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Model uncertainty approach in mortality projection with model assembling methodologies

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Most times the results have driven to a set of questionable decisions linked to those projections. This way to act does not consider the model uncertainty when selecting a specific one. This drawback can be reduced through model assembling. This technique is based on using the results of a set of models in order to get better results.
In this paper we introduce two approaches to ensemble models: a classical one, based on the Akaike information criterion (AIC), and a Bayesian model averaging method.
The data are referred to a Spanish male population and they have been obtained from the Human Mortality Database. We have used four of the most widespread models to forecast mortality rates (Lee-Carter, Renshaw-Haberman, Cairns-Blake-Dowd and its generalization for including cohort effects) together with their respective Bayesian specifications. The results suggest that using assembling models techniques gets more accurate predictions than those with the individual models.

***Keywords*:** AIC model averaging, Bayesian model averaging, bootstrap, Cairns-Blake-Dowd model, Lee-Carter model, longevity risk, projected life tables, Renshaw-Haberman model.

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Abstract

Forecasting mortality rates has become a key task for all who are concerned with payments for non-active people, such as Social Security or life insurance firms managers. The non-ending process of reduction in the mortality rates is forcing to continuously improve the models used to project these variables. Traditionally, actuaries have selected just one model, supposing that this model were able to generate the observed data. Most times the results have driven to a set of questionable decisions linked to those projections. This way to act does not consider the model uncertainty when selecting a specific one. This drawback can be reduced through model assembling. This technique is based on using the results of a set of models in order to get better results. In this paper we introduce two approaches to ensemble models: a classical one, based on the Akaike information criterion (AIC), and a Bayesian model averaging method. The data are referred to a Spanish male population and they have been obtained from the Human Mortality Database. We have used four of the most widespread models to forecast mortality rates (Lee-Carter, Renshaw-Haberman, Cairns-Blake-Dowd and its generalization for including cohort effects) together with their respective Bayesian specifications. The results suggest that using assembling models techniques gets more accurate predictions than those with the individual models.

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1 Introduction and background

The increase of the human lifetime has been one of the most relevant improvements reached during the last century. This process has been specially intense after the end of II World War in developed countries and nobody knows if it has ended or not. So, the ability to predict the right mortality rates has become to be overriding for all the activities linked with third age people, such as social and medical services or payments for retirement (public pensions or private annuities). From this point of view, although the reduction of mortality rates can be considered as an improvement for the mankind, it is also true that it opens the door to a lot of financial troubles for states and private insurance firms due to the unbalance between expected inflows and outflows. Lower mortality rates suppose more years to live and so, more money to spend in retirement pensions and social and medical services. This is the reason to look for models able to give more accurate mortality projections.

Focusing in the insurance business, mortality reduction affects to the annuities because the promised payments depends on the amounts paid (called *premiums*) by the insured people. Premiums amount depends on the age of the customer and the expected number of years receiving the annuities. They are calculated using probabilities included in a life table. In a situation where the mortality rates for each age do not change across years, the premiums received by the insurer should be enough to cover the expected payments and the administrative expenses in the future. The firms would obtain a profit

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and they would guarantee their solvency. However, the diminishing mortality rates make that incomes cannot cover the future expected expenses. The results are worst financial results and even bankruptcy. The potential losses that the annuities business can suffer derived to greater payments linked to the enlargement of human life is called *longevity risk*.

Considering the nature of this risk, the strategy of increasing the portfolio size became wrong because the reduction of mortality affects to all contracts included in the portfolio. There are two possible ways to reduce the impact of longevity risk. The first one is close to financial activity and it supposes the development of financial assets linked to human longevity that can protect insurance firms from losses generated by wrong calibration of life tables (see Cox et al. (2010), Blake et al. (2006) and Lin and Cox (2005)). The second one is linked to the development of statistical methodologies for elaborating life table able to estimate the future reductions in the mortality rates (see Lee and Carter (1992), Cairns et al. (2006) and Renshaw and Haberman (2006)). This paper focuses on the last proposal, and it tries to introduce appropriate methodologies in order to obtain robust life tables accounting for improvements in mortality.

In the recent literature, mortality has been modelled as a stochastic process. Lee-Carter model (Lee and Carter (1992)) is probably the most widespread approach. The authors proposed a model for describing the change in mortality as a function of a latent factor evolving throughout time. On the other hand, some authors have argued that the Lee-Carter age-period model does not always fit accurately empirical data (see Renshaw and Haberman (2003)), and they extended the methodology introducing age-period-cohort effects. Accordingly, Renshaw and Haberman (2006) presented an extension of the Lee-Carter model by adding a term about cohort effects. The CBD model (see Cairns et al. (2006)) is another widespread alternative to the Lee-Carter model, that includes a two-latent factor model, where age is included in the model as an independent variable. Its main assumption is that the effects linked to age and period are respectively different and they both affect the future death rate. Further developments of this model can be found in Cairns et al. (2009b) and in Dowd et al. (2010), that introduce cohort effects and quadratic terms that fit particular US data. From a semi-parametric approach, models that use smoothing techniques have been proposed in Currie et al. (2004), who applied B-splines and P-splines for fitting mortality surfaces, as well as age-period-cohort models also based on splines (see Currie et al. (2006)).

There are several papers with reviews and comparatives of various stochastic mortality models (see Haberman and Renshaw (2011), Booth and Tickle (2008), Cairns et al. (2009b)), Cairns et al. (2011). Recently, Fung et al. (2016) conducted a study aiming to reinterpret the former models as a general state-space model, as well as developing a novel class of Bayesian state-space models, accounting for heteroscedasticity and stochastic volatility to the period effect.

In this paper, we consider several models as well, but the target is to reduce the uncertainty linked to any specific model, concept that is frequently forgotten in statistical practice (Hoeting et al. (1998)). The consequences to choose just a model is that researchers work as if data were generated by the selected model, leading to inferences and decisions based on it, without considering the uncertainty linked to the selection of the proper model.

Lutz et al. (1998), Lutz et al. (2004) and Lutz et al. (2008) proposed some expert-based probabilistic projections. Nevertheless, the method is not based on available data, but on a set of experts opinions and their capacity to specify probabilistic bounds, which are not necessary accurate (Alho (2005)). Bayesian models are based on historical information or in skilled opinions to improve the quality of the estimations. As examples, Girosi and King (2008), Czado et al. (2005), Li et al. (2014), Pedroza (2006) and Gerland et al. (2014) show relevant applications o this methodology. In particular, Pedroza (2006) proposed a Bayesian approach to Lee-Carter model assuming the uncertainty in the age parameters, as well as in the mortality index, and Dellaportas et al. (2011) used a Bayesian version of the Heligman and Pollard (1980) model to predict mortality rates.

Bearing in mind the difference between classical and Bayesian approaches, there are also two main approaches for model assembling. First, a model averaging method based on the Akaike information criterion (AIC) (see Buckland et al. (1997)), consisting on a classical or frequentist methodology. Secondly, a *Bayesian Model Averaging* (BMA) technique proposed by Hoeting et al. (1998). Both approaches have been widely used for model uncertainty in many areas up today. The first one has been used in ecology (see Cade (2015)), medicine (see Schorning et al. (2016)) or finance (see Liu and Kuo (2016)). Whereas the BMA approach has also been applied in econometrical time series (see Kleijn (2016)), energy studies (see Culka (2016)) or management of health (see Pannullo et al. (2016)). But, to the best of our knowledge, none of the former approaches have been used in the actuarial field.

The structure of this paper is as follows. Section 2 presents the main mortality projection models.

Section 3 introduces the model assembling approaches, those based on Akaike weights and the Bayesian model averaging. Section 4 shows the application of these assembling techniques to mortality projection models. Finally, Section 5 summarizes the main conclusions and final remarks.

