INTEGRATION OF PRODUCTION, MAINTENANCE AND QUALITY: Modelling and Solution Approaches

Thèse

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Résumé

Dans cette thèse, nous analysons le problème de l'intégration de la planification de production et de la maintenance préventive, ainsi que l'élaboration du système de contrôle de la qualité. Premièrement, on considère un système de production composé d'une machine et de plusieurs produits dans un contexte incertain, dont les prix et le coût changent d'une période à l'autre. La machine se détériore avec le temps et sa probabilité de défaillance, ainsi que le risque de passage à un état hors contrôle augmentent. Le taux de défaillance dans un état dégradé est plus élevé et donc, des coûts liés à la qualité s'imposent. Lorsque la machine tombe en panne, une maintenance corrective ou une réparation minimale seront initiées pour la remettre en marche sans influer ses conditions ou le processus de détérioration. L'augmentation du nombre de défaillances de la machine se traduit par un temps d'arrêt supérieur et un taux de disponibilité inférieur. D'autre part, la réalisation des plans de production est fortement influencée par la disponibilité et la fiabilité de la machine. Les interactions entre la planification de la maintenance et celle de la production sont incorporées dans notre modèle mathématique. Dans la première étape, l'effet de maintenance sur la qualité est pris en compte. La maintenance préventive est considérée comme imparfaite. La condition de la machine est définie par l'âge actuel, et la machine dispose de plusieurs niveaux de maintenance avec des caractéristiques différentes (coûts, délais d'exécution et impacts sur les conditions du système). La détermination des niveaux de maintenance préventive optimaux conduit à un problème d'optimisation difficile. Un modèle de maximisation du profit est développé, dans lequel la vente des produits conformes et non conformes, les coûts de la production, les stocks tenus, la rupture de stock, la configuration de la machine, la maintenance préventive et corrective, le remplacement de la machine et le coût de la qualité sont considérés dans la fonction de l'objectif.

De plus, un système composé de plusieurs machines est étudié. Dans cette extension, les nombres optimaux d'inspections est également considéré. La fonction de l'objectif consiste à

minimiser le coût total qui est la somme des coûts liés à la maintenance, la production et la qualité.

Ensuite, en tenant compte de la complexité des modèles préposés, nous développons des méthodes de résolution efficaces qui sont fondées sur la combinaison d'algorithmes génétiques avec des méthodes de recherches locales. On présente un algorithme mimétique qui emploi l'algorithme Nelder-Mead, avec un logiciel d'optimisation pour déterminer les valeurs exactes de plusieurs variables de décisions à chaque évaluation. La méthode de résolution proposée est comparée, en termes de temps d'exécution et de qualités des solutions, avec plusieurs méthodes Métaheuristiques.

Mots-clés

Planification de la production, Maintenance préventive imparfaite, Inspection, Qualité, Modèles intégrés, Métaheuristiques

Abstract

In this thesis, we study the integrated planning of production, maintenance, and quality in multi-product, multi-period imperfect systems. First, we consider a production system composed of one machine and several products in a time-varying context. The machine deteriorates with time and so, the probability of machine failure, or the risk of a shift to an out-of-control state, increases. The defective rate in the shifted state is higher and so, quality related costs will be imposed. When the machine fails, a corrective maintenance or a minimal repair will be initiated to bring the machine in operation without influencing on its conditions or on the deterioration process. Increasing the expected number of machine failures results in a higher downtime and a lower availability rate. On the other hand, realization of the production plans is significantly influenced by the machine availability and reliability. The interactions between maintenance scheduling and production planning are incorporated in the mathematical model. In the first step, the impact of maintenance on the expected quality level is addressed. The maintenance is also imperfect and the machine conditions after maintenance can be anywhere between as-good-as-new and as-bad-as-old situations. Machine conditions are stated by its effective age, and the machine has several maintenance levels with different costs, execution times, and impacts on the system conditions. High level maintenances on the one hand have greater influences on the improvement of the system state and on the other hand, they occupy more the available production time. The optimal determination of such preventive maintenance levels to be performed at each maintenance intrusion is a challenging problem. A profit maximization model is developed, where the sale of conforming and non-conforming products, costs of production, inventory holding, backorder, setup, preventive and corrective maintenance, machine replacement, and the quality cost are addressed in the objective function.

Then, a system with multiple machines is taken into account. In this extension, the number of quality inspections is involved in the joint model. The objective function minimizes the total cost which is the sum of maintenance, production and quality costs.

In order to reduce the gap between the theory and the application of joint models, and taking into account the complexity of the integrated problems, we have developed an efficient solution method that is based on the combination of genetic algorithms with local search and problem specific methods. The proposed memetic algorithm employs Nelder-Mead algorithm along with an optimization package for exact determination of the values of several decision variables in each chromosome evolution. The method extracts not only the positive knowledge in good solutions, but also the negative knowledge in poor individuals to determine the algorithm transitions. The method is compared in terms of the solution time and quality to several heuristic methods.

Keywords

Multi-period production planning, Imperfect preventive maintenance, Inspection, Quality, Integrated model, Metaheuristics

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Glossary of abbreviations

ABAO	As bad as old
AGAN	As good as new
CLSP	Capacitated lot-scheduling problem
CM	Corrective maintenance
EA	Evolutionary algorithm
EPQ	Economic production quantity
GA	Genetic algorithm
IC	In control state
JPMQ	Joint production, maintenance, and quality (models and problems)
LP	Linear program
MA	Memetic algorithm
MAPM	Memetic algorithm with population management
MILP	Mixed integer linear program
MR	Minimal repair
NM	Nelder-Mead method
OOC	Out of control state
PM	Preventive maintenance
QC	Quality control
TS	Tabu search

Acknowledgements

First of all, I would like to express my deep and sincere gratitude to my supervisor Prof. Mustapha Nourelfath for the continuous support of my Ph.D. study and this research, for his patience, motivation, and immense knowledge. His guidance helped me in all the time of research and writing of this thesis. I could not have imagined having a better advisor and mentor for my Ph.D. study.

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Foreword

This thesis has been realized under the supervision of Professor Mustapha Nourelfath at the Mechanical Engineering Department, Laval University and Professor Michel Gendreau, the Mathematics and Industrial Engineering Department, Polytechnic School of Montreal. It has been prepared as an article insertion thesis and includes three articles, co-authored by Professor Mustapha Nourelfath and Professor Michel Gendreau. In all of the presented articles, I have acted as the principal researcher. As the first author, I have performed the mathematical modelling, coding the algorithms, analysis and validation of the results, as well as writing the first drafts of the articles. Professors Nourelfath and Gendreau have revised the articles to obtain the final versions.

The main subject of this thesis is on integration of production, maintenance, and quality. The main goal is to enhance the planning and scheduling skills in interactive contexts.

The first article (chapter Two) entitled “Joint maintenance scheduling and production planning for imperfect processes: A profit-maximization model” co-authored with Prof. Mustapha Nourelfath and Prof. Michel Gendreau, is submitted to the *European Journal of Operational Research*.

The second article (chapter Three) entitled “A cost minimization model for joint lot-sizing and maintenance planning under quality constraints” co-authored with Prof. Mustapha Nourelfath and Prof. Michel Gendreau, is under final revision for publication in the *International Journal of Production Research*.

The third article (chapter four) entitled “A memetic algorithm with population management to solve an integrated production, maintenance, and quality planning problem” co-authored with Prof. Mustapha Nourelfath and Prof. Michel Gendreau, is submitted to *IIE Transactions*.

The solution approach discussed in the third paper is able to evaluate different instances of the models developed in chapters Two and Three. The analysis and numerical results presented in chapter Four correspond to the profit maximization model (chapter Two); however, the modifications in chromosome evaluation to apply the proposed solution method on the cost minimization model (chapter three) and the related numerical results are presented in appendix B.

The list of papers presented in conferences is as follows:

1. Beheshti Fakher, H., Nourelfath, M., & Gendreau, M. (2014). Profit maximization by integrating production planning, maintenance scheduling, quality aspects and sale decisions. *Proceedings of the International Conference on Industrial Engineering and Operations Management*, Bali, Indonesia, 1281–1292.
2. Beheshti Fakher, H., Nourelfath, M., & Gendreau, M. (2015). Hybrid genetic algorithm to solve a joint production-maintenance model. 15th IFAC Symposium on Information *Control Problems in Manufacturing,* Ottawa, Canada, IFAC-PapersOnLine, 48 (3), 747–754.
3. Beheshti Fakher, H., Nourelfath, M., & Gendreau, M. (2015). Joint production-maintenance planning in an imperfect system with quality degradation. *Proceedings of the International Conference on Industrial Engineering and Systems Management (IEEE-IESM'2015)*, Seville, Spain, 910–919.

Chapter One: General Introduction and Literature Review

1.1. Introduction

During the recent decades, and as the result of industrial globalization, development of logistics means, expansion of information structures, and increased competency between organizations, the demand for improved efficiency and higher productivity in manufacturing sectors, is raised. The market is forcing manufacturers, especially mass producers, to trim back prices and to improve the quality, in other words, to address higher customer satisfaction levels. In such business conditions, managers have no choice but to look increasingly for cost reductions and quality improvements in order to sustain and enlarge their market shares.

Production and maintenance planning, sale scheduling, and quality systems are the key functions in all manufacturing organizations. Generally, these systems are conflicting because of being connected to the same resources and subjects. Weak integration of interrelated decisions is said as the obstacle between design and application. To improve the planning and scheduling operations, the cooperation between different departments is extremely important.

In competitive markets, the managers make use of several approaches, including the pricing strategies in order to increase their share, to step into new markets, or to discourage competitors entering certain areas. However, such strategies have significant impacts on profitability or on the internal processes of the manufacturing systems among them the production, maintenance and quality system. Sometimes, the decision makers have a couple of choices for production methods, warehousing approaches and marketing strategies with different costs and various outcomes, and a good practice to study their effects is the integration of those processes with marketing schemes. Time varying costs and prices in planning and scheduling models increase the flexibility of decision systems to incorporate such strategies in the low-level decisions. The need for explicit inclusion of time-varying components (quality level, costs, demands…) in the lot-sizing problem is a vital issue in addressing such uncertainties. Fig. 1.1 shows the scope of the problem. As this figure shows, the objective is to help the planning departments to deal with the combined uncertainties and to improve the predictability of the process.

In the literature review, we will show that in the most of the existing papers, such strong interactions are not considered, or only some limited aspects are taken into account. We believe that the joint scheduling and considering the mutual influences results in a more efficient decision system and helps the practitioners to accrue the profitability and achieve the organizational goals.

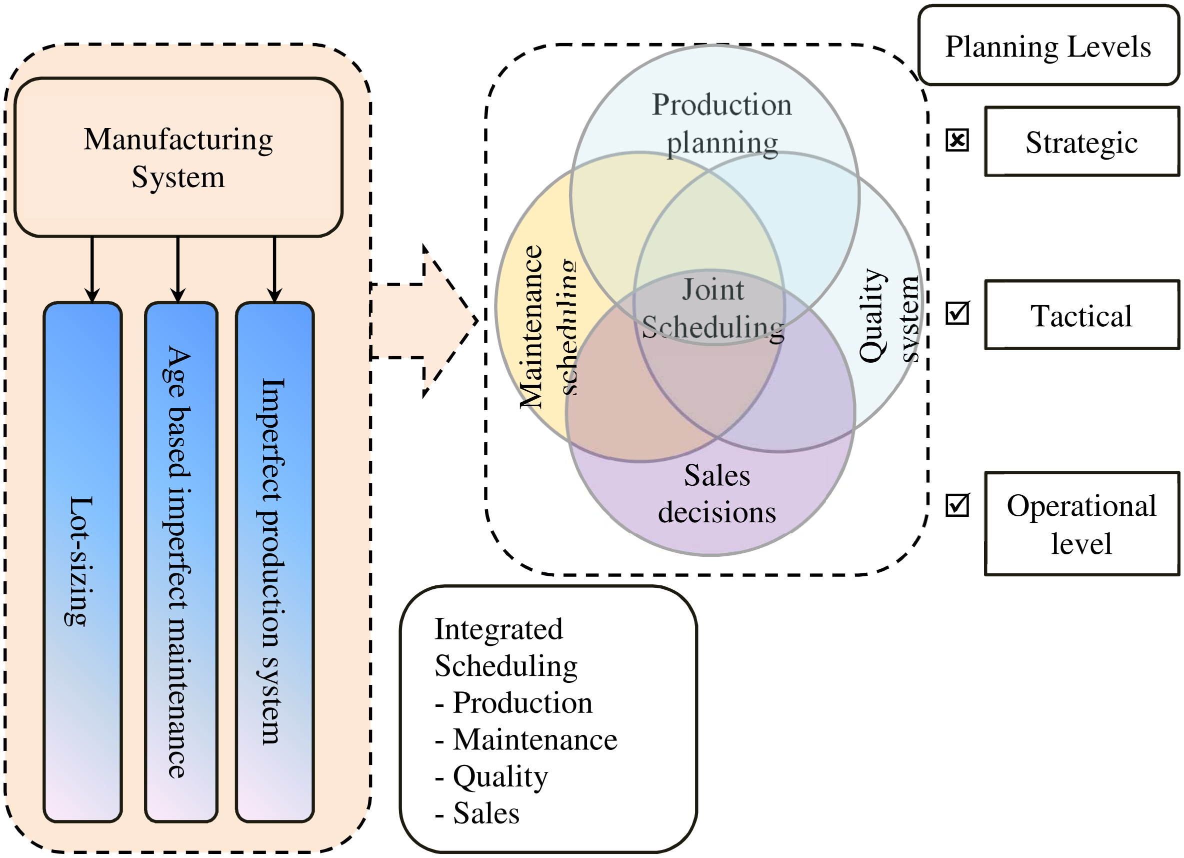

Fig. 1.1: The scope of the research.

1.2. Problem definition

This section is dedicated to the problem definition, first, the coordination of production and maintenance planning and quality system are presented, and the interactions between them are explained. Our objective is to address the mathematical formulation, the cost evaluations, and the solution methods for the "*Joint Production, Maintenance, and Quality (JPMQ)*" planning problem. In section 1.2.2, a profit maximization problem in a single-machine system is addressed. The model integrates the four functions; lot-sizing, age-based imperfect maintenance planning, and sale scheduling taking into account the quality aspects of the system. In section

1.2.3, a cost minimization model is considered that incorporates decisions concerning the optimal design of a quality inspection system. The evaluations and mathematical formulations developed for the two problems are very complicated and so, the problem cannot be solved in a reasonable time. Needless to say that achieving the objective of the research in suggesting efficient decision models is accompanied with the applicability and utility of the proposed approaches. To realize this goal, and in order to reduce the gap between the theory and the application, in section 1.2.4, we discuss the problem of developing efficient solution methods for the proposed JPMQ models.

1.2.1. Process functions and their interactions

This section introduces the production and maintenance planning as well as the quality control system, and addresses several models in each subject. Finally, the interactions between these functions are explained.

1.2.1.1. Production and sales planning problem

Production planning is defined as the assignment of resources, including machines, inventories, financial resources, manpower, etc. to the manufacturing operations such that a given objective function is optimized. A good production plan results in production quality and on time delivery of the demand with the least cost. In the literature, the production planning hierarchy includes capacity planning for strategic level, aggregate planning for long term programs, master production scheduling to split the aggregate plans into smaller midterm elements, and production planning and control that deals with the short-term and shop-floor issues. These levels are demonstrated in Fig. 1.2.

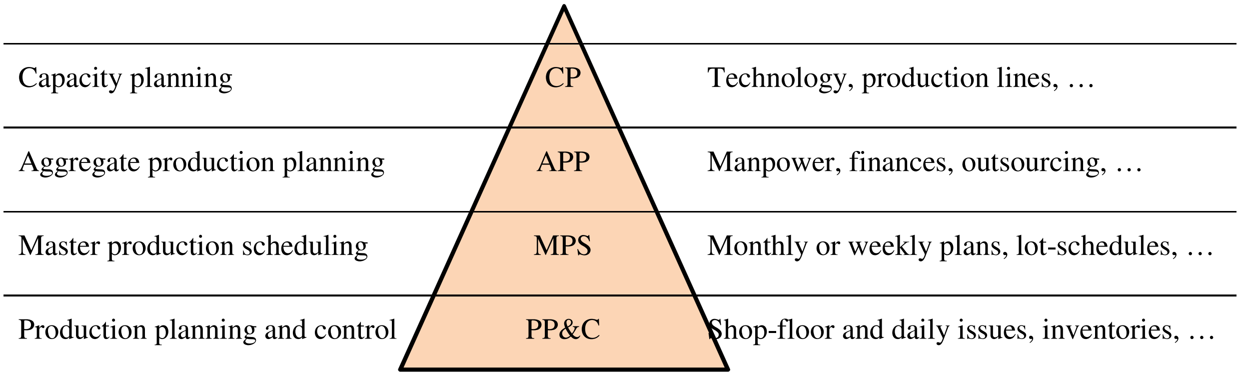

Fig. 1.2: Hierarchy of production planning.

The lot-sizing problem that is considered in this thesis corresponds to the master production scheduling in the hierarchy, where the objective is to determine the optimal value of lot-sizes, inventories and backorder levels to minimize the total cost. In MPS, the aggregate plans are split into monthly or weekly objectives. Therefore, in a planning horizon composed of several periods with given demands, our objective in production planning is to determine the lot-sizes to be produced on each machine in each period. A setup cost is associated with production decisions and inventory holding cost is charged.

The lot-scheduling problem is deeply studied in the literature. Drexel and Kims (1997) list about 120 papers about capacitated lot-sizing problem (CLSP) that indicates the importance of the subject. The general form of a CLSP as a mixed integer linear programs is presented in Fig. 1.3.

Maximize $Z_I = PX_S - (\pi \cdot X_P + h \cdot X_I + b \cdot X_B + s \cdot X_{SET})$

Subject to

Balance and production system constraints

Fig. 1.3: Generic model of a CLSP.

In this figure, Z_1 is the total profit, the decision variables $\boldsymbol{X_S}$, $\boldsymbol{X_P}$, $\boldsymbol{X_I}$, $\boldsymbol{X_B}$, and $\boldsymbol{X_{SET}}$ indicate the sale levels, lot-sizes, inventory levels, backorders, and setups, and $\boldsymbol{P}$, $\boldsymbol{\pi}$, $\boldsymbol{h}$, $\boldsymbol{b}$, $\boldsymbol{s}$ are the unit prices or costs for each product in each period. The constraints establish the link between production, sale, demands, inventories, and backorder levels, as well as the model constraints such as available times on machines and capacity constraints. Despite the approaches used in the majority of lot-scheduling problems, the capacity constraint and the production quality may change subject to internal and external causes. For example, a machine failure and consecutive corrective maintenance operations will diminish the availability of machine. This issue increases the complexity of the CLSP. Karimi et al. (2003) reviewed the CLSP models and solution approaches. They categorized the models according to the length of the planning horizon, the number of levels in the product structure, the number of products, type of the constraint, demand type, setup structure, and some of the successful solution methods for the CLSP (including exact methods, specialized heuristics, mathematical programming methods, branch and bound, and evolutionary algorithms).

Sales planning and management covers several topics including the demand predictions, pricing and marketing strategies, advertisement programs, organization and direction of the sales department. Incorporation of sale decisions in production planning on the one hand, increases the problem size and its difficulty, and on the other hand, establishes the link between external parameters such as price variations and internal operations. Such a combination contributes in maximizing the profit and improving the system performance. Haugen et al. (2007) incorporated prices in CLSP in order to handle the capacity violations with implementation of a market mechanism.

1.2.1.2. Maintenance scheduling problem

Dhillon (2003) defined maintenance as "All actions appropriate for retaining an item/part/equipment in, or restoring it to, a given condition." *Preventive maintenance* (PM) is the set of operations performed to keep an item/equipment in required conditions. These actions are aimed to reduce the failure probabilities and to improve the process quality. *Corrective maintenance* (CM) definition matches with the unscheduled maintenance or repair in order to bring back a deficient or failed item/equipment to a normal state. On significance of maintenance in the costs, he mentioned that about $300 billion are spent on plant maintenance and operations by the U.S. industry, and it is estimated that approximately 80% of this is spent to correct the chronic failure of machines, systems, and people. According to the British Ministry of Technology Working Party, approximate maintenance cost in 1970 in the United Kingdom, was annually £3000 million. 11% of the total operating cost of a military jet, about 5 to 10 percent of operating force in manufacturing industry, and approximately $12 billion (15%) of the annual budget of the U.S. Department of Defense (in 1997) are spent in preventive and corrective maintenance.

The PM is intended to improve the conditions by lubrication, cleaning, performing certain adjustments, replacement of parts and overhaul of machines. Generally, maintenance operations can be performed before or after a failure. PM actions can be divided into the following categories (Pham, 2003):

- Clock-based maintenance
- Age-based maintenance
- Usage-based maintenance
- Condition-based maintenance
- Opportunity-based maintenance
- Design-out maintenance

Production systems deteriorate with time and an old machine has smaller expected time-to-failure and lower quality level. Random failures of machines are of significant importance because they incur higher costs and can result in stoppage of upstream and downstream processes.

In PM planning and scheduling, the main questions to be addressed are the time and the type of maintenance task to be performed on each machine, inventory of spare parts, and the time for overhauls and replacements. The effect of periodic PM on failure rate and system reliability is illustrated in Figures 1.4 and 1.5.

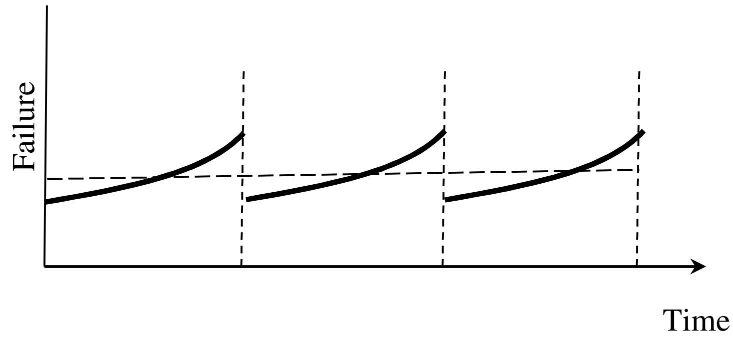

Fig. 1.4: Effect of PM on failure rate.

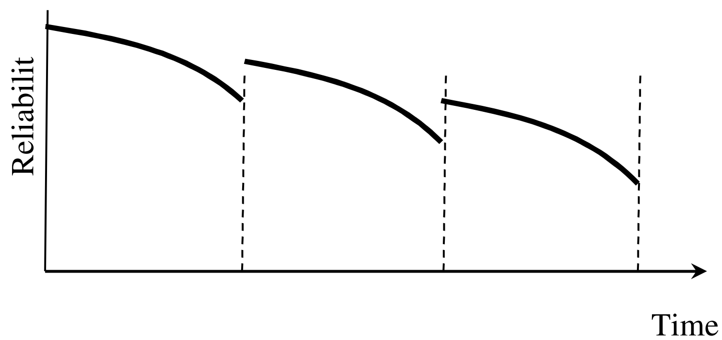

Fig. 1.5: Effect of PM on reliability.

The age based maintenance is a PM strategy in which the maintenance decisions are related to the machine age. In a large number of organizations, maintenance time is not a continuous parameter and there are several previously-known opportunities to perform PM. For example, some firms schedule their PM in weekends, vacations, before starting a mission, when the equipment enters in a station, etc. Therefore, the PM time for them is not a decision variable; instead the type of maintenance task subject to several conditions needs to be optimally determined. Each component may have several PM options ranging from *no-PM* to *complete replacement* or *overhaul* so; the PM planning corresponds to the determination of PM levels for each machine in PM interferences. Denoting ***Y*** as the set of PM alternatives for all machines and all maintenance times, the objective is to minimize the maintenance cost such that the required conditions related to the system reliability and availability is satisfied. Fig. 1.6 shows the general form of a discrete time age-based maintenance with multiple PM levels.

Minimize $Z_2 = \mathbf{C_{PM}}(\mathbf{Y}) + \mathbf{C_{CM}}(\mathbf{Y}) + \mathbf{C_R}(\mathbf{Y})$

Subject to

Maintenance system constraints

Fig. 1.6: Generic form of a discrete time age-based maintenance with multiple levels.

In this figure, Z_2 is the cost of maintenance system, $\boldsymbol{C_{PM}}$ and $\boldsymbol{C_{CM}}$ are nonlinear functions of the preventive and corrective maintenance, and replacement costs, and PM_{mt} is the PM level for machine m in j^{th} interference. The constraints link the PM levels to the system conditions, failure rates, reliabilities, and availabilities.

Mathematical modeling of the deterioration process and extracting the optimal PM decisions according to the equipment state in a multi-machine system in which the machines are initially in different conditions are addresses in this research.

1.2.1.3. Quality control systems

One of the outcomes of the system deterioration is the variations in production that causes higher nonconformity rates. Despite the machine failures in which the processing operations halt, the quality deteriorations are not self-announcing and so, quality control tools are intended to detect such degradations. Optimal planning of quality system improves its performance in signaling the shifted states (and getting less false signals in normal operational states) while the total cost (including the cost of running the quality system) is minimized. Statistical process control (SPC) is a well-known approach to eliminate the process variations by means of control charts. A typical control chart is illustrated in Fig. 1.7. After calculation of upper and lower control limits, the quality control process using a control chart includes taking samples from the process in certain instants of time and plotting the average of the characteristic on the graph. Points situated above or below the control limits and certain rules in these charts are considered as a signal of process variation. In this case, a search is initiated to determine the cause of the change and to fix the problem. Sometimes, the control chart may generate false signals when the process is in-control or no-signal when the process is out-of-control. The capability of the quality tool to signal the changes and lower the probability of no-signal in degraded state are important factors that impacts on economic efficiency of the quality system. The important factors of almost all quality control plans are the sampling frequency, sample size, and the width of in-control range on the chart (*UCL-LCL*). However, upper and lower control limits should conform also with upper and lower specification limits (*USL*, and *LSL*) that define the technically acceptable variation range of the characteristic. The process capability indicator; C_P for symmetric processes is defined as the width of specification limits divided by the width of the control limits $C_p = (USL\text{-}LSL) / (UCL\text{-}LCL)$. The objective of quality control system is to increase the process capability while minimizing the costs.

We may summarize the model of the quality control process as shown in Fig. 1.7. In this figure, $\boldsymbol{Z}$ denotes the set of decision variables in the quality system. These decisions are the sample sizes, the sampling frequencies, and the width of the control limits for each machine. $\boldsymbol{C_{REW}}$ $(\boldsymbol{Z})$ is the reworking cost (or the price difference between conforming and non-conforming products), $\boldsymbol{C_{INS}}$ $(\boldsymbol{Z})$ is the cost of quality inspection, and $\boldsymbol{C_Q}$ $(\boldsymbol{Z})$ is the other costs may involve the rejection cost, guarantee, cost, etc. The set of decisions indicated by $\boldsymbol{Z}$ are related not only to the maintenance system, but also to the production decisions.

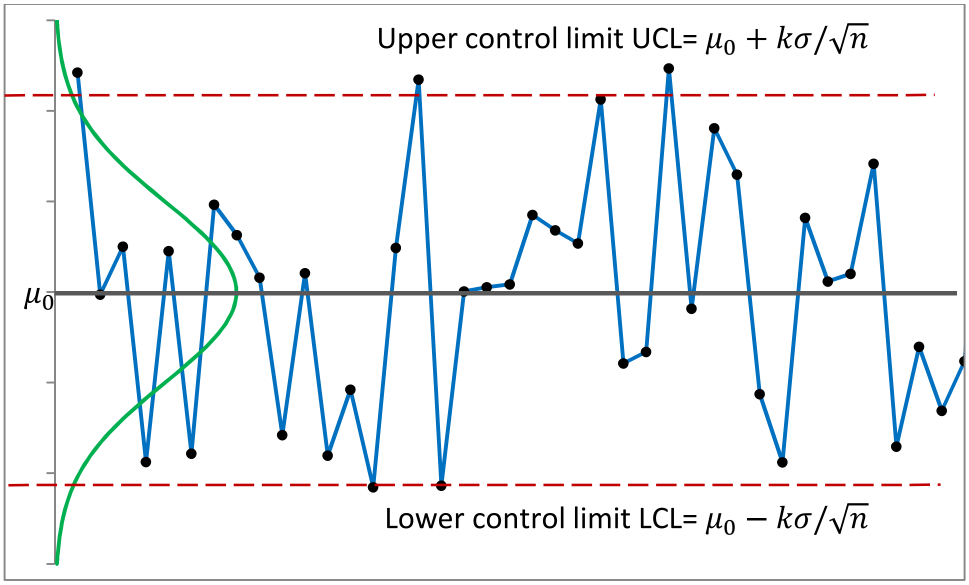

Fig. 1.7: A sample control chart for process mean.

Minimize $Z_3 = \boldsymbol{C_{REW}}(\mathbf{Z}) + \boldsymbol{C_{INS}}(\mathbf{Z}) + \boldsymbol{C_Q}(\mathbf{Z})$

Subject to

Quality system constraints

Fig. 1.8: Typical model of quality control process.

1.2.2. Profit maximization by joint scheduling in single machine systems

In the previous sections we introduced the key functions of manufacturing system and we presented the basic models. The decision variables and constraints are also introduced. The main motivation of this research is to address the interactions between these functions and to propose the framework for an efficient planning-scheduling system that incorporates interacting decision in the same model. Availability and reliability of the machine (that is the main constraint in production planning) are related to (1) the maintenance system that improves the machine conditions and (2) the quality system that signals the degraded states of machines. The cost of quality system and its constraints deal with (1) the production decisions, and (2) with maintenance planning, which manages the machine conditions. In the same way, the performance of maintenance system depends on production and quality parameters. These links are depicted in Fig. 1.9.

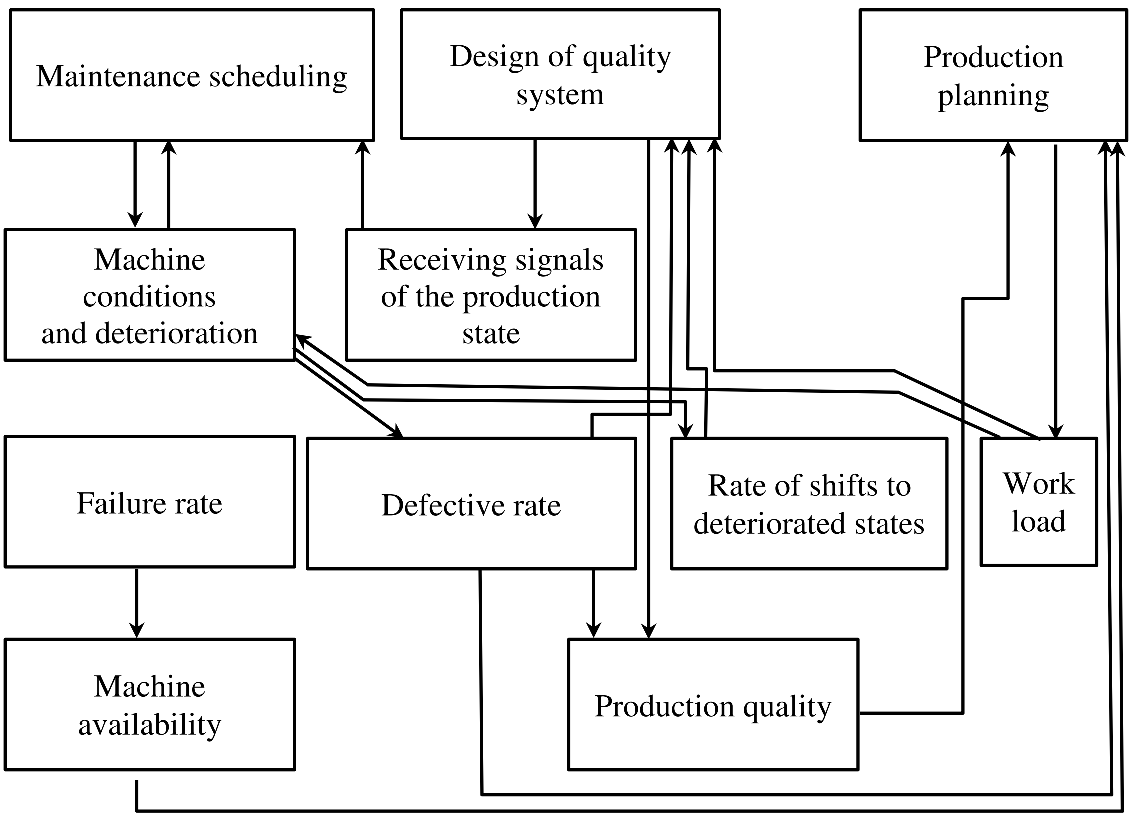

Fig. 1.9: Interactions between functions.

Therefore, instead of optimizing three functions separately, we are interested in the joint production, maintenance, and quality (JPMQ) models as shown in Fig. 1.10. The efficient breakdown of aggregate plans in MPS scheduling requires precise information about the production system. For example, available production time for each machine, processing times, and expected quality levels are the most important questions to be addressed. Fluctuations in demands and processing times, uncertain quality levels, availability and reliability of the machines, etc. are some complexity resources in production planning.

In the first phase, we are interested in imperfect production systems with one machine assigned to production of multiple products in which, the maintenance is imperfect. We assume several maintenance levels of machines. The model involves the sale planning and the cost units and prices change from one period to another. Time varying costs and prices are in line with our objective for developing a flexible model that can consider different scenarios for each system. The scope of this research is summarized in Fig. 1.11.

System	Production and sales planning	Imperfect maintenance scheduling	Quality control system
Variables	$X_S, X_P, X_I, X_B, X_{SET}$	Y	Z
Maximize Z =	Z_1	$- Z_2$	$- Z_3$
Subject to	$Z_1 = PX_S - (\pi \cdot X_P + h \cdot X_I + b \cdot X_B + s \cdot X_{SET})$ Balance and production system constraints	$Z_2 = \boldsymbol{C_{PM}(Y)} + \boldsymbol{C_{CM}(Y)} + \boldsymbol{C_R(Y)}$ Maintenance system constraints	$Z_3 = \boldsymbol{C_{REW}}(\boldsymbol{Z}) + \boldsymbol{C_{INS}}(\boldsymbol{Z}) + \boldsymbol{C_Q}(\boldsymbol{Z})$ Quality system constraints

Fig. 1.10: Typical JPMQ problem.

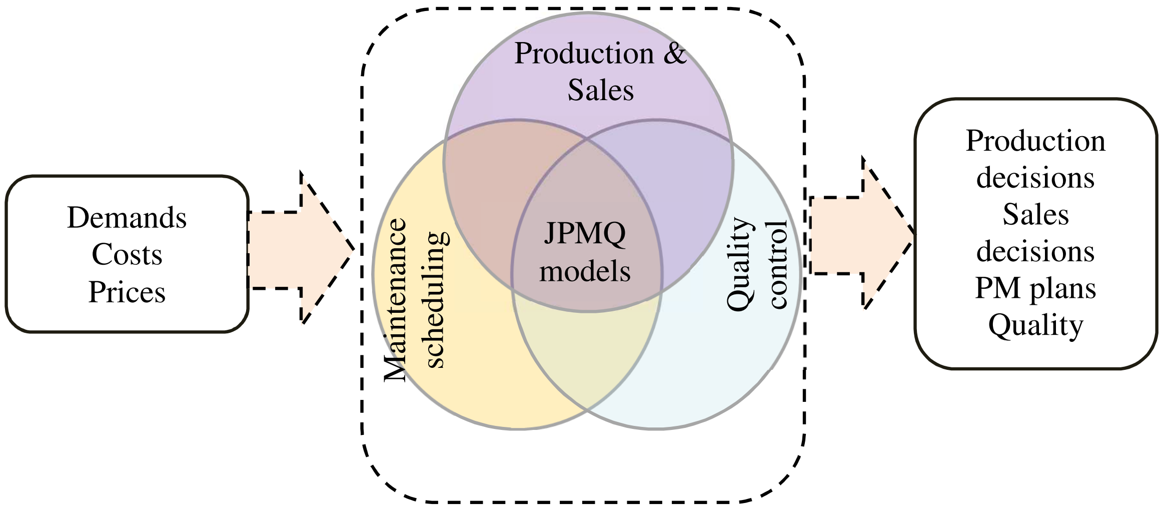

Fig. 1.11: The problem scope.

1.2.3. JPMQ model in multi-machine systems

In this extension, we develop the previous model for the case of multiple machines, and we also consider the sampling frequency from the quality system as a decision variable. Each machine has several PM levels with different costs and execution times. But, a new important constraint (stating the economic dependencies between maintenance decisions of different machines) applies to the choice of maintenance levels. The financial limitations always play the most significant role in decision systems. Solution of the model addresses the assignment of maintenance resources to the system components. The machines are initially in different conditions and we assume a linear relationship between the cost of PM level and the reduction in the machine age. Therefore, in each period, the system state is a function of all previous PM decisions. Considering multi-purpose machines that can process several product types results in a more complicated type of CLSP. Moreover to the inspection frequency, the length of inspection intervals plays a significant role in the performance and cost of the system. Banerjee and Rahim (1988) proposed to maintain a constant integrated hazard over the inspection intervals. Quality inspections are normally performed in a shorter time slots compared to the preventive maintenance operations, so we assume One PM possibility at the beginning of each period and several quality inspections during the periods. After detecting a shifted state by an inspection, a search for its cause will be initiated, and in case of machine degradation, an adjustment (or process calibration) will be performed to bring the machine in its normal state.

