MASOUMEH KAZEMI ZANJANI

MULTI-PERIOD, MULTI-PRODUCT PRODUCTION PLANNING IN AN UNCERTAIN MANUFACTURING ENVIRONMENT

Thèse présentée
à la Faculté des études supérieures de l'Université Laval
dans le cadre du programme de doctorat en génie mécanique
pour l'obtention du grade de Philosophiae Doctor (Ph.D.)

DÉPARTEMENT DE GÉNIE MÉCANIQUE
FACULTÉ DES SCIENCES ET DE GÉNIE
UNIVERSITÉ LAVAL
QUÉBEC

2009

© Masoumeh Kazemi Zanjani, 2009

Résumé

Les travaux de cette thèse portent sur la planification de la production multi-produits, multi-périodes avec des incertitudes de la qualité de la matière première et de la demande. Un modèle de programmation stochastique à deux étapes avec recours est tout d'abord proposé pour la prise en compte de la non-homogénéité de la matière première, et par conséquent, de l'aspect aléatoire des rendements de processus. Ces derniers sont modélisés sous forme de scénarios décrits par une distribution de probabilité stationnaire. La méthodologie adoptée est basée sur la méthode d'approximation par moyenne d'échantillonnage. L'approche est appliquée pour planifier la production dans une unité de sciage de bois et le modèle stochastique est validé par simulation de Monte Carlo. Les résultats numériques obtenus dans le cas d'une scierie de capacité moyenne montrent la viabilité de notre modèle stochastique, en comparaison au modèle équivalent déterministe. Ensuite, pour répondre aux préoccupations du preneur de décision en matière de robustesse, nous proposons deux modèles d'optimisation robuste utilisant chacun une mesure de variabilité du niveau de service différente. Un cadre de décision est développé pour choisir parmi les deux modèles d'optimisation robuste, en tenant compte du niveau du risque jugé acceptable quand à la variabilité du niveau de service. La supériorité de l'approche d'optimisation robuste, par rapport à la programmation stochastique, est confirmée dans le cas d'une usine de sciage de bois. Finalement, nous proposons un modèle de programmation stochastique qui tient compte à la fois du caractère aléatoire de la demande et du rendement. L'incertitude de la demande est modélisée par un processus stochastique dynamique qui est représenté par un arbre de scénarios. Des scénarios de rendement sont ensuite intégrés dans chaque nœud de l'arbre de scénarios de la demande, constituant ainsi un arbre hybride de scénarios. Nous proposons un modèle de programmation stochastique multi-étapes qui utilise un recours complet pour les scénarios de la demande et un recours simple pour les scénarios du rendement. Ce modèle est également appliqué au cas industriel d'une scierie et les résultats numériques obtenus montrent la supériorité du modèle stochastique multi- étapes, en comparaison avec le modèle équivalent déterministe et le modèle stochastique à deux étapes.

Abstract

In this thesis, we study a multi-product, multi-period (MPMP) production planning problem with uncertainty in the quality of raw materials and consequently in processes yields, as well as uncertainty in products demands. We first consider the randomness of processes yields. A two-stage stochastic program with recourse is proposed to address the MPMP production planning problem with random yield. The random yields are modeled as scenarios with stationary probability distributions. The solution methodology is based on the sample average approximation scheme. The proposed two-stage stochastic model is applied as a novel approach for sawmill production planning. The stochastic sawmill production planning model is validated through Monte Carlo simulation. The computational results for a medium capacity sawmill highlight the significance of using the stochastic model as a viable tool for production planning instead of the deterministic model. Next, we study the robustness of customer service level in the MPMP production planning problem with random yield. We propose two robust optimization (RO) models with different service level variability measures. A decision framework is provided to select among the two RO models based on the decision maker's risk aversion level about the variability of service level for different yield scenarios. The superiority of robust optimization approach in generating more robust production plans, compared with stochastic programming, is confirmed through implementation of the proposed RO models for a realistic scale sawmill. Finally, we study the random demand as another uncertain parameter, in addition to the random yield. Demand uncertainty is modeled as a dynamic stochastic data process during the planning horizon which is presented as a scenario tree. Yield scenarios are then integrated in each node of the demand scenario tree, constituting a hybrid scenario tree. Based on the hybrid scenario tree for the uncertain yield and demand, a multi-stage stochastic programming model is proposed which is full recourse for demand scenarios and simple recourse for yield scenarios. We conduct a case study with respect to a realistic-scale sawmill. Numerical results indicate that the solution to the multi-stage model is far superior to the solution to the mean-value deterministic and the two-stage stochastic models.

Preface

This thesis has been realized under the co-direction of Professor Mustapha Nourelfath and Professor Daoud Ait-Kadi, both professors at the Mechanical Engineering Department of Université Laval. It has been prepared as an article insertion thesis. The activities related to this thesis have been carried out at the Forest E-business Research Consortium (FOR@C) of Université Laval.

The thesis includes three articles, co-authored by Pr. Mustapha Nourelfath and Pr. Daoud Ait-Kadi. In all of the presented articles, I have acted as the principal researcher. As the first author, I have also performed the mathematical models development, coding the algorithms, analysis and validation of the results, as well as writing of the first drafts of the articles. Professors Mustapha Nourelfath and Daoud Ait-Kadi have revised the articles to obtain the final version.

The first article entitled: "A stochastic programming approach for production planning with uncertainty in the quality of raw materials: A case in sawmills", co-authored by Pr. Mustapha Nourelfath and Pr. Daoud Ait-Kadi, is submitted to "*Journal of Operations Research Society*" on March 2009.

The second article entitled: "Robust production planning in a manufacturing environment with random yield: A case in sawmill production planning", co-authored by Pr. Daoud Ait-Kadi and Pr. Mustapha Nourelfath, is accepted (article in press) by the "*European Journal of Operational Research*" on February 2009 (doi: 10.1016/j.ejor.2009.03.041).

The third article entitled: "A multi-stage stochastic programming approach for production planning with uncertainty in the quality of raw materials and demand", co-authored by Pr. Mustapha Nourelfath and Pr. Daoud Ait-Kadi, is accepted by the "*International Journal of Production Research*" on February 2009.

To my beloved parents and my dear husband,
Mohammad

Acknowledgements

First, I would like to express my sincere gratitude to my advisor, Professor Mustapha Nourelfath and my co-advisor, Professor Daoud Ait-Kadi, for their endless encouragement, guidance and support. Their insight and knowledge and their valuable and pertinent comments guided me continuously through my Ph.D. journey and taught me how to conduct high quality research. I also truly appreciate their collaboration to arrange the scientific meetings, despite their busy schedules.

I am grateful to Professor Sophie D'Amours, Professor Michel Gendreau, and Professor Robert Pellerin for serving on my thesis committee and for their constructive comments on the final thesis.

This thesis has been realized by the financial support and the productive collaboration of research professionals of the Forest E-business Research Consortium (FOR@C), Université Laval. I would like to acknowledge and thank Professor Sophie D'Amours, the director of FOR@C, and all FOR@C research professionals for their technical assistance, particularly Jonathan Gaudeault, Philippe Marier and Sébastien Lemieux.

My deep love and appreciation goes to my parents for their support, kindness and patience in every stage of my life. They always tried to provide the best opportunities for my education. A especial thank to my dear husband, Mohammad, whose gentle and kind character was an endless source of support and encouragement and his scientific experience was an important source of inspiration for me. Finally, I would like to thank my sisters and friends for all their support and encouragement.

Table of Contents

List of tables

List of figures

Chapter 1

General introduction and literature review

1.1. Introduction

Production planning, i.e., deciding the optimal production level of products by different processes in order to maximize customer demand fulfillment in the presence of limited resources, plays an important role in operations management in manufacturing. In the real world, there are many forms of uncertainty, presented by random events, that affects production processes. Ho (1989) categorizes them into two groups: 1) environmental uncertainty and 2) system uncertainty. Environmental uncertainty includes uncertainties beyond the production process, such as raw material quality variations and demand fluctuations. System uncertainty is related to uncertainties within the production process, such as production lead time uncertainty and random machines failures. Uncertainties make the outputs of the production process be different from the planned production quantities. As a consequent, the realized inventory or backorder sizes would be different from the planned quantities.

Customer orientation is the center of attention in many manufacturing environments, especially those who are active in international markets. Promising a high customer service level that maintains robust in the presence of uncertainties is a crucial component of competitiveness in such markets. Due to the potential significance of uncertain parameters on the accuracy and robustness of production plans, appropriate mathematical programming models that accounts for the uncertainty are required. Such production planning models should be designed to allow the decision maker to adopt a production plan that can respond to uncertain events as they unfold during the time. Moreover, the proper types of adjustments (recourse actions) that are available to the decision maker as a response to different scenarios of the uncertain parameters should also be defined in these models.

In this chapter, we first describe the problem we are addressing in this thesis. As sawmill production planning is considered as a case study, a brief description of sawmills processes and characteristics is also presented. A comprehensive review of literature on the existing approaches to address the production planning models with uncertain parameters, optimization models with random parameters, as well as the existing approaches for sawmill production planning are provided in the following sections. The outline of the thesis is given at the end of this chapter.

1.2. Problem description

To motivate our research, consider a multi-period, multi-product (MPMP) production planning problem in a manufacturing environment with the following characteristics. A mix of products are produced simultaneously (co-production) through alternative processes, from several classes of raw materials. Raw materials classification is performed based on different characteristic attributes. Moreover, the raw materials in each class own non-homogeneous and variable characteristics (as in the case where the raw materials are from natural resources). As a consequent, the quantity of products that can be produced by each process (process yield) becomes a random variable. Furthermore, the product demand during the planning horizon is also characterized as a random non-stationary event. In this production planning problem, in order to fulfill the products demands, we are looking for the number of times each process should be run in each period, as well as the quantity of raw materials in each class that should be consumed by each process. The part of the demand that cannot be fulfilled on time will be postponed to the following period(s) by considering a unit backorder cost. The objective is to minimize the raw material consumption cost, as well as products inventory and backorder costs, subject to machine capacities and materials inventory.

The problem we are addressing can be considered as the combination of several classical production planning problems in the literature. Product mix problem and a special case of process selection problem (simultaneous production of multiple products by a single activity) (see Johnson and Montgomery (1974); Sipper et al. (1997)) are its two main building blocks. Linear programming (LP) models are the traditional tools for addressing such production planning problems. The random processes yields and products demands can be represented as the random coefficients of the constraints matrix and the random right-hand-side vector in such LP models, respectively. As the LP models include the assumption of deterministic parameters, they are not appropriate to generate robust production plans in the presence of random parameters. In other words, if the production plan is determined by a LP model where the expected values of random yield and demand a risk that the demand is not fulfilled with the right products and the backorder size is increased. As a consequent, customer service level, which is defined as the proportion of demand that can be fulfilled on time, will be decreased. Moreover, in service sensitive manufacturing environments, where the service level promised to the customer should be maintained as much as possible, determining a robust production plan with minimum backorder size (service level) variability in the presence of random yield and demand is also very crucial. Finally, in a planning problem, where the uncertain parameters have a non-stationary behavior during the time (e.g. random demand) the decisions should be conform to the availability of random data during the time.

One of the applications of the problem we are addressing is sawmill production planning, while considering non-homogeneous characteristics of logs and uncertain lumber demand. As sawmill production planning is considered as the case study in this thesis, a brief description of sawmills processes and characteristics is presented in the following.

Case study: Sawmill production planning

There are a number of processes that occur at a sawmill: log sorting, sawing, drying, planing and grading (finishing). Raw materials in sawmills are the logs which are transported from different districts of forest after bucking the felled trees. The finished and graded lumbers (products) are then transported to the domestic and international markets. Figure 1.1 illustrates the typical processes.

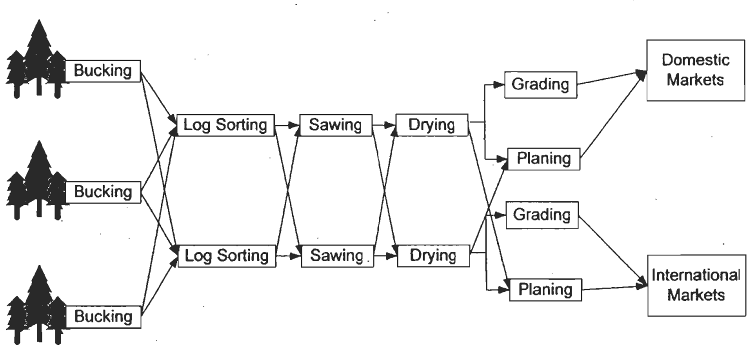


Figure 1.1 - Illustration of sawmills processes (after Rönnqvist, 2003)

As a case study in this thesis, we consider the sawing units in sawmills. In the sawing units, logs are classified according to some attributes namely: diameter class, species, length, taper, etc. Logs are broken down into different dimensions of lumbers by means of different cutting patterns. See figure 1.2 for three different cutting patterns. Each cutting pattern is a combination of activities that are run on a set of machines. From each log, several pieces of sawn lumber (e.g. $2(\text{in})\times 4(\text{in})\times 8(\text{ft})$, $2(\text{in})\times 4(\text{in})\times 10(\text{ft})$, $2(\text{in})\times 6(\text{in})\times 16(\text{ft}),\ldots$) are produced depending on the cutting pattern. The type of lumbers and their quantities produced by each cutting pattern depend on the quality of input logs. The characteristics of logs in each class are non-homogeneous and variable (in terms of diameter, number of knots, internal defects, etc.) due to natural and uncertain conditions that occur during the growth period of trees in the forest. Regarding that in most of sawmills (namely, Quebec sawmills) the logs are not scanned through an X-ray scanner before planning, the exact quantity of lumbers that can be produced by each cutting pattern (process yield) cannot be determined in priori.

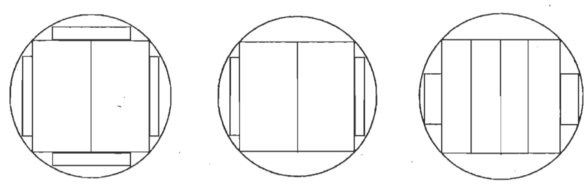


Figure 1.2 - Cutting patterns in sawmills

Lumber demand in the market is another uncertain parameter which has a non-stationary behavior during the planning horizon. In the sawmill production planning problem, we are looking for the optimal combination of log classes and cutting patterns that best fit against lumber demand. The objective is to minimize log consumption cost, as well as products inventory and backorder costs, regarding log inventory and machine capacities. Regarding that the lumber market is very competitive, the need for a robust production plan with maximum service level (minimum backorder size), as well as minimum service level variability, in the presence of uncertain yield and demand, is very crucial in sawmills.

1.3. Literature review

In this section, a review of the existing approaches in the literature to address production planning in the uncertain environments is provided. Moreover, a review of approaches to address optimization models with random parameters is also presented. Finally, sawmill production planning approaches already proposed in the literature are reviewed.

1.3.1. Literature review on production planning in the uncertain manufacturing environments

Mula et al. (2006) provided the existing approaches proposed in the literature (from 1983 to 2004) to address different uncertain production planning problems, including aggregate planning, hierarchical production planning, material requirement planning, etc. In this section, we focus on the existing literature on the production planning problems similar the problem we are addressing.

Thompson and Davis (1990) proposed an integrated approach for modeling uncertainty in selling price, cost, demand, and capacity in aggregate production planning problems. Integrated modeling approach consists of a Monte Carlo simulation model randomly sampling and obtaining numerical values for coefficients and bounds in a mathematical program. The optimal solution for the mathematical program is then determined via the optimization algorithms. This process is repeated until a very large number of mathematical programs have been specified and solved. The resulting collection of optimal solutions is then statistically analyzed to construct empirical distributions describing global optimality. It would be worth mentioning that the integrated modeling approach is a "wait and see" solution for stochastic production planning, i.e. this approach can solve the problem based on the realization of random parameters.

Stochastic programming (Dantzig, 1955; Kall and Wallace, 1994; Birge and Louveaux 1997; Kall and Mayer, 2005) has seen several successful applications in uncertain production planning. Escudero et al. (1993) proposed a multi-stage stochastic programming (MSP) model for addressing a multi-product, multi-period production planning problem with random demand. A scenario modeling method was proposed for solving the MSP model, where solutions are obtained for each scenario and then the individual scenario solutions are aggregated to yield a non-anticipative or implementable policy. Sox and Muckstadt (1996) provided a formulation and solution algorithm for the finite-horizon capacitated production planning problem with random demand for multiple products. Using Lagrangian relaxation, they developed a sub gradient optimization algorithm to solve the problem. Bakir and Byrne (1998) developed a two-stage recourse stochastic LP model to address a multi-product, multi-period production planning problem with stochastic demand. The normally distributed stochastic demand is approximated by a discrete distribution. Alfieri and Brandimarte (2005) reviewed multi-stage stochastic models applied in multi-period production and capacity planning in the manufacturing systems. Huang K. (2005) proposed multi-stage stochastic programming models for production and capacity planning. Brandimarte (2006) proposed a multi-stage programming approach for multi-item capacitated lot-sizing with uncertain demand. The uncertain demand was represented as a scenario tree. Khor et al. (2007) proposed a two-stage stochastic programming with fixed recourse and robust optimization models for capacity expansion planning in petroleum refinery under uncertainty. The uncertain parameters are modeled via a set of scenarios.

Robust optimization (Mulvey et al., 1995) is another approach that has been applied in several applications in uncertain production planning. Leung and Wu (2004) proposed a robust optimization model for stochastic aggregate production planning. Wu (2006) applied the robust optimization approach to uncertain production loading problems with import quota limits under the global supply chain management environment. Leung et al. (2007) developed a robust optimization model to address a multi-site aggregate production planning problem with uncertain data. Stochastic parameters are modeled by introducing different scenarios which are defined for different economical growth scenarios.

In order to deal with planning problems which involve uncertain and fuzzy data, fuzzy linear programming (Rommelfanger, 1996) is proposed. Wang and Fang (2001) proposed a fuzzy linear programming model to address the aggregate production planning problem with multiple objectives, where the product price, workforce level, production capacity and market demand are uncertain and fuzzy in nature. Fuzzy parameters are modeled as fuzzy intervals in trapezoidal form based on current data. An interactive method is implemented to find a "compromise solution" which is satisfactory for managers.

In this thesis, the uncertain parameters in the manufacturing environment (i.e. processes yields and products demands) are considered as the random events. In the following, we focus on the existing approaches in the literature to address optimization models including random parameters in their coefficient matrix and/or right-hand-side vector.

1.3.2. Literature review on the optimization models including random parameters

In order to deal with optimization problems involving random variables in their right-hand-side, their technological coefficients, and/or their objectives coefficients, stochastic programming (Dantzig, 1955; Kall and Wallace, 1994; Birge and Louveaux 1997; Kall and Mayer, 2005), and robust optimization (Mulvey et al., 1995) were proposed. In the following, we discuss about the general characteristics of optimization models with random parameters and the difficulties to solve them.

As we are addressing a multi-period production planning problem in this thesis, we begin by abstracting the statement of a multi-period LP model with random parameters:

$$
\begin{aligned}
&\textit{Minimize } c_1x_1 + c_2x_2 + \ldots + c_Tx_T, \qquad (1.1)\\
&\textit{Subject to}\\
&\qquad A_{11}(\underset{\sim}{\xi})x_1 = b_1(\underset{\sim}{\xi}),\\
&\qquad A_{21}(\underset{\sim}{\xi})x_1 + A_{22}(\underset{\sim}{\xi})x_2 = b_2(\underset{\sim}{\xi}),\\
&\qquad \vdots\\
&\qquad A_{T1}(\underset{\sim}{\xi})x_1 + \ldots + A_{TT}(\underset{\sim}{\xi})x_T = b_T(\underset{\sim}{\xi}),\\
&\qquad x_1 \geq 0, x_2 \geq 0, \ldots, x_T \geq 0.
\end{aligned}
$$

Let $A(\underset{\sim}{\xi}) = (A_{11}(\underset{\sim}{\xi}), \ldots, A_{TT}(\underset{\sim}{\xi}))$ and $b(\underset{\sim}{\xi}) = (b_1(\underset{\sim}{\xi}), \ldots, b_T(\underset{\sim}{\xi}))$ denote the random technological coefficients and right-hand-side vector, respectively; and decision $x = (x_1, \ldots, x_T) \in \mathrm{R}^{n_1} \times \ldots \times \mathrm{R}^{n_T}$ corresponds to a setting of all the decision variables. Problem (1.1) can also be presented as follows.

$$\begin{aligned}&\textit{Minimize } c^T x,\\&\textit{Subject to}\\&\qquad A(\underset{\sim}{\xi})x = b^T(\underset{\sim}{\xi}),\\&\qquad x \geq 0,\end{aligned} \tag{1.2}$$

where $\underset{\sim}{\xi}$ denotes a random vector varying over a set $\Xi \subset \mathbb{R}^k$. We assume that a family F of "events", i.e., subset of Ξ corresponding to the random parameters in model (1.2) with the probability distribution P are given. Furthermore, we assume that the probability distribution P is independent of x. However, problem (1.2) is not well defined, since the meaning of "*minimize*" as well as the constraints are not clear at all, if we think of taking a decision on x before knowing the realization of $\underset{\sim}{\xi}$. Therefore a revision of the modeling process is necessary, leading to various stochastic programming problems.

If we present the stochastic constraint of (1.2) as a constraint that holds with a certain probability (α), a *chance-constraint problem* will be appeared by this general form:

$$\begin{aligned}&\textit{Minimize } c^T x,\\&\textit{Subject to}\\&\qquad pr\{A(\underset{\sim}{\xi})x \geq b^T(\underset{\sim}{\xi})\} \geq \alpha,\\&\qquad x \geq 0.\end{aligned} \tag{1.3}$$

Model (1.3) requires additional refinements of linearization or a nonlinear form to obtain the *deterministic equivalent*. *Chance-constrained programs* are among the most intractable in mathematical computation. In the case that only the right-hand-side vector is random, it can be solved using nonlinear programming methods (Kall and Wallace, 1994). Bound approximation for probabilistic constraints (Birge and Louveaux, 1997) is another approach for solving these models, especially when the deterministic equivalence for constraints can not be simply found.

If we model the random parameters as discrete *scenarios*, then model (1.2) can be transformed into its *deterministic equivalent* which is an ordinary linear program (LP). The *deterministic equivalent* of (1.2) can be introduced in various ways. Depending on how the random parameters are modeled during the planning horizon and whether a risk measure is included in the objective function, the resulting *deterministic equivalent* model will be the *two-stage stochastic program with recourse*, *multi-stage stochastic program with recourse*, and *robust optimization.*

In the following, we first discuss different approaches to model the random parameters in multi-period planning models; then we provide the general concepts as well as mathematical formulations of *two-stage* (static recourse model) and *multi-stage* (dynamic recourse model) *stochastic programs with recourse*, as well as *robust optimization.*

1.3.2.1. Modeling the random parameters in multi-period planning models

As we mentioned before, in order to transform the optimization models with random parameters into a *deterministic equivalent* model, the underlying random data should be modeled as discrete *scenarios*. In multi-period planning problems, random data can be modeled either as a random variable with a stationary probability distribution, or as a non-stationary and dynamic data process.

Stationary random data

Whenever the random data has a stationary behavior during the planning horizon and the uncertainty can be characterized by some noises, we model the random data as random variables with stationary probability distributions. In such cases, the random data are represented as a number of *scenarios* with known probabilities. *Scenarios* can be defined as the atoms of the discrete probability distribution P that is used to approximate the underlying probability distribution P_0. At the moment of decision making (period 0) no information on the random data is available; however at the beginning of the planning horizon, one of the *scenarios* can take place. This approach for random data representation in stochastic models is illustrated in figure 1.3. As it can be observed in figure 1.3, the *scenarios* do not depend on time periods and are defined for the whole planning horizon. As an example of random parameters with a stationary behavior, we can refer to the quality of raw materials which would not change dramatically in different time periods if materials are supplied from the same source, during the planning horizon. It should be noted that the origin of *scenarios* can be very diverse; they can come from a truly discrete known distribution, can be obtained in the course of discretization of a continuous known distribution (see for example Miller and Rice, 1983), or they can result from a preliminary analysis of the problem with probabilities of their occurrence that may reflect an ad hoc belief or a subjective opinion of an expert. A review of approaches for generating *scenarios* in stochastic models, based on the underlying random data is provided in Dupačová (1996).

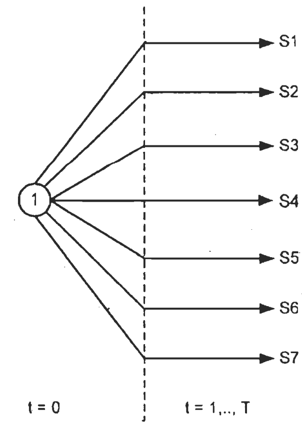


Figure 1.3 - Scenarios in stochastic optimization models

Dynamic random data

Whenever the random data in different time periods are dependent or the random data during the planning horizon are characterized by cycles, trends or temporal patterns, they should be modeled as dynamic stochastic data. As an example of dynamic stochastic data we can refer to products demands which might be characterized by different patterns during the planning horizon. A computationally viable way of discretizing the underlying dynamic stochastic data over time in a problem is *scenario tree*. An illustration of the *scenario tree* is provided in Figure 1.4. To represent the random data as a *scenario tree*, we divide the planning horizons into some stages. Each stage denotes the stage of the time when new information on the random data is available to the decision maker. Thus, the stages do not necessarily correspond to time periods. They might include a number of periods in the planning horizon. A *scenario tree* consists of a number of nodes and arcs at each stage. Each node n in the *scenario tree* represents a possible state of the world, associated with a set of data (stochastic demand, stochastic cost, etc.) in the corresponding stage. The root node of the tree represents the current state of the world. The branches (arcs) in the *scenario tree* denote the *scenarios* for the next stage. A probability is associated to each arc of the *scenario tree* which denotes the probability of the corresponding *scenario* to that arc. It should be noted that, the probability of each node in the *scenario tree* is computed as the product of probabilities of the arcs from the root node to that node. Furthermore, the sum of probabilities of nodes at each stage is equal to 1. A path from the root node to a node *n* describes one realization of the stochastic process from the present time to the period where node *n* appears. A full evolvement of the stochastic process over the entire planning horizon, i.e., the path from the root node to a leaf node, is called a *scenario*. In the *scenario tree* example of figure 1.4, we have 4 stages. Each node *n* in the tree has two branches to the next stage which denote two possible *scenarios* for the next stage, when we are in stage *n*.

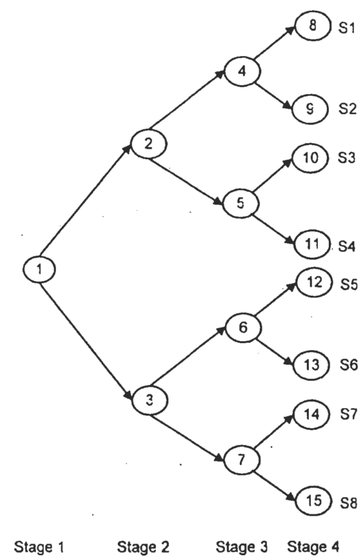


Figure 1.4 - A scenario tree for stochastic optimization problems with dynamic random data

At each stage of the *scenario tree*, either a probability distribution corresponding to the behavior of random parameters at that stage can be considered, which then can be discretized into a number of *scenarios*, or a number of subjective *scenarios* can be directly taken into account. A review of the approaches for generating the *scenario trees* for stochastic programs based on the underlying random data process is provided in Dupačová et al. (2000).

In optimization models with more than one random parameter (e.g. random yield, price, demand, etc.), modeling the random data depend on whether the parameters are dependent or independent. If the random parameters are dependent and are of the same nature (e.g. demand and price which both depend on market conditions), they can be simultaneously represented as a number of *scenarios* or a *scenario tree*, depending on their behavior over time. Otherwise, each random parameter should be modeled independently, and then the *scenarios* or *scenario trees* corresponding to each parameter should be integrated.

In the following, we provided the *deterministic equivalents* of stochastic optimization models with stationary and dynamic random data, including *two-stage stochastic program with recourse*, *robust optimization*, and *multi-stage stochastic program with recourse*.

