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"An Essay on Aggregation Theory and Practice"

by

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Summary: An Essay on Aggregation Theory and Practice

Edwin Kuh

This paper shows how aggregation can improve the efficiency of econometric estimation. The new theorems depend on the random coefficient regression model, Theil's aggregation analysis, and assumptions about the stability of shares held by individual economic entities in macroeconomic totals. Section 1 summarizes existing views held by econometricians about aggregation problems and their solution. Sections 2 and 3 offer an exposition of existing aggregation analysis and the basic new theorems which show precisely how the macro parameter variances will tend to zero as the number of elements in the aggregate increase. Section 4 presents a simple investment model estimated from data for 105 manufacturing firms divided among four industries, to test underlying assumptions and also to test directly predictions of aggregation gain generated by the theory. The test outcomes are sufficiently favorable so that the existence of aggregation gain and its measurability seem now to be clearly established. Section 5 takes up the question of potential aggregation gains for all Census four digit industries. Section 6 discusses the implications for research strategy when micro coefficients differ, contrary to a basic assumption of the random coefficient model employed throughout this paper.

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An Essay on Aggregation Theory and Practice

Edwin Kuh

1. Introduction

One of the most perplexing problems in the design of econometric research, and one that has remained largely unsolved is how aggregated data can be in order to obtain valid estimates. The main purpose of this paper will be to show that aggregation gains not only exist, but how one can provide operational tests of their extent.

Theil's aggregation theorems [21] show that behavior as reflected in postulated population regression coefficients must be nearly identical for all members in a given population if estimates based on aggregated data are to reflect behavior in a meaningful sense.* These implications of Theil's analysis have recently been reinforced by Orcutt, Watts and Edwards [15] whose simulation results suggest that estimates from aggregate data generated from one particular simplified macro-economic model can be inefficient and sometimes inconsistent, even when the micro parameters are identical. There exists a conflicting position, composed of three strands, that is favorable to greater aggregation. First, Griliches and Grunfeld [6] showed that error variances for aggregate regression equations will be less than the sum of the micro variances if negative error covariance terms are sufficiently great; furthermore, costs are normally less for estimates

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*Certain special data configurations permit heterogeneous coefficients, yet avoid what Theil calls contradictions between the micro hypothesis and the macro hypothesis.

based on macro data. They restricted themselves to a simplified aspect of forecasting, neglecting altogether the problem of aggregation bias. Second, there is a metaphysical view most vigorously espoused by Friedman and Meiselman [5] that the relevant behavior of the economy can be most efficiently represented by a single aggregated model; "truth will out" even in the aggregates, so that complex, disaggregated models of behavior are ineffectual ways to test macro economic propositions. This view is a matter of faith unrelated to statistical hypotheses. The third, pragmatic view of perhaps a majority of practicing econometricians is that micro data probably are better, but that it is either unavailable or excessively expensive, so that all one can do practically is use macro data.

From a somewhat different aspect, many cross-sectional studies based on micro data (individual firms, families or individuals) have proven so disappointing that many researchers prefer to avoid this data base. The major source of failure is the high error variances i.e. low correlations, even though the statistical significance tests may be highly affirmative in the sense of decisively rejecting the no-relationship null hypothesis. Yet it is exceedingly cold comfort to find a "very significant" multiple correlation which in a large sample may explain no more than 5% or 10% of the dependent variable's fluctuations, so that the systematic component of the explanation, while "significant", is negligible. I have shown [13] that a strong presumption exists that a component of cross-sectional error variances can and should be imputed to individual effects which vary across individuals but are constant over time. The basic problem of explanatory power still remains, however.

Finally, as with Goldilocks, something that seems "just right" can be found in sub-aggregates such as two digit industries, or sub-catagories of the labor force broken down by age, sex, and skill. While the last remark is mere obiter dictum at this juncture, an analytical basis for it will soon be provided:that is the central purpose of this paper. In short, "proper" aggregation is not merely neutral, nor just a major cost saving (which it usually is), but proper aggregation causes major reductions in the estimated parameter error variances, relative to the underlying, individual error variances.

2. One Resolution - A Sketched Proof

Because the theorems advanced later necessitate an involved notation, even though the results are straightforward, it is convenient to introduce the notation here in the context of a simplified example. The basic analytical tools are twofold. The first is the random coefficient model, that was brought to the attention of practicing econometricians by Klein [10] in the context of cross section models, much as we are doing here. The complex maximum liklihood estimator proposed by Klein discouraged subsequent application. Nevertheless, the random coefficient model itself has much to recommend it. Casual introspection (the kind most of us excel in) suggests that in making our individual consumer expenditure decisions, we do not go through an exact calculation involving many relative prices and income and then tack on a random error to this painstaking result; rather, we often buy haphazardly, on impulse, even though on "average" our purchases do reflect more basic consumer preferences and economic parameters. This latter behavior is more compatible with a random coefficient model than it is with the standard shock model. The random coefficient model, whether or not it is inspired by the motivational scheme above, also helps to rationalize the dilemma posed by the extremely low variance explanation of so much cross-section analysis. The basic economic behavior is stable and explicable on the average, even though it is not in one given time slice when the variances of the individual coefficients are large. Indeed, a related alternative explanation is the errors-in-variables model where the argument has been made that cross-sectional consumption and income data contain transient i.e. error components which will lead to biased least square estimates. The effects on error variances and estimated standard errors of the regression coefficients have normally been given less attention.

C. R. Rao [16] has shown that ordinary least squares is an unbiased, efficient estimator in the random coefficient case. That article contains the statistical basis for the application of ordinary least squares in the aggregation context, to which we now turn.

Theil's aggregation theory[21], the second analytical device on which this paper depends, has been translated into compact matrix notation by Kloeck[11]. I shall reproduce the main results (not complete proofs) in a slightly extended notation.

We will suppose that there are N individual behavior equations of the form:

$$y_i = x_i \underline{\beta}_i + \varepsilon_i \qquad i = 1, 2, \ldots N \tag{1}$$

where y_i is a column vector with T rows for the i^{th} individual; x_i is a (T x K) set of exogenous variables for the i^{th} individual, $\underline{\beta}_i$ is a (K x 1) parameter vector for the i^{th} individual, and ε_i is the column vector of errors for the i^{th} individual.* Corresponding to these

*The time subscript is inessential in what follows, so we have suppressed it from the outset.

micro equations is the macro equation:

$$Y = X\underline{\beta} + \varepsilon \tag{2}$$

where the variables X and Y are aggregates of the microdata x_i and y_i. In the context of Theil-type aggregation, $\underline{\beta}$ is defined as the mathematical expectation of the least squares estimator. When each micro series for each exogenous variable is regressed on all the macro exogenous variables (a useful step essential to a clear interpretation of the macro parameters in terms of the micro parameters), the resulting regression coefficients will be called the aggregation weights. These are presented in (3). The precise nature of these weights will be made clear in the example which follows immediately.

$$(B_{ip}^{(1)} B_{ip}^{(2)} \ldots . B_{ip}^{(K)}) \qquad p = 1, 2, \ldots K \tag{3}$$

This vector obviously has K components and for each individual there exist K such vectors.

Theil has shown (using a modification of Kloeck's notation) that a typical macro coefficient β_ℓ is a B-weighted sum of the micro parameters, where the aggregation weights have certain properties, e.g. a typical macro parameter is:

$$\beta_\ell = \sum_{i=1}^{N} \underline{B}_i^{(\ell)} \underline{\beta}_i = (B_{11}^{(\ell)} B_{12}^{(\ell)} \ldots B_{1K}^{(\ell)}) \begin{pmatrix} \beta_{11} \\ \beta_{12} \\ \cdot \\ \cdot \\ \beta_{1K} \end{pmatrix} + \ldots + (B_{N1}^{(\ell)} B_{N2}^{(\ell)} \ldots B_{NK}^{(\ell)}) \begin{pmatrix} \beta_{N1} \\ \beta_{N2} \\ \cdot \\ \cdot \\ \beta_{NK} \end{pmatrix} \tag{4}$$

There would be K such equations, one for each macro parameter.

The B aggregation weights satisfy the following conditions as a direct implication of least squares:

$$\sum_{i=1}^{N} B_{ip}^{(\ell)} = 1 \text{ for } p = \ell$$
$$= 0 \text{ for } p \neq \ell \tag{5}$$

In words: the weights for corresponding parameters sum to unity, and sum to zero for non-corresponding parameters.

Having set out much of the general notation, I wish to illustrate the main point of this article with a simple example. We wish to estimate the parameters of the following macro equation:

$$Y = (X_1 X_2)\begin{pmatrix}\beta_1\\ \beta_2\end{pmatrix} + \varepsilon \tag{6}$$

$$\beta_1 = \sum_{i=1}^{N} \underline{B}_i^{(1)} \underline{\beta}_i = \sum_{i=1}^{N} B_{i1}^{(1)}\beta_{i1} + \sum_{i=1}^{N} B_{i2}^{(1)}\beta_{i2}$$
$$\tag{7}$$
$$\beta_2 = \sum_{i=1}^{N} \underline{B}_i^{(2)} \underline{\beta}_i = \sum_{i=1}^{N} B_{i1}^{(2)}\beta_{i1} + \sum_{i=1}^{N} B_{i2}^{(2)}\beta_{i2}$$

Suppose now that the micro parameters are mutually independent identically distributed random variables. Then the expected values of the macro parameters:

$$E(\beta_1) = \left(\sum_{i=1}^{N} B_{i1}^{(1)}\right) E(\beta_{i1}) = \bar{\beta}_1, \text{ and similarly for } \beta_2. \tag{8}$$

This result follows immediately from the fact that the aggregation weights are fixed numbers which sum to unity for corresponding parameters and zero for noncorresponding parameters.* Theil, in a related context, has indicated how critical the assumption is that the x's are fixed

*Zellner [22] proved this same proposition. A concluding section relates the material of this paper to papers by Theil[20] and Zellner [22].

variates. It is worth quoting his observations on this matter in full:

> These results indicate that there are no problems of aggregation bias if one works with a random-coefficients micromodel. In fact, the macrocoefficients contrary to the microcoefficients are not random at all when N is sufficiently large. Note, however, that the assumption of non-stochastic x's is not at all innocent. If we select households at random, not only their β's but also their x's become stochastic. This does not mean that we cannot treat the x's as if they are fixed. But it does mean that if we do so, we operate conditionally on the x's and we assume implicitly that the conditional distribution of the β's given the x's is independent of these x's. Thus, the analysis is based on the condition that over the set of individuals who are aggregated, there is stochastic independence between the factors determining their behavior (the x's) and the way in which they react given these factors (the β's).*

The great convenience of the random coefficient model in the aggregation context (initially recognized by Zellner [22], see the last footnote) is that it allows individual parameters to be different at each point of time, which seems realistic, yet be the same on average---a necessary condition for successful aggregation which should be realizable through appropriate grouping of the data.

The main contribution of this paper is to show that the variance of the estimated macro coefficient goes to zero as the number of individuals in the aggregate increases, although this result would appear to be counterintuitive since the variance of the sum of independently distributed random variables tends to infinity as the number of items in the sum tends to infinity. Thus the efficiency of estimation increases as the aggregate grows. In one limiting case, the variance diminishes as 1/N, i.e. it is as if the parameters were averaged instead of being made up of aggregated data for which the underlying error variances cumulate toward an indefinitely large sum as they are subsumed in the aggregation.

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*Theil,[20], pp.5-6.

The variance of β_1 can be written by inspection of the first row in (7) as:

$$V_N(\beta_1) = (\sum_{i=1}^{N} B_{i1}^{(1)2})\sigma_1^2 + (\sum_{i=1}^{N} B_{i2}^{(1)2})\ \sigma_2^2 + 2(\sum_{i=1}^{N} B_{i1}^{(1)} \cdot B_{i2}^{(1)})\sigma_{12} \qquad (9)$$

where σ_1^2 is the variance of the first micro parameter, σ_2^2 is the variance of the second micro parameter and σ_{12} their covariance assuming the micro parameters to be identically and indeoendently distributed for all N individuals.

We must now evaluate the nature of the B weights more closely, for it is their behavior that basically determines what happens to the variance of the macro parameter as N increases. The $B_{i1}^{(1)}$ which sum to one by the aggregation weight rules, are the partial regression coefficients of the i^{th} micro series x_{i1} on the macro series X_1 in $x_{i1} = B_{i1}^{(1)}X_1 + B_{i1}^{(2)}X_2$ and the $B_{i2}^{(1)}$ are the partial regression coefficients of micro series x_{i2} on macro series X_1 in the i^{th} multiple regression $x_{i2} = B_{i2}^{(1)}X_1 + B_{i2}^{(2)}X_2$.

While it is true, as Theil and Kloeck have emphasized, that these are arbitrary weights, it is nevertheless reasonable to suppose that under a wide variety of circumstances that the $B_{i1}^{(1)}$ will be approximately the proportion of X_1 that originates with x_{i1}. One rigid assumption with many interesting implications is that these are exact proportions, a matter to which we return in later empirical sections. Furthermore, one would suppose that the $B_{i2}^{(1)}$ weights would tend to be small, much smaller than the $B_{i1}^{(1)}$. The arbitrary nature of the entire estimation procedure for the B's leads one to suppose that the net regression coefficient of x_{i1} on X_2 can be expected to be small term by term. Furthermore, we know that $\sum_{i=1}^{N} B_{i1}^{(1)} = 1$ and $\sum_{i=1}^{N} B_{i2}^{(1)} = 0$.

While in theory the individual B's could be anything, we shall suppose that in most empirical applications that the $B_{i1}^{(1)}$ are positive fractions, while the $B_{i2}^{(1)}$'s may be positive or negative, but are considerably smaller than the $B_{i1}^{(1)}$'s.

From this assumption the following highly significant and equally simple assertion emerges:

The share of each component in an aggregate will remain unchanged or decrease when the aggregate increases, and the corresponding B weights will behave in corresponding fashion. The sum of squares, $\sum_{i=1}^{N} B_{i1}^{(1)2}$, will thus gradually decline as the squared fractions and their sum decrease.

Consider the case where the following simplified (but illuminating) conditions are correct:

(a) The $B_{i1}^{(1)}$ weights represent shares which are equally divided among members in the aggregate,

(b) The $B_{i2}^{(1)}$ are individually negligible <u>or</u> the variance of β_{i2} is negligible relative to that of β_{i1} and similarly for the covariance σ_{12} among the micro parameters β_{i1}, β_{i2}.

Then,

$$V_N(\beta_1) \cong (\sum_{i=1}^{N} B_{i1}^{(1)2})\sigma^2_{\beta_{i1}} \cong (\sum_{i=1}^{N} (\frac{1}{N})^2)\ \sigma^2_{\beta_{i1}} \cong \frac{1}{N}\sigma^2_{\beta_{i1}} \qquad (10)$$

In this illustrative case, the variance of the macro parameter approximately obeys the law of large numbers, decreasing as $\frac{1}{N}$, i.e. as if we were averaging parameters of individually distributed variables rather than taking weighted sums of independent random variables. The more unequal the weight distribution, the more slowly does the variance of the macro parameter decrease, but it will decrease. Empirical results confirm that, even with skewed distributions that exist in many manufacturing industries, the sums of the squared shares to which corresponding B weights are close analogues tend rapidly to very small magnitudes in most four digit industries.

The remainder of this paper proceeds as follows. A general proof for the full regression model is presented. Then several sets of empirical results will be examined to validate or refute the basic postulates which, after all, are assertions about measurable reality.

3. General Proof

Let for i = 1, 2,...,N individuals and p = 1, 2,...,ℓ,..., K parameters:

$$\underline{\beta}_i = (\beta_{i1}\beta_{i2}\ldots\beta_{iK})^t, \tag{11}$$

$$\underline{B}_i^{(\ell)} = [B_{i1}^{(\ell)}\ B_{i2}^{(\ell)}\ldots\ldots B_{iK}^{(\ell)}], \tag{12}$$

$$\underline{\underline{B}} = \begin{bmatrix} \underline{B}_1^{(1)} & \underline{B}_2^{(1)} & \cdots\cdots & \underline{B}_N^{(1)} \\ \vdots & & & \vdots \\ \underline{B}_1^{(K)} & \underline{B}_2^{(K)} & \cdots\cdots & \underline{B}_N^{(K)} \end{bmatrix}; \tag{13}$$

also $$\beta_\ell = \sum_{i=1}^{N} \underline{B}_i^{(\ell)} \underline{\beta}_i \ , \tag{14}$$

and $$\underline{\beta} = (\beta_1\beta_2\ldots\beta_K)^t \ . \tag{15}$$

Here $\underline{\beta}_i$ is the (K x 1) vector of the micro parameters for individual i, $B_i^{(\ell)}$ is the (1 x K) vector of the B-weights for individual i and macro parameter ℓ, $\underline{\underline{B}}$ is the (K x KN) matrix of all B-weights, β_ℓ is the typical macro parameter and finally, $\underline{\beta}$ is the (K x 1) vector of macro parameters.

We make the

<u>Assumption</u>: $\{\underline{\beta}_i,\ i = 1, 2, \ldots, N\}$ is a sequence of mutually independent identically distributed $(K \times 1)$ random variables, each with mean vector $\overline{\underline{\beta}}$ and positive definite symmetric covariance matrix $\underline{\underline{\Omega}}$.

$$\text{Then } E(\underline{\beta}) = \underline{\underline{B}}\, E[\underline{\beta}_1^t, \ldots, \underline{\beta}_N^t]^t = \underline{\underline{B}}[\overline{\underline{\beta}}^t \overline{\underline{\beta}}^t \ldots \overline{\underline{\beta}}^t]^t = \overline{\underline{\beta}} \qquad (16)$$

so that the estimator is unbiased. Furthermore, $\underline{\beta}$ has covariance matrix

$$\underline{\underline{V}}_N(\underline{\beta}) = \underline{\underline{B}} \begin{bmatrix} \underline{\underline{\Omega}} & & & \\ & \underline{\underline{\Omega}} & & \underline{\underline{0}} \\ & \underline{\underline{0}}^t & \ddots & \\ & & & \underline{\underline{\Omega}} \end{bmatrix} \underline{\underline{B}}^t = \sum_{i=1}^{N} \begin{bmatrix} \underline{B}_i^{(1)} \underline{\underline{\Omega}} \underline{B}_i^{(1)t} & \cdots & \underline{B}_i^{(1)} \underline{\underline{\Omega}} \underline{B}_i^{(K)t} \\ \vdots & \ddots & \vdots \\ & & \underline{B}_i^{(K)} \underline{\underline{\Omega}} \underline{B}_i^{(K)t} \end{bmatrix} \qquad (17)$$

A generic diagonal term of $\underline{\underline{V}}_N(\underline{\beta})$ can be written as:

$$V_N(\beta_\ell) = \sum_{i=1}^{N} \underline{B}_i^{(\ell)}\, \underline{\underline{\Omega}}\, \underline{B}_i^{(\ell)t} = \text{tr}\ [\ \underline{\underline{\Omega}} \cdot \{\sum_{i=1}^{N} \underline{B}_i^{(\ell)t} \underline{B}_i^{(\ell)}\}] \qquad (18)$$

$$= \text{tr}\ \underline{\underline{\Omega}} \cdot \begin{bmatrix} \sum_{i=1}^{N} [B_{i1}^{(\ell)}]^2 & \cdots & \sum_{i=1}^{N} B_{i1}^{(\ell)} B_{iK}^{(\ell)} \\ & \ddots & \vdots \\ & & \sum_{i=1}^{N} [B_{iK}^{(\ell)}]^2 \end{bmatrix} \doteq \text{tr}[\underline{\underline{\Omega}} \cdot \underline{\underline{\Phi}}_N^{(\ell)}] \qquad (19)$$

It must now be shown what restrictions on the B-weights will lead to a decrease in the macro parameter variances as the aggregate population grows. A lemma concerning a sequence of fractions which sum to unity is needed at this point:

Lemma: For $i = 1, 2, \ldots, N$ and $N = 1, 2, \ldots$ let $\{p_{iN}\}$ be such that $-1 < p_{iN} < 1$ and $\sum_{i=1}^{N} p_{iN} = 1$. Then a necessary and sufficient condition that

$$\lim_{N \to \infty} \sum_{i=1}^{N} p_{iN}^2 = 0 \text{ is that}$$

$$\lim \rho_N \to 0 \text{ where } \rho_N \equiv \max \{|p_{iN}|,\ i = 1, 2, \ldots N\} .$$

Proof: Necessity is obvious. To prove sufficiency, examine the sum

$$\sum_{i=1}^{N} (p_{iN}^2 - \frac{1}{N^2}) = \sum_{i=1}^{N} (p_{iN} + \frac{1}{N})(p_{iN} - \frac{1}{N}).$$

Clearly,

$\rho_N \geq |p_{iN}| - \frac{1}{N}$ for each i, so $\sum_{i=1}^{N} (p_{iN}^2 - \frac{1}{N^2}) \leq \rho_N \sum_{i=1}^{N} (p_{iN} + \frac{1}{N}) = 2\rho_N$

By hypothesis $\lim \rho_N \to 0$, so that

$$\lim_{N \to \infty} \sum_{i=1}^{N} (p_{iN}^2 - \frac{1}{N^2}) = \lim_{N \to \infty} [(\sum_{i=1}^{N} p_{iN}^2) - \frac{1}{N}] \leq 0.$$

But the minimum value of $\sum_{i=1}^{N} p_{iN}^2$ is $\frac{1}{N}$, implying for each N that:

$$(\sum_{i=1}^{N} p_{iN}^2) - \frac{1}{N} \geq 0, \text{ whereupon } \lim_{N \to \infty} \sum_{i=1}^{N} p_{iN}^2 = \lim_{N \to \infty} \frac{1}{N} = 0.$$

This lemma enables us to prove Theorem 1.

Theorem 1. Provided that:

(1) $|B_{i\ell}^{(\ell)}| < 1$ and $\sum_{i=1}^{N} B_{i\ell}^{(\ell)} = 1,$

(2) When $j \neq \ell$, $|B_{ij}^{(\ell)}| \leq |B_{i\ell}^{(\ell)}|$, and

(3) $\lim_{N \to \infty} \max \{B_{i\ell}^{(\ell)};\ i = 1, 2, \ldots, N\} \to 0$

then $\lim_{N \to \infty} \underline{\underline{V}}_N(\underline{\beta}) = \underline{\underline{0}}$

Proof: that $\lim_{N \to \infty} \sum_{i=1}^{N} [B_{i\ell}^{(\ell)}]^2 = 0$ follows from condition (3) and the Lemma.