1.2.4. Optimization methods for JPMQ problems

The literature review (section 1.3) underlines a significant gap between the theory and the application of integrated approaches. Some reasons of this issue are:

- Complexity of the models: The current models are nonlinear problems with troublesome interactions that are too complicated to be exploited in most of the manufacturing organizations
- Limited scope of the current researches: Concentration of the research on a limited scope, and disregarding the shop floor facts and limitations that hinder the utilization of the models, and so a large variety of industrial needs and several aspects of the problem are not studied yet
- Insufficiency of managerial insights: Neglected the importance of interactions between the three functions and the lack of general insights and heuristics in joint planning is a significant source of the existing gap.

In the first and second contributions, we will address the JPMQ problem and, as the third one; we develop efficient solution methods, where the aim is to help the implementation of the integrated models and to bridge the gap between theory and application. We first study the approaches used in existing models and the heuristics developed for solving production and maintenance problems. Also, the application of evolutionary algorithms (EAs) and general purpose heuristics are considered, then, we make use of advanced strategies to enhance the algorithm performance. In the literature review, the goodness of hybrid genetic algorithms in solving large problems is shown. We integrate genetic algorithm with tabu search, the Nelder-

Mead (NM) simplex, while employing a state-of-the-art optimization package to solve a part of the main problem. In most of the EA implementations, the information in good and poor solutions is neglected. Extracting the properties of solutions in the population based algorithms helps in determination of improvement direction. In the proposed NM, we implement a method to extract positive knowledge (properties of good solutions) and negative knowledge (from poor solutions) in a memetic algorithm (MA). Despite the tabu search method that the algorithm just moves to one of the evaluated neighbor points, NM has the advantage of moving to a solution potentially better than the best existing alternative. This method uses the neighbor solutions to guess the improvement direction and to propose a contracted, an expanded, and an extended point in the optimization direction. Therefore, the NM is more aggressive than the tabu search and finds the local optima in a smaller number of evaluations. To prevent the algorithm being trapped in a local optima, the population management strategies are implemented in the MA. The goal of the population management is to maintain a desired level of genetic diversity in the population. The selection of survivors at the end of iteration is based on both the objective function and the contribution of the solutions in the heterogeneity. In the meantime, we replace the mutation operator in genetic algorithms with *intensification* and *diversification* operators. Intensification uses the best solutions in the population to introduce new individuals that are likely better than the best current solutions. This process is in line with the extraction of positive knowledge implemented in the NM. Diversification is intended to increase the diversity in case it is dropped below the desired level. It forces the algorithm to investigate the solution space with fewer numbers of delegates in the population; therefore it works better than a random mutation operator. Moreover, we use an adaptive approach in which some algorithm parameters are related to the execution time from the one hand and to the diversity level of the population from the other hand to efficiently balance between the intensification and diversification processes.

1.3. Literature review

Our first objective in the literature review is to address the previous researches in the contexts of production planning, maintenance scheduling, and quality control. We also investigate existing articles in integration of these three functions. The last part of the review is dedicated to the optimization methods in joint problems. The second objective is to establish the theoretical fundamentals of the thesis along with the deployed methodologies.

1.3.1. Lot-scheduling problem

Production planning is the assignment of resources over time to operations in order to best satisfy the constraints and customer needs. The constraints may concern to tradeoffs between operations, setup and inventory holding costs, quality and reworking costs, etc. (Graves, 1981). Subject to complexities originated from variations and uncertainties, maintenance planning can be a very challenging issue. To address the uncertainties and complexities, different production planning models as sub-classes of manufacturing systems, including the lot-sizing problem have been developed (Kenné et al., 2007). In the capacitated lot-sizing problem (CLSP), the optimal quantity of products to be processed on each machine and during each period should be determined subject to several constraints. The periods represent the time-slots in the planning horizon of say, one week to one month in real cases. The CLSP (sometimes called as a large bucket problem) is usually considered as tactical level scheduling that bridge the strategic long-term goals to the operational short-term plans (Fitouhi and Nourelfath, 2012). Uncertainties in CLSP can originate from different sources, but, we are interested in those that are related to the system and internal conditions, including processing times, machine availabilities, and quality of processed items. Giri (2005) dealt with the system unreliability issue considering the processing time as a function of the system conditions in a reactive fashion. Then, assuming the processing times as decision variables, they developed an EPQ model with and without safety stocks to evaluate lot-sizes as well as the production rates in a joint approach.

Lee and Chen (2000) studied the problem of scheduling a set of jobs on a set of parallel machines, where each machine must be maintained once during the planning horizon. The objective was to schedule jobs and maintenance activities so that the total weighted completion time is minimized. They proposed a branch and bound algorithm based on the column generation approach for solving both cases of the problem.

1.3.2. Maintenance planning problem

Organizations are engaged increasingly in improvement of the systems availability and machines reliability, since they play a crucial role in performance, safety, organizational success, and economic efficiency. Therefore, maintenance and PM planning are considered as key functions in manufacturing systems. In the traditional approach, PM planning corresponds to the selection of an optimal maintenance strategy such as scheduled inspection, preventive maintenance, corrective or opportunistic maintenance, group or block replacement, etc. (Ruiz et al., 2007). In real systems, the process conditions deteriorate with time, and preventive maintenance is aimed to prevent or slow down such deterioration.

An ideal preventive maintenance is expected to completely restore the machine conditions to an as-good-as-new state, but in real cases, only a part of the performance can be re-vitalized. After such a PM that is called imperfect maintenance, the system age lies somewhere between as-good-as-new and as-bad-as-old conditions. Pham and Wang (1996), stated that in most cases the maintenance is imperfect and they reviewed the related results. Nakagawa (1980) developed three imperfect maintenance models for a single unit system (Fig. 1.12).

With a certain probability, the unit after PM is perfect or unchanged
PM reduces the age (and the failure rate) of the machine (independent from the PM cost)
The age and the failure rate are reduced in proportion to the PM cost (PM level)

Fig. 1.12: The imperfect maintenance models.

According to the third model, the expected cost of system per time unit is given by:

$$C(T) = \frac{\left[c_1 + c_2 \int_{\left({c_0}/{c_1}-1\right)T}^{\left({c_0}/{c_1}\right)T} r(t)dt\right]}{T} \tag{1.1}$$

In this equation, c_0 and c_1 are the cost of preventive maintenance and replacement, c_2 is the cost of corrective maintenance, T is the length of inspection interval, and $r(t)$ is the hazard function describing the machine failures. Here, we consider this model with multiple PM options and we assume a linear relation between the PM cost and the reduction in the machine age. So, a maintenance schedule would determine the type of PM for each machine in each maintenance intrusion. El-Ferik (2008) studied the optimality condition in imperfect maintenance in a joint lot-sizing and maintenance planning problem, where the PM task is performed either in failure or when the age reaches a pre-determined value.

Two well-known PM strategies are age-based and condition-based maintenance models. In the first approach, PM operations are performed according to the time (or usage), while in the condition based maintenance, PM tasks are scheduled based on the condition monitoring data such as oil analysis, machine vibration data, acoustic measures, electrical properties (voltage, resistance, signal processing,…).

1.3.3. Design of quality system

Statistical process control (SPC) and quality sampling are conventional approaches to control and improve the production quality and the firm's productivity. The primary tools of SPC; the control charts have been exploited for more than 50 years to reduce the process variability, and during the recent years, we are witness of an increased tendency in employing them. Engineering design of a control chart incorporates the determination of certain decision variables; most important among them are the sample size, width of control limits, and the length of sampling intervals. These parameters influence on the long-term quality of the firm's products, cost efficiency of the quality department, and the implementation simplicity of the approach to be used in shop-floor level (Saniga, 1989). Among SPC tools, the $\bar{x}$-chart is one of the frequently used tools to control the variations of process mean. Duncan (1956) was the first who studied the economic design of an $\bar{x}$-chart and he developed the evaluation of sample size, sampling intervals, and the control chart parameters. In (Bouslah et al., 2013), the authors suggest an acceptance sampling plan to control the quality in an unreliable imperfect system.

Lin (1991) extended the model of Lee and Rosenblatt (1987) to the case where the process follows a general distribution with an increasing failure rate. They established the total cost function and found that a periodical inspection policy is the best under some conditions.

1.3.4. Integrated models

This section addresses joint scheduling articles, including production-and-maintenance, quality-and-maintenance, and the JPMQ models.

1.3.4.1. Integrated maintenance-quality planning

The interactions between maintenance and quality have been an interesting research subject in the literature. On the one hand, maintenance influences on production quality and on the other hand, the feedbacks received from the quality system are a significant basement in PM scheduling. Quality inspections are crucial in detecting internal causes of low quality issues, including system state and the need for process calibration, therefore the design of quality system (decisions such as sampling interval, acceptance-rejection conditions…) impact on the performance of the PM system. Most of the existing papers do not deal with these mutual effects in a collaborative approach. Lee and Ni (2013) examined the link between maintenance and quality in a multi-stage multi-product deteriorating system. The decisions are to clean the machine, to maintain it, or to perform a production operation from a set of tasks in a series of workstations. They use PM planning to improve the quality, but no decision variable from the quality system is taken into account. In the papers of Duncan (1956) and Banerjee and Rahim (1988), the optimal design of a quality system is addressed, however, they use the process quality and the function of the "time-to-shift to a deteriorated state" as a given and unchangeable parameter dictated by the system. In (Panagiotidou and Tagaras, 2012) the authors formulated a cost minimization model for the joint optimization of process control and perfect preventive maintenance. The PM is performed either at machine failures or at some critical ages, regardless of the quality state of the equipment. The quality system decision variables are sample size, acceptance control limit, and sampling frequency and the maintenance system decision is the age of performing the scheduled perfect PMs. In fact, perfect PMs that bring the machine in an AGAN state (in some papers called as *machine replacement*) can be very costly, and in real implementation of maintenance with sophisticated and expensive machines, the perfect PM cannot be justified. So, we believe that an imperfect PM with several maintenance levels along with deterioration of machine conditions are closer to real-life problems. Xiang (2013) considered the same quality decision variables in the context of imperfect maintenance. The process deteriorations are modeled as a discrete time Markov chain and he assumed that with known probabilities, the PM brings the machine into a superior state. Cassady et al. (2000) suggested a strategy for monitoring single-machine manufacturing process through the simultaneous implementation of an $\bar{x}$-chart and an age-based PM to uncover the link between "failure monitoring" property of control chart and "failure prevention" property of PM. The variation of the hazard rate is important in evaluating the link between PM and quality. Ben-Daya and Rahim (2000) addressed the effect of maintenance on economic design of a $\bar{x}$-chart and consider a Weibull shock model with increasing hazard rate. The lengths of inspection intervals are determined such that a constant integrated hazard is maintained over all intervals. They showed how the quality cost increases with the selected PM level and the reduction in the age of system is proportional to the PM level. Fitouhi and Nourelfath (2012) integrated noncyclical preventive replacements and corrective maintenance with lot-sizing problem. They developed a model for planning, production, and noncyclical preventive maintenance for a single machine, subjected to random failures and minimal repairs. Noncyclical PM relaxes the limitation of PM time and is the general form of joint scheduling. The approach is appropriate for the cases with large variations in demands.

1.3.4.2. Integrated maintenance-production planning

To cope with the current tough competition, many manufacturing companies have invested in highly automated production systems with sophisticated equipment. To be economically sustainable, such costly equipment should be exploited to the last instant of their maximum possible productive time. When an unplanned downtime, caused by a production line failure, occurs, it often trims down the system's productivity and renders the current production plan obsolete. Revising the production plan in an emergency situation is usually very expensive and often causes increased variability in product quality and in service level. It is, therefore, essential that production planning and preventive maintenance activities be carried out in an integrated way to hedge against these often avoidable failures and re-planning occurrences (Aghezzaf, 2008). The effect of machine failure and corrective maintenance on lot-sizing problem is studied by Groenevelt et al. (1992). They considered both non-resumption (NR) and abort-resume (AR) strategies in a failure prone system, where the repair time is negligible and failure occurs according to a Markovian process. Fitouhi and Nourelfath (2014) showed that the integration of non-cyclical maintenance and production planning and allowing PM at the beginning or during production periods reduces the total cost. Lin and Gong (2006) developed an economic production quantity model for a deteriorating process, where the machine is subject to random breakdowns. They determined an optimal production uptime that minimizes the expected total costs per unit time, consisting of setup, corrective maintenance, inventory carrying, deterioration, and lost sales costs. Lee and Ni (2013) developed an advanced job dispatching/maintenance policy based on both online condition-monitoring information and the dynamic relationship between machine degradation and product quality. In a joint production-maintenance approach, Lee and Rosenblatt (1986) assumed that the machine restoration cost at the end of a mission depends on the delay of detecting faulty state of the machine. More the machine operates in a degraded condition, more is the damage and maintenance cost. Solving the model determines the number of PM interferences in the planning horizon. In (Porteus, 1986), the author addressed the general form of EPQ models in deteriorating systems taking into account the effect of investment on the production system by linking the setup time to investment plans. Lee and Ni (2013) presented a decision-making architecture to determine maintenance and product dispatching policies based on condition monitoring information and the relationship between machine degradation and the associated product quality. They assumed a Markov decision process for long-term decision making and the integer programming for short-term decision making with a multi-product, multi-station system. They demonstrate the advantage of the proposed approach by comparing the proposed policy with the conventional decision-making approaches. Tagaras (1988) formulated a cost model for the simultaneous optimization of process control and maintenance activities. The model allows the evaluation of hybrid process control and maintenance planning. Its comparison with two disjoint policies, i.e. non-integrated planning of preventive maintenance and process control, they underlined the benefits of integration. Xiang (2013) presented an integrated model for the joint optimization of statistical process control (sampling intervals, sample size and control limit) and preventive maintenance. Aghezzaf et al. (2007) assumed that random failures are inherent in a production system. They showed that expected total production and maintenance costs are minimized in integrated planning of maintenance and lot-sizing. Nourelfath et al. (2012b) dealt with the problem of joint preventive maintenance and tactical production planning, for a production system composed of a set of parallel components, in the presence of economic dependence and common cause failures. Lee and Rosenblatt (1987) considered the simultaneous determination of production cycle and inspection schedules in a production system. They proposed a relationship approach to determine whether maintenance by inspection is necessary or not. They also showed that when maintenance by inspection is adopted, the optimal inspection schedules are equally spaced throughout the production cycle. Lee and Rosenblatt (1989) also extended their work to consider the case where no immediate knowledge of the system's state is available unless inspection of produced items is performed. Maintenance activities are subsequently initiated based on the inspection results. The stochastic deterioration process considered in their model follows an exponential failure distribution. Lee and Rosenblatt succeeded in determining the optimal lot size and inspection schedule.

1.3.4.3. Joint production-maintenance-quality; JPMQ models

Regarding the imperfectness of deterioration systems, understanding the relationship between production, quality, process inspection, and maintenance can assist managers to perform production control and quality assurance in a more effective manner. Chen (2013) developed an integrated profit model for imperfect production system incorporating the imperfect rework process, scrapped items, and PM errors. He stated that integrating the reworking process into the EPQ model was beneficial to the total profit, furthermore, PM was shown to raise the expected total profit, and the maximum level of PM would yield the highest expected profit by the production system. Ben-Daya (1999) presented an integrated model for the joint optimization of production quantity, design of quality control parameters, and maintenance level. The model was developed for a process having general shift distribution with increasing hazard rate. The PM schedule is coordinated with the quality control inspections. In his model, PM activities reduce the rate of process shifts to out of control states in proportion to the selected PM level. This change in the quality shifts affects directly on quality control costs and the length of the production cycle. Kazzaz and Sloan (2013) examined single-stage production system that deteriorates with production actions, and improves with maintenance. The conditions of the process can be in any of several discrete states, and transitions from one state to another one follow a semi-Markov process. The firm can produce multiple products, which differ by profit earned, expected processing time, and impact on equipment deterioration. It can also perform different maintenance actions, which differ by their cost, expected down time, and impact on the process condition. The firm needs to determine the optimal production and maintenance choices in each state in a way that maximizes the long-run expected average reward per unit time. In Their paper they show how the critical ratios can be combined in order to determine the optimal policy, simultaneously accounting to the trade-offs involving production profits, maintenance costs, and the impact on the process condition. Also, they demonstrate the impact of market demand conditions on the optimal policy. The set of sufficient conditions that lead to monotone optimal policies are also discussed. Various maintenance policies to prevent system failure and to improve system reliability have been studied intensively over the past several decades. Ben-

Daya, and Duffuaa (1995) discussed the relationship and interaction between maintenance, production and quality, and two different approaches were proposed. Lee and Rosenblatt (1989) suggested a plan for integrating maintenance and inspection with restoration costs depending on the detection delay in a production system that allows product shortages. In the joint approach developed in (Nourelfath, Nahas and Ben-Daya, 2016), the authors presented a model for multi-state system and s-independent components for which, the states are considered to degrade with use, and these degradations may lead to production of nonconforming items.

1.3.5. Solution Methods

Another course of literature review goes to the development and utilization of optimization methods used in joint production, maintenance, and quality (JPMQ) problems, and the strategies in improving their efficiency.

1.3.5.1. Solution approaches used in joint problems

Lee and Chen (2000) proposed a branch and bound algorithm based on the column generation approach for solving two different cases of the problem. Bouslah et al. (2013) considered a modified hedging point policy along with simulation based approaches to determine the approximate control parameters in a lot-sizing problem formulated as a stochastic dynamic programming where the decision variables are the lot-sizes and the production rates. Scholl et al. (2012) investigates a simulation approach to solve long and short term maintenance scheduling problem in a manufacturing line. Subramanian et al. (2012) modeled an Artificial Bee Colony (ABC) algorithm to solve the problem of generating maintenance scheduling and they compared it to a Discrete Particle Swarm Optimization (DPSO). They stated that the ABC outperforms the latter approach in terms of the performance and the solution quality. The NM algorithm (Nelder and Mead, 1965) is a direct search method for optimization of functions where their derivatives are unknown or difficult to evaluate. The method is also hybridized with various approaches and has yielded very good results in solving benchmark functions with large search spaces. Khojaste Sarakhsi et al. (2016) proposed a hybrid Nelder-Mead (NM) and scatter search algorithm for a joint economic lot-sizing problem, where the demands are price-sensitive. They reported a great performance of the solution approach for sample problems with different sizes. Fitouhi and Nourelfath (2014) developed a simulated annealing (SA) algorithm for an integrated lot-sizing and preventive maintenance scheduling problem. Introduced by Kirkpatrick et al. (1983), the SA method is based on the principles of thermodynamics annealing.

1.3.5.2. Evolutionary algorithms and hybrid methods

Evolutionary algorithms have been successfully used in optimization of large classes of problems as well as the JPMQ problems. Compare et al. (2015) offered a multi-objective genetic algorithm to maintenance optimization problem where, the model parameters are uncertain. Sortrakul et al. (2003) declared efficient performance of genetic algorithm applied to a joint production maintenance problem.

A practice in improving the performance of evolutionary algorithms is the combination of population based methods with local search and problem specific heuristics. Sastry et al. (2005) stated that "*Hybridization can be an extremely effective way of improving the performance and effectiveness of Genetic Algorithms. The most common form of hybridization is to couple GAs with local search techniques and to incorporate domain-specific knowledge into the search process*". Sarker et al. (2013) employed a hybrid approach (combination of an EA with a problem specific local search) in solving a joint job-scheduling and maintenance planning problem. Memetic algorithms (Moscato and Norman, 1989) are a good example of successful hybridization of GA and local search methods. Massaro and Benini (2015) solved multi-objective optimization problems using a surrogate-assisted memetic algorithm that preserves the genetic diversity within the population. They use artificial neural network in refinement of the solutions. The examples of exploiting hybrid methods in optimization problems are noticeable, but, because of the novelty of the JPMQ models, as of best of our knowledge, such approaches are not proposed for the JPMQ problems.

1.3.5.3. Diversity management in EAs

A possible problem in GA is the fast convergence and being trapped in a local optima. To deal with this problem, researchers propose to maintain the population diversity during the solution process. Some of the population diversity-maintaining methods, to name, are the sharing method (Goldberg and Richardson, 1987), the DCGA (diversity control oriented genetic algorithm) (Shimodaira, 1997), the ranked space method (Winston, 1992), and the restricted mating method (Eshelman ad Schaffer, 1991).

In (Vidal et al., 2013) the authors offered a hybrid genetic search with advanced diversity control (HGSADC) for a large class of vehicle routing problems (VRP). They compared HGSADC with other heuristics, including variable neighborhood search, tabu search, and state-of-the art VRP heuristics, and reported an impressive efficiency of the algorithm. According to them, incorporation of "*the contribution of solutions in the population diversity*" in the objective function improves the algorithm performance. One of our objectives is to develop hybrid methods while implementing population management schemes to efficiently solve our JPMQ problems.

However, to implement a population management strategy in genetic algorithm, one should address several issues, including the measurement of population diversity, its good value during the solution process, the approaches to increase or decrease it when needed, contribution of solutions in the heterogeneity, selection for mating or survivor selection, etc. for example, the Hamming distance method is offered in binary encoding chromosome structures and for natural or real encoding approaches, other distance measurements such as Minkowski method (Yoon and Kim, 2013) can be considered.

Jassadapakorn and Chongstitvatana (2011) afforded a self-adaptation mechanism to control the population diversity in GA without needing problem specific parameter settings information.

To implement the idea, they introduced the *difference-factor* in the selection process that measures the dissimilarity of chromosomes, and used *contribution-measurement* defined as the rate of *at-least-one-better-child* in crossover operator to select the type of the crossover operator (or crossover configuration). Population diversity is measured by $Diversity = \frac{\sum_{j=1}^{n}\sum_{i=1}^{n} h(I_i, I_j)}{n^2 \cdot l}$, where, n is the population size, I_i is the i^{th} individual, l is the chromosome size, and h is the Hamming distance function between two individuals. They communicated better performance of the algorithm in the test problems compared to the traditional genetic algorithm with the best parameter setting.

1.4. Outline of the thesis

The main question of the project is "How to improve the efficiency of production and maintenance planning taking into account the quality system?"

To answer this general question, the following issues are considered:

1. Modeling and evaluation of the interactions between the key functions
2. Incorporation of process deterioration in a multi-level imperfect maintenance planning system
3. Effect of quality control parameters on the performance of maintenance and production
4. Efficient methods for solving JPMQ problems and exploiting the benefits of integration

In this thesis we develop joint models integrating the key functions; production and sales planning, maintenance scheduling, and quality control systems. Therefore, the research objectives, contributions, and the implemented methodologies can be summarized as follows:

Contribution I: For the first contribution, in chapter Two, we develop a profit maximization model for JPMQ planning problem in a system composed of one machine, multiple periods, and multiple products in imperfect systems, where the nonconformity rate in degraded state of the machine is subject to the PM plans. For the production planning, we take into account a capacitated lot-sizing problem. The production capacity defined as the available production time in each period is impacted mainly by the maintenance system. The time of preventive and corrective maintenance is deduced from the period. The demands are varying from one to another period that distinguishes the model from the well-known economic production quantity models. Preventive maintenance is also imperfect. Therefore, just a part of the machine conditions after preventive maintenance is restored. The improvement is linked to the cost of the selected maintenance task. Assuming various possibilities in PM scheduling, a maintenance plan addresses the operation types for all interferences. In the first contribution, the PM levels for each possible interference in a discrete-time, multi-level maintenance context is addressed in which, the choice of no-PM may cancel maintenance in some intrusions. The model also covers sales planning taking into account time-varying costs and prices.

Contribution II: In chapter Three, we develop the integrated approach for a system with multiple machines and we incorporate decision variables concerning the quality system. In multi-machine processes, the maintenance planning is subject to financial limitations or the availability of other resources. Therefore, economic dependencies between different components of the system are incorporated in the problem. The JPMQ model addresses the optimal assignment of these resources to the system components while determining the economic number of quality inspections to be performed in on each machine in each period. Moreover, using the concept of non-uniform inspections, the time interval between consequent inspections will be determined such that the integrated hazard over all intervals in a period is a constant. Non-uniform inspection policy is in line with the the economic design of quality systems in systems with an increasing hazard rate. Other parts of this contribution are similar to the previous one incorporating the multi-product capacitated lot-sizing problem with time-varying demands.

Contribution III: As the third contribution, chapter Four addresses efficient solution methods for the JPMQ problems to reduce the existing gap between theory and application of the joint models. We employ hybridization and population management strategies in population based heuristics where, the algorithm exploits useful information from the population in order to determine the improvement direction, and to force the algorithm to examine un-visited regions of the search space. In this contribution we propose a memetic algorithm with state-of-the-art population management strategies that can solve different sizes of joint problems presented for the previous contributions. As the local search algorithm, we exploit a tabu-search and a Nelder-Mead method with implemented tools guiding the algorithm moves.

1.5. Conclusions

In this chapter, the need for optimization of decision systems concerning the planning of production, maintenance, and quality is discussed and the role of these functions in the organizational success is demonstrated. Then, the interactions between the related systems are highlighted and some of their mutual interactions are illustrated. We discussed that production planning, preventive maintenance scheduling, and decision parameters of the quality systems are generally dealt separately in the literature as well as in industry. In problem definition, the interactions between these features are highlighted and the need for joint scheduling of the decisions is justified. The literature review demonstrated the current state of the integrated planning and also, we addressed some properties of the simultaneous scheduling problem. Different aspects of the problem are inspected and the benefits of integration are cited. Our review showed that the joint scheduling has receiving more interest of researchers and several aspects of the problem need to be studied. Various important features corresponding to different industries are not incorporated in joint problems. In this thesis we address the incorporation of lot sizing and imperfect preventive maintenance planning for the imperfect manufacturing systems.

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Chapter Two: A Profit Maximization Model

This chapter is dedicated to the article entitled “Joint maintenance scheduling and production planning for imperfect processes: A profit-maximization model” submitted to the *European Journal of Operational Research* in November 2015.

The titles, figures, and mathematical formulations are modified to keep the consistency through the thesis.

Solution methods based on quadratic and mixed integer formulations of this model are presented in appendix A.

Résumé

Cet article intègre l'ordonnancement des lots et la planification de la maintenance préventive imparfaite basée sur l'âge, en tenant compte la détérioration de la qualité. L'état de la machine se dégrade avec le temps et il peut passer de façon aléatoire, d'un état normal à un état détérioré. Par ailleurs, la machine est assujettie à des pannes aléatoires qui sont suivis de réparations minimales. Au cours de chaque période, le processus est inspecté à plusieurs reprises et des activités de maintenance préventive sont effectuées. L'amélioration de l'état de la machine est proportionnelle au niveau de la maintenance préventive. Les variations des coûts et des prix, ainsi que les différentes alternatives de maintenance, offrent un modèle flexible qui permet de prendre en compte plusieurs scénarios d'entretien et différentes stratégies de production. La formulation mathématique repose sur des concepts tirés de la fiabilité, de l'ordonnancement de la production par lots et du contrôle de la qualité, pour l'évaluation des coûts et des facteurs en interaction afin de maximiser le profit. Trois méthodes de résolution sont proposées et des comparaisons sont effectuées, en utilisant des plusieurs instances de problèmes aléatoires. Une analyse de sensibilité est effectuée par rapport aux paramètres clés du modèle dont l'effet sur la rentabilité est expliqué. Les résultats obtenus montrent que des améliorations substantielles sont possibles grâce à l'utilisation de l'approche intégrée proposée dans ce chapitre.

Mots clés

Planification de la production, Maintenance imparfaite, Qualité, Intégration, Maximisation de la rentabilité

Abstract

This paper integrates lot-scheduling, sales planning and age-based imperfect maintenance with respect to quality deterioration in imperfect systems. The machine condition degrades with time and it may randomly switch from an in-control state to a deteriorated condition with increased nonconformity rate. Also, the machine is subject to random failures that initiate a minimal repair. During each period, the process is inspected in several instants of time and the imperfect preventive maintenance activities are performed. Improvement of the machine conditions is proportional to the preventive maintenance level. Time-varying costs and prices in the periods as well as the several maintenance possibilities for each interference, affords a flexible model and allows the consideration of multiple production maintenance scenarios in the system. The mathematical formulation employs the concepts of reliability, capacitated lot-scheduling and quality inspections to evaluate the average of several cost components, where the objective function is to maximize the profit. Three solution methods are proposed and comparisons are performed using several random problems and a detailed numerical example. Sensitivity analysis is conducted with respect to the key parameters of the model and their effect on the profitability is discussed. The results underline potential improvements with the joint approach.

Keywords

Capacitated lot scheduling, Imperfect maintenance, Quality, Integration, Profit maximization

2.1. Introduction

Production and sales planning, preventive maintenance (PM) scheduling and quality control are the key functions of every manufacturing system. Because of using common resources (financial, human, time, etc.) and due to mutual influences, these functions have important interactions and they need to be incorporated in the same model. The value of joint scheduling is reported in several papers. For example, Ben-Daya and Duffuaa (1995) stated that integrated planning is a beneficiary of the whole system or Cassady et al. (2000) and Kenné et al. (2007) reported improved productivity. The significance of integration in the cost effective planning and scheduling of industrial operations is proved, but the majority of the existing models deal separately with them. Lot scheduling, age based preventive maintenance and economic design of quality control systems individually are studied in the literature, for example, one can refer to Haugen et al. (2007), Drexl and Kimms (1997), Salvietti and Smith (2008) for lot scheduling; Chen (2011), Shafiee and Finkelstein (2015) and Liu et al. (2014) for maintenance planning; and Duncan (1956), Banerjee and Rahim (1988), Chen and Yang (2002), and Lee et al. (2012) in design of the quality systems.

Such integration requires an increased level of coordination between the related departments (Lee et al., 1999; Linderman et al., 2005), and joint models are generally of higher complexity levels. Some of the papers on integration of production planning and maintenance scheduling are Rahim (1994), and Chareonsuk et al. (1997). Hadidi et al. (2012), Suliman and Jawad (2012), and Sung and Ock (1992) studied integration in the context of single machine systems, justifying the benefits of integration. Nourelfath and Châtelet (2012) considered economic dependencies and common-cause failures of parallel components. Kenné et al. (2007) proposed an integrated model for an unreliable system with sale returns.

In general, production systems may deteriorate with time and so, preventive maintenance is aimed to improve the system conditions and the quality. Process deterioration in a joint scheduling system is examined by Kazaz and Sloan (2013). Xiang (2013) showed that PM improves the quality. Rivera et al. (2013) investigated production planning problems with overhaul scheduling in deteriorating systems. By deterioration of the conditions, probability of a shift to an out-of-control state with higher nonconformity rate increases. Ho and Quinino (2012) studied the shifts between *in-control* and *out-of-control* states using an integrated model. Several PM models including perfect and imperfect maintenance are considered in joint scheduling problems. Perfect maintenance assumes that the system is “as good as new” following a PM.

However, this assumption may not be true in practice (Nakagawa, 1980; Pham and Wang, 1996). A more realistic assumption is that, upon maintenance, the system lies in a state somewhere between ‘as good as new’ and its pre-maintenance condition, *i.e.*, maintenance is imperfect. In the existing imperfect maintenance literature, various modeling methods have been used. Adopting the concept of age-based maintenance, we assume that the machine age after PM reduces proportional to cost of maintenance (Nakagawa, 1980; Pham and Wang, 1996). Imperfect maintenance is investigated in several joint models (Tagaras, 1988; Ben-Daya, 1999, and Ben-Daya and Rahim, 2000). One of the essential questions to be addressed in quality control is the time interval between the consecutive inspections. Rahim (1994) showed that non-uniform inspections are more efficient. Chareonsuk et al. (1997) determined the PM intervals taking into account the cost and reliability of the system. Selecting between constant and variable maintenance intervals for complex repairable systems is studied by Percy and Kobbacy (2000). Lee and Rosenblatt (1987) stated that equidistant intervals are cost effective. It is generally admitted that while periodic PM can be more convenient to schedule, sequential or non-periodic PM is more realistic when the system requires more frequent maintenance as it ages. Duncan (1956) first established a criterion that measures approximately the average net income of a process under surveillance of a control chart when the process is subject to random shifts in the process mean. He showed how to determine the sample size, the interval between samples, and the control limits that will yield approximately maximum average net income. According to him, these PM-inspections should be equally spaced for an exponential failure time function. Banerjee and Rahim (1988) extended the Duncan's model to non-Markovian shock models by choosing the length of sampling intervals such that the *integrated hazard* over each interval is the same for all intervals. This means that the length of the sampling intervals is defined so as to keep the probability of a shift in an interval, given no shift up to its start, constant for all intervals.

Regarding the link between maintenance and production planning, Bouslah et al. (2013) incorporated lot-scheduling with a quality plan in imperfect systems and developed a heuristic method to solve the problem. Aghezzaf (2007) and Pal et al. (2013) considered lot-scheduling in an imperfect system with reliability parameters. Ji et al. (2007) studied the single machine scheduling with several PM periods and showed that the LPT algorithm is the best method for minimizing the total completion time. Fitouhi and Nourelfath (2012) investigated the problem of non-cyclical maintenance scheduling in a single-machine.

Wang (2013) proposed a model to incorporate minimal repair and rework possibilities in order to determine the optimal number of inspections, the inspection interval, the EPQ, and the PM level. Pinjala et al. (2006) proposed that higher quality levels can be achieved by proactive maintenance policies integrated with efficient production and control systems. Pandey et al. (2011) integrated preventive maintenance with production scheduling in order to determine the optimal batch sequence that minimizes the delay in the primary schedule. In Nourelfath et al. (2016), the authors integrated the three functions in a multi-product imperfect system. Dhouib et al. (2012) modeled the quality aspects of an imperfect deteriorating system with production and inventory control in the context of age-based PM. Chen (2013) investigated the optimal inspection interval, PM policy, and production quantity in order to maximize the unit profit assuming one PM level to be applied in all interferences. In his approach, a correct PM reduces the failure rate and an incorrect PM may shift the system to an out-of-control state.

The literature review underlines that only a few papers have studied the integration of lot-scheduling, maintenance, and quality in a unite model and to the best of our knowledge, sales planning with multiple PM levels in a joint model with production, maintenance, and quality decisions is not addressed in the literature. The existing papers are mainly limited to integration of maintenance and production, or maintenance and quality, and there is a significant gap in theory and application of the joint models. One of the reasons of such a gap is the lack of studies addressing the properties of real systems such as the existence of multiple alternatives for PM interferences, the imperfectness of maintenance, availability of the system subject to PM, and the lack of efficient solution methods for the joint problems. Incorporation of sale planning in the model establishes the missing link between several external parameters (demands, prices…) and internal decisions. This study is the first one addressing profit maximization by integration of production-sales planning, maintenance scheduling, and quality control in a multi-period multi-product system with time-varying costs and prices. A large number of PM alternatives, restoration cost related to the detection delay as we see in real mechanical systems, and variable costs, prices, and demands are the other characteristics of our model. Thus, our approach goes one step beyond the conventional profit maximization of the existing models. By means of a sensitivity analysis, we provide insights on the importance of the integrated approach, and on the significance of the related parameters on the optimality of the solutions. Since the studied problem is inspired from the industrial needs, the proposed model makes contributions not only to methodology, but also to knowledge for improving industrial practices. Section 5 presents several numerical examples to show that potential improvements in the total profit can be achieved with the proposed model, while discussing the effect of the model data on the profitability.

The remainder of this paper is organized as follows: Section 2.2 describes the problem and its specifications. Section 2.3 develops an evaluation method for the costs and interacting factors. The profit maximization model and solution algorithms are presented in section 2.4. Section 2.5 provides several numerical illustrations and a comparison of the solution methods. Section 2.6 is dedicated to sensitivity analysis and finally, section 2.7 presents our concluding remarks.

