



# **Maximizing power output of heat engines through design optimization: Geothermal power plants and novel exhaust heat recovery systems**

**Thèse**

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# Résumé

Le design de machines thermiques menant à une puissance maximale dépend souvent des températures de la source chaude et de la source froide. C'est pourquoi dégager des lignes directrices à partir des designs optimaux de ces machines selon diverses températures d'opération peut faciliter leur conception. Une telle étude est proposée par cette thèse pour deux types de systèmes thermiques.

En premier lieu, le cycle de Rankine organique (ORC) est un cycle thermodynamique de puissance utilisé entre autres dans les centrales géothermiques exploitant des réservoirs à basse température. Depuis quelques années, ce type de centrales suscite un vif intérêt à travers le monde, étant un des modes de production de puissance parmi les plus respectueux de l'environnement. Il s'agit de pomper un géofluide du sol pour transférer sa chaleur à un fluide de travail qui opère en cycle fermé, et de le réinjecter ensuite dans le bassin géologique. Les chercheurs tentent actuellement de mieux caractériser le potentiel géothermique de divers environnements géologiques. Le sous-sol du Québec est relativement froid, alors des études essaient de déterminer s'il serait possible d'y exploiter de manière rentable des centrales géothermiques. Une autre question de recherche importante est de savoir, pour un contexte donné, quel est le design optimal d'une centrale géothermique et quelle est la puissance que l'on peut espérer produire.

Pour répondre à cette question, les cycles de Rankine organiques de base (de type sous-critique ou transcritique) sont dans un premier temps simulés et optimisés pour des températures du géofluide de 80 à 180°C et pour des températures de condensation du fluide de travail de 0.1 à 50°C. Trente-six (36) fluides purs sont investigués pour toutes les combinaisons de températures.

Par la suite, des cycles de Rankine organiques plus avancés sont aussi investigués (ajout d'une tour de refroidissement, d'un système de récupération, et d'une contrainte sur la température de réinjection du géofluide). Les ORCs avec deux pressions de chauffage sous-critique et transcritique sont aussi simulés et optimisés. Les optimisations sont faites pour

20 fluides de travail selon la même plage de température du géofluide et selon des températures du thermomètre mouillé de l'air ambiant de 10 à 32°C.

En second lieu, le cycle de Brayton inversé (IBC) est un cycle thermodynamique qui pourrait être utilisé comme système de récupération de la chaleur perdue dans les gaz d'échappement de moteurs. Il s'agit d'un cycle ouvert comprenant dans sa configuration de base une turbine à gaz, un échangeur de chaleur et un compresseur. Il existe une configuration où l'eau qui se condense lors du refroidissement des gaz est évacuée avant le compresseur pour réduire le débit massique et améliorer le rendement global du système. Le Powertrain and Vehicle Research Centre (PVRC) de l'University of Bath s'est intéressé à savoir si certaines variantes de l'IBC découlant de cette configuration seraient des options viables.

Ces variantes ont mené à la création de trois nouveaux cycles thermodynamiques couplant l'IBC avec (i) une turbine à vapeur, (ii) un cycle de réfrigération, et (iii) ces deux ajouts. En comptant les deux cycles déjà existants décrits au paragraphe précédent, cinq configurations de l'IBC sont simulées et optimisées pour des températures de gaz d'échappement de 600 à 1200 K et températures de la source froide de 280 à 340 K.

La finalité de cette thèse est d'offrir un outil aidant les ingénieurs à concevoir les systèmes introduits précédemment (ORC et IBC) de sorte qu'ils aient un travail spécifique net maximisé. Sous forme d'un ensemble de diagrammes, cet outil peut ainsi être utilisé pour une large plage de température de la source chaude (géofluide ou gaz d'échappement) et de température de la source froide.

## Abstract

Heat engines design leading to maximum power output often depends on the hot source temperature and the cold source temperature. This is why drawing guidelines from optimal designs of these machines according to diverse operating temperatures may facilitate their conception. Such a study is proposed by this thesis for two types of heat engines.

In the first instance, the Organic Rankine Cycle (ORC) is a power thermodynamic cycle used among others in geothermal power plants exploiting low-temperature reservoirs. This type of power plants raises keen interest around the world for being one the most environmentally friendly power production modes. In these power plants, a geofluid is pumped from the ground to transfer its heat to a working fluid operating in a closed cycle. The geofluid is then reinjected in the geological basin. Researchers are currently attempting to characterize in a better way the geothermal potential of diverse geological environments. Considering the province of Québec's relatively cold underground, studies try to determinate whether it is possible to profitably operate geothermal power plants. Another important research question is to determine, for a given context, the optimal geothermal power plant design, and the amount of power that could be generated.

To answer this question, Organic Rankine Cycles (subcritical and transcritical) are first simulated and optimized for geofluid temperatures from 80 to 180°C and for condensing temperatures of the working fluid from 0.1 to 50°C. Thirty-six (36) pure fluids are investigated for each temperature combination.

Next, cycles models are improved by adding a cooling tower, a recuperative system and a constraint on the minimum reinjection temperature. ORCs with dual-pressure heater are simulated and optimized as well. Optimization runs are performed considering 20 working fluids for the same range of geofluid temperature and for ambient air wet bulb temperature from 10 to 32°C.

In the second instance, the Inverted Brayton Cycle (IBC) is a thermodynamic cycle that could be used as a waste heat recovery system for engines exhaust gases. This is an open

cycle which includes a gas turbine, a heat exchanger and a compressor as a basic layout. There is a configuration where the water condensed during the cooling of the gases is evacuated upstream of the compressor in order to reduce the mass flow rate and improve the system global efficiency. The Powertrain and Vehicle Research Centre (PVRC) of the University of Bath is interested in finding out whether particular IBC variants arising from this configuration could be viable options.

These variants led to the creation of three novel thermodynamic cycles that couple the IBC with (i) a steam turbine, (ii) a refrigeration cycle, and (iii) both additions. Including both already existing cycles described in the preceding paragraph, five IBC layouts are simulated and optimized for exhaust gases temperatures from 600 to 1200 K and for heat sink temperatures from 280 to 340 K.

The purpose of this thesis is to offer a tool that help engineers designing the systems previously introduced (ORC and IBC), so that they produced a maximized specific work output. As a set of charts, this tool can be used for a large range of hot source temperature (geofluid or exhaust gases) and of heat sink temperature.

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# Nomenclature

## *Variables*

$c_p$	specific heat, $\text{kJ kg}^{-1} \text{K}^{-1}$
$F$	fluid
$h$	enthalpy, $\text{kJ kg}^{-1}$
$h_f^\circ$	enthalpy of formation, $\text{kJ kg}^{-1}$
$M$	molar mass, $\text{g mol}^{-1}$
$\dot{m}$	mass flow rate, $\text{kg s}^{-1}$
$mf$	mass fraction
$N$	number of moles, mol
$P$	pressure, kPa
$q$	specific heat transfer rate, $\text{kJ kg}^{-1}$
$\dot{Q}$	heat transfer rate, kW
$R$	gas constant, $\text{kJ kg}^{-1} \text{K}^{-1}$
$s$	entropy, $\text{kJ kg}^{-1} \text{K}^{-1}$
$s_f^\circ$	entropy of formation, $\text{kJ kg}^{-1} \text{K}^{-1}$
$sh$	specific humidity
$T$	temperature, $^\circ\text{C}$ , K
$U_R$	refrigeration utilization rate
$V$	volume, $\text{m}^3$
$W$	work, kJ
$w$	specific power output or specific work output, $\text{kJ kg}^{-1}$
$x$	vapor quality
$y$	molar fraction

## *Greek letters*

$\varepsilon$	heat exchanger effectiveness
$\phi$	relative humidity
$\eta$	efficiency
$\rho$	density

### *Subscripts*

<i>atm</i>	atmospheric
<i>B</i>	Baumann
<i>b</i>	brine
<i>bf</i>	best-fit
<i>c</i>	condenser
<i>cr</i>	critical
<i>db</i>	dry bulb
<i>env</i>	environment
<i>f, g</i>	saturated liquid and saturated gas states
<i>fg</i>	evaporation (change form liquid to vapor)
<i>H</i>	high pressure
<i>hex</i>	heat exchangers
<i>i</i>	stage number, species
<i>in</i>	inlet
<i>j</i>	increment
<i>j</i>	iteration number
<i>liq</i>	liquid state
<i>M</i>	medium pressure
<i>max</i>	maximum
<i>min</i>	minimum
<i>opt</i>	optimal
<i>out</i>	outlet
<i>pp</i>	pinch point
<i>ref</i>	reference
<i>s</i>	isentropic
<i>sat</i>	saturated
<i>sw</i>	supplied liquid water
<i>tol</i>	tolerance
<i>tot</i>	total
<i>vap</i>	vapor state
<i>w</i>	cooling fluid

*wb* wet bulb  
*wf* working fluid

*Abbreviations*

CO condenser  
CP compressor  
D dual-pressure heater  
D related to drainage  
EC economizer  
EV evaporator  
HE heat exchanger  
HP high pressure  
GT gas turbine  
IBC Inverted Brayton Cycle  
IC internal combustion  
MP medium pressure  
ORC Organic Rankine Cycle  
PP pump  
R related to refrigeration cycle  
RE recuperator  
S single-pressure heater  
S related to open Rankine cycle  
SC subcritical  
SH superheater  
ST steam turbine  
TB turbine  
TC transcritical cycle  
VA valve  
WCT wet cooling tower

*Personne n'ignore que la chaleur peut être la cause du mouvement, qu'elle possède même une grande puissance motrice : les machines à vapeur, aujourd'hui si répandues, en sont une preuve parlant à tous les yeux. [...]*  
*Développer cette puissance, l'approprier à notre usage, tel est l'objet des machines à feu. L'étude de ces machines est du plus haut intérêt, leur importance est immense, leur emploi s'accroît tous les jours. Elles paraissent destinées à produire une grande révolution dans le monde civilisé.*

- Sadi Carnot, 1824

*It is time that we engineers reclaim our own field – thermodynamics – so that we may expand its deterministic powers in the direction of naturally organized, living and not living systems. We are the ones to do this work because nature is engineered.*

- Adrian Bejan, 1996

## Remerciements

La première personne qui a rendu possible cet accomplissement est celle qui l'a initiée. Merci à Louis Gosselin, mon codirecteur, pour m'avoir tout bonnement proposé une maîtrise lorsque j'étais allée lui quémander de l'aide à l'été 2013 dans le cadre de mon stage au REGAL. Merci de m'avoir fait confiance malgré mes notes du baccalauréat, et de m'avoir laissé choisir le projet qui était le moins sûr de fonctionner. À l'entente du mot géothermie, il était impossible de me faire changer d'avis !

La deuxième personne que je tiens à remercier est mon directeur François Mathieu-Potvin. Ton écoute, ta perspicacité et ta rigueur m'ont toujours fait sentir bien encadrée et apte à faire le travail nécessaire. Nos réunions n'ont pas seulement servi à faire le suivi des avancements et répondre à mes questions. Elles se sont aussi traduites en discussions enrichissantes sur le « comment » et le « pourquoi » de curieux phénomènes physiques. En fin de compte, mon projet est devenu un terrain de jeu où tout était possible pour satisfaire ma curiosité insatiable sur la thermodynamique. François et Louis, vous avez été les meilleurs directeurs qui soient : compétents, respectueux, drôles, compréhensifs et motivants.

Merci à mes parents Sylvie et Denis pour avoir toujours été là pour moi. Dans mes moments de « syndrome de l'imposteur », vous m'avez fait sentir que j'étais assez intelligente, assez compétente pour mériter le titre de « docteur » en génie mécanique. Vous êtes également des modèles pour moi et m'avez appris à développer ma pensée critique, à persévérer et à garder l'esprit ouvert.

Merci à Laurent, mon partenaire de vie que j'ai rencontré à mi-parcours de cette (longue) étape de vie. Ta bonne humeur, tes encouragements, les soirs où tu as fait la vaisselle tout seul, et tes bruits de chèvre m'ont clairement aidé à passer au travers, surtout pendant les derniers miles. Merci de m'avoir écoutée et rappelée à l'ordre après la catastrophe du concours de vulgarisation et la réception des innombrables commentaires des évaluateurs du deuxième article (chapitre 3 de la thèse).

Merci à Mireille, qui était en rédaction en même temps que moi, et avec qui j'ai pu me changer les idées et partager mes inquiétudes par rapport aux études graduées. Sans elle, je n'aurais sûrement pas eu la motivation de travailler autant les soirs et les fins de semaine.

Dans un autre ordre d'idées, ce projet a été financé par l'Institut de recherche d'Hydro-Québec (IREQ) et par le Conseil de recherches en sciences naturelles et en génie du Canada (CRSNG). De plus, le programme des Bourses canadiennes du jubilé de diamant de la reine Elizabeth II (BRE) et le Bureau international de l'Université Laval (BI) m'ont donné des bourses qui m'ont permis d'effectuer un stage de recherche à l'University of Bath durant l'été 2017. J'aimerais souligner l'apport de Colin Copeland qui m'a supervisé durant ce stage et m'a offert d'ajouter à ma thèse un axe de recherche très intéressant.

# Avant-propos

Cette thèse présente les travaux de recherche réalisés lors de la maîtrise et du doctorat en génie mécanique de mai 2014 à juillet 2019. Le passage accéléré au doctorat a été effectué à l'été 2016. Les différents sujets abordés sont mis en contexte dans l'introduction, suivi de leurs objectifs respectifs. Trois publications pour des journaux scientifiques internationaux (Chapitres 1, 2 et 3) forment le corps de la thèse. Ils sont listés ci-dessous, avec des informations concernant leur statut de publication en date du 7 février 2020, leur contexte de recherche, mes contributions et les modifications par rapport à la version publiée.

## CHAPITRE 1 :

N. Chagnon-Lessard, F. Mathieu-Potvin and L. Gosselin, “Geothermal power plants with maximized specific power output: Optimal working fluid and operating conditions of subcritical and transcritical Organic Rankine Cycles,” *Geothermics*, vol. 64, pp. 111–124, DOI: 10.1016/j.geothermics.2016.04.002, Novembre 2016 (publié).

Notes : Article rédigé par N. Chagnon-Lessard (moi-même) et F. Mathieu-Potvin et révisé par L. Gosselin. Le travail a été réalisé sous la supervision de F. Mathieu-Potvin et L. Gosselin au Laboratoire de Transfert Thermique et d'Énergétique (LaTTÉ) de l'Université Laval. J'ai participé à l'élaboration des modèles de cycles thermodynamiques et à l'analyse d'ordre de grandeur. J'ai écrit les scripts de simulation numérique sur MATLAB, effectué les optimisations et réalisé les figures. Finalement, j'ai aidé à dégager les conclusions. Les modifications par rapport à la version publiée concernent le changement de système de références pour l'uniformisation de la thèse et quelques changements mineurs.

## CHAPITRE 2 :

N. Chagnon-Lessard, F. Mathieu-Potvin and L. Gosselin, “Optimal design of geothermal power plants: A comparison of single-pressure and dual-pressure organic Rankine cycles,” *Geothermics*, vol. 86, DOI: 10.1016/j.geothermics.2019.101787, Juillet 2020 (disponible en ligne).

Notes : Article rédigé par N. Chagnon-Lessard (moi-même) et révisé par L. Gosselin et F. Mathieu-Potvin. Le travail a été réalisé sous la supervision de F. Mathieu-Potvin et L. Gosselin au LaTTÉ de l'Université Laval. J'ai élaboré les modèles de cycles, écrit les scripts de simulation numérique et d'optimisation sur MATLAB, effectué les optimisations, réalisé les figures et dégagé les conclusions. Les modifications par rapport à la version publiée concernent le changement de système de références pour l'uniformisation de la thèse et certaines sections conservées de la première version soumise.

### **CHAPITRE 3 :**

N. Chagnon-Lessard, C. Copeland, F. Mathieu-Potvin and L. Gosselin, "Maximizing specific work output extracted from engine exhaust with novel inverted Brayton cycles over a large range of operating conditions" *Energy*, vol. 191, DOI: 10.1016/j.energy.2019.116350, Janvier 2020 (publié).

Notes : Article rédigé par N. Chagnon-Lessard (moi-même) et révisé par l'ensemble des co-auteurs. Le travail a été réalisé en premier lieu sous la supervision de C. Copeland au Powertrain and Vehicle Research Center (PVRC), lors d'un stage à l'University of Bath au Royaume-Uni. Il a ensuite été complété sous la supervision de F. Mathieu-Potvin et L. Gosselin au LaTTÉ de l'Université Laval. L'article se base sur des cycles déjà existants dans la littérature et propose des nouvelles variantes. C. Copeland a proposé ces variantes originales et j'ai aidé à améliorer leur concept. J'ai écrit les scripts de simulation numérique et d'optimisation sur MATLAB, effectué les optimisations, réalisé les figures et participé à tirer les conclusions. La version présentée comprend les modifications apportées après la révision par les évaluateurs. Cependant, la version publiée a été écourtée pour répondre aux exigences de l'éditeur. Il s'agit donc de la version antérieure à la réduction du nombre de mots.

# Introduction

## *Mise en contexte*

L'énergie est un pilier sur lequel repose notre société. Afin de réduire son coût et son impact sur l'environnement, de plus en plus d'efforts sont consacrés à améliorer la manière dont elle est convertie. La chaleur est une importante forme d'énergie permettant de produire de l'énergie mécanique pouvant directement servir dans l'industrie et comme moyen de déplacement ou pour être ensuite convertie en électricité par des centrales thermiques. Les efforts de recherche se concentrent entre autres sur les modes alternatifs de production d'électricité, ainsi que sur des systèmes permettant de récupérer la chaleur perdue lors de procédés impliquant une conversion d'énergie.

Les normes de plus en plus sévères sur la réduction des gaz à effet de serre ouvrent la voie au développement de centrales électriques plus propres. Parmi les options sérieusement considérées et déjà implémentées dans certaines parties du monde se trouve la production d'électricité à partir de réservoirs géothermiques à basse température. L'exploitation de la chaleur de la Terre à haute température se fait de manière efficace avec l'utilisation de centrales géothermiques de types « flash » et « dry », notamment en Islande et en Californie, régions comportant des bassins naturels d'eau chaude à haute pression sous forme liquide ou gazeuse. Pour les régions ayant un sous-sol à plutôt basse température (en bas de 200°C), il est parfois impraticable d'exploiter une centrale géothermique de manière rentable, compte tenu de l'état actuel des connaissances.

Au Canada en 2019, il n'y a pas encore de centrale géothermique en opération. En effet, la majorité de son territoire comporte des roches chaudes, mais elles sont à plusieurs kilomètres de profondeur. Il faudrait donc recourir à la géothermie profonde stimulée par fracturation hydraulique pour exploiter cette ressource énergétique. Afin de développer les connaissances et expertises nécessaires à la réalisation d'un projet expérimental sur la géothermie profonde au Québec, l'Institut de recherche d'Hydro-Québec (IREQ) a initié le projet intitulé *Intégration de la géothermie profonde dans le portefeuille énergétique canadien*. Bénéficiant du financement du programme Initiative écoÉnergie sur l'innovation

du gouvernement du Canada, ce projet multidisciplinaire concerne l'évaluation et l'exploration des ressources géothermiques, l'ingénierie des réservoirs, les aspects sociaux et environnementaux, la fracturation et l'ingénierie de la production de puissance [1].

Dans le cadre de cette initiative, l'aspect de la production de puissance s'articule autour du projet INGÉOPRO : *Développement de modèles avancés pour l'ingénierie de la production de la puissance à partir de la géothermie profonde*. Le volet A de ce projet s'intéresse à la simulation numérique et l'optimisation de centrales géothermiques. Les questions apportées par ce projet sont :

- a) Quels sont les types de centrales géothermiques les mieux adaptés aux conditions de la province de Québec ?
- b) Quelles sont les meilleures configurations de centrales ?
- c) Quels sont les paramètres d'opération optimaux de la centrale, et ce, pour les diverses possibilités de température de l'eau géothermale ?
- d) À quelles valeurs de puissance nette produite devrait-on s'attendre en considérant le climat froid de la province ?

En effet, puisque les cycles thermodynamiques utilisés pour convertir la chaleur à basse température s'avèrent relativement inefficaces, le choix du modèle de centrale est crucial. Dans le présent cas, il faut fracturer le sol pour y injecter un fluide géothermique (« brine » en anglais, ou géofluide en français) sous pression pour recevoir la chaleur de la roche sèche en se déplaçant du puits d'injection aux puits de récupération. Le géofluide entre ensuite dans une centrale dite « binaire » pour transférer sa chaleur à un fluide de travail qui effectue un cycle thermodynamique fermé de puissance, et est ensuite réinjecté dans le sol.

Le cycle de puissance en question peut être choisi parmi plusieurs options. Cependant, seulement deux sont typiquement considérés aptes à être appliqués à la géothermie [2] : le cycle de Kalina [3] et le cycle de Rankine organique. Le cycle de Kalina est un cycle combiné utilisant un mélange eau-ammoniac qui s'évapore et se condense avec un important glissement de température [4], c'est-à-dire avec une hausse/baisse de température

pendant un changement de phase liquide-gaz. Cependant, le cycle de Rankine organique lui est préféré dans ce projet puisque qu'il a une performance énergétique généralement plus élevée pour des réservoirs à basse température [5] et comporte un vaste choix de fluides de travail pouvant être étudiés.

Le cycle de Rankine, un cycle thermodynamique se rapprochant du cycle de Carnot [6], correspond à une centrale thermique élémentaire [7] utilisant habituellement l'eau comme fluide moteur. Il comprend quatre composantes de base : (i) une pompe pour amener l'eau à la pression désirée, (ii) une chaudière (ou échangeur de chaleur) pour chauffer et évaporer l'eau à pression constante, (iii) une turbine à vapeur qui détend l'eau gazeuse, et (iv) un condenseur qui évacue la chaleur latente de l'eau à pression constante. L'utilisation de fluides organiques comme le R134a au lieu de l'eau permet d'exploiter une source de chaleur à plus basse température de manière plus efficace grâce à leur température d'ébullition généralement plus faible [8]. Le cycle se nomme alors le cycle de Rankine organique, ou ORC (Organic Rankine Cycle). La première centrale géothermique comportant un ORC a été construite en 1967 dans la Kamchatka Peninsula en Russie et utilisait le R12 comme fluide de travail [9].

Selon les dernières études sur le sujet ainsi qu'en se basant sur les centrales en opération à travers le monde, la question (a) est répondue d'emblée par la centrale binaire comportant un ORC. Cependant les questions (b), (c) et (d) requièrent une étude plus approfondie. Le rendement thermique de l'ORC étant tout de même plutôt faible, le design de la centrale doit être soigneusement optimisée pour produire un maximum de puissance et être économiquement viable. Le design comprend la configuration du cycle, les paramètres d'opérations (pressions et débits par exemple) et le choix du fluide de travail.

Les questions apportées par Hydro-Québec concernant le design optimal d'une centrale géothermique ont permis, dans le cadre de cette thèse, le développement d'une méthodologie globale d'analyse de cycle thermodynamiques (i.e., modélisation mathématique, implémentation numérique, optimisation numérique, et finalement présentation des résultats sous forme de chartes de conception). Cette méthodologie peut

être appliquée à d'autres machines thermiques pour maximiser leur performance. Par exemple, le Powertrain and Vehicle Research Centre (PVRC) de l'University of Bath au Royaume-Uni s'est intéressé au Cycle de Brayton Inversé (IBC pour Inverted Brayton Cycle) comme alternative au turbo chargeur. L'IBC est un cycle ouvert qui utilise la chaleur des gaz de produits de combustion afin de créer de la puissance mécanique. L'IBC comporte trois composantes principales (une turbine à gaz, un échangeur de chaleur et un compresseur), et diverses variantes peuvent être appliquées à cette configuration de base. Des questions similaires à celles postulées pour les centrales géothermiques sont soulevées :

- e) Quels sont les types de système IBC les mieux adaptés aux diverses températures possibles des gaz d'échappement et du puits de chaleur ?
- f) Quelles sont les meilleures configurations du système IBC ?
- g) Quels sont les paramètres d'opération optimaux du système IBC, et ce, pour les diverses possibilités de températures d'opération ?
- h) À quelles valeurs de puissance nette produite devrait-on s'attendre en considérant les diverses possibilités de températures d'opération ?

Les questions pour les systèmes IBC (questions e à h) sont similaires aux questions pour les centrales géothermiques (questions a à d), et elles pourront donc être traitées par la même méthodologie dans le cadre de cette thèse.

