth. Choice of a site class (or age-diameter relationship) implies a yield table. Applying the yield table and an estimate of normality to the survey data implies acreage and growth. Thus, one can choose the site class index that most closely matches the reported growth and acreage. After playing with many different site class normality assumptions, I chose to assume the growth function shown on table 4.3.1.

Using index 140 overpredicts growth, but gives a reasonable estimate for normality. Using a higher site index would predict even more growth and would lose normality. A lower index would raise normality (which wouldn't be at all belieavable) but would bring growth more in line. The table above is clearly a compromise. It is hoped that it is a reasonable approximation for an aggregative growth function.

Table 4.3.1

Age	M bd. ft. Volume	Acreage (millions)	Bd. ft./acre int. $\frac{1}{4}$"
0	0	1.71	0
30	0	3.6	0
60	35,500	3.91	9,100
90	49,059	2.37	20,700
120	35,584	1.28	27,800
150	51,675	1.59	42,500
180	284,477	4.40	64,653

Site Index $\stackrel{o}{=}$ 140 Normality = .8

Footnotes

1 U.S. Department of Agriculture,Forest Service, Forest Resource Report 18, Timber Trends in the U.S.; Forest Resource Report 20, Outlook for Timber in the U.S.

2 U.S. Government, Department of the Census, Current Industrial Reports.

3 Richard McArdle, Walter Meyer and Donald Bruce, U.S. Department of Agriculture, Technical Bulletin 201, 1949.

Section 4.4

Demand Model:

There is considerable difficulty in estimating the demand function for Douglas Fir stumpage. Conceptually stumpage demand is derived from the demand for lumber and plywood, and, ultimately, the demand for construction. Seventy-two percent of all lumber and fifty percent of plywood is used in construction.[3] Douglas Fir is more likely to be used for construction than most types of wood. The other major end use of Douglas Fir is furniture, or home furnishings. My intentions were to break output down to residential and nonresidential construction as well as furnishings and specify stumpage demand to be the sum of the relevant Diewert factor demands. This fails. I aggregated the value of output data of furnishings and consumption by the share of all materials in each sector (.645 for construction and .570 for furnishings). I denote this new number as value and use it as my income variable. I estimated the demand equation in many different forms; the best fit, in terms of asymptotic t statistics, believability of estimated elasticity of demand with respect to own price, and classification of other goods into complements and substitutes, was a simple log linear form with the income elasticity of demand set equal to one. This assumption corresponds to a conditional factor demand derived from a constant returns to scale technology. (This assumption was tested: A 99% confidence interval on the value coefficient includes 1.0). The equation was specified with a moving average of prices other than own price, own price, and a moving average of past own prices. Own price and the price of other lumber were considered to be jointly determined. All other prices were exogenous. Two stage least squares was used throughout. The equation was Equation 4.4.1.

Table 4.4.1

List of Variables and Their Classifications:

PDFL	en	price of Douglas Fir lumber in dollars/ thousand bd. ft., mill tally
LPROD	en	production of Douglas Fir lumber, million bd. ft., mill tally
VCON	p	value of construction put in place
MWAG	p	mill wages
PDFS	en	price of Douglas Fir stumpage, dollars/ thousand bd. ft., international 1/4" log rule
PS	p	moving average of PDFS for three years, lagged once
PDFP	en	price of plywood: WPI
REMO	en	removals of Douglas Fir in million bd. ft., international 1/4" log rule
QDFSP	en	plywood requirements of Douglas Fir, million bd. ft., international 1/4" log rule
VFURN	p	value of furnishings, millions of dollars: national product accounts
PEXA	p	price of autos WPI
PEXB	p	WPI iron and steel
PEXC	p	WPI nonferrous metals
PEXD	p	WPI nonmetalic structural minerals
PEXE	p	WPI rubber and plastic products
PEXF	en	WPI all lumber
PEXG	p	wages in construction

