

Obfuscation, Learning, and the Evolution of Investor Sophistication*

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Abstract

Investor sophistication has lagged behind the growing complexity of retail financial markets. To explore this, we develop a dynamic model to study the interaction between obfuscation and investor sophistication in mutual fund markets. Taking into account different learning mechanisms within the investor population, we characterize the optimal timing of obfuscation for financial institutions who offer retail products. We show that educational initiatives that are directed to facilitate learning by investors may induce providers to increase wasteful obfuscation, further disorienting investors and decreasing overall welfare. Obfuscation decreases with competition among firms, since the information rents from obfuscation dissipate as each institution attracts a smaller market share.

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1 Introduction

The menu of offerings for retail financial products has grown steadily over the last twenty years, and the sheer number of choices is now daunting. For example, as of 2007, there were 8,029 mutual funds to choose from and 21,631 different share classes (Investment Company Institute 2008). While such proliferation may add value in completing markets, it may also adversely affect investor sophistication.¹ Newcomers in the market have more to learn when they make their initial allocations, and incumbent participants bear a higher burden to keep up with developments in the investment market. Moreover, it remains unclear whether having access to more options leads to better decisions, as participants often make suboptimal choices in the face of too much information (e.g. Iyengar, Huberman, and Jiang, 2004; Salgado, 2006; Iyengar and Kamenica, 2008).

The interaction between the number/attributes of mutual fund offerings and the evolution of investor sophistication, therefore, introduces an externality that new funds and changing product offerings slow learning and may preserve industry rents for providers of financial services. Using this practice strategically has been termed obfuscation by Ellison and Ellison (2008).² Indeed, many new funds do not depart much from old ones, and are redundant even within specific fund families. There are straightforward strategic considerations at play: as Christoffersen and Musto (2002) point out, financial institutions often offer several classes of investment products to price discriminate among investors of varied levels of sophistication. As they document in the money fund industry, discrimination through such purposeful distortions in transparency is an important source of value to providers.

The purpose of this paper, then, is to explore the dynamic relationship between obfuscation and sophistication in retail financial markets, taking into account that learning mechanisms within the investor population play an important role. We specifically address the following questions: How often do providers of financial services optimally practice obfuscation, given that investors learn over time? How do specific learning processes (e.g. learning from others, learning from periodicals) affect these dynamics? What effect do competition and participation have on obfuscation? How does obfuscation affect portfolio allocation decisions? What are the policy implications for educational initiatives and regulation in financial markets?

To address these questions, we begin by analyzing a retail market in continuous time in which a

¹Many participants in the market have limited sophistication regarding the products in the market (e.g. NASD Literacy Survey 2003). See also Capon, Fitzsimons, and Prince 1996; Alexander, Jones, and Nigro 1998; Barber, Odean, and Zheng 2005; Agnew and Szykman 2005.

²For static theoretical models of obfuscation, see Spiegler (2006), Carlin (2009), and Ellison and Wolitzky (2010).

provider who is a monopolist markets a family of mutual funds to a heterogeneous group of investors. Investors are either experts, in which case they always choose the optimal fund within the offering, or they are non-experts, in which case their sophistication waxes and wanes based on learning mechanisms and changes in the fund family. When non-experts are informed (i.e., sophisticated), either through access to experts or public signals, they pick the optimal fund. However, when they are uninformed, they may either pay excessive fees or not get the best quality. As such, the provider earns higher rents from investors who are unsophisticated.

Sophistication evolves according to a general learning process, which is a differential equation with commonly known initial conditions. When the provider changes his mutual funds, the population is “refreshed” so that sophistication returns to its initial level and learning begins again. In essence, then, there are three groups of investors in the market: experts who are always sophisticated, non-experts who become sophisticated transiently, and non-experts who remain unsophisticated. Controlling the balance of sophisticated and unsophisticated non-experts is at the heart of the provider’s problem. As such, the provider maximizes profits by deciding how frequently to alter his mutual fund offerings, affecting the learning process.

The problem that the provider faces is stationary and therefore, in equilibrium there exists a unique optimal time to change the fund offerings. The optimal time is strictly decreasing in the extra rents gained from unsophisticated investors and strictly increasing in the cost incurred in doing so. The intuition is that the more the provider gains from unsophisticated investors, the higher is the benefit from refreshing the population and keeping non-expert investors in the dark. On the other hand, the more costly it is for the provider to do this, the lower is the benefit from refreshing the population. These comparative statics have straightforward cross-sectional empirical implications. For example, our analysis predicts that we should observe more product changes and redundancy among classes of mutual funds with higher price dispersion.³ To our knowledge, this prediction is novel and has yet to be tested.⁴

The relationship between the frequency of obfuscation and the speed at which non-experts become sophisticated is non-monotonic, however. The intuition is as follows. If non-experts learn very quickly, then the gains to refreshing the population are short-lived. Given that there is a fixed cost of changing the funds, it may not be worthwhile to change them as frequently. Moreover,

³As Hortacsu and Syverson (2004) note, the amount of price dispersion varies among different classes of funds, which likely indicates a difference in the sophistication (i.e. search costs) among consumers within each asset class. See Table 1 in Hortacsu and Syverson (2004).

⁴Testing this prediction might involve correlating price dispersion within groups of homogeneous offerings with number of share classes offered, either cross-sectionally or in a time-series.

when non-experts are very slow to learn and the extra rents gained from unsophistication are long-lived, the provider will optimally choose to refresh the population less frequently, again because there is a cost to doing so. This phenomenon has welfare implications for educational initiatives that the government may undertake. If the population learns relatively slowly, improving the learning process marginally will actually decrease welfare, as it increases the frequency of wasteful obfuscation.⁵ Small educational initiatives that increase the speed of learning are only likely to increase welfare when non-experts already learn sufficiently fast.

The type of learning that takes place in the market also affects obfuscation and the policies that are set. Based on the information percolation model of Duffie and Manso (2007), we analyze two specific settings: one in which investors learn by themselves (e.g. by reading periodicals) and one in which they learn from each other. In the former setting, as the initial set of experts rises, the frequency of obfuscation by the provider decreases. This occurs because there is less to gain from refreshing the population. Educational initiatives that improve the level of expertise in the market are always welfare enhancing in this case because increasing expertise lowers obfuscation. When investors learn from each other, however, we obtain different results. The comparative statics and policy considerations are very similar to the effect of learning speed. This non-monotonic relationship can be appreciated as follows. When no one is an expert initially, there is no one to learn from, and therefore the provider never changes their mutual fund offerings. When everyone is an expert initially, then there is no gain to refreshing the population. Therefore, obfuscation only takes place when a fraction of the population has expertise and there is a non-monotonic relationship between expertise and the frequency of obfuscation.

We extend our analysis to consider other welfare effects of obfuscation. Indeed, in the base model, the only cost to society is the cost that the provider incurs when he refreshes. There are many other potential costs to society such as non-participation, the cost of learning, and the misallocation of resources. We extend the model to consider these other sources of welfare loss. First, we show that participation increases obfuscation, which might give policy makers pause before encouraging people to increase their investments in mutual funds. Second, we quantify how obfuscation causes distortions in investors' portfolio allocations. Third, we quantify the opportunity cost of becoming an expert in the first place: we analyze how expertise arises endogenously in the market, given that the provider has an incentive to obfuscate and maximize profits.

⁵Policies designed to assist investors have the adverse effect to induce the provider to refresh more frequently. This causes non-expert investors to fall further behind, making the task of keeping up more costly. Absent such marginal initiatives, the provider would also save on the costs of wasteful innovation.

Finally, we consider the effect of competition on obfuscation. We show that increased competition should slow obfuscation. The reason for this is that there is less to gain for each provider when they refresh the population. In essence, the information rents that financial institutions gain by refreshing dissipate with more competition. This improves welfare as the side effects from obfuscation decrease: fewer investors pay the opportunity cost of becoming an expert, more investors participate, and the misallocation of resources decreases.

The focus of this paper is on mutual fund allocation and investment. Yet, our analysis applies to other financial decisions: credit card financing, life annuities, mortgages, life insurance, education savings plans. The disparity between sophistication and complexity in these other markets is also striking. This not only degrades personal welfare, but also affects the economy as a whole. Indeed, participation without sophistication is frequently cited as a root cause of the recent financial crisis: many home owners did not appreciate the variable-rate clauses in their mortgages and their explicit exposure to interest rate risk. At the same time, failure of many home owners to appreciate the fees and interest rate schedules used commonly in credit cards has led to a record-setting amount of household debt and a growing number of personal defaults in the U.S.

What to do about the disparity between sophistication and complexity has recently received much attention, though the optimal solution remains hotly debated. While Lusardi and Mitchell (2007) argue that education improves investor welfare, Choi, Laibson, and Madrian (2008) show that investors appear to ignore fees when making decisions, even when they are given salient information about the importance of taking fees into consideration. In lieu of a large-scale educational effort, there is now growing support for the use of default options to assist retail investors and improve welfare (e.g., Choi, Madrian, Laibson, and Metrick 2004). Our analysis adds to this debate: while not specifically modeled in our paper, default options would in essence make more investors experts (by proxy) and may decrease obfuscation, especially when used on a grand scale or in markets in which people learn on their own. Libertarian paternalism, as posed by Thaler and Sunstein (2003), makes sense in our model: a large scale increase in the fraction of experts would slow obfuscation and encourage participation. Investigating this debate further is the subject of future research (Carlin, Gervais, and Manso, 2010).

Finally, our paper may be readily applied in consumer markets, especially those in which service is a key component and there is a large discrepancy between sophistication and complexity (e.g., health care). As such, our contribution is also of general economic interest as it adds a new and important dimension to an already extensive literature on oligopoly competition with consumer search (e.g. Diamond, 1971; Salop and Stiglitz, 1977; Varian, 1980; Stahl, 1989; Gabaix

and Laibson, 2006). Indeed, in many existing theoretical models, consumers search for the best alternative, but the firms are unable to affect the search environment except through the prices they choose. Few notable exceptions are papers by Robert and Stahl (1993), Carlin (2009), and Ellison and Wolitzky (2010). Robert and Stahl (1993) analyze a model of sequential search in which firms may advertise to consumers in the population. Carlin (2009) and Ellison and Wolitzky (2010) analyze a static model in which firms simultaneously choose whether to add complexity to their pricing schedules. Our analysis departs from the above papers in several ways. First, our model is fully dynamic so that we can characterize how obfuscation evolves over time. Second, our model is general in that it can account for innovation in both prices and product characteristics. Finally, in our analysis we are able to characterize how different types of learning affects both obfuscation and the policies that govern these markets.