2.2. Problem definition

Problem parameters and indices

b_t^p	Backorder cost of product p in period t
CMR	Cost of minimal repair
$CPM(q)$	Cost of PM level q
d_t^p	Demand of product p in period t
g^p	Production rate of product p
h_t^p	Holding cost of product p in period t
L	Length of the periods
M	Number of PM in each period
NF_t^k	Number of machine failures in interval k of period t
P, p	Number and index of products
PC_t^p	Price of conforming item of product p in period t
PN_t^p	Price of nonconforming item of product p in period t
Q, q	Number and index of PM levels

s_t^p	Set-up cost for product p in period t
T, t	Number and index of periods
$TPM(q)$	Time of PM level q
TMR	Time of minimal repair
α^p	Nonconformity rate of product p in a shifted state
β	Cost of the process inspection
η	PM imperfectness factor
π_t^p	Unit production cost for product p in period t
ξ_0, ξ_1	Parameters of the restoration cost
$\zeta(y)$	Hazard function of machine failures at age y
δ^p	Cost of quality check of product p

Associate parameters……………………………………..………………………………

APT_t	Available production time in period t
NF_t^k	Number of machine failures in interval k of period t
PS_t^k	Probability of shift in interval k of period t
QC_t	Cost of quality checking of products in period t
w_t^k, y_t^k	Age of the machine at beginning and end of the k^{th} interval in period t
XN_t^p, XC_t^p	Number of nonconforming and conforming items of product p in period t

Decision variables…………………………………………………………..…………………

B_t^p	Backorder level of product p in period t
IC_t^p	Inventory of conforming product p at the end of period t
IN_t^p	Inventory of nonconforming product p at the end of period t
M_t^k	PM level at the end of interval k of period t
S_t^p	Set-up decision variable for product p in period t
SC_t^p	Number of conforming items of product p sold in period t

SN_t^p	Number of nonconforming items of product p sold in period t	x_t^p	Lot size for product p in period t

The objective of this paper is to develop a profit maximization model for joint determination of maintenance schedules and production quantities for imperfect processes. A process is said to be imperfect in the sense that it may produce nonconforming items. Production planning includes a multi-period multi-product lot-scheduling problem that lies in the context of medium-term production planning, and its aim is to satisfy customer demands by addressing the trade-offs between various costs. The machine has two types of hazards that are machine failure and shift to out-of-control (OOC) state. In the case of failure, the machine cannot perform production operations and a minimal repair is instantly initiated. In out-of-control state, the rate of nonconformity is increased. It is assumed that shift and failure probabilities are linked to the effective age of the machine. Several PM options are considered to improve the machine age. The production system is composed of one machine and P products. The planning horizon H includes T periods each of length L. Note that in this paper we are using the iscrete-time PM because it is the common strategy in most real systems. Most organizations schedule their PM to be performed in predefined instants of time (weekends, vacations, between consequent missions, or certain seasons, etc.). For example, in aviation and rail transportation, and even in automotive industries, a preventive maintenance is not considered while the system is performing a mission. The discrete-time, multi-level maintenance strategy is widely used in industries because, instead of stopping the function of a machine for a maintenance during a mission, most organizations prefer to postpone or to advance PMs such that they coincide with low-load or not-working time periods. In these cases, the PM time and the number of maintenance per period is not a question, rather the decisions such as to perform or not to perform PM and the optimal maintenance level for each intrusion should be determined. PM levels can range from *no-PM* to a complete overhaul of the system. The periods are divided into several equidistant intervals. Excluding the last interval, process inspection and preventive maintenance activities are to be performed at the end of each interval. Production rate of item p is g^p. The machine conditions deteriorate with time and so, the probability of a shift to *out-of-control* state or the probability of machine failure increases. In out-of-control state, the nonconformity rate of product p increases to α^p. A minimal repair in case of a failure brings the machine in line without influencing on its age. Demands, prices, and cost components of the system are given and for the sake of flexibility, we assume that they may change from one period to another. Conforming items are considered to satisfy the demands, whereas nonconforming items can be sold in the second market. Solving the model should determine the lot sizes and the sale levels in the sense of maximizing the total profit.

Maintenance includes a) several PM levels to be performed at the end of each interval; b) corrective maintenance in the case of machine failure; and c) machine restorations (or replacement) at the end of each period (M + 1 intervals) . At the beginning of each period, the machine is in *as-good-as-new* condition and its age is *zero.* By progressing the time, the machine age increases and it is more prone to fail or to shift to an out-of-control state. Preventive maintenance reduces the machine age and improves its condition. There are several PM options labeled from *1* to Q, of which Q is the index of the lowest PM level. $CPM(i)$ and $TPM(i)$ are respectively the cost and the time of i^{th} PM level. The inspections divide the periods into M + 1 interval. Let's consider that the machine age in the k^{th} interval of period t changes from w_t^k to y_t^k. Let also, m_t^k denote the PM level at the end of the k^{th} interval of period t. If an inspection results in detection of a shift, the items processed in that interval should be quality checked. The inspection also ensures that the machine starts the intervals in a normal state and the efficiency of PM decreases as the machine age increases. The cost of PM determines the capability of that PM level in improving the conditions. So, we presume that PM efficiency corresponds to either cost of the related PM option and the rank of PM in the period. This is in line with the existing literature on age-based imperfect maintenance (Nakagawa, 1980; Lin et al., 2001; Lai et al., 2001). With η $(0 < \eta \leq 1)$ as the PM imperfectness factor, the machine age at the beginning of the k^{th} interval (k = 2… M + 1) is:

$$w_t^k = y_t^{k-1}\left(1 - \frac{CPM(M_t^{k-1})}{CPM(1)} \cdot \eta^{k-2}\right). \tag{2.1}$$

The equation states that the age of machine at the beginning of interval k of period t is calculated by multiplying its age at the end of the previous interval by the age reduction factor $(1 - \frac{CPM(M_t^{k-1})}{CPM(1)} \cdot \eta^{k-2})$. Nakagawa (1980) proposed the link between the PM cost of imperfect maintenance model and the improvement in the system conditions. The linear relationship between the cost of the selected PM level ($CPM\left(M_t^{k-1}\right)$) and the reduction in the age of machine is a general assumption that can be simply replaced with any other function for different systems. Ben-Daya (2002), Chen (2013), Nakagawa (1980) and Nourelfath, Ben-Daya, and Nahas (2016) are some examples from the literature that assume linear link between PM cost and age reduction factor.

The machine failures occur according to a non-homogeneous Poisson Process (NHPP) having intensity function $\zeta(y)$ as the failure rate of machine at age y. So, the expected number of machine failures in the k^{th} interval of period t is:

$$NF_t^k = \int_{w_t^k}^{y_t^k} \zeta(y)dy. \tag{2.2}$$

Note that the cost and the time of minimal repairs are considerably higher than the preventive maintenance. At the end of $(M + 1)^{\text{th}}$ interval, the machine will be restored to its perfect condition. We consider that the cost of the machine restoration linearly depends on the delay in detecting the out-of-control state. The number of process inspections in each period (M) is a parameter of the model and it can be easily changed to find its best value.

Fig. 2.1 shows a sample period with three PM and four intervals. In the 2nd and the 4th intervals, a shift has occurred and the machine has started to produce defective items, therefore a quality check at the end of these intervals is initiated to separate conforming and nonconforming products. The effect of age reduction by PM is not shown in this figure.

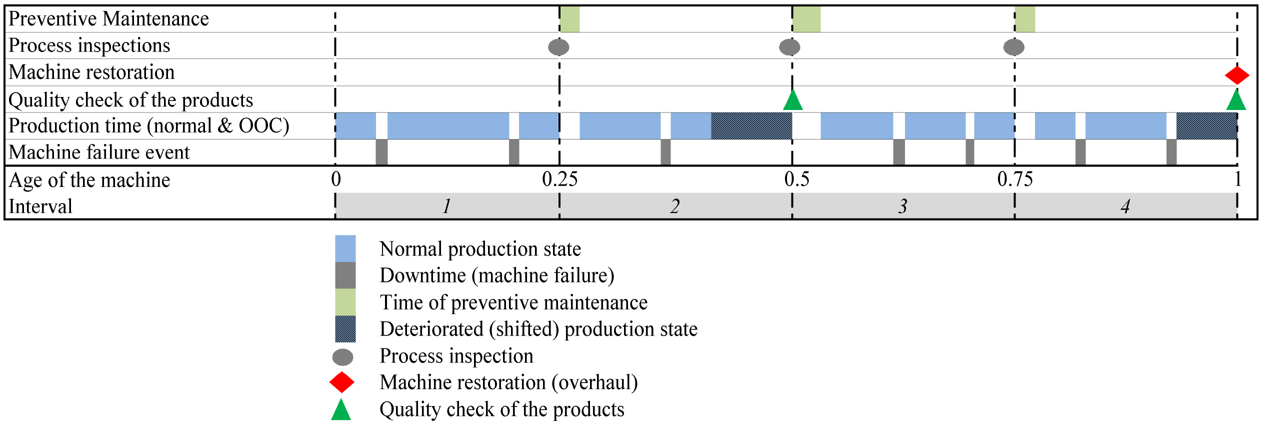

Fig. 2.1: Sample period with three PMs and four intervals.

2.3. Evaluation of costs and interacting factors

This section presents the evaluation of costs, the machine conditions as a function of maintenance plans, nonconformity costs, etc. used in mathematical formulation of the model.

2.3.1. Age of the machine in the intervals and available production time

The most important part of our evaluation concerns the determination of the machine age in intervals. Adopting the idea of linear link between age reduction and PM cost (Ben-Daya, 1999; Ben-Daya and Rahim, 2000), we have:

$$w_t^1 = 0,\ t = 1,\dots,T, \tag{2.3}$$

$$y_t^k = w_t^k + \frac{L}{M+1} - TPM\left(M_t^k\right) - NF_t^k \cdot TMR,\ k = 1,\dots,M+1;\ t = 1,\dots,T, \tag{2.4}$$

$$y_t^{M+1} = w_t^{M+1} + \frac{L}{M+1} - NF_t^M \cdot TMR,\ t = 1,\dots,T, \tag{2.5}$$

$$w_t^k = y_t^{k-1}\left(1 - \frac{CPM\left(M_t^k\right)}{CPM(1)} \cdot \eta^{k-2}\right),\ k = 2,\dots,M+1;\ t = 1,\dots,T. \tag{2.6}$$

Equation (3) indicates that the machine age at the beginning of the first interval of each period is $zero$. Equations (2.4) and (2.5) correspond to the machine age at the end of the k^{th} interval where, $NF_t^k \cdot TMR$ indicates the expected time of minimal repair (See, equation 2.2) and finally (2.6) represents the age at the beginning of intervals. The simultaneous solution of these equations yields all the age values.

The available production time in interval k of period t is $y_t^k - w_t^k$. Note that the PM time and the time of minimal repairs are reduced from the length of the interval. The available production time in period t is:

$$APT_t = \sum_{k=1}^{M+1}\left(y_t^j - w_t^j\right). \quad (2.7)$$

2.3.2. Probability of the shift to out-of-control state and the realization probabilities

Considering *F(t)* cumulative distribution (*CDF*) of the time-to-shift function, the conditional probability of a shift in interval k of period t is:

$$PS_t^k = \frac{F\left(y_t^k\right) - F\left(w_t^k\right)}{1 - F\left(w_t^k\right)}. \quad (2.8)$$

It is assumed that the backorder costs (b_t^p) are larger than the production costs (π_t^p), therefore the model forces the solution to take positive values for lot sizes in order to minimize the sum of the costs. Also, because the cost of inventory holding is included in the objective function; the profit maximization model will not consider positive inventories at the end of the last period. But since there are no limitations on the demand levels, it is probable to have some backorders at the end of each period (including the last one).

2.3.3. Cost of production, inventory holding, setup, and backorder

Production, inventory holding, backorder, and setup costs are respectively:

The total production cost: $\sum_{t=1}^{T}\sum_{p\in P}\pi_t^p x_t^p$, (2.9)

The total inventory holding cost: $\sum_{t=1}^{T}\sum_{p\in P}h_t^p\left(IC_t^p+IN_t^p\right)$, (2.10)

The backorder cost: $\sum_{t=1}^{T}\sum_{p\in P}b_t^p B_t^p$, (2.11)

The total setup cost: $\sum_{t=1}^{T}\sum_{p\in P}s_t^p S_t^p$, (2.12)

where, IC_t^p and IN_t^p are respectively inventory levels of conforming and nonconforming products at the end of period *t*.

2.3.4. Preventive and corrective maintenance cost

The cost of PM in period *t* is:

$$CPM_t=\sum_{k=1}^{M}CPM\left(M_t^k\right). \tag{13}$$

Taking into account the expected number of machine failures, the average cost of corrective maintenance in period *t* is:

$$CMR_t=CMR\cdot NF_t^k. \tag{2.14}$$

2.3.5. Restoration cost

It is assumed that the restoration cost corresponds linearly to the delay in detecting the shift. If a shift occurs at age $t\left(w_t^k\le t\le y_t^k\right)$, detection delay will be y_t^k-t and the conditional probability of shift at *t* given no shift has occurred at the beginning of the interval is $f\left(t\middle| w_t^k\right)=\frac{f(t)}{1-F\left(w_t^k\right)}$. So, the detection delay or the expected duration of time that the machine operates in an out-of-control state in period *t* is:

$$\tau_t=\sum_{k=1}^{M+1}\left(\int_{w_t^k}^{y_t^k}\left(y_t^k-\tau\right)\cdot f\left(t\middle| w_t^k\right)\cdot d\tau\right)\cdot PS_t^k. \tag{2.15}$$

The restoration cost in period t is:

$$RC_t = \xi_0 + \xi_1 \cdot \tau_t, \tag{2.16}$$

where ξ_0 and ξ_1 are some constants.

2.3.6. Cost of process inspection and quality checking of products

The cost of process inspections in all the T periods is simply:

$$IC = \beta \cdot (M+1) \cdot T. \tag{2.17}$$

It is assumed that by detecting a shift, all the items produced in that interval will be quality checked. To evaluate the quality checking cost and the number of conforming and nonconforming products we assume that the production is smoothly distributed over all intervals in the periods; otherwise we would needs to consider the exact job orders and the instants that the processing of each item will be performed. Job scheduling and detailed time of processing each job is not in the scope of this problem. With this assumption, the amount of production in an interval is proportional to the length of the interval divided by the available production time in the period. So, $x_t^p \cdot (y_t^k - w_t^k)/APT_t$ is the average number of items of product p produced in k^{th} interval of period t and PS_t^k is the probability of a shift in this interval. Therefore, taking into account δ^p; the unit cost of quality check of product p, the total expected quality checking cost for all products in period t is:

$$QC_t = \sum_{p \in P} \frac{\delta^p \cdot x_t^p}{APT_t} \cdot \sum_{k=1}^{M} PS_t^k \cdot (y_t^k - w_t^k). \tag{2.18}$$

2.3.7. Number of conforming and nonconforming items

If a shift occurs in interval k of period t (with probabilityPS_t^k), then the average time that the machine remains in this state before being fixed (in inspection) is $\int_{w_t^k}^{y_t^k} (y_t^k - \tau) \cdot f(t | w_t^k) d\tau$.

With the assumption of smooth distribution of productions over all intervals, the expected number of items processed in such a state (for product *p* in period *t*) is $x_t^p \cdot PS_t^k \cdot \int_{w_t^k}^{y_t^k} (y_t^k - \tau) \cdot f(t|\, w_t^k) \cdot d\tau \,/ APT_t$. The nonconformity rate in a shifted state is α^p, so the expected number of nonconforming items produced in period *t* (for product *p*) will be:

$$XN_t^p = x_t^p \frac{\alpha^p}{APT_t} \cdot \sum_{k=1}^{M} \int_{w_t^k}^{y_t^k} (y_t^k - \tau) \cdot f(t|\, w_t^k) \cdot d\tau \cdot PS_t^k \qquad (2.19)$$

Thence, the number of conforming items would be:

$$XC_t^p = x_t^p - XN_t^p. \qquad (2.20)$$

2.3.8. Expected sales

Taking into account the sale prices, the revenue by selling product *p* in period *t* would be $RN_t^p = SC_t^p . PC_t^p + SN_t^p . PN_t^p$, where SC_t^p is the quantity of conforming product and SN_t^p is the quantity of imperfect items sold in period *t*. Then the total income in period *t* is:

$$RN_t = \sum_{p=1}^{P} RN_p^t = \sum_{p=1}^{P} \left(SC_t^p . PC_t^p + SN_t^p . PN_t^p \right). \qquad (2.21)$$

In each period, selling levels are bound to the sum of demand and backorders in the period *i.e.* $SC_t^p \leq d_t^p + B_{t-1}^p$.

2.4. Profit maximization model and solution approach

2.4.1. The mathematical model and its complexity

The profit maximization model is as follows:

$$Max\ Z = \sum_{t\in T}\sum_{p=1}^{P}\left(SC_t^p \cdot PC_t^p + SN_t^p \cdot PN_t^p\right) - \sum_{t\in T}\sum_{p\in P}\left(\pi_t^p \cdot x_t^p + h_t^p\left(IC_t^p + IN_t^p\right) + s_t^p \cdot S_t^p + b_t^p \cdot B_t^p\right)$$

$$-\sum_{t\in T}\sum_{k=1}^{M} CPM\left(M_t^k\right) - CMR \cdot \sum_{t\in T}\sum_{k=1}^{M+1} NF_t^k - \beta \cdot (M+1) \cdot T - \sum_{t\in T} QC_t - T \cdot \xi_0 - \xi_1 \cdot \sum_{t\in T} \tau_t \qquad (2.22)$$

Subject to

$IC_t^p = IC_{t-1}^p - SC_t^p + \left(x_t^p - XN_t^p\right)$, $p = 1, \ldots, P$, $t = 1, \ldots, T$, (2.23)

$IN_t^p = IN_{t-1}^p - SN_t^p + XN_t^p$, $p = 1, \ldots, P$, $t = 1, \ldots, T$, (2.24)

$B_t^p = B_{t-1}^p + d_t^p - SC_t^p$, $p = 1, \ldots, P$, $t = 1, \ldots, T$, (2.25)

$x_t^p \leq g^p \cdot S_t^p$, $p = 1, \ldots, P$, $t = 1, \ldots, T$, (2.26)

$\sum_{p\in P} \frac{x_t^p}{g^p} \leq APT_t$, $t = 1,\ldots, T$, (2.27)

$NF_t^k = \int_{w_t^k}^{y_t^k} \zeta(y)dy$, $k = 1, \ldots, M + 1$; $t = 1, \ldots, T$, (2.28)

$QC_t = \sum_{p\in P} \frac{\delta^p \cdot x_t^p}{APT_t} \cdot \sum_{k=1}^{M+1} PS_t^k \cdot (y_t^k - w_t^k)$, $t = 1, \ldots, T$, (2.29)

$\tau_t = \sum_{k=1}^{M+1} \left(\int_{w_t^k}^{y_t^k} \left(y_t^k - \tau\right) \cdot f\left(t \middle| w_t^k\right) \cdot d\tau\right) \cdot PS_t^k$, $t = 1, \ldots, T$, (2.30)

$XN_t^p = x_t^p \frac{\alpha^p}{APT_t} \cdot \sum_{k=1}^{M} \int_{w_t^k}^{y_t^k} \left(y_t^k - \tau\right) \cdot f\left(t \middle| w_t^k\right) \cdot d\tau \cdot PS_t^k$, $p = 1, \ldots, P$, $t = 1, \ldots, T$, (2.31)

$APT_t = \sum_{k=1}^{M+1} \left(y_t^k - w_t^k\right)$, $t = 1,\ldots, T$, (2.32)

$PS_t^k = \left(F\left(y_t^k\right) - F(w_t^k)\right) / \left(1 - F(w_t^k)\right)$, $k = 1, \ldots, M + 1$; $t = 1, \ldots, T$, (2.33)

$w_t^k = \left(1 - \eta^{k-2} \frac{CPM\left(M_t^k\right)}{CPM(1)}\right) \cdot y_t^k$, $k = 2, \ldots, M + 1$; $t = 1, \ldots, T$, (2.34)

$$y_t^k = w_t^k + \frac{L}{M+1} - TPM(M_t^k) - NF_t^k \cdot TMR,\ k = 1, \ldots, M+1;\ t = 1, \ldots, T, \tag{2.35}$$

$$y_t^{M+1} = w_t^{M+1} + \frac{L}{M+1} - NF_t^k \cdot TMR,\ t = 1, \ldots, T, \tag{2.36}$$

$$w_t^1 = 0,\ t = 1, \ldots, T, \tag{2.37}$$

$$IC_0^p = 0,\ p = 1, \ldots, P \tag{2.38}$$

$$IN_0^p = 0,\ p = 1, \ldots, P \tag{2.39}$$

$$B_0^p = 0,\ p = 1, \ldots, P \tag{2.40}$$

$$S_t^p \in \{0,1\}, x_t^p, XN_t^p, IC_t^p, IN_t^p, B_t^p, y_t^k,\ w_t^k, APT_t,\ PS_t^k, NF_t^k, QC_t \in \mathbb{R}^+, M_t^k \in \{1, \ldots, Q\}. \tag{2.41}$$

Terms of the objective function (2.22) respectively indicate the total income by conforming and nonconforming products, cost of production, inventory holding, backorders and setup, the cost of preventive maintenance, cost of minimal repairs, inspection cost, quality checking cost, and finally the restoration cost. The flow constraints (2.23) and (2.24) link the inventory, backorder and sales to the expected number of products for conforming and nonconforming items. Constraint (2.25) links the backorders to the demands and sales. Constraint (2.26) forces $x_t^p = 0$ if $S_t^p = 0$, and $x_t^p \geq 0$ if $S_t^p = 1$, where g^p is the production rate of product *p*. Constraint (2.27) indicates the capacity limitations. Equation (2.28) calculates the expected number of machine failures, and (2.29) approximates the quality checking cost. Equation (2.30) is the expected time that the machine operates in a out-of-control state and (2.31) is the average number of nonconforming items in relation with the production and maintenance decisions. Equation (2.32) calculates the available production time in periods as a function of PM plans and (2.33) is the probability of a shift in interval *k* of period *t*. Equations (2.34) to (2.36) determine the age of the machine in all intervals and equations (2.37) to (2.40) state the initial conditions of the system. Finally, (2.41) shows the bounds and type of the decision variables and associate parameters.

The model presented in this paper corresponds to a complicated nonlinear mixed-integer program. The size of the solution space, even for small problems, is too large. Each evaluation of the problem needs the calculation of several integrals and solving different algebraic equations to find the age values. These values are then used to compute the shift and failure probabilities and to evaluate the related parts of the model. In fact, four sets of original decision variables are production lot-sizes, sales levels, PM-schedule. These variables and the model parameters such as the number of process inspections interact nonlinearly with each other.

2.4.2. Search space and solution methods

For developing a solution approach, we first need to select the probability distribution of time-to-shift and time-to-failure functions. Weibull distribution has the flexibility to cover different failure functions and it can model different cases of the hazard function. This function is broadly used in reliability and failure models. In this paper, we use the Weibull distribution to model the time-to-shift and time-to-failure functions.

With any arbitrary PM-schedule, the machine age and the shift and failure probabilities can be calculated and the interacting factors can be evaluated. Thence, the problem reduces to a linear mixed integer program (MIP), which can be solved with existing solvers. Considering T the number of periods, M the number of maintenance opportunities per period, and Q the number of PM options, the total number of maintenance plans is $Q^{M \cdot T}$. For example, for the case of T = 3, M = 3, and Q = 4 (The numerical example of section 2.5), the size of the search space is about 2.6×10^5. Three solution methods are presented as follows.

2.4.2.1. Method 1: The integrated approach

Considering that each PM plan results in a complete solution of the problem, this property is used in our first approach. After generating a PM plans, the expected cost of PM plan, expected number of machine failures, the cost of corrective maintenance, and the cost of machine inspections for each period can be computed. Calculation of the age values and shift probabilities yields the available production time and the average nonconformity rate for each period. These factors can be used to form the related lot-scheduling sub-problem which can be solved with existing methods. The approach is programmed in Visual Basic 2010 that generates PM plans and calls CPLEX to solve the lot-scheduling problem. Since CPLEX is used to solve the lot-scheduling problem, and all the PM plans are examined in the algorithm, the results of this method are the exact and the optimal solution of the problem. Of course, CPLEX can be replaced with any existing heuristic within the structure of the algorithm. On our system (Intel core i7 – 3.4 GHz, 16 GB of RAM), each evaluation of the sample problem of section 2.5 takes about 0.6 Sec and the solution time using the integrated method is about 1.8 days. In tactical level planning, the maximum scheduling time and so, the size of the problem that can be evaluated to its optimality is limited according to the organization needs.

2.4.2.2. Method 2: nonintegrated method

We propose a non-integrated approach in which, the maintenance problem is solved separately from the production planning problem. In this process, the PM planning is optimized independently such that the total cost of the maintenance system is minimized subject to several predefined machine availability conditions. It also comprises the case of PM plans with the least cost. We can evaluate the conditions of the production system (i.e. the available production time and the expected nonconformity rate for each period) for each plan. This information can then be used to establish the production planning sub-problem, which is a mixed integer program. Solving the model (with CPLEX) yields the lot sizes, inventory levels and backorders for all the products in the planning horizon.

2.4.2.3. Method 3: Time decomposition method

In this approach, the model is decoupled into T single period integrated problems, where T is the number of periods. The solution process starts from the first period. The PM plan for a single period is called a *partial PM plan* (a complete PM plan is composed of T partial plans). The number of possibilities in a partial PM plan (size of the partial solution space) is much less than the cases of integrated approach, therefore, we can enumerate all partial plans. These plans are evaluated to have a partial solution. The solution results of the first period (inventory levels of conforming and nonconforming products and backorders) are used to update the data of the second period. Then, the solution process continues with solving the updated problem related to the second period. New inventory and backorder levels are sifted to the third period and this process continues to the end of the planning horizon.

2.4.2.4. Method 4: Uniform PM levels

In this approach, uniform PM levels are considered for all the maintenance intrusions and the best solution (with the highest profit) is considered as the result of this approach.

2.5. Numerical example

In this example, the time-to-shift function follows a Weibull distribution with parameters λ and φ ($f(t) = \lambda\varphi t^{\varphi-1}e^{-\lambda t^{\varphi}}$). Similarly, the time-to-failure is another Weibull function with parameters θ and ρ ($g(t) = \theta\rho t^{\rho-1}e^{-\theta t^{\rho}}$). We consider that the shifts and the failures belong to the same deterioration process ($\varphi = \rho$). Price of nonconforming product is set at 25% of the price of conforming item. The problem data are provided in Tables 2.1 and 2.2, where (*T, M, Q, P*) = (3, 3, 4, 3). The unit quality costs (δ^p) are respectively 4, 5, and 6.

Table 2.1: Production data.

Product		Demand			Production cost			Backorder cost			Holding cost		
		1	2	3	1	2	3	1	2	3	1	2	3
Period	1	45	50	100	30	50	70	110	130	180	3	5.5	2.2
	2	30	40	150	26	47	74	110	130	170	2.5	6.1	2.5
	3	60	70	50	33	49	68	120	130	170	3.2	6.5	2.4

Product		Setup cost			Price Conforming		
		1	2	3	1	2	3
Period	1	500	800	450	170	320	65
	2	550	780	420	150	300	70
	3	530	830	400	180	340	68

Table 2.2: Maintenance data.

Cost of PM options	*CPM*	5000, 500, 200, 0	Time of minimal repair	*TMR*	0.02	Length of period	*L*	1
Time of PM options	*TPM*	0.05, 0.003, 0.001, 0	Inspection cost	β	40	Time-to-shift parameters	λ	40
Production rates	g	450, 400, 350	Imperfectness factor	η	0,9		φ	2.5
Nonconformity rates	α	0.7, 0.7, 0.7	Restoration parameters (constant and variable)	ξ_0	200	Time-to-failure parameters	θ	20
Cost of minimal repair	*CMR*	500		ξ_1	3000		ρ	2.5

2.5.1. Optimal solution

The optimal solution of the problem is evaluated with the first solution approach and its optimality is confirmed by comparing it with the best solution of enumeration method. The solution time in method 1 is about 2500 minutes for evaluating all the PM alternatives. Of course, one can use other methods such as heuristic approaches or genetic algorithm to get a promising solution in a very shorter time, but these methods do not guarantee the optimality of the results. Since we are interested in the best solution of the integrated approach, the enumeration of all PM possibilities is a time-consuming but reliable way to find the best solution. The detailed results, PM plans, and the objective value are shown in Table 2.3. The machine availability rates in the three periods under this plan are respectively 84%, 84%, and 78%. The PM plan shows the level of maintenance to be performed in each interval and period, e.g. 3, 0, and 3 are the PM levels are the first period.

Table 2.3: Decision variables.

		Lot-sizes ($x^p{}_t$)			Setup ($S^p{}_t$)			Conforming inventory ($IC^p{}_t$)			Nonconforming inventory ($IN^p{}_t$)		
Period→		1	2	3	1	2	3	1	2	3	1	2	3
Products	1	105	0	103	1	0	1	30	0	0	0	0	0
	2	71	56	120	1	1	1	0	0	0	0	16	0
	3	151	246	31	1	1	1	0	7	32	0	0	0

		Conforming sale ($XS^p{}_t$)			Nonconforming sale ($XN^p{}_t$)			Backorder ($B^p{}_t$)		
Period→		1	2	3	1	2	3	1	2	3
Products	1	45	30	60	31	0	43	0	0	0
	2	50	40	70	20	0	67	0	0	0
	3	100	150	50	44	72	13	0	0	0

Optimal PM plan: (3, 0, 3), (3, 0, 3), (3, 3, 3)	Objective value (total profit): **24059**

2.5.2. Benefits of the joint scheduling

Table 2.4 shows the results obtained from the three solution methods as well as the solution data in case of uniform PM levels for all maintenance intrusions. The solution quality in integrated method is higher than the two other approaches. The second method has yielded the worst results. In this Table, "*Mfg. Cost*" is the sum of production cost, inventory holding cost, backorder cost, and setup cost. The cost of preventive and corrective maintenance and the restoration costs is reported as the "*PM cost*", and the "*QC. Cost*" is the sum of quality checking and process inspection. The sale amount corresponds to both conforming and nonconforming items.

Table 2.4: Solution results.

	Total Profit (Optimality gap)	Mfg. Cost	PM cost	QC. Cost	Total sale	PM plan	Solution time (minutes)
Method 1	24059 (Optimal)	54602	24805	4445	107911	(3,0,3),(3,0,3),(3,3,3)	2500
Method 2	5233 (360%)	78284	21078	4698	109293	(3,3,3),(3,3,3),(3,3,3)	< 2
Method 3	22116 (8.8%)	52199	26668	4197	105180	(3,0,3),(3,0,3),(3,0,3)	3
First PM level	5010 (380%)	40870	49408	2532	97820	(0,0,0), (0,0,0), (0,0,0)	-
Second PM level	8525 (182%)	73862	23584	4790	110761	(1,1,1), (1,1,1), (1,1,1)	-
Third PM level	6616 (263%)	76379	22098	4740	109833	(2,2,2), (2,2,2), (2,2,2)	-
Forth PM level	5233 (360%)	78284	21078	4698	109293	(3,3,3), (3,3,3), (3,3,3)	-

As shown in this Table, the profit is highly sensitive to the maintenance and applying the same level for all PM events, even in the best case is very far from the best solution. As expected, minimizing the maintenance cost independent from the production and quality systems has resulted in a very poor solution.

To study the robustness of the algorithm, 40 random problems are uniformly generated and solved by the three solution methods (as well as applying uniform PM levels). We have considered different problem sizes (up to 8 periods, 8 PM options for the machine, and 6 maintenance opportunities per period). The combinations are selected such that the generated problems can be solved in a reasonable time (not more than 2 days). The problem data (cost units, prices, shift and failure parameters, etc.) are selected between ±50% the average values given in Tables 2.1 and 2.2 (To avoid trivial and unreasonable instances).

Table 2.5 shows the minimum and the average gaps between the integrated and the other methods. It also shows the number of instances in the given optimality ranges for the 40 sample problems.

Table 2.5: Optimality gaps to the best solution.

	Min. Gap	Average Gap	0 to 10%	10-50%	>50%
Method 2	1.3%	53.1%	4	12	24
Method 3	0%	31.1%	13	17	10
Uniform levels	0.2%	37.4%	15	21	4

The above table underlines the significance of the joint scheduling of production, sales, maintenance and quality. The time-decomposition method performs better than the two other approaches because it still employs integrated solution but in multiple periods. The improvement in integration is very significant which justifies the implementation of joint approaches in planning the production, maintenance, and quality.

2.6. Sensitivity analysis

This section discusses the effect of several parameters on the problem and presents a sensitivity analysis.

2.6.1. Effect of maintenance budget

The total maintenance cost in the planning horizon or "*maintenance budget*" is the sum of the cost of all PM levels in the plan and is an appropriate indicator of the maintenance quality. Fig. 2.2 represents the effect of maintenance budget on the profit. It shows that increasing the PM budget, first improves the profit because it diminishes the cost of minimal repairs and quality. But, more increase in the PM budget cannot be compensated by the decrease in the cost of corrective maintenance and the quality. The presented approach can determine the optimal PM budget in periods. Fig. 2.3 illustrates the variations of the rate of out-of-control state and the machine availability rate with the maintenance budget. The proportion of time that the machine operates in a degraded state influences on the quality. This parameter almost linearly diminishes with the increase in the PM budget. Selecting higher PM levels results in a fewer number of the machine failures and smaller corrective maintenance time. The availability rate of the machine first improves with the PM budget, but since the high level maintenance requires more time to be performed, in case of very high PM levels, its time may not be compensated with the reduction in the corrective maintenance time. These graphs represent the effect of PM plans on different performance measures. The availability rate of the machine is important in production planning and smaller rates can cause considerable losses and capacity issues. In the same way, cost of the corrective maintenance, cost of the machine restorations and the quality cost also reduce with the PM budget. These facts are shown in Figures 2.4 and 2.5. Again, selecting the higher PM levels, the expected number of the machine failures and the expected time of operating in a shifted state decrease, so the cost of corrective maintenance and the restoration cost diminish. The shape of these functions depends on the distribution of the time-to-shift, and the time-to-failure functions.

In Fig. 2.5, the decrease in the expected time of operating in an out-of-control state directly influences on the cost of quality.

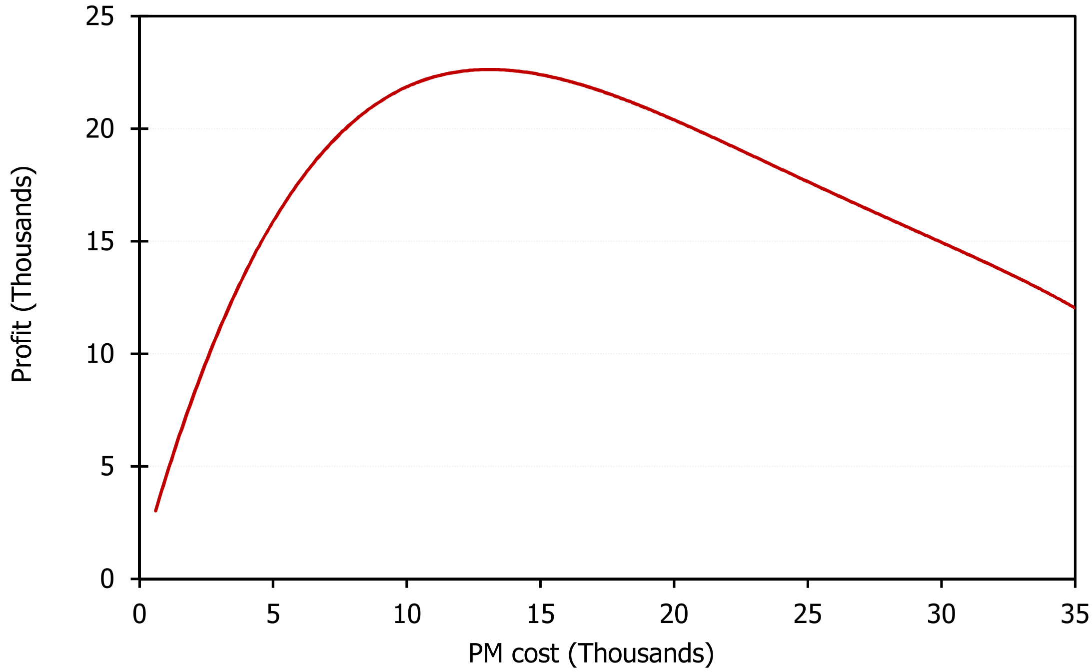

Fig. 2.2: Profit as a function of maintenance

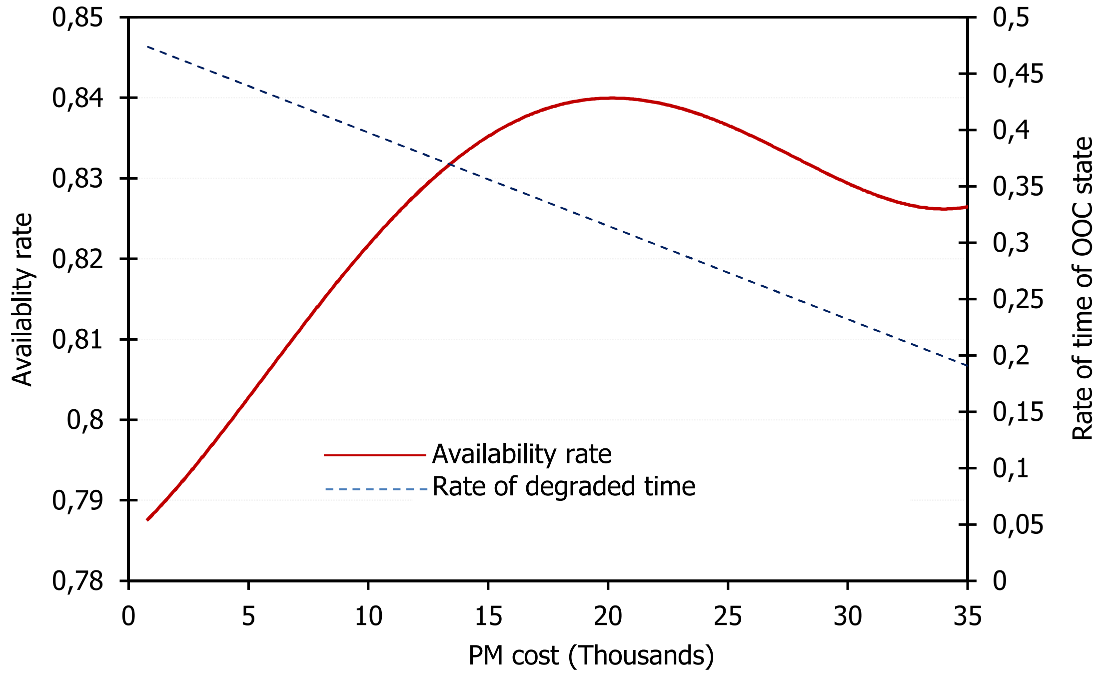

Fig. 2.3: Variations of availability and rate of OOC state with maintenance

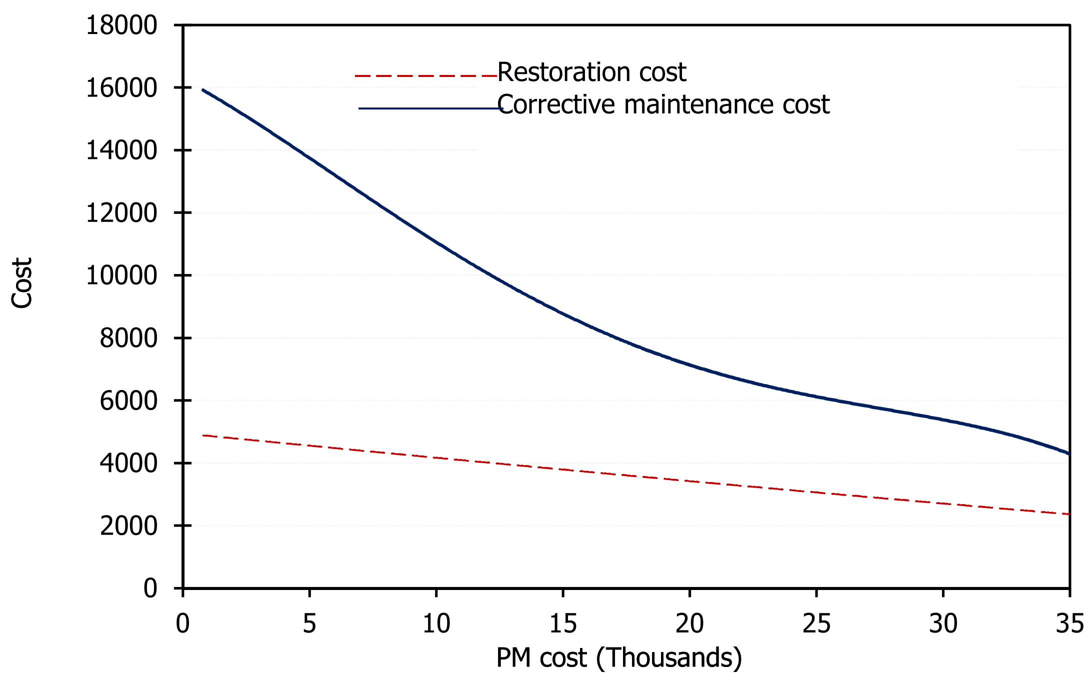

Fig. 2.4: Restoration and corrective maintenance cost with the PM budget.

