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The Distribution of Inventory Holdings in a
Pure Exchange Barter Search Economy

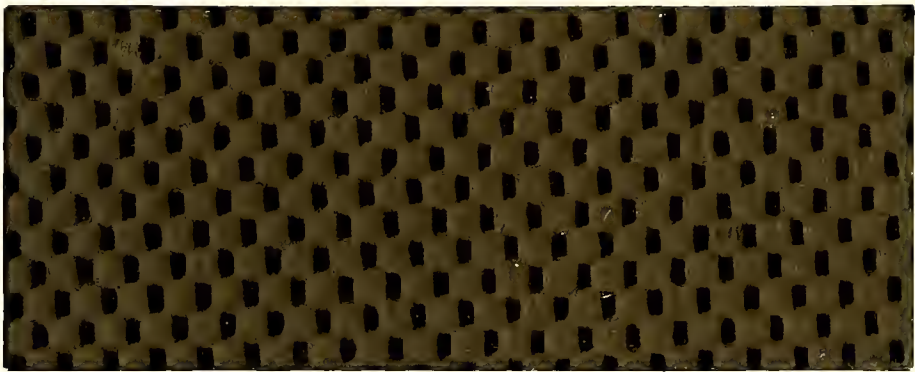
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1. Introduction

Analyses of general equilibrium with imperfect transactions take two different forms. The first posits a schedule of transaction costs and makes no further modifications in the Walrasian mechanism for attaining equilibrium. (For examples, see Foley (1970), Hahn (1971) and more recently Fischer (1982).) The second focuses on the problem of coordinating transactions, rather than upon their costs. Studies of search equilibria (e.g., Diamond (1982, 1984, forthcoming), Mortensen (1982a,b), Weibull (1982)) are examples of this second approach.

This paper continues the exploration of transactions coordination models.¹ In the models discussed here, transactions occur only at meetings between a buyer and seller selected at random.² As in the earlier work cited above, we make the crucial simplifying assumption that individuals explore transactions opportunities one at a time. In particular, economic

¹Diamond (1982) presented a search model of a production economy with inventory levels restricted to 0 and 1. Here, we consider continuous rather than discrete levels of inventory. The introduction of continuous inventories enables us to analyze price setting behavior. To deal with the mathematical complexities that continuous inventories entail, we restrict ourselves to discussion of an exchange economy. An interesting feature of the analysis is the absence of the multiple equilibria that appear in the production economy.

²We make the simplifying assumption that traders pair at random in the belief that the results of our analysis would not change significantly under a regime of systematic search, such as the one discussed by Salop (1973). We briefly consider matching technologies better than random ones in Section 5. We do not consider repeat transactions.

agents experience a Poisson process in which transactions opportunities arrive at a mean rate that depends on the transactions technology and on the collective behavior of all agents in the economy. We confine our analysis to a pure exchange economy without credit, and to circumstances in which there is an incentive to make transactions as rapidly as possible. A deeper understanding of economies with this simple structure may help in analyzing more complicated and realistic models in which credit, production opportunities and intertemporal preferences play their appropriate roles.

Our objective is to describe price setting in a barter economy. We elicit conditions for the existence of steady-state equilibria and analyze optimal price setting by sellers. We show that when equilibria exist, they are unique. Moreover, we conclude that the greater the capacity of the transactions technology to allocate consumer goods, the lower the equilibrium price. We also consider equilibrium distributions of inventory for different arbitrarily set prices. We find that higher equilibrium prices are associated with larger inventories.

In Section 2 we set out a simple, deterministic Robinson Crusoe economy that introduces our basic technological assumptions. Throughout this paper, we assume that inventory accumulates smoothly, and we model consumption as a process in which discrete bundles of goods are consumed at discrete times. In Section 3, we modify the Crusoe economy by introducing a stochastic technology for converting inventory to consumable goods. We model the arrival of consumption opportunities as a Poisson process and calculate the steady state inventory distribution. Section 4 exhibits the expected lifetime utility in our stochastic Crusoe world as a function of

the level of initial inventories. Section 5 begins an equilibrium analysis of a model of stochastic pairwise trading. We relate an individual's ability to trade successfully to two factors controlled by the assumed search technology: the rate at which trading partners are located; and the equilibrium probability that a randomly selected partner has an inventory sufficient to enable him to consummate a transaction. We discuss the distribution of inventories and the aggregate lifetime utility in a steady state equilibrium. The analysis in Section 5 assumes that trading occurs at a relative price of one. In Section 6, we distinguish between buying for consumption and selling for the purpose of increasing inventory. This permits introduction of a (uniform) trading price different from unity and enables us to compare steady states of the economy associated with different trading prices. Higher trading prices result in greater stocks of inventories and are in this sense less efficient. In Section 7 we describe optimal individual price setting in an economy with many profit maximizing firms. We show that price is higher the greater the incoming flow of consumption goods relative to the potential capacity of the economy to distribute goods to consumers. We conclude in Section 8 with a discussion of directions for further research.

2. Robinson Crusoe Economy

To introduce the technology and display our notation, we begin with a simple Robinson Crusoe world. Crusoe is immortal. He receives consumable goods continuously from nature at a rate a . His consumption is not

continuous, however. He derives utility from consumption of discrete bundles of size y . Each consumption bundle carries u units of instantaneous utility.³ Crusoe has a lifetime utility that depends on his consumption path, from the present to the indefinite future. If consumption occurs at discrete times t_1, t_2, \dots , lifetime utility is

$$W = u \sum_i e^{-rt_i}, \quad (2.1)$$

where r is a positive utility discount factor. Given positive r , it is desirable that consumption take place as early in history as possible.

Our assumptions enable us to specify Crusoe's inventory of goods as a function of time. Let Crusoe's initial endowment of goods, x_0 , lie between 0 and y . On our assumptions, inventory grows linearly at the constant rate a until it reaches y . Crusoe then consumes a bundle of size y , and his inventory drops to zero. Crusoe's inventory at time t is therefore

$$x(t) = x_0 + at - y \text{Int}((x_0+at)/y), \quad (2.2)$$

where $\text{Int}(z)$ is the integer part of z . On the other hand, if Crusoe's initial inventory exceeds y , he draws it down (at an infinite rate) until it drops below y , after which the pattern described by (2.2) is repeated. From (2.2), observe that in the Crusoe universe there is an equal probability of an inventory level x at any value between 0 and y , at any time picked at random in the distant future. We may choose the equivalent

³In later sections, we introduce transactions and assume utility in an amount u accrues when there is a successful purchase.

interpretation that in a steady state, there is a population of Crusoes (normalized to one) whose inventories are distributed uniformly between 0 and y , with mean $y/2$.

One may also calculate Crusoe's lifetime utility $W(x_0)$ as a function of his initial inventory, x_0 . Let $x_0=0$. Because it takes y/a units of time to accumulate a consumable bundle of size y , and consumption is repeated at equal intervals y/a into the future, (2.1) tells us Crusoe's lifetime utility is

$$W(0) = u e^{-ry/a} [1 - e^{-ry/a}]^{-1}. \quad (2.3a)$$

If x_0 lies between 0 and y , the consumption path is the same as it would be if Crusoe started with zero inventory at a time x_0/a earlier. Thus,

$$W(x_0) = W(0) \exp(rx_0/a) \quad x_0 < y, \quad (2.3b)$$

and W is convex between 0 and y . If x_0 is greater than y , we have

$$\begin{aligned} W(x_0) &= u \text{Int}(x_0/y) + W(x_0 - y \text{Int}(x_0/y)) \\ &= u \text{Int}(x_0/y) + W(0) \exp[r(x_0 - y \text{Int}(x_0/y))/a]. \end{aligned} \quad (2.3c)$$

From (2.3c), we observe that $W(x_0) - u \text{Int}(x_0/y)$ is a periodic function. Lifetime utility $W(x_0)$, as specified by (2.3), is therefore neither concave nor convex in x_0 , a characteristic of the models analyzed in this paper.

As in the standard continuous consumption model, the one-person Crusoe economy is equivalent to an Arrow-Debreu economy. Assume trade is instantaneous. Let the population be composed of identical agents who do not choose to consume what they themselves are capable of producing. Then, under the assumption that each consumption bundle is composed of y units from the same supplier, trade takes place in discrete

bundles of size y . Accumulation of y units of inventory then leads to the inventory and consumption behavior described by (2.1-3). The presence of a present discounted value, lifetime budget constraint does not change this story, but effectively introduces an interest rate r as a consequence of linear intertemporal indifference between present and future consumption.

