

# The Sound of Silence: Observational Learning in the U.S. Kidney Market

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**Abstract**

Mere observation of others' choices can be informative about product quality. This paper develops an individual-level dynamic model of observational learning, and applies it to a novel data set from the U.S. kidney market where transplant candidates on a waiting list sequentially decide whether to accept a kidney offer. We find strong evidence of observational learning: patients draw negative quality inferences from earlier refusals in the queue, thus becoming more inclined towards refusal themselves. This self-reinforcing chain of inferences lead to poor kidney utilization despite the continual shortage in kidney supply. Counterfactual policy simulations show that patients would have made more efficient use of kidneys had the concerns behind earlier refusals been shared. This study yields a set of marketing implications. In particular, we show that observational learning and information sharing shape consumer choices in markedly different ways. Optimal marketing strategies should take into account on how consumers learn from others.

**Keywords:** observational learning; learning models; informational cascades; herding; quality inference; Bayes' rule; dynamic programming; kidney allocation

## 1 Introduction

Maciej Lampe declared for the NBA draft at the perfect time. He was the rarest commodity in an NBA draft—a tall, young, European big man with a sweet shooting stroke. He was seen as raw but full of potential, which made him a top ten pick in most experts’ projections, and as high as number five overall ([www.nba.com](http://www.nba.com), June 27, 2003). Unfortunately, on draft day, the Miami Heat passed on Lampe at number five, and the bad news started to snowball ([sports.ESPN.go.com](http://sports.ESPN.go.com), June 26, 2003). Teams grossly overestimated the risks in investing a first round pick on Lampe, allowing him to slip all the way to the second round, at number 30 overall. Subsequently playing in BC Khimki Moscow, Lampe was awarded as the MVP in the Russian Cup final in February 2008.

Maciej Lampe is not alone. In labor markets, an episode of unemployment is known to dampen the success of job search, beyond what is justified by the job candidate’s qualification. In housing markets, skepticism accumulates around the value of a property as its “time on market” increases, forcing some sellers to relist their properties to break this chain of negative inferences. In general, people frequently engage in “observational learning,” drawing quality inferences from mere observation of peer choices: Restaurants that maintain a sizable waiting list are often perceived to be of high quality; book buyers pursue bestsellers; internet surfers swarm high click-volume contents. Marketers too have woken up to the prevalence of observational learning, and have created innovative promotional tactics to harness its magic. For example, to introduce the T68i phones to the U.S. in 2002, Sony Ericsson sent trained actresses to bars and lounges with the phones, in hopes that onlookers would notice and believe that they stumbled onto a hot new product (*Wall Street Journal*, July 31, 2002). The goal of this paper is to empirically model observational learning behavior and its impact on choices.

It is challenging, however, to empirically identify the existence and isolate the impact of observational learning. First, observation of choices often coexists with other sources of quality information such as word-of-mouth communication (e.g. Ellison and Fudenberg 1995, Godes and Mayzlin 2004, Mayzlin 2006), payoff experiences (e.g., Nelson 1970, Erdem and Keane 1996, Camerer and Ho 1999, Villas-Boas 2004 and 2006, Hitsch 2006, Narayanan, Chintagunta and Miravete 2007), and the supplier’s selection of marketing mix variables (e.g., Moorthy and Srinivasan 1995, Wernerfelt 1995, Desai 2000, Anderson

and Simester 2001, Guo and Zhao 2008). Second, even in markets where observational learning plays a dominant role, the choice dynamics are often complex. For example, a potential restaurant patron may not know whether those waiting in line had all independently chosen this restaurant, or some had been attracted by the line itself. Depending on the construction of the choice sequence, the quality inference can be vastly different.

This paper meets these challenges by studying observational learning in perhaps its cleanest environment—the U.S. market of transplant kidneys. When a deceased-donor kidney is procured, compatible transplant candidates are sorted into a queue following a nationally implemented priority system. The kidney travels down the queue until a patient is willing to accept it for transplantation. It is ideal to study observational learning in this kidney market for the following reasons. First, decisions are sequential, and the sequence is constructed through a commonly known process. Second, privacy concerns and the limited decision time minimize the chance for between-patient communication. Meanwhile, observational learning is fully enabled in that all previous decisions are observable—the fact that a patient is offered a kidney unambiguously implies that all preceding patients on the queue have turned down this kidney. Third, the kidney market is unlikely to be influenced by other primary mechanisms behind uniform social behavior, such as sanctions of deviants, preference for social identification (e.g., Kuksov 2007), and network effects (e.g., Yang and Allenby 2003, Nair, Chintagunta and Dubé 2004, Sun, Xie and Cao 2004). In particular, kidneys do not contain the “public appearance value” that partly explains the urge for possessing the right cell phone, choosing the right restaurant, or sporting the right fashion gear.

This paper adopts a structural Bayesian approach to modeling observational learning. While all patients on a queue observe the objective kidney quality measures (e.g., donor age), each patient also receives a private quality signal (e.g., her physician’s recommendation). If a kidney is passed on to the second patient, she knows that the first patient’s private signal must have failed to reach a threshold determined by the first patient’s utility function. The second patient can then apply Bayes’ rule to update her quality perception of this kidney. *Ceteris paribus*, the first patient’s rejection decision lowers the second patient’s perception of the kidney’s quality and hence her propensity to accept. The second patient’s likely refusal in turn lowers the quality perception for subsequent patients, triggering a herd of refusals down the queue. As a result, a kidney’s chance of

acceptance critically depends on its choice history as well as its intrinsic quality.

There are several advantages to the structural modeling approach.<sup>1</sup> The pioneering works of Banerjee (1992) and Bikhchandani, Hirshleifer and Welch (1992) have theoretically proven that observational learning may lead to *informational cascades* and *herd behavior*, where individuals rationally ignore their private information and repeat their predecessors' actions. Empirically documenting observational learning therefore often relies on evidence of convergence in actions (e.g., Anderson and Holt 1997, Çelen and Kariv 2004).<sup>2</sup> As the first study to structurally model observational learning at an individual level, this paper does not require action convergence to identify observational learning. In fact, by embedding sequential Bayesian updating in a choice model, we are able to quantify the impact of observational learning from the continuous changes in posterior valuation, which we recover from the discrete variation in observed choices. Furthermore, this individual-level approach allows us to explicitly model how observational learning of common values (such as kidney quality) is moderated by private values (such as patient-donor tissue match). Last, the structural framework enables a set of policy experiments, especially counterfactual comparison of an array of learning mechanisms.

The most common reason for patients to reject a kidney offer is that the kidney is believed to be of marginal quality and that patients choose to wait for better kidneys (United Network for Organ Sharing (UNOS) 2002 Annual Report). That is, kidney adoption decisions involve dynamic tradeoff. For example, even if kidneys are believed to be of poor quality when they reach the back of the queue, patients at the back of the queue are also less likely to receive good kidneys in future. To model this inter-temporal tradeoff, we cast quality learning in a dynamic choice setting where forward-looking patients seek to maximize their expected discounted present value. This dynamic model allows us to capture how patients' decisions depend on the progression of their health conditions, their chance of getting kidney offers in future, and the quality of these future kidney offers, which in turn depends on other patients' decision rule.

We find significant evidence of observational learning. At the first glance, even iden-

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<sup>1</sup>Please see Chintagunta, Erdem, Rossi, and Wedel (2006) for discussion of the development and application of structural models in marketing.

<sup>2</sup>Please see Bikhchandani, Hirshleifer and Welch (1998) for a review of the observational learning literature.

tical kidneys from the same donor are received much differently. While some kidneys are accepted early on in the queue, their identical counterparts have to go far down the line to find a transplant recipient. In other words, early rejections seem to considerably influence subsequent decisions. After further controlling for patient-donor match, deterioration of kidney quality when traveling down the line, patients' option value of waiting, and patients' risk attitudes, model estimation confirms the significant impact of observational learning—on average, the further a kidney travels down the queue, the lower its perceived quality. A competing explanation is that negative information about kidney quality, although unobservable to the researcher, has lowered the acceptance propensity of all patients. This explanation is modeled, estimated, and ruled out.

Another primary learning mechanism in social contexts is information sharing. Policy permitting, a patient could have obtained private quality signals from her predecessors who have evaluated and rejected the kidney. Observational learning and information sharing have distinct choice implications. To see this, suppose a patient receives a favorable signal but decides to reject the kidney due to her higher standards. A unique prediction of observational learning is that a rejection always (weakly) decreases subsequent patients' quality perception. However, if this favorable private signal is shared with subsequent patients, it may help them evaluate the kidney positively despite the rejection decision. If the average of private signals reveals the true underlying value of a kidney, when more signals aggregate, choices will converge to an efficient level. Indeed, policy experiments show that patients would have made much more efficient decisions were they able to communicate the reasons behind rejection decisions. This finding may help the U.S. organ allocation system alleviate the urgent inefficiency problem, where “most of the refused kidneys are of acceptable clinical value” despite the significant shortage of transplant kidney supply (UNOS 2002 Annual Report).

An important message to marketers in general is that a product's market performance is more than a simple sum of sales. A small number of choices can be critical in determining product success, especially in categories with highly visible choices but limited information sharing. Early adopters and marginal consumers are likely to be such pivotal influencers. Optimal marketing strategies should take into account whether and how consumers learn from others.

The rest of the paper is organized as follows. §2 overviews the U.S. kidney transplant market and presents the data. §3 models three learning mechanisms—no social learning, information sharing, and observational learning, and embeds these learning mechanisms in a dynamic choice model of forward-looking patients. These models are estimated in §4, where we find that the observational learning model explains the data best. A competing model of public (i.e., available to all patients) quality information is ruled out. §5 simulates and compares patient decisions under different learning mechanisms. §6 discusses how the insights would apply to general markets. §7 concludes the paper and suggests directions for future research.

## 2 The U.S. Kidney Market and Data

### 2.1 Overview of the U.S. Kidney Market

Each year more than 40,000 people in the United States develop end-stage renal diseases. The two major treatments are dialysis and kidney transplantation. Dialysis requires at least 9 to 12 hours of treatment at a dialysis center each week. Transplantation frees patients from the inconveniences of dialysis and, if successful, offers a quality of life comparable to one without kidney disease. Transplant kidneys come from either living donors or deceased donors. While the former source is superior, the supply is limited in the United States. As a result, more than half of donated kidneys are procured from deceased donors.

Patients waiting for deceased-donor kidneys are placed on a waiting list administered by the United Network for Organ Sharing (UNOS). When a kidney is procured, blood-type compatible patients within the same organ procurement organization (OPO) are sorted into a queue based on a UNOS point system. The Appendix provides details on the queuing scheme, which is largely first-come-first-serve with local perturbations caused by tissue match, high peak panel reactive antibody (PRA) measures, and juvenility. The kidney is offered sequentially to patients in the queue until someone accepts it for transplantation. During the search for transplant recipients, kidneys are kept frozen and accumulate cold ischemia time. A long cold ischemia time may lead to inferior transplant outcomes. Therefore, kidneys are normally discarded if not accepted within 48 hours.

There has been an acute shortage of deceased-donor kidneys in the United States. According to the 2006 Annual Report of the Organ Procurement and Transplantation Network (OPTN), an organization administered by UNOS under contract with the U.S. Department of Health and Human Services, 32,381 new end-stage renal diseases patients in the U.S. joined the transplant waiting list in 2006, while only 10,659 deceased-donor kidneys were transplanted in that year. Between 1992 and 2006, the number of people on the national kidney waiting list grew from 22,063 to 65,199. Despite the short supply, more than 10% of deceased-donor kidneys are discarded after being repeatedly refused by transplant candidates. OPTN has identified the low kidney acceptance rate as a major challenge to kidney allocation efficiency.

