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AN ECONOMETRIC MODEL OF THE WORLD COPPER INDUSTRY

by

Franklin M. Fisher and Paul H. Cootner,
in association with Martin Neil Baily

Massachusetts Institute of Technology
and Stanford University

No. 70   April, 1971

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1. Introduction: The General Form of the Model

This study presents an econometric model of the world copper industry, relatively disaggregated to incorporate different supply equations for each of the major producing countries and different demand equations for each of the principal consuming areas. Largely for reasons of data, the model is an annual one and there is no breakdown into types of copper use. Attention in the construction and use of the model centers on the geographic differences in the industry and on the two-price system generally prevailing wherein much of the copper in the United States moves at the U.S. producer price whereas most other copper moves at the London Metal Exchange (LME) price which, in recent years, has often been well above the U.S. producer price.

A general simplified outline of the structure of the model is as follows.

We begin with the U.S. market. The U.S. producers set the U.S. producer price to reflect what they believe to be a sustainable (and profitable) long-run level of copper prices, taking into account their own resulting supply decisions. They then take that price as given for the time being, believing it in their interest to have a relatively stable price, and decide on the amount of copper they will supply at that price. Other countries selling at the U.S. producer price do the same. This determines the supply of primary copper in the United States (except for imports).

Consumers of copper, on the other hand, take price as given. Some of those consumers will be able to buy at the U.S. producer price; some of them may not. The latter consumers can import copper at the LME price or can purchase scrap. Total demand for copper is also influenced (and very largely) by indicators of general industrial activity and by the free market price of aluminum, the principal substitute.

The supply of secondary copper in the United States depends in the model on the total amount of primary copper produced in the past, on the ease with which that copper can be gathered as scrap and on the scrap price which is very highly correlated with the LME price (as one would expect[1]). The ease of collection is taken as measured by the relative amount of previously produced copper collected in the previous year, being low when that was high and a great deal of the readily available copper already collected. A fuller description is given below.

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[1] Despite the fact that the two prices are very highly correlated, they do not seem to be equal after allowances for transportation and conversion costs.

The difference between U.S. total copper consumption and U.S. available supplies (primary, secondary, and imports) is the decrease (or increase) in U.S. inventories. If inventories tend to increase for some period of time and the LME price is below the U.S. producer's price, then producers gradually conclude that they set too high a price and that the proper long-run price is below the U.S. producer price. On the other hand, if inventories are being depleted over some period of time and the LME price is well above the U.S. producer price, then the producers gradually conclude that the proper long-run price is above the U.S. producer price. They then readjust the U.S. producer price in the indicated direction and the process begins again.

The model for the rest of the world operates in a similar fashion except that the LME price to which it responds is a free market price (most of the time). Given expectations about the LME price, producers selling at that price make output decisions and consumers buying at that price make consumption decisions (which are, of course, influenced by other variables as well). Scrap supply depends on factors similar to those operating in the U.S. Given the total supply of primary from producing countries, total supply of secondary, and net exports to the U.S., the total supply of copper available for consumption outside the U.S. is determined. The difference between this and total copper actually consumed gives the additions to inventories. The LME price responds to the size of inventories relative to total use of copper.

The two markets are linked in three ways. First, as already stated, the U.S. scrap price is highly correlated with the LME price, as both markets are generally free. Second, there is some flow of copper between the U.S. and

the rest of the world, Finally, producers take a large difference between the LME and the U.S. producer price as a signal that the U.S. market is probably out of long-run equilibrium and the U.S. producer price should be adjusted.

2. How and Why Does the Two-Market System Work?

The two-market system described above obviously requires further discussion. In particular, when the U.S. producer price is substantially below the LME price, as has often been the case in recent years, what keeps arbitrage from reducing the LME price and depleting U.S. stocks? Further, why do the U.S. producers consider it in their interest to maintain such a situation?

Part of the answer to the first question is that there do exist forces tending to bring the two prices closer together, although these forces are not fast-acting and strong. When the LME price is above the U.S. price for an extended period because demand is high, U.S. stocks do get drawn down and the producer price adjusted upward. Nevertheless, a high degree of arbitrage does not take place, presumably because in such a situation, when U.S. producers ration their sales, favored customers are unwilling to jeopardize their long-run relations with the producers for the sake of the short-run gains from arbitrage. Presumably, producers can fairly readily discover if a large customer is reselling on the LME.

The question remains, however, as to why it is in the interests of the producers to act in this way rather than adjusting the price upward immediately to the market clearing level and reaping the short-run returns from selling to more customers at higher prices. The basic answer seems to lie in the lag

structure of behavior on both sides of the copper market. It will be a repeated theme of this study and is well borne out in the results below that speeds of adjustment in the copper industry are very low. On the supply side this comes from the length of time necessary to bring new mines into production and the high investment in the working of existing mines. On the demand side, it occurs, at least in part, because of the length of time required to readapt some copper-using equipment to use aluminum. Such long lags mean that the price-elasticities of demand and supply are both substantially higher in the long than in the short run, and, as we shall see, supply elasticity, especially, is quite small in the short run and rather substantial in the long run. This leads to a fairly unstable situation in the short run where a small increase in the exogenous factors affecting demand can easily raise short-run market-clearing prices very considerably, even though the price which equilibrates long-run supply and long-run demand may not be much affected.

In this situation, the major U.S. producers may hesitate to raise price to reap short-run rewards for fear of doing two things. First, a large rise might encourage customers who can use aluminum to invest in aluminum-using machinery. Both the experience gained with that machinery and the time required to change back to copper would mean that such customers would not switch back again for a long time when prices fell. Indeed, it is not hard to see that, given the fact that the changeover is costly, copper producers may gain more by keeping copper prices to such customers just below the point at which the changeover to aluminum becomes profitable than they would be going above that point and later trying to regain those customers as price fell toward its long-run equi-

librium, since the regaining of those customers would require a fall in price not merely to its original level but to a level low enough to compensate for the cost of switching back to copper.

Since an inability to obtain copper would also provide an incentive to switch to aluminum, this argument suggests that the producers operate by rationing only those customers who cannot switch or for whom the cost of switching is low. This suggests that manufacturers of wire are generally able to purchase at the U.S. producer price, but it is difficult to obtain direct evidence on this point.[1]

A second reason for not raising the U.S. producer price to the short-run market clearing level may be that such a rise would be taken by other producers, particularly Canadian ones, as a signal that the long-run price was rising. This would begin to bring new mines into production and, given that the lags are so long, the resulting effects on supply would not disappear when the circumstances substantially raising the short-run market-clearing price ceased to operate. In this situation, the producers may prefer to forego the possible short-run profits rather than encourage new production which will appear on the market when the situation has changed.

Obviously, this second argument assumes that the short-run nature of the given situation is seen differently by the U.S. producers and the operators of potential new mines. However, once having established a system whereby the U.S. producer price is an index of what the U.S. producers believe a sustainable long-run price to be, those producers may hesitate to change it because a new, higher U.S. producer price may be taken to signal a high long-run price and may be acted on as such.

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[1] This implication has been drawn by David McNicol. See [9].

These considerations are difficult to support with hard evidence, although the producers are explicitly disturbed about irreversible substitution effects. They at least make plausible a situation which clearly does exist, the existence of which must be recognized in any adequate model of the copper industry.

3. Distributed Lags and Estimation Methods

Since, as stated, many of the crucial reactions in the copper industry take a good deal of time, the corresponding equations of the model are formulated in terms of distributed lags, with the dependent variable being influenced by past as well as present independent variables. The simplest and best-known formulation of distributed lags, and the one we have employed, is the Koyck or geometric lag[1], which can also be formulated as a stock adjustment model, along the following lines.

We take as an example, the supply curve of mine-produced copper in some country. Denote the amount of copper supplied in year t by S_t and the price received by the producers in question by P_t. Given the price, the producers would like to supply an amount S_t^*, which depends on P_t according to the long-run supply equation:

$$(3.1) \qquad S_t^* = \alpha + \beta P_t \quad ,$$

say. However, since it takes time to adjust supply, they do not immediately

[1] See Koyck [6] and Nerlove [10].

move to a new value for S_t^* in response to a new value for price, but rather begin to move in that direction. If we assume that it is only possible to move some fixed fraction, μ, of the desired distance in any year, then:

$$(3.2) \qquad S_t - S_{t-1} = \mu(S_t^* - S_{t-1}) \qquad (0 < \mu < 1) \quad .$$

Substituting (3.1) into (3.2) and rearranging terms:

$$(3.3) \qquad S_t = \mu\alpha + \mu\beta P_t + (1 - \mu)S_{t-1} \quad .$$

Lag (3.3) and obtain an expression for S_{t-1}, then lag it again and obtain one for S_{t-2}, and so forth. Repeated substitution of the lagged versions into (3.3) yields (where $\lambda = 1 - \mu$):

$$(3.4) \qquad S_t = \alpha + \mu\beta \sum_{\theta=0}^{\infty} \lambda^\theta P_{t-\theta} \quad ,$$

so that supply in year t depends on present and past prices with the weights given to lagged prices declining geometrically with the length of the lag. In this model, the short-run effect of price on supply is given by $\mu\beta$, but the long-run effect is given by β itself. If μ is relatively small, so that adjustments take place fairly slowly, then the long-run effect can be much larger than the short-run one. We would expect this in the case of copper supply.

