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New Insights in Portfolio Selection Modeling

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"An investment in knowledge pays the best interest."

- Benjamin Franklin

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Abstract

Recent advancements in the field of network theory commence a new line of developments in portfolio selection techniques that stands on the ground of perceiving financial market as a network with assets as nodes and links accounting for various types of relationships among financial assets. In the first chapter, we model the shock propagation mechanism among assets via network theory and provide an approach to construct well-diversified portfolios that are resilient to shock propagation and contagion issues in the volatile and crisis periods. The second chapter analyzes the influence of the hedging network among assets on the portfolio diversification attributes. Building on the hedging network perspective of the market, we propose a network centrality measure to find stocks that are most suitable to form a well-diversified portfolio. The results clearly shows that our diversification strategy performs better than the conventional diversification strategies both in-sample and out-of-sample. In the third chapter, we analyze the market network constructed by adopting assets as nodes and their returns' correlation as links. We theoretically show that there is a negative relationship between the centrality of assets in such a network and the weights assigned to them in the Markowitz prescription. Based on our theoretical findings, we propose a portfolio selection strategy that out-performs well-known benchmarks while presenting positive and significant Carhart alphas.

Resumen

Los recientes avances en el campo de la teoría de redes suponen una nueva línea de la evolución de las técnicas de selección de carteras que se basa en plantear el mercado financiero como una red donde los nodos constituyen cada uno de los activos y los enlaces representan diversos tipos de relaciones (o conexiones) entre los activos financieros. En el primer capítulo, vamos a representar el mecanismo de propagación de shocks entre los activos a través de la teoría de redes y proporcionar un enfoque para construir carteras bien diversificadas que sean resistentes a los shocks y problemas de contagio en periodos de elevada volatilidad y/o crisis. El segundo capítulo se analiza la influencia de las redes de cobertura de los activos sobre los atributos de diversificación de la cartera. A partir de la perspectiva de una red de cobertura, se propone una medida de centralidad de la red para encontrar aquellas acciones que sean más adecuadas para formar una cartera bien diversificada. Los resultados muestran claramente que nuestra estrategia de diversificación se comporta mejor que las estrategias de diversificación convencionales tanto en la muestra como fuera de ella. En el tercer capítulo, se analiza el caso de una red construida mediante la adopción de activos como nodos y sus correlaciones como enlaces entre los nodos. Los resultados empíricos nos muestran una relación negativa entre la centralidad de los activos en esta red y los pesos asignados a en el marco del modelo de carteras clásico de Markowitz. En base a los resultados teóricos, se propone una estrategia de selección de cartera que supera los resultados bajo los modelos clásicos, permitiendo obtener alfas (bajo modelo de Carhart) positivos y significativos.

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Chapter 1: Introduction

The process of selecting a portfolio is an ongoing field of research in the finance litreture. The main argument in theorizing the portfolio selection process is built on the diversification rule that advises upon including a large number of assets in the portfolio for the purpose of minimizing the risk. Building on this argument, Markowitz provide a delicate mathematical perscription where he develops a static framework for investors to optimally allocate their wealth across a set of assets considering only the first and second moments of the returns' distribution. Markowitz perscriptions can be categorized as a specific type of diversification rule where apart from the portfolio risk, the return is also considered as influential in allocating wealth among assets.

Despite the logical reasoning behind the diversification argument and the Markowitz profound mathematical prescrption, these strategies face several challenges in real-world application. In one hand, diversified portfolios result in losses in the crisis and volatile periods. In particular, in the 2007-2008 financial crisis, investors who counted on diversification to protect them against shocks, suddenly find their investment plunging in value. Hence, practitioners criticize diversification strategy, pointing to "the end of diversification''.

In addition, although diversification advocates including a high number of stocks in the portfolio to minimize risk, there is no consensus on the optimum number of stocks needed in the portfolio to achieve the maximum diversification benefits. The litreture advises upon including a specific number of stocks as low as ten up to three hundred to obtain the maximum diversifiction benefits. This creates a confusion for investors as how to construct a well-diversified portfolio.

On the other hand, with regard to the profound Markowitz perscriptions, the out-of-sample performance of his strategies is not as promising as expected. The poor performance of Markowitz's rule stems from the large estimation errors on the vector of expected returns and on the covariance matrices leading to the well-documented error-maximization property. The magnitude of this problem is evident when we acknowledge the modest improvements achieved by those portfolio rules specifically designed to tackle the estimation risk. In addition, the evidence indicates that the simple yet effective equally-weighted portfolio rule has not been consistently out-performed by more sophisticated alternatives.

To tackle these issues, we adopt a network perspective of the financial market in this thesis. Recently, researchers from different fields have characterized financial markets as networks in which securities correspond to nodes and links relate to different types of relationships among securities (Barigozzi and Brownlees, 2014; Acemoglu et al., 2012; Billio et al., 2012; Bonanno et al., 2004; Diebold and Yilmaz, 2014; Hautsch et al., 2015; Mantegna, 1999; Onnela et al., 2003; Tse et al., 2010; Vandewalle et al., 2001; Zareei, 2015). In spite of the novel and interesting insights obtained from these network-related papers, most of their results are fundamentally descriptive and lack concrete applications in portfolio selection process. This thesis contributes to this line of research by investigating the extent to which the underlying structure of the financial market network can be used as an effective tool in enhancing the portfolio selection process.

In the first chapter, we address the poor performance of diversified portfolios in the crisis and volatile periods. In general, portfolio diversification overlooks the presence of lead-lag relationships among assets performing as a shock propagation mechanism, which reduces the gains from diversification in the volatile and crisis periods. This paper investigates the influence of shocks propagation among assets on diversification benefits relying on insights from network theory. We introduce Granger network as a directed graph with nodes corresponding to individual assets and links accounting for lead-lag relationships. Analyzing this network, we show that (i) the influence of shocks on portfolio return is proportionate to the centrality ranking of assets in the Granger network and (ii) an asymmetrical network structure hinders diversification benefits by depleting portfolio volatility decay rate. Furthermore, we show that diversifying among assets with star-like network structure where a central asset leads all other assets in the portfolio result in the lowest diversification benefit. Finally, we empirically demonstrate that two distinct datasets of U.S. industries and international stock markets greatly resemble star-like network structures whose central nodes are financial industry and the U.S. market, respectively. Based on the findings in this paper, investors are able to construct well-diversified portfolios that are resilient to shocks propagation and contagion issues in crisis periods.

The second chapter addresses the optimum number of stocks needed to gain the most of diversification benefits. We develop a new framework for portfolio diversification based on network theory and propose a new type of network, a hedging network, with nodes comprising stocks and links accounting for hedging relations. We then analyze different types of network structures (full, star and individual) and the relevance of their components in portfolio diversification. In addition, we propose a centrality measure in hedging networks to determine the best stocks that improve portfolio diversification. Employing this centrality measure, we build well-diversified portfolios with a low number of stocks that perform better than a naïve equally weighted portfolio or portfolios based on classical portfolio theory.

In the third chapter, we address the poor out-of-sample performance of Markowitz rule using a network perspective of the financial market. In this study a financial market is conceived as a network where the securities are nodes and the links account for returns' correlations. We theoretically prove the negative relationship between the centrality of assets in this financial market network and their optimal weights under the Markowitz framework. Therefore, optimal portfolios overweight low-central securities to avoid the large variances that result when highly influential stocks are included in the investor's opportunity set. Next, we empirically investigate the major financial and market determinants of stock's centralities. The evidence indicates that highly central nodes tend to coincide with older, larger-cap, cheaper and financially riskier securities. Finally, we explore by means of in-sample and out-of-sample analysis the extent to which the structure of the stock market network can be employed to improve the portfolio selection process. We propose a network-based investment strategy that outperforms well-known benchmarks while presenting positive and significant Carhart alphas. The major contribution of the paper is to employ the financial market network as a useful device to improve the portfolio selection process by targeting a group of assets according to their centrality.

This thesis contributes to several strands of literature. First, it relates to the papers trying to take advantage of cross-dependencies among returns in order to construct profitable portfolio strategies. A well-known article in this category is DeMiguel et al. (2014) in which they provide an extensive analysis investigating whether they can profit from the cross-dependencies across assets and find that a strategy formed on returns' cross-dependencies would perform well out-of-sample when the transaction cost is low (lower than ten basis points). Moreover, Menzly and Ozbas (2010) find that trading strategies based on cross predictability of returns result in economically and statistically significant premiums and are significantly correlated with the return series of hedge funds that are presumed to exploit return predictability effects. Rapach et al. (2014) construct portfolios using industry returns' cross-predictability and gain 11.37% annualized abnormal returns. We complement this strand of litreture by modeling the cross-dependencies among returns in a network perspective and analyze the influence of this network on portfolio diversification attributes. In abstract, we provide theoretical background for this strand of empirical litreture.

In addition, this thesis is also strongly related to the emerging literature concerning macroeconomic fluctuations stemming from microeconomic idiosyncratic shocks (Horvath, 1998; Dupor, 1999; Shea, 2002; Gabaix, 2009; Acemoglu et al., 2015). In an allied work, Acemoglu et al. (2012) show how an economy's aggregate volatility decay rate is determined by the topology of an industrial input-output network. In a closely related article, Acemoglu et al. (2015) determine the fundamental elements characterizing the contagion process on a network, enabling us to rank them in terms of their aggregate performance. This framework sheds light on the commonalities among different contagion-related papers and provides a sort of coherence to their results.

On the other hand, this thesis complements the vast literature on the underlying attributes of portfolio diversification. This litreture starts by the classical article by Samuelson (1969), where he explores the diversification benefits among a fixed number of stocks when they are negatively correlated. Afterwards, researchers address this issue from an empirical standpoint and provide empirical results as to how many stocks determine the most of diversification benefits (Evans and Archer, 1968; Mao, 1970; Elton and Gruber, 1977; Bird and Tippett, 1986, Evans and Archer, 1968; Statman, 1987; Domian et al., 2007).

In abstract, this thesis is the first comprehensive study on the application of network theory in improving portfolio selection process. Investigating from both theretical and empirical standpoints, we prescribe several portfolio selection rules that out-perform the conventional portfolio selection strategies relying on network theory.

Chapter 2: Network Origins of Portfolio Risk

1. Introduction

In the 2007/2008 financial crisis, investors counting on their diversified portfolio to protect them against shocks, suddenly find their investment plunging in value. Hence, practitioners (e.g. Jaeger and Taraporevala, 2009) and researchers (e.g. James, Kasikov, and Edwards, 2012; Ilmanen and Kizer,2012) criticize diversification strategy, pointing to “the end of diversification”. This paper provides a remedy for poor performance of diversified portfolios in the volatile periods by investigating the contagion notion originating from interdependencies among assets.

The standard theory in finance depicts a decreasing relationship between a portfolio's risk and its size where this negative association lingers until the aggregate variance reaches its asymptotic lower limits, commonly known as non-diversifiable or systematic risk. However, this assertion overlooks the presence of lead-lag relationships[1] among assets performing as a shock propagation mechanism. This paper examines the influence of lead-lag relationships on portfolio volatility relying on insights from network theory. In particular, we show that lead-lag structures with asymmetrical spirit diminish diversification benefits and slow down portfolio volatility convergence.

There is substantial empirical evidence of the presence of lead-lag relationships among returns.[2] Lo and MacKinlay (1990); Brennan, Jegadeesh, and Swaminathan (1993); Badrinath, Kale, and

[1] There is lead-lag relationship between asset i and j if the asset ’s return at time t influences asset i *’s* return at time $t+1$ (in this case, asset j is said to *lead* asset i). In the literature, other terms such as serial-dependency (DeMiguel, Nogales, and Uppal, 2014) and cross-predictability (Menzly and Ozbas, 2010; Rapach, Strauss, Tu, and Zhou, 2014) are also used to point to lead-lag relationships among returns.

[2] The returns cross-predictability literature is a subset of the well-established literature on the predictability of returns supported by many papers e.g. Fama and Schwert (1977); Campbell (1987); Lewellen (2004); Breen, Glosten, and Jagannathan (1989);Fama and French (1988, 1989); Ferson and Harvey(1991); Lettau and Ludvigson (2001); Campbell and Thompson (2008);Cochrane (2008); Pastor and Stambaugh (2009). Welch and Goyal (2008) nurtured doubts regarding the out-of-sample predictive power of returns, and in response, Cochrane(2008) criticized the reliability of their out-of-sample test and back up the return predictability evidence. Campbell and Thompson (2008) argue in favor of including restrictions in regressions to increase their predictive power. Subsequently, several robust out-of-sample evidence on the predictive ability of returns was provided by Campbell and Yogo(2006); Rapach, Strauss, and Zhou (2009); Henkel, Martin, and Nardari (2011);Ferreira and Santa-Clara(2011), and Dangl and Halling (2012). Moreover, following successful evidence of returns predictability, several prominent asset pricing models have incorporated return predictability, such as those of Campbell and Cochrane (1999); Bansal and Yaron (2004). Finally, a branch of literature embodies return predictability as given

Noe (1995); Chordia and Swaminathan (2000); Hou (2007); Cohen and Frazzini (2008), and DeMiguel et al. (2014) substantiate lead-lag relationships among stock returns; Menzly and Ozbas (2010); Hong, Torous, and Valkanov (2007), and Rapach et al. (2014) verify cross-predictability among industry portfolios and Eun and Shim (1989) and Rapach, Strauss, and Zhou (2013) document serial dependency in the international market. Building on this extensive evidence, we construct a network that captures how shocks propagate among assets by summarizing the lead-lag relationships among asset returns, termed as Granger network, with assets as nodes and directed links signifying the serial-dependencies between assets.[3]

Investigating the influence of Granger network, as a shock propagation mechanism, on portfolio diversification, we make four contributions to the literature. First, we characterize the portfolio return of a naive investor by the centrality of assets in the Granger network and show that shocks to the central assets have a high ex-post influence on portfolio return, while the unconditional expected return of the portfolio is insusceptible to the structure of the network. Accordingly, the structure of Granger network have direct influences on the portfolio risk of a diversified portfolio.

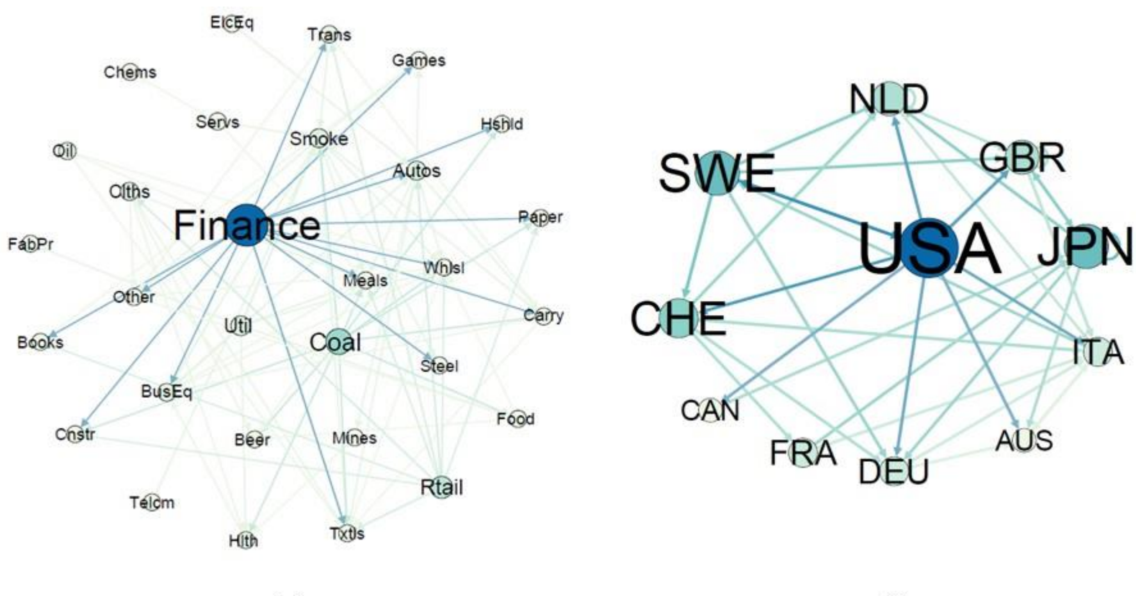
Second, we decompose the portfolio variance to two components: contemporaneous and Granger components where the former measures the risk associated with the simultaneous movements in the asset returns and the latter explains the risk stemming from the structure of Granger network. Subsequently, we theoretically show that higher heterogeneity in the structure of Granger network, as motivated by fat-tail centrality distribution, increases portfolio risk and cuts back portfolio diversification benefits. In particular, we argue that diversification in a stylized star network structure, in which one central asset cross-effects other assets, results in the lowest level of diversification benefits. Third, we empirically demonstrate that two distinct sets of assets, U.S. industry portfolios and international stock markets, resemble star-like network structures whose central nodes are the financial industry and U.S. stock market, respectively. Moreover, we show that the Granger component of portfolio risk, when diversifying across industries or international markets, increases in periods of crisis when there is a higher propensity for shocks to influence the central nodes. Fourth, we build well-performing diversification strategies based on the information in the Granger network. Knowing that high central assets in the portfolio would

in asset allocation approaches, leading to a series of papers such as Campbell and Viceira (2002); Campbell, Chan, and Viceira (2003).

[3] We use a vector auto-regression model (VAR) to capture the lead-lag relationships among returns (DeMiguel et al., 2014; Rapach et al., 2014). The element in row i and column j in Granger network's adjacency matrix signifies the cross-effect of asset j at time t on asset i at time $t+1$. It should be noted that Rapach et al. (2014) consider a similar network representation derived from VAR estimation.

extensively propagate shocks in the portfolio and decrease our diversification benefits, we expect diversification among the lowest central assets to lead to lower portfolio volatility and higher diversification benefits. Our in-sample and out-of-sample analyses clearly demonstrate better performance from diversifying among lowest central assets. Moreover, we also compare our strategy to naive diversification among all assets and assets with lowest average correlation, and, in both cases, diversification among the lowest central assets in the Granger network demonstrates superior performance in-sample and out-of-sample. Therefore, based on our findings, we are able to improve diversification benefits and construct diversified portfolios robust to shocks propagation among assets in the volatile periods.

In order to give a picture of how Granger network looks like in real-world, we proceed by presenting two examples that have been already established in the literature. The net- work in Figure 1(a) summarizes lead-lag relationships across 30 industry portfolios and is already established in Rapach et al. (2014). The nodes in this network represent the industry portfolios and the links account for the lead-lag relationships. The financial industry is clearly positioned as the highest central node in the network. The network in Figure 1(b) illustrates the cross-dependencies among 11 industrialized countries with a clear central placement of the U.S. market (Rapach et al., 2013). If we suppose an investor decides to diversify among assets in these networks (performing industrial diversification or international diversification), an important question arises: To what extent does Granger network structure influence diversification benefits?



(a) (b)

Figure. 1. Granger network examples (a) 30 value-weighted industry portfolios from Kenneth French's Data Library (Monthly excess returns from 1980:01 to 2014:12); (b) Total return indices (Stocks) from Global Financial Data for 11 industrialized countries (Monthly excess returns from 1980:12 to 2014:12 adjusted for difference in closing times across countries); The nodes in the network represent the assets and the directed lines account for lead-lag relationships. The darker and bigger is the node, the more central it is in the network or the more leading characteristic the asset has in the market.

To answer this question, we consider two types of investors both relying on naive diversification as their main strategy: a myopic investor that only cares about tomorrow and a long-term investor who holds a portfolio for a long period of time. Regarding the myopic investor, we find threatening-degree centrality of assets in the Granger network to characterize the impact of idiosyncratic shocks to his-her portfolio return. Threatening-degree centrality is simply the aggregate one-period impact of an asset on other assets in the network. However, the unconditional expected portfolio return is resilient to the structure of the Granger network. Next, we decompose myopic portfolio variance into two parts: contemporaneous and Granger components where the granger component quantifies the impact of Granger network on portfolio variance. Disregarding the contemporaneous component, we provide a proposition showing that fat-tail distribution of threatening-degree centrality results in higher portfolio variance. This is to be expected as shocks to a low number of highly central assets would transmit rapidly across the whole portfolio and drive up the portfolio risk.

In an additional comprehensive analysis, we discuss the portfolio volatility convergence in several stylized network structures. First, we consider a case with no lead-lag relationships among assets, termed as no-dynamic structure, and we find it to follow the standard diversification tenet, where the portfolio risk converges to mean correlations in the contemporaneous component. Next, we study a case in which each asset impacts itself; called disconnected network, this case only takes into account the presence of autocorrelations among assets. Diversification in this case leads to a higher portfolio volatility than in no-dynamic structure. Proceeding with network structural analysis, we consider the cases of an inverse-star network, where every asset cross-predict one asset in the center; a circle network, where each asset leads only the one asset following it in a circle format; and a full network, in which all connections mutually exist. In these structures, we find that diversification converges to the same level as in the disconnected network. On the contrary, the star network, where one asset in the center cross-predicts all other assets, results in the highest portfolio volatility level, and diversification in this structure leads to higher asymptotic systematic risk. Moreover, disregarding contemporaneous correlation across assets, the risk suffered by a myopic investor remains positive, even for an extremely large portfolio size while in other stylized network structures, it converges to zero. This concludes our proposition on the adverse effect of fat-tail centrality distribution on the portfolio risk.

Analyzing the long-term case, we find Katz-Bonacich centrality to be a determinant factor on verifying the influence of shocks on portfolio performance. This makes sense for the long-run, as Katz-Bonacich centrality measures the aggregate long-term influence of an asset on the whole portfolio. We find the same results as the myopic diversification, with Katz-Bonaich centrality acting as a determinant of long-term shock influence on the portfolio. We also show that Katz-Bonaich centrality does not influence the portfolio return of a static portfolio (unconditional portfolio return) and higher heterogeneity of the centrality distribution results in lower diversification benefits. Summing up, we conclude that, for both short-term and long-term investors, large concentration on the effects that an asset imposes on the rest of the system (measured by the corresponding centrality) undermines the benefits of diversification, not only in terms of risk's asymptotic limit but also for the intermediate sized portfolios following risk’s convergence rate modifications.

Finally in the empirical part of the paper, we discuss the network characteristics for industry and international Granger networks and demonstrate that both networks show high propensity in resembling star network structure. This high conformity makes diversification highly exposed to shocks propagation in the volatile periods. We put this into a test. Considering an investor diversifies among industry portfolios (industry diversification) or international stock markets (international diversification), we find the Granger component of portfolio risk through time to be higher in the crisis periods (for example 2007-2008 financial crisis). This is reasonable, as Granger network captures how shocks propagate among assets, and in the crisis periods, shocks are more prone to occur in the market. In addition, we compare diversification benefits from diversifying among lowest and highest central assets in the Granger network with full sample diversification and also diversifying among stocks with lowest average correlation. For this analysis, we consider two datasets residing 100 stocks in S&P500 and FTSE500. We find lowest-centrality diversification to be more beneficial in terms of lower portfolio risk and higher diversification benefits. This is reasonable as lower central stocks are more resilient to shock propagation and according to findings, are less intimidating for our diversification benefits.

Our results are consistent with Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) which show that the structure of industrial input-output linkages influences the rate at which the aggregate volatility decays. Moreover, Rapach et al. (2014) find Granger network and the economy's production network mimicking each other; thus, our findings regarding the influence of Granger network on portfolio variance convergence are in accordance with economy's production interconnections influencing the economy's aggregate volatility decay rate. However, we provide a novel framework to assess the extent to which cross-effect structures influence portfolio performance from an investor point of view.

This paper complements Rapach et al. (2014). They find links in the industrial portfolios' Granger network to represent a more general notion of economic links in real economy and employ these links' information to construct a portfolio strategy that result in 11.37% annualized abnormal returns. We build on their findings by investigating the influence of Granger network by employing network measures. In abstract, we analyze how Granger network impacts portfolio performance from a structural perspective.

This paper complements several papers in the literature. We relates to Billio, Caporin, Panzica, and Pelizzon (2015). They provide a framework to take into account the influence of market structure on portfolio variance using spatial statistics considering CAPM model to be a reduced form of a more general model that includes assets interconnections. However, having included the impact of network structure on portfolio variance, our approach is different; we use a VAR model that is already established in the literature for decomposing portfolio variance and extracting the portion originating from assets' cross-dependencies. Our work builds on Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015). They provide a general framework to establish a relationship between network structure underlying shocks' amplification and an aggregate function. Additionally, we also relate to Rapach et al. (2013) and Eun and Shim (1989) where in our international Granger network, we find U.S. market leading the international market.

Our findings have several important implications. Via the Granger network and centrality measures, we can track the threatening assets in a diversified portfolio where a shock to them would propagate rapidly through the portfolio. Obviously, this provides the investors with a unique approach to diminish the influence of threatening assets in volatile periods. In this regard, this study provides a response to the ongoing discussion on the end of portfolio diversification due to its inability to respond to contagion and shocks propagation issues. Employing the findings in this paper, investors are able to construct diversified portfolios immune to poor performance in crisis periods. Furthermore, our framework can help to find systemic assets in the market if we look at the market as a diversified portfolio. Consequently, we can identify the most threatening assets in the economy as systemic, in that a shock to them would propagate extensively in the market.

2. Literature Review

Our paper is inspired by different branches of literature. First, we discuss the literature on the presence of lead-lag relationships among asset returns. On the one hand, there are papers identifying the determinants of lead-lag relationships. A first study in this regard is done by Lo and MacKinlay (1990), where they find the profitability of contrarian and momentum strategy to be partially caused by lead-lag relations among returns. Size is also found to be a determinant of lead-lag relations, with large firms leading small firms (their results are thoroughly investigated and confirmed by Lewellen (2002)). Exploring other determinants of lead-lag relationships, Brennan et al. (1993), Badrinath et al. (1995) and Chordia and Swaminathan (2000) find analyst coverage, institutional ownership, and trading volume to be determinants of lead-lag effects, respectively.

On the other hand, there are several papers trying to take advantage of cross-dependencies among returns in order to construct profitable portfolio strategies. With this aim in mind, they realized serial-dependencies among returns. DeMiguel et al. (2014) provide an extensive analysis investigating whether they can profit from the cross-dependencies across assets and find that a strategy formed on returns' cross-dependencies would perform well out-of-sample when the transaction cost is low (lower than ten basis points). Moreover, Menzly and Ozbas (2010) find that trading strategies based on cross predictability of returns result in economically and statistically significant premiums and are significantly correlated with the return series of hedge funds that are presumed to exploit return predictability effects. Rapach et al. (2014) construct portfolios using industry returns' cross-predictability and gain 11.37% annualized abnormal returns.

Furthermore, there are several papers discussing the presence of serial-dependencies in the international market. In an early study, Eun and Shim (1989) document lead-lag relation- ships in the international market demonstrating the leading role of U.S. market. Moreover, in a recent study, Rapach et al. (2013), take advantage of up-to-date developments in statistical analysis to explore the cross-dependencies in the international market and clearly show the leading role of the U.S. market among 11 industrialized countries.

The next question that we try to answer is: What are the explanations behind the presence of lead-lag relationships? Various papers provide explanations for the existence of cross-predictability among returns, including slow diffusion of information originating from investor specialization, limited market participation and market segmentation (Allen and Gale, 1994; Hong et al., 2007; Hou, 2007; Cohen and Frazzini, 2008; Menzly and Ozbas, 2010). Moreover, as discussed by Chordia and Swaminathan (2000); Hong et al. (2007) and Billio, Getmansky, Lo, and Pelizzon (2012), even when traders decide to take action using their knowledge of market information, the cross-predictability among returns remains ex- tant due to the presence of market frictions. Therefore, even in the equilibrium, we are able to find cross-effects among returns (Hong et al., 2007), or in other words, as Chordia and Swaminathan (2000) indicate, discovering returns' cross predictability does not necessary imply market inefficiency.

Another important question discussed in the literature is: What is the economical meaning of lead-lag relationships among returns? Several papers discuss the economic reasons be- hind returns' cross dependencies. Cohen and Frazzini (2008) and Menzly and Ozbas (2010) document cross-predictability across supplier-customer firms, finding it to be an adequate explanation of return cross-predictability. On the other hand, Rapach et al. (2014) define the cross-predictability across firms to signify a generalized notion of economic links. This is consistent with Billio et al. (2012), who consider returns to contain all the information in the market and therefore, returns' granger-causality summarizes various types of relationships (e.g. supplier-consumer, contractual agreements) among firms.

Our paper also follows the branch of literature on the aftermath of the last financial crisis that contributed to the elevated interest of the econometric and financial research communities in studying financial networks. In this category, there is a particular strand of literature whose purpose is to gain new insights on systemic risk issues by efficiently encapsulating cross-sectional dependency structure in network form. For example, Billio et al. (2012) build a directed Granger-causality network that relies on lead-lag relationships among returns in order to capture the market interconnections. They consider centrality of firms in this net- work, conveying their systemic importance.

We aim to disentangle the underlying attributes of portfolio diversification. The literature on portfolio diversification is enormous; here we just cover its salient findings. In a classical article, Samuelson (1969) explores the diversification benefits among a fixed number of stocks when they are negatively correlated. From an empirical standpoint, Evans and Archer (1968) focus on the negative relationship between portfolio volatility and the number of stocks in a randomly selected and equally weighted portfolio. They argue that most of diversification benefits are obtained with a small number of stocks (around 10 stocks). Mao (1970) provides theoretical support for this finding. However, Elton and Gruber (1977) and Bird and Tippett (1986) argue that the parametric relationship estimated by Evans and Archer (1968) is incorrectly specified, finding room for risk reduction well beyond the 10- stock rule. Consistent with this evidence, Statman (1987) relies on a Security Market Line approach to conclude that well-diversified portfolios must include at least 30 or 40 stocks if no leverage is permitted.

The studies regarding beta coefficients are also fundamental in the diversification literature. The so-called beta effect, documented by Klemkosky and Martin (1975), states the positive association between portfolio systematic risk (beta) and idiosyncratic risk (variance of the residuals from the market model). Thus, high beta portfolios require a larger number of securities to achieve approximately the same level of diversification as low beta ones. The dynamic nature of the diversification process is considered by Chen and Keown (1981), who provide evidence of the non-stationarity of both the beta coefficients and the idiosyncratic risk. Campbell, Lettau, Malkiel, and Xu (2001) report a noticeable increment in firm-level volatility relative to market volatility for the period of 1962-1997. This evidence led to a reduction in the correlation between the returns of individual stocks and the market. As a consequence, the number of stocks needed to obtain a given level of portfolio diversification rose in that period.

This paper is also strongly related to the emerging literature concerning macroeconomic fluctuations stemming from microeconomic idiosyncratic shocks (Horvath, 1998; Dupor, 1999; Shea, 2002; Gabaix, 2009; Acemoglu et al., 2015). In an allied work, Acemoglu et al. (2012) show how an economy's aggregate volatility decay rate is determined by the topology of an industrial input-output network. In a closely related article, Acemoglu et al. (2015) determine the fundamental elements characterizing the contagion process on a network, enabling us to rank them in terms of their aggregate performance. This framework sheds light on the commonalities among different contagion-related papers and provides a sort of coherence to their results.

To our knowledge, there is no paper investigating the influence of lead-lag relationships among assets on diversification benefits. Our paper examines this issue and extend our perspective on to how diversification in volatile periods becomes inefficient. We proceed to define a Granger network that summarizes the cross-effects among assets and then, considering two types of investors, myopic and long-term, we elaborate on the influence of Granger network on portfolio diversification.

3. Formation of Granger Network

In this section, we define the Granger network of returns that encompasses all the in- formation on the autocorrelations and serial-dependencies of asset returns and next, we introduce measures of network centralities that help us analyze the role of Granger network on an investment portfolio.

The VAR model developed by Sims (1980) is proven to be particularly suitable to capture dynamic relations among economic and financial time series. With regard to a VAR model for capturing returns serial dependencies, we can mention Hodrick (1992), Campbell and Shiller (1988), Kandel and Stambaugh (1990), Campbell (1990), Campbell et al. (2003), Chordia and Swaminathan (2000), Barberis (2000), Campbell and Ammer (1993), Eun and Shim (1989), Rapach et al. (2013), DeMiguel et al. (2014). Following the literature, we use the VAR model to search for the lead-lag relationships among returns.

Let us assume asset returns follow an n-dimensional and stationary VAR(1) process as follows:

$$\boldsymbol{r_t} = \boldsymbol{B}\boldsymbol{r_{t-1}} + \boldsymbol{u_t} \tag{1}$$

Where r_t is an n-dimensional demeaned vector of returns in period t, and $\boldsymbol{B} = [b_{ij}]$ is an $n \times n$ matrix where the element b_{ij} represents the impact of asset j in period t on asset i in period $t+1$ (presenting a granger-causal relationship; Lupkepohl (2005), Chordia and Swaminathan (2000)). Since there is not a constant term in expression (1), the implicit assumption is that $E\,(\boldsymbol{r_t}) = \emptyset$. Furthermore, $\boldsymbol{u_t}$ is a Gaussian white noise process with zero mean vector and positive definite covariance matrix $\boldsymbol{\Sigma_u} = [\sigma_{u,ij}]$. We allow the shocks to be cross-sectionally correlated but we assume that they are homoscedastic.[4]

Assuming the VAR model adequately captures the lead-lag relationships, we define the Granger network of returns as a weighed directed network where nodes corresponds to assets and the links account for return's lead-lag relationships summarized in the matrix $\boldsymbol{B}$ of VAR(1) model. Formally:

Definition 1. The Granger network of returns corresponding to the process in (1) is $\boldsymbol{\Omega} = (\boldsymbol{N}, \boldsymbol{B})$ where $\boldsymbol{N}$ is the set of assets and $\boldsymbol{B} = \{(j, i) \in \boldsymbol{N} \times \boldsymbol{N} : b_{ij} \neq 0\}$, the set of links.

Accordingly, this network accounts for the overall cross-sectional dynamical interactions between assets. Note that in general $\boldsymbol{B} \neq \boldsymbol{B^T}$ and thus, the impact of asset i on j is not necessarily the same as the impact from j to i, $b_{ji} \neq b_{ij}$.[5]

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[4] For a detailed explanation of VAR models, we refer to Luptkepohl (2005).

[5] In the paper by Campbell et al. (2003), they consider a VAR model with various stock returns and some state variables. It should be noted that even including state variables, we can still subtract the Granger network among returns and our theoretical findings that we derive in the following still holds.

The networks in Figure 1 represent the Granger network of 30 Fama and French industry portfolios in the U.S. and also 11 international countries examined by Rapach et al. (2013). One main advantage of a network depiction of serial-dependencies is the relatively easy identification of main players in the network. In the Industry network, we easily identify financial industry as the most central industry and in the International network, the U.S. stock market is the central one. Presumably, we expect any shock to these central nodes to propagate rapidly through the network. In the network theory context, there are several centrality measures that capture the relative centrality of nodes in the network. Since our Granger network is a directed network[6], a node can be central in impacting other nodes in the network (threatening centrality) or it can be central by being impacted by a large number of nodes (vulnerable centrality). Due to the importance of threatening centrality on the notion of risk attribute of a portfolio, hereby, we formally define two measures: threatening-degree centrality and Katz-Bonacich centrality.

Definition 2. Considering directed network, $\boldsymbol{\Omega} = (\boldsymbol{N}, \mathbf{B})$, the *threatening-degree centrality* of asset j is defined as follows:

$$d_j = \sum_{\boldsymbol{i=1}}^{\boldsymbol{N}} B_{ij} \qquad for\ j = 1,2, \dots, N \tag{2}$$

Threatening-degree centrality is simply the summation of weights in the columns of matrix $\boldsymbol{B}$. In our industry and international Granger networks, financial industry and U.S. stock market are the most central nodes based on threatening-degree centrality. In the next step, we define a centrality measure that captures not only the direct impact of an asset on other assets but also its indirect influence.

Definition 3. Considering directed network, $\boldsymbol{\Omega} = (\boldsymbol{N}, \mathbf{B})$, the *Katz-Bonacich Centrality* of asset j is defined as follows[7]:

$$v_j = \mathbf{1}'(I - B)^{-1} e_j = (\mathbf{1}'L) e_j \qquad for\ j = 1,2, \dots, n \tag{3}$$

Where $\boldsymbol{e}_j$ is the $r - th$ unit vector and $\mathbf{1}$ is an n-dimensional vector of ones. Note that the existence of $\boldsymbol{L} = [l_{ij}] = (\boldsymbol{I} - \boldsymbol{B})^{-1}$ is guaranteed by the assumption of stationarity in process (1). It is worth mentioning that Katz-Bonacich centrality has a particular interpretation in the time

[6] In a directed network, there is directions in links connecting the nodes. In our Granger network context, node i is connected to node j when asset i is leading asset j in our VAR estimation.

[7] *Katz-Bonacich Centrality* firstly developed by Katz (1953). We follow Newman (2010) for its derivation.

series literature. As it is clearly explained in Lütkepohl (2007), in impulse response analysis, the coefficient l_{ij} accounts for the long term and cumulative effect of asset i given a unit shock on asset j. Then, v_i could be interpreted as the cumulative long term effect of a unit shock on asset i taking into account higher order connections upon the entire investment set.

In the next section, we continue to investigate the role played by Granger network on portfolio risk of a myopic and long-term investor. Moreover, we also analyze the impact of Granger network on the outcome of diversification in several stylized structures of Granger network.

4. Granger Network and Portfolio Risk

We consider a naive investor that divides his wealth equally among assets. In order to analyze the impact of Granger network on portfolio variance, we further consider two cases: A myopic investor and a long-term investor. In each case, first we provide general propositions on how shocks to assets in Granger network influence portfolio risk.

4.1. The case of a myopic investor

We consider an investor that only cares about tomorrow, a myopic investor (DeMiguel et al. 2014), and study the role of Granger network in determining her portfolio performance.

Equation (1) can be written as the summation of innovations (a moving average representation). Due to the imposition of stationarity, $\boldsymbol{B}^i$ converges to zero rapidly with increasing i.[8] Therefore, a convenient approximation of equation (1) is given by equation (4) where the term $\boldsymbol{B^i u_{t-i}}$ is negligible for $i \geq 2$.

$$\boldsymbol{r_t} = \sum_{i=0}^{\infty} \boldsymbol{B^i u_{t-i}} = \boldsymbol{u}_t + \boldsymbol{B u_{t-1}} \qquad (4)$$

Where u_{t-1} represents a shock to return series in the last period and this shock propagates through the Granger network in proportion to the elements in matrix B. Let us assume that the myopic investor diversifies naively among assets in her portfolio. In this case, she diversifies among the shocks in the current period and also the last period. Studying the portfolio return leads to the following proposition.

[8] In order to demonstrate how fast it converges to zero, we provide several numerical examples in Appendix A.

Proposition 1: Given the return process in (1), and assuming $\Sigma_u = \sigma_u^2 \mathrm{I}$, the portfolio return of a naive myopic investor is:

$$\boldsymbol{r}_{\boldsymbol{p}(\boldsymbol{m})} = \frac{1}{n}\mathbf{1}'\boldsymbol{u_t} + \frac{1}{n}\mathbf{1}'\boldsymbol{B}\boldsymbol{u_{t-1}} = \frac{1}{n}\sum_{i=1}^{\boldsymbol{n}} u_t^i + \frac{1}{n}\sum_{i=1}^{\boldsymbol{n}} d_i u_{t-1}^i \tag{5}$$

Where $\boldsymbol{d} = \mathbf{1}'B$ represents the threatening-degree centrality of assets in the Granger network. We conclude that the higher is the absolute value of threatening-degree centrality of an asset in the network, the higher impact a shock on this asset would have on the portfolio return.[9]

Corollary 1: Given the return process in (1), the expected portfolio return of a naive myopic investor is:

$$\mu_{p(m)} = \emptyset \tag{6}$$

Corollary 1 shows that the expected portfolio return is not a function of the Granger network. Therefore, the pattern of interactions among assets in the Granger network does not present any economic consequences upon the myopic investor portfolio return in expected terms.

Next, we delve into examining portfolio variance. Assuming our myopic investor divides her wealth among assets according to weight vector, $\boldsymbol{w}$, the portfolio variance is as follows:

$$\sigma^2_{p(m)} = \boldsymbol{w}'\boldsymbol{\Sigma_u}\boldsymbol{w} + \boldsymbol{w}'\boldsymbol{B}\boldsymbol{\Sigma_u}\boldsymbol{B}'\boldsymbol{w} \tag{7}$$

$$\sigma^2_{p(m)} = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{u,ij} + \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j (\sum_{k=1}^{n}\sum_{l=1}^{n} B_{ik} B_{jl} \sigma_{u,kl}) \tag{8}$$

[9] Canceling the diagonally assumption of $\boldsymbol{\Sigma_u}$, we can introduce this diagonality via Cholesky decomposition of covariance matrix of error terms. With regard to expression (1), we can decompose the contemporaneous covariance as $\boldsymbol{\Sigma_u} = \boldsymbol{P}\boldsymbol{P}'$ and consider a new error term $\boldsymbol{w_t} = \boldsymbol{P^{-1}}\boldsymbol{u_t}$ and the coefficients become $\boldsymbol{B}^* = \boldsymbol{B}\boldsymbol{P}$. Thereby, we would have $\boldsymbol{r_t} = \boldsymbol{P}\boldsymbol{w_t} + \boldsymbol{B}^*\boldsymbol{w_{t-1}}$. In this case, the errors covariance is diagonal and we should compute the centralities for matrix $\boldsymbol{B}^*$. However, this does not depreciate the importance of Granger network. We compute the centrality values in both matrices $\boldsymbol{B}$ and $\boldsymbol{B}^*$ and compute the Pearson and Kendall rank correlations between these two centralities. Our results in Appendix B, for industry and international datasets, show that these are highly correlated.

The right-hand side of equation (7) is composed of two terms, the first one corresponds to the traditional portfolio variance when there is no Granger network effect. We name this term the *contemporaneous component* of portfolio variance. According to Eun and Shim (1989), the contemporaneous correlation between residual returns, in our case the contemporaneous component, represents the degree of economic integration or in other words, the degree to which new information generating an abnormal return in one asset is shared by other assets.

The second term is associated with the dynamical interactions among returns which captures the impact of the Granger network topology, $\mathbf{\Omega}$. This term is characterized as the *Granger component* of portfolio variance. In order to focus on the specific effects exerted by the topology of the Granger network, we assume $\boldsymbol{\Sigma_u} = \sigma_u^2\mathbf{I}$ and thereby assuming the correlations in the contemporaneous component to be zero. The next proposition states the relationship between portfolio variance and the distribution of threatening-degree centrality of assets in the Granger network.

Proposition 2: Given the return process in (1) and assuming $\boldsymbol{\Sigma_u} = \sigma_u^2\mathbf{I}$, the portfolio variance of a naive myopic investor is:

$$\sigma_{p(m)}^2 = \frac{\sigma_u^2}{n} + \left(\frac{\sigma_u}{n}\right)^2 \|\boldsymbol{d}\|_2^2 \tag{9}$$

$$\sigma_{p(m)}^2 = \frac{\sigma_u^2}{n} + \frac{\sigma_u^2}{n}\bar{\boldsymbol{d}}^2\left(1 + \boldsymbol{CV_d^2}\right) \tag{10}$$

Where $\|\boldsymbol{d}\|_2^2$ accounts for the square of the Euclidean norm of threatening-degree centrality vector. Furthermore, $\bar{\boldsymbol{d}}$ and $\boldsymbol{CV_d}$ are, respectively, the mean and the coefficient of variation of centrality vector elements. Proposition 2 states that, controlling for the mean centrality, those types of networks presenting fat-tail threatening-degree centrality distribution present larger portfolio volatility.

In the next step, we proceeds to investigate the consequences of various stylized Granger network structures on portfolio variance and diversification benefits for a myopic investor.

4.1.1. *Portfolio diversification and special cases of network topology*

Figure 2 demonstrates the full list of network motifs (specific patterns of interactions) underlying Granger network that fundamentally determines portfolio risk.