### ***Objectifs***

L'objectif principal de cette thèse est d'offrir un outil permettant aux ingénieurs de concevoir des systèmes de récupération de la chaleur présentant un travail spécifique net maximisée. Sous forme d'un ensemble de diagrammes, cet outil peut être utilisé pour une large plage de température de la source chaude (géofluide ou gaz d'échappement) et de température du puits de chaleur (la source froide). La thèse se concentre sur deux axes :

A1 : Le cycle de Rankine organique (ORC) employé dans une centrale géothermique

A2 : Le cycle de Brayton inversé (IBC) employé comme système de récupération de la chaleur des gaz d'échappement d'un moteur

Les objectifs secondaires communs aux deux axes sont les suivants :

1. Simuler numériquement plusieurs configurations pour chacun des cycles thermodynamiques avec le logiciel MATLAB.
2. Optimiser le design des cycles pour maximiser leur travail spécifique net selon une large gamme de température de la source chaude et une large gamme de température de la source froide.
3. Organiser les résultats sous forme de diagrammes présentant le travail spécifique net maximisé et les variables de design optimisées.
4. Déterminer les variantes des cycles les plus performantes en fonctions des températures d'opération.

Les objectifs spécifiques à l'axe 1 sont :

- A1.1. Développer de nouvelles corrélations permettant de prédire le travail spécifique net d'une centrale géothermique avec ORC.
- A1.2. Évaluer l'impact de la charge auxiliaire (pompes et système de refroidissement) sur le travail spécifique net en fonction des températures d'opération.

Finalement, les objectifs spécifiques à l'axe 2 sont :

- A2.1 Comparer la performance des nouveaux IBCs avec celle des meilleurs systèmes utilisés à ce jour pour récupérer la chaleur des gaz d'échappement de moteurs.
- A2.2 Déterminer les applications les plus appropriées pour ces nouveaux cycles.

**CHAPITRE 1. GEOTHERMAL POWER PLANTS WITH  
MAXIMIZED SPECIFIC POWER OUTPUT: OPTIMAL  
WORKING FLUID AND OPERATING CONDITIONS  
OF SUBCRITICAL AND TRANSCRITICAL ORGANIC  
RANKINE CYCLES**

## **1.1. Résumé**

Dans cet article, la conception d'un cycle de Rankine organique (ORC) est optimisée au moyen de simulations numériques. Les systèmes d'intérêt sont les cycles thermodynamiques sous-critique et transcritique. Des optimisations sont exécutées avec l'objectif de déterminer la conception maximisant le travail spécifique net. Les variables de design incluent les paramètres d'opération (pressions, débits massiques), et les meilleurs fluides de travail sont déterminés en comparant la performance de 36 réfrigérants. Les optimisations sont réalisées pour une large gamme de températures du géofluide (de 80 à 180°C), et pour une large gamme de température de condensation (de 0.1 à 50°C). Les résultats sont consignés sous forme de diagrammes pouvant être utilisés comme des outils efficaces pour concevoir des centrales géothermiques optimales. Finalement, une analyse d'ordre de grandeur a permis de développer des nouvelles corrélations pour prédire le travail spécifique net maximal d'un ORC.

## **1.2. Abstract**

In this paper, the design of an Organic Rankine Cycle (ORC) is optimized by means of numerical simulations. The systems of interest are the subcritical and transcritical thermodynamic cycles. Optimizations are performed with the objective of determining the design that maximizes the specific power output. The design variables include the operating parameters (pressures, mass flow rates), and the best working fluid is determined by comparing the performance of 36 refrigerants. Optimization runs are performed for a wide range of geofluid temperatures (from 80 to 180°C), and for a wide range of condenser temperature (from 0.1 to 50°C). The results are summarized in charts that may be used as efficient tools for designing optimal geothermal power plants. Finally, an approximate analysis allowed to develop new correlations for predicting the maximal specific power output of an ORC.

## **1.3. Introduction**

The energy sector raises important issues in modern societies. With growing demand, depletion of fossil fuel reserves and global warming, more and more efforts are devoted to

the development of renewable sources of energy. Among the various solutions, geothermal energy is one of the candidates for reducing GES emissions in the context of power generation. Indeed, the exploitation of the Earth's heat could provide 3.5% of the global generated electricity by 2050, according to the International Energy Agency. The race to secure and diversify the energy portfolio paves the way to Organic Rankine Cycle (ORC) geothermal power plants using low-temperature reservoirs.

Organic Rankine Cycles have a relatively low thermodynamic efficiency, and as a consequence, their designs must be carefully optimized to be economically viable. Indeed, Organic Rankine Cycles have been the subject of several studies in literature. For example, Wei et al. [10] analysed and optimized an ORC system using HFC-245fa as a working fluid. Dai et al. [11] optimized the performance of ORC with 10 different working fluids, for a single fixed value of heat source temperature. A multicriteria approach has been used by Toffolo et al. [12] in order to perform the optimal selection of design parameters in ORC. The impact of turbine efficiency on ORC net power output calculations was investigated by Pan and Wang [5]. Binary ORC power plants exploiting low-temperature geothermal heat sources have been studied through design analysis [13]; [14]; [15] and through design optimization [16], [17], [18]. A comparison of a few working fluids was performed by Hung [19] and also by Drescher and Brüggemann [20]. Transcritical cycles are analysed for a case of high brine inlet temperature using three working fluids in [21]. A transcritical cycle using R125 is compared with three other fluids in subcritical cycle for a case with low brine temperature in [22]. Performance and economical comparison of optimized subcritical and transcritical cycles is made in [23] using 12 working fluids for one low brine temperature. An exhaustive review of thermodynamic cycles used to perform the conversion of low-grade heat have been performed by Chen et al. [24], and a world-wide review of geothermal power plant efficiency has been done by Zarrouk and Moon [25].

By looking at the abovementioned references and in literature, it can be seen that there are few texts that provide clear guidelines or rules of thumbs to design an ORC geothermal plant providing maximal power output. However, Clarke and McLeskey Jr. [26] recently

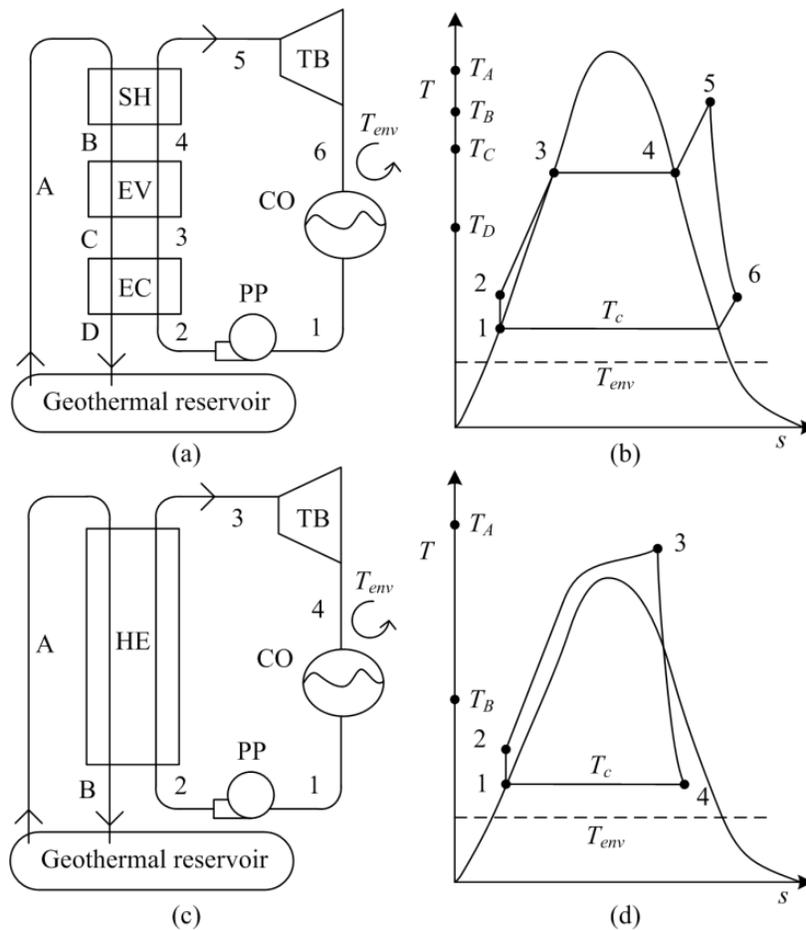
performed a multi-objective optimization of subcritical ORC geothermal power plants by considering 17 working fluids. More specifically, they identified optimal designs for three fixed values of dry air temperatures and six fixed values of brine temperatures. Their results allow to identify the best working fluid and operating conditions in order to get the highest specific power output from a geothermal power plant. Hence, it can be seen that there is a call to develop design guidelines for ORC geothermal power plants. In the present paper, the authors propose to develop new design charts for the preliminary design of geothermal power plants, by extending the work performed by Clarke and McLeskey Jr. [26]. More specifically, several additional features are included in this study, such as: (i) the investigation of transcritical cycles, (ii) the comparison of 36 different working fluids, (iii) the analyses of a continuous set of combination of condenser temperatures and brine temperatures, and (iv) the development of a new approximate theoretical approach for optimal ORC design.

The main goals of the work presented here are: (i) to develop a numerical simulation tool that predicts subcritical and transcritical ORC performance, (ii) to develop optimization algorithms to identify the best ORC designs, and (iii) to establish new charts that provide guidelines for designing optimal ORC geothermal power plants. The analysis presented in this paper covers a large set of operating conditions, i.e., a geofluid temperature from 80 to 180°C, and a condenser temperature (condensing temperature of the working fluid) from 0.1 to 50°C. The objective function to maximize is the specific power output  $w$ , and the design variables are the pressure  $P_2$  that prevails in the heat exchangers, the mass flow rate ratio  $R_{b,w}$  between the geofluid and the working fluid, and the superheater efficiency  $\varepsilon_{SH}$ . The analysis involves 36 different working fluids.

The paper is structured as follows: Section 1.4 describes the methodology for calculating the thermodynamic performance of two ORCs of interest; Section 1.5 describes the optimization problem; Sections 1.6 – 1.9 present results of the optimization runs and provide new charts for designing optimal ORC; Section 1.10 presents the development of new correlations; and finally, discussions and conclusions are provided in Sections 1.11 and 1.12.

## 1.4. Problem statement

A description of the thermodynamic cycles of interest is provided in this section. The systems considered in this paper are two variants of the Organic Rankine Cycles (ORC): (i) a subcritical cycle (turbine inlet pressure smaller than the working fluid critical pressure), and (ii) a transcritical cycle (turbine inlet pressure larger than the working fluid critical pressure). The subcritical and transcritical systems are illustrated in Fig. 1.1a and 1.1c, while their corresponding temperature-entropy diagrams ( $T - s$ ) are illustrated in Fig. 1.1b and 1.1d, respectively.



**Figure 1.1.** Organic Rankine Cycle designs. (a) Subcritical cycle equipment architecture. (b) Subcritical cycle thermodynamic diagram. (c) Transcritical cycle equipment architecture. (d) Transcritical cycle thermodynamic diagram.

#### 1.4.1. *Subcritical cycle*

The subcritical ORC possesses two circuits (see Fig. 1.1a), i.e., the primary and secondary circuits. The primary circuit is dedicated to the geothermal brine. For instance, the geofluid is pumped from the reservoir and enters the plant at state {A}. Then, it passes through three heat exchangers, each with exit conditions corresponding to states {B}, {C} and {D}, respectively. It is also possible to use only one heat exchanger [27] instead of three separated heat exchangers; however, only the configuration with three heat exchangers (as shown in Fig. 1.1a) is considered in this paper. Finally, the geofluid is reinjected in the ground. It should be noticed that the geothermal fluid typically contains dissolved gas and calcite. However, it is assumed in this paper that geothermal fluid properties may be approximated as equal to those of pure water, which is in line with recent literature (e.g., [28], [29]). The geofluid is considered to remain in a compressed liquid state, and it never undergoes evaporation during the whole process. The temperature of the geofluid at states {A}, {B}, {C} and {D} are identified by black dots on the temperature axis of Fig. 1.1b for comparison purpose.

The secondary circuit (a closed loop) is dedicated to the working fluid (see states {1} to {6} in Figs. 1.1a and 1.1b). The working fluid receives heat from the primary circuit by means of three heat exchangers that act as a boiler for the working fluid. Typically, the working fluid is an organic fluid having a low boiling point, and it is made of refrigerant, alkane or other hydrocarbons [17]. The working fluid enters a pump (PP) as saturated liquid at the condenser pressure (state {1} in Fig. 1.1b), and it leaves that pump at the desired subcritical pressure (state {2}). It is then heated at constant pressure in an economizer (EC) to obtain a state of saturated liquid (state {3}). An evaporator (EV) brings the working fluid to a saturated vapor state (state {4}) and involves a thermodynamic evolution at constant temperature and constant pressure. Furthermore, a superheater (SH) provides the necessary amount of heat to reach state {5} at constant pressure. In limiting cases, states {4} and {5} can overlies, which means that the working fluid enters the turbine as saturated vapor and the superheater transfers no heat. The minimal temperature difference for heat exchange between the geofluid and the working fluid (pinch-point) has been set to be larger or equal to 5°C in the present work. After state {5}, the working fluid produces work in the turbine

(TB), and leaves it at state {6}. Finally, the fluid rejects heat to the environment at constant pressure by means of a condenser (CO), and leaves it at state {1}.

#### **1.4.2. *Transcritical cycle***

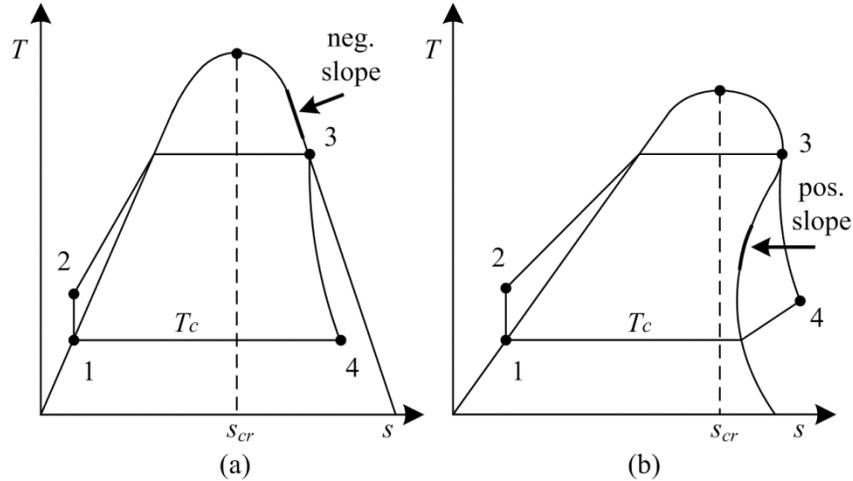
The transcritical cycle is similar to the subcritical cycle, see Figs. 1.1c and 1.1d. The difference lies in the heat transfer between the two fluids. For instance, the hot geofluid in the primary circuit enters the plant at state {A} and transfers its heat to the working fluid through one exchanger (HE) instead of three. The cycle is equipped with only one heat exchanger because there is no definite difference between liquid and vapor phases when the working fluid is heated. The geofluid is then reinjected in the reservoir at state {B}. The relative temperature of the geofluid at states {A} and {B} are identified by black dots on the temperature axis of Fig. 1.1d for comparison purpose.

The working fluid in the secondary circuit begins its path as saturated liquid at condenser pressure (state {1} in Fig. 1.1c) and passes through a pump (PP) to obtain the desired supercritical pressure (state {2}). Then, the working fluid is heated at constant pressure by the geofluid in the heat exchanger (HE) so as to reach the turbine inlet as superheated vapor (state {3}). The fluid expands in the turbine (TB), leaves it at state {4}, and enters the condenser (CO) to reject heat to the surroundings at constant pressure. While there is no definite location for a pinch point in the heat exchanger (HE), it is required to impose a minimal temperature difference in that piece of equipment to ensure that the relative temperature of both fluids is valid for heat transfer. The method used to verify the relative temperatures in the heat exchanger is presented in Section 1.4.5.

#### **1.4.3. *Classes of working fluids***

The working fluids used in this paper are pure fluids (i.e., no mixture) and they can be categorized by their shape in a  $T - s$  diagram [24]. More specifically, fluids are labelled as “normal” when the slope of the saturation line is negative (e.g. propane) for all entropy  $s$  values larger than the entropy at the critical point ( $s_{cr}$ ), see an illustration in Fig. 1.2a. On the other hand, fluids are labelled as “retrograde” when the slope of the saturation line is positive (e.g. octane) for some values of entropy  $s$  larger than the entropy at the critical

point ( $s_{cr}$ ), see Fig. 1.2b. The labels “normal” and “retrograde” are often used in literature (e.g., [2]). It should be noticed that other authors use the labels “wet” and “dry” instead of “normal” and “retrograde”, respectively.

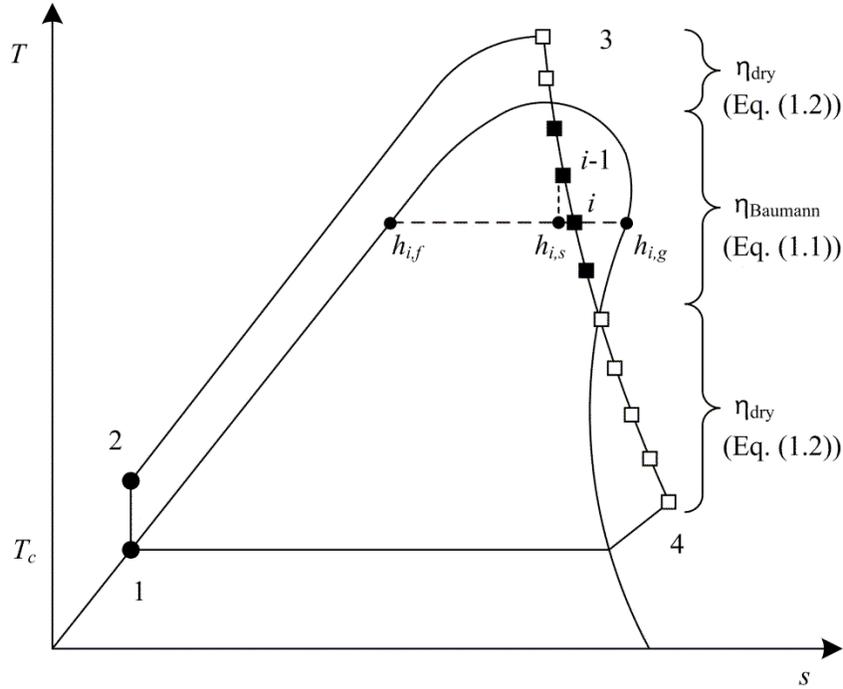


**Figure 1.2.** Simplified thermodynamic diagrams. (a) Normal fluid. (b) Retrograde fluid.

In practice, *normal* fluids typically have to be superheated so as to reduce liquid droplets appearance during expansion in turbines, whereas this is usually not a requirement for the *retrograde* fluids. Indeed, retrograde fluids in turbines typically remain in superheated state during expansion in turbines, as it can be seen in Fig. 1.2b. It should be emphasized that the categories “normal” or “retrograde” correspond to a fluid *property* that is independent of the thermodynamic cycle itself. The thermodynamic evolutions {1}-{2}-{3}-{4} are present in Fig. 1.2 for the sole purpose of illustrating the behaviour of the fluid in a typical turbine (between states {3} and {4}). Normal and retrograde fluids will be both considered in the analysis performed in this paper.

#### 1.4.4. Turbine differential modeling

A schematic representation of the working fluid thermodynamic evolution in the turbine (between states {3} and {4}) is provided in Fig. 1.3. Notice that while that figure illustrates the use of a retrograde fluid in a transcritical cycle, the methodology explained here is also systematically used for any kind of cycles (subcritical or transcritical) and any category of fluid (normal or retrograde) investigated in this paper.



**Figure 1.3.** Calculation principle of turbine efficiency on thermodynamic diagram of a retrograde fluid.

It can be seen in this  $T - s$  diagram that for some ranges of pressure between states {3} and {4}, the working fluid is at superheated vapor state, which requires the use of the turbines dry efficiency  $\eta_{dry}$  [2] for calculating the intermediary thermodynamic states in the turbine (see open squares in Fig. 1.3). However, the fluid may be in saturated mixture states for other ranges of pressure (see black squares in Fig. 1.3), which requires the use of the Baumann efficiency  $\eta_{Baumann}$  [30]; [2] to take into account the decrease of the turbine efficiency due to the presence of liquid droplets (a decrease inversely proportional to the vapor quality). Thus, a method based on differential thermodynamic evolution has been developed to model the evolution from state {3} to state {4}. It consists in representing the turbine into several virtual small turbines (i.e., into several stages). Each stage has its own efficiency expression (i.e., dry efficiency or Baumann efficiency expressions), and deals with a small part ( $\Delta P_i$ ) of the total pressure drop ( $P_3 - P_4$ ). In other words, the pressure drop is discretized so as to calculate the enthalpy at the end of each stage. The liquid content is calculated at each stage, which allows to identify the appropriate efficiency expression to use.

Thus, when the fluid that enters a turbine stage  $i$  contains a saturated mixture, the enthalpy  $h_i$  at the outlet of that stage can be calculated by using the Baumann expression as follows:

$$h_i = \frac{h_{i-1} - A \left( x_i - h_{i,f} / (h_{i,g} - h_{i,f}) \right)}{1 + A / (h_{i,g} - h_{i,f})} \quad (1.1)$$

$$A = \frac{\eta_{dry}}{2} (h_{i-1} - h_{i,s})$$

where subscript  $i$  indicates the actual stage, and subscript  $i-1$  the previous stage. The variable  $h_{i,s}$  is the enthalpy after a small pressure drop when the expansion in a stage is assumed to be isentropic.

In the case where the fluid that enters a turbine stage  $i$  does not contain liquid phase, the enthalpy  $h_i$  at the outlet of that stage can be expressed as:

$$h_i = h_{i-1} - \eta_{dry} (h_{i-1} - h_{i,s}) \quad (1.2)$$

The enthalpy calculated at the last pressure stage corresponds to the enthalpy at state {4}, and it is used in the calculation of the power plant specific output power. The variables  $h_{i,f}$ ,  $h_{i,g}$  and  $h_{i,s}$  are identified in Fig. 1.3.

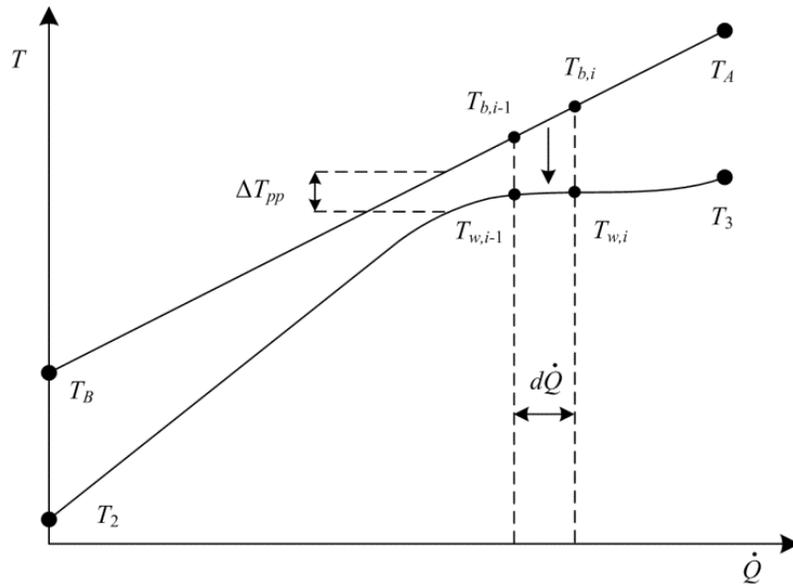
#### **1.4.5. Heat exchanger modeling**

The heat transfer process in the heat exchangers has to be carefully investigated to ensure the thermodynamic and physical validity of a design. As explained earlier, it is necessary to verify the temperature of both fluids throughout the *entire* length of the heat exchanger in transcritical cycles, because there is no preferred position for the occurrence of the pinch point. Even for subcritical designs (Fig. 1.1a), the pinch point may not be located at the evaporator inlet when the pressure  $P_3$  is close to the critical pressure. Hence, a general methodology for identifying the pinch point in the heat exchangers was developed and applied to subcritical and transcritical designs.