The alarming inefficiency of the current kidney allocation system has attracted substantial attention in academia. Studies suggest a number of solutions including paired kidney exchange (e.g., Roth, Sönmez, and Ünver 2004) and restructuring the queuing mechanism (e.g., Su and Zenios 2004). These studies have focused on system optimization from the policy-maker’s perspective, and have left unexplored the micro-level patient decision processes. While the most common reason for kidney refusal is that the current offer is believed to be of marginal quality such that patients choose to wait for a better kidney (UNOS 2002 Annual Report), it remains unknown how patients form this quality perception. In fact, OPTN laments the fact that medical measures alone are insufficient in predicting patient decisions:

“Although the effects of donor and recipient characteristics on kidney graft survival have been documented, the relationship of these characteristics and center-specific practices on organ acceptance rates is not well understood. We hypothesized that variation in acceptance rates, beyond that which can be explained by recipient and donor characteristics, exists among transplant programs, and that metrics could be developed to quantify these behaviors.” (OPTN/SRTR 2006 Annual Report).

In this study, we investigate the underlying drivers of patient decisions, identify observational learning as an important factor behind the “variation in acceptance rates,” and suggest policy changes to promote efficient kidney usage.



## 2.2 Data

The data set for this study combines the national waiting list data from the UNOS 2002 Annual Report and the transplant detail data from the United States Renal Data System 2001 Annual Report. All analyses focus on the TXGC OPO, a major OPO in Texas and one of the largest OPOs in the United States. Kidneys of different blood types normally enlist different queues of patients due to blood-type compatibility screening. This paper presents the statistics for blood-type A kidneys. The resulting sample includes 338 patients and 275 accepted kidneys. Kidneys arrive at the OPO at an average rate of one per six days, which does not vary significantly over time ( $p = 0.141$ ). An observation is defined as one decision occasion where a patient is presented with the choice of whether to accept a kidney. The sample contains 9,384 observations.

Table 1 presents the summary statistics of three classes of variables in the data. Patient-specific variables include patient age, gender, race, employment status, income, PRA measure, and number of years on dialysis. Kidney-specific variables include donor age, gender, race, and queue information (e.g., queue position of the accepting patient).<sup>3</sup> The most important patient-kidney interactive variables are the tissue match measures. The dummy variables “0 Mismatch,” “0 Mismatch at DR,” and “1 Mismatch at DR” indicate perfect, second-best, and third-best tissue match respectively (see the Appendix for details), where perfect tissue match occurs only 0.4% of the time. Another important patient-kidney interactive factor is the cold ischemia time a kidney has accumulated when offered to a patient. The quality of a kidney may deteriorate as its cold time increases.

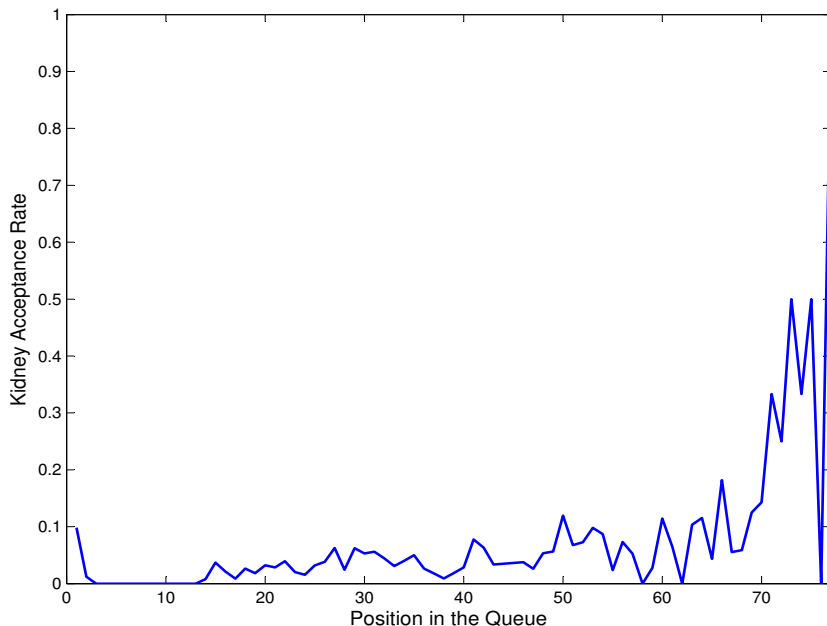
Notably, only 2.9% of kidney offers are accepted. In this data, a kidney can be accepted by as late as the 77<sup>th</sup> patient in the queue. On average, a kidney is accepted by the 34<sup>th</sup> patient, who has already turned down 15 previous offers and has waited 209 days at the time of acceptance. Figure 1 shows kidney acceptance rates across positions in the queue. Approximately 10% of patients at the top of the queue accept the kidney offer. Subsequent analyses reveal that this acceptance rate is largely explained by perfect tissue match, which advances a patient to the top of the queue. Patients from position 2 to

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<sup>3</sup>Other clinical measures include patient body surface area, dialysis modality, comorbidities, donor body surface area, and cause of death. Inclusion of these clinical measures does not significantly alter the estimation results.

position 13 almost always reject the offer. The acceptance rate then increases moderately, remains flat for most part of the queue, and rises sharply at the end. The larger variance near the end of the queue results from a smaller number of observations falling in that range: only 0.35% of observations fall beyond position 70.

Figure 1: Kidney Acceptance Rates across Queue Positions



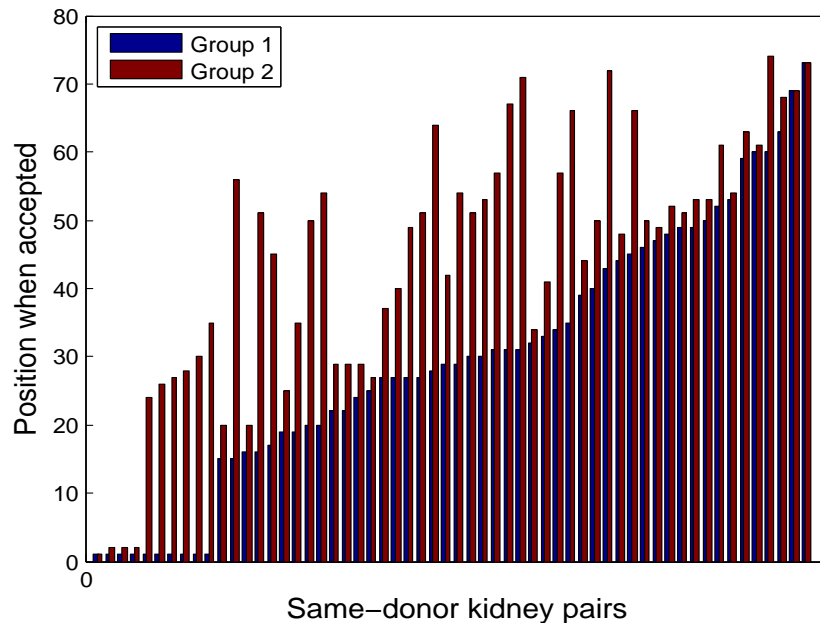
### 2.3 A First Evidence of Observational Learning: Acceptance of Same-Donor Kidneys

A special feature of deceased-donor kidney donation is that sometimes both kidneys can be retrieved from the same donor. Out of 275 kidneys in the sample, there are 58 pairs of same-donor kidneys, each pair containing identical kidney-specific clinical measures and therefore enlisting the same pool of eligible patients. If acceptance decisions are mainly driven by these observable kidney and patient characteristics, same-donor kidneys should exhibit close acceptance patterns.

To see if this is true, we separate the same-donor kidneys into two groups: Group 1 contains the 58 kidneys that are accepted earlier in the queue, and group 2 contains their 58 identical counterparts. Figure 2 illustrates the divergence in acceptance patterns between same-donor kidneys. The 58 pairs of same-donor kidneys are listed along the horizontal axis, each pair adjacently placed. The vertical axis is the queue position of the

accepting patient for each kidney. Even kidneys with identical clinical measures seem to fare differently in their search for transplant recipients. On average, kidneys in group 1 are accepted by the 30<sup>th</sup> patient, while those in group 2 are accepted by the 45<sup>th</sup> patient. The difference in the queue position of the accepting patient is significant ( $t = -4.212$ ,  $p = .000$ ).

Figure 2: Divergence in Acceptance for Same-Donor Kidneys



The distinct acceptance paths for same-donor kidneys suggest that patient decisions may be systematically influenced by a force other than observable kidney and patient characteristics. The data pattern is particularly suited to an observational learning explanation: if patients infer inferior kidney quality from a rejection decision, refusals will be self-reinforcing and will delay acceptance even further. This can be true even if a patient turned down the kidney only due to momentary unavailability (which can be modeled as an idiosyncratic utility shock). As an initial test of whether rejections are self-reinforcing, we estimate a logit model where the dependent variable is whether each patient accepts a kidney offer, and the independent variables include the number of times the kidney has been rejected so far, as well as all observable patient and kidney characteristics (including the kidney's cold time). Consistent with the observational learning hypothesis, the coefficient for the number of previous rejections is negative ( $-0.0138$ ) and

significant ( $p = 0.000$ ).<sup>4</sup>

In fact, an ideal way to identify observational learning in the field is to compare the adoption paths of two identical products and test for path dependence. Same-donor kidneys represent one of the few commodities that satisfy this identicalness condition in naturally occurring markets, and their diverging acceptance paths serve as a first evidence of observational learning. In the following sections, we model observational learning, identify its existence, and quantify its impact on choices.

### 3 A Dynamic Choice Model

This section develops a choice model where patients engage in observational learning, and compares it with two other learning mechanisms: learning from private signals (no social learning), and learning through information sharing. These learning models are cast in a dynamic setting where patients make optimal tradeoff between accepting the current kidney and waiting for future kidneys, given forecast of their future states of being.

#### 3.1 Patients' Dynamic Optimization Problem

Consider a discrete-time infinite-horizon dynamic optimization problem where a patient chooses whether to accept a kidney offer in order to maximize her expected present discounted value.<sup>5</sup> Let  $i$  index patients and  $t = 1, \dots, \infty$  index the kidney arrival time. We consider the Markov Perfect Equilibrium where patients' decisions only rely on payoff-relevant state variables. Let  $S_{it}$  be a vector of all these state variables that are payoff-relevant to patient  $i$  at time  $t$ , and  $d_{it}$  be the decision variable that equals 1 if patient  $i$  accepts kidney  $t$  and 0 if she rejects this kidney offer.

Once she accepts a kidney, a patient moves to the absorbing state of transplantation and receives an expected utility of  $EU(S_{it})$  which captures her expected present dis-

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<sup>4</sup>Although identical kidneys typically have an identical set of eligible patients, those who accept one kidney drop out of the queue for its identical counterpart that arrives later. The logit model including all observable attributes helps to control for such changes in queue composition.