There is another model which leads to the form (3.4) and its equivalent (3.3) (other than postulating (3.4) directly). This is to suppose that adjustments can in fact be completely made but that supply depends upon expected price, with expectations being formed in an adaptive way, as follows. Suppose that the supply curve is given by:

$$(3.5) \qquad S_t = \alpha + \beta P_t^* \quad ,$$

where P_t^* denotes expected long-run price. Suppose further that price expectations are formed by revising earlier expectations in the direction of actual

prices, according to the relation:

(3.6) $\qquad P_t^* - P_{t-1}^* = \mu(P_t - P_{t-1}^*)$ $\qquad (0 < \mu < 1)$.

Here μ is no longer the speed of adjustment of supply, but rather the speed of adjustment of price expectations. Equation (3.6) is equivalent to:

(3.7) $\qquad P_t^* = \mu P_t + (1 - \mu)P_{t-1}^*$,

so that expected price this year is a weighted average of expected price last year and actual price. Repeated lagging of (3.7) and substitution for lagged values of P_t^* yields

(3.8) $\qquad P_t^* = \mu \sum_{\theta=0}^{\infty} \lambda^{\theta} P_{t-\theta}$ $\qquad \lambda = 1 - \mu$

so that expected price is also a weighted sum of present and lagged prices, with the weights declining geometrically with the length of the lag. Substitution of (3.8) into (3.5) now yields (3.4).

Despite the fact that the two models are equivalent as regards the final supply equation, the stock-adjustment version makes much more sense than the adaptive expectations version for relations in the copper industry. It is perfectly clear that adjustments do take a considerable time; moreover, price expectations are not likely to be formed in the way described by (3.6) to (3.8), since, especially in the U.S. market, participants are likely to take current price as a much better index of relatively long-run price than is a weighted average of past prices, with λ much different from zero. Our interpretation will run in terms of the stock-adjustment model, therefore, and we have made no attempt simultaneously to incorporate the adaptive expectations feature.

Estimation of the stock-adjustment model requires some care. The easiest form in which to estimate it is (3.3), but merely estimating this by ordinary least squares (aside from the fact that P_t is an endogenous variable) will lead to inconsistency if the error term is autocorrelated. Such autocorrelation is quite likely in such models. It is not possible satisfactorily to treat a very general model of autocorrelation, and we have settled for a model in which the error term in question is first-order autocorrelated, an assumption which probably comes as close to the truth as any which the data will support. Thus, denoting the error term in (3.3) by u_t, and assuming it enters additively, we assume:

$$(3.9) \qquad u_t = \rho \, u_{t-1} + e_t$$

where e_t is assumed to have expectation zero and variance-covariance matrix $\sigma^2 I$, thus not itself being autocorrelated. For cases (not the supply equations) in which the right-hand side variables of such equations are predetermined (being either exogenous to the model or lagged endogenous variables), we proceed by choosing estimates of ρ and the other parameters to minimize the sum of squares of e_t. This is a consistent estimator and, if e_t is normally distributed, it is also maximum likelihood. On the normality assumption, asymptotic standard errors can be computed.[1]

When (as in the supply equations) one or more of the right-hand side variables are endogenous, an adaptation of the same technique, due to R. Fair [3] is used.[2] This is an instrumental variables technique which takes care of simulta-

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[1] See Cooper [1] for details.

[2] Fair observes that the asymptotic standard errors are difficult to calculate, but this is not the case, since instead of replacing sample values by probability limits, one can calculate the sample version of the asymptotic standard errors, relying on the fact that the formulae are only asymptotic in any case.

neous equations bias as well as of autocorrelation of the sort (3.9). It re-
quires that among the instruments be the current and once lagged values of all
predetermined variables in the equation and the lagged values of all endogenous
variables in the equation. In the results below, we indicate the instrumental
variables used in estimating each equation.

Before closing this section, we may remark that the autocorrelation assump-
tion (3.9) is restrictive, not merely because it is first-order, but also because
it assumed that, given u_{t-1}, u_t is not correlated with the past values of distur-
bances from other equations in the model. This is far better than assuming it is
uncorrelated with the current values of such disturbances, but it is still pretty
strong. More general assumptions are very difficult to handle, however.

Estimation of the supply equations for scrap copper presents certain special
difficulties, since those equations, as will be seen below, are nonlinear in the
parameters. We shall discuss the methods used in the section on the scrap supply
equations.

4. Supply of Primary Copper

In this section, we report the results for estimation of supply curves for
primary copper (mine production). Separate supply curves were estimated for the
United States, Chile, Zambia, Canada, and the Rest of World. The relative impor-
tance of the various suppliers can be seen from the figures for 1963[1]:

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[1] Data sources are listed and discussed in the appendix.

TABLE 4.1

Mine Production of Copper, 1963 (thousand metric tons)

United States	1100.6
Chile	601.1
Zambia	588.1
Canada	410.6
Rest of World[a]	1174.4
Total[a]	3874.8

[a] Excludes Eastern-bloc countries

The principal suppliers for which separate supply curves were not estimated are Peru, Republic of Congo, and Japan. Data problems were insuperable as regards the Congo; Japanese production did not seem large enough to warrant a separate effort; and an attempt to estimate a separate equation for Peru failed.

It will be noticed that a supply curve for Chile was estimated. As part of the purpose of the model is to examine the likely effect of alternative Chilean policies, that supply curve is used only for purposes of comparison. Forecasts will be made which examine the effects of departures from the supply relationship which has obtained in the past.

For the United States, the price of copper used in the supply equation was the Engineering and Mining Journal price deflated by the U.S. wholesale price index. The EMJ price is a weighted average of the U.S. producer price and the LME price, with the U.S. producer price getting more than 97.5% of the weight (this equation is reported in the section on prices). It apparently reflects

the prices at which copper is actually traded in the U.S. a bit more accurately than does the U.S. producer price. Nevertheless, according to the description of the model given in the opening section, it ought to be the U.S. producer price itself which enters the supply relation. Experimentation shows that the results obtained using the EMJ price are uniformly better and more reasonable than those obtained using the U.S. producer price directly and we have proceeded on that basis. This phenomenon can be rationalized by observing that when the EMJ and U.S. producer price get substantially out of line, the chances are pretty good that the latter price will be revised so that the EMJ price may capture long-run expectations a bit better than does its principal component.

Denoting U.S. mine production (thousands of metric tons) by USMP and the EMJ price (dollars per long ton) divided by the U.S. wholesale price index (1957-59 = 1.00) by USP_{EMJ}, the estimated U.S. supply equation is:

(4.1)
$$USMP_t = -160.04 + \underset{\substack{(.2129) \\ (2.9964)}}{.6372} USP_{EMJt} + \underset{\substack{(.2043) \\ (3.5545)}}{.7261} USMP_{t-1}$$

$$\rho = 0.5 \qquad \text{Years: 1949-58, 1962-66.}$$

In this, as in all later equations (save where noted), the figures in the first line of parentheses are the asymptotic standard errors of the corresponding estimated coefficients; the figures in the second line of parentheses are the ratios of the estimated coefficients to their asymptotic standard errors. Small sample significance tests are not known for such estimates, but a good rule of thumb is that a coefficient at least twice its asymptotic standard error indicates a statistically significant relationship.

The figure denoted as ρ is the estimated first-order autocorrelation coefficient of the disturbance in the equation; see equation (3.9), above. The final estimates are obtained by searching over alternate values of ρ ranging from -1.0 to +1.0 by steps of .1 and choosing those results (for all parameters) for which the sum of squares being minimized is least.

As indicated, the years 1959-61 and 1967-68 have been omitted (our data run through 1968 in other equations). This has been done to eliminate the effect of major copper strikes in the U.S. in 1959 and 1967-68.[1]

The slow speed of adjustment in U.S. copper supply is indicated by the coefficient of lagged mine production. Only a little more than a quarter (1.00 - .73) of the gap between desired production and actual production is closed each year (see the discussion in the preceding section). This is naturally reflected in a fairly large difference between short- and long-run supply elasticities. At the point of means for the period, the short-run price elasticity of supply is approximately .453, while the long-run elasticity is approximately 1.67. U.S. adjustment speed, while slow, however, appears faster than the adjustment speed of some of the other producing countries. To the results for those other countries we now turn.

There is some difficulty in deciding on the appropriate price variable to use in the Chilean supply curve because of the effect of the special exchange rates which have been used to tax the copper producers and because of the Chilean inflation. Fortunately, Mamalakis and Reynolds [7] calculate a series for the

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[1] The data begin before 1949, but the lags involved in the equation and the estimation method used mean the loss of two years at the start of the time period. Similarly, eliminating 1959 means eliminating 1960 and 1961 from direct use as observations.

price received by the producers and, while this only goes through 1959, it has been brought up to date by Vittorio Corbo Lioi who kindly made it available to us. We denote that price (the money price is taken as 1.00 is 1965 and deflated by the Chilean wholesale price index which is 1.00 in 1958) by ChP. Denoting Chilean mine production (thousands of metric tons) by ChMP, the results were as follows:

$$(4.2) \qquad ChMP_t = 91.37 + 415.4 \; ChP_t + .7206 \; ChMP_{t-1}$$
$$\qquad\qquad\qquad\qquad (164.9) \qquad\quad (.1309)$$
$$\qquad\qquad\qquad\qquad (2.520) \qquad\quad (5.505)$$

$$\rho = - \, 0.1 \qquad\qquad \text{Years:} \quad 1948\text{–}68.$$

The speed of adjustment is just about the same as in the U.S. At the point of means for the period, the short-run elasticity of supply is approximately .112, rather less than for the U.S. Long-run elasticity, moreover, is approximately .402, considerably less than the comparable U.S. figure. Chilean mine production thus does not appear to have been very price sensitive.[1]

For Canada, we used the EMJ price converted to Canadian dollars and deflated by the Canadian wholesale price index (1958 = 1.00) as the price variable.[2] Denoting this by $CanP_{EMJ}$ and denoting Canadian mine production (thousands of metric tons) by CanMP, the results are:

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[1] For comparison, a similar equation using the EMJ price and the U.S. wholesale price index was estimated. It differed from (4.2) in having a _very_ slow speed of adjustment and hence a high long-run (but not short-run) price elasticity. The price term was smaller relative to its asymptotic standard error than in (4.2), however and we accept (4.2) as the superior equation. The alternate results were:

$$(4.2') \qquad ChMP_t = - \, 54.43 + 274.0 \; USP_{EMJt} + .9517 \; ChMP_{t-1}$$
$$\qquad\qquad\qquad\qquad (156.6) \qquad\qquad (.0736)$$
$$\qquad\qquad\qquad\qquad (1.750) \qquad\qquad (12.921)$$

$$\rho = -0.2 \qquad\qquad \text{Years: } 1948\text{–}68.$$

[2] Some fraction of Canadian supply is sold at the LME price. Some preliminary attempts were made to include the LME price in the equation, but they were unsuccessful.

