The pricing of embedded lease options*

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Abstract

Office leases are generally agreed upon for extended terms, with possible options to leave or to renew in favour of the tenant. Tenants who have no options during the life of their lease expect to pay a lower rent than those who do. In this letter, we built up a conceptual framework based on binomial tree for the pricing of options embedded in a lease contract. Results show that lease options are dependent upon market rents volatility.

Keywords: Leases, Lease Options, Binomial tree, Tenants.

JEL codes: C60, G32, R30, R33.

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1 Introduction

The rent paid by a tenant is the primary cash flow in commercial real estate and is thus the most influential building block in the valuation of office properties. Office leases vary with property types and countries: length, expenses and services are non-standard and can be subject to local practices, market specificities and bargaining power. Due to the difficulty to predict future market rents or space requirements beyond a lease term, rental agreements often include options that accommodate the tenant when the contract expires, such as the option to renew or the option to expand in the same building. The renewal option, which is the focus of this paper, allows a tenant to re-lease the premises at the end of the term at a pre-specified rent. Since the value of such an option hinges on expected variations in the market rent which, in turn, is related to the value of the property, analysing lease options may help characterize the dynamics of real estate prices. As leases are agreed upon at a much greater frequency than properties are bought and sold, studying leases somehow bypasses the lack-of-liquidity problem associated with real estate data.

In exchange for more flexibility, a tenant may accept to pay a higher rent (see Miceli & Sirmans, 1999; Amédée-Manesme et al., 2013), and the value of an option can then be expressed as the difference between the rent of a plain-vanilla lease and that of an option-embedded lease. The value of the option represents the price paid in each period by the tenant in order to get the flexibility to leave or to renew on an agreed date, and thus corresponds to an option-adjusted lease spread. With this option, the tenant may decide - at a specified date - to continue the current lease or to enter a new contract if market rents drop.

In this letter, using a binomial model, we extract an analytical solution is designed for valuing an option embedded in a lease contract. In line with the work of Kulatilaka & Marcus (1994), we do not assume the possibility to build replicating portfolios generating a risk-free world. In particular, we highlight how the value of the option is affected by the volatility of market rents. We explicitly distinguish between contractual rents and market rents.

Despite their importance, options embedded in office lease contracts have received little attention in the literature[1]. Grenadier (1995), Buetow & Albert (1998) and Stanton & Wallace (2009) develop

[1]This is not the case though in other fields of research such as automobile lease contracts (see Gamba & Rigon, 2008 and Giaccotto et al., 2007) or options to expand businesses (see Agliardi, 2006) where the literature proves to be quite abundant.

partial differential equations models to value lease-embedded options but they rely on numerical methods in order to characterize the equilibrium. Clapham (2003), on the other hand, obtain an analytical solution using partial deifferential equations by modifying the approach of Buetow & Albert (1998). However, Clapham (2003) provides a model to value an option in a risk-free, Black-Scholes-Merton, setup. By relying on a binomial model, we define a more flexible environment with regards to both lease-setting and users' conditions wherein an analytical solution can also be found. Beside, our proposal allows, among other things, to account for the risk aversion of the tenant, the negotiation games between landlord and tenants or the possibility for the former to offer rent free periods.

In practice, the proposed model may be used to compute market rents' implied volatility. Indeed, embedded lease options are not traded and thus the pricing of such options is useless for trading activities. However the implied volatility of related rents can be derived from the prices of embedded lease options on the market place. This information may be mostly relevant to practitioners. First, this information is not easily accessible due to both the drawbacks of published indices from which volatility is usually extracted (appraisal based, sampling errors,...) and the limited number of observable transactions (see Patel & Sing, 2000). Second, the implied volatility of related rents contain idiosyncratic information that cannot be obtained from the sole transaction prices of the underlying properties (As shown by Fleming et al., 1995, predictions based on the implied volatility prove to outclass those that are based on the price process alone). Therefore, the volatility derived from our model may be a better estimate of the volatility of direct real estate asset values than the one derived from traditional methods.

2 The model

Time is discrete and represented by $t = 1, 2, \ldots$, measured in years, and $\Delta t = 1/12$ represents one month. The one-month market rent prevailing at the beginning of period t is denoted R_t and evolves according to a binomial model with time steps Δt. Over a period Δt, the market rent may go up by a factor $u = e^{\sigma\sqrt{\Delta t}}$ or down by a factor $d = e^{-\sigma\sqrt{\Delta t}}$, σ being the volatility of the market rent. Let p_u denote the actual probability of an up movement and let μ denote the expected growth rate of market rents. Then $E_t[R_{t+\Delta t}] = p_u R_t u + (1 - p_u) R_t d = R_t e^{\mu\Delta t}$, with $p_u = \frac{e^{\mu\Delta t} - d}{u-d}$. Note that to have $p_u \leq 1$, we need $\mu \leq \sigma/\sqrt{\Delta t}$.