<sup>5</sup>Practically, either the patient or the doctor can make the acceptance decision. This distinction, however, does not conceptually alter the model. Throughout the paper, we refer to the decision-maker as the patient.

counted post-transplant payoffs. If she turns down the kidney, she incurs one period’s cost of waiting  $C(S_{it})$ . Let  $\delta$  denote the discount factor,  $V(S_{it})$  denote a patient’s maximum expected present discounted value given her current state  $S_{it}$ , and  $\mathcal{P}(S_{i,t+1}|S_{it}, d_{it} = 0)$  denote the transition probability of patient  $i$ ’s state from time  $t$  to  $t + 1$  given she rejects kidney  $t$ . The Bellman equation for patient  $i$ ’s dynamic optimization problem at time  $t$  is:

$$V(S_{it}) = \max\{EU(S_{it}), -C(S_{it}) + \delta \int_{S_{i,t+1}} V(S_{i,t+1}) \mathcal{P}(S_{i,t+1}|S_{it}, d_{it} = 0) dS_{i,t+1}\} \quad (3.1)$$

## 3.2 Utility Function and Quality Inference

### 3.2.1 Patients’ Utility Function

In this section, we specify the state variables contained in  $S_{it}$  and formulate  $EU(S_{it})$ , the expected payoff from accepting a kidney offer. Let  $U_{it}(S_{it})$  denote the utility for patient  $i$  to accept the kidney arriving at time  $t$ :

$$U_{it}(S_{it}) = X_{it}\beta + \alpha\theta_t - \alpha\rho\theta_t^2 + \epsilon_{it} \quad (3.2)$$

$X_{it}$  is observable to both patient  $i$  and the econometrician, and contains a constant term, the characteristics of patient  $i$  at time  $t$ , the attributes of kidney  $t$ , and the patient-kidney match measures.  $\beta$  consists of the utility weight parameters associated with  $X_{it}$ .<sup>6</sup> Observable characteristics may not capture the kidney quality completely. Let  $\theta_t$  represent any unobservable (to both the patient and the econometrician) quality component of kidney  $t$ , and let  $\alpha$  be the associated utility weight. Note that since tissue match is the only clinically significant “horizontal” match factor after blood-type compatibility screening (Su, Zenios, and Chertow 2004),  $\theta_t$  is conceptualized as a “vertical” quality component that is of common interest to patients. Patients are allowed to be risk averse towards quality uncertainty. Following Erdem and Keane (1996), we introduce the quadratic term  $\alpha\rho\theta_t^2$  to capture patients’ risk attitudes, where the risk coefficient  $\rho$  is positive if and only if the patient is risk averse. For example, if  $\rho$  is positive, a patient’s utility function will be concave in unobservable kidney quality. Her utility derived from the mean value of

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<sup>6</sup>To keep the model computationally tractable, we do not estimate “parameter heterogeneity” among patients but rely on the individual-level data to capture observable “attribute heterogeneity”.

unobservable kidney quality is thus greater than the mean of the utilities derived from all possible values of unobservable kidney quality. Last,  $\epsilon_{it}$  denotes the idiosyncratic utility shock encountered by patient  $i$  when evaluating kidney  $t$ . For example, a patient may experience momentary inconveniences such as unfavorable physiological conditions which prevent her from accepting instant transplant. Privately observed by patient  $i$ ,  $\epsilon_{it}$  is assumed to follow an i.i.d. Gumbel distribution across patients and across kidneys.

We assume that patients know the distribution of  $\theta_t$  across kidneys, which is assumed to be i.i.d. normal with mean  $\mu$  and variance  $\sigma_\theta^2$ :

$$\theta_t \sim N(\mu, \sigma_\theta^2) \tag{3.3}$$

In addition, patient  $i$  receives a private signal  $s_{it}$  of the unobservable quality  $\theta_t$ . One example of such private signal could be the physician’s quality judgment drawing upon her expertise. Indeed, although organ sharing societies in the United States have published certain policies guiding the kidney allocation process, they have also stated that “this policy, however, does not nullify the physician’s responsibility to use appropriate medical judgment” (UNOS 2002 Annual Report). Without actual data on the signal content, we assume the private signals to follow a conditional i.i.d. normal distribution around  $\theta_t$ , although the model can be extended to incorporate alternative signal distributions.<sup>7</sup> In other words, although private signals vary across individuals, a large-sample average of

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<sup>7</sup>The assumption that private signals are continuous allows for the possibility that physicians communicate a fine gradation of quality judgment. For example, physician recommendations may convey various levels of preferences. Alternatively, physicians may recommend patients to either accept a kidney or reject it. Such binary signals can be modeled as a discrete manifestation of physicians’ latent evaluation of the kidney. Correspondingly, in the learning models presented in this paper, the conditional probability of continuous signals given kidney quality is replaced by the conditional tail probability that a physician’s latent evaluation exceeds or falls below her recommendation threshold given kidney quality. The essence of Bayesian inferences underlying the learning models remains the same.

these signals would be an unbiased indicator of the true quality:<sup>8</sup>

$$s_{it}|\theta_t \sim N(\theta_t, \sigma_s^2) \quad (3.4)$$

As will be discussed later,  $\alpha$ ,  $\sigma_\theta$  and  $\sigma_s$  cannot all be identified. However, we will keep the notation separate throughout to trace the different role each parameter plays in the learning process.

A patient’s inferred value of  $\theta_t$  varies with the information accessible to her. In the rest of this section, we model and compare this quality inference process corresponding to three representative information structures: (1) no social learning, where a patient updates her quality perception based on her knowledge of the prior distribution of  $\theta_t$  and her private signal  $s_{it}$ ; (2) social learning through information sharing, where in addition to the prior distribution and her own signal, a patient also acquires other patients’ private signals through, for example, truthful word-of-mouth communication; and (3) observational learning, where besides the prior distribution and the patient’s private signal, others’ choice decisions contain information about the unobservable quality. Let  $I_{it}$  be the set of aforementioned information that helps patient  $i$  infer the value of  $\theta_t$ . Let  $O_{it}$  be a dummy variable that equals 1 if patient  $i$  is offered a kidney at time  $t$  and 0 otherwise. Lastly, let  $Z_{it}$  denote patient characteristics that affect their cost of waiting.  $Z_{it}$  will be operationalized in §3.4. Patient  $i$ ’s state variables at time  $t$  are therefore decomposed as follows:

$$S_{it} = \{O_{it}, X_{it}, Z_{it}, I_{it}, \epsilon_{it}\} \quad (3.5)$$

The expected payoff for patient  $i$  to accept kidney  $t$  is

$$EU(S_{it}) = E(U_{it}|S_{it}) = X_{it}\beta + \alpha E(\theta_t|I_{it}) - \alpha\rho E(\theta_t^2|I_{it}) + \epsilon_{it}, \quad \text{if } O_{it} = 1 \quad (3.6)$$

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<sup>8</sup>The variance of the private signals  $\sigma_s^2$  may in theory change across kidney episodes. For example, by evaluating kidneys repeatedly, a doctor’s precision in judgment may improve over time. To explore this possibility, we stratify the sample into two subsamples based on a median split of the number of previous offers a patient has received until her current decision. We estimate the model allowing the signal variance for “experienced” patients ( $\sigma_{se}^2$ ) and “inexperienced” patients ( $\sigma_{si}^2$ ) to be different. The likelihood-ratio test fails to reject the null hypothesis that  $\sigma_{se} = \sigma_{si}$  ( $\chi^2(1) = 0.398, p = 0.528$ ). In addition, it is possible that unobservable quality and therefore private signals are correlated across identical kidneys from the same donor. In the estimation we report, unobservable quality and private signals are treated as independent across identical kidneys. A robustness check restricting unobservable quality and private signals to be the same for identical kidneys yields close estimation results.

where  $E(\theta_t^2|I_{it})$  can be decomposed as  $E(\theta_t|I_{it})^2 + E[(\theta_t - E(\theta_t|I_{it}))^2|I_{it}]$ . Therefore, calculating  $EU(S_{it})$  boils down to inferring the posterior distribution of  $\theta_t$  given  $I_{it}$ , which will be modeled in the rest of this section. To complete the utility specification, we normalize the deterministic part of patient  $i$ 's expected payoff to 0 when she does not receive a kidney offer. That is,

$$EU(S_{it}) = \epsilon_{it}, \quad \text{if } O_{it} = 0 \quad (3.7)$$

### 3.2.2 Quality Inference without Social Learning

A patient's expected value of the unobservable quality  $\theta_t$  is equal to the prior mean  $\mu$  if all she knows is the prior distribution of  $\theta_t$ . However, she can fine-tune her quality perception if she also receives a private signal. By Bayes' rule (DeGroot 1970), the posterior expectation of  $\theta_t$  is a weighted average of the prior mean  $\mu$  and the private signal:

$$E(\theta_t|I_{it}) = \frac{\sigma_\theta^2 s_{it} + \sigma_s^2 \mu}{\sigma_\theta^2 + \sigma_s^2}, \quad I_{it} = \{s_{it}\} \quad (3.8)$$

Intuitively, the less accurate the private signal is, the more weight is assigned to the prior quality perception.

### 3.2.3 Quality Inference through Information Sharing

A patient can further update her quality perception when she engages in social learning and obtains private signals from other decision-makers.<sup>9</sup> Let  $r_{it}$  denote patient  $i$ 's position in the queue for kidney  $t$ . For simplicity of presentation, we drop the subscript  $it$ . Suppose a patient acquires private signals from all her  $r - 1$  predecessors, the posterior expectation of  $\theta_t$  is a weighted average of the prior mean  $\mu$  and the sample average of these  $r$  signals:

$$E(\theta_t|I_{it}) = \frac{\sigma_\theta^2 \sum_{j=1}^r s_{jt} + \sigma_s^2 \mu}{r \cdot \sigma_\theta^2 + \sigma_s^2}, \quad I_{it} = \{s_{1t}, \dots, s_{rt}\} \quad (3.9)$$

The weight given to the prior decreases in  $r$ . That is, the more doctors a patient consults, the more likely it is for her to trust the consensus. An analogy in new product diffusion is

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<sup>9</sup>We assume truthful sharing of signals. However, this model can be extended to capture untruthful communication if we can specify a structure for any signal distortion.



that while innovators rely more on their prior quality knowledge, imitators may pay more attention to product reviews. When  $r$  approaches infinity, the posterior expectation of  $\theta_t$  equals the average of all observed signals which, by the law of large numbers, approaches the true value of  $\theta_t$ . This convergence property is consistent with the common notion of “the wisdom of crowds.”

Note that although patients can also share other information such as decisions, in this setup only private signals matter to subsequent patients. Once a patient shares her signal, her actual choice does not add information regarding the quality of this particular kidney. It is possible though that a patient learns more about her *predecessors* by watching their decisions, in which case previous decisions should be part of the information set. Such dynamics are interesting to model in future research.

### 3.2.4 Information Sharing vs. Observational Learning

When communication is costly and others’ private signals unaccessible, mere observations of others’ actual choices can be informative too. Before presenting the observational learning model, we first intuitively describe two key differences between (truthful) information sharing and observational learning.

First, with information sharing, a rejection does not always lower expected quality perceived by subsequent decision-makers. To see this, suppose the second patient is offered a kidney. If the first patient does receive a good signal but rejects the kidney due to poor tissue match, information sharing may actually increase the second patient’s inferred quality. With observational learning, however, the second patient’s inferred quality can only be lowered by the first patient’s rejection. This is because the first patient is more likely to reject the kidney with worse private signals, which are more likely to occur with worse kidneys. The second patient would therefore assign higher probabilities to low kidney qualities by Bayes’ rule. Property 3 in the next section states this result formally.

Second, with information sharing, previous signals enter a patient’s quality evaluation continuously (Equation 3.9). Therefore, extreme values of private signals are diluted in a large sample, eliminating the existence of “pivotal” patients. In contrast, marginal patients can be crucial in shaping subsequent choices with observational learning. This is because a patient’s quality inference is *discontinuous* in her predecessors’ signals under

observational learning due to the discrete nature of choices. To see this, suppose patient one is on the margin but chooses acceptance over rejection. Patient two would then infer that patient one’s private signal must have been “favorable enough.” Suppose alternatively that patient one receives an infinitesimal negative perturbation in her private signal and therefore marginally prefers rejection. This new decision only changes patient one’s own utility infinitesimally. However, patient two’s inferred region of the first signal now becomes the lower tail of the distribution, which decreases patient two’s quality expectation discontinuously. If patient two in turn switches to rejection, patient one’s marginal decrease in private signals can be amplified into chain of rejections down the queue.

These fundamental differences lead to the prediction that choices are ultimately driven by quality with information sharing, but are sensitive to initial choices and marginal choices with observational learning. In the kidney market, the queue ends whenever the kidney is accepted. Therefore, observational learning is asymmetrical in the sense that only observations of rejections influence subsequent patients. Such a market is likely to generate excessive rejections. In the following sections we model observational learning and explore whether it indeed triggers excessive rejections of kidneys. In §6 we discuss a set of aggregate predictions that distinguish between information sharing and observational learning in general markets.

### 3.2.5 Quality Inference through Observational Learning

In this section, we formally model quality inferences when a patient observes all her predecessors’ decisions, but does not know the precise reason behind each decision. The information set for patient in position  $r$  becomes  $I_{it} = \{d_{1t}, \dots, d_{r-1,t}, s_{rt}\}$ . In the kidney market, the fact that the patient in position  $r$  is offered the kidney implies that  $\{d_{1t} = \dots = d_{r-1,t} = 0\}$ . However, the model below can be extended to accommodate a generic permutation of acceptance/rejection decisions in the sequence, and apply to other markets where a product can be accepted by multiple consumers.

