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The JDTDOA algorithm applied to signal recovery: a performance analysis

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Abstract This article suggests a novel method to retrieve a narrowband signal sent in a multipath environment with a delay spread considering ISI between symbols. The proposed method does not require any preamble nor known signal. Using the joint direction and time delay of arrivals (JDTDOA) estimation algorithm developed in prior work, the directions and time delays of arrival in the multipath channel are jointly estimated and associated while keeping a low computational cost. In this process, a MVDR beamformed copy of each arriving signal is created. The quality of these "pseudo copies" is evaluated and compared to the original direct and reflected signals in this work. Another beamforming method, the Moore-Penrose pseudoinverse, with better retrieval of the direct and reflected signals is also proposed. Using a simple delay-and-sum operation on the previously beamformed copies, it is possible to substantially improve the the system's performance in terms of bit error rate. An approach using oversampling on the array antenna is introduced to improve performance. Numerical simulations are discussed to support theory.

Keywords DOA · TDOA · joint estimation · signal processing · performance analysis

1 Introduction

Environments featuring multipath propagation produce at the receiver a superposition of multiple delayed in time and faded copies of the same signal. When a multipath channel has a large delay spread, the incoming signals can be superposed in a destructive manner causing important fading. If these incoming signals are not despread, this superposition leads to poor bit error rate (BER) performance. To meet the increasing needs of our wireless networks, we must therefore develop methods to improve the performance in terms of BER and spectral efficiency that are adapted to multipath environments.

Since array antennas offer a significant gain and multiple beamforming techniques, their use in such condition is advantageous. For this reason, many have shown interest in smart antennas and their beamforming algorithms such as [3] or [14]. For these antennas, in order to constructively recover the incoming signals, an accurate and precise estimation of the channel parameters, direction of arrival (DOA) and time delay of arrival (TDOA), is necessary.

This idea is used as well in the space-time Rake receiver seen in [2] or [5]. However, some limitations appear in those receivers such as the need of a known sequence of symbols, a preamble, to retrieve the delay spread between different paths. This requirement can become problematic in mobile communications where the channel estimation must be done frequently and where the channel's bandwidth is a constraint.

Another method suitable for multipath environments is using diversity reception. With diversity reception, the different paths must be completely uncorrelated to achieve the optimal constructive recovery of the signal. This might not be attainable with line-of-sight (LOS) positioning. Moreover, in LOS multiple-input multiple-output (MIMO) systems, the orthogonality of the received signals must be achieved by a very large inter-element distance between the antennas [11].

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In this paper, we present a novel method to recover each incoming narrowband signal at an array antenna while simultaneously estimating multipath channel parameters. This method does not need any preamble and covers delay spread with interference intersymbol (ISI) between symbols. The channel parameters estimation is done using a recent developed algorithm called JDTDOA [4]. In this published work, we proposed an accurate and precise method to estimate and associate the time delay and direction of arrivals in multipath environments without the use of any preamble.

The JDTDOA algorithm uses the well known multiple signal classification (MUSIC) algorithm to estimate the DOAs and the Capon beamforming [12] along with cross-correlations to estimate the TDOAs. Channel estimation, in particular time delay and direction estimation of each arrival, is a flourishing research subject as seen in [6–8, 13].

The purpose of the prior paper was to measure and evaluate the precision of both time and direction estimations. The performance in terms of complexity was also discussed and it was said that the overall computational complexity of the JDTDOA algorithm was lower than 2D MUSIC.

The purpose of this current paper is however to use the actual beamformed signals from JDTDOA and the estimated parameters to retrieve a better copy of the source signal. The channel spread considered in this work is for intersymbol interference (ISI) between symbols, we therefore do not cover ISI within symbol. An alternative beamforming technique, using the Moore-Penrose pseudoinverse, is suggested instead of the Capon beamforming and both are compared. We also propose a simple delay-and-sum operation on the pseudo copies in order to increase the quality of the recovered signal, a principle also used in Rake receivers.

In the following sections, the system model and the modified algorithm are explained. Afterward, the quality of the pseudo copies and of the final recovered signal are discussed using bit error rate analysis. The use of oversampling on the array for an augmented performance is also introduced.

2 System Model

An array antenna composed of N identical and uniformly distributed elements is considered in this work. Each one of these N elements captures a linear combination of M incident signals. Since we treat propagation in multipath environments, the M signals are composed of one direct and $M-1$ reflected signals coming from a single user.