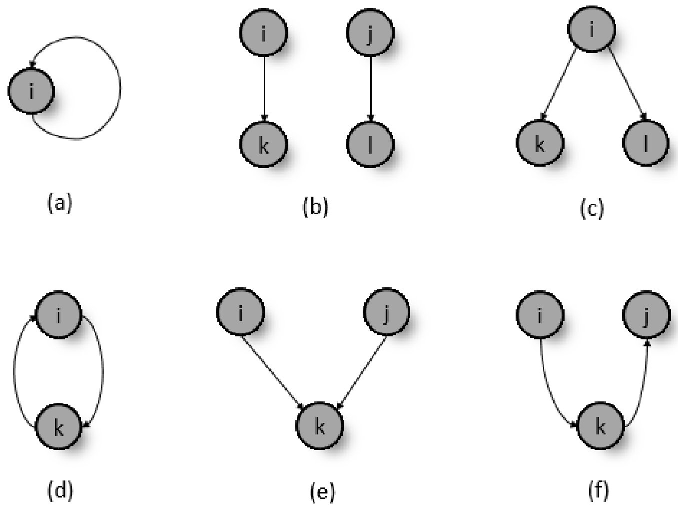

Figure 2. Specific network motifs influencing portfolio variance (Detailed explanation in Appendix B)

In total, the weight invested in the stocks with *in-going* links (vulnerable stocks), and the individual variance and covariance between the stocks with *out-going* links (threatening stocks) are the major determinants of portfolio variance. This is reasonable as any variation in a vulnerable stocks comes from the threat exerted by threatening stocks.

In the next step, we continue to examine various special network topologies and investigate the portfolio volatility in these structures. Hereby, we also cancel the assumption of diagonality of contemporaneous covariance in order to see how the correlations influence portfolio diversification benefits. Throughout this section, we impose simplifying assumptions following Mao (1970) to make our analysis more tractable. We consider a naive investor that allocates his wealth equally among assets in the investor's portfolio set. As a consequence, the (column) vector of portfolio's weights is $\boldsymbol{w} = \frac{1}{n}\mathbf{1}$ where $\mathbf{1}$ is a column vector of ones. Additionally, with the aim to isolate any other possible effect different from the network topology, we assume $\sigma_{u,ij} = \sigma_u^2$ for $i = j$ and $\sigma_{u,ij} = \rho\sigma_u^2$ for $i \neq j$ where ρ accounts for the equal pair correlation of returns. Additionally, we consider $b_{ij} = b < 1; \forall ij$. We say that formula is under *SSA*, when all these sets of simplifying assumptions are in place for a particular formula.

***Case 0**: No-dynamic structure*

When there is no dynamical structure in the return process, $\boldsymbol{B}$ is a null matrix and as a consequence, portfolio variance follows its traditional formulation as in Markowitz 1952. Therefore,

$$\sigma^2_{p(m)} = \boldsymbol{w}'\Sigma_u\boldsymbol{w} = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{u,ij} \tag{11}$$

Consistent with Mao 1970, equation (12) under *SSA* is written as follows stressing the fact that portfolio variance is a function of the number of assets in the portfolio.

$$\sigma^2_{p(m)}(n) = \left[\frac{1}{n} + \rho\left(1 - \frac{1}{n}\right)\right]\sigma^2_u \tag{12}$$

Let us consider two extreme cases, i) none diversification where $n = 1$ leading to $\sigma^2_{p(m)}(1) = \sigma^2_u$ and ii) extreme diversification in which $n = \infty$ (when every asset in the market is included into the investment basket) leading to $\sigma^2_{p(m)}(\infty) = \rho\sigma^2_u$. Note that since $\sigma^2_{p(m)}(1) > \sigma^2_{p(m)}(\infty)$, there exist diversification benefit.[10]

When there is a dynamical structure, the variance of portfolio, in comparison with the no-dynamical case increases by the element $\boldsymbol{w}'\boldsymbol{B}\boldsymbol{\Sigma_u}\boldsymbol{B}'\boldsymbol{w}$ which captures the effect of the network topology, Ω. Next, we assume that the set of assets included in the investment opportunity set form stylized network topologies.

***Case I**: Disconnected Network*

The first case under analysis assumes each stock is affected only by itself as it is depicted in Figure 3 and thus there is no Granger interaction among assets beyond their own autocorrelation process.

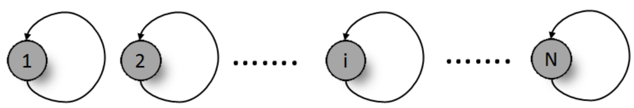

Figure 3. Disconnected network: special case where each stock is threatening only itself

[10] Note that according to Mao (1970), ρ represents the average of correlations of assets in the portfolio set and it is originally positive.

In this case, matrix B is diagonal and since there is no cross-interaction, $b_{ij} = 0$ for $i \neq j$, portfolio variance is as follows:

$$\sigma^2_{p(m)} = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{u,ij} + (\sum_{i=1}^{n}(w_i)^2 {b_{ii}}^2 {\sigma_{u,i}}^2 + \sum_{k=1}^{n}\sum_{l=1\, k\neq l}^{n} w_k w_l b_{kk} b_{ll} \sigma_{u,lk}) \tag{13}$$

The first term of expression (13) is the standard portfolio variance and the second and third terms represent the impact from network topology. In the second and third terms, the weights, variance and covariance of each asset in the investment set affect the portfolio variance. With regard to variance, the impact is positive for all of stocks. Under *SSA*, as long as $|b| < 1$, stationarity holds and the simplified portfolio variance is:

$$\sigma^2_{p(m)}(n) = \left[\frac{1}{n} + \rho\left(1 - \frac{1}{n}\right)\right][1 + b^2]\sigma_u^2 \tag{14}$$

As before, two extreme cases are mentioned. For $n = 1$, the portfolio variance is $\sigma^2_{p(m)}(1) = [1 + b^2]\sigma_u^2$ and for the case of maximal diversification, $n = \infty$, we have $\sigma^2_{p(m)}(\infty) = [1 + b^2]\rho\sigma_u^2$. Apparently, portfolio variance is higher than no-dynamic case by $b^2\rho\sigma_u^2$. Knowing ρ is positive, the higher is b, the higher would be the asymptotic variance of the portfolio.

Assuming the correlation between assets to be zero, we compute the portfolio variance with regard to the distribution of the *threatening-degree centrality* following equation (10). In this regard, the threatening-degree centrality of each asset is equal to b, and thereby, mean degree centrality, $\bar{d}$, is equal to b and the coefficient of variance, CV_d, is zero. Therefore, the variance of portfolio is $\sigma_u^2(\frac{1}{n^2} + \frac{b^2}{n})$ that converges to zero as we include a large number assets in the portfolio. We clearly observe that in this case where the threatening-degree distribution is homogenously distributed or in other words, it does not present any fat-tail characteristics (where some assets take highly central positions), the portfolio variance consistently diverges towards zero as we increase the number of assets.

Case II: *Star Network*

Case (I) assumes no interaction among assets in the Granger network. Another interesting case is when there is one asset influencing all other assets. We termed this case as *star network* and it is presented in Figure 4.

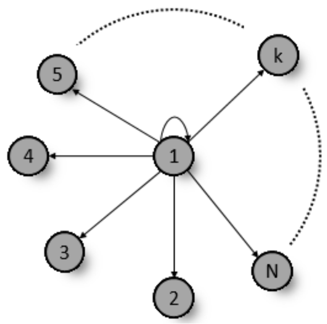

Figure 4. Star Network

Evidently, ***B*** is a zero matrix expect for the first column. The variance of the portfolio is given by expression (15). Considering the second term on the influence of network topology, we observe that variance of the central stock, stock *1*, and also the weights allocated to stocks in the periphery of the network, w_i for $i = \{2,3, \dots, n\}$, are relevant in determining portfolio volatility.

$$\sigma_{p(m)}^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{u,ij} + \sum_{k=2}^{N}\sum_{l=2}^{N} w_k w_l b_{k1} b_{l1} {\sigma_{u,1}}^2 \quad (15)$$

When *SSA* is imposed, matrix ***B*** has all its entries equal to zero except for the components of column *1* which are equal to b. Matrix ***B*** has all its eigenvalues equal to zero except for one which is equivalent to b. Therefore, as long as $|b| < 1$, stationarity holds. In such situation, the portfolio variance is:

$$\sigma_{p(m)}^2(n) = \left[\frac{1}{n} + \rho\left(1 - \frac{1}{n}\right) + b^2\right]\sigma_u^2 \quad (16)$$

When $n = 1$, we have $\sigma_p^2(1) = (1 + b^2)\sigma_u^2$ and for the case of maximal diversification where $n = \infty$,we have $\sigma_p^2(\infty) = (\rho + b^2)\sigma_u^2$. The Granger component inserts the expression $b^2\sigma_u^2$ to the asymptotic variance of the portfolio. Obviously, portfolio variance in this case is higher than it for no-dynamic and Disconnected network structures.

In this network topology, the central asset, asset *1*, has threatening-degree centrality equal to nb, and thereby, it is equal to zero for all other assets in the system. Therefore, the mean centrality, $\bar{d}$ is equal to b and the coefficient of variation of centralities, CV_d, is $\sqrt{n}$. This value of CV_d clearly demonstrates fat-tail characteristic in degree centrality (with some assets taking highly central positions) that increases as we include more assets in the portfolio. Consequently, assuming contemporaneous covariance between stocks, ρ, to be equal to zero, the portfolio variance is $\sigma_u^2(\frac{1}{n} + \frac{b^2}{n} + b^2)$. This evidently demonstrates the role of fat-tail characteristic of threatening-degree centrality distribution on portfolio volatility where the portfolio variance converges to a higher level than no-dynamic and also disconnected network cases.

Case III: *Inverse Star Network*

The inverse of case II is presented in Figure 5 where the center of the network receives impacts from other assets in the investment set.

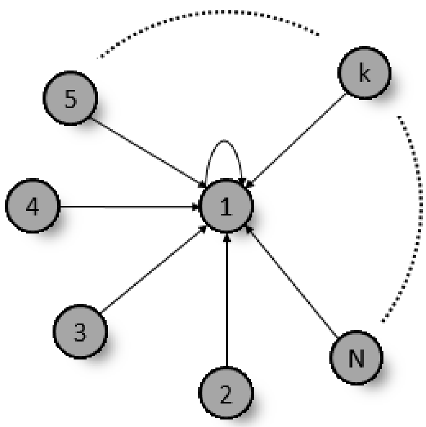

Figure 5. Inverse Star Network

In this case, $\boldsymbol{B}$ is a zero matrix expect for the first row. The corresponding portfolio variance is given by equation (18). Focusing on the granger component, we see that the only weight involved in the formula is the one from the most vulnerable asset, asset *1*. However, all of the correlations between pairs of peripheral assets are crucial players in equation (18).

$$\sigma^2_{p(m)} = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{u,ij} + (w_1)^2 \sum_{k=2}^{N}\sum_{l=2}^{N} b_{1k} b_{1l} \sigma_{u,kl} \tag{17}$$

Since this network topology is the transpose of star network, the eigenvalues for matrix ***B*** is exactly the same. Therefore, Under *SSA,* as long as $|b| < 1$, stationarity holds. Moreover, the portfolio variance with *SSA* conditions is exactly the same as the case of disconnected network and it is lower than the case of star network. This highlights the importance of concentration in the degrees pointing from a particular node (threatening centrality) in comparison to the concentration of the degrees pointing to a particular node (vulnerability centrality). It is clear that a highly out-degrees concentration in the network undermines the benefits of diversification. Moreover, threatening-degree centrality is equal to b for each asset and consequently, the variation of degree centralities is zero. Thereby, we get the same results as in disconnected network structure when we neglect the impact of covariance between assets in contemporaneous component of portfolio variance implying portfolio variance decay to zero in extreme diversification.

Case IV*: Circle Network*

Another interesting case to study regards to the circle network which is depicted in Figure 6. In this symmetric structure asset i affects asset $i+1$ and it is being affected by stock i - 1 (Each asset is influencing only one asset and is being impacted by only one asset in the network).

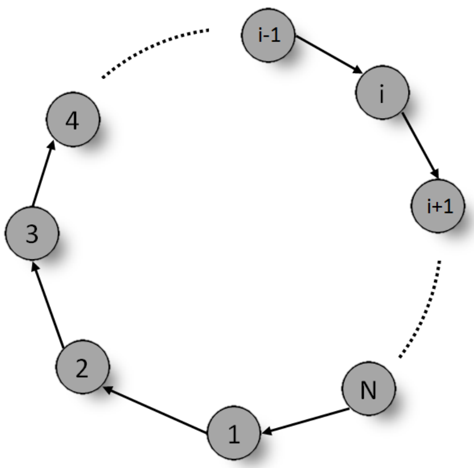

Figure 6. Circle Network

The elements of matrix $\boldsymbol{B}$ are zero except for those located in first diagonal below the main diagonal and for the one located in the upper right corner. The portfolio variance in this case is given by equation (18). Note that the variance of portfolio originated from the Granger network structure is determined by the weights and covariances of the consecutive assets.

$$\sigma^2_{p(m)} = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{u,ij} + \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j b_{i(i+1)} b_{(j)(j+1)} \sigma_{u,(i+1)(j+1)} \tag{18}$$

Under *SSA*, statitionary is preserved as long as $|b| < 1$. We observe that variance is exactly equal to the one from the disconnected network. In this case, the threatening degree centrality for each asset is equal to b and consequently, the coefficient of variation is zero. Thereby, we get the same results as in disconnected network structure by neglecting the impact of covariance between assets in contemporaneous component of portfolio variance.

Case V: *Fully Connected Network*

Finally, the case in which the network is fully connected is depicted in Figure 7 for the case of $n = 4$. In this situation, each asset is connected with the rest of the assets in the investment set implying a reciprocal relations in the network. Thus, all the cases depicted in Figure 2 regarding motifs come to play their role in determining portfolio risk.

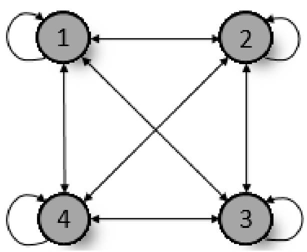

Figure 7. Fully Connected Network

This topology is somehow different since in order to preserve stationary under *SSA*, the parameter b must be sufficiently small in relation to n. Therefore, it is assumed that $b = \frac{1}{n}\delta$ for $0 < \delta < 1$.

It could be proved that the eigenvalues of B are all equal to zero expect for the largest which is equal to δ. The portfolio variance is as follows:

$$\sigma^2_{p(m)}(n) = \left[\frac{1}{n} + \rho\left(1 - \frac{1}{n}\right)\right][1 + \delta^2]\sigma_u^2 \tag{19}$$

As before, two extreme cases regarding n are mentioned. For $n = 1$, the portfolio variance is $\sigma_p^2(1) = [1 + \delta^2]\sigma_u^2$. For the case of maximal diversification characterized by $n = \infty$, we have $\sigma_p^2(\infty) = [1 + \delta^2]\rho\sigma_u^2$. The reader should note that expression (19) presents the same analytical structure than those prevailing for the disconnected network and it is identical as long as $b = \delta$. In this case, the threatening-degree centrality of each asset is equal to $n\delta$ and the coefficient of variation is zero. Therefore, it follows the same routine as in the disconnected case with regard to the impact of the threatening-degree centrality distribution on the portfolio variance level.

In conclusion, after the careful analysis of different stylized networks, we close this subsection with a concluding proposition as follows.

Proposition 3: Given the return process in (1), the portfolio variance of a naive myopic investor ,assuming $\Sigma_u = \sigma_u^2 \mathrm{I}$, as n tends to infinite is zero for the disconnected, inverse-star, circle and full Granger networks. For the Star network case, it is $\sigma_u^2 b^2$.

Proposition 3 establishes an important result regarding the benefit of diversification. Consistent with the standard theory, ignoring contemporaneous correlation, the variance of the portfolio converge to zero expect for the star network. In this case, the risk suffered by a myopic investor remains positive even for an extremely large portfolio size. This results is align with the findings in Acemoglu et al. (2012) and stresses the disproportional effect of some central nodes on the aggregate performance of the system.

4.2. The case of a long-term investor

In this section, we continue to analyze the impact of Granger network on the portfolio performance for a long-term investor[11]. We assume an investor who decides to invest naively on

[11] Several papers have investigated the case of long-horizon portfolio selection (Cochrane (2014), Barberis (2000), Campbell et al. (2003), Campbell and Viceira (2002). To be precise, we are considering an infinitely lived investor. Portfolio decisions for an infinitely lived investor is also investigated by Campbell and Viceira (1999).

a set of assets for a long period and our purpose is to analyze the influence of Ganger network on her portfolio performance. Since we are analyzing the long run effects of shocks on portfolio performance, we drop the time subscript in (1) leading to:[12]

$$\boldsymbol{r}_{(\boldsymbol{LT})} = (\boldsymbol{I} - \boldsymbol{B})^{-1}\boldsymbol{u} \tag{20}$$

Assume an initial scenario in which the system is in its steady state $\boldsymbol{r} = \emptyset$ and suddenly a shock in asset *i* happens. Taking derivatives on (20) with respect to u_i, we get:

$$\frac{d\boldsymbol{r}}{du_i} = (\boldsymbol{I} - \boldsymbol{B})^{-1}\boldsymbol{1}_{\boldsymbol{i}} \tag{21}$$

Where $\boldsymbol{1}_{\boldsymbol{i}}$ is an *n*-dimensional vector whose i^{th}-component equals one and zero otherwise. The first order Taylor approximation of (21) around $\boldsymbol{\varepsilon} = \emptyset$ is thus given by (22). The element $\boldsymbol{r}_{(\boldsymbol{LT})}$ accounts for the long-run vector of returns after all the intermediate adjustments have taken place.

$$\boldsymbol{r}_{(\boldsymbol{LT})} = (\boldsymbol{I} - \boldsymbol{B})^{-1}\emptyset + \sum_{i}^{n} \frac{dr}{du_i} u_i = (\boldsymbol{I} - \boldsymbol{B})^{-1}\boldsymbol{u} \tag{22}$$

In what follows, we consider a naive investor that proportionally distributes her wealth following the $1/n$ rule. Therefore, the *n*-dimensional vector of portfolio weights is given by $\boldsymbol{w} = \frac{1}{n}\boldsymbol{1}$. Thus, the long-term portfolio return is given by:

$$r_{p(LT)} = \frac{1}{n}\boldsymbol{1}'\boldsymbol{r}_{(\boldsymbol{LT})} \tag{23}$$

The relationship between the long-term portfolio return in terms of the threatening Katz-Bonacich centrality of stocks is documented as follows:

Proposition 4: Given the return process in (1), the long-term portfolio return of a naive investor, assuming $\boldsymbol{\Sigma_u} = \sigma_u^2\mathbf{I}$, is:

[12] With $R_t = BR_{t-1} + u_t$, in the long term, we have: $R = BR + u$. This will conclude to the expression (21).

$$r_{p(LT)} = \frac{1}{n}\mathbf{1}'(\boldsymbol{I}-\boldsymbol{B})^{-1}\boldsymbol{u} = \frac{1}{n}\boldsymbol{v}'\boldsymbol{u} = \frac{1}{n}\sum_{i=1}^{n} v_i u_i \tag{24}$$

The above results highlight that, the higher the Katz-Bonacich centrality of asset i, the larger is its effect on the performance of the entire portfolio. Therefore, negative (positive) shocks to those extremely central assets strongly reduces (increases) the long-run portfolio return. By denoting the expected portfolio return as $\mu^{p}_{(LT)} = E\left(r^{p}_{(LT)}\right)$, proposition 4 leads to the next corollary:

Corollary 2: Given the return process in (1), the expected long-term portfolio return of a naive investor is:

$$\mu_{p(LT)} = \emptyset \tag{25}$$

Corollary 2 shows that the expected portfolio return is not a function of the Granger network, a result consistent with the certainty equivalence property found in Acemoglu, Ozdaglar, and Thabaz-Salehi (2015). Thereby, the pattern of interaction between assets in the Granger network does not present any economic consequence upon the long-term portfolio return in expected terms.

Let us move on to consider the effect of the Stock Granger network upon the portfolio volatility. The next theorem states the relationship between this variable and the distribution of Katz-Bonacich centrality of assets.

Proposition 5: Given the return process in (1), assuming $\boldsymbol{\Sigma_u} = \sigma_u^2\mathbf{I}$, the long-term portfolio variance of a naive investor is:

$$\sigma^2_{p(LT)} = \left(\frac{\sigma_u}{n}\right)^2 \boldsymbol{v}'\boldsymbol{v} = \left(\frac{\sigma_u}{n}\right)^2 \|\boldsymbol{v}\|_2^2 \tag{26}$$

$$\sigma^2_{p(LT)} = \frac{\sigma_u^2}{n}\bar{\boldsymbol{v}}^2(1+\boldsymbol{CV}_{\boldsymbol{v}}^2) \tag{27}$$

Where $\|\boldsymbol{v}\|_2^2$ accounts for the square of the Euclidean norm of centrality vector, $\boldsymbol{v}$. Moreover, $\bar{v}$ and CV_v^2 are, respectively, the mean and the coefficient of variation of centralities. The main result from proposition 5 states that, controlling for the mean centrality, those types of network presenting fat-tail centrality distribution lead to larger portfolio volatility. Next, we provide a proposition summarizing the portfolio diversification benefits in different stylized network topologies.[13]

Proposition 6: Given the return process in (1), assuming $\boldsymbol{\Sigma_u} = \sigma_u^2 \mathbf{I}$, the long-term portfolio variance of a naive investor when n tends to infinite is zero for the inverse Star, circle and disconnected Granger Network. For the star network case, it is $\frac{\sigma_u^2 b^2}{(1-b)^2}$.

Proposition 6 posits the same consequence of Granger network topology on portfolio variance as the myopic investor. When there is homogeneity in the threatening centrality distribution, diversification force portfolio variance to converge to zero. However, in the star network, when there is a high heterogeneity in Katz-Bonacich centrality distribution, diversification result in higher level of portfolio volatility. More importantly, this result is in line with Acemoglu et al. (2012) where existence of highly central nodes would have major impact on the aggregate performance of the system.

Moreover, we observe that in both cases, myopic (short-term) and long-term investors, the influence of Granger network topology on the portfolio risk has the same implication. In both cases, the star network structure deviates from other structures with respect to portfolio risk. We know that a determinant factor for analyzing the impact of Granger network on portfolio variance is the centrality measures: threatening-degree centrality for a myopic investor and Katz-Bonacich centrality for a long-term investor. The Katz-Bonacich centrality can be written as follows:

$$\boldsymbol{v} = \mathbf{1}'(\boldsymbol{I} - \boldsymbol{B})^{-1} = \mathbf{1}'(I + \boldsymbol{B} + \boldsymbol{B}^2 + \boldsymbol{B}^3 + \cdots) \quad (28)$$

Where the powers of matrix $\boldsymbol{B}$ higher than 1 take into account higher order interactions in the Granger network. For example, suppose asset *j* impacts asset *i* and asset *i* impacts asset *k* in the Granger network; therefore, the element in row *j* and column *k* in matrix $\boldsymbol{B}^2$ would include this second order influence from asset *j* to asset *k*. Due to the stationarity of return process, we expect this influence to be lower and therefore, we can re-write the above equation by $\boldsymbol{v} \cong \mathbf{1}'(\boldsymbol{I} + \boldsymbol{B}) = \mathbf{1}' + \boldsymbol{d}$. It can be concluded that if we neglect the higher order influences, both centrality measures provide the same ranking.[14] Moreover, in the empirical part, we show that both of these

[13] The details of network structures and portfolio variance parameters for the short-term and long-term cases are summarized in Appendix D.

[14] In an empirical analysis, we document that both of these centrality measures provide a close rankings of the assets and they are highly correlated. We analyze the correlation between centrality measures, threatening-degree centrality and Katz-Bonacich centrality. The correlation in the industry network is 0.9999 and in the international network is

centrality measures provide a close rankings of the assets and they are highly correlated. In this regard, they follow the same pattern in defining the portfolio volatility. In other words, regardless of investing in short-time or long-term, Granger network has the same influence on the portfolio diversification benefits.[15]

4.3. Numerical experiment

Next, we provide empirical evidence for portfolio variance convergence in different network topologies. We assume SSA. Two numerical analysis is implemented. In the first case, we simply compute the portfolio variance for different stylized network structures and discuss the portfolio variance convergence. We proceed by examining portfolio variance elasticity.

Let us consider the case of a myopic investor diversifying naively among all the assets in the stylized networks. Figure 8 presents the results for no-dynamical, disconnected network and star network structures. We plot four graphs employing different values for two fundamental parameters, specifically $\rho \in \{-0.04, 0.04\}$ and $b \in \{0.4, 0.6\}$[16]. We consider positive values for b and we expect the negative values of b leading to the same behavior for portfolio variance convergence in different network structures. Additionally, since ρ is the mean correlation among n assets, such parameter is not bounded from above but it is bounded from below since the minimum negative correlation coefficient among the set of n assets is given by $-1/(n-1)$. Since the maximum portfolio size in Figure 8 is 20, this explains the values given to ρ.[17]

0.9961. Moreover, we also consider two daily datasets, SP100 with daily excess returns (split and divided adjusted) for the 100 most capitalized stocks in the SP500 index residing between 10/1/2002 and 12/31/2012 and FTSE100 with 100 stocks in FTSE250 index between 2/27/2006 to 10/25/2013. The correlations between threatening-degree centrality and Katz-Bonacich centrality for these two datasets are 0.9797 and 0.9976, respectively. The scatter plot for centralities in all of these datasets are presented in the Appendix J.

[15] This view on the same portfolio specification by short and long horizon investors is shared also in the classic works of Samuelson (1969) and Merton (1969).

[16] We consider cases where $\rho \in \{-0.04, 0.04\}$ and $b \in \{0.01, 0.6\}$. The corresponding plots is presented in the Appendix E.

[17] Note that in the limiting case of $n \to \infty$, ρ should not be lower than zero. (Mao, 1970).

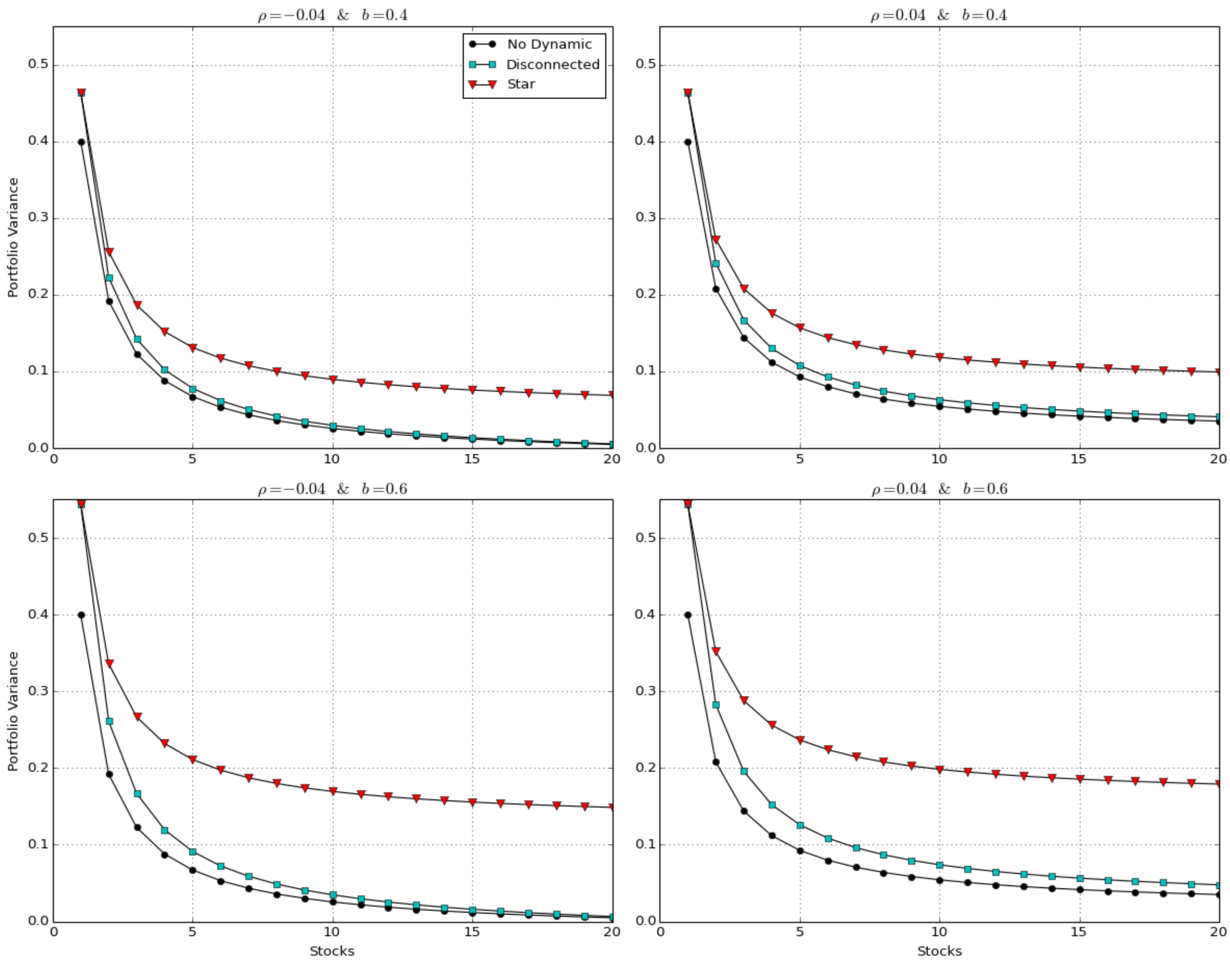

Figure 8. Portfolio variance for no-dynamical, disconnected and star network structures for $\rho \in \{-0.04, 0.04\}$ and $b \in \{0.4, 0.6\}$.

There are three deductions that are worth to highlight. First, diversification benefits are evident given the negative slope shown by $\sigma^2_{p(m)}$ in any of the network configurations and for any parameter specification. Thereby, larger portfolio size provides lower portfolio variance disregarding the network topology in place. Second, there is a clear ordering in terms of $\sigma^2_{p(m)}$ for any given level of portfolio size that prevails irrespectively of the parameter specification. The worst performance is assigned to the star network and the best one corresponds to the case of no-dynamic structure. The rest of the network configurations are located between these two extremes.

In order to get further insights on the effects that Granger network has on the diversification benefits, the portfolio variance elasticity $\xi(n)$ is defined as follows:

$$\xi(n) = \frac{\partial \sigma^2_{p(m)}(n)}{\partial n} \frac{n}{\sigma^2_{p(m)}(n)} \tag{29}$$

Expression (30) and (31) provide the formulas for elasticity in the disconnected and star networks, respectively.[18]

$$\xi^{D}(n)=\frac{\rho-1}{\rho(n-1)+1} \tag{30}$$

$$\xi^{S}(n)=\frac{\rho-1}{\rho(n-1)+b^{2}n+1} \tag{31}$$

Figure 9 draws diversification elasticity for the same parameter specifications as before[19]. Consistent with Figure 8, four aspects should be mentioned. First, ξ is negative stressing the benefit of diversification for any parameter specification and network configuration. Second, the elasticity corresponding to star network is always lower (in absolute terms) than for the rest of the structures which relates to the existing potential of diversification embedded in different network architectures. Third, for $\rho>0$, ξ shows a positive slope representing the decreasing marginal benefit of diversification. However, for $\rho<0$, this behavior is preserved for the star network but this is not the case for the rest of the structures depicting increasing benefit of diversification. Finally, higher b increases the difference between the benefit of diversification among the two types of structures.

[18] The cases of no dynamics, circle and inverse star networks show the same elasticity given by equation (20).
[19] Other cases of $\rho\in\{-0.04,0.04\}$ and $b\in\{0.01,0.6\}$ are provided in the Appendix F.

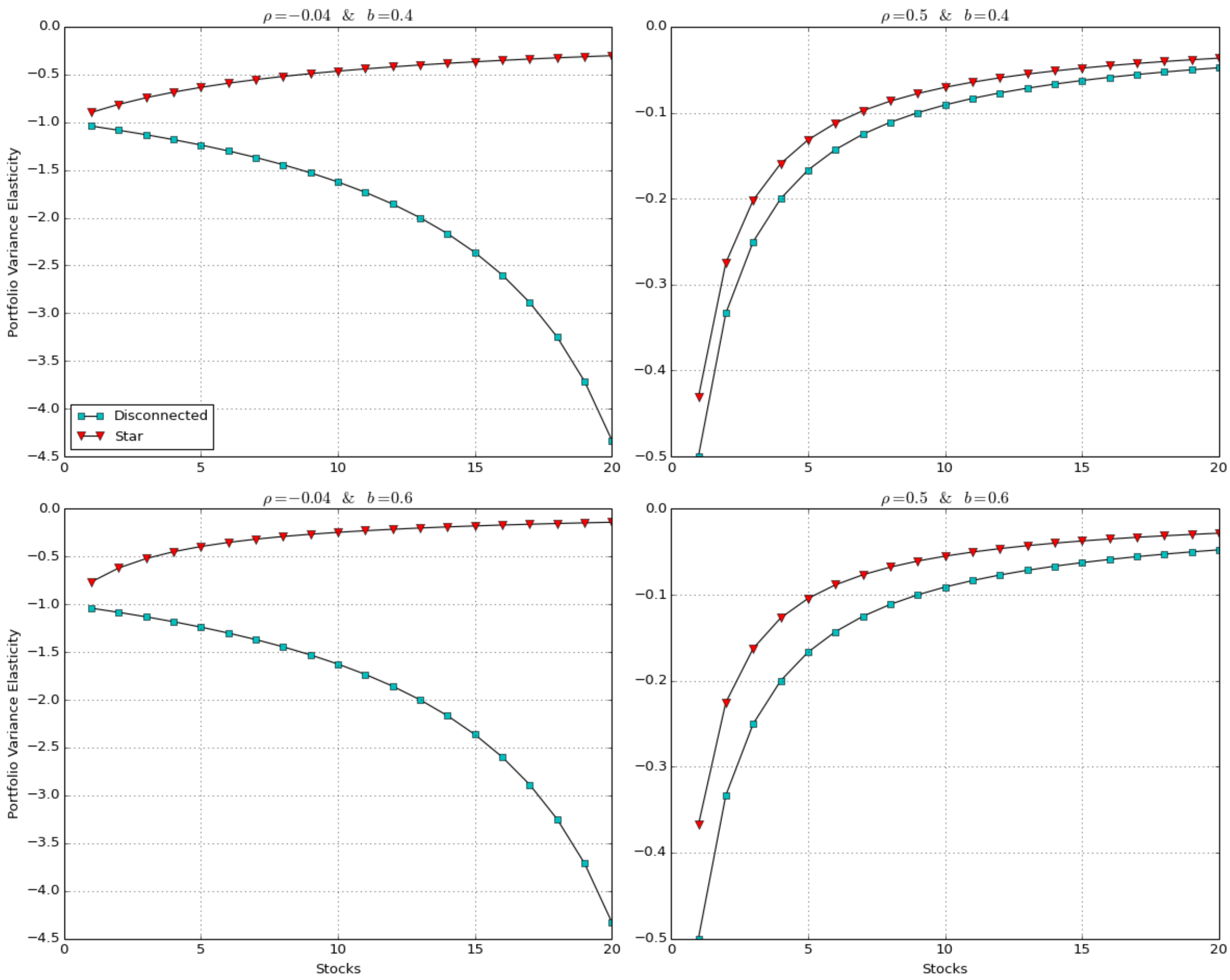

Figure 9. Portfolio variance for No-Dynamical, Disconnected and Star Network structures

As a summary it could be said that large concentration on the effects that an asset imposes to the rest of the system undermines the benefits of diversification, not only in term of its asymptotic limit but also for intermediate size portfolios. Additionally, this effect combined with negative mean correlation drastically changes the behavior of portfolio variance for different values of n. Thereby, special attention should be put on the evolution of the network structures as a way to monitor the potential advantage of diversification.

5. Empirical Analysis

In this section, we start by describing the datasets. Next, we explain the VAR estimation procedure and we continue to compute the Granger network for our datasets. We discuss the network characteristics of estimated Granger networks and examine their proximity to a star-like network structure. Moreover, we discuss the evolution of the granger component of portfolio variance through time. Finally, we close this section with additional analysis mainly focused on statistical significance of Granger network, and optimal diversification along Granger network.

5.1. Datasets

We use two datasets. The first one, termed as *industry dataset*, includes monthly 30 Fama and French industry portfolio returns from 1980:12 to 2014:12. We compute excess returns using one month Treasury bill rates from Ibbotson Associates[20]. The statistical and economical cross-dependency across assets in this dataset is already established in Rapach et al. (2015).

The second dataset, named as *international dataset*, is monthly excess returns from 1980:12 to 2014:12 for 11 industrialized countries: Australia, Canada, France, Germany, Italy, Japan, the Netherlands, Sweden, Switzerland, the United Kingdom, and the United States. The returns are from Global Financial Data denoted as "Total return indices: Stocks" series[21]. The excess returns are computed using each country's three month Treasury bill rate. This dataset was constructed by Rapach et al. (2013) to analyze the leading role of U.S. in the international market. In this paper, however, we evaluate the impact of cross-dependency on international portfolio diversification.

To estimate a VAR model on this dataset and verify Granger network, we need to correct for difference in closing times across international equity markets.[22] The stock markets in Australia and Japan close before markets in Europe and North America. Next, the European markets open and about two hours before they close, the market in U.S. and Canada opens. We adopt the same approach in Rapach et al. (2013) to take these differences into account. Thereby, if market A is still open while another market closes, we exclude the last trading day of the month for market A in computing its monthly returns and use this series of returns in the VAR estimation.[23]

5.2. Estimation procedure

To estimate a VAR model of asset returns, various methodologies have been employed in the literature. Eun and Shim (1989) use OLS to estimate a VAR model of international markets;

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[20] We get these datasets from Keneth French website (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html)

[21] We get this dataset from Dave Rapach website (http://sites.slu.edu/rapachde/home/research). As Rapach et al. (2013) mention, the selection of countries and sample under analysis is bound to the availability of data and their appeal to consider a relatively large number of countries.

[22] Another concern with respect to our VAR estimations is the impact of non-synchronous trading. This issue has been already discussed in various papers (Lo and MacKinlay, 1990; Cohen and Frazzini, 2008; DeMiguel et al., 2014). Based on findings in the literature, we can conveniently conclude that it is unlikely to impact our results. However, Lo and MacKinlay (1990) explains clearly the concern for non-synchronous trading as follows "Although some of the correlation observed in the data may be due to non-synchronous trading, to attribute all of it to the thin trading would require unrealistically thin markets."

[23] However, as Rapach et al. (2013) indicate, this correction does not impact the VAR estimations in the international market.

Rapach et al. (2013) argue presence of correlated regressors as a major drawback of VAR model of returns and apply pooled OLS regression; moreover they also use Elastic Net to estimate a general VAR specification of international countries' returns. DeMiguel et al. (2014), responding to regressors' correlation problem, employ ridge regression and in a recent paper, Rapach et al. (2015) apply adaptive Lasso to estimate lead-lag relationships among U.S. industry portfolios.

Due to the large number of regressors (for our industry dataset), the conventional estimation methods result in uninformative inferences and noisy estimates. Moreover, we also need to tackle the problem of correlated asset returns. In this regard, we employ a modified version of Adaptive Lasso as in Rapach et al. (2015). In the statistical learning literature, Lasso (Tibshirani (1996)) performs both variable selection and shrinkage. However, it does not satisfy oracle properties that are (i) consistent variable selection, (ii) optimal estimation rate (Zou, 2006; Fan and Li, 2001). To address this issue, we employ adaptive Lasso, proposed by Zou (2006), that apply a weighting mechanism to the Lasso penalty terms and moreover, it satisfies the oracle properties. Rapach et al. (2015) employ OLS estimates as weighting mechanism in the Adaptive lasso estimation. In this paper, we consider ridge regression estimates since this method is already employed by DeMiguel et al. (2014) to estimate a VAR model to deal with the correlation among the regressors. This weighting system encourage Lasso to penalize less the regressors found influential by ridge regression[24]. The statistical significance of the estimates are computed via bootstrap 90% confidence intervals (approach explained in details in Chatterjee and Lahiri (2011)).[25]

5.3. Result discussion

This subsection explains the adaptive Lasso estimates of Granger network for our industry and international dataset. The granger interactions among industry portfolios are presented in Appendix H and the corresponding Granger network is illustrated in Figure 1[26]. The results are consistent with Rapach et al. (2015). Looking at the industries via their position in the stages of production process, we find the industries in the first stages, like Coal and Oil industries to negatively influence other industries in the market. This is sensible since any positive shock on

[24] The estimation procedure for adaptive Lasso is explained in details in the Appendix G.

[25] We report the values according to the structure of Granger network adjacency matrix where the element in row i and column j specifies the impact of asset j on asset i. The results are consistent with Rapach et al. (2013) and Rapach et al. (2015). To conserve space, we provide these results in the Appendix H.

[26] Threatening-degree and Katz-Bonacich centrality values for Industry and International Datasets are presented in Appendix L.

these industries results in higher price and returns that would drive up the price of elementary materials and thereby, cut down the profit margin of industries in the later stages of production process. Moreover, for the industries located at the final stage of production, Retail industry, a positive shock would also exacerbate positively across the other industries. This is valid as Retail industry positively impact other industries in the Granger network. Moreover, Financial industry positively influence other industries in the market and it is reasonable, as a positive shock to this industry lead to more financial backing of other industries and a downturn in financial industry would impact negatively other industries. The R^2 statistics are economically significant for all of the industries following Campbell and Thompson (2008) that advocate R^2 statistics higher than 0.5% to be economically significance.[27]

In the international case, we provide the results in the Appendix H and the Granger network is depicted in Figure 1. The results are consistent with Rapach et al. (2013). Obviously, the U.S. market is the central node leading the international market. It impacts eight out of ten other countries positively and significantly and clearly, it has the strongest predictive power. As discussed by Rapach et al. (2013), the central position of U.S. market is attributed to its largest size (Consistent with Lo and MacKinalay (1990) that large-cap firms lead small-cap ones), its global partnership with many other countries, and high investors' attention to the U.S. market (Following gradual information diffusion hypothesis). Moreover, Sweden and Switzerland influence positively other countries situated in Europe. This is consistent with institutional ownership in these markets (The largest ten firms have 52% and 68% in the Swedish and Switzerland markets, respectively). As explained in Rapach et al. (2013), the Swedish market is highly concentrated residing high institutional ownership and thus, exhibit higher pricing efficiency and a rapid reaction to shocks. Moreover, we also find the R^2 statistics to be economically significant.[28]

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[27] The in-sample R2 statistics are provided in the appendix. As explained by Campbell and Thompson (2008), the R2 statistics should be large enough relative to squared Sharpe ratios for the investor to use the information in the predictive regression to obtain increase in portfolio return. A monthly R2 statistic more than %0.5 characterize economic significance. It should be noted that if we estimate the model using OLS, the R2 would be higher however since OLS suffers a major statistical drawback, we use Adaptive Lasso.

[28] In an additional analysis, we test the null hypothesis of $B = 0$ for both industry and international datasets by adopting the procedure proposed by DeMiguel et al. (2014). In short, they consider the test statistic, $M = -(H - N)\ln(|\widehat{\boldsymbol{\Theta}}|/|\widehat{\boldsymbol{\Gamma}_0}|)$; where H is the estimation window used to estimate Granger network, N is the number of stocks. The distribution is estimated through a bootstrap procedure. We consider 100 bootstrap errors from the residuals, $\hat{u}$ and generate recursively bootstrap returns and estimate the VAR model on these returns. Following this procedure, we estimate 100 bootstrap replicates of the covariance matrix of residuals, $\widehat{\boldsymbol{\Theta}}$. Moreover, following the

5.4. Network characteristics

In this subsection, we analyze in details the Granger network characteristics of industries and international markets. The network metrics for both networks are presented in Table 1. Among the most basic network measures, *nodes* and *links* account for the number of nodes and links, respectively, in the network. In industry network, we have 30 nodes and 85 links and in the international network, there are 11 nodes and 36 links. The low number of links comes from the sparcity associated with our estimation procedure where we force the insignificant values to zero. The sparsity is more evident via *density* that measures the fraction of links that actually exist relative to the maximum possible links in the structure[29].For industry and international networks, it is 0.33 and 0.10 respectively.

A *path* between nodes *i* and *j* is a sequence of successive links $(i_1i_2), (i_2i_3), \ldots, (i_{t-1}i_t)$ such that each $(i_si_{s+1}) \in a$ for $s \in \{1, \ldots, t-1\}$ with $i_1 = i$ and $i_t = j$. The length of such a path is the number of links traversed along that path. The *geodesic path* between nodes *i* and *j* is the shortest path between those nodes. The *diameter* of the network is the longest geodesic distance between any two nodes and the *mean distance* is the average over geodesic paths (note that the average distance is bounded above by the diameter). These two measures are helps us to investigate how shocks to an asset moves through the network to get to another asset. The diameter for industry network is 4 meaning the longest a shock travels through the network to influence another node is 4 periods. For international network, it is 2. Moreover, the mean distance for both are 2.12 and 1.45, respectively that are approximately in the level. This points to the average of period a shock takes to get from one asset to another asset in the Granger networks.