STOCK	p	gross estimate of the stock of Douglas Fir, million bd. ft., international 1/4" log rule
PEXH	p	WPI board
TREN	p	natural log of a linear trend -- 1972 = 40, 1949 = 15

p = predetermined   en = endogenous

Equation 4.4.1

REMO - VALUE =	- .83	- .283 PDFS	.365 MWAG	3.37 PEXA
	(.234)	(.129)	(1.934)	(2.43)
	(3.5)	(2.203)	(.189)	(1.39)
	3.79 PEXB	1.18 PEXC	-11.11 PEXD	.608 PEXE
	(2.47)	(.844)	(6.20)	(.930)
	(1.54)	(1.395)	(1.80)	(.654)
	- .101PEXG	- .935 PS	1.32 PEXF	
	(1.606)	(.395)	(1.10)	
	(.0627)	(2.366)	(1.2)	

R-squared = .9932

Durbin-Watson Statistic = 2.3488

Standard Error of the Regression = .448961 E-01

Number of Observations = 26

NOTE: MWAG, PEXA, ... PEXG are logs of three years moving averages.

The implied own price elasticity of demand, in the long run, (that is, through both the current price and moving average terms) is -1.2. As expected, increases in the price of most other inputs increases the demand for lumber. There is no good explanation for structural minerals (PEXD) being a compliment. In regressions done by William McKillop[4] construction board (PEXH) shows up as a complement. Slight changes in the specification will change those results dramatically. None of the t statistics on the other materials reject the hypothesis that the coefficient really has the opposite sign from what was reported. Besides materials, labor is also used as a construction or milling input. Mill wages (MWAG) enters with the wrong sign but it is insignificant. Construction wages (PEXG) enters with the right sign, but it too is insignificant. McKillop thinks wage increases in construction should decrease lumber demand because lumber and plywood are relatively labor intensive materials to use. Though individually the other materials have no statistical significance, an asymtotic F test rejects the hypothesis that they are all zero at once.

Equations two and three are similar to equation one. Construction wages (PEXG) again show with a negative sign, mill wages (MWAG) shows with the proper sign and significant in equation three and wrong sign and insignificant in equation two. The sign of the autos (PEXA) coefficient can be ignored in both regressions because the asymptotic t value is so small. Board (PEXH), which is included in equation two and not in equation thre also has a small asymptotic t value. Rubber and plastic (PEXE) show as complements to Douglas Fir in both regressions and minerals (PEXD) goes from a substitute and significant to a complement and significant. Iron and Steel (PEXB), nonferrous metals (PEXC), and other lumber (PEXF) are all substitutes as expected.

I have also estimated the demand in factor share form using the translog specification. In this specification the use of moving averages was of no help. The equation is shown as Equation 4.4.2. The major problem with this equation is that the estimated elasticity of demand is near zero. The elasticity of demand for a translog form is

$$e = (-share + share * share + c)/share$$

where share is the estimated factor share and c is the coefficient associated with the own price term. For the translog function reported above the elasticity is negative at two thirds of the points and positive at one third of the points. This is not acceptable. Even though the appropriate t statistic is seven, I have restricted the demand curve to slope down by restricting the coefficient on own price to be zero. This gives Equation 4.4.3.

The elasticity of demand in Equation 4.4.3 is (as in Cobb-Douglas) -1+share, which is very close to one. This equation was used instead of the log linear equation because it does not depend on lagged prices. Lagged prices would complicate the rational expectations model that follows. I regret the decision.

Equation 4.4.2

SHRS =	.201E-1	+ .302E-2 * PDRS	- .953-3 * PEXA
	(.114E-1)	(.403E-3)	(.290E-2)
	(1.7)	(7.4)	(.32)

+ .265E-02 * PEXB	+ .907E-3 * PEXC	- .120E-3 * PEXD
(.181)	(.100E-2)	(.450E-2)
(1.4)	(.9)	(2.7)

- .42E-2 * PEXE	+ .160E-2 * PEXF	- .622E-2 * PEXG
(.124E-2)	(.116E-2)	(.228E-2)
(3/4)	(1.4)	(2.7)