The idea is to represent the heat exchanger in a large number of virtual heat exchangers, each one involving a small part  $d\dot{Q}$  of the total heat transfer  $\dot{Q}$  (see an example of the heat exchanger in a transcritical cycle in Fig. 1.4). Then, the temperature  $T_b$  of the geofluid (subscript  $b$  for “brine”) and the temperature  $T_w$  of the working fluid may be calculated at each stage  $i$ :

$$\begin{aligned} T_{b,i} &= T_{b,i-1} + \frac{d\dot{Q}}{\dot{m}_b c_b} \\ h_{w,i} &= h_{w,i-1} + \frac{d\dot{Q}}{\dot{m}_w} \\ T_{w,i} &= T(h_{w,i}, P_{hex}) \end{aligned} \quad (1.3)$$

where subscript  $i$  indicates the actual stage, subscript  $i-1$  the previous stage,  $d\dot{Q}$  the differential heat transfer, and  $P_{hex}$  the pressure in the heat exchangers. The variable  $c_b$  is the liquid specific heat of the geofluid,  $\dot{m}_b$  the brine mass flow rate and  $\dot{m}_w$  the working fluid mass flow rate. With this approach, it is possible to identify the pinch-point temperature difference  $\Delta T_{pp}$ , and to verify the constraint associated to that parameter (i.e.,  $\Delta T_{pp}$  must be larger or equal than  $5^\circ\text{C}$ ). That method was used for analysing heat transfer in the heat exchangers shown in Figs. 1.1a and 1.1c.



**Figure 1.4.** Temperature evolution of both fluids in heat exchangers.

#### **1.4.6. Numerical simulations**

Several numerical tools exist for simulating thermodynamic cycles, such as AxCYCLE<sup>TM</sup> [31] or Thermoptim<sup>TM</sup> [4], and many software allow the calculation of thermodynamic properties, such as CoolProp [32], and XSteam [33]. The option selected in this work for modeling the thermodynamic cycles of interest consists in using a commercial software REFPROP [34] along with the programming script language MATLAB<sup>®</sup> [35]. More specifically, the commercial software is used to evaluate fluid properties and to calculate thermodynamic states, while our in-house script computes energy balances and the specific power output of the thermodynamic cycles. The numerical model developed in this paper has been validated by simulating a binary cycle already analysed in [2]. For instance, the required brine and isopentane mass flow rates obtained from the numerical model differed by less than 0.3% with those presented in [2]. Hence, the numerical model was assumed to be valid.

### **1.5. Optimization methodology**

The objective of this paper is to maximize the geothermal power plants performance. The definitions of the objective function, of the design variables, and of the constraints for both cycles are provided in the following sections. Various algorithms have been used to optimize thermodynamic cycles (e.g. [36]). In the present work, the optimization problems are solved by using the Optimization Toolbox<sup>TM</sup> of the programming software MATLAB [37]. More specifically, the function *fmincon.m* is used with an “interior-point” algorithm [38]. That algorithm has already been used in the context of thermodynamic cycle optimization in literature (e.g., [39]).

#### **1.5.1. Subcritical cycle optimization**

The objective function selected in this study is the power plant specific power output  $w$ . Specific power output has been used for various analyses of ORC in literature (e.g., [16]; [5]; [17]), and it represents the amount of energy (kJ) produced for each kg of geofluid extracted from the ground. The three design variables are the pressure  $P_2$  that prevails in the heat exchangers, the mass flow rate ratio  $R_{b,w} = \dot{m}_b / \dot{m}_w$  between the geofluid and the

working fluid, and the efficiency  $\varepsilon_{SH}$  of the superheater (see SH in Fig. 1.1a). Indeed, by considering this efficiency ( $\varepsilon_{SH}$ ) as a design variable, it is possible to obtain a wide range of possible temperature values for state {5} in the superheated state. The optimization statement of the subcritical cycle can be summarized as:

$$\begin{aligned}
& \text{maximize}(w) \\
& \text{optimizing}(P_2, R_{b,w}, \varepsilon_{SH}) \\
& \text{respecting} \begin{cases} h_5 \geq h_{g@P_2}, & \varepsilon_{EV} \leq \varepsilon_{\max} \\ x_{\min} \geq x_{tol}, & T_b - T_w \geq \Delta T_{tol} \\ \varepsilon_{EC} \leq \varepsilon_{\max}, \end{cases} \quad (1.4) \\
& \text{fixed values} \begin{cases} T_A, T_c, x_{tol}, \Delta T_{tol} \\ \eta_p, \eta_{dry}, \varepsilon_{\max} \end{cases}
\end{aligned}$$

Five constraints are defined in Eq. (1.4). First, the working fluid at state {5} must be superheated ( $h_5 \geq h_{g@P_2}$ ). Next, the minimal vapor quality reported in the turbine  $x_{\min}$  has to be greater than a tolerance quality ( $x_{tol} = 0.85$ ) during the entire expansion process in the turbine, so as to avoid excessive blade wear [7]. The value of  $x_{\min}$  is obtained with the methodology explained in Section 1.4.4. Furthermore, efficiencies of the evaporator  $\varepsilon_{EV}$  and of the economizer  $\varepsilon_{EC}$  are not fixed but determined by post-treatment and they are not allowed to exceed an imposed maximum value ( $\varepsilon_{\max} = 0.85$ ). Finally, the difference between the geofluid temperature  $T_b$  and the working fluid temperature  $T_w$  at any stage (see Fig. 1.4) must be greater than a minimum value ( $\Delta T_{tol} = 5^\circ\text{C}$ ). That last constraint can be verified by using the methodology explained in Section 1.4.5. The value of the pump efficiency is assumed to be  $\eta_p = 0.80$ , and that of the turbine dry efficiency is assumed to be  $\eta_{dry} = 0.85$ .

### 1.5.2. Transcritical cycle optimization

A transcritical cycle can be used when the brine temperature is sufficiently high and when enough heat is provided to the working fluid, so as to obtain a vapor state at the turbine

inlet at a pressure greater than the working fluid critical pressure. The objective function is the specific power output  $w$ , and the number of design variable decreases from three to two, because the heat exchange now occurs in only one heat exchanger with a single fixed efficiency  $\varepsilon_{HE}$ . The optimization problem of a transcritical cycle may be stated as:

$$\begin{aligned}
& \text{maximize}(w) \\
& \text{optimizing}(P_2, R_{b,w}) \\
& \text{respecting} \begin{cases} T_3 \geq T_{cr} \\ x_{\min} \geq x_{tol} \\ T_b - T_w \geq \Delta T_{tol} \end{cases} \\
& \text{fixed values} \begin{cases} T_A, T_c, x_{tol}, \Delta T_{tol} \\ \eta_p, \eta_{dry}, \varepsilon_{HE} \end{cases}
\end{aligned} \tag{1.5}$$

This optimization is simpler than the previous one because the constraints concerning heat exchanger efficiencies are not needed. Indeed, in this paper, the efficiency of the heat exchanger (HE) shown in Fig. 1.1c is set to a fixed value  $\varepsilon_{HE} = 0.85$ , which is a typical value for preliminary analyses of ORC heat exchangers [40].

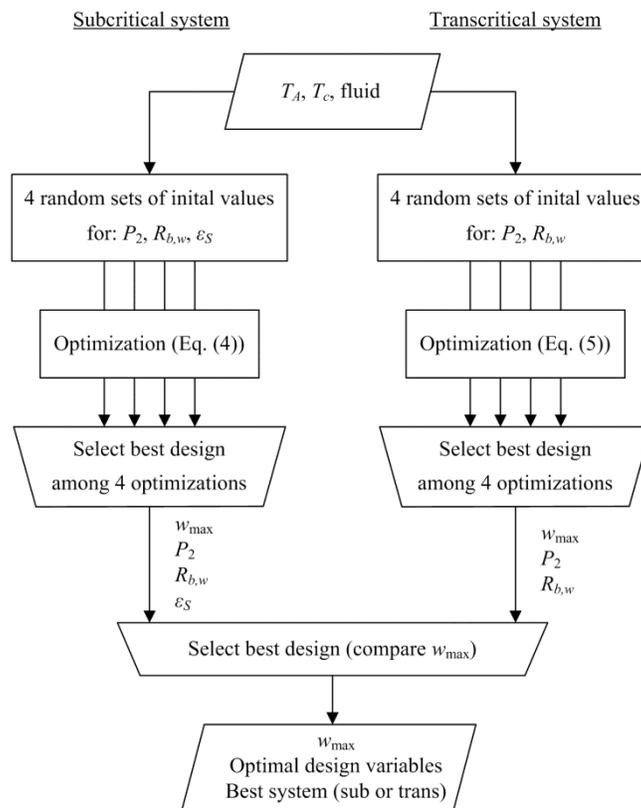
The optimization problem stated in Eq. (1.5) involves three constraints. First, the working fluid at state {3} must be superheated; second, the minimal vapor quality reported in the turbine  $x_{\min}$  has to be greater than  $x_{tol}$  in the entire expansion in this turbine; and third, the temperature difference between the two fluids in the heat exchanger has to be larger than a minimum value  $\Delta T_{tol}$ . The values of  $\Delta T_{tol}$ ,  $x_{tol}$ ,  $\eta_p$  and  $\eta_{dry}$  are the same as those provided in Section 1.5.1. It is worth mentioning that no minimum limit is imposed on the reinjection temperature for both optimization problems (state {D} or state {B}).

### 1.5.3. Starting points

It was observed by the authors that the optimization results may depend on the initial values of the design variables (i.e., the optimization starting points). Each optimization is systematically performed four times, i.e., with four different starting points. The initial values of the design variables are determined randomly, but it is verified that a starting

point respects the constraints stated in Eqs. (1.4) and (1.5). Launching four optimization runs with their own starting point increases the odds of obtaining a result that: (i) is close to the global optimal design, and (ii), that respects the constraints stated in Eqs. (1.4) or (1.5). The highest value of the four maximized specific output  $w$  was considered as the best performance.

To summarize, for a given set of fluid, condenser temperature  $T_c$  and geofluid temperature  $T_A$ , the optimization of the subcritical design (Eq. (1.4)) is performed with four different initial points, and the optimization of the transcritical design (Eq. (1.5)) is also performed with four different initial points. Finally, the best system (either subcritical or transcritical system) is identified by comparing the maximal specific output obtained from the optimization of both systems. The algorithm describing the optimization is illustrated in Fig. 1.5.



**Figure 1.5.** Algorithm for optimizing ORC designs.

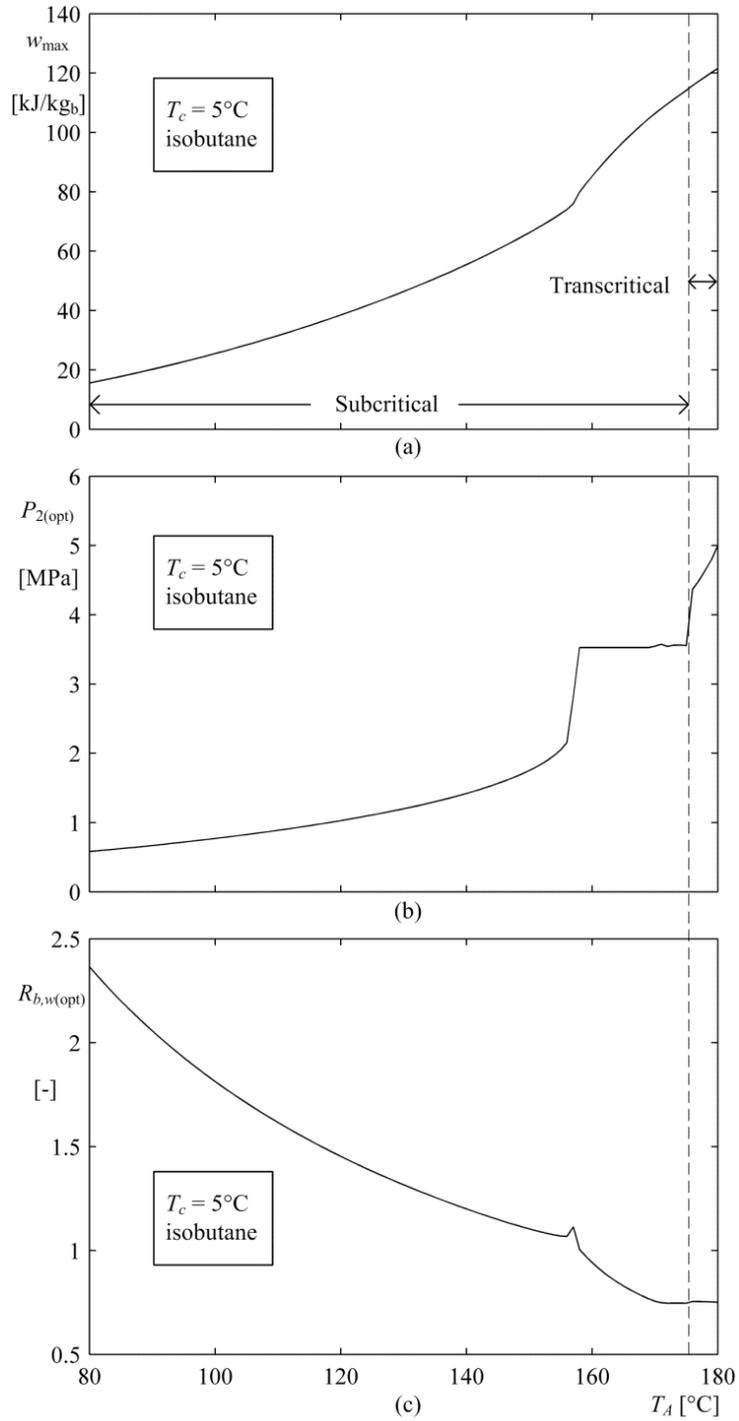
## 1.6. One-fluid optimization results

The optimization methodology presented in Fig. 1.5 is performed with one fluid at a time. More specifically, Section 1.6.1 presents optimization with isobutane as working fluid, and shows the behavior of the maximized objective function and of the corresponding optimized design variables with respect to the geofluid temperature. Section 1.6.2 presents the thermodynamic state of the working fluid at the turbine inlet, with respect to the geofluid temperature. For the sake of illustration, optimization results reported in this section were performed by using a condenser temperature fixed to 5°C.

### 1.6.1. Optimization with isobutane

Optimization runs were performed by using isobutane as the working fluid, for brine temperature between 80 and 180°C. The results are reported in Fig. 1.6. More specifically the maximized specific power is shown in Fig. 1.6a, the optimal pressure  $P_{2(\text{opt})}$  is shown in Fig. 1.6b, and the optimal mass flow ratio  $R_{b,w(\text{opt})}$  in Fig. 1.6c, with respect to  $T_A$ . In other words, each point in this figure is the results of a full optimization.

First of all, it can be seen in Fig. 1.6a that the maximized specific power increases with  $T_A$ , which is expected because there is more available energy in the geofluid as its temperature increases. The simulation results showed that the best system for geofluid temperature in the range  $T_A \in [80,176]^\circ\text{C}$  was the subcritical cycle (see the ‘subcritical’ label in Fig. 1.6a). Nonetheless, it can be seen that the shape of the curve in Fig. 1.6a is different between the interval  $T_A \in [156,176]^\circ\text{C}$ . For the higher temperature range, i.e.,  $T_A \in [176,180]^\circ\text{C}$ , the best system was the transcritical system (see the ‘transcritical’ label in Fig. 1.6a). Indeed, it is expected that transcritical systems only work for the higher range of geofluid temperatures, because such cycles require a higher level of energy from the geofluid.



**Figure 1.6.** Optimization results for a condenser temperature of  $5^{\circ}\text{C}$  with isobutane with respect to brine inlet temperature. (a) Maximal specific power. (b) Optimized heat exchangers pressure. (c) Optimized mass flow ratio.

In Fig. 1.6b, it can be seen that the optimized pressure has a continuous trend until  $T_A = 156^\circ\text{C}$ , where it undergoes a sharp increase. Indeed, the thermodynamic toolbox used in this work is very sensitive to pressure values that approach the critical pressure  $P_{cr}$ . It was observed that for temperature approaching  $156^\circ\text{C}$ , the optimal pressure is very close to the critical pressure (see Fig. 1.6b), and the optimal pressure value of the subcritical design is thus limited by the critical pressure (the plateau observed in Fig. 1.6b). Reaching higher pressure was not possible for  $T_A \in [156, 176]^\circ\text{C}$  because it would involve a transcritical system, and transcritical system as defined in Eq. (1.5) did not provide better results than the subcritical system for that temperature range  $T_A \in [156, 176]^\circ\text{C}$ . Finally, a sudden increase in pressure occurs at  $176^\circ\text{C}$ , because the transcritical system is selected and it requires supercritical pressures.

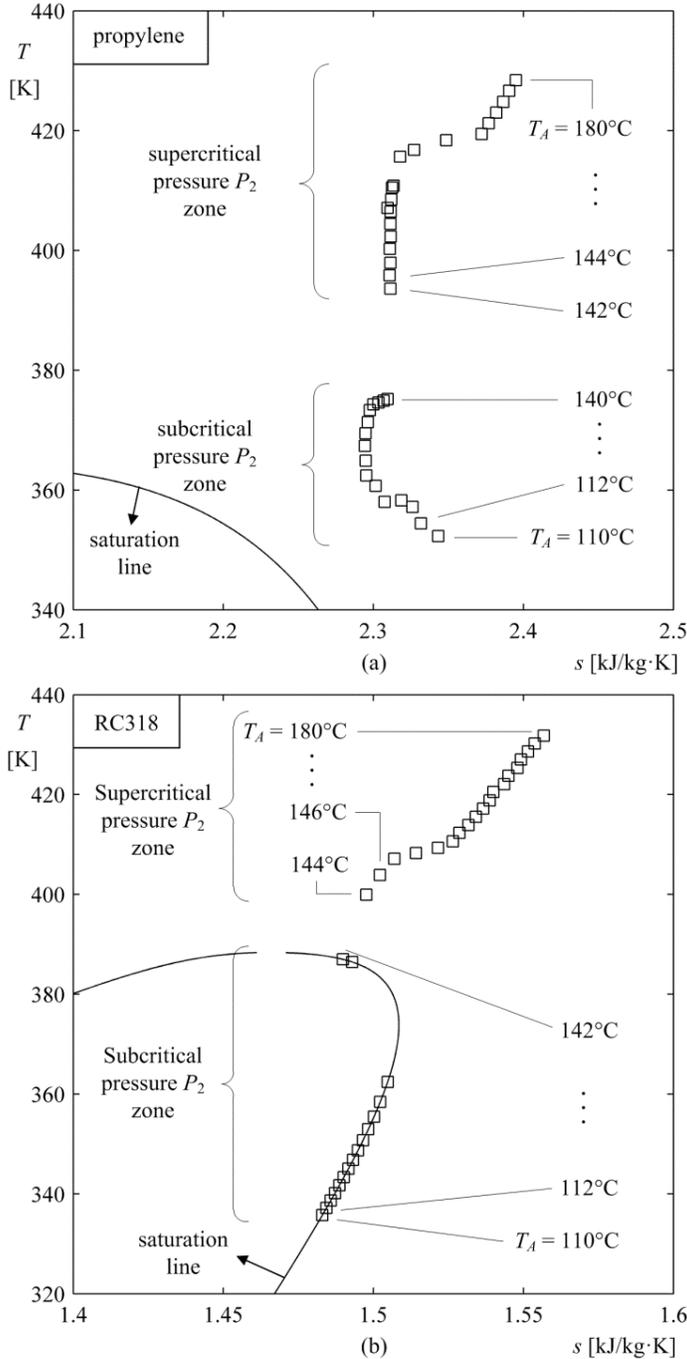
The optimized mass flow rate ratio  $R_{b,w(\text{opt})}$  is shown in Fig. 1.6c. It can be seen that  $R_{b,w(\text{opt})}$  decreases steadily until  $T_A = 156^\circ\text{C}$ , where its trend changes due to the abrupt change of pressure shown previously in Fig. 1.6b. It is also observed that the optimized mass flow ratio becomes almost constant for temperature higher than  $176^\circ\text{C}$ , i.e., when the transcritical cycle provides the highest value of specific power.

### 1.6.2. Optimization with Propylene and RC318

A similar optimization was then performed with two other fluids, i.e., with propylene (a normal fluid), and then with RC318 (a retrograde fluid). However, the attention is now focused on the thermodynamic states of the working fluid at the entrance of the turbine (i.e., at state {5} for the subcritical system, and at state {3} for the transcritical system). An important question is whether the working fluid at the turbine inlet is saturated vapor or superheated vapor. In the former case, a superheater (SH in Fig. 1.1a) is not required, while in the latter case, the superheater is required in the system.

Figure 1.7a shows the turbine inlet states of the optimized design when propylene is used. That state is illustrated for various values of the brine inlet temperature  $T_A$ . Each square corresponds to the thermodynamic state after a full optimization. Indeed, it can be seen in

that figure that the propylene is superheated when subcritical pressures are used, which means that a superheater is required to reach maximal power output. Furthermore, it can be seen in that figure that the temperature at the turbine inlet increases drastically when the best system uses supercritical pressures.



**Figure 1.7.** Thermodynamic states at turbine inlet in T – s diagram. (a) Propylene. (b) RC318.

The behaviour of the fluid at the turbine inlet differs widely between the various possible working fluids. For example, the optimization was also performed by using RC318 as the working fluid, and the results are reported in Fig. 1.7b. Indeed, it can be seen in that figure that the fluid at the turbine inlet is always saturated vapor for geofluid temperature up to 142°C, i.e., when the best system involves subcritical pressures. In other words, these designs do not require the superheater equipment to reach maximal specific power output. Finally, it can be seen in Fig. 1.7b that the temperature at the turbine inlet gradually increases with the geofluid temperature ( $T_A$ ), which was also the case in Fig. 1.7a for the propylene. It is expected that the highest temperature in the thermodynamic cycle increases with the temperature of the geofluid, because more energy is available for heat transfer in the heat exchanger.

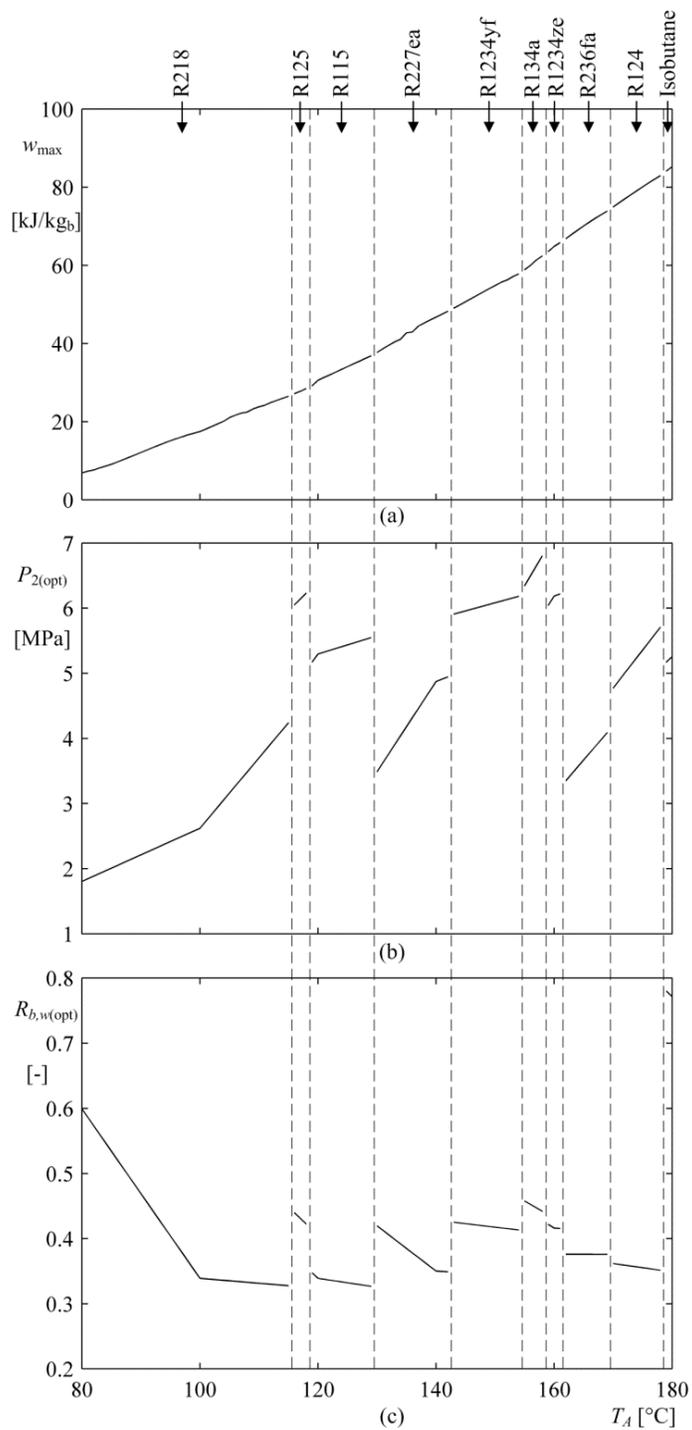
## 1.7. Multiple-fluid optimization results

The optimization methodology described in Fig. 1.5 was repeated for a total of 36 fluids (see Table 1.1), for geofluid temperature  $T_A$  between 80 and 180°C. The condenser temperature was assumed to be 30°C. For each value of geofluid temperature  $T_A$ , the best working fluid was identified (i.e., the fluid with the highest specific power output), and the corresponding objective function and design variables were reported in Fig. 1.8. More specifically, the maximum specific power  $w_{\max}$  among all 36 working fluids is presented in Fig. 1.8a, the optimized pressure  $P_{2(\text{opt})}$  is presented in Fig. 1.8b, and the optimized mass flow ratio  $R_{b,w(\text{opt})}$  is presented in Fig. 1.8c. Each point in this figure represents the comparison of the optimization results for 36 fluids. The best fluid for each temperature range is identified above the figure.