1.3.2.2. Two-stage stochastic program with recourse

In two-stage stochastic programs with recourse, we assume that the random data has a stationary probability distribution during the time, as it is illustrated in figure 1.3. The decision variables are explicitly classified according to whether they are implemented before or after an outcome (*scenario*) of the random data is observed. In other words, we have a set of decisions to be taken without full information on the random parameters. These decisions are called *first-stage* decisions, and are usually represented by a vector (x). Later, full information is received on realizations (*scenarios*) of the random vector $\underset{\sim}{\xi}$. Then, *second-stage* or *recourse* actions (y) are taken under the full insight on the random data. These *second-stage* decisions allow us to model a response to each of the observed outcomes (*scenarios*) of the random variable, which constitutes our *recourse*. In general, this response will also depend upon the *first-stage* decisions. We also consider a unit penalty cost for the *recourse* actions. The objective of the two-stage stochastic model with recourse would be to minimize the *first-stage* cost in addition to the expected *second-stage* (*recourse*) cost for all the *scenarios* of random parameters. We assume that the original probability distribution of $\underset{\sim}{\xi}$ is approximated by a finite number of *scenarios* $\{(s, p^s), s = 1, \ldots, N\}$, where p^s denotes the probability of scenario s ($\sum_{s=1}^{N} p^s = 1$). In mathematical programming terms, we define the two-stage stochastic linear program with recourse (2-stage SLP), corresponding to model (1.2), as follows:

2-stage SLP formulation

$$\textit{Minimize } c^T x + \sum_{s=1}^{N} p^s d^{sT} y^s \tag{1.4}$$

Subject to

$$A^s x + W y^s = b^{sT}, \qquad s = 1,\ldots,N, \tag{1.5}$$

$$x, y^s \geq 0, \qquad s = 1,\ldots,N, \tag{1.6}$$

where, W denotes the recourse matrix, d^{sT} denotes the vector of penalty cost of *second-stage* (*recourse*) variables, A^s, b^{sT}, and y^s denote the technological coefficients matrix, right-hand-side vector, and *recourse* action vector under scenario *s,* respectively. It would be worth mentioning that the *first-stage* decision (x) cannot anticipate the *scenarios* of random parameters, and should be feasible for all *scenarios*.

Solving methods for two-stage stochastic programs with recourse

Model (1.4)-(1.6) can be solved by LP solvers, namely CPLEX. If a huge number of *scenarios* for the random parameters are taken into account, this model can become (very) large in scale; making it impossible to be solved by the existing LP solvers. However, its particular block structure is amenable to specially designed algorithms. Decomposition methods including the L-shaped method (Kall and Wallace, 1994; Birge and Louveaux, 1997; Kall and Mayer, 2005), and regularized decomposition method (Ruszczyński and Świetanowski, 1996) are among the exact methods to solve large scale two-stage stochastic models. The novel architectures of high performance computers and recent developments in parallel and distributed computing can be exploited when applying the mentioned algorithms.

In order to solve two-stage stochastic programs with a huge number of *scenarios* that cannot be solved by the exact methods, due to the present computational capacities, approximate methods based on Monte-Carlo sampling have been proposed in the literature. Monte Carlo solution procedures can use "internal sampling" or "external sampling". The "internal sampling" procedures include stochastic decomposition algorithm (Higle and Sen, 1996) and stochastic quasi-gradient algorithms (Ermoliev, 1983). In the "external sampling" procedures, sampling is performed external to (prior to) the solution procedure. The sample average approximation (SAA) scheme (cf. Shapiro and Hommem-de-Mello, 1998; Mak et al., 1999) is an "external sampling" procedure. The SAA scheme is focused on taking a random sample of *scenarios* through Monte Carlo sampling scheme, instead of considering all the possible *scenarios* for random parameters. Thus, the original large scale stochastic program is approximated by one of a manageable size. Statistical confidence intervals can then be constructed to measure the quality of approximate solutions. Shapiro and Hommem-de-Mello (2000) proved that under some mild regularity conditions, by increasing the sample size the optimal (approximate) solution of the SAA model converges with probability 1 to the optimal solution of the original two-stage stochastic model.

1.3.2.3. Robust optimization

As it was mentioned in 1.3.2.2, two-stage stochastic programming approach focuses on optimizing the expected performance (e.g. minimizing the expected cost) over a range of possible *scenarios* for the random parameters. We can expect that the system would behave optimally in the mean sense if the stochastic programming model solution was implemented. However, the system might perform poorly at a particular realization of *scenarios*, such as the worst-case *scenario*. This means that the stochastic model cannot reflect the variability of performances for each *scenario* realization and it can yield solutions that are not very robust. The robust optimization (RO) method developed by (Mulvey et al., 1995) is a special class of two-stage stochastic programming. It extends stochastic programming with the introduction of higher moments of the objective function. In other words, traditional expected cost minimization objective is replaced by one that explicitly addresses cost variability and a series of solutions are generated that are progressively less sensitive to the *scenarios* of random data.

The optimal solution of model (1.4)-(1.6) will be robust with respect to optimality if it remains close to optimal for any of the *scenarios s*. This is termed *solution robustness*. In other words, the solution robustness measures the variability of the *recourse* cost in a 2-stage SLP model for any of the *scenarios s*. The solution is also robust with respect to feasibility if it remains almost feasible for all *scenarios*. This is termed *model robustness*. The robust optimization (RO) framework introduced by (Mulvey et al., 1995) is a goal programming approach to balance the tradeoffs between *solution robustness* and *model robustness*. Hence, the RO approach is to modify the objective function in 2-stage SLP as follows:

Robust optimization (RO) formulation

$$\text{Minimize } c^T x + \sum_{s=1}^{N} p^s d^{sT} y^s + \lambda\sigma(y^1,\ldots,y^N) + \omega\rho(\delta^1,\ldots,\delta^N) \tag{1.7}$$

Subject to

$$A^s x + W y^s + \delta^s = b^{sT}, \qquad s = 1,\ldots,N, \tag{1.8}$$

$$x, y^s \geq 0, \qquad s = 1,\ldots,N, \tag{1.9}$$

The term $(\sum_{s=1}^{N} p^s d^{sT} y^s + \lambda\sigma(y^1,\ldots,y^N))$ in the objective function denotes the *solution robustness* measure, where $\lambda \geq 0$ is a goal programming weight and $\sigma(y^1,\ldots,y^N)$ denotes the *recourse* cost variability measure. By changing λ, the relative importance of the expectation and variability of the *recourse* cost in the objective function can be controlled. The last term in the objective function ($\rho(\delta^1,\ldots,\delta^N)$) is the *model robustness* measure. It is a feasibility penalty function, which is used to penalize the violation of constraints (1.8) (denoted by δ^s) under some *scenarios*. $\omega \geq 0$ is a goal programming weight which measures the relative importance of *solution robustness* and *model robustness*. In the following, the *recourse* cost variability measures, existing in the literature, are provided.

Variability measures in robust optimization (RO) models

The classical approach to model the tradeoff between the expectation and the variability of the *recourse* cost in RO models is to use mean-variance model of Markowitz (1959). Mean-variance measure has been implemented in many applications, namely capacity expansion of power systems (Malcolm and Zenios, 1994), stochastic logistic problems (Yu and Li, 2000), stochastic aggregate production planning (Leung and Wu, 2004; Leung et al., 2007). However, there are some exceptions against using mean-variance in some applications: variance is a symmetric risk measure, penalizing the cost both above and below the expected *recourse* cost, equally. As in the case of production planning, it is more convenient to use an asymmetric risk measure that would penalize only costs above the expected value. *Scenarios* whose costs are below the expected *recourse* cost should not be penalized, as their occurrence leads to lower actual costs than the one expected. Shabbir and Shahinidis (1998) proposed to use upper partial mean of the *recourse* cost as the measure of variability in a robust optimization model for process planning under uncertainty. The upper partial mean is defined as the partial expectation of costs above the expected value. In List et al. (2003) an upper partial moment (UPM) of order 1 (i.e., the partial expectation of costs above a threshold value) was used in a robust optimization model for fleet planning under uncertainty. Takriti and Shabbir (2004) used the upper partial moment of order 2 for robust optimization of two-stage stochastic models.

Solving methods for robust optimization (RO) models

Robust optimization models with quadratic robustness terms can be solved by the CPLEX quadratic programming solver. In order to solve robust optimization problem with a large number of *scenarios*, decomposition algorithms namely, a modified L-shape method (Takriti and Shabbir, 2004) or diagonal quadratic algorithm (DQA) (Berger et al., 1994) can be applied. When applying the mentioned algorithms, one can exploit the novel architectures of high performance computers and recent developments in parallel and distributed computing.

1.3.2.4. Multi-stage stochastic program with recourse

In a multi-period production planning model with dynamic random parameters, the decision model should be designed to allow the user to adopt a decision policy that can respond to events as they unfold. The specific form of the decisions depends on assumptions concerning the information that is available to the decision maker, when (in time) is it available and what adjustments (*recourses*) are available to the decision maker. The multi-stage stochastic programming (MSP) approach (Kall and Wallace, 1994; Birge and Louveaux 1997; Kall and Mayer, 2005) addresses multi-period optimization models with dynamic stochastic data during the time. It was demonstrated in Escudero et al. (1993) that multi-stage stochastic programming models can be designed to model the information availability, and the decisions produced by these models are optimal with respect to the zero-regret criterion - that which could be achieved if the course of future events could be perfectly predicted. In multi-stage stochastic programming (MSP), a lot of emphasis is placed on the decision to be made today, given present resources, future uncertainties and possible *recourse* actions in the future. Multi-stage stochastic programming models with *recourse* extend the two-stage stochastic models by allowing revised decisions at each stage, based upon the uncertainty realized so far. The uncertainty is represented through a *scenario tree* (see figure 1.4) and an objective function is chosen to represent the risk associated with the sequence of decisions to be made and the whole problem is then solved as a large scale linear or quadratic program. In most situations, the number of possible *scenarios* is so great that it is impossible to model them explicitly. The success of multi-stage stochastic programming depends on the extent to which a relatively simple *scenario tree* can be constructed that captures the risk inherent in making one decision today. In the following, we provide the multi-stage programming formulation.

Multi-stage stochastic program (MSP) formulation

Consider a multi-period LP model with random parameters (model (1.1)). In order to formulate the *deterministic equivalent* of a multi-stage stochastic model (1.1), we assume that the random vector $\underset{\sim}{\xi}$ is represented as a *scenario tree* (see figure 1.4). Recall from 1.3.2.1 that a *scenario* is defined as a path from the root node to each leaf in the *scenario tree*. Let the *scenario s* correspond to a single setting of all data in model (1.1),

$$s = \{ A_{tt'}, b_t : t = 1,\dots,T, t' = 1,\dots,T\},$$

and a decision x corresponds to a setting of all the decision variables

$$x : (x_1,\dots,x_T) \in \mathrm{R}^{n_1} \times \dots \times \mathrm{R}^{n_T}.$$

Solving the deterministic LP model (1.1) for a given *scenario s* of the data is equivalent to solving the following problem for a certain function:

$$\min f(x,s) \qquad \text{over all } x,$$

where

$$f(x,s)=\begin{cases}\sum_{t=1}^{T} c_t x_t, & \text{if } x \text{ satisfies all constraints in (1.1)},\\ +\infty, & \text{otherwise.}\end{cases}$$

The function $f(.,s)$ is called the essential objective function for the LP model (1.1). By setting its value to plus infinity for all points that violate the constraints, we ensure that minimizers of $f(.,s)$ will be feasible for the LP model (1.1).

We next develop the stochastic model. Let us suppose that we are given a set S of *scenarios* on a *scenario tree*. We first, set a policy that makes different decisions under different *scenarios*. Mathematically, a policy X that assigns to each scenario $s \in S$ is a vector

$$X(s) := (X_1(s),\ldots,X_T(s)),$$

where $X_t(s)$ denotes the decision to be made at stage t if encountered by *scenario s*. Decisions that depend on individual *scenarios* do not hedge against the possibility that the *scenario* may not occur, leaving one vulnerable to disastrous consequences if some other scenario does happen. As we mentioned before, the decision process must conform to the flow of available information, which basically means the decisions must be non-anticipative (or implementable). A decision is said to be implementable if for every pair of *scenarios* s and s' that are indistinguishable up to stage t then

$$(X_1(s),\ldots,X_T(s)) = (X_1(s'),\ldots,X_T(s')).$$

As the examples of indistinguishable *scenarios*, we can refer to *scenarios* 1, 2, 3, and 4 in node 2 at stage 2 of the *scenario tree* in figure 1.4. Implementability guaranties that policies do not depend on information that is not yet available. The multi-stage stochastic programming can be formulated as:

$$\min\left\{\sum_{s\in S} p^s f(X(s),s) \mid X \text{ is an implementable policy}\right\},$$

where p^s denotes the probability of scenario s. There are two approaches to impose the non-anticipativity constraints in the multi-stage stochastic programs which lead to the *split variable* formulation and the *compact* formulation.

Split variable formulation

In the *split variable* formulation, we introduce a set of decision variables for each stage and each *scenario*, and then we enforce non-anticipativity constraints explicitly based on the shape of the *scenario tree*. Model (1.1) can be represented by the *split variable* formulation as follows:

$$\text{Minimize} \sum_{s\in S} p^s[c_1x_1(s)+c_2x_2(s)+\ldots+c_Tx_T(s)] \tag{1.10}$$

Subject to

$$\begin{array}{lll}
A_{11}(s)x_1(s) & =b_1(s), & s\in S,\\
A_{21}(s)x_1(s)+A_{22}(s)x_2(s) & =b_2(s), & s\in S,\\
\vdots & & \\
A_{T1}(s)x_1(s)+\ldots \quad +A_{TT}(s)x_T(s) & =b_T(s), & s\in S,\\
x_1(s)\geq 0, x_2(s)\geq 0,\ldots, x_T(s)\geq 0, & & s\in S,
\end{array}$$

non-anticipativity constraints

$$\begin{array}{ll}
x_1(s)=x_1(s'), & s,s'\in\{s\}_1,\\
x_2(s)=x_2(s'), & s,s'\in\{s\}_2,\\
\vdots & \\
x_T(s)=x_T(s'), & s,s'\in\{s\}_T,
\end{array}$$

where $\{s\}_t$ denotes the set of all indistinguishable *scenarios* at stage t of the *scenario tree*. It should be noted that, if the stages in the *scenario tree* do not correspond to time periods, the non-anticipativity constraints should be written for decision variables corresponding to all time periods at each stage. As it can be observed in model (1.10), the *split variable* formulation increases the problem dimensions, significantly.

Compact formulation

In the *compact* formulation, we associate decision variables to the nodes of the *scenario tree* and build non-anticipativity in an implicit way. Moreover, the redundant variables and constraints for partially identical *scenarios* are deleted. We also need the probability $p(n)$ of getting to node *n*.

To represent model (1.1) by the *compact* formulation, consider a *scenario tree* with $t = 1,...,T$ stages, where the nodes for stage *t* are indexed by k_t. There are $K_t - K_{t-1}$ nodes indexed by $k_t = K_{t-1}+1,...,K_t$ for stage t ($K_1 = 1$); particularly, the $K_T - K_{T-1}$ leaves indexed by k_T correspond to *scenarios*. For example, in the *scenario tree* in figure 1.4, T=4, and there are $K_2 - K_1 = 2$ nodes at stage 2, $K_3 - K_2 = 4$ at stage 3, and $K_4 - K_3 = 8$ nodes at stage 4. Furthermore, $k_2 = 2,3$, $k_3 = 4,...,7$, and $k_4 = 8,...,15$. We also denote the probability of node k_t by p_{k_t}. Model (1.11) is the *deterministic equivalent* of multi-stage stochastic model (1.1) represented by *compact* formulation, based on a given *scenario tree*.

$$\text{Minimize } c_1^T x_1 + \sum_{k_2=2}^{K_2} p_{k_2} c_2^T x_{k_2} + \sum_{k_3=K_2+1}^{K_3} p_{k_3} c_3^T x_{k_3} + \ldots + \sum_{k_T=K_{T-1}+1}^{K_T} p_{k_T} c_T^T x_{k_T} \tag{1.11}$$

Subject to

$$\begin{array}{llll}
A_{11}x_1 & & = b_1, & \\
A_{k_2 1}x_1 + A_{k_2 2}x_{k_2} & & = b_{k_2}, & k_2 = 2,...,K_2, \\
& A_{k_3 2}x_{a(k_3)} + A_{k_3 3}x_{k_3} & = b_{k_3}, & k_3 = K_2+1,...,K_3, \\
& \quad \ddots & & \\
& \quad A_{k_T,T-1}x_{a(k_T)} + A_{k_T,T}x_{k_T} & = b_{kT}, & k_T = K_{T-1}+1,...,K_T, \\
x_{k_t} \geq 0, & & & k_t = k_{t-1}+1,...,K_t,\ t = 1,...,T.
\end{array}$$

It should be noted that in multi-stage stochastic model (1.11), $a(k_t)$ denotes the immediate predecessor of node k_t, $A_{k_t t}$ and b_{k_t} denote the coefficient matrix and right-hand-side vector values in node k_t at stage *t*, respectively. For example, in the *scenario tree* of figure 1.4, $a(4) = 2$. For each node of the *scenario tree* at stage *t*, an entire set of decision variables corresponding to that stage is introduced; for instance the vector of the *first-stage* decision variables x_1 corresponds to the root and sub-vectors x_{k_t} of the *t*th stage decision variables are assigned to the node k_t, respectively. At each stage, the sub-vectors of decision variables exploit only the information that comes from the previous stages (preceding nodes of the tree) and the choice of decisions are based on the available and past information and at the same time allow for the continuation of the decision process at the subsequent stages. It can also be observed that the non-anticipativity constraints are implicit in this formulation. It should be noted that, if the stages in the *scenario tree* do not correspond to time periods, each constraint in model (1.11) should be repeated for all time periods at each stage.

Solving methods for multi-stage stochastic programs (MSP) with recourse

Multi-stage stochastic programs are among the most intractable in numerical computations due to dimensionality and complexity of their *deterministic equivalent* models. Not only does the size of the problem grow as a quadratic function of the number of *scenarios*, but also the problem structure is difficult to take advantage.

Split variable formulation of multi-stage stochastic models yields a sparsity structure that is well suited to the interior point algorithms. Alternatively, it is possible to use a decomposition approach on the *split variable* formulation. If the implementability constraints are relaxed, by adding a penalty (possibly, nonlinear) term of these constraints to the objective function, the problem decomposes into several smaller optimization problems. Several strategies have been published in the literature to solve large-scale multi-stage stochastic models, such as the progressive hedging algorithm proposed by Rockafellar and Wets (1991), the augmented Lagrangian decomposition method of Ruszczynski (1989), and the decomposition methods (Mulvey and Ruszczyński, 1995; Liu and Sun, 2004).

Compact formulations are computationally cheaper than *split variable* formulations, when using for solving by the Simplex methodology in standard solvers. Moreover, they lend themselves to generalizations of the L-shaped method such as nested Benders decomposition (Birge and Louveaux, 1997).

1.3.3. Literature review on sawmill production planning

Among different approaches proposed in the literature for sawmill production planning, two of the most applied ones are reviewed in the following. The first approach is focused on combined optimization type solutions linked to real-time simulation sub-systems (Mendoza et al., 1991; Maness and Adams, 1991; Maness and Norton, 2002). In this approach, the stochastic characteristics of logs are taken into account, by assuming that all the input logs are scanned through an X-ray scanner, before planning. The model developed by (Mendoza et al., 1991), consists of a log inventory model with an optimization capability and a real-time process simulation model. The system works as follows: first, an optimization model determines the "best" log input mix to be processed, in order to satisfy the periodic lumber demand. This log mix is optimized under constraints based on resource capacities. Information on log input mix then becomes an input to the process simulation model to create production schedules. The sawmill process simulator receives, through its interface, the sawmill layout file and machine specifications, the current product specifications and database file containing the log input schedule generated by the optimizer. The systems output include the simulated number and volume of logs processed (by species and grade), sawmill operating time, lumber output by species, grade and volume, etc. Maness and Norton (2002) developed an integrated multi-period production planning model which is the combination of an LP model and a log sawing optimizer (simulator). The LP acts as a coordinating problem that allocates limited resources. A series of dynamic programming (DP) sub problems, titled in the literature as "log sawing optimization models" are used to generate activities (columns) for the coordinating LP based on the shadow prices for lumber products. The log sawing optimization model is a sawing algorithm for lumber grade, based on data collected from an X-ray scanner. The inputs to the model are a description of the logs and values, grade rules and dimensions of lumber products. The program processes each log, determines the optimal sawing pattern based on value recovery, and calculates sawing time required per cube meter of saw logs, the conversion volume of lumber products and the conversion volume of byproducts. Although the stochastic characteristics of logs are considered in this approach, it includes the following limitations to be implemented in real capacity sawmills: logs, needed for the next planning horizon, are not always available in sawmills to be scanned before planning. Furthermore, to implement this method, the logs should be processed in production line in the same order they have been simulated, which is not an easy practice. Finally scanning logs before planning is a time consuming process in high capacity sawmills, which delays the planning process.

In the second approach, the randomness of the processes yields as well as demand is simplified and their expected value is considered in a MPMP linear programming model (Gaudreault et al., 2004). However, as the randomness of yield and demand are not taken into account, the production plan proposed by this approach is not robust. Implementing such plans results usually extra inventory of products with lower quality and price, while extra backorder of products with higher quality and price.

There are other works in the literature related to softwood lumber supply chain that are reviewed as follows. Vila (2006) proposed the mathematical programming models to design the production-distribution network for softwood lumber industry. The proposed generic model maps the industry manufacturing process onto potential production-distribution facility locations and capacity options. Furthermore, a two-stage stochastic program with recourse was proposed to find a robust design for the production-distribution network by taking into account three different market opportunities (spot markets, contracts and Vendor Managed Inventory (VMI) agreements), which improve the competitive position of the company or companies involved.

In Frayret et al. (2007), a software architecture for development of an experimentation environment to design and test distributed advanced planning and scheduling systems in the forest products industry was proposed. This architecture enables combination of agent-based technology and operations research-based tools in order to first take advantage of the ability of agent technology to integrate distributed decision problems, and, second, to take advantage of the ability of operations research to develop and exploit specific normative decision models.

Gaudreault et al. (2009) addressed distributed operations planning and scheduling in a softwood lumber supply chain, made of three planning units (sawing unit, drying unit and finishing unit). Each production unit is presented as an agent and a mathematical model is proposed for production planning or scheduling of each agent. Then, in order to coordinate the plans between the agents, different coordination mechanisms were proposed. Using these developments, they showed how an agent-based simulation tool can be used to integrate planning models and evaluate different coordination mechanisms.

1.4. Outline of the thesis

The MPMP production planning problem, described in 1.2, is first studied by considering only the non-homogeneous characteristics of raw materials, and consequently random processes yields. Two-stage stochastic programming and robust optimization approaches are proposed to address the MPMP production planning problem with random yield. The random demand is then studied as another uncertain parameter in the problem. The MPMP production planning problem with uncertain yield and demand is addressed by the multi-stage stochastic programming approach. This thesis includes three original contributions (presented as three articles), which are provided through chapters II to IV as follows.

In chapter II, we study a multi-period, multi-product (MPMP) production planning problem in a manufacturing environment with non-homogeneous raw materials, and consequently random processes yields. A two-stage stochastic linear program with recourse is proposed to address the problem. The random yields are modeled as scenarios with stationary probability distributions during the planning horizon. The objective of the two-stage stochastic model is to find a production plan with the minimum expected inventory and backorder size for all yield scenarios. The two-stage stochastic model is solved by the sample average approximation (SAA) scheme. As a case study, we address the sawmill production planning problem by the proposed approach. Moreover, the stochastic sawmill production planning model is validated through Monte Carlo simulation. The objective of simulation is to compare the realized backorder size after implementation of plans, as well as the plan precision in the mean-value deterministic and the stochastic models.

In chapter III, we study the robustness of service level in a multi-period, multi-product (MPMP) production planning problem where the yields of processes are random variables due to non-homogeneous quality of raw materials. Two robust optimization models with different service level variability measures are proposed. The objective of robust optimization (RO) models is to find a production plan with the minimum expected inventory and backorder size as well as minimum inventory and backorder size variability (service level variability) for all yield scenarios. A decision framework is also provided to select among the two RO models based on the tradeoff between the expected backorder/inventory cost and the decision maker risk aversion level about the variability of customer service level.

In chapter IV, we study a multi-product, multi-period (MPMP) production planning problem with uncertainty in the quality of raw materials and consequently in processes yields, as well as uncertainty in products demands. As demand and yield own different uncertain natures, they are modeled separately and then integrated. Demand uncertainty is considered as a dynamic stochastic data process during the planning horizon which is modeled as a scenario tree. The uncertain yield is modeled as scenarios with stationary probability distributions during the planning horizon. Yield scenarios are then integrated in each node of demand scenario tree, constituting a hybrid scenario tree. Based on the hybrid scenario tree for the uncertain yield and demand, a multi-stage stochastic programming (MSP) model is proposed which is full recourse for demand scenarios and simple recourse for yield scenarios. The objective of the multi-stage model is to find a production plan with minimum expected raw material consumption cost as well as expected product inventory and backorder costs, for all the demand and yield scenarios. We conduct a case study with respect to a realistic scale sawmill.

We summarize the major contributions of the thesis in chapter V, along with a discussion of possible research perspectives.

1.5. Conclusions

In this chapter, we introduced the problem we are addressing, as well as an industrial application which is considered as a case study in this thesis. We also presented the literature review. Finally, the outline of thesis was provided. The three following chapters present the three original contributions of the thesis.

References

Alfieri, A. and Brandimarte, P., 2005. Stochastic programming models for manufacturing applications: A tutorial introduction. *In*: Matta, A. and Semeraro, Q., eds. *Design of Advanced Manufacturing Systems, Models for Capacity Planning in Advanced Manufacturing Systems.* Netherlands: Springer, 73-124.

Bakir, M.A. and Byrne, M.D., 1998. Stochastic linear optimization of an MPMP production planning model. *International Journal of Production Economics*, 55, 87-96.

Berger, A. J., Mulvey, J.M. and Ruszczyński, A., 1994. An extension of the DQA algorithm to convex stochastic programs. *SIAM Journal on Optimization*, 4, 735-753.

Birge, J.R. and Louveaux, F., 1997. *Introduction to stochastic programming.* New York: Springer.

Brandimarte, P., 2006. Multi-item capacitated lot-sizing with demand uncertainty. *International Journal of Production Research*, 44(15), 2997-3022.

Dantzig, G.B., 1955. Linear programming under uncertainty. *Management Science*, 1, 197-206.

Dupačová, J., 1996. Stochastic programming: Approximation via scenarios, *Apportaciones mathematicás, Ser. Communicationes*, 24, 77-94, Also available via http://www.emis.de/proceedings/index.html.

Dupačová, J., Consigli, G. and Wallace, S.W., 2000. Scenarios for multi-stage programs. *Annals of Operations Research*, 100, 25-53.

Ermoliev, Y., 1983. Stochastic quasigradient methods and their applications to system optimization. *Stochastics*, 9, 1-36.

Escudero, L.F., Kamesam, P.V., King, A.J. and Wets, R.J-B., 1993. Production planning via scenarios. *Annals of Operations Research*, 34, 311- 335.

Frayret J.-M., D'Amours S., Rousseau A., Harvey S. and Gaudreault J., 2007. Agent-based supply-chain planning in the forest products industry. *International Journal of Flexible Manufacturing Systems*, 19, 358-391.

Gaudreault, J., Rousseau, A., Frayret, J.M. and Cid, F., 2004. Planification opérationnelle du sciage. *For@c technical document*; Quebec, Canada.

Gaudreault J., Forget P., Frayret J.-M., Rousseau A. and D'Amours S., 2009. Distributed operations planning in the lumber supply chain: Models and coordination. *CIRRELT 2009-07 working document*; Québec, Canada.

Higle, J. L. and Sen, S., 1996. *Stochastic decomposition: an algorithm for two-stage linear programming*. Boston: Kluwer Academic Publishers.

Ho, C., 1989. Evaluating the impact of operating environments on MRP system nervousness. *International Journal of Production Research*, 27, 1115-1135.

Huang, K., 2005. *Multi-stage stochastic programming models for production planning.* Thesis (PhD). School of Industrial and Systems Engineering: Georgia Institute of Technology.

Johnson, L. A. and Montgomery, D. C., 1974. *Operations research in production planning, scheduling, and inventory control*. New York: John Wiley & Sons.

Kall, P. and Wallace, S.W., 1994. *Stochastic programming*. New York: John Wiley & sons.

Kall, P. and Mayer, J., 2005. *Stochastic linear programming*. New York: Springer's International Series.