In Sections 5-7, we explore models in which trade is not instantaneous, and the transactions rate is endogenous. Our assumption of an endogenous transactions rate distinguishes the work presented here from conventional Arrow-Debreu analyses.⁴ As a prelude, we consider in Section 3 a stochastic variant of the Crusoe economy that leads conveniently to the more complex analysis that follows.

3. Stochastic Crusoe Economy

We now change the underlying technology to allow for a preparatory stochastic process before consumption can occur and again explore inventory behavior over time. As in Section 2, we let inventories increase continuously at a constant rate, a . We add, however, the new assumption that a bundle of y units of a good must be processed in home production before it is ready for consumption. The home production process has the following form: each bundle of y units is set aside for ripening; only one bundle can be processed at a time; ripening is stochastic; and the arrival of ripeness obeys a Poisson law with rate b . Consumption takes place immediately after a bundle is ripe. This stochastic Crusoe economy is

⁴Not every model with a finite transactions time differs from an Arrow-Debreu model. With an exogenous distribution of the time needed to deliver goods to the market, one can distinguish produced from marketed goods and apply the standard analysis in terms of marketed goods. An interesting model of this sort has been studied by Lucas and Prescott (1974).

equivalent to an Arrow-Debreu economy in which ripening represents a stochastic technology for delivering goods to consumers, and "trade" is instantaneously coordinated once an agent reaches a market.

In the stochastic Crusoe economy, an inventory less than y grows at the rate a . Once y is exceeded, there is a probability per unit time, b , of a stepwise drop by an amount y . We introduce scarcity into the economy by assuming

$$a < by. \tag{3.1}$$

Equation (3.1) tells us individuals are capable of consuming more rapidly than the rate at which inventories grow. It will be shown explicitly below that this scarcity condition is necessary if the economy is to possess a stationary equilibrium. The quantity a/by is the ratio of the rate of inflow of endowment to the potential rate at which goods can be ripened if sufficient capacity is available. In a steady state, the rate of ripening must equal the rate at which endowment arrives. Therefore, we shall refer to a/by in the discussion below as the capacity utilization ratio.

Assume the economy consists of a large number of individuals, distinguished from each other only by their inventory holdings. Denote by $F(x)$ the steady-state equilibrium distribution of inventories, with associated density $F'(x)$. Consider the flow of agents with given inventories along the (positive) x -axis. By definition, all contributions to this flow sum to zero in a steady state. We shall derive the equilibrium condition and solve for $F(x)$.

At inventory level x , the flow of agents to the right is $aF'(x)$. On the other hand, the flow to the left is given by the rate b times the total number of agents to the right of x with inventories capable of ripening and

therefore of dropping below x . This is $b[F(x+y) - \max[F(x), F(y)]]$, the number of inventory holdings at least as large as y , lying in the interval $(x, x+y)$. Equating rightward and leftward flows, we observe that the equilibrium distribution of inventories satisfies⁵

$$aF'(x) = bF(x+y) - bF(y); \quad x \leq y \quad (3.2a)$$

$$aF'(x) = bF(x+y) - bF(x). \quad x > y \quad (3.2b)$$

These are linear, first-order differential equations with advanced arguments. We seek a continuous solution $F(x)$ such that $F(0)=0$, and $F(\infty)=1$. Given these boundary conditions, the solution is unique.

Equation (3.2b) is independent of (3.2a). Yet the solution of (3.2a) depends on the solution of (3.2b). Thus, we attack (3.2b) first. Substituting e^{kx} for $F(x)$ in (3.2b), we see that $F(x)$ has exponential form provided the rate constant k satisfies the characteristic equation

$$ak = b(e^{ky} - 1). \quad (3.3)$$

By using Rouché's Theorem (e.g., Titchmarsh, 1932, chapt. 3), or simply by diagramming the complex k -plane, we find that (3.3) has only one (real) root k^* with negative real part. This suggests the trial solution

⁵More general non-steady paths are described by a Fokker-Planck (or forward Kolmogorov) equation of the form

$$\frac{\partial f(x,t;x_0)}{\partial t} = -a \frac{\partial f(x,t;x_0)}{\partial x} + b[f(x+y,t;x_0) - \Theta(x-y)f(x,t;x_0)],$$

where $f(x,t;x_0)$ is the probability that an agent with initial inventory x_0 has inventory x at time t , and where $\Theta(z)=1$ for positive z , and $\Theta(z)=0$ for z negative. This equation is analyzed in Yellin and Diamond (1983), where we consider the welfare consequences of an improvement in the search technology.

$F(x) = 1 + (\text{constant}) \exp(k^*x)$. Making this substitution in (3.2b), we obtain

$$F(x) = 1 - [1-F(y)] \exp[k^*(x-y)], \quad x \geq y \quad (3.4)$$

The boundary value $F(y)$ is set by solving (3.2a) and enforcing continuity. Reading off $F(x+y)$ from (3.4) and substituting the result in (3.2a), we have the density

$$F'(x) = (b/a)[1-F(y)][1-e^{k^*x}], \quad x \leq y \quad (3.5)$$

Integrating (3.5), the associated distribution is

$$F(x) = (b/(ak^*)) [1-F(y)] [1-e^{k^*x+k^*y}], \quad x \leq y \quad (3.6)$$

where we have used $F(0)=0$. Setting $x=y$ in (3.6) and using (3.3), one observes that continuity of $F(x)$ requires

$$1-F(y) = a/(by). \quad (3.7)$$

The quantity $1-F(y)$ is the probability that a randomly chosen individual is not stocked out. Equation (3.7) therefore restates our scarcity assumption (3.1) in probability terms and confirms that (3.1) is required for the existence of a steady state equilibrium.⁶

In Figure 1, we have plotted the density $F'(x)$. From (3.3) and (3.8), one observes that apart from an overall scale factor y , $F'(x)$ is a one-parameter family of density functions specified by the capacity utilization ratio $z=a/(by)$. Differentiating (3.3) with respect to z , we have

⁶Indeed, an intuitive argument that assumes there is a steady state equilibrium leads directly to (3.7). One observes that in a steady state, (3.7) is equivalent to the statement that the flow of goods into inventory, a , equals the expected flow of goods out of inventory, $by[1-F(y)]$. Equation (3.7) can also be derived formally by integrating (3.2) over the interval $(0, \infty)$.

$$d(k^*y)/dz = k^*y(1-z+zk^*y)^{-1} > 0. \quad (3.8)$$

Equation (3.8) implies that if the inflow of goods, a , increases, the rate constant k^* decreases in absolute value, and the distribution of inventory holdings acquires a flatter right-hand tail. This behavior is shown in Figure 1, where F' is plotted for two different values of z . On the other hand, an increase in the expected outflow, by , increases the absolute value of k^* and shrinks the right-hand tail. A change in y accompanied by a proportional change in a corresponds to a change in the units in which goods are measured. From (3.3), such a change leads to an inverse proportional change in k^* , in which k^*y remains constant.

If we use (3.4,5) to compute the mean inventory, we obtain

$$\bar{x} = y/2 - 1/k^*. \quad (3.9)$$

The term $y/2$ in (3.9) represents goods in process. If b were indefinitely large, an agent's inventory level would drop to zero immediately on reaching y , and the steady-state mean inventory would be $y/2$, as in the deterministic world of Section 2. Equation (3.9) shows that the consequence of assuming a finite transaction time is lower efficiency, manifested by a shift of mean inventory upward by $-1/k^*$ from the deterministic value. From (3.8) and (3.9) we observe that the more rapid ripening caused by a larger meeting rate b decreases average inventories, while a greater input rate of goods, a , increases average inventories.

4. Expected Lifetime Utility

We turn now to an evaluation of consumption patterns in terms of

lifetime utilities. We discount utility as before at a constant rate r . We will show that, as in Section 2, the expected present discounted value of lifetime utility is neither concave nor convex as a function of initial inventories. We give an explicit expression for lifetime utility for completeness. No further use is made of this expression in this paper; we reserve detailed welfare analysis for separate treatment.⁷

To fix ideas, we first compute the present discounted value of the lifetime utility of an agent whose initial inventory is infinite. Given our assumptions, such an individual takes advantage of every consumption opportunity. The probability of an act of consumption in an infinitesimal time interval dt is $b dt$, and the payoff is $u e^{-rt}$. Assuming an initial infinite endowment, the expected present discounted value of utility is

$$W(\infty) = bu \int_0^{\infty} e^{-rt} dt = bu/r. \quad (4.1)$$

To compute $W(x_0)$ for a finite initial endowment x_0 , we express lifetime utility as the summed product of the Poisson jump probability $b dt$, the payoff $[u e^{-rt}]$, and the probability,

$$\int_y^{\infty} f(x,t;x_0) dx,$$

that an agent with initial endowment x_0 has an inventory greater than y at time t :

$$W(x_0) = bu \int_0^{\infty} e^{-rt} \int_y^{\infty} f(x,t;x_0) dx dt. \quad (4.2)$$

⁷A derivation of the results of this section is presented in Yellin and Diamond (1983).