**Table 1.1.** List of considered working fluids

Fluid	$T_{crit}$ [K]	$P_{crit}$ [kPa]	Fluid	$T_{crit}$ [K]	$P_{crit}$ [kPa]
Ammonia	405.40	11333.00	R1234yf	367.85	3382.20
Butane	425.13	3796.00	R1234ze	382.51	3634.90
Butene	419.29	4005.10	R124	395.43	3624.30
CF3I	396.44	3953.00	R125	339.17	3617.70
COS	378.77	6370.00	R134a	374.21	4059.28
Isobutane	407.81	3629.00	R141b	477.50	4212.00
Isobutene	418.09	4009.80	R152a	386.41	4516.75
Isohexane	497.70	3040.00	R218	345.02	2640.00
Isopentane	460.35	3378.00	R22	369.30	4990.00
Pentane	469.70	3370.00	R227ea	374.90	2925.00
Propane	369.89	4251.20	R236fa	398.07	3200.00
Propylene	364.21	4555.00	R245fa	427.16	3651.00
R11	471.11	4407.64	R32	351.26	5782.00
R113	487.21	3392.20	R365mfc	460.00	3266.00
R115	353.10	3129.00	RC318	388.38	2777.50
R12	385.12	4136.10	RE245cb2	406.81	2886.40
R123	456.83	3661.80	RE245fa2	444.88	3433.00
R1233zd	438.75	3572.60	RE347mcc	437.70	2476.20

It can be seen in Fig. 1.8a that each value of geofluid temperature (abscissa axis) is associated to a working fluid that provides maximal power output. For instance, for geofluid temperature  $T_A$  between 80 to 180°C, 10 different fluids are reported as optimal fluids. Moreover, for each fluid, a distinctive behaviour of the optimal design parameters can be observed in Fig. 1.8b and 1.8c. It was verified by the authors that most of the 10 fluids are actually retrograde, except the R125 and R134a. This is an anticipated result because a retrograde behavior allows to avoid the liquid/vapor mixture state and thus improves the turbine efficiency. For most of the temperature range ( $T_A$ ) shown in Fig. 1.8, the optimal systems selected was the transcritical design (Fig. 1.1c). This design does not involve the superheater efficiency ( $\varepsilon_{SH}$ ) as design variable (see Eq. (1.5)), and as a consequence, that parameter is not shown in Fig. 1.8. Only fluids R218 and R227ea involved the subcritical system, and the corresponding value of  $\varepsilon_{SH}$  was always below 0.33.

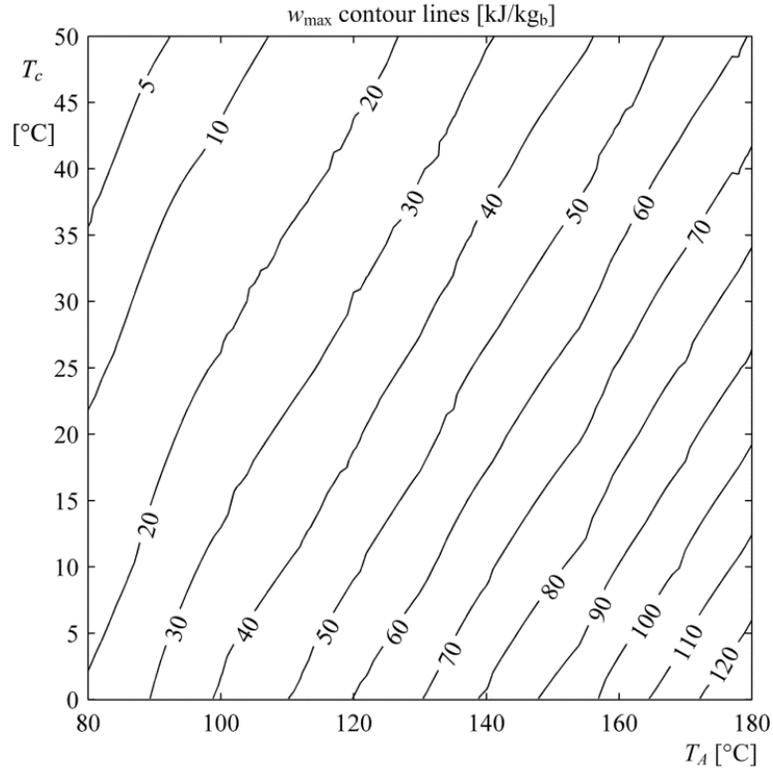


**Figure 1.8.** Optimization results with best fluids identified on top of the figure with respect to brine inlet temperature. (a) Maximal specific power. (b) Optimized heat exchangers pressure. (c) Optimized mass flow ratio.

## 1.8. Impact of condenser temperature

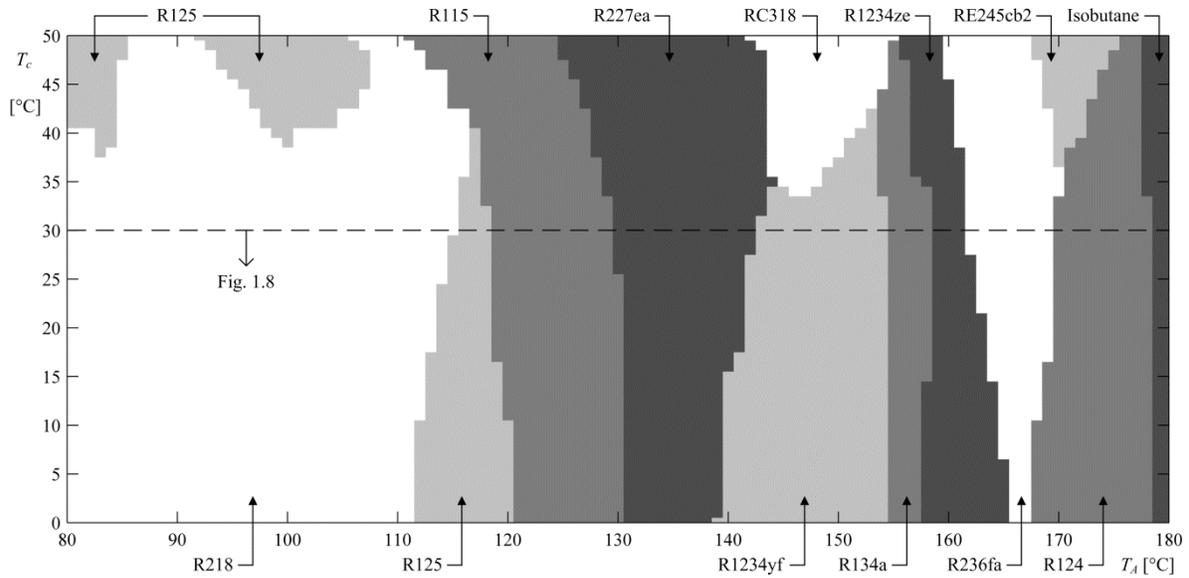
The temperature of the condenser (working fluid condensing temperature) in geothermal power plants may have different values depending on its location in the world. Indeed, the climate around a geothermal power plant dictates the outdoor temperature, which then dictates the temperature that is reachable in the condenser. For instance, outdoor temperature in California may be as high as 42°C (AccuWeather, 2015b), while the temperature in the province of Quebec (Canada) may be as low as -32°C (AccuWeather, 2015a). Hence, the optimization performed in Section 1.7 (with 36 fluids) was repeated for a wide range of condenser temperatures, and the results are reported in Figs. 1.9 to 1.12. This analysis allowed to develop design charts that are useful for a wide range of outdoor temperature and of geothermal fluid temperature.

The maximized specific power output  $w_{\max}$  was reported with respect to the geofluid temperature  $T_A$  (abscissa) and to condenser temperature  $T_c$  (ordinate) in Fig. 1.9. Each point in that figure was obtained by optimizing the subcritical and transcritical designs for 36 different fluids, and then by selecting the fluid that provided the highest specific output value. It can be seen in this figure that for any fixed value of condenser temperature (ordinate axis), the maximized specific output  $w_{\max}$  increases with the geofluid temperature. Moreover, for any fixed value of geofluid temperature (abscissa axis), the specific output increases when the condenser temperature decreases. These two behaviours are in line with the trends predicted by the ideal Carnot cycle [6]; [7], i.e., the performance can be increased by increasing the temperature of the hot reservoir and by reducing the temperature of the cold reservoir.



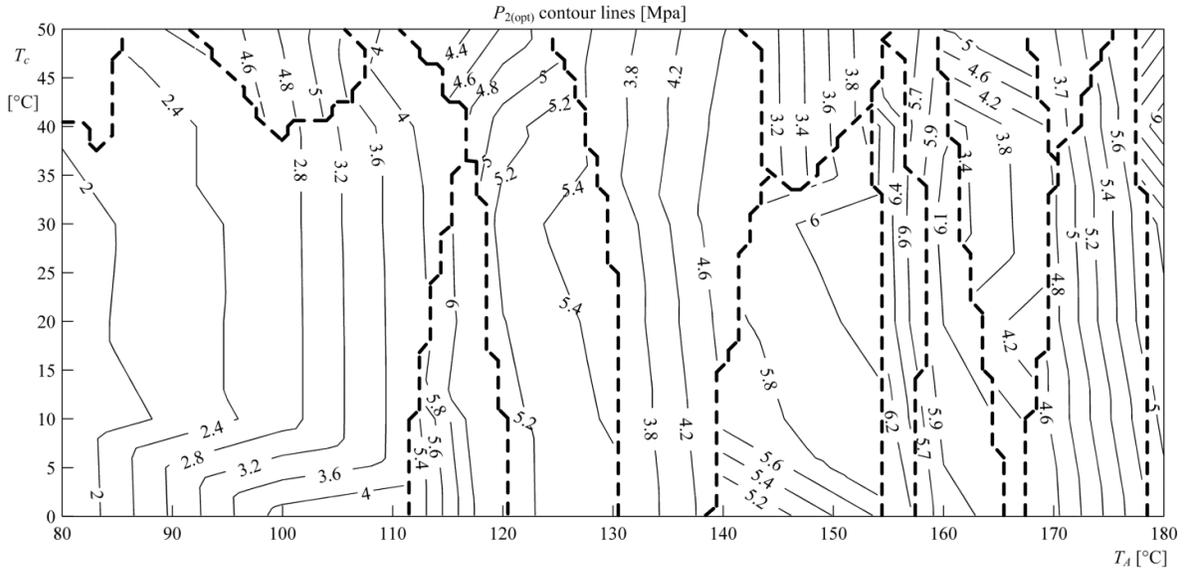
**Figure 1.9.** Maximal specific power contour lines with respect to brine temperature (x – axis) and to condenser temperature (y – axis).

The best fluids associated to each set of condenser temperature (ordinate) and of geofluid temperature (abscissa) are shown in Fig. 1.10. The best fluids are identified at the top and at the bottom of the figure. It can be seen that a total of 12 fluids (among the 36 listed in Table 1.1) are required to obtain the maximized specific power output for the various geofluid and condenser temperatures investigated in Fig. 1.10. This is a larger number of working fluid than that obtained in Section 1.7 (i.e., 10 fluids in Fig. 1.8). Indeed, the results presented previously in Fig. 1.8 were obtained for a condenser temperature of 30°C, which corresponds to the dashed line in Fig. 10. It can be seen that the 2 additional fluids present in Fig. 1.10 (i.e., RC318 and RE245cb2) are only involved for condenser temperatures higher than 30°C, which explains why they were not present in Fig. 1.8.

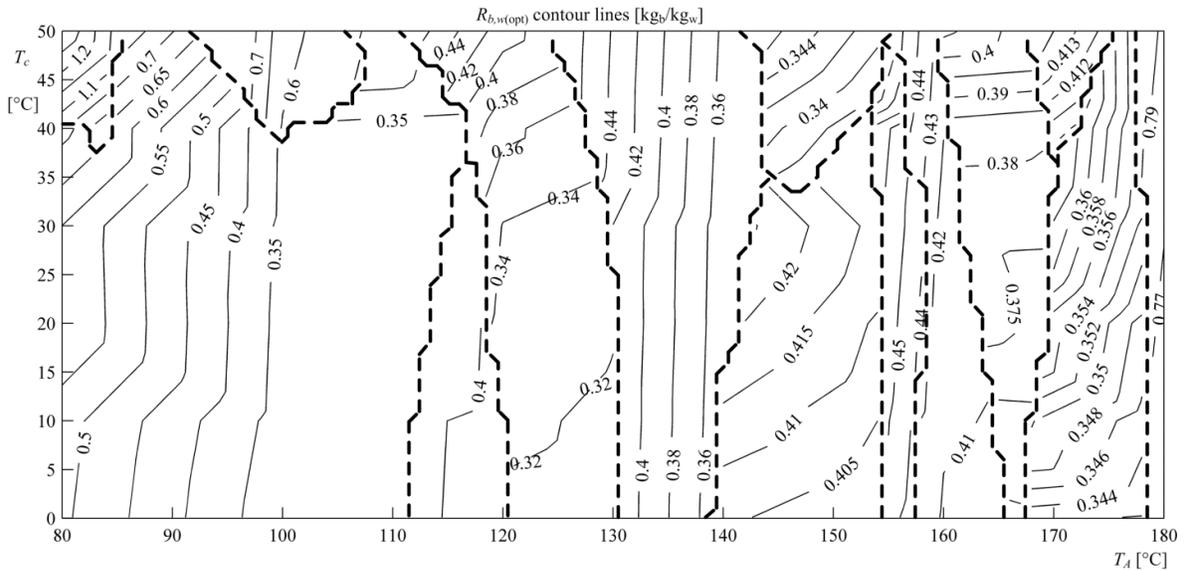


**Figure 1.10.** Diagram of optimal fluids with respect to brine temperature (x – axis) and to condenser temperature (y – axis).

Figures 1.11 and 1.12 present the contour lines of the corresponding values of the optimized heat exchangers pressure  $P_{2(\text{opt})}$  and of the optimized mass flow ratio  $R_{b,w(\text{opt})}$ , respectively. In both figures, bold dotted lines define the boundaries between the different working fluids displayed in Fig. 1.10. It is therefore expected that the operating conditions strongly vary from one side to the other of these lines because they correspond to a change of working fluid. For most of the temperature ranges for  $T_A$  and  $T_c$  shown in Figs. 1.10 – 1.12, the optimal systems selected was the transcritical cycle (Fig. 1.1c). Since this design does not involve the superheater efficiency ( $\varepsilon_{SH}$ ) as design variable (see Eq. (1.5)), that parameter is not shown in this section. Only fluids R218 and R227ea involved the subcritical system, and the corresponding value of  $\varepsilon_{SH}$  was always below 0.4.



**Figure 1.11.** Optimized heat exchangers pressure contour lines with respect to brine temperature (x – axis) and to condenser temperature (y – axis).



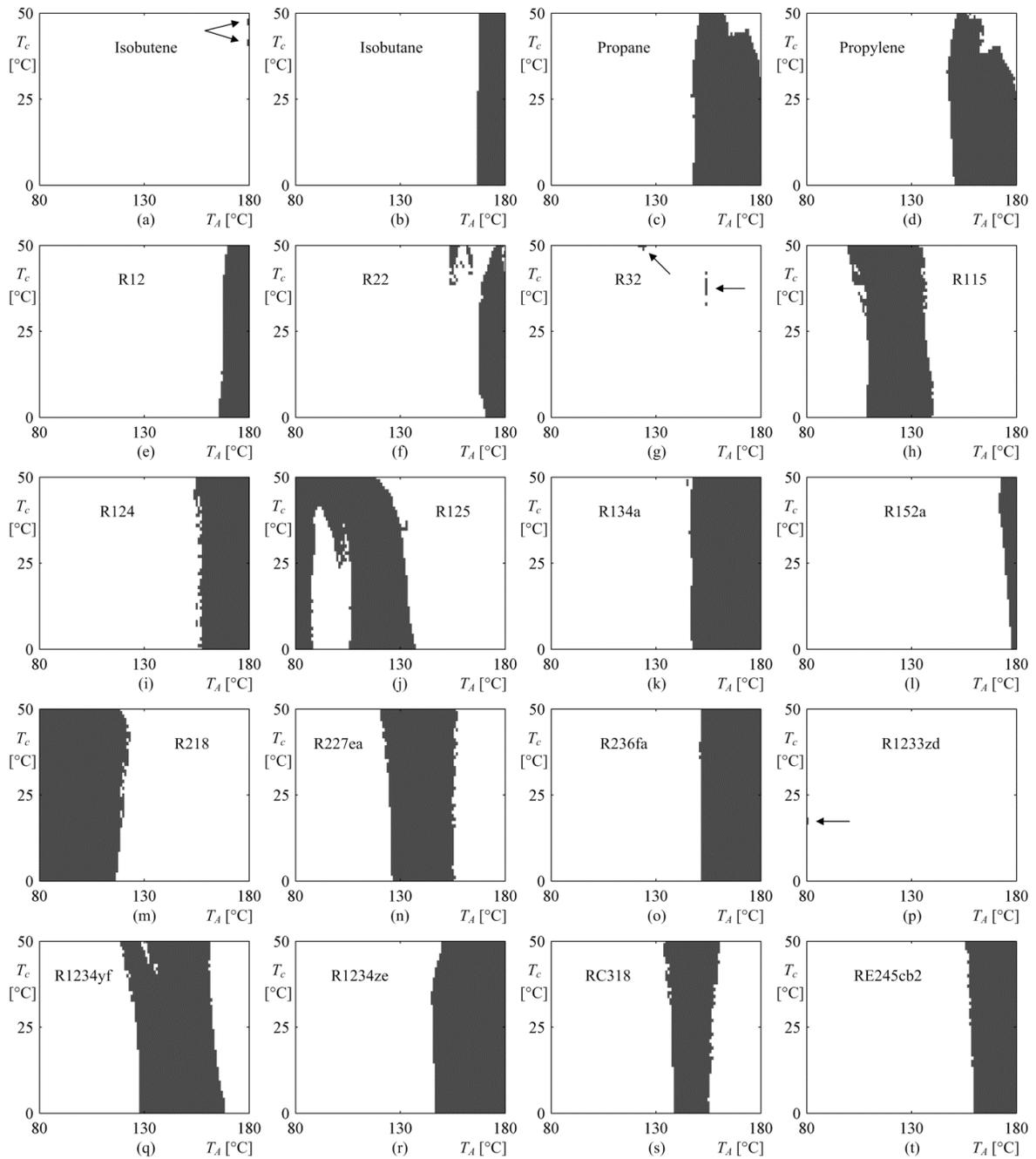
**Figure 1.12.** Optimized mass flow ratio contour lines with respect to brine temperature (x – axis) and to condenser temperature (y – axis).

Figures 1.10 to 1.12 represent efficient tools for the design of geothermal power plant in various climate conditions and for various geothermal fluid temperatures. They can provide the maximal power output that is achievable for specific conditions, the best choice of working fluid, and a good estimate of the design variable values that would be involved.

## 1.9. Near-optimal design

In practice, there may be several reasons for not being able to use the optimal fluid presented in Figure 1.10. Indeed, the fluid selection may not be only based on the criterion of geothermal power plant performance. For example, there can be environmental laws that forbid or discourage the use of some fluids. Other fluids may be dangerous for health in case of leaks or may be flammable. Some fluids may be scarce and their price may not be affordable. For these reasons, it is interesting to have a wider working fluid choice without sacrificing too much performance. Fig. 1.13 shows the conditions (i.e., geofluid temperature  $T_A$  and condenser temperature  $T_c$ ) for which a working fluid can provide at least 95% of the maximal specific power reported in Figure 1.9. Among the 36 fluids considered in this paper, 20 respected that criterion for the range of geofluid and condenser temperatures investigated. The conditions at which each fluid allows to obtain 95% of the maximal power output are shaded in each graphics.

It can be seen that some fluids are only relevant for a very restricted range of operating conditions (see Figs. 1.13a, 1.13g, 1.13p). Furthermore, among the 20 fluids that can provide 95% of the maximal power output, only two are efficient for low geofluid temperatures ( $T_A$  close to  $80^\circ\text{C}$ ), i.e., R125 (Fig. 1.13j) and R218 (Fig. 1.13m). On the other hand, 11 fluids are efficient at higher geofluid temperatures ( $T_A$  close to  $180^\circ\text{C}$ ), i.e., Figs. 1.13b, 1.13c, 1.13d, 1.13e, 1.13f, 1.13i, 1.13k, 1.13l, 1.13o, 1.13r, and 1.13t. Hence, further work could involve optimization with a wider set of fluids, so as to obtain more choice for the selection of a working fluid at low geofluid temperatures. Finally, it can be seen that some of the most commonly analysed working fluids in literature (i.e., ammonia, pentane and R245fa) are not present in Fig. 1.13. Hence, the results provided in this section allow to identify fluids that have the potential to outperform the fluid commonly studied and used in practice.



**Figure 1.13.** Identification of working fluids that can provide more than 95% of the specific power produced by the optimal fluid.

## 1.10. Theoretical analysis

In many cases, *approximate analyses* may help to explain tendencies observed in optimization results. This has been done for example by DiPippo [2] for identifying optimal

separator temperatures that maximize the specific power output of a single-flash steam plant. In this section, it is proposed to use that kind of approximate analysis to develop a function that estimates the maximal possible specific power produced by an ORC, as a function of the operating conditions. Section 1.10.1 shows the theoretical method of the approximate analysis; interpretation of the results is provided in Section 1.10.2; and new correlations are proposed in Section 1.10.3.