(4.3)
$$\text{CanMP} = -43.73 + \underset{(.03901)}{.09505} \text{ CanP}_{\text{EMJt}} + \underset{(.03932)}{.9873} \text{ CanMP}_{t-1}$$
$$\underset{(2.4367)}{} \qquad \underset{(25.11)}{}$$

$$\rho = -0.4 \qquad \text{Years: } 1948\text{-}67.$$

Clearly, the speed of adjustment is extremely low, far lower than for the U.S. or Chile. Less than 2% of desired adjustments take place in any one year. At the point of means for the period, the short-run elasticity of Canadian supply is approximately .188 and the long-run elasticity is far greater, being estimated at 14.84, although the exact figure is not very reliable.

The situation is rather similar for the estimated supply curve for Zambia. Here we used the LME price ($£$ per long ton) deflated by an index of the cost of living for Europeans in Zambia.[1] Denoting this price by ZP_{LME} and Zambian mine production (thousands of metric tons) by ZMP, the results are:

(4.4)
$$\text{ZMP}_t = -69.19 + \underset{(.4446)}{.1269} \text{ ZP}_{\text{LMEt}} + \underset{(.3138)}{1.103} \text{ ZMP}_{t-1}$$
$$\underset{(.2832)}{} \qquad \underset{(3.547)}{}$$

$$\rho = -0.3 \qquad \text{Years: } 1955\text{-}57, 1961\text{-}65.$$

The poorer quality of these results compared to the others doubtless reflects the poor data and the number of observations that had to be dropped because of strikes and political troubles. At the point of means, the short-run price elasticity of Zambian supply is approximately .0684, while the long-run elasticity is far greater. Indeed, as estimated, the effects of past prices never die out, although the coefficient of lagged mine production is not significantly above unity,[2] but the speed of adjustment is obviously extremely slow, although ultimately the

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[1] There is not much choice in terms of available data. There is also an incomplete series on the cost of living of Africans in Zambia.

[2] In forecasting with the model, that coefficient is set at unity.

Zambian supply curve is nearly flat. The very high long-run elasticities ought to be expected where new mines are developing and old ones far from exhausted.

The remaining supply curve estimated was for the rest of the world. The price variable used was the LME price expressed in dollars per long ton deflated by the U.S. wholesale price index (1957-59 = 1.00).[1] Denoting this by USP_{LME} and Rest-of-World mine production (thousands of metric tons) by RWMP, the results are:

$$(4.5) \qquad RWMP_t = -\ 28.44 + \underset{(2.3239)}{\underset{(.09561)}{.2222}} USP_{LMEt} + \underset{(9.1991)}{\underset{(.09601)}{.8832}} RWMP_{t-1}$$

$$\rho = 0.5 \qquad \text{Years: } 1948\text{--}68.$$

The speed of adjustment here is still low, but is higher than in Zambian and Canadian results. At the point of means for the period, the short-run elasticity of Rest-of-World supply is approximately .1963 and the long-run elasticity is approximately 1.680.

This completes the results for primary supply. In the estimation of the equations in this section, the instrumental variables used in addition to those required for the Fair method (see the preceding section) were as follows: the lagged ratio of non-U.S. stocks of copper to non-U.S. total use of copper (this appears in the price equations below); the lagged value of whichever of the LME and EMJ price did not appear in the particular equation being estimated; the lagged value of separately accounted for Western Hemisphere total mine production, taken as moving primarily at the EMJ price (U.S., Chile, and Canada); and the lagged value of the remaining mine production taken as moving primarily at the LME price (Zambia and Rest-of-World).[2] In the case of Chile, both the lagged

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[1] This is not a very satisfactory deflator, but it is difficult to see how to improve it much.

[2] In the case of the Zambian supply curve (4.4), where there were relatively few observations, only the sum of the last two variables was used.

LME and the lagged EMJ price were used as were the two ratios of the Chilean

price (ChP) to the other two prices, since presumably those ratios reflect

exogenous governmental actions in Chile.[1]

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[1] In a few cases in which data on instrumental variables were missing for a year or so at the beginning of the period, we extrapolated backwards to construct them. Note that this was not done for the variables actually appearing in the equation.

5. Secondary Supply

We now consider the supply of copper from scrap. Here we divide the world into the United States and the rest of the world. The breakdown available in the data is somewhat different for the two regions. Some idea of the order of magnitudes involved may be gained from Table 5.1 which reports figures for 1963.

TABLE 5.1

Secondary Supply of Copper, 1963 (thousand metric tons)

United States Old Scrap Collection	382.7
United States New Scrap	629.3
Rest of World Total Scrap[a]	1328.9
Total[a]	2340.9

[a] Excludes Eastern-bloc countries

Old scrap is scrap created by the destruction of old fabricated products; new scrap, on the other hand, is scrap created in the fabricating process itself by cutting or similar actions. It is either returned to the refiner or used again by the fabricator. While it is clear that both types of scrap are part of secondary supply, it is not quite so clear whether the copper scrap which is created by the fabricator should also be included in the variable representing the demand for copper or whether new scrap should be subtracted. While it is possible to argue on both sides of that question, the results for the demand equations (reported in the next section) are far better when direct use of scrap is included in demand and so all consumption figures in this study include such scrap.[1]

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[1] This follows usage in the data sources which call such a variable "total use". Note that "consumption" in the sources does not include direct use of scrap as do our "consumption" variables. (This is not the same issue as that discussed in the text.) See the data appendix for the way in which new scrap figures were calculated and for further discussion.

Obviously, the two different ways in which scrap is generated suggest two different models, and for the United States, the data permit their construction. We begin with the more complex case of old scrap.

The prime determinants of old scrap collections in our model are: the amount of copper available for collection in the broadest sense; the ease with which that copper can be collected; and the price paid for it.

In the broadest sense, the amount of copper available for old scrap collection in the United States is the total amount of copper embodied in already produced copper products. The change in the available scrap supply during year t is given by the identity:

(5.1)   Change in available scrap supply = primary production + net imports

of refined copper + net imports of fabricated copper

- increases in stocks of copper.

If we knew the amount of copper available for collection in any given base year, then the amount available in any other year could be calculated from (5.1). We do not have such a benchmark figure, however, and, as a result, we have estimated that figure as a parameter in our old scrap equation. Thus letting K_t be the copper available at the beginning of year t, we write:

(5.2) $K_t \equiv K_{1948} + K_t^*$,

where K_t^* is obtained by cumulating (5.1) from the beginning of 1948 and K_{1948} is to be treated as a parameter (all figures below are in thousands of metric tons).[1]

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[1] 1948 is used as a benchmark because changes in copper stocks are unknown before that date. The estimated equation begins with 1950 because K_{t-2} is required by the estimation method. A similar statement holds for the rest of the world (below) where K_{1950} is taken as a benchmark and estimation starts with 1952.

Obviously, it is only in the broadest sense that all copper in the United States is available for collection. In part, we account for this in our model by explaining what fraction of K_t is in fact collected. It must be recognized, however, that K_{1948} does not represent the cumulation of (5.1) from time immemorial, but rather represents the amount of copper in a rather narrower sense available for scrap collection at the beginning of 1948. Indeed, this narrower sense applies, of course, to copper produced or imported since 1948, so that K_t^* also is broader than is strictly appropriate. Fortunately, it is easy to see that the fact that not all produced or imported copper is really available for collection matters very little to our model or results.

Denote United States old scrap collections (thousands of metric tons) in year t by $USOS_t$.[1] The dependent variable in our equation below (its lagged value will also appear) will be Log $(USOS_t/K_t)$. Now suppose that instead of K_t being available, some constant fraction of K_t, say δ, is available. Then the true dependent variable should be Log $(USOS_t/\delta K_t)$ = Log $(USOS_t/K_t)$ − Log δ. Hence the only effect of the difference between using K_t and δK_t is to place a term in Log δ in the constant term of our results.

Obviously, this argument makes it convenient to use a logarithmic model here; moreover, it is natural to do so, since it is natural to think in terms of explaining the fraction of available scrap which is actually collected.

As a proxy for a measure of the difficulty of collection, we use the fraction of available scrap that was collected as such in the preceding year. This is not

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[1] The figure actually used was collection of old scrap. This includes a small amount of _imported_ scrap which does not precisely fit the model being discussed.

a very direct measure of difficulty, but it makes sense if we think of available copper as in forms and locations of differing ease of collection. One expects the copper easiest to collect to be collected first; hence if a relatively large amount of the available copper was collected last year, copper is likely to be relatively hard to collect this year. Accordingly (and quite unusually) we should expect the coefficient of the lagged dependent variable to be _negative_.

