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THE EFFECT OF A PROPORTIONAL SUBSIDY OR A TAX
ON THE QUALITY AND QUANTITY OF OUTPUT *

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In an earlier paper I suggested that a bonus expressed as a linear function of profit and sales can induce a socialist (or any) manager to forgo his monopolistic powers.[1/] Here I propose to investigate whether such a bonus might also prompt him to change, and particularly to lower, the quality of his product: the latter effect would be most unwelcome in socialist countries. But because, as was shown earlier, this bonus is equivalent to a subsidy proportional to price, this investigation can be broadened to include the effects of such a subsidy, not restricted to its optimal value, on the quality and also on the quantity of the product.[2/] And finally, since the subsidy can be less than unity, it can be interpreted as a proportional tax as well. It will be shown that this subsidy (or tax) can indeed affect both quality and quantity (and of course the price) of output, and sometimes in a rather unexpected manner.

LIST OF SYMBOLS

(in order of appearance)

$\underline{p}$ = price of output

$\underline{x}$ = quantity of output

$\underline{k}$ = index of quality of output

$\underline{C}$ = total cost

H = profit (including the subsidy or the tax) = $\underline{zR} - \underline{C}$

$\underline{R}$ = revenue (without the subsidy or the tax) = $\underline{px}$

$\underline{z}$ = subsidy (if $\underline{z} > 1$), or tax (if $\underline{z} < 1$)

$\underline{H}_x = \frac{\partial H}{\partial x}$; $\underline{H}_{xx} = \frac{\partial^2 H}{\partial x^2}$; $\underline{H}_{xk} = \frac{\partial^2 H}{\partial x \partial k}$.

Similar notation is used for other derivatives.

$\underline{E}^k_{R_x}$ = elasticity of $\underline{R}_x$ in respect to $\underline{k}$.

Similar notation is used for other elasticities.

We assume that both the price and the cost of output are functions of quantity and of quality:

$$(1) \qquad p = p(x, k), \qquad C = C(x, k),$$

where $\underline{k}$ is some quality index. It can stand for some easily quantifiable characteristic, such as the strength of a material or the longevity of a machine, or for something more elusive, like the variety of dresses or the taste of wine. Quality is of course multidimensional, but no attempt of dealing with the general complex case will be made here.

The firm (or the manager) will maximize

$$(2) \qquad H = zR - C$$

in respect to $\underline{x}$ and to $\underline{k}$:

$$(3) \qquad H_x = zR_x - C_x = 0,$$

$$(4) \qquad H_k = zR_k - C_k = 0.$$

The second order conditions are:

$$(5) \qquad H_{xx} = zR_{xx} - C_{xx} < 0,$$

$$(6) \qquad H_{kk} = zR_{kk} - C_{kk} < 0,$$

and

$$(7) \qquad H_{xk}^2 = (zR_{xk} - C_{xk})^2 < (zR_{xx} - C_{xx})(zR_{kk} - C_{kk}).$$

We now want to find the signs of $\frac{dx}{dz}$ and $\frac{dk}{dz}$. The differentiation of (3) and (4) results in the following system of equations:

$$(8) \qquad H_{xx}\frac{dx}{dz} + H_{xk}\frac{dk}{dz} = -R_x$$

$$(9) \qquad H_{xk}\frac{dx}{dz} + H_{kk}\frac{dk}{dz} = -R_k,$$

with the determinant

$$(10) \qquad D = \begin{vmatrix} H_{xx} & H_{xk} \\ H_{xk} & H_{kk} \end{vmatrix} = H_{xx}H_{kk} - H_{xk}^2 > 0$$

by second order conditions (7).

$$(11) \qquad \frac{dx}{dz} = \frac{-R_xH_{kk} + R_kH_{xk}}{D},$$

$$(12) \qquad \frac{dk}{dz} = \frac{-R_kH_{xx} + R_xH_{xk}}{D}.$$

Since we can readily assume that the firm operates in the region where $R_x > 0$, $R_k > 0$, while $H_{xx} < 0$, $H_{kk} < 0$ by (5) and (6), $\frac{dx}{dz} > 0$, $\frac{dk}{dz} > 0$ if $H_{xk} \geq 0$. Thus it only remains to explore the case when $H_{xk} < 0$. A change in the signs in (8) and (9) gives us two equations with all positive coefficients. We can immediately conclude that:

$$(12) \qquad \text{if} \quad \frac{dx}{dz} > 0, \quad \frac{dk}{dz} \gtreqless 0;$$

$$(13) \qquad \text{if} \quad \frac{dx}{dz} = 0, \quad \frac{dk}{dz} > 0;$$

$$(14) \qquad \text{if} \quad \frac{dx}{dz} < 0, \quad \frac{dk}{dz} > 0,$$

and by symmetry the same relations hold for $\frac{dx}{dz}$ as a function of $\frac{dk}{dz}$. We should note that for either $\frac{dx}{dz} < 0$ or $\frac{dk}{dz} < 0$ the other derivative must be positive and large.

Propositions (12) - (14) can also be established by examining the second order conditions. This method would give us more restricted results, but on the whole it would hardly justify the effort and the space. It may be worth while to examine in detail just two cases, say when $\frac{dx}{dz} < 0$, or when $\frac{dk}{dz} < 0$ though the results will be expressed in such unfamiliar elasticities that, I suspect, they will add little to our understanding of the problem.

