working paper
department
of economics

IS THE PRICE SYSTEM OR RATIONING MORE EFFECTIVE IN GETTING A COMMODITY TO THOSE WHO NEED IT MOST?

Martin L. Weitzman

Number 140

October, 1974

massachusetts
institute of
technology

50 memorial drive
cambridge, mass. 02139

IS THE PRICE SYSTEM OR RATIONING MORE EFFECTIVE IN GETTING A COMMODITY TO THOSE WHO NEED IT MOST?

Martin L. Weitzman

Number 140 October, 1974

1. Introduction

The question of whether it would be better in various circumstances to use quotas or market clearing prices to allocate resources is a debate of long standing. From time to time it has flared up as a policy issue of genuine importance.

While each specific debate in this series is about a particular issue, and therefore has its own special features, it does seem to me that the same general themes reappear again and again. The purpose of the present paper is to single out and clarify one of the common strands.

A favorite argument for relying on the market to allocate a particular good or service concerns what might be called its built in selectivity. The price system, it has been said, is really quite a sophisticated mechanism for matching up a scarce commodity with those who need it most. And this is done automatically, simply by giving consumers a chance to express their preferences in the market place. By contrast, rationing is seen as a crude allocation device which cannot effectively take account of individual differences. Any rationing scheme typically ends up over-delivering goods to some people who don't really want them so much, at the same time that it will be withholding from others with a genuine need for more.

The rejoinder is that using rationing, not the price mechanism, is in fact the better way of ensuring that true needs are met. If a market clearing price is used, this guarantees only that it will get driven up until those with more money end up with more of the deficit commodity. How can it honestly be said of such a system that it selects out and fulfills real needs when awards are being made as much on the basis of income as anything else? One fair way to make sure that everyone has an equal chance to satisfy his wants would be to give more or less the same share to each consumer independent of his budget size.

Both of these arguments are right, or at least each contains a strong element of truth. With the aid of a very simple model, I hope to indicate how the two effects just described interact in determining which allocation system is actually more effective for meeting real needs.

2. A Way of Formulating the Problem

In what follows, the abstraction is going to be on an heroic scale. Because the primary goal of this paper is to capture sharply the interplay of issues discussed in the introduction, a high premium will be placed on the use of analytically convenient functional forms which do not at the same time grossly violate reality. I am well aware that an especially simple case is being treated and that is precisely my purpose. Many essential features of the model would remain under more general representations of the economic environment. But since the basic message would then tend to get diluted, such an approach is not taken.

Suppose the population under consideration consists of n consumers. Any person in this population is endowed with a particular set of tastes and a certain level of income. Let ε be a variable which quantifies intensity of preferences toward the deficit commodity. ε is intended to be some measure of how much a consumer "wants", "needs", or "enjoys" the deficit commodity. It is operationally measured by how much the consumer purchases relative to what other consumers <u>would</u> be buying <u>if</u> they belonged to his same income category. Levels of income will be indirectly quantified by λ, a variable meant to represent the marginal utility of income. As an aggregate approximation, it is being assumed that the marginal value of an extra dollar is identical for all consumers having the same income.

Each consumer endowed with traits (ε,λ) is postulated to demand

$$x(p;\varepsilon,\lambda) = A - B\lambda p + \varepsilon \tag{1}$$

units of the deficit commodity when it is offered for sale at price p. The above form might be defended as a first order approximation to a demand curve if incomes and tastes did not vary significantly. At any rate, choosing a demand function of this type will considerably simplify the analysis.

Different tastes are parameterized in expression (1) by various values of ε. A higher value of ε denotes a more intense desire for x and is represented by a rightward shift in the underlying linear demand curve. Note that the demand schedule is written as a function of λp, the real price of the commodity. It seems to me an allowable partial equilibrium assumption in the present context is that people with the same preferences but different incomes should have the same demand for x when it is expressed as a function of a price normalized to measure the opportunity loss of income foregone. In effect I am assuming that the consumer (ε,λ) picks $x(p;\varepsilon,\lambda)$ to maximize