Khor, C.S., Elkamel, A., Ponnambalamb, K. and Douglas, P.L., 2007. Two-stage stochastic programming with fixed recourse via scenario planning with economic and operational risk management for petroleum refinery planning under uncertainty. *Chemical Engineering and Processing*, doi:10.1016/j.cep.2007.09.016.

Leung, S.C.H. and Wu, Y., 2004. A robust optimization model for stochastic aggregate production planning. *Production Planning & Control*, 15(5), 502-514.

Leung, S.C.H., Tsang, S.O.S., Ng, W.L. and Wu, Y., 2007. A robust optimization model for multi-site production planning problem in an uncertain environment. *European Journal of Operational Research*, 181(1), 224-238.

List, G.F., Wood, B., Nozick, L.K., Turnquist, M.A., Dean, J.A., Kjeldgaard, E.A. and Lawton, C.R., 2003. Robust optimization for fleet planning under uncertainty. *Transportation Research part E* 39, 209-227.

Liu, X. and Sun, J., 2004. A new decomposition technique in solving multistage stochastic linear programs by infeasible interior point methods. *Journal of Global Optimization* 28, 197-215.

Mak, W.K., Morton, D.P. and Wood, R.K., 1999. Monte Carlo bounding techniques for determining solution quality in stochastic programs. *Operations research letters*, 24, 47-56.

Malcolm, S.A. and Zenios, S.A., 1994. Robust optimization of power systems capacity expansion under uncertainty. *Journal of Operational Research Society*, 45, 1040-1049.

Maness, T.C. and Adams D.M., 1991. The combined optimization of log bucking and sawing strategies. *Wood and Fiber Science*, 23, 296-314.

Maness, T.C. and Norton, S.E., 2002. Multiple-period combined optimization approach to forest production planning. *Scandinavian Journal of Forest Research*, 17, 460-471.

Markowitz, H., 1959. *Portfolio selection, efficiency diversification of investments*. New York: John Wiley & Sons.

Mendoza, G.A., Meimban, R.J., Luppold, W.J. and Arman, P.A., 1991. Combined log inventory and process simulation models for the planning and control of sawmill operations. In: *Proceeding 23rd GIRP International Seminar on Manufacturing Systems*; Nancy, France.

Miller, A.C. and Rice, T.R., 1983. Discrete approximations of probability distributions. *Management Science*, 29(3), 352-362.

Mula, J., Poler, R., Garcia-Sabater, J.P. and Lario, F.C., 2006. Models for production planning under uncertainty: A review. *International Journal of Production Economics*, 103, 271-285.

Mulvey, J.M., Vanderbei, R.J. and Zenios, S.A., 1995. Robust optimization of Large-scale systems. *Operations Research*, 43(2), 264-281.

Mulvey, J.M. and Ruszczyński, A., 1995. A new scenario decomposition method for large-scale stochastic optimization. *Operations Research*, 43(2), 477- 490.

Rockafellar, R.T. and Wets, R.J-B., 1991. Scenarios and policy aggregation in optimization under uncertainty. *Mathematics of Operations Research*, 16(1), 119-147.

Rommelfanger H., 1996. Fuzzy linear programming and applications. *European Journal of Operational Research*, 92, 512-527.

Rönnqvist M., 2003. Optimization in forestry. *Mathematical programming, Ser. B*, 97, 267-284.

Ruszczyński, A., 1989. Regularized decomposition and augmented Lagrangian decomposition for angular programming problems. *In*: Levandowski, A. and Wierzbicki, A.P., eds. *Aspiration Based Decision Support Systems*. Berlin: Springer.

Ruszczyński, A. and Świetanowski, A., 1996. On the regularized decomposition method for two-stage stochastic linear problems. IIASA working paper; Laxenburg, Austria.

Shabbir, A. and Shahinidis, N.V., 1998. Robust process planning under uncertainty. *Industrial & Engineering Chemistry Research*, 37, 1883-1892.

Shapiro, A. and Hommem-de-Mello, T., 1998. A simulation based approach to two-stage stochastic programming with recourse. *Mathematical programming*, 81, 301-325.

Shapiro, A. and Hommem-de-Mello, T., 2000. On the rate of convergence of optimal solutions of Monte Carlo approximations of stochastic programs. *SIAM Journal on Optimization*, 11(1), 70-86.

Sipper, D. and Bulfin, R., 1997. *Production: planning, control, and integration.* McGraw-Hill.

Sox, CH.R. and Muckstadt, J. A., 1996. Multi-item, multi-period production planning with uncertain demand. *IIE Transactions*, 28(11), 891-900.

Takriti, S. and Shabbir, A., 2004. On robust optimization of two-stage systems. *Mathematical Programming*, 99, 109-126.

Thompson, S.D. and Davis, W.J., 1990. An integrated approach for modeling uncertainty in aggregate production planning. *IEEE transactions on systems, man, and cybernetics*, 20(5), 1000-1012.

Vila D., 2006. La modélisation mathématique des réseaux logistiques : Procédés divergents et positionnement par anticipation (Applications à l'industrie du bois d'œuvre). Thesis (Ph.D), Université Laval.

Wang, R.C. and Fang, H.H., 2001. Aggregate production planning with multiple objectives in a fuzzy environment. *European Journal of Operational Research*, 133, 521-536.

Wu, Y., 2006. Robust optimization applied to uncertain production loading problems with import quota limits under the global supply chain management environment. *International Journal of Production Research*, 44(5), 849-882.

Yu, C. and Li, H., 2000. A robust optimization approach for stochastic logistic problems. *International Journal of Production Economics*, 64, 385-397.

Chapter 2

A stochastic programming approach for production planning with uncertainty in the quality of raw materials: A case in sawmills

This chapter is dedicated to the article entitles "*A stochastic programming approach for production planning with uncertainty in the quality of raw materials: A case in sawmills*". It has been submitted to the *Journal of Operations Research Society* on March 2009. The titles, figures and mathematical formulations have been revised to the keep the coherence through the thesis.

2.1. Abstract

Motivated by sawmill production planning, this paper investigates a multi-period, multi-product (MPMP) production planning problem in a manufacturing environment with non-homogeneous raw materials, and consequently random processes yields. A two-stage stochastic linear program with recourse is proposed to address the problem. The random yields are modeled as scenarios with stationary probability distributions during the planning horizon. The solution methodology is based on the sample average approximation (SAA) scheme. The proposed two-stage stochastic model can be considered as a novel approach for sawmill production planning. The stochastic sawmill production planning model is validated through Monte Carlo simulation. The objective of simulation is to compare the realized backorder size after implementation of plans, as well as the plan precision in the mean-value deterministic and the stochastic models. Through the simulation, the production plans proposed by the mentioned models are implemented virtually, by considering yield scenarios similar to those that might be realized during the plan implementation in real sawmills. The computational results for a real medium capacity sawmill highlight the significance of using the stochastic model as a viable tool for production planning instead of the mean-value deterministic model, which is a traditional production planning tool in many sawmills.

2.2. Introduction

Most of the production environments are characterized by multiple types of uncertainties. When planned production quantities are released, the outputs are often variable. These uncertainties affect and complicate the production plan and control.

The goal of this work is to address a multi-period, multi-product (MPMP) production planning problem in a manufacturing environment, where alternative processes can produce simultaneously multiple products with random yields. In other words, the quantities of products that can be produced by each process are random variables. Besides, the randomness in processes yields arises from the random and non-homogeneous quality of raw materials. In this production planning problem, we are looking for the number of times each process should be run, as well as the quantity of each class of raw material that should be consumed by each process in each period in the planning horizon. The objective is to minimize products inventory/backorder and raw material costs, regarding fulfillment of products demands, machine capacity, and raw material inventory. This work is motivated by production planning for sawing units in sawmills. In sawmills, raw materials (logs) are classified based on some attributes namely: diameter class, species, length, taper, etc. Logs are broken down into different pieces of lumbers (products) by means of different cutting patterns. Moreover, different combinations of log classes and cutting patterns produce simultaneously different mix of products (lumbers). However, due to non-homogeneity in the quality of logs, each cutting pattern yields a random quantity of corresponding products after processing a known quantity of each log class. Production planning in a sawing unit is to decide about the optimal quantity of log consumption from different classes and the selection of corresponding cutting patterns to fit against products demands. The part of the demand that cannot be fulfilled on time due to machine capacities, log inventory, and random yield will be postponed to the following periods by considering a backorder cost. The objective is to minimize log consumption cost, as well as products inventory/backorder costs. We are studying a customer orientated manufacturing environment that wishes to fulfill the demand as much as possible. Regarding the potential significance of yield uncertainty on the production plan and consequently on the realized total backorder size, obtaining the plans with minimum total backorder size is an important goal of production planning in sawmills.

This production planning problem can be considered as the combination of several classical production planning problems in the literature which have been modeled by linear programming (LP). The product mix problem and a special case of process selection problem (Johnson and Montgomery, 1974; Sipper and Bulfin, 1997) are the two main building blocks of this problem. However, the LP models include the assumption of deterministic parameters. It has been shown in the literature that failure to include uncertainty in optimization models can cause expensive, even disastrous consequences if the anticipated situation is not realized. In Gaudreault et al. (2004), a deterministic LP model was proposed for sawmill production planning by considering the expected values of random processes yields. The production plan proposed by the deterministic model results usually extra inventory of products with lower quality and price, while backorder of products with higher quality and price. Another approach for sawmill production planning is focused on combined optimization type solutions linked to real-time simulation sub systems (Mendoza et al., 1991; Maness and Adams, 1991; Maness and Norton, 2002). In this approach, the stochastic characteristics of logs are taken into account by assuming that all the input logs are scanned through an X-ray scanner, before planning. Maness and Norton (2002) developed an integrated multi-period production planning model which is the combination of a LP model and a log sawing optimizer (simulator). The LP model acts as a coordinating model that allocates limited resources. A series of dynamic programming sub-problems, titled in the literature as "log sawing optimization models" are used to generate activities (columns) for the coordinating LP based on the products' shadow prices. Although the stochastic characteristics of logs are considered in this approach, it includes the following limitations to be implemented in many sawmills: logs, needed for the next planning horizon, are not always available in sawmills to be scanned before planning. Furthermore, to implement this method, the logs should be processed in the production line in the same order they have been simulated, which is not an easy practice. Finally, scanning logs before planning is a time consuming process in high capacity sawmills, which delays the planning process. It has been shown in the literature (see for example Kall and Wallace, 1994; Birge and Louveaux, 1997; Kall and Mayer, 2005) that in mathematical programming models which include random parameters in their right-hand–side and/or technological coefficients, the stochastic programming approach leads to higher quality solutions compared with the mean-value deterministic model. Most of the works in the literature on uncertain production planning are focused on considering random products demands. In Escudero et al. (1993), a multi-stage stochastic programming approach was proposed to address a MPMP production planning model with random demand. In Bakir and Byrne (1998), demand uncertainty in a MPMP production planning model was studied. They developed a demand stochastic LP model based on the two-stage deterministic equivalent problem. In Kazemi et al. (2007), stochastic programming was proposed as one of possible methodologies to address sawmill production planning, while considering random characteristics of logs.

In this paper, a two-stage stochastic program with recourse (Kall and Wallace, 1994; Birge and Louveaux, 1997; Kall and Mayer, 2005) is proposed for MPMP production planning while considering random characteristics of raw materials and consequently random processes yields. As the proposed model is applied for sawmill production planning, we propose an approach to model the random processes yields in sawmills. Due to the astronomic number of scenarios for random yields in the two-stage stochastic model, a Monte Carlo sampling strategy, the sample average approximation (SAA) scheme (cf. Shapiro and Hommem-de-Mello, 1998, 2000; Mak et al., 1999) is implemented to solve the stochastic model. The confidence intervals on the optimality gap for the candidate solutions are constructed based on common random number (CRN) streams (Mak et al., 1999). We also propose a validation approach to compare stochastic and deterministic sawmill production planning models plans, which is based on Monte Carlo simulation. More precisely, we simulate the implementation of plans proposed by the stochastic and deterministic sawmill production planning models, by considering the yield scenarios that might be observed through plan implementation in realistic scale sawmills. The objective is to compare the realized total backorder size, after implementation of the stochastic and deterministic models. Furthermore, the precision of plans, in terms of the gap between the planned and the realized total backorder size, for the stochastic and deterministic models are also compared. Our computational results involving one medium capacity sawmill, with different demand levels, indicate that the proposed stochastic programming approach can be served as a viable tool for sawmill planning by considering the random characteristics of logs.

The remainder of this paper is organized as follows. In the next section, sawmill processes and characteristics are introduced. In section 2.4, a theoretical framework for two-stage stochastic linear programming is provided; in section 2.5 we propose a two-stage stochastic linear program for MPMP production planning under uncertainty of processes yields. In section 2.6, the proposed approach for modeling random processes yields in sawmills is provided. In section 2.7, we provide the solution methodology for the two-stage stochastic model. In section 2.8, the proposed validation approach to compare the stochastic and deterministic sawmill production planning models is presented. In section 2.9, the implementation results of the stochastic model and solution strategy for a realistic scale sawmill are presented. The results of comparison between the plans of stochastic and mean-value deterministic LP models, through Monte Carlo simulation, are also reported in this section. Our concluding remarks are given in section 2.10.

2.3. Sawmill processes and characteristics

There are a number of processes that occur in a sawmill: log sorting, sawing, drying, planing and grading (finishing). Raw materials in sawmills are the logs which are transported from different districts of forest after bucking the felled trees. The finished and graded lumbers (products) are then transported to the domestic and international markets. Figure 2.1 illustrates the typical processes. In this paper we focus on operational level production planning in the sawing units of sawmills. In the sawing units, logs are classified according to some attributes namely: diameter class, species, length, taper, etc. Logs are broken down into different dimensions of lumbers by means of different cutting patterns. See figure 2.2 for three different cutting patterns. Each cutting pattern is a combination of activities that are run on a set of machines. From each log, several pieces of sawn lumber (e.g. $2(in)\times 4(in)\times 8(ft)$, $2(in)\times 4(in)\times 10(ft)$, $2(in)\times 6(in)\times 16(ft),\ldots$) are produced depending on the cutting pattern. The lumber quality (grade) as well as its quantity yielded by each cutting pattern depends on the quality and characteristics of the input logs. Despite the classification of logs in sawmills, variety of characteristics might be observed in different logs in each class. In fact, natural variable conditions that occur during the growth period of trees make it impossible to anticipate the exact yields of a log. Moreover, as it is not possible in many sawmills to scan the logs before planning, the exact yields of cutting patterns for different log classes cannot be determined in priori.

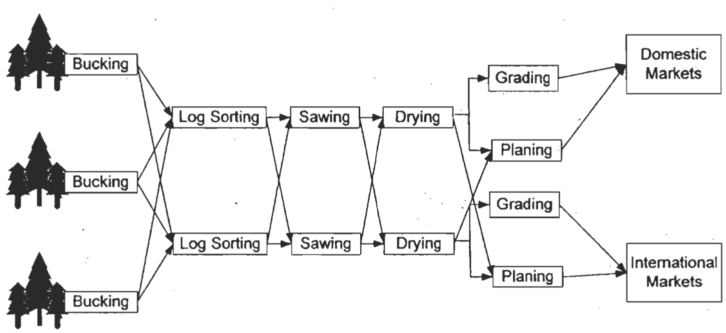


Figure 2.1 - Illustration of sawmills processes (after Rönnqvist, 2003)

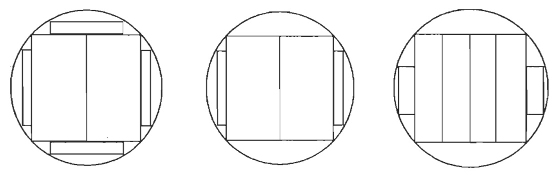


Figure 2.2 - Cutting patterns in sawmills

2.4. A theoretical framework for two-stage stochastic LP

This section gives a brief review on two-stage stochastic LP; for more details, the reader is referred to Kall and Wallace (1994), Birge and Louveaux (1997) and Kall and Mayer (2005). When one or more of the parameters in a linear program is represented by a random variable, a stochastic linear program (SLP) results. Model (2.1)-(2.3) is an example of a SLP.

$$\textit{Minimize} \quad c^T x \tag{2.1}$$

Subject to

$$Ax = b, \tag{2.2}$$

$$T(\underset{\sim}{\xi})x \geq h^T(\underset{\sim}{\xi}), \tag{2.3}$$

$$x \geq 0,$$

where $\underset{\sim}{\xi}$ is the vector of random parameters, $T(\underset{\sim}{\xi})$ and $h(\underset{\sim}{\xi})$ are random technological coefficient matrix and right-hand side vector, respectively. In the above model, constraints (2.2) and (2.3) represent the set of deterministic and stochastic constraints, respectively. In two-stage stochastic models, we explicitly classify the decision variables according to whether they are implemented before or after an outcome of the random variables is observed. In other words, we have a set of decisions to be taken without full information on the random parameters. These decisions are called first-stage decisions, and are usually represented by a vector (x). Later, full information is received on realizations (scenarios) of some random vector $\underset{\sim}{\xi}$. Then, second-stage or recourse actions (y) are taken. These second-stage decisions allow us to model a response to each of the observed outcomes (scenarios) of the random variables, which constitutes our recourse. In general, this response will also depend upon the first-stage decisions. In mathematical programming terms, this defines the so-called two-stage stochastic program with recourse of the form:

$$\text{Minimize} \quad c^T x + E_{\underset{\sim}{\xi}}[Q(x,\underset{\sim}{\xi})] \tag{2.4}$$

Subject to

$$\begin{aligned} &Ax = b, \\ &x \geq 0. \end{aligned} \tag{2.5}$$

where $Q(x,\underset{\sim}{\xi}) = \min\left\{q^T(\underset{\sim}{\xi})y \mid Wy = h^T(\underset{\sim}{\xi}) - T(\underset{\sim}{\xi})x\right\}$, W is the recourse matrix, $q^T(\underset{\sim}{\xi})$ is the vector of penalty cost of second-stage (recourse) variables, and $E_{\underset{\sim}{\xi}}$ denotes mathematical expectation with respect to $\underset{\sim}{\xi}$.

In the case of continuous distribution for random variables in model (2.4)-(2.5), the calculation of the expected value $E_{\underset{\sim}{\xi}}[Q(x,\underset{\sim}{\xi})]$ requires the calculation of multiple integrals with respect to the measure describing the distribution of $\underset{\sim}{\xi}$. The computational effort increases with the dimension of the stochastic variables vector and this leads to tremendous amount of work. On the other hand, if $\underset{\sim}{\xi}$ has a finite discrete distribution $\left\{(\underset{\sim}{\xi}^i, p^i),\, i = 1, \ldots, n\right\}$, then (2.4)-(2.5) can be transformed into its *deterministic equivalent* which is an ordinary linear program as follows.

$$\text{Minimize} \quad c^T x + \sum_{i=1}^{n} p^i q^{iT}(\underset{\sim}{\xi}) y^i \tag{2.6}$$

Subject to

$$\begin{aligned} &Ax = b, \\ &Wy^i = h^{iT}(\underset{\sim}{\xi}) - T^i(\underset{\sim}{\xi})x, \qquad i = 1, \ldots, n, \\ &x \geq 0. \end{aligned} \tag{2.7}$$

Model (2.6)-(2.7) can be solved by the LP solvers. Although this model can become (very) large in scale, its particular block structure is amenable to specially designed algorithms. Solution methods for large-scale two-stage stochastic programs can be divided into two main categories: 1) exact methods including decomposition methods, namely L-shaped method (Kall and Wallace, 1994; Birge and Louveaux, 1997; Kall and Mayer, 2005), and regularized decomposition method (Ruszczyński and Świetanowski, 1996) 2) approximate methods based on Monte Carlo sampling: sample average approximation (SAA) (cf.

Shapiro and Hommem-de-Mello, 1998, 2000; Mak et al., 1999), and stochastic decomposition method (Higle and Sen, 1996).

2.5. Problem formulation by mathematical programming

In this section we first describe the deterministic linear program (LP) formulation for MPMP production planning considered in this paper. Then we develop the proposed stochastic model to address the problem by considering the uncertainty of processes yields.

2.5.1. The deterministic LP model for MPMP production planning

Consider a production unit with a set of products P, a set of classes of raw materials C, a set of production processes A, a set of resources (machines) R, and a planning horizon consisting of T periods. It should be noted that in sawmill production planning model, we define a process as a combination of a log class and a cutting pattern, for modeling simplicity. To state the deterministic linear programming model for this production planning problem, the following notations are used:

2.5.1.1. Notations

Indices

p product

t period

c raw material class

a production process

r resource (machine)

Parameters

h_{pt} inventory holding cost per unit of product p in period t

b_{pt} backorder cost per unit of product p in period t

m_{ct} raw material cost per unit of class c in period t

I_{c0} the inventory of raw material class c at the beginning of planning horizon

I_{p0} the inventory of product p at the beginning of planning horizon

s_{ct} the quantity of material of class c supplied at the beginning of period t

d_{pt} demand of product p by the end of period t

ϕ_{ac} the units of class c raw material consumed by process a (consumption factor)

ρ_{ap} the units of product p produced by process a (yield of process a)

δ_{ar} the capacity consumption of resource r by process a

M_{rt} the capacity of resource r in period t

Decision variables

X_{at} the number of times each process a should be run in period t

I_{ct} inventory size of raw material of class c by the end of period t

I_{pt} inventory size of product p by the end of period t

B_{pt} backorder size of product p by the end of period t

2.5.1.2. The LP model

$$\text{Minimize } Z = \sum_{p\in P}\sum_{t=1}^{T}(h_{pt}I_{pt} + b_{pt}B_{pt}) + \sum_{c\in C}\sum_{t=1}^{T}\sum_{a\in A} m_{ct}\phi_{ac}X_{at} \tag{2.8}$$

Subject to

Material inventory constraint

$$I_{ct} = I_{ct-1} + s_{ct} - \sum_{a\in A}\phi_{ac}X_{at}, \qquad t=1,\dots,T,\ c\in C, \tag{2.9}$$

Product inventory constraint

$$I_{p1} - B_{p1} = I_{p0} + \sum_{a\in A}\rho_{ap}X_{a1} - d_{p1},$$

$$I_{pt} - B_{pt} = I_{pt-1} - B_{pt-1} + \sum_{a\in A}\rho_{ap}X_{at} - d_{pt}, \qquad t=2,\dots,T, p\in P, \tag{2.10}$$

Production capacity constraint

$$\sum_{a\in A}\delta_{ar}X_{at} \le M_{rt}, \qquad t=1,\dots,T, r\in R, \tag{2.11}$$

Non-negativity of all variables

$$X_{at} \ge 0, I_{ct} \ge 0, I_{pt} \ge 0, B_{pt} \ge 0, \qquad t=1,\dots,T,\ p\in P, c\in C, a\in A. \tag{2.12}$$

The objective function (2.8) is a linear cost minimization equation. It consists of total inventory and backorder costs for all products and raw material cost for all classes in the planning horizon. Constraint (2.9) ensures that the total inventory of raw material of class c

at the end of period t is equal to its inventory in the previous period plus the quantity of material of class c supplied at the beginning of that period (s_{ct}) minus its total consumption in that period. It should be noted that the total consumption of each class of raw material in each period is calculated by multiplying the material consumption factor of each process (ϕ_{ac}) by the number of times that process is executed in that period. Constraint (2.10) ensures that the sum of inventory (or backorder) of product p at the end of period t is equal to its inventory (or backorder) in the previous period plus the total production of that product in that period, minus the product demand for that period. Total quantity of production for each product in each period is calculated as the sum of the quantities yielded by each of the corresponding processes considering the yield (ρ_{ap}) of each process. Finally, constraint (2.11) requires that the total production does not exceed the available production capacity. In other words, the sum of capacity consumption of a machine r by corresponding processes in each period should not be greater than the capacity of that machine in that period.

2.5.2. The two-stage stochastic model with recourse for MPMP production planning with random yield

To include the random nature of processes yields in MPMP production planning, we expand the model (2.8)-(2.12) to a two-stage stochastic linear program with recourse. It is assumed that the random processes yields are modeled as scenarios with known probability distributions. We represent the random yield vector by $\underset{\sim}{\xi}$, where $\underset{\sim}{\xi} = \{\rho_{ap} \mid a \in \mathrm{A}, p \in \mathrm{P}\}$. We also represent each realization (scenario) of random processes yields by $\rho_{ap}(\underset{\sim}{\xi})$. We denote the total number of yield scenarios by N, and the probability of each scenario i by p^i, respectively. It should be emphasized that the stages of the two-stage recourse problem do not refer to time periods. They correspond to steps in the decision making. In the first-stage (planning stage), the decision maker does not have any information about the processes yields due to lack of complete information on the characteristic of raw materials. However, the production plan should be determined before the complete information is available. Thus the first-stage decision variable is the production plan. In the second stage (plan implementation stage) when the realized yields are available, based on the first-stage decision, the recourse actions (inventory or backorder sizes) can be computed. The objective of the two-stage stochastic program with recourse would be to minimize the material consumption cost, plus the expected inventory and backorder costs (recourse costs) for all the yield scenarios. The resulting deterministic equivalent formulation for the two-stage stochastic model is as follows:

$$Minimize\, Z = \sum_{c\in C}\sum_{t=1}^{T}\sum_{a\in A} m_{ct}\phi_{ac}X_{at} + \sum_{i=1}^{N}\sum_{p\in P}\sum_{t=1}^{T} p^i[h_{pt}I^i_{pt} + b_{pt}B^i_{pt}] \tag{2.13}$$

Subject to

$$I_{ct} = I_{ct-1} + s_{ct} - \sum_{a\in A}\phi_{ac}X_{at}, \qquad t=1,\dots,T,\ c\in C, \tag{2.14}$$

$$I^i_{p1} - B^i_{p1} = I_{p0} + \sum_{a\in A}\rho_{ap}(\underset{\sim}{\xi}^i)X_{a1} - d_{p1},$$

$$I^i_{pt} - B^i_{pt} = I^i_{pt-1} - B^i_{pt-1} + \sum_{a\in A}\rho_{ap}(\underset{\sim}{\xi}^i)X_{at} - d_{pt}, \quad t=2,\dots,T,\ p\in P,\ i=1,\dots,N, \tag{2.15}$$

$$\sum_{a\in A}\delta_{ar}X_{at} \le M_{rt}, \qquad t=1,2,\dots,T,\ r\in R, \tag{2.16}$$

$$X_{at}\ge 0,\ I_{ct}\ge 0,\ I^i_{pt}\ge 0,\ B^i_{pt}\ge 0, \qquad c\in C,\ p\in P,\ t=1,\dots,T,\ a\in A,\ i=1,\dots,N. \tag{2.17}$$

In the two-stage stochastic program (2.13)-(2.17), I^i_{pt} and B^i_{pt} denote the inventory and backorder sizes of product p in period t under yield scenario i, respectively. In this model there are $|A|\times T$ first-stage decisions, whereas there are $2\times|P|\times T\times N$ second-stage decisions, where $|A|$, $|P|$, and N denote the sizes of process, product, and scenario sets, respectively. The first-stage decisions X_{at} cannot anticipate the yield scenarios and must be feasible for all scenarios.

2.6. Modeling the random processes yields in sawmills

To apply the proposed two-stage stochastic model (2.13)-(2.17) for sawmill production planning, as the first step, we should generate the scenarios for random processes yields. A scenario for the yields of process (*a*) (combination of a log class (*c*) and a cutting pattern (*s*)) in a sawing unit is defined as possible quantities of lumbers that can be produced by cutting pattern (s) after sawing each log of class (c). As an example of the uncertain yields in sawmills, consider the cutting pattern (s) that can produce 6 products (P1, P2, P3, P4, P5, P6) after sawing the logs of class (c). Table 2.1 represents four scenarios among all possible scenarios for the uncertain yields of this process.

Table 2.1 - Scenarios for yields of a process in a sawing unit

Scenarios	Products					
	P1	P2	P3	P4	P5	P6
1	1	0	1	0	1	1
2	2	1	1	0	1	0
3	1	0	0	1	1	1
4	2	0	0	1	0	1

In this work, we assume that all the logs that will be processed in the next planning horizon are supplied from the same discrete of forest. Hence, a stationary probability distribution can be considered for the quality of logs and uncertain processes yields during the planning horizon. In the following we explain how the scenarios for the yields of each process can be generated, and also how their probability distribution can be estimated.