Using (4.2) and the homogeneity and additivity of the underlying stochastic process, one derives differential equations that determine $W(x)$:

$$rW(x) = aW'(x); \quad x < y \quad (4.3a)$$

$$rW(x) = aW'(x) + b[u + W(x-y) - W(x)]. \quad x \geq y \quad (4.3b)$$

Equations (4.3) are dynamic programming relations. They equate the discount rate times expected lifetime utility to the expected flow of utility plus the expected value of capital gains from changes in inventory. The asymptotic limit $W(\infty)$ given by (4.1) is reconfirmed on inspection of (4.3b). In Figure 2, we exhibit the shape of $W(x)$, as given by (4.3). Note the monotone increasing behavior as x increases, and also the upper bound on W that results from the scarcity condition (3.1).

For completeness, we give the explicit solution of (4.3) for $W(x)$. By inspection, the solution of (4.3a) is the simple (convex) exponential

$$W(x) = W(0)e^{rx/a}, \quad x \leq y \quad (4.4a)$$

just as in (2.3b). Since W is convex over the region $x \leq y$, increasing and bounded above, it is neither convex nor concave. For x greater than y , we may integrate (4.3b) stepwise to obtain

$$W(x) = A_j + W(0)e^{r(x-jy)/a} + (-b/a)^j \sum_{m=0}^{j-1} (m!)^{-1} C_{j-m} (x-jy)^m e^{(r+b)(x-jy)/a},$$

$$jy \leq x \leq (j+1)y. \quad (4.4b)$$

In (4.4b),

$$A_j = (bu/r) [1 - (b/(r+b))^j]. \quad (4.4c)$$

The continuity of $W(x)$ in x yields

$$-(b/a)C_1 = W(0)(e^{ry/a} - 1) - ub/(r+b); \quad (4.4d)$$

$$\begin{aligned} (-b/a)^{j+1}C_{j+1} &= -u[b/(r+b)]^{j+1} + W(0)[e^{ry/a} - 1] \\ &\quad + e^{(r+b)y/a} (-b/a)^j \sum_{m=0}^{j-1} C_{j-m} y^m/m! \quad . \quad (4.4e) \end{aligned}$$

There remains the unknown lifetime utility, $W(0)$, of an agent whose initial endowment is zero. From (4.2) we obtain after some calculation (Yellin and Diamond, 1983)

$$W(0) = e^{-ry/a} W(y) = u e^{-ry/a} [1 - e^{-ry/a}]^{-1} [(1 - e^{-Q}) / (r/b + 1 - e^{-Q})], \quad (4.5)$$

where Q is the unique positive root of the characteristic equation

$$e^{-Q} - 1 = r/b - (a/by)Q \quad (4.6)$$

of (4.3b). On comparing (4.5) with (2.3a), we observe that introduction of a finite transactions rate results in the appearance of the factor $1 - e^{-Q} / (r/b + 1 - e^{-Q})$, which lowers $W(0)$ below the deterministic value (2.3a). This is consistent with the efficiency loss that must follow from the introduction of a stochastic process in which transactions are not instantaneous. From (4.6), as the meeting rate b increases, Q increases, the new stochastic factor approaches 1, and $W(0)$ tends asymptotically to the Crusoe value (2.3a).

5. Search Economy

Rather than placing Robinson Crusoe in an Arrow-Debreu economy where trade is perfectly and instantaneously coordinated, let us now place him in an exchange economy where the problem of finding a trading partner replaces the ripening process. To begin, let us assume that meetings between pairs of agents represent two independent random draws from the population, and that such meetings occur at the rate $b'/2$ per capita. Each such trade permits both traders to consume. Then each individual experiences meetings at a rate b' -- the number of agents per meeting, 2, times the meeting rate per capita, $b'/2$. On the other hand, not every transaction can be consummated, for each individual meets with a partner who has sufficient inventory to trade in a fraction $[1-F(y)]$ of his meetings. Thus, each agent experiences an effective arrival rate of trading partners

$$b = [1-F(y)]b'. \quad (5.1)$$

Equation (5.1) shows that while b depends on the exogenous technology for bringing agents together, it also depends on the endogenous distribution of inventory holdings. In particular, in the Arrow-Debreu economy analogous to the Robinson Crusoe economy analysed in Sections 3 and 4, the Poisson parameter b , governing the time it takes to deliver goods to market, is exogenous. In a search model, however, success in finding a trading partner is sensitive to the availability of partners, and the Poisson rate b is partly endogenous.

If we assume, as in Sections 2 and 3, that agents continuously receive goods at the rate a , then the steady-state distribution of inventories

satisfies (cf. (3.2))

$$aF'(x) = b'[1-F(y)][F(x+y) - F(y)]; \quad x \leq y \quad (5.2a)$$

$$aF'(x) = b'[1-F(y)][F(x+y) - F(x)]. \quad x \geq y \quad (5.2b)$$

We analyze (5.2) below.

One can also generalize to search technologies that lead to higher probabilities for successful transactions than does random pairing. Let the fraction of meetings that result in successful transactions be h . In general, h will be an increasing function of $[1-F(y)]$, the fraction of the population holding inventory greater than the minimum necessary for entering into transactions. If all $[1-F(y)]$ individuals capable of trading have equal probabilities of a successful pairing, then each agent experiences trading opportunities at an arrival rate

$$b = b'h(1-F(y))/[1-F(y)]. \quad (5.3)$$

Given random pairings, $h = [1-F(y)]^2$. Thus, the presence of agents unable to trade slows the trading process. If, on the other hand, the search technology is such that those unable to trade do not slow the search for trading partners, $h = [1-F(y)]$. These are polar cases. We may plausibly expect $h(x)$ to be a function (defined on the unit interval) that lies below x and above x^2 . In terms of h , the equilibrium relations (5.2) become

$$aF'(x) = b'h(1-F(y))[1-F(y)]^{-1}[F(x+y) - F(y)]; \quad x \leq y \quad (5.4a)$$

$$aF'(x) = b'h(1-F(y))[1-F(y)]^{-1}[F(x+y) - F(x)]. \quad x \geq y \quad (5.4b)$$

We now analyze the random search model described by (5.2). We will show that the solution of (5.2) is identical to the Crusoe solution (3.4,6), provided that the rate parameter b in Section 3 is replaced by the parametric combination $[ab'/y]^{\frac{1}{2}}$, and that the scarcity condition $a < b'y$ is imposed. It is natural to ask whether by proper choice of parameters one can find a solution of the linear equations (3.2) that satisfies the nonlinear equations (5.2). Recall from (3.7) that continuity of $F(x)$ requires

$$1 - F(y) = a/(by). \quad (5.5)$$

Combining (5.5) with the rate relation (5.1), we obtain

$$b = [ab'/y]^{\frac{1}{2}}. \quad (5.6)$$

Making the replacement (5.6) in (3.3, 3.4, 3.6), and substituting the resulting distribution $F(x)$ into (5.2), one confirms that a solution for the nonlinear random matching problem has been obtained, provided $a < b'y$. We state without proof that any nontrivial solution of (5.2) is unique. Other search technologies may be analyzed by combining (5.5) with the rate relation (5.3).

One may extend this analysis to aggregate lifetime utility. In steady state equilibrium, the aggregate per capita consumption rate equals a , the aggregate per capita incoming flow of goods. Thus, the aggregate per capita flow of utility is ua/y , a quantity that is independent of the meeting rate b' and, more generally, of the pairing technology $h(x)$ introduced in (5.3). The present discounted value of aggregate instantaneous utility, W_0 , equals expected lifetime utility summed over all agents. Therefore, in a steady state equilibrium we have

$$W_0 = ua/(yr) - \int_0^{\infty} W(x) F'(x) dx, \quad (5.7)$$

where $W(x)$ is the individual expected lifetime utility given an initial inventory x , and $F(x)$ is the solution of (5.4).