#### The First Patient

The first patient decides whether to accept kidney  $t$  based on her own signal  $s_{1t}$ . Her

posterior expectation of  $\theta_t$  is

$$E(\theta_t | s_{1t}) = \frac{\sigma_\theta^2 s_{1t} + \sigma_s^2 \mu}{\sigma_\theta^2 + \sigma_s^2}$$

Note that the expected utility from accepting the kidney increases with the private signal  $s_{1t}$ . At the same time, a patient's current private signal does not affect the utility she can derive from accepting a future kidney offer. This is because private signals are drawn independently around the true unobservable quality (by Assumption 3.4), which in turn is drawn from an independent pool (by Assumption 3.3). Therefore, the first patient accepts kidney  $t$  if and only if  $s_{1t} \geq B_{1t}$ , where  $B_{1t}$  is the cutoff signal that solves the indifference condition:

$$EU(S_{1t}) = -C(S_{1t}) + \delta \int_{S_{1,t+1}} V(S_{1,t+1}) \mathcal{P}(S_{1,t+1} | S_{1t}, d_{1t} = 0) dS_{1,t+1}$$

with  $EU(S_{1t})$  given by Equation 3.6.

### The Second Patient

The second patient infers  $\theta_t$  based on two pieces of information: the rejection decision of the first person  $d_{1t} = 0$ , and her private signal  $s_{2t}$ . By Bayes' Rule, the posterior density of  $\theta_t$  is proportional to the product of the conditional (on  $\theta_t$ ) density of the observed data and the prior density of  $\theta_t$ :

$$p(\theta_t | d_{1t} = 0, s_{2t}) \propto p(d_{1t} = 0, s_{2t} | \theta_t) \cdot p(\theta_t)$$

The first patient's cutoff  $B_{1t}$  determines the informativeness of her decision. However,  $B_{1t}$  is not directly observed by the second patient. For example, she does not observe whether the first patient has turned down the kidney due to poor tissue match or despite good match, even though the quality implications are vastly different. Fortunately, the nationally publicized queuing policies provide patients with "distributional" knowledge of the queue. In fact, a patient is often on a queue with the same set of peer patients. For instance, patients would know that the top of the queue tends to be associated with better tissue match and longer waiting time. Therefore, we assume the second and all subsequent patients to know the distribution of  $B_{1t}$ , denoted as  $G(B_{1t})$ . One sufficient condition for this assumption to hold is common knowledge of the distribution of patient

and kidney attributes among the first patients in the line, of the distribution of patients' idiosyncratic utility, and of the transition probability  $\mathcal{P}(\cdot | \cdot)$ . It follows that

$$p(d_{1t} = 0, s_{2t} | \theta_t) = \int p(s_{1t} < B_{1t}, s_{2t} | \theta_t) dG(B_{1t})$$

Since the private signals  $s_{1t}$  and  $s_{2t}$  are conditionally (on  $\theta_t$ ) independent, the conditional probability of the joint event that the first signal is below  $B_{1t}$  and the second event equals  $s_{2t}$  is the product of the conditional probabilities of these two events:

$$p(s_{1t} < B_{1t}, s_{2t} | \theta_t) = p(s_{1t} < B_{1t} | \theta_t) p(s_{2t} | \theta_t) = \Phi\left(\frac{B_{1t} - \theta_t}{\sigma_s}\right) \phi\left(\frac{s_{2t} - \theta_t}{\sigma_s}\right)$$

where  $\Phi(\cdot)$  and  $\phi(\cdot)$  are the c.d.f. and p.d.f. of the standard normal distribution respectively. Consequently, the posterior density of  $\theta_t$  can be written as

$$p(\theta_t | d_{1t} = 0, s_{2t}) \propto \phi\left(\frac{s_{2t} - \theta_t}{\sigma_s}\right) \phi\left(\frac{\theta_t - \mu}{\sigma_\theta}\right) \int \Phi\left(\frac{B_{1t} - \theta_t}{\sigma_s}\right) dG(B_{1t}) \quad (3.10)$$

The second patient's posterior expectation of quality  $\theta_t$  is

$$E(\theta_t | d_{1t} = 0, s_{2t}) = \frac{\int p(\theta_t | d_{1t} = 0, s_{2t}) \theta_t d\theta_t}{\int p(\theta_t | d_{1t} = 0, s_{2t}) d\theta_t}$$

where the denominator serves as a normalizing factor to ensure that the posterior density of  $\theta_t$  integrates to one.

Other things being equal, the higher  $s_{2t}$ , and the lower  $G(B_{1t})$  in the sense of first-order stochastic dominance, the higher the second patient's expected quality of kidney  $t$ . This can be seen from equation 3.10: both a larger  $B_{1t}$  and a larger  $s_{2t}$  shift more weight to  $\theta_t$  values towards the upper tail of its posterior distribution. The intuition is that the second patient will infer higher kidney quality when she receives a more favorable private signal, and when she knows that the first patient rejected the kidney only due to her high standards. Since  $E(\theta_t | d_{1t} = 0, s_{2t})$  increases in  $s_{2t}$ , the second patient's decision rule can also be characterized by a cutoff strategy. She accepts the kidney if and only if  $s_{2t} \geq B_{2t}$ , where  $B_{2t}$  is the private signal value that makes her just indifferent between acceptance and rejection:

$$EU(S_{2t}) = -C(S_{2t}) + \delta \int_{S_{2,t+1}} V(S_{2,t+1}) \mathcal{P}(S_{2,t+1} | S_{2t}, d_{2t} = 0) dS_{2,t+1}$$

## A Generic Patient

The third patient draws quality inference in the same way as the second patient, knowing that the second patient's rejection decision had been partially triggered by the first patient's rejection. In general, after observing  $r - 1$  previous rejection decisions and her own signal, patient  $r$ 's posterior expected value of  $\theta_t$  is

$$E(\theta_t | d_{1t} = \dots = d_{r-1,t} = 0, s_{rt}) = \frac{\int p(\theta_t | d_{1t} = \dots = d_{r-1,t} = 0, s_{rt}) \theta_t d\theta_t}{\int p(\theta_t | d_{1t} = \dots = d_{r-1,t} = 0, s_{rt}) d\theta_t} \quad (3.11)$$

where

$$p(\theta_t | d_{1t} = \dots = d_{r-1,t} = 0, s_{rt}) = \phi\left(\frac{s_{rt} - \theta_t}{\sigma_s}\right) \phi\left(\frac{\theta_t - \mu}{\sigma_\theta}\right) \int \dots \int \prod_{j=1}^{r-1} \Phi\left(\frac{B_{jt} - \theta_t}{\sigma_s}\right) dG(B_{1t}, \dots, B_{r-1,t}) \quad (3.12)$$

The patient in position  $r$  accepts kidney  $t$  if and only if  $s_{rt} \geq B_{rt}$ , where  $B_{rt}$  solves the indifference condition

$$EU(S_{rt}) = -C(S_{rt}) + \delta \int_{S_{r,t+1}} V(S_{r,t+1}) \mathcal{P}(S_{r,t+1} | S_{rt}, d_{rt} = 0) dS_{r,t+1} \quad (3.13)$$

The posterior expected quality has a set of clean properties. For simplicity of presentation, let  $h_{rt} = E(\theta_t | d_{1t} = \dots = d_{r-1,t} = 0, s_{rt})$  represent the posterior expected quality from observational learning:

**Property 1** *The higher a patient's private signal, the higher her expected quality:  $\frac{\partial h_{rt}}{\partial s_{rt}} > 0$ .*

**Property 2** *The higher previous patients' acceptance standard, the higher the expected quality: Let  $G$  and  $G'$  be any two cumulative distribution functions of previous patients' acceptance standards.  $h_{rt}(G) > h_{rt}(G')$  if  $G$  first-order stochastically dominates  $G'$ .*

**Property 3** *Other things being equal, a rejection decision always (weakly) decreases subsequent patients' expected quality: If  $s_{rt} = s_{r+1,t}$ , then  $d_{rt} = 0 \Rightarrow h_{r+1,t} \leq h_{rt}$ .*

Properties 1 and 2 can be shown in the same way as for the second patient. To see why Property 3 holds, notice from Equation 3.12 that when  $s_{rt} = s_{r+1,t}$ ,  $p(\theta_t | d_{1t} =$

$\dots = d_{r,t} = 0, s_{r+1,t}$ ) differs from  $p(\theta_t | d_{1t} = \dots = d_{r-1,t} = 0, s_{rt})$  in the integrand by  $\Phi(\frac{B_{rt} - \theta_t}{\sigma_s})$ , which gives more weight to lower values of  $\theta_t$  for any  $B_{rt} < \infty$ . Therefore,  $h_{r+1,t}$  is lower than or equal to  $h_{rt}$  when patient  $r$  rejects kidney  $t$ . Intuitively, if both patients have witnessed the  $r - 1$  previous decisions, the additional rejection decision seen by patient  $r + 1$  can only (weakly) decrease her expected quality of the kidney unless she receives a sufficiently favorable private signal. It can be similarly shown that, other things being equal, an acceptance decision always (weakly) increases subsequent decision-makers' expected quality.

Note that Property 3 pertains to contexts such as the kidney market where match-related attributes (in particular, tissue type) are observable to patients. Property 3 may not hold if choices are driven by match and if match attributes are yet to be learned. For example, suppose two decision-makers are known to have opposite taste preferences. One person's rejection signals that the product is more likely to match the other person's tastes. In those scenarios, rejection may subsequently spur more acceptance.

It can be seen from the derivation so far that patients' inter-temporal tradeoff affects kidney adoption in at least two ways. A patient's option value of waiting depends on her chance of receiving future kidney offers and the quality of these kidneys. Meanwhile, the same patient's quality perception of the current kidney offer depends on the acceptance standards of her predecessors, which in turn depend on their forecast of the future. To precisely model the dynamics, next we develop the transition probability of patients' dynamic optimization problem.

### 3.3 Transition Probability

The overall transition probability of patients' dynamic optimization problem is decomposed as  $\mathcal{P}(S_{i,t+1} | S_{it}, d_{it} = 0) = \mathcal{P}(O_{i,t+1}, X_{i,t+1}, Z_{i,t+1}, I_{i,t+1}, \epsilon_{i,t+1} | O_{it}, X_{it}, Z_{it}, I_{it}, \epsilon_{it}, d_{it} = 0)$ . The following three features of the state space help simplify this transition probability.

First, since the idiosyncratic utility  $\epsilon_{it}$  is i.i.d. across both patients and time, it is exogenous to the choice variable and orthogonal to all other state variables. Therefore, its transition is independent of the transition of all other state variables:  $\mathcal{P}(S_{i,t+1} | S_{it}, d_{it} = 0) = \mathcal{P}(\epsilon_{i,t+1}) \cdot \mathcal{P}(O_{i,t+1}, X_{i,t+1}, Z_{i,t+1}, I_{i,t+1} | O_{it}, X_{it}, Z_{it}, I_{it}, d_{it} = 0)$ .

Second, since private signals are drawn from an i.i.d. distribution around  $\theta_t$ , which in turn is distributed independently over time, private signals are uncorrelated over time. Therefore, without social learning  $I_{i,t+1}$  is independent of  $I_{it}$ . With information sharing,  $I_{it}$  contains  $r_{it}$  private signals. With observational learning,  $I_{it}$  contains  $r_{it} - 1$  rejections and one private signal. Therefore, for both information sharing and observational learning, given  $O_{i,t+1}$ , the statistical dependence between  $I_{i,t+1}$  and  $I_{it}$  is transmitted entirely through the statistical dependence between  $r_{i,t+1}$  and  $r_{it}$ :  $\mathcal{P}(O_{i,t+1}, X_{i,t+1}, Z_{i,t+1}, I_{i,t+1} | O_{it}, X_{it}, Z_{it}, I_{it}, d_{it} = 0) = \mathcal{P}(O_{i,t+1}, X_{i,t+1}, Z_{i,t+1}, s_{i,t+1}, r_{i,t+1} | O_{it}, X_{it}, Z_{it}, r_{it}, d_{it} = 0)$ .