+ .816E-3 * PEXH	- .316E-2 * MWAG
(.125E-2)	(.120E-2)
(.64)	(2.6)

N = 25

R-squared = .9681

Durbin-Watson statistic = 2.04

Equation 4.4.3

$$SHRS = \underset{\substack{(.385E-1)\\(.7)}}{.288E-1} + 0 * PDFS + \underset{\substack{(.75E-2)\\(.76)}}{.576E-2} * PEXA$$

$$+ \underset{\substack{(.714E-2)\\(2.5)}}{.78E-1} * PEXG + \underset{\substack{(.214E-2)\\(2.33)}}{.500E-2} * PEXC - \underset{\substack{(.103E-1)\\(3.33)}}{.346E-1} * PEXD$$

$$- \underset{\substack{(.280E-2)\\(1.9)}}{.541E-2} * PEXE + \underset{\substack{(.211E-2)\\(4.10)}}{.868E-2} * PEXF - \underset{\substack{(.653E-2)\\(.99)}}{.649E-2} PEXG$$

$$+ \underset{\substack{(.918E-2)\\(.565)}}{.519E-2} * MWAG$$

N = 25

R-squared = .776

Durbin-Watson statistic = 1.99

Data:

Prices of other goods are the wholesale price index reported by the Bureau of Labor Statistics. Gordon, in his thesis, points out that these prices are not particularly good, but no better ones exist. Note: At least part of the problem with modelling demand is the quality of the price series for other structural products. Stumpage price is from Outlook. Removals are calculated from data in Current Industrial Reports. Lumber production is known in every year, as is plywood production of both hard and softwood plywood. Current Industrial Reports also gives the stumpage requirements of plywood (hardwood plywood uses softwood for its core). I have pieced this data together with an estimate of wasteage (12%) to produce a removals series. For the years reported in Trends and Outlook, the numbers are in agreement.

The reported stumpage prices are from sales on national forest land. In the Douglas fir region these sales are held by open bidding. Walter Mead [4] argues that only a few bidders attend each sale and they agree beforehand how to split the sale. His evidence is personal observation.[5] This view is not generally accepted by the forest community. I choose to believe that the sales are competitive and the prices that result are average stumpage prices.

Average needs to be explained. The terms of each sale are different in quantity sold, location, and species mix. Location determines logging costs and transport costs to the mill. These costs are mainly labor and are a large portion of processing costs: processing costs are a large portion of final price. See equation 4.4.4. Thus changes in site location or terrain will change the bid price.

Another problem with the published stumpage price is that the vast majority of stumpage never gets sold on the market. Internal transfers account for much of removals and private deals on which there is no data account for another section. Darius Adams[6] chooses not to consider the stumpage market for this reason. Presumably the large firms are able to bid for Forest Service timber, so public timber can have a price no lower than the internal transfer price of stumpage. Similarly, the firms can sell their stumpage to other mills, so the external price can be no higher than the shadow price. In short, that a major amount of the commodity is not traded is no bar to using the competitively determined price.

Equation 4.4.4

$$PDFL = \underset{\substack{(55.7)\\(4.9)}}{271.4} + \underset{\substack{(9.00)\\(5.3)}}{48.1}\ MWAG + \underset{\substack{(.143)\\(5.9)}}{.842}\ PDFS - \underset{\substack{(21.9)\\(4.24)}}{92.6}\ TREN$$

$$\rho = \underset{\substack{(.165)\\(3.4)}}{.563}$$

$R^2 = .9406$ $\quad$ $N = 24$ $\quad$ d.w. $= 1.8$

Cochrane Orcutt Interative Technique Using Instrumental Variables

Footnotes

1 Stanford Research Institute, America's Demand for Wood: A Report to the Weyerhaueser Timber Company, Stanford Research Institute, 1954.

2 William McKillop, "Supply and Demand for Forest Products," Hilgardia 38, No. 1, March 1967.

3 Robert J. Gordon, Problems of Measurement of Real Investment in the U.S. Private Economy, Ph.D. Thesis, M.I.T., 1967.

4 Walter Mead, Competition and Oligopsony in the Douglas Fir Lumber Industry, Berkeley, University of California Press, 1966.