2 Main mortality projection models

There is fairly consensus regarding what are the main mortality projection models in the actuarial literature, and these models are commonly ordered and labelled from M1 to M8. Some of them are specified by means of the central death rate, while others are specified in terms of the logit of the death probability: Table 1 summarizes their main specifications.

By convention (see Cairns et al. (2009b)), the following notation is used in all models: the sets of parameters $\beta_x^{(i)}$, κ_t, $\kappa_t^{(i)}$ and γ_{t-x} reflect age, periods and cohort effects, respectively. In M4, $B_{i,j}^{ay}(x;t)$ are B-Splines basis functions and $\theta_{i,j}$ are weights associated to each basis function. In M7 and M8, $\overline{x}$ and $\hat{\sigma}^2$ are respectively the average and estimated variance of age in the data set. Finally, in M8, x_c is a centrality parameter.

Table 1: Main mortality projection models

Model	Specification
LC or M1	$\log[m_x(t)] = \beta_x^{(1)} + \beta_x^{(2)}\kappa_t$
RH or M2	$\log[m_x(t)] = \beta_x^{(1)} + \beta_x^{(2)}\kappa_t + \beta_x^{(3)}\gamma_{t-x}$
M3	$\log[m_x(t)] = \beta_x^{(1)} + \kappa_t + \gamma_{t-x}$
M4	$\log[m_x(t)] = \Sigma_{i,j}\theta_{i,j}B_{i,j}^{ay}(x;t)$
CBD or M5	$\text{logit}\ [q_x(t)] = \kappa_t^{(1)} + \kappa_t^{(2)}(x-\overline{x})$
M6	$\text{logit}\ [q_x(t)] = \kappa_t^{(1)} + \kappa_t^{(2)}(x-\overline{x}) + \gamma_{t-x}$
M7	$\text{logit}\ [q_x(t)] = \kappa_t^{(1)} + \kappa_t^{(2)}(x-\overline{x}) + \kappa_t^{(3)}((x-\overline{x})^2 - \hat{\sigma}_x^2) + \gamma_{t-x}$
M8	$\text{logit}\ [q_x(t)] = \kappa_t^{(1)} + \kappa_t^{(2)}(x-\overline{x}) + \gamma_{t-x}(x_c - x)$

M1 was introduced by Lee and Carter (1992) and M2 is a generalization of M1 including cohort effects, proposed by Renshaw and Haberman (2006). M3 is a particular case of M2 proposed by Currie (2006) with $\beta_x^{(2)} = \beta_x^{(3)} = 1$, and they used P-splines in order to estimate the parameters and to introduce smoothness. Currie, Durban, and Eilers (2004) put forward M4 by using B-Splines and P-Splines to fit the mortality surface, with smoothing of the θ_{ij} coefficients of the age and cohort measures. M5 was proposed by Cairns, Blake, and Dowd (2006), and M6 is a generalization of M5 including cohort effects. On the other hand, M7 is a generalization of M6, which adds a quadratic term into the age effect in order to include some possible curvature in the logit $[q_x(t)]$ of some US analysed data. Finally, M8 is another generalization of M5, justified by the possibility that the cohort effect may be a decreasing function of age, instead of being constant.

In what follows, we present the following models in detail: LC, RH, CBD and M6, which we will consider in the model averaging methods that we show. The reason for selecting these four models, lies on the fact that LC is the most extended mortality predictive model, while CBD is the most widespread alternative to LC model. On the other hand, RH and M6 are their respective specifications accounting for cohort effects. Regarding M3 and M4, the innovation is related to using B-Splines and P-Splines to ensure smoothness, and therefore it follows a different approach. These techniques are adequate for fitting mortality, but they are not well suited for forecasting aims.

Lee-Carter model (LC)

Lee and Carter (1992) proposed a model to forecast the mortality as a function of a time-varying index. This paper was the origin for further developments in the estimation of future mortality (see for a review Booth et al. (2006)) such as e.g. Lee (2000), Booth et al. (2002), Li and Lee (2005), Czado, Delwarde, and Denuit (2005), Li et al. (2014) and Pedroza (2006), among others.

Let $m_x(t)$ denote the central death rate for age x in year t, the LC specification is as follows

$$\log[m_x(t)] = \beta_x^{(1)} + \beta_x^{(2)}\kappa_t + \varepsilon_{xt}, \tag{1}$$

where terms $\beta_x^{(1)}$ describe the pattern of the average mortality at each age x, κ_t is a time series that models changes in the level of mortality over time, and terms $\beta_x^{(2)}$ describe deviations from this average pattern when κ_t varies. Finally, the error term ε_{xt} models the random deviations with 0 mean and constant variance σ_ε^2.

Usually, the LC model assumes that κ_t is a random walk with drift

$$\kappa_t = \kappa_{t-1} + \theta_1 + \omega_t^{(1)},$$

where θ_1 is the drift parameter that models linear trends and $\omega_t^{(1)}$ is another error term.

In order to avoid problems of identifiability of (1), Lee and Carter (1992) proposed two additional constraints on parameters $\beta_x^{(2)}$ and κ_t, such as $\sum_x \beta_x^{(2)} = 1$ and $\sum_t \kappa_t = 0$. Under this set of constraints, $\beta_x^{(1)}$ is the average of the log-central death rate for age x over time, whereas $\beta_x^{(2)}$ is the percent change of the natural logarithm of the central death rate at a given age, due to changes in the mortality index in a certain year.

Renshaw-Haberman model (*RH*)

Renshaw and Haberman (2006) proposed an extension of the Lee-Carter model by adding an extra parameter that accounts for the cohort effects. This model is established nowadays as a reference in stochastic models with cohort effects.

The model specification is

$$\log\left[m_x(t)\right] = \beta_x^{(1)} + \beta_x^{(2)}\kappa_t + \beta_x^{(3)}\gamma_{t-x} + \varepsilon_{x.t},$$

where $\beta_x^{(1)}$ parameters have the same meaning as in Lee-Carter model, κ_t is a time series accounting for the period effect, and the added term γ_{t-x} is a time series that accounts for the cohort effect. The RH model assumes that κ_t and γ_t can be modelled respectively as two random walks with drifts,

$$\begin{aligned} \kappa_t &= \kappa_{t-1} + \theta_1 + \omega_t^{(2)}, \\ \gamma_t &= \gamma_{t-1} + \theta_2 + \omega_t^{(3)}. \end{aligned}$$

As in Lee-Carter model, it is necessary to impose some constraints on the parameters in order to estimate them (see e.g. Cairns, Blake, Dowd, Coughlan, Epstein, Ong, and Balevich (2009a)): $\sum_t \kappa_t = 0$, $\sum_x \beta_x^{(2)} = 1$, $\sum_t \gamma_{t-x} = 0$ and $\sum_x \beta_x^{(3)} = 1$.

Cairns-Blake-Dowd model (*CBD*)

The most popular alternative in the demographical literature to Lee-Carter model is the CBD specification proposed by Cairns et al. (2006).

Let us denote $q_x(t)$ as the death probability at age x and time t; then based on the empirical fact that $\log\left[q_x(t)/\left[(1-q_x(t)\right)\right]$ is approximately a linear function of x for fixed t (except for young ages),

$$\log\left[\frac{q_x(t)}{1-q_x(t)}\right] = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \overline{x}),$$

where $\overline{x}$ is the average calendar age and the components of vector $(\kappa_t^{(1)}, \kappa_t^{(2)})'$ are modelled as univariate time series. Accordingly,

$$\begin{cases} \kappa_t^{(1)} = \kappa_{t-1}^{(1)} + \theta_1 + \omega_t^{(1)}, \\ \kappa_t^{(2)} = \kappa_{t-1}^{(2)} + \theta_2 + \omega_t^{(2)}, \end{cases}$$

where θ_1 and θ_2 are drift parameters and $(\omega_t^{(1)}, \omega_t^{(2)})'$ are independent normal variables with 0 mean.