There is a separate scrap price in the United States, but it is very highly correlated with the LME price, as one would expect, since both are free market prices.[1] We found that it made little difference to the results which price was used, and, accordingly, we simplified the model slightly by using the LME price expressed in dollars per long ton and deflated by the U.S. wholesale price index (1957-59 = 1.00). We denote that price by USP_{LME}.

Unfortunately, the estimation of the old scrap supply equation runs into a minor but rather annoying difficulty. The equation is not linear in K_{1948}. Accordingly, we proceeded by choosing alternate values for K_{1948} and then estimating the resulting equation, intending to choose the version in which the final sum of squares was minimized.[2] As it turns out, however, the results are quite insensitive to the value of K_{1948} chosen, suggesting that any estimate thereof will have a very

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[1] As already mentioned, however, it does not appear to be the case that the two prices are equal after allowance for transportation and conversion costs. We are unable to explain this.

[2] The actual estimation procedure used was fairly complicated. First, USP_{LME} was regressed on its own lagged value, the lagged value of the EMJ price, the lagged change in the ratio of stocks outside the U.S. to total consumption outside the U.S. (see the section of prices, below), the lagged sum of mine production in the U.S., Chile, and Canada, and the similar lagged sum for all other countries. This yielded a predicted value of USP_{LME}, which we shall call $\widehat{USP}_{LME}$. Then, for each choice of K_{1948}, Fair's method was used to estimate the final equation with the instrumental variables being those required by the method plus the logarithm of $\widehat{USP}_{LME}$. Note that different Fair method instruments arise for different choices of K_{1948} and that each such estimation requires a search over values of ρ.

large asymptotic standard error. (The asymptotic standard errors reported below are all conditional on the values of K_{1948} chosen.) This would perhaps not be surprising -- although it does not occur in the results for the rest of the world, reported below -- since it indicates that scrap collections are not very dependent on copper produced before 1948; however, the sum of squares to be minimized continues to decline slowly as a function of K_{1948} until that parameter is well beyond any reasonable value.

Fortunately, this makes essentially no difference to our estimate of the price sensitivity of old scrap supply, our estimate of the effect of former collections, or our implied forecast of old scrap collections. We therefore present results for high and for low values of K_{1948} as well as for a value chosen to bear roughly the same relation to U.S. consumption as the similarly estimated figure for the rest of the world (below) bears to rest-of-world consumption. That value of K_{1948} is 60,000 thousand metric tons. When it is used, we obtain:

$$(5.3) \qquad \text{Log}\left(\frac{USOS_t}{60,000 + K^*_t}\right) = \underset{\substack{(.1261) \\ (-2.960)}}{-8.187 - 0.3731} \; \text{Log}\left(\frac{USOS_{t-1}}{60,000 + K^*_{t-1}}\right)$$

$$+ \underset{\substack{(.1064) \\ (3.968)}}{0.4222} \; \text{Log } USP_{LMEt}$$

$\rho = 0.9$ Years: 1950-68   Sum of Squared Residuals = 0.08338

By way of comparison, if we use alternate estimates of K_{1948} of 20,000 and 140,000 thousand metric tons, we obtain, respectively:

$$(5.4) \qquad \text{Log}\left(\frac{USOS_t}{20,000 + K^*_t}\right) = -7.3427 - 0.3761 \; \text{Log}\left(\frac{USOS_{t-1}}{20,000 + K^*_{t-1}}\right)$$

$$+ 0.4371 \; \text{Log } USP_{LMEt}$$

$\rho = 0.9$ \qquad\qquad Sum of Squared Residuals = 0.08755

and

$$(5.5) \quad \text{Log}\left(\frac{USOS_t}{140,000 + K_t^*}\right) = -9.116 - 0.3822 \ \text{Log}\left(\frac{USOS_{t-1}}{140,000 + K_{t-1}^*}\right)$$

$$+ 0.4417 \ \text{Log} \ USP_{LMEt}$$

$$\rho = 0.7 \qquad\qquad \text{Sum of Squared Residuals} = 0.07208 \ ^{[1]}$$

In all these equations, the short-run elasticity of old scrap supply with respect to the LME price is about +.42 to +.44. The implied long-run elasticity is lower because high scrap collections in one period mean lower ones in the next period, other things equal), being about +.31 to +.32. The different choices for K_{1948} affect only the second decimal place, and that only in a very minor way.

Our model for new scrap is much simpler; we estimated it as a linear function of total consumption. Denoting U.S. new scrap collections by $USNS$[2] and total consumption (total use) by USC (both in thousands of metric tons), the estimated equation is:

$$(5.6) \quad USNS_t = -277.9 + 0.3961 \ USC_t$$
$$(0.0524)$$
$$(7.555)$$

$$\rho = 0.2 \qquad\qquad \text{Years:} \quad 1947\text{--}68 \ ^{[3]}$$

---

[1] Asymptotic standard errors are not presented in (5.4) and (5.5) since they are for comparative purposes only and the extra computation (which is indeed extra) does not seem warranted. They would be much the same as in (5.3).

[2] Our new scrap figures, for reasons of consistency, were taken as Direct Use of Scrap plus Secondary Refined less Old Scrap. See the Data Appendix.

[3] Estimation was by Fair's method with additional instrumental variables, the lagged EMJ and LME prices, lagged U.S. mine production, and U.S. industrial production.

We could find no evidence of a significant price effect here. The elasticity

with respect to USC is 1.48 at the point of means.

Outside the U.S., the data do not permit a breakdown into old and new scrap

and we estimated a single equation for total scrap supply. This was necessarily

a hybrid of the two types estimated for the U.S. Denoting rest-of-world secondary

supply by RWS and rest-of-world total consumption by RWC^* (both in thousands of

metric tons)[1], the results were as follows:

$$(5.7) \quad \text{Log}\left(\frac{RWS_t}{53,000 + K^*_t}\right) = -4.221 - \underset{\substack{(0.4729)\\(-1.328)}}{0.6278} \text{Log}\left(\frac{RWS_{t-1}}{53,000 + K^*_{t-1}}\right)$$

$$+ \underset{\substack{(0.09879)\\(2.557)}}{0.2546} \text{Log USP}_{LME\,t} + \underset{\substack{(0.3204)\\(2.978)}}{0.9534} \text{Log}\left(\frac{RWC^*_t}{53,000 + K^*_t}\right)$$

$$\rho = 0.2 \qquad\qquad\qquad \text{Years: } 1952\text{--}68 \text{ [2]}$$

Short-run price elasticity is about +.25 and short-run elasticity with

respect to total copper consumption about +.95. The corresponding long-run

figures are about +.16 and +.52. In comparing these to the U.S. figures, the

hybrid nature of the equation should be recalled, whence it is clear that they

are not very different.

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[1] "Rest-of-world" in any section of this study means the countries not
explicitly studied in that section. Thus, here, the term refers to all countries
outside the U.S. (excluding Eastern-bloc countries). In the equation, K^* is, of
course, for those countries, not for the U.S. and is cumulated from 1950, rather
than 1948.

[2] The estimation method was similar to that described above for U.S. old
scrap, except that we formed not only $\widehat{USP}_{LME}$ and used its logarithm as an instru-
mental variable, but also $\widehat{RWC}^*$ and used its logarithm as an instrumental variable.
In this first stage of the procedure, the same regression was used as before to
form $\widehat{USP}_{LME}$. In the formation of $\widehat{RWC}^*$, the regressors were the lagged EMJ and LME
prices, lagged RWC^*, and the two lagged sums of mine productions.

6. Demand

We come now to the estimation of demand[1] equations for copper. Separate demand equations were estimated for the United States, Europe, Japan, and the rest of the world. Attempts to estimate separate equations for individual European countries did not result in very satisfactory demand equations, although the results for Europe as a whole were reasonably satisfactory. Why this should be true is hard to say. Aside from purely statistical reasons, such as the possibility that the disturbances in the demand equations in two neighboring countries may be negatively correlated, a partial explanation may lie in either of two facts. First, individual European countries may differ in the skill with which they manage to buy on the LME. If the success of their buying agents varies over time, the use of an annual LME figure may be better for Europe as a whole than for individual countries. Second, we have treated industrial production as a single aggregate. If intra-European exports have a different copper-using component than European industrial production generally, then an aggregate relation in which such exports are netted out may do better than individual ones. Neither of these explanations seems wholly satisfactory, however.

The relative importance of the various areas in copper consumption can be seen from the figures for 1963:

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[1] As used in this study, "demand" means "total use". See above.

TABLE 6.1

Total Use of Copper, 1963 (thousand metric tons)

United States	2320.4
Europe	**2631.8**
Japan	592.1
Rest of World[a]	526.0
Total[a]	6070.3

[a] Excludes Eastern-bloc countries.

We begin with the United States, for which data are best.

It is clear that the principal elements of a copper demand equation must be the price of copper, the price of substitutes, and measures of industrial activity. For the first, we used USP_{EMJ}, the EMJ price deflated by the U.S. wholesale price index. The remaining variables require some discussion, however.

The principal substitute for copper is, of course, aluminum. The obvious choice for a price to use is the deflated U.S. price of aluminum (deflation of course takes care of more general substitutes for copper). When this was attempted, the attempt failed, but it failed in an informative way. Without the inclusion of any aluminum price, a reasonably satisfactory looking demand equation was obtained. When the U.S. aluminum price was included, however, not only did its coefficient fail to be positive (as should be the case for a substitute), but it was significantly negative and the whole equation changed radically. One can not properly conclude from this merely that aluminum substitution fails to show up in the U.S. demand equation for copper; it obviously shows up in a way which can not be ignored, but which does not make economic sense.