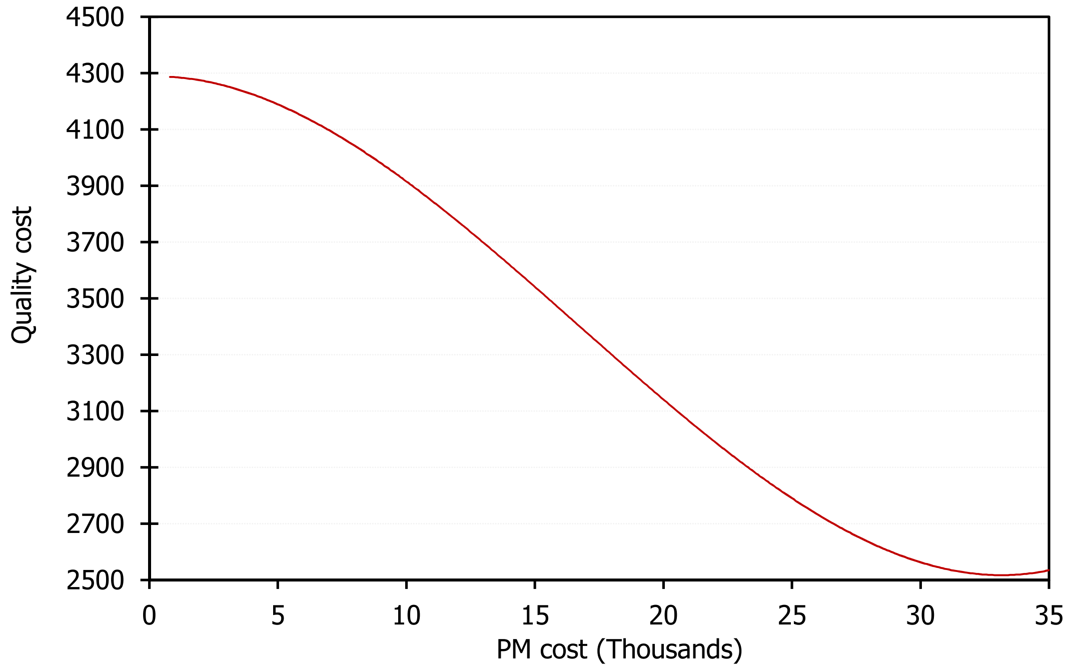

Fig. 2.5: Quality cost as a function of the PM budget.

2.6.2. Sensitivity analysis

Table 2.6 shows the effect of changes in some of the problem parameters on the profit, maintenance, production and the quality cost. The values of parameters are increased or decreased by 25% compared to their initial values in the sample problem and the changes in the cost components are listed in the table. The results obtained by increasing the parameter are reported in the "**+**" column, and the results by decreasing the parameter are listed in the "**–**" column. The first row of this table shows the different cost components for the optimal solution of the original problem. The other data indicate the variations (in percent) from the optimal solution. The three parameters with the highest impacts on each cost component are as follows:

Profit

- Selling price of conforming items

- Unit processing costs
- Time-to-shift function

Total maintenance cost

- Time-to-shift function
- Time-to-failure function
- Cost of minimal repair

Total cost of the production planning (lot-sizing) problem

- Unit processing cost
- Time-to-shift function
- Time-to-failure function

Total quality cost

- Time-to-shift function
- Time-to-failure function
- Time of minimal repair

Total sales

- Price of conforming products
- Price of nonconforming products
- Time-to-shift function

As presented in this table, the time-to-shift and time-to-failure functions are present in the list of the most influencing three parameters for all the cost components.

Table 2.6: Effect of changing parameters on the solution.

	Profit		PM cost		Mfg. Cost		QC. Cost		Sales	
	+	−	+	−	+	−	+	−	+	−
Optimal solution	**24059**		**25285**		**54602**		**3965**		**107911**	
Unit processing cost	-52%	52%	0%	0%	23%	-23%	0%	0%	0%	0%
Restoration constants	-4%	4%	4%	-4%	0%	0%	0%	0%	0%	0%
Cost of minimal repair	-11%	11%	11%	-11%	0%	0%	0%	0%	0%	0%
Time of minimal repair	-6%	5%	5%	-4%	2%	-1%	3%	-3%	1%	-1%
Cost of PM levels	-10%	10%	10%	-10%	0%	0%	0%	0%	0%	0%
Time of PM levels	-2%	2%	1%	-1%	1%	-1%	1%	-1%	0%	0%
Price of conforming	99%	-99%	0%	0%	0%	0%	0%	0%	22%	-22%
Price of nonconforming	13%	-13%	0%	0%	0%	0%	0%	0%	3%	-3%
Inspection cost	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Shift parameter	19%	-48%	-4%	31%	-14%	10%	-29%	13%	-5%	2%
Failure parameter	11%	-18%	-12%	21%	3%	-5%	5%	-10%	1%	-2%

In each column, the maximum increase and decrease in the cost component are highlighted, for example, the PM cost is more influenced by the cost of minimal repair and the parameters of the shift and failure function. The manufacturing cost is more changed with the unit production cost and the time-to-shift function, whereas the probability functions show greater impact on the quality cost. Finally, the prices of conforming and time-to-shift function are the most striking parameters affecting on the sales. The profit as the difference of sales and costs is impacted mostly by the price of conforming items and the unit production cost.

2.6.3. Effect of process inspection and process deterioration

The number of process inspections mostly effects on the quality cost defined as the sum of the costs of inspection and quality checking of the products. Fig. 2.6 shows the variations of QC cost and the expected number of defective items (In case of the best PM level and production of the first product with the highest possibility) as a function of the number of process inspections. Considering different number of process inspections in the joint model and evaluating the related profit can be conveniently used in determination of M.

In the time-to-failure function, the integrated hazard over a period is θt^{ρ}, so the expected number of failures in a period in which the machine age changes from 0 to 1, is θ. Similarly, the number of times that the machine shifts in such a period is λ. We call $\lambda + \theta$ the deterioration rate. The share of machine failures in the total deterioration rate defines the proportion of time that machine failures occur compared to the quality shifts. In some systems, most of deteriorations are quality shifts, but, in some cases, the failures can be dominant deteriorating. In our sample problem, $\theta / (\lambda + \theta) = 57\%$ indicates the rate of machine failures in the deterioration process. Fig. 2.7 shows the variations of the quality checking cost and the corrective maintenance cost as a function of the state of deterioration process ($\theta / (\lambda + \theta)$), while $\lambda + \theta = 60$ is kept constant. Larger values of this parameter indicate that the system is more prone to failures rather than to quality degradations, therefore, the number of machine failures and its cost increased from 47 to 891. In the meantime, the quality cost has decreased from 40 to 375. This information are useful in deciding the best maintenance levels and the production strategies because PM alternatives may result in various deterioration processes.

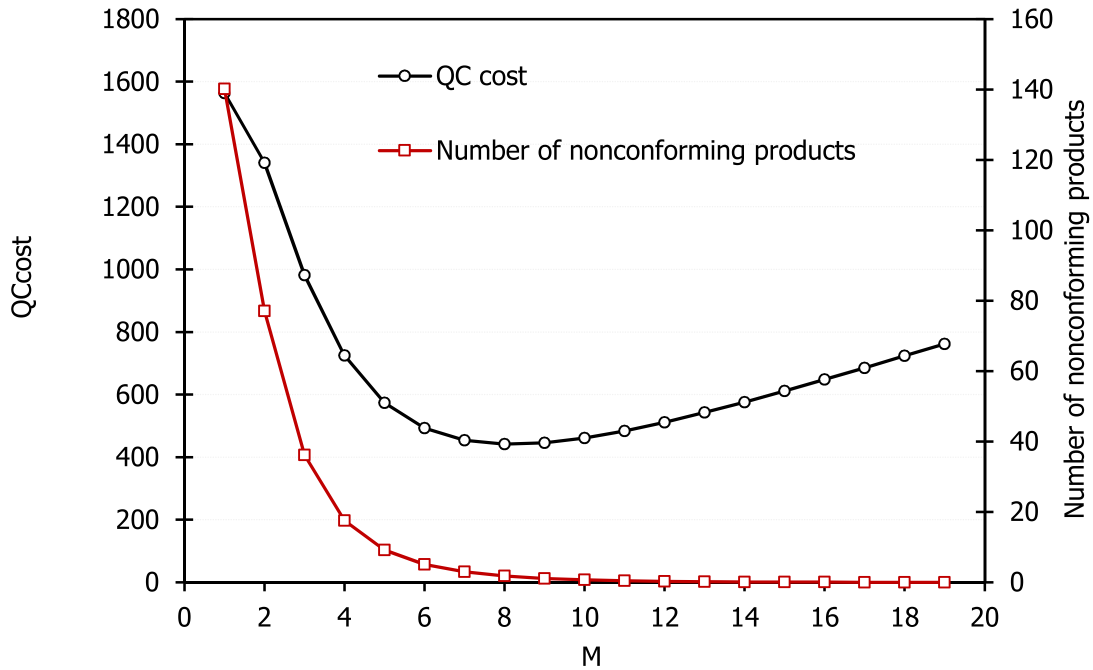

Fig. 2.6: Effect of the number of quality inspections on cost and quality of production.

Higher number of inspections per period increases diminishes the expected time of the out-of-control state, so first the quality cost is reduced. But, more increase in this parameter increases the inspection cost and so, the quality cost starts to increase.

The average number of defective items explicitly decreases with the number of inspections per period.

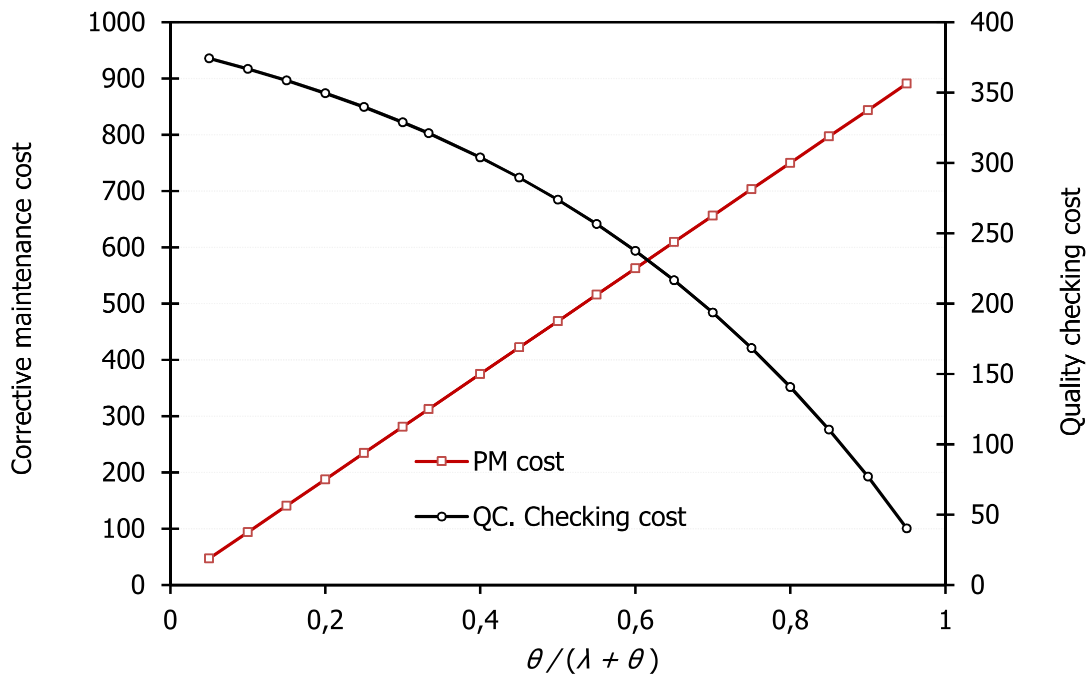

Fig. 2.7: System costs as a function of the state of deterioration process.

The distribution governing the time-to-shift and the time-to-failure functions are important in the shape of all figures the sensitivity analysis results, but in case of the given sample problem, the PM cost shows a linear increase with the rate of failure in the total shift and failure occurrences.

2.7. Conclusions

Production, maintenance, and quality systems are mutually in conflict because they use shared resources (Budget, personnel, available time, *etc.*) and influence on each other. The offline preventive maintenance strategy effects on the quality and the availability of machine either by occupying the processing time or by affecting the rate of machine failures. Therefore, their integration in a single model would benefit the system as a whole. These decisions can be influenced by certain external parameters such as variable costs and demands. Here, a multi-period multi-product capacitated lot-scheduling model is developed to maximize the profit by addressing simultaneous production-sale planning, maintenance scheduling, and the quality inspections. Evaluation of different costs and interacting factors are discussed and a mathematical model is developed. Assumption of time-varying costs and prices yields the required flexibility to examine different alternatives (in the production process, inventory management, etc.) and to study the pricing-marketing strategies and the effect of external parameters in relation to the internal decisions. On the other hand, we suggested a solution method to find the optimal PM plan and related lot-schedule. In this paper, a specific property of the model is employed that can guarantee the optimality of the solution. Results obtained from solving the illustrative examples indicate that the model is consistent and integration has improved the average optimality gaps by 53.1% in method 2 and 31.1% in method 3. Sensitivity analysis suggests that machine restoration cost, CPM, CMR and failure rate are the most influential factors in the maintenance cost, whereas, the unit production cost and the prices are the most significant parameters influencing on the profit and the manufacturing cost. From the robustness point of view, we found that the optimal solution corresponding to the lot-sizing problem is highly sensitive and it changed with variations of all of the parameters, but the optimal PM plan mainly varies with the maintenance-related parameters (deterioration function, failure function, and time and cost of preventive and corrective maintenance). The processing rates, machine availabilities, and deterioration and failure functions had the greatest impacts on the lot-sizing problem, while the variations in selling prices and setup costs had the smallest effect.

Taking into account the very large search space and the complexity of the problem, it is important to develop efficient solution methods to exploit the benefits of integration in a short solution time. We are currently developing meta-heuristic approaches and testing their capability in solving large-size problems. For future research, the model may consider complicated cases of multi-machine systems, and it may deal with the determination of optimal sequence of processing the jobs in a deteriorating system.

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Chapter Three: A Cost Minimization Approach

This chapter is dedicated to the article entitled “A cost minimization model for joint lot-scheduling and maintenance planning under quality constraints” submitted to the *International Journal of Production Research* in November 2015.

The titles, figures, and mathematical formulations are modified to keep the consistency through the thesis.

Résumé

Dans cet article, nous étudions la planification intégrée de la production, de la maintenance préventive imparfaite en tenant compte de contraintes de qualité. On considère un système de production multi-machine, multi-produit sur un horizon de plusieurs périodes. Les machines se détériorent avec le temps. La planification de la production correspond à un problème de détermination des tailles de lots capacitaires, où la disponibilité de la machine est soumise aux plans de la maintenance préventive. La qualité de la production est incertaine et dépend à la fois, du système de qualité et de la maintenance. La planification de la maintenance correspond à un modèle de la maintenance imparfaite à des temps discrets. Les inspections de processus sont conçues pour détecter l'état actuel du système. L'inspection est accompagnée de la vérification de la qualité des sous-lots pour séparer les produits défectueux, et les retravailler avant de les envoyer aux clients. On propose une approche intégrée qui coordonne les décisions des trois systèmes, où la fonction de l'objectif minimise le coût total de système. L'évaluation des coûts et des facteurs en interaction est présentée et un algorithme génétique est proposé pour résoudre le problème d'optimisation combinatoire résultant. Une analyse de sensibilité est effectuée et l'intégration de la maintenance, la production et la qualité est illustrées par plusieurs exemples numériques.

Mots clés

Ordonnancement de lots, Maintenance préventive, Qualité, Inspection, Processus imparfait

Abstract

In this paper, production planning, imperfect preventive maintenance scheduling and design of the quality system are simultaneously investigated. The production system is multi-machine, multi-product and multi-period model in which, the machines deteriorate with time and so, they may fail or shift to a quality degraded state with higher defective rate. The production planning corresponds to a capacitated lot-scheduling problem, where the machine availability is subject to the preventive maintenance. The production quality is uncertain and it depends on both maintenance and quality systems. Maintenance scheduling corresponds to a discrete time imperfect model with a large number of alternatives for PM interferences. The process inspections are considered to detect the current state of the system. Detecting a shift in the process mean by inspection initiates certain courses of actions, including the quality check of the related sub-lots in order to separate and rework the defective items before sending to a customer. We propose a joint approach that coordinates the decisions of the three systems, where the objective function minimizes the total cost of the system. Evaluation of costs and interacting factors is presented and a genetic algorithm is proposed to solve the resulting huge combinatorial optimization problem. A sensitivity analysis is conducted, and the integration of maintenance, production and quality is illustrated using numerical examples.

Keywords

Capacitated lot-scheduling, Preventive maintenance, Quality, Inspection, Imperfect process

3.1. Introduction

Fulfillment of customer demands on time, with higher quality levels, and affordable prices is a challenging issue in all production systems. In spite of strong interactions, production, maintenance, and quality decisions are generally treated separately in the literature. Benefits of integration are reported in several papers. Linderman et al. (2005) reported 0.7% to 52.7% reduced total cost in coordinated design of maintenance and quality systems. Liu et al. (2015) declared higher benefit in the joint determination of product lot-sizes and PM policies. Aramon et al. (2014) reported 21% cost savings by integration. Colledani and Tolio (2012) presented a model linking the production rate to machine deterioration and maintenance plans and showed that the performance can improve by integrating these functions in the system level. Aghezzaf et al. (2007) studied the integration of preventive maintenance and lot-scheduling problem and showed that the two systems are linked. Nourelfath et al. (2010) investigated the integration of lot-scheduling and preventive maintenance (PM) scheduling in multi-state systems. Machani and Nourelfath (2012) solved the integrated problem introduced in (Nourelfath et al., 2010) using a variable neighborhood search. In (Mokhtari et al., 2012), the authors investigated the link between maintenance and availability of machines and developed a mixed integer program for joint scheduling. They solved the problem using population based variable neighborhood search. Also, Xiang (2013) cited considerable savings by integration in all the studied cases. Alfares et al. (2005) incorporated inventory management in joint scheduling of production and maintenance in a deteriorating system. Radhoui et al. (2009) proposed optimal buffer size in a single machine failure-prone system. In (Aghezzaf et al., 2016) the authors investigate the link between operating age of machine influencing on the production capacity and the system reliability in a joint model. The production capacity in a deteriorating system is also studied by Yalaoui et al. (2014), where a hybrid model is proposed to deal with the bi-objective production-maintenance problem.

In order to establish the link between maintenance and quality, Ben-Daya (2002) considered the normal and degraded state where, the defective rate in out-of-control state is higher. Chen (2013) considered the same idea with two imperfect states and established the link between imperfect maintenance and quality degradations. Ponser and Tapiero (1987) developed quality measures for a system that deteriorates with time. Such quality deteriorations are not self-announcing, so process inspection and quality control tools are used to detect the machine conditions. The design of such quality processes needs to be integrated with maintenance scheduling. Xiang (2013) considered a discrete-time Markov chain model to incorporate the design of $\bar{x}$-control chart and maintenance decisions.

Nourelfath et al. (2014) dealt with the integration of production, maintenance and quality in a multi-state production system in which the machines may randomly switch to a degraded state with lower quality. The tradeoff between two systems is also addressed in (Berrichi et al., 2010), where a bi-objective formulation of the model is presented, and an Ant Colony optimization is developed to solve the problem.

In (Panagiotidou and Tagaras, 2007), the authors considered two quality states that are distinguished with different hazard rates and various degradation functions. They showed that an aggressive policy whereby the process is maintained as soon as a shift is detected is not always optimal.

The maintenance time and optimal length of PM intervals is investigated in several studies. Duncan (1956) was the first who studied approximate measure of average net income in a process under surveillance of a control chart. He showed that for an exponential failure time function, the process inspections should be equally spaced. Banerjee and Rahim (1988) extended the Duncan's work to non-Markovian models by choosing the length of sampling intervals such that the integrated hazard over all intervals is constant. Rahim (1994) showed that non-uniform inspections are more efficient in joint scheduling of economic production quantities, process inspections, and determination of control chart parameters. Levrat et al. (2008) modelled a discrete time production-maintenance planning in which, the maintenance operations are performed at machine failures. Van Dijkhuizen and Van Harten (1998) considered perfect maintenance in a two-stage planning model, where first, a maintenance interval is determined based on the technical information. Then, exact PM time is determined using the operational data. Aghezzaf and Najid (2008) developed both cyclic and acyclic maintenance models, while considering the production capacity as a function of maintenance.

Existing literature shows the significance of joint models and reveals that only limited cases of real-life applications are addressed in the literature. Most of the existing papers integrate only two functions (maintenance and production or maintenance and quality), and even in these papers, usually the consequences of one system in a reactive manner are considered in optimal design of the other system. For example, the defective rate in deteriorated state is incorporated in maintenance scheduling. Inclusion of decision variables from the three functions in one model is scarce in the literature. A large number of organizations perform their maintenance operations at specific times, where the production is naturally stopped (vacations, between shifts…). For these cases, the choice of operations in each maintenance opportunity, especially in case of limited resources, is a perplexing decision. Integration of discrete time multi-choice maintenance with resource limitations and capacitated lot scheduling, taking into account the optimal number of process inspections related to the design of a quality system is the main contribution of this research. Each machine has multiple maintenance options and in a system involving more than one machine, the number of maintenance alternatives is very large. Limitation of maintenance resources (here, budget limitation is considered) results in a very difficult model. Sampling interval or the number of inspections during a period is a decision variable from the quality system that is linked to the maintenance and production decisions. Furthermore, a genetic algorithm (GA) is proposed to solve the huge combinatorial problem.

The rest of this paper is organized as follows. In Section 3.2, the problem definition is provided. Section 3.3 develops the formulations and evaluations. Section 3.4 discusses the mathematical model, as well as the proposed solution method. A sample problem and a sensitivity analysis are presented in section 3.5. Section 3.6 concludes the study.

3.2. Problem definition

In the following sections, the production, maintenance, and quality systems and their features and interactions are explained.

3.1.1. Production system

A capacitated lot-scheduling problem in a multi-machine, multi-product, multi-period context is considered. Demand for the products in each period is known and unsatisfied orders will charge the backorder cost to the system. Over production imposes the inventory holding cost and switching to a new product type on a machine is accompanied with a setup charge. Defective rates and expected availability production times depend on the preventive maintenance (PM) plans. The total cost of the system to be minimized is the sum of production, maintenance, and quality costs.

3.1.2. Maintenance system

At the beginning of each period the machines are in normal state, but by progressing the time, the machines deteriorate and so, the failure probability increases. Machine failures are instantly detected and a minimal repair is initiated upon a failure to bring the machine in operation. This type of repairs does not influence on the machine age. Financial consequences of failures and the time to bring a failed machine in operation are large, so the preventive maintenance is aimed to improve the conditions by reducing the effective age of the machines. The imperfect preventive maintenance with multiple PM options for each machine is considered and it is assumed that, reduction in the machine age linearly depends on the cost of the selected maintenance level. Several organizations schedule their PM activities at specific times (between shifts, in weekends, vacations, between consequent missions, etc.), so, in these cases, the maintenance time is not a decision variable. Instead, in each maintenance opportunity, they should decide the PM level to be performed on each machine (the lowest PM level can concern to a no-PM alternative). This is the concept of discrete time maintenance problem with several maintenance scenarios. We assume a maintenance opportunity at the beginning of each period and the solution of the model determines the optimal PM level for each machine in each period.

3.1.3. Quality system

Two possible states of each machine are (1) the normal (or in-control) state with negligible defective rate and (2) the shifted (or out-of-control) state with higher nonconformity. During each period, several error-free process inspections are considered to detect the machine's state. These inspections divide the period into several inspection-intervals. At the beginning of each inspection-interval, the machine is in normal state, but by progressing the time, the machine conditions deteriorate and the probability of a shift to a degraded state increases. By detecting a shifted state upon an inspection (1) the machine will be corrected that imposes an adjustment cost, and (2) the sub-lots produced in that interval will be quality checked to separate the defective items. Such nonconforming products will be reworked before sending to a customer. The last inspection will be executed at the end of the period and at least one process inspection in each period is required. The optimal number of quality inspections depends on the machine conditions and the costs of machine inspection, quality checking, and rework. Given the number of process inspections, the length of intervals will be determined such that the integrated hazard over all intervals is constant (Banerjee and Rahim 1988). A sample period with a PM and four inspection intervals are illustrated in Fig. 3.1.

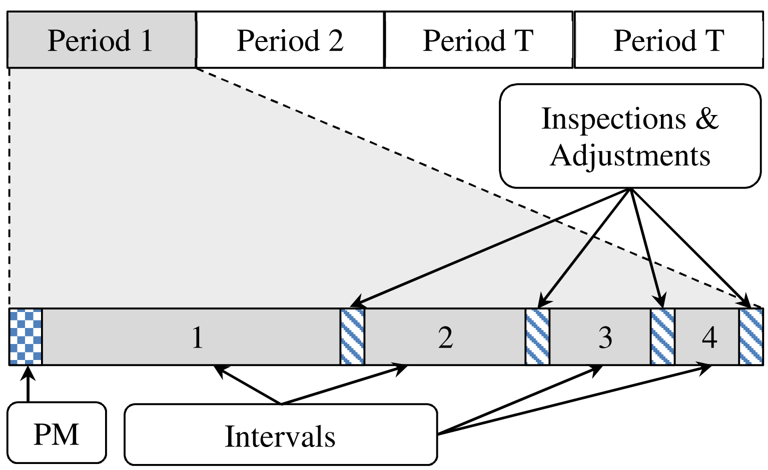

Fig. 3.1: Production periods and intervals.

3.2. Evaluations

In this section, the notations are summarized and the evaluation of costs and interacting factors are explained.

Notations

Indices and parameters

m, M Index and number of machines

p, P Index and number of products

t, T Index and number of periods

j Index of intervals in periods

b_p Unit backorder cost

AC_m Process adjustment cost

CMR_m Cost of minimal repair

$CPM_m(k)$ Cost of k^{th} PM level

d_{pt} Customer demand

g_{pm} Production rate

h_p Inventory holding cost

L Fixed length of periods

PMB_t Available PM budget

Q_m Number of the preventive maintenance levels

R_p Unit reworking cost

s_{pm} Setup cost

TMR_m Time of minimal repair

W_{m0} Initial age of machine

α_{pm} Defective rate in out-of-control state of the machine

β_p Unit cost of quality check

θ_m, ρ_m Parameters of Weibull distribution for time-to-failure function

λ_m, φ_m Parameters of Weibull distribution for time-to-shift function

π_{pm} Manufacturing cost

ν_m Cost of process inspection

Dependent variables………………………………………………………………………

APT_{mt} Available production time

$ESST_{mt}$ Expected duration of a shifted state

IC_{mt}	Process inspection cost	TAC_{mt}	Total process adjustment cost
LS_{pt}	Lot size	TC_{MS}	Total maintenance system cost
NF_{mt}	Number of machine failures	TC_{PS}	Total production system cost
PMC_t	Preventive maintenance cost	TC_{QS}	Total quality system cost
ps_{mjt}	Probability of shift in the j^{th} interval	TRC_{mt}	Total reworking cost
PS_{mt}	Probability of shift in period	W_{mt}	Age at the beginning of a period
QC_{pmt}	Cost of quality checking	y_{mjt}	Age at the end of an interval
		Y_{mt}	Age at the end of a period

Decision variables……………………………………………………………………

B_{pt}	Backorder level	x_{pmt}	Production level
I_{pt}	Inventory level	NI_{mt}	Number of process inspections
S_{pmt}	Setup variable	PM_{mt}	Preventive maintenance level

B_{pt}, I_{pt}, S_{pmt} and x_{pmt} are the production planning variables, NI_{mt}concerns to the quality system, and PM_{mt} to the maintenance system.

3.2.1. Cost of production system and its constraints

Total cost of the production system (TC_{PS}) is the sum of manufacturing cost, setup cost, inventory holding costs, and backorder cost for all combinations of products and machines. We have:

$$TC_{PS} = \sum_{t\in T}\left(\sum_{p\in P}\sum_{m\in M}\left(x_{pmt}\pi_{pm} + S_{pmt}s_{pm}\right) + \sum_{p\in P}\left(I_{pt}h_p + B_{pt}b_p\right)\right) \tag{3.1}$$

Binary variable of setup (S_{pmt}) and production level (x_{pmt}) are linked together, so

$$x_{pmt} \leq S_{pmt} g_{pm} \tag{3.2}$$

Total lot-size of product p in period t is:

$$LS_{pt} = \sum_{m \in M} x_{pmt} \tag{3.3}$$

The balance equation between production, inventory, backorder, and demand in capacitated lot - scheduling problem is given by:

$$I_{pt} - B_{pt} = I_{pt-1} - B_{pt-1} + LS_{pt} - d_{pt} \tag{3.4}$$

Inventory and backorder levels will take their appropriate values because their coefficients in the objective function are positive, and the objective is a minimization function, so only one of each pair of these variables can be non-zero. If production and backorder costs are assigned properly (i.e. $b_p > \pi_{pm}$), production levels will be positive values, otherwise no-production case is preferable. Also, the inventory levels at the end of the last period will be zero because of the cost minimization, and the backorder levels will be minimized according to the production capacities. The available production time of machine m in period t is the remaining time in period after performing minimal repairs, so:

$$APT_{mt} = L - NF_{mt} TMR_m \tag{3.5}$$

In (3.5), NF_{mt} is the expected number of machine failures in period t, TMR_m is the time of minimal repair of machine m, and L is the length of periods. Total production of a machine is constrained to the available production time.

$$\sum_{p \in P} x_{pmt} \leq APT_{mt}\, g_{pm} \tag{3.6}$$

3.2.2. The machine age in periods and intervals

Let us assume that the time-to-shift function follows a Weibull distribution with parameters λ_m and φ_m i.e. $s_m(t) = \lambda_m \cdot \varphi_m \cdot t^{\varphi_m - 1} \cdot e^{-\lambda_m t^{\varphi_m}}$ and its cumulative function is $S_m(t) = 1 - e^{-\lambda_m t^{\varphi_m}}$, so the conditional probability of a shift at age *t* given the machine was in control at age t_0 is:

$$s_m(t|t_0) = \frac{s(t)}{1-S(t_0)} = \lambda_m \cdot \varphi_m \cdot t^{\varphi_m - 1} \cdot e^{-\lambda_m(\cdot t^{\varphi_m} - \cdot t_0{}^{\varphi_m})}, t > t_0 \quad (3.7)$$

To determine the machine age in periods (W_{mt}, Y_{mt}), we adopt the imperfect maintenance concept (Nakagawa, 1988) in which the machine age after the PM is somewhere between as-good-as-new and as-bad-as-old conditions. Using his model, we suppose that the reduction in the machine age by PM linearly depends on the cost of maintenance task, so the machine age *m* at the beginning of period *t* + 1 is:

$$W_{m,t+1} = \left(1 - \frac{CPM_m(PM_{mt})}{CPM_m(1)}\right) Y_{mt} \quad (3.8)$$

where, PM_{mt} is the PM level for machine *m* in period *t*, $CPM_m(k)$ is the cost of k^{th} PM level for machine *m*, and $CPM_m(1)$ is the cost of the highest PM level. Let us assume that the time-to-failure function is also a Weibull function with parameters θ_m and ρ_m, i.e. $f_m(t) = \theta_m \cdot \rho_m . t^{\rho_m - 1} \cdot e^{-t^{\rho_m}}$ and its cumulative distribution is $F_m(t) = 1 - e^{-t^{\rho_m}}$. The instantaneous hazard function is $f_m(t)/\big(1 - F_m(t)\big) = \theta_m \cdot \rho_m \cdot t^{\rho_m - 1}$, and so, the expected number of machine failures in period *t* is:

$$NF_{mt} = \theta_m \cdot (Y_{mt}{}^{\rho_m} - W_{mt}{}^{\rho_m}) \quad (3.9)$$

The expected minimal repair time is $NF_{mt} \cdot TMR_m$, where TMR_m is the average minimal repair time of machine *m*. During minimal repairs, the machine is not operational and its age is not increasing, so, the machine age at the end of period *t* given it is operational while it is available will be:

$$Y_{mt} = W_{mt} + APT_{mt} = W_{mt} + L - \theta_m \cdot (Y_{mt}{}^{\rho_m} - W_{mt}{}^{\rho_m}) \cdot TMR_m \quad (3.10)$$

Since the initial age of machine *m* at the beginning of the first period is $\mathrm{W_{m1}}$, one can use equations (3.8) and (3.10) to recursively calculate the machine ages in the periods.