The rest of the paper is organized as follows. In Section 2, we pose a dynamic model of obfuscation and investor sophistication, given that the provider in the market is a monopolist. In Section 3, we characterize optimal obfuscation by the provider, and evaluate the effect that different learning models have on welfare and policy considerations. In Section 4, we consider participation in the market and how obfuscation affects portfolio allocation by non-experts. In Section 5, we analyze how expertise arises endogenously. In Section 6, we consider the effect that competition has on obfuscation. Section 7 concludes. All of the proofs are contained in the Appendix.

2 The Model

Consider a financial institution (i.e., a provider) that markets a family of heterogeneous mutual funds to a unit mass of household investors over an infinite horizon. Time evolves continuously and future cash flows are discounted at an interest rate r . The funds that are offered may differ along several dimensions (e.g., fees, target benchmarks, turnover, or alphas), but at any particular time one of the funds is clearly superior to the others. For example, within a group of money funds, one of them has lower fees than the others. We begin by considering that the provider is a monopolist, but extend the analysis to competitive markets in Section 6.

Each investor $i \in I$ in the market has unit demand for a mutual fund. Based on the information that they have, each investor chooses within the fund offerings to maximize their expected payoff. At $t = 0$, investors are divided into two groups: experts x_0 and non-experts $y_0 = 1 - x_0$. Experts costlessly acquire information about all of the funds offered, and are able to quickly adapt to changes in the market and the fund family. Experts always make the best choice. In contrast,

non-experts are less discriminating. When they are uninformed, they choose one fund from the product offering randomly.⁶ When they become informed transiently through learning, they mimic the experts and choose the optimal fund. This knowledge is fragile, however, as non-experts are unable to keep up with changes that occur in the market or fund family. As such, the sophistication level of non-experts may wax and wane, whereas experts are always sophisticated. For now, we take the values of x_0 and y_0 as given exogenously, but we consider their endogenous determination in Section 5.

As time evolves, non-experts may learn either through interaction with experts or through access to public signals (e.g., reading periodicals). Therefore, at any time t , the fraction of sophisticated investors x_t is composed of the initial fraction of experts plus non-experts who have become informed through interaction. The remaining population y_t is the group of non-experts who remain unsophisticated.

Learning takes place as follows. Without obfuscation (to be specified shortly), the fraction x_t of sophisticated investors evolves according to the differential equation

$$dx_t = f(\lambda, x_t)dt, \tag{1}$$

with initial values x_0 and y_0 . At each instant in time, the rate at which investors become informed is a function of the fraction x_t of informed investors in the population and λ , which measures the speed of learning. That is, λ parameterizes how easy it is for investors to access public information or learn from each other. By construction, $dy_t = -dx_t$. We assume that $f(\lambda, x_t)$ is strictly positive, continuously differentiable, and increasing in λ . Therefore, without a change in the product offerings, x_t increases over time while y_t decreases. The evolution equation (1) is flexible enough to incorporate several important forms of learning. For now, however, we consider a general learning process and characterize the optimal behavior by the provider. Later, in Section 3.2, we consider more specific models of learning and contrast the effects that they have on the provider's behavior and economic welfare.

The provider earns profits $\pi(x_t, y_t)$ from his investors at each instant in time. Because unsophisticated investors are less able to choose the most advantageous fund, the rents that the provider captures are larger for these investors. To capture this mathematically, we consider that at any

⁶Non-experts are either fully informed or completely uninformed. Therefore, our set-up is a dynamic version of a standard all-or-nothing search model (e.g. Salop and Stiglitz, 1979; Varian, 1980). Random purchase by non-experts is typical in this literature, but an all-or-nothing search process is not necessary to derive the results that follow. For example, using a sequential search model as in Stahl (1989) would generate the same lack of discriminating behavior.

instant, the provider's profit is

$$\pi(x_t, y_t) = ax_t + by_t,$$

where $b > a$. The profit a represents the rent that is gained from selling a fund to informed investors, whereas $b - a$ is the added gain from selling to unsophisticated investors.⁷ For example, experts always choose the money fund with the lowest fees, whereas non-experts may sometimes invest in funds with higher costs, and therefore not get the best deal. As sophistication increases via (1), the added gains to the provider diminish.

The provider may prohibit learning, though, by changing the attributes of the funds in his offering (i.e. obfuscation). This may involve changing the mix of funds offered or altering the fees charged. Mathematically, we assume that the provider can at any time t refresh population learning by returning the fraction of sophisticated and unsophisticated investors to their initial levels x_0 and y_0 . The provider pays a cost c to do so, and chooses his timing optimally to maximize lifetime profits. As such, the times in which the provider obfuscates is given by a vector $T = (t_1, t_2, t_3, \dots)$ with $t_i \leq t_{i+1}$ for all i . For any $t \in [t_i, t_{i+1}]$, the dynamics of sophistication evolves according to the differential equation (1) with $x_{t_i} = x_0$. The fraction x_0 catches on quickly when the provider makes changes, but the fraction y_0 does not: whereas non-experts may become informed for a while, their sophistication is fragile as they depend on the learning process in (1).

The problem that the provider faces is

$$\sup_{\substack{T=(t_1, t_2, t_3, \dots) \\ \text{s.t. } t_i \leq t_{i+1}}} \int_0^\infty e^{-rt} \pi(x_t, y_t) dt - C(r, T), \quad (2)$$

where x_t evolves according to the differential equation (1) in the interval $[t_i, t_{i+1})$ with initial condition $x_{t_i} = x_0$ for all i . The quantity $C(r, T)$ sums the discounted lifetime costs of obfuscation according to the plan T , and is computed as

$$C(r, T) = \sum_{i=1}^{\infty} e^{-rt_i} c. \quad (3)$$

We characterize the solution to this problem in the next section.

⁷In Section 6, when we study competition, the rents obtained from sophisticated and unsophisticated investors arise endogenously. The assumption that $b > a$ can also be considered a Nash bargaining solution in which the provider gains a larger fraction of the trade surplus from unsophisticated investors than from sophisticated investors. In this sense, both experts and non-experts are rationally willing to participate in the market. This assumption is consistent with the search literature (e.g. Varian, 1980; Stahl, 1989) and the literature in which the timing of sales is used to discriminate between well-informed and ill-informed consumers (Salop, 1977; Conlisk, Gerstner, and Sobel, 1984; Rosenthal, 1982; Sobel, 1984).

Before doing so, though, it is instructive to consider a practical example of obfuscation, based on the dynamic fee-setting behavior observed in the money fund industry (Christoffersen and Musto, 2002). Consider a provider that offers two money funds with different fee structures. Sophisticated investors choose the one with lower fees, whereas unsophisticated investors pick one of them randomly. As time progresses and unsophisticated investors learn about the fund with lower fees, fewer investors demand the fund with higher fees. The provider may then add a third money fund that has low fees and progressively raise the fees of the “old” low-fee fund. The result is that expert investors will catch on quickly to this strategy and switch funds, while a new breed of unsophisticated investors evolves. What our model captures, therefore, is the idea that investors need to keep up with innovation in prices and quality in order to continue to get the best deal. Other examples might include different funds that sell for the same price, but have quality differences (e.g., less turnover or better monitoring of managers) that require different investment on the part of the provider. Our model as posed is general to consider heterogeneity on multiple dimensions.

3 Obfuscation and Sophistication

We begin by solving the provider’s obfuscation problem and characterize its solution in generality, given that learning proceeds according to (1). Then, we consider specific examples of learning processes and contrast the obfuscation that takes place in each case. Following this, we discuss several welfare and policy considerations that arise based on the type of learning that takes place in the market.

3.1 Optimal Obfuscation

The provider’s problem is stationary. That is, after the provider obfuscates, he faces a problem that is isomorphic to the one he faced at $t = 0$. The following proposition relies on dynamic programming techniques to simplify the provider’s problem.

Proposition 1. *An optimal $T = (t_1, t_2, t_3, \dots)$ that solves (2) is such $t_{i+1} - t_i = t_i - t_{i-1}$ for any i . Therefore, the provider’s problem reduces to choosing the duration t^* of each cycle:*

$$\max_t \frac{\int_0^t e^{-rs} \{ax_s + b(1 - x_s)\} ds - e^{-rt}c}{1 - e^{-rt}}, \quad (4)$$

where x_s evolves according to (1), the learning process without obfuscation.

According to Proposition 1, we may focus on the solution to the problem in (4) to derive the optimal plan $T^* = (t^*, 2t^*, 3t^*, \dots)$, which is stationary. As such, the population of sophisticated investors evolves according to (1) from x_0 to x_{t^*} , until the provider refreshes and the process begins again. In the proof of Proposition 1 in the appendix, we show that such a stationary plan is superior to any arbitrary control sequence that changes over time (i.e., a non-stationary plan).

The next proposition characterizes the solution to (4).

Proposition 2. (*Optimal Obfuscation*) *There exists a unique optimal stopping time $t^* > 0$ that solves the provider's problem. If*

$$c < \bar{c} \equiv \int_0^\infty e^{-rs} \{ax_s + b(1 - x_s)\} ds - \lim_{t \rightarrow \infty} \frac{\{ax_t + b(1 - x_t)\}}{r}, \quad (5)$$

then the optimal stopping time t^* is finite and solves

$$(1 - e^{-rt})\{ax_t + b(1 - x_t)\} - r \int_0^t e^{-rs} \{ax_s + b(1 - x_s)\} ds + rc = 0. \quad (6)$$

Otherwise, $t^* = \infty$. Moreover, for $t^* < \infty$,

$$(i) \frac{\partial t^*}{\partial b} < 0$$

$$(ii) \frac{\partial t^*}{\partial a} > 0$$

$$(iii) \frac{\partial t^*}{\partial c} > 0$$

According to Proposition 2, obfuscation takes place more frequently when the additional rents that are gained from unsophisticated investors are higher ($b - a$ high) and when the cost is lower. That is, if resetting the learning process is more valuable because unsophisticated investors forfeit significant surplus, then the provider will wish to capture these rents as frequently as possible. Since $c > 0$, though, they will not optimally do this continuously, that is, $t_{i+1} - t_i > 0$ for all i . When the cost is higher, the provider will wait longer before he incurs this cost, *ceteris paribus*.