Third, the current offer status  $O_{it}$  and the current decision  $d_{it}$  do not affect  $X_{i,t+1}$  or  $Z_{i,t+1}$ , which contains exogenous variables. Neither do they affect  $s_{i,t+1}$ , which will be independently redrawn in period  $t + 1$ . In addition, since the UNOS priority system does not punish kidney refusals, future queue position  $r_{i,t+1}$  does not depend on  $O_{it}$  or  $d_{it}$ . Last, the chance for patient  $i$  to receive a kidney offer in period  $t + 1$  is sufficiently determined by  $X_{i,t+1}$ ,  $Z_{i,t+1}$  and  $r_{i,t+1}$ , and does not directly rely on her state or decision at time  $t$ . Altogether,  $\mathcal{P}(O_{i,t+1}, X_{i,t+1}, Z_{i,t+1}, s_{i,t+1}, r_{i,t+1} | O_{it}, X_{it}, Z_{it}, r_{it}, d_{it} = 0) = \mathcal{P}(O_{i,t+1}, s_{i,t+1} | X_{i,t+1}, Z_{i,t+1}, r_{i,t+1}) \cdot \mathcal{P}(X_{i,t+1}, Z_{i,t+1}, r_{i,t+1} | X_{it}, Z_{it}, r_{it})$ .

In combination, the overall transition probability of the state space can be written as

$$\begin{aligned} & \mathcal{P}(S_{i,t+1} | S_{it}, d_{it} = 0) = \\ & \mathcal{P}(\epsilon_{i,t+1}) \cdot \mathcal{P}(O_{i,t+1}, s_{i,t+1} | X_{i,t+1}, Z_{i,t+1}, r_{i,t+1}) \cdot \mathcal{P}(X_{i,t+1}, Z_{i,t+1}, r_{i,t+1} | X_{it}, Z_{it}, r_{it}) \end{aligned} \quad (3.14)$$

The first component  $\mathcal{P}(\epsilon_{i,t+1})$  is simply the p.d.f. of the Gumbel distribution. The second component depends on individual equilibrium choice probabilities, which will be developed in Section 3.4. The last component can be estimated from the data (see the Online Appendix for details).

### 3.4 Choice Probabilities

Assume a patient's cost of waiting is determined by her current state and an idiosyncratic utility shock  $\epsilon_{iot}$ . That is

$$C(S_{it}) = Z_{it}\gamma + \epsilon_{iot}, \quad Z_{it} \subseteq S_{it} \quad (3.15)$$

where  $Z_{it}$  contains patient  $i$ 's number of years on dialysis, income, and employment status. These variables may affect the patient's health status and well-being while waiting, and

capture her opportunity cost of time.<sup>10</sup>

Given the i.i.d. Gumbel assumption of the idiosyncratic utility shocks, the probability of patient  $i$  accepting kidney  $t$  given her current state is

$$Pr(d_{it} = 1|S_{it}) = \frac{e^{EU(S_{it})}}{e^{EU(S_{it})} + e^{-C(S_{it}) + \delta \int V(S_{i,t+1}) \mathcal{P}(S_{i,t+1}|S_{it}, d_{it}=0) dS_{i,t+1}}} \quad (3.16)$$

Data on patients' private signals, such as the physician's recommendations, would be ideal to have but is often unavailable to the researcher. To circumvent this problem, the private signals are integrated out to evaluate the acceptance probabilities of a kidney. Given quality  $\theta_t$ , signals about kidney  $t$  are conditionally independent, so are patients' acceptance probabilities for kidney  $t$ . Denote as  $Pr(R_t|\theta_t)$  the conditional probability that kidney  $t$  of true unobservable quality  $\theta_t$  is accepted by the patient in position  $R_t$ :

$$Pr(R_t|\theta_t) = \prod_{i=1}^{R_t-1} \int Pr(d_{it} = 0|S_{it}) \phi\left(\frac{s_{it} - \theta_t}{\sigma_s}\right) ds_{it} \int Pr(d_{R_t,t} = 1|S_{R_t,t}) \phi\left(\frac{s_{R_t,t} - \theta_t}{\sigma_s}\right) ds_{R_t,t} \quad (3.17)$$

where  $Pr(d_{it} = 0|S_{it}) = 1 - Pr(d_{it} = 1|S_{it})$ .

Meanwhile, neither the patients nor the researcher knows the true unobservable quality  $\theta_t$ . Therefore, the unconditional probability of kidney  $t$  being accepted at position  $R_t$  is

$$Pr(R_t) = \int Pr(R_t|\theta_t) \phi\left(\frac{\theta_t - \mu}{\sigma_\theta}\right) d\theta_t \quad (3.18)$$

It remains to develop the second probability component on the right-hand side of equation 3.14. Assume patients have rational expectations so that  $\mathcal{P}(O_{it}, s_{it}|X_{it}, Z_{it}, r_{it})$  equals the equilibrium joint probability for the patient in position  $r_{it}$  to reach an offer status  $O_{it}$  and to receive a private signal  $s_{it}$ . Importantly, the chance of being offered a kidney and the chance of receiving signal  $s_{it}$  are correlated through the unobservable quality  $\theta_t$ :

$$\begin{aligned} \mathcal{P}(O_{it} = 1, s_{it}|X_{it}, Z_{it}, r_{it}) = \\ \int [\prod_{j=1}^{r_{it}-1} \int Pr(d_{jt} = 0|S_{jt}) \phi\left(\frac{s_{jt} - \theta_t}{\sigma_s}\right) ds_{jt}] \phi\left(\frac{s_{it} - \theta_t}{\sigma_s}\right) \phi\left(\frac{\theta_t - \mu}{\sigma_\theta}\right) d\theta_t \end{aligned} \quad (3.19)$$

Also, the higher the unobservable quality of the kidney, the less likely that the kidney will reach a patient far down the queue. This idea is captured by a patient's probability

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<sup>10</sup>Inclusion of other patient characteristics as waiting cost determinants does not change the estimation results qualitatively.



of not receiving a kidney offer:

$$\begin{aligned} \mathcal{P}(O_{it} = 0 | X_{it}, Z_{it}, r_{it}) = \\ \int [1 - \prod_{j=1}^{r_{it}-1} \int Pr(d_{jt} = 0 | S_{jt}) \phi(\frac{s_{jt}-\theta_t}{\sigma_s}) ds_{jt}] \phi(\frac{\theta_t-\mu}{\sigma_\theta}) d\theta_t \end{aligned} \quad (3.20)$$

## 4 Model Estimation

### 4.1 Estimation Procedure

The dynamic choice model is estimated using the nested fixed point algorithm (Rust 1987). For each set of parameter values, an “inner” algorithm computes the value function and evaluates the likelihood function. An “outer” algorithm then searches for the set of parameters that maximize the likelihood function.

#### 4.1.1 Computing the Value Function

Let  $EV(S_{it})$  denote the total future discounted value patient  $i$  expects to receive when she turns down kidney  $t$ . That is,

$$EV(S_{it}) = \int_{S_{i,t+1}} V(S_{i,t+1}) \mathcal{P}(S_{i,t+1} | S_{it}, d_{it} = 0) dS_{i,t+1} \quad (4.1)$$

The Bellman’s equation becomes  $V(S_{it}) = \max\{EU(S_{it}), -C(S_{it}) + \delta EV(S_{it})\}$  accordingly. Given the i.i.d. Gumbel assumption of the idiosyncratic utility shocks,  $EV(S_{it})$  can be rewritten as (Rust 1987):

$$EV(S_{it}) = \int_{S_{i,t+1}} \ln[e^{EU(S_{i,t+1})} + e^{-C(S_{i,t+1}) + \delta EV(S_{i,t+1})}] \mathcal{P}(S_{i,t+1} | S_{it}, d_{it} = 0) dS_{i,t+1} \quad (4.2)$$

As discussed in the Online Appendix, the state space relevant to solving  $EV(\cdot)$  is discrete and can be much simplified thanks to the high degree of independence among the state variables in this data. Let  $K$  denote the dimension of the state space, and  $\Pi$  a  $K \times K$  Markov transition matrix in which the  $(r, c)$  element represents the transition probability from state  $r$  to state  $c$ . (Please see the Online Appendix for the construction of  $\Pi$ .) The discrete representation of the value function becomes

$$EV(\cdot) = \Pi \cdot \ln[e^{EU(\cdot)} + e^{-C(\cdot) + \delta EV(\cdot)}] \quad (4.3)$$

where  $EV(\cdot)$ ,  $EU(\cdot)$ , and  $C(\cdot)$  are all  $K \times 1$  vectors with the  $r^{th}$  element being the function value evaluated at the  $r^{th}$  state. The value function  $EV(\cdot)$  is then solved iteratively using standard fixed point algorithms.

### 4.1.2 Evaluating the Log-likelihood Function

Given  $EV(\cdot)$  for each state, the choice probability in Equation 3.16 can be rewritten as

$$Pr(d_{it} = 1|S_{it}) = \frac{e^{EU(S_{it})}}{e^{EU(S_{it})} + e^{-C(S_{it}) + \delta EV(S_{it})}}$$

The probability of kidney  $t$  being accepted in position  $R_t$ ,  $Pr(R_t)$ , thus follows as given by Equation 3.18. Note that the value function and these probabilities are derived for a given set of parameters. Let  $\Delta$  denote the parameter vector to be estimated. The log-likelihood associated with kidney  $t$  is a function of  $\Delta$ :

$$LL_t(\Delta) = \ln Pr(R_t|\Delta) \quad (4.4)$$

Last, let  $T$  denote the total number of kidneys offered in the sample, the log-likelihood function for the entire sample is

$$LL(\Delta) = \sum_{t=1}^T LL_t(\Delta) \quad (4.5)$$

The log-likelihood function includes high dimensional integrals, and is evaluated using the simulated maximum likelihood method. (Please see the Appendix for detailed procedures to formulate the simulated likelihood function.)

## 4.2 Identification

**Parameter Identification:** To summarize, the parameters to estimate include patients' utility weights associated with the patient and/or kidney characteristics that determine the utility from accepting the kidney offer ( $\beta$ ), patients' utility weights associated with the cost of waiting ( $\gamma$ ), patients' utility weight associated with the unobservable quality ( $\alpha$ ), patients' risk aversion coefficient ( $\rho$ ), the prior mean of the unobservable kidney quality ( $\mu$ ), the prior standard deviation of the unobservable kidney quality ( $\sigma_\theta$ ), the standard deviation of the private signals ( $\sigma_s$ ), and the discount factor ( $\delta$ ).

The utility weight parameters  $\beta$  and  $\gamma$  are identified from the exogenous variation in patient, kidney, and patient-kidney interactive characteristics.  $\alpha$  is identified from the systematic variation in choice decisions after the observable patient/kidney characteristics are controlled for. The identification of  $\rho$  relies on the functional form restrictions in the model: by assuming a functional form for the prior unobservable quality distribution and for the conditional signal distribution, we are able to specify the posterior variance in the unobservable quality, and isolate the effect of  $\rho$  from the magnitude of the impact of this posterior variance on risk-adjusted preferences (see also Coscelli and Shum 2004).

The parameters  $\mu$ ,  $\sigma_\theta$ , and  $\sigma_s$  together shape the learning process. The prior mean  $\mu$  affects the choices among patients on the top of the queue who do not engage in observational learning. However, since  $X_{it}$  includes a constant term, the intercept term in  $\beta$  cannot be separately identified from  $\mu$ . We set  $\mu$  to zero. The idea is to capture the fixed value of transplantation through the intercept and to measure the mean value of a particular kidney from the other observable attributes, with the unobservable quality adding fluctuations around this mean. Note that  $\alpha$ ,  $\sigma_\theta$  and  $\sigma_s$  cannot be all identified simultaneously. The intuition is that the relative precision of prior quality and signals determines the shape of the learning path, while  $\alpha$  captures the remaining scaling effect. Therefore, we restrict  $\sigma_\theta$  to be 1 and estimate  $\alpha$  and  $\sigma_s$  as free parameters.

