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ON THE OPTIMAL COMPENSATION OF A SOCIALIST MANAGER

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The idea for this paper was suggested, unwittingly to be sure, by the Soviet Premier Alexei Kosygin in his famous speech of September 27, 1965 inaugurating the Soviet Economic Reforms. Of the several changes in directives given to enterprises which he announced, two are relevant here: (1) The greater emphasis to be placed on profits, and (2) the replacement of the output target by sales.[1)]

Taking advantage of the theorist's inherent right of simplification, I would say that the enterprise manager (or director, as he is usually called) was instructed to maximize an unspecified function of profits and sales, subject to certain planning directives and several constraints which, though important in themselves, need not be considered here.[2)] I will argue in Part II that the maximization of a weighted sum of profits and sales makes excellent sense when the enterprise is allowed to set the prices of its outputs. It is not needed, however, if prices are set by the State, as indeed they are in the Soviet Union. Under these conditions, why was not the Manager given freedom of decision and instructed to maximize profits only, in accordance with good old economic theory, and without the additional directives and constraints?

I suspect that Mr. Kosygin's solution was not based on fine theoretical considerations.[3)] Even if he sympathized with them (for which there is little, if any, evidence), he would certainly be reluctant to abolish the planning mechanism and give complete freedom to Soviet enterprise managers. There was no telling in what kind of wild ventures these managers, unused to the freedom of the market, might get involved, and through how many perturbations the economy would have to pass until some reasonable equilibrium was achieved. Besides, Mr. Kosygin, like everyone else, must have known that Soviet prices, based on a mark-up system and usually unchanged for a number of years, do not equate demand and supply.[4)] When such prices are combined with excess demand, still common in the Soviet economy, the maximization of profits by enterprises can lead to all sorts of weird results.

The defects of the Soviet price system, like those of practically any system of controlled prices, are too well known to require a long discussion here. Let me merely mention two: (1) Unless all dimensions of a commodity or of a service are specified explicitly -- a costly and a laborious process -- its numerous characteristics cannot be controlled by the single dimension of a price; its quality will deteriorate. (2) The infrequency of Soviet price revisions discourages the introduction of new products and of new models. A price set for a new commodity normally covers the average cost of production (when large-scale output begins), plus a modest markup. With time, the cost of production declines due to the learning process and similar reasons -- there is little wage inflation in the Soviet Union. The old product becomes highly profitable. The manager has no incentive to replace it with a new one, subject to that modest profit margin.[5)] More frequent price revisions are of course costly. In spite of the present trend toward re-centralization, a day will surely come when Soviet planners will have to delegate at least some price-setting rights to the producers. Hungary has already made some progress in this direction.

But price-setting by producers involves at least two dangers: inflation and monopoly. On inflation I have little to say here, except to suggest that it can be avoided if the planners achieve a reasonable macro-balance and retain some control over wages, or at least prevent their labor unions from behaving like ours. It is not that I underestimate the difficulties of controlling inflation: this paper simply deals with a different subject. It is concerned with the second hazard -- monopoly power. The high concentration of control over industry in Eastern Europe, the strong affection for large-scale enterprises by socialist planners, and the small size of European socialist countries, with the single exception of the Soviet Union, would allow the producers and their organizations to exercise monopoly powers beyond the fondest dream of any Wall-Street operator. Of course, the control over industry could be reorganized; perhaps even anti-trust departments could be set up in the respective ministries of justice -- it takes some imagination to visualize that -- and imports could be used to break monopoly power. But this last weapon, perhaps the most effective of them all, requires ample supplies of foreign exchange. Even then, would socialist managers and workers welcome foreign competition any more enthusiastically than do their capitalist colleagues?

I think it is safe to conclude that if socialist managers are given freedom of decision and are encouraged to maximize profits under a market system of prices set by themselves, monopolistic and oligopolistic practices will abound. But perhaps Mr. Kosygin's suggestion can be utilized to express their instructions in some other, still reasonably practical way, to make them behave in a more socially-desired manner.

II

We shall first consider an enterprise producing only one output and then proceed to the general case with any number of inputs and outputs. We shall assume that the Manager has the ability and all the necessary information about demand and cost schedules to maximize total profit within his time horizon if that was his objective. That is, for every planned range of output he would choose the lowest-cost technology and input combination, and then proceed to the intersection of the marginal cost and marginal revenue curves, as shown by the solid lines on the three diagrams which we all learned in our first course in economics. (The schedules are represented by straight lines in Figs. 1 and 2 only because straight lines are easier to draw.) How he gets this information, whether he takes the mode or the mean or some other moment of a probability distribution, how he protects himself against uncertainty in general, and how he deals with the complexities of oligopolistic strategy is none of our concern, though we'll have to return to oligopoly briefly in Part IV. The important point is that the change in his instructions to be suggested presently will not call for any additional information or any extra ability on his part.

Now, the Planner, as we shall call the official who determines the rules and who desires an optimal allocation of resources, wants the Manager to set his output price at point <u>A</u> where marginal cost equals price.[6] The trouble is that the Planner does not know the position of <u>A</u>. (Even if he did, he would still have to find some method, hopefully other than a direct order, to induce the Manager to move there.) The Planner does know, however, that if the Manager was maximizing total profit he would be at some point <u>D</u>, to the left of <u>A</u>. He also