Though steady state aggregate lifetime utilities are independent of the arrival rate b' , economies that have different search technologies (and therefore different arrival rates b') differ from each other in important ways. In particular, the equilibrium distribution of inventory holdings, $F(x)$, differs among such economies. Recalling (3.8-9), we confirm from (5.6) that average inventories, $y/2-1/k^*$, increase as the exogenous part, b' , of the rate of meetings decreases. The expected lifetime utility from holding a given initial inventory level also varies with the meeting rate. In particular, expected lifetime utility increases with an increased meeting rate, at each level of initial inventory. This can be reconciled with the constant average utility (5.7) by observing that the steady state distribution of inventory holdings shifts to the left for successively larger meeting rates.

6. Trading Prices

We now modify the model of Section 5 so as to allow consideration of price-setting behavior. In this section we calculate the steady state distribution of inventories for arbitrary uniform prices. In Section 7 we analyze equilibria with many profit maximizing firms.

Thus far we have assumed that the only reason for a failure to trade is that either potential partner to a transaction is stocked out.⁸ Under

⁸Therefore we have assumed the double coincidence of wants is always satisfied.

that assumption, we have imposed the rule that all trade takes place on a one-for-one basis -- with each partner trading and receiving y units of the universal consumption good. If, however, we impose instead the rules of specific bargaining theories such as those of Raiffa or Nash (cf. Luce and Raiffa (1957)), then trade cannot be one-for-one except under special circumstances. In particular, given the Raiffa bargaining solution, both parties to any trade have equal gains of utility. However, in one-for-one trade between two agents with inventories x_1 and x_2 , equal utility gains imply

$$u + W(x_1 - y) - W(x_1) = u + W(x_2 - y) - W(x_2). \quad (6.1)$$

The function W is non-linear. For any x_1 , (6.1) therefore can be satisfied only for exceptional values of x_2 , in particular for $x_1 = x_2$.

The straightforward generalization of our model of one-for-one trade is to allow different prices in different trades. This would reflect the reality of price distributions. Such a generalization greatly complicates the analysis, however, and we take an alternative approach. We explore a simple institutional setting in which equilibrium is achieved at a uniform relative price different from unity. When trade occurs, y units are "purchased" for consumption, and the buyer "pays" p units which are added to the inventory of the seller. The inventories of any two traders are therefore assumed to be perfect substitutes in future trades. To maintain the price interpretation, we further assume p is greater than y . Any trade now involves one randomly selected buyer and one randomly selected seller. Thus, one can think of the population of economic agents as a set of pairs, with one member of each pair available to buy and one to sell.

With only these changes in the model, individual inventories would be subject to a Poisson process compounded from distinct stochastic buying and selling behaviors. Inventories would have jump increases by steps $p-y$, and jump decreases by steps p , in addition to increases at a constant rate due to the arrival of endowment. The presence of jump increases greatly complicates the analytic problems discussed in Sections 3-5, and we make no attempt to analyze such a two-jump process. Instead, to retain the picture of uniform inventory growth and discrete inventory outflows, we simplify further by introducing an intermediary that smooths increases in inventory by providing insurance against the random proceeds of the selling process.

We shall refer to our intermediating institution as the "firm." It operates under the following rules. Agents receive inventory from nature at a constant rate a' . Also, every agent is employed by the firm and receives "wages" at a rate $a-a'$. Each agent therefore experiences a continuous growth of inventory at the overall rate a , which is now endogenous. The firm sets the price at which its workers offer goods for trade. It also sets its wage rate $a-a'$ equal to the flow of profits per worker. Wages are paid independent of an agent's success in selling and whether or not she or he possesses the minimum inventory necessary to consummate transactions.⁹ Furthermore, inventories are made available for trade, even though there is no return to making them available.¹⁰ In

⁹If wages depend on whether inventories are above or below y , purchase and consumption by an agent with inventory between y and $y+p$ will lower subsequent wages. Therefore, in order to compute the equilibrium distribution of inventories, it becomes necessary to determine the set of inventory levels at which consumption opportunities are taken.

¹⁰To compensate separately for labor and for inventories would introduce an interest rate into the model.

addition, provided his inventory exceeds p , an agent experiences opportunities to purchase bundles of size y for consumption at the price p , at times set by a Poisson process characterized by the rate b , which is determined endogenously.¹¹ Each act of consumption, as before, results in instantaneous utility u . Finally, we shall assume that profits from a sale, $p-y$, are instantly transmitted to the firm's central accounting office for redistribution as part of the wage flow $a-a'$. Then the distribution of inventory available for sale is identical to the distribution of inventory available for purchase.¹² Indeed, the distributions of inventory and the levels of expected utility are the same as those analyzed in Sections 3 and 4, except that price p plays the role of commodity bundle size y . Inventories grow continuously at the rate a , while decreasing in jumps of size p at times determined by a Poisson process with rate b .

To proceed with the steady state analysis, we consider the determination of b , which is the endogenous arrival rate of purchase opportunities. The expected rate of successful purchases is $[1-F(p)]b$. In general, this rate depends on the percentage of agents who are able to buy, $[1-F(p)]$, and the percentage able to sell, $[1-F(y)]$. As discussed in Section 5, we may write the fraction of meetings that are successful as $h(1-F(p),1-F(y))$, and define the rate of meetings per capita as b' . Then we

¹¹This model can be reinterpreted as a monetary structure in which paper money is backed one hundred percent by loans collateralized on inventories, and no interest is paid on these "deposits." We leave the analysis of a more general economy with interest rates for later work. The introduction of interest greatly complicates the equations for the distribution of inventory holdings.

¹²In a more general model with delays in transmitting goods within the firm which are different from delays in transmitting purchasing ability, the equilibrium conditions determining inventory available for sale differ from those determining inventory available to finance purchases.

have the rate relationship

$$[1-F(p)]b = b'h(1-F(p),1-F(y)). \quad (6.2)$$

Consider the following two polar cases. First, the ability to find a supplier may be independent of the distribution of available inventories. This occurs if demanders are instantly redirected to suppliers who are adequately stocked. Then $h = 1-F(p)$, and

$$(A) \quad b = b'. \quad (6.3)$$

On the other hand, meetings between pairs of agents may be purely random, without redirection to adequately stocked sellers. In this case, h is the product of $1-F(p)$ and $1-F(y)$, and we have

$$(B) \quad b = [1-F(y)]b'. \quad (6.4)$$

It is plausible that the dependence of b on $F(x)$ lies between these two polar extremes, with some delay in finding an adequately stocked supplier, but better than random search.

The two polar possibilities, (A,B) lead to different relationships between inventories and prices. We will now show that the more efficient technology A described by (6.3) results in mean inventory proportional to price. On the other hand, the less efficient technology B described by (6.4) results in less than proportional growth of mean inventory as a function of price. We begin by analyzing the dependence of the distribution of inventories on the endogenous rate b . The resulting equations hold for both search technologies.

The trading rules of the firm introduced above tell us that the price p plays the role of the consumption bundle size y in the equilibrium analysis [cf.(3.2-8)] that fixes the inventory distribution $F(x)$. In

particular, the condition (3.7) for the continuity of $F(x)$ tells us that the fraction of agents who are able to buy satisfies

$$1-F(p) = a/(bp). \quad (6.5)$$

Replacing y by p and x by y in (3.6) and using (6.5), the fraction of the population able to sell is

$$1-F(y) = 1-[pk^*]^{-1}[1-e^{k^*y+k^*y}], \quad (6.6)$$

where k^* is the unique real, negative root of the characteristic equation

$$ak^* = b(e^{k^*p}-1). \quad (6.7)$$

Furthermore, from (3.8), mean inventory becomes

$$\bar{x} = p/2 - 1/k^*. \quad (6.8)$$

We shall use (6.7) to compute the change in mean inventory (6.8) -- in particular in $-1/k^*$ -- as p increases.

To proceed, we first eliminate the endogenous growth rate a from (6.7). The rate of growth of inventories satisfies the firm's budget constraint. This is the requirement that wages equal profits, which here takes the form

$$a-a' = (p-y)b[1-F(p)] = a(p-y)/p, \quad (6.9)$$

where we have used (6.5). From (6.9), we see that the ratio

$$a/p = a'/y \quad (6.10)$$

is independent of price.¹³ Therefore, the real wage,

¹³Equation (6.10) is equivalent to the assertion that in a steady state the aggregate accumulation of goods equals the aggregate consumption rate: $a' = b[1-F(p)]y = ay/p$.