Fortunately, the reason for this is fairly easy to find. During some of the period, the U.S. price of aluminum was a controlled price and aluminum was rationed. The U.S. aluminum price, therefore, is not a proper index of the cost of shifting to aluminum. Since data on the extent of rationing are hard to come by, a reasonably satisfactory way to take account of this is to use a free market aluminum price as an index of the real cost of obtaining aluminum. Since the London price was also controlled for part of the period, the most readily available price is the German price of aluminum in Deutschmarks per 100 kilograms and we used this converted to dollars and deflated by the U.S. wholesale price index (1957-59 = 1.00). It is denoted by USA1P, but the initials represent the deflator and not the source of the money price.[1]

Since even a short-run adjustment to prices is likely to be delayed, we used both copper and aluminum prices lagged by one year, so that demand depends on last year's (and previous years') prices.

There is more than one possible choice for an index of copper-using industrial activity. The two most obvious are the Federal Reserve Board Index of Industrial Production and the FRB Index of Construction Materials. Ideally, one would want to use both, but they are so correlated that no sensible result is obtained when both are included. We present results below using each. The Industrial Production Index (1963 = 100) is denoted by USIP and the Construction Materials Index (1957-59 = 100) by USCM.

One more matter needs to be discussed before proceeding to the results. A demand equation should not include changes in inventories of copper. Changes in

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[1] We have treated the aluminum price as exogenous to the copper market (in the short run) which is somewhat questionable but perhaps not too bad an approximation.

inventories of copper held as such are not included in the consumption data; they are treated separately below. There is another way in which copper inventories can be held, however; that is in the form of fabricated products. Unfortunately, there are no data on the inventories of copper fabricated products. There are data, however, on inventories of durable goods. If we assume that the change in the copper content of such inventories is reasonably linearly related to the total change in the inventories themselves, then that total change should be included in the demand equation. We denote it as $\Delta USID$. (It is measured in billions of dollars deflated by the U.S. wholesale price index, 1957-59 = 1.00.)

This argument has an important further and testable consequence, however. We are using a Koyck distributed lag in copper demand, as described in section 3, above, and already used for supply. The logic of the adjustment model leading to that lag suggests that the appropriate lagged variable to use is not lagged copper consumption as measured, but lagged copper consumption corrected for the (lagged) change in durable inventories. This, however, leads to the following.

The basic assumption on which we include the change in inventories in the equation is that the change in copper inventories held as finished goods is linearly related to the total change in inventories, that is, that the change in such copper inventories in year t is well approximated by a term $(\alpha + \beta\Delta USID_t)$. Denoting U.S. copper consumption in year t as measured (thousands of metric tons) by USC_t,[1] the stock adjustment model implies that instead of having USC_t on the right (where $\mu = 1 - \lambda$ is the speed of adjustment; see section 3), we should have $(USC_t - \alpha - \beta\Delta USID_t)$ on the left and $(USC_{t-1} - \alpha - \beta\Delta USID_{t-1})$ on the right, so that the full demand equation reads (letting γX_t stand symbolically for all the other variables already discussed):

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[1] This includes copper that will become new scrap. See above.

(6.1) $USC_t - \alpha - \beta\Delta USID_t = \delta + \gamma X_t + \lambda(USC_{t-1} - \alpha - \beta\Delta USID_{t-1})$,

where δ is a parameter. Rearranging this, we obtain:

(6.2) $USC_t = (\delta + \alpha - \lambda\alpha) + \gamma X_t + \beta\Delta USID_t - \lambda\beta\Delta USID_{t-1} + \lambda USC_{t-1}$,

so that not only should $\Delta USID_{t-1}$ be included, but its coefficient should turn

out to be minus the product of the coefficients of $\Delta USID_t$ and USC_{t-1}. It would,

of course, be possible (although slightly cumbersome) to impose this constraint

in estimating the equation, but it is far preferable not to impose it and to see

whether it is approximately satisfied in the results. If it is, we have gained

a check on a joint implication of the adjustment model assumed and the argument

as to inventories of copper held in finished form. In the event, the constraint

is indeed approximately satisfied, thus reinforcing our faith in the assumptions

made.

We are now ready for the results. When the Index of Construction Materials

is used, the estimated demand equation is:

(6.3) $USC_t = -194.7 - 1526\ USP_{EMJt-1} + 972.1\ USAIP_{t-1} + 7.029\ USCM_t$
                    (190.3)   (479.4)   (1.265)
                    (-8.020)   (2.028)   (5.557)

          $+ 54.11\ \Delta USID_t - 39.97\ \Delta USID_{t-1} + 0.7363\ USC_{t-1}$
             (5.904)   (7.667)   (0.1218)
             (9.166)   (-5.213)   (6.046)

          $R^2 = 0.991$ [1] $\rho = -0.8$ Years: 1950-58, 1962-66 [2]

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[1] In equations estimated by the Hildreth-Lu technique, $1 - R^2$ is the sum of squares of errors in the original equation to be estimated divided by the centered sum of squares of the dependent variable in that equation.

[2] As with all the equations the years' figures indicate the observations on the dependent variable directly used in the final regression. Two earlier years of data are used in the estimation method.

When the Index of Industrial Production is used, the results are similar, being:

$$(6.4) \qquad USC_t = -14.75 - \underset{(175.2)}{1237} \underset{(-7.060)}{USP_{EMJt-1}} + \underset{(464.2)}{829.0} \underset{(1.786)}{USAIP_{t-1}} + \underset{(0.9134)}{5.078} \underset{(5.559)}{USIP_t}$$

$$+ \underset{(6.413)}{60.49} \underset{(9.431)}{\Delta USID_t} - \underset{(7.102)}{44.40} \underset{(-6.251)}{\Delta USID_{t-1}} + \underset{(0.1126)}{0.7910} \underset{(7.024)}{USC_{t-1}}$$

$$R^2 = 0.991 \qquad \rho = -0.8 \qquad \text{Years: } 1950\text{-}58, 1962\text{-}66$$

In either case, the results are spectacularly good. Most of the estimated coefficients are several times their asymptotic standard errors, the only exception being the coefficient of aluminum price in (6.4). It is particularly noteworthy that this is true of the coefficient of copper price, an unusual feature for econometric estimates of demand equations.

More important than this, every coefficient has the expected sign and the constraint on the coefficient of $\Delta USID_{t-1}$ is very closely satisfied. In (6.3), that coefficient is -39.97, whereas the product of the coefficients of $\Delta USID_t$ and USC_{t-1} is 39.84; in (6.4), the coefficient is -44.40, whereas the corresponding product is 47.84.

In terms of elasticities, the two equations are very close. From (6.3), at the point of means for the period, the elasticity of copper consumption[1] with respect to copper price is $-.2131$ in the short run and $-.9002$ in the long run.

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[1] These are elasticities of USC_t with respect to the several variables, not of copper consumption after the implicit correction for the change in durable goods inventories. The latter cannot be computed without some assumption about the constant term, α, which appears in (6.1).

From (6.4), the corresponding figures are - .1727 and - .8168, respectively. The elasticity with respect to the price of aluminum implied by (6.3) is + .2392 in the short run and + 1.010 in the long run; the corresponding figures from (6.4) are + .2040 and + .9759, respectively. Finally, the elasticity with respect to the Index of Construction Materials is .3318 in the short run and 1.402 in the long run, from (6.3). The comparable elasticities from (6.4) with respect to the Index of Industrial Production are .1529 and .7317 respectively. Note that the last set of elasticities measures roughly the same thing in both equations, since in either equation, the variable in question is serving as a proxy for economic activity in general.[1]

When we turn to consuming areas outside the United States, the results are less strikingly good. In large part, this is probably due to the better quality of U.S. data. In particular, much better data on changes in stocks of copper exist for the U.S. than for most other countries; moreover, only for the U.S. was it possible to utilize data on stocks of durable goods in general to perform the correction for copper held in the form of finished goods which proved so spectacularly successful in (6.3) and (6.4).

The demand equation for Europe is similar to that for the United States, in that it includes copper price, aluminum price, lagged copper consumption, and an index of industrial activity. The copper money price used was, of course, the LME price (£ per long ton), and the aluminum money price used was again the German

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[1] The linear form used for the demand equation must be considered only an approximation, especially where such activity variables are concerned, so these last elasticities have perhaps less meaning than do the elasticities with respect to copper and aluminum prices.

aluminum price (DM per 100 kilograms). Both of these were then converted to dollars. Construction of an appropriate deflator requires some discussion, however.

For each European country, we formed a dollar-equivalent wholesale price index, by taking the country's own wholesale price index (1958 = 1.00) and dividing it by an index of the country's exchange rate vis-à-vis the dollar. The latter was an index of local currency per dollar, scaled to be 1.00 in 1958. These individual country indices, being now in a common unit, were then combined in a weighted average, the weights being proportional to 1963 industrial production (these are the weights given by the OECD to its European members in its industrial production index[1]). The weights are as follows:

TABLE 6.2

Weights of Individual Countries in European Price Index (percent)

Germany	26.6
United Kingdom	21.3
France	19.6
Italy	10.0
Belgium and Luxembourg	3.5
Sweden	3.5
Switzerland	3.5
Netherlands	3.3
Spain	3.3
Austria	2.0
Finland	1.3
Portugal	0.8
Greece	0.5
Ireland	0.5
Total	$\overline{100.0}$[a]

[a] Individual weights as given do not sum to 100.0 because of rounding.