Another measure worth defining is the *degree of assortativity.* If the correlation between the degrees of connected nodes is positive, high (low)-degree nodes tends to be connected with other high (low)-degree nodes. This tendency is called positive assortativeness or just *assortativity* for short. An assortative network is expected to be arranged as a *core/periphery* structure where the core is composed by highly connected nodes and the periphery by poorly connected ones surrounding the core. For the case in which *high-degree* nodes tends to be connected with *low-degree* ones, the correlation between the degrees of connected nodes is negative and we called this tendency as negative assortativeness or *disassortative*. In this case, the general configuration

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same procedure and assuming $B = 0$, we estimate the 100 bootstrap replicates of covariance matrix of returns, $\widehat{\boldsymbol{\Gamma}}_{\mathbf{0}}$. Finally, these bootstrap replicates are used to estimate the distribution of M, and its corresponding p-value regarding the hypothesis test, $B = 0$. We find this test rejects the null hypothesis of $B = 0$ for our industry and international datasets at a 1% significance level.

29 Assuming node size, n and link size, m, the density mathematically computed as $d = m/\binom{n}{2}$.

of the network presents star-like features. In both networks, we observe negative values that is the first indication of these networks resemblence to a star-like structure.

In mathematics, a relationship is said to be transitive when $A \rightarrow B$ and $B \rightarrow C$, then $A \rightarrow C$. In a network context, this relationship means that if node i is connected to j and j is connected to k, then i is connected to k. The level of *network transitivity* captures the likelihood that any given pair of nodes shares another common neighbor. Mathematically, it is calculated as the ratio between the number of triangles in the network divided by the total number of connected triples of nodes. This value is high in the international network showing the transitive nature of lead-lag relationships in the international market. However, this is not true for the industry portfolios. For the directed network case, *reciprocity* is measured accounting for the fraction of edges that are reciprocated (there is a link from node i to node j and another one from node j to node i). Therefore, this metric quantifies the probability of a reciprocal relationship in the structure. In the industry network, reciprocal probability is low, 0.02, pointing to one direction nature of production path and in the international market is 0.12.

Table 1. Granger network metrics

This table reports the network metrics for industrial and international granger networks. The corresponding network adjacency matrix is reported in Appendix C.

		Industry Network	International Network
Basics			
	Nodes	30	11
	Links	85	36
	Density	0.10	0.33
Distance			
	Diameter	4	2
	Mean Distance	2.12	1.45
Patterns of Connectivity			
	Assortativity	-0.28	-0.29

Transitivity	0.13	0.51
Reciprocity	0.02	0.12

From negative *assortativity* measure, we get a first look on the star-like feature of the networks. Next, we discuss *Freeman centralization measure* to verify how close the networks are to the star network. Freeman (1978) propose a measure to capture the proximity of networks to a star like structure. Basically, the *Freeman centralization* measures how central is the most central node (Financial industry in the Industry network, and U.S. in the international network) with regard to the centrality of other nodes. Accordingly, first, we compute the sum of the difference between the centrality of the most central node to other nodes in the network. Next, we assume a hypothetical star network with our most central node as the center of this network and we compute the summation of centrality differences to the most central one in this hypothetical network. This measure gives us the maximum possible value of sum of centrality differences in any network. The division of these two numbers tells us how close our network is to a star-like structure.

We consider two centrality measures, degree centrality and the Katz-Bonacich centrality as our centrality measures. Moreover, we consider two types of Granger network, unweighted network where its adjacency matrix adopt values equal to one when there is a connection between any two nodes and zero otherwise, and weighted network as computed directly from VAR model. The results are presented in Table 2.

Table 2. Freeman centralization values for Granger networks

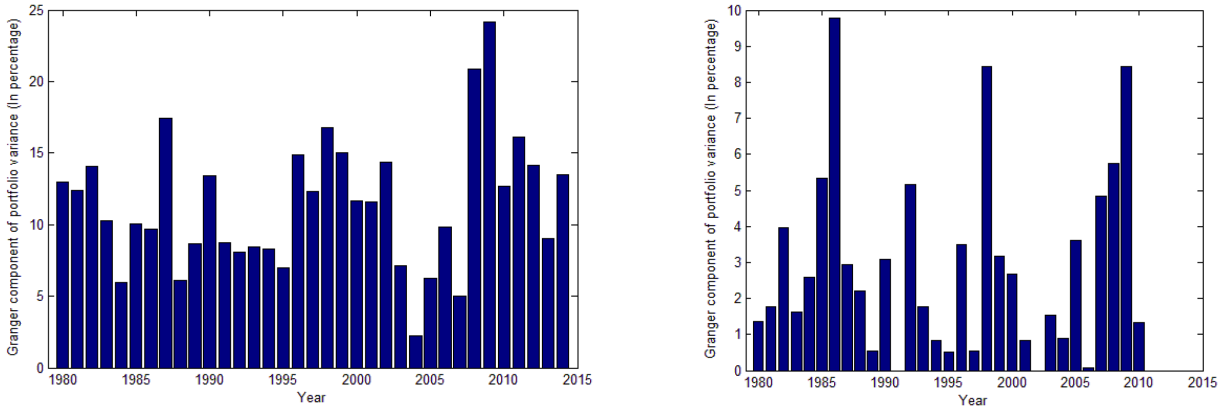
This table reports the Freeman centralization values in percentages for industry and international Granger networks. We consider an un-weighted and weighted form of Granger networks and two measures of centrality: degree centrality and Katz-Bonacich centrality. We consider financial industry as the central node in industry network and the U.S. market as the central node in the international network. The values are reported in percentages with values between 0% and 100% where a centralization of 100% represent an exact star-like structure.

Centralization	Industry Network	International Network
Panel A:Un-weighted		
Threatening-degree centrality	78.43%	60.0%
Katz-Bonacich centrality	87.65%	65.8%
Panel B: Weighted		
Threatening-degree centrality	94.39%	88.1%
Katz-Bonacich centrality	94.09%	87.0%

Clearly, we observe that the industry network with centralization values between 78.43% to 94.39% is close to a star network structure with financial industry as the central node; moreover, in the international market, with U.S. market as the central node, it is close to a star network with values between 60% to 87%.

5.5. Granger component through time

Following this evidence on the resemblance of our networks to the star-like structure, the danger of diversifying among industry and international markets is prominent when a shock occurs. In these cases, we ought to observe a higher variance transcending from the Granger component of the risk. To assess the economic importance of Granger component, we analyze the percentage of Granger component of portfolio volatility through time. In each year, we diversify among all assets available in the industry network and form a diversified portfolio. Next, we estimate the Granger network and compute the percentage of whole portfolio volatility coming from the Granger network. We do the same analysis in the international market. Figure 10 presents the results.



(a) (b)

Figure 10. Percentage of the Granger network to the whole portfolio volatility for a naive diversification strategy through time for (a) Industry portfolios and (b) International market

We observe that the periods where the shocks occur to the central nodes, the granger component of portfolio variance increases. For the industrial portfolios, during the 2007-2008 financial crisis, where shocks strike the financial industry, the Granger component of portfolio variance increases up to 24% of the total variance. Moreover, with regard to international market, during the 2007-2008 financial crisis, the Granger component of portfolio variance increases up to 10%. This value is high knowing how low it is in the last periods. Moreover, high values of Granger component is also evident with regard to 1987 U.S. stock market and 2000 Dot-com crashes.

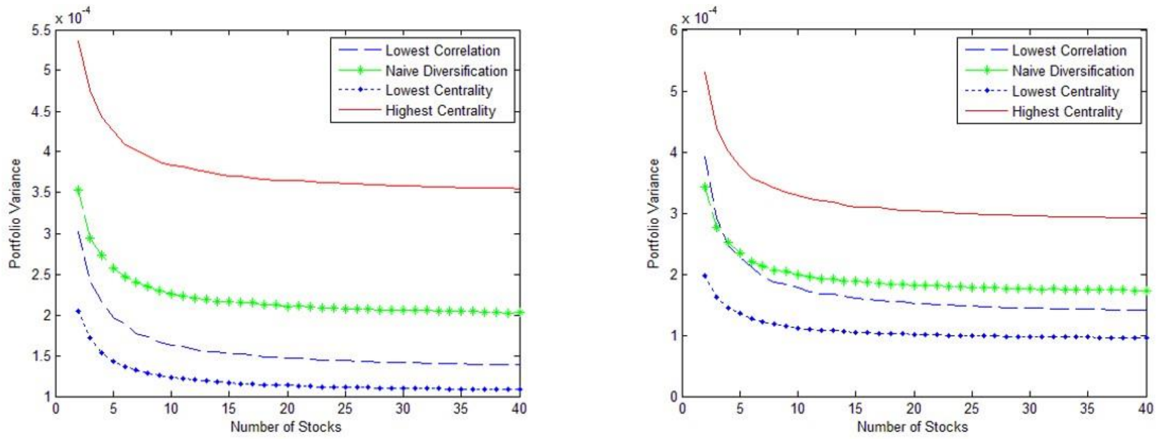
5.6. Portfolio Diversification Based on Granger Network

In this subsection, we aim to increase diversification benefits by using the information in the Granger network. The centrality ranking of assets in the Granger network shows their power for propagating shocks through the portfolio. In particular, high-central assets are extremely prone to spread shocks through the portfolio while low-central assets have a lower propensity to propagate shocks and decrease diversification benefits. Following this argument, we consider two strategies: Lowest-centrality, where we invest in the lowest central assets, and Highest-centrality, where we invest in the highest central assets. We expect the Lowest-centrality strategy to result in higher diversification benefits or in other words, lower portfolio volatility for any number of assets in the portfolio.

In order to make sure the performance of Lowest-centrality strategy is not simply attributed to investing in low beta assets or assets with lowest average correlation, we consider another strategy in which we invest in assets with the lowest average correlation, called Lowest-correlation. Subsequently, if this strategy performs poorly compared to Lowest- centrality strategy, it shows the additional information our Granger network provides in order to improve diversification benefits.

We proceed to our in-sample and out-of-sample analysis. We consider two daily datasets: SP100 with daily excess returns (split and divided adjusted) for the 100 most capitalized stocks in the S&P 500 index between 10/1/2002 and 12/31/2012 and FTSE100 with 100 stocks in FTSE 250 index between 2/27/2006 and 10/25/2013. These two datasets are considered for two reasons. First, these datasets are closer to real-world diversification decisions taken by investors. Second, they give us access to a large number of stocks, 100 stocks in each dataset to be precise and thereby make our analysis more tractable.

We proceed by presenting our in-sample results. In the first step, we compute the Granger network for all observations in the daily datasets, SP100 and FTSE100. We then calculate the Katz-Bonacich centrality of the stocks and rank them according to their centrality level. Next, we divide the 100 stocks of each dataset to two sets of 40 stocks according to their centrality level and label them, high-central and low-central. Subsequently, we do a simulation study by randomly selecting a specific number of stocks from the low- and high- central sets and compute the risk of diversifying among these stocks. We also consider all of the stocks in the dataset and compute the risk of diversifying among a specific number of randomly selected stocks. Additionally, we consider a control set of 40 stocks with lowest average correlations and compute the portfolio variance of naively investing in a randomly selected set of stocks. The results are presented in the Figure 11.[30]



(a) SP100 (b) FTSE100

Figure. 11. Portfolio diversification convergence for diversification strategies up to 40 stocks in the portfolio (a) in SP100 dataset with daily excess returns (split and divided adjusted) for the 100 most capitalized stocks in the SP500 index between 10/1/2002 and 12/31/2012 (b) FTSE100 dataset with 100 stocks in FTSE250 index between 2/27/2006 to 10/25/2013. *Naive Diversification* implies the average performance of diversification strategy by diversifying naively among randomly selected stocks in the full dataset of 100 stocks. *Lowest Centrality* and *Highest Centrality* illustrate naive diversification among 40 randomly chosen highest and lowest central stocks, respectively. *Lowest Correlation* refers to naive diversification between stocks with lowest average correlation in the datasets.

We can clearly observe that investing in low-central assets results in lower portfolio variance on average. Moreover, diversification among high-central stocks gains higher portfolio variance than naively diversifying among all the available stocks. This is consistent with our findings in which low central stocks are less prone to shocks. Additionally, we also notice that Lowest-centrality strategy performs better than a strategy based on diversifying among stocks with the lowest average correlation, Lowest-correlation strategy. This shows that using information in Granger network for diversification purposes is different from simply employing information in stock beta values or their correlation to other stocks in the market.

Moreover, we also consider an out-of-sample analysis. In order to compare the out-of-sample

[30] The results for 20 and 60 stocks are presented in the Appendix K.

performance of these strategies, we employ the "rolling window" approach provided in DeMiguel et al.(2014). Assuming a total T periods of stock's returns, we set an M-period estimation window. Next, we consider a sample of M-period returns (1000 first daily returns in daily datasets), and we calculate the Granger network and compute the Katz-Bonacich centrality of the stocks. According to the centrality values, we categorize stocks into two sets of high and low central sets with 20, 40 and 60 stocks in each. Next, we hold the constructed portfolios for the next $T - M$ periods and calculate the out-of-sample returns. This process is repeated using the computed centrality measures from the first estimation window and finally, we would have $(T - M)$ vectors of portfolio's returns for each strategy. We consider M to be equal to 1000 and hold the portfolios for 200 days in both datasets.

We compare the strategies performance using variance of returns and present the results in Table 3. We consider Naive Diversification strategy as the basis for computing the statistical significance between the portfolio variances among strategies following the procedure in Ledoit and Wolf (2011). The p-values are computed based on studentized circular block bootstrap with block size equal to 5 and the number of bootstrap samples equal to 5000. For portfolio variance comparison, stationary bootstrap is employed from (Politis and Romano, 1994).[31]

We observe that the out-of-sample variance for lowest-centrality strategy is lower than highest-centrality for both datasets with different number of stocks. Moreover, lowest- centrality has better performance in decreasing out-of-sample portfolio risk compared to diversification among all of stocks. For example, lowest-centrality strategy results in annualized portfolio volatility of 0.00087 when diversifying among the 20 lowest central stocks, which is significantly lower than the 0.0111 of Naive-diversification strategy.

Additionally, our results show that this better performance from diversifying on lowest- central stocks is not related to simply diversifying among stocks with the lowest betas (lowest average correlation). In both datasets and across any portfolio size, portfolio volatility for lowest-centrality strategy is lower than lowest-correlation strategy, which implies gain in performance coming from using information from Granger network. In addition, we also compute the Sharpe ratios for the considered strategies, and did not find any statistically significant difference between the Sharpe ratios of these strategies (The results are presented in Appendix L). In total, we conclude that taking into account the structural impact from Granger network, we are able to decrease portfolio risk level.

[31] The corresponding Sharpe ratios are presented in Appendix L.

Table 3. Annualized out-of-sample portfolio variance

Naive Diversification is naively diversifying among all the 100 stocks in both datasets. Lowest-correlation is diversifying among stocks with lowest average correlations. In Lowest-centrality and Highest-centrality strategies, we compute the cen- trality ranking of stocks from Granger network estimated using the first 1000 daily returns and diversify naively on the lowest and highest central stocks. We hold the portfolios for 200 days. Naive Diversification is taken as the benchmark strategy for computing the p-values (following the procedure inLedoit and Wolf(2011)). *** denotes significance at 1%, ** at 5% and * at 10%.

	SP100			**FTSE100**		
Naive-diversification	0.0111			0.0323		
Number of Stocks	**20**	**40**	**60**	**20**	**40**	**60**
Lowest-correlation	0.0096	0.0107	0.0110	0.0319	0.0316	0.0315
	(0.1399)	(0.5375)	(0.7752)	(0.8511)	(0.4805)	(0.3846)
Lowest-centrality	0.0087	0.0098	0.0106	0.0248	0.0277	0.0273
	(0.004)***	(0.011)**	(0.049)**	(0.001)***	(0.001)***	(0.004)***
Highest-centrality	0.0144	0.0130	0.0127	0.0381	0.0423	0.0367
	(0.003)***	(0.005)***	(0.007)***	(0.004)***	(0.001)***	(0.001)***

6. Implication

We provide insights on how the topology structure of lead-lag relationships among returns influences the portfolio variance and moreover, on how shocks impact the portfolio return in short and long-terms in different network topologies. Our findings have several implications. First, portfolio managers are able to build portfolios considering not only the variance-covariance matrix but also the Granger network structure. In this way, they can increase the diversification benefits in their portfolios by making sure the underlying Granger network structure does not resemble a star network structure. Second, portfolio managers can build up portfolios that are less prone to sudden shocks on to the assets by not including highly central assets in their portfolios.

Third, since we consider a general notion of portfolios in our analysis; therefore, we can consider portfolios to be representing a general stock exchange index taking into account all of the stocks in the stock exchange in our portfolio. In this regard, we can analyze the impact of Granger network structure on this market portfolio. Furthermore, our calculations on the relation between network structure and portfolio variance can be extended to quantify the relationship between expected shortfall and network structure. We can compute the expected shortfall for the market assuming the market to be a portfolio of stocks. Expected shortfall has been considered as a measure of systemic risk (Acharya et al., 2010). Expected shortfall of the market is computed as the expected loss in the index conditional on this loss being greater than C (where C represents an α level of portfolio return distribution). Subsequently, since the portfolio return follows a normal distribution, the expected shortfall for the portfolio would be as follows:

$$ES_t^{R_P}(\alpha) = \frac{\varphi(\Phi^{-1}(\alpha))}{1-\alpha} \times (\frac{\sigma_u^2}{n} + \frac{\sigma_u^2}{n}\bar{d}^2(1 + CV_d^2)) \quad (32)$$

Where $\varphi(x)$ is the density of standard normal distribution. Accordingly, we clearly observe that there is a direct relationship between expected shortfall of the index and threatening-degree centrality distribution. Thereby, fat tail centrality distribution leads to higher expected shortfall of the market index. In this regard, we clearly observe how a star network structure would be threatening to the market stability and thereby, we can clearly observe the notion of systemic risk transmitted by Granger network. A market that resembles a star-like structure caries higher systemic risk than a full network or an individual network structure (This is in concordance with Allen and Gale (2000) and Freixas et al. (2000)).

7. Conclusion and Future Research

This paper investigates the influence of lead-lag relationships among returns on portfolio risk from a network perspective. In the first step, we construct a network encompassing all the lead-lag information among returns, termed as Granger network. Next, we consider two types of investor, a myopic investor who cares about tomorrows' return and a long-term investor who invest for the long-run. For these two investors, we analyze the influence of Granger network structures on their portfolio risk.

Starting with myopic investor, we decompose her portfolio variance into contemporaneous and Granger components. The first results show that assets' threatening degree centrality in the Granger network are the main players in attributing the influence of shocks on portfolio return and variance of a myopic investor. In particular, the higher is the threatening degree centrality of an asset, the more would her portfolio return react to shocks to that asset. Moreover, a heavy-tailed threatening-degree centrality distribution translates into higher portfolio variance for the myopic investor. Furthermore, we continue to analyze the diversification benefits in various stylized network structures. We find star-like network structures depleting diversification benefits and causing slow portfolio variance decay. Moreover, for the case of long-term investor, we find Katz-Bonacich centrality to be the main player explaining the influence of Granger network on portfolio variance. Our results show that the fat-tail centrality distribution is a threat to diversification benefits.

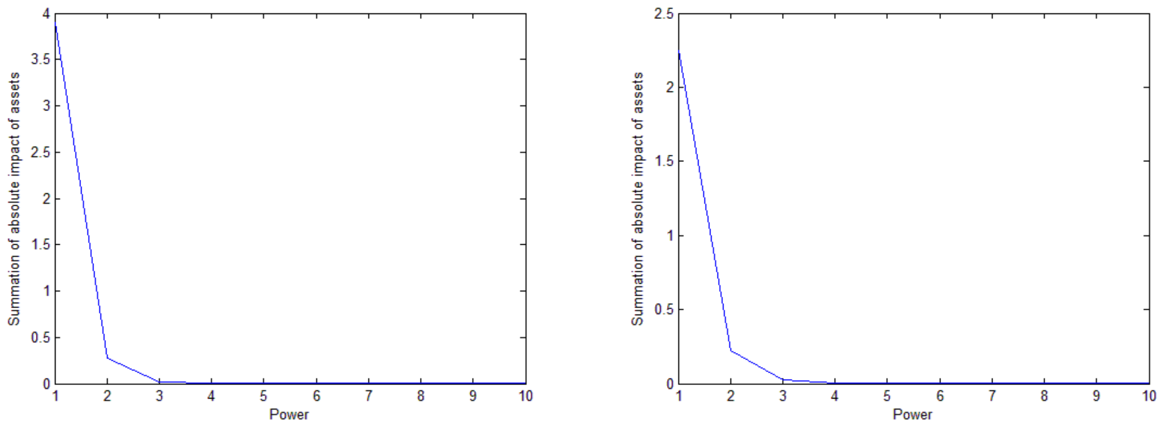
In the empirical part, we show that the Granger network among U.S. industries and also among international stock markets resemble a star network structure. Moreover, the Granger component comes to threaten the portfolio performance in the crisis periods where shocks are more prone to occur.

One main implication of this paper is providing investors with tools to construct well- diversified portfolios robust to shocks propagation and contagion issues in crisis periods. This study provides a new research avenue in several directions. First, the mathematical framework in this paper can be extended to account for asset pricing models. The size- and value-effect that is not captured by CAPM, can be attributed to the cross-dependencies among returns. Second, future research can also follow the path of Barberis(2000) to ac- count for estimation risk in Granger effect via the VAR specification. Third, we can extend our model to analyze the influence of Granger network on the other portfolio strategies such as mean-variance strategy.

Appendices

A. Convergance of serial-dependence matrix *B* under stationarity condition

In order to demonstrate how rapidly the powers of matrix B converge to zero, we consider the industrial portfolios and international markets and calculate the Granger weighted matrix, B, using Adaptive Lasso. Next, we compute the sum of mean absolute impact of stocks that is $\sum_{i=1}^{n}[\frac{\sum_{j=1}^{n}|B_{ij}|}{n}]$. This value shows the average absolute impact of each stock on other stocks in the Granger network. The results for the three datasets are presented in the following figure.



(a) Industral portfolios (b) International market

Figure A1. Summation of average absolute impact of stocks in different powers of granger weighted network ***B***

B. Influence of underlying motifs in Granger network on portfolio variance

Figure 2 demonstrates the full list of network motifs (specific patterns of interactions) underlying Granger network that fundamentally determines portfolio risk. Next, we discuss the cases in details. In case (a), an asset is affecting itself.

This contributes to portfolio variance with the following expression: $(w_i)^2 {B_{ii}}^2 {\sigma_i}^2$ where ${\sigma_i}^2$ is the variance of stock i. In case (b) in figure C2, two different stocks affect two separate stocks contributing to the portfolio variances as follows:

$$w_k w_l B_{ki} B_{lj} \sigma_{ij} + (w_k)^2 B_{ki} B_{ki} {\sigma_i}^2 + (w_l)^2 B_{lj} B_{lj} {\sigma_j}^2 \tag{B1}$$

Intuitively, the weights invested in stocks with in-going links and also the covariance between out-going stocks and their individual variance are the determinant of influence on portfolio variance. In case (c), stock i is affecting both stocks l and k. The impact of this interaction on portfolios variance is:

$$w_k w_l B_{ki} B_{li} {\sigma_i}^2 + (w_k)^2 B_{ki} B_{ki} {\sigma_i}^2 + (w_l)^2 B_{li} B_{li} {\sigma_i}^2 \tag{B2}$$

Under this pattern of connectivity, the variance of the initiator stock plays the important role. This is straightforward as we can see that any perturbation in the prices of both stocks k and l comes from changes in return of the stock that is threatening them. In case (d), both stocks are impacting each other. The influence of this dynamic structure is:

$$w_i w_k B_{ki} B_{ik} \sigma_{ik} + (w_k)^2 B_{ki} B_{ki} {\sigma_i}^2 + (w_i)^2 B_{ik} B_{ik} {\sigma_k}^2 \tag{B3}$$

The individual variance of both stocks and also their covariance is determinant in quantifying the portfolio variance.

In case (e), two stocks i and j are affecting one stock k. The notion signifying this interaction in portfolio's variance is as follows:

$$(w_k)^2 B_{ki} B_{kj} \sigma_{ij} + (w_k)^2 B_{ki} B_{ki} {\sigma_i}^2 + (w_k)^2 B_{kj} B_{kj} {\sigma_j}^2 \tag{B4}$$

The weight allocated to stock *k* and also the variance and covariance of out-going stocks are the major players in this motif. Finally, in case (f), the underlying portfolio variance impact would be:

$$(w_k)^2 B_{ki} B_{ki} {\sigma_i}^2 + (w_j)^2 B_{jk} B_{jk} {\sigma_k}^2 \tag{B5}$$

C. Correlation between centrality ranking for VAR model with and without Cholesky decomposition

Table C.1. Pearson and Kendall-rank correlation between asset centralities.

This table report the Pearson and Kendall-rank correlation between centralities assuming matrices B and B^{*}. Matrix B is computed from direct estimation of equation (1). To compute matrix B^{*}, we decompose the contemporaneous covariance via Cholesky decomposition as $\Sigma_u = P\,P'$ and consider a new error term $w_t = P^{-1}u_t$. Thereby, we have $B^{*} = BP$.

	Industry		International	
	Pearson	**Kendall**	**Pearson**	**Kendall**
Threatening-degree centrality	0.8758	0.6701	0.9351	0.8545
Katz-Bonacich centrality	0.8694	0.6652	0.9502	0.8182

D. Centrality attributes for myopic and long-term investors

Table D1. Threatening-degree centrality for different network structures

Network	***B***	***d***
Disconnected	$\begin{pmatrix} b & 0 & \dots & 0 \\ 0 & b & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & b \end{pmatrix}$	$[b, b, \dots, b]'$
Star	$\begin{pmatrix} b & 0 & \dots & 0 \\ b & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ b & 0 & \dots & 0 \end{pmatrix}$	$[nb, 0, \dots, 0]'$
Inverse Star	$\begin{pmatrix} b & b & \dots & b \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix}$	$[b, b, \dots, b]'$
Circle	$\begin{pmatrix} 0 & 0 & \dots & b \\ b & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & b & 0 \end{pmatrix}$	$[b, b, \dots, b]'$

Table D2. Myopic portfolio variance in terms of the size of the portfolio

Network	$\bar{d}$	CV_d	$\sigma^2_{p(m)}$
Disconnected	b	0	$\frac{\sigma_u^2}{n}(1+b^2)$
Star	b	$\sqrt{n}$	$\sigma_u^2(\frac{1}{n}+\frac{b^2}{n}+b^2)$
Inverse Star	b	0	$\frac{\sigma_u^2}{n}(1+b^2)$
Circle	b	0	$\frac{\sigma_u^2}{n}(1+b^2)$

Table D3. Katz-Bonacich centrality for different network structures

Network	$\boldsymbol{B}$	$\boldsymbol{L}$	$\boldsymbol{v}$
Star	$\begin{pmatrix} b & 0 & \dots & 0 \\ b & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ b & 0 & \dots & 0 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{1-b} & \frac{b}{1-b} & \dots & \frac{b}{1-b} \\ 0 & 1 & \dots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \dots & 1 \end{pmatrix}$	$\left[\frac{1+(n+1)b}{b}, 1, \dots, 1\right]'$
Inverse Star	$\begin{pmatrix} b & b & \dots & b \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{1-b} & 0 & \dots & 0 \\ \frac{b}{1-b} & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \frac{b}{1-b} & 0 & \dots & 1 \end{pmatrix}$	$\left[\frac{1}{1-b}, \frac{1}{1-b}, \dots, \frac{1}{1-b}\right]'$
Circle	$\begin{pmatrix} 0 & 0 & \dots & b \\ b & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & b & 0 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{1-b^n} & \frac{b}{1-b^n} & \dots & \frac{b^{n-1}}{1-b^n} \\ \frac{b^{n-1}}{1-b^n} & \frac{1}{1-b^n} & \dots & \frac{b^{n-2}}{1-b^n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{b}{1-b^n} & \frac{b^2}{1-b^n} & \dots & \frac{1}{1-b^n} \end{pmatrix}$	$\left[\frac{\sum_{i=1}^n b^{i-1}}{1-b^n}, \frac{\sum_{i=1}^n b^{i-1}}{1-b^n}, \dots, \frac{\sum_{i=1}^n b^{i-1}}{1-b^n}\right]'$
Disconnected	$\begin{pmatrix} b & 0 & \dots & 0 \\ 0 & b & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & b \end{pmatrix}$	$\begin{pmatrix} \frac{1}{1-b} & 0 & \dots & 0 \\ 0 & \frac{1}{1-b} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{1}{1-b} \end{pmatrix}$	$\left[\frac{1}{1-b}, \frac{1}{1-b}, \dots, \frac{1}{1-b}\right]'$

Table D4. Long-term portfolio variance in terms of the size of the portfolio

Network	$\overline{v}$	CV_v	$\sigma^2_{p(LT)}$
Star	$\frac{1}{1-b}$	$b\sqrt{n-1}$	$\frac{\sigma_u^2}{n}\frac{\left(1+b^2(n-1)\right)}{(1-b)^2}$
Inverse Star	$\frac{1}{1-b}$	0	$\frac{\sigma_u^2}{n}\frac{1}{(1-b)^2}$
Circle	$\frac{\sum_{i=1}^{n} b^{i-1}}{1-b^n}$	0	$\frac{\sigma_u^2}{n}\left(\frac{\sum_{i=1}^{n} b^{i-1}}{1-b^n}\right)^2$
Disconnected	$\frac{1}{1-b}$	0	$\frac{\sigma_u^2}{n}\frac{1}{(1-b)^2}$

E. Portfolio variance convergence with $\rho \in \{-0.04, 0.04\}$ and $b \in \{0.01, 0.6\}$

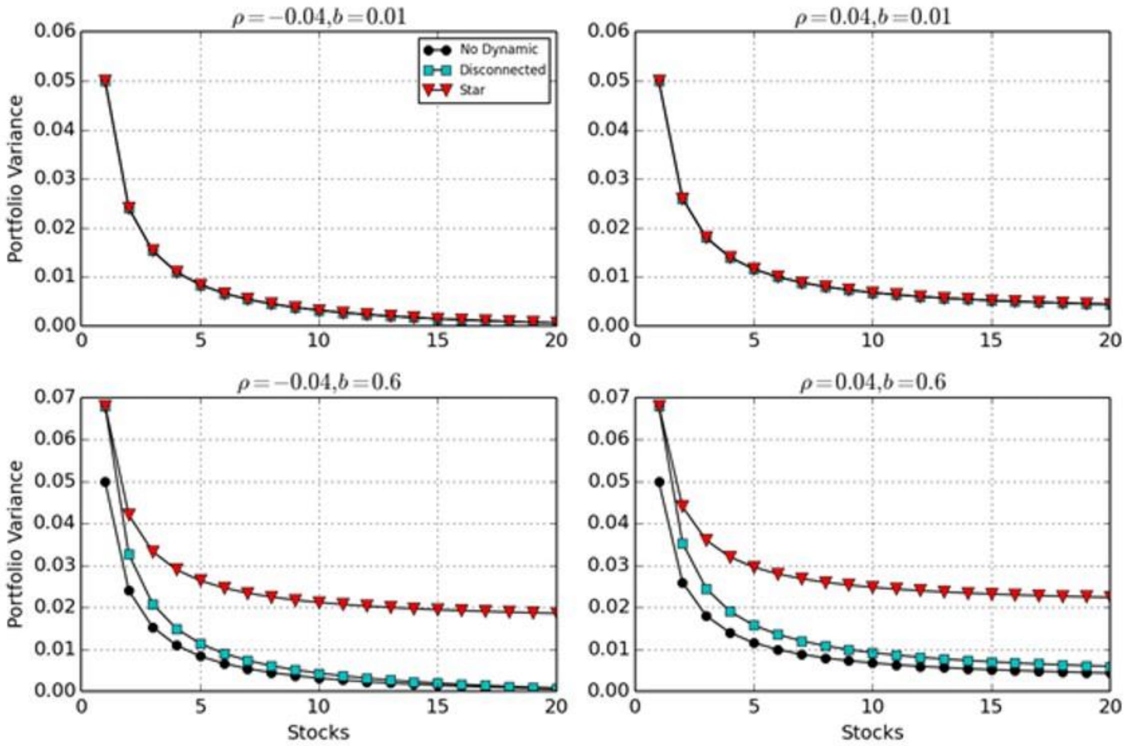


Figure. E.1. Portfolio variance for no-dynamic, disconnected and star network structures for $\rho \in \{-0.04, 0.04\}$ and b $\in \{0.01, 0.6\}$

F. Portfolio variance elasticity with $\rho \in \{-0.04, 0.04\}$ and $b \in \{0.01, 0.6\}$

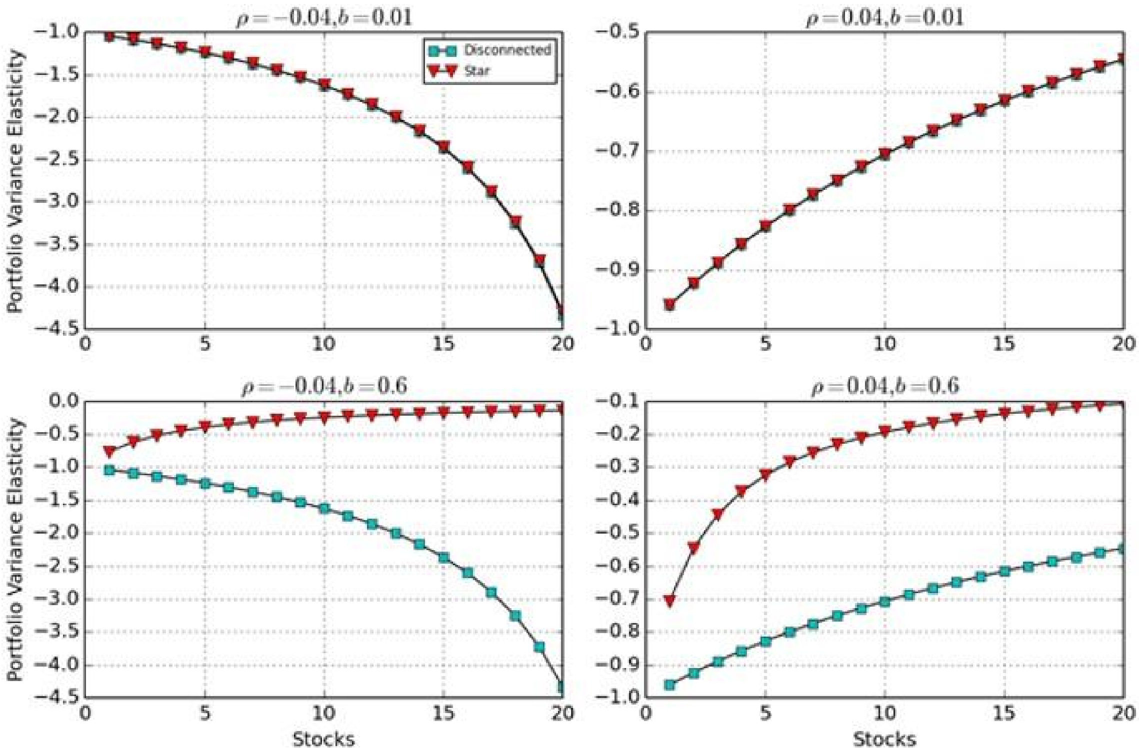


Figure. F.1. Portfolio diversification elasticity for disconnected and star network structures for $\rho \in \{-0.04, 0.04\}$ and b $\in \{0.01, 0.6\}$

G. Adaptive Lasso estimation procedure

We estimate each row of *Granger network*, *B*, via adaptive Lasso. Consider the following expression:

$$r_{i,t} = \sum_{j=1}^{N} b_{i,j} r_{j,t-1} + u_{i,t}$$

Where $r_{i,t}$ is standardize return by sample mean and standard deviation. Let $b_i = [b_{i,1}, b_{i,2}, \dots, b_{i,N}]$ account for row *i* of matrix B, adaptive Lasso estimate is as follows:

$$\hat{b}_i^* = arg\min \parallel r_{i,t} - \sum_{j=1}^{N} b_{i,j} r_{j,t-1} \parallel^2 + \lambda_i \sum_{j=1}^{N} \hat{w}_{i,j} |b_{i,j}|$$

Where λ_i is the regularization parameter and $\hat{w}_{ij}$ is the weight corresponding to coefficient$|b_{i,j}|$. The additional of this weighting mechanism makes adoptive Lasso conforming to Oracle properties. We estimate these weights via Ridge regression and following Zou (2006), we use the weighting function:

$$\hat{w}_{ij} = |\hat{b}_{i,j}|^{-\gamma_i}$$

Where $\gamma_i > 0$. The $\hat{b}_{i,j}$ are estimated via ridge regression to make sure in the estimation mechanism penalize less the important coefficients verified from ridge regression. Both values of λ_i and γ_i are estimated via two-dimensional cross-validation.

H. The VAR estimation of industrial portfolios and international markets

Table H1. Adaptive Lasso estimates of industry portfolios

Regression results for monthly excess returns of industry portfolios from Kenneth French's Data Library from 1980:12 to 2012:14. Adaptive Lasso with penalties adjusted to Ridge regression coefficient estimates is used to estimate the following:

$$r_{i,t+1} = a_i + \sum_{j=1}^{30} b_{ij} r_{j,t} + u_{i,t+1}$$

Where we normalize the returns before estimation and afterwards, adjust the coefficients to the standard deviation of returns. The following table represent matrix B, where element in row i and column j is the element of the matrix B. * point to statistical significance via bootstrapped 90% confidence intervals.

	Food	Beer	Smoke	Games	Books	Hshld	Clths	Hlth	Chems	Txtls
Food	0	0	0	0	0	0	0	0	0	0
Beer	0.013*	0	0	0	0	0	0.027	0	0	0
Smoke	0	0	0	0	0	0	0	0	0	0.006
Games	0	0	-0.025	0	0.037*	0	0	0	0	0
Books	0	0	0	0	0	0	0	0	0	0
Hshld	0	0	0	0	0	0	0	0	0	0
Clths	0	0	0	0	0	0	0	0	0	0
Hlth	0	0	0	0	0	0	0.037	0	0	0
Chems	0	0	0	0	0	0	0	0	0	0
Txtls	0	0	0	0	0	0	0.009*	0	0	0
Cnstr	0	0	0	0	0	0	0	0	0	0
Steel	0	0	-0.015	0	0	0	0	0	0	0
FabPr	0	0	0	0	0	0	0	0	0	0
ElcEq	0	0	0	0	0	0	0	0	0	0
Autos	0	0	0	0	0	0	0	0	0	0
Carry	0	0	0	0	0	0	0	0	0	0
Mines	0	0	0	0	0	0	0	0	0	0
Coal	0	0	0	0	0	0	0	0	0	0
Oil	0	0	0	0	0	0	0	0	0	0
Util	0	0	0	0	0	0	0	0	0	0
Telcm	0	0	0	0	0	0	0	0	0	0
Servs	0	0	0	0	0	0	0	0	0	0
BusEq	0	0	-0.081*	0	0	0	0	0.071*	0	0
Paper	0	0	0	0	0	0	0	0	0	0
Trans	0	0	0	0	0	0	0	0	0	0
Whlsl	-0.014*	0	-0.078	0	0	0	0	0	0	0.002*
Rtail	0	0	0	0	0	0	0	0	0	0
Meals	0	0	-0.003	0	0	0	0.112*	0	0	0
Fin	0	0	0	0	0	0	0	0	0	0
Other	0	0	-0.023	0	0	0	0.058	0	0	0

Table 2 (Continued)

	Cnstr	Steel	FabPr	ElcEq	Autos	Carry	Mines	Coal	Oil	Util	Telcm
Food	0	0	0	0	0	0	0	-0.080*	0	0.008*	0
Beer	0	0	0	0	0	0	0	-0.066*	0	0	0
Smoke	0	0	0	0	0	0.073*	0	-0.050	0	0.105*	-0.011
Games	0	0	0	0	0.027	0	0	0	0	0	0
Books	0	0	0	0	0	0	0	0	0	0	0
Hshld	0	0	0	0	0	0	0	-0.088*	0	0	0
Clths	0	0	0	0	0	0	0	0	0	0	0
Hlth	0	0.000	0	0	0	0	-0.013*	-0.081*	0	0.059*	0
Chems	0	0	0	0	0	0	0	0	0	0	0
Txtls	0	0	0	0	0.056*	-0.008	0	-0.075*	-0.050*	0	0
Cnstr	0	0	0	0	0	0	0	-0.035*	0	0	0
Steel	0	0	0	0	0	0	0	0	0	0	0
FabPr	0	0	0	0	0	0	0	0	0	0.051	0
ElcEq	0	0	0	0	0	0	0	0	0	0	0
Autos	0	0	0	0.031*	0	0	0	0	0	0	0
Carry	0	0	0	0	0.021	0	0	-0.040*	0	0	0
Mines	0	0	0	0	0.045*	0	0	0	0	0	0
Coal	0	0	0	0	0	0	0	0	0	0	0
Oil	0	0	0	0	0	0	0	0	0	0	0
Util	0	0	0	0	0	0	0	0	0	0	0
Telcm	0	0	0	0	0	0	0	0	0	0	0
Servs	0	0	0	0	0	0	0	0	0	0	0
BusEq	0	0	0	0	0	0	0	0	0	0.031*	0
Paper	0	0	0	0	0	0	0	-0.025	0	0	0
Trans	0	0	0	0	0	0	0	0	0	0	0
Whlsl	0	0	0	0	0.004	0	0	-0.011*	-0.086*	0.113*	0
Rtail	0	0	0	0	0	0	0	0	0	0	0
Meals	0	0	0	0	0.002	0	0	-0.101*	-0.106*	0.059	0
Fin	0	0	0	0	0	0	0	0	0	0	0
Other	0	0	0	0	0	0	0	0	0	0	0

Table 2 (Continued)

	Servs	BusEq	Paper	Trans	Whlsl	Rtail	Meals	Fin	Other	R^2
Food	0	0	0	0	0	0	0	0	0.024	4.66%
Beer	0	0	0	0	0	0	0.028*	0	0	4.46%
Smoke	-0.085*	0	0	0.041	0	0	0	0	0	3.50%
Games	0	0	0	0	0	0	0	0.076*	0	1.80%
Books	0	0	0	0	0	0.093*	0	0.001*	0	1.39%
Hshld	0	0	0	0	0	0	0	0.014*	0	3.11%
Clths	0	0	0	0	0	0.099*	0	0	0	2.00%
Hlth	0	0	0	0	0	0	0	0	0	3.58%
Chems	0.005	0	0	0	0	0	0	0	0	1.67%
Txtls	0	0	0	0	0	0.109*	0	0.129*	0	1.91%
Cnstr	0	0	0	0	0	0.075	0	0.068*	0	1.40%
Steel	0	0	0	0	0	0	0	0.094*	0	0.43%
FabPr	0	0	0	0	0	0	0	0	0	0.95%
ElcEq	0	0	0	0	0	0	0	0	0	2.24%
Autos	0	0.018*	0	0	0	0.099*	0	0.003*	0	0.95%
Carry	0	0.047	0	0.058*	0	0	0	0.039	0	2.46%
Mines	0	0	0	0	0	0	0	0	0	0.28%
Coal	0	0	0	0	0	0	0	0	0	0.38%
Oil	0	0	0	0	0	0	0	0	0	1.64%
Util	0	0	0	0	0	0	0	0	0	2.65%
Telcm	0	0	0	0	0	0	0	0	0	1.78%
Servs	0	0	0	0	0	0	0	0	0	1.67%
BusEq	0	0	0	0	0	0	0	0.005*	0	1.00%
Paper	0	0	0	0	0	0.041*	0	0.022*	0	2.06%
Trans	0	0	0	0	0	0.054	0	0.005*	0	2.05%
Whlsl	0	0.025*	0	0	0	0.053	0	0.026*	0.085*	2.26%
Rtail	0	0	0	0	0	0	0	0	0	2.65%
Meals	0.037*	0.014	0	0	0	0.038	0	0.023*	0	3.24%
Fin	0	0	0	0	0	0	0	0	0	1.93%
Other	0	0	0	0.015*	0	0	0	0.027*	0	0.70%

Table H2. Adaptive Lasso estimate of VAR model for international market

This table reports the Adaptive Lasso estimates of coefficients in the following regressions for monthly excess returns of 11 industrialized countries in the international market from 1980:12 to 2012:14. Adaptive Lasso with penalties adjusted to Ridge regression coefficient estimates is used to estimate the following:

$$r_{i,t+1} = a_i + \sum_{j=1}^{11} b_{ij} r_{j,t} + u_{i,t+1}$$

Where we normalize the returns before estimation and afterwards, adjust the coefficients to the standard deviation of returns. The following table represent matrix B, where element in row i and column j is the element of the matrix B. * point to statistical significance via bootstrapped 90% confidence intervals.