In realistic scale sawmills, thousands of logs are sawn in each period in the planning horizon. The total production size, and consequently the inventory or backorder size of each product in each period, depend on the sum of yields of logs sawn in that period. Thus, we propose to consider the average yield of a number of logs in each log class as a scenario and to estimate the probability distribution for the average yields. Such scenarios with their probability distribution in sawmills can be determined as follows.

1) Take a sample of logs in each log class (e.g., 3000) and let them be processed by each cutting pattern. Compute the average yield for the sample.

2) Repeat step 1 for a number of replications (e.g., 30).

3) By the Central Limit Theorem (CLT) in statistics, the average yield has a normal distribution. Thus, based on the average yields computed for each replication in step 2, estimate the mean and variance of normal distribution corresponding to the average yield of each process.

It should be noted that, the implementation of step 1 in this approach is very difficult in sawmills. In fact, the high production speed in the sawing units makes it almost impossible to track the logs through the line and to observe the result of sawing individual logs. As a more feasible alternative, we propose to use the set of yield scenarios generated by a log sawing simulator (Optitek). "Optitek" was developed by a research organization for Canada's solid wood products industry (FP Innovation). It was developed based on the characteristics of a sample of logs in different log classes, as well as sawing rules available in Quebec sawmills. The inputs to this simulator include log class, cutting pattern, and the number of logs to be processed. The simulator considers the logs in the requested class with random physical and internal characteristics; afterwards it generates different quantity of lumbers (yields) for each log based on the sawing rules of the requested cutting pattern. Thus, in order to implement step 1 in the proposed scenario generation approach, a sample (e.g. 3000) of yields can be randomly selected among the set of scenarios already generated by Optitek, and the average yield for the sample can be computed.

2.7. Solution strategy

The two-stage stochastic model (2.13)-(2.17) can be solved by the linear programming solvers, namely CPLEX solver. However, regarding the wide variety of characteristics in each log class in sawmills, a huge number of scenarios for processes yields can be expected. Thus, solving this model would be far beyond present computational capacities. We can however use Monte Carlo sampling techniques, which consider only randomly selected subsets of the set $\left\{\underset{\sim}{\xi}^1,\underset{\sim}{\xi}^2,\ldots,\underset{\sim}{\xi}^N\right\}$ to obtain approximate solutions. Monte Carlo solution procedures for solving stochastic programs can use "internal sampling" or "external sampling". The "internal sampling" procedures include sampling-based cutting plane methods (e.g., Higle and Sen, 1996) and stochastic quasi-gradient algorithms (e.g., Ermoliev, 1998). In the "external sampling" procedures, sampling is performed external to (prior to) the solution procedure. The sample average approximation (SAA) scheme (cf. Shapiro and Hommem-de-Mello, 1998, 2000; Mak et al., 1999), which is selected as the solution approach in this work, is an "external sampling" procedure. In the following, the SAA scheme is described.

Sample average approximation (SAA) scheme

In the SAA scheme, a random sample of n scenarios of the random vector $\underset{\sim}{\xi}$ is generated and the expectation $\sum_{i=1}^{N}\sum_{p\in P}\sum_{t=1}^{T} p^i[h_{pt}I^i_{pt} + b_{pt}B^i_{pt}]$ is approximated by the sample average function $\frac{1}{n}\sum_{i=1}^{n}\sum_{p\in P}\sum_{t=1}^{T}[h_{pt}I^i_{pt} + b_{pt}B^i_{pt}]$. In other words, the "true" problem (2.13)-(2.17) is approximated by the sample average approximation (SAA) problem (2.18).

$$\text{Minimize } \hat{Z} = \sum_{c\in C}\sum_{t=1}^{T}\sum_{a\in A} m_{ct}\phi_{ac}X_{at} + \frac{1}{n}\sum_{i=1}^{n}\sum_{p\in P}\sum_{t=1}^{T}[h_{pt}I^i_{pt} + b_{pt}B^i_{pt}] \qquad (2.18)$$

Subject to
Constraints (2.14)-(2.17).

It can be shown that under mild regularity conditions, as the sample size n increases, the optimal solution vector $\hat{X}_n$ and optimal value $\hat{Z}_n$ of the SAA problem (2.18) converge with probability one to their true counterparts, and moreover $\hat{X}_n$ converges to an optimal solution of the true problem with probability approaching one exponentially fast (Shapiro and Hommem-de-Mello, 1998 and 2000). This convergence analysis suggests that a fairly good approximate solution to the true problem (2.13)-(2.17) can be obtained by solving an SAA problem (2.18) with a modest sample size. The mentioned regularity conditions include: 1) the objective function of the stochastic model has finite mean and variance, 2) the independent identically distributed (i.i.d.) observations of vector $\underset{\sim}{\xi}$ can be generated, 3) instances of SAA problem can be solved for sufficiently large n to result "good" bounding information, and 4) the objective function of the stochastic model can be evaluated exactly for specific values of X_{at} and realizations of vector $\underset{\sim}{\xi}$. It is evident that the mentioned regularity conditions are satisfied for our problem, especially due to considering a normal distribution for the random yields.

In practice, the SAA scheme involves repeated solutions of the SAA problem (2.18) with independent samples. Statistical confidence intervals are then derived on the quality of the approximate solutions (Mak et al., 1999). According to the work of Mak et al. (1999), an obvious approach to test solution quality for a candidate solution ($\bar{X}$) is to bound the optimality gap, defined as $E_{\underset{\sim}{\xi}}[f(\bar{X},\underset{\sim}{\xi})]-z^*$ using standard statistical procedures, where $f(\bar{X},\underset{\sim}{\xi})$ and z^* are the true objective value for $\bar{X}$ and the true optimal solution to the problem (2.13)-(2.17), respectively, and $E_{\underset{\sim}{\xi}}[f(\bar{X},\underset{\sim}{\xi})]$ is the expected value of $f(\bar{X},\underset{\sim}{\xi})$. In our work, a sampling procedure based on common random numbers (CRN) is used to construct the optimality gap confidence interval, which provides significance variance reduction over naive sampling, as shown in (Mak et al., 1999). This approach is described next.

The SAA algorithm (with common random number streams)

Step 1- Generate n_g independent identically distributed (i.i.d.) batches of samples each of size n from the distribution of $\underset{\sim}{\xi}$, i.e., $\left\{\underset{\sim}{\xi}_j^1, \underset{\sim}{\xi}_j^2, \ldots, \underset{\sim}{\xi}_j^n\right\}$ for $j=1,\ldots,n_g$. For each sample solve the corresponding SAA problem (2.18). Let $\hat{Z}_n^j$ and $\hat{X}_n^j$, $j=1,\ldots,n_g$, be the corresponding optimal objective value and an optimal solution, respectively.

Step 2- Compute

$$\bar{Z}_{n,n_g} = \frac{1}{n_g}\sum_{j=1}^{n_g}\hat{Z}_n^j, \text{ and} \tag{2.19}$$

$$s^2_{\bar{Z}_{n,ng}} = \frac{1}{n_g(n_g-1)}\sum_{j=1}^{n_g}(\hat{Z}^j_{n_g}-\bar{Z}_{n,n_g})^2. \tag{2.20}$$

It is well known that the expected value of $\hat{Z}_n$ is less than or equal to the optimal value z^* of the true problem (see e.g., Mak et al., 1999). Since $\bar{Z}_{n,n_g}$ is an unbiased estimator of $E[\hat{Z}_n]$, we obtain that $E[\bar{Z}_{n,n_g}] \le z^*$. Thus $\bar{Z}_{n,n_g}$ provides a lower statistical bound for the optimal value z^* of the true problem (2.13)-(2.17) and $s^2_{\bar{Z}_{n,ng}}$ is an estimate of the variance of this estimator.

Step 3- Choose a candidate feasible solution $\bar{X}$ of the true problem, for example, a computed $\hat{X}^j_{n'}$ by using a sample size (n') larger than used for lower bound estimation (n). Estimate the true objective function value $f(\bar{X})$ for all batches of samples ($j=1,\dots,n_g$) as follows.

$$\tilde{f}^j_n(\bar{X}) = \sum_{c\in C}\sum_{t=1}^{T}\sum_{a\in A} m_{ct}\phi_{ac}\bar{X} + \frac{1}{n}\sum_{i=1}^{n}\sum_{p\in P}\sum_{t=1}^{T}[h_{pt}I^i_{pt} + b_{pt}B^i_{pt}]. \tag{2.21}$$

Step 4- Compute the observations of the optimality gap G^j_n for the candidate solution $\bar{X}$ for all $j=1,\dots,n_g$ as follows:

$$G^j_n = \tilde{f}^j_n(\bar{X}) - \hat{Z}^j_n. \tag{2.22}$$

It has been shown in Mak et al. (1999) that:

$$E\underbrace{\left[\tilde{f}_n(\bar{X}) - \hat{Z}_n\right]}_{G_n} \geq E_{\underset{\sim}{\xi}}[f(\bar{X},\underset{\sim}{\xi})] - z^*. \tag{2.23}$$

where $f(\bar{X},\underset{\sim}{\xi})$ and z^* are the true objective value for $\bar{X}$ and the true optimal solution to the problem (2.13)-(2.17), respectively, and ($E_{\underset{\sim}{\xi}}[f(\bar{X},\underset{\sim}{\xi})] - z^*$) is the true optimality gap for the candidate solution $\bar{X}$. We also have:

$$\sqrt{n_g}\left[\bar{G}_{n_g} - E[G_n]\right] \Rightarrow \mathrm{N}(0,\sigma_g^2) \quad \text{as } n_g \to \infty$$

where $\sigma_g^2 = \operatorname{var} G_n$.

Step 5- Compute the sample mean and sample variance for the optimality gap G^j_n as follows.

$$\bar{G}_{n_g} = \frac{1}{n_g}\sum_{j=1}^{n_g} G^j_n, \text{ and} \tag{2.24}$$

$$s_{G_n^j}^2 = \frac{1}{n_g(n_g-1)}\sum_{j=1}^{n_g}(G_n^j - \bar{G}_{n_g})^2. \tag{2.25}$$

Step 6- Compute the approximate $(1-\alpha)$-level confidence interval for the optimality gap of $\bar{X}$ as $\left[0, \bar{G}_{n_g} + \tilde{\varepsilon}_g\right]$, where $\tilde{\varepsilon}_g = \dfrac{t_{n_g-1,\alpha} s_{G_n^j}}{\sqrt{n_g}}$.

In order to solve the SAA problem (2.18) for each of the n_g batches of n randomly sampled scenarios, we propose either to solve directly its deterministic equivalent, or in the cases where the number of scenarios is very large and the deterministic equivalent model cannot be solved in a reasonable amount of time, to implement the regularized decomposition method (Ruszczyński and Świetanowski, 1996) with l_∞ trust region as it is used in Linderoth and Wright (2003) and Linderoth and Shapiro (2006).

2.8. Validation of the stochastic sawmill production planning model by Monte Carlo simulation

In this section, the proposed approach to validate the two-stage stochastic sawmill production planning model is described. To validate the stochastic model, we propose to compare the plans proposed by the stochastic and deterministic sawmill production planning models. As we mentioned before, we assume that the company is very service sensitive, i.e., the realized total backorder size after implementation of production plan is more crucial than the realized inventory size. Thus, the following key indicators of performance are considered to compare the deterministic and stochastic models:

1) Backorder gap (BO gap): the gap between the expected realized total backorder size of the deterministic and the stochastic models' plans, after implementing the plans proposed by the mentioned models.

2) Plan precision: the gap between the planned total backorder size determined by the production planning model and the expected realized total backorder size, after implementing the model's plan. This indicator evaluates also the extent to which the

yield scenarios considered in the stochastic model are close to the scenarios that can be observed in the real production process.

In order to compute the expected total backorder size after implementing the plans, we propose to use Monte Carlo simulation. The main objective of this simulation is to implement the production plans virtually, by considering the yield scenarios that might be realized during the plan implementation in realistic scale sawmills. Hence, the following features are considered for the simulator:

1) To get the production plans proposed by the deterministic and stochastic models as well as the products demand, as the inputs.

2) To simulate the production plan implementation based on the received production plan as follows:

   2.1) To determine a sample size equal to the number of times each process should be run in each period (production plan).

   2.2) To generate randomly a sample of scenarios (with the size determined in 2.1) for the yields of each process, from the set of possible scenarios for the yield of that process. It should be mentioned that this step is equivalent to select a random sample of logs in each class to be sawn by each cutting pattern, while implementing the production plan in sawmills.

3) To compute the total production size of each product at the end of that period, after simulating the plan implementation for each period (step 2).

4) To compute the backorder or inventory size of each product in each period based on the total production size of that product (computed in step 3) and its demand for that period.

Figure 2.3 illustrates the main features of the simulator which is designed to simulate the plans implementation in sawmills.

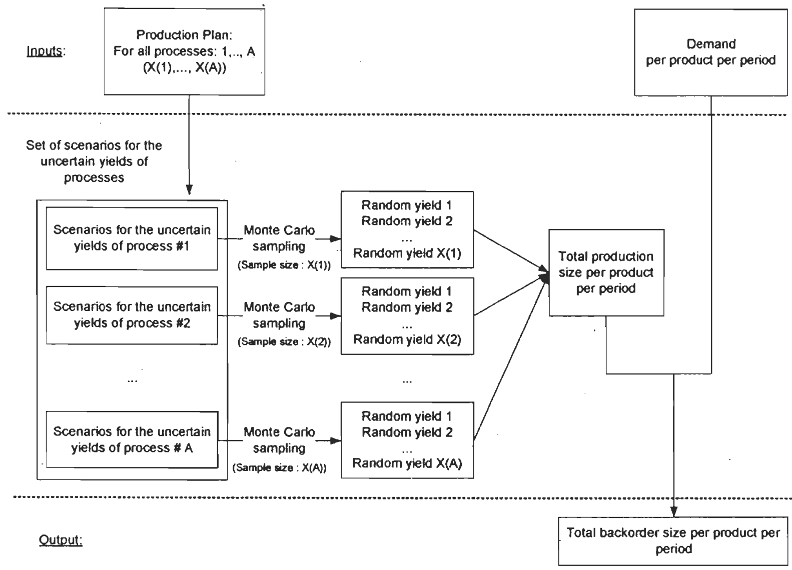


Figure 2.3 - Simulation of the production plans implementation in a sawmill

2.9. Computational results

In this section, we describe the numerical experiments using the proposed two-stage stochastic model to address a medium capacity sawmill production planning problem. We first describe the characteristics of the test industrial problem and some implementation details; then, we comment on the quality of the stochastic model solutions determined by the SAA scheme; finally, we compare the stochastic and mean-value deterministic models' plans by the proposed Monte Carlo simulation approach (see section 2.8), for different demand levels.

2.9.1. Data and implementation

The proposed two-stage stochastic program with recourse in this paper is applied for a prototype sawmill. The prototype sawmill is a typical medium capacity softwood sawmill located in Quebec (Canada). The sawmill focuses on sawing high-grade products to the domestic markets as well as export products to the USA. It is assumed that the input bucked logs into the sawing unit are categorized into 3 classes. 5 different cutting patterns are available. The sawing unit produces 27 products of custom sizes (e.g. 2(in)×4(in), 2(in)×6(in) lumbers) in four lengths. In other words, there are 15 processes; all can produce 27 products with random yields. We consider two bottleneck machines: Trimmer and Bull.

The planning horizon consists of 30 periods (days). Product demand in each period is assumed to be deterministic and is determined based on the received orders. Lumbers that remain from one period to the next are subject to a unit holding cost. The unsatisfied demand is penalized by a unit backorder cost. We assume that the company is very service sensitive and wishes to fulfill customer demands on time as much as possible. Hence, the inventory costs of products are considered much lower than their backorder costs. The inventory holding cost is computed by multiplying the interest rate (per period) by the lumber price; the lumber price is considered as the backorder cost. It would be worth mentioning that the data used in this example are based on the data gathered from different sawmills in province of Quebec (Canada). As the list of custom sizes, machine parameters and prices are proprietary; they are not reported in this paper.

Recall from section 2.7 that the SAA method calls for the solution of n_g instances of the approximating stochastic program (2.18), each involving n sampled scenarios. Statistical validation of a candidate solution is then carried out by evaluating the objective function value using the same n sampled scenarios in each batch. In our implementation test, we used n=30 and 100; and $n_g = 30$. Our candidate solutions are computed by solving the SAA problem (2.18) with $n' = 100$ and 150. To illustrate the complexity of solving (2.18) within the SAA scheme, we present the sizes of the deterministic equivalents of the SAA problems corresponding to different values of n in Table 2.2.

Table 2.2 - Size of the deterministic equivalent of the SAA problem

n	Number of constraints	Number of variables
1	960	2160
30	24450	49140
100	81150	162540
150	121650	243540

The SAA scheme was implemented in OPL Studio 3.7.1. CPLEX 9 LP solver is used for solving the deterministic equivalents for different instances of SAA problems as well as for calculating the true objective function value for the candidate solutions. The simulator is programmed in Java. All computations were carried out on a Pentium(R) IV 1.8 GHz PC with 512 MB RAM running Windows XP.

2.9.2. Quality of the stochastic solutions

In this section, we present the results of applying the SAA scheme for our test problem and the evaluation of quality of candidate approximate solutions. Point estimates (see (2.19) and (2.20)) of the lower statistical bound for the optimal value of the stochastic problem are reported in table 2.3. They are computed based on 30 batches of sampled scenarios with 2 different batch sizes. Table 2.4 displays the quality of 2 candidate solutions and contains the 95% confidence intervals on their optimality gaps based on CRN method (see section 2.7). The candidate solutions $\bar{X}^{100}, \bar{X}^{150}$ for the RCN strategy are computed by solving the approximating problem (2.18) that includes 100 and 150 scenarios. The CPU times for computing each candidate solution are also reported in table 2.4.

Table 2.3 - Lower bound estimation results for the optimal value ($n_g = 30$ batches)

Batch size (n)	30	100
Average ($\bar{Z}_{n,n_g}$)	1,923,901	1,924,380
SD ($s_{\bar{Z}_{n,n_g}}$)	4,730	4,068

Table 2.4 - Optimality gaps for candidate solutions

Candidate solution	$\bar{X}^{100}$	$\bar{X}^{150}$
Batch size (n)	30	100
No. of batches (n_g)	30	30
Point estimate ($\bar{G}_{n_g}$)	1710	918
Error estimate ($\alpha = 95\%$) ($\tilde{\varepsilon}_g$)	737	284
Confidence interval (95%)	[0 , 2447]	[0 , 1202]
CPU time (minutes)	20	25

As it can be observed from Table 2.4, by increasing the sample size, the quality of approximate solutions improves and tighter confidence intervals for the optimality gaps of candidate solutions are constructed. Finally we can conclude that, by considering a moderate number of scenarios (150 scenarios) among the potential enormous number of scenarios, we obtain an approximate solution in a reasonable amount of time with an optimality gap of [0, 1202] which is about 0.006% of the lower bound of the real optimal value (see Tables 2.3). Thus, this solution can be accepted as a good approximation to the optimal solution of the original stochastic model (2.13)-(2.17).

2.9.3. Comparison between the stochastic and deterministic sawmill production planning models

In this section, the results of comparison between the two-stage stochastic and mean-value deterministic sawmill production planning models, through Monte Carlo simulation (see section 2.8), are provided. The comparison is carried out for the sawmill example described in 2.9.1. Four different demand levels (D1, D2, D3, D4) are considered, where D2=2 $\times$ D1, D3=3 $\times$ D1, D4=4 $\times$ D1. For each demand level, 60 demand scenarios are generated randomly which are distinguished by the distribution of total demand between different products. Hence, a total of 240 (4 $\times$ 60) test problems are solved by the deterministic and stochastic models. The simulation of implementing the production plans proposed by the deterministic and the stochastic models is run for 1000 replications. The expected total backorder size computed in 1000 replications is used to compute the key indicators of performances (see section 2.8) for the test problems.

Table 2.5 includes the mean and standard deviation (SD) of the backorder gap (BO gap) as well as the plan precision computed for the 60 test problems, corresponding to the 60 demand scenarios, in each of the 4 demand levels. It would be worth mentioning that the values of BO gap and plan precision presented in table 2.5 are computed as follows:

BO gap $= 100 \times (BO_D - BO_s) / BO_D$

Plan precision $= 100 \times (BO_{sim} - BO_{plan}) / BO_{sim}$

where

BO_D : The expected realized total backorder size of the deterministic model after plan implementation (computed through Monte Carlo simulation)

BO_S : The expected realized total backorder size of the stochastic model after plan implementation (computed through Monte Carlo simulation)

BO_{sim} : The expected realized total backorder size after plan implementation (computed through Monte Carlo simulation)

BO_{plan} : Total backorder size determined by the production planning model

To illustrate how the results in table 2.5 can be interpreted, the following examples are provided. The mean of BO gap for 60 demand scenarios in the demand level D1, which is determined as 75% in table 2.5 can be interpreted as follows: the expected total backorder size of stochastic model plan (computed through Monte Carlo simulation) is 75% smaller than the expected total backorder size of deterministic model plan (computed through Monte Carlo simulation). Thus, it can be concluded that the stochastic model outperforms the deterministic model in proposing the production plans with lower total backorder size.

Table 2.5 - Comparison between the deterministic and stochastic sawmill production planning models

Sawmill production planning model	D1				D2				D3				D4			
	BO gap		Plan precision		BO gap		Plan precision		BO gap		Plan precision		BO gap		Plan precision	
	Mean (%)	SD (%)	Mean (%)	SD (%)	Mean (%)	SD (%)	Mean (%)	SD (%)	Mean (%)	SD (%)	Mean (%)	SD (%)	Mean (%)	SD (%)	Mean (%)	SD (%)
Deterministic	-	-	145	350	-	-	35	138	-	-	7	93	-	-	-13	40
Two-stage Stochastic	75	21	-40	19	58	33	-34	15	39	25	-26	16	31	21	-19	10

The mean of plan precision for 60 demand scenarios in the demand level D3 in the stochastic model, which is indicated as -26%, can be interpreted as follows: The expected total backorder size of production plan proposed by the stochastic model (computed through Monte Carlo simulation) is 26% smaller than the expected total backorder size that was determined by this model. A positive value for plan precision indicates that the realized total backorder size through Monte Carlo simulation is larger than the planned total backorder size, as in the case of the deterministic model for demand levels D1, D2 and D3.

We next analyze the results provided in table 2.5 to compare the performance of the deterministic and the stochastic sawmill production planning models in terms of their expected total backorder size as well as their plan precision. Figure 2.4 compares the mean backorder gap (BO gap) between the stochastic and the deterministic models, for the four demand levels. As it can be observed in table 2.5 and figure 2.4, the production plan proposed by the stochastic model results smaller expected total backorder size (after implementing the plan) than the deterministic model plan, for the four demand levels. However, the gap between the expected total backorder size of stochastic model plan and the deterministic model plan decreases, as demand increases. This should make no surprise. As we mentioned before, the sawmill example is a medium capacity sawmill where thousands of logs are sawn in each period in the planning horizon. By the law of large numbers (LLN) in statistics, as demand increases, the average yield of each process in each period, observed through Monte Carlo sampling in the plan implementation simulator, will be closer to its expected value, which is considered in the deterministic model.

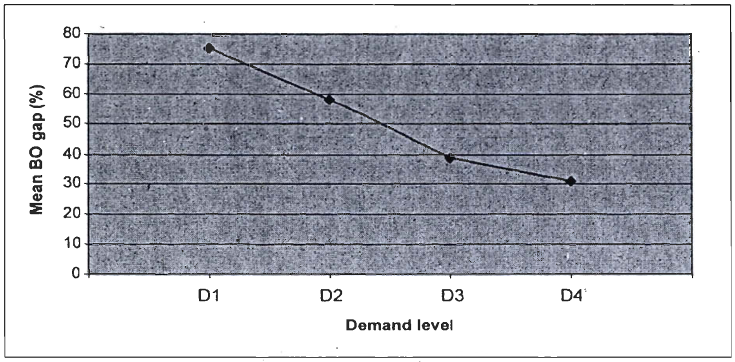


Figure 2.4 - Mean backorder gap (BO gap) of the stochastic and deterministic sawmill production planning models

Figure 2.5 compares the mean of plan precision of the stochastic model and the deterministic model, for the four demand levels. As it can be observed in table 2.5 and figure 2.5, the precision of the production plan proposed by the stochastic model is higher than the deterministic one, for the four demand levels. As the demand increases, the average yield, observed after implementing the plan through Monte Carlo simulation, gets close to the average yield scenarios considered in the stochastic model. Hence the precision of plans of the stochastic model improves for the larger volumes of demand. For the lower demand levels, the stochastic model proposes relatively pessimistic plans. On the other hand, the deterministic model provides the optimistic plans for demand levels D1, D2 and D3, since it does not take into account different scenarios for random yield. However, as the demand increases, the average yield of each process in each period, observed through Monte Carlo simulation, gets closer to its expected value which is used in the deterministic model. Thus, the precision of deterministic model plan increases, as the demand increases.

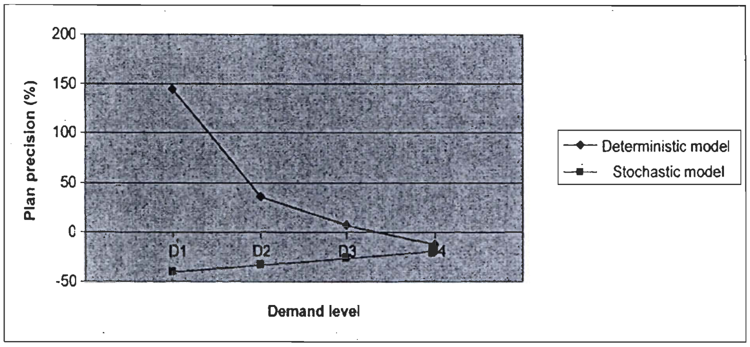


Figure 2.5 - Plan precision comparison of the deterministic and stochastic sawmill production planning models

Regarding to the above comparisons, it is clear that the two-stage stochastic model provides more realistic production plans in sawmills, in terms of the realized backorder size, than the mean value deterministic model. The deterministic model provides optimistic plans, since it considers the deterministic yields (expected values). As the stochastic model considers different scenarios for random yields and finds a production plan with minimum expected backorder and inventory size for all the yield scenarios, the production plans provided by this model are more realistic.

2.10. Conclusions

In this paper, we developed a two-stage stochastic programming model for MPMP production planning under the uncertainty of processes yields. The proposed model was applied for sawmill production planning by considering random characteristics of logs. The SAA method was implemented to solve the stochastic model which provided us an efficient framework for identifying and statistically testing a variety of candidate production plans. We also proposed a validation approach to compare the plans proposed by the stochastic and deterministic sawmill production planning models, which is based on Monte Carlo simulation. We provided the empirical results for production planning in a medium capacity prototype sawmill and we identified the candidate plans in a reasonable amount of time by solving the approximate SAA problem. Furthermore, the confidence intervals for the optimality gap of candidate solutions were constructed by common random number (CRN) streams. The comparison between the two-stage stochastic and deterministic sawmill production planning models was carried out for 4 demand levels. Our results revealed that the production plans proposed by the stochastic model are more realistic than those obtained by traditional mean-value deterministic model. Although these results are found for sawmill production planning, the proposed approach in this work can be applied for production planning in other manufacturing environments where non-homogeneous and random characteristics of raw materials result in random yield. Future research will consider in the stochastic model the decision maker's risk preferences towards the cost of different scenarios in addition to their expected cost. Furthermore, by considering also the products demands as random variables, more realistic production plans can be obtained.

2.11. Acknowledgments

The authors would like to acknowledge the financial support provided by the Forest E-business Research Consortium (FOR@C) of université Laval, and would like to thank specially, Jonathan Gaudreault, Philippe Marier, Sébastien Lemieux, and Christian Rouleau, for their technical support.

References

Bakir, M.A. and Byrne, M.D., 1998. Stochastic linear optimization of an MPMP production planning model. *International Journal of Production Economics*, 55, 87-96.

Birge, J.R. and Louveaux, F., 1997. *Introduction to stochastic programming*. New York: Springer.

Ermoliev, Y., 1983. Stochastic quasigradient methods and their applications to system optimization. *Stochastics*, 9, 1-36.

Escudero, L.F., Kamesam, P.V., King, A.J. and Wets, R.J-B., 1993. Production planning via scenario modeling. *Annals of Operations Research*, 43, 311-335.