Since $|B_{ij}^{(\ell)}| \leq |B_{i\ell}^{(\ell)}|$ implies that $\sum_{i=1}^{N} [B_{ij}^{(\ell)}]^2 < \sum_{i=1}^{N} [B_{i\ell}^{(\ell)}]^2$, convergence of the right hand side of the inequality to zero as $N \to \infty$ implies that all diagonal terms in $\underline{\underline{\Phi}}_N^{(\ell)}$ converge to zero as $N \to \infty$. By the Cauchy-Schwartz inequality convergence of all diagonal terms in $\Phi_N^{(\ell)}$ to zero as $N \to \infty$ implies that $\lim_{N \to \infty} \underline{\underline{\Phi}}_N^{(\ell)} = \underline{\underline{0}}$ and hence $V_N(\beta_\ell) \to 0$ as $N \to \infty$. Since for each N, $\underline{\underline{V}}_N(\underline{\beta})$ is positive definite symmetric, $\lim_{N \to \infty} \underline{\underline{V}}_N(\beta_\ell) = 0$ for $\ell = 1, 2, \ldots, K$ implies that $\lim_{N \to \infty} \underline{\underline{V}}_N(\underline{\beta}) = \underline{\underline{0}}$.

Some few comments may be in order on conditions 1 - 3 of the theorem. Much of the motivation for them has been provided in the previous section and empirical support will be forthcoming in subsequent sections. That all B weights are less than 1 in absolute value follows from the generally proportion-like character of the corresponding B weights; $\Sigma\, B_{i\ell}^{(\ell)} = 1$ follows identically from the least squares restriction.

The second condition, that non-corresponding B weights be smaller than corresponding B weights is an implication of the greater systematic correspondence that $x_{i\ell}$ has with X_ℓ than with X_j. Available empirical evidence supports this presumption. Finally, the last and most critical assumption for the Lemma and hence for the proof, depends on how increasingly large aggregates are made up. Suppose that the $B_{i\ell}^{(\ell)}$ <u>are</u> proportions:

then, by construction, the addition of another amount decreases the proportional share of every single antecedent member of the aggregate. Unless the aggregate happened to be constructed systematically to make the N^{th} member a larger fraction than the largest preceding one, the condition will be satisfied. Aggregates are usually built up in two ways: by combining various sub-populations (e.g. two digit into one digit industries), or expanding a given sample. In the first case, one can only suppose that the share of U.S. Steel in total manufacturing is less than it is in the steel industry. In the second case, supposing sampling to be random, the share of each member can be expected to halve, if the sample is doubled, irrespective of the shape of the underlying population. In general, then, we suppose that the B weights will behave in similar fashion and that this condition will hold in the vast majority of cases.

The preceding theorem and discussion lead directly to an extension of the assumptions to permit unequal variance matrices.* When the $\underline{\beta}_i$ are mutually independent but do not have common variance matrices, setting $\underline{\underline{V}}(\underline{\beta}_i) = \underline{\underline{\Omega}}^{(i)}$, we may rewrite (17) as

$$V_N(\underline{\beta}) = \sum_{i=1}^{N} \begin{bmatrix} \underline{B}_i^{(1)} \underline{\underline{\Omega}}^{(i)} \underline{B}_i^{(1)t} & \cdots & \underline{B}_i^{(1)} \underline{\underline{\Omega}}^{(i)} \underline{B}^{(K)t} \\ & \ddots & \vdots \\ & & \underline{B}_i^{(K)} \underline{\underline{\Omega}}^{(i)} \underline{B}_i^{(K)t} \end{bmatrix} \qquad (20)$$

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*The task of also proving that the limiting variances are zero when the individual parameter variance matrices are correlated has not been attempted. One may conjecture that when the tedious algebra is done, the basic results obtained here will not be altered. Highly complex product chains of fractions under the assumptions I have used throughout will surely converge to zero in the limit. Swamy [19] derived related results in the heteroschedastic case to be considered in Theorem 2.

and (18) as

$$V_N(\beta_\ell) = \sum_{i=1}^{N} \underline{B}_i^{(\ell)} \underline{\underline{\Omega}}^{(i)} \underline{B}_i^{(\ell)t} = \text{tr} \sum_{i=1}^{N} \underline{\underline{\Omega}}^{(i)} \underline{B}_i^{(\ell)t} \underline{B}_i^{(\ell)} \tag{21}$$

Let $\Omega_{rc}^{(i)}$ be a generic element of $\underline{\underline{\Omega}}^{(i)}$ and $\theta_{rc} = \max \{\Omega_{rc}^{(i)}, i = 1, 2, \ldots, N\}$

By setting $\underline{\underline{\Xi}} = [\theta_{rc}]$,

$$V_N(\beta_\ell) \leq \text{tr}[\underline{\underline{\Xi}}\{\sum_{i=1}^{N} \underline{B}_i^{(\ell)t} \underline{B}_i^{(\ell)}\}] \quad \text{tr}[\underline{\underline{\Xi\Phi}}_N^{(\ell)}] \tag{22}$$

Provided that elements of are bounded and that conditions (1)-(3) hold, the proof that $V_N(\beta_\ell) \to 0$ as $N \to \infty$ when $\underline{\underline{\Omega}}^{(i)} = \underline{\underline{\Omega}}^{(j)}$ all i, j applies in this more general case. This is a direct consequence of the fact that if $\underline{\underline{\Phi}}^{(\ell)} \to 0$ as $N \to \infty$, so does tr $\underline{\underline{\Xi\Phi}}_N^{(\ell)}$. Also, the Cauchy-Schwartz inequality again implies that if all diagonal terms of $\underline{\underline{V}}_N(\underline{\beta})$ approach zero as N approaches infinity, so do the off-diagonal elements of $\underline{\underline{V}}_N(\underline{\beta})$. Hence we have:

<u>Theorem 2</u>:

Let $\{\underline{\beta}_i,$ i = 1, 2,..., N be mutually independent random vectors with common mean $\overline{\underline{\beta}}$, but not necessarily identical positive definite symmetric variance matrices $\underline{\underline{V}}(\underline{\beta}_i) = \underline{\underline{\Omega}}^{(i)}$, i = 1, 2,..., N. If conditions (1), (2) and (3) of Theorem 1 obtain, then $\lim_{N \to \infty} \underline{\underline{V}}_N(\underline{\beta}) = \underline{\underline{0}}$.

4. Empirical Evidence -- Firm Investment Relations

Manufacturing investment equations will provide some evidence on the underlying assumptions and the aggregation properties contained in the basic theorem. I have chosen investment behavior for several reasons, not the least of which is an earlier interest in its substantive aspects as well as its aggregation characteristics [13]. In addition, the earliest empirical study of aggregation theory was that of Boot and de Wit [2] using investment behavior and based on a study by Grunfeld [7]. Among the most recent contributions to testing alternative models of investment behavior, research by Jorgenson and Siebert [9] placed heavy reliance on individual firm investment equations. In nearly all the studies cited, a basic equation form was the simple Chenery-Koyck distributed lag [3][12] in which investment is a linear function of sales or output, and the lagged capital stock.

Since my present aim is to study aggregation, I shall adopt the Chenery-Koyck formulation without theoretical motivation (available however, in the sources cited) and without a detailed critique of its merits. The basic data have similar origins to the previous firm studies, except that none of the series on investment, sales and gross fixed assets were price corrected. The cost of rectifying the data did not seem warranted in light of the predominant methodological thrust of this study. There is little reason to believe that failure to correct for price variations will seriously affect the aggregation characteristics of the underlying series, although comparability with previous studies is diminished.

The raw data came from the COMPUSTAT tape which contains financial information from 868 companies listed on the New York Stock Exchange, American Stock Exchange, and, in a few instances, regional exchanges or those with securities traded over the counter. While in most cases the numbers are identical with corporate records, some disparities exist. According to the COMPUSTAT manual [18]:

> Differences will frequently occur between the financial statistics included in COMPUSTAT and those in Corporation Records. These differences may be due to one of the following considerations.
>
> Some restatement of company-reported information has been made in COMPUSTAT in accordance with the definitions outlined in this manual for the purpose of increasing comparability, both within a single company and within a single industry.
>
> COMPUSTAT includes some material taken from SEC and outside sources....
>
> Although almost any item of information in COMPUSTAT may differ from that in the Annual Reports in accordance with the definitions given, a few specific notes are listed below in some of the most frequently used statistics in the system....
>
> Annual data in COMPUSTAT for years prior to a merger are not stated on a pro forma basis (see page 3-1). Both pro forma amd reported data are normally given in Corporation records.
>
> Employment and capital expenditure data may vary slightly because of differing sources of information....

From this larger universe, several industries were selected whose aggregation properties will be reported. These were chosen on the basis of size and relative internal homogeneity (two digit industries are often composed of several heterogeneous three-digit industries, each containing only a few firms) as well as different production characteristics between industries. Since computer processing costs exceeded one dollar per firm, calculations were restricted to four reasonable sized industries that have different production processes, in order to restrain costs within reasonable bounds.

Table 1

Sample Composition

Industry	Years	Firms	Observations
Steel	1949-1967 (19)	23	437
Retail	1949-1966 (18)	19	342
Petroleum	1950-1967 (18)	22	396
Machinery	1950-1967 (18)	41	738
	Total	105	1913

We thus have data beginning in 1949 and ending in 1967 for 105 firms. A total of 1,913 observations on investment are to be explained by corresponding data vectors for sales and gross fixed assets.

Since interest may attach to the individual micro results, these, together with their industry macro regressions, have been reported in Appendix Table 1-A. They bear a strong resemblance to similar firm regressions reported by other investigators.

A. Model Assumptions - B-Weights and Proportions

One basic assumption of the theorem is that proportions resemble B-weights. By calculating proportions as the sample period sum of each firm's sales or assets divided by the corresponding sample aggregate sum, various comparisons with estimated B-weights are possible. One comprehensive comparison is shown in Table 2-I which records the simple regression of the B-weights on proportions, with and without the intercept. Since the basic hypothesis is homogeneous, the most pertinent regression for each industry pair is the one that has been forced through the origin.

For some reason, the regressions for asset parameters are very close or within plausible range of the slope value of unity required by the hypothesis in every case, although the hypothesis is refuted for each sales coefficient except for steel. Considering the extent of collinearity in all but steel (on which a table will be presented shortly) the net outcome is mixed, but not unsatisfactory. Starting from the initial extreme proposition that no economic content can be read into the auxillary regression equation parameters, we have come a long way.

A related measure is shown in Table 2-II. The average absolute difference between B-weight and proportion divided by the average corresponding B-weight or proportion* is a sensible normalization for comparative purposes. The steel industry as before shows the smallest relative difference while Retail is the worst. Petroleum is fair for both coefficients while Machinery does poorly on sales and adequately on assets. Order of magnitude zero is good and order of magnitude unity or larger is bad according to the basic hypothesis.

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*These averages are identical since each series sums to unity.

Table 2

I. Regression of B-Weight on Proportion

Industry		Sales Slope	Sales Intercept	Assets Slope	Assets Intercept
Steel	Coeff.	1.0575	-0.0025	0.8356	0.0071
	T-stat	(31.8271)	(-0.9767)	(31.3322)	(3.0041)
	Coeff.	1.0391	forced through	0.8747	forced through
	T-stat	(37.9362)	origin	(32.1531)	origin
Retail	Coeff.	-0.3114	0.0690	0.6174	0.0201
	T-stat	(-1.1032)	(2.7318)	(2.0663)	(0.8785)
	Coeff.	0.1421	forced through	0.7974	forced through
	T-stat	(0.5339)	origin	(3.6911)	origin
Petroleum	Coeff.	-0.1220	0.0510	1.4166	-0.0189
	T-stat	(-0.7194)	(4.2623)	(15.4912)	(-3.1438)
	Coeff.	0.3439	forced through	1.2182	forced through
	T-stat	(1.9665)	origin	(15.4298)	origin
Machinery	Coeff.	0.3136	0.0167	1.0240	-0.0006
	T-stat	(1.6463)	(2.1568)	(11.0235)	(-0.1426)
	Coeff.	0.5596	forced through	1.0168	forced through
	T-stat	(3.5102)	origin	(13.2754)	origin

II. Average Absolute Difference Between B-Weight and Proportion Divided by Average B-Weight or Average Proportion

Industry	Sales	Assets
Steel	0.158	0.191
Retail	0.889	0.984
Petroleum	0.419	0.369
Machinery	0.793	0.478

Table 3 presents detailed information bearing on another basic assumption, which is that corresponding B-weights are large relative to noncorresponding B-weights. For the Steel industry, the assumption held up for all 23 asset auxillary regressions and for 19 out of 23 sales auxillary regressions. The Machinery industry had eight and nine failures to conform, or success on the order of 80%. A similar average success rate holds up in Petroleum, while Retail had under 50% correspondence to this particular assumption.*

A comparison of relative statistical significance between corresponding and non-corresponding B-weights in Table 4 reinforces the impression obtained from comparing only magnitudes. Retail again excepted, corresponding B-weights have the greater statistical significance most of the time, overwhelmingly so for Steel and Petroleum with somewhat less, though clearly evident superiority in Machinery.

Table 4

Number of Times Absolute Value of t-Statistic for Corresponding B-Weight Exceeds Absolute Value of t-Statistic for Non-Corresponding B-Weight

	No. of Firms	Sales	Assets
Steel	23	18	23
Retail	19	10	13
Petroleum	22	19	18
Machinery	41	35	29

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*Table 2-A (Appendix) reports the auxillary regressions in full detail.

TABLE 3

COMPARISON OF B-WEIGHTS WITH NON-CORRESPONDING B-WEIGHTS AND WITH PROPORTIONS -- SALES

STEEL

ID	CORRES - NON-COR	REL DIFF	PROPOR - B-WEIGHT	REL DIFF	T-STAT
1	0.021825	0.927109	-0.003854	-0.195775	-0.856398
2	-0.000732	-0.022001	0.029997	0.473892	2.966211
3	0.172132	1.056772	0.001056	0.006447	0.065080
4	0.031259	1.320298	-0.005388	-0.294644	-0.680712
5	-0.001068	-0.295238	0.001945	0.349696	1.015854
6	0.000227	0.187826	0.002362	0.661443	3.087941
7	0.006588	0.846138	-0.000132	-0.017347	-0.059032
8	0.024833	1.051719	-0.005919	-0.334586	-2.489863
9	0.005343	0.658537	0.001185	0.127494	0.427019
10	0.026445	0.615431	0.012176	0.220805	3.616642
11	0.012110	0.734722	-0.006923	-0.724266	-0.933082
12	0.040752	0.801892	0.006197	0.108697	1.077853
13	0.004029	0.642162	0.002343	0.271889	2.022107
14	0.007261	0.912326	-0.000243	-0.031603	-0.144590
15	0.001752	0.188081	0.002746	0.227687	0.558390
16	0.023912	0.592160	0.015744	0.280516	1.193431
17	0.019324	1.234083	-0.003900	-0.331751	-3.309688
18	0.117085	1.170985	-0.014154	-0.164903	-2.598701
19	0.022825	1.279430	-0.006403	-0.559876	-2.001394
20	0.008644	0.431532	0.001865	0.083437	0.399295
21	0.383166	1.214609	-0.025881	-0.089373	-1.817422
22	0.017809	1.088780	0.001566	0.087397	0.841964
23	0.054271	1.038819	-0.006384	-0.139226	-2.066426
MEAN		0.768529		0.000697	0.013586
MEAN ABSOLUTE			0.006875		1.488308

COMPARISON OF B-WEIGHTS WITH NON-CORRESPONDING B-WEIGHTS AND WITH PROPORTIONS -- ASSETS

STEEL

ID	CORRES - NON-COR	REL DIFF	PROPOR - B-WEIGHT	REL DIFF	T-STAT
1	0.008967	0.853226	0.000427	0.030069	0.490423
2	0.075782	1.101927	-0.021751	-0.462579	-6.269340
3	0.126427	0.948627	0.011305	0.078192	1.860880
4	0.001686	0.174255	0.009016	0.591122	2.239370
5	0.005568	1.163965	-0.001904	-0.661255	-5.544679
6	0.001462	0.885516	0.000433	0.207752	5.741718
7	0.006902	1.283479	-0.002044	-0.613463	-4.488410
8	0.008254	0.817331	0.002862	0.220828	7.675146
9	0.014976	0.958451	-0.005343	-0.519630	-4.733144
10	0.080151	1.157938	-0.022167	-0.471118	-6.606749
11	0.005419	0.499937	-0.001556	-0.167683	-0.544870
12	0.060798	0.986182	0.000554	0.008920	0.205567
13	0.008048	1.261444	-0.001626	-0.342029	-6.619955
14	0.007544	1.232871	-0.002015	-0.491222	-4.238501
15	0.013095	0.886968	-0.004610	-0.454086	-2.565733
16	0.074820	1.082622	-0.011418	-0.197912	-4.266295
17	0.005299	0.606746	0.002334	0.210883	2.112992
18	0.079611	1.082871	-0.001979	-0.027665	-0.862096
19	0.007357	0.907696	-0.001503	-0.227795	-1.367371
20	0.028397	0.822719	-0.011110	-0.474675	-2.619945
21	0.308621	0.995284	0.065458	0.174302	4.715212
22	0.011544	0.723765	0.003121	0.163646	1.873409
23	0.059857	1.094810	-0.006479	-0.134452	-4.825121
MEAN		0.936036		-0.154364	-1.245195
MEAN ABSOLUTE			0.008305		3.585605

COMPARISON OF B-WEIGHTS WITH NON-CORRESPONDING B-WEIGHTS AND WITH PROPORTIONS -- SALES

RETAIL

ID	CORRES - NON-COR	REL DIFF	PROPOR - B-WEIGHT	REL DIFF	T-STAT
1	-0.243752	20.008101	0.067091	1.221872	3.148525
2	-0.150692	7.911712	0.031600	2.493398	1.679365
3	0.052712	2.837545	-0.014221	-3.265335	-1.789764
4	-0.006165	-0.315396	0.013158	0.402326	0.914363
5	0.402651	3.662428	-0.096839	-7.391738	-2.586722
6	-0.149559	39.684897	0.042107	1.098299	2.314985
7	0.139675	4.029567	-0.014660	-0.732970	-1.464776
8	-0.103071	-7.005342	0.035309	0.705870	1.050008
9	-0.025793	-12.003788	0.011305	0.840283	2.875260
10	-0.082366	-1.022532	0.040466	0.334382	1.128440
11	1.248555	4.069539	-0.200943	-1.898159	-2.896805
12	-0.152931	-0.630682	0.086090	0.262010	0.768009
13	0.433432	3.366305	-0.090756	-2.388360	-1.195917
14	0.033357	0.684943	-0.027049	-1.249293	-0.338739
15	-0.102486	9.989346	0.028709	1.556069	2.324868
16	-0.011652	19.195610	0.006547	1.102180	2.949430
17	-0.143756	10.412506	0.033375	1.705479	1.936979
18	-0.143864	18.211979	0.030435	1.350522	2.163189
19	0.005675	0.093541	0.018084	0.229621	0.835078
MEAN		6.483172		-0.190712	0.727146
MEAN ABSOLUTE			0.046787		1.808485

COMPARISON OF B-WEIGHTS WITH NON-CORRESPONDING B-WEIGHTS AND WITH PROPORTIONS -- ASSETS

RETAIL

ID	CORRES - NON-COR	REL DIFF	PROPOR - B-WEIGHT	REL DIFF	T-STAT
1	-0.026806	6.803535	0.055027	1.077125	0.292692
2	0.031849	1.157645	-0.016915	-1.596161	-1.267838
3	0.017302	1.174448	-0.011691	-3.844128	-1.318746
4	0.037088	1.043681	0.008507	0.193153	0.365841
5	0.019453	1.170693	-0.011605	-2.315760	-2.617667
6	-0.020593	2.647163	0.054461	1.170432	1.692208
7	0.048570	1.145132	-0.004933	-0.130402	-0.220965
8	0.013909	0.468534	0.055761	0.652567	0.626746
9	0.059784	1.166645	-0.033690	-1.919282	-1.592777
10	0.173399	1.140896	-0.098201	-1.825882	-2.416573
11	-0.129705	1.537039	0.151521	2.256974	1.511012
12	0.033346	0.410048	0.158783	0.661302	1.586595
13	0.207079	1.184416	-0.083772	-0.919922	-1.368040
14	0.043232	0.959588	-0.013003	-0.405724	-0.136828
15	0.080318	1.176333	-0.040839	-1.488405	-1.777023
16	0.010865	1.172673	-0.000703	-0.082202	-0.154827
17	0.075177	1.206708	-0.036988	-1.461337	-1.488788
18	0.001814	0.467068	0.008163	0.677584	2.334834
19	0.323920	1.151753	-0.139885	-0.989593	-1.191342
MEAN		1.430737		-0.541561	-0.375867
MEAN ABSOLUTE			0.051813		1.261123

COMPARISON OF B-WEIGHTS WITH NON-CORRESPONDING B-WEIGHTS AND WITH PROPORTIONS -- SALES

PETROLEUM

ID	CORRES - NON-COR	REL DIFF	PROPOR - B-WEIGHT	REL DIFF	T-STAT
1	0.055451	1.418238	-0.028113	-2.559133	-3.157916
2	0.088222	1.493521	-0.037865	-1.785686	-1.975309
3	0.034155	1.270533	0.006611	0.197391	0.978032
4	0.076157	1.199286	-0.036173	-1.323646	-1.612048
5	0.030504	1.179837	0.000859	0.032174	0.085066
6	-0.007904	20.089769	0.005522	1.076710	1.759146
7	0.010045	0.884270	0.000409	0.034816	0.078200
8	0.024261	0.911555	0.009160	0.256313	1.026425
9	0.001621	1.171644	0.000534	0.278436	0.747976
10	0.071886	1.106029	-0.005947	-0.100731	-1.026053
11	0.047000	1.163909	-0.030922	-3.269329	-1.358106
12	0.038419	1.289129	0.007262	0.195936	0.483286
13	0.076717	1.252655	0.004586	0.069668	0.905113
14	0.023555	1.274182	-0.004501	-0.321892	-1.542216
15	0.040753	1.344396	-0.005197	-0.206951	-1.486148
16	-0.015304	3.871021	0.015310	1.348139	1.506813
17	0.110055	1.401762	-0.060645	-3.394410	-2.268332
18	0.063266	0.927523	0.014231	0.172627	0.525791
19	0.040982	0.552431	0.028686	0.278854	0.981785
20	0.044426	0.868231	0.005379	0.095128	0.341367
21	0.157579	0.786149	0.056828	0.220865	0.864604
22	-0.011868	-0.361669	0.053963	0.621849	2.585607
MEAN		2.049746		-0.367402	-0.070768
MEAN ABSOLUTE			0.019033		1.240697