5 Personal discussion with Henry Vaux and Dennis Teeguarden.

6 Darius M. Adams, The Impact of Changes in Federal Timber Sales Policies on the Douglas Fir Region Forest Economy: An Econometric Simulation, Ph.D. Thesis. Wildlife Resource Science, University of California, Berkeley, 1972.

Section 4.5

Empirical Results:

The multimarket equilibrium model was estimated for six years and two interest rates. The period structure of the model is as follows: period one is own year, period two is own year plus one, period three is year 30 and periods four, five, six and seven are years 60, 90, 120, and 150 respectively. The point of using both the present and one year into the future was to try to capture the producers' response to the fickle nature of demand. As can be seen from the table below, the predicted first period outputs and second period outputs fluctuate far more than the actual series. This happens because the timber producers are not able to guess demand in time to adjust to it and because the model is numerically unstable in its first and second periods: one thirtieth the demand of other periods is felt in these periods -- thus the gradient associated with a mistake in these periods is small, compared to other periods. There are at least four estimates of the first period output: the amount demanded or the amount supplied; first period estimates or one thirtieth third period estimates. Because of the numerical stability problem discussed above, the third period estimates divided by thirty or the first period demand estimates would seem to be the estimates of choice.

The salient features of these estimates are that they predict less well than the mean and that they predict large cuts at the beginning of the sample period and smaller cuts toward the end while the actual cutting was pretty much even over the whole period.

	1948	1956	1960	1964	1968	1972
Actual price in dollars per thousand bd. ft.	14.5	27.6	23.4	27.8	44.7	52.4
Calculated removals in billion bd. ft.	10.28	11.69	11.17	12.17	11.94	11.81
PREDICTIONS WITH INTEREST RATE OF 1.05						
1st period price	5.78	5.96	7.37	8.54	8.27	10.1
3rd period price, discounted to yr. 1	6.42	6.15	7.83	9.10	8.73	10.6
1st period supply	--	5.09	7.55	7.42	6.50	8.76
1/30 of 3rd period supply	21.8	15.3	14.0	13.0	12.0	11.0
1st period demand	23.4	54.0	32.6	34.4	59.7	61.3
1/30 of 3rd period demand	13.0	15.3	11.0	--	12.0	11.0
PREDICTIONS WITH INTEREST RATE OF 1.02						
1st period price	10.6	13.8	14.3	14.8	17.2	16.5
3rd period price, discounted to yr. 1	15.4	18.5	19.9	21.4	21.6	23.5
1st period supply	--	12.1	15.5	18.2	21.7	24.4
1/30 of 3rd period supply	21.0	18.0	16.0	11.0	11.0	11.0
1st period demand	12.7	22.4	16.8	19.9	28.7	37.4
1/30 of 3rd period demand	13.0	12.0	12.0	12.0	12.0	11.0

This model aims at predicting the cut from behavioral considerations. The mean specifies only that people did whatever they did. Since the series has no long run trend, the model can be viewed as an attempt to predict the mean. In this sense it is successful. From purely behavioral considerations it was possible to predict that 11 billion board feet of lumber would be harvested.

The problem of the time trend of the cuts is probably inherent in the demand equation. If a near-zero demand elasticity were used, then the predicted cuts would have no time trend. Perhaps the estimate of demand using a moving average would give better results. It was too expensive to find out.

The table shows, as expected, that the lower the interest rate, the smaller the cut in the first period. Prices increase faster than the rate of interest until the old growth is gone and then increase at a slower rate (sometimes decreasing slightly) and tending towards a zero rate of change by the seventh period. The rotation age in the long run appears to be on the order of 90 years and the competitive market will reach that point after 30 to 60 years. Below is the output for the 5 percent interest rate, starting from year 1972 equilibrium.