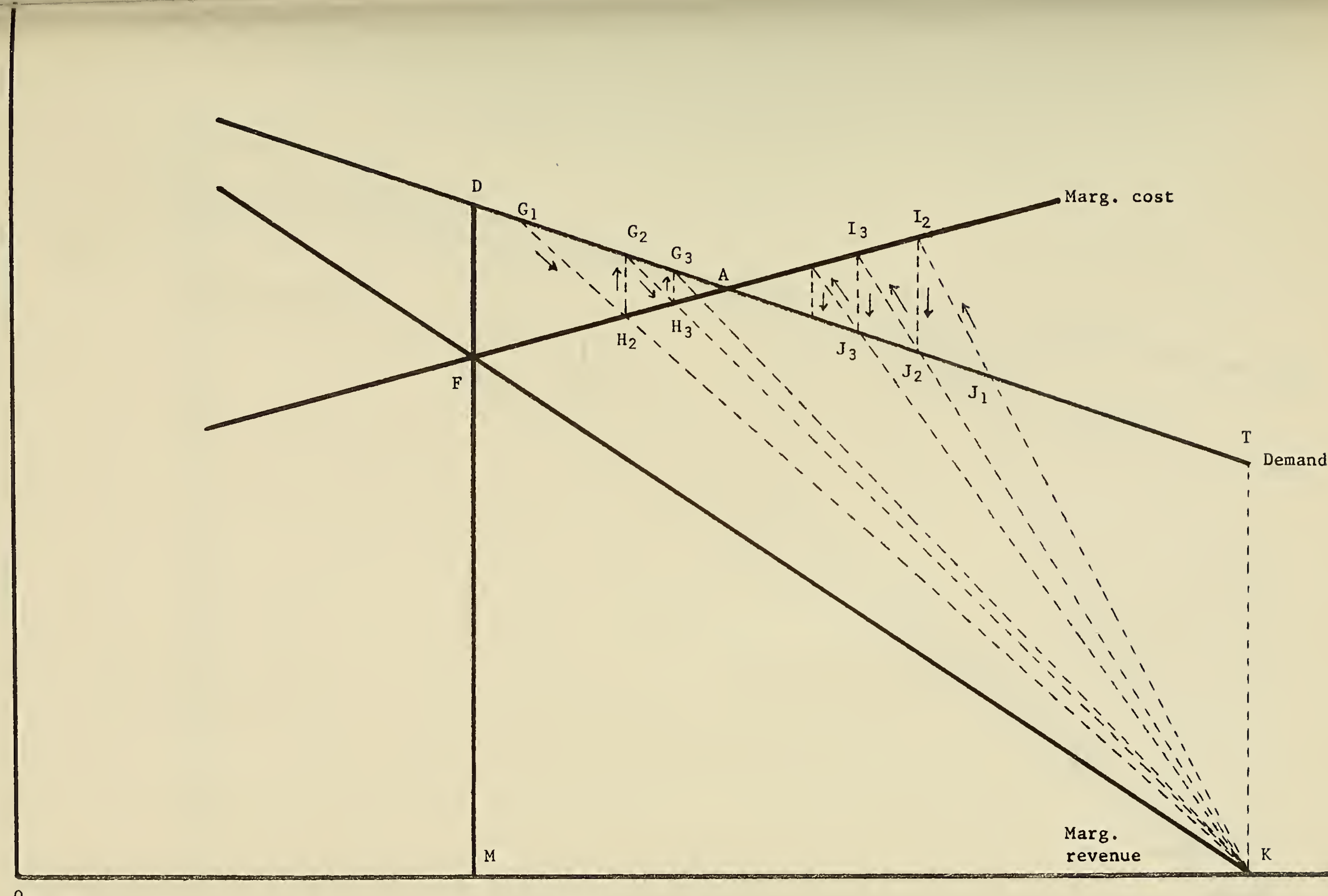
D
G_1
G_2
G_3
A
I_3
I_2
Marg. cost
H_2
H_3
J_3
J_2
J_1
F
T
Demand
M
Marg.
revenue
K
0
Output

Fig. 1

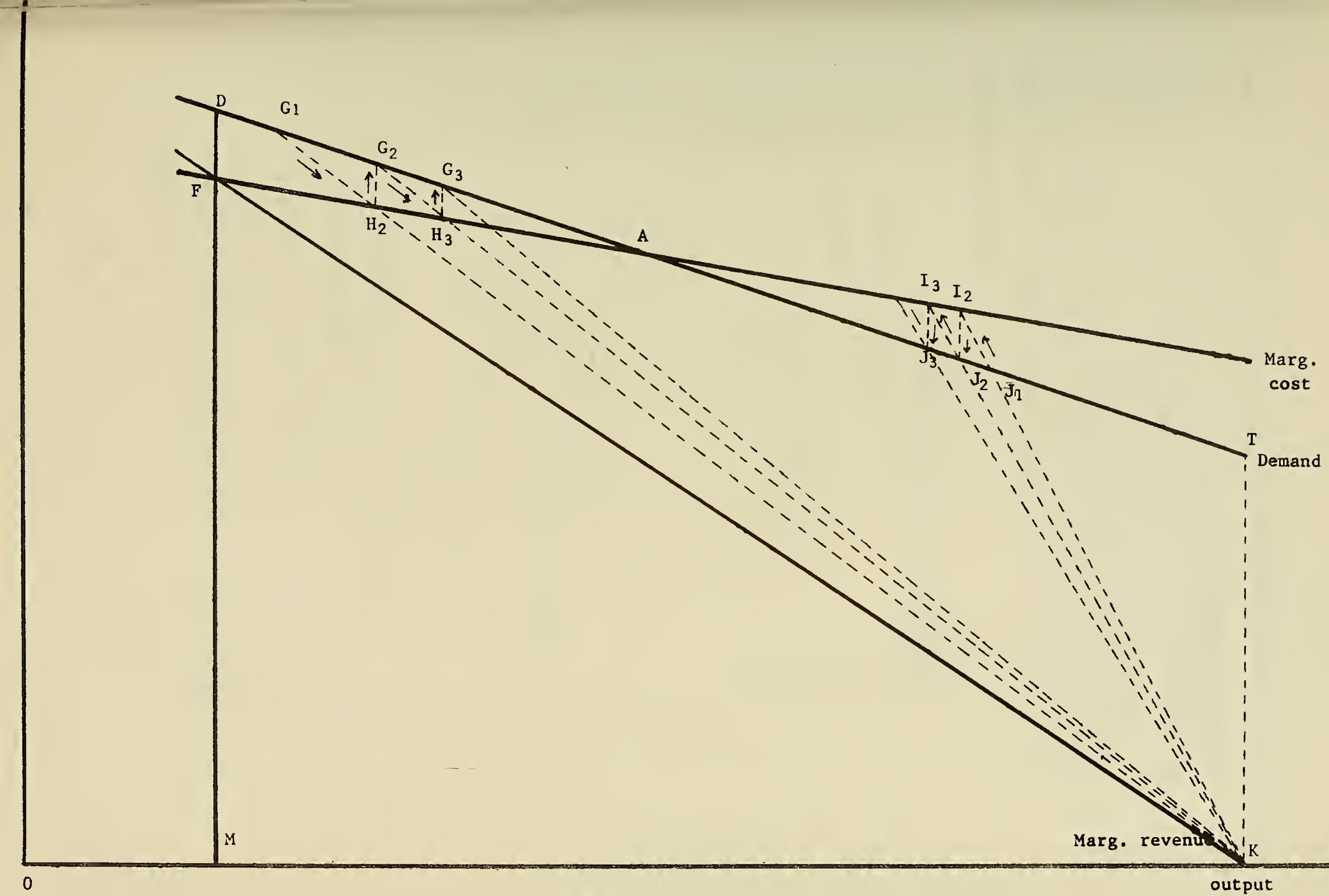
D
G1
G2
G3
F
H2
H3
A
I3
I2
J3
J2
J1
Marg. cost
T
Demand
M
Marg. revenue
K
0
output