The CBD model specification does not have any identifiability problem as in the case of the Lee-Carter model and therefore no constraints are imposed.

In the CBD model, unlike the Lee-Carter one, age is treated as an explanatory variable and it is included linearly in the model on a logit scale. The intercept $\kappa_{t-1}^{(1)}$ and the slope $\kappa_{t-1}^{(2)}$ are modelled as a bivariate time series.

The CBD model includes two time factors $\kappa_{t-1}^{(1)}$ and $\kappa_{t-1}^{(2)}$ that affect to different ages. This is a substantial difference with the Lee-Carter model, where only a time series induces correlation between death rates at different ages in two consecutive years. According to Pitacco et al. (2009), there is empirical evidence that suggests that changes in the death rates are highly correlated.

Cairns-Blake-Dowd model accounting for cohort effects (*M6*)

Cairns et al. (2009b) proposed an extension of the original CBD model accounting for cohort effects. Its specification is

$$\log\left[\frac{q_x(t)}{1-q_x(t)}\right] = \kappa_t^{(1)} + \kappa_t^{(2)}(x-\overline{x}) + \gamma_{t-x}$$

$$\begin{cases} \kappa_t^{(1)} = \kappa_{t-1}^{(1)} + \theta_1 + \omega_t^{(1)}, \\ \kappa_t^{(2)} = \kappa_{t-1}^{(2)} + \theta_2 + \omega_t^{(2)}, \end{cases}$$

$$\gamma_t = \gamma_{t-1} + \theta_3 + \omega_t^{(3)},$$

where γ_{t-x} is the latent times series that includes the cohort effect as in the RH model.

3 Model assembling

Researchers usually choose the *best* model among a set of candidates according to some information criteria, discarding the remaining models. This practice, known as *model selection*, in fact ignores model uncertainty and the corresponding uncertainty associated with the projection is underestimated and it leads to over–confident inferences.

A proposal to overcome this problem is to *average* models according to some assembling criteria. Thus, the final model is a mixed one that takes the whole set of candidates models into consideration. Specifically, a weight is assigned to each model that determines the contribution of each of them to the assembling model.

Model assembling solves the problem of having to select a best model among a set of candidates, and the associated loss of information that implies the discard of other possible models, and often it obtains better predictions and more accurate inferences.

3.1 Method based on Akaike weights

The Akaike Information Criterion (AIC) (Akaike 1974) is a classical measure that compares the quality of different models given a set of data. It considers the likelihood of a model and the corresponding number of parameters, in such a way there is a trade–off between the goodness of fit and the complexity of a model, following the principle of *parsimony*. In general, AIC provides a very popular tool for model selection (see Burnham and Anderson (2003)) .

On the other hand, in the case of small sample sizes, a corrected version of the AIC can be defined

$$AIC_c = AIC + \frac{2k(k+1)}{n-k-1}.$$

As the sample size n increases faster than the number of parameters k, the bias–adjustment term becomes smaller and it is even negligible for large sample sizes.

In order to compute the weights for each model, the first step is the identification of the best one which has the lowest AIC_c. Then, it is computed the difference between the AIC_c of each model i and the lowest one

$$\delta_i = AIC_{c\ (i)} - AIC_{c\ (best)}.$$

It can be shown that the likelihood of a model given some data, is proportional to $e^{\frac{1}{2}\delta_i}$ (see Burnham and Anderson (2003)). This expression is known as the *relative likelihood* of the model given data. An alternative approach is to work with rescaled values (see Burnham and Anderson (2003))

$$\delta_i = \frac{AIC_i - AIC_{best}}{AIC_{best}}.$$

Finally, after rescaling the relative likelihoods, the *AIC weights* for each model i are obtained as

$$w_i = \frac{e^{\frac{1}{2}\delta_i}}{\sum_{j=1}^{M} e^{\frac{1}{2}\delta_j}},$$

where M is the number of the proposed models. Thus, w_i is the relative weight that model i has in relation with the best one, given both data and the the proposed models.

3.2 Bayesian model averaging

Bayesian model averaging makes inferences based on a weighted average over the model space. In this way, the model uncertainty is also included in both predictions and parameter estimates (see for a basic introduction Hoeting et al. (1999) and Hoeting (2002)).

Let $\mathcal{M} = (M_1, \ldots, M_K)$ be the set of models under consideration. If Δ is the quantity of interest, such as future observations or a vector of parameters, then the posterior distribution of Δ given data Z is

$$P(\Delta|Z) = \sum_{k=1}^{K} P(\Delta|Z, M_k)P(M_k|Z).$$

This is an average of the posterior predictive distribution for Δ under each of the included models, weighted by the corresponding posterior model probability. The posterior probability for model M_k is given by

$$P(M_k|Z) \propto P(Z|M_k)P(M_k)$$

where

$$P(Z|M_k) = \int \cdots \int P(Z|\theta_k, M_k)P(\theta_k|M_k)d\theta_k$$

is the integrated likelihood of model M_k, θ_k is the vector of parameters of model M_k, $P(\theta_k|M_k)$ is the prior density of the parameters under model M_k, $P(Z|\theta_k, M_k)$ is the likelihood, and $P(M_k)$ is the prior probability that M_k is the true model. All probabilities are assumed to be conditional on $\mathcal{M}$, the set of all models taken into consideration.

From a practical point of view, the posterior probability for each model M_k can be computed by a MCMC approach (see Chib (1995)) and the predictions of each model are weighted by the posterior probability in order to obtain the ensemble ones. In this work, the MCMC algorithm has been programmed using `Jags` ((Plummer et al. 2003)); one of its advantages is that it constructs the full conditional distributions and it carries out the Gibbs sampling from the model specifications.

4 Model assembling in mortality projection

In this section we apply the model averaging techniques by using four of the most relevant and popular mortality parametric predictive models. Notice that it is the first time that model averaging is applied to this kind of predictive models, to the best of our knowledge. Specifically, we apply the model averaging based on Akaike weights and the Bayesian model averaging (BMA) methodologies on some Spanish mortality data located in the *Human Mortality Database* (see `www.mortality.org`). We have focused on male gender data, a timespan between single calendar years 1960 and 2009, and a range between 60 and 100 years old.

4.1 Classical Approach: AIC-based assembling

First, we have estimated the models LC, RH, CBD and M6 with a training sample, including data from years 1960 to 1999, and then we have obtained predictions for the period of years 2000 until 2009. We have fitted the four models using the packages `demography` (Hyndman et al. (2014)) and `StMoMo` (Villegas et al. (2015)) from the `R` project (R Core Team (2015)). In table 2, the corresponding AIC values and weights are shown.

Table 2: AIC values and model weights

Model	AICc	δ_i	w_i
LC	1318.552	0.02612246	0.2567122
RH	1497.384	0.16529290	0.2394562
CBD	1284.986	0.00000000	0.2600872
M6	1451.768	0.12979312	0.2437444

In order to validate the predictive performance of the assembled model, we have computed forecasts of the central death rates for the period 2000 to 2009, and we have compared them with the real observed

rates. The assembled projected central death rates were computed as

$$\hat{m}_x(t) = \sum_i w_i \hat{m}_x^{(i)}(t),$$

where $\hat{m}_x^{(i)}(t)$ is the estimated central death rate for age x in year t under model i.

By the other hand, the variance of predictions

$$var(\hat{m}_x(t)) = \sum_i w_i^2 var(\hat{m}_x^{(i)}(t)) + \sum_{i \neq j} w_i w_j cov(\hat{m}_x^{(i)}(t)), \hat{m}_x^{(j)}(t)),$$

is not easy to compute, since it includes the covariances of projected central death rates obtained with different models.