Gaudreault, J., Rousseau, A., Frayret, J.M. and Cid, F., 2004. Planification opérationnelle du sciage. *For@c technical document*, Quebec (Canada).

Higle, J.L. and Sen, S., 1996. *Stochastic decomposition: an algorithm for two-stage linear programming*. Boston: Kluwer Academic Publishers.

Johnson, L.A. and Montgomery D.C., 1974. *Operations research in production planning, scheduling, and inventory control*. New York: John Wiley & Sons.

Kall, P. and Wallace, S.W., 1994. *Stochastic programming*. New York: John Wiley & sons.

Kall, P. and Mayer J., 2005. *Stochastic linear programming*. New York: Springer's International Series.

Kazemi Zanjani, M., Nourelfath, M. and Ait-Kadi, D., 2007. Production planning for simultaneous production of multiple products by alternative processes with random yield. In: *Proceeding of 7e Congrès international de génie industriel (2007);* Trois-Rivières, Canada.

Linderoth, J. and Wright, S., 2003. Decomposition algorithms for stochastic programming on a computational grid. *Computational optimization and applications*, 24, 207-250.

Linderoth, J. and Shapiro A., 2006. The empirical behavior of sampling methods for stochastic programming. *Annals of Operations Research*, 142, 214-241.

Mak, W.K., Morton, D.P. and Wood R.K., 1999. Monte Carlo bounding techniques for determining solution quality in stochastic programs. *Operations research letters*, 24, 47-56.

Maness, T.C. and Adams, D.M., 1991. The combined optimization of log bucking and sawing strategies. *Wood and Fiber Science*, 23, 296-314.

Maness, T.C. and Norton, S.E., 2002. Multiple-period combined optimization approach to forest production planning. *Scandinavian Journal of Forest Research*, 17, 460-471.

Mendoza, G.A., Meimban, R.J., Luppold, W.J. and Arman P.A., 1991. Combined log inventory and process simulation models for the planning and control of sawmill operations. In: *Proceeding 23rd GIRP International Seminar on Manufacturing Systems;* Nancy, France.

Rönnqvist M., 2003. Optimization in forestry. *Mathematical programming, Ser. B*, 97, 267-284.

Ruszczyński, A. and Świetanowski, A., 1996. On the regularized decomposition method for two-stage stochastic linear problems. *IIASA working paper;* Laxenburg, Austria.

Shapiro A. and Hommem-de-Mello T., 1998. A simulation based approach to two-stage stochastic programming with recourse. *Mathematical Programming*, 81, 301-325.

Shapiro A. and Hommem-de-Mello T., 2000. On the rate of convergence of optimal solutions of Monte Carlo approximations of stochastic programs. *SIAM Journal of Optimization*, 11 (1), 70-86.

Sipper D. and Bulfin R., 1997. *Production: planning, control, and integration*. New York: McGraw-Hill.

Chapter 3

Robust production planning in a manufacturing environment with random yield: A case in sawmill production planning

The article entitled "*Robust production planning in a manufacturing environment with random yield: A case in sawmill production planning*" is included in this chapter. It has been accepted on March 2009 for publication in the "*European Journal of Operational Research*". The version presented in the thesis is identical to the final corrected version sent to the editor for publication.

3.1. Abstract

This paper addresses a multi-period, multi-product sawmill production planning problem where the yields of processes are random variables due to the non-homogeneous quality of raw materials (logs). In order to determine the production plans with robust customer service level, robust optimization approach is applied. Two robust optimization models with different variability measures are proposed, which can be selected based on the tradeoff between the expected backorder/inventory cost and the decision maker risk aversion level about the variability of customer service level. The implementation results of the proposed approach for a realistic-scale sawmill example highlights the significance of using robust optimization in generating more robust production plans in the uncertain environments compared with stochastic programming.

3.2. Introduction

Production planning in many manufacturing environments is based on some parameters with uncertain values. Uncertainties might arise in product demand, yield of processes, etc. Thus, the robustness of a production plan, in term of fulfillment of product demand, is dependent on incorporating the uncertain parameters in production planning models.

This study is concentrated on multi-period, multi-product (MPMP) production planning in the sawing units of sawmills, where possible combinations of log classes and cutting patterns produce simultaneously different mix of lumbers. As logs are grown under uncertain natural circumstances, non-homogeneous and random characteristics (in terms of diameter, number of knots, internal defects, etc.) can be observed in different logs in each class. Consequently, the processes yields (quantities of lumbers that can be produced by each cutting pattern) are random variables. Lumber demand in each period is assumed as a deterministic parameter which is determined based on the received orders. That is, we do not deal with "seasonality" or "trend-based" demand in this work. In the sawmill production planning problem, we are looking for the optimal combination of log classes and cutting patterns that best fit against lumber demand. The part of demand that cannot be fulfilled on time, due to machine capacities and/or uncertain yield, will be postponed to the next period by considering a backorder cost. The objective is to minimize products inventory and backorder costs and raw material consumption cost, regarding fulfillment of product demand, machine capacities, and raw material (log) inventory. The uncertainty in the yields of cutting patterns in sawmills can be represented as uncertain yield coefficients in the coefficients of constraints matrix. Regarding the potential significance of yield uncertainty on the production plan, and customer orientation which is at center of attention in the sawmills which are dependent on the export markets, obtaining robust plans with minimum backorder size (service level) variability is an important goal of production planning in sawmills.

Sawmill production planning problem can be considered as the combination of several classical production planning problems in the literature which have been modeled by linear programming (LP). This problem was formulated by a deterministic LP model and was solved based on the average values for processes yields in Gaudrault et al. (2004). However, if decisions are made based on the deterministic model, there is a risk that the demand might not be met with the right products. Consequently, this results in high inventory levels of products with low quality and price, as well as extra levels of backorder of products with high quality and price (decreased customer service level). The other approach in the literature for sawmill production planning is focused on combined optimization type solutions linked to real-time simulation sub-systems (Maness and Norton, 2002; Maness and Adams, 1991; Mendoza et al., 1991). In this approach, the stochastic characteristics of logs are taken into account by assuming that all the input logs are scanned through an X-ray scanner, before planning. Maness and Norton (2002) developed an integrated multi-period production planning model which is the combination of an LP model and a log sawing optimizer (simulator). The LP model acts as a coordinating problem that allocates limited resources. The log sawing optimization models are used to generate columns for the coordinating LP based on the products' shadow prices. Although the stochastic characteristics of logs are considered in this approach, it includes the following limitations: logs, needed for the next planning horizon, are not always available in sawmills to be scanned before planning. Furthermore, to implement this method, the logs should be processed in production line in the same order they have been simulated, which is not an easy practice. Finally scanning logs before planning is a time consuming process in the high capacity sawmills which delays the planning process.

There are several techniques to incorporate uncertainty in optimization models, including stochastic programming (Kall and Wallace, 1994; Birge and Louveaux, 1997; Kall and Mayer, 2005), and robust optimization (Mulvey et al., 1995). Bakir and Byrne (1998) developed a stochastic LP model based on the two-stage deterministic equivalent problem to incorporate demand uncertainty in a multi-period multi-product (MPMP) production planning model. In Escudero et al. (1993) a multi-stage stochastic programming approach was proposed for solving a MPMP production planning model with random demand. Kazemi et al. (2007b; 2008) proposed a two-stage stochastic model for sawmill production; it was shown in Kazemi et al. (2007b; 2008) that the production plans proposed by stochastic programming approach results a considerably lower expected inventory and backorder cost than the plans of the mean-value deterministic model. It is important to note that stochastic programming approach focuses on optimizing the expected performance (e.g., cost) over a range of possible scenarios for the random parameters. We can expect that the system would behave optimally in the mean sense. However, the system might perform poorly at a particular realization of scenarios such as the worst-case scenario. More precisely, unacceptable inventory and backorder size might be observed for some scenarios when implementing the solution of two-stage stochastic model. To handle the tradeoff associated with the expected cost and its variability in stochastic programs, Mulvey et al. (1995) proposed the concept of robust optimization. Leung and Wu. (2004) proposed a robust optimization model for stochastic aggregate production planning. In Leung et al. (2007) a robust optimization model was developed to address a multi-site aggregate production planning problem in an uncertain environment. In Kazemi et al. (2007a) robust optimization approach was proposed as one of the potential methodologies to address MPMP production planning in a manufacturing environment with random yield.

In this paper, a robust optimization (RO) approach is proposed for multi-period sawmill production planning while considering the random characteristics of raw materials (logs) and consequently random processes yields. The random yields are modeled as scenarios with a stationary discrete probability distribution during the planning horizon. We are studying a service sensitive company that wants to establish a reputation for always meeting customer service level. We also define the customer service level as a proportion of the customer demand that can be fulfilled on time, and we use the expected backorder size as a measure for evaluating the service level. Thus, the need for robustness has been mainly recognized in term of determining a robust customer service level by minimizing the products backorder size variability in the presence of different scenarios for random yields. The robustness in the products inventory size is also considered in this problem. Two alternative variability measures are used in the robust optimization model which can be selected depending on risk aversion level of decision maker about backorder/inventory size variability and the total cost. The proposed robust optimization (RO) approach is applied for a realistic-scale sawmill production planning example. The resulting large-scale quadratic programming models are solved by CPLEX 10 in a reasonable amount of time. A comparison between the backorder/inventory size variability in the two-stage stochastic model and the two robust optimization models is provided. Finally, the tradeoff between the backorder/inventory size variability and the expected total cost in the two RO models is discussed and a decision framework to select among them is proposed.

The main contributions of this paper can be summarized as follows. Applying robust optimization approach as a robust tool for sawmill production planning, regarding to the limitations of the existing approaches for sawmill production planning; comparing the performance of two different robust optimization models in controlling the robustness of production plans through applying them for a prototype sawmill; proposing a framework for selecting the most appropriate robust optimization model depending on the risk preferences of the decision maker about service level robustness and total expected cost of plans.

The rest of this paper is organized as follows. In section 3.3, sawmill processes and specific characteristics are introduced. In section 3.4, the robust optimization formulation for two-stage stochastic programs is provided. In section 3.5, the proposed robust optimization model for multi-period sawmill production planning is presented. In section 3.6, the scenario generation approach for random yields is described. In section 3.7, the computational results of implementing the proposed robust optimization models for a prototype sawmill are provided. Our concluding remarks are given in section 3.8.

3.3. Sawmill processes and specific characteristics

There are a number of processes that occur at a sawmill: log sorting, sawing, drying, planing and grading (finishing). Raw materials in sawmills are the logs which are transported from different districts of forest after bucking the felled trees. The finished and graded lumbers (products) are then transported to the domestic and international markets. Figure 3.1 illustrates the typical processes. In this paper we focus on operational level production planning in the sawing units of sawmills. In the sawing units, logs are classified according to some attributes namely: diameter class, species, length, taper, etc. Logs are broken down into different dimensions of lumbers by means of different cutting patterns. See figure 3.2 for three different cutting patterns. Each cutting pattern is a combination of activities that are run on a set of machines. From each log, several pieces of sawn lumber (e.g. $2(in)\times 4(in)\times 8(ft)$, $2(in)\times 4(in)\times 10(ft)$, $2(in)\times 6(in)\times 16(ft),\ldots$) are produced depending on the cutting pattern. The lumber quality (grade) as well as its quantity yielded by each cutting pattern depends on the quality and characteristics of the input logs. Despite the classification of logs in sawmills, a variety of characteristics might be observed in different logs in each class. In fact, natural variable conditions that occur during the growth period of trees make it impossible to anticipate the exact yields of a log. As it is not possible in many sawmills to scan the logs before planning, the exact yields of cutting patterns for different log classes cannot be determined in priori.

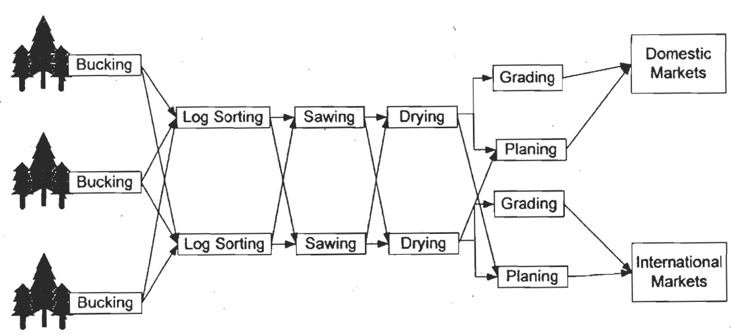


Figure 3.1 - Illustration of sawmills processes (after Rönnqvist, 2003)

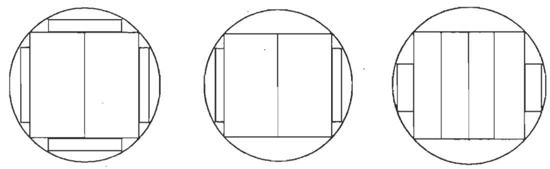


Figure 3.2 - Cutting patterns in a sawmill

3.4. Robust optimization formulation for two-stage stochastic programs

The robust optimization method developed by (Mulvey et al., 1995) extends stochastic programming by replacing traditional expected cost minimization objective by one that explicitly addresses cost variability.

Consider the following LP model that includes random parameters:

$$\textit{Minimize } c^T x \tag{3.1}$$

Subject to

$$Ax = b, \tag{3.2}$$

$$Bx \leq e, \tag{3.3}$$

$$x \geq 0, \tag{3.4}$$

where x denotes the vector of decision variables that should be determined under the uncertainty of model parameters. B and e represent the random technological coefficient matrix and the right-hand side vector, respectively. Assume a finite set of scenarios $\Omega = \{1,2,...,S\}$ to model the uncertain parameters; with each scenario $s \in \Omega$ we associate the subset $\{d^s, B^s, C^s, e^s\}$ and the probability of the scenario p^s, ($\sum_{s=1}^{S} p^s = 1$). The standard two-stage stochastic linear program is formulated as follows.

SP:

$$\textit{Minimize } c^T x + \sum_{s \in \Omega} p^s d^{sT} y^s \tag{3.5}$$

Subject to

$$Ax = b, \tag{3.6}$$

$$B^s x + C^s y^s = e^s, \qquad s \in \Omega, \tag{3.7}$$

$$x, y^s \geq 0, \qquad s \in \Omega, \tag{3.8}$$

where x denotes the vector of first-stage decision variables whose optimal value is not conditioned on the realization of uncertain parameters, y^s denotes the vector of second-stage (recourse) decision variables, corresponding to scenario s, that are subject to adjustment once the uncertain parameters are observed. C^s and d^s denote the recourse matrix and the penalty recourse cost vector corresponding to scenario s, respectively. The optimal solution of model (3.5)-(3.8) will be robust with respect to optimality if it remains close to optimal for any of the scenarios $s \in \Omega$. This is termed *solution robustness*. In other words, the solution robustness measures the variability of the recourse cost in model SP for any of the scenarios $s \in \Omega$. The solution is also robust with respect to feasibility if it remains almost feasible for all scenarios. This is termed *model robustness*. The robust optimization (RO) framework introduced by Mulvey et al. (1995) is a goal programming approach to balance the tradeoffs between solution robustness and model robustness. Hence, the RO approach modifies the objective in SP as follows.

RO:

$$\textit{Minimize } c^T x + \sum_{s\in\Omega} p^s d^{sT} y^s + \lambda\sigma(y^1,...,y^s) + \omega\rho(\delta^1,...,\delta^s) \qquad (3.9)$$

Subject to

$$Ax = b, \qquad (3.10)$$

$$B^s x + C^s y^s + \delta^s = e^s, \qquad s \in \Omega, \qquad (3.11)$$

$$x, y^s \geq 0, \qquad s \in \Omega. \qquad (3.12)$$

The term ($\sum_{s\in\Omega} p^s d^{sT} y^s + \lambda\sigma(y^1,...,y^s)$) in the objective function denotes the solution robustness measure, where $\lambda \geq 0$ is a goal programming weight and $\sigma(y^1,...,y^s)$ denotes the recourse cost variability measure. By changing λ, the relative importance of the expectation and variability of the recourse cost in the objective can be controlled. The last term in the objective function $\rho(\delta^1,...,\delta^s)$ is a feasibility penalty function, which is used to penalize the violation of constraints (3.11) (denoted by δ^s) under some of the scenarios. ω is a goal programming weight. In the following, the recourse cost variability measures existing in the literature, as well as the measures that we use in this work, are presented.

3.4.1. Variability measures in robust optimization models

The classical approach to model the tradeoff between the expectation and the variability in RO models is to use mean-variance model of Markowitz (1959) which has been implemented in many applications, namely capacity expansion of power systems (Malcolm and Zenios, 1994), stochastic logistic problems (Yu and Li , 2000), stochastic aggregate production planning (Leung and Wu., 2004; Leung et al., 2007). However, there are some exceptions against using mean-variance in some applications: variance is a symmetric risk measure, penalizing equally the cost both above and below the expected recourse cost. As in the case of production planning it is more convenient to use an asymmetric risk measure that would penalize only costs above the expected value. Shabbir and Shahinidis (1998) proposed to use the upper partial mean of the recourse cost as the measure of variability in a robust optimization model for process planning under uncertainty. In List et al. (2003) an upper partial moment (UPM) of order 1 was used in a robust optimization model for fleet planning under uncertainty. Takriti and Shabbir (2004) used the upper partial moment of order 2 for robust optimization of two-stage stochastic models.

3.4.2. Proposed variability measures

As we have already mentioned, in the production planning problem that we are addressing, using the symmetric mean-variance tradeoff for recourse cost can generate solutions that are inefficient and which would not be considered by a rational manager. Regarding the proposed asymmetric variability measures in the literature and the recent developments in optimization solvers, namely CPLEX 10, which have made it possible to solve large-scale quadratic programs in a reasonable amount of time, we propose two variability measures of recourse costs in this problem, namely the upper partial moment of order 2 (UPM-2), and the upper partial variance (UPV).

3.4.2.1. Upper partial moment of order 2 (UPM-2)

The upper partial moment of order 2 (used also in Takriti and Shabbir, 2004) is defined as follows.

$$\bar{\Delta}_+^2 = \sum_{s\in\Omega} p^s \Delta_+^{s^2}, \tag{3.13}$$

where

$$\Delta_+^s = \max\left\{0, (d^{sT} y^s - R^*)\right\}, \tag{3.14}$$

and R^* is the target recourse cost. For scenario s, Δ_+^{s2} is the squared positive deviation of that scenario's recourse cost from the target recourse cost. In this way, $\bar{\Delta}_+^2$ is defined as the expectation of the squared positive deviations over all scenarios.

3.4.2.2. Upper partial variance (UPV)

The upper partial variance is the quadratic version of upper partial mean (UPM) of Shabbir and Shahinidis (1998). It is defined as follows.

$$\bar{\Delta}_+^2 = \sum_{s\in\Omega} p^s \Delta_+^{s2}, \tag{3.15}$$

where

$$\Delta_+^s = \max\left\{0, (d^{sT} y^s - \sum_{s\in\Omega} p^s d^{sT} y^s)\right\}. \tag{3.16}$$

For scenario s, Δ_+^{s2} is the squared positive deviation of that scenario's recourse cost from the expected recourse cost. In this way, $\bar{\Delta}_+^2$ is defined as the expectation of the squared positive deviations over all scenarios. It should be mentioned that the advantage of UPV variability measure over the (UPM-2) is that UPV does not require a priori specification of a target recourse cost and therefore is more flexible.

3.5. Robust optimization model for multi-period sawmill production planning

Consider a sawing unit with a set of products (lumbers) P, a set of classes of raw materials (logs) C, a set of production processes A, a set of machines R, a planning horizon consisting of T periods, and a scenario set $\Omega = \{1, 2, ..., N\}$ for random processes yields. For modeling simplicity, we define a production process in a sawing unit as a possible combination of a log class and a cutting pattern. The (first-stage) decision variable is the number of times each process should be run in each period (production plan X_{at}). This is equivalent to finding log consumption of each log class as well cutting pattern selection for each log class in each period. The production plan X_{at} cannot anticipate the yield scenarios and must be feasible for all of the scenarios. Inventory (I^i_{pt}) and backorder (B^i_{pt}) size of each product in each period are the recourse decision variables that can be determined based on the first-stage production plan and the realized scenarios for processes yields. To state the robust optimization model for this production planning problem, the following notations are used:

3.5.1. Notations

Indices

p product (lumber)

t period

c raw material (log) class

a production process (combination of a log class and a cutting pattern)

r machine

i scenario

Parameters

h_{pt} inventory holding cost per unit of product p in period t

b_{pt} backorder cost per unit of product p in period t

m_{ct} raw material cost per unit of class c in period t

I_{c0} the inventory of raw material class c at the beginning of planning horizon

I_{p0} the inventory of product p at the beginning of planning horizon

s_{ct} the quantity of raw material of class c supplied at the beginning of period t

d_{pt} demand of product p by the end of period t

ϕ_{ac} the units of class c raw material consumed by process a (consumption factor)

ρ^i_{ap} the units of product p produced by process a (yield of process a) for scenario i

p^i the probability of scenario i

δ_{ar} the capacity consumption of resource r by process a

M_{rt} the capacity of resource r in period t

N number of yield scenarios

λ goal programming parameter ($\lambda \geq 0$)

R^* target inventory/backorder cost

Decision variables

X_{at} the number of times each process a should be run in period t

I_{ct} inventory size of raw material of class c by the end of period t

I^i_{pt} inventory size of product p by the end of period t for scenario i (recourse decision variable)

B^i_{pt} backorder size of product p by the end of period t for scenario i (recourse decision variable)

Δ^i_+ the variability measure of inventory and backorder cost for scenario i

3.5.2. The robust optimization model

$$Minimize\ Z = \sum_{c\in C}\sum_{t=1}^{T}\sum_{a\in A} m_{ct}\phi_{ac}X_{at} + \sum_{i=1}^{N}\sum_{p\in P}\sum_{t=1}^{T} p^i[h_{pt}I^i_{pt} + b_{pt}B^i_{pt}] + \lambda\sum_{i=1}^{N} p^i\Delta^{i^2}_+ \qquad (3.17)$$

Subject to

Material inventory constraint

$$I_{ct} = I_{ct-1} + s_{ct} - \sum_{a\in A}\phi_{ac}X_{at}, \qquad t=1,\dots,T,\ c\in C, \qquad (3.18)$$

Production capacity constraint

$$\sum_{a\in A}\delta_{ar}X_{at} \leq M_{rt}, \qquad t=1,2,\dots,T,\ r\in R, \qquad (3.19)$$

Product inventory constraint

$$I^i_{p1} - B^i_{p1} = I_{p0} + \sum_{a\in A}\rho^i_{ap}X_{a1} - d_{p1}, \qquad (3.20)$$

$$I^i_{pt} - B^i_{pt} = I^i_{pt-1} - B^i_{pt-1} + \sum_{a\in A}\rho^i_{ap}X_{at} - d_{pt}, \qquad t=2,\dots,T,\ p\in P,\ i=1,\dots,N,$$

Recourse cost variability

$$\Delta_+^i \geq \sum_{p\in P}\sum_{t=1}^{T}(h_{pt}I_{pt}^i + b_{pt}B_{pt}^i) - \sum_{i'=1}^{N}\sum_{p\in P}\sum_{t=1}^{T} p^{i'}(h_{pt}I_{pt}^{i'} + b_{pt}B_{pt}^{i'}), \quad i=1,\dots,N, \quad \text{(RO-UPV)} \quad (3.21)$$

$$\Delta_+^i \geq \sum_{p\in P}\sum_{t=1}^{T}(h_{pt}I_{pt}^i + b_{pt}B_{pt}^i) - R^*, \quad i=1,\dots,N, \quad \text{(RO-(UPM-2))}$$

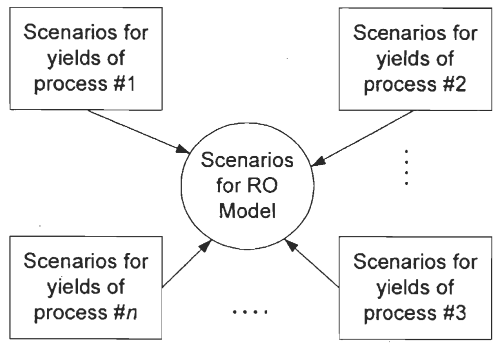
Non-negativity of all variables

$$X_{at} \geq 0,\ I_{ct} \geq 0,\ I_{pt}^i \geq 0,\ B_{pt}^i \geq 0, \Delta_+^i \geq 0, \quad c\in C,\ p\in P, t=1,\dots,T, a\in A,\ i=1,\dots,N. \quad (3.22)$$

The objective function (3.17) is to minimize the raw material consumption cost, the expected inventory and backorder costs, in addition to inventory and backorder cost variability for all yield scenarios in the planning horizon. The inventory and backorder costs are computed by multiplying the inventory and backorder unit cost by the inventory and backorder size, respectively. As it was mentioned in section 3.4, λ is a goal programming parameter that models the tradeoff between the expectation and variability of the recourse cost in the objective function. For $\lambda = 0$, model (3.17)-(3.22) would be the two-stage stochastic model in Kazemi et al. (2007b; 2008). Constraint (3.18) ensures that the total inventory of raw material of class c at the end of period t is equal to its inventory in the previous period plus the quantity of material of class c supplied at the beginning of that period (s_{ct}) minus its total consumption in that period. It should be noted that the total consumption of each class of raw material in each period is calculated by multiplying material consumption factor of each process (ϕ_{ac}) by the number of times that process is executed in that period. Constraint (3.19) requires that the total production do not exceed the available production capacity. In other words, the sum of capacity consumption of a machine r by the corresponding processes in each period should not be greater than the capacity of that machine in that period. Constraints (3.20) ensure that the sum of inventory (or backorder) of product p at the end of period t is equal to its inventory (or backorder) in the previous period plus the total production of that product in that period, minus the product demand for that period. Total quantity of production for each product in each period is calculated as the sum of the quantities yielded by each of the corresponding processes regarding the yield (ρ_{ap}) of each process. Due to the randomness of process yields (ρ_{ap}), these constraints are defined for each scenario of the processes yields. Constraints (3.21) compute the inventory and backorder cost variability for each scenario. Depending on the type of variability measure that is used in the RO model, the mentioned cost variability is defined as follows. In the RO-UPV model (see section 3.4) it is defined as the difference between the total inventory and backorder cost of each scenario and the expected inventory and backorder cost, while in the RO-(UPM-2) (see section 3.4) it denotes the difference between the total inventory and backorder cost of each scenario and a target inventory and backorder cost (R^*). Note that constraints (3.21) and the non-negativity of Δ_+^s together with the minimization in the objective function satisfy the definition of upper partial moment of order 2 ((3.13) and (3.14)) and upper partial variance ((3.15) and (3.16)).

3.6. Scenario generation

In this section, we explain how the scenarios for random processes yields (ρ_{ap}^i) can be generated in the RO model. We define a scenario in this model as the combinations of scenarios for yields of individual processes. We suppose that the yields of different processes are independent. Therefore, as the first step, all possible scenarios for yields of each process should be determined and then these scenarios should be aggregated to generate the scenarios for the RO model. This approach is illustrated in figure 3.3. A scenario for the yields of process (*a*) (combination of a log class (*c*) and a cutting pattern (*s*)) in a sawing unit is defined as possible quantities of lumbers that can be produced by cutting pattern (*s*) after sawing each log of class (*c*). As an example of the uncertain yields in sawmills, consider the cutting pattern (*s*) that can produce 6 products (P1, P2, P3, P4, P5, P6) after sawing the logs of class (*c*). Table 3.1 represents four scenarios among all possible scenarios for the uncertain yield of this process.



n: number of individual processes

Figure 3.3 - Scenario generation approach in the robust optimization model

Table 3.1 - Scenarios for yields of a process in a sawing unit

Scenarios	Products					
	P1	P2	P3	P4	P5	P6
1	1	0	1	0	1	1
2	2	1	1	0	1	0
3	1	0	0	1	1	1
4	2	0	0	1	0	1

In this work, we assume that all the logs that will be processed in the next planning horizon are supplied from the same discrete of forest. Hence, a stationary probability distribution can be considered for the quality of logs and uncertain processes yields during the planning horizon. Regarding the limited volume of logs and dimensions of lumbers, we assume a discrete probability distribution for processes yields. Furthermore, due to the wide variety of characteristics in each log class a huge number of scenarios for processes yields can be expected. The scenarios for processes yields with their probability distributions in sawmills can be determined as follows.

(1) Take a sample of logs in each class (e.g. 300 logs) and let them to be processed by each cutting pattern.