Fig. 2

knows that if the Manager was instructed to maximize total sales he would move to the right of A.[7] Thus -- and this is the central point of this paper -- profits in the objective function move the Manager to the left, and sales -- to the right of A along the demand curve. Surely, there must exist some combination of profit and sales which would induce the Manager to operate at point A. But first, a few mathematical symbols.

LIST OF SYMBOLS (In order of Appearance)

B	the Manager's bonus
u, v	parameters
N	net profit (before the bonus)
R	revenue or sales
p	price
x	output
C	total cost
E	elasticity, usually of demand
n	number of outputs, or of inputs and outputs
E_s	elasticity of supply
z	$= \frac{u+v}{u}$
N^*	adjusted net profit (with shadow prices)
q	corporate income tax rate
t	time (in adjustment units)
$\hat{}$	means optimal

The symbols $'$ and $''$ indicate first and second derivatives in respect to x.

Let us assume that the Planner offers the Manager a bonus which the Manager is absolutely determined to maximize (both for the sake of income and as a success indicator). Let this bonus consist of a weighted sum of profits and of sales:

$$(1) \quad B = uN + vR = u(px-C) + vpx = (u+v)px - uC.$$

To maximize it, differentiate B in respect to x and equate the derivative to zero:

$$(2) \quad \frac{dB}{dx} = (u+v)(p + x\frac{dp}{dx}) - uC' = 0,$$

which yields

$$(3) \quad p = \frac{C'}{\left(\frac{u+v}{u}\right)\left(1+\frac{1}{E}\right)},$$

where $E = \frac{dx}{dp} \cdot \frac{p}{x}$ is of course the elasticity of demand.[8)]

But the Planner wants the price to equal marginal cost:

$$(4) \quad p = C'.$$

Hence u and v should be chosen in such a way that

$$(5) \quad \frac{C'}{\left(\frac{u+v}{u}\right)\cdot\left(1+\frac{1}{E}\right)} = C',$$

which, after a few simple manipulations, reduces to

$$(6) \quad \frac{u}{v} = -(E+1),$$

the E indicating here the demand elasticity at the optimal point A the location of which is still unknown.

If the enterprise produces several outputs $x_1, x_2, \ldots, x_n$ and sets the corresponding prices of $p_1, p_2, \ldots, p_n$, the bonus should be expressed as

$$(7) \quad B = uN + \sum_{i}^{n} v_i x_i p_i .$$

By taking partial derivatives in respect to x_i and equating them to zero we again obtain the result that

$$(8) \qquad \frac{u}{v_i} = -(E_i+1),$$

but with the very important qualification that the cross-elasticities of demand are sufficiently small to be disregarded. Otherwise the C_i' refuse to cancel out, and the mathematical solution is too complex for practical use. This means that the parameters v_i cannot be set separately for each product, but must be applied to the total output of each department of the enterprise, the departments being arranged in such a way as to make the inter-departmental cross elasticities of demand negligible. From an administrative point of view this may be even an advantage: the Planner would undoubtedly prefer not to have to compute demand elasticities for each model of say, General Motors cars, to give an American example. But it might be difficult to divide General Motors into proper departments because of the continuous characteristics, so to speak of its outputs. It is unlikely that Chevrolets compete with Cadillacs directly. But Chevrolets compete with Pontiacs, Pontiacs with Buicks, and Buicks with Cadillacs. Where are we to draw the line? It may be necessary to put all General Motors cars into one department, while trucks, Diesel engines, refrigerators, etc., each comprise a separate one. As a result, the ratios $\frac{u}{v_i}$ will correspond not to the actual demand elasticities for specific commodities but to their weighted average. Hence, products whose elasticities are higher than the average for the department, will be over-produced, and the others -- produced below the optimum. It is highly unlikely, however, that demand elasticities can be estimated with much precision even under the best of circumstances. So all we can expect from our bonus scheme is a movement to some approximation of the optimal output.[9)]

Expressions (6) and (8) give only the relative magnitudes of $\underline{u}$ and $\underline{v}$: they do not of course determine the absolute size of the bonus which the Planner will presumably set according to some other considerations.

The whole scheme will make no sense if $|\underline{E}| \leq 1$. Direct price regulation (perhaps similar to that practiced in our public utilities) would be required. Actually, many demand elasticities need not be particularly low because they pertain not to the demand for the whole industry but only to that for the individual enterprise.