As an alternative technique to calculate confidence intervals, we can resort to a resampling based approach in order to obtain approximate confidence intervals for the projected central death rates $\hat{m}_x(t)$.

The resampling bootstrap technique was introduced by Efron (1979) as an alternative to the jackknife procedure (see Quenouille (1956)), and it is based on resampling with replacement from an original sample of observations, in order to obtain approximate standard errors and confidence intervals. In the demographical and actuarial fields there are many papers that use bootstrap techniques, such as Koissi, Shapiro, and Högnäs (2006), who applied bootstrap methods for forecasting life expectancies under Lee-Carter models, England and Verrall (1999) for prediction errors in claims reserving, Hoedemakers, Beirlant, Goovaerts, and Dhaene (2003) for estimating confidence intervals for loss reserves and Caswell (2001) who presents a description of resampling methods for confidence intervals related with demographic estimates.

There are several methods for determining bootstrap confidence intervals. In this paper we use bootstrap percentile intervals since they show good properties and they are straightforward to compute without assuming any parametric model. In what follows, we introduce the basic ideas about the residual bootstrap percentile interval applied to model assembling, in order to obtain confidence intervals for the predicted central death rates.

The bootstrap procedure for the Spanish data can be shown in the next scheme:

Step 1 We have split the original data in two sets: a training and a validation samples. We have taken calendar years $t = 1960, \ldots, 1999$ as a training sample, and calendar years $t = 2000, \ldots, 2009$ as a validation sample.

Step 2: We have obtained the estimates of all parameters of each model by using the training sample. Then, for each model, a matrix of residuals R is obtained.

Step 3: We have generated N (here we fix $N = 1000$) replicates $R^{(n)}, n = 1, \ldots, N$, by sampling with replacement the elements of matrix R. Then, we have added each matrix $R^{(n)}$ to the original matrix of data, obtaining the corresponding N different bootstrap samples.

Step 4: We have computed $\hat{m}_x^{(n)}(t) = \sum_i w_i \hat{m}_x^{(i)(n)}(t)$ for each age $x = 60, \ldots, 100$, each calendar year $t = 2000, \ldots, 2009$ and $n = 1, \ldots, N$ bootstrap replicates.

Step 5: Finally, we have constructed prediction intervals for the central death rates by computing the corresponding percentiles at a given confidence level. Accordingly, in regards to 95% prediction intervals, we have taken the 2.5 and 97.5 percentiles of $\hat{m}_x^{(n)}(t)$ for each age $x = 60, \ldots, 100$ and each calendar year $t = 2000, \ldots, 2009$.

In Appendix A, we show the 95% assembling bootstrap predictive intervals for the averaged central death rates, and the corresponding actual values for ages $x = 60, 70, 80, 90$ and 100. Notice that the assembled model yields narrow intervals, and the actual values of the central death rates lie within the intervals bounds in the 90% of cases, which suggests an accurate and unbiased projection.

After having assessed the predictive ability of the Akaike weighted model, which yielded accurate projections, we consider to project the mortality rate for a long period of time. Since life insurance firms usually need long-run predictions for their survival products, we have projected the central death rates of the male Spanish population according to the assembled model based on Akaike weights. Considering data from 1960 to 2009, we have obtained predictions for the following 50 years, i.e., from 2010 until 2059. In table 3, the corresponding AIC values and weights are shown.

Table 3: AIC values and model weights

Model	AICc	δ_i	w_i
LC	1566.115	0.000000000	0.2578872
RH	1764.097	0.126415923	0.2420911
CBD	1570.151	0.002576591	0.2575552
M6	1759.245	0.123317415	0.2424665

Appendix B shows the surface of both historic and projected central death rates $\hat{m}_x(t)$, according to the Akaike weights method. On the left hand side of the figure, the observed central death rates are represented by the height of the surface for $t = 1960, \ldots, 2009$, followed by the projected central death rates for $t = 2010, \ldots, 2059$.

By the other side, Appendix C shows the projected mortality profiles for ages 60, 70, 80, 90 and 100 years old for the next 50 years. Results suggest significant decreasing mortality rates.

4.2 Bayesian model averaging (BMA)

Alternatively to the classical analysis, we apply a Bayesian model averaging approach in the same Spanish data that were analysed by means of the classical (frequentist) procedure.

We first consider the Bayesian specifications of the former models, LC, RH, CBD and M6. For the sake of brevity we do not show the complete mathematical details. We may assume conjugate prior distributions for all the parameters of the four models. Then, following the notations of Section 2,

$$\begin{aligned}
\beta_x^{(1)} &\sim N(0, \sigma_\beta) \\
\beta_x^{(2)}, \quad \beta_x^{(3)} &\sim \text{Dirichlet}(1, 1, \ldots, 1) \\
\kappa_1 &\sim N(0,\ \sigma_\omega) \\
\kappa_1^{(1)} &\sim N(0,\ \sigma_{\omega_1}) \\
\kappa_1^{(2)} &\sim N(0,\ \sigma_{\omega_2}) \\
\gamma_1 &\sim N(0,\ \sigma_\gamma) \\
\theta_1, \quad \theta_2, \quad \theta_3 &\sim N(0,\ \sigma_\theta) \\
\sigma_\varepsilon^2, \quad \sigma_{\omega_3}^2 &\sim \text{InvGamma}(\alpha_1,\ \alpha_2) \\
(\omega_t^{(1)}, \omega_t^{(2)})' &\sim N_2((0,0)', \Omega) \\
\Omega &\sim \text{Wishart}(\mathbf{I}_2, 2)
\end{aligned}$$

We assume, in the Spanish data example, vaguely informative prior distributions. In this way we can assume a Dirichlet distribution for $\beta_x^{(j)}$, for $j = 1, 2$, in such a way that the constraints $\sum_x \beta_x^{(j)} = 1$ are imposed, and inverse-gamma distributions for variances, as they are conjugate distributions in normal models. The values of standard deviations of the normal distributions are chosen to be large, and therefore α_1 and α_2 will be fixed in order that the inverse-gamma distributions have mean equal to 1 and large variances. In the case of CBD and M6 models, whose specifications include a bivariate latent series $(\omega_t^{(1)}, \omega_t^{(2)})'$, we consider a prior bivariate normal distribution with $(0, 0)'$ mean and, for the covariance matrix, a Wishart distribution as prior distribution with the identity matrix $\mathbf{I}_2$ and 2 degrees of freedom as parameters.

Regarding the model assembling, the prior distribution of the model weights is chosen to have a discrete distribution with equal probability for each model, that is, $P(M_i) = \frac{1}{4}$, for $i = 1, \ldots, 4$. The MCMC algorithm has been programmed using `Jags` ((Plummer et al. 2003)) by means of the package `R2jags` ((Su and Yajima 2015)) from the `R` project ((R Core Team 2012)).

In order to validate the predictive performance of the averaged model, we obtained forecasts for the central death rates $m_x(t)$ for the period 2000 to 2009 and we compared them with the observed rates. In Appendix D, 95% HPD of the central death rates and the corresponding actual values for ages $x = 60, 70, 80, 90$ and 100 are shown. Notice that all the real values lie within the HPD intervals bounds, which suggests a good fit.

Appendix E shows the surface of both historic and projected central death rates $\hat{m}_x(t)$ according to the Bayesian model averaging. On the left hand side of the figure, the observed central death rates are represented by the height of the surface for $t = 1960, \ldots, 2009$, followed by the projected central

death rates for $t = 2010, \ldots, 2059$. By the other side, Appendix F shows the values of the the projected mortality profiles for ages 60, 70, 80, 90 and 100 years old for the next 50 years. As in the classical approach, results with the Bayesian methodology suggest significant decreasing mortality rates.