VNEW and VOLD are the values of the objective function before and after the current iteration. The objective function is the difference between the integral of the demand curve and profits. In this iteration the objective function has changed by about 7 E-9, which is a very small number indeed. 125 iterations earlier, the objective function was changing by about .01 each iteration. Periods one and two refer to 1972 and 1973 respectively. Periods three through seven are 2002, 2032, 2062, 2092, and 2122. PSUPPLY is the expected price in dollars per board foot. XSUP and XDEM are the expected quantities demanded and supplied. They must be multiplied by ten to the twelvth (E12) to be read in board feet. X, C, LAM are the stock and cut, each measured in acres, and the shadow price, measured in dollars per acre (multiplying by ten to the third gives the proper scale -- E3). ALF is one half unless the age class in question is a tie -- that is, it can either be cut or saved with no change in the level of profits. Then ALF is adjusted to minimize the gradient squared, or what is the same, the distance between the demand and supply points. The seven numbers give the values for the seven age classes. Each age class is separated from its neighbor by thirty years. Consider age class one in period one. There are 1.53 million acres in this age class. From the growth function displayed earlier in this section we know that there is no timber on this land. In fact, it is bare land with tree seeds. None of this land is cut in period one. In period two this land is still in age class one because only one year has passed. In period three, this land is promoted to age class two. In period four it is in age class three and in period five it is in age class four and it is cut, along with age class three. They reappear together as age class one land in the next period and so on. Until the land was cut, the shadow price (LAM) column contained the same number .875E-4. This is because the present value shadow price of class two land in period three is the same as that of class one land in period one. Class one land in period land is allowed to mature to be class two in period three. All reported prices are discounted at the rate of interest shown (5%).

VNEW = - .323769450E-1

VOLD = - .323769525E-1

PERIOD = 1 PSUPPLY= 0.100818500 E-01

XSUP = .8756E-1 XDEM = .6126E-1

X	C	LAM	ALF	IALF
0.153E+01	0.0	0.375E-04	0.500E+00	1
0.360E+01	0.0	0.169E-03	0.500E+00	1
0.391E+01	0.0	0.226E-03	0.500E+00	1
0.237E+01	0.0	0.301E-03	0.500E+00	1
0.128E+01	0.0	0.351E-03	0.500E+00	1
0.159E+01	0.810E+00	0.366E-03	0.491E+00	0
0.458E+01	0.947E+00	0.690E-03	0.733E+00	0

PERIOD = 2 PSUPPLY= 0.100817606 E-01

XSUP = .6906E-1 XDEM = .3087E-1

X	C	LAM	ALF	IALF
0.153E+01	0.0	0.875E-04	0.500E+00	1
0.360E+01	0.0	0.169E-03	0.500E+00	1
0.391E+01	0.0	0.226E-03	0.500E+00	1
0.237E+01	0.0	0.301E-03	0.500E+00	1
0.128E+01	0.0	0.351E-03	0.500E+00	1
0.780E+00	0.730E+00	0.366E-03	0.500E+00	2
0.364E+01	0.537E+00	0.690E-03	0.852E+00	0

PERIOD = 3 PSUPPLY= 0.105612502 E-01

XSUP = .3455 XDEM = .3169

X	C	LAM	ALF	IALF
0.307E+01	0.0	0.385E-04	0.500E+00	1
0.153E+01	0.0	0.875E-04	0.500E+00	1
0.360E+01	0.0	0.169E-03	0.500E+00	1
0.391E+01	0.182E+01	0.226E-03	0.534E+00	0
0.237E+01	0.237E+01	0.301E-03	0.500E+00	2
0.128E+01	0.128E+01	0.351E-03	0.500E+00	2
0.310E+01	0.310E+01	0.690E-03	0.500E+00	2

PERIOD = 4 PSUPPLY= 0.903881884 E-02

XSUP = .1326 XDEM = .1170

X	C	LAM	ALF	IALF
0.857E+01	0.0	0.747E-05	0.500E+00	1
0.307E+01	0.0	0.385E-04	0.500E+00	1
0.153E+01	0.0	0.375E-04	0.500E+00	1
0.360E+01	0.360E+01	0.169E-03	0.500E+00	2
0.209E+01	0.209E+01	0.226E-03	0.500E+00	2
0.0	0.0	0.264E-03	0.500E+00	2
0.0	0.0	0.522E-03	0.500E+00	2