$$U(x;\varepsilon) - \lambda px$$

where $U(\cdot)$ is a quadratic utility function of the form

$$U(x;\varepsilon) = C + \frac{(A+\varepsilon)x}{B} - \frac{x^2}{2B} . \tag{2}$$

Let $f(\varepsilon)$ denote the number of consumers of preference type ε and let $g(\lambda)$ be the number with marginal utility of income equal to λ. Naturally

$$\sum_{\varepsilon} f(\varepsilon) = \sum_{\lambda} g(\lambda) = n \ .$$

As a norm of sorts, and to make the distinction between their roles especially clear, it will be assumed that tastes and income are independently distributed.[1] In other words, the number of consumers possessing the trait combination (ε,λ) is

$$h(\varepsilon,\lambda) = \frac{f(\varepsilon)\ g(\lambda)}{n} \ . \tag{3}$$

Purely for notational convenience and without loss of generality, ε and λ are normalized in (1) so that their average values are respectively zero and one:

$$E[\varepsilon] \equiv \sum_{\varepsilon} \frac{\varepsilon\ f(\varepsilon)}{n} = 0 \tag{4}$$

$$E[\lambda] \equiv \sum_{\lambda} \frac{\lambda\ g(\lambda)}{n} = 1 \ . \tag{5}$$

The variance of ε

$$V[\varepsilon] \equiv E[(\varepsilon - E[\varepsilon])^2] = \sum_{\varepsilon} \frac{\varepsilon^2 f(\varepsilon)}{n} \tag{6}$$

can be interpreted from equation (1) as the mean square deviation in the demand for x when income is held constant. This is by contrast with the mean square deviation in the overall demand for x (without controlling for income). The latter variance will be denoted

---

[1] It should not be difficult for the reader to carry through the analysis when ε and λ have a non-zero correlation. Formula (18) or (19) will just end up with a covariance term tacked on.

$$\sigma^2(p) \equiv E[(x(p;\varepsilon,\lambda) - E[x(p;\varepsilon,\lambda)])^2] , \tag{7}$$

where the symbol E[·] as usual indicates the average per capita value of the variable in question. From (1), (3), (4), (5),

$$E[x(p;\varepsilon,\lambda)] \equiv \sum_{\varepsilon}\sum_{\lambda} \frac{x(p;\varepsilon,\lambda)\ h(\varepsilon,\lambda)}{n} = A - Bp . \tag{8}$$

Using (8), (7) becomes

$$\sigma^2(p) = \sum_{\varepsilon}\sum_{\lambda} \frac{(Bp(\lambda - 1) + \varepsilon)^2\ h(\varepsilon,\lambda)}{n} .$$

By (3), (4), (5), this reduces to

$$\sigma^2(p) = B^2p^2V[\lambda] + V[\varepsilon] , \tag{9}$$

where

$$V[\lambda] \equiv E[(\lambda - E[\lambda])^2] = \sum_{\lambda} \frac{(\lambda - 1)^2\ g(\lambda)}{n} . \tag{10}$$

3. Alternative Allocation Mechanisms

Suppose there is a fixed supply $\bar{X}$ of the deficit commodity which is available to be allocated among the n consumers.[2] The average x available per capita is

$$\bar{x} \equiv \frac{\bar{X}}{n} .$$

---

[2] This is also an abstraction since the supply may be variable and might itself depend on the allocation system, e.g., through the price. Tobin [ ], stresses that the elasticity of supply can be an important determinant of whether to use a price or quota system.

A distribution plan or allocation mechanism which gives $\chi(\varepsilon,\lambda)$ to each person of type (ε,λ) is <u>feasible</u> if