### 1.10.1. Approximate development

The development begins with the formula that expresses the specific output power produced in the turbine of a subcritical cycle (see Fig. 1.1b):

$$w = \frac{\dot{W}}{\dot{m}_b} = \frac{\dot{m}_w (h_5 - h_6)}{\dot{m}_b} = \frac{1}{R_{b,w}} (h_5 - h_6) \quad (1.6)$$

In order to obtain an expression that depends on temperature, a few approximations have to be made. First, it is supposed that the working fluid enters the turbine at a state that is close to the saturated vapor state, so state {5} is assumed to be approximately equal to state {4}. Furthermore, state {6} is assumed to be relatively close to the saturated vapor state. That kind of assumption may appear to be very imprecise for various types of fluid, but it offers sufficient precision to perform reliable theoretical analyses of thermodynamic cycles (e.g., [2]). Moreover, enthalpy of state {2} is assumed to be approximately equal to that of state {1}. The working fluid enthalpy at the input and output of the turbine can now be expressed in terms of the evaporation enthalpy  $h_{fg}$  and in terms of  $h_3$  and  $h_1$ , respectively.

$$h_5 \approx h_4 = h_3 + h_{fg} \quad (1.7)$$

$$h_6 \approx h_{g@P_c} = h_1 + h_{fg} \quad (1.8)$$

It is assumed that  $h_{fg}$  does not vary much with the pressure. Next, relying on the fact that  $h_3 = h_1 + (h_3 - h_1)$ , and assuming that the change of enthalpy between states {1} and {3} may be expressed as  $c_w(T_3 - T_1)$ , Eq. (1.7) becomes:

$$h_5 = h_1 + c_w(T_3 - T_c) + h_{fg} \quad (1.9)$$

where  $T_1$  is equal to the condenser temperature  $T_c$ . Hence, by using Eqs. (1.7) and (1.8), the enthalpy drop present in Eq. (1.6) can be expressed in terms of temperature and specific heat of the working fluid:

$$\begin{aligned} h_5 - h_6 &\approx h_1 + c_w(T_3 - T_c) + h_{fg} - h_1 - h_{fg} \\ h_5 - h_6 &\approx c_w(T_3 - T_c) \end{aligned} \quad (1.10)$$

Hence, Eq. (1.6) becomes

$$w \approx \frac{c_w}{R_{b,w}}(T_3 - T_c) \quad (1.11)$$

Subsequently, for the purpose of expressing  $T_3$  in terms of  $T_A$ , an energy balance in the evaporator (EV in Fig. 1.1a) is invoked. Indeed, the evaporation enthalpy between states {3} and {5} (approximately equal to  $h_{fg}$  by virtue of Eq. (1.7)) matches with the enthalpy drop of the geofluid from state {A} to state {C}, i.e.,

$$\begin{aligned} h_{fg} &= R_{b,w}(h_A - h_C) \\ h_{fg} &= R_{b,w}c_b(T_A - T_C) \end{aligned} \quad (1.12)$$

The value of  $T_C$  may be expressed in terms of  $T_3$  by involving the pinch-point temperature difference, i.e.,

$$T_C \approx T_3 + \Delta T_{pp} \quad (1.13)$$

Combining Eq. (1.12) and Eq. (1.13), it is now possible to express  $T_3$  in terms of the fluids properties and input parameters, i.e.,

$$\begin{aligned} h_{fg} &\approx R_{b,w}c_b(T_A - T_3 - \Delta T_{pp}) \\ T_3 &\approx T_A - \Delta T_{pp} - \frac{h_{fg}}{c_b R_{b,w}} \end{aligned} \quad (1.14)$$

The expression of  $T_3$  in Eq. (1.14) can be included in Eq. (1.11), and the specific power output can be expressed by the following function:

$$w \approx \frac{c_w}{R_{b,w}} \left( T_A - \Delta T_{pp} - \frac{h_{fg}}{c_b R_{b,w}} - T_c \right) \quad (1.15)$$

$$w \approx \frac{c_w}{R_{b,w}} (T_A - T_c - \Delta T_{pp}) - \frac{c_w}{c_b} \frac{h_{fg}}{R_{b,w}^2}$$

This expression only depends on one design variable,  $R_{b,w}$ . Hence, to have an optimum, it is necessary to differentiate Eq. (1.15) with respect to  $R_{b,w}$ :

$$\frac{dw}{dR_{b,w}} = -\frac{c_w}{R_{b,w}^2} (T_A - T_c - \Delta T_{pp}) + \frac{2c_w h_{fg}}{c_b R_{b,w}^3} = 0 \quad (1.16)$$

The optimal value of the mass flow ratio is obtained solving for  $R_{b,w}$ :

$$R_{b,w(\text{opt})} = \frac{2h_{fg}}{c_b (T_A - T_c - \Delta T_{pp})} \quad (1.17)$$

The final step is to include that optimal value of  $R_{b,w}$  in the specific output power function Eq. (1.15):

$$w_{\max} = \frac{c_w c_b}{4h_{fg}} (T_A - T_c - \Delta T_{pp})^2 \text{ [J/kg]} \quad (1.18)$$

Equation (1.18) provides an approximate expression for predicting the maximal specific output  $w_{\max}$  in terms of the geofluid and condenser temperatures ( $T_A$  and  $T_c$ ), and in terms of the fluids properties ( $c_w$ ,  $c_b$ ,  $h_{fg}$ ).

### 1.10.2. Interpretation

The approximate function obtained in Section 1.10.1 (Eq. (1.18)) predicts some interesting trends. First, the expression is divided in two parts: (i) a coefficient representing the properties of the brine and of the working fluid, and (ii) a squared temperature difference.

The coefficient shows that the maximal specific output power is proportional to the working fluid  $c_w$  at liquid state; the greater it is the more power is produced by the plant. Furthermore, it may be observed that the working fluid evaporation enthalpy  $h_{fg}$  has an inversely proportional influence on power output in Eq. (1.18). This indicates that a working fluid having a “thin” bell shape in its  $T-s$  diagram may provide better geothermal power plant performance. That statement is corroborated by the results of Fig. 1.10, where the majority of working fluids identified are retrograde, i.e., have thin bell shape (Fig. 1.2b) compared to normal fluid (Fig. 1.2a). Overall, from Eq. (1.18), the relevant fluid properties of the working fluid are its specific heat at liquid state and its evaporation enthalpy. Furthermore, from Eq. (1.18), it can be observed that the most influential property of the geofluid is its specific heat, due to its importance during the heat transfer in the heat exchanger EV in Fig. 1.1a.

The second term of the approximate function (the squared temperature difference) involves the temperatures  $T_A$  and  $T_c$ . Indeed, Eq. (1.18) shows that increasing the condenser temperature  $T_c$  for a fixed value of  $T_A$  results in a decrease of  $w_{\max}$ , and that increasing the geofluid temperature  $T_A$  for a fixed value of  $T_c$  results in an increase of  $w_{\max}$ , which is corroborated by the numerical results obtained in Fig. 1.9. Overall, the approximate function Eq. (1.18) is able to capture important tendencies observed in the optimization results.

### 1.10.3. Correlation development

The *form* of the approximate equation presented in Eq. (1.18) is used in order to develop a correlation that can predict the trends shown in Fig. 1.9. The idea then consists in

performing a best-fit on the numerical results that were used to generate Fig. 1.9. To make the work easier, a change of variable is performed:

$$\begin{aligned}x &= T_A - T_c \\a &= \frac{c_b c_w}{4h_{fg}} \\b &= \Delta T_{pp}\end{aligned}\tag{1.19}$$

Equation (1.18) can now be expressed in a quadratic form:

$$w_{\max} = a(x-b)^2 = ax^2 - 2abx + ab^2\tag{1.20}$$

Then, a second change of variable is performed, i.e.,  $A = a$  and  $B = ab$ , which leads to

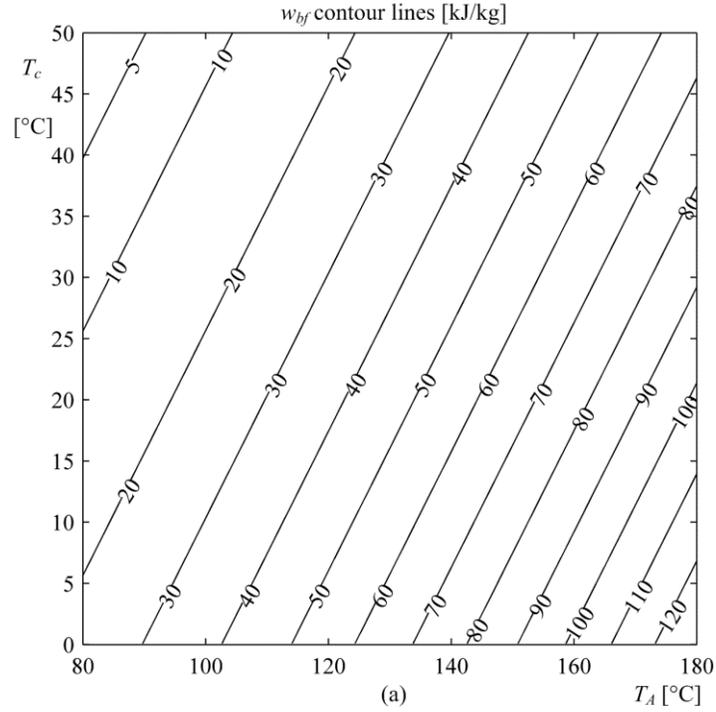
$$w_{\max} = Ax^2 - 2Bx + B^2/A\tag{1.21}$$

Equation (1.21) represents the expected form of a correlation that matches with the numerical results shown in Fig. 1.9. Hence, a best fit is done by finding the value of coefficients  $A$  and  $B$  that leads to the lowest difference with the optimization results shown in Fig. 1.9. It is carried out by using the Optimization Toolbox<sup>TM</sup> of MATLAB [37]. The values of the coefficients found and their corresponding physical meaning are presented below:

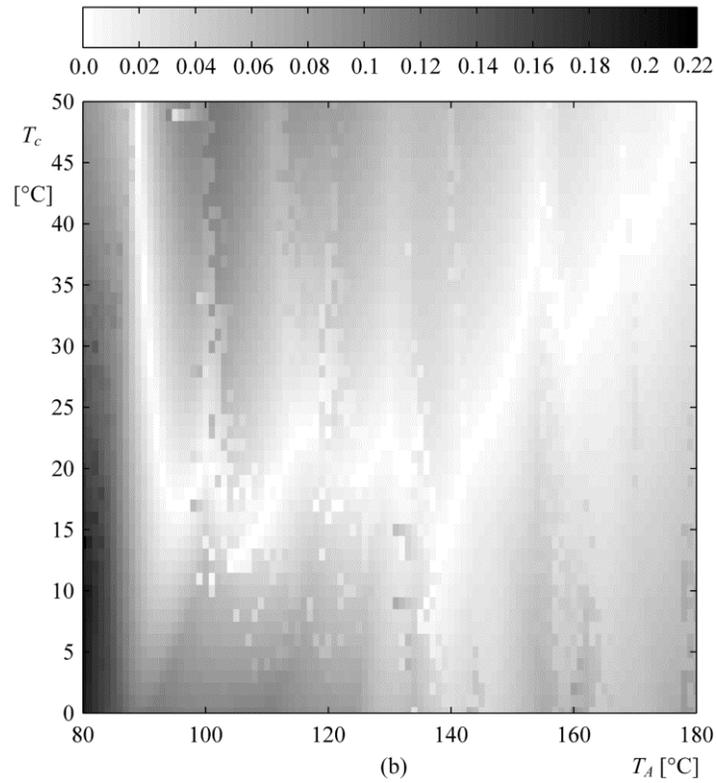
$$\begin{aligned}A_{bf} &= a = \frac{c_w c_b}{4h_{fg}} = 4.3016 \text{ [J/K}^2 \cdot \text{kg]} \\B_{bf} &= ab = 26.410 \text{ [J/K} \cdot \text{kg]} \\b_{bf} &= \Delta T_{tot} = 6.140 \text{ [K]}\end{aligned}\tag{1.22}$$

The relative error between the best-fit function  $w_{bf}$  and the numerical results  $w_{\max}$  is 4%, with a maximum error of 21% for the lowest value of brine temperature.

It should be observed that a best-fit value of 6.140 K is found for the minimum imposed temperature difference  $\Delta T_{tot}$ , which is very close to the assumed minimum value of 5.0 K that was imposed in the numerical optimization.



(a)



(b)

**Figure 1.14.** Illustration of the best-fit correlation (Eqs. (1.21)-(1.22)) obtained from approximate analysis. (a) Specific power output best-fit contour lines with respect to brine temperature (x – axis) and to condenser temperature (y – axis). (b) Relative error with numerical results.

Figure 1.14a illustrates the best fit function  $w_{bf}$  (Eq. (1.21)) obtained with the values of  $A_{bf}$  and  $B_{bf}$  reported in Eq. (1.22), and Fig. 1.14b illustrates the relative error with the original optimization results (Fig. 1.9), both with respect to the brine inlet temperature  $T_A$  on the x – axis and to the condenser temperature  $T_c$  on the y – axis. The values shown in Fig. 14a are clearly very similar to those shown in of Fig. 1.9, which shows that the approximate analysis can provide a correct form of correlation for expressing the maximal power output  $w_{\max}$ . In other words, the approximate analysis performed in Section 1.10.1 correctly captured the relevant phenomena present in the thermodynamic cycle investigated in this paper.

## 1.11. Discussion

Diagrams presented in Sections 1.8 and 1.9 (i.e., Figs. 1.9 – 1.13) are meant to be used as tools to design the Organic Rankine Cycle of a geothermal power plant. They provide some “rules of thumb” to select the optimal design that maximizes the specific output power of a geothermal power plant. For instance, the optimal working fluid and operating parameters ( $P_{2(\text{opt})}$ ,  $R_{b,w(\text{opt})}$ ) can be determined by knowing the operating conditions ( $T_A$  and  $T_c$ ).

To use the results of this paper is quite simple. The brine inlet temperature and the condenser temperature become coordinates of a point in Figs 1.9 – 1.13 that gives the estimated maximal specific power, the best working fluid, the optimized turbine inlet pressure, the optimized mass flow ratio, and the alternatives working fluids, respectively. For example, if a region has a geothermal reservoir with brine that reaches 165°C at the surface, and a climate that leads to a condenser temperature of 30°C, the maximum specific power is 70 kJ/kg (see Fig. 1.9). The best fluid would be R236fa (see Fig. 1.10), while its optimal operating conditions would be a pressure of 3.67 MPa (see Fig. 1.11) and a mass flow ratio of 0.376 (see Fig. 1.12). At the operating conditions mentioned above ( $T_A = 165^\circ\text{C}$  and  $T_c = 30^\circ\text{C}$ ), Fig. 1.13 shows that there are 7 alternatives for R236fa among the 36 fluids analysed: R1234ze (Fig. 1.13r), R124 (Fig. 1.13i), R134a (Fig. 1.13k), R236fa (Fig. 1.13o), RE245cb2 (Fig. 1.13t), propane (Fig. 1.13c) and propylene (Fig. 1.13d).

Naturally, if one of these fluids is chosen instead of the optimal one shown in Fig. 1.10, the optimal operating parameters will differ from those shown in Figs. 1.11 and 1.12.

## **1.12. Conclusion**

In this paper, subcritical and transcritical Organic Rankine Cycles for geothermal power plant are numerically simulated and then optimized with respect to brine inlet temperature and condenser temperature. New elements of modeling were introduced: a method based on differential thermodynamic evolution to calculate the turbine efficiency, and a technique to assert that the minimal pinch point constraint is respected in heat exchangers. Thirty-six (36) working fluids were considered in the optimization. The objective function was the power plant specific output, and the design variables were the turbine inlet pressure, the mass flow ratio between the geofluid and the working fluid, and the superheater efficiency (only for the subcritical cycle). Optimization was performed numerically for a range of brine inlet temperature from 80 to 180°C and a range of condenser temperature from 0.1 to 50°C.

The work described in this paper proposes two new results. First, it introduces a predesign decision-making tool (Figs. 1.9 – 1.13) for thermodynamic cycle design of low-temperature reservoir geothermal plants, which takes the form of charts. These new charts widen the guidelines provided in a previous work by Clarke and McLeskey Jr. [26]. Second, a new correlation allows to predict the maximal specific output (Eqs. (1.21)-(1.22)).

Further studies could extend the analysis performed in this paper. Other cycle configurations can be modeled and optimized, such as an ORC with dual-pressure heating, resuperheating, and steam extraction. Moreover, the cooling system could be added to the model so that optimization could involve the thermodynamics of cooling tower instead of a simple condenser temperature. The performance of the geothermal thermodynamic cycle could be assessed in the context of off-design operating conditions during the whole lifespan of the power plant. Finally, the list of working fluid candidates (Table 1.1) could be extended by taking account more fluids and mixtures of fluids.

**CHAPITRE 2. OPTIMAL DESIGN OF GEOTHERMAL POWER  
PLANTS: A COMPARISON OF SINGLE-PRESSURE  
AND DUAL-PRESSURE ORGANIC RANKINE  
CYCLES**

## 2.1. Résumé

Quatre variantes du cycle de Rankine organique (ORC) appliqué à des centrales géothermiques sont optimisées au moyen d'outils numériques. Ces variantes sont : (i) l'ORC sous-critique avec chauffage à pression unique (ORC/S/SC), (ii) l'ORC transcritique avec chauffage à pression unique (ORC/S/TC), (iii) l'ORC sous-critique avec chauffage à deux pressions (ORC/D/SC), et (iv) l'ORC transcritique avec chauffage à deux pressions (ORC/D/TC). Tous les systèmes incluent un système de récupération interne de la chaleur et une tour de refroidissement à voie humide. La fonction objectif est le travail spécifique net et les variables de design comprennent les pressions d'opération, les ratio de débits massiques entre le géofluide et le fluide de travail, l'efficacité des surchauffeurs et l'écart de température dans la tour de refroidissement. Les systèmes sont optimisés pour chacun des 20 fluides de travail potentiels. L'optimisation est effectuée pour des températures d'entrée du géofluide de 80 à 180°C, et pour des températures du thermomètre mouillé de l'air ambiant de 10 à 32°C. Les résultats montrent : (i) la supériorité de l'ORC/D/TC pour la plupart des cas, (ii) la pertinence d'utiliser un chauffage à deux pressions lorsque la température du puits de chaleur est haute et la température du géofluide est basse, et (iii) l'importance du choix de l'écart de température de la tour de refroidissement pour un design de centrale optimal.

## 2.2. Abstract

Four variants of the Organic Rankine Cycle (ORC) applied to geothermal power plants are optimized by means of numerical tools. These variants are: (i) the subcritical ORC with single-pressure heater (ORC/S/SC), (ii) the transcritical ORC with single-pressure heater (ORC/S/TC), (iii) the subcritical ORC with dual-pressure heater (ORC/D/SC), and (iv) the transcritical ORC with dual-pressure heater (ORC/D/TC). All the systems are recuperative and include a wet cooling tower. The objective function is the specific work output and design variables include operating pressures, mass flow ratios between the brine and the working fluid, superheaters effectiveness and cooling tower range. The systems are optimized for 20 different potential working fluids. The optimization is performed for inlet brine temperatures from 80 to 180°C, and for ambient air wet bulb temperatures from 10 to

32°C. The results show: (i) the superiority of ORC/D/TC for most of the cases, (ii) the relevance of using a dual-pressure heater at high sink temperature and low brine temperature, and (iii) the importance of choosing the right cooling tower range for an optimal power plant design.

### **2.3. Introduction**

Finding energy alternatives to fossil fuels is required to reduce greenhouse gases emissions. In recent years, research on geothermal power plants generated a lot of interest worldwide. Increasing their efficiency gives the opportunity to regions of the world with low-temperature geothermal reservoirs to produce power from Earth's heat in a cost-effective way. In this context, the Organic Rankine Cycle (ORC) is the most appropriate system to convert low-grade heat to electricity [43]. The ORC has become a relatively mature technology over the last decade, but there is still a challenge to improve its thermodynamic performance and competitiveness [44].

Various studies were performed on the ORC to determine optimal design guidelines, like Astolfi et al. [17], Clarke and McLeskey Jr. [26], Chagnon-lessard et al. [45], Park et al. [46] and Uusitalo et al. [47], just to name a few. Cycle variants are also investigated in order to improve performance. First, more and more investigations are devoted to transcritical regime [48]–[51], in which a greater enthalpy drop could be achieved in the turbines. Then, energy recovery within the system can be set up to increase the overall thermal efficiency, which can be accomplished by using regeneration [52]–[54] or recuperation [55]–[57], for example. Even if both techniques help preheating the working fluid, they are significantly different: energy recovery is accomplished by mass transfer (vapor extracted from the turbine at an intermediate pressure) when using regeneration, whereas it relies on heat transfer (from the turbine outlet) when using recuperation. Concerning the latter, Oyewunmi et al. [58] concluded that for maximum power production, a recuperator is necessary for ORCs with constraints imposed on their evaporation and condensation pressures. Moreover, Astolfi et al. [17] determined that recuperation is profitable if a constraint on reinjection temperature is assumed. Furthermore, a dual-pressure heater may be employed, in which the working fluid is

divided in two distinct flows to evaporate them at two different pressures, which may lead to a better heat utilization. Recent research development on this ORC variant demonstrated potential benefits when considering low and medium-grade heat sources.

For example, a notable study on dual-pressure heater ORC was made by Li et al. [59], where the authors optimized evaporation pressures and turbines inlet temperatures considering a general heat source temperature from 100 to 200°C with a fixed inlet cooling water temperature. Based on the design optimization for nine pure fluids, they found that the net power of dual-pressure heater ORCs can increase by 21.4 to 26.7% compared to an ORC with a single-pressure heater. Manente et al. [60] suggested a guideline stating that a dual-pressure heater yields no power gain when the heat source inlet is more than 40°C above the working fluid critical temperature. When the latter is similar or higher than the heat source inlet temperature, the gain in power output is in the order of 20% and more. Employing isobutane, Wang et al. [61] determined that a dual-pressure heater can significantly increase the ORC net power output, without decreasing its thermo-economic performance, though there is no power gain when heat source temperature is above 177.2°C (which corresponds to 42.54°C above its critical temperature). Dual-pressure heater ORC presents two typical turbine layouts, i.e., the separate turbine layout and the induction turbine layout: Li et al. [62] concluded that the induction turbine layout was the one leading to the greater power output.

Finally, one of the most recent developments in the field of ORC with multi-pressure heaters was done by Li et al. [63] who proposed a novel ORC configuration where the highest evaporating pressure is supercritical while the lowest one stays subcritical. Using R1234ze(E), the maximum net power output of this new cycle is the largest for heat source temperatures above approximately 135 °C. The authors wrote that “it can increase by 19.9%, 49.8%, and 20.4% at most compared with those of the conventional subcritical, transcritical, and dual-pressure evaporation cycles, respectively.”

The paper mentioned above provided important insights on the potential benefits of using a multi-pressure ORC over the single-pressure ORC, and to the authors' opinion, new

questions arise from this body of work: (i) What is the best design of ORC (single or dual-pressure) when considering various possible combinations of geothermal fluid temperature and outdoor conditions? (ii) What is the corresponding optimal working fluid? (iii) What are the corresponding optimal design variable values? Similar questions were raised in a previous work [45], and the results were synthesized in the form of design charts. However, that paper was only investigating single pressure ORC, cooling system was not explicitly considered, there was no constraint on the reinjection temperature, and energy recuperation system was not present.

Hence, in this paper, the ideas presented in Chagnon-Lessard et al. [45] (i.e., performing numerical optimization of ORC cycles and presenting the results in the form of charts) is improved by considering the most recent developments in ORC designs. Upgrades include the addition of the subcritical and transcritical dual-pressure heater ORC (in the ORC/D/SC and ORC/D/TC systems), the implementation of a wet cooling tower as cooling system, a minimum reinjection temperature, and the addition of a recuperator in the cycle.

In other words, the goal of this work is to optimize and compare four variants of ORC: (i) the subcritical ORC with single-pressure heater (ORC/S/SC), (ii) the transcritical ORC with single-pressure heater (ORC/S/TC), (iii) the subcritical ORC with dual-pressure heater (ORC/D/SC), and (iv) the transcritical ORC with dual-pressure heater (ORC/D/TC). All the systems are recuperative and include a wet cooling tower. The objective function is the specific work output and design variables include operating pressures, mass flow ratios between the brine and the working fluid, superheaters effectiveness and cooling tower range. A total of 20 different potential working fluids are considered. The optimization is performed for inlet brine temperatures from 80 to 180°C, and for ambient air wet bulb temperatures from 10 to 32°C.

The paper is organized as follows: Section 2.4 describes the four ORCs of interest and the approach to perform the numerical simulations; Section 2.5 formulates the optimization problems; Section 2.6 presents the results of this work with discussions; and Section 2.7 provides the conclusions.

## 2.4. Problem statement

The four power cycles are described in Sections 2.4.1 to 2.4.4. The systems considered in this paper are (i) a subcritical Organic Rankine Cycle (ORC) with single-pressure heater (ORC/S/SC), (ii) a transcritical ORC with single-pressure heater (ORC/S/TC), (iii) a subcritical ORC with dual-pressure heater, and (iv) a transcritical ORC with dual-pressure heater (ORC/D/TC). The selected cooling system is presented in Section 2.4.5 and details on the numerical simulations are provided in Section 2.4.6.

### 2.4.1 *Single-pressure heater subcritical Organic Rankine Cycle (ORC/S/SC)*

A binary geothermal power plant possesses two main circuits, i.e., the primary and secondary circuits (see Fig. 2.1a). The primary circuit is an open cycle where the geothermal brine, or geofluid, is first pumped from the reservoir and enters the plant at state {A}. Then, it flows through either one heat exchanger [27] or a series of three heat exchangers. The latter configuration is considered in this paper for subcritical regime, where the geofluid exit conditions of each exchanger correspond to states {B}, {C} and {D}, respectively. The circuit ends with the geofluid reinjection in the reservoir at state {D}. Although the geothermal fluid typically contains dissolved gas and calcite, it is assumed in this work that its properties may be approximated as equal to those of pure water, which is in line with recent literature, e.g., [28], [29].