(2) Register the yield of the process (the corresponding products with their quantity) for each individual log and consider the result as a scenario.

(3) Having observed all the scenarios, calculate their probabilities as their proportion in the population of scenarios.

It should be noted that the implementation of the above approach is very difficult in sawmills. In fact, the high production speed in the sawing unit makes it difficult to track the logs through the line and to observe the result of sawing individual logs. In this paper we use yield scenarios generated by a log sawing simulator which will be discussed more in 3.7.2.

3.7. Computational results

In this section, we describe the characteristics of the prototype sawmill, scenario generation approach for uncertain processes yields, and some implementation details. We also provide the results of implementing the proposed robust optimization models for the sawmill example. We compare the recourse cost variability in the two RO models and two-stage stochastic one. We also discuss about the performance of the two robust optimization models in controlling backorder/inventory size variability and provide a framework to choose among them depending on the decision maker's risk preference.

3.7.1. Example description

A prototype sawmill is used to illustrate the use of the two robust optimization models. The prototype sawmill is a typical softwood sawmill located in Quebec (Canada). The sawmill focuses on sawing high-grade products to the domestic markets as well as export products to the USA. It is assumed that the input bucked logs into the sawing unit are categorized into 3 classes based on their two ends diameters. 5 different cutting patterns are available. The sawing unit produces 27 products of custom sizes (e.g. 2(in)$\times$4(in), 2(in)$\times$6(in) lumbers) in four lengths. In other words, there are 15 processes all can produce 27 products with random yields. We consider two bottleneck machines: Trimmer and Bull. The planning horizon consists of 30 periods (days). Product demand in each period is assumed to be deterministic which is determined based on the received orders. Lumbers that remain from one period to the next are subject to a unit holding cost. The unsatisfied demand is penalized by a unit backorder cost. We assume that the company is very service sensitive and wishes to fulfill customer demands on time as much as possible. Hence, the inventory costs of products are considered much lower than their backorder costs. The inventory holding cost is computed by multiplying the interest rate (per period) by the lumber price;

the lumber price is considered as the backorder cost. It would be worth mentioning that the data used in this example are based on the gathered data from different sawmills in Quebec province (Canada). As the list of custom sizes, machine parameters and prices are proprietary, they are not reported in this paper.

3.7.2. Scenario generation for the uncertain processes yields

In the prototype sawmill that we considered in this work, due to the lack of historical data on the yields of processes, the yield scenarios already generated by a log sawing simulator (Optitek) were used. "Optitek" was developed by a research company for Canada's solid wood products industry (Forintek Canada Corp.). "Optitek" simulates the sawing process in the sawing units of Quebec sawmills. It was developed based on the characteristics of a sample of logs in different log classes, as well as sawing rules available in Quebec sawmills. The inputs to this simulator include log class, cutting pattern, and the number of logs to be processed. The simulator considers the logs in the requested class with random physical and internal characteristics; afterwards it generates different quantity of lumbers for each log based on the sawing rules of the requested cutting pattern. The yielded lumbers of each log can then be considered as a scenario for the yields of the corresponding process.

Recall from section 3.5 that a yield scenario in the RO model is the combination of yield scenarios of all the processes in the problem. In this example we have 15 processes, each can produce 27 products. Thus, the RO model (3.17)-(3.22) includes 405 (27×15) yield coefficients ρ_{ap}. If we assume that each yield coefficient can take 5 different values, the number of scenarios for random yields in the RO model can be estimated as $5^{405} \approx 1.2 \times 10^{283}$. As solving the robust optimization model (3.17)-(3.22) for all scenarios of random yields is far beyond present computational capacities, a random sample of such scenarios is considered. Thus, we generated 250 scenarios by Monte Carlo sampling among the scenarios generated by "Optitek" for the same log classes and cutting patterns that we considered in this example. It should be noted that the same sample size for yield scenarios was used in a two-stage stochastic model for production planning in the same prototype sawmill in Kazemi et al. (2007b; 2008). Based on the sample average approximation (SAA) scheme which was applied in Kazemi et al. (2007b; 2008), by considering 250 scenarios a good approximate solution with an acceptable optimality gap can be obtained.

3.7.3. Implementation details

By considering 250 scenarios for processes yields in this example, the quadratic programming model (3.17)-(3.22) consists of 202900 constraints and 405790 decision variables. Both of the quadratic robust optimization models (RO-UPV and RO-(UPM-2)) were solved by CPLEX 10 barrier solver and all the calculations of the recourse cost variability for different values of goal programming parameter (λ) as well as threshold values (R^*) were performed by the scripts in OPL Studio 5.1. All computations were carried out on a Pentium(R) IV 1.8 GHz PC with 512 MB RAM running Windows XP.

3.7.4. Results of robust optimization approach for the sawmill example

In this section, we report the results of the implementation of the two robust optimization models for the prototype sawmill described in 3.7.1.

3.7.4.1. RO-(UPM-2) model results

Remember from section 3.4 that RO-(UPM-2) model requires a target recourse cost R^*. It should be noted that the target cost can be determined based on the desired service level. In this sawmill example, we provide the target cost as a percentage of the optimal expected backorder and inventory cost when $\lambda = 0$ (the standard two-stage stochastic program). For example, the expected backorder and inventory cost for this prototype sawmill, by considering 250 yield scenarios, without any penalty on recourse cost variability is 379367. As we mentioned before, a range of robust optimal solutions can be generated in the robust model as we change the robustness parameter λ. This parameter reflects the decision maker level of concern with exceeding the target cost for all scenarios of random yield. Table 3.2 presents the results of RO-(UPM-2) model for various $R^* - \lambda$ combinations for the sawmill example.

Table 3.2 - Results of RO-(UPM-2) model for the sawmill example - The values of λ are in multiples of 10^{-5}.

R^*	λ	Raw material cost	Expected recourse (backorder/inventory) cost	Expected backorder cost	Expected inventory cost	Recourse (backorder/inventory) cost variability ($\bar{\Delta}_+$)	CPU time (min.)
-	0	155473	379367	378560	807	-	3.3
60%	1	218392	343974	342942	1032	178009	4.5
	2	236984	340742	339662	1080	176136	4.5
	5	259574	339013	337895	1118	175070	4.5
	10	275420	338259	337121	1138	174592	4.5
	20	285092	338041	336900	1141	174474	4.5
80%	1	210971	346041	345031	1009	137648	4.5
	2	228366	342169	341110	1059	135700	4.5
	5	252038	339526	338420	1106	134252	4.5
	10	269560	338756	337640	1116	133738	4.5
	20	280942	338406	337285	1121	133533	4.5
100%	1	201327	349348	348369	979	98539	4.5
	2	219352	343931	342896	1035	96124	4.5
	5	242070	340381	339292	1089	94284	4.5
	10	258540	339528	338435	1093	93611	4.5
	20	272871	338812	337702	1110	93195	4.5
120%	1	191259	354018	353074	944	62844	4.5
	2	205886	348158	347165	993	60523	4.5
	5	227558	342977	341925	1052	58490	4.5
	10	243688	340873	339787	1085	57644	4.5
	20	257247	340047	338962	1085	57145	4.5
140%	1	181544	359445	358536	951	42515	4.5
	2	194034	353158	352207	1007	41033	4.5
	5	213167	346798	345790	1041	39364	4.5
	10	227432	344132	343091	1045	38532	4.5
	20	240147	342989	341944	951	38091	4.5

When table 3.2 provides the value of 80% in column "R^*", the target cost is $379367 \times 80\% = 303494$. In the second column in table 3.2 the values of λ are provided in multiples of 10^{-5}, since R^* =379367 and a quadratic variability measure is used in the RO-(UPM-2) model. The recourse cost variability measure in column 7 is presented as the square root of (3.13) used in RO-(UPM-2) model. The last column of table 3.2 includes CPU time (on minutes) for finding the optimal solution of RO-(UPM-2) model by CPLEX 10. As expected, for a given value of R^*, increasing λ reduces the backorder/inventory cost (size) variability. Thus, we can expect more control on the exceeding of each scenario's backorder/inventory cost over the target cost (R^*), as well as decreased expected backorder cost (size), although at the expense of increased raw material cost and the expected inventory cost (size). In other words, by enforcing the backorder/inventory cost variability measure in the objective function of model (3.17)-(3.22) (see section 3.5), the production level and consequently raw material consumption is increased in order to minimize the exceeding of backorder/inventory cost of all scenarios over the target cost. Furthermore, the increased inventory cost (size) is also the result of increasing the production level and raw material consumption. Figure 3.4 illustrates better the tradeoff between the backorder/inventory cost variability and raw material cost for different values of λ for each R^*. Figure 3.5 illustrates the tradeoff between the expected backorder and inventory cost by enforcing the robustness parameter in RO-(UPM-2) model for $R^* = 100\%$.

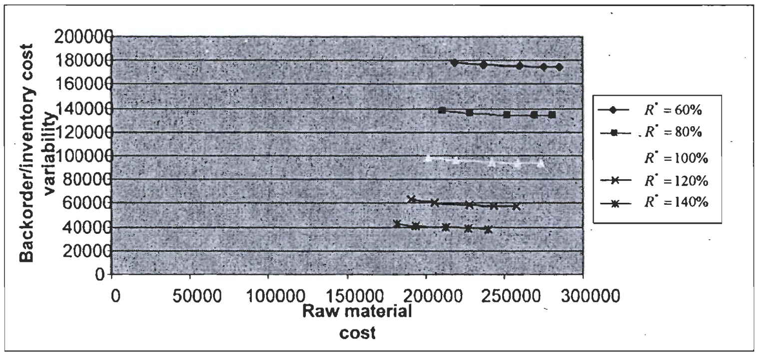


Figure 3.4 - Raw material cost and backorder/inventory cost variability tradeoff in RO-(UPM-2) model for different values of λ and R^*

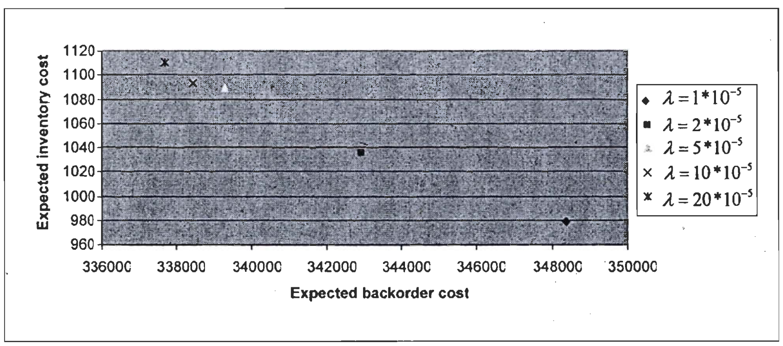


Figure 3.5 - Tradeoff between the expected backorder and inventory costs for different values of λ ($R^* = 100\%$) in RO-(UPM-2) model

As it can be observed from the results presented in table 3.2, decreasing the target recourse cost R^* in this example does not necessarily decrease the variability measure. This implies that the control on the exceeding of the backorder/inventory cost of scenarios over a target cost might be limited depending on the yield scenarios, as well as problem constraints (i.e. raw material inventory and machine capacity constraints). In other words, by imposing a target cost on the variability measure in the RO model, it might not be feasible to achieve a plan with small recourse cost variability. In this example, for the target costs R^* below or equal to the two-stage stochastic model expected recourse cost, by enforcing the value of λ the recourse cost variability can be decreased to a limited value. On the other hand, for higher values of R^* (120% and 140%) more robust production plans with less variable backorder/inventory cost (size) can be achieved at the expense of lower service level (higher expected backorder size).

From the above discussions, it can be concluded that, if the decision maker wishes to use RO-(UPM-2) model to obtain a robust production plan, he/she should choose a value of λ which reflects his/her risk aversion about backorder/inventory cost (size) variability, as well as increased raw material consumption and expected inventory cost (size). Moreover, it might not be feasible to achieve a completely robust production plan by considering any desirable service level (target cost R^*), depending on the yield scenarios and problem constraints.

3.7.4.2. RO-(UPV) model results

Table 3.3 presents the results of RO-(UPV) models, as well as the two-stage stochastic LP for the sawmill example. In the second column in table 3.3, the values of λ are provided in multiples of 10^{-5}, since a quadratic variability measure is used in the RO-(UPV) model. The recourse cost variability measure in column 7 is presented as the square root of (3.15) used in RO-(UPV) model. The last column of table 3.3 includes CPU time (in minutes) for finding the optimal solution of RO-(UPV) model by CPLEX 10. In the RO-(UPV) model, as it can be observed from table 3.3, by increasing the value of parameter λ, the backorder/inventory cost variability decreases significantly, while the expected backorder cost is augmented considerably and the expected inventory cost and the raw material cost is decreased. In other words, by enforcing the backorder/inventory cost variability measure in the objective function of model (3.17)-(3.22) (see section 3.5), a higher expected backorder/inventory cost is determined by the model to minimize the exceeding of backorder/inventory cost of all scenarios over the expected backorder/inventory cost. Thus, the expected backorder size is increased and consequently production level and raw material consumption are decreased. Furthermore, the decreased expected inventory cost is also the result of decreasing the production level and raw material consumption.

Table 3.3 - Results of RO-(UPV) model for the sawmill example - The values of λ are in multiples of 10^{-5}.

λ	Raw material cost	Expected recourse (backorder/inventory) cost	Expected backorder cost	Expected inventory cost	Recourse (backorder/inventory) cost variability ($\bar{\Delta}_+$)	CPU time (min.)
0	155473	379367	378560	807	111627	3.3
1	134286	537009	536298	711	50000	12
2	137009	628833	628118	715	25000	12
5	129062	751170	750492	678	10000	12
10	118655	831630	831004	626	5000	12
20	102452	923633	923089	544	2500	12

Figure 3.6 illustrates the tradeoff between the backorder/inventory cost variability and the expected backorder/inventory cost in RO-(UPV) model.

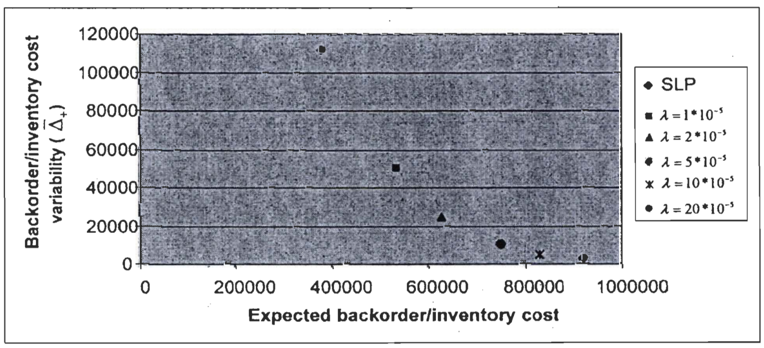


Figure 3.6 - Expected backorder/inventory cost and backorder/inventory cost variability tradeoff in RO-(UPV) model for different values of λ

From the above discussions, it can be concluded that if the decision maker wishes to have a robust production plan by using RO-UPV model, he/she should choose a value of λ which reflects his/her risk aversion about backorder size variability as well as increased expected backorder cost (size). Since the customer service level is defined in this work as the proportion of customer demand that can be fulfilled, the increased expected backorder cost (size) leads to decreased customer service level.

It should be noted that, setting a value for λ and R^* in the above robust optimization models requires explicit managerial input regarding the degree of risk aversion that is appropriate for a given situation. In practical sense, it is probably most effective to run the model with a substantial range of λ, and R^* values, creating a set of solutions like the set graphed in table 3.2, and 3.3 and figures 3.4 and 3.6 and let the manager pick a desired solution from that set, rather than trying to specify the most appropriate value of λ and R^* a priori.

3.7.4.3. Comparison between RO-(UPM-2) and RO-UPV models performances

As the expected recourse cost is not limited by a target value in RO-UPV model, the backorder/inventory cost (size) variability can be controlled as much as possible by increasing the value of λ. On the other hand, in RO-(UPM-2) model, the control over recourse cost variability depends on the target cost (target service level), as well as yield scenarios and problem constraints. Figure 3.7 illustrates the difference between the robustness of optimal solutions in RO-(UPM-2) model and RO-UPV model for different values of R^* and λ. In figure 3.7, as the target cost R^* increases (the service level decreases) in RO-(UPM-2) model, the robustness of plans proposed by this model gets closer to those of RO-UPV model. However, the more robust solution of RO-UPV model might own larger expected backorder cost (size) (lower customer service level) compared to those of the RO-(UPM-2) model. The comparison between the total costs of both RO models is presented in figure 3.8. Finally, as it is shown in tables 3.2 and 3.3, the execution time of the RO-UPV model is also larger than that of the RO-(UPM-2) one.

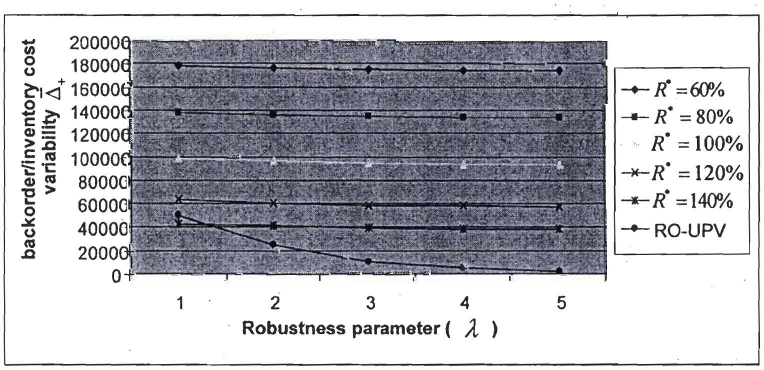


Figure 3.7 - Comparison between the performance of two robust optimization models in controlling recourse cost variability

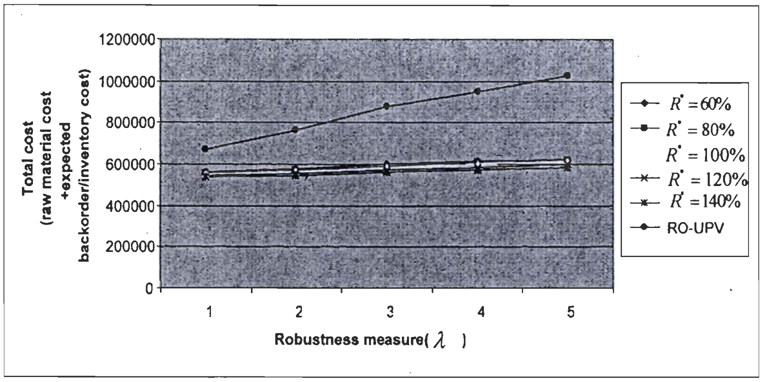


Figure 3.8 - Comparison between the total cost resulted by two robust optimization models

In a very service sensitive company that wants to establish a reputation for always meeting customer service level, the robust optimization formulation allows a decision maker to see explicitly what possible tradeoffs between backorder/inventory cost (size) variability and the expected cost exists, and to choose a solution that is consistent with his/her willingness to accept risk. In the sawmill example, the following decision framework can be proposed. If the decision maker prefers to determine a robust customer service level that remains near optimal as much as possible for all scenarios of random yield, he/she should select the solution of RO-UPV model which results in less backorder/inventory cost (size) variability. However, the solution of this model might result high expected backorder cost (size) which reduces customer service level. Thus, in the case of choosing the RO-UPV model, a value of robustness term λ should be selected that reflects appropriately the tradeoff between the risk aversion level of the decision maker about the robustness of customer service level and the expected backorder cost (size). By choosing the smaller values of λ, lower expected backorder size and consequently better service level can be promised to the customer while this service level is not completely robust. On the other hand, larger values of λ result in promising a lower service level to the customer, which is considerably more robust. In a company where the variability of backorder size (customer service level) is less crucial for the decision maker, the solution of RO-(UPM-2) can be selected which results an expected backorder size (service level) close to a target one. However the desired robustness level of the plans might not be necessarily achieved depending on the problem constraints and yield scenarios. In this case, λ should be selected that reflects appropriately the tradeoff between the risk aversion level of the decision maker about the robustness of customer service level and raw material and expected inventory cost.

3.8. Conclusions

In this paper, two robust optimization models with different variability measures were proposed to address multi-period sawmill production planning by considering the uncertainty in the quality of raw materials (logs). The computational results of addressing a prototype sawmill by this approach provided evidence supporting the advantages of robust optimization approach in generating more robust production plans over the 2-stage stochastic programming approach. Furthermore, the tradeoff between the plan's robustness (backorder/inventory cost (size) variability) and raw material consumption and expected backorder/inventory cost (size) for different values of robustness term was discussed for both models. The robust optimization models were compared in terms of their performance in controlling backorder size (customer service level) variability for all scenarios in addition to their total cost (raw material and the expected backorder/inventory cost). A decision framework was also proposed to select among two RO models based on risk aversion level of the decision maker for the robustness of backorder size (customer service level) and the increased total cost. Although these results are found for sawmill production planning, the proposed approach in this work can be applied for production planning in other manufacturing environments where non-homogeneous and random characteristics of raw materials result in random yield. Future research will consider also the products demands as random variables in order to obtain more realistic production plans.

3.9. Acknowledgment

This work was supported by For@c research consortium of Université Laval.

References

Bakir, M.A. and Byrne, M.D., 1998. Stochastic linear optimization of an MPMP production planning model. *International Journal of Production Economics*, 55, 87-96.

Birge, J.R. and Louveaux, F., 1997. *Introduction to stochastic programming*. New York: Springer.

Escudero, L.F., Kamesam, P.V., King, A.J. and Wets, R.J-B., 1993. Production planning via scenarios. *Annals of Operations Research*, 34, 311- 335.

Gaudreault, J., Rousseau, A., Frayret, J.M. and Cid, F., 2004. Planification opérationnelle du sciage. *For@c technical document*, Quebec (Canada).

Kall, P. and Wallace, S.W., 1994. *Stochastic programming*. New York: John Wiley & sons.

Kall, P. and Mayer, J., 2005. *Stochastic linear programming*. New York: Springer's International Series.

Kazemi Zanjani, M., Nourelfath, M. and Ait-Kadi, D., 2007a. Production planning for simultaneous production of multiple products by alternative processes with random yield. *Proceedings of 7e Congrès international de génie industriel (2007)*, Trois-Rivières (Canada).

Kazemi Zanjani, M., Nourelfath, M. and Ait-Kadi, D., 2007b. A stochastic programming approach for production planning in a manufacturing environment with random yield. *CIRRELT 2007-58 working document*, Québec (Canada).

Kazemi Zanjani, M., Ait-Kadi, D. and Nourelfath M., 2008. A stochastic programming approach for sawmill production planning. *Proceedings of 7e Conférence Internationale de MOdélisation et SIMulation (MOSIM'08)*, Paris (France).

Leung, S. C. H. and Wu, Y., 2004. A robust optimization model for stochastic aggregate production planning. *Production Planning & Control*, 15(5), 502-514.

Leung, S.C.H., Tsang, S.O.S., Ng, W.L. and Wu, Y., 2007. A robust optimization model for multi-site production planning problem in an uncertain environment. *European Journal of Operational Research*, 181(1), 224-238.

List, G.F., Wood, B., Nozick, L.K., Turnquist, M.A., Dean, J.A., Kjeldgaard, E.A. and Lawton, C.R., 2003. Robust optimization for fleet planning under uncertainty. *Transportation Research part E*, 39, 209-227.

Malcolm, S.A. and Zenios, S.A., 1994. Robust optimization of power systems capacity expansion under uncertainty. *Journal of Operational Research Society*, 45, 1040-1049.

Maness, T.C. and Adams D.M., 1991. The combined optimization of log bucking and sawing strategies. *Wood and Fiber Science*, 23, 296-314.

Maness, T.C. and Norton, S.E., 2002. Multiple-period combined optimization approach to forest production planning. *Scandinavian Journal of Forest Research*, 17, 460-471.

Markowitz, H., 1959. *Portfolio selection, efficiency diversification of investments*. New York: John Wiley & Sons.

Mendoza, G.A., Meimban, R.J., Luppold, W.J. and Arman, P.A., 1991. Combined log inventory and process simulation models for the planning and control of sawmill operations. *Proceeding 23rd GIRP International Seminar on Manufacturing Systems*, Nancy (France).

Mulvey, J.M., Vanderbei, R.J. and Zenios, S.A., 1995. Robust optimization of Large-scale systems. *Operations Research*, 43(2), 264-281.

Rönnqvist M., 2003. Optimization in forestry. *Mathematical programming, Ser. B*, 97, 267-284.

Shabbir, A. and Shahinidis, N.V., 1998. Robust process planning under uncertainty. *Industrial & Engineering Chemistry Research*, 37, 1883-1892.

Takriti, S. and Shabbir, A., 2004.On robust optimization of two-stage systems. *Mathematical Programming Ser. A*, 99, 109-126.

Yu, C. and Li, H., 2000. A robust optimization approach for stochastic logistic problems. *International Journal of Production Economics*, 64, 385-397.

Chapter 4

A multi-stage stochastic programming approach for production planning with uncertainty in quality of raw materials and demand

The forth chapter consists the article entitled "*A multi-stage stochastic programming approach for production planning with uncertainty in the quality of raw materials and demand*" accepted on February 2009 by the "*International Journal of Production Research*". The version presented here corresponds to the revised version sent to the editor.

4.1. Abstract

Motivated by the challenges encountered in sawmill production planning, we study a multi-product, multi-period production planning problem with uncertainty in the quality of raw materials and consequently in processes yields, as well as uncertainty in products demands. As the demand and yield own different uncertain natures, they are modeled separately and then integrated. Demand uncertainty is considered as a dynamic stochastic data process during the planning horizon, which is modeled as a scenario tree. Each stage in the demand scenario tree corresponds to a cluster of time periods, for which the demand has a stationary behavior. The uncertain yield is modeled as scenarios with stationary probability distributions during the planning horizon. Yield scenarios are then integrated in each node of the demand scenario tree, constituting a hybrid scenario tree. Based on the hybrid scenario tree for the uncertain yield and demand, a multi-stage stochastic programming (MSP) model is proposed which is full recourse for demand scenarios and simple recourse for yield scenarios. We conduct a case study with respect to a realistic scale sawmill. Numerical results indicate that the solution to the multi-stage stochastic model is far superior to the optimal solution to the mean-value deterministic and the two-stage stochastic models.

4.2. Introduction

Production planning is a key area of operations management. The plans have to be determined in the face of environmental and system uncertainties, namely uncertain products demands, processes yields, etc. An important methodology for production planning is mathematical programming. Traditional mathematical programming models for production planning are deterministic, and may result unsatisfactory production plans in the presence of uncertainties.

The goal of this work is to address a multi-period, multi-product (MPMP) production planning problem in a manufacturing environment where alternative processes produces simultaneously multiple products from several classes of raw materials. Besides, raw materials own non-homogeneous and random characteristics (e.g. logs in sawmills, or crud oil in refineries). Thus, the quantities of products that can be produced by each process (processes yields) are random variables. Moreover, the market demand for products is also uncertain and non-stationary during the planning horizon. The production planning problem we are studying includes deciding how many times each process should be run and which quantity of each class of raw materials should be consumed by each process in each period in the planning horizon. The objective is to minimize products inventory/backorder and raw material consumption costs, regarding fulfillment of products demands, machine capacities, and raw material inventory. This work is motivated by production planning for sawing units in sawmills, where the processes yields are random variables due to non-homogeneity in the characteristics of logs, and lumber demand is also uncertain.

A review of some of the existing literature of production planning under uncertainty is provided in Mula et al. (2006). Stochastic programming (Dantzig, 1955; Kall and Wallace, 1994; Birge and Louveaux 1997; Kall and Mayer, 2005) and robust optimization (Mulvey et al., 1995) has seen several successful applications in production planning. In Escudero et al. (1993) a multi-stage stochastic programming approach was used for addressing a MPMP production planning model with random demand. Bakir and Byrne (1998) developed a stochastic LP model based on the two-stage deterministic equivalent problem to incorporate demand uncertainty in a multi-period multi-product (MPMP) production planning model. Huang K. (2005) proposed the multi-stage stochastic programming models for production and capacity planning under uncertainty. Alfieri and Brandimarte (2005) reviewed the multi-stage stochastic models applied in multi-period production and capacity planning in the manufacturing systems. Brandimarte (2006) proposed a multi-stage stochastic programming approach for multi-item capacitated lot-sizing with uncertain demand. Kazemi et al. (2007) proposed a two-stage stochastic model for addressing MPMP production planning with uncertain yield. Khor et al. (2007) proposed a two-stage stochastic programming model as well as robust optimization models for capacity expansion planning in petroleum refinery under uncertainty. Leung and Wu (2004) proposed a robust optimization model for stochastic aggregate production planning. Wu (2006) applied the robust optimization approach to uncertain production loading problems with import quota limits under the global supply chain management environment. In Leung et al. (2007) a robust optimization model was developed to address a multi-site aggregate production planning problem in an uncertain environment. Kazemi et al. (2009) proposed two robust optimization models with different recourse cost variability measures to address MPMP production planning with uncertain yield.