If the cost is sufficiently high, that is, if $c > \bar{c}$, the provider will never change his fees or mutual funds. The first term for \bar{c} in (5) represents the present value of the provider's profits if he never refreshes the fund family. The second term in \bar{c} represents the present value of a stream of constant payments equal to the instantaneous profit the provider earns once sophistication reaches its upper limit. Thus, the difference between the two terms in (5) computes the excess rents that the provider collects when sophistication is subpar in this market absent any obfuscation.

Not surprisingly, \bar{c} changes in an intuitive way based on the other parameters in the model. As we show in Proposition 9 in the appendix, \bar{c} is increasing in $(b - a)$, which implies that it is more

attractive to engage in obfuscation when the extra rents gained from unsophisticated investors are high. We also show, under fairly general conditions, that \bar{c} is decreasing in λ and x_0 .⁸ This means that, if learning is slower and the fraction of experts is lower, it is more worthwhile for the provider to engage in obfuscation.

Now, we consider the relationship between t^* and λ , which is a bit trickier. We first state the proposition that characterizes this relationship and then describe it intuitively.

Proposition 3. *Suppose that $c < \sup_{\lambda} \bar{c}(\lambda, x_0)$. If*

$$\lim_{\lambda \rightarrow 0} x_t = x_0 \quad \text{and} \quad \lim_{\lambda \rightarrow \infty} x_t = 1 \quad \forall t > 0, \quad (7)$$

then

$$\lim_{\lambda \rightarrow 0} t^*(\lambda) = \infty \quad \text{and} \quad \lim_{\lambda \rightarrow \infty} t^*(\lambda) = \infty, \quad (8)$$

and the function $t^(\lambda)$ is non-monotone in λ .*

Proposition 3 can be appreciated as follows. The conditions about a particular learning process in (7) are sufficient for $t^*(\lambda)$ to be non-monotone in λ . The first condition says that as the rate of learning converges to zero, then the fraction of sophisticated investors remains the same (i.e., the initial level) for any fixed time. The second condition says that as the rate of learning converges to infinity, the entire population becomes sophisticated for any arbitrarily small time.

If these two conditions hold for a particular learning process, then $t^*(\lambda)$ is non-monotone in λ . Intuitively, if learning occurs very slowly, the provider will make changes to his mutual funds infrequently. In fact, if $\lambda \rightarrow 0$, then the provider will never make any changes because there is no benefit to paying the cost c . As learning occurs at a higher rate (higher λ), then the provider might want to make changes more quickly to keep resetting the process and stay ahead of investor sophistication. However, as learning becomes sufficiently fast, there is a diminishing benefit to obfuscation. To see this, consider a limiting case when $\lambda \rightarrow \infty$, that is when sophistication evolves instantaneously. In this case, the rents that are gained compared to the monopoly rent a are negligible because they are short-lived. Since, $c > 0$, the optimal strategy for the provider is never to make changes. Therefore, when $\lambda \rightarrow 0$ and when $\lambda \rightarrow \infty$, we expect $t^* = \infty$, whereas for values of λ in between we may observe a finite t^* . Thus, there exists a non-monotonic relationship between λ and t^* .

⁸Specifically, if there exists a constant κ such that for all λ and x_0 , $\lim_{t \rightarrow \infty} x_t = \kappa$, then the mentioned comparative statics hold. This condition is sufficient and implies that if the provider never obfuscates, the sophistication in the population reaches the same limit, which is independent of x_0 and λ . The specific learning processes that we study in Section 3.2 satisfy this condition.

Now, we consider the relationship between t^* and x_0 .

Proposition 4. *Suppose that $c < \sup_{x_0} \bar{c}(\lambda, x_0)$. If*

$$\frac{\partial^2 x_t}{\partial x_0 \partial t} < 0, \tag{9}$$

then $\frac{\partial t^*}{\partial x_0} > 0$. If

$$\lim_{x_0 \rightarrow 0} x_t = 0 \quad \forall t > 0 \tag{10}$$

then

$$\lim_{x_0 \rightarrow 0} t^*(x_0) = \infty \quad \text{and} \quad \lim_{x_0 \rightarrow 1} t^*(x_0) = \infty, \tag{11}$$

and the function $t^*(x_0)$ is non-monotone in x_0 .

Condition (9) means that the rate at which investors become sophisticated is decreasing in the initial fraction x_0 of experts in the population. If the learning process has this property, the provider obfuscates less frequently when x_0 is higher: the rents that are collected from unsophisticated investors are strictly lower so that the benefit to paying the cost c is decreased. As we will soon illustrate by example, the condition in (9) usually exists when investors learn independently from each other, but does not hold generally for group learning processes.

If the condition in (10) holds, however, then the relationship between t^* and x_0 is non-monotonic. Intuitively, the limit in (10) says that if the fraction of experts tends to zero, that no matter how much time passes, all investors remain unsophisticated. This condition will hold generally when investors learn from each other. If no one is sophisticated in the first place, then there is no one to learn from. In contrast, if investors learn on their own from accessing information from outside sources, it may be that x_t will increase over time, despite the fact that $x_0 = 0$ initially.

To better appreciate how the conditions in Propositions 3 and 4 affect obfuscation, it is instructive to consider a few examples.

3.2 Learning and Information Percolation

Based on the information percolation model of Duffie and Manso (2007), we analyze two specific settings: one in which investors learn by themselves and one in which they learn from each other.

Let us first consider a learning process in which investors learn from their own research. This may occur by reading periodicals, accessing news through the media, or reading a prospectus. More specifically, we assume that an uninformed non-expert learns what the optimal fund is at a Poisson

arrival time with a mean arrival rate (intensity) λ , which is common across uninformed investors. After this time, the investor becomes transiently informed and is able to choose the optimal fund until there are further changes in the fund family.

Relying formally on the law of large numbers, (1) takes the form

$$dx_t = \lambda(1 - x_t)dt. \tag{12}$$

In this process, a fixed proportion λ of unsophisticated investors become sophisticated at each point in time. This learning process is a degenerate case of the information percolation model studied in Duffie and Manso (2007): it can be obtained from equation (9) in that paper when each of the signals observed by people is either uninformative or fully informative and the intensity of meetings between people is set to zero. Integrating (12) and using the initial condition x_0 yields the solution

$$x_t = 1 - (1 - x_0)e^{-\lambda t}. \tag{13}$$

Now, we can consider (13) in terms of the conditions in Propositions 3 and 4. For this process,

$$\frac{\partial^2 x_t}{\partial x_0 \partial t} = -e^{-\lambda t} \lambda, \tag{14}$$

which is always negative. From Proposition 4, we have that the optimal time to obfuscate is increasing in x_0 . This makes intuitive sense, since in this example the ability to learn is independent of other investors' sophistication, so that as x_0 rises the rents available to the provider decrease. Obfuscation is less attractive and occurs with decreased frequency.

To show that the relationship between t^* and λ is non-monotone, we compute that

$$\lim_{\lambda \rightarrow 0} x_t = x_0$$

and

$$\lim_{\lambda \rightarrow \infty} x_t = 1.$$

Consequently, $t^* = \infty$ when λ approaches 0 or ∞ . Intuitively, this implies that if there are no sources of information and investors do not learn, then there is no incentive for the provider to refresh the investor population. In contrast, if access to media or periodicals allows investors to educate themselves quickly, then the provider will avoid obfuscation because refreshing the investor population is a futile effort.

Consider the example in Figure 1, in which investors learn without the help of others. The series of subfigures plot t^* versus the underlying parameters, while holding all else fixed. As predicted by

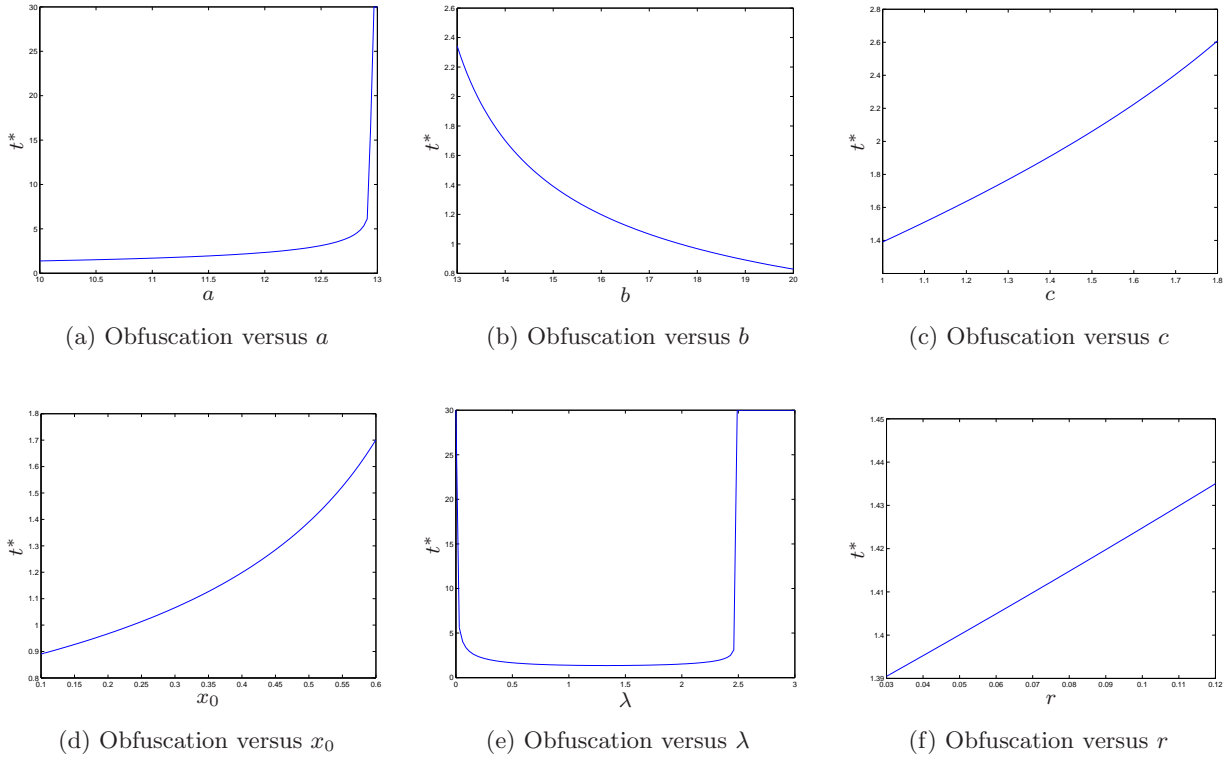


Figure 1: Learning On Your Own: The series of figures (a)-(f) plot t^* versus the fundamental parameters in the model, when the learning process involves learning about mutual funds through the use of periodicals or media. The time t^* is monotonically decreasing in b and increasing in a , c , x_0 , and r . The relationship between t^* and λ is U-shaped. Parameters, when held fixed, are $r = 0.03$, $\lambda = 1$, $x_0 = 0.5$, $a = 10$, $b = 15$, and $c = 1$.