The number of process inspections (and so, the number of intervals) for machine *m* in period *t* is NI_{mt}, and the machine age at the end of each interval is y_{mjt}. Banerjee and Rahim (1988) showed that in a Weibull shock model, the optimal strategy in determining the length of inspection intervals is to maintain a constant integrated hazard over them. The integrated hazard over the j^{th} interval (considering the time-to-shift function) is as follows:

$$y_{m,j,t}{}^{\varphi_m} - y_{m,j-1,t}{}^{\varphi_m} \tag{3.11}$$

Using the concept of constant integrated hazard over all intervals in a period and considering NI_{mt}; the number of process inspections for machine *m* in period *t*, these quality inspections divide the period into NI_{mt} intervals, the integrated hazard over each interval should be $(Y_{mt}{}^{\varphi_m} - W_{mt}{}^{\varphi_m})/NI_{mt}$. Therefore, for interval *j* in period *t*, we have $y_{m,j,t}{}^{\varphi_m} - y_{m,j-1,t}{}^{\varphi_m} = \frac{(Y_{mt}{}^{\varphi_m} - W_{mt}{}^{\varphi_m})}{NI_{mt}}$. Starting from the first interval (*j* = 1), $y_{m1t}{}^{\varphi_m} - y_{m0t}{}^{\varphi_m} = \frac{(Y_{mt}{}^{\varphi_m} - W_{mt}{}^{\varphi_m})}{NI_{mt}}$. Note that with the given notation $y_{m,0,t}$ is the machine age at the beginning of the first interval and it is equal to W_{mt}, so $y_{m1t}{}^{\varphi_m} = W_{mt}{}^{\varphi_m} + \frac{(Y_{mt}{}^{\varphi_m} - W_{mt}{}^{\varphi_m})}{NI_{mt}}$. For the next interval (case *j* = 2), again the constant integrated hazard indicates that $y_{m2t}{}^{\varphi_m} - y_{m1t}{}^{\varphi_m} = \frac{(Y_{mt}{}^{\varphi_m} - W_{mt}{}^{\varphi_m})}{NI_{mt}}$. Substitution of $y_{m1t}{}^{\varphi_m}$ from the previous equation yields $y_{m2t}{}^{\varphi_m} = W_{mt}{}^{\varphi_m} + 2\,\frac{(Y_{mt}{}^{\varphi_m} - W_{mt}{}^{\varphi_m})}{NI_{mt}}$.Continuing this approach gives the following general equation:

$$y_{m,j,t}{}^{\varphi_m} = W_{mt}{}^{\varphi_m} + j \cdot \frac{(Y_{mt}{}^{\varphi_m} - W_{mt}{}^{\varphi_m})}{NI_{mt}n} \tag{3.12}$$

Therefore, the machine age at the end of interval *j* (beginning of interval *j*-1) can be calculated from the following equation:

$$y_{m,j,t}{}^{\varphi_m} = (j/NI_{mt}) \cdot Y_{m,t}{}^{\varphi_m} + (1 - j/NI_{mt}) \cdot W_{mt}{}^{\varphi_m} \tag{3.13}$$

3.2.3. Probability of shift in periods and intervals

Given the time-to-shift function and the machine age in periods and intervals, the conditional probability of a shift occurs in period *t* given it was initially in a normal state is:

$$PS_{mt} = 1 - e^{-\lambda_m({Y_{mt}}^{\varphi_m} - {W_{mt}}^{\varphi_m})} \tag{3.14}$$

Similarly, the conditional probability of a shift in interval j is $ps_{mjt} = 1 - e^{-\lambda_m\left({y_{mjt}}^{\varphi_m} - {w_{mjt}}^{\varphi_m}\right)}$. Substitution of (3.11) and (3.14) in the latter gives:

$$ps_{mjt} = 1 - (1 - PS_{mt})^{1/NI_{mt}} \tag{3.15}$$

3.2.4. Cost of maintenance system and its constraints

The total cost of preventive maintenance in period *t* is:

$$PMC_t = \sum_{m \in M} CPM(PM_{mt}) \tag{3.16}$$

where, PM_{mt} is the PM level of machine m in period t and $CPM(PM_{mt})$ is its cost. Considering the limitation of PM budget, following constraint holds.

$$PMC_t \leq PMB_t \, , \forall t \in T \tag{3.17}$$

Finally, with equation (3.9), the expected minimal repair cost in period *t* will be:

$$MRC_t = \sum_{m \in M} NF_{mt} \cdot CMR_m \tag{3.18}$$

The total cost of the maintenance system is the sum of preventive and corrective maintenance costs.

$$TC_{MS} = \sum_{m \in M} (PMC_t + MRC_t) \tag{3.19}$$

3.2.5. Cost of quality system

If process inspection shows that the quality degradation has occurred (a) the process will be adjusted, (b) all the sob-lots produced in the related interval will be quality checked, and (c) the defective items will be reworked. The inspection cost for machine *m* in period *t* is:

$$IC_{mt} = NI_{mt} \cdot v_m \tag{3.20}$$

Since, the job-scheduling is not in the scope of this problem, and for simplification purposes, we suppose that the production is smoothly distributed over the periods. The sub-lot or the part of product *p* processed in interval *j* depends on the length of the interval. So, size of the sub-lot is $x_{pmt} \cdot \left(y_{m,j,t} - y_{m,j-1,t}\right)/APT_{mt}$. Therefore, the expected quality checking cost for product *p* over all intervals is $\sum_{j=1}^{NI_{mt}} x_{pmt} \cdot ps_{mjt} \cdot \beta_p \cdot \left(y_{m,j,t} - y_{m,j-1,t}\right)/APT_{mt}$. Since $\sum_{j=1}^{NI_{mt}}\left(y_{m,j,t} - y_{m,j-1,t}\right)$ is APT_{mt} and ps_{mjt} is constant in all intervals, the quality checking cost of product *p* will be $x_{pmt}.\beta_p \cdot ps_{mjt}$. By substitution of $ps_{mjt}$from (3.15), and considering all products, the total expected quality checking cost is:

$$QC_{mt} = \left(1 - (1 - PS_{mt})^{1/NI_{mt}}\right) \cdot \sum_{p\in P} x_{pmt} \cdot \beta_p \tag{3.21}$$

If a shift occurs at age τ in interval *j*, from this instant of time until the end of the interval, the machine will work in a shifted state. Given the number of process inspections (NI_{mt}), the expected shifted state time that machine *m* operates in out-of-control conditions in period *t*, is:

$$ESST_{mt} = \left(1 - (1 - PS_{mt})^{1/NI_{mt}}\right) \cdot \int_{W_{mt}}^{Y_{mt}} (Y_{mt} - \tau) \cdot s_m(\tau|W_{mt}) d\tau \tag{3.22}$$

Note that $s_m(\tau|W_{mt})$ is the conditional probability of a shift at τ given it was in normal state at age W_{mt}. Therefore, the proportion of time that machine *m* works in out-of-control state in period *t* is $ESST_{mt}/APT_{mt}$, the number of defective items of product *p* in this period will be $ESST_{mt}/(Y_{mt} - W_{mt}) \cdot \alpha_{pm} \cdot x_{pmt}$, where α_{pm} is the nonconformity rate in shifted state of the machine. Hence, the total expected reworking cost for machine *m* is given by:

$$TRC_{mt} = \frac{ESST_{mt}}{APT_{mt}} \cdot \sum_{p\in P} x_{pmt} \cdot \alpha_{pm} \cdot R_p \tag{3.23}$$

Finally, we assumed that detecting a shift, initiates the process adjustment. Considering the probability of shift in intervals, total expected cost of machine adjustments in period *t* is:

$$TAC_{mt} = NI_{mt} \cdot AC_m \cdot \left(1 - (1 - PS_{mt})^{1/NI_{mt}}\right) \tag{3.24}$$

Total cost of the quality system is the sum of inspection cost, quality checking cost, reworking cost, and the process adjustment cost for all machines in all periods.

$$TC_{QS} = \sum_{t\in T} \sum_{m\in M} (IC_{mt} + QC_{mt} + TRC_{mt} + TAC_{mt}) \tag{3.25}$$

All these cost components depend on the number of process inspections. To find the optimal number of process inspections, taking into account the worst case product with the maximum production level, we need to minimize the cost of the quality system for each machine in each period. Then, the optimal NI_{mt} values can be evaluated using numerical analysis or existing solvers.

3.3. Mathematical model and the search space

The integrated model minimizes the total cost of the system.

$$Min\, TC_{PS} + TC_{MS} + TC_{QS} \tag{3.26}$$

The components of the objective function are explained is sections 3.1, 3.4, and 3.5, and the model constraints are (a) the link between setups and production levels; inequality (3.2), (b) the balance equation between lot-sizes, inventories, backorders, and demands; equation (3.4), (c) the production capacity; inequality (3.6), and (d) the maintenance budget constraint; inequality (3.17).

The integrated model presented in this paper is a nonlinear problem with complicated evaluations. The sources of its difficulty are originated from the challenging interactions between maintenance and production from the one hand, and between maintenance and quality from the other hand. Also, determining the optimal number of process inspections minimizing the quality cost (eq. 4.25) is very hard. The number of possible scenarios for maintenance is $\prod_{m=1}^{M} (Q_m)^T$, where, Q_m is the number of PM levels for machine m, and T is the number of periods. With decision variables related to production and quality systems, size of the solution space is very huge, which justifies the need for efficient solution methods.

Meta-heuristic methods and genetic algorithms are able to deal with large, non-linear problems, and they can find promising solutions in a reasonable resolution time. In this paper, a standard form genetic algorithm is adopted. For more information about genetic algorithms, one can refer to existing papers including Reeves (2003).

3.4. Solution method

Genetic algorithms are widely employed in the literature for solving production and maintenance problems. Because of good global optimization capabilities, flexibility in adapting to our model, and successful implementations in similar problems, we adopted a genetic algorithm to solve the problem. For more details on genetic algorithms one can refer for instance to Reeves (2003). The first step in a GA based approach is to define the chromosome structure. Considering that with a PM and a process inspection plan, the exact solution of the lot-scheduling problem can be evaluated, we encode only maintenance and quality decision variables (PM_{mt} and NI_{mt}) in the chromosome. The exact value of other decision variables (x_{pmt}, S_{pmt}, I_{pt}, B_{pt}) will be found using existing solvers (In this study, we use CPLEX optimization package). Therefore, a chromosome is a vector of size $2 \times M \times T$. The first $M \times T$ elements (integers between 1 and a maximum upper limit) indicate the number of process inspections for each machine in each period. The second part of the vector corresponds to the PM levels (integers between 1 to Q_m) for each machine-period combinations. The first population is generated by randomization (population size is p_s), then, using tournament method, two parents are selected for uniform crossover (Reeves, 2003). Selection of parents is based on the objective function and the crossover execution is controlled by parameter p_c. To keep the best solutions, n_e elites are directly copied into the next generation. To prevent premature convergence, mutation is considered which assigns new values to some genes. Probability of mutation of a gene is controlled by parameter

p_m. As the population size reaches p_s, the first population is replaced by the new one, and the process continues until the total solution time T_{max} is satisfied. The algorithm parameters, p_s, p_c, p_m, and n_e are calibrated using the meta-calibration method developed by Mercer and Sampson (1978). Eiben and Smith (2011), state that this method works very well in calibration of genetic algorithms. The parameters of the algorithm are the population size, the selection pressure (or the number of individuals that are randomly selected from the population for the tournament selection method), the mutation probability, and the number of elites to be directly transferred to the next generation. The first step in our genetic algorithm is the algorithm calibration using the meta-calibration method. The performance criteria is defined as the average of several (in this case, 5) replications of the algorithm with a given vector of the algorithm parameters executed for a limited time (50 Sec.). After generating a random set of such vectors, a Meta algorithm is executed and the average objective value for 5 replications is calculated as the fitness of each vector of parameters. Then, using the genetic operators (selection, crossover, and mutation) a new set of parameter values are generated and incorporated in the population. The process continues for a limited time (1000 Sec.), and the best vector is considered as the calibrated values of the algorithm parameters. The solution process and the procedure of chromosome evaluation are illustrated in Fig. 3.2.

3.5. Experimental results

Let us first consider a problem with M = 3 machines, P = 2 products, T = 6 periods, and the length of periods L = 1. Each machine has Q_m = 4 PM levels and the maintenance budget limitation for all periods is PMB_t = 1000. The problem data are given in Table 3.1.

Solution method

1. Parameter calibration
2. Initialization of the first population; P_1 and evaluate the chromosomes
3. Do
   3.1. Copy n_e elite solutions from P_1 to P_2
   3.2. Do
      3.2.1. Select two parents p_1 *and* p_2 form P_1 (tournament method)
      3.2.2. Crossover: Generate a child p with uniform crossover
      3.2.3. Mutation: Replace randomly selected genes of p with new values
      3.2.4. Evaluate p
      3.2.5. Add p to P_2
   3.3. Loop until the size of P_2 is p_s
   3.4. Replace P_1 by P_2
4. Loop until the stop condition is satisfied (solution time = T_{max})
5. Return the best solution

Chromosome evaluation procedure

1. Calculate the age reduction factors γ_{mt}
2. Compute the age values; w_{mjt} and y_{mjt}
3. Evaluate the maintenance system cost (sections 3.3.2, 3.3.3, and 3.3.4.
4. Evaluate the quality system cost (section 3.3.5)
5. Solve the lot-scheduling problem using CPLEX (Section 3.3.1)
6. Return the decision variables and the objective function

Fig. 3.2: Solution method and procedure of chromosome evaluation.

Table 3.1: Problem data.

	Product	s_{pm} 1	s_{pm} 2	α_{pm} 1	α_{pm} 2	CMR_m	TMR_m	v_m	λ	φ	θ	ρ	W_{m0}	AC_m	CPM (Thousands)
Machine	1	40	-	0.6	-	800	0.02	50	1	2.5	1	2.5	2	40	3000,500,200,0
	2	30	10	0.4	0.5	700	0.01	30	0.177	2.5	0.177	2.5	2	20	5000,500,200,0
	3	-	35	-	0.8	900	0.015	40	0.064	2.5	0.064	2.5	2	30	4000,600,300,0

	Product	g_{pm} 1	g_{pm} 2	π_{pm} 1	π_{pm} 2	p	h_p	b_p	β_p	R_p	d_{pt} 1	d_{pt} 2	d_{pt} 3	d_{pt} 4	d_{pt} 5	d_{pt} 6
Machine	1	2500	-	6	-	1	2	25	1	2	3500	4000	1500	2500	1000	5000
	2	1000	1500	8	9	2	3	40	2	3	2500	2000	1500	1500	3500	3500
	3	-	3000	-	10											

After algorithm tuning with the meta-calibration method (execution time is 1000 Sec.), the selected values are population size = 25, selection pressure = 5, n_e = 4, and p_m = 0.04. The problem is solved with the genetic algorithm and the results are presented in Table 3.2, where the value of the objective function is 462,832. The solution time was set to 30 minutes and the solution reported in this Table, is the best one among 30 replications. Note that on longer runs up to 5 hours, no better solution was found. The violation penalty for the PM budget is 10.

Table 3.2: The best solution found by the genetic algorithm.

	T→	1	2	3	4	5	6
PM levels	*M*=1	1	1	1	1	1	1
	M=2	2	2	1	2	1	2
	M=3	2	2	3	2	3	2
Number of process Inspections	*M*=1	20	21	22	23	23	23
	M=2	13	17	20	23	24	26
	M=3	9	12	16	18	21	23
Production Levels	*P*=1, *M*=1	2168	2080	2013	1961	1922	1893
	P=1, *M*=2	983	974	967	957	951	630
	P=1, *M*=3	0	0	0	0	0	0
	P=2, *M*=1	0	0	0	0	0	0
	P=2, *M*=2	0	0	0	0	0	467
	P=2, *M*=3	2500	2000	1500	2238	2904	2891
Setups	*P*=1, *M*=1	1	1	1	1	1	1
	P=1, *M*=2	1	1	1	1	1	1
	P=1, *M*=3	0	0	0	0	0	0
	P=2, *M*=1	0	0	0	0	0	0
	P=2, *M*=2	0	0	0	0	0	1
	P=2, *M*=3	1	1	1	1	1	1
Backorders	*P*=1	349	1295	0	0	0	1
	P=2	0	0	0	0	0	0
Inventories	*P*=1	0	0	185	603	2476	0

P=2	0	0	0	738	142	0

Table 3.3 shows the effect of the different PM budget levels on the production, maintenance, and quality systems.

Table 3.3: The effect of different maintenance cost limitations on the system.

PMB_{mt}	Total cost	TC_{PS}	TC_{MS}	TC_{QS}	Optimal PM plan
250	507470	333846	93493	80131	2,2,3,2,2,2,3,3,2,3,3,3,3,3,3,3,3
500	474715	314332	82036	78347	1,1,1,1,2,2,3,3,3,3,2,3,3,3,3,2,3
750	466906	311765	78219	76923	1,1,1,1,1,1,2,2,2,2,2,2,3,3,3,3,3
1000	462832	311384	77075	74373	1,1,1,1,1,1,2,2,1,2,1,2,2,2,3,2,3,2
1250	462680	311403	77416	73861	1,1,1,1,1,1,2,3,1,2,2,2,2,1,3,2,2,2
1500	459190	310889	76130	72170	1,1,1,1,1,1,1,1,1,1,2,2,2,2,2,2,1
2000	456547	310648	76060	69839	1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1
2500	456547	310648	76060	69839	1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1
3000	395486	290681	51473	53332	1,0,1,1,0,1,1,3,1,1,3,1,1,3,1,1,3,1

Increasing the maintenance budget reduces all cost components. However, in some cases, higher PM levels may not be compensated by the decrease in the other costs and so; there would be an optimal maintenance point that the proposed model is able to find it. The large savings for small PMB values are mainly because of the significance of the corrective maintenance in the process. The economic consequences of downtime are very big and even low-cost preventive maintenances have chief impacts on the expected number of failures. In Fig. 3.3, the effect of the initial age and the number of quality inspections on the duration of time that the machine remains in a normal state is presented (for the first machine). We see that by increasing the initial age of the machine, the expected duration of the normal operating state diminishes. The process inspections help to maintain the machine in an in-control state, but increasing the number of quality inspections cannot be justified because of its cost. Such an impact is presented in Fig. 3.4.

The graph is based on data of the first machine. When the machine age before PM is higher, more process inspections are required.

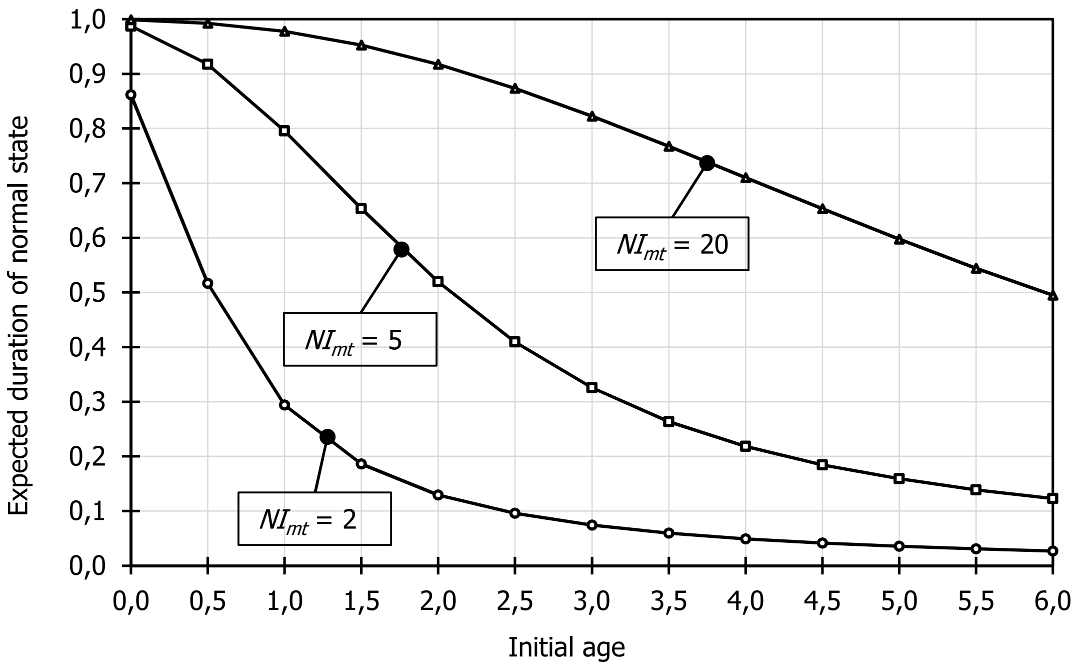

Fig. 3.3: Effect of the initial age of machine on the duration of normal working state.

In this figure, more frequent quality inspections diminish the average time of out-of-control state. However, increasing the residual life after PM quickly diminishes the time of the normal operational state.

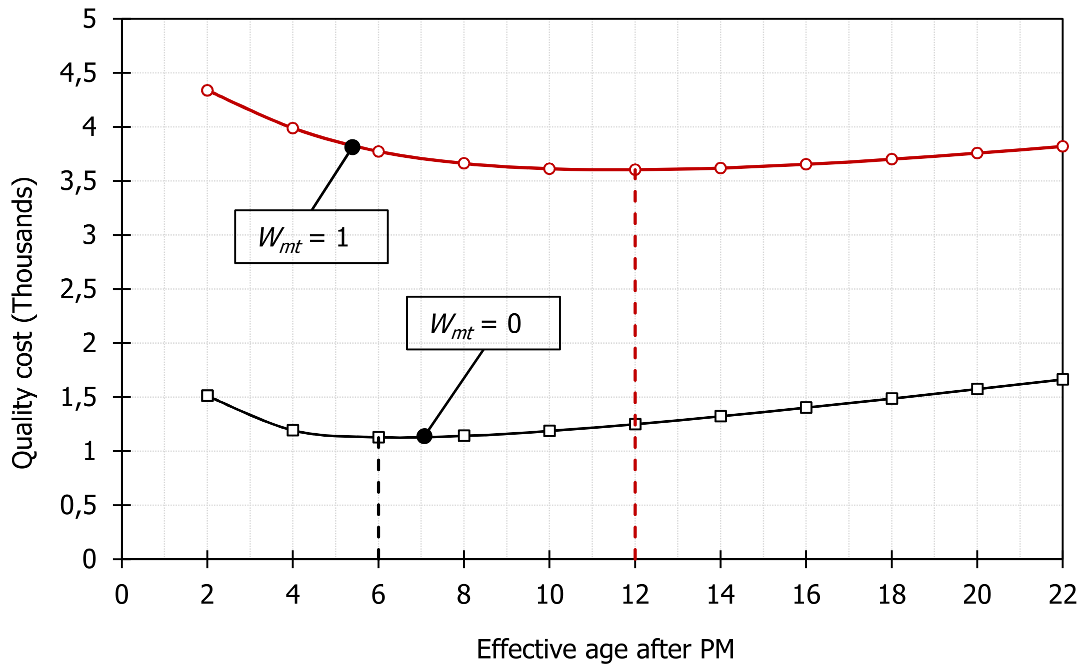

Fig. 3.4: The effect of process inspections on the quality cost.

According to the obtained results, the quality cost quickly increases when the number of quality inspections reduces to the very small numbers (like 0 and 1 at the left ide of the optimal number of process inspections), but at the right-side, the quality cost increases slightly. This graph states that a conservative inspection plan with fewer numbers of inspections may be very risky.

As illustrated in Fig. 3.5, older machines are more responsive to maintenance as they show a higher cost reduction by the same PM costs. Therefore, in a system with resource limitations, the priority of high level maintenances is yielded to more risky components.

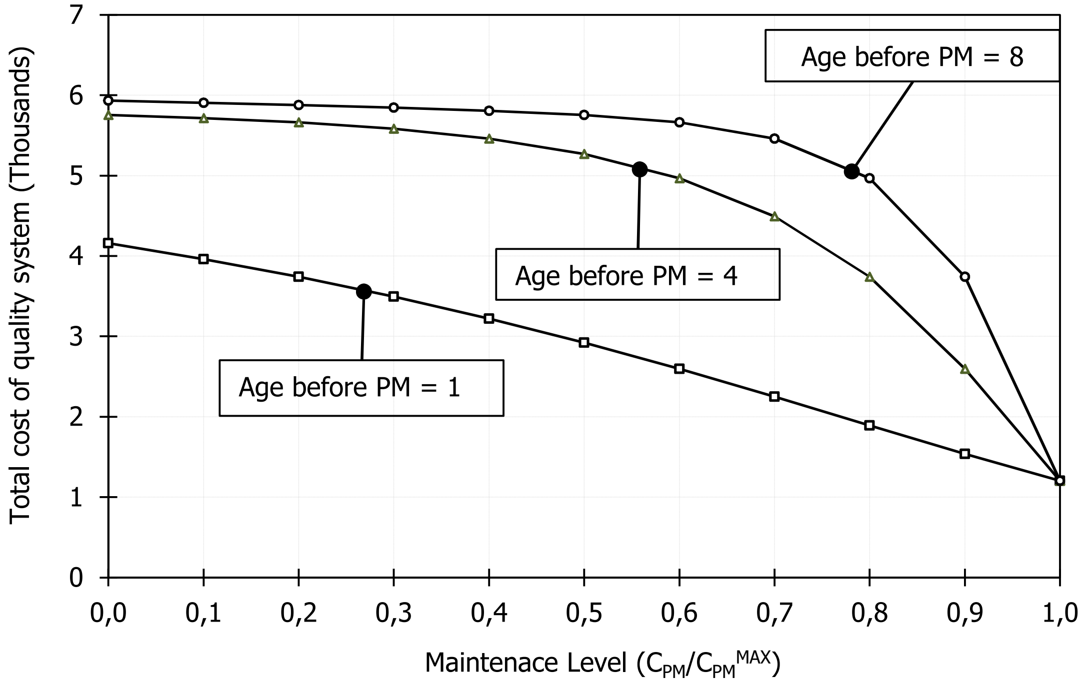

Fig. 3.5: Impact of maintenance level on the total cost of the quality system.

According to Fig. 3.5, when the residual life after PM is high, the machine is less responsive to low-level maintenances. Reductions in the total quality cost with low-level PM (smaller PM costs) in old machines are almost negligible. But, younger machines are good choices for lowlevel maintenances.

Availability of the machine defined as the rate of remaining time after minimal repairs, and its reliability is significantly influenced by the maintenance. Fig. 3.6 shows the machine availability rate as a function of its age. In machine 3, increasing the age has resulted in completely unavailable machine that should be considered in production capacity and lot-scheduling.

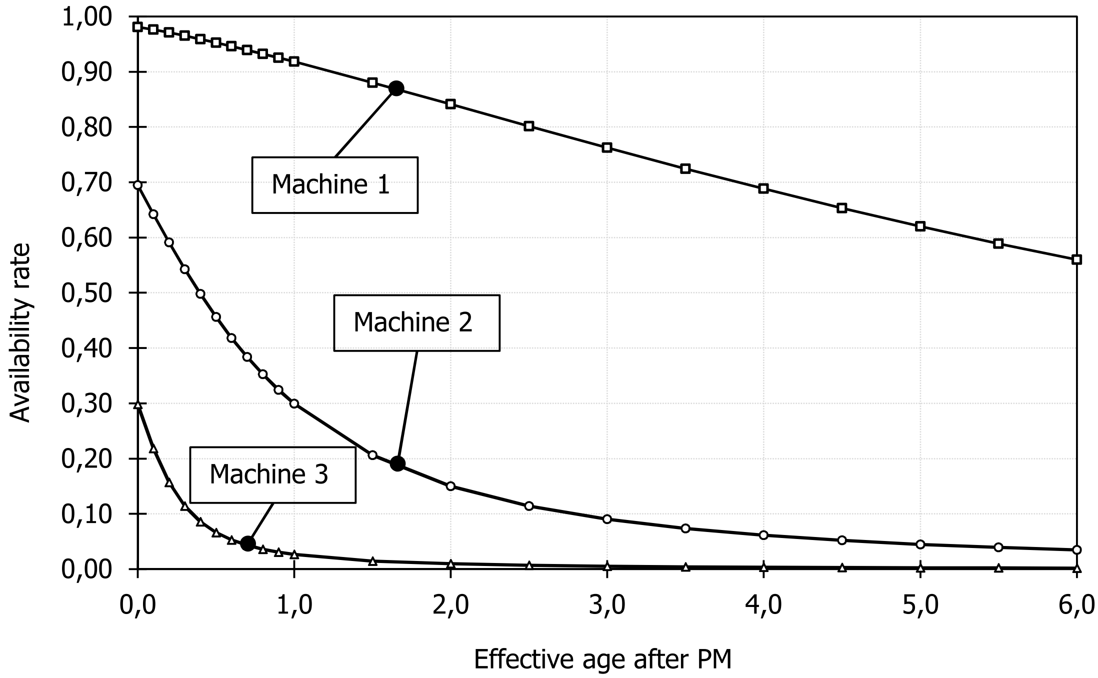

Fig. 3.6: Effect of the machine age on its availability rate.

Several factors, such as reworking cost, quality checking costs, inspection and adjustment costs influence on the optimal number of process inspections. Machine deteriorations are followed by the higher number of process inspections. This issue is illustrated in Fig. 3.7.

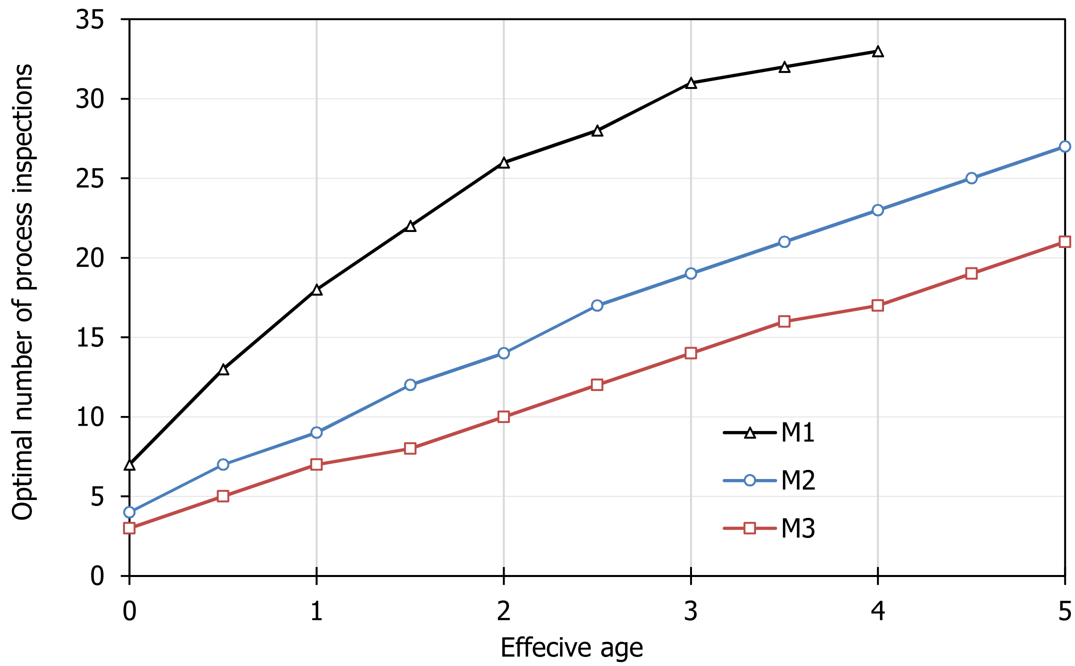

Fig. 3.7: The optimal number of process inspections as a function of the machine age.

Process inspections can relief the effect of aging in processes. The optimal number of inspections linearly increases with the residual age after PM.

3.6. Conclusions

Production planning, maintenance scheduling and quality systems are three key functions in manufacturing systems. There are strong interactions between these systems, but this link is generally neglected in the literature. Considering the large diversity of models in real-life applications concerning production, maintenance, or quality, just a limited number of industrial applications are addressed in joint scheduling. In this paper, an age-based imperfect maintenance is integrated with lot-scheduling problem while considering the number of process inspections of the quality system as a decision variable. The production system is composed of several machines and multiple products an each machine has a set of maintenance levels with different costs and various impacts on the system. In our discrete time maintenance model, a PM can be performed at the beginning of each period and finding the optimal maintenance plan in a constrained system is a difficult problem and has real-life applications. The model is formulated and evaluation of costs and interacting factors is explained. Also, a solution method based on genetic algorithms is presented and a sensitivity analysis is conducted. According to the numerical analysis, we saw that the maintenance limitation (PM budget), the deterioration and failure functions, and the time and the cost of preventive and corrective maintenance respectively have the greatest effects on the optimal PM plan. These results interpret the robustness and sensitivity of the solutions and indicate how the optimal solution changes with them. In relation to the lot-sizing part of problem, the processing rates and the availability times completely change the optimal solution of this problem.

Considering the complexity of the integrated models and the huge size of the solution spaces, developing efficient solution methods is important in real applications of the model. Also, the model can be extended to involve the cases of assembly and disassembly systems in which the products are linked together as elements of a sub-assembly.

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Chapter Four: Optimization Methods

This chapter is dedicated to the article entitled “A memetic algorithm with population management to solve an integrated production, maintenance, and quality planning problem” is submitted to *IIE Transactions* in January 2016.

The titles, figures, and mathematical formulations have been revised to keep the text consistency through the thesis. In the submitted version, the mathematical model and the data of the sample problem are included as an appendix.

The sample problem and solution results presented in this section correspond to the profit maximization model of chapter II. However, similar results (in terms of the algorithm efficiency) are obtained when applying the algorithm to the model of chapter III. These results are summarized in appendix B.

Résumé

Au cours des dernières années, la planification intégrée de la production, de la maintenance, et la qualité ont suscité l'intérêt de plusieurs chercheurs. Ces problèmes intégrés sont très difficiles à résoudre. Par conséquent, afin d'exploiter les avantages des modèles intégrés, il faut développer des méthodes de résolution efficaces. La gestion de la population dans les algorithmes évolutionnaires a un impact sur la qualité des algorithmes. Dans cet article, nous proposons un algorithme mimétique avec gestion de la population. Le modèle étudié permet d'intégrer l'ordonnancement des lots et la planification de la maintenance imparfaite basé sur l'âge avec des aspects de qualité en considérant des systèmes imparfaits qui se détériorent avec le temps. La méthode de résolution proposée se compose d'un algorithme génétique bénéficiant des méthodes de recherche locale et, plusieurs outils de gestion de la population sont considérés pour éviter la convergence prématurée, ou pour améliorer la qualité des solutions. La mesure de la diversité de la population, l'utilisation des stratégies d'intensification et de diversification, l'exploitation d'un logiciel d'optimisation comme une méthode exclusive de problème, ainsi que l'intégration de l'algorithme génétique avec des méthodes de recherche avec tabous et Nelder-Mead sont les principales caractéristiques de l'approche proposée. Des comparaisons de l'algorithme avec des heuristiques existantes montrent que l'approche proposée est plus performante en termes de temps de résolution et de qualité des solutions.

Mots clés

Production, Maintenance, Qualité, Algorithme mimétique, Gestion de la population, Diversité

Abstract

During the recent years, integration of production planning, maintenance scheduling, and the quality system parameters in a single model has drawn the interest of several researchers, but generally, such joint problems are very difficult to be solved in a reasonable time or effort. Therefore, to exploit the benefits of joint models, we need to develop efficient solution methods. Population management in evolutionary algorithms impacts on the quality of the algorithms. In this paper, we propose a memetic algorithm with population management to solve a joint model integrating lot-sizing and age based imperfect maintenance with quality aspects in deteriorating systems. More specifically, a genetic algorithm is hybridized with local search methods and in the meantime; several population management tools are considered to avoid premature convergence or to improve the quality of the solutions. Measuring the population diversity, using intensification and diversification strategies, exploiting an optimization package as a problem specific method in memetic algorithm, and integrating genetic algorithm with Nelder-Mead and Tabu-search methods are the main specifications of the proposed approach. Comprehensive experiments and comparison of the algorithm with existing heuristics are provided. The proposed approach represents better performance in terms of the solution time and quality.

Keywords

Production, Maintenance, Quality, Memetic algorithm, Population management, Diversity

4.1. Introduction

In this paper, first, the value of integration of production planning and maintenance scheduling is addressed. Then, the solution methods used in solving joint models are discussed. Finally, the objectives and the proposed algorithm are introduced.

4.1.1. Integration of production and maintenance in imperfect systems

The idea of integrating production planning and maintenance scheduling with imperfect systems is motivated by the need to coordinate the complex interactions between different functions. Production, maintenance and quality systems strongly interact (Nourelfath et al., 2016). Existing literature on integration highlights the significance of improvements achieved by the joint models (Xiang, 2013; Cassady et al., 2000; Kenné, Gharbi, and Beit, 2007). This approach is a beneficiary of the whole system (Ben-Daya and Duffuaa, 1995; Ben-Daya, 2007; Lou, Cheng, and Ji, 2015; Hadidi, Al-Turki, and Abdur-Rahim, 2012; Suliman and Jawad, 2012; Sung and Ock, 1992; Cassady and Kutanoglu, 2003 and 2005; Pandey, Kulkarni, and Vrat, 2011; and Nourelfath, Nahas, and Ben-Daya, 2016) and improves the productivity (Cassady et al., 2000; Kenné, Gharbi, and Beit 2007) or the profitability (Chen, 2013; Beheshti, Nourelfath, and Gendreau, 2014). The model developed by (Brandolese et al., 1996) uses integrated production-maintenance scheduling to minimize the total cost and plant utilization in a system composed of flexible parallel machines assigned to production of several products in a single stage.

However, such joint models are generally non-linear and difficult to be solved in a reasonable time or with rational computational effort. Exploiting the benefits of integrated models necessitates the development of efficient solution methods. In the next section, some solution approaches used in solving the joint models are discussed.

4.1.2. Solution methods of integrated models and application of memetic algorithms

Maravelias and Sung (2009) classified the solution methods in solving integrated problems into: *(a) hierarchical methods*, *(b) iterative methods*, and *(c) full-space methods*. In the first two methods, the problem is decomposed into two smaller (master and slave) sub-problems. Full-space methods try to solve directly the integrated problem. Some examples of the solution methods used in solving the integrated problems are genetic algorithms (Sortrakul, Nachtmann, and Cassady, 2005), tabu-search method (Pineyro and Viera, 2010), approximation of non-linear functions (Lee and Rosenblatt, 1989), iterative solutions (Aghezzaf, Jamali, and Ait-Kadi, 2007; Dhouib, Gharbi, and BenAziza, 2012; Suliman and Jawad, 2012), approximate algorithm based on lagrangian decomposition (Aghezzaf and Najid, 2008; Alaoui-Selsouli, Mohafid and Najid, 2012), variable neighborhood search (Machani and Nourelfath, 2012), simulation based methods (Roux et al., 2013; Liao and Chen, 2003), hybridization of genetic algorithm with tabu-search (Gopalakrishnan, Mohan, and He, 2001), chaotic partial swarm optimization (Leng, Ren, and Gao, 2006), memetic algorithms (Layegh, Jolai, and Amalnik, 2009; Sörensen and Sevaux, 2006; Franca, Mendes, and Moscato, 2001), simulated annealing (Loukil, Taghem, and Fortemps, 2007), harmony search (Zammori, Braglia, and Castellano, 2014), and ant colony optimization (Samrout et al., 2005, Berrichi et al., 2010). Moghaddam and Usher (2011) compared the efficiency and accuracy of exact and heuristic algorithms and concluded that the solution time of exact methods exponentially increases by the problem size, whereas the solution time in heuristic methods is almost constant and considerably low. They proposed heuristic methods for large problems. Iravani and Duenyas (2002) formulated the joint production inventory system in a single machine deteriorating system using a Markov decision structure and showed that the performance of dis-integrated policies can be inadequate. Modeling with Markov decision process and using integer programming formulation is also considered in the paper of (Aramon Bajestani et al., 2014) for joint scheduling in a multi-machine deteriorating system. According to them, online condition monitoring in maintenance and production planning has reduced the costs to 21%.