In this study, the secondary circuit of the first system (ORC/S/SC) corresponds to the ORC with recuperation, which consists of a closed cycle performed by the working fluid (see states {1} to {6'} in Figs. 2.1a and 2.1b). Fluid at state {1}, a saturated liquid at the condenser pressure, enters the pump (PP) to reach the desired subcritical pressure at turbine inlet  $P_H$  (state {2}). Then, it receives heat from the turbine outlet (state {6}) in the recuperator (RE) to achieve state {2'}. Recuperation in power cycles must not be confused with regeneration, e.g. see Chapter 3 of [64] for more explanations on the recuperator. Next, the working fluid is heated at constant pressure by the geofluid circulating in the primary circuit: the fluid becomes a saturated liquid (state {3}) in the economizer (EC), then it is fully evaporated to reach the state of saturated vapor (state {4}) in the evaporator (EV), and finally it turns to superheated vapor (state {5}) at the superheater outlet (SH).

See Fig. 2.1c for the T-Q diagram of this heat exchange. States {4} and {5} overlie (as well as states {A} and {B}) when there is no heat transfer in the superheater, and as a consequence, the fluid enters the turbine as saturated vapor. The fluid then leaves the turbine (TB) at the condensing pressure (state {6}) and passes through the recuperator so as to transfer heat to the fluid leaving the pump outlet when the temperature difference is sufficient. Lastly, the working fluid rejects heat to the water used in the cooling system and returns to state {1} as saturated liquid. See Appendix A for the step-by-step calculation model used to determine the specific work output.

#### **2.4.2 Single-pressure heater transcritical Organic Rankine Cycle (ORC/S/TC)**

Figures 2.1d and 2.1e show the ORC/S/TC, i.e., the second system studied in this work. Its only difference with the ORC/S/SC is that its pressure  $P_H$  in the heater is supercritical, making the cycle transcritical. The fluid at state {2'} leaving the recuperator (RE) is heated at constant pressure by the geofluid in a single heat exchanger (HE) instead of three, and it becomes superheated (state {3}). From state {A}, the geofluid is thus cooled to state {B} before being reinjected in the reservoir (see Fig. 2.1f for the T-Q diagram).

This system can be used solely if the working fluid critical temperature is sufficiently lower than the geofluid inlet temperature (state {A}) and also lower than its maximum temperature of applicability (identified as  $T_{\max}$  in Table 2.6, where values are given by the National Institute of Standards and Technology [34]). Above a certain temperature, organic fluids may experience chemical decomposition due to the loss of their thermal stability. Therefore, some working fluids cannot be safely used in a transcritical cycle, like R236fa, with its critical temperature being practically the same as its maximum temperature of applicability. Finally, the calculation method for predicting the specific work output of this system is similar to that shown previously in Appendix A.

#### **2.4.3 Dual-pressure heater subcritical Organic Rankine Cycle (ORC/D/SC)**

Having a multi-pressure heater in an ORC can improve the use of the geothermal energy by achieving a smaller average temperature difference between the two fluids, thus reducing the thermodynamic losses in the heat exchangers [65]. The ORC/D/SC integrates this

concept by including a second set of heat exchangers, pump, and turbine or turbine stage (see Figs. 2.1g and 2.1h). The working fluid route begins at state {1} where it enters in the first pump (PP/MP) to reach the medium subcritical pressure  $P_M$  at state {2} and passes through the recuperator (RE) to reach the conditional state {2'}. It receives the geofluid heat in the medium-pressure economizer (EC/MP) to attain the state of saturated liquid (state {3}). At the outlet, the fluid is split into two streams. One of them pursues its way in the medium-pressure evaporator (EV/MP) leaving it at saturated vapor (state {4}) and then the medium-pressure superheater (SH/MP) brings it to state {5}. The other stream is compressed to state {6} at the subcritical heating pressure  $P_H$  by means of the second pump (PP/HP). Then, it is heated by the higher-grade geothermal energy in the second heat exchangers set. The high-pressure economizer (EC/HP) brings this working fluid fraction to saturated liquid (state {7}), the high-pressure evaporator (EV/HP) to saturated vapor (state {8}), and the high-pressure superheater (SH/HP) to the high-pressure turbine (TB/HP) inlet at state {9}, see Fig. 2.1i for the T-Q diagram. From state {A}, the geofluid passes through six heat exchangers, which are assumed to be in series, and is reinjected at state {G}. The high-pressure working fluid fraction leaves the turbine at the medium pressure  $P_M$  (state {10}) to be mixed with the other fraction at state {5}, creating state {11}. The total working fluid flow is then admitted in the medium-pressure turbine (TB/MP), exiting at the condensing pressure (state {12}). This turbine arrangement corresponds to an induction turbine layout [62]. Finally, it reaches the conditional state {12'} in the recuperator and returns to state {1} as saturated liquid in the condenser (CO), cooled at constant pressure by the water from the cooling system.

Figure 2.1i illustrates how the geofluid heat can be used more efficiently in the ORC/D/SC than in the ORC/S/SC. Evaporating the working fluid at two different temperatures allows each fraction to absorb a portion of the heat with a smaller average temperature difference. In other words, the geofluid line in a T-Q diagram (the dashed line) is closer to the working fluid line (the solid line). The global area between the brine and working fluid in a T-Q diagram is therefore reduced and less exergy is lost in the heat exchange process.

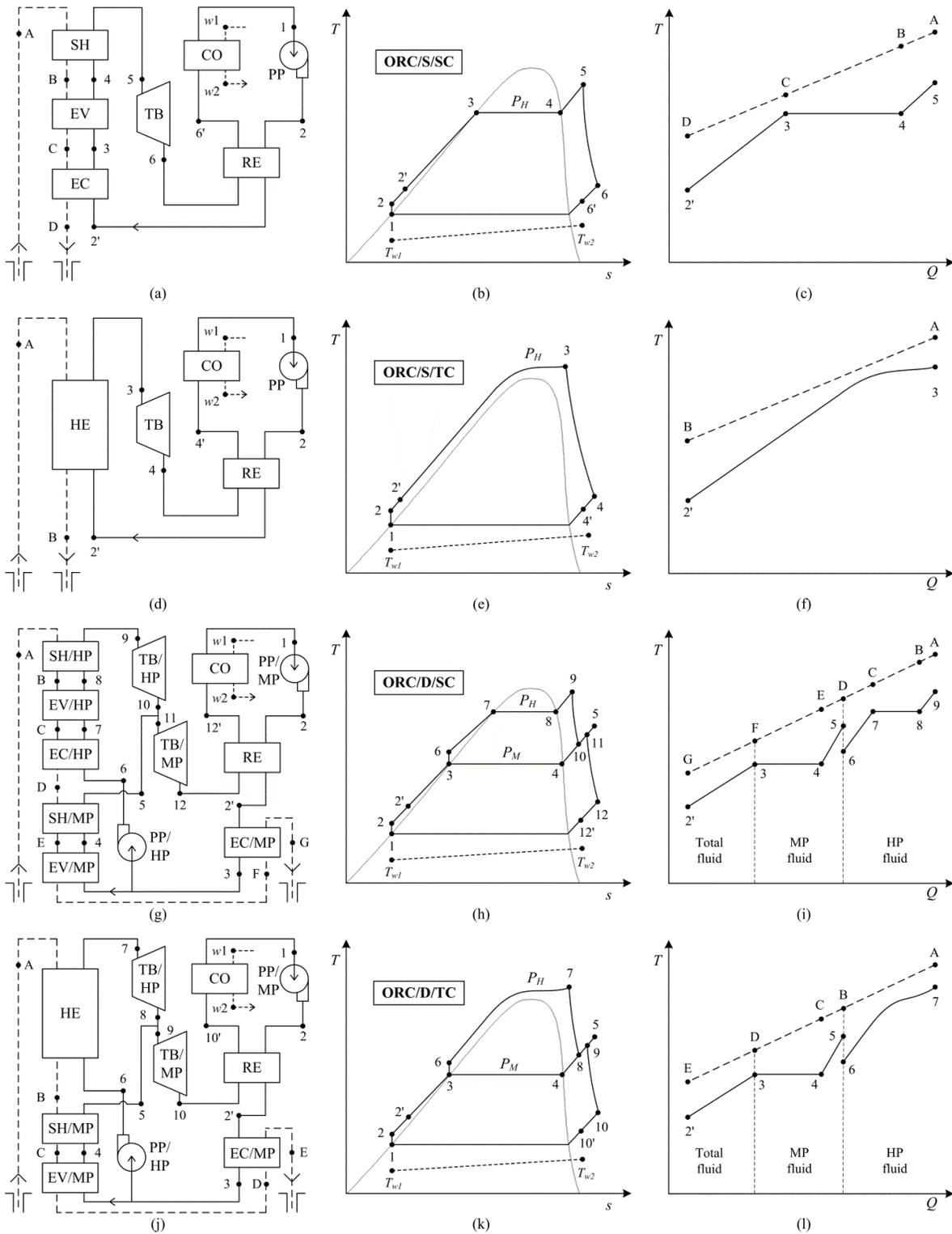
Finally, the calculation method for predicting the specific work output of this system follow the same ideas as those shown previously in Appendix A, but it is not included here for conciseness of the paper.

#### **2.4.4 Dual-pressure heater transcritical Organic Rankine Cycle (ORC/D/TC)**

The ORC/D/TC shares most of its states with the ORC/D/SC, see Figs. 2.1j and 2.1k. Their difference resides in the replacement of the high-pressure series of heat exchangers (EC/HP, EV/HP, SH/HP in Fig. 2.1g) by one supercritical heat exchanger (HE). The working fluid portion at state {6} receives the higher-grade heat from the geofluid at the desired supercritical pressure  $P_H$  and is admitted in the high-pressure turbine (TB/HP) at state {7}. Figure 11 shows the geofluid going from state {A} to state {B} while circulating in the supercritical heat exchanger, and to states {C}, {D} and {E} after passing through each medium-pressure heat exchanger, before being reinjected in the ground. Note that the fluid is not supercritical at the inlet of the second turbine stage (state {9}), since the medium pressure was assumed to be subcritical.

As for the ORC/S/TC, the conditions of utilization are a geofluid inlet temperature (state {A}) and a working fluid's maximum temperature of applicability sufficiently higher than the working fluid's critical temperature.

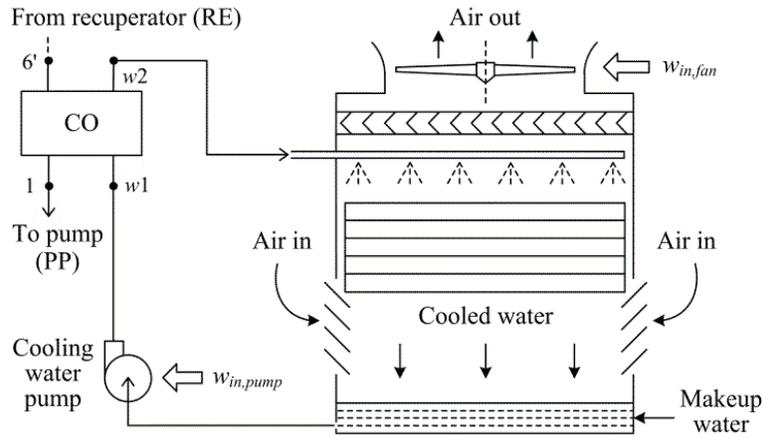
Again, the calculation method for predicting the specific work output of this system follow the same ideas as those shown previously in Appendix A, but it is not included here for conciseness of the paper.



**Figure 2.1.** Organic Rankine Cycle designs. ORC/S/SC (a) equipment architecture. (b) thermodynamic diagram. (c) heat exchange. ORC/S/TC (d) equipment architecture. (e) thermodynamic diagram. (f) heat exchange. ORC/D/SC (g) equipment architecture. (h) thermodynamic diagram. (i) heat exchange. ORC/D/TC (j) equipment architecture. (k) thermodynamic diagram. (l) heat exchange.

### 2.4.5 Cooling system

The simulated cooling system used for all cycles presented in this work is a wet cooling tower (WCT) with induced draught, conceptualized in Fig. 2.2. A cooling tower discharges heat in the surrounding air in the form of sensible heat and latent heat by increasing the moisture content of the air draught (Chapter 15.8 in [4]). This cooling system was chosen for its compactness, its close control of cold water temperature and its assured supply of required air.



**Figure 2.2.** Representation of the cooling system layout.

The WCT performance depends on the ambient air wet bulb temperature  $T_{wb}$ , as well as on two design parameters, i.e., the approach  $A$  and the range  $r$ . This work thus employs  $T_{wb}$  as the “cold source temperature”, because other cold temperatures (such as the condensing temperature or the surrounding dry bulb temperature) would be less convenient in the calculations and in the presentation of the results. Here, the working fluid condensing temperature is determined by the evolution of the cold water temperature, as depicted by the dotted line in Figs. 2.1b, 2.1e, 2.1h and 2.1k, where minimum temperature difference (pinch point) and condenser effectiveness constraints are applied.

Figure 2.2 shows the cooling water trajectory in the ORC/S/SC, as an example. Starting from state  $\{w1\}$ , where  $T_{w1} = T_{wb} + A$ , the cooling water enters the condenser to absorb the working fluid heat and leaves it at state  $\{w2\}$ , with  $T_{w2} = T_{w1} + r$ . It then reaches the tower where it is sprayed in fine droplets. The fan at the tower top induces an upwards air flow in

which the heat is discharged. The remaining liquid water falls in the basin to be mixed with makeup water and pumped to the condenser again. A WTC requires auxiliary energy consumption due to its fan and cooling water pump, and the calculation method used for predicting the operating specific work is provided in Appendix A.

#### **2.4.6 Numerical simulations**

The numerical simulations in this work are performed with in-house scripts coded with the programming software MATLAB<sup>®</sup> [35]. The commercial software REFPROP [34] was used to calculate fluids thermodynamic states. Thorough verifications of the numerical models have been done by calculating specific cases of the four cycles. More specifically, the verifications were performed by comparing the results obtained from the numerical code with those obtained manually. Furthermore, validation of the numerical code was achieved by comparing both subcritical cycles (ORC/S/SC and ORC/D/SC) with other authors' results using the same context.

Table 2.1 lists the inputs required to calculate the specific work output for each cycle and shows the relative difference between the numerical and manually calculated results. Sources of discrepancy include the less precise iterative technique to find the condensing pressure and the states properties during the manual calculation, i.e., thermodynamic tables (Appendix A in [7]) and the “refrigerant calculator” from the software CoolPack [66]. Relative differences of the specific work output  $\delta w$  of less than 4% are obtained for all cycles, justifying that the numerical model works as expected.

**Table 2.1.** Verification of numerical scripts

Cycle		ORC/S/SC	ORC/S/TC	ORC/D/SC	ORC/D/TC
Input	Fluid	R134a	R134a	R134a	R134a
	$T_A$ [°C]	150	150	150	150
	$T_{wb}$ [°C]	20	20	20	20
	$P_H$ [MPa]	4	5.5	4	5.5
	$R_H$ [kg <sub>b</sub> /kg <sub>wf</sub> ]	0.55	0.55	0.65	0.65
	$\varepsilon_H$ [-]	0.75	N/A	0.75	N/A
	$P_M$ [MPa]	N/A	N/A	2	2
	$R_M$ [kg <sub>b</sub> /kg <sub>wf</sub> ]	N/A	N/A	3.5	3.5
	$\varepsilon_M$ [-]	N/A	N/A	0.6	0.6
	$r$ [°C]	7	7	7	7
	$A$ [°C]	5	5	5	5
	$\eta_{dry}$ [-]	0.85	0.85	0.85	0.85
$\eta_{PP}$ [-]	0.8	0.8	0.8	0.8	
Output	$w_{num}$ [kJ/kg <sub>b</sub> ]	37.1169	33.8048	33.4232	32.7357
	$w_{hand}$ [kJ/kg <sub>b</sub> ]	36.9262	32.6382	33.2041	33.6412
	$\delta w$ [%]	0.51	3.5	0.66	2.8

The model validation (see Table 2.2) is done by comparing model results of Guzović et al. [65] for the example of the potential geothermal field Velika Ciglena (Croatia). The last four inputs of the current model are for the WCT calculations, so they are not used in the work of Guzović et al. (they used a dry cooling system). The mean value of Bjelovar's (main city near the field) relative humidity [67] is chosen to calculate the ambient air wet bulb temperature for the validation. Cooling system power is not considered in the current model specific output for the sake of comparison, since it was not accounted for in [65]. Relative differences between outputs are 22% and 28% for the ORC/S/SC and ORC/D/SC, respectively. The large difference can be explained by the different cooling system and by

using the condenser effectiveness  $\varepsilon_{CO}$  in addition to a minimal approach to dictate  $P_{CO}$ . The last hypothesis was confirmed by employing in the present model the same  $P_{CO}$  as in [65]: relative differences between models of 2% and 5% for the ORC/S/SC and ORC/D/SC are then calculated. The concordance between the present model and that from the work of Guzović et al. can thus be qualified of acceptable.

**Table 2.2.** Comparison with the work of Guzović et al., 2014

Cycle		ORC/S/SC	ORC/D/SC	
Input	Fluid	Isopentane	Isopentane	
	$T_A$	[°C]	175	175
	$T_{db}$	[°C]	15	15
	$P_H$	[MPa]	0.9	1.0864
	$P_M$	[MPa]	N/A	0.3
	$\dot{m}_{wf,HP}$	[kg <sub>wf</sub> /s]	80.13	71.9
	$\dot{m}_{wf,MP}$	[kg <sub>wf</sub> /s]	N/A	28.5
	$\dot{m}_b$	[kg <sub>b</sub> /s]	83	83
	$\eta_{dry}$	[-]	0.85	0.85
	$\eta_{PP}$	[-]	0.75	0.75
	$\phi$	[%]	75	75
	$T_{wb}$	[°C]	12.46	12.46
	$r$	[°C]	7	7
	$A$	[°C]	5	5
Output (Guzović et al.)	$P_{CO}$	[MPa]	0.11	0.1092
	$w$	[kJ/kg <sub>b</sub> ]	63.494	76.759
Output (current model)	$P_{CO}$	[MPa]	0.169	0.168
	$w$	[kJ/kg <sub>b</sub> ]	49.654	55.575

## 2.5. Optimization

The objective function selected in this study is the power plant specific work output  $w$ . It represents the net amount of energy (kJ) produced for each kg of geofluid extracted from the ground. The previous work of Chagnon-lessard et al. [45] uses the term ‘specific *power* output’ to designate the same physical quantity, but this work employs ‘specific *work* output’ to be more in line with previous literature. This is the net specific work, where the working fluid feed pump(s) and cooling tower work are subtracted from the turbine(s) work.

Section 2.5.1 provides the definition of the design variables and of the constraints for the four systems, while Section 2.5.2 presents the variables bounds and fixed values and Section 2.5.3 describes the selected optimization algorithm.

### 2.5.1 Optimization problem

Each studied system has its own optimization problem summarized in Table 2.3. The number of design variables varies from three (ORC/S/TC) to seven (ORC/D/SC). They are: (i) the pressure at first turbine inlet  $P_H$ , (ii) the mass flow ratio  $R_H$  between the brine and the high-pressure working fluid, (iii) the high-pressure superheater effectiveness  $\varepsilon_H$ , (iv) the pressure at second turbine inlet  $P_M$ , (v) the mass flow ratio  $R_M$  between the brine and the medium-pressure working fluid, (vi) the medium-pressure superheater effectiveness  $\varepsilon_M$ , and (vii) the cooling tower range  $r$ . Making  $\varepsilon_H$  and  $\varepsilon_M$  design variables instead of turbines inlet temperature gives more latitude on the enthalpy of states {5} and {9} and ensures that the value of the effectiveness of the heat exchanger is physically possible.

Several constraints limit the design optimization. Five of them are applied for all ORC designs. First, approach temperatures (pinch point)  $\Delta T_{pp,H}$  in the high pressure heater and  $\Delta T_{pp,CO}$  in the condenser must be greater than a minimum value ( $\Delta T_{tol} = 5$  K). Next, the effectiveness of the condenser  $\varepsilon_{CO}$  needs to be lower than a maximum value ( $\varepsilon_{max} = 0.85$ ). Then, the vapor quality in the turbine must remain above a tolerance value ( $x_{tol} = 0.9$ ) in

order to prevent excess blade wear [2]. Finally, the temperature of the geofluid leaving the cycle at state {D}, {B}, {G} or {E} has to be greater than a minimum reinjection temperature ( $T_{inj} = 60^\circ\text{C}$ ).

Other constraints concern specific systems. For cycles comprising a subcritical heater, the effectiveness of economizer(s)  $\varepsilon_{EC}$  and evaporator(s)  $\varepsilon_{EV}$  needs to be lower than a maximum value ( $\varepsilon_{\max} = 0.85$ ). For transcritical cycles, a restriction is added to only admit designs with a superheated vapor at the turbine inlet ( $x_3 = 1$  and  $x_7 = 1$ ). Last, cycles with dual-pressure heater must have a minimum temperature difference in the medium-pressure heater (EC/MP, EV/MP and SH/MP)  $\Delta T_{pp,M}$  greater than  $\Delta T_{tol}$ .

**Table 2.3.** Optimization problem of each system

System	Optimization problem	Eq.
ORC/S/SC	$\left\{ \begin{array}{l} \text{maximize}(w) \\ \text{varying}(P_H, R_H, \varepsilon_H, r) \\ \text{respecting} \left\{ \begin{array}{l} \Delta T_{pp,H} \geq \Delta T_{tol}, \quad \varepsilon_{EV} \leq \varepsilon_{\max} \\ \Delta T_{pp,CO} \geq \Delta T_{tol}, \quad x_{\min} \geq x_{tol} \\ \varepsilon_{CO} \leq \varepsilon_{\max}, \quad T_D \geq T_{inj} \\ \varepsilon_{EC} \leq \varepsilon_{\max} \end{array} \right. \\ \text{fixed parameters: see Table 2.4} \end{array} \right.$	(1)
ORC/S/TC	$\left\{ \begin{array}{l} \text{maximize}(w) \\ \text{varying}(P_H, R_H, r) \\ \text{respecting} \left\{ \begin{array}{l} \Delta T_{pp,H} \geq \Delta T_{tol}, \quad x_3 = 1 \\ \Delta T_{pp,CO} \geq \Delta T_{tol}, \quad x_{\min} \geq x_{tol} \\ \varepsilon_{CO} \leq \varepsilon_{\max}, \quad T_B \geq T_{inj} \end{array} \right. \\ \text{fixed parameters: see Table 2.4} \end{array} \right.$	(2)

---

ORC/D/SC	{	maximize ( $w$ ) varying ( $P_H, R_H, \varepsilon_H, P_M, R_M, \varepsilon_M, r$ ) respecting $\begin{cases} \Delta T_{pp,H} \geq \Delta T_{tol}, & \varepsilon_{ECs} \leq \varepsilon_{max} \\ \Delta T_{pp,M} \geq \Delta T_{tol}, & \varepsilon_{EVs} \leq \varepsilon_{max} \\ \Delta T_{pp,CO} \geq \Delta T_{tol}, & x_{min} \geq x_{tol} \\ \varepsilon_{CO} \leq \varepsilon_{max}, & T_G \geq T_{inj} \end{cases}$	(3)
		fixed parameters: see Table 2.4	

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ORC/D/TC	{	maximize ( $w$ ) varying ( $P_H, R_H, P_M, R_M, \varepsilon_M, r$ ) respecting $\begin{cases} \Delta T_{pp,H} \geq \Delta T_{tol}, & \varepsilon_{EVs} \leq \varepsilon_{max} \\ \Delta T_{pp,M} \geq \Delta T_{tol}, & x_7 = 1 \\ \Delta T_{pp,CO} \geq \Delta T_{tol}, & x_{min} \geq x_{tol} \\ \varepsilon_{CO} \leq \varepsilon_{max}, & T_E \geq T_{inj} \\ \varepsilon_{ECs} \leq \varepsilon_{max} \end{cases}$	(4)
		fixed parameters: see Table 2.4	

---

### 2.5.2 Fixed parameters and design variables

Table 2.4 presents the values of the fixed parameters, and the range of values for the operating conditions. These values are discussed in this paragraph. First, values for turbomachinery efficiencies, maximum heat exchanger effectiveness and minimum temperature difference are taken from [45]. The minimum tolerated vapor quality has been increased to 0.9 based on the selected value in [68] and on the limit usually applied to turbines [69]. The minimum reinjection temperature  $T_{inj}$  is often around 70°C, but this value is specific to the brine composition. A minimal reinjection temperature of 60°C was chosen in the present work to allow reasonable brine heat utilization at low  $T_A$ . For example, the Neustadt-Glewe power plant in Germany ( $T_A = 99^\circ\text{C}$ ) reinjects the brine at 60°C [70]. As stated in [71], the size of a cooling tower increases significantly when the approach is reduced. Hence, in this paper, the value of the approach was fixed to 5°C so as to avoid cooling tower that would be too large in practice. The range of the geofluid temperature investigated here corresponds to what could be expected in a low-grade geothermal reservoir. The range of wet bulb temperatures investigated here are the ones

recommended by the Cooling Technology Institute (CTI) [72] for the operation of a wet cooling tower. Other assumptions concerning the WCT are not required for the calculation of its power consumption (see Appendix A).