$$E[\chi] = \bar{x} \ , \tag{11}$$

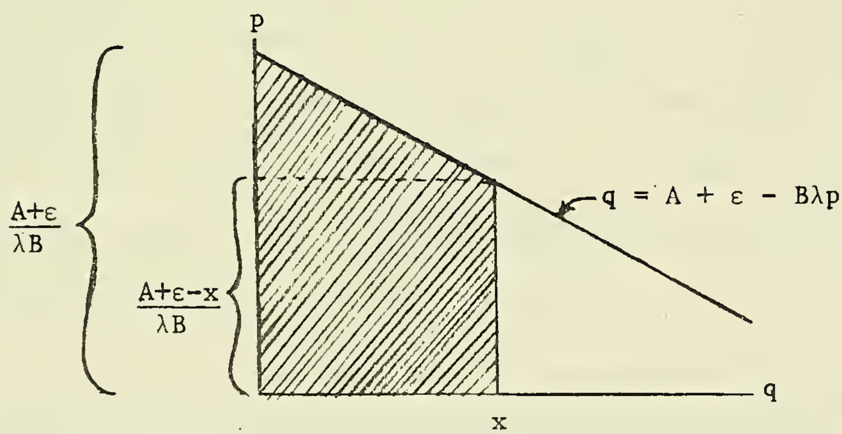
where

$$E[\chi] = \sum_{\varepsilon} \sum_{\lambda} \frac{\chi(\varepsilon,\lambda)\ h(\varepsilon,\lambda)}{n} \ .$$

Note that without loss of generality we are excluding trivial inefficiency from our definition of feasibility by only considering feasible plans of the equality rather than inequality form in (11).

An allocation of x to a person of type (ε,λ) yields him benefits in money terms $\beta(x;\varepsilon,\lambda)$ equal to the shaded $\int p\ dq$ area of figure 1:

$$\beta(x;\varepsilon,\lambda) = \frac{(A+\varepsilon)\ x}{B\lambda} - \frac{x^2}{2B\lambda} \ . \tag{12}$$



<u>Figure 1</u>

The <u>effectiveness</u> of a plan $\{\chi(\varepsilon,\lambda)\}$ in meeting true needs for the deficit commodity is defined as the <u>weighted</u> (by the marginal utility of income) <u>average</u> benefit which it yields,

$$E[\beta(\chi(\varepsilon,\lambda);\varepsilon,\lambda)\cdot\lambda] = \sum_{\varepsilon}\sum_{\lambda} \frac{\beta(\chi(\varepsilon,\lambda);\varepsilon,\lambda)\ \lambda\ h(\varepsilon,\lambda)}{n} . \tag{13}$$

It seems to me that expression (13) provides a not unreasonable partial equilibrium measure of the <u>ceteris partibus</u> social welfare obtainable from any given distribution of the scarce commodity.

The most effective possible distribution plan $\{\chi^*(\varepsilon,\lambda)\}$ would maximize (13) subject to (11). Such an ideal plan is not in general attainable because the government typically lacks the information, authority, or inclination to identify people of type (ε,λ) for all ε and λ. Even when approximate grouping distinctions can be made, the same problem of not being able to accurately demarcate types will crop up again <u>within</u> each group.

Our point of departure is a collection of consumers with similar enough observable characteristics to make further subdivision of types too expensive or downright infeasible. In this context only allocation mechanisms can be employed which do not depend on the ability of anyone to screen out types. Furthermore, we limit ourselves to <u>simple</u> distribution rules.[3]

One simple rule to follow in the absence of identifiable type-features is to give[4] every consumer the same ration $\bar{x}$. The effectiveness of rationing in meeting true needs for the deficit commodity is the weighted average benefit which the allocation $\chi(\varepsilon,\lambda) = \bar{x}$ yields

---

[3] More complicated rules than we will consider are certainly possible, for example an entire schedule of prices as a function of the quantity purchased, or even just a two-tiered version. Naturally an optimal schedule can't help but be better than a single price or quantity, which are just special cases of it. But a schedule is hard to institute and lacks the important quality of simplicity, which the two special cases provide.

[4] This is what might be called a <u>pure</u> rationing system. We are abstracting away from prices doing any of the allocating by in effect assuming that the price charged is so low it deters no one from purchasing his allotted quota.

$$E[\lambda\cdot\beta(\bar{x};\varepsilon,\lambda)] \equiv \sum_{\varepsilon}\sum_{\lambda} \frac{\beta(\bar{x};\varepsilon,\lambda)\ \lambda\ h(\varepsilon,\lambda)}{n} .$$