Last, we fix the value of  $\delta$  at 0.95 due to the usual difficulties in estimating the discount factor in forward-looking dynamic models (see Erdem and Keane 1996). Altogether, the set of parameters to be estimated are  $\Delta = \{\beta, \gamma, \alpha, \rho, \sigma_s\}$ .

**Observational Learning and Queue Position:** Since the amount of (negative) observational learning monotonically increases down the queue, it is crucial to isolate observational learning from other queue-position-related factors. We try to keep the identification of observational learning clean in the following ways. First, the same kidney may be of different quality when it reaches the 30<sup>th</sup> patient than when it was with the 1<sup>st</sup> patient. We capture this within-kidney quality variation across positions by the “cold time” variable, which measures the time from when a kidney was retrieved from the donor until when it reaches the patient. Second, queue position is completely determined by a set of exogenous variables, which are observable to the econometrician and are controlled for in the analyses. Third, due to the queuing policy, a patient’s queue position fluctuates

across kidney episodes. This variation enables us to observe choices of the same patient with different amounts of observational learning, and thus separately identify observational learning from patient-specific heterogeneity. Fourth, as information accumulates along the queue, the precision of the posterior quality varies across queue position. This may create additional cross-position variation in utilities if patients are not risk neutral. We capture this variance by adding a flexible risk adjustment component in the utility specification. Last, patients in different positions of the queue may have systematically different prospects of future kidney offers. Modeling patients' dynamic tradeoff helps to rule out potential confounds from the inter-temporal dimension.

### 4.3 Alternative Models

In addition to observational learning, we specify four alternative models, each corresponding to a different behavioral account of patients' decision making processes. All five models are embedded in the dynamic choice setting.

**No Quality Uncertainty:** In this basic model, patients make decisions based on observable attributes only, either because quality is fully certain, or because quality uncertainty does not affect their utilities. This is equivalent to restricting  $\alpha$  in the full observational learning model to 0. As a result,  $\sigma_s$  and  $\rho$  cannot be identified in this model.

**Public Quality Information:** Causality claims for socially correlated choices demand extra caution (see Manski 1993). If there exist common contextual factors which the econometrician neither observes nor accounts for, choice conformity can be spuriously attributed to social contagion.<sup>11</sup> For example, Van den Bulte and Lilien (2001) re-analyze the classic diffusion study *Medical Innovation* (Coleman, Katz, and Menzel 1966) and discover that the adoption of tetracycline turns out to be driven by marketing efforts rather than social contagion as previously speculated. Manchanda, Xie and Youn (2008) separate the effects of marketing communication and interpersonal communication, and find that both affect adoption. In the NBA draft example at the start of the paper, inferences could coexist with rumors about the player's caliber that spread among teams. In our data, one major competing explanation for repeated kidney refusals is the exis-

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<sup>11</sup>See Villas-Boas and Winer (1999) for a general discussion of how the correlation between independent variables and the error term can bias parameter estimates in choice models.

tence of (negative) kidney quality information which is publicly known to patients but is unobserved by the researcher. This competing explanation can be modeled by restricting  $\sigma_s$  to 0 in the full observational learning model. It follows that  $\theta_t$  represents the public quality information unobserved by the researcher, and the model essentially becomes one with random kidney effects. Therefore, given the functional form assumption, the test between public quality information and observational learning becomes the parameter test of whether  $\sigma_s = 0$ . Note that since there is no quality uncertainty,  $\rho$  is not identified in this model.

**No Observational Learning:** In this competing account of the decision process, patients ignore previous rejections and infer kidney quality using the prior and their private signal only as if they were the first in the queue, as specified in Equation 3.8. Note that since every patient updates the prior only once, the variance of the posterior is identical across patients. Therefore, the risk adjustment in the acceptance utility cannot be identified separately from the intercept. We do not estimate  $\rho$  as a free parameter but fix its value at 0.

**Information Sharing:** Although information sharing does not exist in the data by institutional design, we estimate this model for comparison purpose. The quality updating rule is specified in Equation 3.9.

## 4.4 Estimation Results

### 4.4.1 Goodness of Fit and Model Selection

Table 2 reports the parameter estimates and model fit statistics of the observational learning model and the four alternative models. Observational learning fits the data best with the highest log-likelihood. In particular, the nested models “no quality uncertainty” and “public information” are both rejected (likelihood-ratio statistic = 31.602,  $p = 0.000$ ; likelihood-ratio statistic = 27.164,  $p = 0.000$  respectively). Indeed, the estimate of  $\alpha$  in the observational learning model differs from 0 at the  $p = 0.000$  level, which means that uncertain kidney quality does affect patients’ decisions. The estimated  $\sigma_s$  in the observational-learning model is also significantly different from 0 ( $p = 0.000$ ), which rules out the competing explanation of public kidney quality information, given the functional

form assumption.

The “no social learning” and “information sharing” models are not nested models of observational learning. The Akaike information criterion (AIC) selects observational learning as the best model. In fact, due to the significant signal variance, quality inference by simply observing one’s own signal is noisy, which necessitates social learning. The information sharing model fits better than no social learning. Note that information sharing does not exist in the data. The better fit comes from the additional risk component; because the posterior variance under information sharing declines with queue position and because the corresponding risk coefficient is negative (meaning patients are risk seeking by definition), other things being equal, the back of the queue would have lower acceptance utility—a pattern in the same direction of observational learning. The estimated utility weight associated with the unobservable quality,  $\alpha$ , is more significant in the observational learning model than in the alternative models. One explanation is that since the quality inference processes specified in the observational learning model is more consistent with the data, it assumes higher explanatory power.

#### 4.4.2 Parameter Estimates

All five models yield similar parameter estimates for the observable attributes. In particular, older patients are more likely to accept a kidney offer. There is no significant effect of patient’s number of years on dialysis, which is included to control for medical urgency, need for transplant, and dialysis-induced status quo bias. As expected, good tissue match increases the acceptance propensity; perfect issue match increases it dramatically. Interestingly, a longer cold time is associated with higher acceptance rates across all models. This coefficient should better be interpreted as a correlation rather than a causal effect. One possibility is that patients take longer time to evaluate “marginally acceptable” kidneys, but are able to reject obviously poor kidneys immediately. Consistent with this interpretation, cold time and queue position are negatively correlated (correlation coefficient =  $-0.127$ ,  $p = 0.035$ ).

Figure 3 illustrates at the micro level how inferred quality changes along the queue. For illustrative purpose, we take one representative kidney, fix unobservable quality at zero, draw random signals and calculate each patient’s inferred unobservable quality us-

ing the parameters estimates from the observational learning model. In the absence of social learning, inferred unobservable quality fluctuates with private signals, but shows a smaller variance due to the stickiness of the prior quality perception. With information sharing, inferred unobservable quality quickly converges to the true value. With observational learning, inferred unobservable quality still responds to private signals, but declines noticeably towards the end of the queue.

Figure 4 shows the impact of observational learning at the aggregate level. It plots the average inferred unobservable quality across queue position. As expected, overall the inferred quality declines down the queue, as doubts about quality accumulate with repeated refusals. Interestingly, the shape of patients' inferred quality curve shows how heterogeneity in acceptance standards create heterogeneity in the pace of learning. Among patients at the top of the queue, 10.91% have perfect tissue match, compared to 0.35% across all patients. A rejection in spite of perfect match contains a strongly negative message, lowering the inferred quality significantly from position 1 to position 2. After that inferences slow down. This is because patients near the top of the queue tend to have longer waiting time, and are likely to keep their priority in the queue at the next offer. Therefore, they can afford to wait for the "ideal kidney," and their refusal reveals little information about their private signals. This is consistent with the fact that patients in positions 2 to 12 almost always reject (see Figure 1). Moving down the line, when the kidney keeps being rejected by patients with lower queue priority and lower acceptance standards, negative quality inference escalates. However, as more patients reject partly because their predecessors have done so, refusals become less informative. Consequently, observational learning slows down again near the end of the queue.

The impact of acceptance standards on quality inference calls for rethinking of the conventional need-based allocation mechanisms for scarce resources. By giving priority to people with the most need, efficiency is enhanced *conditional on acceptance*. However, in the possible case of refusal despite urgent need, others may draw strongly negative quality inferences which slow down the utilization of scarce resources.

## 5 Counterfactual Simulations of Alternative Learning Mechanisms

In this section, we use parameter estimates obtained from the observational learning model to simulate patients’ kidney acceptance decisions under two counterfactual learning mechanisms. One is if there were no social learning and each patient only followed her private signal. The other is if each patient were able to share the private signals of all her predecessors. We then compare the decision quality of these mechanisms and observational learning.

We make 10,000 random draws from the distribution of unobservable kidney quality, and match each up with one random draw of observable kidney attributes from the data. Each simulated kidney draw is assigned a queue of eligible patients based on the UNOS point system. These patients then receive independent private signals conditional on the draw of unobservable kidney quality. Finally, each decision is assigned a random idiosyncratic utility shock. We use the “first best” case of complete information as the benchmark to assess the decision quality of each learning mechanism. That is, we define optimal patient decisions as those dictated by true kidney quality, assuming it is observable to patients. We then simulate patients’ decisions under different learning mechanisms.

We first compare the prescriptive accuracy of these learning mechanisms. We define “hit rate” as the percentage of decisions consistent with those indicated by complete information, assuming each patient has the choice over each kidney.<sup>12</sup> Information sharing achieves a hit rate of 97.26%, higher than the 89.10% with observational learning ( $p = 0.000$ ), which in turn is higher than the 88.17% without social learning ( $p = 0.000$ ). Out of all decisions, the percentage of type I errors, where a patient rejects a kidney while complete information prescribes acceptance, is 10.08% with observational learning,

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<sup>12</sup>Alternatively, we can remove a simulated kidney draw from the queue once it is accepted. However, this may lead to biased measures of decision accuracy. For example, suppose complete information indicates that a kidney is accepted at position 20, while observational learning delays acceptance until position 40. If we truncate the queue after position 20, it will appear that observational learning achieves a hit rate of 95%, which can be an overstatement because any decision mistakes after position 20 are not captured.



3.89% without social learning, and 1.33% with information sharing. The percentage of type II errors, where a patient accepts a kidney while complete information prescribes rejection, is 7.94% without social learning, 1.41% with information sharing, and 0.82% with observational learning.

One limitation of hit rate is that it does not measure the valence of decision mistakes. Also, since the above hit rate analysis is conditional on each patient receiving the current kidney offer, it does not capture the possibility that better kidneys might have been accepted early in the queue. To address both problems, we study patients' *ex ante* expected utility under different learning mechanisms. For each learning mechanism, a kidney is removed from the queue once it is accepted. If a patient accepts a simulated kidney offer, she earns the acceptance utility based on true kidney quality; if she rejects an offer or does not receive one, she earns the discounted value of her future expected utility net of waiting costs, taking her transition probabilities into account. The average of a patient's utility (given her choice and offer status) across simulated kidney draws yields the *ex ante* expected utility of this patient for this learning mechanism.

Figure 5 plots patients' average *ex ante* expected utility across queue positions. Generally, patients' *ex ante* expected utility decreases along the queue, as good kidneys are less likely to reach the back of the line. The only exception is in position one. Because some patients are advanced to the top due to perfect tissue match with the current kidney, they are not guaranteed the same priority when the next kidney arrives. These patients therefore enjoy lower rejection utilities, which in turn reduces the average *ex ante* expected utility at the top of the queue. Among the three learning mechanisms, information sharing generates the highest expected utility. In fact, the expected utility curve with information sharing is almost identical to that with complete information. Patients are worse off with observational learning, and are the worst off without social learning.<sup>13</sup> As a measure of aggregate patient welfare, the total *ex ante* expected utility across all patients is 102.641 with complete information, 102.179 with information sharing, 88.550 with observational learning, and 64.911 without social learning. The difference in average

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<sup>13</sup>Importantly, in simulating decisions without social learning, we assume that patients know their positions in the queue. That is, although patients judge the quality of the current kidney offer as if they were first in the queue, they rationally know that in future they are less likely to receive good kidneys if they are far down the line. Alternatively, if patients naïvely believe that they will be the first in the queue in future, they may overestimate their *ex ante* expected utility.