PERIOD = 5 PSUPPLY= 0.422640086 E-02

XSUP = .5959E-1 **XDEM = .5822E-1**

X	C	LAM	ALF	IALF
0.569E+01	0.0	0.260E-05	0.500E+00	1
0.857E+01	0.0	0.747E-05	0.500E+00	1
0.307E+01	0.307E+01	0.385E-04	0.500E+00	2
0.153E+01	0.153E+01	0.875E-04	0.500E+00	2
0.0	0.0	0.117E-03	0.500E+00	2
0.0	0.0	0.137E-03	0.500E+00	2
0.0	0.0	0.273E-03	0.500E+00	2

PERIOD = 6 PSUPPLY= 0.320502406 E-03

XSUP = .7799E-1 **XDEM = .7571E-1**

X	C	LAM	ALF	IALF
0.460E+01	0.0	0.0	0.500E+00	1
0.569E+01	0.0	0.260E-05	0.500E+00	1
0.857E+01	0.857E+01	0.747E-05	0.500E+00	2
0.0	0.0	0.179E-04	0.500E+00	2
0.0	0.0	0.228E-04	0.500E+00	2
0.0	0.0	0.267E-04	0.500E+00	2
0.0	0.0	0.530E-04	0.500E+00	2

PERIOD = 7 PSUPPLY= 0.286084600 E-03

XSUP = .5176E-1 **XDEM = .5371E-1**

X	C	LAM	ALF	IALF
0.857E+01	0.857E+01	0.0	0.500E+00	2
0.460E+01	0.460E+01	0.0	0.500E+00	2
0.569E+01	0.569E+01	0.260E-05	0.500E+00	2
0.0	0.0	0.592E-05	0.500E+00	2
0.0	0.0	0.795E-05	0.500E+00	2
0.0	0.0	0.930E-05	0.500E+00	2
0.0	0.0	0.185E-04	0.500E+00	2

Section 5

Section 5.1

Forest Service Policy:

Although the U. S. Forest Service controls 38 percent of the land in the Douglas Fir region and 44 percent of the land bearing saw-timber, it is not at all clear what their policy is. The intended policy of the Forest Service is expressed in both the Multiple Use Sustained Yield Act of 1960 and the writings of Forest Service Officials. Forest Service policy is revealed in the cut and inventory statistics.

The best place to start is the Multiple Use Sustained Yield Act . "National forests ... shall be administered for outdoor recreation, range, timber watershed, and wildlife and fish purposes." Moreover this shall be done with regard to "multiple uses" which means "the management of the resources ...in the combination that will best meet the needs of the American poeple" and not necessarily maximize dollar of physical output. "Sustained yield means the achievement and maintenance in perpetuity of a high-level annual or regular periodic output without impairment of the productivity of the land." [1]

Thus multiple use directs the Secretary of Agriculture to do what he thinks best while sustained yield cautions him to produce a lot of whatever is produced without "impairing the productivity of the land." The act allows the National Forests to be used mostly as playgrounds or mostly for timber production (and hopes the uses will be simultaneous and compatible). Since the Sustained Yield Act gives so little guidance, one must look at the statements of Forest Service policy.