COMPARISON OF B-WEIGHTS WITH NON-CORRESPONDING B-WEIGHTS AND WITH PROPORTIONS -- ASSETS

PETROLEUM

ID	CORRES - NON-COR	REL DIFF	PROPOR - B-WEIGHT	REL DIFF	T-STAT
1	-0.004670	-2.045602	0.003414	0.599268	2.367514
2	-0.035452	-98.420906	0.025285	0.985954	1.553079
3	0.024088	1.022977	0.011148	0.321318	1.412450
4	-0.006447	-0.353055	0.009494	0.342081	0.779483
5	0.037821	1.543955	-0.006796	-0.383963	-0.905341
6	0.006446	0.971721	-0.002834	-0.745854	-1.390233
7	0.015505	1.050958	0.001359	0.084352	0.634695
8	0.005399	0.216157	0.018364	0.423687	1.664038
9	-0.001266	5.325720	0.001211	1.244456	5.429408
10	0.032449	0.659706	0.013084	0.210112	2.730298
11	0.019082	1.425164	-0.007172	-1.153787	-3.102609
12	0.031665	1.043509	0.008131	0.211328	1.033567
13	0.066476	1.066373	0.015096	0.194959	1.838011
14	0.004915	0.689429	0.003632	0.337551	1.431257
15	-0.003633	-0.340090	0.010222	0.488941	2.545100
16	0.028547	1.405605	-0.003605	-0.230592	-0.404348
17	-0.129900	4.015257	0.058788	2.223760	2.485393
18	0.180504	1.299768	-0.041899	-0.432074	-2.714926
19	0.081769	0.991917	0.000791	0.009514	0.081461
20	0.065047	0.923557	0.004528	0.060413	0.565859
21	0.452906	1.452441	-0.096042	-0.445088	-3.891869
22	0.128727	1.069921	-0.025994	-0.275594	-1.429294
MEAN		-3.406432		0.185033	0.577863
MEAN ABSOLUTE			0.016777		1.835920

COMPARISON OF B-WEIGHTS WITH NON-CORRESPONDING B-WEIGHTS AND WITH PROPORTIONS -- SALES

MACHINERY

ID	CORRES - NON-COR	REL DIFF	PROPOR - B-WEIGHT	REL DIFF	T-STAT
1	0.077923	1.647965	-0.005728	-0.137843	-0.570309
2	0.073699	1.430699	-0.017955	-0.535053	-0.987602
3	0.047406	1.609331	-0.008718	-0.420383	-0.768253
4	0.187029	2.387539	-0.009470	-0.137523	-1.084549
5	0.103684	2.328421	-0.023954	-1.164192	-2.364165
6	0.085588	0.645662	0.063173	0.322752	1.386774
7	-0.064361	-2.434111	0.030886	0.538763	2.703085
8	0.014357	0.630986	-0.009289	-0.689914	-1.786735
9	0.037858	2.530100	-0.004595	-0.443215	-1.631633
10	0.186376	1.409304	-0.038151	-0.405459	-2.212262
11	0.103843	1.815269	-0.031577	-1.232117	-5.247604
12	-0.280794	3.222977	0.113661	4.282833	3.058110
13	0.028774	1.694535	-0.008043	-0.899938	-5.125203
14	0.053539	1.411841	-0.009612	-0.339576	-0.840898
15	0.197407	1.999657	-0.067230	-2.134965	-5.172372
16	0.074417	2.303938	-0.015478	-0.920150	-2.394865
17	0.016520	3.245607	-0.003269	-1.795362	-2.167292
18	0.024666	2.766032	-0.004766	-1.148467	-2.657683
19	0.005534	1.408207	-0.000893	-0.294360	-1.405312
20	0.001254	0.146340	0.000324	0.036499	0.051063
21	0.130377	3.182072	-0.029997	-2.733096	-2.875571
22	0.036320	2.057771	-0.009732	-1.229154	-4.793523
23	-0.237560	5.515065	0.075361	2.334129	5.273817
24	0.020503	1.409782	-0.006328	-0.770326	-2.940155
25	-0.295214	4.876884	0.084469	3.529000	6.208041
26	0.003243	0.770716	0.000042	0.010064	0.016527
27	0.003821	0.251545	-0.005225	-0.524242	-0.748490
28	0.019578	1.118987	-0.000784	-0.046960	-0.137308
29	0.038528	2.042051	-0.005910	-0.456106	-1.453824
30	0.053810	1.540554	-0.017761	-1.034570	-2.307479
31	0.061124	1.422143	-0.017028	-0.656137	-2.428953
32	-0.035492	-28.289500	0.014231	0.918983	1.865431
33	0.022324	1.644787	0.000672	0.047201	0.316378
34	-0.015373	-12.016931	0.007845	0.859795	3.165202

COMPARISON OF B-WEIGHTS WITH NON-CORRESPONDING B-WEIGHTS AND WITH PROPORTIONS -- ASSETS

MACHINERY

ID	CORRES - NON-COR	REL DIFF	PROPOR - B-WEIGHT	REL DIFF	T-STAT
1	0.026721	0.804736	0.008632	0.206337	2.399830
2	0.022497	0.975657	-0.000455	-0.020165	-0.073190
3	0.009076	1.055487	0.000565	0.061706	0.321297
4	0.081204	1.137625	-0.007231	-0.112725	-0.720012
5	0.012826	1.055489	0.011434	0.484783	3.189642
6	0.241123	1.137080	0.011114	0.049800	0.460844
7	0.061558	0.981031	-0.014776	-0.308021	-1.928306
8	-0.000351	-0.163442	0.003709	0.633182	2.149510
9	0.019546	1.279316	-0.004015	-0.356529	-0.932355
10	0.104464	0.919958	-0.005858	-0.054393	-0.521847
11	-0.015280	4.568923	0.023430	1.166504	4.424334
12	0.161970	1.361859	-0.093044	-3.593976	-3.361808
13	0.004964	0.787385	0.003366	0.348085	2.515411
14	0.046108	1.207453	-0.011936	-0.454729	-1.398928
15	0.012282	0.479226	0.018620	0.420803	2.818349
16	0.018731	1.040992	0.000824	0.043819	0.351754
17	0.000896	0.865184	0.001150	0.526209	2.372868
18	0.000458	0.491427	0.002093	0.691829	5.609064
19	0.000525	0.510095	0.001258	0.549805	3.526775
20	0.003011	0.607592	0.002732	0.355353	1.189612
21	-0.029984	1.602748	0.032246	2.381869	3.159180
22	0.011351	1.000667	-0.000955	-0.091933	-0.773366
23	0.134434	1.201383	-0.069217	-1.621689	-5.611798
24	0.004534	0.732355	0.001545	0.199698	1.341131
25	-0.000331	-0.036038	0.006864	0.427709	0.645899
26	0.001231	0.642256	0.003690	0.658060	6.259383
27	-0.010397	4.382961	0.014766	1.191396	2.424442
28	0.029734	0.947909	-0.003391	-0.121216	-0.849392
29	0.008823	0.905744	0.002080	0.175994	1.163398
30	-0.006617	-1.314854	0.013226	0.724376	1.051259
31	-0.031538	1.991085	0.032685	1.940298	4.833408
32	0.041049	1.179632	-0.018895	-1.188173	-5.270933
33	0.009644	0.995157	0.003653	0.273781	2.948917
34	0.011874	1.105837	-0.003577	-0.499684	-2.380428

35	0.015076	1.141336	-0.002222	-0.202246	-1.290500
36	-0.001052	-0.8431[illegible]	0.006324	0.835174	5.713184
37	0.010447	1.06883[illegible]	-0.001853	-0.234046	-2.230207
38	-0.026108	2.475[illegible]	0.025912	1.686306	3.686078
39	0.008768	1.095773	0.001207	0.131103	0.881046
40	0.002718	1.076066	-0.001420	-1.283128	-3.000223
41	0.004001	0.807177	0.005718	0.535632	7.480245
MEAN		1.055160		0.159925	1.038379
MEAN ABSOLUTE			0.011651		2.518540

Once again, there appears to be a definite relation to collinearity. The quality of the results is inverse to the extent of linear dependence: when there is not excessive correlation among the explanatory variables, the assumptions of the statistical aggregation model are more nearly validated.

Table 5

Industry	Sales-Asset Simple Correlation
Steel	.8378
Retail	.9978
Petroleum	.9902
Machinery	.9696

The individual t-statistic on the difference between corresponding B's and proportions has been calculated using the B weight's standard error and treating the proportion as a known parameter. This understates the true variability so that the outcome is relatively unfavorable to a finding of no significant difference. The arithmetic mean of the t statistics tends to be small, amounting to less than unity for all sales coefficients (and nearly zero in two of the four industries) and about unity for two industry asset coefficients and magnitude one-half for the other two. Calculated t statistics rank ordered from largest positive to most negative appear in Appendix Table 3-A. Casual inspection is enough to indicate that there is too much density in the tails for these tabulations to represent an identically distributed random drawing from a t distribution with mean zero. Retail for both coefficients, and Steel for the sales coefficient are comparatively tightly bunched about zero, while the others are much less so. One problem with these inferences is apparent from examination of Appendix Table 2-A showing proportions and auxillary regressions. Goodness of fit is nearly always very high and the corresponding coefficient standard errors are very small. As a consequence, even though a visual scan reveals that the two series are close to each other most of the time (with several glaring exceptions in every industry, to be sure) "significant" differences arise with substantial frequency.

The upshot of this section will now be summarized. Qualified support has been found for the assumptions relating corresponding B-weights to proportions, and the magnitudes of corresponding and non-corresponding B-weights. When there is excessive collinearity among the explanatory macro variables, however, the assumptions tend to break down. Yet under this very circumstance it is difficult or impossible to obtain sensible macro estimates anyway. In short, when estimation feasibility breaks down most, so do the aggregation assumptions. Finally, the comprehensive tests summarizing information about all pairs of proportions and B-weights stand up somewhat better than pair-wise tests based on the t-statistic.

B. Model Predictions: Micro Variances, Macro Variances and Aggregation Gain

This theory's main innovation is to predict how aggregation gain can be expected to occur, and its magnitude. That there is any aggregation gain whatsoever until recently would have been a puzzling result of itself. Table 6 reports the major empirical results most revealing on this central feature.

Aggregation gain is measured by the reduction of the estimated regression coefficient variances. In the macro relative to micro estimates, the theoretical gain can be calculated in two ways. Assuming, first, that proportions are good approximations to the corresponding B-weights, their

Table 6

Summary Table on Industry Aggregation

	Steel		Retail		Petroleum		Machinery	
	Sales	Assets	Sales	Assets	Sales	Assets	Sales	Assets
Coefficient								
Mean of micro	0.08417	0.02094	0.03314	-0.03773	0.30074	-0.08810	0.07345	-0.05550
Macro	0.13407	0.00858	0.07539	-0.16308	0.39543	-0.16154	0.10657	-0.11744
Variances								
Mean micro variance	0.00745	0.00321	0.00114	0.01641	0.01432	0.00478	0.00123	0.00873
Macro variance	0.00200	0.00047	0.00137	0.02169	0.00168	0.00070	0.00015	0.00085
Sum of squared weights								
H Index	0.13634	0.18295	0.15219	0.11187	0.10946	0.09546	0.06806	0.08044
Sum of squared B-weights	0.14947	0.14296	0.19717	0.16510	0.08325	0.15416	0.09050	0.10204
Theoretical aggregation gain								
From proportions	7.33458	5.46584	6.57088	8.93919	9.15383	10.47594	14.69215	12.43115
From B-weights	6.69025	6.99507	5.07180	6.05694	12.01257	6.48672	11.04968	9.80046
Actual aggregation gain								
Mean micro var. ÷ macro var.	3.71689	6.85962	0.83455	0.75629	8.54690	6.80369	8.01489	10.24131

squared sum designated as H (for reasons given in the next section) shows the expected reduction in the macro variance as a consequence of aggregation. Its reciprocal is therefore termed the aggregation gain. Second, and in parallel fashion, the sum of squared corresponding B-weights and its reciprocal provides another relevant estimate of potential aggregation gain. Since the postulated empirical relation between the two measures has been considered already at length, it is noteworthy that the two sums of squares turn out to be highly similar figures.

Retail bombed out, as in every other aspect it least complied with the theoretical assumptions of the basic model. The remaining industries:

1. All show positive aggregation gain;
2. All show gains of rather similar magnitude to the theoretical prediction.

I consider these findings to be the most significant quantitative result in the paper.

C. Microproportion Stability

Some proportions grow rapidly, others decline swiftly, while some barely change at all. Since the stability of proportions was assumed at the outset, some tests were made to evaluate an assumption we knew would be violated in actuality. A straightforward test is presented in Appendix Table 4-A from which Table 7 has been extracted. The two quartiles and median relative growth rate (defined as the linear trend slope of the proportion divided by the ratio of variable means, $dP/dt \div \overline{x}/\overline{X}$) summary statistics are presented here. The top quartile rate of growth is about 2 1/2 - 4 1/2% per annum in most industries while the median trend rate of change is in the neighborhood of 1% - 2 1/2%, so that for most firms - not all - relative shares did not radically change over the twenty year sample period.

Table 7

Quartiles for Relative Rate of Growth of Proportions

	Sales	Assets
Steel		
First Quartile	0.026	0.030
Median	0.011	0.014
Third Quartile	0.009	0.008
Retail		
First Quartile	0.031	0.039
Median	0.011	0.025
Third Quartile	0.005	0.007
Petroleum		
First Quartile	0.034	0.021
Median	0.024	0.013
Third Quartile	0.008	0.003
Machinery		
First Quartile	0.045	0.045
Median	0.025	0.031
Third Quartile	0.014	0.013

5. Empirical Evidence -- All Manufacturing H Indexes

According to the previously developed theory, expected gains from aggregation depend on the number of elements in the aggregate and their size distribution. Maximum aggregation gain arises when all elements are the same size. This is evident, since the variances of proportions will be zero when all shares are identical and the correlative fact that the sum of squared proportions (or variances) is nearly one for very skewed shares. The sum of squares for equal shares is 1/N.

Data on the sum of squared shares exist for manufacturing industries because this market parameter has begun to acquire descriptive and analytical significance in the study of industrial organization. It has been named the H concentration measure or index after the two originators who independently promoted it, O.C. Herfindahl and A.P. Hirschmann. The most interesting interpretation is that of M.A. Adelman [1], who shows that the H index can be viewed as the weighted average slope of the cumulative concentration curve. In a second interpretation, suppose that there were N equal sized firms in an industry; then N turns out to equal 1/H. Adelman suggests that the reciprocal of H can properly be viewed as a "numbers equivalent".

Since its introduction into industrial organization literature by Herfindahl [8], the H index has been calculated for various firm attributes for four (and some five) digit industries in a 1963 volume on concentration in manufacturing by Ralph L. Nelson [14]. Resort to four digit industries is stringent but useful. It is stringent in the sense that such a fine subdivision has comparatively few firms in it, useful because one might reasonably expect that substantial homogeneity of markets and production methods will prevail at this classification level, in contrast to the more common econometric category of two-digit aggregates. The exhaustive tabulations of Nelson yield several impressions on which we briefly comment. First, concentration and H measures by value added, by shipments, by production manhours and by employees yield highly similar results, so that estimates of aggregation gain can be equally well derived from any of them.* A second impression is that these various measures are quite stable over time. These census groupings are much more realistic than the arbitrarily formed industries selected out of COMPUSTAT, which, for the most part, is composed of large New York Stock Exchange listed firms. Several quantitative tabulations bearing aggregation properties in manufacturing deserve explicit presentation. The first is a tabulation of ranked H indexes for 1947 shown in the first column of Table 8. While these data are now twenty-two years old, they are more complete, but still similar to the latest census information available to Nelson for 1954, plus annual survey information for 1955, 1956 and 1957. Later tabulations indicate that these measures are stable according to visual impressions, so that broad inferences drawn from 1947 will be reliable, even though some industries now have widely different H and concentration indexes than they had in 1947.

An experiment with these data turned out to be a partial success. The purpose of the exercise was to determine whether a simple one parameter function like the exponential can be used to derive H indexes from

*It is generally true that establishment based indexes show considerably less concentration than company data. Thus, subsequent evaluation of these tabulations are restricted to the series listed only.

usually more readily available data. More concretely, if the concentration density curve could be adequately represented by an exponential density function with exponential parameter designated $\underline{c}$ it is a simple matter to use concentration indexes to estimate $\underline{c}$.* Furthermore, the sum of squares of the density function will then be $\underline{c}/2$. If the initial hypothesis of equivalency is correct, $\underline{c}/2$ will then be equal to the industry H index and in addition, estimates of $\underline{c}$ for the top four, eight and twenty firms in each industry will turn out to be approximately the same. While the implicit estimates of the squared sum of the hypothesised exponential were indeed similar in magnitude to the actual H index, the second implication of the exponential form was not supported by the data. For all but a few four digit industries, $\underline{c}(4) > \underline{c}(8) > \underline{c}(20)$ where $\underline{c}$ has been estimated for each of the four, eight and twenty largest firms in Table 8.** This result

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*Suppose $f(i) = ce^{-ci}$ gives the i'th firm's proportion of total industry shipments. Then the proportion of total shipments of the M largest firms is given by the cumulated density function,

$$s(M) = \int f(i)\,di = 1 - e^{-cM}$$

Thus, if $s(M)$ is known, the last expression may be solved for c to give

$$c = \frac{1}{M} \ln \left(\frac{1}{1 - s(M)}\right).$$

**This table is based on all census industries in Nelson's Table A:1, Company Concentration Indexes, Industries, 1947, 1954, 1955, and 1956, with the exception of the following which could not be used because sufficient data were not available, or were not reported to avoid disclosing figures for individual companies:

2224, 2233, 2651, 2824, 3211, 3331, 3332, 3334, 3429, 3461, 3497, 3519, 3553, 3555, 3616, 3821.

suggests that typically industry is less concentrated than exponential curves derived in this fashion. Yet it is still possible to use concentration indexes to obtain an approximate notion of potential aggregation gain when, as usual, these data are available but H indexes are not. For industries with high to medium concentration $\underline{c}(4)/2$ provides the best estimate of H (actual H >.07); $\underline{c}(8)/2$ is closest to H for medium to low concentration (.02 < actual H < .07) and $\underline{c}(20)/2)$ is closest to H for the least concentrated industries (actual H < .02). While close estimates of H were obtained for medium and low concentration industries, the most highly concentrated industries usually provided the poorest information about H from the concentration indexes, as an examination of the top 15 - 20 industries in Table 8 indicates.

H indexes in manufacturing four digit industries have a median value of .09 according to Table 8. Translated into the terms of our approach to aggregation, the gain in terms of variance reduction from aggregation is twelve fold. Even at the first quartile mark, above which lie 25% of the most concentrated industries, a seven fold gain will occur from using aggregated data instead of micro information directly. At the third quartile, an H index of about .04 implies that a twenty-eight fold aggregation gain may be expected.

To what extent does the existing degree of concentration in American industry reduce estimation efficiency? To recall the earlier statement of the problem, a highly skewed size distribution of elements in an aggregate reduces the beneficial aggregation effects of collapsing

TABLE 8

H INDEXES FOR SHIPMENTS OF FOUR DIGIT INDUSTRIES IN 1947, COMPANY BASIS, AND ESTIMATES OF SUM OF SQUARES OF EXPONENTIAL DENSITY FUNCTION

INDUSTRY	H	C(4)/2	C(8)/2	C(20)/2
3352	*******	0.3516	0.2414	N.A.
3583	*******	0.1831	0.1279	0.0804
3612	*******	0.2531	0.1662	0.0954
3651	*******	0.3096	0.2011	0.1085
3664	*******	0.3876	0.2414	0.1067
3511	0.2997	0.2599	0.2043	N.A.
3272	0.2971	0.2322	0.1718	0.1049
3021	0.2843	0.2049	0.1635	0.1207
2898	0.2837	0.2024	0.1439	0.1049
2141	0.2628	0.2629	0.1768	0.0838
2111	0.2461	0.2916	0.3197	N.A.
3411	0.2447	0.1870	0.1206	0.0675
3615	0.2102	0.1622	0.1141	0.0634
3692	0.2059	0.1794	0.1790	0.1177
3861	0.2013	0.1177	0.0748	0.0401
2052	0.1941	0.1560	0.0926	0.0479
2825	0.1847	0.1909	0.1779	N.A.
3572	0.1800	0.1962	0.1996	N.A.
3562	0.1794	0.1229	0.0806	0.0445
2062	0.1775	0.1492	0.1284	N.A.
3011	0.1746	0.1804	0.1385	0.1004
3571	0.1745	0.1451	0.1078	0.0922
3221	0.1736	0.1229	0.0966	0.0613
3717	0.1692	0.1378	0.0846	0.0466
2043	0.1609	0.1717	0.1385	0.0885
3715	0.1603	0.1026	0.0692	0.0428
3722	0.1584	0.1560	0.1309	0.0824
3662	0.1583	0.1636	0.1540	0.1018
2073	0.1520	0.1504	0.1251	0.0922
2812	0.1518	0.1500	0.1594	N.A.
2085	0.1514	0.1698	0.1228	0.0683
3641	0.1488	0.1370	0.1024	0.0625
2063	0.1472	0.1432	0.1680	N.A.
3425	0.1391	0.1323	0.0863	0.0545

3593	0.1362	0.1199	0.0963	0.0619
3292	0.1329	0.1066	0.0788	0.0527
3914	0.1326	0.1180	0.0791	0.0412
3275	0.1309	0.1034	0.0822	0.0545
3842	0.1223	0.1090	0.0689	0.0371
3261	0.1220	0.1075	0.1047	0.1067
3613	0.1197	0.1167	0.0837	0.0487
3691	0.1181	0.1196	0.0932	0.0484
3614	0.1158	0.1096	0.0666	0.0394
3312	0.1129	0.0933	0.0700	0.0384
3351	0.1112	0.1145	0.0897	0.0607
2092	0.1096	0.1114	0.1024	0.1018
3576	0.1090	0.0959	0.0765	0.0554
3742	0.1085	0.1017	0.0795	0.0601
3262	0.1040	0.1009	0.0854	0.1067
3333	0.1004	0.0946	0.1058	N.A.
3359	0.1001	0.1078	0.0891	0.0691
3253	0.0998	0.0884	0.0750	0.0780
3565	0.0949	0.1034	0.0721	0.0396
3721	0.0936	0.0946	0.0894	0.0804
3229	0.0934	0.0878	0.0645	0.0396
3491	0.0860	0.0914	0.0830	0.0631
2522	0.0853	0.0856	0.0666	0.0481
3631	0.0795	0.0648	0.0527	0.0396
2271	0.0777	0.0876	0.0744	0.0519
2011	0.0733	0.0795	0.0527	0.0271
2823	0.0731	0.0720	0.0609	0.0538
3392	0.0710	0.0740	0.0596	0.0407
3264	0.0699	0.0720	0.0633	0.0540
3751	0.0687	0.0685	0.0706	0.0616
3493	0.0651	0.0689	0.0626	0.0525
2952	0.0630	0.0670	0.0645	0.0498
3581	0.0629	0.0630	0.0640	0.0573
3263	0.0627	0.0585	0.0523	0.0493
2661	0.0602	0.0727	0.0546	0.0357
2045	0.0598	0.0644	0.0560	0.0371
3871	0.0597	0.0648	0.0557	0.0420
3293	0.0594	0.0683	0.0554	0.0386
2911	0.0556	0.0581	0.0552	0.0425
3621	0.0550	0.0548	0.0389	0.0291
2121	0.0515	0.0644	0.0521	0.0345
3431	0.0503	0.0528	0.0407	0.0277
2295	0.0490	0.0513	0.0471	0.0399

3585	0.0489	0.0595	0.0489	0.0319
3564	0.0489	0.0611	0.0475	0.0308
3579	0.0483	0.0538	0.0417	0.0293
3589	0.0475	0.0506	0.0402	0.0300
3322	0.0460	0.0538	0.0408	0.0285
3943	0.0417	0.0447	0.0449	0.0362
2234	0.0394	0.0454	0.0307	0.0200
3241	0.0357	0.0431	0.0369	0.0291
3561	0.0348	0.0429	0.0299	0.0199
3554	0.0330	0.0412	0.0338	0.0263
3399	0.0328	0.0400	0.0335	0.0266
2041	0.0322	0.0424	0.0322	0.0204
2834	0.0317	0.0407	0.0357	0.0249
2031	0.0308	0.0422	0.0329	0.0234
3661	0.0267	0.0391	0.0285	0.0205
3611	0.0261	0.0305	0.0270	0.0216
3141	0.0257	0.0405	0.0262	0.0144
2893	0.0251	0.0338	0.0297	0.0218
3566	0.0246	0.0351	0.0280	0.0194
2033	0.0242	0.0383	0.0264	0.0146
3323	0.0231	0.0318	0.0275	0.0202
3542	0.0227	0.0317	0.0263	0.0213
2432	0.0221	0.0304	0.0252	0.0197
3591	0.0220	0.0333	0.0240	0.0170
3391	0.0218	0.0333	0.0266	0.0187
3494	0.0184	0.0277	0.0231	0.0177
3489	0.0171	0.0280	0.0231	0.0160
3541	0.0171	0.0268	0.0218	0.0165
3531	0.0169	0.0246	0.0224	0.0166
2082	0.0160	0.0299	0.0220	0.0139
2671	0.0140	0.0243	0.0197	0.0126
3463	0.0127	0.0235	0.0178	0.0111
2071	0.0111	0.0232	0.0174	0.0118
3321	0.0109	0.0219	0.0164	0.0105
2261	0.0093	0.0179	0.0148	0.0111
2252	0.0078	0.0169	0.0128	0.0091

******* - H INDEX WITHHELD TO AVOID DISCLOSING
FIGURES FOR INDIVIDUAL COMPANIES

N.A. - NOT AVAILABLE

parameter variances. Subtracting H's minimum theoretical value 1/N (which occurs when all elements are equal) from actual H offers a simple measure of this loss. This difference, H - 1/N, also has significance for the analysis of industrial organization. The results are arrayed in Table 9. The median within manufacturing is about .07, which is almost the same as the value of H itself. While the H - 1/N distribution is somewhat more concentrated than the H distribution and some rearrangement in the rank orderings do occur, the two distributions strongly resemble each other. Since for all but a handful of industries 1/N tends to be negligible, we may conclude that it is not fewness of firms but the skewness of their size distribution that limits potential aggregation gain.