*ex ante* expected utility is insignificant between complete information and information sharing ( $p = 0.191$ ), but significant between all other learning mechanisms ( $p = 0.000$ ).

The best decision quality generated by information sharing is anticipated because it represents the most informative mechanism among the three. With observational learning, patients also draw information from previous rejections. However, repeated observations of rejections may bias quality inferences downwards. This explains the frequent type I decision errors with observational learning. Without social learning, patients further ignore the information contained in previous rejections. In particular, patients ignore the fact that good kidneys are *ex ante* less likely to remain available. As a result, they make frequent type II decision errors.

An imperative of organ allocation in the U.S. is to improve the usage efficiency of kidneys. The dominant problem is the high volume of type I decision errors, where “most of the refused kidneys are of acceptable clinical value.” The policy experiments suggest that facilitating information sharing among patients can help achieve this goal. A platform could be set up where patients exchange their concerns for turning down the kidney offer, should confidentiality regulations permit. This enhanced decision transparency can limit over-interpretation of previous refusals, and prevent excessive rejections down the line. Note that although suppressing social learning also increases acceptance, it creates the opposite problem of overusing low quality kidneys. Whether organ allocation authorities should suppress social learning (for example, by offering kidneys simultaneously to a batch of patients) depends on whether they aim to maximize kidney usage or maximize aggregate patient utility.

## 6 Discussion: Implications for Other Markets

This paper models and finds evidence of observational learning from the kidney market. The results bear direct relevance to other markets of single non-divisible goods which can be consumed by a single buyer. Examples include labor markets, housing markets, auctions, business-to-business contracting, journal publications, child adoptions, and marriages. In these markets, mere “availability” signals lesser quality, although the signal may be exaggerated. Credibly communicating the reasons behind availability facilitates future transactions. In particular, marketers of these goods may want to emphasize it if

availability is caused by non-quality reasons such as stringent adoption standards, taste mismatch, high prices, and circumstantial restrictions.

More generally, observational learning affects choices if peer decisions convey relevant quality information. This paper highlights the critical difference between observational learning and information sharing (for example, through truthful word-of-mouth communication) in shaping choices. As two major ways of social learning, observational learning and information sharing are often intertwined in practice with their effects studied in combination. For example, most diffusion models focus on forecasting product adoption paths that are jointly fueled by observations and communications (e.g., Bass 1969, Horsky and Simon 1983, Narasimhan 1989, Talukdar, Sudhir and Ainslie 2002, Golder and Tellis 2004). It is therefore often unclear which force is the main driver of sales and what the optimal marketing strategies should be. This paper suggests two aggregate predictions that differentiate observational learning and information sharing in general marketplaces.

First, if consumers' private information collectively reveals the true value of a product, information sharing as a signal averaging mechanism will ensure that the ultimate success of a product reflects its quality. If choices are instead driven by observational learning, mass behavior can sometimes depart from what the underlying values would prescribe. As a result, the quality of popular products may turn out to be surprisingly low. For example, one major criticism of today's user-moderated web sites such as Digg.com is that stories promoted to the front page for their popularity are frequently found to carry poor content. Indeed, hits and misses can crucially depend on how the product is initially received by the market. The business book *The Discipline of Market Leaders* is believed to have made the bestseller list despite lackluster reviews because the authors secretly bought back 50,000 copies at book release (Bikhchandani, Hirshleifer and Welch 1998). Beyond the ethical debate surrounding such promotional tactics, a general message to marketers is that the early stage can be critical in shaping a product's life cycle, especially in categories such as apparel, automobile, and digital music where choices are highly visible. For these categories, the impact of observational learning should be factored into dynamic marketing decisions such as advertising timing, introductory pricing, and targeting.

Second, market dynamics under observational learning can be sensitive to the choices of a few pivotal consumers. While diffusion paths driven by information sharing tend to

follow a smooth trajectory, those shaped by observational learning can be turbulent with abrupt changes in mass behavior triggered by small events. It is likely, for example, that observational learning has powered the unanticipated rejuvenation of Hush Puppies in the mid 1990s, the sudden rave of text messaging despite little promotion, and the whimsical rise and fall of fashion ideals. While injecting significant unpredictability into the market, observational learning also offers marketers ample opportunities to orchestrate large-scale changes with a limited budget. For example, marketing resources spent on marginal customers and visible users may bring disproportionate returns to the firm, although the exact amount of returns depends on how consumers strategically react to such marketing tactics.

## 7 Concluding Remarks

Mere observation of others' choices can be a quality signal. This paper studies observational learning in the U.S. deceased-donor kidney market, where transplant candidates on a waiting list sequentially decide whether to accept a kidney offer. The fact that a patient receives a kidney offer implies that all patients before her in the queue have turned down the same kidney. However, confidentiality does not allow between-patient communication of the reasons for the refusals.

We model observational learning at the patient level. Kidney quality is not perfectly observed. However, each patient has private information on kidney quality, such as her doctor's opinion. Suppose the second patient is offered a kidney. She can infer that the first patient's private signal is not favorable enough. She then uses this information and her private signal to update her quality perception following Bayes' rule. Without sharing the exact concerns, the first patient's refusal can only (weakly) lower the second patient's inferred quality, thus increasing her probability of refusal as well. Consequently, refusals can be self-reinforcing, causing an otherwise acceptable kidney to be wasted.

The data shows aggregate patterns consistent with observational learning. Even identical same-donor kidneys are received much differently; some of them are accepted early on in the queue while their identical counterparts have to travel far down the line to find a willing recipient. At the same time, the U.S. kidney allocation organizations lament the poor kidney acceptance rate which is lower than what the observable patient and

kidney characteristics could justify. We estimate the observational learning model using disaggregate data, controlling for patient-donor match, deterioration of kidney quality while traveling down the line, unobservable (to the researcher) kidney quality information, patients' risk attitudes and prospects of future kidney offers. We find evidence of observational learning, where inferred quality indeed declines towards the back of the queue. We then simulate patient choices in two counterfactual scenarios, one without social learning, and the other with information sharing. Patients make more efficient decisions with information sharing, and worse decisions without social learning. The findings suggest that facilitating communication among patients can help improve kidney utilization.

A general message to marketers is that mass behavior can be shaped by the choices of a few. Therefore, how to manage observational learning to marketers' benefits becomes an important managerial question, especially in markets where choices are immediately visible while information sharing lags behind. Early adopters, visible lead users, and marginal consumers can all be critical determinants of product success.

This study suggests a way to model observational learning in the field. Technically, observational learning becomes relevant when decisions are at least partially sequential and are not sufficient statistics of decision-makers' private information (Banerjee 1992). Below we discuss several possibilities of extending the observational learning model to more complex marketplaces.

First, decisions may not always be sequential. The pace of learning will vary with the timing of decisions. For example, suppose a new laptop model has achieved success among technology enthusiasts who make independent purchase decisions. The rest of the population can then infer higher quality than if the early wave itself was formed through observational learning. In other words, by delaying observational learning, marketers may subsequently create a fast rising herd. The optimal timing to enable observational learning would be interesting to explore, given that timing itself can signal quality.

Second, in general markets observational learning may drive the herd in both ways. Product success is path-dependent rather than a simple sum of per-period sales. For example, declining sales following an early rave communicates a different quality image than delayed popularity following a slow start. The model we present can be extended

to accommodate any permutation of adoption/rejection decisions along the sequence. An interesting question remains though on how marketers should allocate promotional resources across time, given strategic consumer reactions.

Third, it is often uncertain how many people have actually made a decision. For example, a consumer may not know whether sluggish sales is due to lack of awareness or lack of preference. This is analogous to the “attribution story” of the kidney market, where a patient may not know whether a refusal is due to mismatch or a poor signal. Future studies can model awareness as a moderator of quality inferences and a strategic marketing decision variable.

Last but not least, observational learning often coexists with information sharing (Chen, Wang, and Xie 2009). It would be important to understand how they interact. Also, it would be interesting to distinguish between observational learning and simple mimicking, which may generate similar behavior although they represent distinct behavioral mechanisms.

## 8 Appendix

### 8.1 The Queue Construction Process

UNOS oversees 90 organ procurement organizations (OPOs) throughout the United States. An OPO is an organization which concentrates its organ procurement efforts within a geographic territory. When a kidney is procured by an OPO, blood-type compatible patients within this OPO are selected and sorted into a queue based on a point system that UNOS launched in 1995.<sup>14</sup> Specifically, the UNOS point system constructs the queues based on the following four criteria. First, priority is given to patients with longer waiting time. A patient receives 1 point for each year spent on the waiting list. Second, priority is given to patients who have better tissue match with the donor. The tissue type is determined by six proteins at six loci, namely, A1, A2, B1, B2, DR1 and DR2. A “mismatch” occurs at a locus if the patient and the donor have different protein types there. A patient receives infinite points if there is no mismatch at any of the six loci (perfect tissue match), 2 points if there is no mismatch at the DR loci (second-best tissue match), and 1 point if there is one mismatch at the DR loci (third-best tissue match). Third, priority is given to patients with higher peak panel reactive antibody (PRA) measures, who are subject to higher risk of graft failure. Peak PRA ranges between 0 and 1. 4 points are given to patients whose peak PRA are greater than 80%. Fourth, priority is given to patients below 18 years of age who have higher risk of graft failure. Patients below 11 receive 4 points, and those between 11 and 18 get 3 points.

For each kidney, eligible patients are ranked in descending order of total UNOS points. In practice, the continued shortage of kidneys has lengthened the average waiting time, making it the dominant factor in determining the queue. Meanwhile, only a small fraction of patients qualify for criteria two to four. (See Table 1 for the percentages in the sample of this study.) As a result, the UNOS point scheme is converging to a first-come-first-served priority system (Su and Zenios 2004). In this data, patients’ current queue position and next-period queue position are significantly positively correlated ( $\rho = .803$ ,  $p = .000$ ).

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<sup>14</sup>A small fraction of patients register at multiple OPOs. According to the UNOS 2002 Annual Report, 5.74% of patients on the national waiting list sign up with two OPOs, 0.30% sign up with three, 0.02% four, and none above four. This study does not model multiple registration, but treats each OPO as one separate waiting list.

## 8.2 Formulating the Simulated Log-likelihood Function

The log-likelihood function involves high dimensional integrals. First of all, the cutoff sequence  $\{B_{it}\}$  is only stochastically known to subsequent patients. Therefore, to form her quality inference, a patient needs to evaluate Equation 3.12 by integrating over the joint distribution  $G(B_{1t}, \dots, B_{r-1,t})$ . We approximate this integral by taking  $N$  random draws from the joint distribution of  $B_{1t}, \dots, B_{r-1,t}$ , evaluating the integrand at these draws, and taking the mathematical average:

$$\frac{1}{N} \sum_{n=1}^N \prod_{j=1}^{r-1} \Phi\left(\frac{B_{jt}^n - \theta_t}{\sigma_s}\right)$$

where  $B_{jt}^n$  is obtained by solving patient  $j$ 's indifference condition (Equation 3.13) given an  $n^{th}$  draw from the joint distribution of  $X_{jt}$ ,  $Z_{jt}$ ,  $\epsilon_{jt}$  and  $\epsilon_{jot}$ . (Technical details on how to solve the cutoff sequence recursively are available upon request.) Note that the cutoff sequence only depends on the joint distribution of patient and kidney characteristics and idiosyncratic utility shocks, but not on the actual signals. Therefore,  $\{B_{it}\}$  can be solved recursively independent of  $\{s_{it}\}$ . This property allows us to perform simulation in separate modules: the total number of simulation draws needed to form the log-likelihood is linear in, rather than multiplicative of, the number of signal draws and cutoff draws.