4.3 Comparison between AIC-based assembling and Bayesian model averaging (BMA)

Both AIC-based assembling and Bayesian model averaging (BMA) were able to project the central death rates rates more accurately than what can be obtained, by applying each model independently on the same data. Furthermore, both assembling methods show accurate predicted values for mortality rates and they are close to their actual values, when analysing the test set of observations.

But the corresponding interpretation of the bootstrap confidence intervals an the HPD ones vary, as the Bayesian intervals are based on a prior information that is included in the model. We have used in the Spanish data example weekly informative distributions, although in other real analysis applications, researchers may include any kind of expert-based information in the models.

Notice that, apart from their different theoretical interpretation, frequentist bootstrap intervals seem to be a little narrower than the corresponding Bayesian HPD alternative ones (see in Appendices table A and table D).

5 Conclusions and final remarks

In this paper we have introduced two model assembling methodologies in order to study model uncertainty in mortality projection. The first methodology is based on a classical (frequentist) point of view, and the other one is based on a Bayesian approach. We have applied them by including some of the most extended and popular parametric mortality models. Although model assembly methodologies have been applied to different areas, it is the first time that they are used in the actuarial field, to the extent of our knowledge.

We have presented an example based on some Spanish male mortality data from calendar years 1960 until 2009. Then, we have considered first, a validation study by splitting the data set in two parts: one for estimating parameters (from years 1960–1999) and the other one to test the procedures (from years 2000–2009). Regarding the validation study, both methods (classical and Bayesian) were able to rightly predict the central death rates rates in all cases.

Thereupon, we have estimated the projected mortality rates for a long time period (until year 2059). Remarkably, it is obtained a significant progressive decreasing mortality rates, in both classical (frequentist) or a Bayesian approaches. This is quite relevant, as nowadays one of the most outstanding problems for the insurance and pensions systems is related with wrong estimations of survival probabilities, based on wrong calibration of life tables. New methodologies, like those shown in this work, can be directly used in the management of private and public pension systems.

Future research will involve the implementation of these methodologies in other kind of models used to forecast death rates, such as those based on splines and other semi-parametric models.

References

Akaike, H. (1974). A new look at the statistical model identification. *Automatic Control, IEEE Transactions on 19*(6), 716–723.

Alho, J. (2005). *Statistical Demography and Forecasting*. Springer.

Blake, D., A. Cairns, K. Dowd, and R. MacMinn (2006). Longevity bonds: Financial engineering, valuation, and hedging. *Journal of Risk and Insurance 73(4)*, 647–672.

Booth, H., R. Hyndman, L. Tickle, P. De Jong, et al. (2006). Lee-Carter mortality forecasting: A multi-country comparison of variants and extensions.

Booth, H., J. Maindonald, and L. Smith (2002). Applying lee-carter under conditions of variable mortality decline. *Population Studies 56(3)*, 325–336.

Booth, H. and L. Tickle (2008). Mortality modelling and forecasting: A review of methods. *Annals of actuarial science 3*(1-2), 3–43.

Buckland, S. T., K. P. Burnham, and N. H. Augustin (1997). Model selection: an integral part of inference. *Biometrics*, 603–618.

Burnham, K. P. and D. R. Anderson (2003). *Model selection and multimodel inference: a practical information-theoretic approach*. Springer Science & Business Media.

Cade, B. S. (2015). Model averaging and muddled multimodel inferences. *Ecology 96*(9), 2370–2382.

Cairns, A., D. Blake, and K. Dowd (2006). A two.factor model for stochastic mortality with parameter uncertainty: Theory and calibration. *The Journal of Risk and Insurance 73(4)*, 687–718.

Cairns, A., D. Blake, K. Dowd, G. Coughlan, D. Epstein, A. Ong, and I. Balevich (2009a). A quantitative comparison of stochastic mortality models using data from England and Wales and the United States. *North American Actuarial Journal 13*, 1–35.

Cairns, A. J., D. Blake, K. Dowd, G. D. Coughlan, D. Epstein, and M. Khalaf-Allah (2011). Mortality density forecasts: An analysis of six stochastic mortality models. *Insurance: Mathematics and Economics 48*(3), 355–367.

Cairns, A. J., D. Blake, K. Dowd, G. D. Coughlan, D. Epstein, A. Ong, and I. Balevich (2009b). A quantitative comparison of stochastic mortality models using data from england and wales and the united states. *North American Actuarial Journal 13*(1), 1–35.

Caswell, H. (2001). *Matrix population models*. Wiley Online Library.

Chib, S. (1995). Marginal likelihood from the gibbs output. *Journal of the American Statistical Association 90*(432), 1313–1321.

Cox, S., Y. Lin, and H. Pedersen (2010). Mortality risk modeling: Applications to insurance securitization. *Insurance: Mathematics and Economics 46*, 242–253.

Culka, M. (2016). Uncertainty analysis using bayesian model averaging: a case study of input variables to energy models and inference to associated uncertainties of energy scenarios. *Energy, Sustainability and Society 6*(1), 1.

Currie, I. (2006). Smoothing and forecasting mortality rates with p-splines. *URL http://www. macs. hw. ac. uk/˜ iain/research/talks/Mortality. pdf*.

Currie, I., M. Durbán, and P. Eilers (2006). Generalized linear array models with applications to multidimensional smoothing. *Journal of the Royal Statistical Society, Series B 68*, 259–280.

Currie, I. D., M. Durban, and P. H. Eilers (2004). Smoothing and forecasting mortality rates. *Statistical modelling 4*(4), 279–298.

Czado, C., A. Delwarde, and M. Denuit (2005). Bayesian Poisson log-bilinear mortality projections. *Insurance: Mathematics and Economics 36*(3), 260–284.

Dellaportas, P., A. Smith, and P. Stavropoulos (2011). Bayesian analysis of mortality data. *Journal of the Royal Statistical Society, Series A 164(2)*, 275–289.

Dowd, K., A. Cairns, D. Blake, G. Coughlan, D. Epstein, and M. Khalaf-Allah (2010). Backtesting stochastic mortality models: An ex-post evaluation of multi-period ahead density forecasts. *North American Actuarial Journal 14*, 281–298.

Efron, B. (1979). Bootstrap methods: another look at the jackknife annals of statistics 7: 1–26. *View Article PubMed/NCBI Google Scholar*.

England, P. and R. Verrall (1999). Analytic and bootstrap estimates of prediction errors in claims reserving. *Insurance: mathematics and economics 25*(3), 281–293.

Fung, M. C., G. W. Peters, and P. V. Shevchenko (2016). A unified approach to mortality modelling using state-space framework: characterisation, identification, estimation and forecasting. *Identification, Estimation and Forecasting (May 30, 2016)*.

Gerland, P., A. Raftery, et al. (2014). World population stabilization unlikely this century. *Science 346*(6206), 234–237.

Girosi, F. and G. King (2008). *Demographic forecasting*. Princeton University Press.

Haberman, S. and A. Renshaw (2011). A comparative study of parametric mortality projection models. *Insurance: Mathematics and Economics 48*(1), 35–55.

Heligman, L. and J. Pollard (1980). The age pattern of mortality. *Journal of Institute of Actuaries 107*, 49–80.

Hoedemakers, T., J. Beirlant, M. J. Goovaerts, and J. Dhaene (2003). Confidence bounds for discounted loss reserves. *Insurance: Mathematics and Economics 33*(2), 297–316.

Hoeting, J. A. (2002). Methodology for bayesian model averaging: an update.

Hoeting, J. A., D. Madigan, A. E. Raftery, and C. T. Volinsky (1998). Bayesian model averaging. In *In Proceedings of the AAAI Workshop on Integrating Multiple Learned Models*, pp. 77–83. Citeseer.

Hoeting, J. A., D. Madigan, A. E. Raftery, and C. T. Volinsky (1999). Bayesian model averaging: a tutorial. *Statistical science*, 382–401.