To obtain some idea about the composition of the bonus, let us take a demand elasticity as high as -4. Set $\underline{v}$ = 1 per cent, and $\underline{u}$ = 3 per cent (as given by expression (6)) and assume sales of 1000 and a net profit of 100 (a 10 per cent profit margin seems reasonable). Then the bonus will equal 3% x 100 + 1% x 1000 = 3 + 10 = 13. Note that more than three-quarters of this bonus (77 per cent) are derived from sales. Even with $\underline{E}$ = -6, two-thirds of the bonus still come from that source. And these are high elasticities. If the profit margin was only 5 per cent, the corresponding shares would be even higher -- 87 and 80 per cent. Mr. Kosygin certainly had a point.

Monopsony can be handled in exactly the same manner, except that in expression (1) the parameter $\underline{v}$ is applied not to sales, but to, say, the payroll, if labor is the factor subject to monopsonistic exploitation. But because payroll is an expenditure rather than a receipt the ratio $\underline{u/v}$ takes the form of

$$(9) \qquad \frac{u}{v} = E_s + 1,$$

again on the condition that the cross-elasticities, this time of the supplies of inputs, can be neglected. A solution for any number of inputs and outputs with a generalized production function is given in the Mathematical Note, but because of the close analogy between the cases of monopoly and monopsony the latter will be omitted from the subsequent discussion.

III

It now remains to find the correct elasticity of demand. If this elasticity is constant, at least in the relevant range, let us hope that it can be estimated. But it need not be constant. It is the elasticity at (or near) point $\underline{A}$ that the Planner needs. Yet all available empirical data will pertain to the region around point $\underline{D}$ if sales were not previously included in the bonus function, or around some other point on the demand curve if the bonus was set incorrectly, or if the demand and/or the cost curves shifted since the bonus had been arranged. How can the Planner discover that the bonus, as it is presently composed, is wrong and ascertain in what direction it should be changed? How can he induce the Manager to operate at point $\underline{A}$ when he does not know where this point is?

Two cases will be considered, depending on whether the (absolute magnitude of the) elasticity of demand declines or rises with increasing output.

1. Declining Elasticity of Demand. Assume that the bonus scheme has been in operation for some time, and that the Manager is now at some point $\underline{G}_1$ on the demand curve as shown in Figs. 1 and 2. The Planner knows $\underline{x}_1$ and $\underline{p}_1$ at $\underline{G}_1$ and the current bonus ratio arranged previously which we shall call $\underline{u/v}_0$. By assumption, he can estimate $\underline{E}_1$ as well.

Comparing u/v_0 with the u/v_1 which would correspond to E_1 he finds $u/v_0 > u/v_1$. This tells him that point G_1 is to the left of A, a piece of information which, while not absolutely necessary, is convenient to have.[10)] He now sets a new $u/v_1 = -(E_1 + 1)$.

It will now be convenient to introduce a new parameter $z = \frac{u+v}{u}$. By definition, at any point on the demand curve,

$$(10) \quad z = \frac{1}{1+\frac{1}{E}} = \frac{1}{1+\frac{dp}{dx} \cdot \frac{x}{p}} = \frac{p}{p+ x\frac{dp}{dx}} = \frac{p}{R'} .$$

Thus

$$(11) \quad z_1 R_1' = p_1 ,$$

i.e. z_1R' passes through point G_1.

The maximizing equation (2) can be rewritten as

$$(12) \quad zR' = C'.$$

This expression shows that our bonus scheme is simply a device for inducing the Manager, in his quest for the largest reward, to maximize an adjusted profit $N^* = zpx - C$ which determines his bonus. Mathematically speaking, this is a better scheme because it uses only one parameter z instead of our two -- u and v (see the Mathematical Note). I think, however, that practical people (capitalist or socialist) will be more at home with a bonus expressed in terms of conventional profits and sales rather than with one based on a price adjustment. In either case, the Manager will work with adjusted marginal revenue curves zR' and equate them to C'. A family of such curves for particular values of z is represented by the dotted lines GHK and IJK on Figs. 1 and 2. They all meet the original R' curve (corresponding to $z = 1$) at K where $R' = 0$.

It can be shown that point G_2 will lie between G_1 and A, that is that the method of setting the bonus described here will result in a non-oscillatory movement converging on A. In the linear case represented on Figs. 1 and 2 this is obvious. A general proof is given in the Mathematical Note.

After the Manager moved to point G_2, the Planner, having collected sufficient information about E_2 at G_2 and finding that it does not correspond to z_1 will calculate a new z_2 and change the bonus accordingly. The Manager will now move to point G_3 between G_2 and A, and so on. Only when the Planner ascertains that a newly calculated E corresponds to a previously set z does he know that the Manager has indeed reached the optimal point A where $zR' = p = C'$.

If the Manager's original position was at J_1 to the right of A, the same method of successive bonus adjustments would move him leftward toward A.

The negative slope of the marginal cost curve in Fig. 2 creates no problems in this respect, so long as the stability conditions are satisfied, but it ~~does~~ may call for one more decision: either the enterprise will have to be subsidized or the price will have to be set at some multiple of marginal cost by changing equation (4) accordingly -- a subject amply discussed in welfare economics.