Given the random cutoff draws, the posterior expected quality with observational learning,  $h_{rt}$ , can be approximated as

$$\hat{h}_{rt}(s_{rt}, \Delta) = \frac{1}{D} \int \phi\left(\frac{s_{rt} - \theta_t}{\sigma_s}\right) \phi\left(\frac{\theta_t - \mu}{\sigma_\theta}\right) \frac{1}{N} \sum_{n=1}^N \prod_{j=1}^{r-1} \Phi\left(\frac{B_{jt}^n - \theta_t}{\sigma_s}\right) \theta_t d\theta_t$$

where

$$D = \int \phi\left(\frac{s_{rt} - \theta_t}{\sigma_s}\right) \phi\left(\frac{\theta_t - \mu}{\sigma_\theta}\right) \frac{1}{N} \sum_{n=1}^N \prod_{j=1}^{r-1} \Phi\left(\frac{B_{jt}^n - \theta_t}{\sigma_s}\right) d\theta_t$$

Evaluating  $\hat{h}_{rt}(s_{rt}, \Delta)$  involves one-dimensional integration over  $\theta_t$ , which is numerically implemented using Gaussian quadratures.

After knowing  $\hat{h}_{rt}(s_{rt}, \Delta)$ , the expected utility and hence the probability for patient  $i$  to accept kidney  $t$  can be calculated based on Equation 3.16. Denote this probability as  $\hat{Pr}(d_{it} = 1 | s_{it}, \Delta)$ , which is a function of the draw of private signal  $s_{it}$  and  $\Delta$ . Last, to



evaluate  $Pr(R_t|\Delta)$ , the private signals need to be simulated:

$$\hat{Pr}(R_t|\Delta) = \frac{1}{L} \sum_{l=1}^L \left\{ \prod_{i=1}^{R_t-1} \left[ \frac{1}{M} \sum_{m=1}^M (1 - \hat{Pr}(d_{it} = 1 | s_{it}^{lm}, \Delta)) \right] \frac{1}{M} \sum_{m=1}^M \hat{Pr}(d_{R_t,t} = 1 | s_{R_t,t}^{lm}, \Delta) \right\}$$

The specific procedure is to make  $L$  random draws from the distribution of  $\theta_t$  for each kidney  $t$ . Label the  $l^{th}$  draw  $\theta_t^l$ . Given each  $\theta_t^l$ , the private signals are conditionally independent. Let  $e_{it}$  denote the deviation of actual signal  $s_{it}$  from  $\theta_t^l$ .  $e_{it}$  follows an i.i.d. normal distribution with mean 0 and variance  $\sigma_s^2$ . Make  $M$  draws from the distribution of  $e_{it}$  and label the  $m^{th}$  draw  $e_{it}^m$ . It follows that  $s_{it}^{lm} = \theta_t^l + e_{it}^m$ . This procedure maintains the signal correlation for the same kidney.

Finally, the simulated log-likelihood function to maximize is

$$\hat{LL}(\Delta) = \sum_{t=1}^T \ln \hat{Pr}(R_t|\Delta)$$

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Table 1: Summary Statistics

**Patient-specific variables (N=338)**

Variable	Mean	Std. Dev.	Min	Max
Patient age	47.059	14.342	4	79
Patient age =< 11*	0.018	0.132	0	1
11 < Patient age =< 18*	0.018	0.132	0	1
Patient_female*	0.340	0.474	0	1
Patient_Caucasian*	0.790	0.408	0	1
Patient_unemployed*	0.559	0.497	0	1
Income (\$1,000)	30.733	11.789	6.399	86.254
PRA > 80%*	0.018	0.132	0	1
# Years on dialysis	1.649	2.025	0	13

**Kidney-specific variables (N=275)**

Variable	Mean	Std. Dev.	Min	Max
Donor age	32.186	15.483	0	73
Donor_female*	0.447	0.498	0	1
Donor_Caucasian*	0.895	0.308	0	1
Accepting patient: position in queue	34.124	19.406	1	77
Accepting patient: # previous offers	15.455	23.994	0	166
Accepting patient: # days waiting	209.440	206.311	1	1272

**Patient-kidney interactive variables (N=9384)**

Variable	Mean	Std. Dev.	Min	Max
0 mismatch*	0.004	0.059	0	1
0 mismatch at DR*	0.038	0.190	0	1
1 mismatch at DR*	0.406	0.491	0	1
Cold time	8.877	7.034	0.016	43
Accept*	0.029	0.169	0	1

\* dummy variable which equals 1 if the statement in the variable name is true, and 0 otherwise

Table 2: Estimation Results

Parameters	No Quality Uncertainty ( $\alpha = 0$ )	Public Information ( $\sigma_s = 0$ )	No Social Learning	Information Sharing	Observational Learning
Intercept	0.000	-0.001	-0.001	-0.001	0.000
Patient Age	0.023 **	0.023 **	0.023 **	0.016 **	0.015 **
Patient_Female	0.008	0.018	0.027	0.019	0.009
Patient_Caucasian	-0.245	-0.205	-0.194	-0.283	-0.284
Patient Income	0.000	0.000	0.000	0.000	0.000
Patient_Unemployed	-0.026	-0.006	-0.002	-0.007	-0.039
# Years on Dialysis	-0.014	-0.002	0.000	-0.002	-0.008
PRA > 80%	-0.784	-0.317	-0.284	-0.273	-0.491
Patient Below 11	0.935	0.976	0.991	0.919	0.920
Patient Bw 11 & 18	1.325	1.209	1.218	1.278	0.726
Donor Age	0.000	0.000	0.000	0.000	0.000
Donor_Female	0.159	0.127	0.118	0.043	0.053
Donor_Caucasian	-0.142	-0.113	-0.096	-0.127	-0.284
0 Mismatch	6.396 ****	6.838 ****	6.832 ****	6.858 ****	6.004 ****
0 Mismatch at DR	1.487 ****	1.462 ***	1.441 ***	1.956 ***	1.474 ***
1 Mismatch at DR	0.186	0.158	0.144	0.630	0.442 *
Cold Time	0.075 **	0.078 **	0.078 **	0.093 **	0.104 **
Utility Weight on Unobs. $\alpha$	----	0.001 **	0.003 **	0.172 **	2.260 ****
Signal Noise $\sigma_s$	----	----	0.000	0.208 **	0.524 ****
Risk Coefficient $\rho$	----	----	0 ----	-0.011 *	0.003
# Observations	9384	9384	9384	9384	9384
# Parameters	17	18	19	20	20
LL	-913.732	-911.513	-911.397	-908.323	-897.931
AIC	1861.464	1859.026	1860.794	1856.646	1835.862

\* P < 0.10    \*\* P < 0.05    \*\*\* P < 0.01    \*\*\*\* P < 0.001

Figure 3: Quality Inference—Example of One Kidney

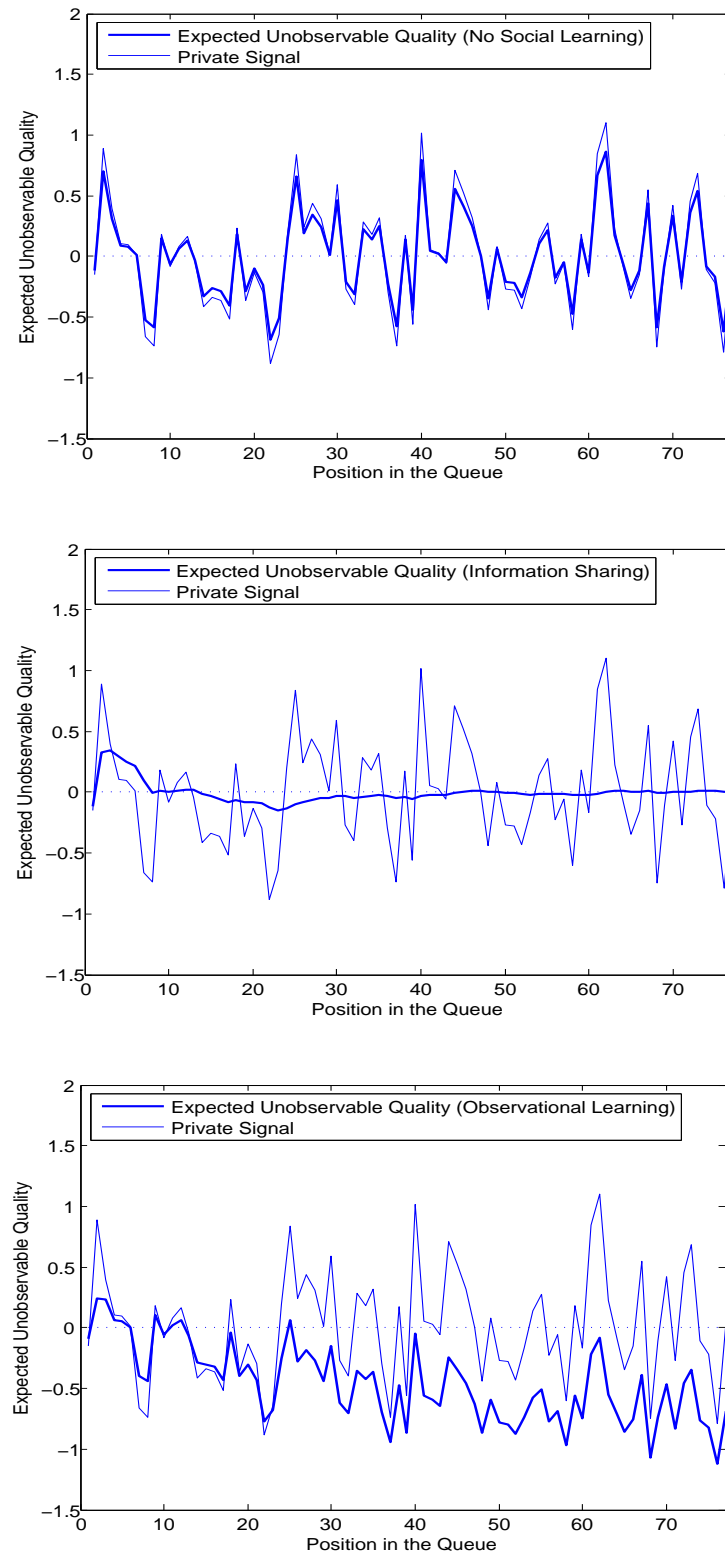
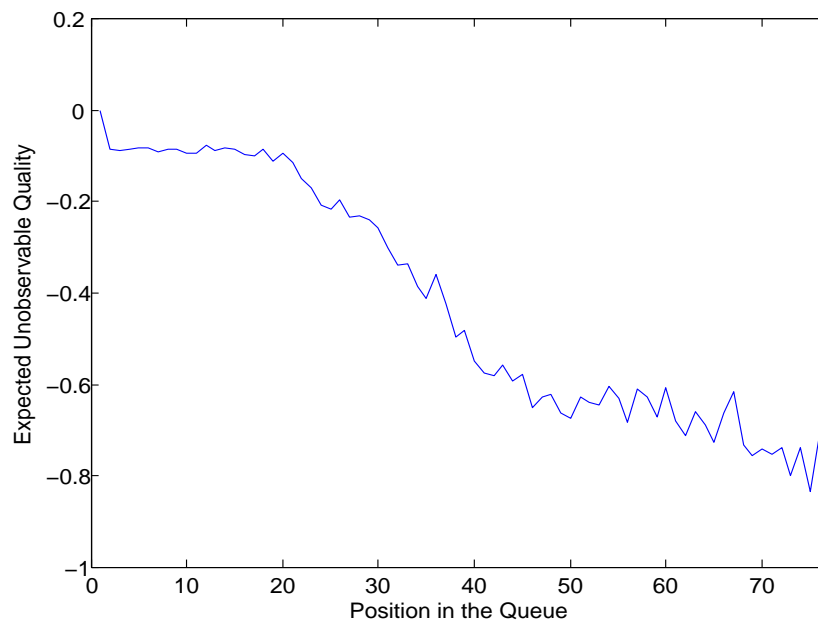




Figure 4: Quality Inference—Aggregate Effect

Figure 5: Policy Experiments: Patients' *Ex Ante* Expected Utility