If $\frac{dx}{dz} < 0$, then from (11)

$$\frac{H_{xk}}{R_x} < \frac{H_{kk}}{R_k} . \tag{15}$$

From (15), (6) and (7)

$$\frac{zR_{xk}}{R_x} - \frac{C_{xk}}{R_x} < \frac{zR_{kk}}{R_k} - \frac{C_{kk}}{R_k} . \tag{16}$$

From (16), (3) and (4)

$$\frac{R_{xk}}{R_x} - \frac{C_{xk}}{C_x} < \frac{R_{kk}}{R_k} - \frac{C_{kk}}{C_k} . \tag{17}$$

Introducing the elasticity of R_x in respect to k

$$E^k_{R_x} = \frac{\partial(R_x)}{\partial k} \cdot \frac{k}{R_x} = \frac{kR_{xk}}{R_x} , \tag{18}$$

and using similar definitions for the other elasticities, we can express (17) as

(19) $$E^k_{R_x} - E^k_{C_x} < E^k_{R_k} - E^k_{C_k}$$

or as

(20) $$E^k_{R_x} - E^k_{R_k} < E^k_{C_x} - E^k_{C_k} ,$$

if $\frac{dx}{dz} < 0$.

Following the same procedure we can find that

(21) $$E^x_{R_k} - E^x_{R_x} < E^x_{C_k} - E^x_{C_x} ,$$

if $\frac{dx}{dz} < 0$.

While it is probable that $E^k_{C_x} > 0$ (because an improvement in quality should raise the marginal cost), $E^k_{C_k} > 0$ (because further quality improvements should be more expensive, and $E^x_{R_x} < 0$ (if demand elasticity declines to the right), I would not venture to predict on a priori grounds the signs of the other elasticities and particularly of the differences between them.

If we recollect that a $H_{xk} \geq 0$ always yields $\frac{dx}{dz} > 0$, $\frac{dk}{dz} > 0$, and that only a large negative H_{xk} (subject to the second order restrictions) can give us $\frac{dx}{dz} < 0$ or $\frac{dk}{dz} < 0$, a negative effect of the subsidy either on quantity or on quality seems unlikely. But since we do not know the probabilities of each configuration it is best to leave the question open. It is possible that a subsidy can improve the quality to such an extent as to reduce the quantity, and vice versa (while exactly the opposite would be true of a tax). All this can happen, but I wonder if it has ever happened in reality?[3/]

It is also possible that a subsidy may have an unexpected effect on price. For

(22) $$\frac{dp}{dz} = p_x \frac{dx}{dz} + p_k \frac{dk}{dz} .$$

Assuming, as usual, that $p_x < 0$ and that $p_k > 0$, we find that only if $\frac{dx}{dz} > 0$, $\frac{dk}{dz} \leq 0$ (one of the "less probable" cases) will $\frac{dp}{dz}$ be definitely negative. If $\frac{dx}{dz} < 0$, $\frac{dk}{dz} > 0$, then $\frac{dp}{dz} > 0$, while in the supposedly most "common" case when $\frac{dx}{dz} > 0$, $\frac{dk}{dz} > 0$ the result is uncertain.

* * *

The introduction of quality as a decision variable may also cast some doubts on the welfare effects of our bonus scheme. No longer can we assert (abstracting from the complex general equilibrium considerations) that the bonus, even if set correctly, will increase social welfare by inducing the manager to move from the usual monopolistic position, point D on the diagrams of the earlier paper, to point A (of the same demand curve) where marginal cost equals price. All we can now claim is that, if quality changes, the manager will move from point D on the demand curve for products of one quality to point A on the demand curve for products of another quality. It is plausible that social welfare will increase, but it is not certain.4/

NOTES

* This paper was prompted by a question raised by one of my graduate students in the Seminar on Economic Development at La Trobe University in Melbourne (June-August, 1974) whether my bonus scheme (see note 1) might not lower the quality of output. Unfortunately, I do not remember who of the several students should be thanked for that.

I am very grateful to my colleague Professor Peter A. Diamond for his gentle guidance through some labyrinths of welfare economics. He is not to be held responsible, however, for any of my remaining mistakes.

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1. "On the Optimal Compensation of a Socialist Manager", *The Quarterly Journal of Economics*, LXXXVIII (February 1974), 1-18.

2. That optimal value of z was the one which induced the manager to move to the point where marginal cost equaled price. *Ibid.*, pp. 10, 16-17.

3. In the last few years there have appeared a number of articles on the effects of monopolization on the quality of output, most of them dealing with the durability of capital goods. For a bibliography see Richard W. Parks, "The Demand and Supply of Durable Goods and Durability", *The American Economic Review*, LXIV (March 1974), 37-55. It seems that the results of that discussion have been rather inconclusive. See also an unpublished paper by Michael Spence, "Product Selection, Fixed Costs and Monopolistic Competition" (1974).

4. I have not proved yet that changes in quality will not affect the convergence of the iterative process (the "Simple Rule") described in the Mathematical Appendix of the earlier paper.

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