2.1 Multipath propagation

A narrowband signal coming from one path in a multipath environment can be expressed as:

$$s_m(t) = \alpha_m s_d(t - T_m) \ , \tag{1}$$

where α_m is the amplitude ratio between the m-th signal and the direct signal and T_m is the time delay of arrival respective to the m-th path. The signal $s_d(t)$ can be referred as the direct path, so that $\alpha_m = 1$ and $T_m = 0$ for $m = d$.

Each signal, direct and reflected, comes from a specific direction of arrival, noted θ_m. We consider the DOAs, θ_m, and the TDOAs, T_m, as independent parameters in this paper as it is usually done.

2.2 Phased array

Like illustrated in Figure 1, a path-length difference between adjacent elements is seen for a given DOA. Since the path-length difference is small with respect to the inverse of the signal bandwidth, the path-length difference can be reported as a phase shift:

$$\varphi_m = 2\pi \frac{f_c}{c} D \sin(\theta_m) \ , \tag{2}$$

where f_c is the carrier frequency, D is the inter-element spacing and the directions θ_m are measured such that the path-length difference is null for a zero value of θ_m. The following model is used to represent the captured signal:

$$\boldsymbol{X} = \boldsymbol{A}\boldsymbol{S} + \boldsymbol{N} \ . \tag{3}$$

The $[M \times K]$ matrix $\boldsymbol{S}$ contains the M arriving signals:

$$\boldsymbol{S} = \left[\boldsymbol{s_1} \ \boldsymbol{s_2} \ ... \ \boldsymbol{s_M}\right]^T \ , \tag{4}$$

where $\boldsymbol{s_m}$ is the m-th incident signal vector captured at the K sampling times. The matrix $\boldsymbol{N}$ is a $[N \times K]$ matrix containing the additive white Gaussian noise (AWGN):

$$\boldsymbol{N} = \left[\boldsymbol{n_1} \ \boldsymbol{n_2} \ ... \ \boldsymbol{n_N}\right]^T \ , \tag{5}$$

where $\boldsymbol{n_n}$ is the AWGN vector captured on the n-th sensor for all K sampling times. The matrix $\boldsymbol{A}$ is the $[N \times M]$ steering matrix which characterizes the phase shift related to each DOA:

$$\boldsymbol{A} = \left[\boldsymbol{a}(\theta_1) \ \boldsymbol{a}(\theta_2) \ ... \ \boldsymbol{a}(\theta_M)\right]^T \ , \tag{6}$$

where $\boldsymbol{a}(\theta_m)$ is the steering vector formed as:

$$\boldsymbol{a}(\theta_m) = \left[1 \ e^{-j\varphi_m} \ e^{-j2\varphi_m} \ ... \ e^{-j(N-1)\varphi_m}\right] \ . \tag{7}$$

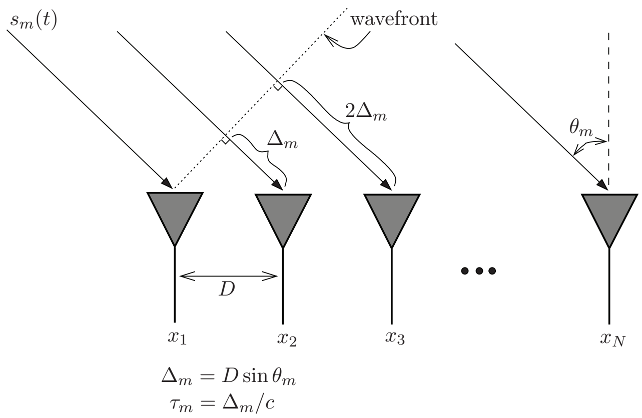

Fig. 1 Uniform array antenna with two impinging signals

The covariance matrix $\boldsymbol{R_{xx}}$ of the captured signal $\boldsymbol{X}$ is written as:

$$\boldsymbol{R_{xx}} = \boldsymbol{A}\boldsymbol{R_{ss}}\boldsymbol{A}^H + \boldsymbol{R_{nn}} \ . \tag{8}$$

If the M signals are considered independent, or uncorrelated, $\boldsymbol{R_{ss}}$ is a diagonal matrix where each element of the diagonal corresponds to the power of each arrival. In this paper, the M signals are considered uncorrelated since the time delays between each signals T_m are larger than the duration of the auto-correlation of the direct path. Therefore, only ISI between symbols is taken into account at this time. However, if some correlation exists between the signals, it would be possible to use spatial smoothing prior to the process. So ISI within symbol could be also considered eventually.