Proposition 2, t^* is strictly decreasing in b and increasing in a and c . As discussed, t^* is strictly increasing in x_0 and is non-monotonic (U-shaped) in λ .

Now, let us consider an alternative learning process in which investors learn from each other. Indeed, as unsophisticated investors meet those who are informed, sophistication within the population rises. Such meetings may occur via friends, relatives, co-workers, or advisors. The key factor in these types of learning processes is that the chance that an unsophisticated investor becomes sophisticated depends directly on the fraction of investors who are already knowledgeable.

Let us consider a particular example in which investors meet each other in bilateral meetings. More specifically, we assume that any particular investor is matched to another investor at each of a sequence of Poisson arrival times with a mean arrival rate (intensity) λ , which is common across investors. At each meeting time, another investor (i.e., the counterparty in a meeting) is randomly

selected from the population. An uninformed investor that meets an informed investor becomes transiently informed about all of the mutual funds offered and can choose the optimal one until there are further changes in the fund family.

Relying formally on the law of large numbers, (1) takes the form

$$dx_t = \lambda x_t(1 - x_t)dt. \quad (15)$$

This process differs from (12) in that the rate at which market participants become informed depends on the proportion of sophisticated investors in the market. This learning process is also a degenerate case of the information percolation model studied in Duffie and Manso (2007): it can be obtained from equation (9) in that paper when each of the signals observed by people is either uninformative or fully informative and the intensity of arrival of new private information is set to zero. Integrating (15) and using the initial condition x_0 yields the solution

$$x_t = \frac{x_0 e^{\lambda t}}{(1 - x_0) + x_0 e^{\lambda t}}. \quad (16)$$

Again, we can consider (16) in terms of the conditions in Propositions 3 and 4. With this process,

$$\frac{\partial^2 x_t}{\partial x_0 \partial t} = \frac{e^{\lambda t} \lambda (1 - x_0 + e^{\lambda t} x_0)}{(1 - (1 - e^{\lambda t}) x_0)^3}, \quad (17)$$

which can be positive or negative depending on t . Therefore, based on Proposition 4, we are not guaranteed that the optimal time to obfuscate is monotonically increasing in x_0 . In fact, we can use the condition in (10) to show that the relation between t^* and x_0 is non-monotone. Specifically,

$$\lim_{x_0 \rightarrow 0} x_t = 0. \quad (18)$$

This follows from the fact that if no one is an expert ($x_0 = 0$), there is no one to learn from. It then follows from Proposition 4 that the relationship between x_0 and t^* is non-monotone.

As before, using conditions in (7), we can show that the relation between t^* and λ is also non-monotone. For that, it is enough to note that

$$\lim_{\lambda \rightarrow 0} x_t = x_0 \quad (19)$$

and

$$\lim_{\lambda \rightarrow \infty} x_t = 1. \quad (20)$$

Consequently, t^* approaches infinity when λ approaches 0 or ∞ .

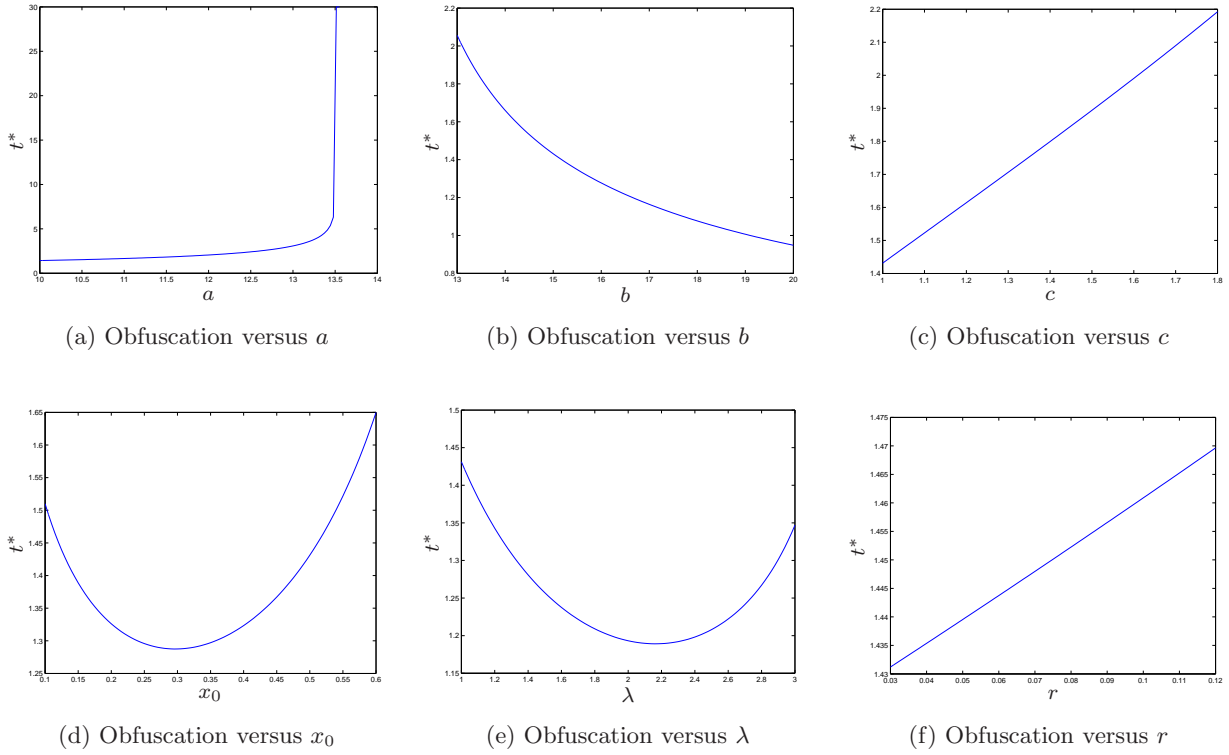


Figure 2: Learning From Others: The series of figures (a)-(f) plot t^* versus the fundamental parameters in the model, when the learning process involves learning about mutual funds through bilateral meetings between investors. The time t^* is monotonically decreasing in b and increasing in a , c , and r . The relationship between t^* and λ is U-shaped, as is the relationship between t^* and x_0 . Parameters, when held fixed, are $r = 0.03$, $\lambda = 1$, $x_0 = 0.5$, $a = 10$, $b = 15$, and $c = 1$.

Consider the example in Figure 2, in which investors learn from each other. The series of subfigures plot t^* versus the underlying parameters, while holding all else fixed. Again, as predicted by Propositions 2-4, t^* is strictly decreasing in b , increasing in a and c , and non-monotonic (U-shaped) in λ and x_0 .

Now, we consider how these different learning process may affect welfare in the market and the efficacy of educational initiatives.

3.3 Welfare and Policy Implications

So far in the model, the only source of welfare loss is the cost that the provider incurs when he changes his fund family. The fees that investors pay and the extra rents gained from unsophisticated investors are transfers between the parties to the transaction. A social planner who wishes to

maximize welfare in this market, therefore, seeks to minimize the quantity

$$\mathcal{L} = C(r, T),$$

where we recall that $C(r, T)$ is the lifetime costs of obfuscation, given the plan T .

The social planner may consider undertaking initiatives to raise the rate of learning λ or alter the fraction of experts in the market x_0 . For example, subsidizing websites to enhance investor education or legislating initiatives to enhance disclosure might increase the ability for people to learn about the market (i.e. increase λ). Requiring that financial education be an integral part of secondary education would be likely to increase the fraction of experts in the first place (i.e. raise x_0).

The discussion in Section 3.2, though, implies that optimal intervention through policies needs to take into account the way in which people learn. Likewise, the magnitude of intervention is equally important as small scale programs might actually decrease welfare. For example, consider in the two examples discussed that the relationship between t^* and λ is non-monotonic. For low λ , small increases in the speed of learning will decrease the time to obfuscation and will thus decrease welfare.

The key here is that when a social planner considers an initiative to improve investor sophistication, they need to consider the natural response on the part of the provider to maximize rents, given the initiative that is undertaken. In this way, for any λ on the decreasing portion of the curve in Figure 1e or 2e, a small supplement to λ will lead to more obfuscation by the provider, which destroys value. Only if the magnitude of intervention is large enough will the market reach the upward sloping portion in which increased access to information leads to lower obfuscation. Counter-intuitively, for low values of λ it may be more effective for the social planner to make learning more difficult, decreasing λ .

The differences in the relationship between t^* and x_0 among the two examples of learning also highlights that the social planner needs to take into account the mechanism by which people learn when they set policy. If people learn from periodicals, increasing education is always welfare enhancing (see Figure 1d) no matter how unsophisticated the population is. This does not hold for learning processes in which people learn from each other (see Figure 2e). If x_0 is low, small positive increments will induce the provider to obfuscate more frequently and destroy welfare. Only larger scale initiatives are able to overcome this loss in value.