$(a-a')y/p = a'(p-y)/p$, rises monotonically from zero at $p=y$. Using (6.10), the characteristic equation (6.7) becomes

$$a'k^*p/y = b(e^{k^*p}-1). \quad (6.11a)$$

Technology A. Given the more efficient technology described by (6.3), b is independent of p and may be replaced by b' in (6.11a), yielding

$$a'/(b'y) [e^{k^*p}-1]^{-1}(k^*p) = 1. \quad (6.11b)$$

Comparing (6.11b) with the Crusoe result (3.3), we observe that with search technology A, the present model differs from the model of Section 3 by the replacement of the Crusoe rate constant k^* by the new value k^*y/p .

Introduction of the uniform relative price, p , therefore shifts the entire inventory distribution to the right. Moreover, (6.11b) tells us that for fixed $a'/(b'y)$, k^*p is fixed and negative, and $(-k^*^{-1})$ is proportional to p . The mean inventory level, $p/2-1/k^*$, therefore is proportional to price.

Reinterpretation in money terms lends insight into the efficiency properties of a steady state economy mediated by search technology A. With either technology, this model can be reinterpreted as describing an economy with commodity-backed money, in which commodities are available for sale, but money, rather than commodities, is used for purchases. With technology A, essentially the same steady state as under the pure inventory model can be achieved by introducing money that is only partially backed by commodities. In particular, with technology A, buyers are instantly redirected to adequately stocked suppliers, and the arrival rate of consumption opportunities therefore is not affected by inventories in excess of $y/2$ per capita -- the minimum mean inventory necessary for the accumulation process prior to "ripening" described in Section 3.

Therefore, with partially unbacked money, the same distribution of money

holdings as given by (6.11b) can be achieved, while restricting mean inventory to $y/2$. This is the familiar gain of efficiency from eliminating stocks of commodities whose only role is to back the money supply.

Technology B. In the model of random meetings described by (6.4), the relationships among price, the shape of the inventory distribution, and the mean inventory level are more complex. The exponential rate k^* decreases with p for low prices sufficiently near the zero-profit point $p=y$, but increases with p for higher prices. Mean inventory increases monotonically in p . In contrast to the behavior that obtains with technology A, average inventories rise less than in proportion to p .

A heuristic explanation is as follows. The introduction of a price p (greater than y) effectively divides the population of agents into three groups. These groups have inventories: (I) less than y ; (II) between y and p ; (III) greater than p . For $p=y$, no agents fall into Group II, and the inventory distribution is therefore identical to the two-group distribution (3.4, 3.6), as modified by the parametric substitution (5.6) derived in Section 5. For p greater than y , two effects increase the net meeting rate b , tending to increase the absolute magnitude of the rate constant k^* and, therefore, to shift the inventory distribution above p to the left. First, wages are positive, speeding up the movement from Group I to Group II. Second, whatever the price level, members of Group II are unable to purchase goods, and are therefore prevented from dropping into Group I. However, for p sufficiently large with respect to y , we pass to a regime in which Group I is negligible. Thus, for large p there is a two-group dynamics in which p effectively plays the role of y in the model of Section

3, and $-1/k^*$ is proportional to p , just as it is when the efficient technology A is in operation.

Lemma 1. When search technology B is in operation, the exponential rate constant $k^*(p)$ decreases as p increases from the zero-profit point $p=y$, reaches a minimum at some value $p=p_0$, and then increases for all greater p . On the other hand, the quantity pk^* decreases monotonically in p , approaching a constant (negative) value for sufficiently large p .

Proof. Lemma 1 can be confirmed analytically by substituting (6.4) and (6.6) into (6.11a), and using (6.7), deriving an implicit equation for k^* :

$$1-F(y) = [a'/(b'y)][e^{k^*p}-1]^{-1}(k^*p) = 1-(1-e^{k^*y+k^*y})/(pk^*). \quad (6.12)$$

Rewrite the last equality in (6.12) in the form

$$R_1(k^*y) - R_2(k^*p), \quad (6.13)$$

where

$$R_1(x) = 1 + x - e^x; \quad (6.14a)$$

$$R_2(x) = x[1 + (a'/(b'y))x(1-e^x)^{-1}]. \quad (6.14b)$$

From (6.14a), we observe that $R_1(x)$ is negative, monotone increasing and concave for nonpositive x , with $R_1(0)=0$. From (6.14b), provided the scarcity condition $z'=a'/b'y < 1$ holds, $R_2(x)$ is convex¹⁴ and has two roots, one at zero, the other at a negative value greater than $-1/z'$. Moreover,

¹⁴By differentiating (6.14b), we obtain

$$R_2''(x) = [a'/(b'y)](1-e^x)^{-3}[2(1-e^x+xe^x)^2+x^2e^x(1-e^x)] \geq 0.$$

$R_2'(x)$ is positive at the zero-profit point $p=y$, where $R_1(x)=R_2(x)$.¹⁵

These characteristics of the $R_i(x)$ are shown in Figure 3. The derivative dk^*/dp obtained from (6.13), $k^*R_2'[yR_1'-pR_2']^{-1}$, is negative (positive) when the derivative $R_2'(k^*p)$ is positive (negative). Therefore, dk^*/dp is negative for p sufficiently near the zero-profit point $p=y$, as Lemma 1 asserts. For sufficiently large p , the solution k^*p of (6.13) occurs where $R_2'(k^*p)$ is negative, and dk^*/dp is positive. The derivative of k^*p with respect to p , $k^*yR_1'[yR_1'-pR_2']^{-1}$, is positive, consistent with the less than proportional growth of $-1/k^*$ anticipated above.

The dependence of k^* and k^*p on p is exhibited in Figure 4.

Lemma 2. The price response of mean inventory,

$$d\bar{x}/dp = 1/2 + k^{*-2}dk^*/dp, \tag{6.15}$$

is positive for all allowed p .

Proof. One confirms this result by differentiating (6.13). We have

$$\begin{aligned} k^{*-2} \frac{dk^*}{dp} &= \frac{R_2'(k^*p)}{k^*y R_1'(k^*y) - k^*p R_2'(k^*p)} \\ &= R_2'(k^*p)[k^*y(1-e^{k^*y}) - k^*pR_2'(k^*p)]^{-1}. \end{aligned} \tag{6.16}$$

To prove that mean inventory increases with price, we must show that $k^{*-2}dk^*/dp$ is greater than $-\frac{1}{2}$. From (6.15), the critical region in p is the

¹⁵One checks the sign of R_2' at $p=y$ by using $R_1(x)=R_2(x)$ to eliminate $(a'/(b'y))$ from the expression for $R_2'(x)$, obtaining

$$R_2'(x) = x^{-1}[x(1+e^x) + 2(1-e^x)]$$

at the zero-profit point.

one in which dk^*/dp is negative. Recalling Figure 3, this is the region in which R_2' is positive. Differentiating (6.16) with respect to k^*y , we have

$$\text{sign } \frac{d}{d(k^*y)} [k^{*-2} \frac{dk^*}{dp}] = \text{sign} [k^*y[(1-e^{k^*y})^2 R_2''/R_1' + e^{k^*y} R_2']]. \quad (6.17)$$

Since R_2' is convex, the sign of (6.17) is negative, and $k^{*-2}dk^*/dp$ takes its lowest value at the zero-profit point $p=y$. From (6.13), we may write $a'/(b'y)=[(1-e^x)/x]^2$ at $p=y$. Substituting this result in (6.16), we have

$$k^{*-2} \frac{dk^*}{dp} = \frac{1+e^x+2(1-e^x)/x}{-2xe^x-2(1-e^x)}, \quad (6.18)$$

where $p=y$, and $x=k^*p$. It is straightforward to confirm that the right hand side of (6.18) exceeds $-1/2$ for all negative x , and thus that mean inventory increases monotonically with p .

Lemma 3. With technology B, the net meeting rate b increases with p .

Proof. From (6.12) and (6.4), we have

$$d/dp \log b(p) = d/dp \log [1-F(y)] = \frac{-(kp)'(1-e^{kp} + kp e^{kp})}{-kp(1-e^{kp})}. \quad (6.19)$$

By inspection, the denominator of (6.19) is positive. From Lemma 1, $-(kp)'$ is also positive. The remaining quantity $(1-e^{kp}+kpe^{kp})$ is positive also: it is monotone decreasing for negative kp and vanishes at $kp=0$.

7. Price Determination

To close this analysis, we study a many-firm model in which we

derive profit maximizing, equilibrium prices.¹⁶ We separately analyze the existence of an equilibrium price for each of the search technologies (A,B), and we also examine the dependence of the equilibrium prices on the capacity utilization ratio $z'=a'/b'y$. It will be shown that the condition on z' for the existence of an optimal price is more stringent than the scarcity condition of Section 3 that the capacity utilization ratio is less than 1.