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[1] General Statistics, Organization for Economic Cooperation and Development (Paris, OECD, 1970).

Three different indices of production were tried, all with very similar results. They were the OECD Europe industrial production index, the UN index of manufacturing, and the UN index of industrial production. The UN indices have somewhat wider coverage than the OECD index. We report the result using the UN index of industrial production.

The dependent variable was total use of copper (thousands of metric tons) for all non-communist Europe. We denote it by EURC. The two prices are denoted by $EURP_{LME}$ and EURA1P, respectively, and the index of industrial production 1958 = 100) by EURIP.

The estimated European demand equation is:

(6.5)

$$EURC_t = -1220 - 2693 \; EURP_{LMEt-1} + 285200 \; EURAIP_{t-1}$$
$$(1961) \qquad\qquad (249200)$$
$$(-1.373) \qquad\qquad (1.144)$$

$$+ \; 904.5 \, EURIP_t + .5426 \; EURC_{t-1}$$
$$(579.5) \qquad (.3395)$$
$$(1.561) \qquad (1.598)$$

$$R^2 = 0.9335 \qquad\qquad \rho = -0.1 \qquad\qquad Years: \; 1952-68$$

The elasticities at the point of means are as follows: with respect to copper price, − .0878 in the short run and − .1920 in the long run; with respect to aluminum price, + .6133 in the short run and 1.341 in the long run; and with respect to industrial production, + .4534 in the short run and + .9913 in the long run.

The results indicate a rather slower speed of adjustment than was found for the United States. Elasticities with respect to copper price are rather lower and with respect to aluminum price rather higher than for the United States.

Elasticities with respect to industrial production are roughly the same.

It must be remembered, however, that the results for Europe are not nearly so reliable as those for the U.S. Moreover, it is not possible to make all the same adjustments for changes in copper stocks as for the U.S., so that, in particular, the effect of changes in the stocks of copper held as finished goods are included rather than being separately accounted for. Given this, it is nontrivial to have obtained a demand equation with all coefficients being of the expected sign and reasonable magnitude and bigger than their respective asymptotic standard errors.

The results for Japanese demand are rather different. Here alone in the model, we could find no evidence of a lagged adjustment process. This may be due to the very rapid growth of the Japanese economy, since a situation in which a significant fraction of industrial capacity is new each year is likely also to be a situation in which a significant part of copper demand does not depend on the relative adjustment of old capacity to new price situations. Nevertheless, it seems doubtful that adjustment is so rapid as to be complete in a single year (although it must be remembered the price entered in the model is last year's price).

Moreover, related also to the rapid growth of Japan is the fact that we would expect the variables with large effect on Japanese copper demand to be those which are related to industrial production. Despite this, we did find a negative effect of copper price; aluminum price, on the other hand, did not appear to play any role.

We present two equations which differ only in the measure of copper-using

activity. (The two measures are too collinear to use in the same equation.) The first of these is an index of total industrial production (1955 = 100) and denoted by JIP. The second is an index of the production of construction materials (1955 = 100) denoted JCM. The price variable used was the LME price expressed in yen per long ton, deflated by the Japanese wholesale price index (1958 = 1.00). It is denoted by JP_{LME}. Japanese consumption of copper (thousands of metric tons) is denoted by JC.

The two equations are as follows:

(6.6)
$$JC_t = 124.2 - \underset{\substack{(.00008916)\\(-1.780)}}{.0001587} \; JP_{LMEt-1} + \underset{\substack{(0.0716)\\(24.06)}}{1.723} \; JIP_t$$

$$R^2 = .982 \qquad \rho = 0.0 \qquad \text{Years: } 1951\text{-}68 \quad .$$

(6.7)
$$JC_t = 66.96 - \underset{\substack{(.0001067)\\(-1.475)}}{.0001574} \; JP_{LMEt-1} + \underset{\substack{(0.1413)\\(17.22)}}{2.433} \; JCM_t$$

$$R^2 = .975 \qquad \rho = 0.1 \qquad \text{Years: } 1954\text{-}68 \quad .$$

The results using the total industrial production index seem somewhat more reliable than those using the index of construction materials, although this may merely reflect the fact that they are based on three more years of data. In any case, the implications of both equations are quite similar. There is no difference between short and long-run elasticities. From (6.6), the elasticity of Japanese consumption with respect to price is -.09428 at the point of means for the period; from (6.7), that elasticity is -.1184. From (6.6), at the point of means, the elasticity with respect to the index of industrial production is .6014; from (6.7), the comparable elasticity with respect to the index of construction materials is .9921.

Finally, a demand equation was estimated for the rest of the world. Here the prices were the LME price, and the German aluminum price, both expressed in dollars and both deflated by the U.S. wholesale price index (1957-59 = 1.00).[1] The former price is denoted (as before) by USP_{LME} and the latter, as before, by USAlP. For a production index, we used that compiled by the UN.[2] It is denoted by RWIP. The dependent variable is rest-of-world total use of copper (thousands of metric tons) and is denoted by RWC. The results are as follows:

(6.8)
$$RWC_t = -11.82 - \underset{\substack{(.03746)\\(-3.237)}}{.1212}\ USP_{LMEt-1} + \underset{\substack{(3.014)\\(0.2929)}}{.8828}\ USAlP_{t-1}$$

$$+ \underset{\substack{(0.8127)\\(2.426)}}{1.971}\ RWIP_t + \underset{\substack{(.1613)\\(4.740)}}{.7646}\ RWC_{t-1}$$

$$R^2 = .939 \qquad \rho = -0.3 \qquad \text{Years: } 1951\text{-}68$$

The result as to the effects of aluminum price is unreliable, the remaining coefficients are considerably larger than their asymptotic standard errors. At the point of means, the estimated elasticities are as follows: with respect to copper price, - .2177 in the short run and - .9248 in the long run; with respect to aluminum price, + .1074 in the short run and + .4561 in the long run; and with respect to industrial production, + .4087 in the short run and + 1.736 in the long run. The price elasticities are fairly similar to those found for the United States.

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[1] This is not a very satisfactory deflator, but it is difficult to see how to improve it much.

[2] Excluding U.S., Europe, and Japan.

In general, the demand results show copper demand to be rather inelastic with respect to copper price, even in the long run. The most important determinants of copper demand, as one would expect, are the levels of industrial activity in the consuming countries.

7. Prices

So far, there are two prices directly used in the model, the LME price and the EMJ price. As already mentioned, however, the latter is a constructed price which is very closely associated with the U.S. producer price. Indeed, expressing all three in common units, and denoting the three prices by P_{EMJ}, P_{LME}, and P_{Prod}, least squares regression (with no constant term) reveals:

(7.1)
$$P_{EMJt} = \underset{(.00842)}{.9762} P_{Prodt} + \underset{(.007045)}{.01538} P_{LMEt}$$
$$(115.9) \qquad (2.183)$$

$$R^2 = .999 \qquad \text{Years: } 1946\text{-}68$$

The sum of the coefficients is not very different from unity.

We have already outlined in the introductory section the way in which the model describes the setting of the U.S. producer price. Essentially, that price is set as a long-run price, being adjusted relatively slowly. Adjustments in it are made in response to indications that the U.S. market is drifting out of equilibrium. One such indication is clearly the accumulation of private stocks of copper in the United States; another is a large difference between the U.S. producer price and the LME price. We find both of these to have a definite effect.

Denoting the change in U.S. private stocks of copper during year t by ΔUSS_t and measuring it in thousands of metric tons, we divide it by U.S. consumption in year t to obtain a measure of the relative size of stocks. Assuming that decisions are taken for this year with an eye to last year's variables, we enter the resulting ratio lagged as well as the lagged difference between the LME and the U.S. producer price. Denoting deflation by the U.S. wholesale price index (1957-59 = 1.00) by the prefix US, as before, the estimated equation is (where all prices are in deflated dollars per long ton):

$$
(7.2) \qquad USP_{Prodt} = 320.8 - \underset{\substack{(825.4)\\(-2.248)}}{1856} \frac{\Delta USS_{t-1}}{USC_{t-1}} + \underset{\substack{(.1461)\\(1.840)}}{.2689} (USP_{LMEt-1} - USP_{Prodt-1})
$$

$$
+ \underset{\substack{(.2452)\\(2.438)}}{.5980} USP_{Prodt-1}
$$

$$
R^2 = .556 \qquad \rho = 0.2 \qquad \text{Years: } 1952\text{-}66 \ [1]
$$

Note that while the speed of adjustment is well below unity, it is high relative to that found for supply adjustments and for U.S. demand adjustment. About forty percent of the gap between desired and actual price is covered in a year. The other two coefficients have the expected sign.[2]

---

[1] 1967 and 1968 were omitted because of the copper strike. Figures on the change in government stocks of copper required to calculate the change in private stocks are not available before 1949.

[2] Elasticity calculations do not make much sense for this equation, since both of the operative variables can (and do) change sign over the period.

We turn now to the equation explaining the LME price. Here also we expect the size of copper stocks to play the chief role (although, of course, these will be stocks outside the U.S.); however, the rationale behind this is a bit differ-ent than in the case of the U.S. producer price. Whereas that price is an admin-istered price and the equation just reported described the behavior of those ad-ministering it, the LME price is basically a free price. The equation describing LME price determination is hence one describing the behavior of a market. In effect, one can think of the LME price as adjusting until holders of stocks are satisfied to hold them.[1] On this view, it should be the size of stocks (relative to consumption) rather than the change in stocks that counts for the LME price; however, as one might expect the relationship to be a long-run one, it is sensible to use a distributed lag and also to investigate the effect of lagged stocks as well as current ones.