Hyndman, R. J., H. Booth, L. Tickle, and J. Maindonald. (2014). *demography: Forecasting mortality, fertility, migration and population data*. R package version 1.18.

Kleijn, R. (2016). Essays on bayesian model averaging using economic time series. Technical report.

Koissi, M.-C., A. F. Shapiro, and G. Högnäs (2006). Evaluating and extending the lee–carter model for mortality forecasting: Bootstrap confidence interval. *Insurance: Mathematics and Economics 38*(1), 1–20.

Lee, R. (2000). The lee-carter method for forecasting mortality, with various extensions and applications. *North American Actuarial Journal 4(1)*, 80–93.

Lee, R. and L. Carter (1992). Modeling and forecasting U.S. mortality. *Journal of the American Statistical Society 87*, 659–675.

Li, J. et al. (2014). An application of MCMC simulation in mortality projection for populations with limited data.

Li, N. and R. Lee (2005). Coherent mortality forecasts for a group of populations and extension of the lee-carter method. *Demography 42*, 575–594.

Lin, Y. and S. Cox (2005). Securitization of mortality risks in life annuities. *Journal of Risk and Insurance 72(2)*, 227–252.

Liu, C.-A. and B.-S. Kuo (2016). Model averaging in predictive regressions. *The Econometrics Journal*.

Lutz, W., W. Sanderson, and S. Scherbov (1998). Expert-based probabilistic population projections. *Population and Development Review 24*, 139–155.

Lutz, W., W. Sanderson, and S. Scherbov (2004). *The end of World population growth in the 21th century: new challenges for human capital formation and sustainable development*. Sterling.

Lutz, W., W. Sanderson, and S. Scherbov (2008). IIasa's 2007 probabilistic world population projections.

Pannullo, F., D. Lee, E. Waclawski, and A. H. Leyland (2016). How robust are the estimated effects of air pollution on health? accounting for model uncertainty using bayesian model averaging. *Spatial and Spatio-temporal Epidemiology*.

Pedroza, C. (2006). A bayesian forecasting model: predicting U.S. male Mortality. *Biostatistics 7(4)*, 530–550.

Pitacco, E., M. Denuit, S. Haberman, and A. Olivieri (2009). *Modeling longevity dynamics for pensions and annuity business*. Oxford University Press.

Plummer, M. et al. (2003). Jags: A program for analysis of bayesian graphical models using gibbs sampling. *URL http://citeseer. ist. psu. edu/plummer03jags. html*.

Quenouille, M. H. (1956). Notes on bias in estimation. *Biometrika 43*(3/4), 353–360.

R Core Team (2012). *R: A Language and Environment for Statistical Computing*. Vienna, Austria: R Foundation for Statistical Computing. ISBN 3-900051-07-0.

R Core Team (2015). *R: A Language and Environment for Statistical Computing*. Vienna, Austria: R Foundation for Statistical Computing.

Renshaw, A. and S. Haberman (2003). Lee–carter mortality forecasting: A parallel generalized linear modelling approach for england and wales mortality projections. *Journal of the Royal Statistical Society: Series C (Applied Statistics) 52*(1), 119–137.

Renshaw, A. and S. Haberman (2006). A cohort-based extension to the Lee-Carter model for mortality reduction factors. *Insurance: Mathematics and Economics 38*, 556–570.

Schorning, K., B. Bornkamp, F. Bretz, and H. Dette (2016). Model selection versus model averaging in dose finding studies. *Statistics in medicine*.

Su, Y.-S. and M. Yajima (2015). *R2jags: Using R to Run 'JAGS'*. R package version 0.5-7.

Villegas, A., P. Millossovich, and V. Kaishev (2015). *Stochastic Mortality Modelling*. R package version 0.3.0.

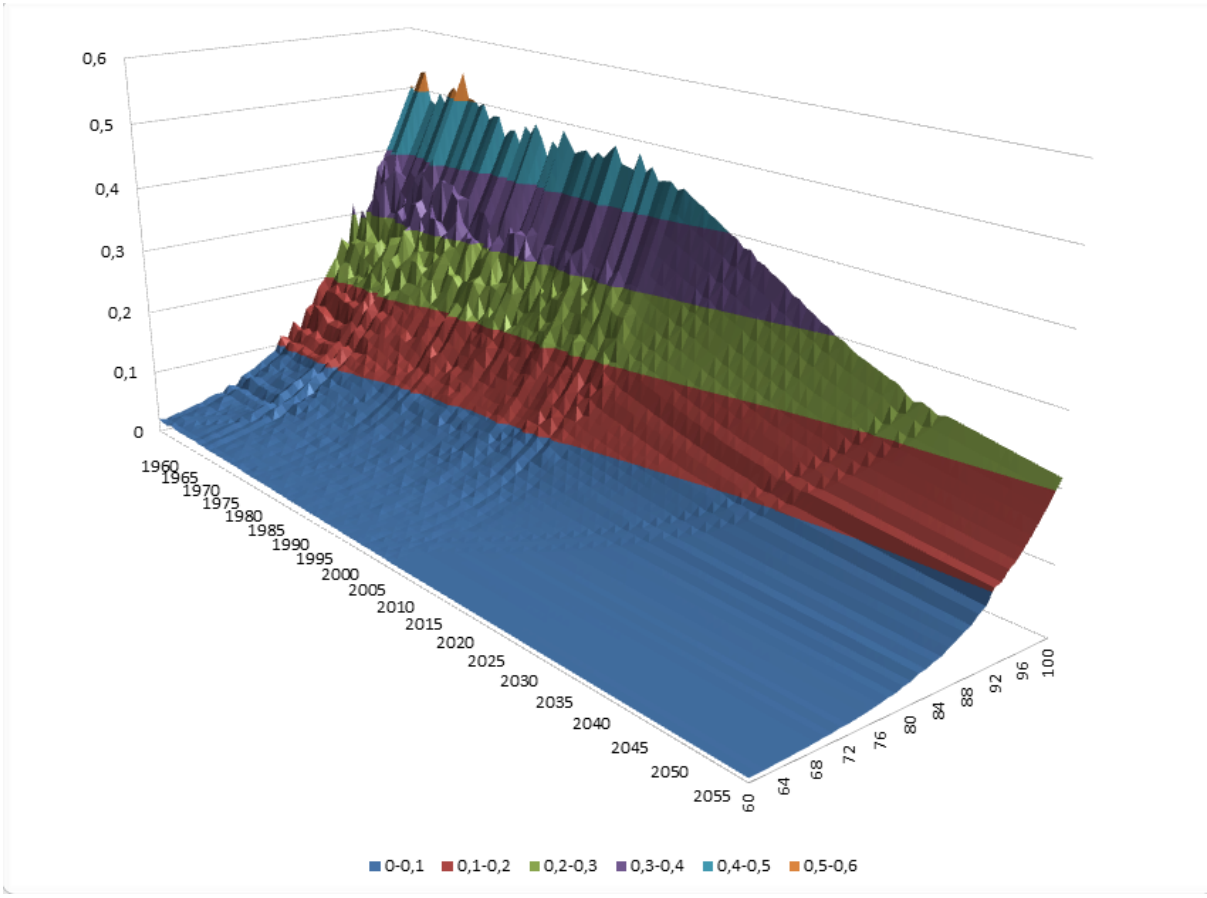
Appendices

A 95% Bootstrap Akaike weighted prediction intervals and actual values of central death rates