Let us assume there are a large number of identical, though independent firms that have no price reputations, and that consumers solicit prices at random, one at a time. With sufficiently many firms, there is no link between profits and perceived demand. Thus, firms will set a price, p' , that maximizes aggregate perceived profits (see, e.g., Burdett and Judd (1983), Diamond (1971)). Perceived profits per arriving customer equal $(p'-y)[1-F(p')]$, where the distribution of purchasing power, F , satisfies (3.4) and (3.6), with p substituted for y . That is, perceived profits equal the product of the profit per sale and the fraction of arriving customers who are potential active buyers.

The first-order condition for profit maximization,

$$1/(p'-y) = F'(p')/[1-F(p')] , \quad (7.1)$$

instructs each firm to set a price that precisely balances its marginal increase in revenue with the marginal reduction in its pool of buyers due to a price increment. Such a balance can be achieved only if the

¹⁶We assume, as above, that wages equal profits and do not consider more complicated wage contracts. Worker inventories must be unobservable in order that wages are uniform. Wages will exhaust profits if firms have free entry and the ability to attract a random selection of workers.

right-hand tail of the inventory distribution falls sufficiently fast. From Section 3, we know that the right-hand tail flattens as the capacity utilization ratio increases. A priori, therefore, we expect that an optimal equilibrium price will exist only if the capacity utilization ratio z' does not exceed a critical maximum. As a corollary, we expect that the higher the capacity utilization ratio, the higher the equilibrium price. The same reasoning allows us to compare equilibrium prices for the two different search technologies. We expect the more efficient technology A, associated with a shorter right-hand tail, to result in a lower equilibrium price.

Since we assume no collusion among firms, the derivative in (7.1) acts only on the inventory level and does not take account of the structural relationships between k^* and z' derived in Section 6. The right hand side of (7.1) is therefore simply minus the exponential rate constant k^* (cf.(3.4)), and we may write

$$k^*p = -p/(p-y). \quad (7.2)$$

The system is in equilibrium when p satisfies (7.2), and k^* simultaneously satisfies (6.11b) for technology A, or (6.12) for technology B. By combining (7.2) with (6.11b) and (6.12) respectively, we obtain pairs of equations that relate the equilibrium price to the capacity utilization ratio z' . For technology A, we have, defining $U=p/(p-y)$,

$$z' = a'/(b'y) = (1-e^{-U})/U. \quad (7.3a)$$

For technology B,

$$z' = (1-e^{-U})(2-e^{1-U})U^{-2}. \quad (7.3b)$$

Lemma 4. For both search technologies, there exists a unique equilibrium

if and only if the parametric condition

$$a'/(b'y) < 1 - 1/e \tag{7.4}$$

is satisfied.

Proof. The proof follows by observing, from (7.3), that z' is monotone decreasing in U for both technologies,¹⁷ and takes its maximum value, $1-1/e$, when $U=1$. From the monotonicity of z' in U , and the monotonicity of $U=p/(p-y)$ in p , it follows immediately that:

Lemma 5. For both search technologies, the optimal price p is a monotone increasing function of the capacity utilization ratio $a'/b'y$.

The monotonicity of p excludes the existence of multiple equilibria characterized by different prices but the same value of z' . We show the behavior of the optimal price as a function of z' in Figure 5. Note the singular behavior of p as z' approaches $1-1/e$ and also the higher equilibrium price for technology B at each value of z' -- the comparative behavior anticipated above.

¹⁷To derive these monotonicity properties, one may sign and bound the logarithmic derivatives of the right-hand sides of (7.3a,b). In the more difficult case (7.3b), we have the logarithmic derivative

$$e^{-U}[1-e^{-U}]^{-1} + e^{1-U}[2-e^{1-U}]^{-1} - 2/U \leq e^{-U}[1-e^{-1}]^{-1} + e^{1-U} - 2/U.$$

The bound on the right side of this inequality is negative over the relevant range $U=p/(p-y) \geq 1$.

8. Conclusions

Using simple stochastic search models that permit explicit solutions, we have explored the role of purchasing power in the determination of steady state equilibria. In particular, we have examined the role of economy-wide price setting as it affects the distribution of inventory holdings, the aggregate level of inventories, and the aggregate transactions rate. Simplicity is achieved by omitting many important economic phenomena.

In sequels, we plan to study monetary models with the same structure, as well as models incorporating interest paid on "deposits" of inventory. In the interest models, we will introduce deaths and births, with "estates" going to the government for redistribution. This will allow us to construct models in which there are equilibrium steady states with a positive interest rate. We will then modify the model of Section 6 to distinguish wages from interest paid on inventories. In this way, we hope to analyze the effects of price-wage-interest setting on the distribution of inventory holdings, and therefore also on aggregate inventory levels and the aggregate transactions rate. Once a model that includes interest rates is constructed and solved, it should be possible to analyze a monetary economy with alternative tradeable assets.

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Figure Captions

Figure 1. Probability density of inventory distribution defined by equations (3.4,6) of text. Note the flattening and extension of the right-hand tail as z increases.

Figure 2. Lifetime utility W as determined by dynamic programming equations (4.3) of text, with parameters $b=r=a/y$. Inventory is measured in units of commodity bundle size y .

Figure 3. Functions $R_{1,2}(x)$, as defined by equations (6.14) of text, for the capacity utilization ratio $z'=1/2$. Shown is zero-profit root $k_0^*y=x$ of $R_1(k_0^*y)=R_2(k_0^*y)$. Note that this root lies to the right of the minimum in $R_2(x)$, where dR_2/dx is positive. Therefore, as shown in distorted scale in inset, for p sufficiently small two values of p correspond to each root x of $R_1(x)=R_2(px/y)$. Note that the derivative dx/dp is negative for p near y and positive for large p , as shown explicitly in Figure 4.

Figure 4. Price behavior of k^*y and k^*p , as given by equation (6.13) of text.

Figure 5. Optimum equilibrium prices as function of capacity utilization ratio in many-firm model described in text. Curve A: efficient search technology with instantaneous redirection of traders to adequately stocked suppliers. Curve B: random search technology.