So far, in the model, the only loss from obfuscation arises from the cost the provider pays to “refresh” the line of mutual funds. Realistically, though, there are other costs of obfuscation in the

market that a social planner needs to consider. First, investors might incur costs to learn or keep up with changes in the market. That is, the learning process in (1) might proceed with a cost to non-expert investors. Any policy that would increase obfuscation would increase investors' reliance on (1), which would cause mounting welfare losses. Likewise, becoming an expert represents an important opportunity cost to society. The more that providers practice obfuscation, the more investors will rationally choose to pay an opportunity cost (of time and resources) to make sure they get a good deal. We characterize this deadweight loss in Section 5 on endogenous expertise.

Obfuscation also has an adverse effect on the willingness of investors to participate in the market. If the market is too confusing, some investors may choose to just drop out, which may induce them to misallocate resources. This represents an important deadweight loss and any policy that increases the incentives for the provider to change his fund family more frequently leads to further welfare loss. We analyze this source of welfare loss next.

4 Participation and Portfolio Allocation

So far, we have assumed that all investors participate in the market and buy shares in a mutual fund. Now, we relax that assumption and consider how participation affects obfuscation in the market, and vice versa. We begin by considering that the fraction of the population that invests in a mutual fund is given exogenously, to see how this changes the provider's incentives to obfuscate. Following that, we consider how participation endogenously arises by analyzing a portfolio allocation problem in which investors choose how much of their wealth to invest in various sectors in the market. Obfuscation causes distortions in the allocations of some non-expert investors, and we quantify the welfare loss this induces.

4.1 Participation and Obfuscation

Consider that the unit mass of investors is split between investors who participate in the market and those who are either unaware of the funds, or who have decided that these funds are not appropriate for their needs. Specifically, in addition to expert and non-expert investors (fraction ϕ), there is a group $1 - \phi$ of investors who do not participate in the market. As such, the parameter ϕ may proxy for the scope of the funds: low ϕ implies specialized funds whereas funds with high ϕ have more widespread use (e.g., precious metal funds versus S&P 500 index funds). The provider receives no demand from investors who do not participate. At any given point in time, then, there is fraction ϕx_t of sophisticated investors and ϕy_t of unsophisticated investors.

Taking ϕ as given, the provider's instantaneous profit is

$$\pi(x_t, y_t, \phi) = \phi\{ax_t + by_t\},$$

where $b > a$. As before, the provider may choose to refresh the population of market participants at any time t . The times in which the provider does this are given by the vector $T = (t_1, t_2, t_3, \dots)$.

The provider, therefore, solves

$$\sup_{\substack{T=(t_1, t_2, t_3, \dots) \\ \text{s.t. } t_i \leq t_{i+1}}} \int_0^\infty e^{-rt} \pi(x_t, y_t, \phi) dt - C(r, T). \quad (21)$$

As in Section 3, the provider's problem is stationary and can be reposed as

$$\max_t \frac{\int_0^t e^{-rs} \phi \{ax_s + b(1 - x_s)\} ds - e^{-rt} c}{1 - e^{-rt}}. \quad (22)$$

The following proposition characterizes the effect that participation has on the solution to the problem.

Proposition 5. (*Obfuscation and Participation*) *There exists a unique optimal stopping time $t^* > 0$ that solves the provider's problem. For $t^* < \infty$, $\frac{\partial t^*}{\partial \phi} < 0$.*

According to Proposition 5, when funds are more specialized (low ϕ), there is lower obfuscation than when the funds have more widespread use (high ϕ). The result is a straightforward application of the analysis in Section 3. As ϕ rises, the aggregate rents to be gained from unsophisticated investors rise, which induces the provider to refresh the population more frequently. The result has important and interesting empirical implications. Comparing two classes of mutual funds with different specialization (e.g. S&P 500 Index funds versus Precious Metals funds), we would expect the product mix of funds to change more frequently with the funds with less specialization. Empirically, one might proxy for ϕ by considering the fraction of total assets in the marketplace invested in a particular class of funds, or by considering the number of people who invest in that class of funds.

From a welfare perspective, it is reasonable to assume that there is a cost associated with having unaware consumers that do not participate in the market. In this case, a social planner might like to increase awareness and participation to reduce those costs. However, as Proposition 5 shows, increasing participation leads to more frequent wasteful obfuscation. From the social planner perspective, there should exist an optimal level of participation that balances the losses from leaving investors out of the market with the losses from more frequent wasteful obfuscation.

4.2 Portfolio Allocation

Let us now consider that all investors face a portfolio allocation problem. Since x_0 -types can easily keep up with changes in any particular market, they allocate their wealth to assets in the market optimally. In contrast, y_0 -types have to take into account ex ante whether to participate in particular markets, given the rents that they anticipate paying and the obfuscation they will face.

Consider that y_0 -types derive a heterogeneous surplus from participating in this particular market, which we denote by s_i for all $i \in I$. Indeed, s_i may be a function of each non-expert's risk aversion, bequest motive, or longevity risk. We suppose that s_i is distributed in the y_0 population according to a twice continuously differentiable function M over the support $[0, \bar{s}]$. We assume that $\bar{s} \geq \frac{b}{r}$, so that at least some non-experts are always willing to allocate some wealth to the market under study.

When y_0 -types participate in this market, they anticipate that they will forfeit rents $R(t^*, \vec{w})$ to the provider. These rents are a function of the underlying parameters $\vec{w} \equiv (a, b, c, \lambda, x_0)$ and the equilibrium level of obfuscation t^* chosen by the provider. Higher obfuscation increases the rents that each y_0 -type can expect to pay, but will in turn affect their willingness to allocate investment resources to this market.

Each non-expert chooses whether to allocate some of their wealth to this market, given the surplus s_i and the rents $R(t^*(\phi, \vec{w}), \vec{w})$ they expect to forfeit, given that the provider subsequently chooses t^* according to (22). As such, a non-expert investor participates in this market iff

$$s_i \geq R(t^*(\phi, \vec{w}), \vec{w}). \quad (23)$$

Participation in this market is a fixed point implicitly defined by

$$\phi = 1 - M(R(t^*(\phi, \vec{w}), \vec{w})) \equiv z(\phi). \quad (24)$$

The following proposition establishes an equilibrium in this market and characterizes the participation by non-expert investors.

Proposition 6. *A unique solution $\phi^* \in (0, 1)$ exists for the expression in (24) for any $M(\cdot)$. Participation by non-experts have the following properties:*

- (i) $\frac{\partial \phi^*}{\partial b} < 0$
- (ii) $\frac{\partial \phi^*}{\partial a} > 0$
- (iii) $\frac{\partial \phi^*}{\partial c} > 0$

Proving that a fixed point in this problem exists is straightforward. Since the function $K(\phi) \equiv z(\phi) - \phi$ is strictly decreasing in ϕ , $K(0) > 0$, and $K(1) < 0$, there exists one, and only one, ϕ^* such that $K(\phi^*) = 0$.

Economically, Proposition 6 implies that higher $(b - a)$ causes fewer non-experts to allocate wealth to this market via two channels. First, as $(b - a)$ rises, non-experts will pay higher rents during times that they are unsophisticated. Second, as $(b - a)$ increases, the provider has a greater incentive to obfuscate, which increases the expected time that a non-expert will spend as an unsophisticated investor. Obfuscation magnifies the destruction of value for non-experts and decreases the attractiveness of allocating their wealth into this market.

When some non-experts misallocate their wealth away from this market, this adds a deadweight loss to the analysis. Since the rents are merely transfers when they do participate, the loss from misallocation can be calculated as

$$\mathcal{NP} = \int_0^{s^*} s dM(s), \quad (25)$$

where $s^* = M^{-1}(1 - \phi^*)$. Since ϕ^* is an increasing function of t^* , higher obfuscation causes ϕ^* to drop, thus raising s^* . By inspection of (25), it is clear that more frequent obfuscation causes a higher deadweight loss through misallocation.

5 Endogenous Expertise

Now, we consider that the fraction of expert investors x_0 arises endogenously. The timing of the game is as follows. First, each investor $i \in I$ chooses whether to pay a cost k_i to become an expert and join the x_0 population. Following this, the rest of the game follows in the same fashion as in Section 2. The cost k_i is a one-time cost, and could be considered to be the decision whether to obtain a financial education. Alternatively, it could represent the decision whether to become familiar with a particular mutual fund sector.⁹ Investors in the x_0 pool are experts and know to keep up with developments as time goes on. Those in the y_0 pool may learn about particular funds over time, but do not have higher levels of sophistication that are required to make sure that they always get the best deal. Learning takes place as before according to the differential equation in (1).

Suppose that investors are heterogeneous and the costs to become an expert are distributed

⁹An alternative specification of the model would be to allow investors to make this choice each time the provider refreshes their product line. Focusing on Markov Perfect Equilibria, the results would be qualitatively similar to what we derive here. Of course, in such a model, other Nash Equilibria that are not stationary might arise, but this is beyond the scope of our analysis.

over the support $[0, \bar{k}]$ according to a twice continuously differentiable function $G(\cdot)$. Define B as the expected benefit of becoming an expert given the actions of other investors and the expected actions of the provider. Therefore, if $k_i \leq B$, investor i becomes sophisticated. It follows then that

$$x_0 = G(B).$$

The value of B will depend on x_0 and on the t^* that is chosen by the provider based on x_0 and the underlying parameters. Going forward, we define $\vec{v} \equiv (a, b, c, \lambda)$ to be the parameters that are given exogenously in the model, and keep t^* and x_0 separate since they are determined endogenously. As such, we express the expected benefit to becoming an expert as $B(x_0, t^*, \vec{v})$. The function B is decreasing in t^* since the rents that the unsophisticated pay decrease when less obfuscation takes place. Also, $\frac{\partial B}{\partial b} > 0$ and $\frac{\partial B}{\partial a} < 0$. Define \underline{B} as the lower limit of B , that is, the benefit to becoming an expert when $t^* = \infty$. We assume that $\underline{B} < \bar{k}$; otherwise, all investors would become experts leading to an uninteresting interaction.