The noise on each element is considered independent from the $N-1$ other elements such that:

$$\boldsymbol{R_{nn}} = \sigma^2 \boldsymbol{I} \ , \tag{9}$$

where σ^2 is the power of the noise and $\boldsymbol{I}$ is the $[N \times N]$ identity matrix.

With a sufficient quantity of samples (K), this covariance matrix of the received signals can be approximated with:

$$\boldsymbol{R}_{xx} \approx \frac{1}{K}\boldsymbol{X}\boldsymbol{X}^H \ . \tag{10}$$

3 Proposed algorithm

Our goal is to recover the original source signal while keeping a reduced computational complexity. To do so, we first propose to estimate the space-time parameters of the incoming signals. For this matter, we use the JDTDOA (Joint Direction and Time Delay of Arrivals) algorithm recently developed [4]. In JDTDOA, the MVDR beamforming is used to retrieve a copy of the source signal. We propose to modify this algorithm by using another beamforming method: the Moore-Penrose pseudoinverse. With the output of the beamforming, we then perform a delay-and-sum operation to recover a more precise and accurate copy of the incoming signal.

3.1 JDTDOA Algorithm Modification

The estimation of the space-time parameters, direction and time delay, is based on the JDTDOA algorithm. This algorithm applies the conventional MUSIC algorithm to retrieve the directions of arrivals (DOAs). For a more straightforward approach, we use Root-MUSIC [1] to estimate the DOAs.

A beamforming is then performed on the phased array towards each DOA found so that an estimation of the upcoming signal from each direction is generated. The output is computed such that:

$$y_m(t) = \boldsymbol{w}^H(\theta_m)\boldsymbol{x}(t) \approx s_m(t) + n_m(t) \ , \tag{11}$$

where $\boldsymbol{w}(\theta_m)$ is the m-th DOA's beamforming weight vector. The noise $n_m(t)$ would have a variance that is N times smaller than the actual captured noise at each N elements, due to the intrinsic gain of the array antenna. These output signals from the beamforming are called pseudo copies.

In the original version of JDTDOA, a Capon beamforming [12], or maximum variance distortion response (MVDR), was proposed to retrieve the pseudo copies since it should maximize the signal-to-interference ratio (SIR).

However, this beamforming only considers one direction at a time and estimates the directions of interferences using the signal's covariance matrix.

Since the MUSIC algorithm already estimated every directions of arrival with high-resolution, a simpler step using the Moore-Penrose pseudoinverse is now proposed in this paper.

Using the M previously estimated DOAs, the steering matrix $\boldsymbol{A}$ can be approximated. The Moore-Penrose pseudoinverse of this estimation is then used to cancel the actual steering matrix. The weight matrix for this Moore-Penrose beamforming is simply :

$$\boldsymbol{W}^H = \hat{\boldsymbol{A}}^{+} = (\hat{\boldsymbol{A}}^H\hat{\boldsymbol{A}})^{-1}\hat{\boldsymbol{A}}^H \ , \tag{12}$$

where $\boldsymbol{W}$ contains each weight vector from each DOA such as : $\boldsymbol{W} = [\boldsymbol{w}(\theta_1)\,\boldsymbol{w}(\theta_2)\,...\boldsymbol{w}(\theta_M)]$. The approximated steering matrix $\hat{\boldsymbol{A}}$ is considered to be a full rank matrix when $M < N$, a condition also required by MUSIC.

With an accurate approximation of DOAs, the Moore-Penrose pseudoinverse beamforming will retrieve the source signal from a direction without any interference from the other $(M-1)$ directions. The pseudo copies without interference are then expressed by:

$$\boldsymbol{Y} = \boldsymbol{S} + \boldsymbol{W}^H \boldsymbol{N} \ . \tag{13}$$

Considering the MVDR weight vector, this perfect signal isolation can only be achieved if the covariance matrix of the sampled signal, $\boldsymbol{R_{xx}}$, is equal to $\boldsymbol{A R_{ss} A}^H$ and if $\boldsymbol{R_{ss}}$ is equal to $\sigma_s^2 \boldsymbol{I}$. In other words, there must be no noise and the source signals must be independent. For this particular case of MVDR beamforming and for the Moore-Penrose beamforming in general, the covariance matrix of the pseudo copies is then given by:

$$\boldsymbol{R_{yy}} = \boldsymbol{R_{ss}} + \sigma^2 \, \boldsymbol{W}^H \, \boldsymbol{W} \ . \tag{14}$$

Since $\boldsymbol{W}^H \, \boldsymbol{W} \neq \boldsymbol{I}$ for both Moore-Penrose and MVDR beamforming, the noise of each pseudo copy is never completely independent from the ones of the other $M-1$ pseudo copies.