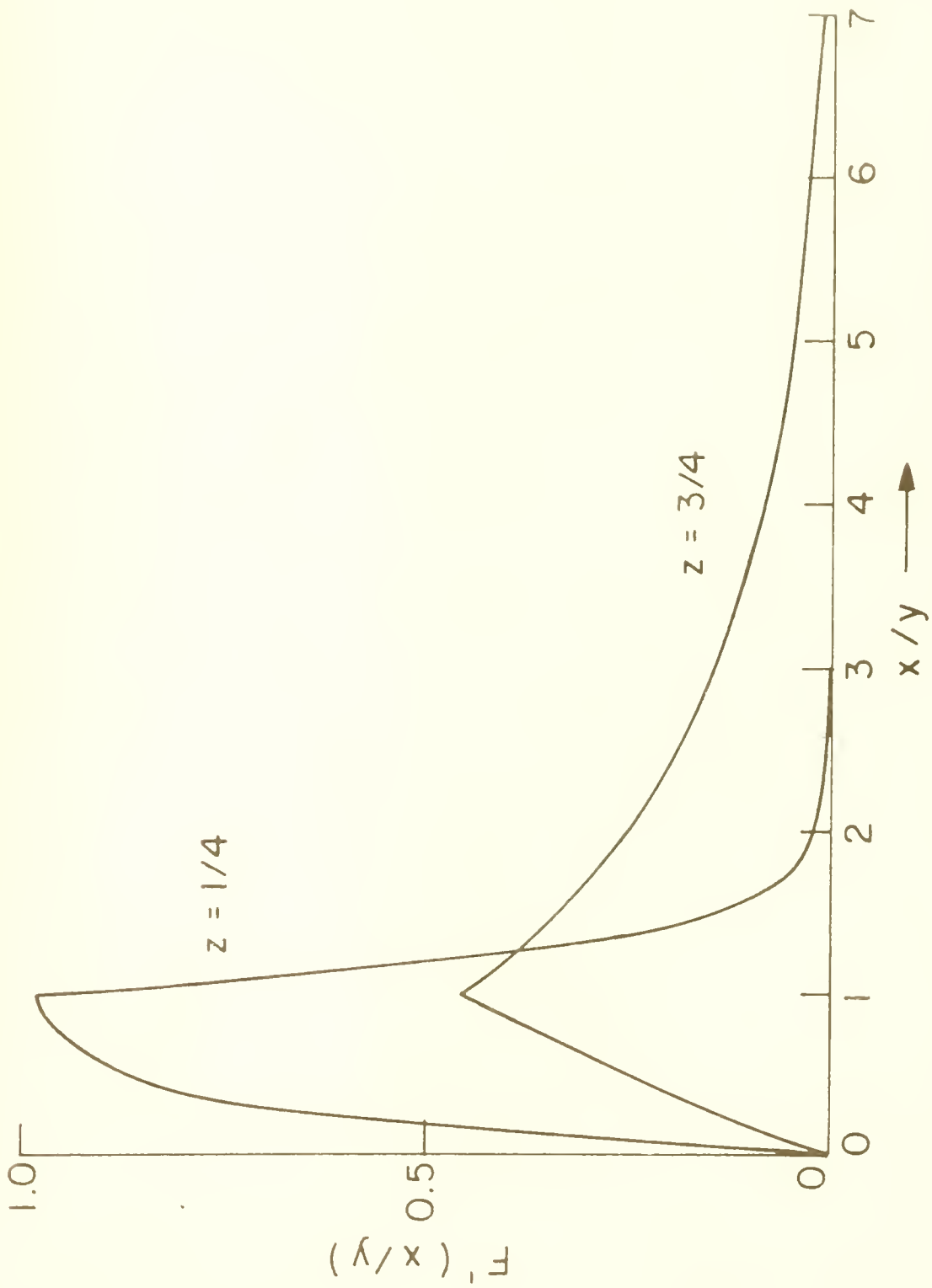


FIG. 1

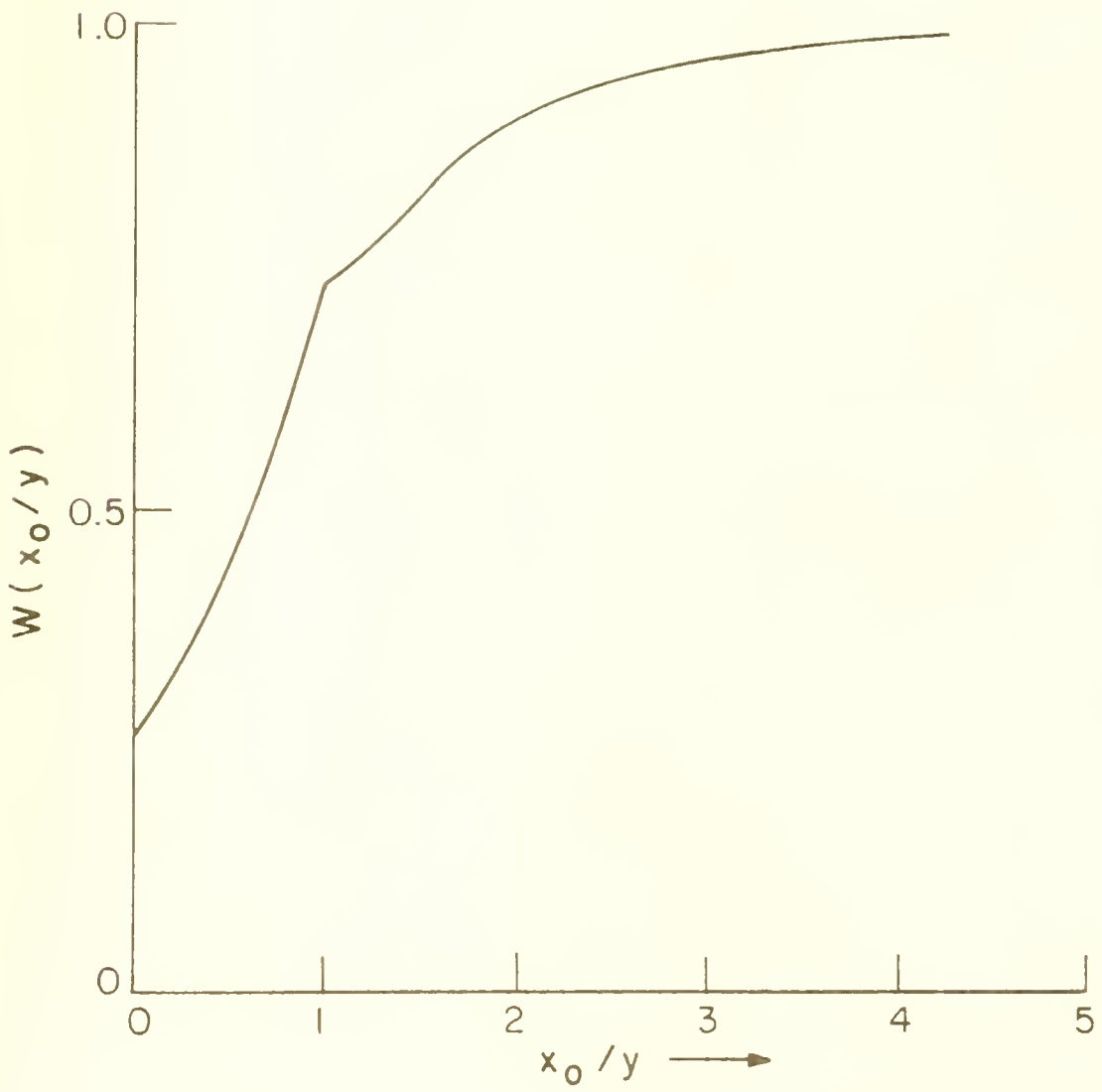


FIG. 2

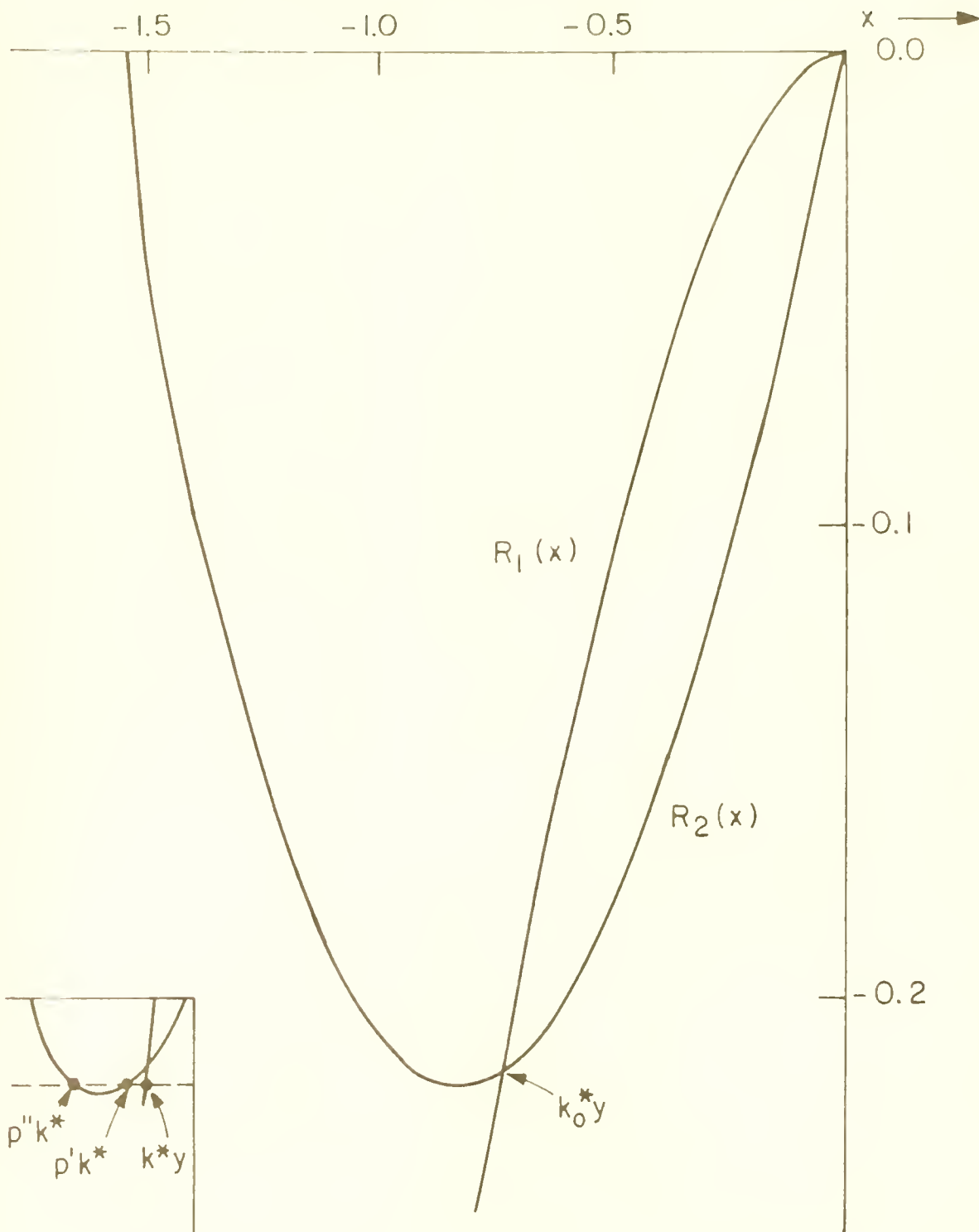


FIG. 3