The stability of concentration measures over time depends on which measure has been selected according to Table 10. H indexes show greater sensitivity than the more standard concentration index. Noticable instability is apparent for one fifth of all manufacturing industry total shipments between 1947 and 1956 for the H index, while a mere three out of 113 industries appeared to deviate significantly from the base share held by the four largest firms over the same period.*

In studying aggregation, the more sensitive H index is the most pertinent magnitude. While far from ideal, the overall picture for stability is quite satisfactory. A batting average of .800 over a decade (what happened in between we cannot say) is good enough to warrant broad confidence to a basic presumption of stability. While these data are more inclusive than the micro data previously analyzed, they are not as powerful,

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*A quick subjective examination reveals that the industries 2031, 2043, 2045, 2073, 2085, 2111, 2825, 3021, 3261, 3272, 3275, 3312, 3331, 3493, 3562, 3576, 3613, 3631, 3717, 3722, and 3861 had marked changes in their H index between 1947 and 1956, while industries 2031, 2045 and 3613 had striking changes in the concentration index for the four largest firms.

TABLE 9

THE QUANTITY (H - 1/N), FROM LARGEST TO SMALLEST

INDUSTRY	(H - 1/N)	INDUSTRY	(H - 1/N)	INDUSTRY	(H - 1/N)
3352	******	3583	******	3612	******
3651	******	3664	******	3272	0.2668
2141	0.2521	2898	0.2437	3411	0.2349
3021	0.2343	3511	0.2331	3615	0.2027
3861	0.1984	2111	0.1934	2052	0.1901
3562	0.1702	3717	0.1680	3692	0.1624
3571	0.1545	3715	0.1510	3221	0.1492
3011	0.1460	2085	0.1445	2043	0.1427
3722	0.1399	2825	0.1393	3641	0.1388
3572	0.1365	3914	0.1280	3662	0.1270
3425	0.1267	2073	0.1234	3593	0.1233
3292	0.1211	3842	0.1206	2062	0.1187
3275	0.1172	3691	0.1132	3613	0.1130
3614	0.1113	3312	0.1071	2812	0.0963
3576	0.0953	2092	0.0949	3742	0.0938
3351	0.0933	3565	0.0897	2063	0.0884
3229	0.0841	3261	0.0835	3359	0.0757
2522	0.0753	3631	0.0728	2011	0.0728
3721	0.0723	3253	0.0704	2271	0.0659
3491	0.0656	3262	0.0655	2823	0.0624
3392	0.0613	3751	0.0554	3871	0.0543
2661	0.0539	3293	0.0536	2952	0.0534
2911	0.0520	3621	0.0518	2045	0.0511
3493	0.0503	2121	0.0502	3263	0.0491
3264	0.0486	3585	0.0471	3431	0.0465
3581	0.0459	3564	0.0436	3579	0.0433
3589	0.0419	2295	0.0396	2234	0.0371
3943	0.0333	3561	0.0324	3322	0.0313
2041	0.0313	2834	0.0308	3399	0.0299
3333	0.0289	2031	0.0264	3554	0.0260
3661	0.0255	3141	0.0247	2893	0.0237
2033	0.0236	3611	0.0229	3566	0.0221
3241	0.0221	3542	0.0202	3591	0.0197
3391	0.0177	3323	0.0174	3489	0.0160
3494	0.0155	2432	0.0150	3531	0.0149
3541	0.0139	2082	0.0135	2671	0.0132
3463	0.0121	2071	0.0105	3321	0.0102

2261	0.0077	2252	0.0060

****** - WITHHELD TO AVOID DISCLOSING FIGURES FOR INDIVIDUAL COMPANIES

TABLE 1?

FRACTION OF TOTAL INDUSTRY SHIPMENTS OF FOUR LARGEST FIRMS
AND H INDEX, 1947 AND 1956

INDUSTRY	YEAR	CONC. INDEX	H
2011	1947	0.471	0.073
	1956	0.382	0.054
2031	1947	0.287	0.030
	1956	0.458	0.074
2033	1947	0.263	0.024
	1956	0.293	0.033
2041	1947	0.288	0.032
	1956	0.354	0.042
2043	1947	0.747	0.160
	1956	0.858	*****
2045	1947	0.403	0.059
	1956	0.875	0.273
2052	1947	0.712	0.194
	1956	0.709	0.183
2062	1947	0.697	0.177
	1956	0.651	0.154
2063	1947	0.682	0.147
	1956	0.670	0.142
2071	1947	0.170	0.011
	1956	0.181	0.013
2073	1947	0.700	0.152
	1956	0.859	0.266
2082	1947	0.213	0.016

	1956	0.250	0.029
2085	1947	0.743	0.151
	1956	0.578	0.100
2092	1947	0.590	0.109
	1956	0.482	0.082
2111	1947	0.903	0.246
	1956	0.813	0.198
2121	1947	0.403	0.051
	1956	0.486	0.073
2141	1947	0.878	0.262
	1956	N.A.	*****
2234	1947	0.305	0.039
	1956	0.326	0.037
2252	1947	0.126	0.007
	1956	0.191	0.013
2261	1947	0.134	0.009
	1956	N.A.	*****
2271	1947	0.503	0.077
	1956	0.536	0.091
2295	1947	0.337	0.049
	1956	0.429	0.062
2432	1947	0.216	0.022
	1956	0.217	0.019
2522	1947	0.496	0.085
	1956	0.410	0.060
2661	1947	0.441	0.060
	1956	0.318	0.036
2671	1947	0.177	0.014
	1956	0.164	0.013

2812	1947	0.699	0.151
	1956	0.726	0.159
2823	1947	0.438	0.073
	1956	0.450	0.068
2825	1947	0.783	0.184
	1956	0.769	0.255
2834	1947	0.278	0.031
	1956	0.253	0.033
2893	1947	0.237	0.025
	1956	0.287	0.032
2898	1947	0.802	0.283
	1956	0.814	0.284
2911	1947	0.372	0.055
	1956	0.317	0.046
2952	1947	0.415	0.063
	1956	0.439	0.078
3011	1947	0.764	0.174
	1956	0.757	0.163
3021	1947	0.806	0.284
	1956	0.744	0.229
3141	1947	0.277	0.025
	1956	0.291	0.026
3221	1947	0.626	0.173
	1956	0.585	0.159
3229	1947	0.505	0.093
	1956	N.A.	*****
3241	1947	0.291	0.035
	1956	0.320	0.041
3253	1947	0.507	0.099

	1956	0.507	0.087
3261	1947	0.577	0.122
	1956	0.548	0.118
3262	1947	0.554	0.104
	1956	0.587	0.111
3263	1947	0.374	0.062
	1956	0.358	0.056
3264	1947	0.438	0.069
	1956	0.487	0.080
3272	1947	0.844	0.297
	1956	0.815	0.263
3275	1947	0.563	0.130
	1956	0.662	0.231
3292	1947	0.574	0.132
	1956	0.587	0.140
3293	1947	0.421	0.059
	1956	0.322	0.039
3312	1947	0.526	0.112
	1956	0.529	0.116
3321	1947	0.161	0.010
	1956	0.253	0.022
3322	1947	0.350	0.046
	1956	N.A.	*****
3323	1947	0.225	0.023
	1956	0.223	0.019
3333	1947	0.531	0.100
	1956	0.576	0.110
3351	1947	0.600	0.111
	1956	0.526	0.088

3352	1947	0.940	*****
	1956	0.892	*****
3359	1947	0.578	0.100
	1956	0.627	0.120
3391	1947	0.234	0.021
	1956	0.308	0.035
3392	1947	0.447	0.071
	1956	0.377	0.053
3399	1947	0.274	0.032
	1956	0.256	0.028
3411	1947	0.776	0.244
	1956	0.823	0.271
3425	1947	0.653	0.139
	1956	0.533	0.109
3431	1947	0.345	0.050
	1956	0.404	0.073
3463	1947	0.172	0.012
	1956	0.126	0.007
3489	1947	0.201	0.017
	1956	0.151	0.010
3491	1947	0.518	0.086
	1956	0.512	0.074
3493	1947	0.424	0.065
	1956	0.365	0.041
3494	1947	0.199	0.018
	1956	0.197	0.020
3511	1947	0.875	0.299
	1956	0.819	*****
3531	1947	0.179	0.016

	1956	0.180	0.015
3541	1947	0.193	0.017
	1956	0.164	0.015
3542	1947	0.224	0.022
	1956	0.210	0.021
3554	1947	0.280	0.033
	1956	0.374	0.053
3561	1947	0.291	0.034
	1956	N.A.	*****
3562	1947	0.626	0.179
	1956	0.475	0.090
3564	1947	0.387	0.048
	1956	N.A.	*****
3565	1947	0.563	0.094
	1956	0.533	0.085
3566	1947	0.245	0.024
	1956	0.214	0.019
3571	1947	0.687	0.174
	1956	0.778	0.163
3572	1947	0.792	0.180
	1956	N.A.	*****
3576	1947	0.536	0.109
	1956	0.433	0.060
3579	1947	0.350	0.048
	1956	0.343	0.046
3581	1947	0.396	0.062
	1956	N.A.	*****
3583	1947	0.769	*****
	1956	0.899	*****

3585	1947	0.379	0.048
	1956	0.338	0.042
3589	1947	0.333	0.047
	1956	0.422	0.078
3591	1947	0.234	0.022
	1956	0.183	0.015
3593	1947	0.617	0.136
	1956	0.621	0.125
3611	1947	0.217	0.026
	1956	0.186	0.019
3612	1947	0.868	*****
	1956	0.892	*****
3613	1947	0.606	0.119
	1956	0.361	0.052
3614	1947	0.583	0.115
	1956	0.487	0.084
3615	1947	0.727	0.210
	1956	0.739	0.168
3621	1947	0.355	0.055
	1956	0.434	0.084
3631	1947	0.405	0.079
	1956	0.503	0.118
3641	1947	0.666	0.148
	1956	0.625	0.137
3651	1947	0.916	*****
	1956	0.923	*****
3661	1947	0.268	0.026
	1956	0.299	0.031
3662	1947	0.730	0.158

	1956	N.A.	*****
3664	1947	0.955	*****
	1956	0.956	*****
3691	1947	0.616	0.118
	1956	0.745	0.152
3692	1947	0.762	0.205
	1956	N.A.	*****
3715	1947	0.560	0.160
	1956	N.A.	*****
3717	1947	0.668	0.169
	1956	0.834	0.293
3721	1947	0.531	0.093
	1956	0.518	0.088
3722	1947	0.712	0.158
	1956	0.621	0.115
3742	1947	0.557	0.108
	1956	N.A.	*****
3751	1947	0.422	0.068
	1956	0.416	0.063
3842	1947	0.582	0.122
	1956	0.552	0.126
3861	1947	0.610	0.201
	1956	0.670	0.281
3871	1947	0.405	0.059
	1956	N.A.	*****
3914	1947	0.611	0.132
	1956	0.585	0.137
3943	1947	0.301	0.041
	1956	0.418	0.064

***** - WITHHELD TO AVOID DISCLOSING FIGURES FOR INDIVIDUAL COMPANIES

N.A. - NOT AVAILABLE

since the stability in an industry aggregate does not preclude shift in the underlying individual distributions.

6. Heterogeneous Parameters amd Research Strategy

A complementary statistical aggregation approach starts with the assumption of heterogeneity, but concludes that structural simplification through clustering of similar objects can lead to increased understanding and manageability, although at some cost in precision. Walter Fisher [4] provides operational solutions to this problem through statistical decision theory.

The present treatment of aggregation uses a quadratic cost function i.e. least squares, on the aggregate relation whose implications were long ago illuminated by Theil. While I have nothing to add to Theil's theory under conditions where micro paramaters differ, the interpretation given here to the B-weights has a definite bearing on this question.

When the B-weights for corresponding parameters resemble proportions, and noncorresponding parameter aggregation weights are negligible term-by-term, ordinary least squares parameter estimates of macro relations will have desirable properties, even when the micro parameters are different. In this exposition, my debt to Theil should be evident, even though it is my inclination to emphasize the economic substance of B-weights and hence the feasibility of using macro relations effectively to predict and study behavior.

Assume the following conditions hold:

1. Corresponding $B_{i\ell}$ weights are proportions i.e.

$$B_{i\ell} = \frac{x_{i\ell}}{X_\ell} = P_{i\ell}$$

2. Noncorresponding B weights are zero.

Then $\beta_\ell = \sum_{i=1}^{N} P_{i\ell}\beta_{i\ell}$

The macro predictions will then be $\hat{Y} = \sum_{\ell=1}^{K} \beta_\ell X_\ell$

$$\hat{Y} = \left(\frac{x_{11}}{X_1}\beta_{11} + \dots + \frac{x_{N1}}{X_1}\beta_{N1}\right) X_1 + \dots + \left(\frac{x_{1K}}{X_K}\beta_{1K} + \dots + \frac{x_{NK}}{X_K}\beta_{NK}\right) X_K$$

$$\hat{Y} = \sum_{i=1}^{N}\beta_{i1}x_{i1} + \dots + \sum_{i=1}^{N}\beta_{iK}x_{iK}$$

In these circumstances the micro predictions and macro predictions are the same. As Theil characterizes this sort of outcome, there is no contradiction between the micro relations and the macro relation. The forecasting benefits from relying on macro equations are evident, even when these conditions hold only approximately. Much interest thus attaches on the stability of shares through time for members of economic population. The law of proportional growth analyzed for instance by Simon and Bonini [17] suggests that stability in shares is not implausible among economic populations. The matter is fundamentally an empirical one that has been examined here for several subsets of manufacturing data.

Furthermore, the optimal properties of least squares estimation apply to the auxillary equation parameter estimates as an estimator of proportions under the conditions stated above. When shares are changing, ordinary least squares provides an efficient estimate of the average proportion over the sample period.

Efficient predictions can be obtained when shares are stable and parameters differ or when parameters are similar and shares are not. Often one, the other or both sets of conditions will hold - this explains much of the modest success that highly aggregative macro models have had.

Many failures have their origins in the failure of one or both conditions. The ideal situation, of course, is one where underlying behavior is relatively homogeneous so that compositional aspects of the data universe become secondary. But macro model building strategy can and should proceed on several fronts simultaneously, relying on aggregates (and hence share stability) more heavily in the short term while searching for significant behavioral homogeneities in the longer run. This observation actually characterizes much econometric model building activity of the past twenty years, beginning with the simplest three to nine equation Keynesian models. Research activity then expanded in several directions to disaggregate behavior equations that obviously contained disparate elements. Thus aggregate investment was separated into inventory, plant, equipment and residential construction. Total consumption has by now come to be divided into at least three components - durables, nondurables and services - in an effort to isolate significant differences in underlying behavior. Differential industry behavior has led to the proliferation of economic sectors. When the disaggregated data can be obtained, it becomes possible to weigh aggregation gains arising from grouping similar behavior units against the losses from subsuming too much disparate behavior in one relationship, and the perils or benefits inherent in shifting or stable shares and, more generally, aggregation weights.

Material in Section 5 is fragmentary, The empirical propositions held up well in numerous situations, poorly in others. While clear advantages accrue from proper aggregation, one noticable instance that failed especially badly was Retail Trade. The main reason seemed to be extreme collinearity. In this case no aggregation gain occured: in fact, if the calculations are to be believed, there were actual aggregation losses. In instances where collinearity is pervasive, we may speculate that the only possibility for successful estimation is to rely more heavily on disaggregated data. This is neither necessary not sufficient to assure success. It is a conjecture based partly on this study and other econometric work with micro data.

Another reason that micro data is desirable is fundamental to the acquisition of a solid basis for using macro equations, namely, the understanding how much heterogeneity exists, where the main tool would be some form of covariance analysis, and in what catagories it is most prevalent. Thus sensible research strategy requires simultaneous exploration of micro and macro data; micro data to test hypotheses and explore homogeneity, macro data for statistical efficiency and as a way to hold expenses down to tolerable levels.

Intensive analysis of aggregation was restricted to manufacturing, using some concocted two digit industries from COMPUSTAT for which all possible parameters were calculated that bear on the theory and "real" four digit cencus data using the sum of squared shares , or H index as a direct indicator of the sort of aggregation gains that can be expected from such industries in future investigations. With the tools developed in Section 6 for translating concentration indexes into H indexes, personal income distribution data should be explored. One may speculate that H indexes are smaller for personal income - income is more evenly distributed than are firm sizes - so that potential aggregation gain is even greater in the study of consumption.

It should now be more possible to explore wider areas of economic behavior in an operational framework that will enable econometricians more readily to adopt a research strategy by choosing aggregation levels on the basis of systematic information rather than hunch or random availability.

7. Related Literature and Acknowledgements

Zellner [22] was first to point out that the random coefficient model provided a natural approach to the problem of consistent aggregation. He demonstrated that least squares estimation in the random coefficient case, where data had been aggregated, was unbiased.

Theil [20] has postulated a random coefficient model with identical properties to those we have adopted. In the regression equation:

$$\bar{y}_t = \frac{\sum_{i=1}^{N} \beta_{i1t} x_{i1t}}{\sum_{i=1}^{N} x_{i1t}} \bar{x}_{1t} + \ldots + \frac{\sum_{i=1}^{N} \beta_{iKt} x_{iKt}}{\sum_{i=1}^{N} x_{iKt}} \bar{x}_{Kt} , \tag{A}$$

the typical aggregate coefficient is a function of time and sample size:

$$\beta_{\ell t} = \left(\frac{\sum_{i=1}^{N} \beta_{i\ell t} x_{i\ell t}}{\sum_{i=1}^{N} x_{i1t}} \right) . \tag{B}$$

Then the covariance of macro coefficient β_ℓ with β_h can be written as follows.

$$\frac{\sigma_{h\ell i}}{N} \left[1 + \frac{\frac{1}{N} \sum_{i=1}^{N} (x_{hit} - \bar{x}_{ht})(x_{\ell it} - \bar{x}_{\ell t})}{\bar{x}_{ht} \bar{x}_{\ell t}} \right] . \tag{C}$$

where $\sigma_{h\ell i}$ is the micro covariance among micro parameters $\beta_{\ell i}$ and β_{hi}. The variance of β_ℓ can thus be written:

$$[\sigma^2_{\beta i\ell t}(1 + c^2_{\ell t})]/N \tag{D}$$

where $c^2_{\ell t}$ is the coefficient of variation of $x_{\ell i}$ during the t^{th} period.

Theil concludes that as the numbers in the aggregate N increase, the variance of the estimated parameters will tend to zero. His result is the same as mine, but there are differences in approach and some possible contradictions that arise. First, Theil's macro coefficient β_{ℓ}, is not an estimated coefficient in the standard sense: it cannot be estimated in the absence of the micro parameters themselves. This may be seen from the fact that the parameters are strictly functions of time (involving both $\beta_{i\ell t}$ and $x_{i\ell t}$) in ways that the more conventional macro parameters I have postulated are not. Second, I have shown that, on admittedly extreme assumptions, it is possible for the variance of the macro parameters not to diminish as N increases. The same is possible in Theil's presentation of the problem, but the precise conditions relating to coefficients of variation for the explanatory variables do not seem to be related to my requirements. This possible contradiction is not readily explicable.

An advantage of my treatment, in addition to its being directly related to standard least squares estimation, is that the macro parameters variance is closely though approximately related to important---and often available---information on size distributions of firms, individuals, or incomes, for instance. The relative stability of these distributions and their properties provide genuine insight into when, and how much, aggregation gain can arise from enlarging the population aggregate. Coefficients of variation do not have the same intuitive significance, are less readily available, and we know less of their stability over time. It is reassuring, however, that two rather different approaches using similar assumptions, do reach the same conclusions.