**Table 2.4.** Values of the fixed parameters in this study

Parameter	Values
Turbines dry efficiency $\eta_{TB}$	0.85
Pumps efficiency $\eta_{PP}$	0.8
Minimum tolerated vapor quality $x_{tol}$	0.9
Maximum heat exchanger effectiveness $\varepsilon_{max}$	0.85
Condenser exchanger effectiveness $\varepsilon_{CO}$	0.85
Supercritical heat exchanger effectiveness $\varepsilon_{HE}$	0.85
Minimum temperature difference $\Delta T_{tol}$	5°C
Minimum reinjection temperature $T_{inj}$	60°C
Cooling tower approach $A$	5°C
Range of geofluid inlet temperature $T_A$	80 – 180°C
Range of ambient air wet bulb temperature $T_{wb}$	10 – 32°C
Working fluid $F$	See Table 2.6

Table 2.5 presents the assigned bounds in brackets for each design variable, for each cycle. Pressures are the variables having the largest amount of conditions to be respected in order to lead to viable designs. First, a pressure range of  $\pm 0.02$  MPa is not allowed around the critical point to guarantee numerical stability in thermodynamic property calculations. Second, the lowest heating pressure inferior bound marked as  $P_{min}$  is the saturated pressure corresponding to 20°C above the heat sink temperature  $T_{wb}$ . Third, a range of 0.1 MPa must be respected between  $P_H$  and  $P_M$  to ensure a minimal amount of work in the high-pressure turbine.

The rest of the bounds are simpler. The ones for the mass flow ratios  $R_H$  and  $R_M$  were chosen based on observation of results from optimization tests. In each simulation, it was verified that the optimized flow ratios were always within these bounds. Superheaters effectiveness  $\varepsilon_H$  and  $\varepsilon_M$  can vary from zero (a saturated cycle) to the maximum value  $\varepsilon_{\max}$  set for heat exchangers. Finally, the inferior bound of the cooling range  $r$  is the one recommended by the CTI [72], 2.2°C, rounded up to 3°C, and its superior bound of 10°C corresponds to what is recommended in Chapter 13 of [4].

**Table 2.5.** Bounds of design variables

Design variable	ORC/S/SC	ORC/S/TC	ORC/D/SC	ORC/D/TC
$P_H$ [MPa]	$[P_{\min} : P_{cr} - 0.02]$	$[P_{cr} + 0.02 : 20]$	$[P_M + 0.1 : P_{cr} - 0.02]$	$[P_{cr} + 0.02 : 20]$
$R_H$ [kg <sub>b</sub> /kg <sub>wf</sub> ]	[0.05 : 4]	[0.05 : 4]	[0.05 : 4]	[0.05 : 4]
$\varepsilon_H$ [-]	[0 : 0.85]	N/A	[0 : 0.85]	N/A
$P_M$ [MPa]	N/A	N/A	$[P_{\min} : P_H - 0.1]$	$[P_{\min} : P_{cr} - 0.02]$
$R_M$ [kg <sub>b</sub> /kg <sub>wf</sub> ]	N/A	N/A	[0.5 : 7]	[0.5 : 7]
$\varepsilon_M$ [-]	N/A	N/A	[0 : 0.85]	[0 : 0.85]
$r$ [°C]	[3 : 10]	[3 : 10]	[3 : 10]	[3 : 10]

### 2.5.3 Optimization algorithm

Two algorithms have been investigated as candidates for the optimization task. The first is the function *fmincon.m* with the "interior-point" algorithm from the Matlab Optimization Toolbox [37]. The second is an in-house function based on the Particle Swarm Optimization (PSO), a method rising in popularity in the field of thermodynamic cycles. The first algorithm (*fmincon.m*) failed to deliver global maxima due to its lack of exploratory capacity in a given situation. The second one (PSO) was able to find near-optimum solutions within three attempts, and thus, it is the method used for this work.

The PSO algorithm is a metaheuristic developed by Kennedy and Eberhart [73] recognized for its searching ability over a large space of contender solutions. This principle has been implemented in MATLAB with the help of Yarpiz tutorials [74]. The technique in Clarke et al. [36] was also used to ensure that constraints are respected in the optimal solution, i.e., by assigning to unfeasible designs a lower specific work output than the worst performing feasible design by post-treatment. The PSO control parameters used in this work are: (i) stop criterion: relative error of  $10^{-5}$  between iterations  $j$  and  $j-3$ ; (ii) maximum number of iterations: 40; (iii) swarm size:  $7 \cdot n_{dv}^2$ , where  $n_{dv}$  is the number of design variables; (iv) inertia coefficient: 1; (v) damping coefficient: 0.75; (vi) personal acceleration coefficient: 1; (vii) social acceleration coefficient 1.25. Three optimization runs were done systematically for each set of geofluid temperature  $T_A$  and wet bulb ambient air temperature  $T_{wb}$ , and in the end, the one with the highest maximized specific work was retained.

## 2.6. Results and discussion

The optimization methodology described in Section 2.5 was applied to the four systems presented in this paper, and for a large amount of operating conditions (i.e., a large amount of combination of  $T_A$  and  $T_{wb}$ ). More specifically, optimization were performed for brine temperature  $T_A$  from 80 and 180°C (by 5°C increment), and for the ambient air wet bulb temperature  $T_{wb}$  from 10 to 32°C (by 2°C increment). The 20 working fluids investigated in this paper are listed in Table 2.6. These 20 fluids were chosen because they were the best performing fluids in a previous work [45]. Table 2.6 also provides their critical pressure, critical temperature, maximum temperature of applicability and Global Warming Potential (GWP-100 years values from the Fifth Assessment Report of the Intergovernmental Panel on Climate Change [75]). To summarize, a total of 15,864 scenarios ( $21 T_A$  values  $\times$  12  $T_{wb}$  values  $\times$  20 fluids  $\times$  4 cycles – 4296 infeasible cases) were optimized to obtain the figures presented below. Optimal designs calculated condensing pressure ranging from 3 to 16 atm, thus appropriate for technical implementation.

**Table 2.6.** Selected fluids and their properties

Fluid	$P_{cr}$ [MPa]	$T_{cr}$ [K]	$T_{max}$ [K]	GWP
Isobutene	4.0098	418.09	550	< 3
Isobutane	3.6290	407.81	575	< 3
Propane	4.2512	369.89	650	< 3
Propylene	4.5550	364.21	575	< 3
R12	4.1361	385.12	525	10200
R22	4.9900	369.30	550	1760
R32	5.7820	351.26	435	677
R115	3.1290	353.10	550	7670
R124	3.6243	395.43	470	527
R125	3.6177	339.17	500	3170
R134a	4.0593	374.21	455	1300
R152a	4.5168	386.41	500	138
R218	2.6400	345.02	440	8900
R227ea	2.9250	374.90	475	3350
R236fa	3.2000	398.07	400	8060
R245fa	3.6510	427.16	440	858
R1234yf	3.3822	367.85	410	< 1
R1234ze(E)	3.6349	382.51	420	< 1
RC318	2.7775	388.38	623	9540
RE245cb2	2.8864	406.81	500	654

### 2.6.1 ORC with single-pressure heater results

This section displays the combined results of ORC/S/SC and ORC/S/TC. In other words, it presents the results of the best performing cycle for each combination of  $T_A$  and  $T_{wb}$  when considering only the single-pressure heater designs. Figure 2.3 shows the maximized specific work output  $w_{max}$  in the form of contour lines (Fig. 2.3a), the best working fluid (Fig. 2.3b) and the cycle leading to the highest  $w_{max}$  (Fig. 2.3c) all with respect to the brine inlet temperature (x-axis) and the ambient air wet bulb temperature (y-axis). Each datapoint or ‘pixel’ in the charts describes the output of the best fluid/cycle scenario.

Figure 2.3a shows an expected trend where more power is produced for a warmer hot source and a cooler cold source. As indicated in Section 8 of [45], it can be shown based on thermodynamics reasoning that  $w_{max}$  has a quadratic tendency with the form:

$$w_{\max} \cong a(x-b)^2 \quad (2.5)$$

where  $x$  is the temperature difference between the hot source and the cold source. In, [45]  $x = T_A - T_{\text{condenser}}$ . However, in the present work, the driving temperature difference is  $x = T_A - T_{wb}$ . A best fit was done by finding the value of coefficients  $a$  and  $b$  in Eq. (5) that leads to the lowest difference with the numerical results of Fig. 2.3a. With a mean relative error of 4%, the following coefficients were found:

$$\begin{aligned} a &= 3.5709 \\ b &= 23.654 \end{aligned} \quad (2.6)$$

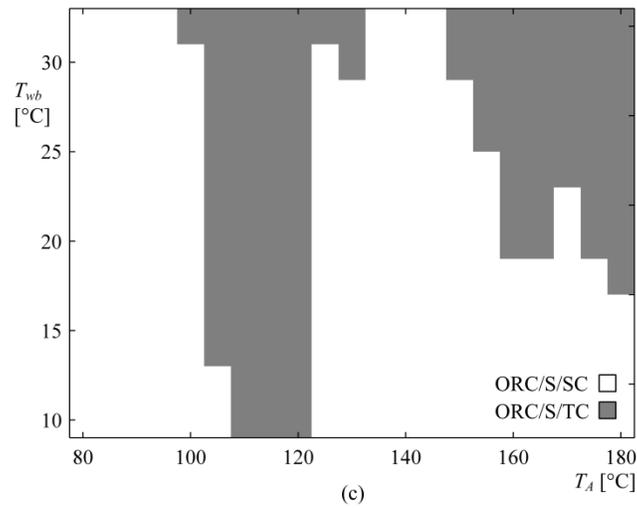
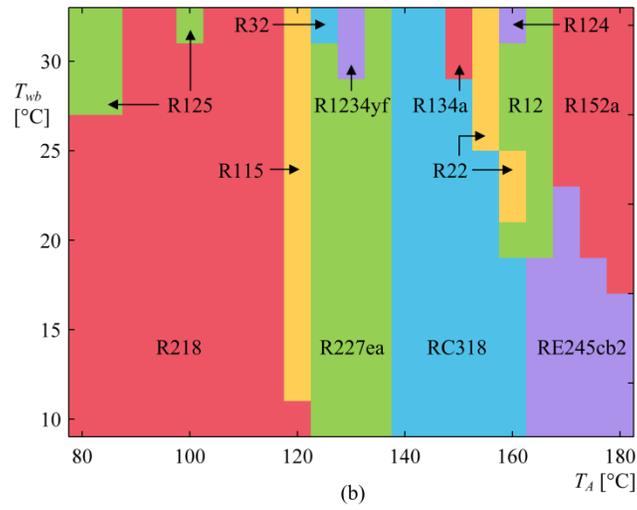
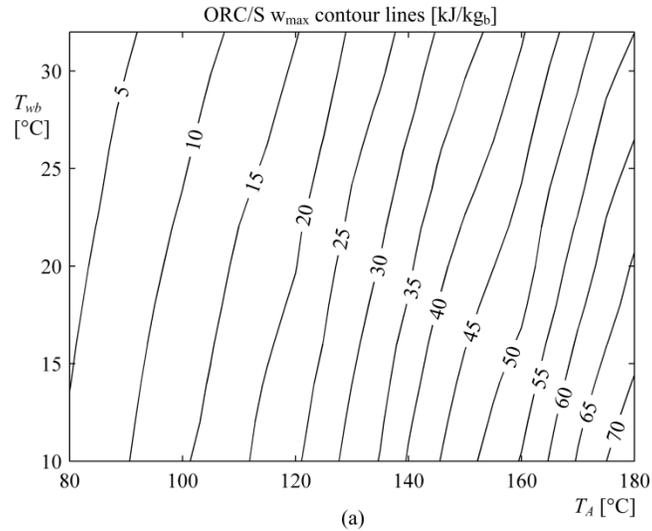
Thus, the maximized specific work output of a single-pressure heater ORC employing a WCT (accounting for feed pump and WCT parasitic losses) can be estimated by the following equation:

$$w_{\max} \cong 3.5709(T_A - T_{wb} - 23.654)^2 \quad (2.7)$$

Among the 20 candidate fluids listed in Table 2.6, Fig. 2.3b indicates that 13 of them lead to the highest specific work for at least one combination of  $T_A$  and  $T_{wb}$ : R218, R125, R115, R227ea, R32, R1234yf, RC318, R134a, R22, R12, R124, RE245cb2 and R152a. Among these 13 fluids, 7 are retrograde fluids (presence of positive saturated gas slope in their T-s diagram), and 6 are normal fluids (saturated gas slope negative everywhere in their T-s diagram). Normal and retrograde fluids are discussed in DiPippo [2], for example. It was observed that the critical temperature associated to the optimal fluid shown in Fig. 3b increases alongside the value of  $T_A$ , the lower being 66.02°C for R125 and the larger being 133.66°C for RE245cb2.

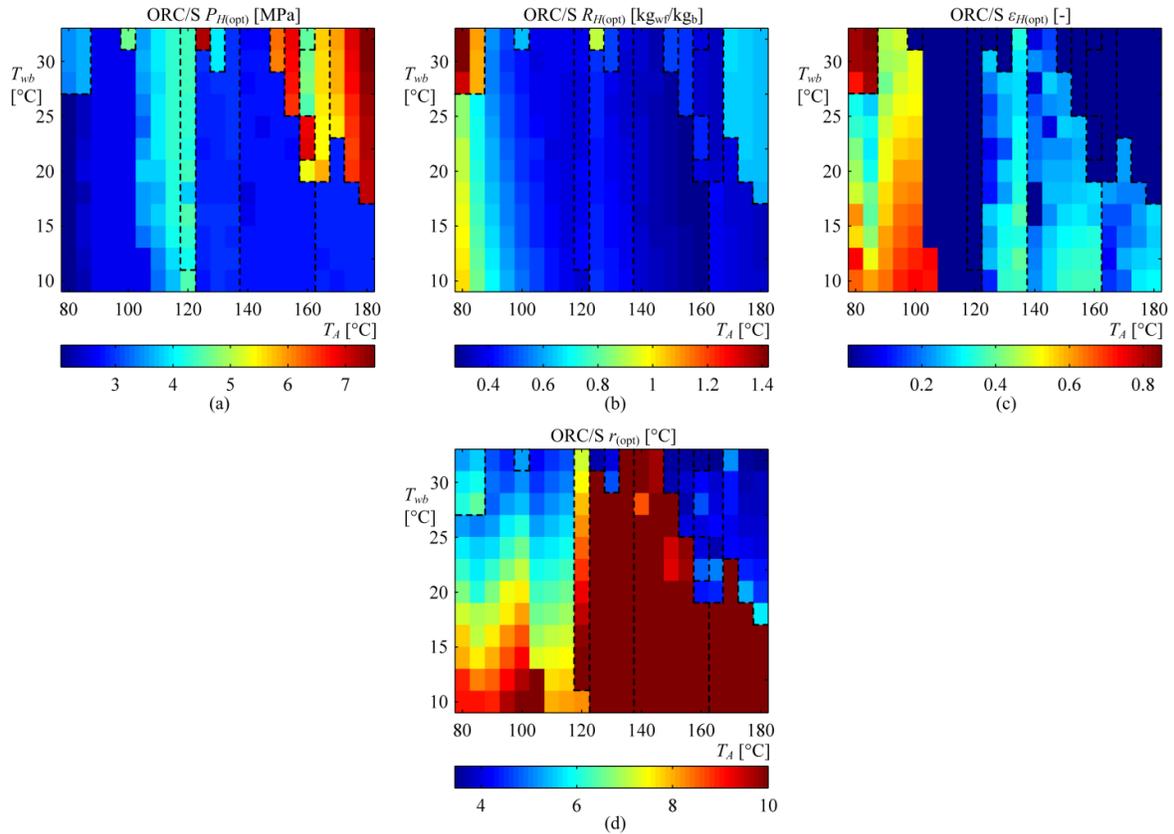
Figure 2.3c presents the best cycle between the ORC/S/SC and ORC/S/TC. The optimal regime for retrograde fluids (except for R115, R1234yf and 124) is subcritical, while normal fluids are best used with a transcritical regime (and also for R125, when  $T_A$  is sufficiently high). This phenomenon could be explained by looking at inlet and outlet turbine enthalpies, since their difference dictates the gross specific work of the plant.

It was generally observed that the optimal design for retrograde fluids in a subcritical cycle includes none to small superheater use. Indeed, keeping the turbine outlet state closer to the saturated gas line and reducing mass flow ratio is more advantageous. In the optimal design of transcritical cycles, the turbine inlet state has a greater temperature and entropy, but a not much higher enthalpy. The little gain in enthalpy then does not compensate for the much higher outlet enthalpy (since the entropy is greater), in comparison with the subcritical cycle. In the case of the three retrogrades optimal fluids used in transcritical cycle (i.e. R115, R1234yf and 124), they are less affected by this phenomenon due to their rather vertical (or isentropic) saturated gas slope in their T-s diagram. For normal fluids, a typical optimal design positions the turbine outlet right on the gas saturated line to avoid the efficiency loss caused by the liquid droplets. The enthalpy drop is then greater with a transcritical regime.



**Figure 2.3.** Optimization results of ORC/S/SC and ORC/S/TC with respect to brine temperature (x – axis) and to cold sink temperature (y – axis). (a) Maximal specific power contour lines. (b) Optimal working fluid. (c) Optimal regime.

Figure 2.4 presents the corresponding values of the four optimized design variables. The purpose of this figure is to reveal their orders of magnitude. Dotted lines mark the change of working fluid, where drastic behavior changes may occur. Other radical changes sometimes indicate the switch of optimal cycle type, which are not delineated for a greater visibility (please refer to Fig. 2.3c for the optimal cycles). Figs. 2.4a, 2.4b and 2.4c display straightforward results data for the optimized inlet turbine pressure, mass flow ratio and superheater efficiency. Figure 2.4d reveals that the best design does not always involve the maximum cooling tower range. For normal fluids (blue zone in the top right corner), or when  $T_{wb}$  is high, a lower range  $r$  allows the working fluid to exit the turbine at a lower pressure, where the additional enthalpy drop compensates the supplementary cooling tower load. Depending on the working fluid and operating temperatures, the range is thus a parameter to choose with care when employing a wet cooling tower.



**Figure 2.4.** Optimized design variables contours lines for ORC/S/SC and ORC/S/TC with respect to brine temperature (x – axis) and to cold sink temperature (y – axis). (a) Turbine inlet pressure  $P_H$ . (b) Mass flow ratio  $R_H$ . (c) Superheater effectiveness  $\varepsilon_H$ . (d) Wet cooling tower range  $r$ .

### 2.6.2 ORC with dual-pressure heater results

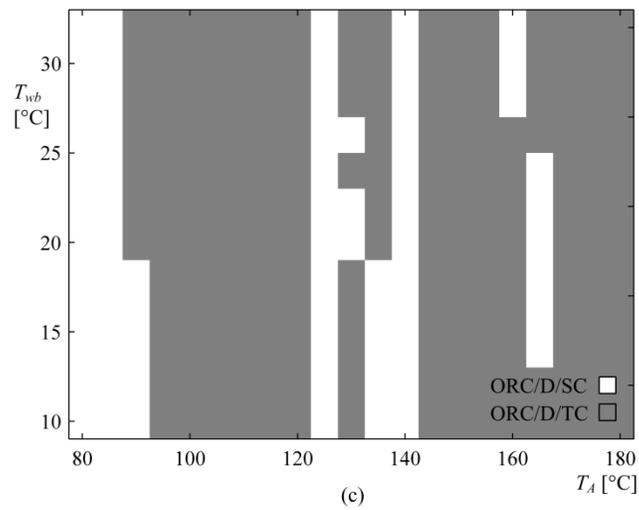
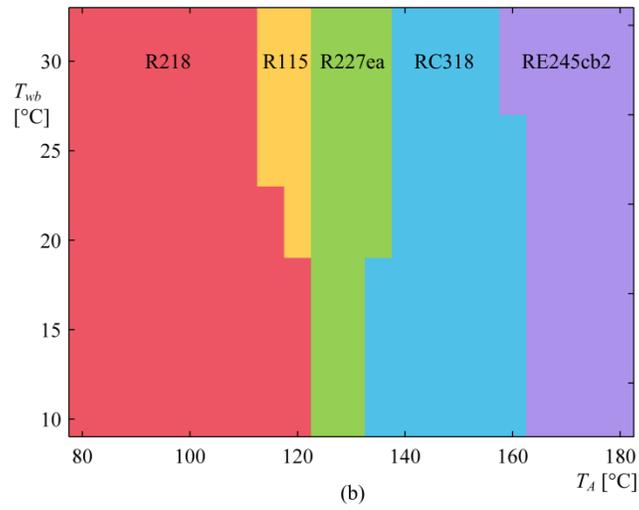
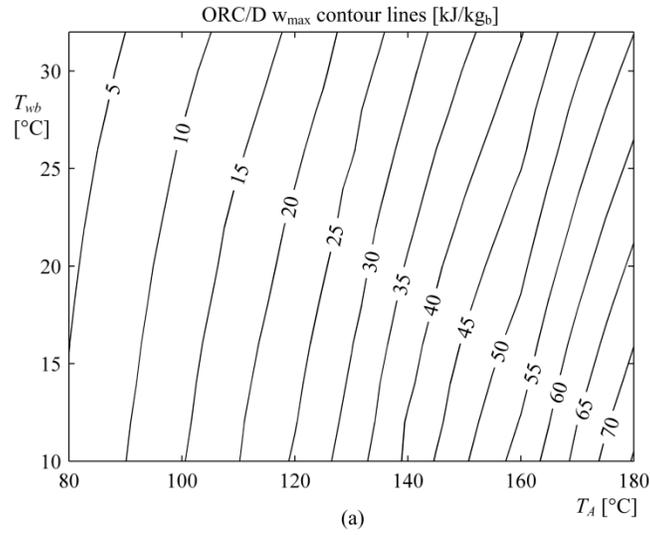
This section displays the combined results of ORC/D/SC and ORC/D/TC. In other words, it presents the results of the best performing cycle for each combination of  $T_A$  and  $T_{wb}$  when considering only the dual-pressure heater designs.

Figure 2.5 presents the maximized specific work output  $w_{\max}$  (Fig. 2.5a), the best working fluids (Fig. 2.5b) and the cycle leading to the highest  $w_{\max}$  (Fig. 2.5c). The tendency of the maximized specific work output is once again quadratic. The best fit done to find coefficients of Eq. (2.5) with a mean relative error of 4% gives:

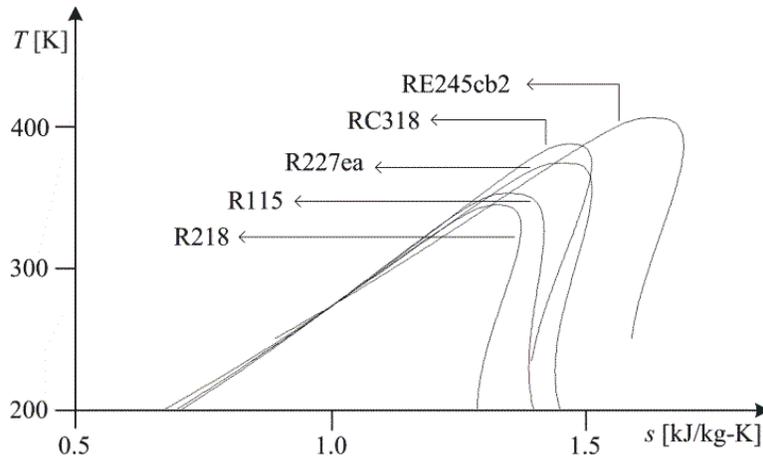
$$w_{\max} \cong 3.4527(T_A - T_{wb} - 20.564)^2 \quad (2.8)$$

Fig. 2.5b shows that only 5 fluids stand out by leading to the highest specific work for at least one combination of  $T_A$  and  $T_{wb}$ : R218, R115, R227ea, RC318 and RE245cb2. They all have a retrograde thermodynamic shape and the corresponding critical temperature increases with respect to the value of  $T_A$ . Their bell shapes in the T-s diagram of Fig. 2.6 (at scale) are overlapping and fairly similar.