It is possible to adapt the JDTDOA algorithm to correlated signals (ISI within symbols) by using a spatial smoothing technique such as the forward/backward one in [9] on the covariance matrix $\boldsymbol{R_{xx}}$. Spatial smoothing has been proved to be efficient for both MUSIC algorithm and Capon beamforming [10].

According to the JDTDOA algorithm, a cross-correlation is subsequently performed between the M pseudo copies. The time delays between each signals (TDOAs) are deduced by a maximization of the cross-correlations.

The performance of the JDTDOA estimation algorithm was discussed in the prior article [4]. It is very precise for both DOAs and TDOAs in case of ISI between symbols and very low SNR. Furthermore, the cross-correlation process can accurately associate each pseudo copy to its time delay.

As discussed, we replace in this current paper the Capon beamforming with a more simple method, the Moore-Penrose pseudoinverse. With MUSIC's high-resolution estimation of DOAs and this modified beamforming step, we recover higher quality pseudo copies while reducing the computational complexity.

3.2 Delay-and-sum Signal Recovery

Up to now, the modified JDTDOA algorithm was used to retrieve the pseudo copies $y_m(t)$, estimated DOAs $\hat{\theta}_m$ and TDOAs $\hat{T}_m$ of the M source signals. We recall that the beamformed pseudo copies are estimates of their respective source signal (11). Since the pseudo copies were accurately associated with their time delay, this additional step consists in delaying each pseudo copy using its respective TDOA and summing them together.

It is possible to enhance the quality of the recovered signal by using oversampling while collecting data: the oversampling increases the resolution of the delay process and therefore offers an improved synchronization between samples and symbols. The sampling rate is then set higher than the symbol rate. This procedure is frequently used in industry.

Considering independent noise and perfectly delayed pseudo copies ($\hat{T}_m = T_m$) without interference and combining (11), the recovered signal would offer a coherent gain compared to the direct signal with AWGN:

$$\hat{s}(t) \approx \sum_{m=1}^{M} (s_m(t+\hat{T}_m) + n_m(t+\hat{T}_m)) \ ,$$

$$\hat{s}(t) \approx \sum_{m=1}^{M} \alpha_m s_d(t) + \sqrt{M} n(t) \ . \tag{15}$$

The SNR gain observed between this recovered signal and the reference direct signal alone with AWGN would be:

$$G = 20 \log(\frac{\sum_{m=1}^{M} \alpha_m}{\sqrt{M}}) \ . \tag{16}$$

The power of the noise is constant for each sensor in the array and is equal to σ^2.

If the M incoming signals have identical amplitudes, the result will give the maximal coherent gain:

$$G = 10 \log(M) \ . \tag{17}$$

This coherent gain also considers that the amplitude of the sampled signal does not depend on the sampling time. In other words, the signals are not filtered. Given a filter with impulse response $h(t)$, the filtered, sampled and delayed signal $s_m(t+\hat{T}_m)$ would be described by the following discrete convolution :

$$s_m(t+\hat{T}_m) = \sum_{n=-\infty}^{\infty} h(t_{m_n}) s_{nf}(t - t_{m_n}) \ , \tag{18}$$

where t_{m_n} is equal to $(T_m \bmod \frac{T_s}{P} + nT_s)$ and s_{nf} is the non-filtered direct signal. The constant P is the oversampling factor defined as the ratio between the symbol period T_s and the sampling period.

Considering (11) and a raised cosine filter in particular, if the argument $(T_m \bmod \frac{T_s}{P})$ is kept close to zero, the pseudo copies tend to approach the reference direct signal with a different amplitude and AWGN in terms of quality. This condition is achieved when the TDOAs are multiples of $\frac{T_s}{P}$ and, consequently, can be approached by increasing the oversampling factor. In other words, the probability of error decreases with a greater oversampling factor.