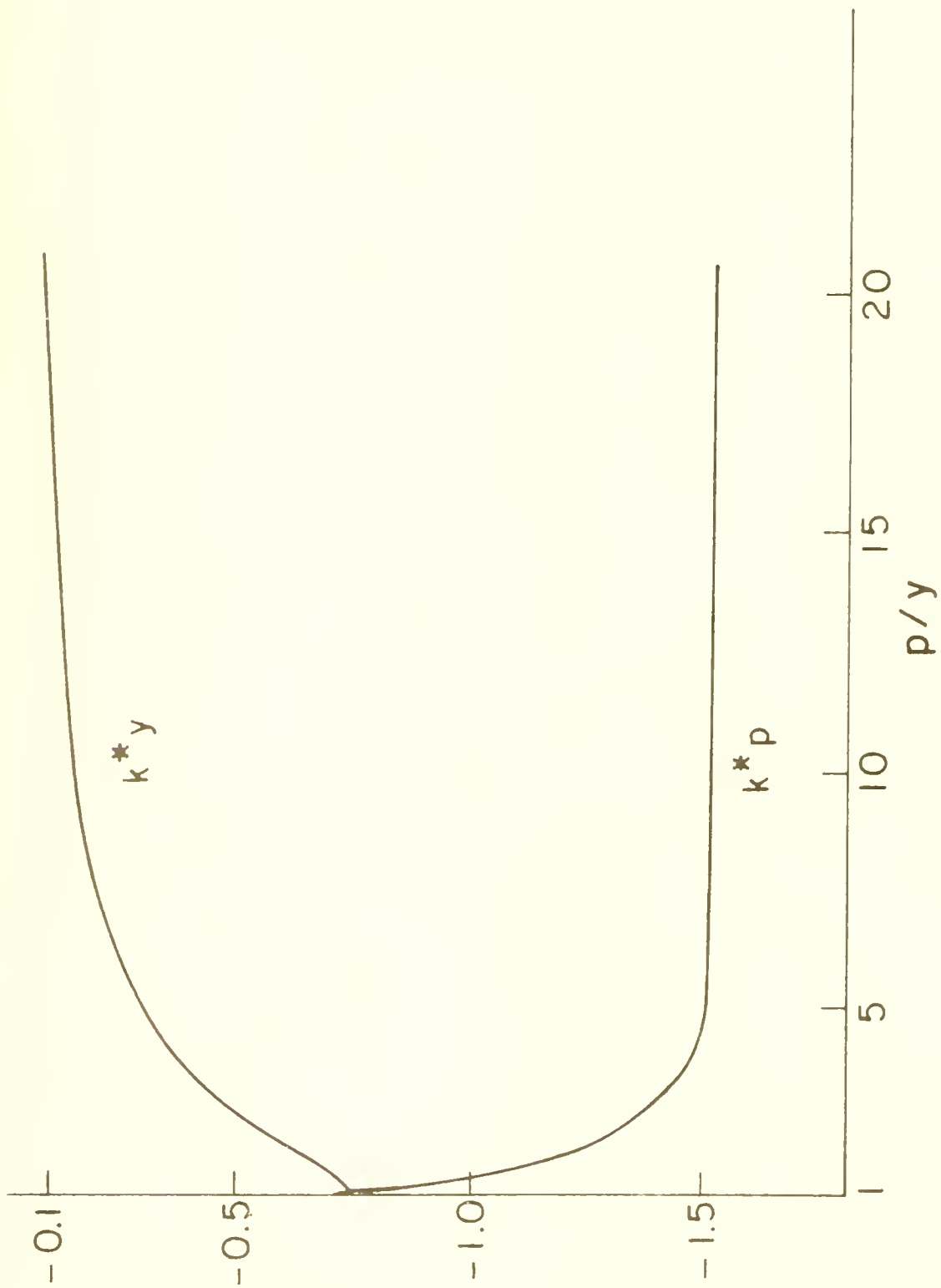


FIG. 4

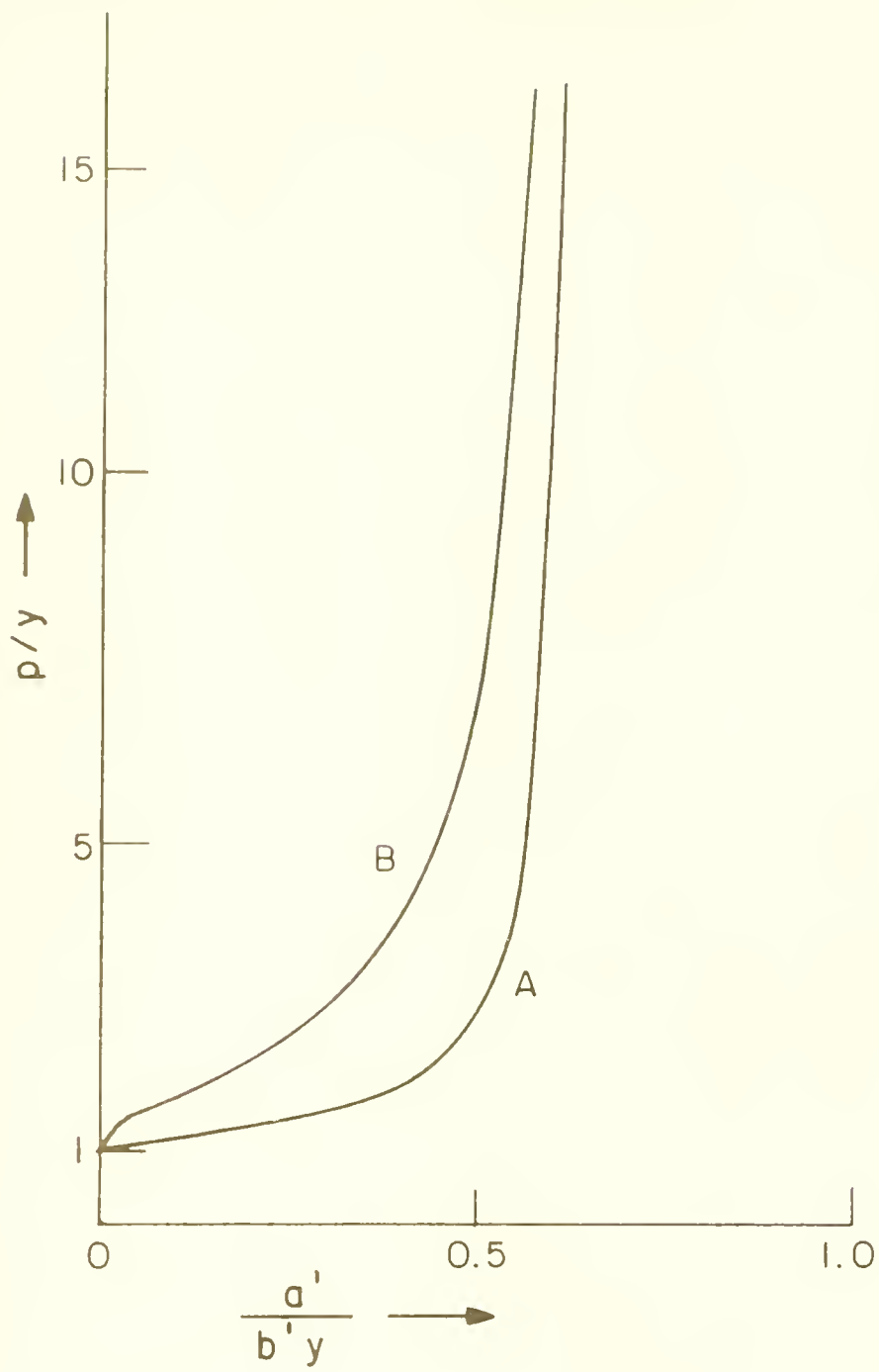


FIG. 5

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