	x=60			x=70			x=80		
t	95% PI		Actual	95% PI		Actual	95% PI		Actual
	Lower B.	Upper B.	Value	Lower B.	Upper B.	Value	Lower B.	Upper B.	Value
2000	0,0108	0,0117	0,0107	0,0277	0,0303	0,0266	0,0767	0,0847	0,0775
2001	0,0105	0,0118	0,0105	0,0266	0,0312	0,0269	0,0724	0,0883	0,0716
2002	0,0102	0,012	0,0105	0,0252	0,0321	0,0253	0,0686	0,0928	0,0722
2003	0,0099	0,0123	0,0107	0,0241	0,0334	0,0255	0,0649	0,0974	0,0729
2004	0,0097	0,0126	0,01	0,0228	0,0345	0,024	0,0614	0,1025	0,0701
2005	0,0094	0,0128	0,0102	0,0215	0,036	0,0235	0,0578	0,1085	0,0734
2006	0,0091	0,0131	0,0102	0,0205	0,0375	0,0223	0,0544	0,1139	0,0643
2007	0,0089	0,0134	0,0098	0,0195	0,0392	0,0228	0,0514	0,1212	0,0661
2008	0,0087	0,0136	0,0099	0,0186	0,0409	0,0217	0,0482	0,128	0,0629
2009	0,0085	0,0139	0,009	0,0177	0,0427	0,0209	0,0452	0,1372	0,0617

	x=90			x=100		
t	95% PI		Actual	95% PI		Actual
	Lower B.	Upper B.	Value	Lower B.	Upper B.	Value
2000	0,2018	0,2285	0,2033	0,4626	0,5388	0,4597
2001	0,19	0,2435	0,2032	0,4305	0,5865	0,4576
2002	0,1782	0,2601	0,1974	0,4008	0,6377	0,4551
2003	0,1677	0,2788	0,2187	0,3746	0,7065	0,4818
2004	0,1573	0,2994	0,1982	0,3477	0,7778	0,4567
2005	0,1475	0,3223	0,2028	0,3233	0,8603	0,4662
2006	0,1365	0,3458	0,1896	0,2987	0,9553	0,4473
2007	0,1276	0,3764	0,1935	0,278	1	0,4503
2008	0,1194	0,4056	0,1924	0,2562	1	0,4516
2009	0,1107	0,4423	0,1837	0,2364	1	0,4414

B Akaike weighted projected mortality surface



C Akaike weighted projected mortality profiles for ages x = 60, 70, 80, 90 and 100

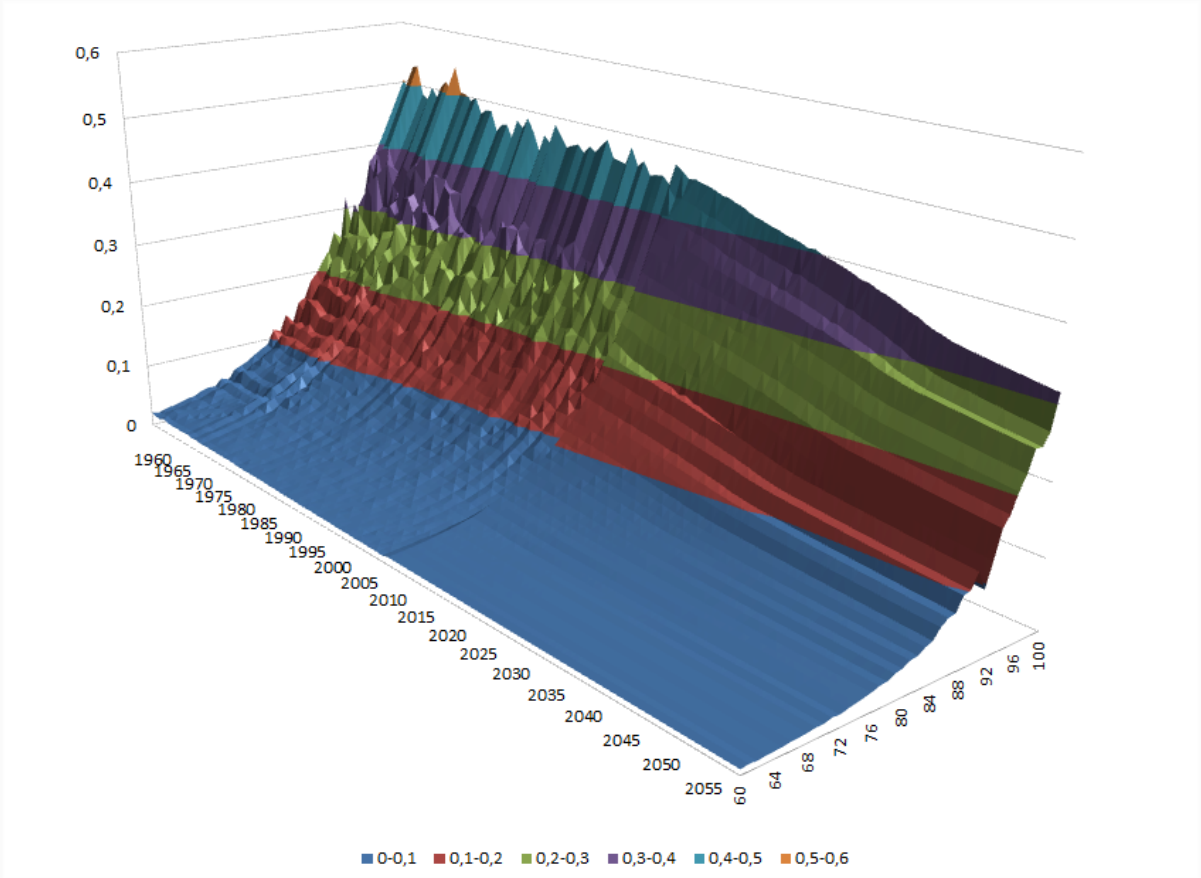
	2010	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**
60	0,0088	0,0087	0,0087	0,0086	0,0085	0,0085	0,0084	0,0083	0,0082	0,0082
70	0,0213	0,0205	0,0198	0,0201	0,0197	0,0192	0,0194	0,0193	0,0191	0,0189
80	0,0606	0,0598	0,0578	0,0572	0,0556	0,0542	0,0534	0,0524	0,0515	0,0505
90	0,1803	0,1741	0,1722	0,1681	0,1653	0,1621	0,1570	0,1553	0,1507	0,1482
100	0,4465	0,4383	0,4318	0,4268	0,4188	0,4134	0,4057	0,3992	0,3950	0,3839
	2020	**2021**	**2022**	**2023**	**2024**	**2025**	**2026**	**2027**	**2028**	**2029**
60	0,0081	0,0080	0,0080	0,0079	0,0078	0,0078	0,0077	0,0076	0,0076	0,0075
70	0,0187	0,0185	0,0182	0,0180	0,0178	0,0176	0,0174	0,0173	0,0171	0,0169
80	0,0499	0,0479	0,0463	0,0469	0,0459	0,0448	0,0453	0,0448	0,0444	0,0439
90	0,1439	0,1420	0,1376	0,1361	0,1324	0,1293	0,1275	0,1251	0,1230	0,1208
100	0,3842	0,3719	0,3684	0,3603	0,3549	0,3486	0,3385	0,3353	0,3258	0,3210
	2030	**2031**	**2032**	**2033**	**2034**	**2035**	**2036**	**2037**	**2038**	**2039**
60	0,0074	0,0074	0,0073	0,0072	0,0072	0,0071	0,0070	0,0070	0,0069	0,0069
70	0,0167	0,0165	0,0163	0,0161	0,0160	0,0158	0,0156	0,0154	0,0153	0,0151
80	0,0434	0,0429	0,0424	0,0419	0,0414	0,0409	0,0404	0,0399	0,0395	0,0390
90	0,1194	0,1149	0,1110	0,1125	0,1103	0,1076	0,1088	0,1079	0,1068	0,1057
100	0,3124	0,3087	0,2995	0,2967	0,2892	0,2829	0,2792	0,2744	0,2702	0,2657
	2040	**2041**	**2042**	**2043**	**2044**	**2045**	**2046**	**2047**	**2048**	**2049**
60	0,0068	0,0067	0,0067	0,0066	0,0066	0,0065	0,0065	0,0064	0,0063	0,0063
70	0,0149	0,0147	0,0146	0,0144	0,0143	0,0141	0,0139	0,0138	0,0136	0,0135
80	0,0385	0,0381	0,0376	0,0372	0,0368	0,0363	0,0359	0,0355	0,0351	0,0346
90	0,1045	0,1034	0,1023	0,1011	0,1000	0,0989	0,0978	0,0968	0,0957	0,0947
100	0,2629	0,2534	0,2454	0,2489	0,2442	0,2387	0,2414	0,2397	0,2376	0,2354
	2050	**2051**	**2052**	**2053**	**2054**	**2055**	**2056**	**2057**	**2058**	**2059**
60	0,0062	0,0062	0,0061	0,0061	0,0060	0,0060	0,0059	0,0059	0,0058	0,0058
70	0,0133	0,0132	0,0130	0,0129	0,0127	0,0126	0,0125	0,0123	0,0122	0,0121
80	0,0342	0,0338	0,0334	0,0330	0,0327	0,0323	0,0319	0,0315	0,0312	0,0308
90	0,0936	0,0926	0,0916	0,0906	0,0896	0,0886	0,0876	0,0866	0,0857	0,0847
100	0,2332	0,2310	0,2288	0,2266	0,2244	0,2222	0,2201	0,2180	0,2159	0,2138