Thus our method, which may be called the "Simple Rule", does result in convergence without oscillations. But the speed of convergence remains unknown. An experienced Planner may improve on it by making stronger adjustments(in either direction), and thus sending the Manager to point A with fewer iterations. These must be disturbing

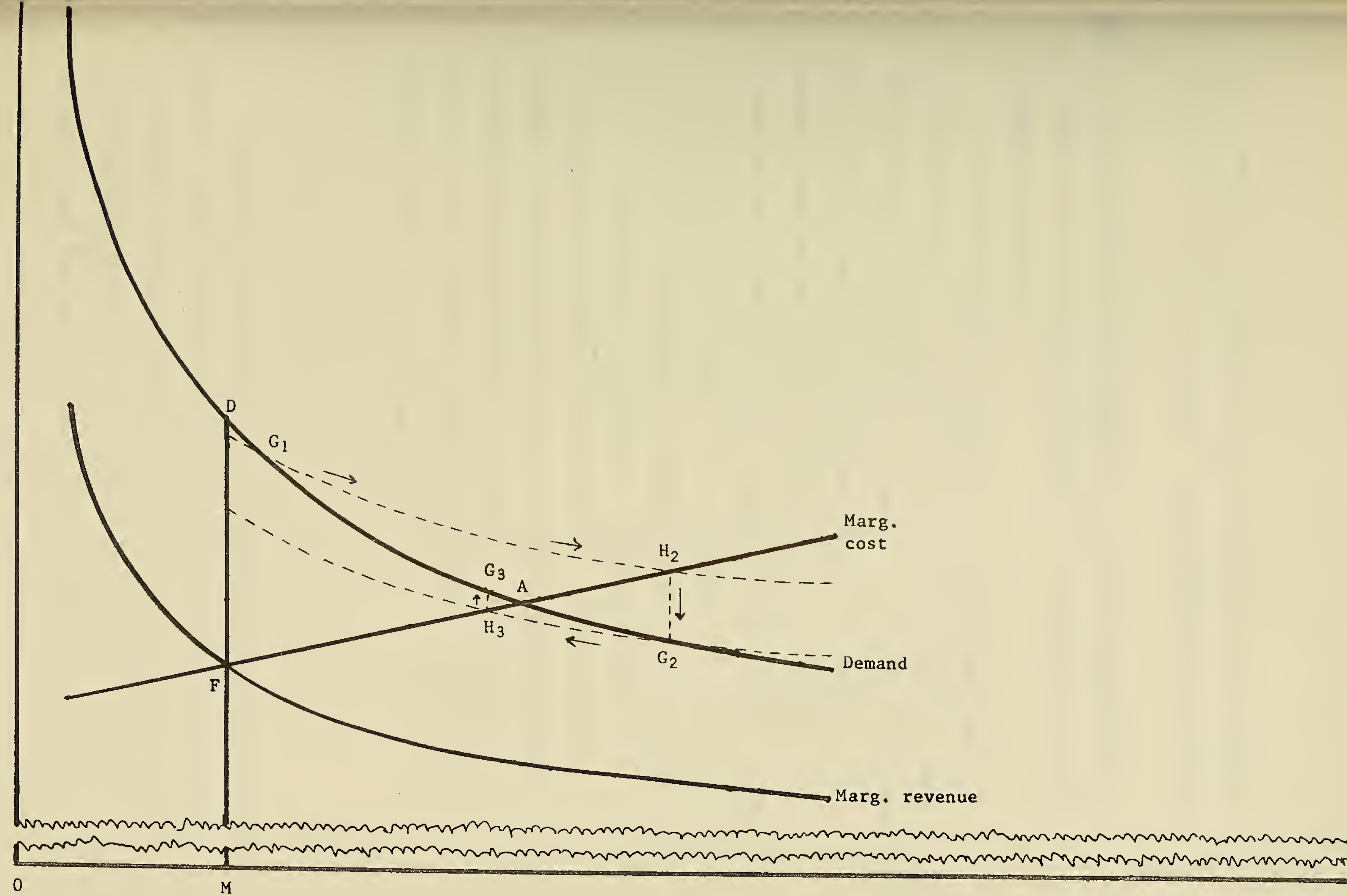

Fig. 3

to the Manager and particularly to his customers. But the Planner should take care not to overshoot. His reputation may be at stake.

2. <u>Increasing Elasticity of Demand</u>. R.G.D. Allen regards this an "abnormal" case, at least by implication.[11)] I hope that it is most uncommon in practice.

A demand curve with an increasing (in absolute magnitude) E is presented on Fig. 3. We again start at point G_1 and try to apply the Simple Rule. Unfortunately, as shown on Fig. 3 and in the Mathematical Note, this Rule results in oscillations around A.[12)] On Fig. 3 the process converges (and rather rapidly at that), but this need not always be true. Even if it does, a practical Planner will be reluctant to use the Simple Rule because of the oscillations. To avoid them, he will have to dilute the Rule. Suppose that the original z_0 was set at 1.10 while the z_1 corresponding to E_1 at point G_1 is 1.18. The Planner sets the new z at 1.13 or even at 1.12 and watches the Manager's moves before making another change. I have not been able to devise a simple general method for dealing with this unusual case and I doubt that further effort is worth while. Let us hope that the Planner learns from experience.[13)]

IV

The practical application of our method merely calls for periodic, perhaps annual, checks whether the <u>u/v</u> ratio (or the z) as it appears in the bonus corresponds to the E of the Manager's present position, and for an adjustment of this ratio when a significant discrepancy is found. More frequent changes would be irritating both to the

Manager and to his customers. On the other hand, if the u/v ratio is changed infrequently, it may pay a group of managers to engage in fictitious sales with one another.[14)] Hence some check of the sales record of the enterprise and of its profit/sales ratio may be required.

It is important to assure the Manager that the absolute size of his bonus is little affected by changes in the u/v ratio. Otherwise, the Manager, whose knowledge and intelligence need not be inferior to the Planner's, and who can readily figure out the Planner's rules, will be tempted to pursue a game strategy against the Planner. It may pay the Manager not to maximize his bonus at a given moment either in the hope that a change in the u/v ratio will increase his bonus or for the fear that it will diminish it. Hence the Manager may not be in the position dictated by the current u/v ratio, a situation that may mislead the Planner and possibly lead to wrong adjustments.[15)] It may even cause instability. Further exploration of this potentially exciting process I will leave to connoisseurs of game theory.