Charles Revier with some help from Dan Luria and Joe Steuart provided admirable research assistance, relying heavily on M.I.T. computational facilities. J. Philip Cooper made useful comments. Support by the National Science Foundation is gratefully acknowledged.

My colleague Gordon Kaufman supplied generous assistance. He devised the lemma in a more ingenious form than I had contemplated as well as helping to set up and prove the theorems in their full generality.

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APPENDIX

APPENDIX

Note: The computer used for calculating summary statistics in this study did not round off, but simply truncated all figures after the ones printed. Therefore, the means and other summary statistics reported here cannot be duplicated exactly by using the numbers in these tables or in Table 3.

TABLE 1-3

INVESTMENT REGRESSIONS -- MICRO AND MACRO

STEEL

ID		SALES	ASSETS	INTERCEPT	R-SQUARED
1	COEFF	0.1317	-0.0690	-5.4816	0.4566
	T-STAT	(3.2808)	(-1.8303)	(-0.9389)	
2	COEFF	-0.0206	0.1215	4.8073	0.6910
	T-STAT	(-0.2900)	(2.2871)	(0.1740)	
3	COEFF	0.1087	0.0599	-181.1280	0.6266
	T-STAT	(1.6883)	(1.7389)	(-1.9951)	
4	COEFF	0.1117	-0.0548	-1.1813	0.5265
	T-STAT	(4.1446)	(-2.3331)	(-0.2496)	
5	COEFF	0.0766	-0.0212	0.0574	0.2759
	T-STAT	(1.1432)	(-0.2194)	(0.0275)	
6	COEFF	-0.0108	0.1070	-0.2519	0.3271
	T-STAT	(-0.1466)	(1.4722)	(-0.1402)	
7	COEFF	0.0911	0.0082	-3.9973	0.5811
	T-STAT	(2.5798)	(0.2136)	(-1.7710)	
8	COEFF	0.0340	0.0375	-1.6319	0.2313
	T-STAT	(0.6577)	(0.7374)	(-0.2253)	
9	COEFF	0.0759	0.0474	2.2678	0.1952
	T-STAT	(0.4907)	(0.6981)	(0.1855)	
10	COEFF	0.2293	-0.0411	-67.4353	0.6314
	T-STAT	(2.8147)	(-0.9162)	(-2.1132)	
11	COEFF	0.0749	-0.0225	2.9551	0.4888
	T-STAT	(2.4894)	(-0.7645)	(1.3847)	
12	COEFF	0.1165	-0.0017	-4.6473	0.2422
	T-STAT	(1.0419)	(-0.0290)	(-0.1132)	

13	COEFF	0.1404	-0.0537	-5.8163	0.4680
	T-STAT	(2.6111)	(-1.1026)	(-1.7350)	
14	COEFF	0.0927	-0.0083	-2.7554	0.0925
	T-STAT	(0.9253)	(-0.0982)	(-0.3860)	
15	COEFF	-0.4028	0.3233	32.3545	0.3531
	T-STAT	(-2.8965)	(2.9162)	(3.4612)	
16	COEFF	0.0987	0.0069	-2.3715	0.3348
	T-STAT	(1.2381)	(0.1564)	(-0.0760)	
17	COEFF	0.0960	-0.0394	2.7376	0.0756
	T-STAT	(1.0945)	(-0.9256)	(0.2577)	
18	COEFF	0.1378	0.0185	-85.0888	0.4326
	T-STAT	(2.2477)	(0.6049)	(-1.6134)	
19	COEFF	0.0635	0.0863	-6.9735	0.3833
	T-STAT	(1.0876)	(2.0804)	(-0.9234)	
20	COEFF	0.2704	-0.0222	-27.9812	0.8279
	T-STAT	(3.3724)	(-0.4976)	(-2.5837)	
21	COEFF	0.0766	0.0126	-11.7819	0.2381
	T-STAT	(1.3653)	(0.6609)	(-0.0673)	
22	COEFF	0.1560	0.0087	-15.5392	0.1353
	T-STAT	(0.8927)	(0.1300)	(-0.5566)	
23	COEFF	0.1849	-0.0024	-46.2750	0.4039
	T-STAT	(2.0708)	(-0.0613)	(-1.3240)	
INDUSTRY	COEFF	0.1340	0.0085	-685.8160	0.6998
	T-STAT	(2.9951)	(0.3966)	(-1.9359)	

INVESTMENT REGRESSIONS -- MICRO AND MACRO

RETAIL

ID		SALES	ASSETS	INTERCEPT	R-SQUARED
1	COEFF	-0.0062	0.0588	11.7245	0.6812
	T-STAT	(-0.9812)	(3.7786)	(2.7701)	
2	COEFF	0.0551	-0.1408	-1.1654	0.2734
	T-STAT	(1.1490)	(-0.5382)	(-0.5341)	
3	COEFF	0.0124	-0.0499	0.6085	0.3695
	T-STAT	(2.6231)	(-1.3079)	(3.0712)	
4	COEFF	0.0728	-0.1954	0.2498	0.3133
	T-STAT	(1.6326)	(-1.2545)	(0.0795)	
5	COEFF	0.0205	-0.1198	0.6262	0.8207
	T-STAT	(4.5792)	(-0.9671)	(0.7145)	
6	COEFF	0.0907	-0.1120	-11.7980	0.4470
	T-STAT	(1.2935)	(-0.5430)	(-1.0287)	
7	COEFF	0.0735	-0.1055	-1.3278	0.1956
	T-STAT	(1.8925)	(-1.4518)	(-0.2582)	
8	COEFF	0.0196	0.0624	-3.6822	0.6355
	T-STAT	(0.7223)	(1.2201)	(-0.4590)	
9	COEFF	0.0486	-0.0558	-1.0839	0.1634
	T-STAT	(0.9282)	(-0.5291)	(-0.2519)	
10	COEFF	0.0666	-0.2505	-46.1061	0.9139
	T-STAT	(5.5175)	(-2.9012)	(-6.0546)	
11	COEFF	0.1005	-0.0673	-90.9502	0.7686
	T-STAT	(2.3867)	(-0.5355)	(-2.6002)	
12	COEFF	-0.0021	0.2358	-46.5286	0.9030
	T-STAT	(-0.0775)	(1.7442)	(-1.3316)	

13	COEFF	0.0151	-0.0182	11.3828	0.5293
	T-STAT	(3.2805)	(-0.8826)	(3.3514)	
14	COEFF	0.0248	-0.0491	2.5660	0.4406
	T-STAT	(1.4083)	(-0.7148)	(1.8302)	
15	COEFF	0.0115	-0.0405	5.3702	0.0503
	T-STAT	(0.3240)	(-0.4625)	(2.2746)	
16	COEFF	0.0179	-0.0607	0.9135	0.0276
	T-STAT	(0.5212)	(-0.6238)	(1.0560)	
17	COEFF	-0.0390	0.2638	0.9621	0.1393
	T-STAT	(-1.3456)	(1.5527)	(0.1762)	
18	COEFF	0.0198	-0.0301	-0.3819	0.6367
	T-STAT	(0.8181)	(-0.1428)	(-0.3515)	
19	COEFF	0.0290	-0.0415	24.0891	0.2789
	T-STAT	(1.3546)	(-0.7538)	(4.0970)	
INDUSTRY	COEFF	0.0753	-0.1630	-247.5520	0.9318
	T-STAT	(2.0402)	(-1.1072)	(-2.4755)	

INVESTMENT REGRESSIONS -- MICRO AND MACRO

PETROLEUM

ID		SALES	ASSETS	INTERCEPT	R-SQUARED
1	COEFF	0.0958	-0.0023	-1.7590	0.0463
	T-STAT	(2.1676)	(-0.0365)	(-0.2581)	
2	COEFF	0.3063	-0.0131	-81.4449	0.8649
	T-STAT	(6.2819)	(-0.5562)	(-4.1547)	
3	COEFF	0.1742	0.0123	-59.0341	0.5622
	T-STAT	(1.3800)	(0.2197)	(-0.9515)	
4	COEFF	0.2131	-0.0384	10.6884	0.9368
	T-STAT	(4.3290)	(-0.9160)	(0.8553)	
5	COEFF	0.3697	-0.2568	-41.8306	0.7652
	T-STAT	(6.8918)	(-6.2456)	(-2.2978)	
6	COEFF	0.1451	-0.0022	3.8648	0.9356
	T-STAT	(2.8003)	(-0.0408)	(1.9314)	
7	COEFF	0.4639	-0.1574	1.2597	0.7767
	T-STAT	(3.1732)	(-1.7427)	(0.0898)	
8	COEFF	1.4227	-0.6571	-212.1310	0.5882
	T-STAT	(3.6813)	(-3.2623)	(-2.3418)	
9	COEFF	0.1733	-0.1481	-1.2002	0.4855
	T-STAT	(3.1688)	(-1.7168)	(-0.6748)	
10	COEFF	0.5504	-0.1978	-201.3100	0.9385
	T-STAT	(5.9659)	(-3.6424)	(-5.0109)	
11	COEFF	0.0289	0.0464	11.7604	0.9121
	T-STAT	(3.0201)	(2.2471)	(4.0054)	
12	COEFF	0.0206	0.0373	70.7422	0.3976
	T-STAT	(0.2324)	(1.0402)	(1.3008)	

13	COEFF	0.2969	-0.0403	-177.3450	0.8438
	T-STAT	(3.3448)	(-1.1115)	(-2.4091)	
14	COEFF	0.2440	-0.1314	0.0553	0.6241
	T-STAT	(3.7996)	(-2.5115)	(0.0056)	
15	COEFF	0.7600	-0.2040	-308.6050	0.7384
	T-STAT	(3.5224)	(-1.9980)	(-3.6013)	
16	COEFF	0.1220	0.0062	13.3638	0.7592
	T-STAT	(1.5713)	(0.1337)	(1.5104)	
17	COEFF	0.1404	0.0270	6.1472	0.9632
	T-STAT	(5.8130)	(1.7128)	(0.8569)	
18	COEFF	0.2879	-0.0334	-185.3940	0.8883
	T-STAT	(3.2555)	(-0.8529)	(-2.1973)	
19	COEFF	0.1118	-0.0314	70.9758	0.8237
	T-STAT	(1.5771)	(-0.4499)	(2.0084)	
20	COEFF	0.2624	-0.0316	-37.2321	0.9267
	T-STAT	(3.2041)	(-0.7111)	(-1.3193)	
21	COEFF	0.2538	-0.1235	-102.8830	0.8936
	T-STAT	(5.2231)	(-3.2499)	(-1.1401)	
22	COEFF	0.1719	0.0021	-46.5913	0.9590
	T-STAT	(1.7752)	(0.0384)	(-0.8260)	
INDUSTRY	COEFF	0.3954	-0.1615	-1451.1400	0.9794
	T-STAT	(9.6606)	(-6.0964)	(-5.6517)	

INVESTMENT REGRESSIONS -- MICRO AND MACRO

MACHINERY

ID		SALES	ASSETS	INTERCEPT	R-SQUARED
1	COEFF	0.0630	0.0038	-7.1030	0.6772
	T-STAT	(1.9594)	(0.0669)	(-1.3643)	
2	COEFF	0.0723	-0.1376	-0.8131	0.6035
	T-STAT	(2.6446)	(-1.1652)	(-0.2711)	
3	COEFF	0.0435	-0.1638	0.2335	0.6222
	T-STAT	(4.0790)	(-2.0325)	(0.1879)	
4	COEFF	0.0554	-0.0127	-11.2305	0.6955
	T-STAT	(4.2630)	(-0.5705)	(-2.2328)	
5	COEFF	0.0617	0.0513	-6.2896	0.7885
	T-STAT	(4.6827)	(0.7918)	(-1.7356)	
6	COEFF	0.0441	-0.0007	-11.3372	0.8656
	T-STAT	(3.8355)	(-0.0255)	(-1.3799)	
7	COEFF	0.1284	-0.1453	-16.8985	0.6259
	T-STAT	(2.3639)	(-1.1426)	(-1.6994)	
8	COEFF	0.0188	0.0240	0.0609	0.6568
	T-STAT	(1.2776)	(0.2123)	(0.0888)	
9	COEFF	0.0051	-0.0368	4.4136	0.0166
	T-STAT	(0.1506)	(-0.5038)	(1.3004)	
10	COEFF	0.0900	0.0424	-25.9829	0.6745
	T-STAT	(1.3978)	(0.3381)	(-1.3682)	
11	COEFF	0.0823	-0.1658	-1.7387	0.7589
	T-STAT	(3.3605)	(-1.8810)	(-1.2249)	
12	COEFF	0.0580	-0.0534	-1.8217	0.5465
	T-STAT	(3.7560)	(-1.4757)	(-0.6652)	

13	COEFF	0.0426	-0.0390	0.7480	0.6886
	T-STAT	(2.4685)	(-0.7398)	(1.3277)	
14	COEFF	0.0778	-0.1554	4.0418	0.7220
	T-STAT	(5.7992)	(-3.2500)	(1.7558)	
15	COEFF	0.0817	-0.0490	10.0890	0.8450
	T-STAT	(3.9382)	(-0.9457)	(3.4894)	
16	COEFF	0.0228	-0.0169	3.0035	0.3715
	T-STAT	(2.3294)	(-0.7188)	(3.1716)	
17	COEFF	0.0491	0.0805	-0.5547	0.3988
	T-STAT	(2.6593)	(1.3952)	(-1.1861)	
18	COEFF	0.0230	0.1176	-1.0333	0.6999
	T-STAT	(2.0300)	(2.6673)	(-3.1070)	
19	COEFF	0.1272	-0.0420	-1.6407	0.6840
	T-STAT	(1.6675)	(-0.1565)	(-2.9140)	
20	COEFF	0.2577	-0.2585	-8.6626	0.7788
	T-STAT	(3.0104)	(-0.9515)	(-3.8769)	
21	COEFF	0.0675	-0.1237	3.3034	0.0788
	T-STAT	(1.1315)	(-0.8444)	(0.6885)	
22	COEFF	0.0844	-0.0609	0.4794	0.6489
	T-STAT	(3.5039)	(-1.1779)	(0.5347)	
23	COEFF	0.0337	-0.0030	9.2993	0.2746
	T-STAT	(0.7310)	(-0.0404)	(2.0519)	
24	COEFF	0.1293	-0.1912	-0.6429	0.8927
	T-STAT	(5.1278)	(-2.6385)	(-1.3623)	
25	COEFF	0.0381	0.0586	-1.0680	0.7644
	T-STAT	(3.2821)	(1.5636)	(-0.5603)	
26	COEFF	0.0094	0.0587	-0.3802	0.7717
	T-STAT	(1.0840)	(1.9386)	(-1.2870)	
27	COEFF	0.0171	0.0456	-0.2480	0.8205

	T-STAT	(1.1069)	(1.0491)	(-0.4326)	
28	COEFF	0.0679	0.0532	0.2159	0.7576
	T-STAT	(1.7925)	(1.1996)	(0.0816)	
29	COEFF	0.0912	-0.0776	-2.8796	0.8513
	T-STAT	(6.4266)	(-2.3869)	(-3.6315)	
30	COEFF	0.0108	0.0915	-1.3466	0.8700
	T-STAT	(0.5036)	(2.2029)	(-1.0827)	
31	COEFF	0.0552	-0.0869	-2.9087	0.7874
	T-STAT	(3.9081)	(-1.6535)	(-2.5454)	
32	COEFF	0.1136	-0.1868	-0.3619	0.5688
	T-STAT	(4.3196)	(-3.4695)	(-0.2197)	
33	COEFF	0.0885	-0.0386	-5.0236	0.9045
	T-STAT	(5.2845)	(-0.9428)	(-6.5267)	
34	COEFF	0.1677	-0.3184	-3.0699	0.4258
	T-STAT	(2.6300)	(-2.0318)	(-1.6839)	
35	COEFF	0.0556	-0.1041	1.6368	0.2242
	T-STAT	(1.9595)	(-1.4529)	(1.4028)	
36	COEFF	0.1112	-0.2069	-0.8829	0.7944
	T-STAT	(4.4831)	(-2.1833)	(-1.1257)	
37	COEFF	0.0882	-0.1809	0.3511	0.9234
	T-STAT	(7.9518)	(-4.9747)	(1.4292)	
38	COEFF	0.0296	0.1240	-3.9538	0.9691
	T-STAT	(2.1742)	(2.9552)	(-2.6957)	
39	COEFF	0.0216	-0.0103	1.1856	0.0114
	T-STAT	(0.3295)	(-0.1066)	(0.4840)	
40	COEFF	0.3319	-0.2510	-0.2337	0.8248
	T-STAT	(6.0058)	(-3.1702)	(-1.5868)	
41	COEFF	-0.0072	0.0914	0.4217	0.5598
	T-STAT	(-0.5793)	(3.0547)	(0.4420)	

INDUSTRY	COEFF	0.1065	-0.1174	-153.1440	0.9627
	T-STAT	(8.6146)	(-4.0220)	(-5.3324)	

TABLE 2-A

ESTIMATED COEFFICIENTS OF AUXILIARY EQUATIONS
AND FIRM PROPORTIONS IN INDUSTRY TOTALS

RANKED BY PROPORTIONS IN INDUSTRY SALES

STEEL

IC		PROP	REGRESSION B-WEIGHTS -- SALES CORRES	NONCORRES	INTERCEPT	R-SQUARED	PROP	REGRESSION B-WEIGHTS -- ASSETS NONCURRES	CORRES	INTERCEPT	R-SQUARED
21	COEFF	0.2895	0.3154	-0.0677	648.3540	0.9790	0.3755	0.0014	0.3100	995.1710	0.9905
	T-STAT		(22.1526)	(-9.8404)	(5.7526)			(0.0508)	(22.3366)	(4.3759)	
3	COEFF	0.1639	0.1628	-0.0092	146.6700	0.9451	0.1445	0.0066	0.1332	72.0205	0.9905
	T-STAT		(10.0290)	(-1.1785)	(1.1410)			(0.5444)	(21.9378)	(0.7236)	
18	COEFF	0.0858	0.0999	-0.0170	64.8424	0.9741	0.0715	-0.0060	0.0735	47.7461	0.9950
	T-STAT		(18.3577)	(-6.4970)	(1.5041)			(-1.2821)	(32.0233)	(1.2695)	
2	COEFF	0.0632	0.0333	0.0340	-110.1070	0.9535	0.0470	-0.0070	0.0687	-243.4920	0.9869
	T-STAT		(3.2930)	(6.9660)	(-1.3756)			(-0.9761)	(19.8223)	(-4.2841)	
12	COEFF	0.0570	0.0508	0.0100	-68.2830	0.9681	0.0622	0.0008	0.0616	-3.3303	0.9910
	T-STAT		(8.8382)	(3.6241)	(-1.5004)			(0.1524)	(22.8390)	(-0.0753)	
16	COEFF	0.0561	0.0403	0.0164	-40.4324	0.8600	0.0576	-0.0057	0.0691	-97.0823	0.9924
	T-STAT		(3.0609)	(2.5839)	(-0.3872)			(-1.0307)	(25.8227)	(-2.2143)	
10	COEFF	0.0551	0.0429	0.0165	-84.5995	0.9902	0.0470	-0.0109	0.0692	-195.9750	0.9874
	T-STAT		(12.7627)	(10.1591)	(-3.1748)			(-1.5746)	(20.6366)	(-3.5665)	
23	COEFF	0.0458	0.0522	-0.0020	-52.4338	0.9812	0.0481	-0.0051	0.0546	-32.3005	0.9969
	T-STAT		(16.9086)	(-1.3586)	(-2.1442)			(-1.8648)	(40.7124)	(-1.4682)	
20	COEFF	0.0223	0.0204	0.0116	-142.5580	0.9461	0.0234	0.0061	0.0345	-253.0640	0.9411
	T-STAT		(4.3862)	(5.1610)	(-3.8547)			(0.6971)	(8.1393)	(-3.6428)	
1	COEFF	0.0196	0.0235	0.0017	-72.4846	0.8802	0.0109	0.0015	0.0105	-16.0452	0.9716
	T-STAT		(5.2430)	(0.7910)	(-2.0397)			(0.8553)	(12.0623)	(-1.1241)	
4	COEFF	0.0182	0.0236	-0.0075	43.5462	0.3813	0.0152	0.0051	0.0062	73.3726	0.4790
	T-STAT		(2.9910)	(-1.9629)	(0.6950)			(0.6179)	(1.5489)	(1.1124)	

22	COEFF	0.0179	0.0163	-0.0014	40.2724	0.9216	*	0.0190	0.0044	0.0159	-6.7624	0.9597
	T-STAT		(8.7917)	(-1.6155)	(2.7349)		*		(1.2777)	(9.5745)	(-0.3217)	
8	COEFF	0.0176	0.0236	-0.0012	-56.8823	0.9450	*	0.0129	0.0018	0.0100	19.7959	0.9944
	T-STAT		(9.9314)	(-1.0631)	(-3.0230)		*		(2.3899)	(27.0809)	(3.2402)	
15	COEFF	0.0120	0.0093	0.0075	-71.8230	0.8332	*	0.0101	0.0016	0.0147	-90.0783	0.9333
	T-STAT		(1.8940)	(3.1830)	(-1.8452)		*		(0.4486)	(8.2160)	(-3.0598)	
17	COEFF	0.0117	0.0156	-0.0036	3.4720	0.9399	*	0.0110	0.0034	0.0087	-7.4502	0.9466
	T-STAT		(13.2861)	(-6.4373)	(0.3722)		*		(1.5022)	(7.9067)	(-0.4116)	
19	COEFF	0.0114	0.0178	-0.0049	-10.0329	0.7043	*	0.0066	0.0007	0.0081	-33.3905	0.9245
	T-STAT		(5.5761)	(-3.2251)	(-0.3962)		*		(0.3279)	(7.3699)	(-1.8537)	
11	COEFF	0.0095	0.0164	0.0043	-150.8160	0.6968	*	0.0092	0.0054	0.0108	-94.8552	0.8154
	T-STAT		(2.2214)	(1.2197)	(-2.5680)		*		(0.9166)	(3.7942)	(-2.0267)	
9	COEFF	0.0093	0.0081	0.0027	-23.7191	0.8277	*	0.0102	0.0006	0.0156	-90.5844	0.9764
	T-STAT		(2.9222)	(2.0654)	(-1.0792)		*		(0.2778)	(13.8418)	(-4.8982)	
13	COEFF	0.0080	0.0062	0.0022	-2.5993	0.9449	*	0.0047	-0.0016	0.0063	-3.0659	0.9912
	T-STAT		(5.4151)	(4.0108)	(-0.2834)		*		(-3.2809)	(25.9749)	(-0.7619)	
14	COEFF	0.0077	0.0079	0.0006	-13.3365	0.8619	*	0.0041	-0.0014	0.0061	-12.1634	0.9662
	T-STAT		(4.7196)	(0.8564)	(-0.9992)		*		(-1.4476)	(12.8670)	(-1.5611)	
7	COEFF	0.0076	0.0077	0.0011	-19.9925	0.8042	*	0.0033	-0.0015	0.0053	-11.3009	0.9584
	T-STAT		(3.4619)	(1.1025)	(-1.1231)		*		(-1.6167)	(11.8049)	(-1.5143)	
5	COEFF	0.0055	0.0036	0.0046	-42.5847	0.9046	*	0.0028	-0.0007	0.0047	-18.9202	0.9726
	T-STAT		(1.8891)	(5.0646)	(-2.8088)		*		(-1.1034)	(13.9297)	(-3.3627)	
6	COEFF	0.0035	0.0012	0.0009	15.5478	0.7768	*	0.0020	0.0001	0.0016	3.7847	0.9910
	T-STAT		(1.5805)	(2.6570)	(2.5676)		*		(1.2110)	(21.8955)	(3.0621)	