D 95% HPD prediction intervals and actual values of central death rates

	x=60			x=70			x=80		
t	95% PI		Actual	95% PI		Actual	95% PI		Actual
	Lower B.	Upper B.	Value	Lower B.	Upper B.	Value	Lower B.	Upper B.	Value
2000	0,0105	0,0126	0,0107	0,0253	0,0313	0,0266	0,0711	0,0887	0,0775
2001	0,0101	0,0128	0,0105	0,0239	0,0319	0,0269	0,0672	0,0897	0,0716
2002	0,0098	0,013	0,0105	0,0223	0,0319	0,0253	0,0644	0,0915	0,0722
2003	0,0096	0,0132	0,0107	0,0217	0,032	0,0255	0,062	0,0921	0,0729
2004	0,0093	0,0132	0,01	0,0202	0,0319	0,024	0,0595	0,0933	0,0701
2005	0,0091	0,0132	0,0102	0,0193	0,032	0,0235	0,0573	0,0932	0,0734
2006	0,0088	0,0134	0,0102	0,0184	0,0323	0,0223	0,0543	0,0939	0,0643
2007	0,0087	0,0134	0,0098	0,0178	0,0324	0,0228	0,053	0,0955	0,0661
2008	0,0085	0,0135	0,0099	0,017	0,0325	0,0217	0,0505	0,0937	0,0629
2009	0,0081	0,0137	0,009	0,0166	0,0328	0,0209	0,0482	0,095	0,0617

	x=90			x=100		
t	95% PI		Actual	95% PI		Actual
	Lower B.	Upper B.	Value	Lower B.	Upper B.	Value
2000	0,1955	0,2348	0,2033	0,4339	0,5153	0,4597
2001	0,189	0,2369	0,2032	0,4253	0,5117	0,4576
2002	0,1829	0,2384	0,1974	0,4098	0,5041	0,4551
2003	0,178	0,241	0,2187	0,4086	0,5153	0,4818
2004	0,1724	0,241	0,1982	0,3969	0,5143	0,4567
2005	0,1688	0,2454	0,2028	0,3914	0,5169	0,4662
2006	0,1632	0,2451	0,1896	0,3817	0,52	0,4473
2007	0,1609	0,2471	0,1935	0,3749	0,5184	0,4503
2008	0,1554	0,2483	0,1924	0,3668	0,52	0,4516
2009	0,1503	0,2498	0,1837	0,3602	0,5257	0,4414

E BMA projected mortality surface



F BMA projected mortality profiles for ages x = 60, 70, 80, 90 and 100

	2010	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**
60	0,0115	0,0114	0,0113	0,0112	0,0111	0,0110	0,0110	0,0109	0,0108	0,0107
70	0,0279	0,0273	0,0264	0,0259	0,0249	0,0243	0,0238	0,0232	0,0229	0,0226
80	0,0797	0,0778	0,0770	0,0755	0,0743	0,0730	0,0712	0,0708	0,0690	0,0677
90	0,2159	0,2118	0,2089	0,2066	0,2043	0,2025	0,1999	0,1983	0,1959	0,1928
100	0,4733	0,4658	0,4566	0,4584	0,4552	0,4534	0,4498	0,4453	0,4440	0,4396
	2020	**2021**	**2022**	**2023**	**2024**	**2025**	**2026**	**2027**	**2028**	**2029**
60	0,0106	0,0105	0,0104	0,0104	0,0103	0,0102	0,0101	0,0100	0,0099	0,0098
70	0,0223	0,0221	0,0218	0,0216	0,0213	0,0211	0,0209	0,0206	0,0204	0,0201
80	0,0663	0,0650	0,0633	0,0621	0,0601	0,0587	0,0576	0,0563	0,0553	0,0547
90	0,1922	0,1892	0,1875	0,1847	0,1825	0,1800	0,1769	0,1757	0,1726	0,1703
100	0,4365	0,4308	0,4266	0,4240	0,4202	0,4181	0,4135	0,4119	0,4090	0,4041
	2030	**2031**	**2032**	**2033**	**2034**	**2035**	**2036**	**2037**	**2038**	**2039**
60	0,0098	0,0097	0,0096	0,0095	0,0094	0,0094	0,0093	0,0092	0,0091	0,0091
70	0,0199	0,0197	0,0194	0,0192	0,0190	0,0188	0,0186	0,0184	0,0182	0,0180
80	0,0540	0,0533	0,0526	0,0519	0,0514	0,0507	0,0501	0,0495	0,0489	0,0482
90	0,1680	0,1651	0,1615	0,1596	0,1560	0,1530	0,1511	0,1484	0,1468	0,1453
100	0,4054	0,3997	0,3973	0,3938	0,3906	0,3864	0,3814	0,3802	0,3749	0,3716
	2040	**2041**	**2042**	**2043**	**2044**	**2045**	**2046**	**2047**	**2048**	**2049**
60	0,0090	0,0089	0,0089	0,0088	0,0087	0,0086	0,0086	0,0085	0,0084	0,0084
70	0,0178	0,0176	0,0175	0,0172	0,0170	0,0169	0,0167	0,0165	0,0163	0,0161
80	0,0477	0,0471	0,0465	0,0460	0,0454	0,0449	0,0443	0,0438	0,0433	0,0428
90	0,1441	0,1426	0,1413	0,1397	0,1385	0,1375	0,1360	0,1349	0,1336	0,1324
100	0,3675	0,3631	0,3577	0,3545	0,3482	0,3437	0,3403	0,3362	0,3342	0,3319
	2050	**2051**	**2052**	**2053**	**2054**	**2055**	**2056**	**2057**	**2058**	**2059**
60	0,0083	0,0082	0,0082	0,0081	0,0080	0,0080	0,0079	0,0078	0,0078	0,0077
70	0,0160	0,0158	0,0156	0,0154	0,0153	0,0151	0,0150	0,0148	0,0146	0,0145
80	0,0423	0,0418	0,0413	0,0408	0,0402	0,0398	0,0394	0,0389	0,0384	0,0379
90	0,1312	0,1298	0,1287	0,1276	0,1263	0,1251	0,1239	0,1229	0,1218	0,1206
100	0,3299	0,3279	0,3263	0,3243	0,3224	0,3211	0,3192	0,3173	0,3154	0,3141