4 Numerical examples

The modulation used for the following simulations is QPSK at a symbol rate of 1 MHz. The symbol period T_s is then 1.0 μs. The sampling rate is $P\times 1$ MHz so that P is the oversampling factor. The oversampling factor controls the resolution over the delay-and-sum process and this resolution corresponds to the sampling period which is T_s/P. A raised cosine filter with a roll-off factor of $r = 0.5$ is applied on the incoming signals.

The phased array is composed of $N = 8$ elements with an inter-element spacing of $D = \lambda/2$. The estimation of the covariance matrix is computed using $K = 400$ snapshots. Also, for simplicity, the four signals arrive at the phased array with the same signal-to-noise ratio:

$$\boldsymbol{SNR} = [snr_x,\ snr_x,\ snr_x,\ snr_x]\ .$$

The value of snr_x follows the x-axis of the BER figures. Since an array antenna of $N = 8$ elements with independent element noise is used here, a intrinsic gain of $10\log(8)$ dB is observed between the output of the array and its corresponding signal with AWGN.

Each one of the following bit error rate simulations is calculated until at least 100 errors are detected in the recovered signal.

(a) The purpose of this example is to evaluate the performance of the system in ideal conditions. Both beamformer, the initially proposed MVDR and the Moore-Penrose pseudoinverse, are tested. The simulated performances are then compared to theoretical expectations.

The $M = 4$ source signals are coming from directions:

$$\boldsymbol{\theta} = [-60,\ -20,\ 20,\ 60]^\circ\ .$$

Their respective time delays are:

$$\boldsymbol{T} = [8,\ 0,\ 4,\ 12]\ \mu\text{s}\ .$$

Thus, the direct signal is $s_2(t)$ coming from direction $\theta_2 = -20°$. Because every $T_m > 1$ μs, these signals show ISI between symbols. No oversampling is used, so that $P = 1$.

The DOAs are calculated using Root-MUSIC and the cross-correlation technique is used to compute the TDOAs. Since an estimation of time delay is done between each pseudo copy $y_m(t)$, a mean of $M = 4$ cross-correlation estimations is computed for each TDOA.

In figure 2, the radiation pattern created by a MVDR beamforming towards each DOA is compared to the one using the Moore-Penrose pseudoinverse. For both techniques, the main lobe associated with the m-th weight vector is in the direction of arrival of the m-th incoming signal. As this figure shows, the gain of $\boldsymbol{w}^H\boldsymbol{a}(\hat{\theta}_m)$ in the estimated desired direction $\hat{\theta}_m$ is always a unity gain. In the $(M-1)$ other directions, the radiation pattern using Moore-Penrose goes to zero. For MVDR however, the radiation pattern does not reach zero, so that interference is not completely eliminated.

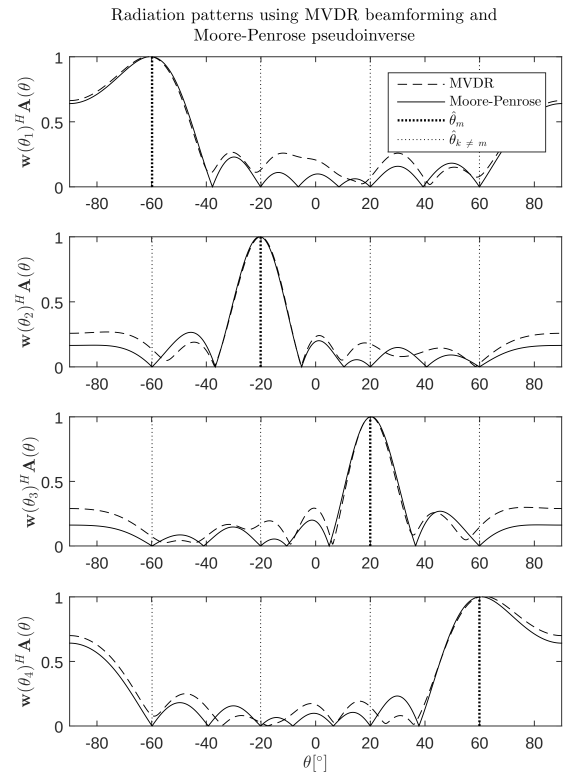

Fig. 2 Radiation patterns of the MVDR beamforming and of the Moore-Penrose pseudoinverse for each estimated DOA and for a SNR of −5 dB.