In any equilibrium of this game, the fraction of expert investors is implicitly defined by

$$x_0 = G\left(B\left(t^*(x_0), \vec{v}\right)\right) \equiv H(x_0). \quad (26)$$

Going forward, we follow the standard approach of Debreu (1970) and Mas-Colell (1985) and focus on the “regular” equilibria of the game.¹⁰ Such equilibria are robust to small perturbations of the set of parameters, and are therefore locally unique, which allows for meaningful comparative statics. Technically, in our setting, a regular equilibrium is defined as a fraction $x_0^* > 0$ such that $h(x_0^*) \equiv H(x_0^*) - x_0^* = 0$ and $\frac{\partial h}{\partial x_0^*} \neq 0$. By straightforward application of Sard’s Theorem and Mas-Colell (1985), other pathologic equilibria may be ruled-out as non-generic. Specifically, it is straightforward to show that, except for a set of parameters having zero measure in the general parameter space, the equilibria that will arise are regular.¹¹

The following proposition proves existence of a fixed point and characterizes the regular equilibria of the game. In doing so, we distinguish equilibria based on their stability in the sense of Vives (1990, 1999, 2005). Specifically, an equilibrium is said to be locally stable at a point x_0^* if there

¹⁰Originally, Debreu (1970) and Mas-Colell (1985) focused on “regular” equilibria to characterize general equilibria in exchange economies. Indeed, there are pathologic situations in which the excess demand function $z(p)$ might lead to an infinite number of equilibria, preventing comparative statics exercises. By limiting the focus to regular equilibria and proving that such pathologic cases are non-generic, local uniqueness and differentiability of the equilibria is guaranteed, thereby allowing for comparative statics to be generated.

¹¹See Chapter 8 in Mas-Colell (1985) for a thorough discussion of genericity analysis and Carlin, Dorobantu, and Viswanathan (2008) for an application of this in finance. A proof of this statement would follow from the same arguments in the proof of Proposition 2 in Carlin, Dorobantu, and Viswanathan (2009).

exists a neighborhood around it such that for any initial position x_0 within that neighborhood, the system converges to the point x_0^* according to the function $H(x_0)$.

Proposition 7. *A solution $x_0^* \in (0, 1)$ exists for the expression in (26) for any $G(\cdot)$. If $\frac{\partial^2 x_t}{\partial x_0 \partial t} < 0$, there exists a unique $x_0^* > 0$ with the following properties:*

$$(i) \frac{\partial x_0^*}{\partial b} > 0$$

$$(ii) \frac{\partial x_0^*}{\partial a} < 0$$

$$(iii) \frac{\partial x_0^*}{\partial c} < 0$$

If $\frac{\partial^2 x_t}{\partial x_0 \partial t} \geq 0$, there may exist multiple regular equilibria. However, in any stable equilibrium, the same properties hold.

According to Proposition 7, if $\frac{\partial^2 x_t}{\partial x_0 \partial t} < 0$, then there exists only one equilibrium for the game. This will generally be the case when people learn on their own from periodicals, which was the case when $f(\lambda, x_t)$ took the form in (12). In other cases, where this condition does not hold (e.g. the process in (15)), we are not guaranteed to have a unique equilibrium. Indeed, two classes of regular equilibria may form: a stable one with the properties specified in Proposition 7 and an unstable variant that has the opposite comparative statics. The proof of Proposition 7 details this distinction.

The comparative statics in Proposition 7 imply that as the rents to the provider $b - a$ increase, more investors will become experts in the first place. This occurs through two channels. First, as $b - a$ increases, unsophisticated investors will pay higher rents over time, so they are more willing to invest in education. Second, and equally important, rising $b - a$ induces the provider to increase the frequency with which they change their product mix, which drives more investors to gain expertise in the first place. The same relationship holds with regard to the cost c .

Based on this analysis, there is another cost of obfuscation to society. Specifically, a proportion of investors will have to allocate resources to gain expertise, which represents an opportunity cost. This cost may then be computed as

$$\mathcal{K} = \int_0^B k dG(k).$$

As in Section 3.3, this has important policy implications when a social planner considers initiatives to change learning through λ . As shown there, a change in λ may induce the provider to change his mix of investment funds more frequently. Looking forward, more investors will expend resources

to gain expertise, which destroys value. If the magnitude of intervention is large enough, though, increased access to information will lead to lower obfuscation and higher welfare.

Of course, throughout the paper so far, we have considered the effects of obfuscation when the provider is a monopolist. Now, we turn our attention to the effects of competition on obfuscation and welfare.

6 Competition and Obfuscation

Consider now that n homogeneous financial institutions, indexed by $j \in N = \{1, \dots, n\}$, each sell a continuum of mutual funds. Each fund in the market is denoted by $w_{j,k}$, where j designates the provider and $k \in [0, 1]$ is their k^{th} fund. There is a unit mass of investors with unit demand who are willing to forfeit rents of $v > 0$ to purchase a fund.¹²

Experts are able to identify the best available funds among all the of products offered in the market, and they efficiently update their decisions as providers change their fund families over time. Each expert at any time thus demands the best available fund as long as the forfeited rents are weakly less than v . If more than one provider offers the best fund, experts will choose randomly among these options. Non-experts can become sophisticated transiently through learning according to the process in (1). When they do so, they learn about the best available fund(s) in the market. Each transiently sophisticated investor gets the best deal in the market, and invests in the same fashion as experts. However, when non-experts are unsophisticated, they purchase randomly from one of the providers as long as the expected forfeited rents from doing so is weakly less than v . As already mentioned, this approach to modeling unsophisticated investors is standard in both the literature on consumer search theory (e.g. Salop and Stiglitz, 1977, Varian, 1980, and Stahl, 1989) and household finance (e.g. Carlin 2009).¹³

The providers, therefore, design fund families and choose refreshing policies to maximize their profits. The following proposition characterizes an equilibrium for this game.

Proposition 8. *(Competition and Obfuscation) There exists a symmetric equilibrium in which at any time t each provider extracts zero rents from sophisticated investors and rents v from unsophis-*

¹²For example, if the funds in each fund family are homogenous so that the only differentiating feature is the fee structure, v is each investor's willingness to pay for the fund.

¹³For example, in models of "all-or-nothing" search (e.g. Salop and Stiglitz 1977 and Varian 1980), unsophisticated consumers are explicitly assumed to choose randomly among firms. In sequential search models, unsophisticated consumers are randomly assigned to their first firm and then choose whether to continue searching for the best alternative. In equilibrium, unsophisticated consumers stop at the first firm, so that they in essence make purchases randomly from the firms. See either Stahl (1989) or Baye, Morgan, and Scholten (2006) for a complete review of consumer search theory.

icated investors. Moreover, for any n , there exists a unique optimal stopping time $t^*(n) > 0$ that solves each provider's refreshing problem

$$\max_t \frac{\int_0^t e^{-rs} v \frac{y_s}{n} ds - e^{-rt} c}{1 - e^{-rt}}. \quad (27)$$

For finite n , $\frac{\partial t^*(n)}{\partial n} > 0$, and as $n \rightarrow \infty$, $t^*(n) \rightarrow \infty$.

Proposition 8 can be appreciated as follows. Because every provider offers a continuum of funds, they can perfectly discriminate between sophisticated and unsophisticated investors. By introducing a measure zero set of funds that yields rents a , and a measure one set of funds that yields expected rents b , a provider can assure that only sophisticated investors will ever be able to find its best available funds, forfeiting rents a . With such a product line, unsophisticated investors, who purchase randomly, will forfeit rents b almost surely.¹⁴

Competition drives the rents a earned from sophisticated investors to zero. Suppose that in equilibrium the best available fund in the market yields rents $a > 0$. One provider could profitably deviate from the equilibrium and produce an even better fund that yields a rent of $a - \epsilon$ per investor, with a small $\epsilon > 0$, capturing the demand of all sophisticated investors. This might involve decreasing the fees associated with the fund, improving quality, or adding an attractive dimension. In classic Bertrand fashion, the ensuing equilibrium involves each provider offering a superior product with the same value, earning zero rents ($a = 0$) from sophisticated investors.

On the other hand, at each time t , each provider faces a captive demand y_t/n of unsophisticated investors. To maximize profits without violating the participation constraint of unsophisticated investors, it is optimal for providers to set $b = v$.

With regard to refreshing policies, each provider's optimal choice is thus qualitatively similar to that when they are a monopolist, except that their problem is a scaled down version of the stationary problem in Section 2. Each provider solves the problem in (27). As compared to the problem in (4), providers obtain zero rents from the sophisticated investors. Because unsophisticated investors purchase their funds randomly from any of the n providers, at any time t each provider receives an equal share of demand from unsophisticated investors, $\frac{y_t}{n}$. Moreover, the rents forfeited by an unsophisticated investor are equal to v , their reservation value for owning the fund.

As in our previous analysis, there exists an optimal time to obfuscate. The solution $t^*(n)$ is strictly increasing in n , and under perfect competition, obfuscation disappears altogether.

¹⁴Alternatively, the providers could each choose a measure one distribution of rents according to a continuous function $F(\cdot)$ with expected rent of b . Since investors in this model are risk-neutral, such equilibria are payoff-equivalent to the one described here.

One concern that may arise is that the results of Proposition 8 depend on the assumption that obfuscation involves only fixed costs c to each provider, while the rents earned by each provider are decreasing in the number n of providers. However, the results of Proposition 8 are robust to an alternative formulation in which the obfuscation cost incurred by providers each time they refresh their product line is $c(\mu) = c_1 + c_2\mu$, where μ is the market share of the providers. In this case, as long as $c_1 > 0$, obfuscation is decreasing in n and disappears altogether when all investors become sophisticated as in the benchmark case studied above.

The decrease in obfuscation associated with competition has straightforward welfare implications based on the discussion in previous sections of the paper. Clearly, as obfuscation decreases the incentive to become an expert (an x_0 -type) decreases, so that investors in aggregate incur lower costs when they participate in the market. It remains ambiguous, however, the effect that lower obfuscation has on the aggregate costs that providers incur. On an individual basis, $C(r, T)$ is lower when there is more competition, but it is unclear whether $nC(r, T)$ is lower in aggregate.

The results here, though, must be taken with some degree of caution as the model does not admit generality to cover all market settings and conditions. It is intuitive that the information rents that accrue due to obfuscation dissipate when there are more providers. This result is consistent with Robert and Stahl (1993), who show that informative advertising increases with competition. However, it is also possible to consider alternative models in which competition may lead to greater obfuscation. Indeed, Carlin (2009) presents a static model in which complexity rises with competition. In the same way, if λ or x_0 were to decrease with competition because learning is more arduous (i.e., consistent with Carlin, 2009), then obfuscation might increase. Therefore, while the results in Proposition 8 are plausible and intuitive, we do not assert that they are general to all market settings. This is the subject of future research.