All these suggestions are based on the optimistic assumption that the elasticity of demand (in the relevant range) can be estimated with some tolerable degree of accuracy and that both the Planner and the Manager arrive at the same estimate. (If the absolute size of the bonus is made reasonably independent of $\underline{u/v}$, there is no reason for excluding consultations.) In respect to simple monopoly this assumption can probably be justified, but surely demand elasticity becomes a rather elusive concept under conditions of oligopolistic competition -- a much more frequent case. Is the Planner to assume that his managers act independently of one another, or that they enter into, possibly secret, collusive agreements? Even if he can estimate the $\underline{E}$ for the industry as a whole, can he really approximate it for each individual enterprise, dependent as its $\underline{E}$ is on the actions of its competitors? If problems of static oligopolistic price-setting are very complex (and I have no desire to discuss them here), do not they become unmanageable in the presence of technological progress?

There are two answers to these objections: First, the needed elasticities pertain not the specific products, but to departments of an enterprise (see Part II). Demand for their outputs in the aggregate should be more stable than that for individual products. Second -- and more important -- do we have better alternatives?

The defects of profit maximization as an instrument for achieving an optimal allocation of resources are well known. Nevertheless, it seems to me that Soviet experience clearly shows that this very imperfect method is still the best available, at least for the normal operations of an ordinary enterprise. (Large investment decisions are a different matter.)[16)] If an effective policy of price control without the usual difficulties (see Part I) could be devised, the Manager should be instructed to maximize profit without much ado. But for most goods and services a satisfactory policy of this kind has not yet been invented. It seems better then to let the Manager (except in special sectors) set his own prices. But if sales are not included in his instructions and/or in his bonus function, this is tantamount to the assumption of an infinite elasticity of demand. Surely more realistic assumptions can be made.

Perhaps the skeptical Planner may be persuaded, at the beginning, to set the $\underline{u/v}$ ratio in each industry in such a manner that the profit component in managerial bonuses approximately equals that of sales. With the profit/sales ratio of 10 percent which we used previously (it varies among industries and enterprises), this amounts to the assumption that $\underline{E} = -11$; with a 5 percent markup, $\underline{E} = -21$, surely generous overestimates. Perhaps the profit component should be made equal to half of that of sales. This would still imply high elasticities: -6 and -11 respectively. In time, differentiated ratios adjusted to the characteristics of the various industries and firms could be worked out, even if the Planner did not follow each step of the fascinating process described in Part III.

But even if the Planner still rejects the bonus idea as being impractical, not everything is lost. He should at least make the Manager understand that his performance is evaluated by the Planner not only on the basis of profit but also of sales. So if a report of rising sales brings about a broad smile on the Planner's face, the Manager, a professional person interested in promoting his career, may well behave as if sales were indeed included in his bonus formula. But the Planner, only a human being, may not always show the right breadth of a smile, and the Manager, another human being, may not always know how to quantify it.

* * *

The inclusion of sales in the bonus need not be limited to a socialist manager. It can be applied to the compensation of any decision-maker who has monopolistic (or monopsonistic) powers, as for instance to that of a head of a department of a vertically integrated capitalist corporation.[17)] The principle can also be used a general anit-monopoly measure by taxing profits at the rate $\underline{q}$ and subsidizing sales (or certain purchases) at the rate of $\underline{v}$, the $\underline{(1 - q)/v}$ ratio being determined by the elasticity of demand (or supply). But even if such a flexible tax-subsidy policy could be used in a country like France, I doubt that the American legal system would tolerate it. For that matter, strange as it may seem, we may not need it. For if Professor Galbraith and others are right in asserting that sales are included in the objective functions of large corporations, the problem of monopolistic pricing may have already been solved, or at least seriously mitigated. In our modern industrial state wonders never cease.[18)]

MATHEMATICAL NOTE

Assumptions: (i) Demand and cost function are monotonic and twice differentiable in the relevant range. (2) $\underline{p}_i' < 0$ if $\underline{x}_i$ is an output, $\underline{p}_i' > 0$ if $\underline{x}_i$ is an input. (3) $\underline{p}$ intersects $\underline{C}'$ in a single point $\underline{A}$ in the region where $|E| > 1$. Hence it follows from the simple monopoly maximizing equation $\underline{p} + \underline{xp}' = \underline{C}'$ that $\underline{p} > \underline{C}'$ at point $\underline{D}$ and between $\underline{D}$ and $\underline{A}$ and $\underline{p} < C'$ to the right of $\underline{A}$.

To Part II. Second-order conditions for profit maximization under ordinary monopoly are $\underline{R}'' < \underline{C}''$. The differentiation of equation (2) in the text expresses these conditions as $\underline{zR}'' < \underline{C}''$. Hence, if $\underline{R}'' < 0$ stability is reinforced. But if $\underline{R}'' > 0$ (which may happen with increasing $|\underline{E}|$), a previously stable situation may become unstable.

In the general case,

$$(13) \qquad N = \sum_{1}^{n} p_i x_i \quad (i = 1, \ldots, n) \; ,$$

and

$$(14) \qquad B = \sum_{1}^{n} (u+v_i) p_i x_i \; ,$$

subject to the production function

$$(15) \qquad f(x_1, \ldots, x_n) = 0.$$

Using Lagrangian multiplier $\underline{\lambda}$ form

$$(16) \qquad Y = \sum_{1}^{n} (u+v_i) p_i x_i - \lambda f(x_1, \ldots, x_n).$$

Differentiate (16) in respect to $\underline{x_i}$ and equate to zero:

(17) $$p(u+v_i)\cdot(1 + \frac{x_i}{p_i} \cdot \frac{\partial p_i}{\partial x_i}) = \lambda f_i ,$$

provided of course that $\frac{\partial p_i}{\partial x_j} = 0$ for $\underline{i} \neq \underline{j}$.