The price or market system gives a consumer with traits (ε,λ) that allocation

$$\chi(\varepsilon,\lambda) = x(\hat{p};\varepsilon,\lambda)$$

which he demands by equation (1). In the above expression $\hat{p}$ is the competitive price of x, satisfying the market clearing condition

$$E[x(\hat{p};\theta,\lambda)] = \bar{x} .$$

Substituting from (8), the above expression becomes

$$A - B\hat{p} = \bar{x} . \tag{14}$$

The effectiveness of the price system in meeting real needs for the deficit commodity is the weighted average benefit it delivers

$$E[\lambda\cdot\beta(x(\hat{p};\varepsilon,\lambda);\varepsilon,\lambda)] \equiv \sum_{\varepsilon}\sum_{\lambda} \frac{\lambda\ \beta(x(\hat{p};\varepsilon,\lambda);\varepsilon,\lambda)\ h(\varepsilon,\lambda)}{n} .$$

The above expression points up how incomplete, limited, and partial is our treatment of welfare. In the price allocation system, and this should never be lost from view, revenues of $\hat{p}\bar{X}$ are collected by someone or other. A full treatment would take account of where those funds are going and who is benefitting from them. But since anything is possible here, we merely evade the more general issue after warning that what happens to collected revenues might just be the most relevant consideration of all.[5] Instead, we continue to concentrate on the narrower technical question of how well the deficit commodity in and of itself alone is being allocated, abstracting away from all else that might be happening.

4. Rationing vs. the Price System

The comparative effectiveness[6] of the price system over rationing in meeting true needs for the deficit commodity is defined as

$$\delta \equiv E[\lambda\beta(x(\hat{p};\varepsilon,\lambda);\varepsilon,\lambda)] - E[\lambda\beta(\bar{x};\varepsilon,\lambda)] \ . \tag{15}$$

The loss function implicit in the definition of δ is the difference in the weighted average benefit obtained under the two simple allocation mechanisms.

Setting $p = \hat{p}$ in (1) and substituting into (12),

$$\beta(x(\hat{p};\varepsilon,\lambda);\varepsilon,\lambda) = \frac{(A + \varepsilon)(A - B\lambda\hat{p} + \varepsilon)}{B\lambda} - \frac{(A - B\lambda\hat{p} + \varepsilon)^2}{2B\lambda} \ . \tag{16}$$

Similarly, plugging $\bar{x}$ from (14) into (12) yields

$$\beta(\bar{x};\varepsilon,\lambda) = \frac{(A + \varepsilon)(A - B\hat{p})}{B\lambda} - \frac{(A - B\hat{p})^2}{2B\lambda} \ . \tag{17}$$

After employing (3), (4), (5), (6), (10), (16), (17), and cancelling out terms, expression (15) reduces to

$$\delta = \frac{1}{2B}\left(V[\varepsilon] - B^2\hat{p}^2 V[\lambda]\right) \ . \tag{18}$$

---

[5] As an example, it certainly matters whether the funds are taxed by the government and go to help relieve flood victims or they end up in the bank account of an oil billionaire. Diamond and Mirrlees [1971] develop a general equilibrium taxation framework which sheds light on the nature of the general problem.

[6] This criterion is analogous to the one employed in Weitzman [1974].

An equivalent but somewhat more useful (because it is more operational) expression for δ is obtained by substituting from (9) into (18) to obtain

$$\delta = \frac{1}{2B} (2V[\varepsilon] - \sigma^2) \ . \tag{19}$$

Here $\sigma^2 \equiv \sigma^2(\hat{p})$ is the mean square deviation in demand for x at its market clearing price $\hat{p}$.

Expression (18) or (19) is the fundamental result of this paper. The last section is devoted to examining it in some detail.

5. Analyzing the Coefficient of Comparative Effectiveness

Starting with an examination of (18), from (2) $\frac{V[\varepsilon]}{B^2}$ is interpretable as the variance of the marginal utility of an extra unit of the deficit commodity. Thus, it is a measure of the heterogeneity of tastes. The larger is $V[\varepsilon]$, the more widely dispersed are "true needs" for the deficit commodity.

Analogously, $\hat{p}^2 V[\lambda]$ is the variance of the marginal utility of income where the basis of account is the money value of one unit of the deficit commodity. Thus, $V[\varepsilon]$ and $B^2\hat{p}^2 V[\lambda]$ are both expressed in terms of the same numeraire, and are therefore comparable. The smaller is $V[\lambda]$, the more uniform is the distribution of income.