As we see, evolutionary algorithms (EAs) have been widely and successfully used in solving difficult and large problems. Lacksonen (2001) compared genetic algorithm, Hooke-Jeeves pattern search, Nelder-Mead algorithm (NM), and simulated annealing in solving different discrete optimization problems and reported that GA was the most robust as it founds near optimal solution of all test problems, but it requires more replications. Among the existing evolutionary algorithms, MAs have been increasingly employed in solving problems in operations research and computer sciences (Hart, Krasnogor, and Smith 2004). Memetic algorithms exploit the benefits of integrating population-based search and local improvement methods. Merz (2000) showed that by such a combination, the power of EA is considerably improved. Efficiency and robustness of MAs in solving complex planning and scheduling problems and their capability in balancing between exploitation and exploration are our motivation in adopting a memetic algorithm to solve the integrated production-maintenance problem.

4.1.3. Objectives

The literature review first, illuminates the value of joint scheduling and shows that just a few papers have addressed the integration of production planning and maintenance scheduling in imperfect systems, and second, underlines the gap between the theory and application of joint models. To the best of our knowledge, very few implementations of joint models have been reported yet. Neglected the importance of interactions between planning functions, the novelty of the research resulted in multiple unaddressed issues in real life problems, complexity of the models resulted in hardness of solving them, and the inadequacy of general insights to help practitioners in shop-floor decisions are the main grounds of the gap between theory and application of joint models. As the main contribution of this paper, we develop a solution method based on memetic algorithms for an integrated problem from the literature. In the meantime, we implement innovative strategies to improve the algorithm efficiency. For the first time in similar problem, we not only make use the properties of good solutions (known as positive knowledge) but also, the characteristics of poor solutions (negative knowledge) in algorithm transitions. The proposed method uses population management to maintain the diversity during the solution process. Hybridization of genetic algorithm with local search methods is carried out in two points. First, using CPLEX to solve a part of the model that is a linear program, and then, we use Nelder-Mead or tabu-search algorithms to enhance its performance.

The rest of this paper is organized as follows. In section 4.2, the integrated model is introduced and the problem of solving definition discusses the integrated production-maintenance scheduling problem. Section 4.3 is presents the solution method and search management tools and section 4.4 provides a comprehensive examination of the algorithm performance and comparisons. Section 4.5 includes our concluding remarks.

4.2. Problem definition and solution method

In this section, first, a general explanation of the joint production and maintenance model in imperfect systems is presented and then, a solution method is introduced.

4.2.1. Joint scheduling model

Integration of multiple decisions in a single model is an appropriate way to deal with the interactions between production planning, maintenance scheduling and quality control. The main objective in this paper is to implement existing, and novel strategies used in combinatorial optimization to develop efficient solution methods to an integrated model proposed in (Beheshti et al., 2014). They proposed a profit maximization model to integrate the three functions in imperfect systems. The model addresses the optimal maintenance schedule as well as the production lot sizes and selling levels, such that the variable demands are met. It takes into account imperfect production system and time-varying costs in the context of age-based imperfect maintenance. Dealing with degrading machines and imperfectness of maintenance as notable properties of real systems, introducing multiple maintenance options, industrial applications of the model, and quantifying the link between maintenance and quality that may be used, for example, in six-sigma projects, are the main motivations of selection the model. The mathematical formulation of the joint model and its explanation can be found in chapter II.

4.2.2. Solution method and its features

In order to exploit the benefits of integration, it is essential to develop efficient solution methods for joint problems. The literature reported the good performance of the memetic algorithm in solving complicated models. Introduced by Moscato and Norman (1989), Memetic algorithm (also called *hybrid genetic algorithms* or *genetic local search*) is a population-based approach that combines GA with problem specific heuristics or local search methods. Some of its recent implementations are Schemeleva et al. (2012), Cheng et al. (2011), Layegh et al. (2009), Boudia and Prins (2009). A literature review of memetic algorithms can be found in Neri and Cotta (2012). Memetic algorithms are also employed in solving production-maintenance problems (França et al., 2001). The literature shows the importance of preserving the population diversity on efficiency and productivity of a population-based heuristics (Laguna et al., 1999, Ferland et al., 2001). Vidal et al. (2013, 2014) reported an impressive increase in the algorithm's efficiency in population management. Hertz and Widmer (2003) discussed that maintaining the population diversity is critical in efficiency of evolutionary algorithms. This concern is even more important in memetic algorithms, because the local search forces the exploration to focus on some restricted regions. The concept of a memetic algorithm with population management (MAPM) was first introduced by Sörensen and Sevaux (2006), where the aim was to dynamically preserve the diversity of a small population of high-quality individuals and to avoid slow or premature convergence. Using numerical comparisons they showed that the MAPM outperforms very similar hybrid genetic algorithms without population management. Vidal et al. (2012) developed a hybrid genetic algorithm (HGA) for the multi-depot and periodic vehicle routing problem. They proposed a mechanism that addresses both the objective value and its contribution to population diversity in the evaluation of individuals. Lozano et al. (2008) took into account a replacement strategy to preserve the heterogeneity. On the other hand, calculating the diversity factor has its own significance. Nsakanda et al. (2007) showed that the method of measuring the population diversity has an important effect on the results and they proposed a new approach based on computing the distance and the similarity of chromosomes. Vidal et al. (2012) used the Hamming distance method to a set of close neighbors. In Lozano et al. (2008), the diversity contribution of a chromosome is defined as the distance of an offspring to its nearest neighbor.

In this article, a memetic algorithm with population management for the integrated production-maintenance problem is proposed. The algorithm employs CPLEX optimization package to solve the linear part of the problem, and in the meantime, it exploits Nelder-Mead or tabu-search methods to improve the performance. Population management strategies are used to well-organize the solution process and both positive and negative knowledge (information extracted from better and worse solutions) are used in the solution process. Intensification of good solutions, diversification of the population, adaptive control of the algorithm parameters, and survivor selection based on the contribution of individuals in the population heterogeneity are the implemented strategies. Performance indicators (the solution time and quality, and the algorithm robustness) are used to compare it to other heuristics or different configurations of MAPM.

4.3. Memetic algorithm with population management

Encoding the solutions in a chromosome structure, generation of the first population, parent selection, crossover operator, a function for propagating the search process in the solution space, and selecting survivors that contribute in the next generation are the standard features of every genetic algorithm that are explained in the next sections. For more details on genetic algorithms, one can refer to Reeves (1997) and Reeves (2010). Memetic algorithms are combination of GA with local search methods. We have implemented this combination in two phases, first, using CPLEX to solve the linear part of the problem and second, using Nelder-Mead method to improve the algorithm efficiency. It also, utilizes population management strategies to maintain the diversity. In this section, first we present the process flow of the MAPM and then, we explain in detail the implemented features and genetic operators.

4.3.1. MAPM algorithm

The process flow of MAPM is presented in Fig. 4.1. The solution starts with parameter calibration. Then, after the creation of the initial generation, the population diversity is evaluated the dynamic parameters are assigned to control the diversity. Then, a child is created either by crossover, intensification of good solutions, or diversification of the population. Each chromosome evaluation comes with solving the linear part of the model with CPLEX. The child goes through an improvement process (education) with Nelder-Mead or tabu-search algorithms. Then, the solution will be added to the population until the population size is doubled. At this time, the survivor selection is initiated to remove some solutions before starting the next iteration. Selection of survivors is based on the biased fitness, which considers both the objective function and the chromosome diversity contribution. A maximum solution time determines the stop condition of the algorithm.

- Parameter calibration (§ 4.3.11)
- Generate the initial population (§ 4.3.4))
- Do (Solution process)
    - Evaluate the population diversity (§ 4.3.5)
    - Update the dynamic parameters (§ 4.3.11)
    - Do (Generation of the next population)
        - Generate a child by
            - Crossover (§ 4.3.7)
            - Intensification of top-half solutions of the population (§ 4.3.8)
            - Diversification of the whole population (§ 4.3.8)
        - Perform local search on the child (§ 4.3.9)
        - Add the child to the population
    - Loop until the size of the population is doubled
    - Perform survivor selection (§ 4.3.10)
- Loop until the stop condition is satisfied
- Return the best solution

Fig. 4.1: Process flow of MAPM.

4.3.2. Solution encoding and search space

Taking into account the mathematical formulation of the integrated profit maximization problem (Chapter II), the decision variables are the maintenance plan (M_t^k), lot-schedules (X_t^p), backorders (B_t^p), setup variables (S_t^p), inventory levels of conforming and nonconforming products (IC_t^p and IN_t^p) And the sales level of conforming and nonconforming items (SC_t^p and SN_t^p), where p is the index of products (from 1 to P), t is the index of periods (from 1 to T), and k is the index of PM intrusions in a period (from 1 to M)., where the elements are the PM options for each maintenance (integers between 1 to Q, where Q is the total number of maintenance levels). Investigation of the model shows that, given a PM schedule, the problem reduces to a linear mixed integer program (MIP) that can be solved using existing methods. Solving this MIP yields the exact value of the other decision variables. Therefore, each PM schedule as a chromosome corresponds to a complete solution. For example, $\hat{p}$ = [2, 0, 1, 0, 0, 2, 1, 2, 0] is a chromosome (a PM schedule or a solution) of a problem with T = 3 periods and M = 3 PM per period, where PM options are {0, 1, and 2}. In this example 2, 0, and 1 are PM levels in the first period, etc.

4.3.3. Evaluation of chromosomes

To extract a full solution out of a chromosome, the generic procedure of Fig. 4.2 is presented.

- Given a chromosome $\hat{p}$, for each interval in each period:
    - Calculate the machine age; w_{mt}, y_{mt}.
    - Evaluate the shift probabilities; PS_t^k.
    - Compute the cost of preventive maintenance, inspection, restoration, and minimal repair (terms of the objective function).
    - Determine the average nonconformity rates and the available production times; APT_{mt}.
- Sub-problem formation; given defective rate and APT_t, model the related lot-scheduling problem using first and second terms of the objective function and the constraints
- Solve the MIP with CPLEX and get the objective and the value of decision variables
- Calculate the total objective value

Fig. 4.2: Evaluation of chromosomes.

4.3.4. Initialization of the population

The population size (λ) is an important factor affecting the scalability and performance of evolutionary algorithms. Experiments show that large population sizes are not necessary and many authors suggest that a small population such as 30 is enough to produce satisfactory results (Reeves, 1997). In this section, the population is determined in the calibration process (between 10 and 50. The initial population is generated according to a uniform random process.

4.3.5. Population diversity and distance measurement methods

To prevent premature convergence of the algorithm, the population diversity is controlled to stay above a minimum allowed range. Each time it falls below the limit, certain parameters of the algorithm are modified such that it generates more diverse solutions. These adjustments and related issues are explained in the next sections. S*hannon entropy* originated from information theory is one of the methods used in the literature to quantify the population diversity in genetic algorithms (San Jose-Revuelta, 2007; Guchait et al., 2013). Furthermore, in intensification and diversification of the solutions, we need to quantify the distance between two chromosomes. In this section, Shannon entropy and the general form of the distance function; *Minkowski distance method* (Yoon and Kim, 2013) is utilized to quantify the population diversity and the distance between chromosomes.

Given the number of possible genetic values (*M*), and the length of chromosome (*L*), the entropy of i^{th} value in location *j* is $E_{ij} = \begin{cases} -v_{ij} \ln v_{ij} & v_{ij} > 0 \\ 0 & v_{ij} = 0 \end{cases}$, where v_{ij} is the occurrence rate of i^{th} value in location *j*, and *ln* is the logarithm function. So, the normalized entropy of population *P* is the average of all entropies divided by the maximum entropy i.e.

$$E_N(P) = \sum_{i=1}^{M} \sum_{j=1}^{L} \frac{E_{ij}}{L.\ln M} \tag{5.1}$$

Note that the maximum entropy in a population (when all the occurrence rates are the same) is $-\frac{1}{M} ln \frac{1}{M}$.

The normalized distance between two solutions *A* and *B*; based on the Minkowski distance method is

$$\Delta(A, B) = \frac{\left(\sum_{i=1}^{L} |a_i - b_i|^r\right)^{1/r}}{\Delta \cdot L^{1/r}} \tag{5.2}$$

where a_i and b_i are i^{th} genes in *A* and *B*, *r* is the parameter of the distance function, *Δ* is the greatest difference between gene values, and $\Delta \cdot L^{1/r}$ is the maximum possible distance between two chromosomes. Note that, $r = 1$ in this function returns the *Manhattan distance*, $r = 2$ corresponds to the *Euclidian* distance, and $r \rightarrow \infty$ is related to the *Chebyshev* distance.

The distance between solution *A* and population *P* is defined as the normalized minimum distance between *A* and the population members, i.e.

$$D(A, P) = \min_{p \in P} \Delta(A, p) \tag{4.3}$$

4.3.6. Neighborhood structure

For a given solution A, a neighbor point B is a chromosome that one of its genes is only one unit different from A. In this case; the distance between A and B (neighborhood circle) is $\Delta(A,B) = \Delta \cdot L^{-1/r}$. In other words, A and B are adjacent if they are in the neighborhood circle. For example, B = [2, 0, **0,** 0, 0, 2, 1, 2, 0] and C = [2, 0, **2,** 0, 0, 2, 1, 2, 0] are two neighbors of A= [2, 0, **1**, 0, 0, 2, 1, 2, 0].

4.3.7. Selection and crossover

The selection process takes the advantage of a *tournament method* (Reeves, 2003) with selection pressure ϕ and picks the parents by a uniform probability function. Selection is based on the fitness of individuals. Then, the two parents are combined to generate a child. Among the existing methods, uniform crossover that is considered in this paper has the flexibility to handle the amount of disruptions by its recombination parameter (P_r). This parameter is used to control the bias of reproduction toward parents. By increasing it, the similarity of child j to parent j (j = 1 or 2) increases. Fig. 4.3 shows an example of uniform crossover. On the other hand, the probability of calling the crossover process is controlled by another parameter P_c. The first offspring is considered as the output of this operator.

Parent *a*	a_1	a_2	a_3	a_4	a_5	a_6
Parent *b*	b_1	b_2	b_3	b_4	b_5	b_6
Child	a_1	b_2	a_3	a_4	b_5	b_6

Fig. 4.3: Uniform crossover.

In crossover, parent selection, and the move strategy in the tabu-search algorithm, other approaches are tested and compared. We did not found significant difference between uniform, one-point, two-point, and three-parent crossover methods. Also, the tournament method in parent selection procedure performed slightly better than the roulette wheel method but there was no significant difference between the tournament and the universal sampling method. The *move* in tabu-search algorithm (the neighborhood selection strategy) has two parameters; the neighborhood radius and the number of randomly generated neighbors that participate in the selection process. The best value of these parameters (along with the other parameters of the algorithm) is selected in the calibration process.

4.3.8. Intensification and diversification strategies

The objectives of intensification and diversification are to ensure that the best solution of the current search space is found and also, to force the search process to investigate not-visited areas. Intensification uses the data of top-half solutions, whereas diversification considers the whole population. The method is based on the distance between the solutions and the current population. In this approach, several random chromosomes are generated and their distance to the current population is calculated. Distance measurement is based on *Minkowski* method that is explained in section 4.3.4. In case of intensification, a solution with the minimum distance, and in case of diversification, a solution with the maximum distance is selected. The intensification-diversification power; ζ is the number of solutions that compete in the selection process.

4.3.9. Local search methods

Memetic algorithms (MA) are integration of the genetic algorithms (GA) and local search approaches. The proposed method exploits this advantage in two phases of the algorithm, first; the evaluation of each chromosome is comprised of solving a mixed integer problem with the existing methods; and second, we make use of one of the two proposed alternatives; the Nelder-Mead algorithm (NM) or the tabu-search method (TS) to improve the performance of the evolutionary algorithm (EA).

4.3.9.1. Using CPLEX to solve the linear part of the problem

As explained in section 4.3.1, just a part of the decision variables are encoded in the chromosome (partial encoding) and the exact value of the other variables would be found using an existing solver. In section 4.3.2, the evaluation of chromosomes is discussed and the CPLEX software is suggested to solve the related MIP model.

4.3.9.2. Using Nelder-Mead algorithm as local search

Nelder-Mead algorithm (NM) introduced by Nelder and Mead (1965) is a search method to optimize functions whose derivatives are difficult to evaluate. The method is used in solving different problems, for example, Khojaste Sarakhsi et al. (2016) utilized it to solve a lot-scheduling problem. Fig. 4.4 shows the algorithm flowchart. The objective in NM is to exploit the information from several neighbor solutions in transitions toward the local optima. In this method, we consider not only better solutions (positive knowledge), but also the worse solutions (negative knowledge) to introduce three candidate moves. Then, the best answer among them is considered as the algorithm transition. The three candidates are (1) The contracted point (X_C) that is the best neighbor, (2) The reflected point (X_R) that is the reflection of the given solution one step in improvement direction, and (3) The expanded point (X_E) which is the expansion of the given solution two steps in the improvement direction.

Given a solution p to be improved:

1. Repeat
   1.1. Generate the sub-population P (N neighbor points of p) and evaluate them
   1.2. Determine the improvement (reflection) vector D

   For each gene position $j \in \{1,..., L\}$

   For each solution $\hat{p} \in P$

   If $\hat{p}$ is better than p then $d_j = \hat{p}_j - p_j$ ($\hat{p}_j$, p_j, and d_j are respectively j^{th} elements of $\hat{p}$, p, and D)

   Else $d_j = p_j - \hat{p}_j$

   1.3. Determine the reflected point; X_R, the expanded point; X_E, and the contracted point; X_C and modify them if needed
      i. XR = p + D
      ii. $XE = p + 2 \times D$
      iii. XC = best solution in P

   1.4. Move to the best solution among the candidates ($p = \max \{X_R, X_E, X_C\}$)
2. Loop until the *Stop Condition* is satisfied (number of attempts that fail to improve the solution)

Fig. 4.4: Nelder-Mead algorithm.

For example, suppose that the given solution p is {1, 1, 3, 0, 2} and the two neighbors are $\hat{p}_1$ = {1, 1, 4, 0, 2} and $\hat{p}_2$ = {1, 1, 3, 0, 1}, where $\hat{p}_1 > p$ and $\hat{p}_2 < p$. So, the reflection vector, reflected point, and expanded point are respectively D = {0, 0, 1, 0, 1}, $X_R = p + D$ = {1, 1, 4, 0, 3}, and $X_E = p + 2 * D$ = {1, 1, 5, 0, 4}. In some cases, the generated solutions can be infeasible (because of the value of genes), accordingly; these genes will be modified to the nearest possible values. As this example shows, both good and bad solutions participate in determination of the next solution. In contrast to tabu-search in which the poor solution are discarded, NM exploits the information from all evaluated points to determine the best move and also, it is not limited to move to one of neighbor points.

4.3.9.3. Using tabu-search method as local search

Tabu-search (TS) algorithm is successfully used to solve a wide variety of optimization problems (Gendreau and Potvin, 2003) and its aggressive capability makes it a good choice of local search in Memetic algorithms. Fig. 4.5 shows the general form of a tabu-search algorithm that is considered in this article.

Given a solution p to be improved:

1. Repeat
   1.1. Generate the sub-population P (N neighbor points of p) and evaluate them
   1.2. Determine the best neighbor solution $\hat{p}$ in P
   1.3. Move to the best non-tabu solution (let $p = \hat{p}$)
   1.4. Add p to the tabu list
   1.5. If the size of tabu list exceeds the allowed number (TLS), remove the oldest member from the list
2. Loop until the *stop condition* is satisfied (number of attempts that fail to improve the solution)

Fig. 4.5: Standard tabu-search algorithm.

4.3.10. Survivor selection

While the algorithm is running, the population size increases until it is doubled (2λ). At this moment, the iteration is completed, and before introducing the next generation, some individuals should be discarded. The selection of survivors in MAPM is based on both their fitness and their contribution in the population diversity. Fitness of a chromosome is the value of the objective function as presented in section 4.3.2, and contribution of a solution in the population diversity is the average distance between the chromosome and the population (section 4.3.4). Let us consider that $R_f(p)$ and $R_d(p)$ are respectively the rank of chromosome p in the population, based on its fitness and its diversity contribution. The biased fitness of p is $B(p) = \gamma \times R_f(p) + (1- \gamma) \times R_d(p)$, where γ is the bias parameter ($0 \leq \gamma \leq 1$). Selection of survivors is based on the biased fitness and the parameter γ increases during the algorithm execution such that, at the final phases of the solution process, higher priorities are yielded to the better solutions. This process helps to keep the diverse solutions, but it cannot introduce new genetic material to the population, so diversification (section 4.3.6) is considered to deal with this issue.

4.3.11. Parameter calibration

Metaheuristics and evolutionary algorithm have multiple parameters that should be assigned properly (parameter tuning) in order to improve the efficiency of the algorithm. Multiple interpretations of the efficiency criteria, tuning methods, structural and parametric tuning, and conceptual framework for parameter tuning and related issues are discussed in Eiben and Smith (2011). They show that the Meta-Calibration method (Mercer and Sampson 1978), works well especially in calibration of genetic algorithms (Smith, 2012). The algorithm has both static and dynamic parameters. The static parameters do not change after initialization, whereas the dynamic parameters will be updated according to the population diversity and the remaining solution time. Static parameters are determined by the Meta - calibration method. We handle it as an optimization problem with real value encoding method in chromosome representation of the parameters. After introducing a random population (of size 5) of parameter vectors, the base level algorithm is executed for a fixed duration of time (200 Sec). The capability of each vector is tested for several times (5 replications) and utility of a vector is considered as the average fitness of the final solutions. Then, using a general GA with the tournament method for parent selection (tournament size 2), one-point recombination (crossover probability 0.5), and swap method for mutation (mutation probability is 0.05), new vectors are introduced and the process is continued. The stop condition is the number of iterations (25). The best parameter vector is then considered as the calibrated parameters of the algorithm. Table 4.1 shows the static parameters and their range of variations.

Table 4.1: Static parameters.

Parameter		From – to
Population size	λ	10 – 50
Selection pressure	Φ	2 – 5
Intensification-diversification power	ζ	0 – 1
Stop condition in local search	It_{NI}	1 – 50
Minkowski *R* value in distance function	r	0 – 3

4.3.12. Adjustment of dynamic parameters

Ye et al. (2010) showed that adaptive GA (dynamically adjusting the algorithm parameters) outperforms standard GA in optimization problems. In MAPM, by controlling the minimum entropy level (ψ) and the parameter of the biased fitness function, we initially force the algorithm to keep higher diversity levels (prohibiting the loss diversity) and then, we let them to linearly decrease by progressing the time. Each time that the population entropy is less than ψ, the algorithm is forced to improve the heterogeneity by decreasing the probability of local search (p_e), crossover (p_c), and intensification (p_i) to the benefit of diversification (p_d). Dynamic parameters, their variations range, and starting values are presented in Table 4.2. To increase a parameter, its value is replaced by (upper limit + current value) / 2 and to decrease a parameter, it is replaced by (lower limit + current value) / 2. Also, for ease of implementation, it is assumed that $p_c = p_i$.

Table 4.2: Dynamic parameters.

Parameter		From – to	Starting value
Bias parameter in survivor selection	γ	0.5 – 1	0.5
Crossover probability	p_c	0 – 1	0.25
Intensification probability	p_i	0 – 1	0.25
Diversification probability	p_d	0 – 0.25	0. 5
Minimum entropy level	ψ	0 – 0.5	0.5

4.4. Computational results

This section illustrates the performance of the proposed algorithm and compares it to existing heuristics. Also, we show the advantage of population management in MAPM. The algorithm is implemented in VB.net and the numerical tests are performed on an Intel core i7 – 3.4 GHz with 16 GB of RAM.

4.4.1. The algorithms and heuristics

Five different algorithms and heuristics used in this section are summarized in Table 4.3. MAPM-NM and MAPM-TS are the main approaches that respectively use NM and TS as local search. The third method is a stand-alone implementation of a tabu - search algorithm as explained in section 4.3.9. If TS fails to improve the solution in a certain number of iterations (defined by It_{NI}), a new start point is selected and the process continues. To show the effect of population management, we consider MAPM and MA, two memetic algorithms with and without population management. In these cases, local search (NM and TS but, not CPLEX) is disabled in order to underline the effect of population management on the performance. MA uses mutation instead of intensification-diversification processes, where the probability of mutation in a gene is controlled by parameter p_m.

Table 4.3: The algorithm variants and heuristics.

Abbreviation	Description
MAPM-NM	A memetic algorithm with population management using the Nelder-Mead for local search
MAPM-TS	A memetic algorithm with population management using tabu-search for local search
TS	A Tabu-search algorithm as presented in section 4.3.9
MAPM	A memetic algorithm with population management (no local search) uses intensification, diversification and biased fitness in survivor selection
MA	A memetic algorithm without population management in which, intensification and diversification processes are replaced by a mutation operator (no local search)

4.4.2. Test problems and size of the search space

The data of a sample problem; called *P1* with T = 12 periods, P = 2 products, M = 3 maintenance per period, and Q = 4 PM levels is presented in Table 4.4. Moreover, a smaller version; *P2* (with the first 6 periods of P1) and P3; a set of 50 random problems are also considered. The random problems are created by uniform randomization, where $T \in (1\ldots12)$, (M, P, and Q) $\in (1\ldots5)$, other data between 0.5 to 1.5 times the average data given for the first product in the first period. These instances are used to study the performance and robustness of the algorithm with different problem sizes.

The specifications of the sample problems P1...P3 are summarized in Table 4.5.

Table 4.4: Problem data (P1).

Product		$d^p{}_t$		$\pi^p{}_t$		$b^p{}_t$		$h^p{}_t$		$s^p{}_t$		$PC^p{}_t$	
(p)→		1	2	1	2	1	2	1	2	1	2	1	2
Period (t)	1	50	116	67	124	31	51	4	11	441	845	182	319
	2	76	83	60	137	32	58	4	11	419	822	174	307
	3	82	95	60	123	30	50	4	12	436	844	172	315
	4	97	109	65	120	32	50	5	11	433	830	181	309
	5	95	87	69	136	31	51	4	10	428	824	170	309
	6	52	123	64	129	31	50	4	12	425	828	176	301
	7	67	86	68	128	30	57	5	10	444	804	176	315
	8	57	126	66	136	34	57	5	10	433	811	183	304
	9	61	109	68	123	31	54	5	10	424	815	173	301
	10	72	120	63	121	34	57	4	12	426	803	183	304
	11	51	128	60	125	34	55	4	12	445	840	177	319
	12	57	128	69	120	31	57	5	12	401	830	170	309

k	1	2	3	4	5	Parameter	Value	Parameter	Value
$CPM(k)$	3000	2000	1000	500	0	CMR	500	L	1
$TPM(k)$	0.02	0.015	0.01	0.005	0	TMR	0.02	λ	50
$g(k)$	200	250				v	40	φ	4
$\alpha(k)$	0.7	0.7				η	0.9	θ	30
$\beta(k)$	4	5				ξ_1	500	ρ	4
						ξ_2	3000		

Table 4.5: Sample problems.

Problem	Description	Problem size
P1	The sample problem presented in Table 4.4	4.7×10^{21}
P2	Data of the first 6 periods of P1	6.9×10^{10}
P3	Set of 50 random problems	$\leq10^{42}$

4.4.3. Parameter settings for the joint scheduling problem

The values of the algorithm parameters are set using the meta-calibration method of section 4.3.11. With slight modifications, the parameter values; (λ, Φ, ζ, It_{NI}, r) related to the chromosome lengths ($T\times M$) are:

- (10, 3, 5, 10, 2) for $T\times M \leq 10$
- (15, 3, 5, 20, 2) for $10 < T\times M \leq 20$
- (20, 5, 10, 30, 2) for $20 < T\times M \leq 30$
- (40, 6, 25, 50, 2) for $30 < T\times M \leq 50$

The size of the tabu list in TS is set to the chromosome length, mutation probability in MA is $p_m = 0.05$, and if not indicated, the considered solution time is $5\times T\times M\times Q$.

4.4.4. The best solution of the test problems with different algorithms

Problems P1 and P2 are solved with MAPM-NM, MAPM, and TS algorithms. The best and the average solution in 30 replications for each problem, the solution time, and the best chromosome (PM schedules) are presented in Table 4.6. The plans found by the MAPM-NM for all the test problems are the best solutions. The methods developed in this article are able to solve various types of the JPMQ problems.

Table 4.6: Solutions of the sample problems with different algorithms.

	Algorithm	Solution time	Min objective	Average objective	Max objective	Best PM plan
P1	MAPM-NM	1080	223032	223935	**224932**	1,1,0,3,0,2,2,0,2,2,0,1,4,0,4,3,0,4,1,1,1,4,0,4,4,0,4,3,0,4,3,0,4,4,0,4
	MAPM-TS		221759	222817	**224932**	
	MAPM		183568	201463	221720	
	TS		220261	222461	224035	
P2	MAPM-NM	540	106806	107998	**109076**	2,0,4,3,0,4,4,0,4,3,0,4,4,0,4,3,0,4
	MAPM-TS		104519	106883	**109076**	
	MAPM		76990	88872	101699	
	TS		103937	106692	**109076**	

4.4.5. Robustness of the algorithm

Capability of the algorithm and the solution quality is also investigated in solving the set of 50 random problems (P3) using MAPM-NM, MAPM-TS, TS, and MA algorithms. The proportion of times that the solution found by an algorithm was the best solution (the success rates) is shown in Table 4.7.

Table 4.7: Average success rates in 50 random problems.

	MAPM-NM	MAPM-TS	TS	MA
Success rate	76%	32%	8%	0

Accordingly, MAPM-NM yields the higher success rate among four algorithms and so, it can be conveniently used to solve the joint problem. Only in 12 out of 50 problems, the solution found by MAPM-NM was very close but not the best solution. By increasing the solution time, the gap between the success rates decrease, but almost in all cases, MAPM-NM yields better solutions.

The results of 30 replications of MAPM-NM, MAPM-TS, and TS for P1 with two different solution times are shown in Figures 4.6 and 4.7.

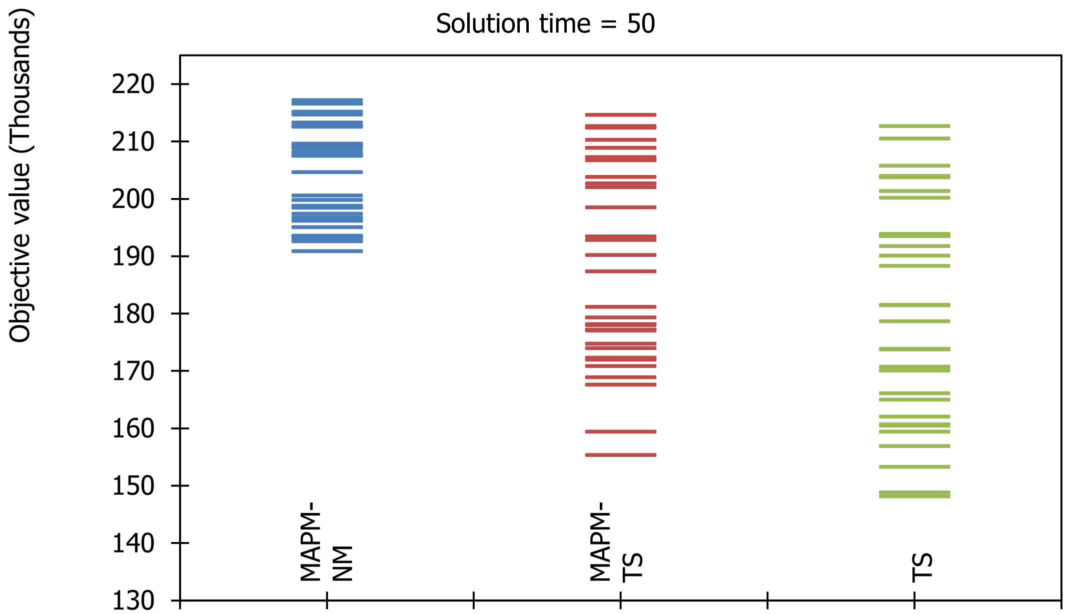

Fig. 4.6: Results of 30 replications in 50 seconds.

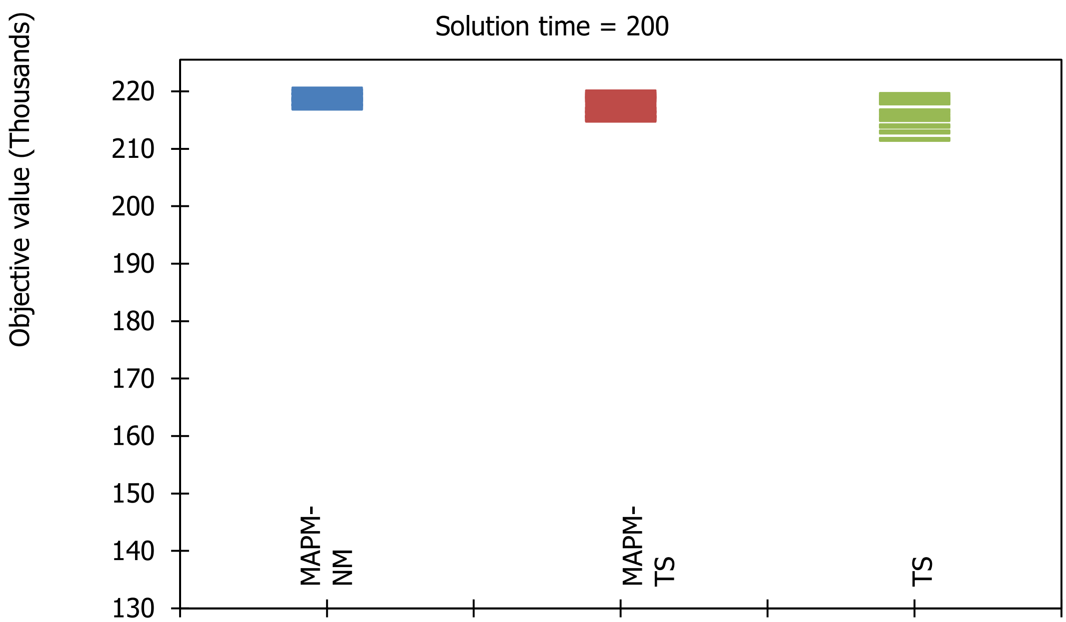

Fig. 4.7: Results of 30 replications in 200 seconds.

In both cases, MAPM-NM outperforms MAPM-TS. The two variants of MAPM perform better than tabu-search algorithm and as expected, increasing the solution time has resulted in the improvement of the final solution (smaller total costs). The minimum, average, maximum and variance of the solutions are listed in Table 4.8. Similar results were obtained in different solution times and problems, therefore, we remark that the proposed algorithm is robust and, integration of genetic algorithm with a Nelder-Mead method has resulted in a better performance.

Table 4.8: Solution data in different execution times (30 replications; *P1*).

	Run time = 50 Sec.			Run time = 200 Sec.		
	MAPM-NM	MAPM-TS	TS	MAPM-NM	MAPM-TS	TS
Min	190859	155342	148174	216988	214902	211564
Max	217227	214620	212720	220502	220000	219564
Average	203841	187664	180018	218678	217160	216262
Success rate	66.67%	16.67%	16.67%	60.00%	16.67%	23.33%
Standard dev.	8413.3	17404.5	19434.3	1008.3	1621.9	2676.7

The standard deviation values indicate the better performance of MAPM-NM, and small standard deviation compared to the average value shows the robustness of the algorithm. The average success rate in MAPM-NM is higher and in 20 out of 30 replications (in 50 Sec results) and in 18 out of 30 replications (in 200 Sec results) the solution of MAPM-NM was very close but not the best solution.

4.4.6. Performance of local search methods

The performance of tabu-search and Nelder-Mead algorithms are compared in 30 replications of the algorithms for 200 Seconds. Fig. 4.8 shows the dispersion of the solutions. In a pairwise comparison, in all the cases (70%) the Nelder-Mead outperformed the tabu-search and as shown in this figure, range of the results in NM is very smaller while the quality of solutions is also higher. Therefore, the Nelder-Mead method and extracting positive and negative knowledge in algorithm transitions has improved the performance.