	AUS	**CAN**	**FRA**	**DEU**	**ITA**	**JPN**	**NLD**	**SWE**	**CHE**	**GBR**	**USA**	R^2
AUS	0	0	0	0.023*	0.026*	0.010*	0	0	0	0	0.067*	2.67%
CAN	0	0	0	0	0	0	0	0	0	0	0.052*	1.91%
FRA	0	0	0	0	0	0	0	0	0.030*	0	0	1.42%
DEU	0	0	0	0	0	0.004*	0	0.068*	0.027*	0	0.057*	3.94%
ITA	-0.016*	0	0.099*	0.009	0	0	-0.188	0	0.139*	0.058*	0.021*	4.74%
JPN	0	0.031*	0.040*	0	0	0.028*	0	0	0	0.012*	0	2.52%
NLD	0	0	0	0	0	0.040*	-0.130	0.094*	0.117*	0	0.123*	8.03%
SWE	0	0	0	0	0.021	0	0	0	0	0	0.106*	4.86%
CHE	0	0	0	0	0	0	0	0.142*	0	0	0.066*	7.12%
GBR	0	0	0	0	0	0.026*	-0.013	0.067*	0	-0.008	0.119*	4.30%
USA	0	0	0	0	0	0	0	0.073*	0	0	0	3.02%

I. Centrality values for industry and international Datasets

Table I.1. Centrality values for Industry dataset

Monthly excess returns of industry portfolios are downloaded from Kenneth French's Data Library from 1980:12 to 2012:14. The Granger networks, B, are computed via adaptive Lasso estimation of a VAR model. Threatening-degree centrality is the summation of columns in the Granger network adjacency matrix, computed as follows: $d_j = \sum_{i=1}^{N} B_{ij}$; Moreover, Katz-Bonacich centrality captures the long-run influence t of assets. Mathematically, it is computed as follows: $\boldsymbol{v}_j = \mathbf{1}\,(\boldsymbol{I} - \boldsymbol{B})^{-1}\boldsymbol{e}_j$ where $\boldsymbol{v}_j$ is the Katz-Bonacich centrality of asset j, $\boldsymbol{e}_j$ is the r_{th} unit vector and $\mathbf{1}$ is an n-dimensional vector of ones.

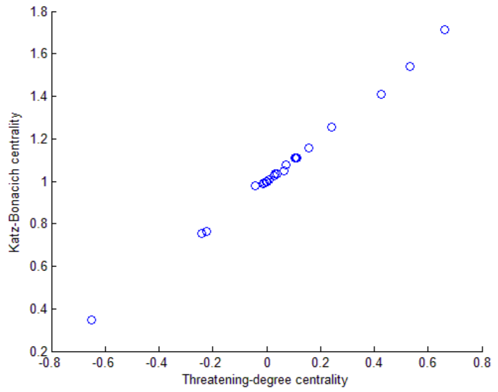
Industry	Threatening-degree centrality	Katz-Bonacich Centrality
Food	-0.001	0.999
Beer	0.000	1.000
Smoke	-0.225	0.764
Games	0.000	1.000
Books	0.037	1.037
Hshld	0.000	1.000
Clths	0.243	1.255
Hlth	0.071	1.079
Chems	0.000	1.000
Txtls	0.008	1.007
Cnstr	0.000	1.000
Steel	0.000	1.000
FabPr	0.000	1.000
ElcEq	0.031	1.036
Autos	0.155	1.156
Carry	0.065	1.048
Mines	-0.013	0.986
Coal	-0.652	0.348
Oil	-0.242	0.755
Util	0.426	1.411
Telcm	-0.011	0.992
Servs	-0.043	0.978
BusEq	0.104	1.109
Paper	0.000	1.000
Trans	0.114	1.109
Whlsl	0.000	1.000
Rtail	0.661	1.713
Meals	0.028	1.028
Fin	0.532	1.540
Other	0.109	1.109

Table I.2. Centrality values in International dataset

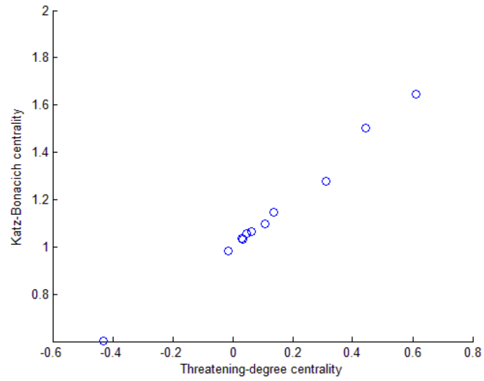
We consider monthly excess returns of 11 industrialized countries in the international market from 1980:12 to 2012:14. The Granger networks, B, are computed via adaptive Lasso estimation of a VAR model. Threatening-degree centrality is the summation of columns in the Granger network adjacency matrix, computed as follows: $d_j = \sum_{i=1}^{N} B_{ij}$; Moreover, Katz-Bonacich centrality captures the long-run influence t of assets. Mathematically, it is computed as follows: $\boldsymbol{v}_j = \mathbf{1}\,(\boldsymbol{I} - \boldsymbol{B})^{-1}\boldsymbol{e_j}$ where $\boldsymbol{v}_j$ is the Katz-Bonacich centrality of asset j, $\boldsymbol{e_j}$ is the r_{th} unit vector and $\mathbf{1}$ is an n-dimensional vector of ones.

Country	Threatening-degree centrality	Katz-Bonacich Centrality
AUS	-0.016	0.983
CAN	0.031	1.034
FRA	0.139	1.149
DEU	0.032	1.032
ITA	0.047	1.057
JPN	0.108	1.096
NLD	-0.431	0.602
SWE	0.444	1.500
CHE	0.313	1.280
GBR	0.062	1.066
USA	0.611	1.645

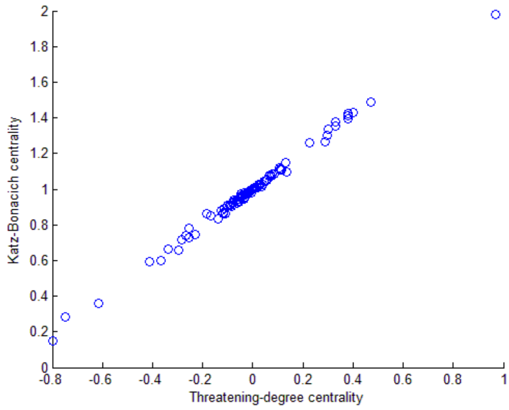
J. Correlation between threatening-degree centrality and Katz-Bonacich centrality



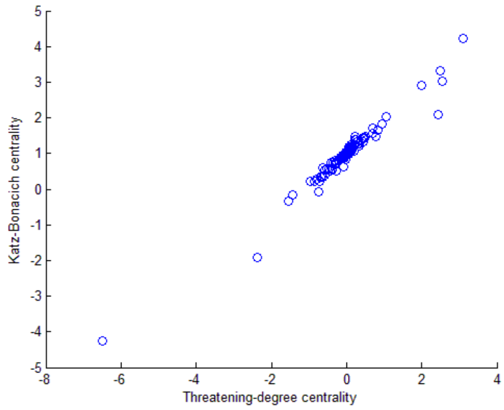
(a) Industry portfolios



(b) International market



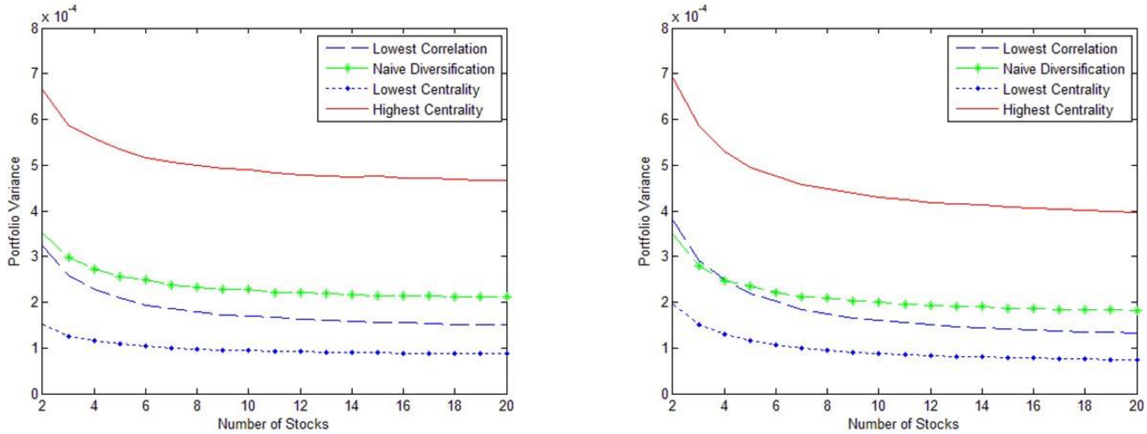
(c) SP100



(d) FTSE100

Figure. J.1. Scatter plot of threatening-degree and Katz-Bonacich centralities

K. In-sample results for portfolio variance convergence



(a) SP100 (b) FTSE100

Figure. K.1. Portfolio diversification convergence for diversification strategies up to 20 stocks in the portfolio (a) In SP100 dataset with daily excess returns (split and divided adjusted) for the 100 most capitalized stocks in the SP500 index between 10/1/2002 and 12/31/2012 (b) FTSE100 dataset with 100 stocks in FTSE250 index between 2/27/2006 to 10/25/2013. Nave Diversification implies the average performance of diversification strategy by diversifying naively among randomly selected stocks in the full dataset of 100 stocks. Lowest Centrality and Highest Centrality illustrate nave diversification among 20 randomly chosen highest and lowest central stocks, respectively. Lowest Correlation refer to nave diversification between stocks with lowest average correlation in the datasets.

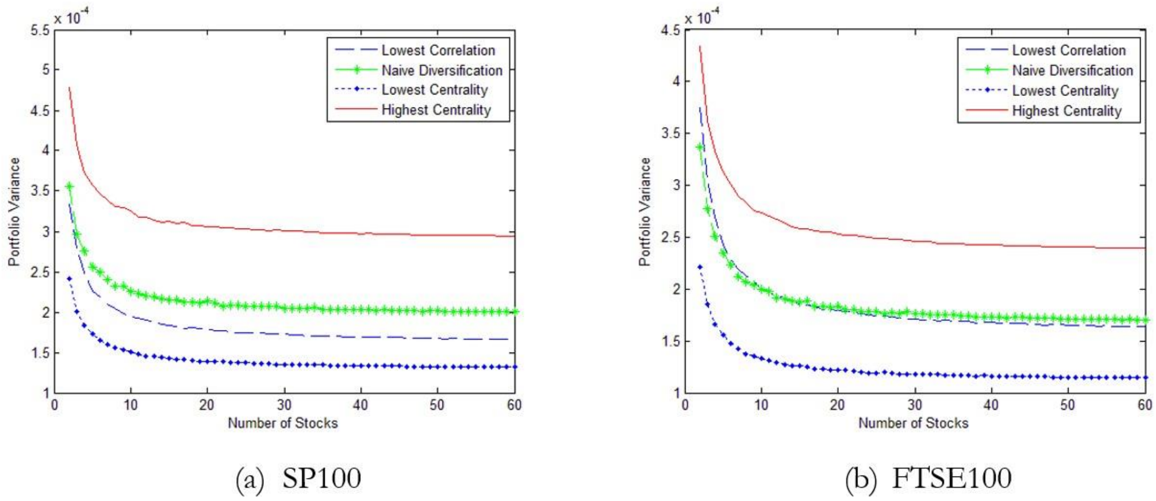


Figure. K.2. Portfolio diversification convergence for diversification strategies up to 60 stocks in the portfolio (a) In SP100 dataset with daily excess returns (split and divided adjusted) for the 100 most capitalized stocks in the SP500 index between 10/1/2002 and 12/31/2012 (b) FTSE100 dataset with 100 stocks in FTSE250 index between 2/27/2006 to 10/25/2013. Nave Diversification implies the average performance of diversification strategy by diversifying naively among randomly selected stocks in the full dataset of 100 stocks. Lowest Centrality and Highest Centrality illustrate nave diversification among 60 randomly chosen highest and lowest central stocks, respectively. Lowest Correlation refer to nave diversification between stocks with lowest average correlation in the datasets.

L. Out-of-sample performance evaluation

Table L.1. Annualized out-of-sample Sharpe ratios

Naive Diversification is naively diversifying among all the 100 stocks in both datasets. Lowest-correlation is diversifying among stocks with lowest average correlations. In Lowest-centrality and Highest-centrality strategies, we compute the centrality ranking of stocks from Granger network estimated using the first 1000 daily returns and diversify naively on the lowest and highest central stocks. We hold the portfolios for 200 days. Naive Diversification is taken as the benchmark strategy for computing the p-values (following the procedure inLedoit and Wolf(2008)). *** denotes significance at 1%, ** at 5% and * at 10%.

	SP100			**FTSE100**		
Naive Diversification	0.0111			0.0323		
Number of Stocks	**20**	**40**	**60**	**20**	**40**	**60**
Lowest-Correlation	1.6857	2.1890	2.0278	0.7871	0.6599	0.8737
	(0.505)	(0.7508)	(0.8704)	(0.9402)	(0.3887)	(0.6279)
Lowest-Centrality	1.3850	1.8763	2.1029	0.1575	0.5036	0.6779
	(0.2425)	(0.4718)	(0.794)	(0.0664)	(0.1462)	(0.3621)
Highest-Centrality	2.0019	1.9169	2.1165	1.0019	0.9579	0.9787
	(0.8771)	(0.5349)	(0.701)	(0.4252)	(0.4186)	(0.1927)

Chapter 3: Hedging Network Structures and Portfolio Diversification

1. Introduction

Under a diversification strategy (understood as reducing portfolio variance by simply adding more assets in a portfolio), a high number of stocks are included in the portfolio to minimize risk; however, the optimum number of stocks needed in the portfolio to achieve the maximum benefits of diversification is not explicitly specified. Mao (1970) provides a theoretical analysis regarding the number of stocks required to gain the greatest level of diversification and, in accord with Evans and Archer (1968), concludes that not a large number of stocks are needed for diversification. Several studies recommend a much larger number of stocks: for example, Bird and Tippett (1986) argue that the relationship provided by Evans and Archer (1968) is biased and that the number of stocks should be more than 10 to achieve the maximum benefits of diversification. Elton and Gruber (1977) derive an analytical expression for the relationship between portfolio size and risk, which also includes the risk associated with the probability that the return on the portfolio held will differ from the return in the market, and they demonstrate that a well-diversified portfolio needs more than 10 assets. In this regard, Bloomfield et al. (1977) propose including 20 stocks, while Statman (1987) proposes having at least 30 stocks. More recently, Statman (2004) finds that the optimal level of diversification, as measured based on the rules of mean–variance portfolio theory, exceeds 300 stocks, whereas Domian et al. (2007) suggests including more than 100 stocks to achieve the greatest benefits of diversification.[32]

However, Desmoulins-Lebeault and Kharoubi-Rakotomalala (2012) show that in non-Gaussian return distributions, increasing the portfolio size cannot reduce monotonically large risk and that a lower number of assets can improve diversification in a bear market. Although the number of stocks required to obtain a perfectly diversified portfolio seems to be high, some studies (Kelly, 1995; Bertaut, 1998; and Goetzmann and Kumar, 2008

[32] From a different perspective, some authors, such as Moreno and Rodríguez (2013), demonstrate that the portfolios of equity mutual funds are not perfectly diversified despite the huge number of stocks that mutual funds hold (more than 100 in many cases). Other authors, such as Bello (2007), have found similar results.

among others) highlight that household investors usually hold equity portfolios that contain a very limited number of different assets. They find that the number of stocks held by households in the American market is between 5 and 10, depending on several factors, such as education. Therefore, in this paper, we focus on a wide range of portfolios regarding the number of stocks, from 2 to 60, in order to contribute to the different studies noted here.

This paper contributes to the literature by investigating diversification from a new perspective.[33] We introduce the notion of a hedging network, where each stock resides as a node and each link specifies the hedging relations among the stocks, and we find that the optimum number of stocks in a diversified portfolio is driven by the specific structure of the hedging network. Moreover, we propose using the Katz centrality measure to rank stocks based on their favorability in order to determine whether they should be included in a diversified portfolio. The tools provided in this paper are very useful instruments for determining the structure of hedging relations in a portfolio network and building a well-diversified portfolio.

Network theory has been employed in finance to analyze the interconnectedness among entities in capital markets.[34] For example, Peltonen et al. (2014) and Brunnermeier et al. (2013) analyze the network structure of counterparties' bilateral notional exposure in the CDS market; Boss (2004), Cont and Moussa (2010), Georg (2013), and Rogers and Veraart (2013) investigate the network of claims and liabilities in the interbank market; and Raffestin (2014) provides a theoretical model where investors compose a network of asset holdings with home bias and studies several issues regarding the systemic risk. In addition, Pozzi et al. (2013) study the network structure of a stock returns correlation matrix. In this paper, for the first time in the literature, we present a new type of network, called a hedging network, which is built on hedging relations (representing the correlation between stocks after the influence of the other stocks in the opportunity set of stocks is

[33] In recent years, some authors have started to call for a change in classical portfolio theory, as new models and theories should be considered. For example, You and Daigler (2010) examine international diversification and show that conclusions concerning diversification based solely on constant correlations across markets can be misleading.

[34] See Newman (2010) for a more extensive description of networks.

eliminated). Accordingly, a primary benefit of employing a hedging network from a network of returns correlations is that only the relationships between stocks are captured via the hedging relations. In other words, if two stocks can hedge for each other, the impact of other stocks on their relation is offset, whereas if two stocks are correlated with each other, their correlation may arise because both stocks are correlated with another stock.

We can distinguish two parts of this paper: a theoretical part presented first and an empirical part presented at the end. In the theoretical part, we start by analyzing certain hedging network structures, such as full, individual and star networks, and we compare these network structures by examining how the portfolio variance decreases as we add more stocks in the portfolio (called the variance decay rate) when we consider a minimum-variance strategy. To analyze the change in the level of variance as we increase the number of stocks in the portfolio, we consider a special case in which all the unhedgeable components and hedging relations are assumed to be constant and equal. For the hedging relations, we consider two possible values in order to capture a low and high intensity connection and then analyze the effect on the variance when we consider negative and positive hedging relations. We find that the level (high or low) and sign of the hedging relation affects the portfolio variance and the potential benefits of diversification. When there is a low intensity of hedging relations among the stocks, the portfolio variance is clearly similar for all the different structures; thus, the type of network is not relevant in such a case. However, if there is a high intensity of hedging relations (positive or negative), we find significant differences in the portfolio variance decay rate among the different network structures, where the full network is the best network structure if the value of hedging relations is negative and the intensity is sufficiently high. Thus, having negative and higher intensity hedging relations among the stocks in an investment opportunity set is most beneficial for portfolio managers (or any investors worried about risk) to maximize the benefits of diversification.

In the empirical part, we analyze the network structure of several datasets, and based on the conclusions from the previous theoretical section, we propose using the Katz centrality measure to rank stocks in a hedging network based on their contribution to the benefits of diversification. This centrality measure takes into account both the hedgeable and unhedgeable components of the stocks in the hedging network. We then demonstrate in both an in-sample and out-of-sample analysis the convenience of using this measure to select stocks to maximize the benefits of diversification, and based on this centrality measure, we show that diversifying in a particular subset of stocks in-sample, we achieve a lower portfolio risk. Moreover, the most interesting result is found in the out-of-sample analysis, where we provide empirical evidence that our centrality measure shows superior performance (in terms of risk) in selecting stocks with the greatest diversification effect. This strategy yields better performance than both naïve diversification (holding an equally weighted portfolio among all stocks in the investment opportunity set) and a portfolio strategy based on classical portfolio theory using a covariance matrix (holding stocks with the lowest average correlation coefficient in an effort to maximize the effects of diversification).

To our knowledge, this is the first time in the literature hedging networks have been investigated with regard to diversification rule. Moreover, we also try to explain the notion of diversification from a network perspective, which provides us with a new way of characterizing diversification strategies. Regarding the optimum number of stocks to include in a portfolio to gain the greatest benefits of diversification, we conclude that the optimal number varies depending on the hedging network structure (and the number and quantity of negative hedging relations) of the investment opportunity set. Moreover, based on the centrality measure we present in this paper, we can achieve a very well-diversified portfolio by investing in only a small portion of stocks in the hedging network, which shows better performance in both the in-sample and out-of-sample analyses.

The rest of the paper is organized as follows. In Section 2, we define a hedging network and present several types of network structures. Section 3 presents a theoretical analysis of the weights allocation for the minimum-variance portfolio in the case of several hedging network structures. In addition, we compare the different networks with respect to the portfolio variance decay rate when we increase the number of stocks. In Section 4, we propose a centrality measure to rank stocks based on their favorability for inclusion in a diversified portfolio and analyze the empirical results with several datasets. Finally, in Section 5, we conclude.

2. Hedging Network

A network is defined as a pair of sets $\varphi = \{V, E\}$, where V corresponds to the set of nodes and E represents the set of links connecting pairs of nodes. We use n to denote the number of nodes, and if a link exists from node i to node j, $(i, j) \in E$. A network is represented by an adjacency matrix, $A_{n\times n} = [A_{ij}]_{n\times n}$, which summarizes the information in set E by setting $A_{ij} \neq 0$ when there a relationship exists between the nodes i and j.

Depending on the structure of the adjacency matrix, we can differentiate different types of networks. Most of the networks identified in physics can be represented by a binary matrix, $A_{ij} \in \{0,1\}$, where $A_{ij} = 1$ if i and j are connected and 0 otherwise. In this case, the network φ is said to be an unweighted network. However, many networks have edges with different weights of connection when $A_{ij} \in \mathbb{R}$, so a network is called a weighted network if it is represented by an adjacency matrix with entries that are not simply zero or one. Finally, the network is undirected if all nodes are bidirectional, i.e., the adjacency matrix is symmetric, $A = A^T$.

Suppose that we aim to hedge stock i with all of the other available stocks in the market. Stevens (1998) shows that the inverse covariance matrix provides us the optimal hedging relations among the stocks. The i-th hedge portfolio consists of taking a long position in the i-th stock and a short position in the other stocks that track the i-th stock return.

Each portfolio can be estimated from the following "hedge regression": [35]

$$r_{i,t} = \alpha_i + \sum_{k=1,k\neq i}^{N} \beta_{ik} r_{k,t} + \varepsilon_{i,t} \tag{1}$$

where $r_{i,t}$ is the return of stock i at time t. β_{ik}, called the hedging relation, refers to the contribution of stock k in hedging stock i beyond the contribution of all other stocks. $\varepsilon_{i,t}$ is the part of $r_{i,t}$ that is not being hedged by other stocks. The variance of $\varepsilon_{i,t}$, denoted by υ_i, is a measure of the risk of the unhedgeable component and hereinafter is called the unhedgeable component of $r_{i,t}$.

The information on the hedging relations and unhedgeable component in Equation (1) is summarized by the inverse covariance matrix. Denoting the inverse covariance matrix by

[35] For more details about hedging regressions and unhedgeable components, see Gotoh et al. (2013).

$\mathbf{\Psi} = \mathbf{\Sigma}^{-1} = [\psi_{ij}]_{n \times n}$, Stevens (1998) proposes the following relation between $\mathbf{\Psi}$ and the hedge regression in Equation (1):

$$\psi_{ij} = \begin{cases} -\frac{\beta_{ij}}{v_i} & if\ i \neq j \\ \frac{1}{v_i} & if\ i = j \end{cases} \quad (2)$$

Subsequently, we can subtract the hedging network from the inverse covariance matrix ψ_{ij} given a measure of marginal hedgeability between the stocks i and j. Therefore, the hedging network is defined as follows:

Definition. The stock hedging network is defined by $\Phi^W = \{N, \boldsymbol{E}\}$ with adjacency matrix $\boldsymbol{\beta}$, where N is the set of stocks and $\boldsymbol{E} = \{(i,j) \in NxN: \beta_{ij} \neq 0\}$ denotes the set of links connecting each stock. The intensity of an edge between node *i* and *j* is equal to β_{ij}, and the self-edge of node *i* in the network is equal to v_i.

Therefore, a weighted link exists between stock *i* and stock *j* as long as β_{ij} differs from zero. We define an unweighted network Φ^U when the adjacency matrix is binary, $\beta_{ij} \in \{0,1\}$. Within these two families of networks, we can differentiate particular structures:

- *Full network structure*

We define a full network as network in which all the nodes are connected to each other as presented in Figure 1.

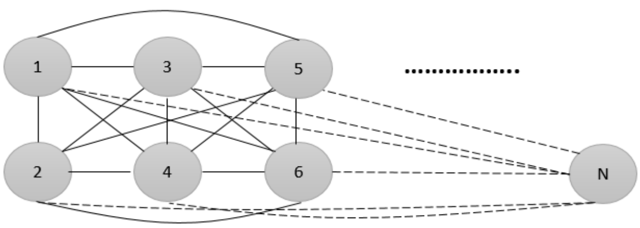


Figure 1. Full hedging network structure

- *Individual network*

We consider a network be to an individual network if no hedging relations exist between the stocks. Such a network structure is presented in Figure 2.

Figure 2. Individual network

- *Star network*

In a star network, there is high asymmetry in the hedging relations in the network. Figure 3 presents a star network structure, where a central stock is connected to all of the other stocks and no hedging relations exist among $N-1$ stocks.

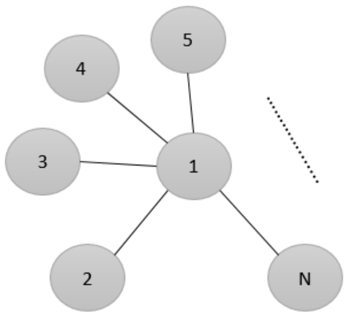


Figure 3. Star network structure

In the next section, we investigate how the minimum risk can be obtained for various hedging network structures and examine the role of hedging relations and unhedgeable components in determining the portfolio risk. This is the first study to consider the implication of hedging network structures on portfolio risk.

3. Minimum Risk in Hedging Networks

The primary goal of diversification is to reduce risk. However, a minimum-variance strategy is designed to minimize risk by employing an optimization framework. Since we aim to examine the role of hedging network structures in determining the benefits of diversification, we start by examining how the greatest benefits of diversification can be achieved by constructing a minimum-risk portfolio under specific hedging network structures. In this regard, we consider the particular hedging network structures defined in section 2 and investigate how the minimum-variance (*minv*) strategy assigns value to stocks in these structures in order to obtain the greatest benefits of diversification. Finally, the findings in this section lead us to propose a centrality measure in the hedging network in order to verify most beneficial stocks for constructing a well-diversified portfolio.

In this section, we have two goals: First, we wish to analyze how the value of components (β, v) in several network structures (full, star or individual) affects the determination of the minimum-variance portfolio. To simplify the analysis, we assume that every hedging component and unhedgeable component is constant and equal, and we distinguish two different values of hedging relations (high and low). Second, we examine the variance decay rate (representing how the portfolio variance decreases as we add more stocks to the portfolio) under these specific network structures, which allows us to determine whether the effect of adding more stocks to a portfolio (popularly known as a diversification strategy) differs depending of the specific hedging network and to identify the most beneficial network structure for investors.

Next, we briefly summarize the mathematics behind the Markowitz (1952) framework and a minimum-variance strategy, as we heavily rely on them. Let us assume n risky assets with a vector of expected returns denoted as μ and a covariance matrix denoted as Σ. Consider the problem of finding the vector of optimal weights, w, that minimizes the variance of such a portfolio subjected to $\mathrm{w}^{\mathrm{T}}\mathbf{1} = 1$, where $\mathbf{1}$, in bold, corresponds to a vector whose components equal one. This strategy is commonly known as a minimum-variance strategy (*minv)*. Formally:

$$Min\,\sigma_p^2 = w^T \Sigma w$$

$$subject\ to:$$

$$w^T\mathbf{1} = 1$$

The solution of this optimization problem is known to be given by:

$$w^{minv} = \frac{\Sigma^{-1}\mathbf{1}}{\mathbf{1}^{\boldsymbol{T}}\Sigma^{-1}\mathbf{1}} \tag{3}$$

In the next section, we use Equation (3) and the general hedging network definition in Equation (2) to study the composition of the minimum-variance portfolio under different network structures. Considering that $\boldsymbol{\Psi} = \boldsymbol{\Sigma}^{-1}$, we can express Equation (3) as:

$$w^{minv} = \frac{\boldsymbol{\Psi 1}}{\boldsymbol{1'\Psi 1}} \tag{4}$$

3.1. *Full network structure*

First, we analyze the portfolio variance for a full network in which all stocks are assumed to hedge each other, as presented in Figure 1. Based on the optimal weights of minimum variance (w^{minv}) in Equation (4), we can compute the minimum-variance portfolio for this hedging network structure as follows:

$$Var_p^{minv} = w'\Sigma w \quad = \frac{1}{\boldsymbol{1'\Psi 1}} = \frac{1}{\sum_{i=1}^{N}\frac{1}{v_i} - \sum_{i=1,i\neq j}^{N}\sum_{j=1}^{N}\frac{\beta_{i,j}}{v_i}} \tag{5}$$

If the hedging relations are taken into account, the denominator in Equation (5) is the summation of all the elements in the inverse covariance matrix $\boldsymbol{\Psi}$. Therefore, the lowest variance of the portfolio is realized when the denominator is highest and, accordingly, when the unhedgeable component is low and the sum of all hedging relations is high and negative. Analyzing this issue more deeply, we observe that a higher negative hedging coefficient would increase the size of the denominator in Equation (5) and consequently lead to a lower level of variance. This observation may be logical, as we know that stocks with reverse return behavior are suitable for a well-diversified portfolio.[36] Regarding positive hedging, we expect an increase in portfolio variance for stocks with a higher positive $\beta_{i,j}$. Hence, a network in which all assets negatively hedge each other ($\beta_{i,j} < 0$) will have lower variance than a network in which all hedging relations are positive, when the level of unhedgeable component remains constant. Additionally, a smaller value for the unhedgeable component (v_i) decreases the portfolio variance. Such a result is expected, as a lower unhedgeable component (v_i) translates into lower unexplained asset

[36] Although a negative hedging relation (β) could be related to a negative correlation coefficient in classical portfolio theory, we must understand they are not exactly the same. As we will demonstrate in the final empirical section, investing in both types of stocks (negative hedging and negative correlation) is not exactly the same.

performance; accordingly, this unexplained performance cannot be restrained by investing in other stocks.

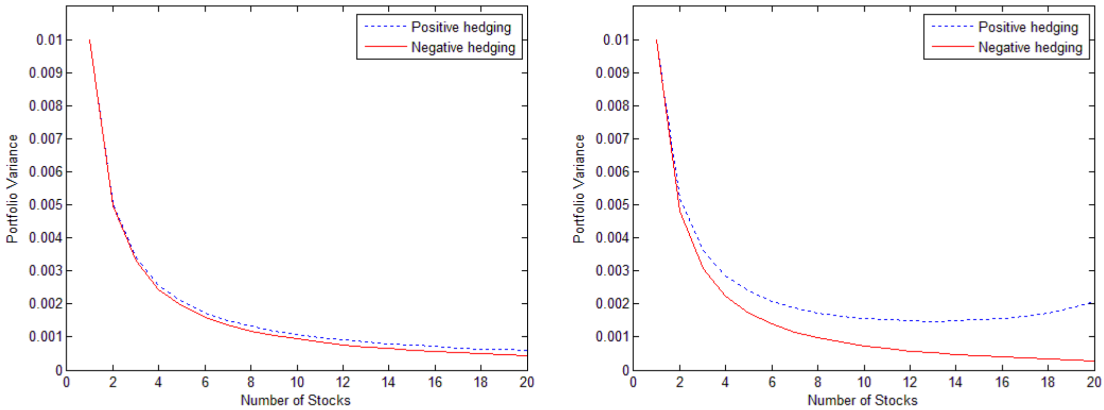
From classical portfolio theory, we know that the variance will decrease as we increase the number of stocks in the portfolio. As Equation (5) shows, increasing the number of stocks would result in a greater number of $\frac{1}{v_i}$ arguments and, accordingly, a lower level of variance. Another important factor explaining whether diversification pays off is the value of the hedging relations coefficient, $\beta_{i,j}$. With negative hedging, increasing the number of stocks would lead to a lower level of portfolio variance, but in the case of positive hedging, increasing the number of stocks may increase the portfolio variance. To explain this phenomenon in detail and without a lack of generality, we consider a special case in which the unhedgeable component and hedging relations are assumed to be constant for every stock ($v_i = v$; $\beta_{i,j} = \beta^*$). In this case, Equation (5) is transformed as follows:

$$Var_p^{minv} = \frac{v}{N - N \times (N-1) \times \beta^*} \quad (6)$$

To be able to study the effect of adding more stocks to a portfolio, we must consider two different cases: first, when the value of hedging relations is negative ($\beta = -\beta^*$ with $\beta^* > 0$) and, second, when the value of hedging relations is positive ($\beta = \beta^*$ with $\beta^* > 0$). In the first case, when the hedging relations are negative, we observe from Equation (6) that the portfolio variance decreases as the number of stocks in the portfolio increases. However, if β^* is positive, the portfolio variance decay rate will differ depending on the value of β^*. For sufficiency low values of β^*,[37] the portfolio variance decreases as we increase the number of stocks, even though the variance decay rate is lower than that when the hedging relations are negative. However, when these coefficients (β^*) are not small, our diversification strategy would not work, and instead, we could increase the portfolio variance as we increase the number of stocks in the portfolio. Such a case would be like investing in a time bomb.

[37] In the positive hedging case, the constraint $\beta^* < \frac{1}{N-1}$ should be held constant to maintain positive portfolio variance.

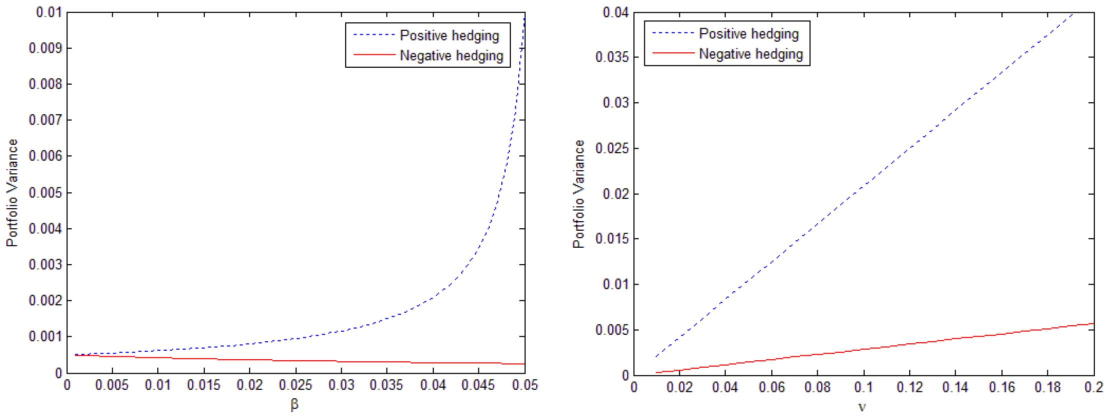
To explain this argument and compare positive and negative hedging relations with respect to the portfolio variance decay rate, we provide a numerical experiment. We consider a case in which the unhedgeable component (υ) is assumed to be equal to 0.01 and the absolute hedging relations (β^*) take two different values: 0.008 (we could think that is a low intensity) and 0.04 (denoting high intensity). The results are shown in Figure 4. In both cases, negative hedging results in a higher portfolio variance decay rate and, more important, a lower level of variance. For positive hedging in the case of high intensity, having more than 13 stocks in the portfolio results in an increase in the portfolio variance. This finding demonstrates the importance of having knowledge about the hedging network structure and composition (β, υ) before investment decisions are made, given that adding more stocks could have a negative effect.



(a) (b)

Figure 4. This figure shows the variance for portfolios of different size (from 2 to 20 stocks) in a full network structure where $\upsilon = 0.01$. Figure (a) shows the results for low intensity hedging relations ($\beta^* = 0.08$), and Figure (b) shows the results for high intensity hedging relations ($\beta^* = 0.04$).

Notably, in a full network structure, a high level of negative or positive hedging relations cannot be achieved. However, a higher intensity of hedging relations is beneficial for negative hedging in a full network since it always results in lower portfolio risk. To analyze the impact of the intensity of hedging relations on the portfolio variance, we next study, in a numerical example, the sensitivity of the portfolio variance to changes in the intensity of hedging relations (β^*) and the value of the unhedgeable component (υ). The results regarding changes in the portfolio variance for positive and negative hedging with a portfolio of 20 stocks in a full network structure are presented in Figure 5.



(a) (b)

Figure 5: Portfolio variance for a portfolio of 20 stocks in a full network structure with respect to (a) changes in the intensity of hedging relations (β) with a constant unhedgeable component, $\upsilon = 0.01$, and (b) changes in the unhedgeable component (υ) with a constant intensity of hedging relations, $\beta^* = 0.04$.

From Figure 5, we observe that as we increase β^*or υ, the difference in the portfolio variance between negative and positive hedging relations in a full network structure increases. Higher intensity of hedging relations is beneficial for negative hedging in a full network since it always results in lower portfolio risk. However, we observe an exponential increase in the portfolio variance in the case of positive hedging as we increase β^*. From Figure 5b, we observe that a higher value of the unhedgeable component generally increases the portfolio variance for both positive and negative hedging relations. Moreover, we find that while the portfolio risk increases for higher values of the unhedgeable component in the case of negative hedging, the increase is much lower in such a case than in the case of positive hedging.

Therefore, in sum, we can conclude that in the case of a full network structure, portfolio managers aiming to minimize the portfolio risk should overweigh stocks with negative and higher intensity hedging relations and a lower unhedgeable component and avoid holding stocks with high intensity and positive hedging relations.

3.2. Individual network

In the next step, we consider an individual network where no hedging relations exist between the stocks. Such a network structure is presented in Figure 2. In this case, the weight allocation for the *minv* strategy is expressed as follows:

$$w_i^{minv} = \frac{1}{\sum_{j=1}^{N} \frac{1}{v_j}} \times \frac{1}{v_i}$$

The *minv* strategy allocates to each stock a weight equal to the reverse of their individual variance, which is in accord with the aim of minimizing the portfolio variance. Hence, stocks with lower variance will be allocated a higher portion of the wealth. The portfolio variance is expressed as follows:

$$Var_p^{minv} = \frac{1}{\sum_{i=1}^{N} \frac{1}{v_i}}$$

The lower the unhedgeable component of each stock is, the lower the portfolio variance is. To explain the behavior of a diversification strategy as we increase the number of stocks, we consider a simplified case where the unhedgeable component of each stock is equal for every stock, v. In this case, we would have:

$$Var_p^{minv} = \frac{v}{N}$$

Accordingly, as we increase the number of stocks, the portfolio variance converges toward zero, and the convergence rate is slower in an individual network than in a full network where all the stocks negatively hedge each other. However, the portfolio variance decay rate is higher in an individual network than in a full network where all stocks show positive hedging.

3.3. Star network

A star network represents a network in which there is high asymmetry in the hedging relations. One stock is connected to all the other stocks, and no hedging relations exist among $N - 1$ stocks (see Figure 3). Under this network structure, the *minv* strategy assigns weights to the stocks as follows:

$$w_1^{minv} = \frac{1}{v_1} \times (1 - \sum_{j=2}^{N} \beta_{1j})$$

$$w_i^{minv} = \frac{1}{v_i} - \frac{\beta_{1i}}{v_1}, \quad i = 1, 2, 3, \dots, N$$

Accordingly, there is asymmetry in allocating weights between the stock in the center of the network (w_1^{minv}) and among the stocks in the peripheries of the network (w_i^{minv}). The portfolio variance of the portfolio of a star network is expressed as follows:

$$Var_p^{minv} = \frac{1}{\sum_{i=1}^{N} \frac{1}{v_i} - \sum_{j=2}^{N} \frac{\beta_{1j}}{v_1}}$$

Under this network structure, if all the hedging relations in the star network are negative, the portfolio variance is lower than in the case where the relationships are positive. Moreover, the unhedgeable component of the stock in the center of the network (v_1) is an important determinant of the portfolio variance. This unhedgeable component on the portfolio variance can increase or decrease the variance depending on the direction of the hedging relations. If the hedging relations are negative, ($\beta_{1j} < 0$), a decrease in the unhedgeable component of the stock in the center of the network decreases the level of portfolio variance, as occurs in a full network or individual network structure. However, if $\beta_{1j} > 0$, a lower v_1 will increase the portfolio variance. This result is interesting in the sense that the influence of positive hedging relations is eliminated when a stock with a high unhedgeable component is in the center of the network.

Next, to analyze the level of variance as we increase the number of stocks in the portfolio, we again consider a special case in which all the hedging relations and the unhedgeable component are assumed to be constant and equal. In this regard, the portfolio variance is expressed as follows:

$$Var_p^{minv} = \frac{v}{N - (N-1) \times \beta}$$

As with the full network, we consider two independent cases: first, with negative hedging ($\beta = -\beta^*$ with $\beta^* > 0$) and, second, with positive hedging[38] ($\beta = \beta^*$ with $\beta^* > 0$). In the case of negative hedging, the portfolio variance decay rate is higher than in the case of positive hedging, with large values of β^*. Interestingly, with a low number of stocks with high intensity negative hedging relations, we can achieve the same level of portfolio variance than with a high number of stocks with positive hedging relations. This result clearly demonstrates the importance of the information underlying the hedging network for portfolio managers.

To analyze the difference in the decline in portfolio variance between negative and positive hedging as the size of the portfolio increases, we conduct a numerical experiment, the results of which are presented in Figure 6. Assuming that the value of υ is 0.01 and that β^* represents a low level of intensity (0.008) or high level of intensity (0.1), we analyze the portfolio variance when the number of stocks changes from 1 to 20. From Figure 6a, we can conclude the portfolio variance for the case of negative hedging is clearly similar to the case in which stocks positively hedge each other when there are a low number of connections of low intensity. However, the difference between the case of negative and positive hedging relation is much greater when we increase the level of intensity (Figure 6b). This result indicates that the direction of hedging relations may be relevant for a star network only when the intensity of hedging relations is sufficiently large.

[38] Moreover, for the case of positive hedging, the value of β^* should be lower than $\frac{N}{N-1}$ for the portfolio variance to be positive.

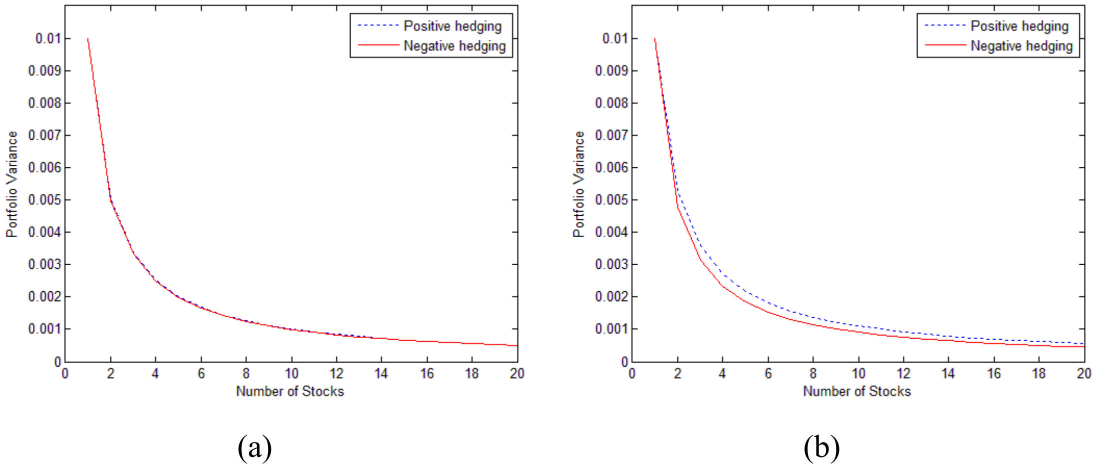
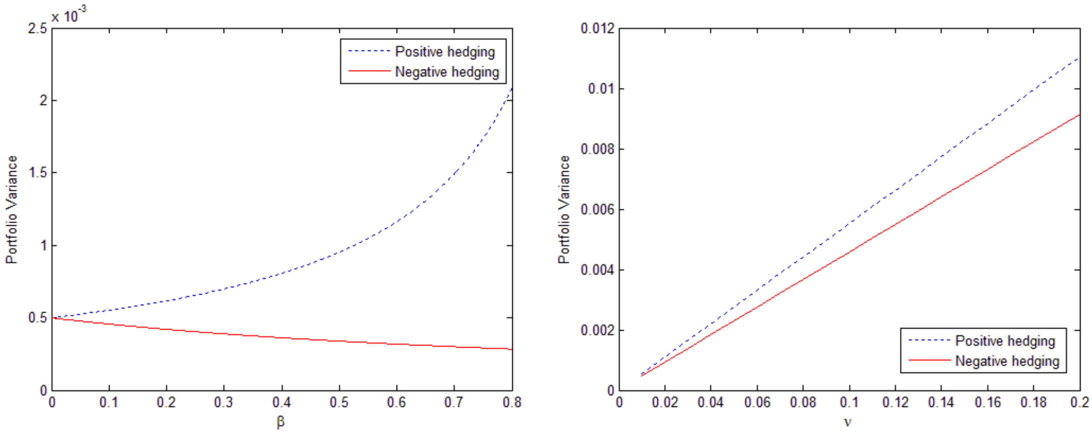


Figure 6. Portfolio variance as number of stocks in the star network structure increases where $\upsilon = 0.01$ and $\beta^* = 0.008$ (Figure (a)) and $\beta^* = 0.1$ (Figure (b))

Next, we investigate the sensitivity of the portfolio variance to the values of β^* and υ by using a numerical simulation; the results are presented in Figure 7. From Figure 7a, we can confirm the conclusions made above, since we observe that the portfolio variance increases for star network with positive hedging relations and decreases for a network with negative hedging relations as we increase the value of β^*. We also observe from Figure 7b that the impact of increasing the unhedgeable component is smaller than the effect observed in 7b when beta is increased under a positive hedging case. Nevertheless, the effect of the unhedgeable component of the central stock is lower if the hedging relations are negative.