Current Forest Service policy depends upon allowable cut. Crudely, allowable cut is determined by choosing a rotation age (90 to 120 years) and an adjustment period (on the order of 40 years) and then finding a policy that will produce an even aged forest (in which trees are cut at the rotation age) at the end of the adjustment period. That is, the Forest Service ideal is a forest containing equal numbers of acres of every age class. Trees are cut at the rotation age. In the long run, a constant supply of lumber results. The adjustment period is the length of time it takes to remove the old growth and set up the even aged forest. During the adjustment period there may be much larger harvests than will obtain during the even rotation regime.[2] The justification for all the even flow or sustained yield statements, at the Forest Service level, seems to be short term economic stability.[3]

Two recent Forest Service studies deserve attention in this respect. The Douglas Fir Supply Study of 1969 calculates increment to present discounted value of a number of management alternatives. The basic finding is that using a 5% rate of discount anything that hurries up the cut will increase net worth. The management alternatives considered in that report are not even flow alternatives. That is, the increase in output cannot be sustained, it is a once and for all increase. A preliminary draft of the Forest Regulation Study done in 1973, but not officially released, is critical of the Douglas Fir Supply Study. The regulation study points to the Multiple Use Sustained Yield Act and says that the management alternatives in the Supply Study do not meet the sustained yield requirements of the act. The point is that maximization of present net worth is not consistent with an even flow constraint; this inconsistency is recognized within the Forest Service and the desirability of the various aims is debated.

(The Douglas Fir Supply Study proceeds on the assumption that the price of stumpage (properly defined) will be at 140% of the 1970 level thirty years hence. Calculations are made in real terms with an interest rate of 5 percent. My prediction uses a 5 percent nominal interest rate and yields a 400% increase in price over the same time interval. Taking account of the difference between nominal and real prices, my estimate and the study's estimate are grossly compatible. The study recommends more intensive management to produce more lumber sooner and increase present value.)

Finally the writings of the Chief Foresters McCardle (1956) Cliff (1968), and McGuire (1974) point to the Forest Service walking a political tightrope between the conservationists and industrial forces. McGuire seems particularly concerned with the conflict between the recreation and timber industry groups: He claims to chart a course "somewhere in between".

A cynical (and essentially correct) summary of the preceeding discussion is: The Forest Service sets its harvest policy according to political pressure and its own sense of what is good for America. The decision gives weight to quantity stabilization, recreational needs, and forestry industry needs.

Objective functions for forest management differ in their use of interest rate, value to standing stock for noncommercial use, even flow considerations, and expected prices. The hard line Sustained Yield School can be characterized as maximizing physical output (price expected constant, zero interest rate, even flow constraint, no explicit stock value) with the constraint that the flow of timber be the same in every period. A soft line Sustained Yield School forester would lessen the even flow constraint. (Perhaps, like Carl Jugenfeld he would attach a penalty for rapid changes in the rate of harvest.)[4] My proposal is to use a positive (in fact, market) discount rate and value the standing stock explicitly. Moreover I would use expected prices and place no inherent even flow constraints on the model. Something approaching the Soft Line Sustained Yield School notion of even flow will result. The demand curves working through expected prices will assure that the period to period change in stumpage sales is not very great. The net result of changing from the amorphous arithmetic of sustained yield to the calculus of present value maximization with rational expectations may well be just a little more than a change in rationalization.

The next section will contend that a moderate valuation of the stock of timber for noncommercial uses is sufficient to produce a policy very much like the Forest Service's current policy. It remains to enumerate the fine details that will differ. The present value model allows correct assessment of management practices. Proposals (of which there are many) for intensified management should be accepted if they increase the present value of the forest with stock valued. Thus this model will differ from sustained yield in which projects it accepts. Sustained yield may require the benefits of a project to be spread over many years, or, in what's called the allowable cut effect, sustained yield may make it possible to harvest more timber in the very first year and that may dominate the objective function.[5] The allowable cut effect needs some explanation. Because of even flow constraints too little timber is cut at the beginning of the planning period. With even moderate interest rates (5% real) what happens in thirty years matters only one quarter as much as what happens today. But if even flow constraints are in effect, then an investment today that will result in say 30,000 extra board feet in thirty years will resutl in an extra cut today of 1,000 board feet. Because of the even flow constraint, management practices that would not be profitable for a profit maximizing firm appear desirable to a maximum sustained yielder.