When this is done, however, a striking result emerges. In every version of the equation tried[2], what appears to matter is the change in the stocks-consumption

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[1] There is a vast literature on this subject. See, for example, Telser [11].

[2] There were a number of these. They included versions with and without lagged price and with current and lagged stocks-consumption ratios being for year t and t-1 or for year t-1 and t-2. In addition, since stocks figures do not exist for the total non-U.S. region, they had to be constructed by cumulating figures for changes in stocks from the estimates given in the next section. This is a satisfactory procedure, given a benchmark estimate of stock level in a particular year, but no such estimate exists except for the U.S. where only the change in stocks enters the model in any case. For the rest of the world, stocks were arbitrarily assumed zero at the end of 1949. The only effect of this is in the equation under discussion where stock levels are divided by consumption. This suggests adding a correction to the equation in the form of the current and lagged reciprocals of consumption. When this was done, the corrections mattered little, but the phenomenon under discussion in the text persisted. Note that the change in the stocks-consumption ratio is very close to the change in stocks divided by consumption so that the correction should not matter if only changes are important; this slightly reinforces that finding.

ratio and not its level. Indeed, when the change and the level of the ratio are both used in the equation (which is equivalent to using both the current and the lagged values of the ratio), not only is the coefficient of the level very close to zero (and small relative to its asymptotic standard error) but also it is slightly positive, which does not make economic sense.

Denoting the level of Rest-of-World stocks at the end of year t by RWS_t, the two best equations were as follows:

$$(7.3) \quad USP_{LMEt} = 285.6 - \underset{\substack{(1565.5) \\ (-2.312)}}{3620} \left(\frac{RWS_t}{RWC_t} - \frac{RWS_{t-1}}{RWC_{t-1}} \right) + \underset{\substack{(.4886) \\ (1.628)}}{.7957} \, USP_{LMEt-1}$$

$$\rho = -0.3 \qquad \qquad \text{Years: } 1952\text{-}68$$

and

$$(7.4) \quad USP_{LMEt} = 415.2 - \underset{\substack{(738.8) \\ (-2.626)}}{1939.7} \left(\frac{RWS_{t-1}}{RWC_{t-1}} - \frac{RWS_{t-2}}{RWC_{t-2}} \right) + \underset{\substack{(.1769) \\ (3.369)}}{.5960} \, USP_{LMEt-1}$$

$$\rho = -0.1 \qquad \qquad \text{Years: } 1953\text{-}68 \, [1]$$

Now, the difficulty in accepting equations such as these is their long-run implications under certain circumstances. In them, the LME price goes down when stocks go up relative to consumption, a perfectly reasonable result. The possibly surprising thing, however, is that if one considers what would eventually happen to price if the stocks-consumption ratio were to remain unchanged indefinitely, the implication is that the LME price would approach an equilibrium level inde-pendent of the level at which the stocks-consumption ratio remained constant.

---

[1] The only instrumental variables used in estimating (7.4) were those required by Fair's method.

What the stocks-consumption ratio had been doing before entering the constant phase would matter for the short and middle run, but in the long run, a constant stocks-consumption ratio would lead to the same price, whatever the constant level.

This is not a result which is instantaneously acceptable, although its importance for short-run or even middle-run forecasting is extremely limited. We can, however, go a long way toward rationalizing it along the following lines.

The LME price not only adjusts according to the desire to hold copper stocks, but, much more fundamentally, it serves as the long-run equilibrator of supply and demand. If, over a very long period, there is no change in the stocks which people desire to hold, then stocks asymptotically fail to affect the price which asymptotically approaches the level at which long-run supply and long-run demand are in balance so that stocks will not in fact change. We have already seen, however, that the supply of copper is extremely elastic in the long run, so that such an asymptotic price, so far as we can predict, is a constant (in real terms) approximating the almost horizontal level of the long-run supply curve.[1]

Indeed, if we recall that the change in stocks is defined as the difference between total supply and total demand, it becomes apparent that the price adjustment equations merely state that price adjusts in the direction of excess demand and does not move (in the long run) if supply and demand are in long-run balance.

It must be emphasized, however, that such arguments and implications are only asymptotic and that speeds of adjustment are slow enough in the copper industry that the long run is very long and not very relevant. Short-run movements in demand (and supply) and consequent short-run changes in stock will have much more to do with the price over any reasonable horizon than will whatever asymptotic level would be approached were everything forever in equilibrium.

For what it is worth, however, the asymptotic equilibrium level of the LME price is $1506/long ton from (7.3), $1164/long ton from (7.4)(in 1969 dollars). This compares to an average price of $1490 (current dollars) in 1969.

---

[1] This is consistent with the argument in Herfindahl [5, pp. 230-235] that long-run marginal costs in copper supply are nearly constant.

It must be sharply emphasized that this statement in no way constitutes a pre-diction that the LME price will approach this level over the next several years. We shall return to this in the section on forecasting.

8. Closing the Model

There remain only three more relationships to close the model. These are the two identities accounting for changes in stocks and the equation describing net exports of copper from the rest of the world into the United States.

The first two are easy to describe. Let $\Delta USGS_t$ be the change in United States government stocks during year t (thousands of metric tons) and let RWX_t denote net exports from the rest of the world to the United States (thousands of metric tons)[1]. Then the change in United States private stocks is given by the identity:

$$(8.1) \qquad \Delta USS_t = USMP_t + USOS_t + USNS_t + RWX_t - USC_t - \Delta USGS_t \quad ,$$

which is to say that the change in U.S. private stocks is the total supply (mine production, old scrap, new scrap, and imports) less consumption and the amount going into U.S. government stocks. Note that new scrap is included in our consumption figures and is netted out of the change in stocks, as it should be, since what matters is copper put through the refining process once during the year less the disappearance of copper into true consumption and government stocks.

A similar identity gives the change in stocks in the rest of the world. As already remarked in an earlier footnote, there are no benchmark figures for the size of stocks outside the U.S., so we have cumulated the change in stocks from the end of 1949. Aside from our inability to account for governmental stocks

[1] Excluding scrap already counted in old scrap collections. See above.

outside the U.S.,[1] therefore, our rest-of-world stock figures differ from correct ones by an unknown constant. This matters only to the LME price equation, and, as discussed in the preceding section, it matters very little since only changes appear to be important in that equation.

The identity giving the change in rest-of-world stocks is:

$$(8.2) \qquad \Delta RWS_t \equiv ChMP_t + CanMP_t + ZMP_t + RWMP_t + RWS_t$$
$$- EURC_t - JC_t - RWC_t - RWX_t$$

which says that the change in rest-of-world stocks is given by the sum of mine production outside the U.S. (Chile, Canada, Zambia, and rest-of-world) plus rest-of-world scrap supply less consumption of copper outside the U.S. (Europe, Japan, and rest-of-world) less net exports to the U.S.

The estimation of the equation explaining net exports to the United States (RWX) is not a simple matter. One naturally expects a prime factor in that equation to be the difference between the producer price and the LME price, but simple attempts to explain net exports by this difference do not get very far. The reason is not hard to find. When a producer price is well below the LME price, there is often rationing of primary copper in the United States and copper may flow into rather than out of that country.

This suggests several devices to try. One of these is to divide the difference between producer price and LME price into two variables, depending on whether that difference is positive or negative. In fact, when this was done, the results were often suggestive, but when the other variables about to be discussed were added to

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[1] Figures on government stocks do not exist. The only non-U.S. government stocks whose existence are known to us were held in small amounts by the United Kingdom during the Korean War.

the equation, the coefficients on the two price-difference variables became almost equal, lending powerful support to the view that the other variables had adequately accounted for the rationing problem.

Those other variables were two. The first is the excess of U.S. consumption over U.S. mine production, an indication of the shortfall which must be filled by secondary copper, stock changes, and imports. The second, denoted XD, is a dummy variable representing the presence or absence of export controls in the United States (recall that RWX is <u>net</u> exports). This variable was given the value of unity for 1949-52, 1955-56, and 1966-68. For 1953, it was set at .75, for 1954 at .25, and for 1957 at .5 (fractional values occurring when controls were imposed or taken off in the middle of a year). For all other years, the variable was set equal to zero.

The resulting equation was as follows (note that the prices are in deflated dollars per metric ton):

$$
(8.3) \quad RWX_t = -795.5 + \underset{\substack{(0.6310)\\(2.2138)}}{1.397} (USP_{Prodt} - USP_{LMEt})
$$

$$
+ \underset{\substack{(0.4238)\\(2.204)}}{.9340} (USC_t - USMP_t) + \underset{(55.22)}{145.8} XD_t
$$
$$
(2.640)
$$

$$
\rho = -0.1 \qquad\qquad \text{Years: } 1952\text{-}68 \ .
$$

The results are strikingly good, particularly since, as mentioned, attempts to explain net exports without the two final variables lead to very poor results. Elasticity calculations do not mean much when the variables lie close to zero, so we do not give them. An increase in the U.S. producer price of one cent per pound, with the LME price constant, increases net exports to the U.S. by about

31 thousand metric tons other things equal. (By way of comparison, net exports were about 201 thousand metric tons in 1963, so there is substantial price sensitivity.) An increase in the gap between U.S. consumption and mine production is filled, other things equal, about 93% by imports into the U.S. This figure seems a little high, but (aside from the fact that a more plausible figure would lie within one asymptotic standard error of the point estimate) it must be remembered that this is <u>not</u> the same statement as the assertion that 93% of the entire gap is filled by imports. Even if the two prices were identical and export controls not present, the large negative constant term would mean that much less of the gap is filled on the average than at the margin.[1]

---

[1] In the estimation of (8.3), the following instrumental variables were used in addition to those required by the Fair method: lagged mine production (including the United States) and lagged stocks/use outside the United States.