ESTIMATED COEFFICIENTS OF AUXILIARY EQUATIONS AND FIRM PROPORTIONS IN INDUSTRY TOTALS

RANKED BY PROPORTIONS IN INDUSTRY SALES

RETAIL

ID		PROP	REGRESSION B-WEIGHTS -- SALES CORRES	NONCORRES	INTERCEPT	R-SQUARED	*	PROP	REGRESSION B-WEIGHTS -- ASSETS NONCORRES	CORRES	INTERCEPT	R-SQUARED
12	COEFF	0.3285	0.2424	0.3954	115.5260	0.9931	*	0.2401	0.0479	0.0813	-212.5970	0.9313
	T-STAT		(2.1632)	(0.8849)	(0.3808)		*		(1.9108)	(0.8126)	(-3.1287)	
10	COEFF	0.1210	0.0805	0.1629	104.9240	0.9944	*	0.0537	-0.0214	0.1519	22.5617	0.9768
	T-STAT		(2.2462)	(1.1397)	(1.0811)		*		(-2.1004)	(3.7400)	(0.8177)	
11	COEFF	0.1058	0.3068	-0.9417	-210.7180	0.9446	*	0.0671	0.0453	-0.0843	-194.6500	0.9351
	T-STAT		(4.4229)	(-3.4059)	(-1.1224)		*		(1.8014)	(-0.8415)	(-2.8589)	
19	COEFF	0.0787	0.0606	0.0549	87.1437	0.9945	*	0.1413	-0.0426	0.2812	192.2680	0.9339
	T-STAT		(2.8016)	(0.6371)	(1.4868)		*		(-1.4486)	(2.3952)	(2.4116)	
1	COEFF	0.0549	-0.0121	0.2315	260.1940	0.9863	*	0.0510	0.0228	-0.0039	-152.7480	0.7697
	T-STAT		(-0.5717)	(2.7263)	(4.5117)		*		(0.4648)	(-0.0209)	(-1.1966)	
8	COEFF	0.0500	0.0147	0.1177	145.4450	0.9641	*	0.0854	0.0157	0.0296	-60.2999	0.9438
	T-STAT		(0.4375)	(0.8787)	(1.5981)		*		(0.7068)	(0.3336)	(-0.9982)	
6	COEFF	0.0383	-0.0037	0.1457	161.0260	0.9806	*	0.0465	0.0130	-0.0079	-28.0529	0.9669
	T-STAT		(-0.2071)	(2.0108)	(3.2711)		*		(1.6178)	(-0.2464)	(-1.2837)	
13	COEFF	0.0379	0.1287	-0.3046	-371.0520	0.8823	*	0.0910	-0.0322	0.1748	198.3520	0.9027
	T-STAT		(1.6966)	(-1.0072)	(-1.8066)		*		(-2.0988)	(2.8551)	(4.7707)	
4	COEFF	0.0327	0.0195	0.0257	94.7634	0.9806	*	0.0440	-0.0015	0.0355	42.1546	0.9612
	T-STAT		(1.3583)	(0.4482)	(2.4330)		*		(-0.2660)	(1.5282)	(2.6699)	
18	COEFF	0.0225	-0.0078	0.1359	45.2354	0.9818	*	0.0120	0.0020	0.0038	-5.7158	0.9946
	T-STAT		(-0.5614)	(2.4243)	(1.1879)		*		(2.3600)	(1.1109)	(-2.4077)	
14	COEFF	0.0216	0.0487	0.0153	-358.3020	0.8705	*	0.0320	0.0018	0.0450	-53.3749	0.8247
	T-STAT		(0.6098)	(0.0482)	(-1.6579)		*		(0.0763)	(0.4740)	(-0.8271)	

7	COEFF	0.0200	0.0346	-0.1050	68.4395	0.9189	*	0.0378	-0.0062	0.0427	66.2990	0.9112
	T-STAT		(3.4632)	(-2.6322)	(2.5265)		*		(-1.1080)	(1.9154)	(4.3737)	
17	COEFF	0.0195	-0.0138	0.1299	95.2112	0.9489	*	0.0253	-0.0128	0.0622	70.6272	0.7713
	T-STAT		(-0.8012)	(1.8920)	(2.0416)		*		(-2.0661)	(2.5075)	(4.1868)	
15	COEFF	0.0184	-0.0102	0.0922	127.7430	0.9444	*	0.0274	-0.0120	0.0682	50.3214	0.9260
	T-STAT		(-0.8308)	(1.8736)	(3.8222)		*		(-2.0881)	(2.9709)	(3.2248)	
9	COEFF	0.0134	0.0021	0.0279	69.2315	0.9882	*	0.0175	-0.0085	0.0512	23.9821	0.9133
	T-STAT		(0.5465)	(1.7829)	(6.5059)		*		(-1.6092)	(2.4226)	(1.6698)	
5	COEFF	0.0131	0.1099	-0.2927	-471.1960	0.9381	*	0.0050	-0.0028	0.0166	6.1022	0.9582
	T-STAT		(2.9366)	(-1.9615)	(-4.6505)		*		(-2.5501)	(3.7480)	(2.0271)	
2	COEFF	0.0127	-0.0190	0.1316	73.7615	0.8956	*	0.0105	-0.0043	0.0275	12.8428	0.9029
	T-STAT		(-1.0058)	(1.7441)	(1.4392)		*		(-1.2958)	(2.0621)	(1.4177)	
16	COEFF	0.0059	-0.0006	0.0110	52.0990	0.9366	*	0.0085	-0.0015	0.0092	18.5807	0.8660
	T-STAT		(-0.2734)	(1.2481)	(8.6710)		*		(-1.4029)	(2.0383)	(6.0199)	
3	COEFF	0.0043	0.0185	-0.0341	-89.4244	0.9612	*	0.0030	-0.0025	0.0147	3.3534	0.8042
	T-STAT		(2.3378)	(-1.0777)	(-4.1582)		*		(-1.1555)	(1.6618)	(0.5571)	

ESTIMATED COEFFICIENTS OF AUXILIARY EQUATIONS
AND FIRM PROPORTIONS IN INDUSTRY TOTALS

RANKED BY PROPORTIONS IN INDUSTRY SALES

PETROLEUM

ID		PROP	REGRESSION B-WEIGHTS -- SALES CORRES	NONCORRES	INTERCEPT	R-SQUARED	*	PROP	REGRESSION B-WEIGHTS -- ASSETS NONCORRES	CORRES	INTERCEPT	R-SQUARED
21	COEFF	0.2572	0.2004	0.0428	43.0803	0.9825	*	0.2157	-0.1410	0.3118	502.0910	0.9964
	T-STAT		(3.0496)	(1.0074)	(0.1044)		*		(-3.7010)	(12.6359)	(2.0997)	
19	COEFF	0.1026	0.0741	0.0332	-451.2650	0.9843	*	0.0832	0.0006	0.0824	26.8896	0.9960
	T-STAT		(2.5390)	(1.7553)	(-2.4620)		*		(0.0443)	(8.4800)	(0.2854)	
22	COEFF	0.0867	0.0328	0.0446	-140.3360	0.9878	*	0.0943	-0.0084	0.1203	-852.5170	0.9927
	T-STAT		(1.5723)	(3.3072)	(-1.0719)		*		(-0.2994)	(6.6155)	(-4.8375)	
18	COEFF	0.0824	0.0682	0.0049	243.7500	0.9642	*	0.0969	-0.0416	0.1388	-422.6300	0.9945
	T-STAT		(2.5200)	(0.2821)	(1.4355)		*		(-1.7462)	(8.9983)	(-2.8260)	
13	COEFF	0.0658	0.0612	-0.0154	748.2990	0.9947	*	0.0774	-0.0041	0.0623	780.0290	0.9945
	T-STAT		(12.0865)	(-4.7171)	(23.5419)		*		(-0.3261)	(7.5896)	(9.8005)	
10	COEFF	0.0590	0.0649	-0.0068	92.2395	0.9967	*	0.0622	0.0167	0.0491	15.6069	0.9981
	T-STAT		(11.2120)	(-1.8363)	(2.5365)		*		(2.2611)	(10.2641)	(0.3360)	
20	COEFF	0.0565	0.0511	0.0067	-107.0910	0.9812	*	0.0749	0.0053	0.0704	22.4780	0.9965
	T-STAT		(3.2471)	(0.6609)	(-1.0833)		*		(0.4355)	(8.8005)	(0.2898)	
12	COEFF	0.0370	0.0298	-0.0086	565.1450	0.8097	*	0.0384	-0.0013	0.0303	395.6630	0.9797
	T-STAT		(1.9832)	(-0.8857)	(5.9953)		*		(-0.1086)	(3.8572)	(5.1902)	
8	COEFF	0.0358	0.0266	0.0023	186.0670	0.9751	*	0.0433	0.0195	0.0249	147.6170	0.9755
	T-STAT		(2.9781)	(0.4068)	(3.3162)		*		(1.1485)	(2.2634)	(1.3803)	
3	COEFF	0.0334	0.0268	-0.0072	88.4500	0.9496	*	0.0346	-0.0005	0.0235	489.9320	0.9674
	T-STAT		(3.9767)	(-1.6618)	(11.5186)		*		(-0.0443)	(2.9833)	(6.4057)	
4	COEFF	0.0273	0.0635	-0.0126	-599.7780	0.9303	*	0.0277	0.0247	0.0182	-396.7650	0.9643
	T-STAT		(2.8299)	(-0.8711)	(-4.2609)		*		(1.3131)	(1.4991)	(-3.3615)	

5	CCEFF	0.0267	0.0258	-0.0046	214.4420	0.9222	*	0.0177	-0.0133	0.0244	144.5460	0.9397
	T-STAT		(2.5588)	(-0.7108)	(3.3833)		*		(-1.1491)	(3.2632)	(1.9871)	
15	CCEFF	0.0251	0.0303	-0.0104	253.7010	0.9833	*	0.0209	0.0143	0.0106	-34.0489	0.9882
	T-STAT		(8.6673)	(-4.6109)	(11.5638)		*		(2.3078)	(2.6602)	(-0.8748)	
2	CCEFF	0.0212	0.0590	-0.0291	1.9758	0.6395	*	0.0256	0.0358	0.0003	-105.8120	0.8778
	T-STAT		(3.0815)	(-2.3491)	(0.0164)		*		(1.4240)	(0.0221)	(-0.6707)	
17	CCEFF	0.0178	0.0785	-0.0315	-591.1550	0.8221	*	0.0264	0.0975	-0.0323	-659.9950	0.8585
	T-STAT		(2.9365)	(-1.8224)	(-3.5247)		*		(2.6698)	(-1.3677)	(-2.8795)	
14	CCEFF	0.0139	0.0184	-0.0050	61.9451	0.9791	*	0.0107	0.0022	0.0071	83.1984	0.9750
	T-STAT		(6.3333)	(-2.6823)	(3.3830)		*		(0.5647)	(2.8088)	(3.3827)	
7	CCEFF	0.0117	0.0113	0.0013	-41.1513	0.9572	*	0.0161	-0.0007	0.0147	81.1750	0.9934
	T-STAT		(2.1678)	(0.3875)	(-1.2518)		*		(-0.2272)	(6.8895)	(3.9118)	
16	CCEFF	0.0113	-0.0039	0.0113	17.8713	0.8607	*	0.0165	-0.0082	0.0203	110.3820	0.8977
	T-STAT		(-0.3891)	(1.7256)	(0.2803)		*		(-0.5666)	(2.1578)	(1.2103)	
1	CCEFF	0.0109	0.0390	-0.0163	-202.6880	0.9013	*	0.0056	0.0069	0.0022	-83.2213	0.9869
	T-STAT		(4.3918)	(-2.8374)	(-3.6294)		*		(3.1213)	(1.5831)	(-5.9550)	
11	CCEFF	0.0094	0.0403	-0.0066	-678.2240	0.8587	*	0.0062	-0.0056	0.0133	-127.4680	0.9839
	T-STAT		(1.7735)	(-0.4490)	(-4.7484)		*		(-1.5940)	(5.7916)	(-5.6900)	
6	CCEFF	0.0051	-0.0003	0.0075	-130.2810	0.9776	*	0.0038	0.0001	0.0066	-135.2250	0.9741
	T-STAT		(-0.1253)	(3.6957)	(-6.6158)		*		(0.0595)	(3.2541)	(-6.8448)	
9	CCEFF	0.0019	0.0013	-0.0002	24.9839	0.8756	*	0.0009	0.0010	-0.0002	18.0000	0.9279
	T-STAT		(1.9383)	(-0.5139)	(5.5765)		*		(2.9866)	(-1.0665)	(8.3281)	

// EJECT

ESTIMATED COEFFICIENTS OF AUXILIARY EQUATIONS
AND FIRM PROPORTIONS IN INDUSTRY TOTALS

RANKED BY PROPORTIONS IN INDUSTRY SALES

MACHINERY

ID		PROP	REGRESSION B-WEIGHTS -- SALES CORRES	NONCORRES	INTERCEPT	R-SQUARED	*	PROP	REGRESSION B-WEIGHTS -- ASSETS NONCORRES	CORRES	INTERCEPT	R-SQUARED
6	COEFF	0.1957	0.1325	0.0469	365.2880	0.9253	*	0.2231	-0.0290	0.2120	295.3790	0.9762
	T-STAT		(2.9099)	(0.4368)	(3.4540)		*		(-2.8450)	(8.7928)	(12.4525)	
10	COEFF	0.0940	0.1322	-0.0541	-146.3570	0.9784	*	0.1076	0.0090	0.1135	-104.7730	0.9937
	T-STAT		(7.6684)	(-1.3297)	(-3.6555)		*		(1.9111)	(10.1158)	(-9.4893)	
4	COEFF	0.0688	0.0783	-0.1086	246.8890	0.9485	*	0.0641	-0.0098	0.0713	64.9452	0.9639
	T-STAT		(8.9708)	(-5.2735)	(12.1783)		*		(-2.3087)	(7.1073)	(6.5745)	
7	COEFF	0.0573	0.0264	0.0908	-18.9797	0.9725	*	0.0479	0.0011	0.0627	-57.3346	0.9878
	T-STAT		(2.3141)	(3.3668)	(-0.7154)		*		(0.3666)	(8.1885)	(-7.6070)	
1	COEFF	0.0415	0.0472	-0.0306	44.5457	0.9306	*	0.0418	0.0064	0.0332	-28.3921	0.9950
	T-STAT		(4.7076)	(-1.2923)	(1.9103)		*		(4.2544)	(9.2308)	(-8.0246)	
2	COEFF	0.0335	0.0515	-0.0221	-79.6020	0.8587	*	0.0226	0.0005	0.0230	-7.4515	0.9445
	T-STAT		(2.8334)	(-0.5170)	(-1.8859)		*		(0.2127)	(3.7027)	(-1.2165)	
23	COEFF	0.0322	-0.0430	0.1944	34.4673	0.9058	*	0.0426	-0.0225	0.1119	-22.3597	0.9654
	T-STAT		(-3.0143)	(5.7681)	(1.0389)		*		(-4.3123)	(9.0722)	(-1.8430)	
15	COEFF	0.0314	0.0987	-0.0986	-251.5740	0.9583	*	0.0442	0.0133	0.0256	-56.0810	0.9879
	T-STAT		(7.5950)	(-3.2166)	(-8.3368)		*		(4.7683)	(3.8792)	(-8.6301)	
14	COEFF	0.0283	0.0379	-0.0156	-30.8758	0.8946	*	0.0262	-0.0079	0.0381	21.8084	0.8862
	T-STAT		(3.3172)	(-0.5787)	(-1.1633)		*		(-2.1914)	(4.4753)	(2.5985)	
38	COEFF	0.0280	0.0626	-0.0689	-75.5748	0.9585	*	0.0153	0.0155	-0.0105	-52.0395	0.9411
	T-STAT		(8.1418)	(-3.8002)	(-4.2322)		*		(5.2252)	(-1.5001)	(-7.5261)	
12	COEFF	0.0265	-0.0871	0.1936	350.9480	0.2706	*	0.0258	-0.0430	0.1189	79.1417	0.6009
	T-STAT		(-2.3440)	(2.2076)	(4.0672)		*		(-3.6703)	(4.2972)	(2.9072)	

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31	COEFF	0.0259	0.0429	-0.0181	-85.4001	0.9663	*	0.0168	0.0156	-0.0158	-31.4443	0.9221
	T-STAT		(6.1308)	(-1.0964)	(-5.2470)		*		(5.4794)	(-2.3423)	(-4.7275)	
11	COEFF	0.0256	0.0572	-0.0486	-116.5640	0.9783	*	0.0200	0.0119	-0.0033	-29.1969	0.9611
	T-STAT		(9.5066)	(-3.2835)	(-8.3438)		*		(5.3198)	(-0.6315)	(-5.6053)	
25	COEFF	0.0239	-0.0605	0.2346	-15.0338	0.9187	*	0.0160	0.0095	0.0091	-59.9776	0.9070
	T-STAT		(-4.4488)	(7.3072)	(-0.4759)		*		(2.1134)	(0.8642)	(-5.7378)	
3	COEFF	0.0207	0.0294	-0.0179	-17.8955	0.8095	*	0.0091	-0.0004	0.0085	5.5266	0.9530
	T-STAT		(2.5957)	(-0.6701)	(-0.6792)		*		(-0.6398)	(4.8855)	(3.1921)	
5	COEFF	0.0205	0.0445	-0.0591	-22.2086	0.8290	*	0.0235	-0.0006	0.0121	41.7641	0.9071
	T-STAT		(4.3949)	(-2.4734)	(-0.9441)		*		(-0.4439)	(3.3898)	(11.8449)	
30	COEFF	0.0171	0.0349	-0.0188	-89.0483	0.9331	*	0.0182	0.0116	0.0050	-59.0433	0.8808
	T-STAT		(4.5378)	(-1.0392)	(-4.9830)		*		(2.1855)	(0.4000)	(-4.7711)	
16	COEFF	0.0168	0.0323	-0.0421	1.9509	0.8668	*	0.0188	-0.0007	0.0179	9.6569	0.9818
	T-STAT		(4.9975)	(-2.7608)	(0.1300)		*		(-0.7426)	(7.6756)	(4.1881)	
28	COEFF	0.0167	0.0174	-0.0020	1.2913	0.9043	*	0.0279	0.0016	0.0313	-24.1543	0.9885
	T-STAT		(3.0612)	(-0.1543)	(0.0973)		*		(0.9659)	(7.8565)	(-6.1507)	
32	COEFF	0.0154	0.0012	0.0367	6.9180	0.8436	*	0.0159	-0.0062	0.0347	-5.6225	0.9740
	T-STAT		(0.1644)	(2.0407)	(0.3906)		*		(-4.1157)	(9.7070)	(-1.5946)	
41	COEFF	0.0146	0.0160	-0.0237	60.5399	0.8725	*	0.0106	0.0009	0.0049	10.3847	0.9898
	T-STAT		(5.8762)	(-3.6791)	(9.5370)		*		(2.9515)	(6.4850)	(13.8101)	
33	COEFF	0.0142	0.0135	-0.0087	31.7892	0.9611	*	0.0133	0.0000	0.0096	11.3227	0.9858
	T-STAT		(6.3863)	(-1.7445)	(6.4427)		*		(0.0894)	(7.8221)	(9.2914)	
8	COEFF	0.0134	0.0227	0.0083	-100.4320	0.9659	*	0.0058	0.0025	0.0021	-9.9427	0.9599
	T-STAT		(4.3765)	(0.6842)	(-8.3204)		*		(3.4196)	(1.2452)	(-5.8577)	
29	COEFF	0.0129	0.0188	-0.0196	10.6982	0.8905	*	0.0118	0.0009	0.0097	-1.2175	0.9799
	T-STAT		(4.6413)	(-2.0490)	(1.1335)		*		(1.2118)	(5.4470)	(-0.6921)	
21	COEFF	0.0109	0.0409	-0.0894	21.5541	0.5153	*	0.0135	0.0112	-0.0187	4.8708	0.4981
	T-STAT		(3.9277)	(-3.6310)	(0.8899)		*		(2.6075)	(-1.8328)	(0.4851)	