Another major difference between sustained yield and profit maximization with stock valued is that the sustained yield school does not try to adjust its sales to current market conditions. The Forest Service should calculate its reservation price for timber sales based on what it thinks it can get for the timber in a year with large construction demand, not on some notion of "fair profit" for the mills. If selling no timber when there is no demand is too unpalatable, the Forest Service could at least extend the period a logger has in which to cut contracted timber and let the logger reap the speculative profits. (Since loggers bid against each other, the expected value of these profits is the logger's payment for accepting the risk involved.)

The last advantage to the profit-maximizing stock valued approach is clarity of thought. The Multiple Use Sustained Yield Act directs the Secretary of Agriculture to manage the forests for both commercial and noncommercial purposes. Choosing a stock value (one would hope by careful research on what people would pay for recreation, and what the nation as a whole should pay for wildlife) and choosing timber value as the discounted market price makes it very clear what products are produced by our forests and how much our public servants value these products. If the logging industry or the conservationists don't like the decision, let them go to Congress; there the discussion can be dominated properly: in the public's money.

Section 5.2

Imputed Value:

Although the writings of the Forest Service shed very little light on the actual tradeoffs the Service makes between recreational and commercial use of the forests, actual Forest Service decisions can be used to impute a value per acre to the Forest Service holdings. At both the Forest Service and the competitive sector hold virgin timber. The multi-market partial equilibrium model (model) implies that the price of stumpage must be rising fast enough to make the holding decision rational for the private sector. A result is that it must also maximize Forest Service revenues. So long as the private sector continues to hold mature timber, there is no implications about Forest Service policy. However, in twenty or thirty years, private sector holdings of old stock will be negligible compared to the public holdings. It is at that time that the Forest Service valuation of its standing stock will be revealed. Using the model with an interest rate of 5%, it is possible to impute a value per acre to Forest Service land held past the time a private operator would cut it. That is, the value per acre is the sum of the acres value for timber plus the acres value for recreation, wildlife, etc. which we shall call noncommercial use. Consider old growth timber in 2002. An acre of land bearing old growth would sell for about $690. A profit maximizing manager would cut the timber. Instead, suppose the Forest Service owned the land and decided to save the trees for one more period (30 years). The policy of holding the trees thirty years longer would give the land an imputed value of 522 dollars. Thus the noncommercial value of old growth must exceed $5.50 per year for it to be worthwhile to the Forest Service to hold the timber. Similarly, in the year 2032, an additional valuation of sixty cents per acre would make the difference between keeping and cutting ninety year old trees for thirty more years.

At first glance these values seem small. Yet one must remember that something on the order of six million acres are involved. Thus a complete no rent policy would cost thirty million dollars per year at the computed expected prices. If the Forest Service were to cut none of its holdings, prices would undoubtably be a good deal higher, so it is erroneous to carry the analysis too far. Similarly, a first guess at what it would cost the Forest Service to use a rotation age longer by thirty years than the one employed in the private sector is three and a half million dollars per year. All of these figures are not cash payout, but own year foregone value of income, or shadow cost.

The previous exercise is meant mostly to be illustrative. The model I have used predicts the gross shape of the forest economy. It cannot distinguish between rotation ages of 80 and 110 years, nor can it say that an allowable cut that liquidates inventories in forty years instead of twenty-five years is off base. What the previous exercise shows is that a relatively modest value for noncommercial use per acre will be enough to justify the broad outline of the Forest Service management policy. By simply choosing a relatively small dollar value per acre per year for noncommercial uses, the Forest Service could easily justify their present policy (or virtually any other policy).

Footnotes

1 Multiple Use Sustained Yield Act of 1960, 74 Stat. 215; 16 USC 582.

2 Neff, Paul E., "Calculation of Allowable Harvest for the National Forests," Journal of Forestry 71, 1973.

3 Guthrie, John A. and George Armstrong, Western Timber Industry: An Economic Outlook, Johns Hopkins Press, 1961, pp. 256-274.

4 Jugenfeld, op. cit.

5 Schweitzer, Dennis L. and Robert A. Sassarman, "Harvest Volume Regulation Affects Investment Value," Forest Industries, March 1973.