The multiplicative factor $\frac{1}{2B}$ merely translates $(V[\varepsilon] - B^2\hat{p}^2 V[\lambda])$ into units of marginal utility. Expression (18) is essentially the difference of two terms -- a "taste distribution effect", and an "income distribution effect". Other things being equal, the price system has greater comparative effectiveness in sorting out the deficit commodity and getting it to those who need it most when wants are more widely dispersed or when the society is relatively equalitarian in its income distribution. Conversely, rationing is more effective as tastes for the deficit commodity are more uniform or as there is greater income inequality.[7]

Expression (19) is really the same as (18), except that in (19) the variables can be operationally interpreted, whereas (18) is defined in terms of variances of marginal utilities. From (19), δ is essentially a difference of two observable mean square deviations - twice the variance of demand among a sub-population controlled for income (see (6)) minus the uncontrolled variance of demand (7).

Thus, if the mean square deviation of demand by the entire consumer population is not much larger than it is by fixed income subgroup, the price mechanism has greater effectiveness in screening the deficit commodity and funneling more of it to those who need it relatively more. Conversely, the greater the dependence of demand on income (as measured by (19)), the larger the comparative effectiveness of a quota system because it essentially prevents those with larger incomes from monopolizing consumption of the commodity in question. Naturally if both variances σ^2 and $V[\varepsilon]$ are small, it doesn't make much difference which system is used.

6. Concluding Remarks

It might be appropriate to end this paper by commenting on its relation to some more standard approaches. Economists sometimes maintain or imply that the market system is a superior mechanism for distributing resources. After all, the argument goes, consider any other allocation. There is always some corresponding way of combining the price system with a specific lump sum

---

[7] Note that a *ceteris paribus* increase in the market clearing price makes rationing relatively more effective because the income distribution effect takes on added importance.

transfer arrangement which will make everyone better off (or at least no worse).[8]

This is true enough in principle, but not typically very useful for policy prescriptions because the necessary compensation is practically never paid. When dispensation is made, the point deserves emphasis. For example, by the standard argument of consumer sovereignty, it certainly makes everyone better off if ration tickets can be sold (and bought) than if resale is not allowed. The "white market" is superior to strict rationing because in redistributing property rights, the losers are being adequately compensated or "bought off". However, this does not necessarily mean that pure direct wants for the deficit commodity are better served in the sense that those who "need" it most actually end up receiving more.

Besides, why stop here? Surely the status quo income distribution is non-optimal. An even better system from an overall social welfare position might be to give all the rationing tickets to the infirm (who maybe don't even consume the commodity in question) or to Bangladesh, or so forth. There is no end to what could be done on the income distribution side.

This is one reason why I have preferred to leave in abeyance income considerations and concentrate in this paper on inquiring about the pure distributive effectiveness of an allocation system in getting the deficit commodity to those who need it most. The other and more substantive reason is that I am not sure but that for many situations such a formulation might not be the most appropriate way of posing the question in the first place.

There is a class of commodities whose just distribution is sometimes viewed as a desirable end in itself, independent of how society may be

---

[8] Given the usual general equilibrium assumptions. See, for example, Arrow and Hahn [1971].

allocating its other resources. Such "natural right goods" as basic food and shelter, security, legal aid, military service, medical assistance, education, justice, or even many others are frequently deemed to be sufficiently vital in some sense to give them a special status. The principle of limited dimensional equity in the distribution of a commodity is an open violation of consumer sovereignty. Yet society doesn't seem to mind, at least sometimes. In cases when this is so, our model may have some relevant things to say about whether prices or quotas are better allocation instruments.

References

Arrow, K.J., and F. Hahn, General Competitive Analysis, San Francisco, 1971.

Diamond, P.A., and Mirrlees, J.A., "Optimal Taxation and Public Production", American Economic Review, March 1971 (pp. 8-27) and June 1971 (pp. 261-278).

Samuelson, P.A., Foundations of Economic Analysis, Cambridge, 1947.

Tobin, J., "On Limiting the Domain of Inequality", Journal of Law and Economics.

Weitzman, M., "Prices vs. Quantities", Review of Economic Studies, forthcoming.