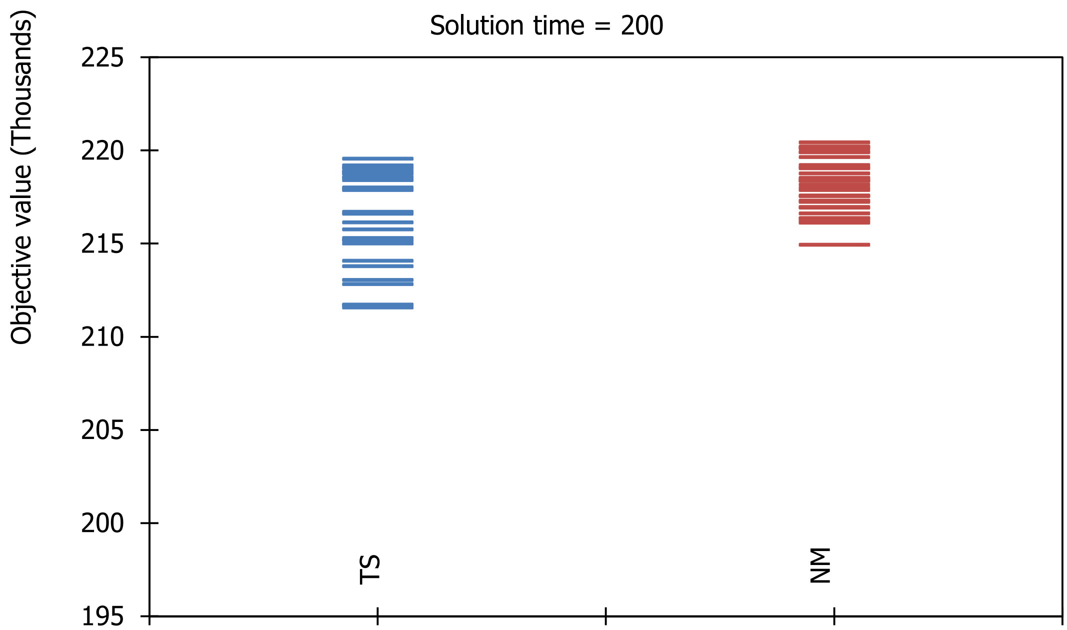

Fig. 4.8: Performance of NM and TS (P1).

4.4.7. Evolution of the best solution and effect of the population management

Figures 4.9 and 4.10 illustrate the evolution of the best solution in memetic algorithm with and without population management. Population diversity in MAPM smoothly reduces with the solution time, whereas in MA without population management, the heterogeneity is quickly lost (mutation probability is p_m = 0.05). In MAPM, the best solution has quickly improved and the final solution is better than the results of MA.

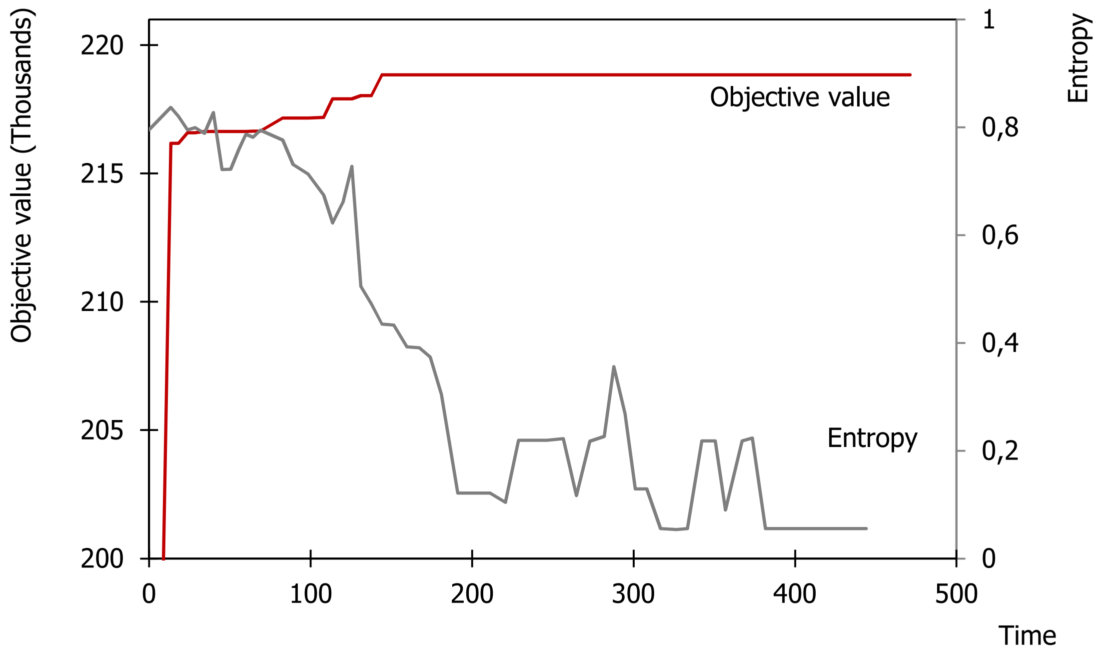

Fig. 4.9: Evolution of the best solution and entropy variations in MAPM.

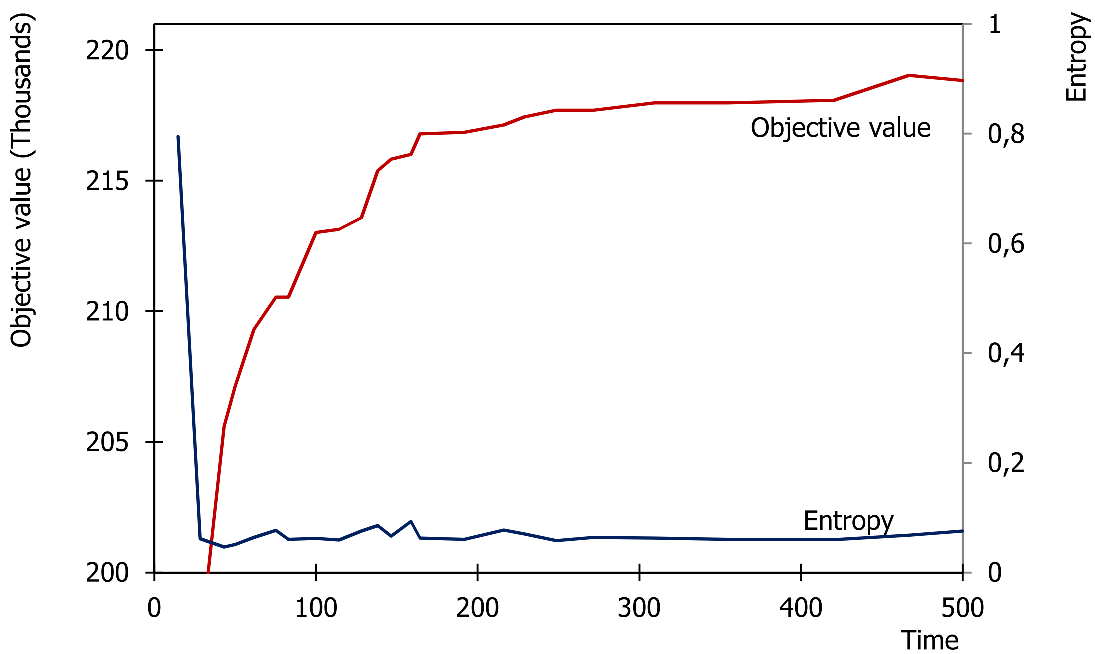

Fig. 4.10: Evolution of the best solution and entropy variations in MA.

Using intensification and diversification strategies and maintaining the population diversity in MAPM has resulted in obtaining better solutions in each instant of time. In 30 replications of the two algorithms (for 200 Sec.), in 93% of the cases, MAPM has outperformed MA, and a statistical analysis (F-test) indicates a significant difference between the means of the two algorithms (F = 83.7 compared to $F_{CRITICAL}$ = 4). This fact can be seen in the distribution of the final solutions illustrated in Fig. 4.11. Good performance of MAPM compared to MA corresponds to the population management strategies, including the biased fitness, intensification, and diversification processes.

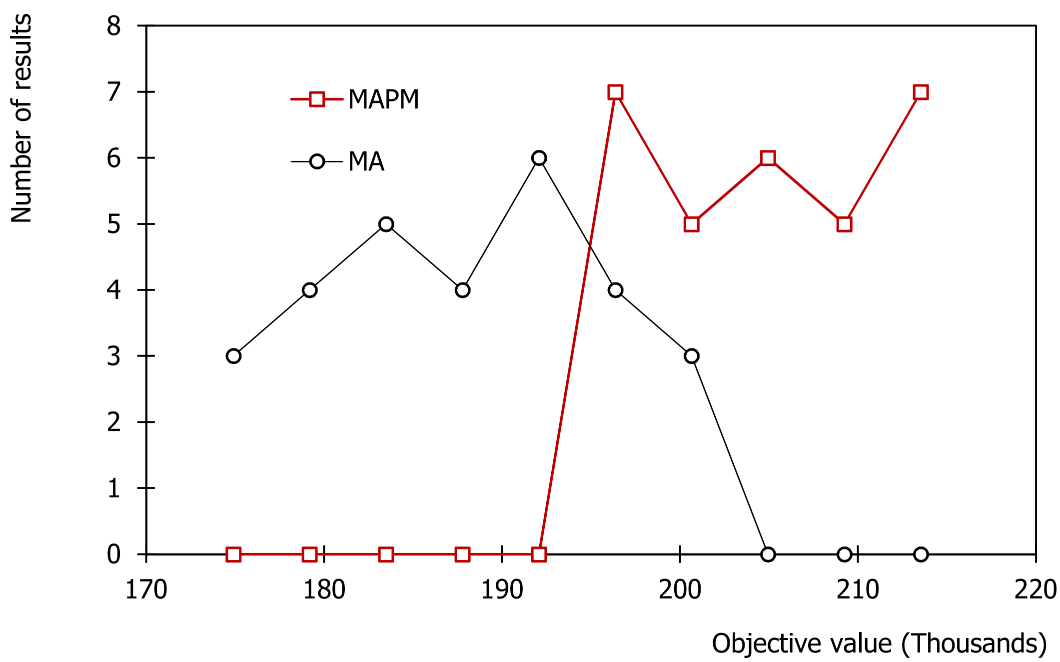

Fig. 4.11: Distribution of solutions in 30 replications of MAPM and MA (problem P1).

4.5. Conclusions

We proposed a memetic algorithm with population management (MAPM) to figure out an integrated, production-maintenance scheduling problem with regard to quality aspects. Since this level of integration is almost new in the literature, similar papers and so, the existing solution methods are very restricted. This study is the first one addressing a MAPM to solve joint problems in imperfect processes. The goal of population management is to balance between exploration and exploitation capabilities of population-based evolutionary algorithms. To carry out this idea, we adopted several strategies, including the diversity contribution of individuals in survival selection, inserting intensified or diversified solutions to maintain the heterogeneity, and adaptive control of diversity related parameters during the execution of the algorithm. Nelder-Mead and tabu-search methods are proposed for local search while, the algorithm makes use of CPLEX to solve the linear part of the model. We also introduced an approach to exploit both positive and negative knowledge of the neighbor points. The results show that the MAPM-NM algorithm outperforms MA, TS and other configurations proposed in this section. Finally, several investigations are conducted and the impact of population management in the algorithm efficiency is demonstrated. The proposed algorithm is efficient in terms of solution time and quality, and it can solve different instances of the problem.

Examining the performance of other heuristic methods such as ant colony optimization compared to the presented algorithm, and developing fast and robust methods for very large problems with multiple machines are proposed for the future studies.

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Chapter Five: General Conclusions

In this thesis the problem of joint production planning and maintenance scheduling in imperfect systems is studied. Production planning, maintenance scheduling, and quality systems decisions interact with each other and so, the optimality of the plans cannot be guaranteed if the interactions are not taken into consideration. Despite such strong relationships, these three systems are generally addressed separately in the literature as well as in industry. The increased level of competency between organizations and the enhanced request for improvement in the manufacturing industry has motivated the incorporation of those inter-relevant decisions in a joint model. integrated approach not only results in reduction of total cost, but also it may bring a higher quality and customer satisfaction levels. The first chapter of this thesis is dedicated to a general statement of the problem and investigation of the existing literature from different points of view. The research questions and the methodology are also discussed.

In chapter two, we considered a single machine production system with a profit maximization objective. The machine is subject to deterioration of its conditions with time. Such degradation increases the risk of machine failure as well as the expected non-conformity rate. The machine has several PM levels that are distinguished by their time, cost, and effects on the machine. The discrete time maintenance models are broadly used in industries. In fact, a lot of maintenance systems schedule their operations at some specific times; between two consequent missions, in vacations, or when the workload is low. Therefore, the maintenance time for them is not a decision variable, but, selecting the most appropriate maintenance alternative among a large number of possible scenarios for each PM intrusion is a challenging issue. Assuming a linear relationship between the cost of PM and the improvement in the machine state, we presented the methods for evaluating different costs and interaction factors and then, the joint model is formulated. The machine availability in influenced by the time of minimal repairs that are initiated after each machine failure and by the preventive maintenance time. Related to a PM plan, the model yields the expected available time in each production period. Then, a mixed integer linear model is solved to determine the exact value of the lot-sizes, inventory and backorder levels, and the setup decision variables. The results obtained from solving several random problems shows significant gaps between the profit of the joint model and the solutions by non-integrated and iterative approaches. In most of the production systems, the consequences of a random failure are much higher than the cost of and time of preventive maintenance. Our sensitivity analysis showed that the solutions are highly influenced by the cost and the time of minimal repair, the deterioration process and the failure function. Also, we presented the importance and the process of determining the optimal PM budget and we showed how deviations from this optimal point may influence on the cost or the profit. Poor maintenance with lower cost increases the rate of machine failures and diminishes the availability of the machine. On the other hand, expending more in maintenance with selecting higher PM levels, the increased preventive maintenance cost cannot be compensated with the decrease in quality and corrective maintenance cost. In our first contribution, the impact of maintenance on the quality system as the expected non-conformity rate in relation to the machine age is taken into account.

Then, in chapter Three, we addressed a joint model that involves decision variables of the quality system. Solving the problem yields the optimal number of quality inspections for each machine in each period. The optimal numbers of process inspections correspond not only to the maintenance plans but also to the production decisions. The production system is composed of several machines, and each machine has its own maintenance options. Considering all PM levels of all the machines, the total number of PM alternatives for each maintenance intrusion is very huge. On the other hand, performing PM is constrained to a limited budget, so the problem is to determine the optimal PM level for each machine such that the total cost of maintenance does not exceed the allowed budget. This maintenance limitation plays the role of economic dependency between different components of a system. Evaluation of the optimal number of quality inspections regarding the effective age of machines and the production plans is discussed. The optimal inspection process is highly sensitive to the machine conditions and the deterioration function. The numerical example shows that reliability and availability of the machine are vastly influenced by the machine age and the maintenance plans. The genetic algorithm presented to solve the problem is able to find good solutions in a relatively short time. However, increasing the size of the problem resulted in slower improvement speed, underlines the need for more efficient solution methods.

The joint models developed in chapters Two and Three correspond to nonlinear complicated programs that are very hard to solve in a reasonable time or computational effort. Specifically, the multi-machine multi-period problem with parameters related to the quality system is too difficult. The literature review revealed a significant gap between theory and application of joint models that one of its reasons returns to the difficulty of solving the joint problems. To exploit the benefits of the proposed approaches we need to bind an efficient solution method with the models. In section Four, we proposed a memetic algorithm with population management; MAPM that uses the Nelder-Mead algorithm. Memetic algorithms are the combination of GA with problem specific and local search methods. The MAPM exploits the benefits of such integration in two phases. First, each chromosome evaluation uses CPLEX to find the exact value of the lot-sizing problem, and it also utilizes a Nelder-Mead method (in MAPM-NM), or a Tabu-search method (in MAPM-TS) to promote the search power of the genetic algorithm. The MAPM-NM method exploits not only the positive knowledge of good solutions, but also the negative knowledge of the poor solutions in the improvement process. The suggested MAPM is the first algorithm to solve the integrated production–maintenance planning problem in imperfect systems. The implemented population management is aimed to control the population diversity during the execution process. Its main objectives are to prevent premature convergence and to force the search process to examine the whole solution space. Diversification of the current population and intensification of the best solutions are designed to replace the blind mutation operator of the genetic algorithms. A set of 50 random problems is used to compare the performance of APM-NM and MAPM-TS with several meta-heuristics. The results show that MAPM-NM outperforms all other approaches and the gaps to the best solution in MAPM-NM and MAPM-TS are respectively 0.5% and 6%. The random problems used in performance analysis belong to a wide range of problem sizes so; we remark that the method is capable to solve different instances of the joint problems. In MAPM-NM, both the solution quality in a limited execution time and the solution time to reach a targeted objective level are better.

In the presented models, the machines and products are statistically and structurally independent from each other, but in real systems, the production decisions are relevant because of the final product structure and bill of materials. We considered the time and budget constraints in the maintenance model, and the production plans were subject to the availability of machines. In a large number of industries, the system components (products, machines, or available resources) have intrinsic correlations. For example, in case of systems with assembly-disassembly operations, the planning departments need to take into account not only the production flow dictated by the product structure, but also the buffers between consequent processes. These organizations are generally characterized with the large number of machines and so, they generally use a condition-based maintenance approach. In this scheme, the machines are inspected in certain instants of time and the PM alternatives are designed according to the machine state and operation-finance limits. These organizations may employ quality sampling plans to verify the production conformity. Integration of decisions in such systems will be an interesting issue for future extensions of this research. Also, developing heuristics and testing other solution methods, comparing their results with the solution methods proposed in this thesis, and taking into account well-known features of real system in joint models can be other windows of future studies in this context.

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Appendix A

Quadratic and mixed integer reformulations and a heuristic method for solving the model presented in chapter II

Abstract

Integration of interrelated decisions in a joint model (proposed in chapter Two) has resulted in a complicated problem that is very difficult to be solved in a rational effort. Exploiting the benefits of integration necessitates the development of efficient solution approaches. Here, the general features of the joint problems and the important considerations to be implemented in the solution methods are discussed. We propose a quadratic and a mixed integer reformulation for the joint problem along with a Nelder-Mead method to solve and compare the performance of the solution methods. These approaches are then used in solving a sample problem and the methods are compared in terms of their efficiency and quality of solutions.

Keywords

Integration, Production, Maintenance, Mixed Integer, Quadratic, Non-Linear Problem.

A.1. Introduction

Joint production, maintenance, and quality (JPMQ) problems, are complicated nonlinear models with discrete, not-differentiable, and non-convex search spaces, and so they generally are very difficult to be solved in a reasonable time. Most of the existing papers in this field are nonlinear optimization models and the main solution methods are heuristics and evolutionary algorithms. Dhouib et al. (2012) studied joint scheduling in imperfect cell and considered a numerical solution procedure to solve the problem. Machani and Nourelfath (2012) proposed a variable neighborhood search for the joint noncyclical preventive maintenance scheduling in a single machine system. Mosinski et al. (2012) investigated a simulation approach to solve long and short term maintenance scheduling problem in a manufacturing line. The proposed model comprises a discrete optimization with several PM choices. Arora and Huang (1994) studied discrete non-linear problems and summarized various methods for solving them. Anandhakumar et al. (2011) modeled an Artificial Bee Colony (ABC) algorithm to solve the problem of generating maintenance schedules and compared it to a Discrete Particle Swarm Optimization. They discuss that ABC outperforms the latter approach in terms of performance and the solution quality.

Our objective in this appendix is to develop linear and quadratic formulations and heuristic methods for the joint production, maintenance scheduling problem in single machine systems (the problem introduced in chapter Two).

In the next sections, we first explain the problem and the mathematical model. In section A.3, the quadratic and mixed integer formulations of the joint model are presented. In section A.4 the solution methods are discussed and in section A.5, a numerical example and comparison of the solution methods are discussed. Finally, in section A.6 some concluding remarks are presented.

A.2. Problem definition

Production, maintenance, and quality decisions are strongly interacting with each other and their integration in one decision model can benefit the whole system and may result in better solutions. But generally, such a combination results in difficult nonlinear optimization problems that existing solution methods are not easily applicable to them. Therefore, exploiting the benefits of integrated models necessitates the development of efficient solution methods. The literature review highlighted the need for development of capable solution methods. In this appendix, we consider the joint model of chapter two, integrating lot-sizing and imperfect preventive maintenance planning taking into account the quality aspects of an imperfect production system.

The problem is the combination of lot sizing and a discrete-time maintenance that takes into account the impact of PM on both non-conformity rates (influencing on the quality system) and machine availabilities (influencing on the production planning problem). The system is composed of one machine and several products in a multi-period horizon, where the objective function maximizes the profit. There are several maintenance intrusions in each period. The cost and specifications of these PM options are different and so, they have various effects on the machine. It is assumed that the preventive maintenance reduces the effective age of the machine. Younger machines (with smaller effective ages) are more reliable, so the probability of machine failure and the probability of a shift to a deteriorated state are smaller. Therefore, the expected cost of corrective maintenance and quality-related problems are linked to the effective age of machine and this latter is subject to the maintenance schedule. By increasing the age of machine in a period, the probability of a shift to an out-of-control state increases. In the out-of-control state, the rate of nonconformity and the risk of a machine failure is higher. In real applications, the consequences of an unforeseen stop can be extremely high and it may result in stoppage of the whole or a part of production system. The evaluation methods and the mathematical model are developed and discussed in the previous section.

It is shown that with an arbitrary PM schedule, one can apply the evaluation method and compute the expected available production time. Also, expected non-conformity rate depending on such a PM plan can be evaluated. Given these interacting factors, the nonlinear joint model reduces to a linear mixed-integer problem that can be solved using existing approaches.

Our objective in this appendix is to propose quadratic programming (QP) and mixed integer linear formulations (MIP) to the discrete nonlinear optimization program discussed in chapter Two. The mixed integer linear models and certain type of the quadratic problems can be solved with existing methods (such as branch and bound) or solvers (such as Cplex). These approaches are not applicable to the larger problems because the problem size and the number of decision variables exponentially increase with the model parameters. To find promising solutions for large problems, we have proposed a Nelder-Mad algorithm in section A.4.

A.3. Quadratic and mixed integer formulations

A.3.1. The general model

The JPMQ model for the single machine systems presented in chapter Two incorporates all the cost components and the decision variables from the lot-sizing, maintenance, and quality systems in an integrated problem.

Considering $\boldsymbol{X}$; the set of all decision variables of the production system, $\boldsymbol{Y}$ the set of variables of the maintenance system, and $\boldsymbol{Z}$ the set of variables concerning to the quality system, the general representation of the integrated models (profit maximization problem) is as follows:

$$\boldsymbol{Max\ F(X) - G(Y,Z) - H(X,Y,Z)} \tag{A.1}$$

Subject to

$$\boldsymbol{\hat{F}(X,Y,Z) <=> 0} \tag{A.2}$$

$$\boldsymbol{\hat{G}(Y,Z) <=> 0} \tag{A.3}$$

$$\boldsymbol{\hat{H}(X,Y,Z) <=> 0} \tag{A.4}$$

In this general model, $\boldsymbol{F(X)}$ stands for the linear part of the objective function related to the lot scheduling problem, $\boldsymbol{G(Y,Z)}$ is the cost of maintenance system as a function of maintenance and quality-related decisions ($\boldsymbol{Y}$ and $\boldsymbol{Z}$), and $\boldsymbol{H(X,Y,Z)}$ is the cost of the quality system as a function of production, maintenance, and quality decision variables. It is assumed that the maintenance cost is independent from the production related decision. The cost of the quality system is subject to the maintenance decisions (it impacts on the production quality), production levels (in case of no production, quality cost is zero), and quality decisions themselves (length of sampling intervals, sample size etc.). First set of the constraint indicated with $\boldsymbol{\hat{F}(X,Y,Z)}$ concerns production planning part of the model, $\widehat{\boldsymbol{G}}(\boldsymbol{Y},\boldsymbol{Z})$ is the set of maintenance system constraints, and $\widehat{\boldsymbol{H}}(\boldsymbol{X},\boldsymbol{Y},\boldsymbol{Z})$ is the set of quality constraints. The presented formulation states that the production system is subject to all the decision variables (***X***, ***Y***, and ***Z***), maintenance system is just related to maintenance and quality decisions (***Y***, ***Z***), and the quality system may be constrained to maintenance and production decisions as well as the quality system parameters. The generic JPMQ model is illustrated in Fig. A.1.

Variables	***X***	***Y***	***Z***
Max profit	**Production System** ***F(X,Y,Z)*** -	**Maintenance System** ***G(Y, Z)*** -	**Quality system** ***H(X, Y, Z)***
	Production cost + Inventory holding cost + Setup cost + Backorder cost	Preventive maintenance cost + Corrective maintenance cost + Restoration cost	Nonconformity cost + Sampling and inspection cost System running cost
Subject to	**Lot sizing constraints** $\widehat{\boldsymbol{F}}(\boldsymbol{X},\boldsymbol{Y},\boldsymbol{Z})=\mathbf{0}$	**Maintenance constraints** $\widehat{\boldsymbol{G}}(\boldsymbol{Y},\boldsymbol{Z})=\mathbf{0}$	**Quality constraints** $\widehat{\boldsymbol{H}}(\boldsymbol{X},\boldsymbol{Y},\boldsymbol{Z})=\mathbf{0}$
Constraints	· Balance equation · Demands · Production and setup · Production capacity	· Relationship between effective ages and maintenance schedule · Maintenance intrinsic constraints	· Quality constraints
Interactions	*Available production time*	*Quality level and customer satisfaction*	*Nonconformity rate*
Problem type	***LP or MIP***	***NLP***	***NLP***

Fig. A.1: General form of integrated problems and interactions between sub-systems.

Appendix A – Quadratic and mixed integer reformulations of the profit maximization model

As shown in Fig. A.1, the objective function maximizes the profit, where the cost function is the sum of the production system cost (processing cost, inventory holding cost, backorder cost, and setup cost), the cost of maintenance system (inspection cost, preventive maintenance cost, and corrective maintenance), and the cost of quality system (cost of nonconformity, sampling cost, cost of running the quality inspection system etc.). There are several constraints concerning each system. For example, balance equation to link lot sizes, demands, backorders, and inventories or the relationship between lot-sizes and setup costs (lot-sizing problem generally is a linear; LP or mixed integer; *MIP* problem). In maintenance system, several constraints are to establish the link between maintenance cost and the effective age of machine or between age of machine and expected nonconformity rate. Maintenance model is generally a complex and nonlinear; NLP program. Also, constraints of the quality system link the cost of quality system to decision variables of the quality system (such as the number of quality inspections and the time of sampling, sample sizes etc.). Similar to the maintenance problem, quality planning mostly results in NLP problems.

With these definitions and considering the model presented in chapter 2, we have:

$$\boldsymbol{X} = \{x_t^p, SC_t^p, SN_t^p, IC_t^p, IN_t^p, S_t^p, B_t^p\} \tag{A.5}$$

$$\boldsymbol{Y} = \{M_t^k, w_t^k, y_t^k, NF_t^k, PS_t^k, APT_t, \tau_t\} \tag{A.6}$$

$$\boldsymbol{Z} = \{\, M\} \tag{A.7}$$

The cost functions of production, maintenance, and quality systems are:

$$\boldsymbol{F}(\boldsymbol{X}) = \sum_{t\in T}\sum_{p=1}^{P}\left(SC_t^p \cdot PC_t^p + SN_t^p \cdot PN_t^p\right) - \sum_{t\in T}\sum_{p\in P}\left(\pi_t^p \cdot x_t^p + h_t^p\left(IC_t^p + IN_t^p\right) + s_t^p \cdot S_t^p + b_t^p \cdot B_t^p\right) \tag{A.8}$$

$$\boldsymbol{G}(\boldsymbol{Y},\boldsymbol{Z}) = \sum_{t\in T}\sum_{k=1}^{M} CPM\left(M_t^k\right) - CMR \cdot \sum_{t\in T}\sum_{k=1}^{M+1} NF_t^k + T \cdot \xi_0 + \xi_1 \cdot \sum_{t\in T}\tau_t \tag{A.9}$$

$$\boldsymbol{H}(\boldsymbol{X},\boldsymbol{Y},\boldsymbol{Z}) = \beta \cdot (M+1) \cdot T + \sum_{t\in T}\left(\sum_{p\in P}\frac{\delta^p \cdot x_t^p}{APT_t} \cdot \sum_{k=1}^{M+1} PS_t^k \cdot \left(y_t^k - w_t^k\right)\right) + x_t^p \frac{\alpha^p}{APT_t} \cdot \sum_{k=1}^{M+1}\int_{w_t^k}^{y_t^k}\left(y_t^k - \tau\right) \cdot f\left(t \middle| w_t^k\right) \cdot d\tau \cdot PS_t^k \cdot \left(PC_t^p - PN_t^p\right) \tag{A.10}$$

In $\boldsymbol{F}$, the first term is the total sale of conforming and nonconforming products with different prices, and the second term is the sum of processing cost, inventory holding costs for conforming and nonconforming products, setup cost, and the backorder cost. In $\boldsymbol{G}$, the terms indicate respectively to the preventive maintenance cost, corrective maintenance cost, and restoration cost and $\boldsymbol{H}$, is the sum of the costs of process inspection, product quality inspection (in case of detecting a deteriorated state to separate defective items) and the cost of the lost profit caused by the production imperfectness. Note that the maintenance cost function ($\boldsymbol{G}$) is independent from the production planning system with $\boldsymbol{X}$ as its decision variables).

A.3.2. Full and partial maintenance schedules

As explained in chapter 2, a complete PM schedule explicitly specifying the maintenance levels to be performed in each PM time, yields a complete solution of the whole problem (in this model, M the number of process inspections is a parameter). Given such a full plan, one can evaluate the interacting factors and nonlinear components of the problem and so, the model reduces to a mixed integer linear program (MILP) that can be solved using existing methods or optimization software. A sample PM schedule for a problem with T = 6 periods and M = 3 PM per period is illustrated in Fig. A.2. The machine has 4 PM levels (0, 1, 2, and 3).

Period	Period 1			Period 2			Period 3			Period 4			Period 5			Period 6		
PM intrusion	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
PM level	2	1	3	0	0	1	2	0	1	3	1	2	2	2	1	0	1	1

Fig. A.2: A sample PM Schedule.

The PM plan illustrated in Fig. A.2 is a vector of integers; that can be split into smaller vectors called as partial PM plans indicating the maintenance plan for each period, thence, 2-1-3 in Fig. A.1 is a partial plan (the maintenance schedule) in period 1.

Since, a restoration takes place at the end of each period, the machine is initially in its as-good-as-new conditions when each mission starts, therefore, the state of the machine in each period just depends on the maintenance plan in that period and it is independent from the previous decisions. With this property, the partial maintenance schedules are statistically and economically independent from each other. Each partial plan corresponds to a specific state of the machine, distinguished by available production time in the period and an expected nonconformity rate. A full PM plan is a vector of $M\times T$ (with $Q^{M\times T}$ possibilities) whereas the length of a partial plan is just M with $N = Q^M$ possibilities.

A.3.3. Quadratic formulation of the model

Let us assume $APT(j)$ is the available production time, subject to the j^{th} partial plan, so the available production time in period t will be:

$$APT_t = \sum_{j=1}^{N} y_{jt} \cdot APT(j) \tag{A.11}$$

Where y_{jt} is the new binary decision variable defined as:

$$y_{jt} = \begin{cases} 1 \text{ partial plan } j \text{ is assigne to period } t \\ 0 \qquad\qquad otherwise \end{cases} \tag{A.12}$$

Since only one partial plan should be assigned to each period, the new constraint holds:

$$\sum_{j=1}^{N} y_{jt} = 1 \tag{A.13}$$

In quadratic formulation using the new variable y_{jt} we first need to replace the factors related to the maintenance plan in $\boldsymbol{H}$ and the constraints as follows. Defining D_l; the expected duration of production time; $U_1(j) = \sum_{k=1}^{M+1} \frac{PS_t^k}{APT_t}\left(y_t^k - w_t^k\right)$ and the rate of shifted operational time under partial plan j; $U_2(j) = \frac{PS_t^k}{APT_t} \sum_{k=1}^{M+1} \int_{w_t^k}^{y_t^k} \left(y_t^k - \tau\right) \cdot f\left(t \middle| w_t^k\right) \cdot d\tau$ corresponding to the j^{th} partial plan, the quality cost $\boldsymbol{H}$ will be:

$$\boldsymbol{H}(\boldsymbol{X},\boldsymbol{Y},\boldsymbol{Z}) = \beta \cdot (M+1) \cdot T + \sum_{p\in P} \sum_{t\in T} \delta^p \cdot x_t^p \cdot \sum_{j=1}^{N} U_1(j) \cdot y_{jt} + x_t^p \cdot \alpha^p \cdot \left(PC_t^p - PN_t^p\right) \cdot \sum_{j=1}^{N} U_2(j) \cdot y_{jt} \tag{A.14}$$

Similarly, the maintenance cost function $\boldsymbol{G}$ would be:

$$\boldsymbol{G}(\boldsymbol{Y},\boldsymbol{Z}) = \sum_{j=1}^{N} y_{jt} \cdot G(j) \tag{A.15}$$

where, $G(j)$ is the maintenance cost function in a period subject to the j^{th} partial plan. Substitution of equations A.11, A.14, A.15 and the interacting factors $U_1(j)$ and $U_2(j)$ as well as the new constraint (A.13) in the original model of chapter 2, yields the quadratic formulation of the model as follows:

$$\text{Max} \quad \boldsymbol{F}(\boldsymbol{X}) - \sum_{j=1}^{N} y_{jt} \cdot G(j) - \beta \cdot (M+1) \cdot T - \sum_{p \in P} \sum_{t \in T} \delta^p \cdot x_t^p \cdot \sum_{j=1}^{N} U_1(j) \cdot y_{jt} - x_t^p \cdot \alpha^p \cdot \left(PC_t^p - PN_t^p\right) \cdot \sum \quad U_2(j) \cdot y_{jt} \qquad \text{(A.16)}$$

Subject to:

$$XN_t^p = x_t^p \cdot \alpha^p \sum_{j=1}^{N} U_1(j) \cdot y_{jt},\ p = 1, \ldots, P,\ t = 1, \ldots, T, \qquad \text{(A.17)}$$

$$IC_t^p = IC_{t-1}^p - SC_t^p + \left(x_t^p - XN_t^p\right),\ p = 1, \ldots, P,\ t = 1, \ldots, T, \qquad \text{(A.18)}$$

$$IN_t^p = IN_{t-1}^p - SN_t^p + XN_t^p,\ p = 1, \ldots, P,\ t = 1, \ldots, T, \qquad \text{(A.19)}$$

$$B_t^p = B_{t-1}^p + d_t^p - SC_t^p,\ p = 1, \ldots, P,\ t = 1, \ldots, T, \qquad \text{(A.20)}$$

$$x_t^p \leq g^p \cdot S_t^p,\ p = 1, \ldots, P,\ t = 1, \ldots, T, \qquad \text{(A.21)}$$

$$\sum_{p \in P} \frac{x_t^p}{g^p} \leq \sum_{j=1}^{N} y_{jt} * APT(j),\ t = 1, \ldots, T, \qquad \text{(A.22)}$$

$$\sum_{j=1}^{N} y_{jt} = 1 \qquad \text{(A.23)}$$

A.3.4. Mixed integer formulation of the main problem

The quadratic terms of the model in A.16 and A.17 are the multiplication of production levels (x_t^p) and the variable corresponding to the assignment of partial plans to the periods (y_{jt}). Since the latter is a binary variable, the term $x_t^p \cdot y_{jt}$ can be replaced with an equivalent decision variable as follows:

$$\Psi_{pjt} = x_t^p \cdot y_{jt} \qquad \text{(A.24)}$$

In this case, the following linear constraints should be added to the model:

$$\Psi_{pjt} \leq x_t^p \tag{A.25}$$

$$\Psi_{pjt} \leq g^p \cdot y_{jt} \tag{A.26}$$

In A.26, $g^p (g^p \leq x_t^p)$ is the processing rate of product p. These constraints will force $\Psi_{pjt} = 0$ if one of the right hand values is 0, considering a proper positive coefficient of Ψ_{pjt} in the objective function will force the variable to take its maximum value (i.e. $\Psi_{pjt} = x_t^p \cdot y_{jt}$), so with these modifications, the quadratic terms of the model can be replaced with Ψ_{pjt}. The model reduces to a mixed integer linear problem and the number of decision variables and the model constraints increase. As an example, the quadratic term $\sum_{p \in P} \sum_{t \in T} \delta^p \cdot x_t^p \cdot \sum_{j=1}^{N} U_1(j) \cdot y_{jt}$ will change to $\sum_{p \in P} \sum_{t \in T} \delta^p \sum_{j=1}^{N} U_1(j) \cdot \Psi_{pjt}$.

A.4. The solution methods

A.4.1. Solution method for MILP and QP formulations

In this appendix, we use Cplex package to optimize the MILP program. The quadratic formulation is more difficult and solving it requires certain conditions mainly the solution space needs to be convex. Moreover to the existing algorithms such as branch and bound and dynamic programming, the MILP formulation can be efficiently solved using several optimization packages. In this section, Cplex is used to evaluate and the MILP.

A.4.2. Nelder-Mead algorithm as a general solution method

Considering a full PM as a complete solution of the problem, a vector of size $M{\times}T$ with real values indicating the PM level for maintenance interferences, these vectors can be considered as the solution structure in a heuristic algorithm. A neighbor solution is a vector of the same length that only one of its components differs from the current solution. We limit the difference to 1, so for example, 2,1,3,0,0,1,2,0,1,3,1,2,2,2,1,**1**,1,1 and 2,1,3,0,0,1,2,0,1,3,1,2,2,**1**,1,0,1,1 are two neighbors of the sample solution presented in Fig. A.2.

Nelder-Mead algorithm (NM) introduced by Nelder and Mead (1965) is a search method to optimize functions whose derivatives are difficult to evaluate. The method is used in solving different problems, for example, Khojaste Sarakhsi et al. (2016) employed it to solve a lot-sizing problem. Fig. A.3 shows the algorithm flowchart. The objective in NM is to exploit the information from several neighbor solutions in transitions toward the local optima. In this method, we consider not only better solutions (positive knowledge), but also the worse solutions (negative knowledge) to introduce three candidate moves. Then, the best solution among them is considered as the algorithm transition. The three candidates are (1) The contracted point (X_C) that is the best neighbor, (2) The reflected point (X_R) that is the reflection of the given solution one step in improvement direction, and (3) The expanded point (X_E) which is the expansion of the given solution two steps in the improvement direction.