9	COEFF	0.0103	0.0149	-0.0228	30.6170	0.8369	*	0.0112	-0.0042	0.0152	24.7837	0.6865
	T-STAT		(5.3130)	(-3.4441)	(4.6825)		*		(-2.3387)	(3.5474)	(5.8502)	
35	COEFF	0.0101	0.0113	-0.0043	3.5611	0.9640	*	0.0109	-0.0018	0.0132	9.5476	0.9682
	T-STAT		(5.8159)	(-0.9443)	(0.7827)		*		(-2.5592)	(7.6713)	(5.6374)	
27	COEFF	0.0099	0.0151	0.0113	-77.0789	0.9005	*	0.0123	0.0080	-0.0023	-21.8229	0.8928
	T-STAT		(2.1762)	(0.6900)	(-4.7556)		*		(3.1100)	(-0.3894)	(-3.6427)	
34	COEFF	0.0091	0.0012	0.0166	14.4425	0.9259	*	0.0071	-0.0011	0.0107	-1.6929	0.9704
	T-STAT		(0.5161)	(2.8464)	(2.5099)		*		(-1.7847)	(7.1442)	(-1.1451)	
37	COEFF	0.0090	0.0071	0.0101	-14.9964	0.9851	*	0.0079	-0.0006	0.0097	-0.0288	0.9909
	T-STAT		(4.8514)	(2.9056)	(-4.3561)		*		(-1.9105)	(11.7591)	(-0.0352)	
36	COEFF	0.0089	0.0170	-0.0156	-18.6599	0.9638	*	0.0075	0.0023	0.0012	-0.3854	0.9757
	T-STAT		(7.7720)	(-3.0308)	(-3.6743)		*		(4.9053)	(1.1275)	(-0.3539)	
13	COEFF	0.0089	0.0169	-0.0117	-30.1864	0.9853	*	0.0096	0.0013	0.0063	-0.8265	0.9821
	T-STAT		(10.8203)	(-3.1838)	(-8.2851)		*		(2.3642)	(4.7109)	(-0.6279)	
20	COEFF	0.0088	0.0085	0.0073	-18.5475	0.7877	*	0.0076	0.0019	0.0049	-10.9521	0.9499
	T-STAT		(1.3479)	(0.4875)	(-1.2563)		*		(1.9988)	(2.1580)	(-4.8475)	
24	COEFF	0.0082	0.0145	-0.0059	-33.9016	0.9724	*	0.0077	0.0016	0.0061	-9.7389	0.9883
	T-STAT		(6.7569)	(-1.1730)	(-6.7843)		*		(3.3953)	(5.3746)	(-8.5947)	
22	COEFF	0.0079	0.0176	-0.0186	-22.4588	0.9654	*	0.0103	-0.0000	0.0113	-2.9793	0.9894
	T-STAT		(8.6933)	(-3.8958)	(-4.7646)		*		(-0.0144)	(9.1855)	(-2.4527)	
39	COEFF	0.0073	0.0033	-0.0005	33.3302	0.6176	*	0.0092	-0.0007	0.0080	10.9117	0.9588
	T-STAT		(1.2934)	(-0.0925)	(5.5031)		*		(-1.3200)	(5.8392)	(8.0956)	
26	COEFF	0.0042	0.0042	0.0009	-3.4654	0.7791	*	0.0056	0.0006	0.0019	6.0666	0.9753
	T-STAT		(1.6256)	(0.1579)	(-0.5767)		*		(2.7464)	(3.2524)	(10.4613)	
18	COEFF	0.0041	0.0089	-0.0157	8.6521	0.7499	*	0.0030	0.0004	0.0009	2.5656	0.9707
	T-STAT		(4.9717)	(-3.7199)	(2.0778)		*		(2.9992)	(2.4985)	(6.9892)	
19	COEFF	0.0030	0.0039	-0.0016	-2.3467	0.9672	*	0.0022	0.0005	0.0010	-0.6662	0.9770
	T-STAT		(6.1794)	(-1.0686)	(-1.5893)		*		(3.3393)	(2.8878)	(-1.8976)	
17	COEFF	0.0018	0.0050	-0.0114	7.8380	0.4328	*	0.0021	0.0001	0.0010	2.4457	0.8973

	T-STAT		(3.3744)	(-3.2103)	(2.2381)		*		(0.6798)	(2.1364)	(5.1262)		
40	COEFF	0.0006	0.0003	0.0019	-4.0813	0.8713	*	0.0011	-0.0001	0.0025	-3.0082		0.9560
	T-STAT		(0.8013)	(1.6807)	(-3.5781)		*		(-0.9584)	(5.3384)	(-6.4620)		

TABLE 3-A

RANK ORDERING OF T-STATISTICS FOR (PROPORTION - B-WEIGHT) -- SALES

STEEL

ID	T-STAT
10	3.616642
6	3.087941
2	2.966211
13	2.022107
16	1.193431
12	1.077853
5	1.015854
22	0.841964
15	0.558390
9	0.427019
20	0.399295
3	0.065080
7	-0.059032
14	-0.144590
4	-0.680712
1	-0.858398
11	-0.933062
21	-1.817422
19	-2.001394
23	-2.066426
8	-2.489863
18	-2.598701
17	-3.309688

RANK ORDERING OF T-STATISTICS FOR (PROPORTION - B-WEIGHT) -- ASSETS

STEEL

ID	T-STAT
8	7.675146
6	5.741718
21	4.715212
4	2.239370
17	2.112992
22	1.873409
3	1.860880
1	0.490423
12	0.205567
11	-0.544870
18	-0.862096
19	-1.367371
15	-2.565733
20	-2.619945
14	-4.238501
16	-4.266295
7	-4.488410
9	-4.733144
23	-4.825121
5	-5.544679
2	-6.269340
10	-6.608749
13	-6.619955

RANK ORDERING OF T-STATISTICS FOR (PROPORTION - B-WEIGHT) -- SALES

RETAIL

ID	T-STAT
1	3.148525
16	2.949430
9	2.875260
15	2.324868
6	2.314985
18	2.163189
17	1.936979
2	1.679365
10	1.128440
8	1.050008
4	0.914363
19	0.835078
12	0.768009
14	-0.338739
13	-1.195917
7	-1.464776
3	-1.789764
5	-2.586722
11	-2.896805

RANK ORDERING OF T-STATISTICS FOR (PROPORTION - B-WEIGHT) -- ASSETS

RETAIL

ID	T-STAT
18	2.334834
6	1.692208
12	1.586595
11	1.511012
8	0.626746
4	0.365841
1	0.292692
14	-0.136828
16	-0.154827
7	-0.220965
19	-1.191342
2	-1.267838
3	-1.318746
13	-1.368040
17	-1.488788
9	-1.592777
15	-1.777023
10	-2.416573
5	-2.617667

RANK ORDERING OF T-STATISTICS FOR (PROPORTION - B-WEIGHT) -- SALES

PETROLEUM

ID	T-STAT
22	2.585607
6	1.759146
16	1.506813
8	1.026425
19	0.981785
3	0.976032
13	0.905113
21	0.864604
9	0.747976
18	0.525791
12	0.483286
20	0.341367
5	0.085066
7	0.078200
10	-1.026053
11	-1.358106
15	-1.486148
14	-1.542216
4	-1.612048
2	-1.975309
17	-2.268332
1	-3.157916

RANK ORDERING OF T-STATISTICS FOR (PROPORTION - B-WEIGHT) -- ASSETS

PETROLEUM

ID	T-STAT
9	5.429408
10	2.730298
15	2.545100
17	2.485393
1	2.367514
13	1.838011
8	1.664038
2	1.553079
14	1.431257
3	1.412450
12	1.033567
4	0.779483
7	0.634695
20	0.565859
19	0.081461
16	-0.404348
5	-0.905341
6	-1.390233
22	-1.429294
18	-2.714926
11	-3.102609
21	-3.891869

RANK ORDERING OF T-STATISTICS FOR (PROPORTION - B-WEIGHT) -- SALES

MACHINERY

ID	T-STAT
25	6.208041
23	5.273817
34	3.165202
12	3.058110
7	2.703085
32	1.865431
39	1.521987
6	1.386774
37	1.261010
40	0.430904
33	0.316378
20	0.051063
26	0.016527
28	-0.137308
41	-0.515313
1	-0.570309
35	-0.625277
27	-0.748490
3	-0.768253
14	-0.840898
2	-0.987602
4	-1.084549
19	-1.405312
29	-1.453824
9	-1.631633
8	-1.786735
17	-2.167292
10	-2.212262
30	-2.307479
5	-2.364165
16	-2.394865
31	-2.428953
18	-2.657683
21	-2.875571
24	-2.940155

36	-3.665755
38	-4.491807
22	-4.793523
13	-5.125203
15	-5.172372
11	-5.247604

RANK ORDERING OF T-STATISTICS FOR (PROPORTION - B-WEIGHT) -- ASSETS

MACHINERY

ID	T-STAT
41	7.480245
26	6.259383
36	5.713184
18	5.609064
31	4.833408
11	4.424334
38	3.686078
19	3.526775
5	3.189642
21	3.159180
33	2.948917
15	2.818349
13	2.515411
27	2.424442
1	2.399830
17	2.372868
8	2.149510
24	1.341131
20	1.189612
29	1.163398
30	1.051259
39	0.881046
25	0.645899
6	0.460844
16	0.351754
3	0.321297
2	-0.073190
10	-0.521847
4	-0.720012
22	-0.773366
28	-0.849392
9	-0.932355
35	-1.290500
14	-1.398928
7	-1.928306

37	-2.230207
34	-2.380428
40	-3.000223
12	-3.361808
32	-5.270933
23	-5.611798

TABLE 8-A

TIME TREND REGRESSIONS FOR FIRM PROPORTIONS IN INDUSTRY SALES

RANKED BY ABSOLUTE VALUE OF RELATIVE RATE OF GROWTH OF PROPORTION

STEEL

ID		TREND	INTERCEPT	R-SQUARED	PROP	REL GROWTH
15	COEFF	0.0006	0.0048	0.7088	0.0120	0.0514
	T-STAT	(6.6200)	(4.3481)			
5	COEFF	0.0002	0.0024	0.8226	0.0055	0.0483
	T-STAT	(9.1368)	(6.9696)			
11	COEFF	0.0004	0.0047	0.4215	0.0095	0.0430
	T-STAT	(3.6215)	(3.4732)			
20	COEFF	0.0008	0.0130	0.9030	0.0223	0.0361
	T-STAT	(12.9502)	(17.3803)			
2	COEFF	0.0016	0.0444	0.7457	0.0632	0.0261
	T-STAT	(7.2662)	(16.2672)			
9	COEFF	0.0002	0.0065	0.5430	0.0093	0.0258
	T-STAT	(4.6247)	(10.4607)			
19	COEFF	-0.0002	0.0140	0.3994	0.0114	-0.0203
	T-STAT	(-3.4598)	(17.4634)			
10	COEFF	0.0008	0.0459	0.8295	0.0551	0.0145
	T-STAT	(9.3572)	(44.7230)			
21	COEFF	-0.0040	0.3358	0.9019	0.2895	-0.0138
	T-STAT	(-12.8654)	(89.7560)			
1	COEFF	0.0002	0.0168	0.3583	0.0196	0.0121
	T-STAT	(3.1705)	(18.5539)			
4	COEFF	-0.0002	0.0206	0.0997	0.0182	-0.0116
	T-STAT	(-1.4121)	(11.4651)			
17	COEFF	-0.0001	0.0132	0.5375	0.0117	-0.0113

	T-STAT	(-4.5727)	(37.8266)			
13	COEFF	0.0000	0.0075	0.5346	0.0086	0.0112
	T-STAT	(4.5452)	(29.4901)			
18	COEFF	-0.0009	0.0964	0.7262	0.0858	-0.0107
	T-STAT	(-6.9067)	(60.0721)			
22	COEFF	-0.0001	0.0199	0.5958	0.0179	-0.0098
	T-STAT	(-5.1510)	(48.7702)			
12	COEFF	0.0005	0.0511	0.5297	0.0570	0.0090
	T-STAT	(4.5008)	(37.4479)			
7	COEFF	0.0000	0.0068	0.0906	0.0076	0.0086
	T-STAT	(1.3382)	(11.6651)			
16	COEFF	0.0004	0.0507	0.1931	0.0561	0.0085
	T-STAT	(2.0757)	(18.2359)			
14	COEFF	0.0000	0.0071	0.1300	0.0077	0.0065
	T-STAT	(1.6385)	(19.3789)			
8	COEFF	0.0001	0.0163	0.1875	0.0176	0.0060
	T-STAT	(2.0375)	(26.0251)			
3	COEFF	-0.0006	0.1718	0.2789	0.1639	-0.0041
	T-STAT	(-2.6293)	(55.1580)			
23	COEFF	0.0000	0.0456	0.0041	0.0458	0.0003
	T-STAT	(0.2584)	(57.1417)			
6	COEFF	0.0000	0.0035	0.0005	0.0035	0.0003
	T-STAT	(0.0767)	(18.6417)			

TIME TREND REGRESSIONS FOR FIRM PROPORTIONS IN INDUSTRY ASSETS

RANKED BY ABSOLUTE VALUE OF RELATIVE RATE OF GROWTH OF PROPORTION

STEEL

ID		TREND	INTERCEPT	R-SQUARED	PROP	REL GROWTH
15	COEFF	0.0004	0.0043	0.6146	0.0101	0.0457
	T-STAT	(5.3584)	(4.1892)			
9	COEFF	0.0004	0.0044	0.8301	0.0102	0.0449
	T-STAT	(9.3786)	(7.5467)			
20	COEFF	0.0009	0.0110	0.8889	0.0234	0.0415
	T-STAT	(12.0035)	(11.3948)			
5	COEFF	0.0001	0.0013	0.8984	0.0028	0.0415
	T-STAT	(12.6199)	(12.0732)			
7	COEFF	0.0001	0.0019	0.7097	0.0033	0.0331
	T-STAT	(6.6343)	(9.7009)			
2	COEFF	0.0013	0.0294	0.9648	0.0470	0.0295
	T-STAT	(22.2195)	(39.3774)			
14	COEFF	0.0001	0.0027	0.7478	0.0041	0.0263
	T-STAT	(7.3063)	(15.4922)			
10	COEFF	0.0012	0.0315	0.9078	0.0470	0.0261
	T-STAT	(13.3174)	(28.5062)			
4	COEFF	-0.0003	0.0201	0.3306	0.0152	-0.0249
	T-STAT	(-2.9820)	(13.1705)			
11	COEFF	0.0002	0.0065	0.3705	0.0092	0.0226
	T-STAT	(3.2547)	(8.5296)			
19	COEFF	0.0001	0.0052	0.5072	0.0066	0.0162
	T-STAT	(4.3039)	(17.5638)			
13	COEFF	0.0000	0.0038	0.7040	0.0047	0.0143

	T-STAT	(6.5413)	(31.0716)			
8	COEFF	-0.0001	0.0150	0.8842	0.0129	-0.0130
	T-STAT	(-11.7247)	(87.1802)			
21	COEFF	-0.0048	0.4372	0.9429	0.3755	-0.0130
	T-STAT	(-17.2538)	(128.9940)			
16	COEFF	0.0006	0.0492	0.7918	0.0576	0.0116
	T-STAT	(8.2715)	(50.7425)			
6	COEFF	-0.0000	0.0023	0.5999	0.0020	-0.0101
	T-STAT	(-5.1945)	(48.3026)			
22	COEFF	-0.0001	0.0212	0.4116	0.0190	-0.0092
	T-STAT	(-3.5489)	(35.7501)			
1	COEFF	0.0000	0.0098	0.2212	0.0109	0.0081
	T-STAT	(2.2596)	(20.9035)			
23	COEFF	0.0003	0.0436	0.7785	0.0481	0.0074
	T-STAT	(7.9529)	(80.1880)			
3	COEFF	-0.0007	0.1537	0.4074	0.1445	-0.0050
	T-STAT	(-3.5100)	(61.2149)			
17	COEFF	-0.0000	0.0115	0.0259	0.0110	-0.0031
	T-STAT	(-0.6889)	(19.3303)			
12	COEFF	0.0000	0.0612	0.0261	0.0622	0.0012
	T-STAT	(0.6855)	(46.0017)			
18	COEFF	-0.0000	0.0720	0.0159	0.0715	-0.0005
	T-STAT	(-0.4446)	(70.7136)			

TIME TREND REGRESSIONS FOR FIRM PROPORTIONS IN INDUSTRY SALES

RANKED BY ABSOLUTE VALUE OF RELATIVE RATE OF GROWTH OF PROPORTION

RETAIL

ID		TREND	INTERCEPT	R-SQUARED	PROP	REL GROWTH
5	COEFF	0.0008	0.0023	0.6109	0.0131	0.0680
	T-STAT	(5.1670)	(1.2016)			
14	COEFF	0.0014	0.0042	0.6278	0.0216	0.0678
	T-STAT	(5.3548)	(1.3689)			
3	COEFF	0.0002	0.0011	0.8348	0.0043	0.0616
	T-STAT	(9.2692)	(3.5793)			
7	COEFF	-0.0007	0.0286	0.9625	0.0200	-0.0374
	T-STAT	(-20.9011)	(70.1734)			
16	COEFF	-0.0001	0.0081	0.9184	0.0059	-0.0313
	T-STAT	(-13.8343)	(53.0027)			
11	COEFF	-0.0028	0.1385	0.7130	0.1058	-0.0270
	T-STAT	(-6.4987)	(27.5412)			
2	COEFF	0.0002	0.0097	0.3980	0.0127	0.0214
	T-STAT	(3.3524)	(10.4444)			
9	COEFF	-0.0001	0.0158	0.8745	0.0134	-0.0147
	T-STAT	(-10.8827)	(75.9311)			
18	COEFF	0.0002	0.0196	0.5602	0.0225	0.0111
	T-STAT	(4.6537)	(31.7685)			
4	COEFF	-0.0003	0.0369	0.6580	0.0327	-0.0110
	T-STAT	(-5.7189)	(51.1454)			
15	COEFF	-0.0001	0.0204	0.3005	0.0184	-0.0086
	T-STAT	(-2.7021)	(30.3966)			
13	COEFF	0.0002	0.0344	0.0877	0.0379	0.0070

	T-STAT	(1.2788)	(14.5266)			
17	COEFF	0.0001	0.0184	0.1781	0.0195	0.0053
	T-STAT	(1.9198)	(29.5553)			
6	COEFF	-0.0001	0.0406	0.2717	0.0383	-0.0049
	T-STAT	(-2.5172)	(46.9212)			
12	COEFF	0.0016	0.3105	0.3562	0.3285	0.0048
	T-STAT	(3.0682)	(52.0778)			
1	COEFF	-0.0002	0.0579	0.1831	0.0549	-0.0043
	T-STAT	(-1.9499)	(41.3765)			
8	COEFF	-0.0001	0.0522	0.1316	0.0500	-0.0035
	T-STAT	(-1.6039)	(41.0505)			
19	COEFF	-0.0001	0.0805	0.1750	0.0787	-0.0019
	T-STAT	(-1.8826)	(88.3261)			
10	COEFF	0.0001	0.1193	0.0575	0.1210	0.0012
	T-STAT	(0.9617)	(66.5834)			

TIME TREND REGRESSIONS FOR FIRM PROPORTIONS IN INDUSTRY ASSETS

RANKED BY ABSOLUTE VALUE OF RELATIVE RATE OF GROWTH OF PROPORTION

RETAIL

ID		TREND	INTERCEPT	R-SQUARED	PROP	REL GROWTH
16	COEFF	-0.0004	0.0139	0.9811	0.0085	-0.0509
	T-STAT	(-29.7180)	(83.3328)			
14	COEFF	0.0015	0.0138	0.5226	0.0320	0.0468
	T-STAT	(4.3144)	(3.4833)			
7	COEFF	-0.0015	0.0565	0.9189	0.0378	-0.0400
	T-STAT	(-13.8806)	(45.4271)			
17	COEFF	-0.0009	0.0376	0.7600	0.0253	-0.0386
	T-STAT	(-7.3376)	(24.7830)			
1	COEFF	0.0019	0.0256	0.4125	0.0510	0.0385
	T-STAT	(3.4549)	(3.9514)			
13	COEFF	-0.0031	0.1300	0.8812	0.0910	-0.0342
	T-STAT	(-11.2303)	(41.1220)			
3	COEFF	0.0001	0.0018	0.3843	0.0030	0.0339
	T-STAT	(3.2578)	(5.0169)			
10	COEFF	0.0015	0.0348	0.7460	0.0537	0.0293
	T-STAT	(7.0666)	(13.6547)			
4	COEFF	-0.0011	0.0585	0.9159	0.0440	-0.0264
	T-STAT	(-13.6152)	(60.0174)			
11	COEFF	0.0016	0.0456	0.3655	0.0671	0.0245
	T-STAT	(3.1293)	(7.5864)			
5	COEFF	0.0000	0.0038	0.3503	0.0050	0.0180
	T-STAT	(3.0277)	(11.4233)			
15	COEFF	-0.0003	0.0325	0.3492	0.0274	-0.0141

	T-STAT	(-3.0204)	(22.1863)			
19	COEFF	-0.0016	0.1634	0.3592	0.1413	-0.0118
	T-STAT	(-3.0856)	(26.4264)			
12	COEFF	0.0021	0.2120	0.5201	0.2401	0.0090
	T-STAT	(4.2890)	(36.7579)			
6	COEFF	-0.0003	0.0503	0.1748	0.0465	-0.0069
	T-STAT	(-1.8979)	(26.0756)			
9	COEFF	0.0001	0.0165	0.0591	0.0175	0.0057
	T-STAT	(1.0327)	(14.7575)			
8	COEFF	0.0003	0.0802	0.0732	0.0854	0.0046
	T-STAT	(1.1563)	(20.6403)			
2	COEFF	0.0000	0.0103	0.0142	0.0105	0.0035
	T-STAT	(0.4930)	(11.9503)			
18	COEFF	-0.0000	0.0120	0.0065	0.0120	-0.0005
	T-STAT	(-0.2948)	(48.1513)			

TIME TREND REGRESSIONS FOR FIRM PROPORTIONS IN INDUSTRY SALES

RANKED BY ABSOLUTE VALUE OF RELATIVE RATE OF GROWTH OF PROPORTION

PETROLEUM

ID		TREND	INTERCEPT	R-SQUARED	PROP	REL GROWTH
11	COEFF	0.0010	-0.0046	0.8091	0.0094	0.1136
	T-STAT	(8.4907)	(-2.9820)			
6	COEFF	0.0004	-0.0003	0.9067	0.0051	0.0831
	T-STAT	(12.8591)	(-0.9579)			
9	COEFF	-0.0000	0.0030	0.7440	0.0019	-0.0454
	T-STAT	(-7.0303)	(19.9696)			
2	COEFF	-0.0009	0.0329	0.6081	0.0212	-0.0434
	T-STAT	(-5.1369)	(14.9579)			
3	COEFF	-0.0012	0.0499	0.9129	0.0334	-0.0378
	T-STAT	(-13.3525)	(42.7849)			
15	COEFF	-0.0008	0.0362	0.9307	0.0251	-0.0343
	T-STAT	(-15.1172)	(51.7703)			
12	COEFF	-0.0012	0.0527	0.8928	0.0370	-0.0325
	T-STAT	(-11.8991)	(42.3182)			
13	COEFF	-0.0021	0.0935	0.9461	0.0658	-0.0324
	T-STAT	(-17.2770)	(61.4917)			
16	COEFF	0.0003	0.0066	0.4089	0.0113	0.0322
	T-STAT	(3.4297)	(5.0991)			
4	COEFF	0.0007	0.0175	0.5693	0.0273	0.0270
	T-STAT	(4.7408)	(9.1641)			
17	COEFF	0.0004	0.0119	0.2484	0.0178	0.0243
	T-STAT	(2.3704)	(5.3076)			
14	COEFF	-0.0003	0.0182	0.7655	0.0139	-0.0234