(a) (b)

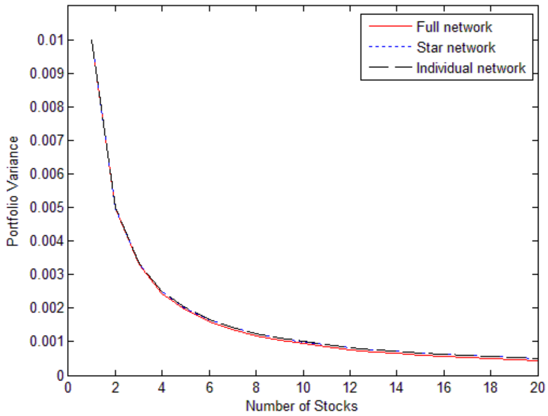
Figure 7: Portfolio variance for a portfolio of 20 stocks in a star network structure with respect to (a) changes in the intensity of hedging relations (β) with a constant unhedgeable component, $\upsilon = 0.01$, and (b) changes in the unhedgeable component (υ) with a constant $\beta^* = 0.1$.

3.4. Comparison of the portfolio variance decay rate in different network structures

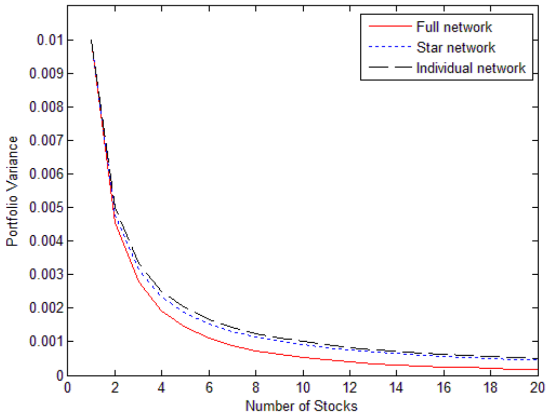
In the previous section, we exhaustively analyzed the benefits of diversification for each network structure depending on the sign of the hedging relations and the value of the unhedgeable component; however, the analysis was performed independently for each network structure, making comparisons among the structures difficult. This section aims to compare the benefits of diversification depending on the type of network (full, star or individual network) from two different perspectives: i) that of a portfolio manager using optimal weights from a minimum-variance strategy (which is the approach followed in the previous section) and ii) that of a portfolio manager using a naïve diversification strategy based on an equally-weighted portfolio among every stocks in the investment opportunity set (w_j=1/n). The latter will allow us to know whether the differences in portfolio variance found in the previous section also hold under naïve diversification or whether they appear only under a minimum-variance strategy. Moreover, we analyze the portfolio variance for both positive and negative hedging relations and for both high and low intensity of hedging relations.

Next, in Figure 8, we show the portfolio variance for each network structure in the case of negative (Panel A) and positive (Panel B) hedging relations, with a constant unhedgeable component (υ) equal to 0.01. Figure 8, Panel A presents the case for negative hedging relations with a low intensity of 0.08 (left figure) and a high intensity of 0.1 (right figure). In Panel B, we show the case of positive hedging relations with a high and low intensity of hedging relations (ranging from 0.008 to 0.04). We present the portfolio variance decay rate for a portfolio holding from 2 to 20 stocks to determine the weights under a minimum-variance portfolio strategy.

Panel A: Negative Hedging Relations

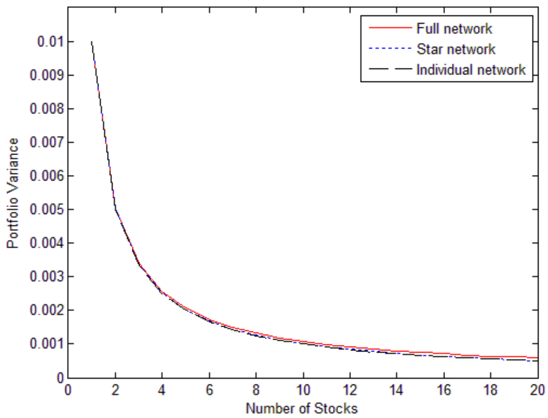


(a) Low Intensity

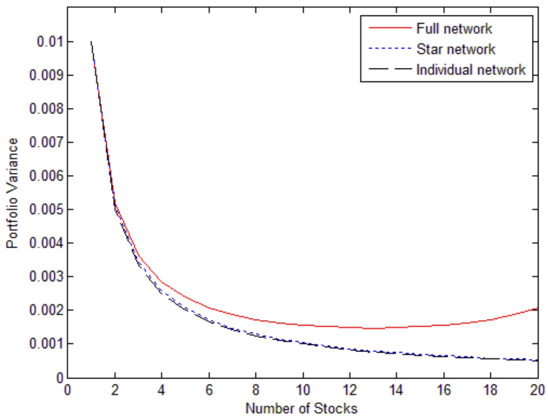


(b) High Intensity

Panel B: Positive Hedging Relations



(a) Low Intensity



(b) High Intensity

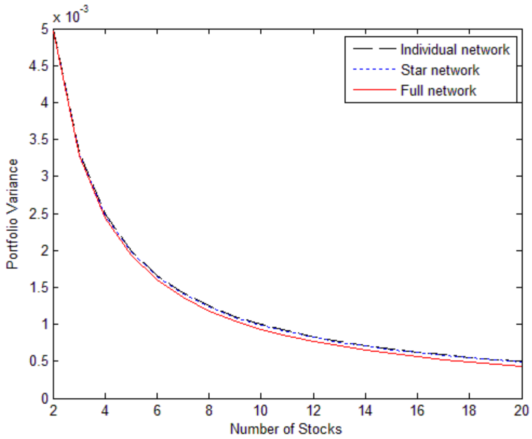
Figure 8. Portfolio variance of portfolios with “n” stocks (where “n” changes from 2 to 20) for negative (Panel A) and positive (Panel B) hedging relations. For each panel, a high (left graph) value of $\beta^*$and a low (right graph) value of β^* is presented. The value of the unhedgeable component remains equal and constant for every stock ($\upsilon = 0.01$). The weights for each portfolio in the figure are computed from a minimum-variance strategy analyzed in the previous section.

From the above figure, we can conclude that if the intensity of hedging relations (positive or negative) is sufficiently low, there are no significant differences between the hedging network structures. However, if the intensity of hedging relations is high, we can observe relevant differences in the benefits of portfolio diversification for an optimal portfolio based on minimizing the variance. The most novel and interesting results may be that there is no network structure that dominates the others; however, the benefits of diversification depend on the sign of the hedging relations, which should investors and portfolio managers should account for to achieve maximum diversification. If β has a high and negative value (Panel A, right figure), the full network structure outperforms the rest in terms of variance, given that the variance decay rate is higher and the portfolio has low variance despite the number of stocks. However, if β is positive and sufficiently high (Panel B, right figure), the full network structure shows the lowest diversification (the lowest variance decay rate), and if the number of stocks in the portfolio is increased up to a limit (up to 14 stocks in our experiments), the portfolio variance can even increase as we add more stocks to our portfolio.

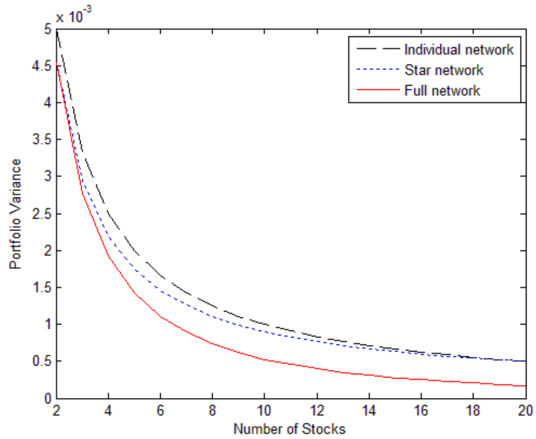
Next, we analyze how the network structure affects the portfolio variance in the case of a naïve strategy based on an equally weighted portfolio instead of a portfolio in which the weights are determined in a *minv* strategy. As with the *minv* strategy, in a first step, we consider the case in which all stocks are negatively hedging each other. We then simulate the inverse covariance matrix by assuming that the unhedgeable component is equal to 0.01 and that all the other inverse covariance matrix entities can take two different values (high and low intensity). We consider a number of stocks in the portfolio ranging from 2 to 20 and apply a naïve portfolio strategy to different simulated structures. The results are presented in Figure 9, which shows the results for the case in which every hedging relations is either negative (Panel A) or positive (Panel B). For each panel, we analyze the case of high (right figure) and low (left graph) intensity, with values of 0.008 and 0.01 in Panel A and values of 0.008 and 0.04 in Panel B.

As shown in Figure 9, differences in the portfolio variance between the networks can be observed when there are high intensity connections between the stocks; in this case, the full and star networks lead to lower portfolio variance, and as we increase the number of stocks, the portfolio variance for these networks decays faster.

Panel A: Negative Hedging Relations

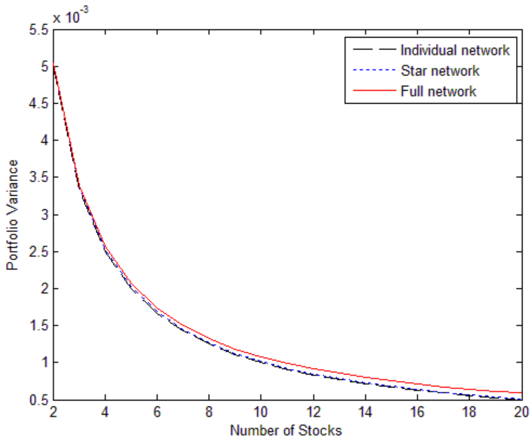


(a) Low Intensity

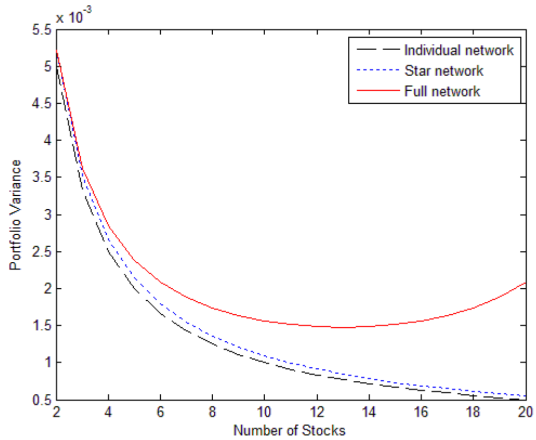


(b) High Intensity

Panel B: Positive Hedging Relations



(a) Low Intensity



(b) High Intensity

Figure 9. Portfolio variance of portfolios with "n" stocks ("n" changes from 2 to 20) for negative (Panel A) and positive (Panel B) hedging relations. For each panel, a high (left graph) value of $\beta^*$and a low (right graph) value of β^* is presented. The value of the unhedgeable component is equal and constant for every stock ($\upsilon = 0.01$). The weights for each portfolio in the figure are computed from a naïve equally weighted strategy.

We observe that the findings are consistent with those from the *minv* strategy, although the differences in portfolio variance among the different network structures are slight more pronounced. Moreover, we find that the full network structure achieves the greatest benefits of diversification if the hedging relations are negative and sufficiently high and that the full network shows higher portfolio variance if the hedging relations are positive and high, as adding more stocks to the portfolio will increase the portfolio variance.

4. Benefits of Diversification and Centrality Measure

In the previous section, we analyze several types of networks from a theoretical perspective and show that both the hedging relations and unhedgeable component are important for achieving the greatest portfolio diversification. In general, for most of the networks structures, we find that higher diversification is achieved when there are higher negative hedging relations and a lower unhedgeable component. However, the problem with applying this strategy (i.e., selecting the stocks with negative hedging) in the real world arises from the fact that when we estimate a hedging network for a set of stocks, we obtain positive and negative hedging relations for each stock. In contrast in the previous theoretical sections, for simplicity, we assumed that every hedging relation for a stock is identical ($\beta_{i|j} = \beta$); however, in the real world, this does not happen, and we need a measure that summarize this information. Therefore, in this section, we identify a measure that accounts for both variables (β, υ) and allow us to select the most suitable stocks to be held in a portfolio in order to increase diversification. We evaluate the convenience of using this measure to achieve a well-diversified portfolio first in an in-sample analysis and second in an out-of-sample experiment.

We use a popular measure in the network literature, the Katz centrality measure[39] (Katz, 1953), to capture information on hedging relations and the unhedgeable component into a single value and then to classify (rank) stocks according to their centrality value. This measure is presented as follows:

$$x_i = (I - \alpha\boldsymbol{\beta})^{-1}\boldsymbol{\upsilon} \tag{7}$$

where $\boldsymbol{\beta}$ is the hedging network adjacency matrix and $\boldsymbol{\upsilon}$ is the unhedgeable component vector. Katz centrality differs from the ordinary eigenvector centrality in the important respect that it has a free parameter α, which governs the balance between the hedging relations ($\boldsymbol{\beta}$) and the unhedgeable component. Low values of α indicate that the unhedgeable components has more importance, whereas higher values of α indicate that the hedging relations matrix has more relevance. We define parameter α as being between 0 and 1.

[39] For more details about centrality measures, see Newman (2010).

Basically, this measure of centrality gives a lower centrality value to both stocks with higher negative hedging relations with the rest of the stocks in the dataset and stocks with a lower unhedgeable component. According to the general conclusions we obtained in the previous sections, these stocks (i.e., those with the lowest centrality value) are the ones with the greatest diversification effects.[40] Using this centrality measure, we can rank stocks based on the benefits of diversification they bring to our diversified portfolio, or in other words, this centrality measure provides a rankings of the benefits of diversification for the stocks.

4.1. Empirical evidence: Estimating a hedging network

In the empirical section, we employ several dataset to provide sufficient robustness for our results. First, we use a monthly dataset (called NYSE100), which includes 100 randomly selected stocks from the New York Stock Exchange (NYSE) for the time period between January 1990 and December 2012. Second, we use another two datasets with a daily frequency: (1) SP100 includes 100 stocks that integrate the S&P500 index for 2103 and that have data available for the whole time period from January 2002 to September 2010 and (2) the last dataset includes 100 stocks from the FTSE 250 index with data available for the selected sample. The datasets are described in detail in Table 1.

[40] In Appendix A, we also analyze other centrality measures (such as degree centrality and the eigenvector centrality measure), but they are not able to identify the stocks with the greatest benefits of diversification.

Table 1. Datasets description

Dataset Description	Source	Dates	Abbreviation	Observations
Monthly returns for 100 randomly selected stocks from NYSE	CRSP	From 1/1/1990 to 12/31/2012	NYSE100	276
Daily returns for the 100 stocks integrating the S&P500	Datastream	From 1/1/2002 to 1/09/2010	SP100	2000
Daily returns of 100 stocks integrating the FTSE 250	Datastream	From 1/3/2006 to 1/11/2013	FTSE100	2000

In the previous theoretical section, we showed that knowing the type of network structure could be relevant for investors to achieve the maximum diversification. When using real data, it is not as easy as in theory to exactly identify the type of network we have; however, there are certain procedures that can help. As this is the first paper in the literature on hedging networks, we next present the results from estimating a hedging network for the NYSE100 and analyze each of its components (hedging relations, unhedgeable components, and structure) in detail.

In Figure 10, we show the hedging network for the 100 stocks in the NYSE100 dataset using the 276 observations in the sample. In this figure, the hedging relations are represented by lines, and the unhedgeable component is represented by nodes or circles, where positive (negative) hedging relations are indicated in blue (red). Note that only examining the presentation of the network does not allow us to correctly identify the type of network we have, as the connections are represented by lines in the figure, but the intensities ($\beta_{i,j}$) are not represented. Thus, an initial conclusion from the presentation of the network could be that it is a full network, as every stock seems to be connected with each other. However, some of the connections may have very low values that could be nonsignificant, which would completely change how the network is classified. Next, we analyze two different methodologies to study the network structure. In addition, we can extract two separate networks, one with positive hedging relations and another with negative hedging relations. This is presented in Figure 11.

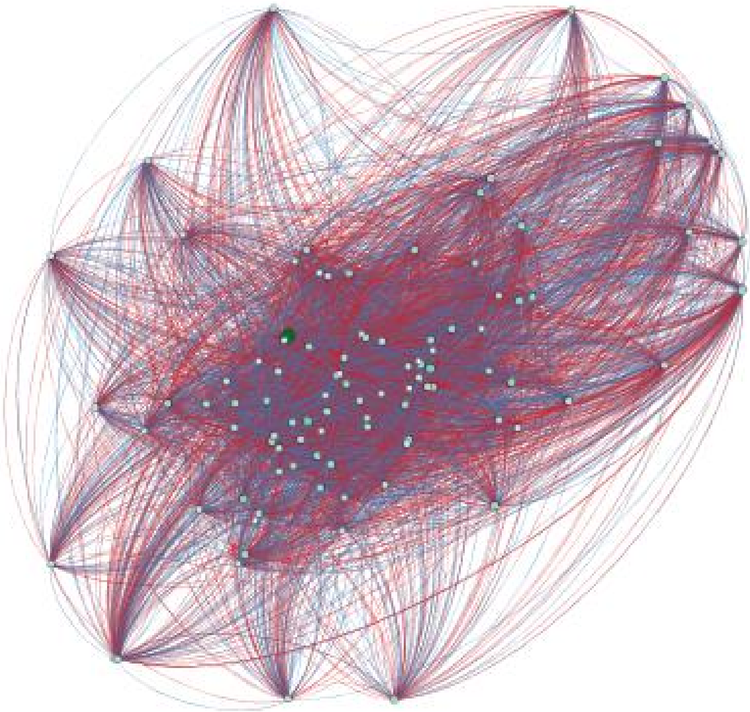
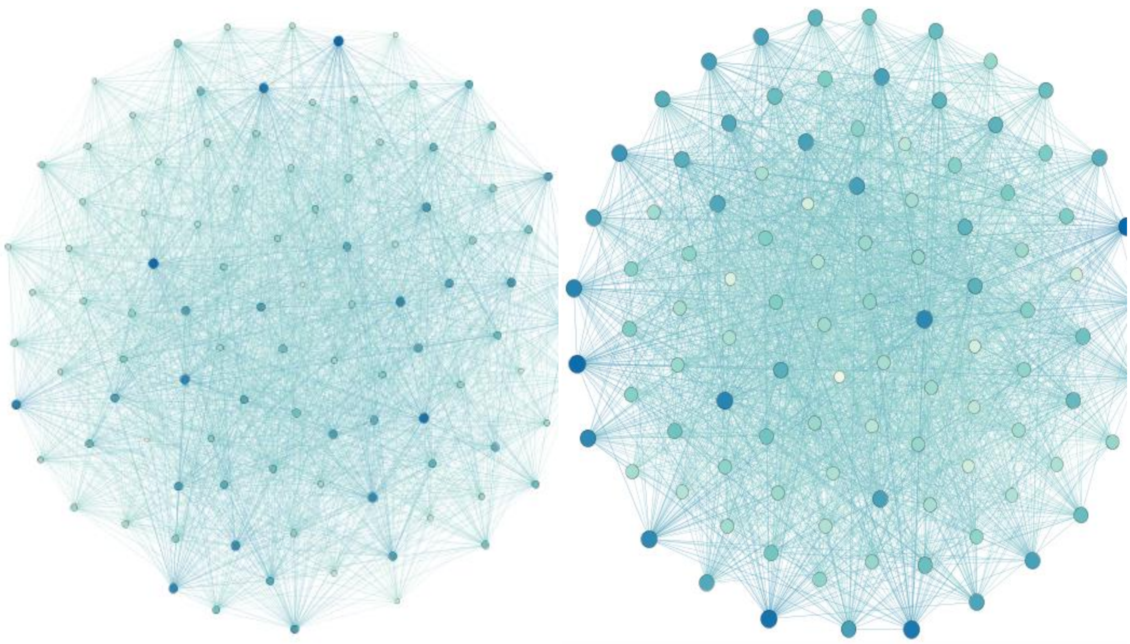


Figure 10: This figure presents a hedging network estimated for the 100 stocks in the NYSE100 dataset. The nodes are stocks, and the lines are hedging relations. The blue lines are positive hedging relations, and the red lines are negative hedging relations. Moreover, the size and darkness of the nodes are related to the value of the unhedgeable component. The higher and darker is the node, the higher the value of the unhedgeable component is.



(a) Only positive hedging relations (b) Only negative hedging relations

Figure 11: This figure presents a hedging network estimated for the 100 stocks in the NYSE100 dataset, when the positive and negative hedging relations are extracted separately. In Figure (a), we show only the positive hedging relations, and in Figure (b), we show only the negative hedging relations.

To shed some light on the structure of the network, we consider the Freeman centralization measure (Freeman, 1979), which measures how close a network is to star-like network structure. This author introduces the idea of network centralization, which can be measure by $\mathrm{C_x}$:

$$C_x = \frac{\sum_{i=1}^{n}[C_x(p^*) - C_x(p_i)]}{\max \sum_{i=1}^{n}[C_x(p^*) - C_x(p_i)]}$$

Where $C_x(p_i)$ is the centrality of node i and $C_x(p^*)$ is the largest centrality in the network. Therefore, in the above definition, we are dividing (a) the sum of the differences in centrality between the highest central node in the actual network and all other nodes from (b) the theoretically largest possible sum of centrality differences in a specific network. We compute this theoretical value while assuming that our network resembles a star network. According to Freeman, $0 < C_x < 1$, where the value is equal to 0 if every node is equally important and equal to 1 if and only if one point (p^*) completely dominates the network with respect to centrality (the underlying network is a star network with a unique central node). Logically, when applying this measure to the empirical data, we usually obtain a value that is not at the extremes (0 or 1), as networks usually have several (more than one) nodes that dominate the network and not every nodes bears the same relevance. The results for the three datasets are presented in Table 2.

Table 2: Values of the Freeman centralization measure for the entire sample for the different dataset

	Positive Hedging	Negative Hedging
NYSE	0.14	0.17
SP100	0.20	0.24
FTSE100	0.17	0.19

The values in Table 2 clearly indicate that the corresponding networks are not a full network where every node has identical importance, as the value is not equal to 0. In addition, we the networks do not have a unique central node that dominates the whole network (as the value is not equal to 1).[41] Thus, we can conclude that our hedging network has some characteristics of a star network structure, where some particular stocks play the role of dominating nodes (as is very common in financial markets, there are central stocks and peripheral ones with a lower relevance), and of a full network, as many of the nodes seems to be linked and Cx is close to zero. Nonetheless, exactly identifying the type of network structure we have is not a critical issue for us, as we know that the diversification works approximately in the same way in both star and full networks (the portfolio variance decreases when the held stocks have more negative hedging relations and a lower unhedgeable component). We must remember that the goal of this section is to provide a measure (centrality measure) that allows investors to choose the stocks to be included in their portfolios in order to achieve maximum diversification. In the next section, we evaluate the performance of this strategy (holding an equally weighted

[41] We also analyze the distribution of the degree centrality in the networks to determine the network structure under the basic idea that if a network is more prone to exhibit a star-like structure, we would expect some nodes to hedge a large portion of other nodes, thereby leading to a fat-tail network structure. Moreover, if the network is close to a full network structure, the distribution should be close to uniformly distributed. On the other hand, an individual network structure would lead to a very low number of hedging relations, thereby moving the entire degree distribution to the left. The results are not showed here to save space, but in general, we observe heavy-tail characteristics; thus, these networks do not show an individual or full network structure, but both networks bear resemblance to a star network structure, which is in accordance with the previous evidence from the Freeman analysis.

portfolio with the lowest value for the Katz centrality measure) in both an in-sample and out-of-sample analysis.

In Table 3, we present the descriptive statistics for the hedging relations and unhedgeable component obtained for the networks estimated with the three datasets. We provide the mean value, standard deviation, 5th percentile, median value and 95th percentile. Based on the three dataset values, we observe that the hedging relations are between -0.1720 and 0.1981 in 90% of the cases, with the mean value ranging between 0.005 and 0.010. From the previous theoretical section, for the parameter beta, we use a value of 0.1 for high intensity and a value of 0.008 for low intensity, which are reasonable values according to this table. Regarding the unhedgeable component (υ), the mean ranges from 0.0002 to 0.0050, and 90% of the values range between 0.000 and 0.0142 (representing 0.0101 and 0.1730, respectively, in annualized terms). In addition, the specific values considered in the simulations of Section 2 (0.01) are also realistic.

Table 3. Summary statistics for $\beta_{i,j}$ and $\upsilon_{i,j}$ in the hedging networks estimated in each dataset

	Mean	**Std**	**5%**	**Med**	**95%**
			NYSE100		
$\beta_{i,j}$	0.0100	0.1200	-0.1720	0.0060	0.1981
$\upsilon_{i,j}$	0.0050	0.0050	0.0012	0.0030	0.0142
$\upsilon_{i,j}$ (annualized)	0.0620	0.0180	0.0080	0.0400	0.1730
			SP100		
$\beta_{i,j}$	0.0050	0.0436	-0.0551	0.0024	0.0696
$\upsilon_{i,j}$	0.0002	0.0002	0.0001	0.0001	0.0004
$\upsilon_{i,j}$ (annualized)	0.0475	0.0000	0.0154	0.0361	0.1111
			FTSE100		
$\beta_{i,j}$	0.0098	0.0550	-0.0665	0.0050	0.0968
$\upsilon_{i,j}$	0.0003	0.0002	0.0000	0.0002	0.0007
$\upsilon_{i,j}$ (annualized)	0.0650	0.0000	0.0101	0.0547	0.1862

4.2. Empirical results: In-sample and out-of-sample analysis

In this section, we evaluate the performance of a strategy based on the Katz centrality measure (which is supposed to take into account the findings from the theoretical section: higher negative hedging relations, lower positive hedging relations or lower unhedgeable components will reduce the portfolio variance). We conduct an in-sample analysis first and an out-of-sample analysis using a rolling-widows procedure second. In addition, we compare the results against some other strategies in order to be able to evaluate the benefits of diversification of this new strategy.

We consider a 1/N naïve strategy (called *Naïve Diversification* in the figures) with an equally weighted portfolio for each number of stocks. In this case, the results in the figures show the average variance from considering a random sampling of 1000 equally weighted portfolios selected from the 100 stocks in each dataset. This is the simplest strategy of diversification analyzed in papers such as Statman (1987) or Domian et al. (2007) to determine, for example, the number of stocks that should held in a well-diversified portfolio. Moreover, we also consider a strategy based on holding the stocks with the lowest average correlation coefficients (called *Lowest Correlation*), where for each number of stocks, N, we rank the 100 stocks in the dataset according to the average correlation coefficient and create an equally weighted portfolio that includes N stocks with the lowest values. We introduce this strategy to be able to compare our strategy based on the Katz centrality measure for different hedging networks against a strategy under classical portfolio theory based on returns correlations, which recommends investing in stocks with the lowest correlation to achieve the maximum benefits of diversification.[42] A strategy based on choosing N stocks with the lowest unhedgeable component is also analyzed (called *Lowest Unhedgeable*). Finally, we also include two strategies based on the Katz centrality measure, *Lowest Centrality* and *Highest Centrality*, which rank the 100 stocks in the dataset by their centrality values (defined in Equation 6) and construct an equally weighted portfolio with the N stocks with the lowest and highest centrality values, respectively, for a portfolio with N stocks. Although we are interested in the strategy that takes the stocks with the lowest centrality, we also introduce the

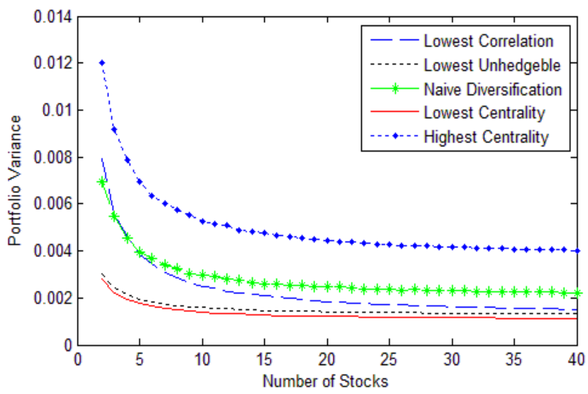
[42] At the same time, this comparison will allow us to demonstrate that negative hedging and negative correlation are not identical concepts. Specifically, we will be able to demonstrate that a portfolio holding the stocks with the lowest average correlation coefficient differs from a portfolio holding the stocks with the lowest centrality values.

strategy with the highest centrality because the stocks with the highest centrality values should be the ones with the worst diversification if our hypothesis is correct. Finding such a result also allows us to be sure about the benefits of diversification when we choose stocks with lowest centrality.

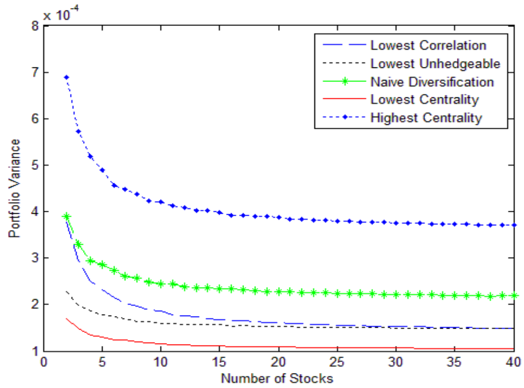
In Figures 12 and 13, we present the results from the in-sample analysis for the three datasets, where the alpha value of the Katz centrality measure equals 0.8 and 0.4, respectively. Remember that parameter α governs the balance between the hedging relations ($\boldsymbol{\beta}$) and the unhedgeable component in the Katz centrality measure (where the relevance of hedging relations matrix decreases as the value of α decreases). As alpha approaches zero, the Lowest Centrality strategy should become more similar to the Lowest Unhedgeable strategy, since the weight of hedging relations in the former strategy is almost zero.

In general, from Figures 12 and 13, we observe that the Lowest Centrality strategy (red line) yields the greatest benefits of diversification regardless of the number of stocks and market analyzed here. This portfolio strategy performs particularly well in the SP100 dataset, and it performs slightly better than the Lowest Unhedgeable strategy in the rest of the markets (NYSE100 and FTSE100). As noted previously, a strategy based on holding the stocks with the highest centrality values should perform the worst in terms of diversification. This result thus confirms that the Katz centrality measure classifies stocks properly according to benefits of diversification. We also observe that a portfolio based on holding the stocks with the lowest average correlation coefficients performs better than a naïve portfolio based on taking a random sampling of N stocks, but it does not outperform the Lowest Centrality and Lowest Unhedgeable strategies based on network theory. The Lowest Correlation strategy nevertheless performs almost as well as the Lowest Centrality and Lowest Unhedgeable strategies if the number of stocks is sufficiently high. For example, in both the NYSE100 and FTSE 100 datasets, the portfolio must include almost 30 stocks to show equivalent performance; however, in the SP100 dataset, the Lowest Correlation strategy never achieves the level of variance of the Lowest Centrality strategy regardless of the number of stocks. If we compare the results in Figure 12 (alpha of 0.8) with those in Figure 13 (alpha of 0.4), we can draw the same conclusions, but the performance of the Lowest Unhedgeable strategy is much closer to that of the

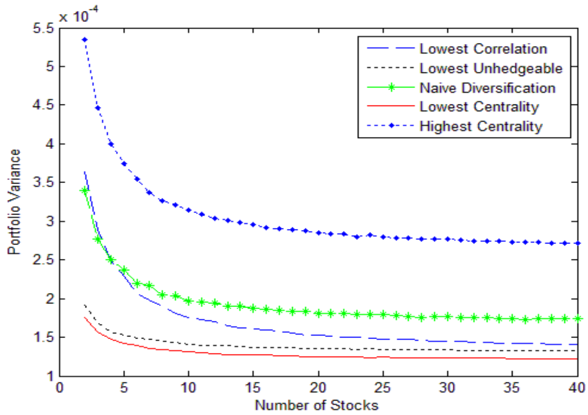
Lowest Centrality strategy because we underweight the hedging relations in the centrality measure.[43] Next, we analyze whether these conclusions hold in an out-of-sample analysis, and therefore, whether they can be used by investors to achieve the maximum benefits of diversification in financial markets.



(a) NYSE100



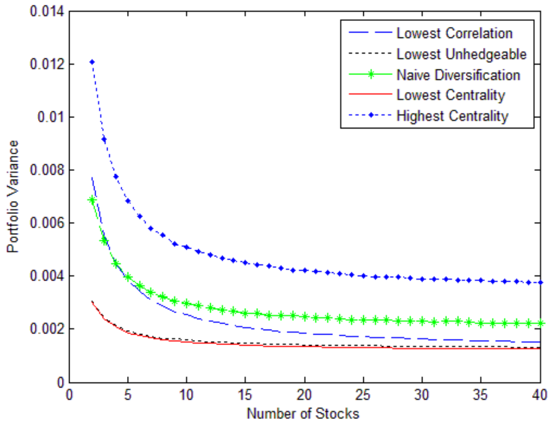
(b) SP100



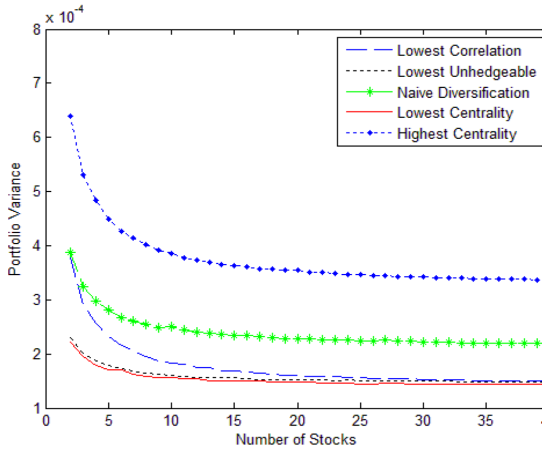
(c) FTSE100

Figure 12: This figure shows the portfolio variance, from an in-sample analysis, for a set of portfolio strategies with from 2 to 40 stocks for different datasets. The alpha parameter for the Katz centrality measure equals 0.8

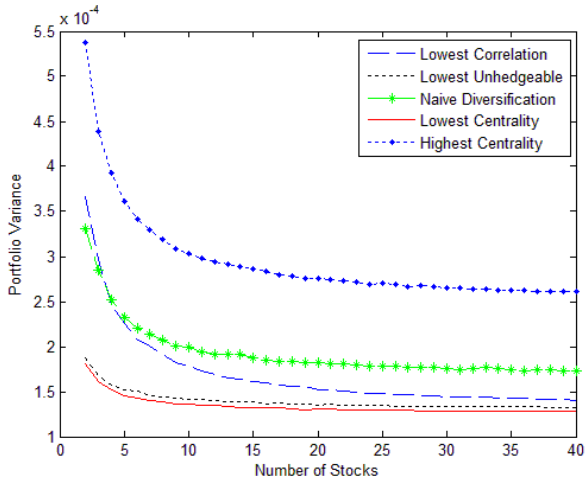
43 We also repeat the analysis with a value of alpha of 0.9, and the results are very similar. We do not present these results to save space.



(a) NYSE100



(b) SP100



(c) FTSE100

Figure 13: This figure shows the portfolio variance, from an in-sample analysis, for a set of portfolio strategies with from 2 to 40 stocks for different datasets. The alpha parameter for the Katz centrality measure equals 0.4

In this subsection, we examine the out-of-sample performance of a portfolio with a specific number of stocks (20, 40 and 60) according to the previously proposed strategies (e.g., in the case of 20 stocks, the Lowest Centrality strategy estimates the hedging network and selects 20 stocks of the 100 in the dataset with the lowest centrality values). In the first step, we explain the rolling-window approach that is employed and the methodology that is used to estimate the hedging network in order to address estimation error problems. Next, we present the results of the out-of-sample analysis.

To compare the performance of these strategies in the out-of-sample analysis, we employ the "rolling-window" approach provided in DeMiguel et al. (2009). Assuming that there are, in total, T periods of a stock's returns, we set an M-period estimation window (120 for the NYSE monthly dataset and 1000 for the other daily datasets), and we estimate the hedging networks (and the correlation coefficients) and identify the stocks with highest and lowest centralities. We maintain the same portfolio (an equally weighted portfolio among the selected stocks) for the next H observations and compute the out-of-sample returns. After these H observations, we reestimate the networks structure and recompute the correlations among the stocks in order to add the new information into the market. We then reestimate the network by including these new H observations and removing the earliest H observations in the M-period sample. In the baseline case, we set H to 12 for the monthly dataset and 20 for the daily datasets; thus, we reestimate and rebalance the portfolios once a year for the monthly dataset and every month for the daily datasets. We also present the results from using a shorter time horizon, where H equals 6 for the monthly dataset and 5 for the daily datasets, to show the robustness of our conclusions.[44]

Note that in the out-of-sample analysis, we may face an estimation error problem, and in the estimation of the inverse covariance matrix, there may be a multicollinearity problem, particularly if the number of stocks is higher than the number of observations (e.g., in our analysis with monthly observations, the estimation window is only 120 observations). Moreover, estimation of an inverse covariance matrix may be subject to high estimation error (Kan and Zhou, 2007). In this regard, several approaches have been proposed in the literature to tackle this problem. Friedman et al. (2008), for example, propose a Graphical Lasso algorithm in which they use a block-coordinate descent method. However, this approach has the drawback that it does not clearly specify the stopping criterion for the Lasso iterations, which affects its convergence capability. In a more recent paper, Hsieh et al. (2011) offer a solution to this problem by proposing a highly convergent algorithm, called the Quadratic Approximation method (QUIC, QUadratic Inverse Covariance), and demonstrate that this algorithm outperforms the Graphical Lasso algorithm. In the QUIC algorithm, the multicollinearity problem is resolved by penalizing the l_1 norm of the

[44] In addition of the figures presented in the paper, we also study the out-of-sample results for other windows and rebalancing sizes, and the results are identical. We do not present the results to save space, but they are available upon request.

estimation parameters. Therefore, this algorithm can yield a sparse inverse covariance matrix where elements that are highly significant are selected.[45]

The QUIC algorithm is used in our out-of-sample analysis to estimate the hedging network. In QUIC algorithm, we should specify the regularization parameter for Graphical Lasso approximations. However, in specifying the regularization parameter, we face a tradeoff. In one case, we can have a dense hedging network that contains the true hedging network with a high probability, whereas in the other case, we may want as sparse a network as possible to focus on only the highly significant elements—or the bone structure of the network. For the following analysis, we set the sparsity of the hedging network to approximately 30% to make sure that we have the true hedging network structure with a high probability and to ensure that we do not face high estimation error.[46]

In Table 4, we present the out-of-sample results measured by the annualized variance. Each row represents a different portfolio strategy previously described in the in-sample analysis, and we present the results for portfolios of different sizes, which include from 20 to 60 stocks. In the case of the Diversification Strategy, the results are the same for each portfolio size across the datasets (e.g., 0.0309 for NYSE100), since this strategy shows the variance of a portfolio holding the totality of stocks in each dataset. The aim of using this strategy was to be able to compare the results with those from a naïve strategy similar to the one used by DeMiguel et al. (2009) in order to confirm if naïve diversification is really better than any other strategy as these authors found. In parentheses and italics, we show the p-values[47] for the differences between each strategy and the naïve Diversification Strategy, which is considered the benchmark. For each column, we indicate the lowest variance in bold.

[45] A more technical and detailed description of the QUIC algorithm is provided in Appendix B.

[46] We also compute the results for other levels of sparsity (10% and 40%), and the conclusions are identical: the Lowest Centrality strategy provides the lowest variance for each portfolio. We do not present these results in the text in order to save space, but they are available upon request.

[47] To observe whether the differences in variance are statistically significant, we follow the approach proposed by Ledoit and Wolf (2011). The p-values are computed based on the studentized circular block bootstrap (Politis and Romano 1994) with a block size of 5 and 5000 bootstrap samples.

Table 4: Portfolio variance in the out-of-sample analysis with M=120 and H=12 for the monthly dataset and M=1000 and H=20 for the daily datasets

	NYSE100			**SP100**			**FTSE100**		
	20	**40**	**60**	**20**	**40**	**60**	**20**	**40**	**60**
Diversification Strategy	0.0309	0.0309	0.0309	0.0872	0.0872	0.0872	0.0265	0.0265	0.0265
Lowest Correlation	0.0236	0.0235	0.0241	0.0628	0.0644	0.0754	0.0242	0.0243	0.0265
	(0.054)	*(0.003)*	*(0.002)*	*(0.001)*	*(0.001)*	*(0.001)*	*(0.007)*	*(0.001)*	*(0.912)*
Lowest Unhedgeable	0.0169	0.0186	0.0220	0.0560	0.0682	0.0786	0.0190	0.0196	0.0195
	(0.001)	*(0.003)*	*(0.003)*	*(0.001)*	*(0.003)*	*(0.001)*	*(0.000)*	*(0.001)*	*(0.001)*
Lowest Centrality	**0.0111**	**0.0149**	**0.0194**	**0.0432**	**0.0560**	**0.0668**	**0.0158**	**0.0173**	**0.0188**
	(0.000)	*(0.000)*	*(0.011)*	*(0.000)*	*(0.002)*	*(0.001)*	*(0.005)*	*(0.000)*	*(0.001)*
Highest Centrality	0.0797	0.0628	0.0499	0.1563	0.1335	0.1178	0.0611	0.0439	0.0357
	(0.001)	*(0.002)*	*(0.001)*	*(0.000)*	*(0.001)*	*(0.001)*	*(0.000)*	*(0.001)*	*(0.001)*

From Table 4, we can conclude that the strategy based on holding the stocks with the lowest centrality performs the best in terms of measuring the risk based on the variance. We observe that for all datasets and portfolio sizes, this strategy yields the lowest variance, and difference from the variance of the naïve Diversification Strategy is significant. We also observe that the lower the number of stocks held in the portfolio is, the lower the variance is. This is a very relevant result for investors, as they would not need a large portfolio and could save in transaction costs. For example, in the case of the monthly dataset (NYSE100), the variance with only 20 stocks is approximately 40% lower than the variance of a larger portfolio with 60 stocks. In addition, we observe that the Highest Centrality strategy shows the worst performance in terms of diversification, confirming that our Katz centrality measure works properly as the stocks with highest centrality should offer the lowest benefits of diversification. We also observe that the Lowest Correlation and the Lowest Unhedgeable strategies generally perform better than the naïve Diversification strategy but not better than the Lowest Centrality strategy.