Adopting a two-stage approach in the uncertain multi-period production planning literature (see e.g. Bakir and Byrne, 1998; Kazemi et al., 2007, 2008, 2009; Khor et al., 2007) cannot model the dynamic decision process in such problems. In a two-stage approach, the plan for the entire multi-period planning horizon is determined before the uncertainty is realized, and only a limited number of recourse actions can be taken afterwards. In contrast, a multi-stage approach allows the revision of the planning decisions as more information regarding the uncertainties is revealed. Consequently, the multi-stage model is a better characterization of the dynamic planning process, and provides more flexibility than does the two-stage model.

In the existing contributions in the literature for production planning with uncertainty, either one uncertain parameter (e.g. either demand or yield) was taken into account (Escudero et al., 1993; Bakir and Byrne, 1998; Brandimarte, 2006; Kazemi et al., 2007, 2008, 2009) or one set of scenarios or a scenario tree is considered for all the uncertain parameters simultaneously (Leung and Wu, 2004; Huang K., 2005; Wu, 2006; Leung et al. 2007; Khor et al., 2007). However, when the uncertain parameters own different dynamics and behavior over time and each might need different sorts of recourse actions, it is more realistic to model them separately and then integrate them to be used in the stochastic programming models.

In this paper, we propose a multi-stage stochastic program for MPMP production planning with uncertain yield and demand. As demand uncertainty originates from market conditions and yield uncertainty is due to non-homogeneity in the quality of raw materials, they are modeled separately and independently. We assume that the uncertain demand evolves as a discrete time stochastic process during the planning horizon with a finite support. This information structure can be interpreted as a scenario tree. Each stage in a demand scenario tree corresponds to a cluster of time periods. It is supposed that the demand has a stationary behavior during the periods at each stage. The uncertain yields are modeled as scenarios with stationary probability distributions during the planning horizon. Finally, the yield scenarios are integrated into the demand scenario tree, forming a hybrid scenario tree with two types of branches in each node. Depending on the availability of information on the uncertain parameters at the beginning of each stage in the scenario tree, different recourse actions are defined for them in the multi-stage stochastic model. We suppose that at the beginning of each stage in the demand scenario tree, the decision maker has a perfect insight on the demand scenario for that stage. Thus, the production plan can be adjusted for demand scenarios (full recourse). On the other hand, as yield scenarios are revealed after the plan implementation, the production plan is constant for yield scenarios (simple recourse). The goal of the multi-stage stochastic model is to determine an implementable plan for production that takes into account the possible demand and yield scenarios, provides for recourse actions in the future, and minimizes the expected costs of raw material consumption, holding inventory, and backorder. It should be noted that the multi-stage stochastic model is represented as compact formulation (see for example Alfieri and Brandimarte, 2005) based on the scenarios of the hybrid scenario tree, in order to have a deterministic equivalent model of manageable size that can be solved by CPLEX. The proposed approach is applied for sawmill production planning under the uncertainty in raw material (log) quality and product (lumber) demand. Regarding the large dimensionality of the resulting deterministic equivalent model for a realistic scale sawmill, based on the experts' insight on the lumber market, the periods in the planning horizon are clustered into three stages. As a result, the original multi-stage model is approximated by a 4-stage one. Numerical results indicate that the solution to the multi-stage model is far superior to the optimal solution of the mean-value deterministic and the two-stage stochastic models. Furthermore, it is shown that the significance of using multi-stage stochastic programming is increased as the variability of random demand is augmented in the scenario tree.

The remainder of this paper is organized as follows. In the next section, a theoretical framework for multi-stage stochastic programming (MSP) is provided. In section 4.4, we provide a multi-stage stochastic linear program for MPMP production planning with random yield and demand. In section 4.5, we describe one of the applications of this problem which is sawmill production planning under the uncertainty in raw material (log) quality and product (lumber) demand. In section 4.6, the implementation results of the multi-stage stochastic model for a prototype realistic scale sawmill are presented. Our concluding remarks are given in section 4.7.

4.3. Multi-stage stochastic programming

In a problem where time and uncertainty play an important role, the decision model should be designed to allow the user to adopt a decision policy that can respond to events as they unfold. The specific form of the decisions depends on the assumptions concerning the information that is available to the decision maker, when (in time) is it available and what adjustments (recourses) are available to the decision maker. Multi-stage stochastic programming (MSP) approach (Kall and Wallace, 1994; Birge, and Louveaux 1997; Kall and Mayer, 2005) was proposed to address multi-period optimization models with dynamic stochastic data during the time. In multi-stage stochastic programming (MSP) a lot of emphasis is placed on the decision to be made today, given present resources, future uncertainties and possible recourse actions in the future. The uncertainty is represented through a scenario tree and an objective function is chosen to represent the risk associated with the sequence of decisions to be made and the whole problem is then solved as a large scale linear or quadratic program. In the following, we first review the characteristics of scenario trees, and then we provide a general formulation for multi-stage stochastic programming.

4.3.1. Scenario tree

A scenario tree is a computationally viable way of discretizing the underlying dynamic stochastic data over time in a problem. An illustration of a scenario tree is provided in Figure 4.1. In a scenario tree, each stage denotes the stage of the time when new information is available to the decision maker. Thus, the stages do not necessarily correspond to time periods. They might include a number of periods in the planning horizon. A scenario tree consists of a number of nodes and arcs at each stage. Each node n in the scenario tree represents a possible state of the world (scenario), associated with a set of data (stochastic demand, stochastic cost, etc.) in the corresponding stage. The root node of the tree represents the current state of the world. The branches (arcs) in the scenario tree denote the scenarios for the next stage. A probability is associated to each arc of the scenario tree which denotes the probability of the corresponding scenario to that arc. It should be noted that, the probability of each node in the scenario tree is computed as the product of probabilities of the arcs from the root node to that node. Furthermore, the sum of probabilities of the nodes at each stage is equal to 1. A path from the root node to a node n describes one realization of the stochastic process from the present time to the period where node n appears. A full evolvement of the stochastic process over the entire planning horizon, i.e., the path from the root node to a leaf node, is called a scenario. In the scenario tree example of figure 4.1, we have 4 stages. Each node n in the tree has two branches to the next stage which denote two possible scenarios for the next stage, when we are at stage n. Consequently, we have 8 scenarios by the end of stage 4. A review of the approaches for generating the scenario trees for multi-stage stochastic programs, based on the underlying random data processes is provided in (Dupačová et al., 2000).

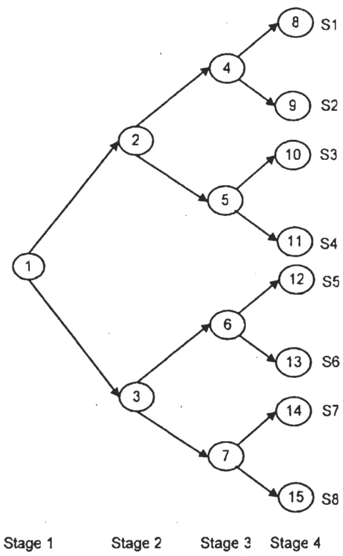


Figure 4.1 - Scenario tree for multi-stage stochastic programming

4.3.2. Multi-stage stochastic programming models

We begin by abstracting the statement of a multi-period deterministic LP model:

$$\begin{aligned}
&\textit{Minimize } c_1x_1 + c_2x_2 + \ldots + c_Tx_T, \\
&\textit{Subject to} \\
&\qquad A_{11}x_1 \qquad\qquad\qquad\quad = b_1, \\
&\qquad A_{21}x_1 + A_{22}x_2 \qquad\quad\; = b_2, \\
&\qquad \cdot \\
&\qquad \cdot \\
&\qquad \cdot \\
&\qquad A_{T1}x_1 + \ldots \quad + A_{TT}x_T = b_T, \\
&\qquad x_1 \geq 0, x_2 \geq 0, \ldots, x_T \geq 0.
\end{aligned} \tag{4.1}$$

Let the scenario s correspond to a single setting of all data in this problem,

$$s = \{\ c_t,\ b_t,\ A_{tt'} : t = 1,\ldots,T, t' = 1,\ldots,T\},$$

and a decision x corresponds to a setting of all the decision variables

$$x : (x_1,\ldots,x_T) \in \mathrm{R}^{n_1} \times \ldots \times \mathrm{R}^{n_T}.$$

Solving the deterministic LP model (4.1) for a given setting s of the data is equivalent to solving the following problem for a certain function:

$$\min f(x,s) \qquad \text{over all } x,$$

where

$$f(x,s) = \begin{cases} \sum_{t=1}^{T} c_t x_t, & \text{if } x \text{ satisfies all constraints in (4.1)}, \\ +\infty, & \text{otherwise.} \end{cases}$$

We next develop the stochastic model. Let us suppose that we are given a set S of scenarios. The decision-maker wishes to set a policy that makes different decisions under different scenarios. Mathematically, a policy X that assigns to each scenario $s \in S$ is a vector $X(s) := (X_1(s),\ldots,X_T(s))$, where $X_t(s)$ denotes the decision to be made at stage t if encountered by scenario s. Decisions that depend on the individual scenarios do not hedge against the possibility that the scenario may not occur, leaving one vulnerable to disastrous consequences if some other scenario does happen. In other words, our decision process must conform to the flow of available information, which basically means the decisions must be non-anticipative (or implementable). A decision is said to be implementable if for every pair of scenarios s and s' that are indistinguishable up to stage t,

$$(X_1(s),\ldots,X_t(s)) = (X_1(s'),\ldots,X_t(s')).$$

As examples of indistinguishable scenarios, we can refer to scenarios 1, 2, 3, 4 in node 2, at stage 2 of scenario tree in figure 4.1. Implementability guaranties that policies do not depend on information that is not yet available. The multi-stage stochastic programming can be formulated as:

$$\min\left\{\sum_{s\in S} p^s f(X(s),s)\,|\,X \text{ is an implementable policy}\right\},$$

where p^s denotes the probability of scenario s. There are two approaches to impose the non-anticipativity constraints in the multi-stage stochastic programs which lead to *split variable* formulation and *compact* formulation.

4.3.2.1. Split variable formulation

In the *split variable* formulation, we introduce a set of decision variables for each stage and each scenario, and then we enforce non-anticipativity constraints explicitly based on the shape of scenario tree. Although this representation increases the problem dimensions, it yields a sparsity structure that is well suited to the interior point algorithms. Alternatively, it is possible to use a decomposition approach on the splitting variables formulation. Several strategies have been published in the literature for solving large-scale multi-stage stochastic programs (Ruszczynski 1989; Rockafellar and Wets 1991; Mulvey and Ruszczyński 1995; Liu and Sun 2004).

4.3.2.2. Compact formulation

In the *compact* formulation, we associate decision variables to the nodes of scenario tree and build non-anticipativity in an implicit way. In other words, the variables such as $X(s)$ for $X(s)=X(s')$ are replaced in the model by one single variable, and redundant constraints for partially identical scenarios are deleted. Compact formulations are computationally cheaper when using for solving by the Simplex methodology in standard solvers.

4.4. Model development

In this section, we first present a deterministic mathematical formulation for the problem under consideration. Then, we provide the multi-stage stochastic formulation to address the problem by considering the uncertain processes yields and products demands.

4.4.1. A deterministic model for multi-product, multi-period production planning

Consider a production unit with a set of products P, a set of classes of raw materials C, a set of production processes A, a set of machines R, and a planning horizon consisting of T periods. To state the deterministic linear programming model for this problem, the following notations are used:

4.4.1.1. Notations

Indices

p product

t period

c raw material class

a production process

r machine

Parameters

h_{pt} inventory holding cost per unit of product p in period t

b_{pt} backorder cost per unit of product p in period t

m_{ct} raw material cost per unit of class c in period t

I_{c0} the inventory of raw material class c at the beginning of planning horizon

I_{p0} the inventory of product p at the beginning of planning horizon

s_{ct} the quantity of material of class c supplied at the beginning of period t

d_{pt} demand of product p by the end of period t

ϕ_{ac} the units of class c raw material consumed by process a (consumption factor)

ρ_{ap} the units of product p produced by process a (yield of process a)

δ_{ar} the capacity consumption of machine r by process a

M_{rt} the capacity of machine r in period t

<u>*Decision variables*</u>

X_{at} the number of times each process a should be run in period t

I_{ct} inventory size of raw material of class c by the end of period t

I_{pt} inventory size of product p by the end of period t

B_{pt} backorder size of product p by the end of period t

4.4.1.2. The deterministic LP model

$$Minimize\ Z = \sum_{p \in P} \sum_{t=1}^{T} (h_{pt} I_{pt} + b_{pt} B_{pt}) + \sum_{c \in C} \sum_{t=1}^{T} \sum_{a \in A} m_{ct} \phi_{ac} X_{at} \tag{4.2}$$

Subject to

$$I_{ct} = I_{ct-1} + s_{ct} - \sum_{a \in A} \phi_{ac} X_{at}, \qquad t = 1, \dots, T,\ c \in C, \tag{4.3}$$

$$I_{p1} - B_{p1} = I_{p0} + \sum_{a \in A} \rho_{ap} X_{a1} - d_{p1},$$

$$I_{pt} - B_{pt} = I_{pt-1} - B_{pt-1} + \sum_{a \in A} \rho_{ap} X_{at} - d_{pt}, \qquad t = 2, \dots, T,\ p \in P, \tag{4.4}$$

$$\sum_{a \in A} \delta_{ar} X_{at} \leq M_{rt}, \qquad t = 1, \dots, T,\ r \in R, \tag{4.5}$$

$$X_{at} \geq 0, I_{ct} \geq 0, I_{pt} \geq 0, B_{pt} \geq 0, \qquad t = 1, \dots, T,\ p \in P,\ c \in C,\ a \in A. \tag{4.6}$$

The objective function (4.2) minimizes total inventory and backorder costs for all products and raw material cost for all classes in the planning horizon. Constraint (4.3) ensures that the total inventory of raw material of class c at the end of period t is equal to its inventory in the previous period plus the quantity of material of class c supplied at the beginning of that period (s_{ct}) minus its total consumption in that period. Constraint (4.4) ensures that the sum of inventory (or backorder) of product p at the end of period t is equal to its inventory (or backorder) in the previous period plus the total production of that product in that period, minus the product demand for that period. Total quantity of production for each product in each period is calculated as the sum of the quantities yielded by each of the corresponding processes, regarding the yield (ρ_{ap}) of each process. Finally, constraint (4.5) requires that the total production do not exceed the available production capacity.

4.4.2. Multi-stage stochastic programming extension

In this section, we first describe our proposed approach to model the uncertain yield and demand, and then provide the production planning formulation by multi-stage stochastic programming.

4.4.2.1. Modeling the uncertain yield and demand

We assume that the uncertain demand evolves as a discrete time stochastic process during the planning horizon with a finite support. This information structure can be interpreted as a scenario tree (see figure 4.1 in section 4.3). The nodes at stage t of the tree constitute the states (scenarios) of demand that can be distinguished by information available up to stage t. For each stage a limited number of demand scenarios are taken into account (e.g., high, average, low). In order to define the scenarios for each stage, we can either use the traditional approach of making distributional assumptions, estimating the parameters from historical data, or use some scenarios proposed by experts. In order to keep the resulting multi-stage stochastic model within a manageable size, we assume that the planning horizon is clustered into N stages, where each stage includes a number of periods. In other words, it is supposed that the uncertain demand is stationary during the time periods in each stage. For example, if the demand scenario for the first period at stage n is high, it remains the same (high) for the remainder of periods in stage n; however the demand scenario might change (e.g., to low) for the first period in the next stage (n+1). It should be noted that, the number of periods that can be considered at each stage depends on the behavior of demand in the industry, which can be determined based on the expert's insight on the market.

On the other hand, we assume that the raw materials are supplied from the same supply source during the planning horizon. Thus, it is supposed that the uncertain yield has a stationary probability distribution. The probability distribution of the random yield is estimated based on historical data in industry. A number of scenarios are taken in to account for yields by discritization of the original probability distribution. Regarding the stationary distribution of yield, only one of the scenarios can take place during the planning horizon.

In order to have a single stochastic production planning model that considers uncertain yield and demand, yield scenarios are integrated with the demand scenario tree, forming a hybrid scenario tree. An example of a four-stage hybrid scenario tree is depicted in figure 4.2, where full line branches denote demand scenarios while dashed line branches denote yield scenarios. At each node of the tree, which denotes one demand scenario for the corresponding stage, different yield scenarios can take place (3 scenarios in the example of figure 4.2). However, regarding the stationary behavior of uncertain yield, only one of the yield scenarios can be observed during the planning horizon. Thus, the total number of scenarios in the hybrid scenario tree can be computed as the number of leaves in demand scenario tree by the number of yield scenarios (in the example of figure 4.2, this number is equal to 24).

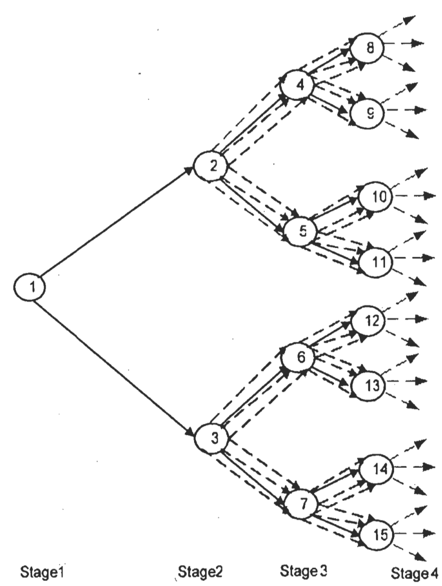


Figure 4.2 - A hybrid scenario tree for uncertain demand and yield

4.4.2.2. Multi-stage stochastic program for MPMP production planning with uncertain yield and demand

Let us now formulate the problem as a multi-stage stochastic (MSP) model based on the hybrid scenario tree for the uncertain yield and demand. The decision (control) variable of deterministic model (4.2)-(4.6) is production plan X_{at}. The inventory and backorder variables I_{pt} and B_{pt} are the consequences (state variables) of the plan. In this problem, we assume that the decision maker can adjust the production plan X_{at} for different demand scenarios at each stage of the demand scenario tree. In other words, it is supposed that at the start of the first period of each stage, enough information is available to the decision maker to know which demand scenario is in force for that stage. Thus he/she can select properly among the plans proposed by the MSP model for different scenarios. The multi-stage stochastic model is full recourse with respect to demand scenarios. As we use compact formulation to represent the problem, the decision variables X_{at} are defined for each node of the demand scenario tree. On the other hand, as the quality of materials is not known before being processed, the yield scenarios can only be revealed after implementation of the production plan. Thus, the production plan for each node of the demand scenario tree should be fixed for all the yield scenarios. In other words, the model becomes simple recourse with respect to yield scenarios. It is evident that the inventory and backorder sizes of products in each period ($I^i_{pt}(n)$ and $B^i_{pt}(n)$), which are the state variables, depend on the demand scenarios as well as yield scenarios, thus they are indexed for yield scenarios as well as demand nodes. Regarding the above discussions, the following notations in addition to those provided in 4.4.1.1 are used in the multi-stage stochastic model. The compact formulation of the multi-stage stochastic model follows the notations.

4.4.2.2.1. Notations

<u>*Indices*</u>

Tree scenario tree.

S number of scenarios for random yields.

i scenario of random yield.

n, m node of the scenario tree.

$a(n)$ ancestor of node n in the scenario tree.

t_n set of time periods corresponding to node n in the scenario tree.

Parameters

$d_{pt}(n)$ demand of product p by the end of period t at node n of the scenario tree.

$p(n)$ probability of node n of the scenario tree.

p^i probability of scenario i for random yield.

Decision variables

$X_{at}(n)$ the number of times each process a should be run in period t at node n of the scenario tree.

$I_{ct}(n)$ inventory size of raw material of class c by the end of period t at node n of the scenario tree.

$I^i_{pt}(n)$ inventory size of product p by the end of period t for scenario i of random yield at node n of the scenario tree.

$B^i_{pt}(n)$ backorder size of product p by the end of period t for scenario i of random yield at node n of the scenario tree.

4.4.2.2.2. Multi-stage stochastic model (compact formulation)

$$Minimize\, Z = \sum_{n\in Tree} p(n)(\sum_{t\in t_n}\sum_{c\in C}\sum_{a\in A} m_{ct}\phi_{ac}X_{at}(n)) + \sum_{n\in Tree} p(n)(\sum_{i=1}^{S} p^i(\sum_{t\in t_n}\sum_{p\in P}(h_{pt}I^i_{pt}(n)+b_{pt}B^i_{pt}(n)))) \quad (4.7)$$

Subject to

$$I_{ct}(n) = I_{ct-1}(m) + s_{ct} - \sum_{a\in A}\phi_{ac}X_{at}(n), \qquad n\in Tree,\, t\in t_n,\, c\in C,$$

$$m = \begin{cases} a(n), & t-1\notin t_n, \\ n, & t-1\in t_n, \end{cases} \quad (4.8)$$

$$\sum_{a\in A}\delta_{ar}X_{at}(n) \le M_{rt}, \qquad n\in Tree,\, t\in t_n, r\in R, \quad (4.9)$$

$$I^i_{pt}(n) - B^i_{pt}(n) = I^i_{pt-1}(m) - B^i_{pt-1}(m) + \sum_{a\in A}\rho^i_{ap}X_{at}(n) - d_{pt}(n),$$

$$n\in Tree,\, t\in t_n,\, p\in P,\, i=1,\ldots,S,$$

$$m = \begin{cases} a(n), & t-1\in t_n, \\ n, & t-1\notin t_n, \end{cases} \quad (4.10)$$

$$X_{at}(n)\geq 0,\ I_{ct}(n)\geq 0,\ I^{i}_{pt}(n)\geq 0,\ B^{i}_{pt}(n)\geq 0,\quad n\in Tree,\ t\in t_n,\ c\in C,\ p\in P,\ a\in A,\ i=1,\dots,S. \tag{4.11}$$

The first term of the objective function (4.7) accounts for the expected material cost for demand nodes of the scenario tree. The second term is the expected inventory and backorder costs for demand nodes and yield scenarios. In model (4.7)-(4.11), the decision variables are indexed for each node, as well as for each time period, since the stages do not correspond to time periods. As it was mentioned in 4.4.2.1, each node at a stage includes a set of periods which is denoted by t_n. In this model, there are coupling variables between different stages and these are the ending inventory and backorder variables at the end of each stage. As it can be observed in this model, two different node indices (n, m) are used for inventory/backorder variables in the inventory balance constraints ((4.8) and (4.10)). More precisely, for the first period at each stage, the inventory or backorder is computed by considering the inventory or backorder of the last period corresponding to its immediate predecessor node, while for the rest of the periods at that stage, the inventory/backorder size of previous period corresponding to the same node are taken into account.

4.5. Case study: sawmill production planning

In this section, we introduce one of the applications of the general problem already described in this paper, which is sawmill production planning. There are a number of processes that occur at a sawmill: log sorting, sawing, drying, planing and grading (finishing). Raw materials in sawmills are the logs which are transported from different districts of forest after bucking the felled trees. The finished and graded lumbers (products) are then transported to the domestic and international markets. Figure 4.3 illustrates the typical processes. As a case study, we consider the sawing units in sawmills. In the sawing units, logs are classified according to some attributes namely: diameter class, species, length, taper, etc. Logs are broken down into different dimensions of lumbers by means of different cutting patterns. See figure 4.4 for three different cutting patterns. Each cutting pattern is a combination of activities that are run on a set of machines. From each log, several pieces of sawn lumber (e.g. 2(in)$\times$4(in)$\times$8(ft), 2(in)$\times$4(in)$\times$10(ft),...) are produced depending on the cutting pattern. The lumber quality (grade) as well as the quantity yielded by each cutting pattern depend on the quality and characteristics of the input logs. Despite the classification of logs in sawmills, variety of characteristics might be observed in different logs in each class. In fact, due to natural variable conditions that occur during the growth period of trees, non-homogeneous and random characteristics (in terms of diameter, number of knots, internal defects, etc.) can be observed in different logs in each class, make it impossible to anticipate the exact yield of a log.

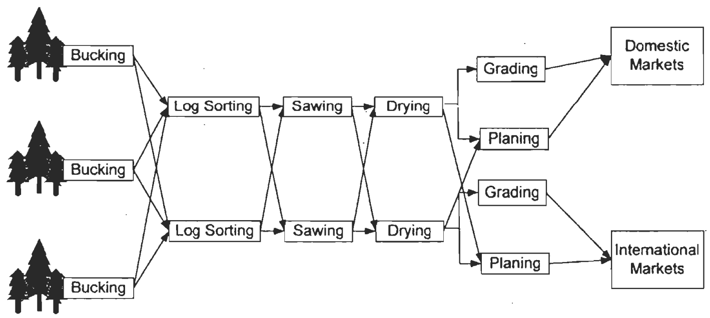


Figure 4.3 - Illustration of sawmills processes (after Rönnqvist, 2003)

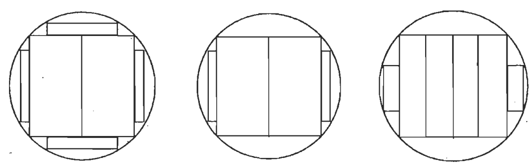


Figure 4.4 - Cutting patterns in a sawmill

As it is not possible in many sawmills to scan the logs before planning, the exact yields of cutting patterns for different log classes cannot be determined in priori. As an example of the uncertain yields in sawmills, consider the cutting pattern (*s*) that can produce 6 products (P1, P2, P3, P4, P5, P6) after sawing the logs of class (*c*). Table 4.1 represents four scenarios among all possible scenarios for the uncertain yields of this process.

Table 4.1 - Scenarios for yields of a process in a sawing unit

Scenarios	Products					
	P1	P2	P3	P4	P5	P6
1	1	0	1	0	1	1
2	2	1	1	0	1	0
3	1	0	0	1	1	1
4	2	0	0	1	0	1

Uncertainty in the market demand for different lumbers is another important parameter that should be taken into account in sawmill production planning. We focus on operational level production planning in a sawing unit. The decision variables include the optimal quantity of log consumption from different classes and selection of best cutting patterns for each log class in each period of the planning horizon, in order to fulfill the demand. The objective is to minimize log consumption cost, as well as products inventory and backorder costs. Regarding the potential significance of yield and demand uncertainty on the production plan, and customer orientation which is at center of attention in the sawmills that are dependent on export markets, obtaining production plans with minimum expected backorder size is an important goal of production planning in sawmills.

Different approaches have been already proposed in the literature to address sawmill production planning. The first approach is focused on combined optimization type solutions linked to real-time simulation sub-systems (Mendoza et al., 1991; Maness and Adams, 1991; Maness and Norton, 2002). In this approach, the stochastic characteristics of logs are taken into account by assuming that all the input logs are scanned through an X-ray scanner before planning. Maness and Norton (2002) developed an integrated multi-period production planning model, which is the combination of an LP model and a log sawing optimizer (simulator). The LP model acts as a coordinating problem that allocates limited resources. The log sawing optimization models are used to generate columns for the coordinating LP based on the products' shadow prices. Although the stochastic characteristics of logs are considered in this approach, it includes the following limitations to be implemented: logs needed for the next planning horizon are not always available in the sawmill to be scanned before planning. Furthermore, to implement this method, the logs should be processed in production line in the same order they have been simulated, which is not an easy practice. Finally scanning logs before planning is a time consuming process in high capacity sawmills, which delays the planning process. In the second approach, the randomness of the processes yields as well as demand is simplified and their expected value is considered in a MPMP linear programming model (Gaudreault et al., 2004). However, the production plans issued by these models result usually in extra inventory of products with lower quality and price, while backorder of products with higher quality and price. In Kazemi et al. (2008) a two-stage stochastic program with recourse is proposed to address sawmill production planning by considering the random yield. The solutions of stochastic model are considerably superior to those of deterministic model in terms of the expected inventory and backorder costs. Among different contributions in the literature for sawmill production planning, we did not succeed to find any contribution that considers simultaneously the random demand and yield. In the next section, computational results of implementing the proposed multi-stage stochastic program for a realistic scale sawmill example are provided.