7 Concluding Remarks

Many retail investors lack sophistication regarding financial products, but choose to participate in the market. Over time, many learn but are required to keep abreast of developments in the market as they occur. Such changes are endogenously induced by producers in the financial market, and must be taken into account when government-sponsored educational initiatives are implemented.

In this paper, we study the interaction between obfuscation and investor sophistication in a dynamic setting. We characterize optimal cycles of obfuscation and demonstrate how they change based on primitives in the market: the extra rents available from unsophisticated investors, the

baseline financial education that investors possess, the speed at which learning takes place, and the underlying mechanism in which sophistication evolves. Strikingly, we show that small educational initiatives may induce further obfuscation by providers, which destroys economic surplus. Such wasteful obfuscation is enhanced as more investors participate in the market. On the other hand, major educational initiatives are effective in protecting investors and reducing wasteful obfuscation, but may entail high implementation costs. Our results suggest that an alternative way to reduce obfuscation and increase welfare is to increase competition among providers.

The analysis in this paper supports the view that education may not be an effective solution in retail financial markets. As Choi, Laibson, and Madrian (2008) show, retail investors do not make improved investment choices when they have better information about the market. There is now growing support for the use of default options to assist retail investors and improve welfare (e.g. Choi, Madrian, Laibson, and Metrick 2004). While not specifically modeled in our paper, default options would in essence make more investors experts (by proxy) and may decrease obfuscation, especially when used on a grand scale or in markets in which people learn on their own. Such libertarian paternalism makes sense in our model, as it would slow obfuscation and encourage participation. Given the welfare impact of such policies, continued exploration appears warranted.

Appendix

Proof of Proposition 1

Let t^* solve (4). Then the discounted profits achieved by the provider under the optimal policy are given by:

$$V \equiv \frac{\int_0^{t^*} e^{-rs} \{ax_s + b(1 - x_s)\} ds - e^{-rt} c}{1 - e^{-rt^*}}. \quad (28)$$

We need to show that there does not exist another policy that achieves a higher discounted profits. Let $T = (t_1, t_2, \dots)$ be an arbitrary policy. Then, by the above definition of t^* ,

$$V \geq \int_{t_i}^{t_{i+1}} e^{-rs} \{ax_s + b(1 - x_s)\} ds - e^{-r(t_{i+1}-t_i)} c + e^{-r(t_{i+1}-t_i)} V, \quad (29)$$

for $i = 0, 1, \dots$ and $t_0 = 0$. Multiplying both sides of the above inequalities by e^{-rt_i} we have

$$e^{-rt_i} V \geq e^{-rt_i} \int_{t_i}^{t_{i+1}} e^{-rs} \{ax_s + b(1 - x_s)\} ds - e^{-rt_{i+1}} c + e^{-rt_{i+1}} V \quad (30)$$

Summing the inequalities from $i = 0$ to $i = I$ causes telescopic cancellation on the left-hand side, leaving only

$$V - e^{-rt_{I+1}} V \geq \sum_{i=0}^I \int_{t_i}^{t_{i+1}} e^{-rs} \{ax_s + b(1 - x_s)\} ds - e^{-rt_{I+1}} c. \quad (31)$$

Taking the limit as I goes to infinity yields the result. ■

Proof of Proposition 2

The derivative of the objective function in (4) with respect to t is:

$$\frac{e^{-rt} \{ax_t + b(1 - x_t)\} + re^{-rt} c}{1 - e^{-rt}} - \frac{re^{-rt} (\int_0^t e^{-rs} \{ax_s + b(1 - x_s)\} ds - e^{-rt} c)}{(1 - e^{-rt})^2}. \quad (32)$$

The first order condition is given by:

$$rc + (1 - e^{-rt}) \{ax_t + b(1 - x_t)\} - r \int_0^t e^{-rs} \{ax_s + b(1 - x_s)\} ds = 0. \quad (33)$$

The left-hand side of (33) is positive at $t = 0$. Moreover, the derivative of the left-hand side of (33) with respect to t is equal to

$$-(1 - e^{-rt})(b - a)f(\lambda, x_t) \quad (34)$$

which is strictly negative since $b > a$ and $f(\lambda, x_t) > 0$.

In equation (5), \bar{c} is defined as the cost that solves (33) when $t = \infty$. Let us first assume that $c < \bar{c}$. When this is the case, the left-hand side of (33) is negative at $t = \infty$. Therefore, there exists

a unique $t^* < \infty$ that solves (33). Moreover, because (34) is negative, the left-hand side of (33) is positive for $t < t^*$ and negative for $t > t^*$. Consequently, t^* is a global maximum.

If, on the other hand, $c \geq \bar{c}$, the left-hand side of (33) is positive at $t = \infty$. Therefore, there does not exist a t^* that solves (33) and the derivative of the objective function with respect to t is positive for all $t \geq 0$. Therefore, the maximum is at infinity, meaning that it is optimal for the monopolist never to innovate.

Because the expression in (34) is negative and $f(\lambda, x_t)$ is continuous, we can apply the implicit function theorem to prove comparative statics.¹⁵ First, we take the derivative of the left-hand side of (33) with respect to b :

$$(1 - e^{-rt})e^{-rt}(1 - x_t) - re^{-rt} \left(\int_0^t e^{-rs}(1 - x_s) ds \right) \quad (35)$$

which is negative if x_t is increasing in t . Therefore, the optimal time t^* to obfuscate is decreasing in b . A similar calculation for a shows that the optimal time t^* to obfuscate is increasing in a .

Next, we take the derivative of the left-hand side of (33) with respect to c :

$$(1 - e^{-rt})re^{-rt} + re^{-2rt} \quad (36)$$

which is always positive. Therefore, the optimal time t^* to obfuscate is increasing in c . ■

Proof of Proposition 3

If the first limit condition in (7) holds, it is easy to see that there is a λ sufficiently small such that \bar{c} as defined in 5 is lower than c and therefore $t^*(\lambda) = \infty$. Using the same argument and the second limit condition in (7), we have that for λ sufficiently large $t^*(\lambda) = \infty$. ■

Proof of Proposition 4

For the first part of the proposition, we take the derivative of (33) with respect to x_0 to obtain:

$$(1 - e^{-rt})e^{-rt}(a - b) \frac{\partial x_t(\lambda, x_0)}{\partial x_0} - re^{-rt} \left(\int_0^t e^{-rs}(a - b) \frac{\partial x_s(\lambda, x_0)}{\partial x_0} ds \right) \quad (37)$$

which is negative if $\frac{\partial x_t(\lambda, x_0)}{\partial x_0}$ is increasing in t and positive if $\frac{\partial x_t(\lambda, x_0)}{\partial x_0}$ is decreasing in t . Therefore, the optimal time t^* to obfuscate is decreasing (increasing) in x_0 if $\frac{\partial x_t(\lambda, x_0)}{\partial x_0 \partial t}$ is positive (negative).

The second part of the proposition is proved with a similar argument as in the previous proposition. ■

¹⁵See, for example, Rudin (1976, p. 224).

Proof of Proposition 5

We take the cross-derivative of the objective function with respect to ϕ and t :

$$(1 - e^{-rt^*})e^{-rt^*}(ax(t^*) + b(1 - x(t^*))) - re^{-rt^*} \left(\int_0^{t^*} e^{-rs}(ax_s + b(1 - x_s))ds \right) \quad (38)$$

which is negative if x_t is increasing in t . Therefore, the optimal time to obfuscate is decreasing in ϕ . ■

Proof of Proposition 6

Define $H(\phi) = z(\phi) - \phi$. Since $\frac{\partial t^*}{\partial \phi} < 0$, $z(\phi)$ is continuous and monotonically decreasing in ϕ . Therefore, $H(\phi)$ is continuous and monotonically decreasing in ϕ . Since $H(0) > 0$ and $H(1) < 0$, $H(\phi) = 0$ at one, and only one, value of ϕ . Therefore, there is a unique fixed point at a particular $\phi \in (0, 1)$.

We now derive the comparative statics with regard to b ; the corresponding relationships for a and c may be derived similarly. Define $w = (a, b, c, \lambda, x_0)$ and $w' = (a, b', c, \lambda, x_0)$ such that $b' > b$. Consider an equilibrium value ϕ^* such that $H(\phi^*, w) = z(\phi^*, w) - \phi^* = 0$. Since $R(t^*(\phi^*, w'), w') > R(t^*(\phi^*, w), w)$, then $z(\phi^*, w') < z(\phi^*, w)$, which implies that $H(\phi^*, w') < 0$. Given that $H(\phi, \cdot)$ is continuous, there exists a $\phi^{*'} < \phi^*$ such that $H(\phi^{*'}, w') = z(\phi^{*'}, w') - \phi^{*'} = 0$. The analysis for $b'' < b$ is performed in the same fashion. Therefore, $\frac{\partial \phi^*}{\partial b} < 0$. ■

Proof of Proposition 7

First, we show that if a solution x_0^* exists, it must be that $x_0^* \in (0, 1)$. Since $\lambda < \infty$ and $b > a$, it must be that $\underline{B} > 0$. There must exist a fraction of investors $G(\underline{B})$ such that $k_i \leq \underline{B}$, which implies that x_0^* cannot be zero. Now, suppose that $x_0^* = 1$. Then, $t^* = \infty$ and $B = \underline{B}$. Since $\bar{k} > \underline{B}$, there exists an investor $i \in I$ such that $k_i > \underline{B}$. Specifically, a fraction $1 - G(\underline{B})$ will not pay the cost k_i . Therefore, it cannot be that $x_0^* = 1$.

Now, we can prove existence of an equilibrium. We know that $H(x_0) > 0$ when $x_0 = 0$ and that $H(x_0) < 1$ when $x_0 = 1$. Therefore, the function $H(\cdot)$ must cross the 45-degree line at least once. Given the continuity of H , there must exist at least one point x_0^* at which (26) holds with equality.