Note that XD was taken as exogenous (indeed it is the only exogenous variable in (8.3)).

It is true, of course, that the imposition of export controls is a function of the situation in the United States copper market. We assume that it depends on lagged rather than current variables, however.

9. General Conclusions

In the first section, we outlined the general way in which the model works. Part II of this study will discuss specific forecasts. Intermediate between such general and such specific discussions, however, are the conclusions to be drawn from the model as estimated and reported in the preceding sections. This section rather briefly discusses some of the more important such conclusions.

The central overriding fact about the copper market is the very high elasti-
city of supply coupled with a low adjustment speed and relatively low short-run
elasticities. This means that while long-run equilibrium price -- in the sense
of that price which would make long-run supply equal long-run demand for current
levels of the various exogenous variables -- may be well below current price,
there is no very marked tendency for price to approach such a long-run equili-
brium. This occurs because of general growth in the activity variables influ-
encing demand. Steady growth in those variables, even at a relatively low rate,
leads to prices forever above long-run equilibrium because of the inelasticity
of short-run supply. In essence, the demand curve is continually shifting out-
ward and new supply coming slowly into production.[1]

On the other hand, this very fact means that prices are very sensitive to a
reduction in the rate of growth of the exogenous activity variables and especially
to a decrease therein. If those variables slacken their growth rate or stop grow-
ing altogether, the new supply induced by previously high prices will come onstream
and prices will drop. Since demand is relatively price inelastic, such a drop
can be quite substantial. Indeed, it is clear that whereas the high long-
run elasticity of supply makes for a stable, if largely irrelevant long-run
price, the low short-run elasticities of supply and demand make for a rela-
tively unstable short-run price which is very sensitive to changes in general
economic conditions. This is more true of the LME price than it is of the
U.S. producer price whose fluctuations are deliberately reduced by the price-
setters, but it is somewhat ture of the latter price as well.

---

[1] This basic result was also found in a preliminary study done some years
ago by one of the authors. See Fisher [4].

The way in which such instability enters the model is, of course, through stock accumulation. A fall in demand, given the low short-run and high long-run elasticity of supply leads to a sharp increase in copper stocks. This in turn depresses the price, which, indeed will remain depressed so long as stock accumulation continues. An upturn in general economic conditions, on the other hand, will lead to an outward shift in demand and a decumulation of stocks, providing an upward push on price. Because of the long time required to make adjustments, however, the actual timepath of prices is not a simple one. Part II of this study discusses forecasts of that path) and of the timepath of the other variables as well) under various assumptions as to Chilean supply.

APPENDIX: Data Used in the Model

Almost all data on copper was obtained from the annual publication

Metal Statistics Metallgesellschaft Akliengesellschaft (Frankfurt Am Main).

This source seems the most reliable,consistent source of data covering the

world industry although for just the U.S. more detailed data are available

from the Bureau of Mines and the Copper Development Association. The data

in Metal Statistics are revised frequently and so the most recent figure

available was used. In order that the reader may check the definition of the

variables used in the model we give, below, page number references to the

1969 edition of Metal Statistics (56th Annual Issue).

Prices

The   **EMJ**   and LME prices were taken from the average figure in Metal

Statistics, page 284. The producer price from Metal Statistics: The Pur-

chasing Guide of the Metal Industries (The American Metal Market Company:

New York). The figure was the average producer price of electrolytic copper

(page 131 of the 1970 edition).

Wholesale price indexes were taken from the UN Statistical Yearbook and

exchange rates from International Financial Statistics (The International

Monetary Fund).

Primary Supply

Mine production for the US (page 190), Chile (page 196), Canada (page

198), Zambia (page 188). The rest of world figure was the Free World

Figure (page 19) less the countries above.

Mine production rather than smelter production was used because some secondary copper is introduced at the smelting stage. Since we estimate secondary production separately using smelter would involve double-counting.

Secondary Supply

Total Scrap supply for the US was taken as the sum of production of secondary refined copper plus direct use of scrap (page 190). Direct use of scrap includes new and old scrap in good condition which does not require re-refining. For the US there is a figure for old scrap (page 190) which we used to estimate the old scrap model. The difference between total scrap supply and old scrap was then used in the new scrap model. Sample figures for 1963 are given below:

United States Scrap

1)	Production of Secondary Refined Copper	274.0
2)	Direct Use of Scrap	738.0
3)	Total Scrap Supply (1) + 2))	1012.0
4)	Old Scrap	382.7
5)	New Scrap (3) – 4))	629.3

The figure for 5) does not correspond to the new scrap figure given on page 190 of Metal Statistics since the latter excludes direct use of new scrap. For the rest of the world the total scrap supply was defined in the same way as direct use plus secondary refined (page 25). There is no breakdown into old scrap for countries other than the US.

Rest of World Scrap

1) Production of Secondary Refined   388.0

2) Direct Use   940.9

3) Total Scrap Supply $\overline{1328.9}$
        (1) + 2))

In computing the figure for K_t (the stock of copper lying around)
we used primary production from page 190, net imports of refined from pages
192 and 193, and change in stocks was the computed figure explained below.
The net imports of fabricated items were from page 194 with each item
weighted approximately by copper content. The weights were based on McMahon
Copper: A Materials Survey (US Bureau of Mines, 1965) and were as follows:

Weights Used in Calculating Net Imports of Copper Fabrications

Copper (Scrap, Alloy Scrap { Copper content }   1.0
        rods, tubes, wire, plate, other)

Brass (rods, tubes, wire)   0.65

Muntz metal   0.60
Zinc (wire, rods, powder)   0.70

Net imports for the rest of the world are the negative of the US figure.

Demand

The total use of copper figure for each country's demand equation was
consumption of refined copper plus direct (or actual) use of scrap. Figures
are given on pages 24 and 25.

The exogenous variables used in the demand equations were obtained as
follows:

1)   Indexes of Industrial production for all countries and groups of
     countries from UN Statistical Yearbook

2)   German price of aluminum from: Metal Statistics, page 277 (average)

3)   US index of Construction Materials from: The Bulletin of the
     Federal Reserve Board

4) US Inventories of Durables from: The Economic Report of the President

5) Japanese Index of Construction Materials from: Monthly Statistics of Japan.

Stocks of Copper

Stocks of copper were computed as a residual and the best explanation
is provided by the sample figures given below:

SAMPLE FIGURES FOR THE UNITED STATES (1963)

		Thousands of Metric Tons
1)	Mine Production	1100.600
2)	Net Imports of Refined, Unrefined, and Ores	201.958
3)	Old Scrap	382.700
4)	New Scrap	629.300
5)	Total Supply (1) + 2) + 3) + 4))	2314.558
6)	Total Use of Copper	2320.400
7)	Decrease in Stocks (6) - 5))	5.842
8)	Decrease in Government Stocks	10.310
9)	Increase in Private Stocks (8) - 7))	4.468

The figures are all from Metal Statistics except i) government stocks which
came from McMahon [8, p. 252] and Copper Development Association [2] and
ii) imports of copper ores which were from [2], up to 1953.

A similar computation was made for the rest of the world except that
figures for government stocks (held by the UK government for a short period)
were not available. Net imports of the rest of the world are the negative
of the US figure.

The figures for changes in stocks do not provide a benchmark. For
the US the benchmark for private stocks was 614.162 thousand metric tons
at the end of 1949. This figure was stocks of copper at primary smelting
plants plus refined copper held by fabricators reported in McMahon [8, p. 250].
For the rest of the world, the benchmark was zero at the end of 1949.

REFERENCES

[1]   Cooper, J.P., "Asymptotic Covariance Matrix of Procedures for Linear
        Regression in the Presence of First-Order Serially Correlated Dis-
        turbances," to be published in Econometrica (March or May, 1972).

[2]   Copper Supply & Consumption 1948-1967 (New York: Copper Development
        Association, 1968).

[3]   Fair, R.C., "The Estimation of Simultaneous Equation Models with Lagged
        Endogenous Variables and First Order Serially Correlated Errors,"
        Econometrica, 38(1970), 507-516.

[4]   Fisher, F.M., "The Price of Copper and the Level of Chilean Copper
        Exports in the Next Decade," unpublished, 1962.

[5]   Herfindahl, O.C., Copper Costs and Prices: 1870-1957 (Baltimore: The
        Johns Hopkins Press, 1959).

[6]   Koyck, L.M., Distributed Lags and Investment Analysis (Amsterdam:
        North-Holland, 1954).

[7]   Mamalakis, M., and C.W. Reynolds, Essays on the Chilean Economy (Homewood,
        Illinois: R.D. Irwin, 1965).

[8]   McMahon, A.D., Copper: A Materials Survey (Washington: U.S. Department
        of the Interior, Bureau of Mines, 1964).

[9]   McNicol, D., "An Econometric Model of the Copper Industry," Ph.D. thesis
        in progress, Massachusetts Institute of Technology.

[10]   Nerlove, M., The Dynamics of Supply: Estimation of Farmers' Response to
        Price (Baltimore: The Johns Hopkins Press, 1958).

[11]   Telser, L.G., "Futures Trading and the Storage of Cotton and Wheat,"
        Journal of Political Economy 66(1958), 233-255.