(18) $$p = \frac{\lambda f_i}{(u+v_i)(1 + \frac{1}{E_i})} .$$

It is desired that prices should be proportional to the respective rates of transformation in production:

(19) $$\frac{p_i}{p_j} = \frac{f_i}{f_j} \qquad \text{for all } \underline{i} \text{ and } \underline{j}.$$

Substituting (19) into (18) yields

(20) $$(u+v_i)(1 + \frac{1}{E_i}) = (u+v_j)(1 + \frac{1}{E_j}) \quad \text{for all } \underline{i} \text{ and } \underline{j}.$$

The solution given by expression (8) in the text is

(21) $$\frac{u}{v_i} = - (E_i + 1).$$

It can be easily ascertained that (21) satisfies equation (20), because (21) implies for all $\underline{i}$ and $\underline{j}$

(22) $$(u+v_i)(1 + \frac{1}{E_i}) = u = (u+v_j)(1 + \frac{1}{E_j}).$$

The condition (21) is not the only solution which satisfies (20). The latter expression says that the manager will produce (or buy) the proper amounts (and charge or pay the proper prices) provided the $(\underline{u} + \underline{v}_i)$ are inversely proportional to the respective $(1 + \frac{1}{E_i})$. As was already mentioned in the text regarding expression (12), $(\underline{u} + \underline{v}_i)$ can be regarded as price-adjusting weights transforming our bonus scheme into an ordinary profit maximization with shadow prices defined as

$$\frac{p_i}{1 + \frac{1}{E_i}} .$$

Note that if $\underline{x}_i$ is an output, $\underline{v}_i \geq 0$, provided $|\underline{E}_i| > 1$. If $\underline{x}_i$ is an input $\underline{v}_i \leq 0$ because inputs have negative signs.

To Part III. 1. Declining $|\underline{E}|$.[19] The Simple Rule implies the following sequence of bonus ratios and outputs:

$$(23) \quad z_o \rightarrow x_1 \rightarrow z_1 \rightarrow x_2 \ldots z_{t-1} \rightarrow x_t \rightarrow z_t \rightarrow x_{t+1} \cdots$$

A larger $\underline{z}$ means a greater relative weight given to sales. Hence if $\underline{z}_t > \underline{z}_{t-1}$, $\underline{x}_{t+1} > \underline{x}_t$, and because of the declining $|\underline{E}|$ if

$\underline{x}_{t+1} > \underline{x}_t$, $\underline{z}_{t+1} > \underline{z}_t$.

By definition of $\underline{z}$ given in (10),

$$(24) \qquad z_t = \frac{p_t}{R'_t} .$$

Since $\underline{z}_t$ determines $\underline{x}_{t+1}$, the maximizing equation (12) can be expressed as

$$(25) \qquad z_t = \frac{C'_{t+1}}{R'_{t+1}} ,$$

and

$$(26) \qquad z_{t-1} = \frac{C'_t}{R'_t} .$$

As explained in the text, the Planner, starting from $\underline{x}_1 < \underline{\hat{x}}$, activates the adjustment process by setting $\underline{z}_1 > \underline{z}_0$. Therefore $\underline{x}_2 > \underline{x}_1$, $\underline{z}_2 > \underline{z}_1$ and in general $\underline{z}_t > \underline{z}_{t-1}$. From (24) and (26),

$$(27) \qquad \frac{p_t}{C'_t} = \frac{z_t}{z_{t-1}} > 1.$$

Thus $\underline{p}_t > \underline{C}'_t$, and no overshooting takes place. Conversely, so long as $\underline{p}_t > \underline{C}'_t$, $\underline{z}_t > \underline{z}_{t-1}$; the process continues until $\underline{p}_t = \underline{C}'_t$ at $\underline{A}$. This means convergence without oscillations.

A similar process, but in reverse takes place when at the start $\underline{x}_1 > \underline{\hat{x}}$.

2. Increasing $|\underline{E}|$. The application of the Simple Rule still gives $\underline{x}_{t+1} > \underline{x}_t$ if $\underline{z}_t > \underline{z}_{t-1}$, but unfortunately $\underline{z}_{t+1} < \underline{z}_t$ if $\underline{x}_{t+1} > \underline{x}_t$ because of increasing $|\underline{E}|$. So if $\underline{x}_t < \underline{\hat{x}}$, $\underline{z}_t > \underline{z}_{t-1}$ and $\underline{x}_{t+1} > \underline{x}_t$, but from (24) and (26),

$$(28) \qquad \frac{p_{t+1}}{C'_{t+1}} = \frac{z_{t+1}}{z_t} < 1,$$

and oscillations around $\underline{A}$ are inevitable. Convergence is not assured.

NOTES

*A number of persons have contributed to the development of this paper. My M.I.T. students and listeners elsewhere have allowed me to try these ideas on them for a number of years. L. Dwight Israelsen helped with the research; John Broome and my colleague Professor Martin L. Weitzman improved the mathematics; Professors Michael Manove and Abram Bergson made many helpful comments. Professor Karl G. Jungenfelt of the Stockholm School of Economics raised a number of questions which made me re-work the whole paper and develop Part III. He also suggested an alternative method of setting the managerial bonus which he may wish to develop on his own. My expression of gratitude to all these persons does not of course make them accomplices in my mistakes.

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1. His speech was originally published in Pravda and Izvestiia on September 28, 1965. English translations can be found in The Current Digest of Soviet Press, Vol. 17, No. 38 (October 13, 1965), pp. 3-12; in Problems of Economics, Vol. 8 (October, 1965), pp.3-28; and in Morris Bornstein and Daniel R. Fusfeld, The Soviet Economy: A Book of Readings, Third Edition (Homewood, Illinois: Richard D. Irwin, Inc., 1970), pp. 387-96. The latter version is somewhat abbreviated.

Many comments and analyses of his speech have been published. See for instance Gertrude E. Schroeder, "Soviet Economic 'Reform': A Study in Contradictions," Soviet Studies, Vol. 20 (July, 1968), pp. 1-21.

For the discussions preceding the reforms, see Jere L. Felker, *Soviet Economic Controversies* (Cambridge, Mass.: The M.I.T. Press, 1966).