These weight vectors are then used to compute the pseudo copies of the M incoming signals. The pseudo copies are delayed using these calculated TDOAs with a resolution of 1.0 μs, since no oversampling is used. In figure 3, the delayed pseudo copies (created with Moore-Penrose pseudoinverse) were summed in order to obtain the recovered signal $\hat{s}(t)$. Only the real part of the QPSK signal, which corresponds to the sent bits $I(t)$, is shown (the imaginary part gives similar results). As this figure shows, the delay-and-sum operation allows an amplification of the original signal while reducing the quantity of errors seen on the pseudo copies.

Afterward, in figures 4 and 5, the following signals are compared to the initial sent bits:

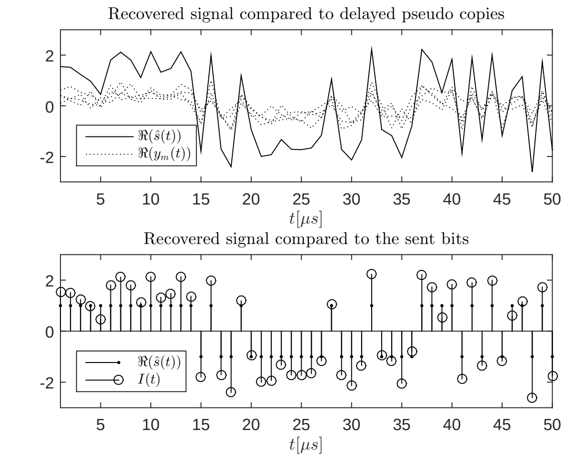

Fig. 3 Delayed pseudo copies $y_m(t+T_m)$, recovered signal $\hat{s}(t)$ and real part of the sent bits $I(t)$ for a SNR of -5 dB.

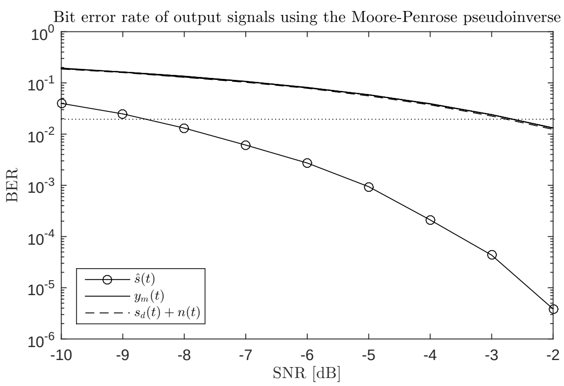

Fig. 4 Bit error rate of the delayed pseudo copies $y_m(t+\hat{T}_m)$ compared to the direct signal with adjusted noise $s_d(t)+n(t)$ and to the recovered signal $\hat{s}(t)$. The pseudo copies were beamformed using the Moore-Penrose pseudoinverse. The exact TDOAs are stated in (19).

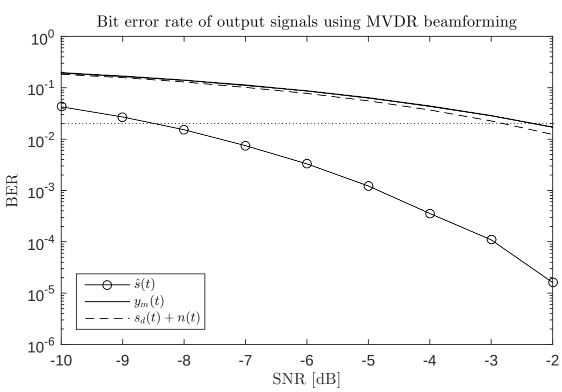

Fig. 5 Bit error rate of the delayed pseudo copies $y_m(t+\hat{T}_m)$ compared to the direct signal with adjusted noise $s_d(t)+n(t)$ and to the recovered signal $\hat{s}(t)$. The pseudo copies were beamformed using the MVDR beamforming. The exact TDOAs are stated in (19).