Given a solution p to be improved:

1. Repeat
   1.1. Generate the sub-population P (N neighbor points of p) and evaluate them
   1.2. Determine the improvement (reflection) vector D

   For each gene position $j \in \{1,..., L\}$

   For each solution $\hat{p} \in P$

   If $\hat{p}$ is better than p then $d_j = \hat{p}_j - p_j$ ($\hat{p}_j$, p_j, and d_j are respectively j^{th} elements of $\hat{p}$, p, and D)

   Else $d_j = p_j - \hat{p}_j$

   1.3. Determine the reflected point; X_R, the expanded point; X_E, and the contracted point; X_C and modify them if needed
      iv. $X_R = p + D$
      v. $X_E = p + 2 \times D$
      vi. X_C = best solution in P

   1.4. Move to the best solution among the candidates (p = max $\{X_R, X_E, X_C\}$)
2. Loop until the *Stop Condition* is satisfied (number of attempts that fail to improve the solution)

Fig. A.3: Nelder-Mead algorithm.

For example, suppose that the given solution p is {1, 1, 3, 0, 2} and the two neighbors are $\hat{p}_1$ = {1, 1, 4, 0, 2} and $\hat{p}_2$ = {1, 1, 3, 0, 1}, where $\hat{p}_1 > p$ and $\hat{p}_2 < p$. So, the reflection vector, reflected point, and expanded point are respectively D = {0, 0, 1, 0, 1}, $X_R = p + D$ = {1, 1, 4, 0, 3}, and $X_E = p + 2 \times D$ = {1, 1, 5, 0, 4}. In some cases, the generated solutions can be infeasible (because of value of genes), accordingly; these genes will be modified to the nearest possible values. As this example shows, both good and bad solutions participate in determination of the next solution. In contrast to tabu-search in which the poor solution are generally discarded, the

Nelder-Mead algorithm exploits the information from all evaluated points to determine the best move and also, it is not limited to move to one of the neighbor points.

A.5. Numerical example

Let us consider a sample problem with $T = 3$ periods and $P = 3$ products, where the machine has $Q = 4$ PM levels and $M = 3$ maintenances per period. The problem data are given in Tables A.1 and A.2. The search space for the original problem is about 2.6×10^5

Table A.1: Production data.

Product		Demand			Production cost			Backorder cost			Holding cost			Setup cost			Price Conforming		
		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Period	1	45	50	100	30	50	70	110	130	180	3	5.5	2.2	500	800	450	170	320	65
	2	30	40	150	26	47	74	110	130	170	2.5	6.1	2.5	550	780	420	150	300	70
	3	60	70	50	33	49	68	120	130	170	3.2	6.5	2.4	530	830	400	180	340	68

Table A.2: Maintenance data.

Cost of PM options	*CPM*	5000, 500, 200, 0	Time of minimal repair	*TMR*	0.02
Time of PM options	*TPM*	0.05, 0.003, 0.001, 0	Inspection cost	β	40
Production rates	g	450, 400, 350	Imperfectness factor	η	0,9
Nonconformity rates	α	0.7, 0.7, 0.7	Restoration parameters (constant and variable)	ξ_0	200
Cost of minimal repair	*CMR*	500		ξ_1	3000
Length of period	L	1			
Time-to-shift parameters	λ	40			
	φ	2.5			
Time-to-failure parameters	θ	20			
	ρ	2.5			

The first step in application of quadratic and MILP formulations is the evaluation of the costs and interacting factors related to the partial PM plans. Using the data of the sample problem, a part of the partial plans and their parameters (for the first five possibilities) are presented in Table A.3. Note that the sample problem has $Q^{MP} = 2.6 \times 10^5$ partial plan.

Table A.3: Maintenance cost and interacting factors of partial PM plans.

J	*Partial plan*	*F(j)*	*APT(j)*	$U_1(j)$	$U_2(j)$
1	0,0,0	16629	0.81	0.14	0.59
2	0,0,1	13203	0.83	0.26	0.68
3	0,0,2	12969	0.83	0.27	0.68
4	0,0,3	12810	0.83	0.27	0.68
5	0,1,0	13203	0.83	0.28	0.73

A.5.1. Solving the quadratic model

As an example, with introduced variable y_{jt}, the cost of the maintenance system in three periods is 16629 ($y_{11} + y_{12} + y_{13}$) + 13203 ($y_{21} + y_{22} + y_{23}$) + 12969 ($y_{31} + y_{32} + y_{33}$) + 12810 (y_{41} +

$y_{42} + y_{43}) + 13203\ (y_{51} + y_{52} + y_{53})$. The available production time in period t is $APT_t = 0.81y_{1t} + 0.83y_{2t} + 0.83y_{3t} + 0.83y4t + 0.83y_{5t}$, where $\sum_{j=1}^{5} y_{jt} = 1$.

Solving this problem with the existing packages such as Cplex is limited to the cases where the problem is convex. For the presented problem, the Cplex fails to evaluate the model with the error of nonconvex model (The matrix is not positive semi-definite).

A.5.1. Solving the mixed integer linear model

As explained previously, the quadratic model can be converted to a mixed integer linear program (MILP). The number of decision variables and constraints is MILP is larger than the quadratic model. Solving the sample problem with all the 64 partial plans takes about 1169 seconds and it finds the optimal solution.

The value of the penalty should be selected properly according to the other terms of the objective function and by testing several values. However, the solution time even for this small problem is very large and increasing the size of the problem results in inefficiency of the MILP method. The solution time of MILP as a function of the number of partial PM plans is presented in Fig. A.4.

A.5.1. Solving with the heuristic method

The Nelder-Mead algorithm is able to solve different instances of the problem and in most cases, the time to find the optimal solution is considerably short. Fig. A.5 shows the evolution of the best solution in a sample run of the Nelder-Mead algorithm. The average time to find the optimal solution in 30 replications was 41 Seconds. However, the optimality of the solutions found by NM algorithm are not guaranteed.

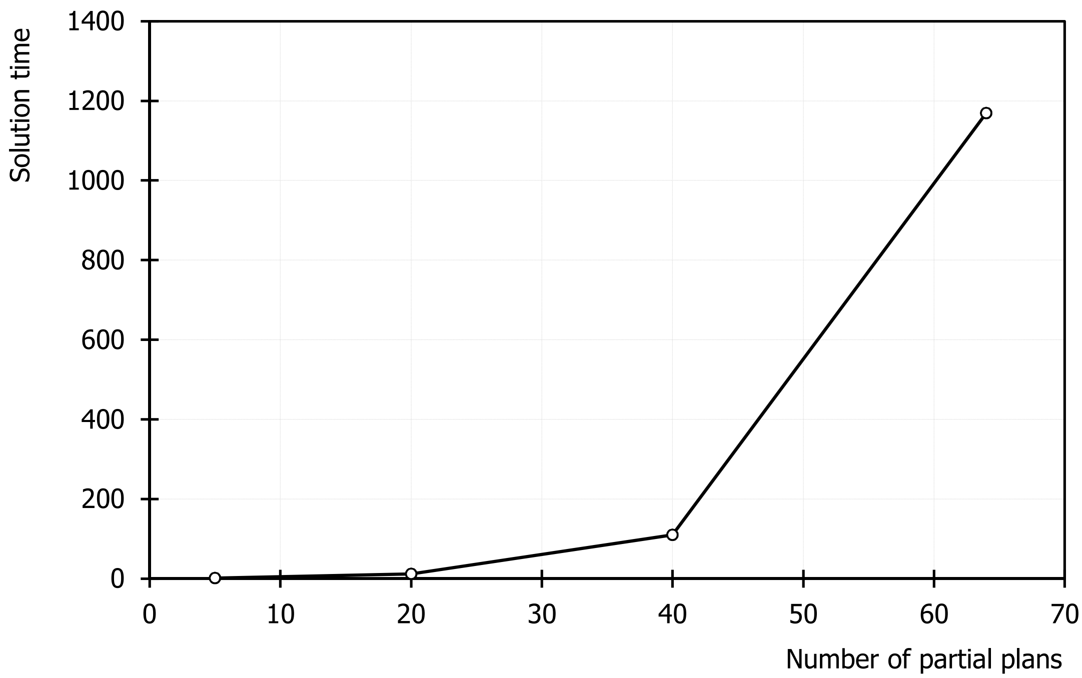

Fig. A.4: Solution time as a function of the number of partial PM plans.

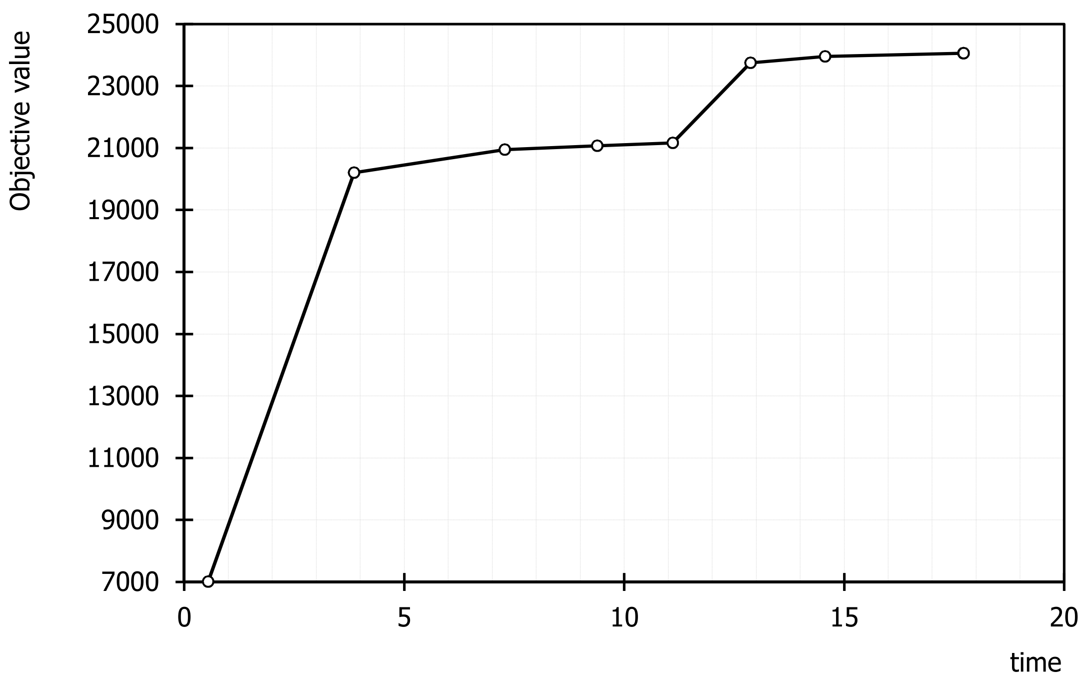

Fig. A.5: Evolution of the best solution in Nelder-Mead method.

A.6. Conclusions

Considering the interactions between maintenance scheduling, production planning, and decision variables related to the quality system, literature shows that their integration in a joint model can result in improvement of profit or reduction in the total system cost. But, such models are generally hard to solve in a reasonable effort. Since the system starts each period in its as-good-as-new conditions, the state of the machine in each period is independent from the maintenance tasks performed in the previous periods. Dividing a large PM plan to several partial schedules and evaluating the costs and interacting factors for each partial plan, the maintenance scheduling problem is reformulated as a quadratic and a mixed integer linear program. The generic form of the joint models is presented and a heuristic method based on Nelder-Mead algorithm is also proposed to solve the original problem. Since the objective and a constraint of the quadratic formulation are of the second order, most of the existing packages are not able to deal with the quadratic program, but, the mixed integer formulation can be evaluated using existing methods or optimization tools. The issue in MILP formulation is the huge size of the problem that exponentially increases with the model parameters and with the number of partial plans. Also, proper selection of the penalties is a challenging issue which influences on the final solution. Despite the problem size, the proposed Nelder-Mead method efficiently finds promising solutions (or the optimal solutions in most cases) in a shorter solution time.

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Appendix B

The results of applying the solution method on the problem of chapter III

This appendix illustrates the performance of the solution method proposed in chapter Four used in solving the cost minimization model of chapter Three. The algorithm is able to solve different instances of the problem with different sizes. Since the performance of the algorithm is similar to the results presented in chapter Four, and to avoid restating them, the results are summarized in this appendix.

B.1. The mathematical model

Notations

Indices and parameters

m, M	Index and number of machines
p, P	Index and number of products
t, T	Index and number of periods
j	Index of intervals in periods
b_p	Unit backorder cost
AC_m	Process adjustment cost
CMR_m	Cost of minimal repair
$CPM_m(k)$	Cost of k^{th} PM level
d_{pt}	Customer demand
g_{pm}	Production rate
h_p	Inventory holding cost
L	Fixed length of periods
PMB_t	Available PM budget
Q_m	Number of the preventive maintenance levels

R_p	Unit reworking cost
s_{pm}	Setup cost
TMR_m	Time of minimal repair
W_{m0}	Initial age of machine
α_{pm}	Defective rate in out-of-control state of the machine
β_p	Unit cost of quality check
θ_m, ρ_m	Parameters of Weibull distribution for time-to-failure function
λ_m, φ_m	Parameters of Weibull distribution for time-to-shift function
π_{pm}	Manufacturing cost
ν_m	Cost of process inspection
APT_{mt}	Available production time
$ESST_{mt}$	Expected duration of a shifted state
IC_{mt}	Process inspection cost
LS_{pt}	Lot size
NF_{mt}	Number of machine failures
PMC_t	Preventive maintenance cost
ps_{mjt}	Probability of shift in the j^{th} interval
PS_{mt}	Probability of shift in period
QC_{pmt}	Cost of quality checking
TAC_{mt}	Total process adjustment cost
TC_{MS}	Total maintenance system cost
TC_{PS}	Total production system cost
TC_{QS}	Total quality system cost
TRC_{mt}	Total reworking cost
W_{mt}	Age at the beginning of a period
y_{mjt}	Age at the end of an interval
Y_{mt}	Age at the end of a period

Decision variables……………………………………………………………………

B_{pt}	Backorder level
I_{pt}	Inventory level
S_{pmt}	Setup variable
x_{pmt}	Production level
NI_{mt}	Number of process inspections
PM_{mt}	Preventive maintenance level

B_{pt}, I_{pt}, S_{pmt} and x_{pmt} are the production planning variables, NI_{mt}concerns to the quality system, and PM_{mt} to the maintenance system.

Appendix B. The results of applying the solution method on the cost minimization problem

$$Min\, TC_{PS} + TC_{MS} + TC_{QS} \tag{B.1}$$

$$TC_{PS} = \sum_{t\in T}\left(\sum_{p\in P}\sum_{m\in M}\left(x_{pmt}\pi_{pm} + S_{pmt}s_{pm}\right) + \sum_{p\in P}\left(I_{pt}h_p + B_{pt}b_p\right)\right) \tag{B.2}$$

$$TC_{MS} = \sum_{m\in M}(PMC_t + MRC_t) \tag{B.3}$$

$$TC_{QS} = \sum_{t\in T}\sum_{m\in M}(IC_{mt} + QC_{mt} + TRC_{mt} + TAC_{mt}) \tag{B.4}$$

$$PMC_t = \sum_{m\in M} CPM(PM_{mt}) \tag{B.5}$$

$$MRC_t = \sum_{m\in M} NF_{mt} \cdot CMR_m \tag{B.6}$$

$$NF_{mt} = \theta_m \cdot ({Y_{mt}}^{\rho_m} - {W_{mt}}^{\rho_m}) \tag{B.7}$$

$$PMC_t \le PMB_t \,, \forall t \in T \tag{B.8}$$

$$IC_{mt} = NI_{mt} \cdot \nu_m \tag{B.9}$$

$$QC_{mt} = \left(1 - (1 - PS_{mt})^{1/NI_{mt}}\right) \cdot \sum_{p\in P} x_{pmt} \cdot \beta_p \tag{B.10}$$

$$TRC_{mt} = \frac{ESST_{mt}}{APT_{mt}} \cdot \sum_{p\in P} x_{pmt} \cdot \alpha_{pm} \cdot R_p \tag{B.11}$$

$$TAC_{mt} = NI_{mt} \cdot AC_m \cdot \left(1 - (1 - PS_{mt})^{1/NI_{mt}}\right) \tag{B.12}$$

$$APT_{mt} = L - NF_{mt}TMR_m \tag{B.13}$$

$$LS_{pt} = \sum_{m\in M} x_{pmt} \tag{B.14}$$

$$x_{pmt} \le S_{pmt}g_{pm} \tag{B.15}$$

$$I_{pt} - B_{pt} = I_{pt-1} - B_{pt-1} + LS_{pt} - d_{pt} \tag{B.16}$$

$$\sum_{p\in P} x_{pmt} \le APT_{mt}\, g_{pm} \tag{B.17}$$

$$s_m(t|t_0) = \frac{s(t)}{1-S(t_0)} = \lambda_m \cdot \varphi_m \cdot t^{\varphi_m - 1} \cdot e^{-\lambda_m(\cdot t^{\varphi_m} - \cdot {t_0}^{\varphi_m})}, t > t_0 \tag{B.18}$$

$$PS_{mt} = 1 - e^{-\lambda_m({Y_{mt}}^{\varphi_m} - {W_{mt}}^{\varphi_m})} \tag{B.19}$$

$$ps_{mjt} = 1 - (1 - PS_{mt})^{1/NI_{mt}} \tag{B.20}$$

$$ESST_{mt} = \left(1 - (1 - PS_{mt})^{1/NI_{mt}}\right) \cdot \int_{W_{mt}}^{Y_{mt}} (Y_{mt} - \tau) \cdot s_m(\tau | W_{mt}) d\tau \qquad \text{(B.21)}$$

$$W_{m,t+1} = \left(1 - \frac{CPM_m(PM_{mt})}{CPM_m(1)}\right) Y_{mt} \qquad \text{(B.22)}$$

$$Y_{mt} = W_{mt} + APT_{mt} \qquad \text{(B.23)}$$

$$y_{m,j,t}{}^{\varphi_m} - y_{m,j-1,t}{}^{\varphi_m} = c \qquad \text{(B.24)}$$

$$y_{m,j,t}{}^{\varphi_m} = W_{mt}{}^{\varphi_m} + j \cdot c \qquad \text{(B.25)}$$

$$y_{m,j,t}{}^{\varphi_m} = (j/NI_{mt}) \cdot Y_{m,t}{}^{\varphi_m} + (1 - j/NI_{mt}) \cdot W_{mt}{}^{\varphi_m} \qquad \text{(B.26)}$$

The objective function (B.1) represents the total cost of the system; the sum of production, maintenance and quality costs. Equations (B.2), (B.3) and (B.4) respectively indicate the costs of production system, maintenance system, and quality system. (B.5) is the cost of preventive maintenance, (B.6) is the cost of corrective maintenance, and (B.7) is the expected number of machine failures in periods. Constraint (B.8) corresponds to the maintenance budget limitation, (B.9) is the process inspection cost, (B.10) is the quality checking cost, (B.11) is related to reworking cost of defective items, and (B.12) evaluates the average machine adjustment cost after detecting quality shifts in the process mean. Equation (B.13) computes the machine availability after corrective maintenances, (B.14) evaluates the lot-sizes produce in each period, (B.15) establishes the link between production and setup variables, and (B.16) is the balance equation between variables and parameters of the lot-scheduling problem. Production limitations are defined by (B.17). Equations (B.18), (B.19), and (B.20) represent the time-to-shift function, the probability of a shift in a period, and the probability of a shift in interval *j* of period *t*. (B.21) is the expected time that machine *m* operates in out-of-control state in period *t*. The other equations (B.22)-(B.26) evaluate the age of machines in periods and intervals.

B.2. Test problems and size of the search space

The data of a problem; called *P1* with T = 6 periods, P = 2 products, M = 3 maintenance per period, and Q = 4 PM levels are presented in chapter III (Section 3.5). Moreover, a larger version *P2* (with T = 12 periods) is considered. *P2* is the replication of the data of *P1* (demand of period 7 is the same as period 1, etc.). A set of 50 random problems is also generated to compare the performance and robustness of the algorithm. These samples are created by uniform randomization, where $T \in (1\ldots12)$, (M, P, and Q) $\in (1\ldots5)$, other data between 0.5 to 1.5 times the average of data given for the first product, first machine, in the first period.

All the test problems and size of the search spaces are summarized in Table B.1.

Table B.1: Sample problems.

Problem	Description	Problem size
P1	The sample problem given in the chapter III (Section 3.5)	6.9×10^{10}
P2	Larger version of *P1* with 12 periods	4.7×10^{21}
P3	Set of 50 problems	$<8.7\times10^{41}$

B.3. Parameter settings for the joint scheduling problem

The values of the algorithm parameters are set using the meta-calibration method of section 4.3.11. With slight modifications, the parameter values; (λ, Φ, ζ, It_{NI}, r) related to the chromosome lengths ($T\times M$) are:

- (10, 3, 5, 10, 2) for $T\times M \leq 10$
- (15, 3, 5, 20, 2) for $10 < T\times M \leq 20$
- (20, 5, 10, 30, 2) for $20 < T\times M \leq 30$
- (40, 6, 25, 50, 2) for $30 < T\times M \leq 50$

The size of the tabu list in TS is set to the chromosome length, mutation probability in MA is $p_m = 0.05$, and if not indicated, the considered solution time is $5 \times T \times M \times Q$.

B.4. The best solution of the test problems with different algorithms

Problems *P1… P3* are solved with MAPM-NM, MAPM, and TS algorithms. The best and the average solution in 30 replications for each problem, the solution time, and the best chromosome (PM schedules) are presented in Table B.2. The plans found by the MAPM-NM for all the test problems are the best solutions. The methods developed in this section are able to solve various types of the JPMQ problems.

Table B.2: Solutions of the sample problems with different algorithms.

	Algorithm	Solution time	Max objective	Average objective	Min objective	Best PM plan
P1	MAPM-NM	360	434799	430718	**428502**	0,1,2,0,2,1,3,2,1,3,1,2,3,2,2 ,3,2,2
	MAPM-TS		436984	432513	**428502**	
	MAPM		486378	453888	432518	
	TS		442132	435914	431103	
P2	MAPM-NM	720	936129	920568	**900212**	0,1,2,0,2,0,3,1,1,2,0,2,3,1,2 ,3,2,3,1,2,2,1,3,1,3,3,1,3,1, 3,1,2,2,2,3,2
	MAPM-TS		957820	934289	916503	
	MAPM		1023466	993364	964520	
	TS		984536	942825	923710	

B.5. Robustness of the algorithm

Capability of the algorithm and the solution quality is also investigated in solving the set of 50 random problems (P5) using MAPM-NM, MAPM-TS, TS, and MA algorithms. For each problem, the gap to the best solution is calculated and the average gaps are shown in Table B.3.

Table B.3: Average gaps to the best solution in 50 random problems.

	MAPM-NM	MAPM-TS	TS	MA
Success rate	73.3%	26.7%	6.7%	20.0%

Accordingly, MAPM-NM yields the smallest gap among four algorithms and so, it can be conveniently used to solve the joint problem. Only in 8 out of 30 problems, the solution found by MAPM-NM was very close but not the best solution. By increasing the solution time, the gap decreases, but almost in all cases, this algorithm yields the better solutions.

The results of 30 replications of MAPM-NM, MAPM-TS, and TS for P1 with two different solution times are shown in Figures B.1 and B.2.

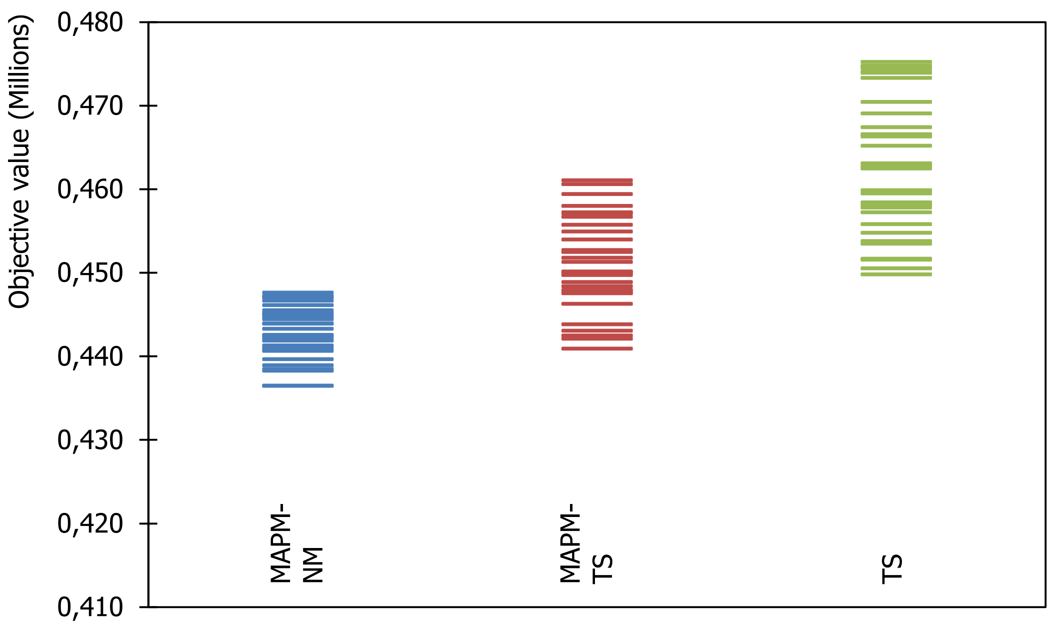

Fig. B.1: Results of 30 replications in 100 seconds.

In both cases, MAPM-NM outperforms MAPM-TS. The two variants of MAPM perform better than tabu-search algorithm and as expected, increasing the solution time has resulted in the improvement of the final solution (smaller total costs). The minimum, average, maximum and variance of the solutions are listed in Table 5.7. Similar results were obtained in different solution times and problems, therefore, we remark that the proposed algorithm is robust and, integration of genetic algorithm with a Nelder-Mead method has resulted in a better performance.

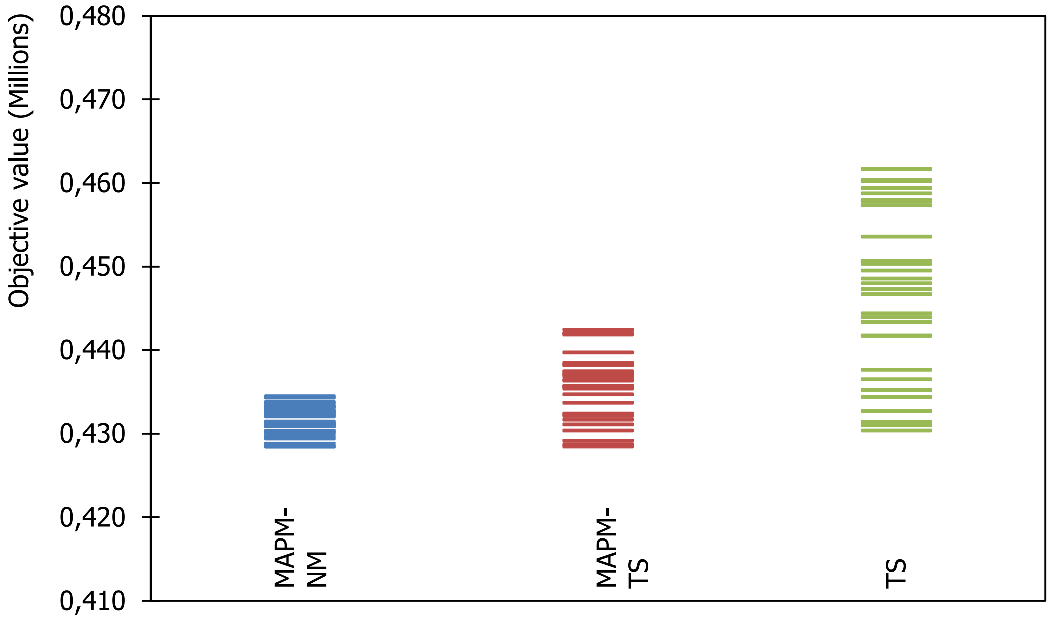

Fig. B.2: Results of 30 replications in 360 seconds.

Table B.4: Solution data in different execution times (30 replications; *P1*).

	Run time = 50 Sec.			Run time = 200 Sec.		
	MAPM-NM	MAPM-TS	TS	MAPM-NM	MAPM-TS	TS
Min	436480	440946	449824	428502	428502	430416
Max	447647	461061	475252	434518	442487	461695
Average	442881	451009	462878	431614	435668	446816
Average gap to the best solution	1.5%	3.3%	6.0%	0.7%	1.7%	4.3%

Standard dev.	3199	5635	8417	1679	3990	9916

The standard deviation values indicate the better performance of MAPM-NM, and small standard deviation compared to the average value shows the robustness of the algorithm. The average gap to the best solution in MAPM-NM is also smaller and in 5 out of 30 replications (in 50 Sec results); the solution of MAPM-NM was very close to but not the best solution.

B.6. Evolution of the best solution and effect of the population management

Figures B.3 and B.4 illustrates the evolution of the best solution in memetic algorithm with and without population management. Population diversity in MAPM smoothly reduces with the solution time, whereas in MA without population management, the heterogeneity is quickly lost (mutation probability is $p_m = 0.05$).

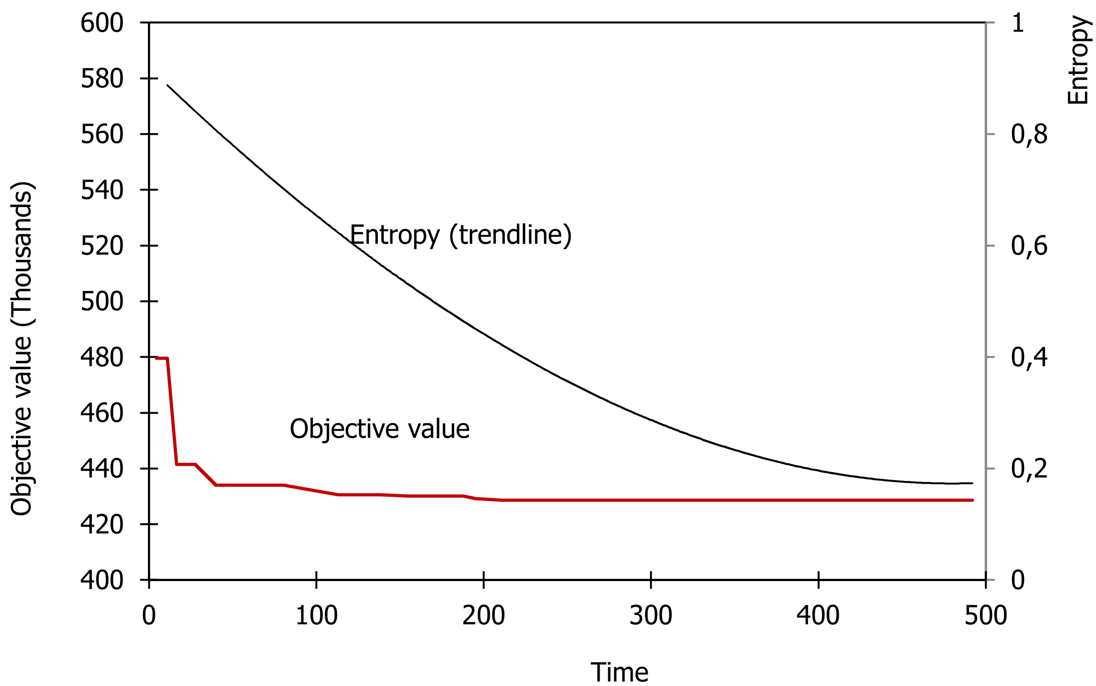

Fig. B.3: Evolution of the best solution and entropy variations in MAPM.

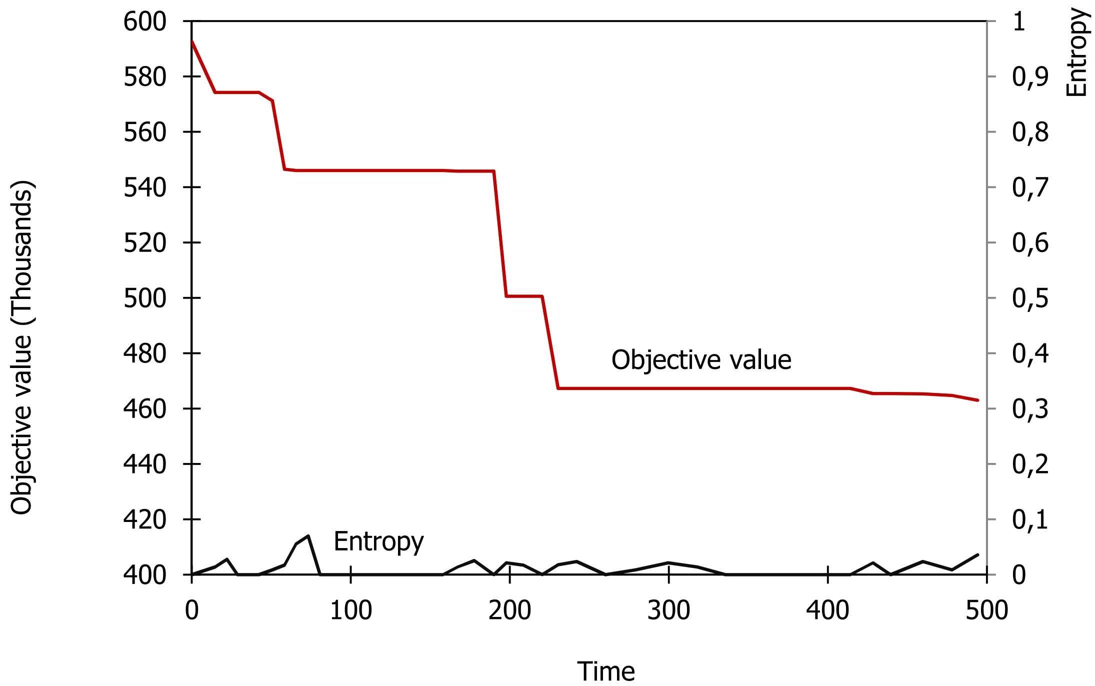

Fig. B.4: Evolution of the best solution and entropy variations in MA.

Using intensification and diversification strategies and maintaining the population diversity in MAPM has resulted in obtaining better solutions in each instant of time. In 30 replications of the two algorithms (for 50 Sec.), in 78% of cases, MAPM has outperformed MA, and a statistical analysis (F-test) indicates a significant difference between the means of the two algorithms (F = 28.3 compared to $F_{CRITICAL}$ = 3.9). This fact can be seen in the distribution of the final solutions illustrated in Fig. B.5.

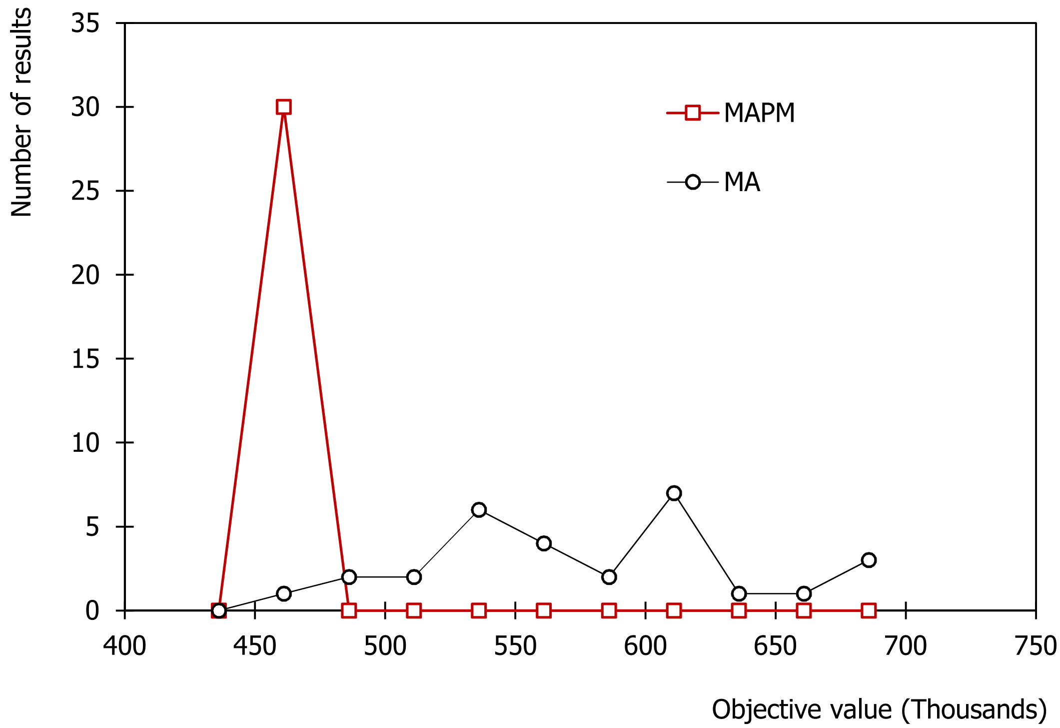

Fig. B.5: Distribution of solutions in 50 replications of MAPM and MA (problem *P1*).

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