2. The most important constraints were the "major assortment" of sales and a maximum payroll limitation which was expected to remain in force until a more adequate supply of consumer goods was achieved. Direct orders from the authorities to the managers have never disappeared and have become more frequent in recent years. There has been a return to centralization.

An investigation of the actual objective function of a Soviet enterprise would require a separate paper, and probably more than one. To put it briefly, much emphasis has been placed in recent years on the Material Incentive Fund from which bonuses not only to the manager but also to his staff, workers and employees are paid. The Fund is calculated by multiplying the wage fund of the enterprise by a ratio obtained from a formula containing a number of variables, such as increase in sales, rate of profit on capital, planned production of new products as a fraction of total production, improvement in labor productivity and so on. It seems that when the authorities decide to correct some particular deficiency, such as low labor productivity, they make a corresponding change in the Incentive Fund formula. It also differs among industries and enterprises. See Michael Ellman, *Soviet Planning Today: Proposals for an Optimally Functioning Economic System* (Cambridge University Press, 1971), pp. 131-62; Bertrand N. Horwitz, *Accounting Controls and the Soviet Economic Reforms of 1966* (American Accounting Association, 1970); S.I. Shkurko, *Material'noe Stimulirovanie v Novy͡kh Uslovii͡akh Khozi͡aĭstvovanii͡a* (Moscow: Misl', 1970).

3. It is quite possible that his decision was simply a compromise, so common in governmental circles, between the advocates of managerial freedom via profits and the proponents of central control via sales.

4. It seems that Mr. Kosygin did not wish prices to clear the market. Instead "Prices ... must cover production and turnover outlays and secure the profits of each normally functioning enterprise." Bornstein and Fusfeld, op.cit., p. 395. Note that President Nixon's Price Control Board also follows this doctrine. Perhaps a governmental body is incapable of regulating prices in any other way.

5. Note that wage inflation, that is, wage rates rising faster than labor productivity, would produce an opposite result: the enterprise would be delighted to produce a "new" product to get a new price higher than the original one -- the usual effect of price control during inflation. In the capitalist world, firms are anxious to produce something that is or looks new in order to enjoy temporary monopoly gains from higher prices until their competitors catch up.

As was stated in Note 2, the proportion of output represented by new products is explicitly included in some of the Incentive Fund formulas.

6. In assuming that the Planner does want the Manager to be at the socially optimal point A I am merely following the tradition. But the Planner may have his own motivations and incentives which may or may not be socially desirable. Perhaps this question deserves greater attention than it has received in the literature so far.

7. On Figs. 1 and 2, where demand elasticity declines to the right, maximization of sales would be achieved at point I because

there demand elasticity equals -1 and the marginal revenue is zero. On Fig. 3, the rising demand elasticity provides no maximum point for sales.

8. The second-order conditions are given in the Mathematical Note.

9. In case of a discriminating monopoly the $\underline{u/v}$ ratios will have to be differentiated among the several markets. This point was made by Lars Jonung of the University of Lund.

10. Another way of ascertaining that the Manager is to the left of point $\underline{A}$ is by finding the $\underline{p/C}'$ ratio from expression (3). But if this ratio, or more exactly, $\underline{C'}$ at $\underline{G}_1$ can be calculated why does not the Planner simply order the Manager to set the price and output accordingly without bothering about a particular bonus scheme? First, $\underline{C'}$ at $\underline{G}_1$ is not the marginal cost that the Planner needs. To get from $\underline{G}_1$ to $\underline{A}$ a number of iterations would be needed which may prove to be oscillatory (on Fig. 1 for instance). Second, such a direct order would merely follow the present Soviet practice of price and quantity controls with all its defects.

11. R.G.D. Allen, Mathematical Analysis for Economists (New York: The Macmillan Co., 1939), pp. 257-58.

12. The $\underline{zR}'$ may not even intersect the $\underline{C}'$ curve at all. See the stability conditions in the Mathematical Note.

13. I have not presented a diagram showing a combination of increasing $|\underline{E}|$ with a declining marginal cost. This combination has a good chance of being unstable.

14. I was once told in Bogota, Colombia about a pair of businessmen, one in Colombia and the other in Peru, who kept sending hides to each other in order to profit from special foreign exchange rates. Of course

the hides never left either country; only papers were sent back and forth.

15. Much will depend on the Planner's ability in estimating _E_ and on his faith in his own estimates. The scheme will miscarry if the Manager's estimates of _E_ are different from those of the Planner (see below). But if the Manager does not maximize the bonus at all our whole scheme should be abandoned.

16. It goes without saying that many enterprises, particularly in such fields as education, public health, cultural activities and perhaps in urban transportation need not make any profits. Qualifications arising from external effects are too well known to require comment.

17. That marginal costs, particularly in the absence of market prices, should be the basis of transfer prices is well recognized in the literature. See for instance Jack Hirshleifer, "On the Economics of Transfer Pricing," _The Journal of Business_, Vol. 29 (July, 1956), pp. 172-84 and his "Economics of the Divisionalized Firm," same Journal, Vol. 30 (April, 1957), pp. 96-108. It would be interesting to find out what specific incentives, if any, are offered to managers of departments to make them adhere to this policy.

In general, the instructions given to managers of branches or departments of capitalist firms, the evaluation of these managers' performance, the nature of their compensation, the delegation of powers to them, and similar subjects should be of great interest to researchers on socialist countries.

18. John Kenneth Galbraith, The New Industrial State (Boston: Houghton Mifflin Co., 1967, 1971, pp. 171-77. See Also William J. Baumol, Business Behavior, Value and Growth, Revised Edition, (New York: Harcourt, Brace & World, Inc., 1967), pp. 45-63, 68-77, 96-103.

19. This is a modified version of a proof suggested by my colleague Professor Martin L. Weitzman. It is clearer and more rigorous than the one contained in an earlier draft of this paper.