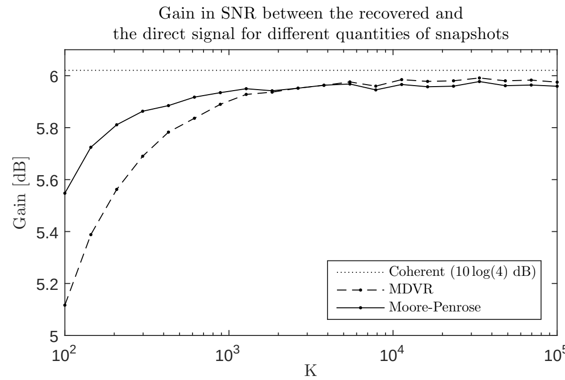

Fig. 6 Gain between the direct signal with adjusted noise $s_d(t)+n(t)$ and the recovered signal $\hat{s}(t)$ evaluated at $BER = 2\times10^{-2}$ for both MVDR and Moore-Penrose beamforming. These gains are compared to the coherent gain ($10\log(M)$ for $M=4$ pseudo copies). The number of snapshots K is increased until both gains are stable.

– The $M = 4$ pseudo copies $y_m(t)$ constructed with the specified beamforming method. The graph of the four pseudo copies are perfectly superposed since the DOAs are relatively far from each other and the TDOAs are multiples of the sampling period $T_s/P = 1.0$ μs.
– The direct source signal with adjusted AWGN ($s_d(t)+n(t)$). The variance of this noise was decreased by a factor $1/N$ where $N = 8$ elements in order to consider the intrinsic gain of the array.
– The recovered signal $\hat{s}(t)$ constructed with the delay-and-sum operation. According to (17), a gain approaching the theoretical coherent gain $10\log(4) = 6.02$ dB should be seen between this signal and the direct source signal with adjusted AWGN.

For the Moore-Penrose method, figure 4 shows that the BER curves of the pseudo copies approach the signal with AWGN as predicted. A difference of 0.11 dB is observed between the pseudo copies and the direct signal alone plus noise. This difference is explained by the dependency between the beamformed noises. A gain of 5.86 dB ($BER = 2\times10^{-2}$) is seen between the recovered signal $\hat{s}(t)$ and the direct signal with AWGN. This gain is close to the theoretical coherent gain.

For the MVDR beamforming, figure 5 illustrates that the BER curves of the pseudo copies diverge from the signal with AWGN with increasing SNR. This phenomena is mainly due to the interferences that are not completely eliminated. These interferences increase with the amplitude of the signals which is dictated by the SNR. Still, a gain of 5.75 dB ($BER = 2\times10^{-2}$) is seen between the recovered signal $\hat{s}(t)$ and the direct signal with AWGN.

The BER curves of both MVDR and Moore-Penrose were computed for different values of K, the number of snapshots. For each curve, the gain in terms of SNR between the recovered signal and the direct signal with adjusted AWGN was evaluated at $BER = 2 \times 10^{-2}$. Figure 6 shows these gains approaching the coherent gain of 6.02 dB while the number of snapshots is increased, as expected. Since the covariance matrix approximation is more precise with a large quantity of snapshots, the DOAs are also more precisely estimated for both beamforming techniques. Moreover, with a large quantity of snapshots, the radiation pattern of the MVDR beamforming approaches the one of Moore-Penrose, which leads to a more efficient interferences' elimination. Thus, while the Moore-Penrose is better for smaller values of K, the MVDR beamforming is equivalent to the Moore-Penrose in terms of gain for large values such as $K \geq 2000$ snapshots. However, as predicted in (14), the MVDR and Moore-Penrose techniques do not reach the coherent gain because the pseudo copies' beamformed noises are not completely independent.

(b) In this example, we focus on showing the benefit of oversampling. For this purpose, we keep the same parameters as the previous example but the fourth source signal's TDOA.
Oversampling does not affect the computational complexity of the JDTDOA algorithm, since the number of snapshots K is kept constant.
The beamforming technique used is the proposed Moore-Penrose pseudoinverse, because it gives results that are closer to the maximal gain.
Since a raised cosine filter is applied on the signals along with a limited resolution in the delay step, the sampled value of a pseudo copy is dictated by the amplitude of the raised cosine's impulse response evaluated at each TDOA modulo T_s/P as described in equation (18).
In order to show the maximum gain oversampling can provide, the fourth TDOA was fixed so that $(T_m \bmod \frac{T_s}{P}) = T_s/(2P)$, which corresponds to the worst situation in terms of probability of error for a chosen oversampling factor P since the value of $(T_m \bmod \frac{T_s}{P})$ is maximal.
In figure 7, the bit error rate of the fourth pseudo copy is computed for different SNR. The oversampling factor is first set to $P = 1$. For this oversampling factor, the fourth TDOA is 12.5 μs, which correspond to the worst case possible considering a sampling period of 1.0 μs. For $P = 2$, the fourth TDOA is set to 12.25 μs and for $P = 4$, the fourth TDOA is fixed at 12.125 μs.