According to Proposition 4, if $c < \sup_{x_0} \bar{c}$ and $\frac{\partial^2 x_t}{\partial x_0 \partial t} < 0$, then we know that $\frac{\partial t^*}{\partial x_0}$ is positive. Since $H(x_0)$ is decreasing in t^* , this implies that once $H(x_0)$ crosses the 45-degree line from above, it never crosses again. Therefore, when $\frac{\partial^2 x_t}{\partial x_0 \partial t} < 0$, the fixed point at x_0^* is unique. Comparative statics follow in the same fashion as in the proof of Proposition 6.

For convenience, we define the function $\omega(t^*, \vec{v})$ as

$$\omega(t^*(x_0), \vec{v}) = H(x_0) - x_0,$$

so that in any equilibrium $\omega(t^*(x_0^*), \vec{v}) = 0$. If $\frac{\partial^2 x_t}{\partial x_0 \partial t} \geq 0$, then $H(x_0)$ may cross the 45-degree line multiple times, sometimes from above and sometimes from below. In such case, there will not exist a unique x_0^* , but rather two classes of equilibria (Class 1 and Class 2). For those that cross from above (Class 1),

- (i) $\frac{\partial x_0^*}{\partial b} > 0$
- (ii) $\frac{\partial x_0^*}{\partial a} < 0$
- (iii) $\frac{\partial x_0^*}{\partial c} < 0$

This follows in the same fashion as in the proof of Proposition 6 and the fact that $\frac{\partial \omega(t^*(x_0^*), \vec{v})}{\partial x_0} < 0$ at the point of equilibrium. For those that cross from below (Class 2),

- (i) $\frac{\partial x_0^*}{\partial b} < 0$
- (ii) $\frac{\partial x_0^*}{\partial a} > 0$
- (iii) $\frac{\partial x_0^*}{\partial c} > 0$

This follows in the same fashion as in the proof of Proposition 6 and the fact that $\frac{\partial \omega(t^*(x_0^*), \vec{v})}{\partial x_0} > 0$ at the point of equilibrium.

Now, we follow the discussion of Vives (2005, pages 440-445) and show that Class 1 equilibria are stable and Class 2 equilibria are unstable. The function $H(x_0) = G(B(x_0))$ is an aggregate best response function. Note that the aggregate best-response function as defined by Vives (2005) is $r(\tilde{a}) = F(g(\tilde{a}))$, where F is our function G , g is our benefit function B , and \tilde{a} is the fraction of players who take a particular binary action.

Consider a particular Class 1 equilibrium x_0^* and the neighborhood $\mathcal{N}_1 = [x_0^* - \epsilon, x_0^* + \epsilon]$. By the definition of this class of equilibrium, $H'(x_0) < 1$ at $x_0 = x_0^*$. First, choose an arbitrary $x_0 \in \mathcal{N}_1$ such that $x_0 - x_0^* = \delta > 0$. By the definition of a Class 1 equilibrium, it follows that $H(x_0) - H(x_0^*) < \delta$. Since $x_0^* = H(x_0^*)$, we know that $x_0^* + \delta > H(x_0^* + \delta)$ or $x_0 > H(x_0)$, which implies that the cost of becoming an expert exceeds the benefit of doing so for the marginal investor. Converging toward equilibrium implies that $x_0 \rightarrow x_0^*$. In words, since the benefit to becoming an expert is lower than x_0 , fewer investors will become experts. The same can be shown for an arbitrary $x_0 \in \mathcal{N}_1$ such that $x_0 - x_0^* = -\delta < 0$: the system will again converge toward the equilibrium point x_0^* . These two observations together assure that Class 1 equilibria are locally stable (Vives 1999).

Now, we show the opposite for a Class 2 equilibrium. Consider a particular Class 2 equilibrium x_0^* and the neighborhood $\mathcal{N}_2 = [x_0^* - \epsilon, x_0^* + \epsilon]$. By the definition of this class of equilibrium, $H'(x_0) > 1$ at $x_0 = x_0^*$. First, choose an arbitrary $x_0 \in \mathcal{N}_2$ such that $x_0 - x_0^* = \delta > 0$. By the definition of a Class 2 equilibrium, it follows that $H(x_0) - H(x_0^*) > \delta$. Since $x_0^* = H(x_0^*)$, we know that $x_0^* + \delta < H(x_0^* + \delta)$ or $x_0 < H(x_0)$, which implies that the cost of becoming an expert is less than the benefit of doing so for the marginal investor. In words, more investors will have an incentive to become an expert. Therefore, the system does not converge back to x_0^* . The same can be shown for an arbitrary $x_0 \in \mathcal{N}_2$ such that $x_0 - x_0^* = -\delta < 0$: the system will again fail to converge toward the equilibrium point x_0^* . Either of these two observations assure us that Class 2 equilibria are not locally stable (Vives 1999). ■

Proof of Proposition 8

Suppose that indeed all providers play the strategy outlined in the statement of the proposition. We will show that there is no profitable deviation available to provider j .

First, the rents a extracted from sophisticated investors must indeed be zero at any point in time. If provider j attempted to extract rents $a > 0$, no sophisticated investor would invest in the

inferior fund from provider j . Instead they would just invest in a fund offered by another provider that offers $a = 0$. Therefore, this is not a profitable deviation for provider j .

Moreover, provider j cannot extract rents higher than v from unsophisticated investors at any point in time. If that is the case, unsophisticated investors would opt for the outside option in which case each of them only forfeits rents v . Also, attempting to extract rents lower than v from unsophisticated investors can only reduce the profits for provider j , since the demand from unsophisticated investors for provider j 's funds at time t is fixed at y_t/n as long as the rents unsophisticated investors forfeit in the market are less than or equal to v .

The problem of setting an optimal obfuscation schedule is symmetric since all n providers earn zero rents from sophisticated investors and face a captive demand from unsophisticated investors. As such, the providers all choose an obfuscation schedule given that they receive demand $\frac{x_t}{n}$ from sophisticated investors and demand $\frac{y_t}{n}$ from unsophisticated investors. The rents to sophisticated investors are zero ($a = 0$) and the rents to unsophisticated investors are positive ($b = v$).

Stationarity of the problem follows in the same fashion as in the proof of Proposition 1. Each provider therefore solves

$$\max_t \frac{\int_0^t e^{-rs} v \frac{y_s}{n} ds - e^{-rt} c}{1 - e^{-rt}}. \quad (39)$$

Uniqueness of a solution $t^*(n)$ is established using the same logic as in the proof of Proposition 2.

Finally, we take the cross-derivative of the objective function with respect to n and t :

$$\left(-(1 - e^{-rt})e^{-rt}(1 - x_t) + re^{-rt} \left(\int_0^t e^{-rs}(1 - x_s) ds \right) \right) \times \left(\frac{1}{n^2} \right) \quad (40)$$

which is positive if x_t is increasing in t . Therefore, the optimal time to obfuscate is increasing in n . Taking the limit as $n \rightarrow \infty$ yields the result that $t^* \rightarrow \infty$. ■

Comparative Statics Results on \bar{c}

We now prove some comparative statics results on \bar{c} .

Proposition 9. *If there exists a constant κ such that for all λ and x_0 , $\lim_{t \rightarrow \infty} x_t = \kappa$, then*

$$(i) \frac{\partial \bar{c}}{\partial b} > 0$$

$$(ii) \frac{\partial \bar{c}}{\partial a} < 0$$

$$(iii) \frac{\partial \bar{c}}{\partial \lambda} < 0$$

$$(iv) \frac{\partial \bar{c}}{\partial x_0} < 0$$

Note: The condition $\lim_{t \rightarrow \infty} x_t = \kappa$ for all λ and x_0 implies that after enough time has passed without any change in the funds the fraction of the population that becomes sophisticated does not depend on λ and x_0 . In the two examples we study in Subsection 3.2, this condition is satisfied, since all investors in the population eventually become sophisticated if the funds are not changed (i.e. $\lim_{t \rightarrow \infty} x_t = 1$).

Proof: If $\lim_{t \rightarrow \infty} x_t = \kappa$, then

$$\lim_{t \rightarrow \infty} \frac{\{ax_t + b(1 - x_t)\}}{r} = \frac{a\kappa + b(1 - \kappa)}{r},$$

which is independent of x_0 and λ .

For the first comparative statics result, we just take the derivative of \bar{c} with respect to b to obtain:

$$\int_0^\infty e^{-rs} \{(1 - x_s)\} ds - \frac{(1 - \kappa)}{r},$$

which can be rewritten as

$$\int_0^\infty e^{-rs} \{(\kappa - x_s)\} ds.$$

Since x_t is increasing in t , $x_t \leq \kappa$ for all t . The above derivative is thus positive, completing the proof. A similar argument can be used to show the second comparative statics result.

For the third comparative statics result, it is enough to show that

$$\int_0^\infty e^{-rs} \{ax_s + b(1 - x_s)\} ds \tag{41}$$

is decreasing in λ . Taking the derivative of (41) with respect to λ we obtain

$$- \int_0^\infty e^{-rs} (b - a) \frac{\partial x_s(\lambda, x_0)}{\partial \lambda} ds.$$

Since $b > a$, to show that the above derivative is negative all we need to show is that $X \equiv \frac{\partial x_t(\lambda, x_0)}{\partial \lambda}$ is positive. From a standard result in ordinary differential equation (see e.g. Hsu (2006, Theorem 2.5.1)), we have that X satisfies the following equation:

$$\begin{aligned}\frac{dX}{dt} &= f_x(x_t, \lambda)X + f_\lambda(x_t, \lambda) \\ X_0 &= 0.\end{aligned}$$

Since $X_0 = 0$ and $\frac{dX}{dt}$ is positive at $X = 0$, X is always positive.

The proof of the fourth comparative statics result is analogous, except that we need to show that $Y \equiv \frac{\partial x_t(\lambda, x_0)}{\partial x_0}$ is positive. Again, using a standard result in ordinary differential equations (see e.g. Hsu (2006, Theorem 2.5.2)), we have that Y satisfies the following equation:

$$\begin{aligned}\frac{dY}{dt} &= f_x(x_t, \lambda)Y \\ Y_0 &= 1.\end{aligned}$$

Since $Y_0 = 1$ and $\frac{dY}{dt}$ is zero at $Y = 0$, Y is always positive.

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