The chart in Fig. 2.5c displays the corresponding optimal regime. When looking at Fig. 2.5b and 2.5c, it may be observed that for each given optimal fluid in Fig. 2.5b, the optimal regime passes from the subcritical regime to the transcritical regime as  $T_A$  (x-axis) increases in Fig. 2.5c. The exception is R115, which has an optimal design with a transcritical regime everywhere. This could be interpreted by its saturated gas T-s line being more isentropic than the others, and as discussed in Section 2.6.1, normal and isentropic fluids tend to perform better with a transcritical regime.



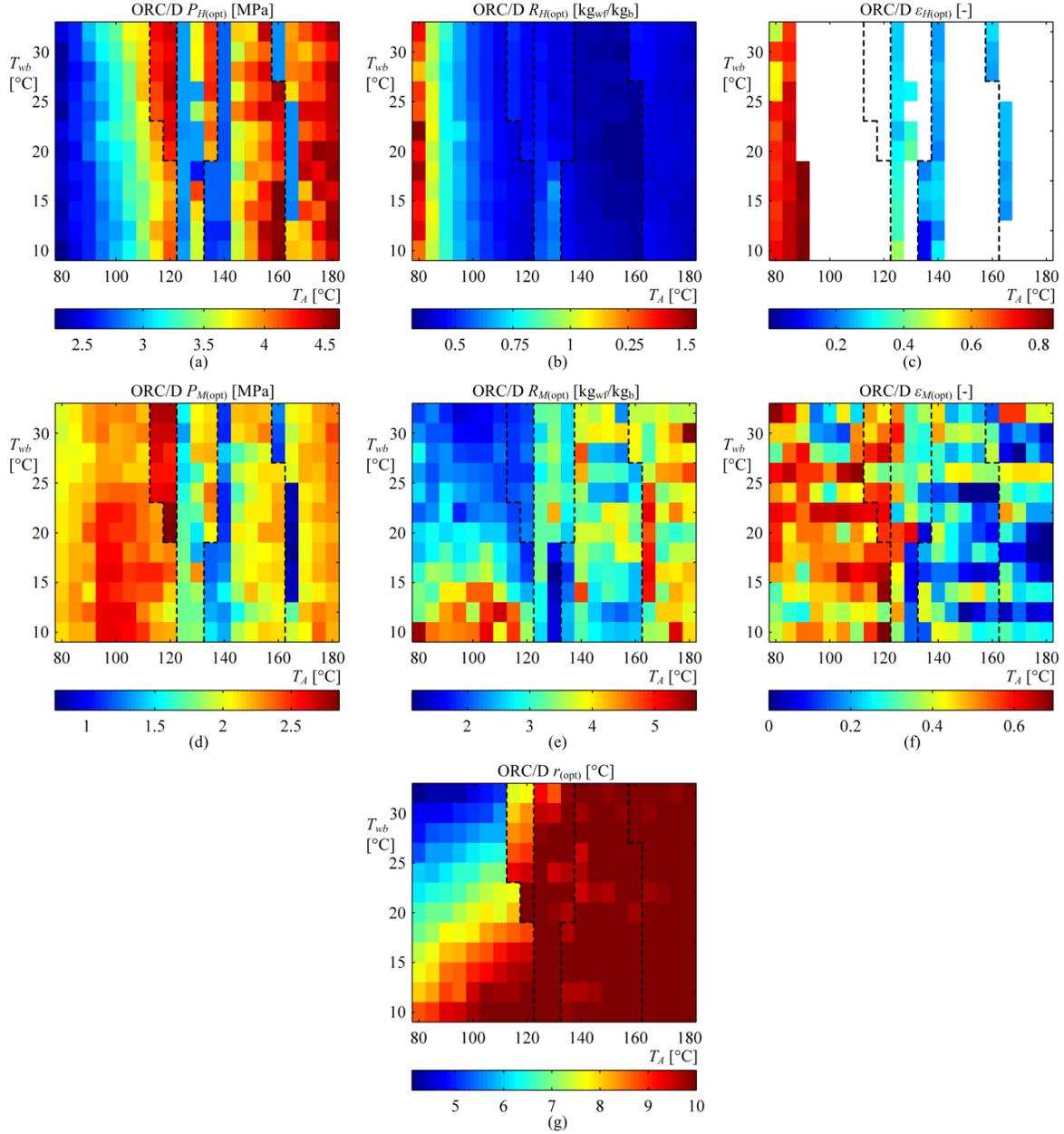
**Figure 2.5.** Optimization results of ORC/D/SC and ORC/D/TC with respect to brine temperature (x – axis) and to cold sink temperature (y – axis). (a) Maximal specific power contour lines. (b) Optimal working fluid. (c) Optimal regime.



**Figure 2.6.** T-s diagram of the five optimal fluids for the dual-pressure ORC cycles of Fig. 2.5b.

Figure 2.7 gathers the corresponding values of the seven optimized design variables. Pressure at first turbine inlet (Fig. 2.7a) and mass flow ratio between brine and high-pressure working fluid (Fig. 2.7b) are linked together, that is, a pressure increase leads to a mass flow ratio increase since more heat from the brine is then needed (as a reminder,  $R = \dot{m}_b / \dot{m}_{wf}$ ). High-pressure superheater effectiveness (Fig. 2.7c) is applicable only for the ORC/D/SC, thus the white area indicates there is no superheater in the cycle.

Figure 2.7f reporting the optimized medium-pressure superheater effectiveness is particularly ‘pixelated’ since this parameter has less impact on  $w_{\max}$  than the other design variables. Indeed, for higher values of  $\varepsilon_M$ , the additional high-temperature heat transferred before the second turbine inlet increases its outlet temperature as well, when considering a retrograde fluid. Therefore, more heat is exchanged in the recuperator to increase state  $\{2'\}$  temperature, which allows a better use of the heat before the brine reaches  $T_{inj}$ . The inverse situation, a lower value of  $\varepsilon_M$ , is preferred when there is no recuperator in order to stay as close as possible to the saturated gas curve. Hence, the impact of the medium-pressure superheater on  $w_{\max}$  is weak when recuperation is included in the cycle. Moreover, its effect is further reduced at high ratio brine/working fluid for the medium-pressure portion (low utilization of the medium-pressure heater), thus the small influence on the state at the second turbine inlet.



**Figure 2.7.** Optimized design variables contours lines for ORC/D/SC and ORC/D/TC with respect to brine temperature ( $x$  – axis) and to cold sink temperature ( $y$  – axis). (a) Pressure at first turbine inlet  $P_H$ . (b) Mass flow ratio  $R_H$  between brine and high-pressure working, (c) High-pressure superheater effectiveness  $\varepsilon_H$ . (d) Pressure at second turbine inlet  $P_M$ . (e) Mass flow ratio  $R_M$  between the brine and the medium-pressure working fluid. (f) Medium-pressure superheater effectiveness  $\varepsilon_M$ . (g) Cooling tower range  $r$ .

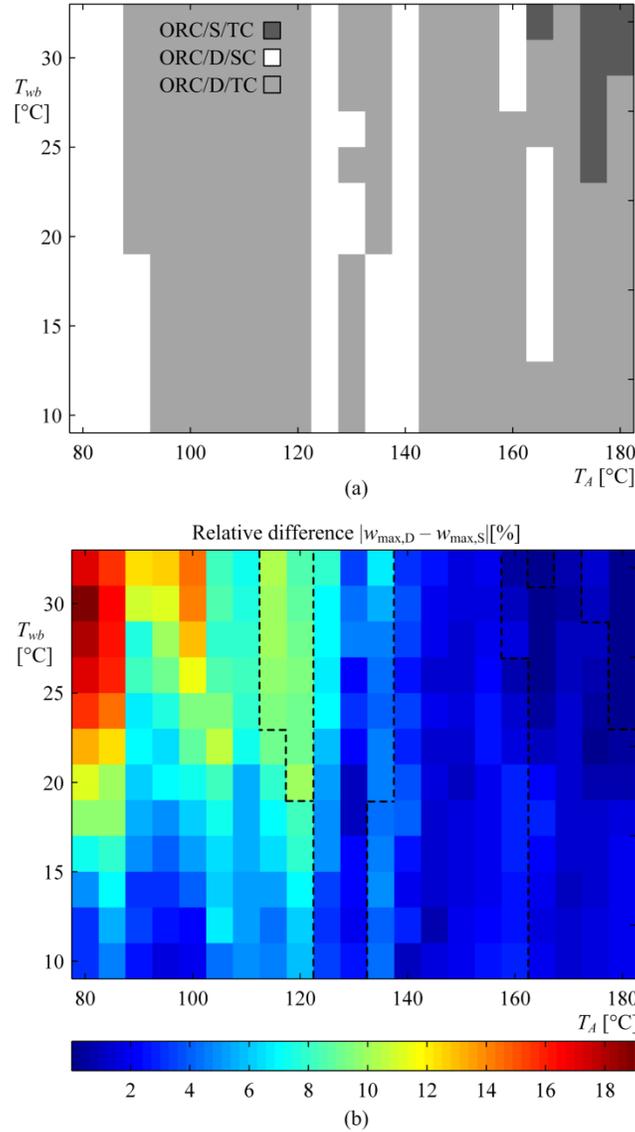
The optimized cooling tower range shown in Fig. 2.7g matches the superior bound for the best fluids starting from 125°C (R227ea, RC318 and RE245cb2). For R218 and R115, it follows a progression depending on both  $T_A$  and  $T_{wb}$  to eventually reach the maximum

value at high  $T_A$  and low  $T_{wb}$ . This behavior encountered at low brine temperatures (see Section 2.6.1 for the interpretation) for dual-pressure heater ORCs confirms as well the pertinence of including the cooling tower range in the list of decision variables.

### 2.6.3 Comparison of single versus dual-pressure results

The purpose of this section is to evaluate the differences between the optimal results of Section 2.6.1 (single-pressure heater) and Section 2.6.2 (dual-pressure heater). First, Fig. 2.8a presents the best cycles by combining Fig. 2.3c and Fig. 2.5c. While the ORC/S/SC is not optimal for any temperature cases, the ORCs with dual-pressure heater dominate the figure, with ORC/D/TC being the best cycle for the majority of the cases. However, to what extent increasing the level of complexity (i.e., adding a set of heat exchangers, pump and turbine stage) is beneficial to the specific work output? Figure 2.8b reveals that for low values of  $T_A$  and high values of  $T_{wb}$  the difference between  $w_{\max}$  of dual-pressure and single-pressure heater ORCs can be as high as 19%, while it drops below 4% for  $T_A = 140^\circ\text{C}$  and above. The lowest differences are at high values of  $T_A$  and  $T_{wb}$ , where the ORC/S/TC just barely surpasses the ORC/D/TC.

As a matter of fact, Fig. 2.7e is a good indicator of how much the dual-pressure system is utilized. The large relative difference in Fig. 2.8b at high values of  $T_{wb}$  and low values of  $T_A$  corresponds to low  $R_{M(\text{opt})}$  (strong utilization). At high values of  $T_A$ ,  $R_{M(\text{opt})}$  is higher (weak utilization), which leads to a smaller relative difference. One could attempt to determine on the  $T_{wb}$  and  $T_A$  coordinates where the dual-pressure and single pressure heater ORCs offset each other, but the studied  $T_A$  range in the present work is not large enough to identify the shift for all  $T_{wb}$  values.



**Figure 2.8.** (a) Best ORC cycles among the ones studied in this paper. (b) Relative difference between maximized specific work output of ORC/D and ORC/S including both regimes.

What stands out when comparing Figs. 2.3b and 2.5b is the change in the number of optimal fluids (13 and 5 respectively). All normal and isentropic fluids in Fig. 2.3b situated at high values of  $T_{wb}$  are not found in Fig. 2.5b, making the dual-pressure heater ORCs optimal only with retrograde fluids. Indeed, what limits their performance in single-pressure heater ORCs is the greater sensible enthalpy of vapor between the turbine output state and the gas saturated state. Having a second turbine or turbine stage where the entropy can be reduced before its input thus shifts the expansion towards the gas saturated line (referring to the T-s diagram) and increases the enthalpy drop. Considering a larger set of

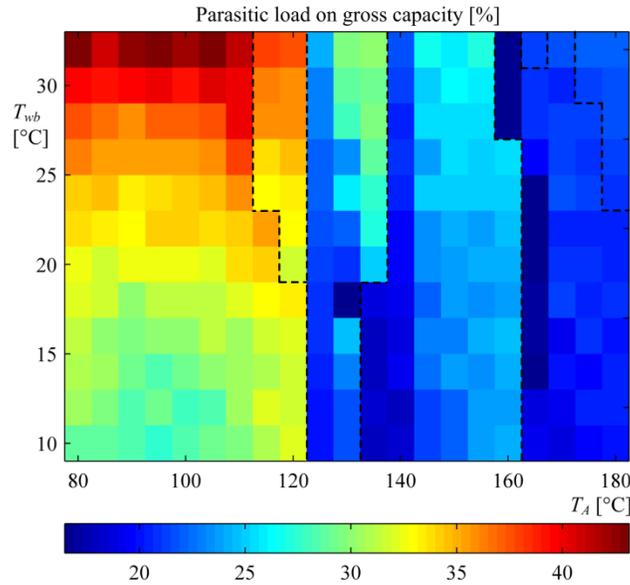
working fluids could potentially lead to alternative results, since the twenty working fluids were chosen based on single-pressure heater ORCs results of a previous work [45]. Fluids with greater critical temperature would likely surpass RE245cb2 at high  $T_A$  values of the studied range, as proposed by Manente et al. [60].

#### **2.6.4 Impact of parasitic load**

Results presented in Figs. 2.3a and 2.5a are based on the net specific work and do not reveal the work consumed by the pump(s) and cooling system. Such work is subtracted from the gross power produced by the turbine(s), thus is called parasitic load. Figure 2.9 displays the proportion of the total parasitic load (includes feed pump(s), cooling water pump and tower fan) on the power plant gross power for the optimal designs. It should be noted that the geofluid pump work is not taken into account in the results. Behavior changes can be observed between fluids (separated by dotted lines) and regimes. For example, employing R227ea leads to relatively low parasitic load at  $T_A = 125^\circ\text{C}$ , but it increases when the optimal regime switches to transcritical due to the rise of the pressure  $P_H$ . A similar behavior is seen for RC318 and RE245cb2. Moreover, for both regimes, the optimal strategy to handle the parasitic loads was highly dependent on the working fluid. Indeed, two opposite tendencies occur when moving up along the y-axis: the cooling system demands more input work while the feed pump(s) work is reduced since the pressure drop decreases as well. For some fluids, the increase of the condensation pressure  $P_{CO}$  when the cold sink is warmer (i.e. high  $T_{wb}$ ) is significant enough to nearly offset the change in cooling load. On the other hand, for fluids with lower average pressure drop (R218 and R125) the increase of the cooling system work is more important than the reduction of the pump(s) work. However, despite their higher cooling load, these fluids remain the optimal choice among the twenty fluids studied for brine temperatures values up to  $120^\circ\text{C}$ .

The model used effectiveness in heat exchangers calculations, but limiting heat transfer areas for economic reasons would change results. Considering pressures losses, which become larger for an increased heat transfer area, would raise the feed pump(s) work

consumption and thus affect the net specific work, particularly for low geofluid temperature cases where the parasitic losses are the most significant.



**Figure 2.9.** Parasitic load over the gross capacity with respect to brine temperature and to cold sink temperature.

## 2.7. Conclusion

In this paper, four different Organic Rankine Cycles applied to geothermal power plants were numerically simulated and optimized for various operating conditions (inlet brine temperature and ambient air wet bulb temperature): the single-pressure heater ORCs, ORC/S/SC (subcritical) and ORC/S/TC (transcritical), and the dual-pressure heater ORCs, ORC/D/SC (subcritical) and ORC/D/TC (transcritical). Each system includes recuperation of the outlet turbine heat and a wet cooling tower as a cooling system. The objective function was the specific work output, and the design variables depended on the cycle, including operating pressures, mass flows ratios, superheaters effectiveness and the cooling tower range. A total of 20 working fluids were tested in the optimization of each case.

The outputs of the optimization are reported in the form of charts, where one can find the maximized specific work output, best fluids, best regimes, and optimized design variables for the single-pressure heater ORCs and dual-pressure heater ORCs. While a mix of retrograde, normal and isentropic fluids are optimal for single-pressure heater cycles,

exclusively retrograde fluids are optimal for dual-pressure heater cycles. Charts presenting the best cycles and the relative difference between a dual-pressure heater and a single-pressure heater in ORCs were also developed. They revealed the dominance of the ORC/D/TC and the pertinence of a dual-pressure heater at low brine temperature and high cold sink temperature. The maximized specific work output was increased by a maximum of 19% compared to ORCs with single-pressure heater, while the difference drops to less than 4% at a brine temperature above 140°C. Finally, the study of the parasitic loads revealed that the best strategy to maximize the net work output varies among working fluids, emphasizing the importance of choosing carefully the cooling tower range.

The work presented in this paper could be extended in various ways. First, the minimum reinjection temperature remained fixed, but other values could be considered to evaluate its impact on the maximized specific work output. While solely a wet cooling tower has been employed here, it would be interesting to simulate natural and dry cooling towers, as well as using complete tower analysis for a more detailed modeling. As seen in Section 2.4.6, a slight difference on the condensing pressure has a great influence on the output, which means the condenser effectiveness is a crucial parameter. A sensitivity analysis could help to find the best cost/performance trade-off. Zeotropic mixtures of fluids could be investigated for more efficient heat absorption, as studied by Noriega Sanchez et al. [76]. Finally, considering other configurations of the heat exchanger network (e.g., parallel heating) could lead to higher maximized work for dual-pressure heater cycles.

## **2.8. Appendix A: Calculation details for single-pressure heater subcritical organic Rankine cycle**

This appendix describes how the net specific output  $w$  of the ORC/S/SC was obtained. It should be noted that  $w$  does not take into account pressure losses in heat exchangers and electrical losses in the generator. The required inputs include two types of data: the imposed parameters (see Table 2.4) and the design variables (see Table 2.5). This system needs the value of four design variables to calculate the objective function  $w$ : (i) the

turbine inlet pressure  $P_H$  , (ii) the mass flow ratio  $R_H$  between the brine and the working fluid, (iii) the superheater effectiveness  $\varepsilon_H$  , and (iv) the cooling tower range  $r$  .

The geofluid enthalpy at state {A} can be obtained from thermodynamic libraries:

$$\text{State \{A\}:} \quad \left. \begin{array}{l} T_A \\ \text{sat. liq.} \end{array} \right\} \rightarrow h_A \quad (2.9)$$

The first working fluid state that may be calculated is state {4} at the evaporator (EV) output where it is saturated vapor at the known pressure  $P_H$  .

$$\text{State \{4\}:} \quad \left. \begin{array}{l} P_4 = P_H \\ \text{sat. liq.} \end{array} \right\} \rightarrow \begin{array}{l} h_4 \\ T_4 \end{array} \quad (2.10)$$

To obtain the enthalpies of fluids leaving the superheater (states {5} and {B}) two hypothetical states are specified. At state {5\*}, the working fluid reaches the maximum theoretical temperature in the superheater (i.e.,  $T_A$ ), and at state {B\*} the geofluid reaches the minimum theoretical temperature in the superheater (i.e.,  $T_4$ ). The enthalpies of these hypothetical states are:

$$\text{State \{5*\}:} \quad \left. \begin{array}{l} T_{5^*} = T_A \\ P_{5^*} = P_H \end{array} \right\} \rightarrow h_{5^*} \quad (2.11)$$

$$\text{State \{B*\}:} \quad \left. \begin{array}{l} T_{B^*} = T_4 \\ \text{sat. liq.} \end{array} \right\} \rightarrow h_{B^*} \quad (2.12)$$

The maximal heat transfer  $\dot{Q}_{\max}$  that could occur in the superheater can now be determined.

On the geofluid side, the expression for  $\dot{Q}_{b,\max}$  is

$$\dot{Q}_{b,\max} = \dot{m}_b (h_A - h_{B^*}) = \dot{m}_{wf} R_H (h_A - h_{B^*}) = \dot{m}_{wf} \tilde{q}_{b,\max} \quad (2.13)$$

and on the working fluid side, the expression for  $\dot{Q}_{wf,\max}$  is

$$\dot{Q}_{wf,\max} = \dot{m}_{wf}(h_{5^*} - h_4) = \dot{m}_{wf} \tilde{q}_{wf,\max} \quad (2.14)$$

The value of  $\dot{Q}_{\max}$  can be determined by selecting the minimal value between  $\tilde{q}_{b,\max}$  and  $\tilde{q}_{wf,\max}$ , and then using the definition of heat exchanger efficiency provides the expression

$$\varepsilon_H = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{\tilde{q}}{\tilde{q}_{\max}} \rightarrow \tilde{q} = \varepsilon_H \tilde{q}_{\max} \quad (2.15)$$

The variable  $h_5$  can be isolated since it is the only unknown and the entropy of state {5} is found as follows:

$$\text{State \{5\}:} \quad \left. \begin{array}{l} h_5 = h_4 + \varepsilon_H \tilde{q}_{\max} \\ P_5 = P_H \end{array} \right\} \rightarrow s_5 \quad (2.16)$$

In this work, the condensing pressure  $P_{CO}$  (pressure at states {6} and {1}) is not given but considered as the lowest pressure respecting the constraints on the condenser: effectiveness lower than  $\varepsilon_{\max}$  and approach temperature difference higher than  $\Delta T_{tol}$  between the working fluid and the cooling water. It means that states {1}, {2}, {2'}, {6} and {6'} must be determined altogether in an iterative or incremental method to find  $P_{CO}$ .

Cooling water inlet and outlet temperatures are determined with the wet cooling tower characteristics and the given wet bulb ambient air temperature  $T_{wb}$ . The cooling water input temperature in the condenser (state {w1} in Fig. 2) is determined with the definition of the approach  $A$ , that to say the difference between this temperature and  $T_{wb}$ :

$$\text{State \{w1\}} \quad T_{w1} = T_{wb} + A \quad (2.17)$$

Then, the cooling water outlet temperature (state {w2} in Fig. 2) is known with the range  $r$ , i.e. the cooling water temperature difference:

$$\text{State \{w2\}} \quad T_{w2} = T_{w1} + r \quad (2.18)$$

Continuing on the working fluid side, state {1} is first determined knowing the condensing pressure  $P_{CO}$ :

$$\text{State \{1\}} \quad \left. \begin{array}{l} P_1 = P_{CO} \\ \text{sat. liq.} \end{array} \right\} \rightarrow \begin{array}{l} h_1 \\ \rho_1 \end{array} \quad (2.19)$$

State {2} is then found calculating the pump work to reach  $P_H$  from state {1}:

$$\text{State \{2\}} \quad \begin{array}{l} h_2 = h_1 + w_{PP} \\ w_{PP} = (P_H - P_{CO}) / (\eta_{PP} \rho_1) \end{array} \quad (2.20)$$

Next is state {6} at the turbine outlet. The method to determine this state with more accuracy consists in dividing the turbine into several virtual stages, where each stage deals with a small part of the total pressure drop with the appropriate efficiency, determined by the liquid content. When the working fluid is superheated vapor, one uses the turbine dry efficiency  $\eta_{dry}$ , and when it is a saturated mixture, the Baumann efficiency is required to take into account the effect of liquid droplets [30]; [2]. See Chagnon-Lessard et al. [45] for more details on this method.

Stage  $i$  outlet enthalpy is thus expressed by Eq. (2.21) when the fluid is superheated vapor, and by the Baumann expression (Eq. (2.22)) when it is a saturated mixture. Stage  $i$  enthalpy where the expansion is assumed isentropic is always obtained with Eq. (2.23).

$$\text{State \{i\} (dry):} \quad h_i = h_{i-1} - \eta_{dry} (h_{i-1} - h_{i,s}) \quad (2.21)$$

$$\text{State \{i\} (mixture):} \quad h_i = \frac{h_{i-1} - A \left( x_{i-1} - h_{i,f} / (h_{i,g} - h_{i,f}) \right)}{1 + A / (h_{i,g} - h_{i,f})} \quad (2.22)$$

$$A = \frac{\eta_{dry}}{2} (h_{i-1} - h_{i,s})$$













































































