In Table 5, we repeat the analysis using a different rebalancing window size, where H equals 6 for the monthly dataset and 5 for the daily datasets, and the results are identical. We again observe that the Lowest Centrality strategy yields the lowest variance for each dataset and portfolio size (20, 40 and 60), and the level of variance is significantly different from that of the Diversification Strategy with a 99% level of confidence. We repeat the out-of-sample analysis by reestimating the hedging network and correlations every month or day (H=1) and conclusions are identical.[48]

Table 5: Portfolio variance in the out-of-sample analysis with M=120 and H=6 for the monthly dataset and M=1000 and H=5 for the daily datasets

	NYSE100			SP100			FTSE100		
	20	**40**	**60**	**20**	**40**	**60**	**20**	**40**	**60**
Diversification Strategy	0.0309	0.0309	0.0309	0.0872	0.0872	0.0872	0.0265	0.0265	0.0265
Lowest Correlation	0.0227	0.0228	0.0240	0.0612	0.0630	0.0750	0.0241	0.0242	0.0264
	(0.063)	*(0.006)*	*(0.005)*	*(0.001)*	*(0.001)*	*(0.001)*	*(0.003)*	*(0.001)*	*(0.847)*
Lowest Unhedgeable	0.0160	0.0186	0.0225	0.0556	0.0674	0.0773	0.0191	0.0195	0.0195
	(0.001)	*(0.001)*	*(0.002)*	*(0.001)*	*(0.001)*	*(0.001)*	*(0.001)*	*(0.001)*	*(0.001)*
Lowest Centrality	**0.0106**	**0.0147**	**0.0191**	**0.0428**	**0.0555**	**0.0663**	**0.0161**	**0.0174**	**0.0188**
	(0.001)	*(0.001)*	*(0.000)*	*(0.001)*	*(0.001)*	*(0.001)*	*(0.001)*	*(0.001)*	*(0.001)*
Highest Centrality	0.0809	0.0634	0.0501	0.1599	0.1344	0.1184	0.0613	0.0439	0.0357
	(0.000)	*(0.001)*	*(0.001)*	*(0.001)*	*(0.000)*	*(0.001)*	*(0.002)*	*(0.001)*	*(0.000)*

5. Conclusion and Future Research

This paper investigates diversification from a hedging network perspective. By considering various stylized hedging network structures, i.e., individual, full and star networks, we investigate how a minimum-variance strategy assigns value among stocks to achieve the greatest benefits of diversification. The results provide insights regarding the optimal approach to diversification in a hedging network in order to gain the lowest portfolio risk.

In addition, we theoretically evaluate the change in the portfolio variance decay rate as we increase the number of stocks, and we find that full network structure performs the best when the hedging relations are negative. However, when the hedging relations are positive, the full network structure performs poorly, and the portfolio variance could even increase if the number of stocks held in the portfolio is sufficiently high. By contrast, the star network structure is found to perform reasonably well in both cases.

48 We do not present these results to save space, but they are available upon request.

In the empirical part of the paper, we first employ a centrality measure for the hedging network that captures stocks' propensity to perform well in terms of risk. Based on a ranking of stocks according to the centrality measure, we can obtain a subset of stocks that offers the greatest benefits of diversification. In evaluating the performance in an in-sample and out-of-sample analysis, we find these portfolios constructed with this centrality measure are superior portfolios in terms of diversification. In evaluating the performance in an in-sample and out-of-sample analysis, we find portfolios constructed with this centrality measure are superior portfolios in which all the available stocks in the opportunity set of stocks are diversified. The findings in this paper suggest that the hedging network structure should be verified before a diversification strategy is implemented in order to obtain the greatest benefits of diversification by simply investing in a small portion of stocks that are well-placed in the network.

Appendices

A. Other centrality measures

In this paper, we use a popular centrality measure in the network literature, the Katz centrality measure; however, other centrality measures have been proposed (see Newman, 2010). For instance, we could also use centrality measures that partially capture the information in hedging network, such as the degree centrality or the eigenvector centrality measures.

The degree centrality is computed as follows:

$$d_i = \sum_{j=1}^{N} \beta_{i|j}$$

This centrality measure performs the same as the *average correlation measure*.

Second, we analyze the eigenvector centrality measure, which is defined as follows for a node i:

$$c_i = \kappa_1^{-1} \sum_{j} \beta_{i|j} c_j$$

The centrality of node i is defined as the weighted sum of the centralities of the adjoining nodes. In this regard, node i is highly central if it is connected to numerous nodes or to another highly central node. In matrix terms, it is defined as follows:

$$\boldsymbol{\beta c} = \kappa_1 \boldsymbol{c}$$

where the vector $\boldsymbol{c}$ corresponds to the eigenvector of the highest eigenvalue (κ_1) of adjacency matrix ($\boldsymbol{\beta}$),as discussed in Newman (2010)**.** Using this centrality measure, we can place greater importance on the benefits of hedging relations. In this case, the in-sample performance comparison is presented in the following figure, where we can observe that neglecting the importance of unhedgeable components would result in lower performance for the *Lowest Centrality* strategy.

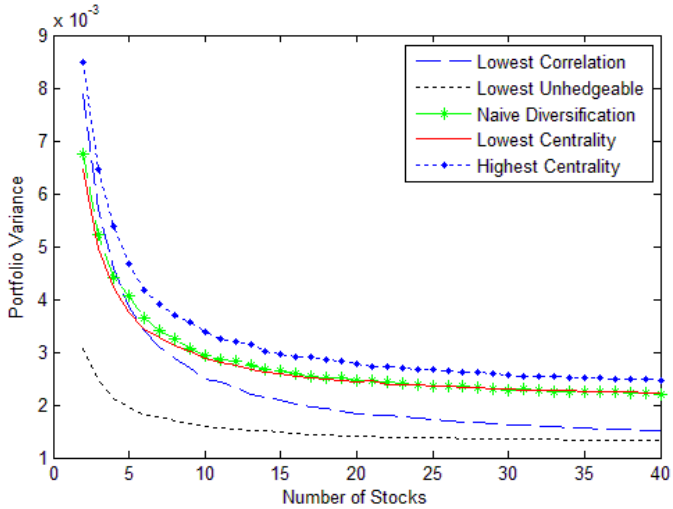


Figure: This figure shows the portfolio variance for a set of portfolio strategies with from 2 to 40 stocks. This figure presents the results of an in-sample analysis for the NYSE100 dataset. The Lowest and Highest Centrality strategies are based on holding the stocks with the lowest and highest values for the eigenvector centrality measure, respectively.

B. QUIC estimation procedure

In this appendix, we explain in detail the QUIC estimation procedure. Assuming $\boldsymbol{\Psi}$ as the inverse covariance matrix for N stocks in the dataset, we estimate this matrix by solving the following constrained quasi-maximum likelihood estimation problem:

$$\min_{\boldsymbol{\Psi}} -\frac{T}{2}\ln\det\boldsymbol{\Psi} + \frac{T}{2}trace(\widehat{\boldsymbol{S}}\boldsymbol{\Psi}) + \lambda\sum_{i=1;i\neq j}^{N}\sum_{j=1}^{N}|\psi_{ij}| \text{ (A1)}$$

where T is the number of observations in the sample and $\widehat{\boldsymbol{S}}$ is the sample covariance matrix. The parameter λ is the regulation parameter.

Hsieh et al. (2011) rewrite the objective function in Equation (A1) in two terms:

$$g(x) = \frac{T}{2}\ln\det\boldsymbol{\Psi} + \frac{T}{2}trace(\widehat{\boldsymbol{S}}\boldsymbol{\Psi}) \text{ and } h(x) = \lambda\sum_{i=1;i\neq j}^{N}\sum_{j=1}^{N}|\psi_{ij}|$$

The first function, $g(x)$, is twice differentiable and strictly convex, and the second function, $h(x)$, is convex but not differentiable. These two functions provide a second-order approximation for $g(x)$ as follows:

$$\bar{g}_x(\Delta) = trace((\widehat{\boldsymbol{S}} - \mathbf{W})\Delta) + \frac{1}{2}trace(\boldsymbol{W}\Delta\boldsymbol{W}\Delta) - \log(\det(\boldsymbol{\Psi})) + trace(\widehat{\boldsymbol{S}}\boldsymbol{\Psi})$$

where $\boldsymbol{W} = \boldsymbol{\Psi}^{-1}$.

Thus, the Newton direction is written as follows:

$$D_t = \min_{\Delta} \bar{g}_x(\Delta) + h(x + \Delta)$$

This Newton direction is used to compute iterative estimates of $\boldsymbol{\Psi}$ in order to solve the optimization problem in A1.

Chapter 4: A Network Approach to Portfolio Selection

1. Introduction

In his seminal paper, Markowitz (1952) laid the foundation of modern portfolio theory. In this static framework, investors optimally allocate their wealth across a set of assets considering only the first and second moment of the returns' distribution. Despite the profound changes derived from this publication, the out-of-sample performance of Markowitz's prescriptions is not as promising as expected. The poor performance of Markowitz's rule stems from the large estimation errors on the vector of expected returns (Merton, 1980) and on the covariance matrices (Jobson and Korkie, 1980) leading to the well-documented error-maximizing property discussed by Michaud and Michaud (2008). The magnitude of this problem is evident when we acknowledge the modest improvements achieved by those models specifically designed to tackle the estimation risk (DeMiguel et al., 2009). Moreover, the evidence indicates that the simple yet effective equally-weighted portfolio rule has not been consistently out-performed by more sophisticated alternatives (Bloomfield et al., 1977; DeMiguel et al., 2009; Jorion, 1991).

Recently, researchers from different fields have characterized financial markets as networks in which securities correspond to the nodes and the links relate to the correlation of returns (Barigozzi and Brownlees, 2014; Billio et al., 2012; Bonanno et al., 2004; Diebold and Yilmaz, 2014; Hautsch et al., 2015; Mantegna, 1999; Onnela et al., 2003; Peralta, 2015; Tse et al., 2010; Vandewalle et al., 2001; Zareei, 2015). In spite of the novel and interesting insights obtained from these network-related papers, most of their results are fundamentally descriptive and lack concrete applications in portfolio selection process. We contribute to this line of research by investigating the extent to which the underlying structure of this financial market network can be used as an effective tool in enhancing the portfolio selection process.

Our theoretical results establish a bridge between Markowitz's framework and the network theory. On the one hand, we show a negative relationship between optimal

portfolio weights and the *centrality* of assets in the financial market network. The intuition is straightforward: those securities that are strongly embedded in a correlation-based network greatly affect the market and their inclusion in a portfolio undermines the benefit of diversification resulting in larger variances. We refer to the centrality of stocks as their systemic dimension. On the other hand, each security is also characterized by an individual dimension such as Sharpe ratio or volatility depending on the specific portfolio formation objective. Next, we theoretically show a positive relationship between the assets' individual performances and their optimal portfolio weights. In a nutshell, optimal weights from the Markowitz framework can be interpreted as an optimal trade-off between the securities' systemic and individual dimensions in which the former is intimately related to the notion of network centrality.

From a descriptive perspective and relying on US data, we present evidence indicating that financial stocks are the most central nodes in the financial market network in accordance with Peralta (2015) and Tse et al., (2010). Additionally, we document a positive association between the centrality of a security and its corresponding beta from CAPM pointing out the large, although not perfect, correlation between this network indicator and the standard measure of systematic risk. In order to identify the salient financial and market features affecting securities' centrality, we estimate several specifications of a quarterly-based panel regression model upon a set of 200 highly capitalized stocks in the S&P500 index from Oct-2002 to Dec-2012. Our results present some empirical evidence indicating that highly central stocks correspond to firms that are older, cheaper, higher capitalized and financially riskier.

Finally, by means of in-sample and out-of-sample analysis, we investigate the extent to which the structure of the financial market network can be used to enhance the portfolio selection process. In order to check the robustness of our results and to avoid data mining bias, four datasets are considered, accounting for different time periods and markets. We propose a network-based investment rule, termed as ρ-dependent strategy, and report its performance against well-known benchmarks. The evidence shows that our network-based strategy provides significant larger out-of-sample Sharpe ratios compared to the ones obtained by implementing the $1/N$ rule or Markowitz-based models. Moreover, these enhanced out-of-sample performances are not explained by large exposures to the standard risk factors given the reported positive and statistically significant Carhart alphas. Finally, it is worth mentioning that our results are robust to different portfolio settings and transaction cost. We argue that our network-based investment policy captures the *logic* behind Markowitz's rule while making more efficient use of fundamental information, resulting in a substantial reduction of wealth misallocation.

The contribution of the paper is twofold. On the one hand, this paper sheds light on the connection between the modern theory of portfolios and the emerging literature on financial networks. On the other hand, our network-based investment strategy attempts to simplify the portfolio selection process by targeting a group of stocks within a certain range of network centrality. As far as we are aware, Pozzi et al. (2013) is the only paper that attempts to take advantage of the topology of the financial market network for investment purposes. They argue in favor of an unconditional allocation of wealth towards the outskirts of the structure. We depart from their results by proposing the ρ-dependent strategy which is contingent on the correlation between the systemic and individual dimensions of the assets comprising the financial market network. Moreover, and in contrast to their synthetic centrality index, we show that our measure of centrality is strongly rooted in the principles of portfolio theory.

The remainder of the paper is organized as follows. Section 2 presents the notion of assets' centrality in the financial network and its connection to Markowitz framework. Section 3 describes the datasets used in the empirical applications. Section 4 provides a detailed statistical description of stocks in accordance to their centrality. Section 5 addresses the interaction between assets' centrality and optimal portfolio weights by relying on an in-sample analysis. Section 6 presents the ρ-dependent strategy and compares its out-of-sample performance to various conventional portfolio strategies. Finally, section 7 concludes and outlines future research lines.

2. A Bridge Between Optimal Portfolio Weights and Network Centrality

The notion of centrality, intimately related to the social network analysis, aims to quantify the influence/importance of certain nodes in a given network. As discussed in Freeman (1978), there are several measurements in the literature each corresponding to the specific definition of centrality. The so-called *eigenvector centrality*, firstly proposed by Bonacich (1972), has become standard in network analysis. This section formally defines this measure and determines its relationship with optimal portfolio weights.

2.1. Defining network centrality

We denote by $G = \{N, \omega\}$, a network composed by a set of nodes $N = \{1,2, \dots, n\}$ and a set of links, ω, connecting pairs of nodes. If there is a link between nodes i and j, we indicate it as $(i, j) \in \omega$. A convenient rearrangement of the network information is provided by the $n \times n$ adjacency matrix $\Omega = [\Omega_{ij}]$ whose element $\Omega_{\mathrm{ij}} \neq 0$ whenever $(i, j) \in \omega$. The network G is said to be undirected if no-causal relationships are attached to the links implying that $\Omega = \Omega^{\mathrm{T}}$ since $(i, j) \in \omega \Leftrightarrow (j, i) \in \omega$. When Ω_{ij} entails a causal association from node j to node i, the network G is said to be directed. In this case, it is likely that $\Omega \neq \Omega^{T}$ since $(i, j) \in \omega$ does not necessarily imply $(j, i) \in \omega$. For unweighted networks, $\Omega_{ij} \in \{0,1\}$ and therefore only on/off relationships exist. On the contrary, when $\Omega_{ij} \in \mathbb{R}$, the links track the intensity of the interactions between nodes giving rise to weighted networks. The reader is referred to Jackson (2010) for a comprehensive treatment of the network literature.

According to Bonacich (1987, 1972), the eigenvector centrality of node i, denoted by v_i, is defined as the proportional sum of its neighbors' centrality.[49] (Newman, 2004) extends this notion to weighted networks for which v_i is proportional to the weighted sum of the centralities of neighbors of node i with Ω_{ij} as the corresponding weighting factors. It is computed as follows.

$$v_i \equiv \lambda^{-1} \sum_j \Omega_{ij} v_j \tag{1}$$

Note that node i becomes highly central (large v_i) by being connected either to many other nodes or to just few highly central ones. By restating equation (1) in matrix terms, we obtain $\lambda v = \Omega v$ indicating that the centrality vector v is given by the eigenvector of Ω corresponding to the largest eigenvalue, λ.[50] More formally:

[49] The terms eigenvector centrality and centrality are used interchangeably throughout this study.

[50] In principle, each eigenvector of Ω is a solution to equation (1). However, the centrality vector corresponding to the largest component in the network is given by the eigenvector corresponding to the largest eigenvalue (Bonacich, 1972).

Definition 1: Consider the undirected and weighted network $\mathrm{G} = \{\mathrm{N}, \Omega\}$, with N as the set of nodes and Ω as the adjacency matrix. The eigenvector centrality of node i in G denoted by v_i is proportional to the i-th component of the eigenvector of Ω corresponding to the largest eigenvalue λ_1.

2.2. Key results from the modern portfolio theory

The mathematical principles of modern portfolio theory were established in (Markowitz, 1952). Given that our theoretical results strongly rely on this framework, this section briefly reviews two of its fundamental results: the minimum-variance and the mean-variance investment rules.

Let us assume n risky securities with expected returns vector, μ, and covariance matrix, $\Sigma = [\sigma_{ii}]$. Consider the problem of finding the vector of optimal portfolio weights, w, that minimizes the portfolio variance subject to $w^T\mathbf{1} = 1$ where $\mathbf{1}$ (in bold) corresponds to a column vector whose components are equal to one. This strategy is commonly known as minimum-variance or *minv* for short. Formally the problem is stated as:

$$\min_{w} \sigma_p^2 = w^T \Sigma w \quad subject\ to \quad w^T\mathbf{1} = 1 \tag{2}$$

The solution of equation (2) is given by:

$$w_{minv}^* = \frac{1}{\mathbf{1}^T\Sigma^{-1}\mathbf{1}}\Sigma^{-1}\mathbf{1} \tag{3}$$

Denoting by Ω the correlation matrix of returns, and by Δ the diagonal matrix whose *ith*-main diagonal element is $\sigma_i = \sqrt{\sigma_{ii}}$, the relationship between Ω and Σ can be written as $\Sigma = \Delta\Omega\Delta$. Then, equation (3) is restated in terms of the correlation matrix as follows.

$$\widehat{w}_{minv}^* = \varphi_{minv}\Omega^{-1}\epsilon \tag{4}$$

where $\widehat{w}_{i,minv}^* = w_{i,minv}^* * \sigma_i$, $\varphi_{minv} = \frac{1}{\mathbf{1}^T\Sigma^{-1}\mathbf{1}}$ and $\epsilon_i = 1/\sigma_i$.

The introduction of a risk-free security whose return is given by r_f allows us to account for the mean-variance investment rules. We denote the excess return of security i by $r_i^e \equiv r_i - r_f$ and the vector of expected excess returns by μ^e. The problem of finding the optimal portfolio weights that minimize the portfolio variance for a given level of the portfolio expected excess return R^e is established as follows:

$$\min_{w} \sigma_p^2 = w^T \Sigma w \ subject\ to\ w^T \mu^e = R^e \tag{5}$$

The strategy implied by equation (5) is commonly known in the financial literature as the mean-variance strategy or *mv* for short. Since an investor's wealth might be partially allocated to the risk-free security and short sales of the risk-free security are allowed, the restriction $w^T \mathbf{1} = 1$ is not included in equation (5).[51] The optimum mean-variance portfolio weights vector is computed as follows:

$$w_{mv}^* = \frac{R^e}{\mu^{e^T} \Sigma^{-1} \mu^e} \Sigma^{-1} \mu^e \tag{6}$$

Following the same reasoning as before, equation (6) is written in terms of the correlation matrix as follows:

$$\widehat{w}_{mv}^* = \varphi_{mv} \Omega^{-1} \hat{\mu}^e \tag{7}$$

where $\widehat{w}_{i,mv}^* = w_{i,mv}^* * \sigma_i$, $\varphi_{mv} = \frac{R^e}{\mu^{e^T} \Sigma^{-1} \mu^e}$ *and* $\hat{\mu}_i^e = \mu_i^e / \sigma_i$.

2.3 The relationship between optimal portfolio weights and asset centralities

By interpreting the correlation matrix of returns as the adjacency matrix of a given network, an overlapping region between portfolio theory and network theory is established. More formally:

Definition 2: Consider N to be a set of securities in a given asset opportunity set and Ω the corresponding returns' correlation matrix. The undirected and weighted financial market network is $FMN = \{N, \Omega\}$, with N as the set of nodes and Ω as the adjacency matrix.

Throughout the study, we set the main diagonal of Ω to zero in order to discard meaningless self-loops in a given financial market network. Since the eigenvectors' structures and the ordering of eigenvalues are the same after performing this operation, our statements in terms of eigenvector centrality remain valid (see appendix A).

[51] Nevertheless, when the tangency portfolio is considered, $w^T \mathbf{1} = 1$ must hold.

Proposition 1 and Corollary 1 establish the negative relationship between security i's optimal portfolio weight and its centrality in the respective financial market networks for both the *minv* and *mv* strategies. The reader is referred to appendix A for a detailed proof.

Proposition 1: Consider a financial market network $FMN = \{N, \Omega\}$ where $\{v_1, \dots v_n\}$ and $\{\lambda_1, \dots \lambda_n\}$ account for the sets of eigenvectors and eigenvalues (in descending order) of Ω, respectively. The optimal portfolio weights in the equations (4) and (7) can be written as:

$$\widehat{w}^*_{minv} = \varphi_{minv}\epsilon + \varphi_{minv}\left(\frac{1}{\lambda_1} - 1\right)\epsilon_M v_1 + \Gamma_{minv}$$

(8)

$$\widehat{w}^*_{mv} = \varphi_{mv}\hat{\mu}^e + \varphi_{mv}\left(\frac{1}{\lambda_1} - 1\right)\hat{\mu}^e_M v_1 + \Gamma_{mv}$$

(9)

where $\epsilon_M = ({v_1}^T\epsilon)$, $\Gamma_{minv} = \varphi_{minv}\left[\sum_{k=2}^{n}\left(\frac{1}{\lambda_k} - 1\right)v_k v_k^T\right]\epsilon$, $\hat{\mu}^e_M = {v_1}^T\hat{\mu}^e$ *and* $\Gamma_{mv} = \varphi_{mv}\left[\sum_{k=2}^{n}\left(\frac{1}{\lambda_k} - 1\right)v_k v_k^T\right]\hat{\mu}^e$.

Note that ϵ_M and $\hat{\mu}^e_M$ in equations (8) and (9) account for weighted averages of the inverted standard deviations of returns and Sharpe ratios, respectively, with weighting factors given by the elements of v_1. From a principal component perspective, we interpret them as the corresponding variables at market level. Moreover, since the empirical evidence indicates that only eigenvector elements corresponding to the largest eigenvalue have informational content (Green and Hollifield, 1992; Laloux et al., 1999; Trzcinka, 1986), we focus on v_1 and λ_1 by defining Γ_{minv} and Γ_{mv} in terms of v_j and λ_j for $j > 1$.

The first term in equations (8) and (9) considers simple investment rules that only take into account the performance of securities as if they were in isolation. Therefore, lower (higher) standard deviations of returns (Sharpe ratios) are consistent with higher optimal portfolio weights. We call this the *individual dimension of securities*. The second term in the same expressions quantifies the extent to which optimal weights deviate from the previously mentioned rule due to the centrality of securities. We call this the *systemic dimension of securities*. Corollary 1 states that, under plausible conditions, there is a negative relationship between optimal portfolio weights and network centralities.

Corollary 1: Assuming that $\lambda_1 > 1$ and $\epsilon_M, \hat{\mu}_M^e \in \mathbb{R}_+$. Then, $\frac{\partial \hat{w}_{r,i}^*}{\partial v_{1,i}} < 0$ for $r = minv, mv$.

Based on Rayleigh's inequality (Van Mieghem, 2011), the assumption $\lambda_1 > 1$ in Corollary 1 requires a positive mean correlation of returns, which is not a strong condition in practical terms.[52] It is worth mentioning that centrality does not necessarily rank assets in the same way as the mean correlation (mean of the row or columns in Ω) does. It is straightforward to show that for a correlation matrix with equal off-diagonal entries, each security is given the same amount of centrality and mean correlation (the leading eigenvector is $\frac{1}{\sqrt{n}}\mathbf{1}$). However, this association breaks as the dispersion in the distribution of Ω_{ij} increases.[53]

Our theoretical results are consistent with Pozzi et al. (2013) in establishing that optimal portfolio strategies should overweight low-central securities and underweight high-central ones. Therefore, optimal investors attempt to benefit from diversification by avoiding the allocation of wealth towards assets that are central in the correlation-based network. However, we depart from Pozzi et al. (2013) in two respects. First, our measure of centrality is derived from the investor's optimization problem, and thus is intimately associated with Markowitz's framework. Secondly, the individual performance of securities is overlooked in their study, leading to an incomplete analysis and potentially impairing the benefits of a network-based portfolio strategy.

3. Dataset Description and Stock Market Network Estimation

In order to avoid data mining bias and to test the robustness of our results, four datasets are considered throughout the empirical sections accounting for different markets and time periods. Unless otherwise stated, split-and-divided-adjusted returns and traded volumes are obtained from CRSP while quarterly financial data comes from COMPUSTAT. The dataset *d-S&P* contains daily returns for 200 highly capitalized

[52] Rayleigh's inequality states a classical lower bound for the largest eigenvalues as follows $\lambda_1 \geq \frac{u'\Omega u}{u'u}$ for $u \in \mathbb{R}^n$. If $u = \mathbf{1}$ then $\lambda_1 \geq 1 + \frac{\sum_i \sum_j \Omega_{ij}}{n} = 1 + (n-1)\bar{\Omega}_{ij}$ where $\bar{\Omega}_{ij}$ is the mean correlation of off-diagonal elements of Ω.

[53] In a non-reported simulated exercise, a positive correlation between centrality and mean correlation is observed for a mild dispersion in the distribution of Ω_{ij}.

constituents of the S&P-500 index at the end of year 2012 showing non-negative total equity in the period Oct-2002 to Dec-2012. The dataset *m-NYSE* considers monthly returns for the 200 firms that remain listed in NYSE normalized for capitalization and cheap stocks in the period Jan-64 to Dec-06. [54] The dataset *d-FTSE* accounts for daily stock returns of the 200 most capitalized constituents of FTSE-250 index during the period Feb-06 to Oct-13. For this particular sample, we rely on Datastream as the data provider. A large dataset named *d-NYSE* is mainly used for simulation purposes and it considers daily returns for 947 firms listed in NYSE (adjusted for capitalization and cheap stocks explain as in *m-NYSE*) in the period Jan-2004 to Jul-2007. Finally, the risk-free rates required to compute excess returns for the US and UK markets are gathered from Kenneth French's website and from Gregory et al. (2013), respectively.

The large estimation error of the sample correlation matrix is well-documented (Jobson and Korkie, 1980). Therefore, we implement the shrinkage estimator the correlation matrix $\hat{\Omega}$ upon excess returns as in Ledoit and Wolf (2004) to estimate the adjacency matrix of the corresponding financial market network.

4. Fundamental Drivers of Stock Centrality: Descriptive Analysis

Given the fundamental role assigned to the notion of centrality in this study, this section provides a set of descriptive results found in the *d-S&P* dataset. Table 1 reports the sample size, market capitalization and traded volume in 2012 (measured in millions of dollars) and the total and mean centrality by economic sectors (classified by firms' SIC codes). Although the manufacturing sector is the largest in terms of capitalization (48%) and traded volume (43%), financial firms are the most central nodes in the stock market network in accordance with reported evidence (Barigozzi and Brownlees, 2014; Peralta, 2015; Tse et al., 2010).

We also notice that the dispersion of the risk-adjusted returns' distribution is not constant across the network. The left panel of figure 1 plots the Sharpe ratio's boxplots conditioning on the low, middle and high terciles of the centrality distribution. Despite no significant difference in means, the Sharpe ratio's distribution shrinks as larger centralities are considered. Moreover, table B.1, included in Appendix B to preserve

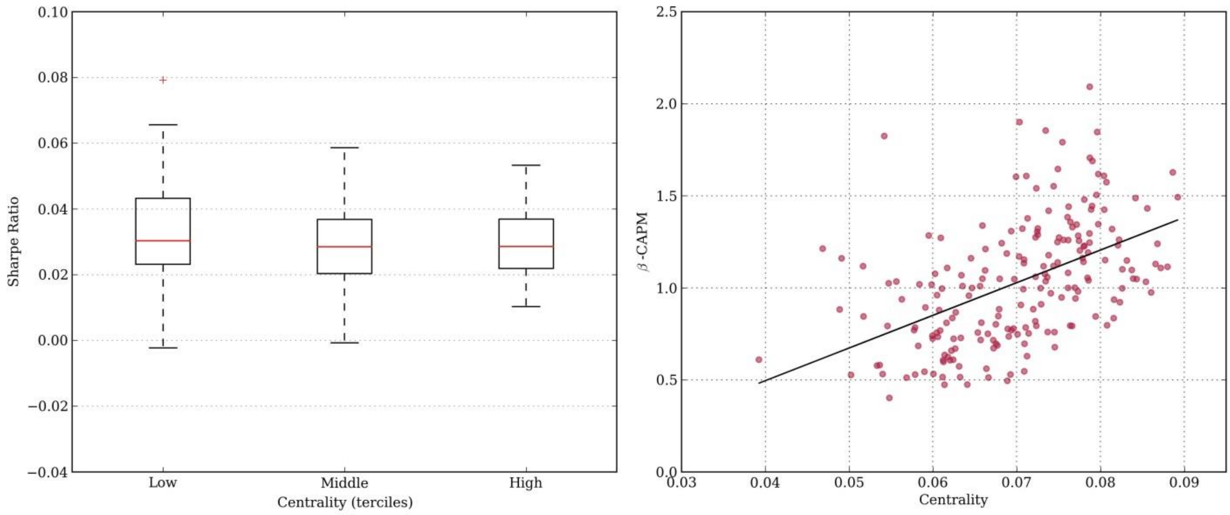
[54] The stocks are chosen to have capitalization more than 20th percentile of market capitalization and prices higher than $5 (Penny stocks with prices lower than $5 are discarded).

space, reports significantly greater mean volatility as we move from the bottom tercile (2.05%) to the middle (2.09%) and top tercile (2.24%) of the centrality distribution demonstrating larger risks among high-central securities. The relationship between the β-CAPM and the centrality of each security in the sample is plotted in right panel of figure 1.[55] This graph shows an upward-sloping relationship that, although not perfect, indicates that central assets tend to correspond to high systematic risk securities.

Table 1. Market capitalization, traded volume, and centrality by economic sectors

This table reports the total and mean firms' centrality by economic sectors in the d-S&P500 dataset. Market capitalization and traded volume are in millions of dollars and correspond to the end of year 2012. The column Firms gives the number of companies in each economic sector. The columns denoted by % present the percentages with respect to the total of each of the preceding variables.

Economic Sector	Firms	%	Market Cap.	%	Traded Vol.	%	Centrality	
							Total	Mean
Finance, Insurance, And R. Estate	37	19%	2,254,158	20%	70,788	19%	2.74	0.0741
Mining	17	9%	656,688	6%	26,178	7%	1.23	0.0722
Transp., Comm., Elect, Gas, Sanit. S.	27	14%	1,145,092	10%	36,988	10%	1.88	0.0696
Manufacturing	87	44%	5,338,584	48%	156,881	43%	6.01	0.0691
Retail Trade	12	6%	745,486	7%	27,130	7%	0.82	0.0682
Services	13	7%	829,671	7%	37,314	10%	0.89	0.0682
Wholesale Trade	5	3%	142,423	1%	6,011	2%	0.33	0.0656
Construction	2	1%	44,905	0%	3,073	1%	0.13	0.0646
Total	200		11,157,008		364,364			



[55] The index S&P 500 is used as the market index for the estimation of the corresponding β from CAPM.

Figure 1. Sharpe ratio distributions for the high, middle and low terciles of securities' centrality (left panel). Relationship between the securities' centrality and the β from CAPM (right panel). Both panels consider the d-S&P dataset.

In order to identify the key financial and market drivers of stocks' centralities, we estimate an unbalanced quarterly-based panel regression largely inspired by Campbell et al. (2008) for the selection of relevant regressors. The study of Green and Hollifield (1992) rejects at a very high level of confidence the hypothesis that the first eigenvector of the covariance matrix is a constant vector. Our empirical exercise attempts to provide an economic content to their result. As before, the dataset used is d-S&P dataset comprising 7,931 firms-quarter data points.

The dependent variable is the centrality of a security (firm) i in quarter t.[56] The financial explanatory variables are ROA_{it}, Lev_{it} and Liq_{it} accounting for the ratio of *net income*, *total liability* and *cash and short term assets* to *total assets*, respectively. As market explanatory variables, we include $ln(MV_{it})$ and $ln(P_{it})$, denoting the logarithms of the market capitalization on a common-shares basis and the stocks' market price at the end of quarter t, respectively. In addition, $ln(TV_{it})$, Ret_{it} and Std_{it}, referring to the logarithm of total trades, the excess return and the daily standard deviation of returns during quarter t, are additional market explanatory variables. The variable M/B_{it} denotes the Market-to-Book ratio on a common-shares basis at the end of quarter t and is incorporated in the regressions as well. Finally, the logarithm of firms' age indicated as $ln(Age_{it})$ comprises the last independent variable of the model. It is computed by counting the number of quarters elapsed since the appearance of the first market price in CRSP until period t as in Fama and French (2004). To control for the effects of outliers, 1% winsorizing is implemented on regressors except for the case of $ln(Age_{it})$. Table 2 reports summary statistics.

Table 2. Descriptive statistics for the quarterly panel regression variables included in the d-S&P dataset

The description of the variables is as follows: *ROA* is net income/total assets at the end of period *t*, *Lev* is total liability/total assets at the end of period *t*, *Liq* is cash and short term assets over the total assets at the end of period *t*, *ln(MV)* is the logarithm of market capitalization on a common-share basis at the end of period *t*, *ln(TV)* is the logarithm of total trades during period *t*, *Ret* is the excess return during period *t*, *ln(P)* is the logarithm of stocks' prices at the end of period *t*, *Std* is the return variance during period *t*, *M/B* is the market-to-book ratio on a common-share basis at the end of period *t* and *ln(Age)* is the logarithm of the firms' age

[56] Only the data corresponding to quarter t is considered for computing the centralities. Additionally, we rescaled centrality by multiplying it by 100 for exposition purposes.

computed as in Fama and French (2004) and corresponding to the end of period *t*. The descriptive statistics are reported after 1% winsorising except for *ln(Age)*.

Variables	Mean	Std	Percentiles					Skew	Kurtosis
			Min	5%	50%	95%	Max		
Independent									
Financial									
ROA	1.70%	1.70%	-5.00%	-0.10%	1.50%	4.70%	7.50%	0.10	2.70
Lev	59.00%	19.50%	12.20%	23.20%	59.60%	90.80%	94.50%	-0.20	-0.40
Liq	12.10%	13.40%	0.20%	0.70%	6.90%	41.20%	67.00%	1.90	3.70
Market									
ln(MV)	16.98	1.02	14.20	15.34	16.88	18.91	19.42	0.10	0.20
ln(TV)	12.47	1.04	9.95	10.83	12.41	14.28	15.21	0.20	0.10
Ret	2.60%	13.60%	-35.10%	-20.50%	2.80%	25.00%	45.10%	0.10	1.00
ln(P)	3.69	0.58	2.23	2.72	3.70	4.58	5.54	0.10	0.60
Std	1.80%	1.10%	0.70%	0.80%	1.50%	3.90%	6.80%	2.40	6.90
M/B	3.32	2.50	0.59	0.95	2.57	8.50	14.71	2.11	5.42
ln(Age)	4.84	0.77	1.79	3.43	4.96	5.81	5.86	-0.66	-0.12
Dependent									
Centrality	7.01	0.93	4.01	5.27	7.11	8.41	8.95	-0.69	0.86

Table 3 presents OLS estimations for various specifications of the panel regression described above. Model I includes dummy variables by quarter and economic sector (first 2 digits of SIC codes) and considers only robust-heteroskedastic standard errors (White, 1980). To tackle the bias induced by autocorrelation and heteroskedasticity in the error term, two-way clustering correction by Cameron et al. (2011) is included in model II. As suggested by Petersen (2009), we also report estimations of model III that considers economic sector dummy variables while clustering the error term by quarters.

Table 3. OLS estimations of three specifications of the quarterly panel regression model
Each specification depends on the particular standard error correction method. *t-statistics* are in parentheses and the statistical significance is as follows: * at 5% level, ** at 1% and *** at 0.1% level. Model I combines economic sector and quarterly dummies with robust-heteroscedastic standard errors (White, 1980). Model II considers two-way clustered standard errors (Cameron et al., 2011). Model III includes economic sector dummies while clustering standard errors by quarters. The regressions are implemented upon the d-S&P dataset.

	Models		
	I	II	III
ROA	-1.128	-0.746	-1.062
	(-1.44)	(-0.41)	(-1.03)
Lev	0.111	0.484	0.111
	(1.31)	(2.30)*	(1.29)
Liq	-0.481	0.200	-0.582
	(-4.36)***	(0.49)	(-4.13)***

M/B	-0.00523	-0.0412	-0.00865
	(-0.90)	(-3.25)**	(-1.15)
ln(MV)	0.239	0.201	0.216
	(8.96)***	(2.23)*	(8.32)***
ln(TV)	-0.230	-0.150	-0.186
	(-7.88)***	(-1.62)	(-6.10)***
Ret	0.401	0.215	0.196
	(4.23)***	(1.15)	(1.16)
ln(P)	-0.239	-0.0106	-0.168
	(-6.40)***	(-0.11)	(-4.02)***
Std	-5.029	5.074	2.399
	(-2.50)*	(1.22)	(0.97)
ln(Age)	0.104	0.0574	0.124
	(5.53)***	(1.23)	(6.78)***
N	7931	7931	7931
R^2	0.174	0.055	0.165
Dummies	*Econ. Sectors and Quarter*	-	*Econ. Sectors*
Std. errors' correction	*Robust Heteroscedastic*	*Clustering by Econ. Sectors and Quarters*	*Clustering by Quarters*

Starting with the analysis of financial explanatory variables, table 3 indicates that ROA's coefficient is negative in each specification, however, it is not significant at the conventional levels. The variable Lev stays positive across models and shows a statistically significant coefficient in Model II comparable to M/B but with the opposite sign. Finally, the coefficient for variable Liq is negative and strongly significant in models I and III. In summary, there is some evidence indicating that highly central stocks are associated with leveraged firms showing less liquid asset positions and low Market-to-Book ratios, suggesting that firms' centrality conveys information on the financial risk profile of the sampled firms.

Among market explanatory variables, $ln(MV)$ shows positive and strongly significant coefficients across specifications evidencing a size effect. The variables $ln(TV)$ and $ln(P)$ present a negative impact on centrality for each specification and remain significant for models I and II. Therefore, low-traded and cheaper securities tend to be highly central in the financial market network. The variable Ret is positive for each model but statistically significant only for the first model. In the case of Std, since it is marginally significant only for model I and changes its sign across specifications, we disregard its effects. Finally, $ln(Age)$ shows positive coefficients that are strongly significant for models I and III.[57]

In a nutshell, our empirical results identify central stocks with greater capitalization, lower price and older firms with riskier financial profiles in terms of leverage, liquid asset positions and Market-to-Book ratios. We argue that the findings in Green and Hollifield (1992), indicating the importance of the first principal component of the covariance matrix, can be explained in terms of the financial and market informational content embedded in assets' centralities.

In an additional analysis reported in Appendix F to save space, we elaborate on the relationships between stocks' centrality and their stability. Specifically, we associate the concept of a stock's stability to the tendency to remain listed in the market without any change in its relative centrality status across time. We find that among the stocks that remain listed in the market, there is a strong tendency to show the same level of centrality through time. Additionally, we investigate the consequences of the period size used in the estimation of the correlation-based network on the ordering of securities provided by centrality. Our results show the large correlations between the rankings of centralities for different lengths of sample periods indicating the robustness of this ordering.

5. Stock Centrality and Optimal Portfolio Weights: In-sample Evaluation

The interaction between the individual and the systemic dimensions of stocks is empirically investigated in this section. The dataset used is *d-S&P* and the results reported below come from both a cross-sectional and a time series in-sample analysis.

5.1. Cross-sectional approach

The detailed pattern of co-movements across stocks is properly captured by $\hat{\Omega}$. This matrix conveys an excessive amount of information and leads to a fully connected stock market network that is difficult to analyze.[58] The Minimum Spanning Trees (MST), first

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[57] We are grateful to the anonymous referee for suggesting the inclusion of $ln(Age)$ in the analysis.

[58] A fully connected network refers to a network structure in which each node is connected with the rest.

introduced in the financial markets by Mantegna (1999), allows us to filter out this adjacency matrix with the aim of uncovering the market skeleton.[59] This technique has been widely applied to several country-specific stock markets including the US (Bonanno et al., 2004; Onnela et al., 2003), Korean (Jung et al., 2006), Greek (Garas and Argyrakis, 2007) and Chinese (Huang et al., 2009) markets among others.

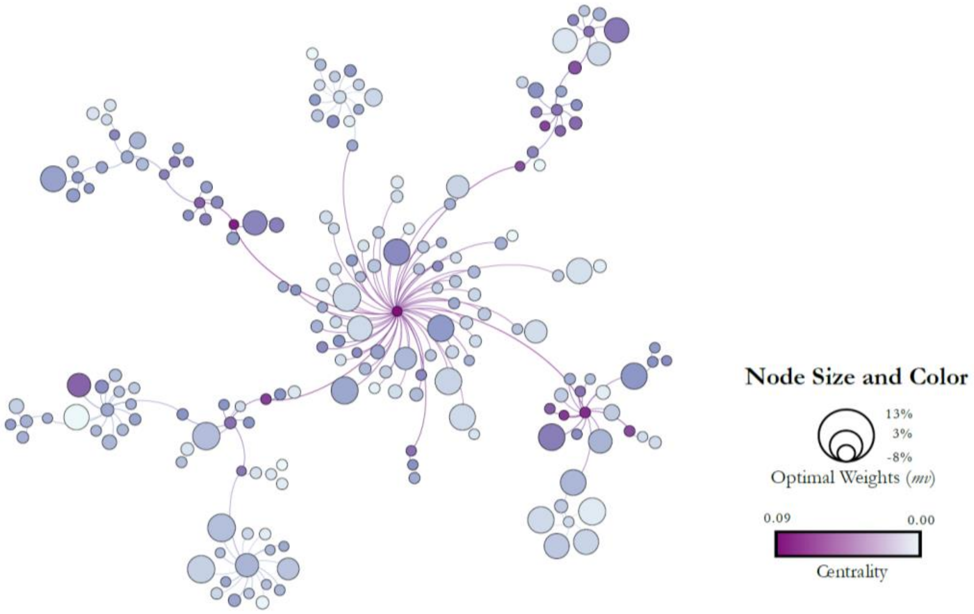

Figure 2. MST stock market network for the d-S&P dataset. The size of nodes corresponds to the optimal weights for the minimum-variance strategy (see equation 3) and the intensity of the colour accounts for the corresponding security's centrality.

Figure 2 plots the MST financial market network for our data where nodes are scaled to the optimal portfolio weights for the *minv* strategy (see equation (3)) while the colors account for the respective security's centrality (darker colors imply greater centrality).[60] Note that investor's wealth is allocated toward lighter nodes (low-central securities) in accordance with Corollary 1. See Appendix C for the full list of stocks with their respective centrality.

The relation between the individual and systemic dimensions of assets is illustrated in figure 3. The horizontal and vertical axes of this plot account for the securities' centralities

[59] MST connects the n stocks in a tree-like network by considering the highest $n-1$ paired correlations of returns as links to the extent that no loops are created.

[60] Short sales are allowed in the computation of optimal weights for both *minv* and *mv* strategies.

and the corresponding standard deviations of returns, respectively. Each security is represented by a bubble whose size and color are given by the optimal portfolio weight in the *minv* rule. Note that most of the investor's wealth is assigned toward stocks located in the bottom-left corner of the graph, thus overweighting low-central-&-low-volatile securities.

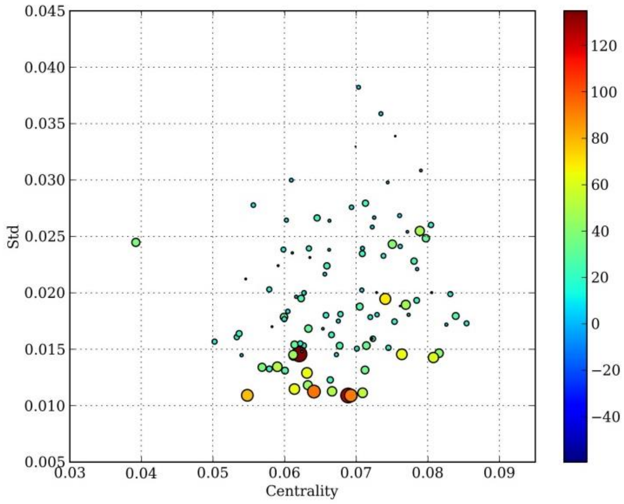

Figure 3. Relationship between standard deviation of returns (Std) and stocks' centrality. Stocks correspond to bubbles whose sizes and colours (colour bar) reflect their optimal portfolio weights in a minimum-variance (*minv*) weights specification. We use d-S&P dataset for this analysis.