	T-STAT	(-7.4502)	(33.6413)			
19	COEFF	0.0015	0.0824	0.6086	0.1028	0.0153
	T-STAT	(5.1410)	(21.7911)			
5	COEFF	-0.0003	0.0314	0.4960	0.0267	-0.0136
	T-STAT	(-4.0902)	(28.8040)			
22	COEFF	0.0008	0.0756	0.4488	0.0867	0.0099
	T-STAT	(3.7186)	(26.4949)			
1	COEFF	0.0000	0.0097	0.0877	0.0109	0.0083
	T-STAT	(1.2782)	(11.0356)			
21	COEFF	0.0020	0.2318	0.3416	0.2572	0.0077
	T-STAT	(2.9687)	(28.0038)			
8	COEFF	-0.0002	0.0391	0.4462	0.0358	-0.0071
	T-STAT	(-3.7001)	(46.1742)			
10	COEFF	-0.0003	0.0637	0.7754	0.0590	-0.0061
	T-STAT	(-7.6582)	(110.1660)			
20	COEFF	0.0002	0.0535	0.1765	0.0565	0.0041
	T-STAT	(1.9039)	(35.5294)			
7	COEFF	0.0000	0.0114	0.0119	0.0117	0.0017
	T-STAT	(0.4450)	(19.9048)			
18	COEFF	-0.0000	0.0830	0.0022	0.0824	-0.0005
	T-STAT	(-0.1748)	(27.8982)			

TIME TREND REGRESSIONS FOR FIRM PROPORTIONS IN INDUSTRY ASSETS

RANKED BY ABSOLUTE VALUE OF RELATIVE RATE OF GROWTH OF PROPORTION

PETROLEUM

ID		TREND	INTERCEPT	R-SQUARED	PROP	REL GROWTH
6	COEFF	0.0003	-0.0002	0.9397	0.0038	0.0804
	T-STAT	(16.2830)	(-1.2677)			
11	COEFF	0.0003	0.0012	0.9322	0.0062	0.0596
	T-STAT	(15.2944)	(4.3137)			
9	COEFF	-0.0000	0.0016	0.8779	0.0009	-0.0523
	T-STAT	(-11.0600)	(29.1246)			
3	COEFF	-0.0012	0.0513	0.7847	0.0346	-0.0364
	T-STAT	(-7.8729)	(26.0534)			
1	COEFF	0.0001	0.0040	0.6716	0.0056	0.0218
	T-STAT	(5.8967)	(15.5516)			
13	COEFF	-0.0016	0.0995	0.9723	0.0774	-0.0214
	T-STAT	(-24.4669)	(119.2920)			
12	COEFF	-0.0007	0.0487	0.8961	0.0384	-0.0199
	T-STAT	(-12.1089)	(62.5768)			
22	COEFF	0.0018	0.0698	0.9304	0.0943	0.0194
	T-STAT	(15.0797)	(46.7857)			
14	COEFF	-0.0002	0.0134	0.8949	0.0107	-0.0188
	T-STAT	(-12.0321)	(64.9059)			
4	COEFF	0.0005	0.0210	0.6313	0.0277	0.0180
	T-STAT	(5.3957)	(18.4760)			
18	COEFF	0.0014	0.0783	0.6993	0.0969	0.0144
	T-STAT	(6.2867)	(28.5288)			
7	COEFF	-0.0001	0.0187	0.7104	0.0161	-0.0123

	T-STAT	(-6.4563)	(49.5307)			
8	COEFF	-0.0004	0.0490	0.5124	0.0433	-0.0098
	T-STAT	(-4.2267)	(39.6605)			
2	COEFF	-0.0001	0.0276	0.0902	0.0256	-0.0064
	T-STAT	(-1.2979)	(17.7740)			
17	COEFF	0.0001	0.0244	0.0245	0.0264	0.0049
	T-STAT	(0.6542)	(9.9426)			
15	COEFF	-0.0000	0.0218	0.1548	0.0209	-0.0033
	T-STAT	(-1.7638)	(44.8595)			
21	COEFF	0.0006	0.2073	0.1816	0.2157	0.0030
	T-STAT	(1.9315)	(49.4320)			
20	COEFF	-0.0001	0.0775	0.2613	0.0749	-0.0026
	T-STAT	(-2.4406)	(78.5395)			
5	COEFF	-0.0000	0.0183	0.0140	0.0177	-0.0023
	T-STAT	(-0.4902)	(17.4034)			
10	COEFF	-0.0001	0.0642	0.2121	0.0622	-0.0023
	T-STAT	(-2.1205)	(74.9841)			
19	COEFF	-0.0001	0.0849	0.1032	0.0832	-0.0015
	T-STAT	(-1.3866)	(75.1311)			
16	COEFF	-0.0000	0.0167	0.0005	0.0165	-0.0006
	T-STAT	(-0.0960)	(11.5850)			

TIME TREND REGRESSIONS FOR FIRM PROPORTIONS IN INDUSTRY SALES

RANKED BY ABSOLUTE VALUE OF RELATIVE RATE OF GROWTH OF PROPORTION

MACHINERY

ID		TREND	INTERCEPT	R-SQUARED	PROP	REL GROWTH
21	COEFF	-0.0012	0.0270	0.5857	0.0109	-0.1145
	T-STAT	(-4.9027)	(8.5971)			
40	COEFF	0.0000	-0.0000	0.7679	0.0006	0.0815
	T-STAT	(7.5015)	(-0.5237)			
25	COEFF	0.0018	0.0008	0.4866	0.0239	0.0755
	T-STAT	(4.0147)	(0.1555)			
17	COEFF	-0.0001	0.0035	0.3484	0.0018	-0.0733
	T-STAT	(-3.0151)	(6.5096)			
8	COEFF	0.0009	0.0006	0.9464	0.0134	0.0718
	T-STAT	(17.3373)	(0.9791)			
27	COEFF	0.0006	0.0013	0.7045	0.0099	0.0650
	T-STAT	(6.3668)	(1.0847)			
39	COEFF	-0.0003	0.0122	0.6982	0.0073	-0.0501
	T-STAT	(-6.2714)	(16.9249)			
23	COEFF	0.0015	0.0125	0.4389	0.0322	0.0485
	T-STAT	(3.6473)	(2.3712)			
15	COEFF	0.0014	0.0117	0.6901	0.0314	0.0463
	T-STAT	(6.1533)	(4.0254)			
9	COEFF	-0.0004	0.0165	0.7725	0.0103	-0.0458
	T-STAT	(-7.5990)	(21.5340)			
4	COEFF	-0.0031	0.1095	0.9084	0.0688	-0.0450
	T-STAT	(-12.9906)	(37.3393)			
41	COEFF	-0.0006	0.0231	0.7232	0.0146	-0.0443

	T-STAT	(-6.6658)	(19.3451)			
18	COEFF	-0.0001	0.0063	0.3910	0.0041	-0.0418
	T-STAT	(-3.3041)	(9.8979)			
5	COEFF	-0.0008	0.0307	0.2794	0.0205	-0.0391
	T-STAT	(-2.5677)	(7.9681)			
20	COEFF	0.0002	0.0051	0.4148	0.0088	0.0325
	T-STAT	(3.4715)	(4.9883)			
32	COEFF	0.0004	0.0097	0.2860	0.0154	0.0291
	T-STAT	(2.6100)	(4.6035)			
11	COEFF	0.0007	0.0157	0.6215	0.0256	0.0283
	T-STAT	(5.2837)	(9.3161)			
37	COEFF	0.0002	0.0058	0.7268	0.0090	0.0273
	T-STAT	(6.7247)	(12.8531)			
26	COEFF	-0.0001	0.0056	0.1065	0.0042	-0.0257
	T-STAT	(-1.4241)	(5.9571)			
24	COEFF	0.0002	0.0053	0.7044	0.0082	0.0254
	T-STAT	(6.3647)	(13.3412)			
30	COEFF	0.0004	0.0111	0.3392	0.0171	0.0254
	T-STAT	(2.9539)	(6.1633)			
33	COEFF	-0.0003	0.0187	0.7612	0.0142	-0.0238
	T-STAT	(-7.3617)	(33.0207)			
6	COEFF	-0.0042	0.2519	0.5245	0.1957	-0.0219
	T-STAT	(-4.3298)	(20.6673)			
7	COEFF	0.0011	0.0429	0.4666	0.0573	0.0196
	T-STAT	(3.8562)	(11.9544)			
31	COEFF	0.0004	0.0192	0.4759	0.0259	0.0191
	T-STAT	(3.9291)	(12.3509)			
13	COEFF	0.0001	0.0066	0.6623	0.0089	0.0188
	T-STAT	(5.7745)	(18.5837)			

22	COEFF	0.0001	0.0059	0.1827	0.0079	0.0181
	T-STAT	(1.9492)	(6.5610)			
12	COEFF	-0.0004	0.0341	0.0176	0.0265	-0.0172
	T-STAT	(-0.5523)	(3.3581)			
2	COEFF	0.0005	0.0261	0.1808	0.0335	0.0166
	T-STAT	(1.9374)	(7.3744)			
29	COEFF	-0.0002	0.0155	0.2860	0.0129	-0.0154
	T-STAT	(-2.6090)	(16.4527)			
1	COEFF	-0.0005	0.0492	0.3101	0.0415	-0.0141
	T-STAT	(-2.7642)	(18.8731)			
28	COEFF	0.0001	0.0141	0.1088	0.0167	0.0119
	T-STAT	(1.4411)	(8.3564)			
10	COEFF	0.0011	0.0792	0.4527	0.0940	0.0117
	T-STAT	(3.7492)	(21.8636)			
14	COEFF	0.0002	0.0243	0.1435	0.0283	0.0104
	T-STAT	(1.6881)	(11.3008)			
34	COEFF	0.0000	0.0083	0.0731	0.0091	0.0072
	T-STAT	(1.1572)	(11.8641)			
16	COEFF	-0.0000	0.0178	0.0103	0.0168	-0.0049
	T-STAT	(-0.4209)	(7.3370)			
3	COEFF	-0.0000	0.0215	0.0075	0.0207	-0.0032
	T-STAT	(-0.3590)	(9.2429)			
38	COEFF	0.0000	0.0269	0.0060	0.0280	0.0020
	T-STAT	(0.3226)	(12.2451)			
35	COEFF	0.0000	0.0099	0.0068	0.0101	0.0014
	T-STAT	(0.3311)	(18.5661)			
19	COEFF	0.0000	0.0029	0.0042	0.0030	0.0013
	T-STAT	(0.2678)	(15.7373)			
36	COEFF	0.0000	0.0088	0.0002	0.0089	0.0001

T-STAT (0.0240) (12.5859)

TIME TREND REGRESSIONS FOR FIRM PROPORTIONS IN INDUSTRY ASSETS

RANKED BY ABSOLUTE VALUE OF RELATIVE RATE OF GROWTH OF PROPORTION

MACHINERY

ID		TREND	INTERCEPT	R-SQUARED	PROP	REL GROWTH
40	COEFF	0.0001	-0.0003	0.9132	0.0011	0.0990
	T-STAT	(13.3782)	(-3.8729)			
25	COEFF	0.0012	-0.0012	0.7659	0.0160	0.0786
	T-STAT	(7.4580)	(-0.5930)			
30	COEFF	0.0011	0.0022	0.6728	0.0182	0.0636
	T-STAT	(5.9124)	(0.9507)			
5	COEFF	-0.0014	0.0428	0.9241	0.0235	-0.0603
	T-STAT	(-14.3886)	(35.2325)			
21	COEFF	-0.0008	0.0243	0.4872	0.0135	-0.0594
	T-STAT	(-4.0194)	(9.8835)			
23	COEFF	0.0022	0.0126	0.5656	0.0426	0.0529
	T-STAT	(4.7052)	(2.1362)			
20	COEFF	0.0004	0.0021	0.6054	0.0076	0.0520
	T-STAT	(5.1078)	(2.2036)			
26	COEFF	-0.0002	0.0094	0.8320	0.0056	-0.0509
	T-STAT	(-9.1772)	(24.7454)			
9	COEFF	-0.0005	0.0190	0.7977	0.0112	-0.0500
	T-STAT	(-8.1877)	(22.4605)			
38	COEFF	0.0007	0.0047	0.5968	0.0153	0.0499
	T-STAT	(5.0166)	(2.5240)			
6	COEFF	-0.0100	0.3588	0.8791	0.2231	-0.0448
	T-STAT	(-11.1225)	(32.4222)			
18	COEFF	-0.0001	0.0047	0.7962	0.0030	-0.0426

	T-STAT	(-8.1507)	(24.4884)			
41	COEFF	-0.0004	0.0165	0.9138	0.0106	-0.0402
	T-STAT	(-13.4283)	(41.9816)			
39	COEFF	-0.0003	0.0142	0.8467	0.0092	-0.0400
	T-STAT	(-9.6894)	(30.3766)			
7	COEFF	0.0018	0.0226	0.8865	0.0479	0.0388
	T-STAT	(11.5279)	(11.4223)			
32	COEFF	0.0005	0.0082	0.5633	0.0159	0.0360
	T-STAT	(4.6833)	(5.4641)			
17	COEFF	-0.0000	0.0032	0.6572	0.0021	-0.0351
	T-STAT	(-5.7091)	(19.5889)			
24	COEFF	0.0002	0.0041	0.9043	0.0077	0.0343
	T-STAT	(12.6749)	(15.9734)			
8	COEFF	0.0001	0.0032	0.7435	0.0058	0.0324
	T-STAT	(7.0205)	(9.7406)			
10	COEFF	0.0033	0.0617	0.9473	0.1076	0.0312
	T-STAT	(17.4890)	(26.1438)			
33	COEFF	-0.0004	0.0189	0.8718	0.0133	-0.0310
	T-STAT	(-10.7557)	(40.0288)			
15	COEFF	0.0013	0.0260	0.7994	0.0442	0.0304
	T-STAT	(8.2332)	(12.9565)			
28	COEFF	0.0007	0.0185	0.8437	0.0279	0.0250
	T-STAT	(9.5801)	(20.5460)			
34	COEFF	0.0001	0.0048	0.4676	0.0071	0.0243
	T-STAT	(3.8644)	(8.7099)			
4	COEFF	-0.0015	0.0853	0.7645	0.0641	-0.0238
	T-STAT	(-7.4285)	(33.6693)			
35	COEFF	-0.0002	0.0139	0.6467	0.0109	-0.0197
	T-STAT	(-5.5781)	(29.2708)			

2	COEFF	0.0003	0.0178	0.3290	0.0226	0.0155
	T-STAT	(2.8872)	(11.8988)			
1	COEFF	0.0006	0.0334	0.8361	0.0418	0.0147
	T-STAT	(9.3146)	(40.9972)			
11	COEFF	0.0002	0.0159	0.2521	0.0200	0.0145
	T-STAT	(2.3942)	(10.6706)			
12	COEFF	0.0003	0.0221	0.0113	0.0258	0.0134
	T-STAT	(0.4425)	(2.2939)			
27	COEFF	0.0001	0.0101	0.0822	0.0123	0.0127
	T-STAT	(1.2338)	(6.4570)			
16	COEFF	-0.0002	0.0220	0.4973	0.0188	-0.0124
	T-STAT	(-4.1007)	(31.4738)			
22	COEFF	0.0001	0.0086	0.5297	0.0103	0.0123
	T-STAT	(4.3752)	(24.0851)			
36	COEFF	-0.0000	0.0087	0.1830	0.0075	-0.0122
	T-STAT	(-1.9511)	(15.0021)			
3	COEFF	-0.0000	0.0105	0.1898	0.0091	-0.0105
	T-STAT	(-1.9952)	(17.5649)			
14	COEFF	0.0002	0.0224	0.0501	0.0262	0.0099
	T-STAT	(0.9473)	(6.6055)			
19	COEFF	0.0000	0.0021	0.0554	0.0022	0.0055
	T-STAT	(0.9990)	(13.5321)			
13	COEFF	-0.0000	0.0103	0.1862	0.0096	-0.0052
	T-STAT	(-1.9661)	(32.4834)			
37	COEFF	0.0000	0.0074	0.1357	0.0079	0.0041
	T-STAT	(1.6280)	(30.5095)			
31	COEFF	0.0000	0.0157	0.0077	0.0168	0.0037
	T-STAT	(0.3625)	(7.3160)			
29	COEFF	0.0000	0.0113	0.0482	0.0118	0.0030

T-STAT (0.9229) (23.3076)

TABLE 5-A

LIST OF FIRMS AND THEIR COMPUSTAT NUMBERS AND ID NUMBERS

STEEL

FIRM	COMPUSTAT INDUS. NO.	COMPUSTAT CO. NO.	ID
ALLEGHENY LUDLUM STEEL CORP	3310	11600	1
ARMCO STEEL CORP.	3310	46800	2
BETHLEHEM STEEL CORP.	3310	76700	3
C F AND I STEEL	3310	105976	4
CARPENTER STEEL CO	3310	127400	5
CONTINENTAL STEEL CORP.	3310	187900	6
COPPERWELD STEEL	3310	189500	7
CRUCIBLE STEEL CORP.	3310	197800	8
GRANITE CITY STEEL CO	3310	322500	9
INLAND STEEL CO.	3310	378800	10
INTERLAKE STEEL CO	3310	381710	11
JONES & LAUGHLIN STEEL CORP.	3310	398600	12
KEYSTONE STEEL WIRE CO DEL	3310	409600	13
LUKENS STEEL CO	3310	441200	14
MCLOUTH STEEL CORP	3310	464750	15
NATIONAL STEEL CORP.	3310	516900	16
PITTSBURGH STEEL CO	3310	589000	17
REPUBLIC STEEL CORP.	3310	616800	18
SHARON STEEL CORP	3310	655400	19
STEEL CO OF CANADA	3310	692700	20
U. S. STEEL CORP.	3310	752200	21
WHEELING STEEL CORP.	3310	786100	22
YOUNGSTOWN SHEET & TUBE CO.	3310	801700	23

LIST OF FIRMS AND THEIR COMPUSTAT NUMBERS AND ID NUMBERS

RETAIL

FIRM	COMPUSTAT INDUS.NO.	COMPUSTAT CO.NO.	ID
ALLIED STORES	5311	14200	1
BROADWAY HALE STORES	5311	94000	2
DIANA STORES CORP	5311	216000	3
GIMBEL BROS., INC.	5311	313800	4
INTERSTATE DEPARTMENT STORES	5311	387100	5
MACY R.H. & CO., INC.	5311	444900	6
MARSHALL FIELD & CO.	5311	455100	7
MAY DEPARTMENT STORES CO.	5311	459600	8
MERCANTILE STORES CO., INC.	5311	467900	9
PENNEY J.C. CO., INC.	5311	571700	10
MONTGOMERY WARD & CO INC	5322	492700	11
SEARS, ROEBUCK & CO.	5322	648200	12
KRESGE S.S. CO.	5331	413700	13
MCCRORY CORPORATION	5331	463200	14
MURPHY G.C. CO.	5331	499600	15
NEISNER BROS., INC.	5331	519600	16
NEWBERRY J.J. CO.	5331	528300	17
WALGREEN CO	5331	766400	18
WOOLWORTH F.W. CO.	5331	796500	19

// XEQ FIRMS

LIST OF FIRMS AND THEIR COMPUSTAT NUMBERS AND ID NUMBERS

PETROLEUM

FIRM	COMPUSTAT INDUS.NO.	COMPUSTAT CO.NO.	ID
ASHLAND OIL REF	2912	46900	1
ATLANTIC RICHFIELD CO	2912	53377	2
CITIES SERVICE CO.	2912	152800	3
CONTINENTAL OIL CO.	2912	187700	4
IMPERIAL OIL LTD	2912	373250	5
KERR MCGEE CORP	2912	407900	6
MARATHON OIL	2912	452180	7
PHILLIPS PETROLEUM CO.	2912	583700	8
QUAKER STATE OIL REFINING	2912	606500	9
SHELL OIL CO.	2912	656500	10
SIGNAL COS	2912	659205	11
SINCLAIR OIL CORP.	2912	662800	12
STANDARD OIL CO. INDIANA	2912	686300	13
STANDARD OIL CO OHIO	2912	686700	14
SUN OIL CO	2912	699600	15
SUNRAY DX OIL CO	2912	701220	16
UNION OIL CO. OF CALIFORNIA	2912	736700	17
GULF OIL CORP.	2913	334600	18
MOBIL OIL CORP	2913	486085	19
STANDARD OIL CO. OF CALIFORNIA	2913	686200	20
STANDARD OIL CO. NEW JERSEY	2913	686500	21
TEXACO INC	2913	711282	22

LIST OF FIRMS AND THEIR COMPUSTAT NUMBERS AND ID NUMBERS

MACHINERY

FIRM	COMPUSTAT INDUS.NO.	COMPUSTAT CO.NO.	ID
BABCOCK & WILCOX CO.	3511	59000	1
COMBUSTION ENGINEERING, INC.	3511	169700	2
FOSTER WHEELER CORP.	3511	285500	3
ALLIS CHALMERS MANUFACTURING CO.	3522	14600	4
CASE J.I. COMPANY	3522	128710	5
INTERNATIONAL HARVESTER CO.	3522	383300	6
MASSEY FERGUSON	3522	457900	7
WICKES CORP	3522	788800	8
BUCYRUS-ERIE COMPANY	3531	98800	9
CATERPILLAR TRACTOR CO.	3531	131000	10
CLARK EQUIPMENT CO.	3531	156800	11
MERRITT CHAPMAN & SCOTT CORP	3531	471400	12
REX CHAINBELT	3531	617760	13
DRESSER INDUSTRIES, INC.	3533	225900	14
HALLIBURTON COMPANY	3533	336700	15
CINCINNATI MILLING MACHINE CO.	3540	151600	16
MONARCH MACHINE TOOL CO	3540	488200	17
NATIONAL ACME CO.	3540	504400	18
SKIL CORP	3540	663900	19
SUNDSTRAND CORP	3540	701100	20
WALLACE MURRAY CORP	3540	767490	21
WARNER & SWASEY	3540	768800	22
AMERICAN MACHINE & FOUNDRY CO.	3550	28100	23
BLACK AND DECKER MANUFACTURING CO	3550	79300	24
BRUNSWICK CORP	3550	97610	25
CROMPTON KNOWLES CORP	3550	195800	26
EMHART CORP	3550	245300	27
EX-CELL-O CORP.	3550	252200	28
JOY MANUFACTURING CO.	3550	399200	29
MIDLAND ROSS CORP	3550	477470	30
OTIS ELEVATOR CO.	3550	555300	31
OUTBOARD MARINE CORP	3550	555700	32
AMERICAN CHAIN & CABLE CO. INC.	3560	21000	33
BRIGGS/STRATTON	3560	89200	34
CHICAGO PNEUMATIC TOOL CO.	3560	147700	35

COOPER INDUSTRIES INC	3560	188500	36
GARDNER-DENVER CO.	3560	294800	37
INGERSOLL-RAND CO.	3560	378400	38
NESTA MACHINE CO	3560	471900	39
NEW HAMPSHIRE BALL BEARINGS	3569	523580	40
STEWART WARNER CORP	3569	694600	41

// XEQ FIRMS