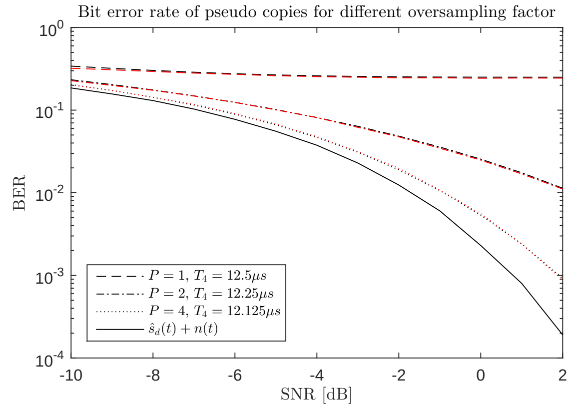

Fig. 7 Bit error rate of the fourth pseudo copy with T_m so that $(T_m \bmod \frac{T_s}{P}) = 1/(2P)$ for different oversampling factor. Their respective theoretical BER curves are illustrated in red. The pseudo copies were beamformed using the Moore-Penrose pseudoinverse.

Figure 7 illustrates that the SNR loss between the perfect direct signal plus noise and the fourth pseudo copy decreases as the oversampling factor increases: while a loss of 3.34 dB is seen for $P = 2$, a smaller loss of 0.76 dB is observed for $P = 4$ ($BER = 2 \times 10^{-2}$).
Moreover, it is very clear that the situation figuring a TDOA at $(T_m \bmod \frac{T_s}{P}) = 0.5$ and $P = 1$ is critical, since the m-th pseudo copy does not contribute to the delay-and-sum gain. For such situation, the overall gain between the summed recovered signal and the direct signal plus noise is $10\log(M-1)$ rather than $10\log(M)$ dB, as this reflection has no effect on integration gain.
It is possible to predict theoretically the probability of error of the pseudo copies by estimating their density probability function :

$$f_{y_m}(x) \approx f_{s_m}(x) * f_{n_m}(x), \tag{19}$$

where $f_{n_m}(x)$ is a normal distribution with zero mean and $\frac{\sigma_n^2}{N}$ variance and $*$ stands for convolution. With (18), the density probability $f_{s_m}(x)$ is found by a normalized histogram of the expected values of the delayed, sampled and raised cosine filtered signal $s_m(t + \hat{T}_m)$.
Since every symbol in QPSK is equiprobable, only the real part of this histogram was computed.
The probability of error obtained by the estimated density probability function follows the bit error rate curves simulated previously. Both theoretical and Monte-Carlo simulations BER curves are superposed in figure 7.

5 Conclusion

We have proposed an efficient method to retrieve a source signal sent in a multipath channel with delay spread considering ISI between symbols. This method does not require any preamble. In order to retrieve each reflected and direct signal, we first estimate the space-time channel parameters using the joint estimation algorithm proposed in [4]. The directions of arrival (DOA) are estimated by the high-resolution MUSIC algorithm which uses projection over a reduced orthogonal space. Using the estimated DOAs, a beamformed copy of each arriving signal is created. The time delays of arrival (TDOA) are estimated using a cross-correlation between these beamformed copies.

An analysis concerning the precision of these estimations and the computational complexity of the original JDTDOA algorithm with MVDR beamforming was previously developed in [4]. In the present paper, we suggest to use another beamforming method, the Moore-Penrose pseudoinverse, to recover a copy of each arriving signal. This method, while offering a lower computational complexity, is more accurate.

The beamformed copies, with both methods, are compared to the original direct signal and it is said that, while their quality is lower in terms of BER, the beamformed copies are a good estimation of the original direct signal. Afterward, the beamformed copies are delayed (or despread) by the estimated time delay to synchronize them. The BER curves of the delayed-and-summed beamformed copies are computed. The proposed system is almost as efficient as the coherent combination of the M signals (direct plus reflected ones) would be with a gain up to $10\log(M)$ when all signal powers are equal. This coherent gain is approached with increasing SNR since the beamformed noises are not completely independent.

The effect of oversampling on the array antenna is also introduced in theory and by a BER comparison. Oversampling is then used to improve the synchronization between the samples and the symbols. By using the same quantity of snapshots at the array but a greater oversampling factor, it is possible to significantly decrease the probability of error.

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