Figure 4 presents a scatter plot of Sharpe ratios and centralities for a portfolio applying *mv* strategy as the investment rule. The left and right panel sets the expected portfolio return, R^e, equal to 10% and 40% of the maximum possible portfolio return, respectively (see equation (6)). As expected, the optimal portfolio for low R^e mainly comprises assets with middle-ranged Sharpe ratios while the investment set moves toward securities with higher Sharpe ratios for larger R^e. Note, however, that the *mv* strategy avoids the allocation of wealth towards high-central stocks, say stocks with $v_i > 0.08$.

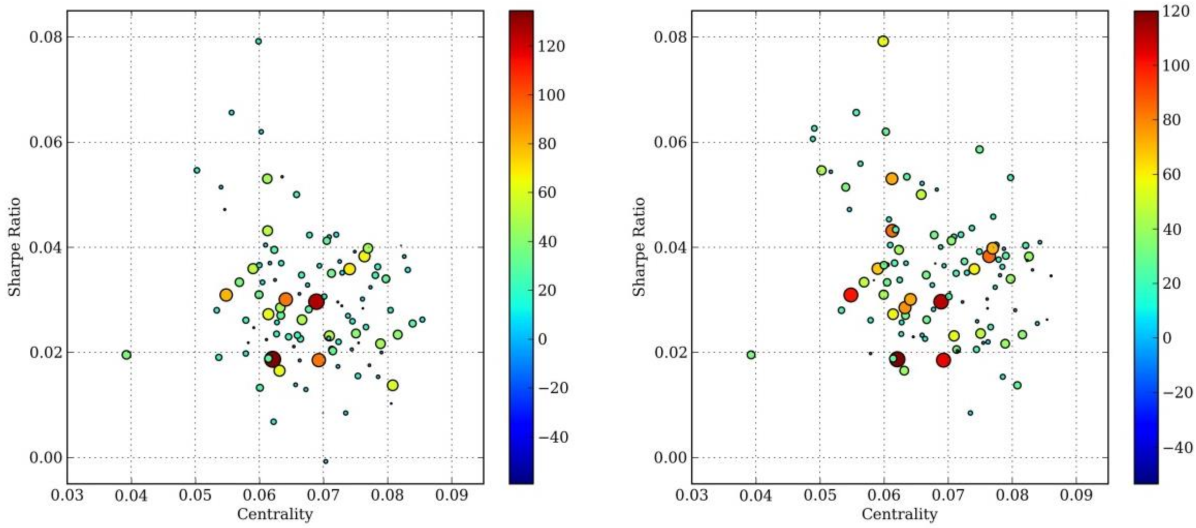

Figure 4. Relationship between Sharpe ratio and stock centrality. Stocks correspond to bubbles whose sizes and colours (colourbar) reflect their optimal portfolio weights in a mean-variance (*mv*) weights specification. The required expected portfolio return R^ein equation 6 is 10% (left panel) and 40% (right panel).of the maximum return in the dataset. We used the d-S&P dataset for this analysis.

To further disentangle the roles of stock dimensions in the determination of optimal portfolio weights, we report in table 4 the results from the OLS estimations of regressions (10) and (11). Equation (10) considers the case of the *minv* strategy where stock i's portfolio weight in a minimum-variance specification, $w^*_{i,minv}$, depends linearly on its centrality, v_i, and on its standard deviation of returns, σ_i. Similarly, equation (11) specifies a linear relationship between stock i's portfolio weight in a mean-variance specification, $w^*_{i,mv}$, with the corresponding stock centrality, v_i, and Sharpe Ratio, SR_i.[61] Therefore, the coefficients β_1 and β_2 stands for the effects of the systemic and individual dimensions of stocks upon optimal portfolio weights.

$$w^*_{i,minv} = \beta_0 + \beta_1 v_i + \beta_2 \sigma_i + \varepsilon_i \qquad (10)$$

$$w^*_{i,mv} = \beta_0 + \beta_1 v_i + \beta_2 SR_i + \varepsilon_i \qquad (11)$$

Table 4. Optimal portfolio weights as a function of the individual and systemic stock's dimensions considering a cross-sectional approach

v_i is the centrality of stock i. σ_i is the standard deviation of stock i. SR_i is the Sharpe ratio of stock i. N is the number of observations (stocks) in the cross-sectional regressions. The regression R^2 is adjusted for degrees of freedom. Each row of the table reports OLS estimations of equations (10) and (11), respectively, where *t*-

[61] In equation 11, optimal weights are obtained assuming R^e equals 40% of the maximum possible return.

statistics are in parentheses and the statistical significance is as follows: * at 5% level, ** at 1% and *** at 0.1% level.

	$\boldsymbol{v_i}$	$\boldsymbol{\sigma_i}$	$\boldsymbol{SR_i}$	$\boldsymbol{N}$	$\boldsymbol{R^2}$
$\boldsymbol{w^*_{i,minv}}$	-0.740	-1.755		200	0.231
	(-4.05)***	(-6.38)***			
$\boldsymbol{w^*_{i,mv}}$	-0.778		0.761	200	0.179
	(-3.64)***		(4.77)***		

The coefficients reported in table 4 are strongly statistically significant and provide support to Proposition 1 and Corollary 1. The coefficient β_1 is negative for both regressions indicating that highly central stocks tend to be underweighted regardless of the specific investment objective. The coefficient β_2 is negative for the *minv* rule and positive for the *mv* case indicating the tendency to optimally allocate wealth towards low-standard deviation and high-Sharpe ratio securities, respectively.

5.2. Time series approach

Contrary to the static description provided by the cross-sectional analysis, this subsection takes a dynamic perspective on the portfolio selection by implementing a time series approach. The entire dataset is divided into 2,522 60-day-long rolling windows considering 1-day displacement steps. The vectors of stock centrality, v_t and portfolio weights for the *minv* and *mv* rules, $w^*_{minv,t}$ and $w^*_{mv,t}$, are computed for each rolling window and indexed by the time subscript t.

We introduce the mean stock centrality $\bar{v}_t$ and the weighted stock centrality $\bar{\bar{v}}_{r,t}$ as follows:

$$\bar{v}_t = \frac{1}{n}\sum_{i=1}^{n} v_{it} \quad (12)$$

$$\bar{\bar{v}}_{r,t} = \sum_{i=1}^{n} w^*_{r,it}\, v_{it} \;\; for\ r = minv,\ mv \quad (13)$$

By construction, the expression $\bar{\bar{v}}_{r,t} = \bar{v}_t$ for $r = minv,\ mv$ is satisfied when $w^*_{r,it} = 1/n$. In accordance with Corollary 1, it would be expected that $\bar{\bar{v}}_{r,t} < \bar{v}_t$ most of the time indicating a bias toward low-central stocks.

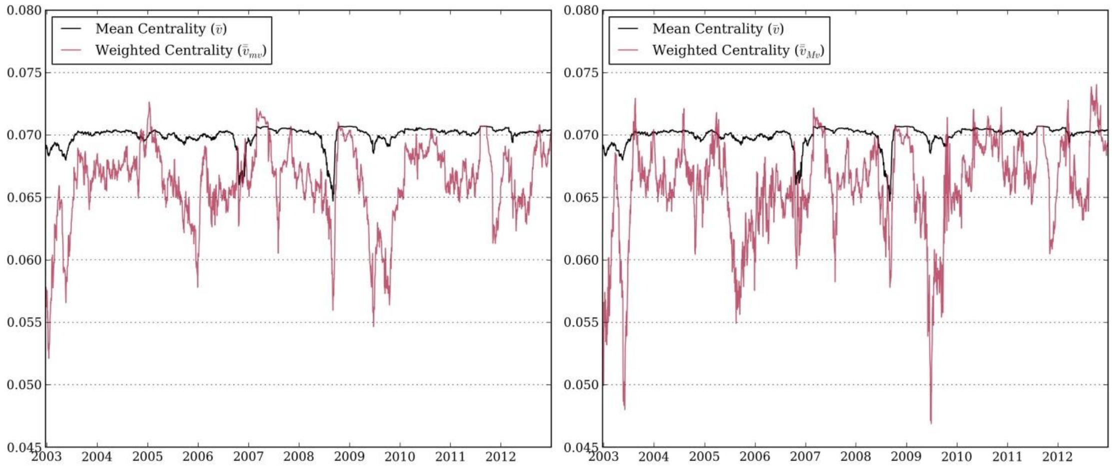

Figure 5. Mean centrality $\bar{v}_t$ and weighted centrality $\bar{\bar{v}}_{r,t}$ through time. We consider the *minv* strategy (left panel) and the *mv* strategy (right panel) for the d-S&P dataset. We divide the dataset into 2,522 of 60-days long rolling windows with 1-day displacement steps. The weight allocations of *minv* and *mv* strategies and the centrality rankings of stocks are computed for each rolling window. The mean centrality $\bar{v}_t$ is the average of centralities among stocks in each 60-days window and the weighted centrality, $\bar{\bar{v}}_{r,t}$, is computed by weighting each stock's centrality by their corresponding weights in *mv* and *minv* strategies.

Figure 5 plots the time series of $\bar{v}_t$ and $\bar{\bar{v}}_{r,t}$ presenting the pattern just described. Note, however, that $\bar{\bar{v}}_{r,t}$ shows values closer to, or even surpassing, $\bar{v}_t$ for some periods. The dynamic of the correlation between the individual and the systemic dimensions of stocks explains this time-dependent behavior of $\bar{\bar{v}}_{r,t}$. Let us denote by π_t the cross sectional correlation between v_{it} and σ_{it} and by ρ_t the cross-sectional correlation between v_{it} and SR_{it}, both at period t. When $\pi_t > 0$, the lowest standard deviation stocks tend to coincide with the weakly systemic ones, and as a consequence, overweighting these securities is certainly the optimal choice under the $minv$ rule. For $\pi_t < 0$, a trade-off arises since low-central stocks also correspond to high-volatility securities. In this case, an optimal portfolios rule should balance these two confronting forces by adapting the investment set to include more central stocks. Considering the mv strategy, for $\rho_t < 0$, the assets with highest Sharpe ratios show the lowest centrality, and therefore, this leads to investing in non-systemic securities as the optimal portfolio choice. When $\rho_t > 0$, a trade-off between assets' dimensions takes place and an optimal wealth allocation should increase portfolio weights towards central securities. Figure 6 plots the time series of ρ_t and π_t and the corresponding 120-days moving averages evidencing the sign-switching nature of these two variables.

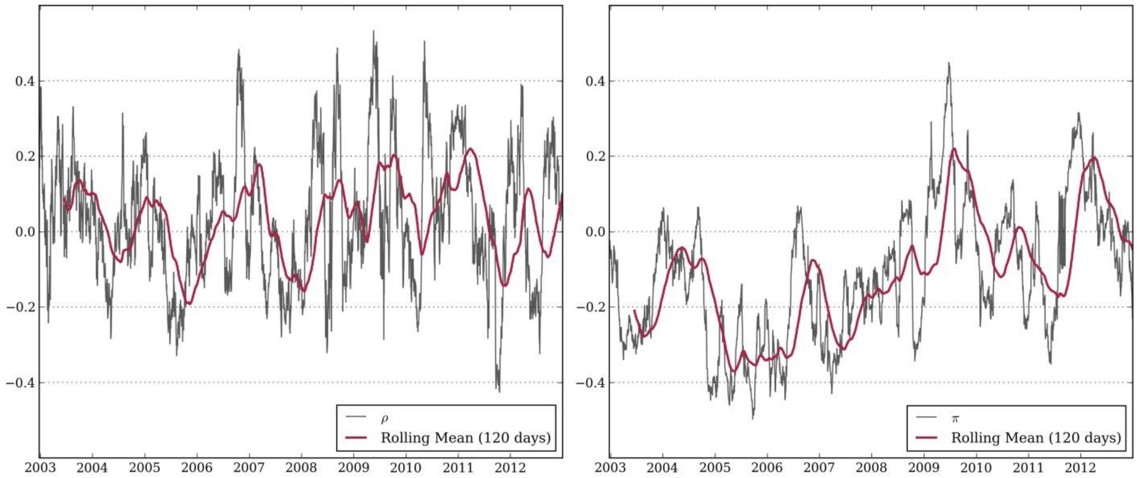

Figure 6. Time series of the cross-sectional correlations between the individual and the systemic dimensions of stocks.

This figure presents the correlation between stocks' centralities and Sharpe ratios, $\rho_t = corr(v_{it}, SR_{it})$ (left panel) and stocks' centralities and standard deviation $\pi_t = corr(v_{it}, \sigma_{it})$ (right panel) for the d-S&P dataset. We also include the corresponding 120-day moving averages of ρ_t and π_t in each panel.

Further insights on the security selection process are gained by estimating the regressions (14) and (15) accounting for the time series versions of expressions (10) and (11). In (14), the optimally weighted centrality $\bar{\bar{v}}_{minv,t}$ for the *minv* strategy in period t is explained by the mean centrality, $\bar{v}_t$, the coefficient of variation of the centrality distribution, $\left(\frac{\sigma_{v,t}}{\bar{v}_t}\right)$, and π_t. Similarly, equation (15) considers the *mv* strategy and therefore, the dependent variable is $\bar{\bar{v}}_{mv,t}$ while ρ_t replaces π_t as explanatory variable.

$$\bar{\bar{v}}_{minv,t} = \beta_0 + \beta_1 \bar{v}_t + \beta_2 \left(\frac{\sigma_{v,t}}{\bar{v}_t}\right) + \beta_3 \pi_t + \varepsilon_t \tag{14}$$

$$\bar{\bar{v}}_{mv,t} = \beta_0 + \beta_1 \bar{v}_t + \beta_2 \left(\frac{\sigma_{v,t}}{\bar{v}_t}\right) + \beta_3 \rho_t + \varepsilon_t \tag{15}$$

Table 5 reports OLS estimations of (14) and (15) noting that all of the coefficients are strongly statistically significant. The coefficient β_1 is negative for both regressions indicating that higher $\bar{v}_t$ results in an overweighting of low-systemic assets as a mean to avoid the undesirable consequences of high-central securities in the portfolio. The negative signs of β_2 are interpreted as the benefits derived from a wider centrality distribution that allows an increased presence of assets with lower centrality scores in the portfolio after controlling for $\bar{v}_t$. The coefficient β_3 is negative for the *minv* rule (see equation (14)). In this case therefore, larger values of π, indicating no trade-off between assets' dimensions, are consistent with optimal wealth allocations away from central securities. In contrast, the coefficient β_3 shows a positive sign for the mv rule (see equation (15)) explained by the trade-off between assets' dimensions arising for large ρ. As a consequence the increments of ρ rise $\bar{\bar{v}}_{mv}$ by moving the investment set toward central nodes.

Table 5. Optimal portfolio weights as a function of the individual and systemic stock dimensions taking a time series approach

We consider a 60-day rolling window estimation procedure for each variable. Each t denotes a 60-day rolling window. $\bar{v}_t$ is the mean centralities at each t. $\left(\frac{\sigma_{v,t}}{\bar{v}_t}\right)$ is the coefficient of variation of the centrality distribution at t. π_t is the correlation between centralities, v_{it} and standard deviations, σ_{it} at t. ρ_t is the cross-sectional correlation between centralities, v_{it}, and Sharpe ratios, SR_{it}, at t. N is the number of observations (stocks) in the cross-sectional regressions. The regression R^2 is adjusted for degrees of freedom. Each row of the table reports OLS estimation of equations (14) and (15). *t-statistics* are in parentheses and the statistical significance is denoted as follows: * at 5% level, ** at 1% and *** at 0.1% level.

	$\bar{v}_t$	$\frac{\sigma_{v,t}}{\bar{v}_t}$	π_t	ρ_t	N	R^2
$\bar{\bar{v}}_{minv,t}$	-3.324	-0.0707	-0.00613		2522	0.573
	(-19.04)***	(-33.80)***	(-26.34)***			
$\bar{\bar{v}}_{mv,t}$	-2.661	-0.0720		0.00627	2522	0.457
	(-10.83)***	(-24.50)***		(17.79)***		

6. Stock Centrality and Optimal Portfolio Weights: Out-of-sample Evaluation

In DeMiguel et al. (2009), naïve strategy, commonly termed as $1/N$[62], is shown not to be consistently outperformed by Markowitz-based rules or their extensions designed to deal with the estimation error problem. This better out-of-sample performance of $1/N$ strategy relative to Markowitz's rule is also investigated and supported by Jobson and Korkie (1980), Michaud (2008) and Duchin and Levy (2009). Moreover, DeMiguel et al. (2009, p. 1936) report that among those models designed to tackle the estimation error problem, the constrained Markowitz rules might be considered as second-best alternatives to naïve diversification. Accordingly, naïve strategy and constrained Markowitz rules portray two reasonable benchmarks for out-of-sample portfolio evaluations.

[62] Naïve strategy assigns a fraction $1/N$ of wealth to each asset out of the N available assets.

Based on the insights obtained mainly from sections 2 and 5, we here propose a network-based investment strategy, termed as *ρ-dependent strategy,* that targets groups of assets in accordance with their centrality rankings. We proceed to define this strategy in detail and evaluate its out-of-sample performance against the benchmarks.

6.1. The out-of-sample evaluation

The out-of-sample returns are computed as follows. For a T-period-long dataset, we implement several portfolio strategies (specified below) upon a set of M-period-long rolling windows indexed by the subscript t. We buy-and-hold the portfolios for H periods and the resulting out-of-sample return is recorded. The rolling window in period t is created by simultaneously adding the next H data points and discarding the H earliest ones from the previous rolling window in order to preserve its length. This process is repeated several times until the end of the dataset is reached thus accounting for $\lfloor(\mathrm{T}-\mathrm{M})/\mathrm{H}\rfloor$ rebalancing periods, vectors of portfolio's weights and out-of-sample returns for each portfolio strategy.

We introduce our network based investment policy, the so-called *ρ-dependent strategy*, as follows. The process estimates the correlation matrix $\hat{\Omega}_{\mathrm{t}}$, the vector of securities' centrality, v_t and the correlation between the systemic and individual dimensions of stocks, ρ_t, upon the rolling window corresponding to period t. Assuming a threshold parameter $\tilde{\rho}$, for a sufficiently large ρ, say $\rho > \tilde{\rho}$, we naively invest in the 20 stocks with the highest centrality. Conversely, for a sufficiently low ρ, say $\rho < \tilde{\rho}$, we naively invest in the 20 stocks with the lowest centrality. This strategy is designed to benefit from investing in low systemic stocks to the extent that the most central ones do not show significant individual performances, thus replicating Markowitz's logic to some extent.

In accordance with the previous literature (DeMiguel et al., 2009), the $1/N$ rule applied upon the entire investment opportunity set is a convenient benchmark with which to evaluate the performance of our network-based strategy. To ensure that the performance of ρ-dependent strategy does not happen by chance, we also includes the *reverse ρ–dependent strategy* that naively invests in the 20 lowest central stocks when $\rho > \tilde{\rho}$ and in the 20 highest central ones when $\rho < \tilde{\rho}$. Moreover, two Markowitz-related rules, the mean-variance and minimum-variance strategies with short-selling constraints are incorporated in the analysis as well. Finally, we consider three more investment rules as "control strategies" that naively invest in the 20 stocks with the Highest Sharpe Ratio, the Highest Centrality and the Lowest Centrality[63].

The investment policies are compared using three out-of-sample performance measures: i) Sharpe ratio, ii) variance of return and iii) turnover. This latter measure averages the amount and size of the rebalancing operations as follows

$$turnover \quad = \frac{1}{T-M-1} \sum_{t=M}^{T-1} \sum_{j=1}^{N} |w_{j,t+1} - w_{j,t^+}| \tag{16}$$

where $w_{j,t+1}$ is the weight of security j at the beginning of period $t+1$ and w_{j,t^+} is the weight of that security just before the rebalancing occurring between the end of period t and the beginning of period $t+1$.[64]

We statistically test the difference in out-of-sample portfolios' Sharpe ratios and variances between each of the investment rules against the $1/N$ strategy following Ledoit and Wolf (2008) and Ledoit and Wolf (2011). More specifically, we implement a studentized circular block bootstrap with block size equal to 5 and bootstrap samples equal to 5.000 to compute the respective p-values.[65]

6.2. Determining the threshold parameter $\tilde{\rho}$

Before the application of our network-based strategies, the value of $\tilde{\rho}$ needs to be specified. In order to do that, we rely on an extensive simulation procedure using artificially created subsamples where ρ spans a broad range of values. Specifically, we generate 120 datasets by randomly selecting 150 stocks from the d-NYSE dataset (see section 3) with ρ ranging from -0.20 to 0.45. Then, we compute the out-of-sample Sharpe ratio where M and H are set to be equal to 500 days and 20 days, respectively.

Panels a, b and c from figure 7 plot the out-of-sample Sharpe ratios from three particular datasets with ρ equal to 0.45, -0.20 and 0.0, arising from a progressive percentage

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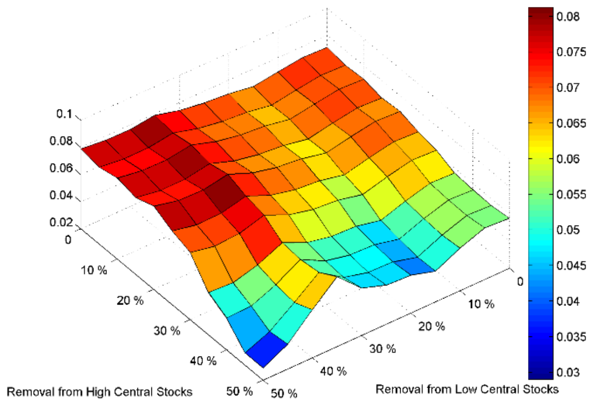
[63] The decision to construct portfolios made up of 20 assets is based on Desmoulins-Lebeault and Kharoubi-Rakotomalala (2012) which highlights the fact that most diversification benefits are gained by investing in such a number of stocks.

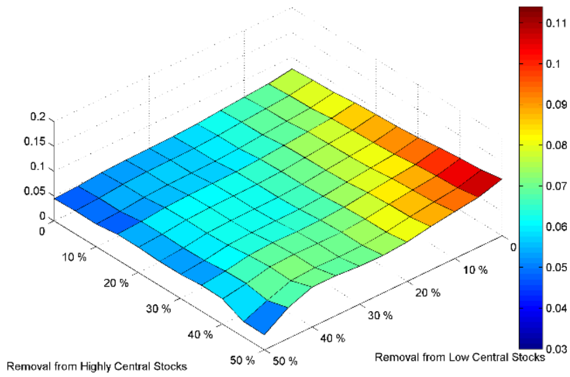
[64] Note that in the expression (16), rebalancing is assumed in each investment period as if $H = 1$. However, with $H > 1$, we only consider the rebalancing periods in our *turnover* calculation.

[65] For the particular case of the variance, a stationary bootstrap is employed as in Politis and Romano (1994).

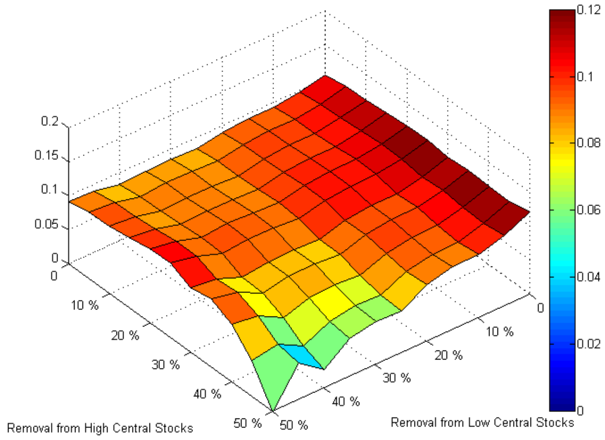
removal of stocks from the upper and lower tails of the centrality distribution (with 5% increments). Panel c from figure 7 (when $\rho = 0$) presents a scenario in which an intensive removal of high central stocks leads to better out-of-sample performance in terms of Sharpe ratio. This result is expected since highly individual performing stocks are randomly disseminated in the stock market network. Panel (b) from figure 7 (when $\rho = -0.20$) shows us the convenience of selecting among low-central stocks as the target region to invest. This is due to the absence of trade-off between the systemic and individual dimension of stocks. The opposite case is observed in panel (a) of figure 7 (when $\rho = 0.45$). In this case, the best performance is achieved by investing in high central stocks with an intense removal of securities from the left tail of the centrality distribution. This is explained given that the positive effect of the individual dimension of stocks over-compensates the negative effect of their systemic dimension.



(a)



(b)



(c)

Figure 7. Out-of-sample Sharpe ratios resulting from progressive percentage removal of stocks from the lower tail and upper tail of the centrality distribution. Three specific randomly generated samples from d-NYSE dataset are considered with ρ values: (a) ρ = 0.4550, (b) ρ =-0.2010, (c) ρ =0.000. The out-of-sample Sharpe ratios are computed with $M = 500$ and $H = 20$.

In the next step, we investigate the break point of ρ that characterizes the region of the market network that is susceptible to being discarded from the investment opportunity set. Let us define the high central stock investment region as the set of securities arising from the deletion of 25% to 45% of assets from the left tail of the centrality distribution (starting from the lowest central security) and no more than 20% from its right tail (starting from the highest central security). In a symmetric fashion, let us define the low central stock investment region as the set of stocks comprising the deletion of 25% to 45% of stock from the right tail of the centrality distribution and no more than 20% of its left tail. Then, from all of the 120 artificially-constructed data sets, we identify the investment region that generates the highest out-of-sample Sharpe ratio. The identification rule simply averages out the out-of-sample Sharpe ratios generated in each of the two investment regions and then selects the one with the highest average performance. Figure 8 plots the distribution of ρ conditional on the investment region that leads to the largest Sharpe ratio. In accordance with figure 8, low values of ρ are more consistent with high Sharpe ratio emerging from the low central investment region. In contrast, for large values of ρ, it is the high central stock investment region that generates the largest risk-adjusted returns.

With the aim of setting the value of $\tilde{\rho}$, note that such a threshold must be high enough to be worthwhile to move the optimal investment region from the low central securities towards more central ones.[66] Taking figure 8 into account, we consider 0.2 as a reasonable value for $\tilde{\rho}$ since it roughly coincides with the 75% percentile of ρ from the low central stock investment region and the 25% percentile of ρ from the high central stock investing region. We acknowledge that the determination of $\tilde{\rho}$ is the weakest point in our procedure since it implies ad-hoc rules. Other methodologies might be investigated in this regard and we leave them as future research lines.

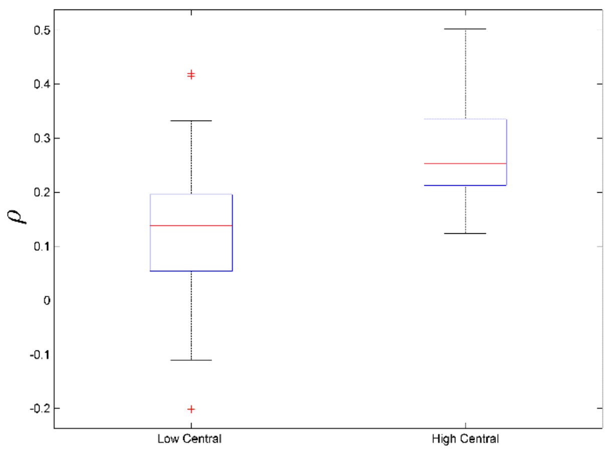


Figure 8. Distribution of ρ conditioning on the investment region generating the highest out-of-sample Sharpe ratio. 120 artificially-created datasets from d-NYSE dataset is considered. *High Central* refers to the investment region comprised of the set of stocks arising from the deletion of 25% to 45% of assets from the left tail of the centrality distribution and no more than 20% from its right tail. *Low Central* refers to the investment region with the set of stocks comprising the deletion of 25% to 45% of stocks from the right tail of the centrality distribution and no more than 20% of its left tail. The out-of-sample Sharpe ratios are computed with $M = 500$ and $H = 20$.

[66] The tendency is to invest in low-central stocks unless high-central ones show good individual performances.

6.3. Out-of-sample performance

The annualized out-of-sample performance measures originated from the two daily datasets d-S&P and d-FTSE and the monthly dataset m-NYSE are reported in table 6. Since the average ρs from these datasets are lower than $\tilde{\rho} = 0.20$, the performance of the ρ-dependent strategy is indistinguishable from an unconditional lowest-central investment rule. Therefore, we also include an additional sample (d-NYSE150) obtained by a random selection of 150 stocks from d-NYSE presenting an average ρ equal to 0.23. The setup for the out-of-sample evaluation assumes M=1000 and H=20 for the daily datasets and M=192 and H=12 for the monthly dataset.[67]

Table 6 shows the noticeable outperformance of the ρ–dependent strategy for the daily datasets. The Sharpe ratio from the $1/N$ naïve strategy reaches 0.471 and 1.153 for d-S&P and d-FTSE, respectively. The same measures rise to 0.724 and 2.584 when the ρ–dependent strategy is in place, showing a statistically significant difference with respect to the benchmark. Similar good performances are observed in terms of portfolio variance, presenting a statistically significant reduction from 0.066 to 0.051 and from 0.025 to 0.014, for d-S&P and d-FTSE. Considering the m-NYSE dataset, the ρ–dependent results are worse in terms of Sharpe ratio and variance in comparison to the benchmark but not to a statistically significant extent as indicated by the p-values. The benefit derived from the switching nature of ρ–dependent rules is explored by means of the d-NYSE150 dataset. In this case, a higher and statistically different Sharpe ratio (2.083) is obtained by following the proposed investment strategy in comparison to the naïve (1.241) and lowest-central (1.942) rules.

Table 6 also shows that ρ–dependent strategy implies distinctive and enhanced investment dynamics when it is compared to the unconditional strategies. In general terms, the Highest Sharpe Ratio, Highest Central Stocks and Lowest Central Stocks strategies present poorer outcomes in a portfolio's Sharpe Ratio in all of the datasets. In order to discard the possibility that our results were driven by chance, table 6 also reports the results for the reverse ρ–dependent strategy. In this case, none of the out-of-sample

[67] Due to data limitation, M=500 for the d-NYSE dataset.

portfolio's Sharpe ratios show better results when compared either with the benchmark or with the ρ–dependent strategy. Moreover, in the d-NYSE150 dataset, we observe that the ρ–dependent rule significantly outperforms other strategies. Since the mean value of ρ in this dataset surpasses the threshold 0.2, the ρ–dependent strategy can exploit the benefit of moving the investment set from high-central to low-central nodes and vice versa.

Table 6. Out-of-sample Performance of Portfolio Strategies

We report the out-of-sample Sharpe ratio, variance and turnover for portfolio strategies. The benchmark strategy is denoted as "*All stocks*", that is naïve strategies applied to all of the stocks in the dataset. Following the *ρ-dependent* strategy, when ρ is higher than 0.2, we diversify among highest central stocks and otherwise, diversify among the lowest central stocks. The *Reverse ρ-dependent* approach takes an opposite investment decision to the ρ-dependent strategy. The *Highest Sharpe Ratio* strategy refers to diversifying naively among stocks with the highest level of Sharpe ratios. In *Lowest Central* and *Highest Central* strategies, we diversifying among lowest and highest central stocks, respectively. The *minv-cc* and *mv-cc* strategies refer to minimum-variance and mean-variance strategies with short-selling constraints. We considered 20 stocks for our portfolios. The p-values are computed following the procedure in Ledoit and Wolf (2008) and Ledoit and Wolf (2011) based on a studentized circular block bootstrap with block size equal to 5 and number of bootstrap samples equal to 5000.

Panel A	**d-S&P (Avg ρ: -0.0556)**			**d-FTSE (Avg ρ: -0.1853)**		
	Sharpe Ratio	**Variance**	**Turnover**	**Sharpe Ratio**	**Variance**	**Turnover**
All Stocks	**0.471**	**0.066**	**0.141**	**1.153**	**0.025**	**0.138**
ρ-dependent	0.724	0.051	0.149	2.523	0.014	0.164
	(0.0125)	(0.0010)		(0.0033)	(0.0009)	
Reverse ρ-dependent	0.315	0.110	0.126	0.549	0.026	0.061
	(0.1523)	(0.0009)		(0.0100)	(0.014)	
Highest Sharpe Ratio	0.438	0.038	0.141	1.201	0.018	0.138
	(0.8239)	(0.0009)		(0.8007)	(0.0009)	
Highest Central	0.315	0.110	0.126	0.549	0.026	0.061
	(0.1523)	(0.0009)		(0.0100)	(0.014)	

Lowest Central	0.724	0.051	0.149	2.523	0.014	0.164
	(0.0125)	(0.0010)		(0.0033)	(0.0009)	
minv_cc	0.448	0.040	0.115	1.589	0.015	0.127
	(0.9568)	(0.0020)		(0.5482)	(0.0010)	
mv-cc	0.573	0.067	0.148	1.089	0.025	0.137
	(0.8339)	(0.8951)		(0.9535)	(0.6563)	
Panel B	**m-NYSE (Avg ρ: 0.1157)**			**d-NYSE150 (Avg ρ: 0.2323)**		
	Sharpe Ratio	**Variance**	**Turnover**	**Sharpe Ratio**	**Variance**	**Turnover**
All Stocks	**0.754**	**0.019**	**0.057**	**1.241**	**0.021**	**0.127**
ρ-dependent	0.591	0.025	0.065	2.083	0.019	0.140
	(0.3997)	(0.2553)		(0.0664)	(0.3367)	
Reverse ρ-dependent	0.552	0.034	0.049	0.968	0.028	0.113
	(0.0233)	(0.0009)		(0.5216)	(0.0020)	
Highest Sharpe Ratio	0.531	0.017	0.039	1.0571	0.018	0.103
	(0.1694)	(0.2248)		(0.5781)	(0.0110)	
Highest Central	0.555	0.032	0.049	1.201	0.033	0.106
	(0.0299)	(0.0009)		(0.8671)	(0.0009)	
Lowest Central	0.583	0.028	0.065	1.942	0.014	0.147
	(0.0831)	(0.0009)		(0.1927)	(0.0009)	
minv_cc	0.777	0.015	0.046	1.310	0.013	0.109
	(0.9302)	(0.1558)		(0.9734)	(0.0110)	
mv-cc	0.456	0.054	0.053	1.353	0.039	0.122
	(0.3422)	(0.0010)		(0.9502)	(0.0010)	

Additionally, table 6 reports the performance of the two Markowitz-related strategies: mean-variance and mean-variance with short selling constraints denoted by *minv-cc* and *mv-cc*, respectively. In comparison with these strategies, the ρ–dependent rule shows a better performance except for the case of *minv-cc* in the m-NYSE. For example, in the case of d-S&P dataset, while the *minv-cc* strategy manages to earn a lower level of portfolio variance compared to the benchmark, it fails to improve the level of the Sharpe ratio. On the other hand, the *mv-cc* strategy results in a non-significant increase in the Sharpe ratio level while maintaining the portfolio risk at the same level as the $1/N$ rule. In contrast, the ρ–dependent strategy manages to simultaneously increase the level of the Sharpe ratio and decrease the portfolio variance level as commented previously. This comparison highlights the poor out-of-sample performance of Markowitz rules reported in the literature and also shows the convenience of implementing the network-based portfolio strategy.[68]

Unfortunately, the major shortcoming of our proposed rule is that it tends to raise the number of rebalancing operations, a phenomenon captured by the increased portfolio turnover compared to the $1/N$ benchmark. The severity of this issue is further investigated in section 6.5 below. Two additional comments are worth mentioning. First, the relatively extreme and negative value of ρ (-0.1853 on average) for the d-FTSE dataset might explain the strikingly good results obtained in the UK market where no trade-off between the individual dimension and systemic dimension exists. Secondly, it should be mentioned that the prescription from Pozzi et al. (2013) in favor of unconditional allocation of wealth only towards the periphery of the stock market network shows clearly inferior results relative to the ρ–dependent rule when ρ assumes large values as in the d-NYSE150 dataset. Nevertheless, in the cases where ρ is relatively low, as in the d-S&P and d-FTSE datasets, our results are in line with theirs.

6.4. Carhart alpha for the ρ–dependent strategy

The large risk-adjusted returns of the ρ–dependent strategy might result from large exposures to systematic risk factors. We investigate this hypothesis by estimating Carhart's alpha from the four risk factor models (Carhart, 1997; Fama and French, 1996, 1993) as in equation (17).

$$R_t^{\rho} - r_t^f = \alpha + \beta_{MKT}\left(R_t^M - r_t^f\right) + \beta_{HML} HML_t + \beta_{SMB} SMB_t + \beta_{MOM} MOM_t + \varepsilon_t \tag{17}$$

where $R_t^{\rho} - r_t^f$ is the excess out-of-sample return from the ρ–dependent strategy, $R_t^M - r_t^f$ is the market risk premium, HML_t is the difference between the returns of high and low book-to-market portfolios, SMB_t is the difference between the returns of small-cap and large-cap portfolios and MOM_t is the momentum factor. The parameter α measures the abnormal risk-adjusted return capturing the excess return above what would be expected based solely on the portfolio's risk profile. The time series of the four risk factors are gathered from Ken French's website for the US market and from (Gregory et

[68] We also evaluate the performance of mean-variance and minimum-variance strategies without the short-selling constraints. However, our main conclusions remain unchanged. These results are available upon request.

al., 2013) for the UK case. We correct the standard errors in the estimation of equation (17) for heteroskedasticity and serial correlation in ε_t following Newey and West (1987).

Table 7 reports the estimated Carhart's alpha for three portfolio sizes (10, 20 and 50) and two holdings periods (1 and 20 days for the daily datasets and 1 and 12 months for the monthly dataset) across the US and UK samples. The reader is referred to Appendix D for a detailed results of the estimation of equation 17 without winsorizing. However, to account for outliers, table 7 reports results after 5% data winsorizing, noting that qualitative similar results are obtained either without winsorizing or by winsorizing at 10% (see table E.1 and E.2 in Appendix E). The estimations show positive and statistically significant alphas for each of the portfolio configurations. The weakest results stem from the monthly dataset where the reduced estimation window might undermine the statistical significance.

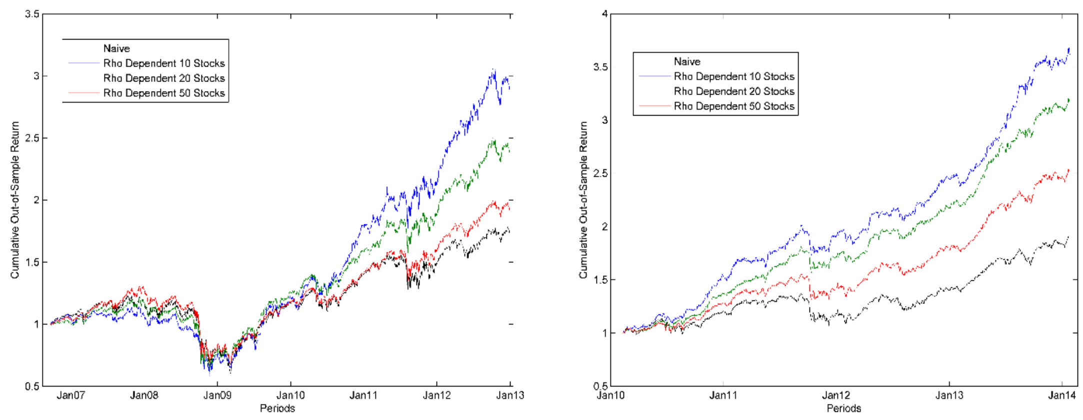
Table 7. Annualized risk-adjusted returns for ρ-dependent strategy with 5% winsorisation

We report annualised risk-adjusted returns for different settings of ρ-dependent strategy on the four Carhart (1997) factors, MKT, HML, SMB, MOM with 5% winsorisation. The estimation window is considered to be 1000 days (192 months) for daily (monthly) datasets. The t-statistics are reported in parentheses. ** and * indicate significance at 1% and 5% levels, respectively.

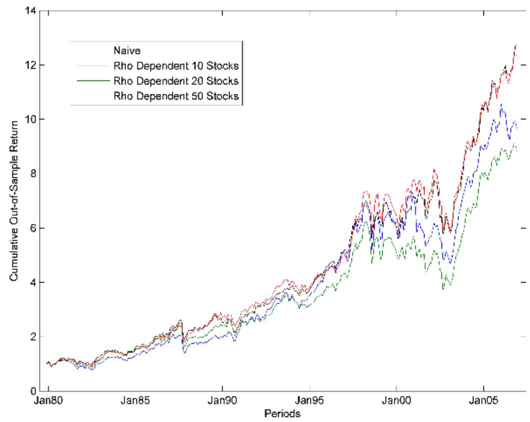
Portfolio Setting		Alpha (5% winsorizing)		
Stocks	**Holding Period**	**d-S&P**	**d-FTSE**	**m-NYSE**
10	1d/1m	14.75	35.97	5.1
		(3.37)**	(6.37)**	(2.26)*
	20d/12m	17.19	36.01	4.49
		(3.97)**	(6.21)**	(1.98)*
20	1d/1m	10.57	33.65	4.42
		(3.02)**	(6.65)**	(2.31)*
	20d/12m	11.97	32.84	4.37

		(3.45)**	(6.52)**	(2.36)*
50	1d/1m	10.52	28.07	5.45
		(3.63)**	(5.17)**	(3.23)**
	20d/12m	10.08	28.08	5.82
		(3.50)**	(5.18)**	(3.40)**

To sum up, the ρ-dependent rule is able to provide enhanced risk-adjusted returns that are not explained by exposure to traditional risk factors. Interestingly, the evidence also indicates a negative relationship between portfolio sizes and alphas for each of the considered holding periods and across datasets. For instance, considering the d-S&P dataset and a 20-day holding period, the Carhart's alphas of portfolios made of 10, 20 and 50 stocks are 17.19, 11.97 and 10.08, respectively. To confirm this regularity, however, further analyses are required.



(a) (b)



(c)

Figure 9. Cumulative return for the ρ–dependent strategy for (a) d-S&P, (b) d-FTSE, (c) m-NYSE datasets in comparison to the naïve strategy for 20 days and 12 month holding periods in daily and monthly datasets, respectively.

Finally, figure 9 plots the cumulative out-of-sample returns of the ρ–dependent strategy for different portfolio configurations. For the d-S&P dataset plotted in panel (a), we observe that investing in 10 stocks results in the largest payoffs while the $1/N$ rule produces the poorest performance. The same observation applies for the d-FTSE dataset captured in panel (b) from the same figure. The case of the m-NYSE dataset presented in panel (c) is different; the time series of cumulative returns of the naïve strategy outperforms the ρ–dependent strategy for portfolios made of 10 and 20 stocks while for a portfolio size equal to 50 they both tend to behave similarly.

6.5 Transaction cost

Since our ρ–dependent strategy is a dynamic strategy that requires rebalancing operations, it is important to investigate the impact of transaction costs. Following the approach provided in Han et al. (2013), we compute the breakeven transaction cost (BETC) that sets the average returns of the ρ–dependent strategy equal to zero. Denoting by R_t the out-of-sample return of the ρ–dependent strategy in period t, the breakeven transaction cost (BETC) is computed as follows:

$$BETC = \frac{\sum_{t=M+1}^{T} R_t}{T-M} \times H \tag{18}$$

As stated in Balduzzi and Lynch (1999), reasonable lower and upper bounds for the transition cost are 1 bp and 50 bps, respectively. Thus, obtaining a BETC higher than 50 bps means that a disproportionally abnormal transaction cost is required to wipe out the returns of the ρ–dependent strategy.

The results for BETC are reported in table 8 considering the d-S&P, d-FTSE and m-NYSE datasets and different portfolio settings (size and holding period). In general terms, we observe that for short holding periods, BETC assumes low values for any investment configuration. For instance, in a setting of a 1 day holding period and 10 stocks from the FTSE dataset, a transaction cost of only 13.79 bps is needed to wipe out the benefit of ρ–dependent strategy. However, for longer holding periods, our calculations indicate large values of BETC which supports the outperformance of the ρ–dependent strategy after controlling for transaction cost. For example, considering again the FTSE dataset, a portfolio size of 10 and a holding period of 20 days, a transaction cost of more than 264.56 bps is needed to wipe out the returns of the proposed strategy.

It should be noted that in this analysis, it is assumed that the investor faces a flat-transaction fee every time he/she wants to rebalance the portfolio regardless the size of the rebalancing operation. Appendix G provides a detailed analysis in a context in which the investor pays in proportion to the changes in portfolio's weights. The results reported in this appendix provide further support to the application of our proposed network-based investment rule.