4.6. Computational results

In this section, we report on the computational experiments with the proposed multi-stage stochastic programming approach for a realistic scale sawmill. The objective of our experiments is to investigate the quality of production plans suggested by multi-stage stochastic programming comparing to those of deterministic LP, and two-stage stochastic programming. The multi-stage stochastic model is also compared to the models which take into account either the random demand or the random yield. Finally, we compute the value of multi-stage stochastic programming (VMSP) for this example. In the following, we first describe our experimental environment and then report on the experimental results in the light of the mentioned objectives.

4.6.1. Experimental environment

A prototype sawmill is selected to illustrate the application of the multi-stage stochastic model. The prototype sawmill is a typical medium capacity softwood sawmill located in Quebec (Canada). The sawmill focuses on sawing high-grade products to the domestic markets as well as export products to the USA. It is assumed that the input bucked logs into the sawing unit are categorized into 3 classes. 5 different cutting patterns are available. The sawing unit produces 27 products of custom sizes (e.g. $2(\text{in})\times4(\text{in})$, $2(\text{in})\times6(\text{in})$ lumbers) in four lengths. In other words, there are 15 processes that can produce 27 products with random yields. We consider two bottleneck machines: Trimmer and Bull. The planning horizon consists of 30 periods (days). It would be worth mentioning that the data used in this example are based on the gathered data from different sawmills in Quebec (Canada). As the list of custom sizes, machine parameters and prices are proprietary, they are not reported in this paper. The hybrid scenario tree for uncertain demand and yield in this example is generated as follows.

Demand Scenario tree

Due to the lack of complete historical data on products demands in this example, we used the existing historical data as well as the insights of the experts on the lumber market to estimate the demand scenario tree. At each stage of the scenario tree, except stage 1, based on the historical data for products demands (per product, per day) in Quebec sawmills, we estimate a normal distribution. The normal distribution is then approximated by a 3 point discrete distribution (*high*, *average*, and *low* demand) by using the Gaussian quadrature method (Miller and Rice, 1983). Since considering each time period as a stage leads to an extremely large scenario tree, we need to approximate the scenario tree by something more manageable. In our computational experiment, we supposed that the demand for the next 10 days has a stationary behavior, which is a realistic assumption in the lumber market. Thus, we clustered the 30-periods planning horizon into 3 stages and hence the multi-stage decision process is approximated by a 4-stage one. The first stage consists of time period zero (present time), the second-stage includes periods 1-10, etc. Moreover, based on the experts' insight, we suppose that the demands for all products are perfectly correlated and all products have the same behavior at each stage of the scenario tree, while each product has its own daily demand (normal) distribution. In other words, if at stage t, the market condition is *good*, the demand scenario for all products can be expected to be *high*. The mentioned approximations results a scenario tree with 27 demand scenarios and 40 nodes. In the following, we illustrate how the demand scenarios corresponding to different nodes of the demand scenario tree are generated. We start with the second stage and the node which denotes the *high* scenario for demand. Based on the 3 point approximation corresponding to the normal distribution of each product for period 1, we select the *high* scenario for that product; and then we repeat the same procedure for all products and for the rest of the periods (2-10) at that stage. Then we go to the other nodes (corresponding to *average* and *low* demand) and we repeat the same procedure, using the *average* and *low* scenarios for the demand of each product per day. At the next stage we start with the first period and we repeat the above procedure for all the nodes and the periods at that stage.

In this experiment, we consider three different normal distributions for the demand of each product (per period) with the same mean but different standard deviations (5% mean, 20% mean, and 30% mean). Thus, three demand trees (DT1, DT2, DT3) and a total of 3 test problems are considered.

Yield Scenarios

As we mentioned in section 4.3, at each node of demand scenario tree a number of yield scenarios are taken into account. It would be worth mentioning that, we define a scenario for the yields of each process in a sawing unit as the average yields of a sample of logs (e.g. 3000 logs) corresponding to that process. By the central limit theorem (CLT), the average yields have a normal distribution. Thus, based on the historical data in Quebec sawmills for the processes yields, the normal distributions were estimated for the random yields. The normal distribution corresponding to the yield of each process was then approximated by three scenarios, by using Gaussian quadrature method (Miller and Rice, 1983). As the randomness of processes yields is the result of non-homogeneity in quality of logs, we consider three scenarios for yield of each log class. As we considered 3 classes of logs in this example, the total number of yield scenarios is equal to $3^3 = 27$. It should be noted that the same yield scenarios are considered in the three test problems.

The above scenario generation approach for uncertain demand and yield in this sawmill production planning example results a hybrid scenario tree similar to the one in figure 4.2 with 40 nodes, where each node includes 3 branches as demand scenarios and 27 branches as yield scenarios. The total number of scenarios at the end of stage 4 is equal to 27×27=729. The compact multi-stage stochastic model (4.7)-(4.11) for this sawmill example is a linear programming (LP) model with nearly 600,000 decision variables and 300,000 constraints.

CPLEX 10 and OPL 5.1 are used to solve the linear program (4.7)-(4.11) and to perform further analysis on the solutions of the test problems. All numerical experiments are conducted on an AMD Athlon$^{\text{TM}}$ 64×2 dual core processor 3800+, 2.01 GHz, 3.00 GB of RAM, running Microsoft Windows Server 2003, standard edition.

4.6.2. Quality of multi-stage stochastic model solution

In this section, for the three test problems mentioned in 4.6.1, we compare the solution of 4-stage stochastic programming model to those of a 3-stage, and 2-stage stochastic programming model as well as mean-value deterministic model. The 4-stage stochastic programming model with random yield and demand is also compared to the models which take into account either the random demand or the random yield. It should be noted that in the 3-stage model, the 30 periods planning horizon is clustered into 2 stages, each includes 15 periods. In other words, in order to reduce the size of the multi-stage model, it was supposed that the random demand has a stationary behavior during 15 days. The 2-stage stochastic model corresponds to considering a static probability distribution for the uncertain demand during the planning horizon. In table 4.2 the solutions of mentioned models for the three test problems are compared with respect to the expected total cost, the expected material consumption cost, as well as the expected inventory and backorder costs. It should be noted that the expected inventory/backorder costs of 3-stage, 2-stage, and mean-value deterministic models are computed by setting the production plan variables (X_{at}) in the 4-stage stochastic model (4.7)-(4.11) as the optimal production plan (X^{*}_{at}) proposed by the mentioned models. In other words, the expected inventory/backorder costs of production plans proposed by the 3-stage, 2-stage and deterministic model are computed for the hybrid 4-stage scenario tree corresponding to the uncertain yield and demand in each test problem. The "relative gap (%)" columns in table 2 include the relative gap between the cost of the 3-stage, 2-stage and the deterministic models by the cost of 4-stage model, which are provided as the percentages. For example, as presented in table 4.2, the expected backorder/inventory cost of the plan proposed by the 2-stage model is 33% higher than that of the 4-stage model, in the test case DT1.

Table 4.2 - Cost comparison of different production planning models

Demand tree	Production planning model	Expected total cost	Relative gap (%)	Expected material cost	Expected inventory/backorder costs	Relative gap (%)	CPU time (minutes)
DT1	4-Stage SLP	1957950	-	1737500	220450	-	29
	3-Stage SLP	1973563	0.8	1746415	227148	3	6
	2-Stage SLP	2118125	7.5	1788142	329983	33	2
	Mean-value deterministic LP	2129030	8	1751675	377355	41	0
DT2	4-Stage SLP	2028777	-	1749677	279100	-	29
	3-Stage SLP	2038543	0.5	1756840	281703	0.9	6
	2-Stage SLP	2391261	15	1889265	501996	44	2
	Mean-value deterministic LP	2432717	16	1751675	681042	59	0
DT3	4-Stage SLP	2163458	-	1766387	397071	-	29
	3-Stage SLP	2206165	2	1763839	442326	10	6
	2-Stage SLP	2836707	24	2025122	811585	51	2
	Mean-value deterministic LP	3095556	30	1751675	1343881	70	0

As it can be observed in table 4.2, in all the tree test problems the solution of the 4-stage stochastic model is significantly superior to those of the deterministic model. Furthermore, if the uncertain demand is considered as a random variable with a static probability distribution during the planning horizon (as in the two-stage stochastic programming model), the expected material cost as well as the expected inventory/backorder costs of the production plan are considerably higher than those of the multi-stage stochastic model's plan. Finally, by clustering the planning horizon into two stages (as in 3-stage stochastic programming model) the expected inventory/backorder costs of the plan is higher than those of 4-stage stochastic model. The last column of table 2 indicates that the high quality of multi-stage stochastic model requires higher computational time compared to those of the deterministic and two-stage ones. Figures 4.5 and 4.6 illustrate better the comparison between the total expected cost as well as expected inventory/backorder costs of different models for the three test problems which are distinguished by the variability of demand at each stage. As the variability of demand increases at each stage, the difference between the expected cost of the multi-stage stochastic model's plan and the deterministic and two-stage stochastic models' plans increases. In other words, the significance of using a multi-stage programming model instead of a two-stage or a deterministic model is increased as the variability of demand increases at each stage of the scenario tree. It would be worth mentioning that by increasing the number of stages in the demand scenario tree, which is equivalent to reducing the number of periods at each stage, the uncertain behavior of demand can be captured more precisely. Thus, a production plan with lower expected cost can be obtained. However, as the difference between the expected cost of the 4-stage and 3-stage models in not very significant in the three test problems (see figures 4.5 and 4.6), we did not consider more stages in the scenario trees in the test problems.

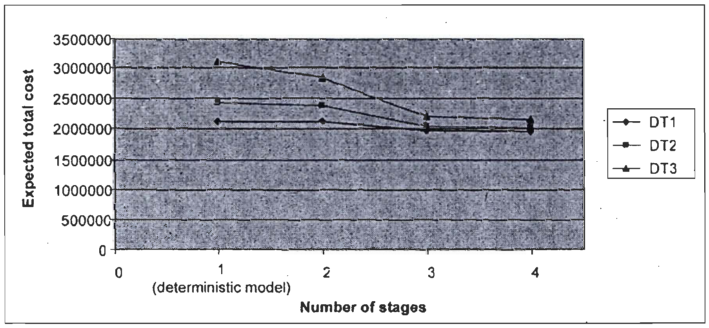


Figure 4.5 - Expected total cost comparison of different production planning models

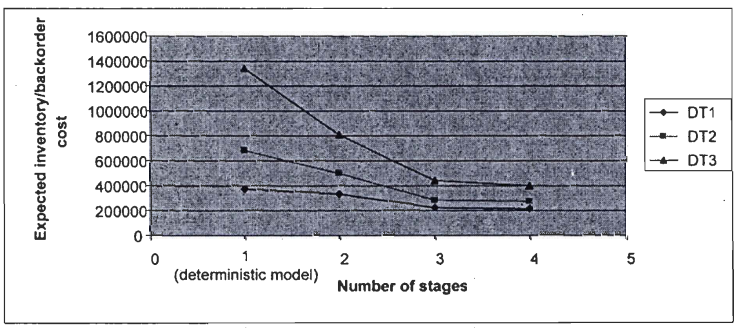


Figure 4.6 - Expected inventory/backorder costs comparison of different production planning models

In table 4.3, we compare the costs of the sawmill production planning model with random yield and demand (model 1) to a model with random demand and deterministic yield (model 2) and the other one with random yield and deterministic demand (model 3). As in this experiment, we modeled the random demand as a scenario tree and the random yield as a set of scenarios, model 1 corresponds to the 4-stage stochastic model with the hybrid demand/yield scenario tree, model 2 corresponds to a 4-stage stochastic model with a demand scenario tree, and model 3 corresponds to a two-stage stochastic model with a set of yield scenarios. The "relative gap (%)" column in table tree includes the relative difference between the cost of models 2 and 3 by model 1.

Table 4.3 - Cost comparison of the production planning models with different uncertain parameters

Demand tree	Uncertain parameters	Expected total cost	Relative gap (%)	Expected inventory/backorder costs	Relative gap (%)
DT1	Random demand and yield (model 1)	1957950	-	220450	-
	Random demand (model 2)	2002777	2	269741	18
	Random yield (model 3)	2063061	5	323320	32
DT2	Random demand and yield (model 1)	2028777	-	279100	-
	Random demand (model 2)	2059657	1.5	315341	11.5
	Random yield (model 3)	2309588	12	569847	51
DT3	Random demand and yield (model 1)	2163458	-	397071	-
	Random demand (model 2)	2185069	1	424004	6
	Random yield (model 3)	2810012	23	1070271	62

As it can be observed in table 4.3 and figure 4.7, failing to take into account either the random yield or random demand in the sawmill production planning example leads to the production plans with increased costs. Moreover, as the random demand has a non stationary and dynamic behavior during the planning horizon, considering it as a deterministic parameter effects more significantly on the production plan's cost than the random yield which owns a stationary behavior. Finally, as the variability of demand increases in the demand scenario tree, the significance of considering the random demand in the production planning model is increased.

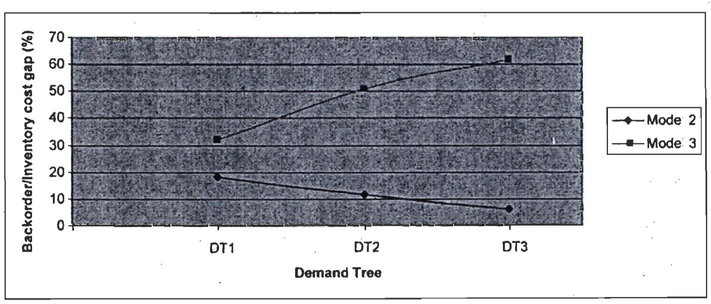


Figure 4.7 - The expected backorder/inventory cost comparison of models 2 and 3 with model 1

4.6.3. Value of multi-stage stochastic programming

As it was mentioned in section 4.4, we considered the production plan (X_{at}) as full recourse with respect to demand scenarios. In other words, we assumed a flexible production plan that can be adjusted based on the demand scenarios, at different stages. However, in some manufacturing environments the production plan is not flexible and should be fixed at the beginning of planning horizon. Thus, a simple recourse multi-stage stochastic model should be used to determine the plan. In this section, we compare the solutions of multi-stage stochastic programs with full recourse and simple recourse, for the three test problems. In table 4.4, it can easily be verified that in all the test problems the total cost of full recourse problem is smaller than that of the simple recourse problem. This should come as no surprise, since the multi-stage model with full recourse offers more flexibility in the production plan decisions with respect to the uncertain states of demand. We denote the optimal objective values corresponding to full recourse and simple recourse multi-stage stochastic programs by v^{FR}, and v^{SR}, respectively. The value of multi-stage stochastic programming (VMSP) is defined as follows (Huang and Shabbir, 2005; Huang 2005): $VMSP = v^{SR} - v^{FR}$.

Table 4.4 - Value of multi-stage stochastic programming in the three test problems

Demand tree	4-stage stochastic model	Objective function value	VMSP
DT1	Full recourse	1957950	79195
	Simple recourse	1973563	
DT2	Full recourse	2028777	174076
	Simple recourse	2037145	
DT3	Full recourse	2163458	300000
	Simple recourse	2202853	

Value of multi-stage stochastic programming (VMSP) indicates the value of allowing the production plan to be adjusted for different scenarios at each stage of decision process instead of fixing its value at the beginning of planning horizon. Figure 4.8 compares the VMSP of the three test problems with different variability levels in demand. As it can be observed in figure 4.8, the value of multi-stage stochastic solution increases with the variability of demand. In other words, as the variability of demand increases at each stage, considering a full recourse multi-stage stochastic model becomes more significant.

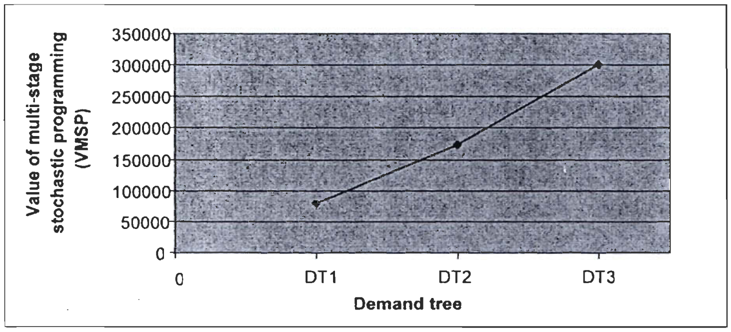


Figure 4.8 - VMSP comparison of different test problems with different demand variability

4.7. Conclusions

In this paper, we addressed a multi-period, multi-product (MPMP) production planning problem under uncertainty in products demands and processes yields. We proposed a multi-stage stochastic model to address the problem. The uncertain demand was modeled as a dynamic stochastic process presented as a scenario tree. The uncertain yield was modeled as a static random variable with a stationary probability distribution during the planning horizon. We integrated the uncertain yield and demand into a hybrid scenario tree. The proposed approach was applied for sawmill production planning under the uncertainty in raw material (log) quality and product (lumber) demand. We presented the computational results using a realistic scale prototype sawmill. Our numerical results indicated that the quality of the 4-stage stochastic model solutions is significantly higher than those of the mean-value deterministic and two-stage stochastic models. Moreover, it was shown that as the variability of demand is augmented at each stage of the scenario tree, the significance of using the multi-stage stochastic programming approach is increased. As further extensions of this work, we can consider seasonal demand and different trends at each stage of the demand scenario tree. Moreover, the proposed approach can be applied for production planning in other manufacturing environments with uncertain demand and non-homogeneous and random characteristics of raw materials which results the random processes yields.

4.8. Acknowledgements

This work was supported by For@c research consortium of Université Laval.

References

Alfieri, A. and Brandimarte, P., 2005. Stochastic programming models for manufacturing applications: A tutorial introduction. *In*: Matta, A. and Semeraro, Q., eds. *Design of Advanced Manufacturing Systems, Models for Capacity Planning in Advanced Manufacturing Systems*. Netherlands: Springer, 73-124.

Bakir, M.A. and Byrne, M.D., 1998. Stochastic linear optimization of an MPMP production planning model. *International Journal of Production Economics*, 55, 87-96.

Birge, J.R. and Louveaux, F., 1997. *Introduction to stochastic programming*. New York: Springer.

Brandimarte, P., 2006. Multi-item capacitated lot-sizing with demand uncertainty. *International Journal of Production Research*, 44(15), 2997-3022.

Dantzig, G.B., 1955. Linear programming under uncertainty. *Management Science*, 1, 197-206.

Dupačová, J., Consigli, G. and Wallace, S.W., 2000. Scenarios for multi-stage programs. *Annals of Operations Research*, 100, 25-53.

Escudero, L.F., Kamesam, P.V., King, A.J. and Wets, R.J-B., 1993. Production planning via scenarios. *Annals of Operations Research*, 34, 311- 335.

Gaudreault, J., Rousseau, A., Frayret, J.M. and Cid, F., 2004. Planification opérationnelle du sciage. *For@c technical document*; Quebec, Canada.

Huang, K., 2005. *Multi-stage stochastic programming models for production planning*. Thesis (PhD). School of Industrial and Systems Engineering: Georgia Institute of Technology.

Huang, K. and Shabbir, A., 2005. The value of multi-stage stochastic programming in capacity planning under uncertainty. *Technical Report*, School of Industrial & Systems Engineering, Georgia Tech.

Kall, P. and Wallace, S.W., 1994. *Stochastic programming*. New York: John Wiley & sons.

Kall, P. and Mayer, J., 2005. *Stochastic linear programming*. New York: Springer's International Series.

Kazemi Zanjani, M., Nourelfath, M. and Ait-Kadi, D., 2007. A stochastic programming approach for production planning in a manufacturing environment with random yield. *CIRRELT 2007-58 working document*; Québec, Canada.

Kazemi Zanjani, M., Ait-Kadi, D. and Nourelfath, M., 2008. A stochastic programming approach for sawmill production planning. In: *Proceedings of 7e Conférence Internationale de MOdélisation et SIMulation (MOSIM'08)*; Paris, France.

Kazemi Zanjani, M., Ait-Kadi, D. and Nourelfath, M., 2009. Robust production planning in a manufacturing environment with random yield: A case in sawmill production planning. *European Journal of Operational Research*, doi: 10.1016/j.ejor.2009.03.041.

Khor, C.S., Elkamel, A., Ponnambalamb, K. and Douglas, P.L., 2007. Two-stage stochastic programming with fixed recourse via scenario planning with economic and operational risk management for petroleum refinery planning under uncertainty. *Chemical Engineering and Processing*, doi:10.1016/j.cep.2007.09.016.

Leung, S.C.H. and Wu, Y., 2004. A robust optimization model for stochastic aggregate production planning. *Production Planning & Control*, 15(5), 502-514.

Leung, S.C.H., Tsang, S.O.S., Ng, W.L. and Wu, Y., 2007. A robust optimization model for multi-site production planning problem in an uncertain environment. *European Journal of Operational Research*, 181(1), 224-238.

Liu, X. and Sun, J., 2004. A new decomposition technique in solving multistage stochastic linear programs by infeasible interior point methods. *Journal of Global Optimization*, 28, 197-215.

Maness, T.C. and Adams D.M., 1991. The combined optimization of log bucking and sawing strategies. *Wood and Fiber Science*, 23, 296-314.

Maness, T.C. and Norton, S.E., 2002. Multiple-period combined optimization approach to forest production planning. *Scandinavian Journal of Forest Research*, 17, 460-471.

Mendoza, G.A., Meimban, R.J., Luppold, W.J. and Arman, P.A., 1991. Combined log inventory and process simulation models for the planning and control of sawmill operations. In: *Proceeding 23rd GIRP International Seminar on Manufacturing Systems*; Nancy, France.

Miller, A.C. and Rice, T.R., 1983. Discrete approximations of probability distributions. *Management Science*, 29(3), 352-362.

Mulvey, J.M., Vanderbei, R.J. and Zenios, S.A., 1995. Robust optimization of Large-scale systems. *Operations Research*, 43(2), 264-281.

Mulvey, J.M. and Ruszczyński, A., 1995. A new scenario decomposition method for large-scale stochastic optimization. *Operations Research*, 43(2), 477- 490.

Mula, J., Poler, R., Garcia-Sabater, J.P. and Lario, F.C., 2006. Models for production planning under uncertainty: A review. *International Journal of Production Economics*, 103, 271-285.

Rockafellar, R.T. and Wets, R.J-B., 1991. Scenarios and policy aggregation in optimization under uncertainty. *Mathematics of Operations Research*, 16(1), 119-147.

Rönnqvist M., 2003. Optimization in forestry. *Mathematical programming, Ser. B*, 97, 267-284.

Ruszczyński, A., 1989. Regularized decomposition and augmented Lagrangian decomposition for angular programming problems. *In*: Levandowski, A. and Wierzbicki, A.P., eds. *Aspiration Based Decision Support Systems*. Berlin: Springer.

Ruszczyński, A. and Shapiro, A., 2003. Stochastic programming models. *In*: Ruszczyński, A. and Shapiro, A., eds. *Stochastic Programming*. Vol. 10 of Handbooks in Operations Research and Management Science. Amsterdam: Elsevier.

Wu, Y., 2006. Robust optimization applied to uncertain production loading problems with import quota limits under the global supply chain management environment. *International Journal of Production Research*, 44(5), 849-882.

Chapter 5

General Conclusion

In this thesis, we investigated a multi-period, multi-product (MPMP) production planning problem with uncertain yield and demand. We focused on a manufacturing environment where multiple alternative processes produces simultaneously a mix of products (co-production) from several classes of raw materials. The uncertain yield originates from the non-homogeneous and random characteristics of raw materials. Our motivation to address this problem was sawmill production planning, while considering random characteristics of logs as well as uncertain lumber demand. The existing approaches in the literature for sawmill production planning, either simplify the uncertainty in problem parameters, by considering their expected values, or own the limitations to be implemented in sawmills. Regarding the results that we obtained through the case studies, we believe the approaches proposed in this thesis can be considered as novel and viable tools for sawmill production planning. We can expect that the proposed stochastic sawmill production planning models result more realistic and more robust plans compared with the traditional deterministic model.

In the first and second articles of the thesis, we addressed the MPMP production planning problem by considering the uncertainty in processes yields. In the third article, we considered the uncertainty in products demands and processes yields, simultaneously. The contributions of the three articles in the thesis can be summarized as follows:

In the first article, we proposed a two-stage stochastic program with recourse to address the MPMP problem by considering the yield uncertainty. We modeled the uncertain yields as scenarios with stationary probability distributions during the planning horizon. The proposed two-stage stochastic model was applied for sawmill production planning, which can be considered as a novel approach for this problem. We also proposed a Monte-Carlo simulation approach to compare the stochastic sawmill production planning model with the deterministic one, by considering yield scenarios similar to those that might be observed during the production process in real sawmills. The simulation results for a real medium capacity sawmill confirmed that the proposed two-stage stochastic model can be considered as a more realistic tool for sawmill production planning, since it proposes a plan with considerably lower backorder size (higher customer service level), in the presence of different yield scenarios, than the mean-value deterministic model.

In the second article, we proposed two robust optimization (RO) models in order to minimize the risk behind the customer service level variability in the presence of different random yield scenarios, in the MPMP production planning problem. The backorder size was considered as a measure of service level. The two robust optimization models are distinguished by their service level variability measures. A decision framework was provided to select among the two RO models based on the tradeoff between the expected backorder/inventory size and the decision maker's risk aversion level about the variability of customer service level. The results of comparison between the robust optimization model and the two-stage stochastic model (proposed in article 1) in a prototype sawmill, confirmed the superiority of robust optimization approach in generating production plans with more robust (less variable) backorder size (service level) in the presence of different yield scenarios.

In the third article, we proposed a multi-stage stochastic programming (MSP) model to address the multi-product, multi-period (MPMP) production planning problem with uncertain yield and demand. As demand and yield are independent and own different uncertain natures, they were modeled separately and then integrated. Demand uncertainty was considered as a dynamic stochastic data process during the planning horizon, which was modeled as a scenario tree. Yield scenarios were then integrated in each node of the demand scenario tree, constituting a hybrid scenario tree. For each uncertain parameter, different types of recourse actions were defined in the multi-stage stochastic programming (MSP) model. We conducted a case study with respect to a medium capacity sawmill. Numerical results indicated that the plan proposed by the multi-stage model owns considerably lower expected raw material consumption cost as well as expected product inventory and backorder costs than the mean-value deterministic and the two-stage stochastic models.

Future work

The study reported here is a first step leading to a series of applications in different manufacturing environments, where the throughputs of the processes are uncertain due to uncertainty of their inputs. As an example, we can refer to industries where raw materials are of natural resources and their exact characteristics cannot be measured in priori. Crude oil refineries, meat & agricultural products industries, and stonecutting industry can be categorized in this group. Recycling industry is another example where the throughputs of processes are uncertain due to the lack of complete information on the state of the recycled items in priori.

The uncertain parameters studied in the thesis are among the environmental uncertainties. Material cost is another environmental uncertainty that can be taken into account. Moreover, most of the manufacturing environments are influenced by several system uncertainties, namely, machine failures. Thus, taking into account such uncertainties in addition to yield and demand uncertainty in production planning models, leads to more realistic production plans.

In this study, we compared the stochastic and deterministic models in a static fashion, i.e. with a finite time window. However, in practice a rolling horizon is used for production planning. Therefore, it is of interest to study the performance of stochastic programming models under a rolling horizon approach.

In chapter IV, we represented the uncertain demand and yield as a limited size scenario tree and a limited number of stationary scenarios, respectively. Our objective was to have a multi-stage stochastic model of manageable size that can be solved by CPLEX solvers. However, a limited size scenario tree, or a limited number of scenarios is only able to model uncertainty quite roughly. It would be interesting to consider larger number of scenarios for uncertain yield and demand in order to model the uncertainties more precisely. In order to solve the resulted large-scale multi-stage stochastic models, the decomposition algorithms which are also amenable to parallel computation can be applied.

In this study, we modeled the uncertain yield as a static random variable with a stationary probability distribution during the planning horizon. As we are addressing a multi-period production planning problem, modeling the uncertain yield as a dynamic data process can increase the precision of plans, although at the expense of increasing the dimensionality of the stochastic model. Further research can also be focused on studying different techniques, namely, different sampling techniques, to model more precisely the uncertainties based on the available data processes.