Table 8. BETC for ρ–dependent strategy

We report the BETC for the ρ–dependent strategy in various strategy settings across the three datasets of d-S&P, d-FTSE and m-NYSE. The results are reported in basis points (bps).

Portfolio Setting		**BETC**		
Stocks	Holding Period	d-S&P	d-FTSE	m-NYSE
10	1d/1m	6.98	13.79	85.69
	20d/12m	156.74	264.56	937.18
20	1d/1m	5.42	12.12	79.69
	20d/12m	130.1	236.59	982.12
50	1d/1m	5.04	9.67	85.5
	20d/12m	102.06	191.76	1040.94

7. Conclusion and Future Research Lines

In this study, a stock market is conceived as a network where securities correspond to nodes while the paired returns' correlations account for the links. The paper establishes a bridge between Markowitz's framework and network theory by stating the tendency to overweight low-central stocks in order to build efficient portfolios. Therefore, optimal portfolio weights of highly influential securities in a correlation-based network are biased downward after controlling for their individual performance measured by either Sharpe ratios or the volatility of returns (depending on the specific portfolio's goal).

From a more descriptive point of view, we find that financial firms are the most central nodes in the market network and that both financial and market variables are major determinants of a stock's centrality. More precisely, we provide some evidence indicating that highly central securities correspond to large-capitalized, cheap and old firms presenting weak financial profiles.

We also investigate the extent to which network-based investment strategies might improve portfolio performance by means of in-sample and out-of-sample analysis. We propose the so-called ρ–dependent strategy and test its performances against the extremely simple yet effective $1/N$ naïve rule and two Markowitz-related policies. Our out-of-sample results show that the ρ–dependent strategy tends to present significant higher portfolio Sharpe ratios and lower portfolio variance relative to these well-known benchmarks. Additionally, this enhanced performance is not explained by large exposures to traditional risk factors as indicated by the reported positive and statistically significant Carhart's alphas. More importantly, our results are robust to several portfolio configurations, time periods and markets even after accounting for transaction costs.

There are several future research lines that could provide novel insights from the interaction between network theory and portfolio selection. However, it seems particularly appealing to extend the approach by considering stock markets as *directed and weighted* networks. This framework may contribute to improve our understanding on the shock-transmission mechanisms across stocks and disentangling the differential role played by specific securities as an absorber or booster of initial impulses.

Appendices

A. Proof of Proposition 1

Let us consider two symmetric square $n \times n$ matrices Ω^0 and $\Omega^1 = \Omega^0 + a * I$ where $a \in \mathbb{R}$ and I is the identity matrix with their corresponding sets of eigenvectors and eigenvalues denoted by $\{v_1^s, \dots, v_n^s\}$ and $\{\lambda_1^s, \dots, \lambda_n^s\}$ for $s = 1{,}0$. By definition of eigenvectors $\lambda_k^s v_k^s = \Omega^s v_k^s$, it follows that $\lambda_k^1 v_k^1 = \Omega^1 v_k^1 = (\Omega^0 + a * I)\, v_k^1$. After some simple algebraic manipulations $(\lambda_k^1 - a) v_k^1 = \Omega^0 v_k^1$ that allows us to conclude that $v_k^1 = v_k^0$ and $\lambda_k^1 = \lambda_k^0 + a$. Therefore, as a preliminary result, we show that the eigenvectors of Ω^0 and $\Omega^1$are exactly equal and the corresponding associated eigenvalues are related as follows: $\lambda_k^1 = \lambda_k^0 + a$ for $k = 1 \dots n$.

The proof of Proposition 1 is stated only for the case of the mean-variance strategy given that the minimum-variance rule follows exactly the same steps. We assume that the correlation matrix Ω is a $n \times n$ diagonalizable symmetric matrix with a set of eigenvectors given by $\{v_1, \dots v_n\}$ and a set of eigenvalues given by $\{\lambda_1, \dots \lambda_n\}$, both sets arranged in descendent order. Then, $\Omega = P \Lambda P^T$, where P is an $n \times n$ orthogonal matrix whose columns are $v_1, \dots v_n$. Let us denote by $\Lambda = diag(\lambda_i)$ a diagonal matrix whose *ith*-main diagonal element is λ_i. Thus the inverse of Ω could be written as

$$\Omega^{-1} = P \Lambda^{-1} P^T = P * diag(1/\lambda_i) * P^T \qquad \text{A.1}$$

$$\Omega^{-1} = \sum_k \left(\frac{1}{\lambda_k} v_k {v_k}^T \right) = \frac{1}{\lambda_1} v_1 {v_1}^T + \frac{1}{\lambda_2} v_2 {v_2}^T + \dots + \frac{1}{\lambda_n} v_n {v_n}^T \qquad \text{A.2}$$

From equation (7) in section 2.2, we have $\widehat{w}^* = \varphi \Omega^{-1} \hat{\mu}^e$. By adding and subtracting $\varphi \hat{\mu}^e$ from this expression we get

$$\widehat{w}^* = \varphi \hat{\mu}^e + \varphi [\Omega^{-1} - I] \hat{\mu}^e \qquad \text{A.3}$$

Using the preliminary results above-mentioned in this appendix, we know that the matrix $\Omega^{-1} - I$ has the same eigenvectors with eigenvalues equal to $\frac{1}{\lambda_k} - 1$ for $k = 1 \dots n$. Therefore, A.3 is stated as follows:

$$\widehat{w}^* = \varphi \hat{\mu}^e + \varphi \left[\sum_k \left(\frac{1}{\lambda_k} - 1 \right) v_k {v_k}^T \right] \hat{\mu}^e \qquad \text{A.4}$$

Given that eigenvector centralities refer to the elements of the eigenvector corresponding to the largest eigenvalue, we define $\Gamma = \varphi \left[\sum_{k=2}^{n} \left(\frac{1}{\lambda_k} - 1\right) v_k {v_k}^T\right] \hat{\mu}^e$. Then, A.4 is stated as

$$\widehat{w}^* = \varphi\hat{\mu}^e + \varphi\left(\frac{1}{\lambda_1} - 1\right) v_1 {v_1}^T \hat{\mu}^e + \Gamma \qquad \text{A.5}$$

$$\widehat{w}^* = \varphi\hat{\mu}^e + \varphi\left(\frac{1}{\lambda_1} - 1\right) ({v_1}^T \hat{\mu}^e) v_1 + \Gamma \qquad \text{A.6}$$

$$\widehat{w}^* = \varphi\hat{\mu}^e + \varphi\left(\frac{1}{\lambda_1} - 1\right) \hat{\mu}_M^e v_1 + \Gamma \qquad \text{A.7}$$

B. Descriptive Statistics of Stock Performance in terms of Centrality

Table B.1. Sharpe ratios, excess returns and standard deviations of returns for the stocks included in the d-S&P dataset conditioning on their centralities

The stocks are categorised in terms of the low, middle and high terciles of the centrality distribution.

	Mean	**Std**	**Percentiles**					**Skew**	**Kurtosis**
Sharpe Ratio			Min	5%	50%	95%	Max		
Low	3.31%	1.57%	-0.22%	1.23%	3.03%	6.16%	7.92%	0.51	0.13
Middle	2.82%	1.13%	-0.07%	0.88%	2.85%	4.35%	5.86%	-0.02	-0.06
High	2.88%	0.92%	1.03%	1.36%	2.86%	4.04%	5.33%	0.00	-0.66
Excess Return									
Low	0.07%	0.05%	-0.01%	0.02%	0.06%	0.18%	0.22%	1.20	0.84
Middle	0.06%	0.03%	0.00%	0.02%	0.06%	0.12%	0.13%	0.58	-0.08
High	0.06%	0.02%	0.02%	0.03%	0.06%	0.09%	0.16%	0.85	2.59
Std									
Low	2.05%	0.70%	1.09%	1.20%	1.95%	3.21%	4.84%	1.30	2.53
Middle	2.09%	0.63%	1.09%	1.15%	2.03%	3.24%	3.82%	0.58	-0.23
High	2.24%	0.50%	1.43%	1.49%	2.23%	3.17%	3.76%	0.60	0.23

C. Securities' description by their Centrality in the d-S&P dataset

Table C.1. Sharpe ratio, CAPM Beta, Market capitalization and Centrality ranking for each stock in the d-S&P dataset

Ticker	Name	Economic Sector	Sharpe Ratio	CAPM Beta	Market Cap.	Centrality
BEN	FRANKLIN RESOURCES INC	Finance, Insurance, And Real Estate	0.554	1.493	26588.88	0.0892
TROW	PRICE (T. ROWE) GROUP	Finance, Insurance, And Real Estate	0.605	1.628	16593.66	0.0886
DD	DU PONT (E I) DE NEMOURS	Manufacturing	0.290	1.115	41936.60	0.0880
PPG	PPG INDUSTRIES INC	Manufacturing	0.568	1.109	20755.92	0.0872
L	LOEWS CORP	Manufacturing	0.451	1.240	16038.30	0.0868
EMR	EMERSON ELECTRIC CO	Manufacturing	0.475	1.130	35205.53	0.0866
UTX	UNITED TECHNOLOGIES CORP	Manufacturing	0.547	0.976	75165.85	0.0860
PCAR	PACCAR INC	Manufacturing	0.614	1.432	15959.13	0.0856
ITW	ILLINOIS TOOL WORKS	Manufacturing	0.415	1.033	28182.33	0.0854
PX	PRAXAIR INC	Manufacturing	0.648	1.048	32520.54	0.0843
AXP	AMERICAN EXPRESS CO	Finance, Insurance, And Real Estate	0.373	1.488	64492.56	0.0842
APD	AIR PRODUCTS & CHEMICALS INC	Manufacturing	0.403	1.051	17508.17	0.0839
HON	HONEYWELL INTERNATIONAL INC	Mining	0.530	1.098	49720.61	0.0837
ETN	EATON CORP PLC	Manufacturing	0.565	1.149	18307.42	0.0831
DIS	DISNEY (WALT) CO	Services	0.527	1.101	94569.29	0.0826
CVX	CHEVRON CORP	Manufacturing	0.605	0.998	211649.54	0.0826
DHR	DANAHER CORP	Manufacturing	0.593	0.923	38721.26	0.0823
WY	WEYERHAEUSER CO	Manufacturing	0.355	1.262	15041.49	0.0822
CAT	CATERPILLAR INC	Manufacturing	0.638	1.232	58598.03	0.0821
XOM	EXXON MOBIL CORP	Manufacturing	0.530	0.937	394610.88	0.0816
MMM	3M CO	Manufacturing	0.370	0.836	64245.78	0.0816
IR	INGERSOLL-RAND PLC	Manufacturing	0.452	1.320	14512.69	0.0814
UPS	UNITED PARCEL SERVICE INC	Transp., Comm., Elect., Gas, Sant. S	0.217	0.797	140381.93	0.0808
BK	BANK OF NEW YORK MELLON CORP	Finance, Insurance, And Real Estate	0.207	1.575	30033.20	0.0807
GE	GENERAL ELECTRIC CO	Manufacturing	0.163	1.152	220107.37	0.0805
VNO	VORNADO REALTY TRUST	Retail Trade	0.443	1.425	14906.33	0.0804
JPM	JPMORGAN CHASE & CO	Finance, Insurance, And Real Estate	0.433	1.609	167063.15	0.0804
PRU	PRUDENTIAL FINANCIAL INC	Finance, Insurance, And Real Estate	0.385	1.847	24746.29	0.0796
SCHW	SCHWAB (CHARLES) CORP	Finance, Insurance, And Real Estate	0.323	1.505	18308.84	0.0795
ECL	ECOLAB INC	Manufacturing	0.599	0.845	21060.08	0.0794
MET	METLIFE INC	Finance, Insurance, And Real Estate	0.316	1.690	35003.59	0.0790

Continue Table C.1.

SPG	SIMON PROPERTY GROUP INC	Finance, Insurance, And Real Estate	0.607	1.445	48904.96	0.0790
GS	GOLDMAN SACHS GROUP INC	Finance, Insurance, And Real Estate	0.343	1.426	62006.90	0.0789
PLD	PROLOGIS INC	Finance, Insurance, And Real Estate	0.352	1.707	16818.13	0.0787
MS	MORGAN STANLEY	Finance, Insurance, And Real Estate	0.214	2.093	151051.07	0.0787
PSA	PUBLIC STORAGE	Finance, Insurance, And Real Estate	0.628	1.246	49470.50	0.0787
DOW	DOW CHEMICAL	Manufacturing	0.286	1.296	38770.45	0.0787
FDX	FEDEX CORP	Transp., Comm., Elect., Gas, Sant. S	0.308	1.041	30527.15	0.0786
ALL	ALLSTATE CORP	Finance, Insurance, And Real Estate	0.243	1.193	19402.11	0.0785
COP	CONOCOPHILLIPS	Mining	0.574	1.055	70393.77	0.0784
DE	DEERE & CO	Manufacturing	0.548	1.229	33464.03	0.0781
CBS	CBS CORP	Transp., Comm., Elect., Gas, Sant. S	0.272	1.480	48475.70	0.0781
JCI	JOHNSON CONTROLS INC	Manufacturing	0.422	1.226	37478.21	0.0780
MHFI	MCGRAW HILL FINANCIAL	Manufacturing	0.361	1.142	15181.86	0.0780
CSX	CSX CORP	Transp., Comm., Elect., Gas, Sant. S	0.595	1.162	20349.09	0.0778
CCL	CARNIVAL CORP/PLC (USA)	Transp., Comm., Elect., Gas, Sant. S	0.300	1.204	30038.82	0.0775
OXY	OCCIDENTAL PETROLEUM CORP	Mining	0.639	1.257	62068.18	0.0774
MRO	MARATHON OIL CORP	Mining	0.597	1.284	21645.96	0.0773
CB	CHUBB CORP	Finance, Insurance, And Real Estate	0.512	0.981	39459.22	0.0773
IP	INTL PAPER CO	Manufacturing	0.278	1.345	17473.82	0.0771
OKE	ONEOK INC	Mining	0.724	0.943	17493.21	0.0770
UNP	UNION PACIFIC CORP	Mining	0.629	1.002	59230.57	0.0769
HCP	HCP INC	Finance, Insurance, And Real Estate	0.491	1.330	19417.90	0.0766
ADP	AUTOMATIC DATA PROCESSING	Services	0.393	0.794	27234.43	0.0766
BBT	BB&T CORP	Manufacturing	0.209	1.358	20363.64	0.0764
IBM	INTL BUSINESS MACHINES CORP	Manufacturing	0.606	0.795	216438.64	0.0764
AFL	AFLAC INC	Finance, Insurance, And Real Estate	0.348	1.441	24898.61	0.0762
BLK	BLACKROCK INC	Finance, Insurance, And Real Estate	0.604	1.260	35561.77	0.0761
NSC	NORFOLK SOUTHERN CORP	Transp., Comm., Elect., Gas, Sant. S	0.507	1.082	19544.10	0.0761
HD	HOME DEPOT INC	Retail Trade	0.449	1.000	100112.32	0.0761
EQR	EQUITY RESIDENTIAL	Finance, Insurance, And Real Estate	0.477	1.385	17152.58	0.0761
SLB	SCHLUMBERGER LTD	Mining	0.508	1.261	91998.73	0.0756

Continue Table C.1.

COF	CAPITAL ONE FINANCIAL CORP	Finance, Insurance, And Real Estate	0.345	1.792	33674.64	0.0754
MSFT	MICROSOFT CORP	Services	0.245	0.948	256956.00	0.0753
USB	U S BANCORP	Finance, Insurance, And Real Estate	0.373	1.274	60049.63	0.0750
STT	STATE STREET CORP	Finance, Insurance, And Real Estate	0.302	1.646	22039.55	0.0749
PCP	PRECISION CASTPARTS CORP	Manufacturing	0.927	1.139	27770.42	0.0749
NBL	NOBLE ENERGY INC	Mining	0.619	1.249	18211.46	0.0749
DTE	DTE ENERGY CO	Transp., Comm., Elect., Gas, Sant. S	0.434	0.678	20665.97	0.0745
NI	NISOURCE INC	Transp., Comm., Elect., Gas, Sant. S	0.410	0.760	15411.73	0.0745
WFC	WELLS FARGO & CO	Finance, Insurance, And Real Estate	0.325	1.553	180799.38	0.0744
INTC	INTEL CORP	Manufacturing	0.298	1.119	102728.84	0.0744
HCN	HEALTH CARE REIT INC	Finance, Insurance, And Real Estate	0.567	0.971	15907.19	0.0741
M	MACY'S INC	Retail Trade	0.431	1.419	15626.20	0.0738
APA	APACHE CORP	Mining	0.427	1.178	30714.85	0.0738
SRE	SEMPRA ENERGY	Transp., Comm., Elect., Gas, Sant. S	0.690	0.761	17167.48	0.0737
ACE	ACE LTD	Finance, Insurance, And Real Estate	0.473	1.058	27110.84	0.0736
BAC	BANK OF AMERICA CORP	Finance, Insurance, And Real Estate	0.134	1.855	125124.05	0.0734
LOW	LOWE'S COMPANIES INC	Wholesale Trade	0.307	1.037	42887.37	0.0734
TWX	TIME WARNER INC	Services	0.344	1.076	45438.50	0.0733
CSCO	CISCO SYSTEMS INC	Manufacturing	0.312	1.118	85858.85	0.0732
TMO	THERMO FISHER SCIENTIFIC INC	Manufacturing	0.556	0.911	22974.50	0.0729
TYC	TYCO INTERNATIONAL LTD	Manufacturing	0.456	0.999	25866.88	0.0728
HAL	HALLIBURTON CO	Mining	0.590	1.323	32192.32	0.0725
APC	ANADARKO PETROLEUM CORP	Mining	0.470	1.289	37132.70	0.0725
COH	COACH INC	Manufacturing	0.683	1.306	16804.75	0.0724
NOV	NATIONAL OILWELL VARCO INC	Manufacturing	0.609	1.541	29177.17	0.0723
GD	GENERAL DYNAMICS CORP	Manufacturing	0.348	0.795	24457.15	0.0723
ORCL	ORACLE CORP	Services	0.555	1.061	159407.81	0.0722
BHI	BAKER HUGHES INC	Mining	0.274	1.275	17932.14	0.0722
T	AT&T INC	Transp., Comm., Elect., Gas, Sant. S	0.468	0.819	192384.94	0.0722
NKE	NIKE INC -CL B	Manufacturing	0.671	0.885	110200.60	0.0720
WM	WASTE MANAGEMENT INC	Transp., Comm., Elect., Gas, Sant. S	0.321	0.753	15650.87	0.0714
PNC	PNC FINANCIAL SVCS GROUP INC	Finance, Insurance, And Real Estate	0.325	1.378	30845.99	0.0713
D	DOMINION RESOURCES INC	Transp., Comm., Elect., Gas, Sant. S	0.554	0.630	29764.79	0.0712

Continue Table C.1.

FCX	FREEPORT-MCMORAN COP&GOLD	Mining	0.592	1.608	32455.80	0.0711
EXC	EXELON CORP	Transp., Comm., Elect., Gas, Sant. S	0.297	0.785	25406.38	0.0711
NEE	NEXTERA ENERGY INC	Transp., Comm., Elect., Gas, Sant. S	0.591	0.697	29281.61	0.0709
PG	PROCTER & GAMBLE CO	Manufacturing	0.366	0.547	167831.49	0.0709
RL	RALPH LAUREN CORP	Manufacturing	0.664	1.154	15356.42	0.0709
DVN	DEVON ENERGY CORP	Mining	0.364	1.135	21076.20	0.0708
TGT	TARGET CORP	Retail Trade	0.359	0.994	39538.35	0.0708
HES	HESS CORP	Manufacturing	0.395	1.322	18088.32	0.0707
VFC	VF CORP	Manufacturing	0.652	0.908	16597.19	0.0705
C	CITIGROUP INC	Retail Trade	-0.012	1.901	116010.54	0.0703
ADBE	ADOBE SYSTEMS INC	Services	0.499	1.171	17122.67	0.0703
VZ	VERIZON COMMUNICATIONS INC	Transp., Comm., Elect., Gas, Sant. S	0.484	0.749	123492.58	0.0701
STI	SUNTRUST BANKS INC	Finance, Insurance, And Real Estate	0.133	1.604	15275.57	0.0699
SBUX	STARBUCKS CORP	Retail Trade	0.599	1.048	38529.45	0.0697
PEG	PUBLIC SERVICE ENTRP GRP INC	Transp., Comm., Elect., Gas, Sant. S	0.483	0.780	15480.32	0.0696
PFE	PFIZER INC	Manufacturing	0.155	0.771	184648.21	0.0694
VLO	VALERO ENERGY CORP	Manufacturing	0.577	1.309	18884.29	0.0693
JNJ	JOHNSON & JOHNSON	Manufacturing	0.293	0.530	193655.32	0.0692
LLY	LILLY (ELI) & CO	Manufacturing	0.164	0.737	54636.25	0.0690
EIX	EDISON INTERNATIONAL	Transp., Comm., Elect., Gas, Sant. S	0.720	0.778	14723.39	0.0689
ED	CONSOLIDATED EDISON INC	Transp., Comm., Elect., Gas, Sant. S	0.469	0.496	32532.20	0.0689
EOG	EOG RESOURCES INC	Mining	0.633	1.187	32715.95	0.0688
CTSH	COGNIZANT TECH SOLUTIONS	Services	0.806	1.243	22160.25	0.0682
QCOM	QUALCOMM INC	Manufacturing	0.584	1.050	106948.64	0.0680
MMC	MARSH & MCLENNAN COS	Finance, Insurance, And Real Estate	0.129	0.885	18743.96	0.0679
TJX	TJX COMPANIES INC	Retail Trade	0.669	0.847	32947.65	0.0678
PPL	PPL CORP	Transp., Comm., Elect., Gas, Sant. S	0.446	0.688	16633.17	0.0677
NOC	NORTHROP GRUMMAN CORP	Manufacturing	0.233	0.697	33207.73	0.0675
AGN	ALLERGAN INC	Manufacturing	0.519	0.803	27553.94	0.0675
SYY	SYSCO CORP	Wholesale Trade	0.205	0.673	17447.53	0.0672
FE	FIRSTENERGY CORP	Transp., Comm., Elect., Gas, Sant. S	0.358	0.717	17464.69	0.0672
KMB	KIMBERLY-CLARK CORP	Manufacturing	0.415	0.513	33121.87	0.0666
COST	COSTCO WHOLESALE CORP	Wholesale Trade	0.549	0.751	42401.49	0.0665
KO	COCA-COLA CO	Manufacturing	0.357	0.562	162617.50	0.0664
MSI	MOTOROLA SOLUTIONS INC	Manufacturing	0.292	1.212	31236.48	0.0663

Continue Table C.1.

EBAY	EBAY INC	Services	0.492	1.095	65991.02	0.0662
TXN	TEXAS INSTRUMENTS INC	Manufacturing	0.368	1.050	34621.60	0.0659
WMB	WILLIAMS COS INC	Construction	0.825	1.339	20527.98	0.0659
SHW	SHERWIN-WILLIAMS CO	Manufacturing	0.791	0.812	15859.92	0.0658
AEP	AMERICAN ELECTRIC POWER CO	Transp., Comm., Elect., Gas, Sant. S	0.400	0.718	41418.02	0.0657
HPQ	HEWLETT-PACKARD CO	Manufacturing	0.219	1.010	27242.95	0.0656
SYK	STRYKER CORP	Manufacturing	0.334	0.758	20842.62	0.0653
GPS	GAP INC	Retail Trade	0.459	1.000	15686.40	0.0647
RIG	TRANSOCEAN LTD	Mining	0.363	1.161	16051.65	0.0645
ADM	ARCHER-DANIELS-MIDLAND CO	Manufacturing	0.398	0.958	19483.20	0.0643
SO	SOUTHERN CO	Transp., Comm., Elect., Gas, Sant. S	0.476	0.475	37420.48	0.0641
MON	MONSANTO CO	Manufacturing	0.845	1.010	46367.59	0.0635
EMC	EMC CORP/MA	Manufacturing	0.585	1.069	53298.37	0.0634
CTL	CENTURYLINK INC	Construction	0.428	0.729	24377.39	0.0633
PEP	PEPSICO INC	Manufacturing	0.451	0.516	106134.93	0.0632
WMT	WAL-MART STORES INC	Retail Trade	0.261	0.574	234892.09	0.0631
INTU	INTUIT INC	Services	0.407	0.868	17076.96	0.0627
BMY	BRISTOL-MYERS SQUIBB CO	Manufacturing	0.372	0.671	53773.50	0.0627
WAG	WALGREEN CO	Retail Trade	0.187	0.724	30698.70	0.0625
DUK	DUKE ENERGY CORP	Transp., Comm., Elect., Gas, Sant. S	0.534	0.610	89830.40	0.0624
ACN	ACCENTURE PLC	Services	0.625	0.837	39386.24	0.0623
MDT	MEDTRONIC INC	Manufacturing	0.108	0.659	47146.80	0.0622
BRK. B	BERKSHIRE HATHAWAY	Finance, Insurance, And Real Estate	0.296	0.607	666986.40	0.0620
PCG	PG&E CORP	Transp., Comm., Elect., Gas, Sant. S	0.686	0.625	17251.56	0.0618
CI	CIGNA CORP	Finance, Insurance, And Real Estate	0.378	1.109	15319.54	0.0617
EL	LAUDER (ESTEE) COS INC -CL A	Manufacturing	0.585	0.811	42074.07	0.0616
LMT	LOCKHEED MARTIN CORP	Manufacturing	0.298	0.636	29863.84	0.0614
K	KELLOGG CO	Manufacturing	0.431	0.475	19994.30	0.0614
BF.B	BROWN-FORMAN -CL B	Manufacturing	0.682	0.599	30119.15	0.0613
MCD	MCDONALD'S CORP	Retail Trade	0.839	0.608	88562.84	0.0612
CL	COLGATE-PALMOLIVE CO	Manufacturing	0.451	0.516	49393.26	0.0611
SYMC	SYMANTEC CORP	Services	0.355	0.997	17009.18	0.0611
GLW	CORNING INC	Manufacturing	0.639	1.272	18652.36	0.0609
MRK	MERCK & CO	Manufacturing	0.214	0.769	124637.90	0.0609
ESRX	EXPRESS SCRIPTS HOLDING CO	Services	0.716	0.880	43718.39	0.0607
CAH	CARDINAL HEALTH INC	Wholesale Trade	0.101	0.735	14532.00	0.0606

Continue Table C.1.

AET	AETNA INC	Finance, Insurance, And Real Estate	0.580	0.961	15490.70	0.0605
MCK	MCKESSON CORP	Wholesale Trade	0.527	0.757	25154.68	0.0604
CCI	CROWN CASTLE INTL CORP	Transp., Comm., Elect., Gas, Sant. S	0.980	1.077	42308.98	0.0602
MDLZ	MONDELEZ INTERNATIONAL INC	Manufacturing	0.210	0.533	45232.57	0.0600
CVS	CVS CAREMARK CORP	Manufacturing	0.579	0.725	60244.10	0.0600
AON	AON PLC	Finance, Insurance, And Real Estate	0.490	0.740	17721.07	0.0599
AAPL	APPLE INC	Manufacturing	1.252	1.019	625254.73	0.0598
BRCM	BROADCOM CORP -CL A	Manufacturing	0.524	1.285	37460.88	0.0595
UNH	UNITEDHEALTH GROUP INC	Finance, Insurance, And Real Estate	0.391	0.894	54890.88	0.0591
BDX	BECTON DICKINSON & CO	Manufacturing	0.568	0.545	15677.04	0.0590
WFM	WHOLE FOODS MARKET INC	Retail Trade	0.533	1.020	17975.56	0.0584
AMGN	AMGEN INC	Manufacturing	0.345	0.686	66210.21	0.0582
ABT	ABBOTT LABORATORIES	Manufacturing	0.413	0.530	103533.75	0.0579
WLP	WELLPOINT INC	Finance, Insurance, And Real Estate	0.313	0.785	19129.61	0.0579
STZ	CONSTELLATION BRANDS - CL A	Manufacturing	0.450	0.771	16242.10	0.0577
HSY	HERSHEY CO	Manufacturing	0.528	0.513	32237.28	0.0568
CELG	CELGENE CORP	Manufacturing	0.884	0.938	33269.00	0.0563
AMT	AMERICAN TOWER CORP	Transp., Comm., Elect., Gas, Sant. S	1.038	1.035	30549.00	0.0556
GIS	GENERAL MILLS INC	Manufacturing	0.489	0.403	30333.64	0.0548
YHOO	YAHOO INC	Services	0.493	1.025	23600.70	0.0547
GILD	GILEAD SCIENCES INC	Manufacturing	0.746	0.794	55670.17	0.0546
AIG	AMERICAN INTERNATIONAL GROUP	Finance, Insurance, And Real Estate	-0.035	1.825	52113.24	0.0542
MO	ALTRIA GROUP INC	Manufacturing	0.813	0.532	63670.15	0.0540
TAP	MOLSON COORS BREWING CO	Manufacturing	0.301	0.581	15507.10	0.0536
BAX	BAXTER INTERNATIONAL INC	Manufacturing	0.443	0.578	36570.27	0.0533
BIIB	BIOGEN IDEC INC	Manufacturing	0.480	0.845	34630.70	0.0517
ISRG	INTUITIVE SURGICAL INC	Manufacturing	0.860	1.118	19516.72	0.0517
RAI	REYNOLDS AMERICAN INC	Manufacturing	0.864	0.528	23157.22	0.0502
PCLN	PRICELINE.COM INC	Transp., Comm., Elect., Gas, Sant. S	0.990	1.161	30936.37	0.0491
ALXN	ALEXION PHARMACEUTICALS INC	Manufacturing	0.958	0.883	18207.68	0.0489
REGN	REGENERON PHARMACEUTICALS	Manufacturing	0.690	1.214	16508.26	0.0468
NEM	NEWMONT MINING CORP	Mining	0.309	0.611	45653.85	0.0392

D. Regressions of returns from ρ-dependent strategy on the four risk factors, MKT, HML, SMB, MOM without winsorizing.

Table D.1. OLS estimation of Annualized Carhart's Alpha from the four factor model in equation (17)

We consider the d-S&P, and d-FTSE, m-NYSE datasets and without winsorising. *t-statistics* are in parentheses and the statistical significance is as follows: * at 5% level, ** at 1% and *** at 0.1% level. Standard errors are corrected following Newey and West (1987). Portfolios of size 10, 20 and 50 are considered with a holding period of one (1d) and twenty (20d) for the daily datasets and one (1m) and twelve (12m) months for the monthly dataset.

Dataset	Obs.	Portfolio Setting		α	β_{MKT}	β_{HML}	β_{SMB}	β_{MOM}
		Stocks	Holding Period					
SP500	1580	10	1d/1m	12.87 (2.74)***	0.87 (53.93)***	-0.15 (-3.32)***	0.06 (1.49)	0.18 (0.76)
			20d/12m	15.06 (3.20)***	0.9 (38.60)***	-0.17 (-3.34)***	0.01 (0.16)	0.04 (1.72)*
		20	1d/1m	3.5 (2.60)***	0.89 (54.95)***	-0.19 (-5.28)***	-0.07 (-2.47)**	0.03 (2.02)**
			20d/12m	11.79 (3.33)***	0.9 (33.82)***	-0.21 (-4.39)***	-0.1 (-2.39)**	0.03 -1.61
		50	1d/1m	8.14 (3.44)***	0.92 (61.96)***	-0.14 (-5.06)***	-0.14 (-5.93)***	0.05 (3.95)***
			20d/12m	8.27 (3.47)***	0.93 (47.79)***	-0.17 (-5.60)***	-0.17 (-5.60)***	0.04 (3.57)
FTSE	1000	10	1d/1m	35.45 (5.37)***	-0.01 (-0.39)	0.19 (0.30)	-0.02 (-0.39)	0.01 (0.14)
			20d/12m	34.03 (5.07)***	-0.01 (-0.07)	0.03 (0.51)	-0.03 (-0.54)	0.02 (0.55)
		20	1d/1m	31.27 (5.13)***	-0.2 (-0.61)	0.04 (0.73)	0.01 (0.18)	0.03 (0.85)
			20d/12m	30.49 (5.02)***	-0.02 (-0.59)	0.03 (0.57)	0.01 (0.11)	0.03 (0.86)
		50	1d/1m	24.77 (3.77)***	-0.02 (-0.54)	0.02 (0.32)	0.03 (0.6)	0.06 (1.37)
			20d/12m	24.59 (3.73)***	-0.02 (-0.54)	0.02 (0.36)	0.03 (0.59)	0.05 (1.35)
NYSE	324	10	1d/1m	0.88 (0.38)	0.91 (17.16)***	0.32 (4.64)***	0.36 (4.31)***	-0.07 (-0.38)
			20d/12m	1.4 (0.62)	0.89 (17.55)***	0.41 (4.81)***	0.32 (4.37)***	-0.1 (-1.64)
		20	1d/1m	1.69 (0.93)	0.95 (23.08)***	0.41 (5.79)***	0.29 (4.91)	-0.14 (-2.64)***
			20d/12m	1.11 (0.63)	0.93 (21.60)***	0.38 (5.63)***	0.27 (4.78)***	-0.14 (-2.99)***
		50	1d/1m	2.01 (1.40)	0.91 (26.68)***	0.38 (5.59)***	0.23 (3.93)***	-0.13 (-2.77)***
			20d/12m	2.25 (1.59)	0.89 (26.04)***	0.38 (5.85)***	0.24 (3.97)***	-0.12 (-2.64)***

E. Annualized risk-adjusted returns for ρ-dependent strategy with 0% and 10% winsorization

Table E.1. Annualized risk-adjusted returns for different settings of ρ-dependent strategy without winsorisation

We report annualized risk-adjusted returns for different settings of the ρ-dependent strategy on the four Carhart (1997) factors, MKT, HML, SMB, MOM. The estimation window is considered to be 1000 days (192 months) for daily (monthly) datasets. The t-statistics are reported in the parentheses. ** and * indicate significance at 1% and 5% levels, respectively.

Portfolio Setting		Alpha (no winsorising)		
Stocks	**Holding Period**	**d-S&P**	**d-FTSE**	**m-NYSE**
10	1d/1m	12.87 (2.74)**	35.45 (5.37)**	0.88 (0.38)
	20d/12m	15.06 (3.20)**	34.03 (5.07)**	1.40 (0.62)
20	1d/1m	3.50 (2.60)**	31.27 (5.13)**	1.69 (0.93)
	20d/12m	11.79 (3.33)**	30.49 (5.02)**	1.11 (0.63)
50	1d/1m	8.14 (3.44)**	24.77 (3.77)**	2.01 (1.4)
	20d/12m	8.27 (3.47)**	24.59 (3.73)**	2.25 (1.59)

Table E2. Annualized risk-adjusted returns for ρ-dependent Strategy with 10% winsorisation

We report annualized risk-adjusted returns for various settings of ϱ-dependent strategy on the four Carhart (1997) factors, MKT, HML, SMB, MOM with 10% winsorisation. The estimation window is considered to be 1000 days (192 months) for daily (monthly) datasets. The t-statistics are reported in the parentheses. ** and * indicate significance at 1% and 5% levels, respectively.

Portfolio Setting		Alpha (10% windsorizing)		
Stocks	**Holding Period**	**d-S&P**	**d-FTSE**	**m-NYSE**
10	1d/1m	14.81 (3.82)**	35.14 (7.06)**	5.16 (2.48)*
	20d/12m	17.85 (4.57)**	34.75 (6.88)**	5.30 (2.55)*
20	1d/1m	13.83 (4.24)**	34.47 (7.77)**	9.98 (4.29)**
	20d/12m	15.72 (4.86)**	33.70 (7.60)**	5.91 (3.39)**
50	1d/1m	12.68 (4.51)**	29.51 (6.19)**	5.78 (3.70)**
	20d/12m	13.14 (4.70)**	29.74 (6.70)**	6.45 (4.03)**

F. The Relationship between Stock Centrality and Stability

We associate the concept of stock stability with the tendency of a particular asset to remain listed in the market through time without any change in its relative centrality status. This appendix analyzes the stability of stocks listed in NYSE from two different perspectives. First, we present results regarding the switching nature of assets in accordance to their different centrality in the stock market network. Second, we investigate whether the period size chosen to compute the correlation matrix influence the ranking of centralities.

We employ an m-NYSE dataset that accounts for all of the NYSE stocks with monthly pricing records in the period starting from April-1968 until April-2012. Thus, we include a full list of companies that have ever existed at some point in this period. We analyze the change in the nature of stocks in terms of centrality by relying on a moving window approach. We specify a 30-year moving window and divide it into two sub-periods, each of 15 years. We use the first and the second period to give an initial and final categorization of stocks in accordance with their centrality. Three exclusive and collectively exhaustive groups of stocks are created: high, medium and low central stocks in accordance to the top, middle and bottom terciles of the centrality distribution. Next, we construct a switching matrix accounting for the distribution of stocks belonging to a particular range of centrality in the initial period, in terms of their centrality in the final period. Since a one-year displacement step is under consideration, 15 individual switching matrices were computed. Figure F1 reports the results for each of those iterations[69].

[69] The sum of vertical height for each line does not sum to one since the proportion of the delisted firms is not included in the graph.

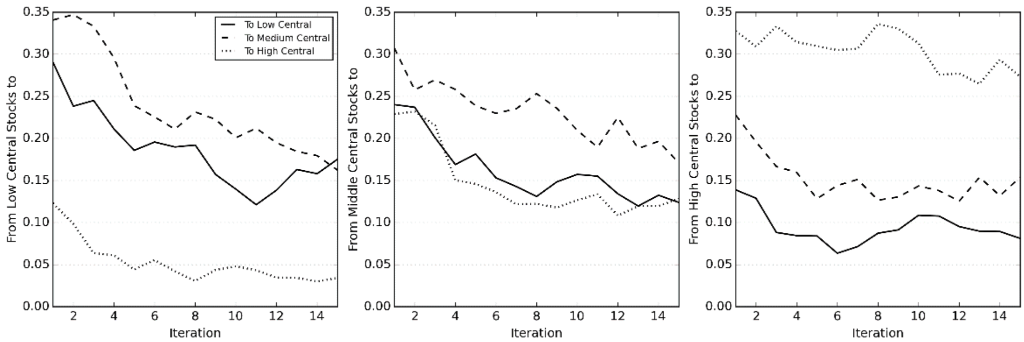


Figure F.1: Change in centrality of securities from an initial tercile of centrality to a final tercile of centrality across 15 iterations in m-NYSE dataset. We specify a 30-year moving window and divide it into two sub-periods, each of 15 years. We use the first and the second period to have an initial and final categorization of stocks securities in accordance with their centrality. Three exclusive and collectively exhaustive groups of stocks are created: high, medium and low central stocks in accordance to the top, middle and bottom terciles of the centrality distribution.

In the left-panel of Figure F1, it is obvious that most of low central stocks tend to change their nature to medium central or stay low central. Additionally, we can see that just a small proportion of low central stocks change to become highly central. The middle-panel of Figure F1 depicts the changing nature of medium central stocks. As it can be inferred, medium central stocks mostly stay in the middle range of centrality and lower percentages of them tend to change towards the low or high centrality bucket. Finally, the right-panel of Figure F1 presents the result of the experiment when stocks were initially classified as highly central assets. We can observe that high central stocks tend to stay central across the iterations and only lower percentage of them tend to become low or medium central. As a summary, Figure F2 presents the average of the percentage of switching in centrality over the 15 interactions. In a nutshell, we find that among the stocks that remain listed, there is a tendency to occupy the same position into the stock market network through time providing a sort of stability to the structure.

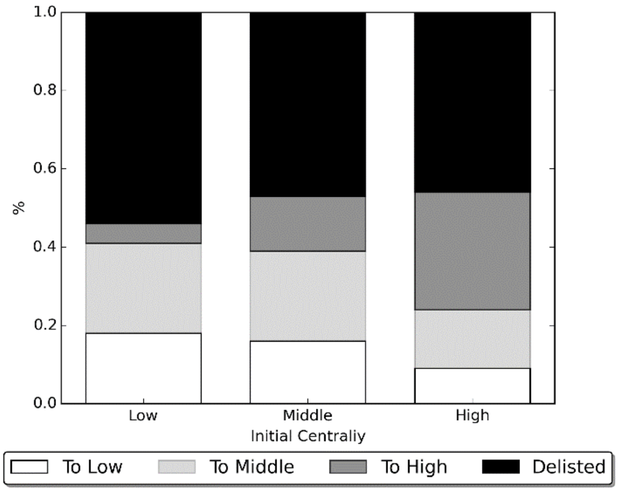


Figure F.2: Average percentage of centrality switching for low, medium and high central securities. A 30-year moving window is specified in m-NYSE dataset and it is divided into two sub-periods, each of 15 years. We use the first and the second period to give an initial and final categorization of securities in accordance to their centrality. Three exclusive and collectively exhaustive groups of assets are created: high, medium and low central stocks, in accordance with the top, middle and bottom terciles of the centrality distribution.

In the second analysis, we investigate the relationship between the ranking of centrality and the size of the period chosen to compute the correlation matrix. We consider the d-S&P dataset for this analysis because several lengths of period can be investigated with a daily frequency.

We split the d-S&P data into several yearly subsamples made of 250 daily returns and compute the centrality rankings for each of these subsamples. In the next step, we further divide the yearly subsamples into shorter datasets accounting for one month (25 daily returns), two month (50 returns) and so on up to the yearly subsamples. Afterward, the mean correlations between the ranking of centralities obtained from these shorter datasets with the initial yearly subsamples are computed and reported in figure F.3. Clearly, the correlation between these centrality rankings is high (on average 0.8) indicating that the ordering it provides is considerably robust to the period size used to estimate the correlation matrix.

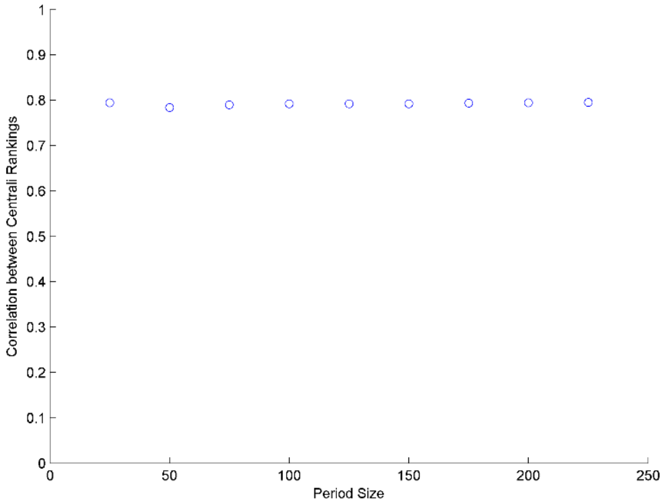


Figure F.3: Average correlation of centrality rankings in the d-S&P500 dataset between yearly subsamples (250 returns) and a set of lower-size subsamples starting with length equal to 25 days and subsequent increments of 25 days up to 250 days (one year).

G. Turnover-driven Transaction Cost

The analysis in section 6.5 assumes that investor faces a fixed transaction cost without accounting for the magnitude of changes in portfolio weights. However, investors may incur proportional costs that severely undermine the performance of their strategy. In order to take this point into account, we compute another version of the breakeven transaction cost (BETC) by solving the next expression:

$$\sum_{t=M}^{T-1}\left[(1+R_t)\left(1-BETC^{*}\times\sum_{j=1}^{N}\left|w_{j,t+1}^{s}-w_{j,t^{+}}\right|\right)-1\right]=0 \tag{G.1}$$

Therefore, the value of $BETC^{*}$represents the corresponding proportional of transaction cost that is required in order to eliminate portfolio return. The results are presented in table G.1 where low values of $BETC^{*}$ are displayed. Nevertheless, considering the low proportional transaction cost that investors face in the current state of the financial markets, we conclude that the ρ–dependent strategy remains profitable even from this perspective.

Table G.1. $BETC^{*}$ for ρ–dependent strategy

We report the $BETC^{*}$ for ρ–dependent strategy in various strategy settings across the three datasets of d-S&P, d-FTSE and m-NYSE. The results are reported in basis points.

Portfolio Setting		BETC		
Stocks	**Holding Period**	**d-S&P**	**d-FTSE**	**m-NYSE**
10	20d/12m	13	20	12
20	20d/12m	12	19	14
50	20d/12m	11	18	13

Since the corresponding $BETC^{*}$s for one day/month holding period are extremely low, table G.1 does not report their values. In these cases, even a small proportional transaction cost might eliminate any benefit which in turn raises the concern derived from the application of the ρ–dependent strategy in a highly frequent rebalancing framework.

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