In what follows, it is assumed that the tenant has the choice between a long-term fixed-lease contract ($2T$) or a short-term renewable lease contract (T), the shorter term contract being half the longer term contract. It is also assumed that rent conditions are fixed throughout the full lease contract and known from start, excluding, more often than not, rent indexation and negotiation dimensions.

When a W-year lease is signed at the beginning of year t with a fixed Δt-payment L_t^W (the contractual rent), the present value of all rent payments is given by

$$V_t^W = L_t^W + e^{-k\Delta t} L_t^W + e^{-k(2\Delta t)} L_t^W + \ldots + e^{-k(W-\Delta t)} L_t^W = \frac{\left(1 - e^{-kW}\right) L_t^W}{1 - e^{-k\Delta t}},$$

where k is the annual discount rate[2]. When a contract is signed over W years, the value of the fixed-lease contract has to be equal to the expected present value of the sum of all market rents[3] over the same period, i.e.

$$\begin{aligned} V_t^W &= E_t \left[R_t + e^{-k\Delta t} R_{t+\Delta t} + e^{-k(2\Delta t)} R_{t+2\Delta t} + \ldots + e^{-k(W-\Delta t)} R_{t+W-\Delta t} \right] \\ &= R_t + e^{(\mu-k)\Delta t} R_t + e^{(\mu-k)(2\Delta t)} R_t + \ldots + e^{(\mu-k)(W-\Delta t)} R_t \\ &= \frac{\left(1 - e^{(\mu-k)W}\right) R_t}{1 - e^{(\mu-k)\Delta t}}, \end{aligned}$$

and thus

$$L_t^W = \frac{\left(1 - e^{(\mu-k)W}\right)\left(1 - e^{-k\Delta t}\right)}{\left(1 - e^{(\mu-k)\Delta t}\right)\left(1 - e^{-kW}\right)} R_t. \tag{1}$$

With $\mu < k$, the lease payment under an infinite contract would be

$$L_t^\infty = \frac{1 - e^{-k\Delta t}}{1 - e^{(\mu-k)\Delta t}} R_t,$$

and the present value of all future expected market rents as of time t is given by

$$V_t^\infty = L_t^\infty + e^{-k\Delta t} L_t^\infty + e^{-2k\Delta t} L_t^\infty + e^{-3k\Delta t} L_t^\infty + \ldots = \frac{L_t^\infty}{1 - e^{-k\Delta t}} = \frac{R_t}{1 - e^{(\mu-k)\Delta t}}. \tag{2}$$

[2]It is straightforward to add indexation (i) to the process. In this case the geometric progression will be $e^{(-k+i)\Delta t}$. In order to facilitate the readability of the results, this factor has been omitted.

[3]Contractual rents (L_t) charged to tenants rarely follow market rents (R_t). Rents are usually contracted at a value close to the market rents at the initiation of the lease. Later on, rents have usually been indexed and do not necessarily represent the current market value, which may collapse in bear markets or raise in bull markets. In uncertain economic times, many previously determined contractual rents may end up being above market rents. Market rents can thus be defined as the most likely lease rate a property would command in an open market.

The solution to our model assumes that a tenant considers the value of the rent paid into infinity when considering different types of contract terms (the tenant will never own its real estate). The equilibrium also assumes that an infinite sequence of W-year contracts has to be equal to the present value of expected market rents into infinity, i.e.

$$V_t^W \; + \; e^{-kW} E_t \left[V_{t+W}^W \right] \; + \; e^{-2kW} E_t \left[V_{t+2W}^W \right] \; + \; \ldots \; = \; V_t^\infty, \tag{3}$$

and thus the present value of an infinite sequence of W-year contracts can be written as

$$V_t^W \; + \; e^{-kW} E_t \left[V_{t+W}^\infty \right] \; = \; V_t^W \; + \; e^{(\mu-k)W} V_t^\infty. \tag{4}$$

Consider now a T-year renewable contract with a Δt-payment $\tilde{L}_t^T$. After T years, the tenant can renew the lease at the same rate or enter into another T-year renewable contract at a lower rate. If, at time $t+T$, the available renewable contract demands $\tilde{L}_{t+T}^T > \tilde{L}_t^T$, then the tenant will renew the contract signed at time t for a lease rate $\tilde{L}_t^T$. If, on the other hand, $\tilde{L}_{t+T}^T \leq \tilde{L}_t^T$, then the tenant will exit the current contract and sign a new renewable contract[4]. Note that equation (3) must hold with renewable leases as well, and thus the present value of an infinite sequence of T-year renewable leases at time $t+j$ must equal V_{t+j}^∞ for all j. The present value of an infinite sequence of T-year renewable contracts is then equal to

$$\begin{aligned}\frac{\left(1-e^{-kT}\right)\tilde{L}_t^T}{1-e^{-k\Delta t}} \; &+ \; \Pr\left(\tilde{L}_{t+T}^T > \tilde{L}_t^T\right) e^{-kT} \left(\frac{1-e^{-kT}}{1-e^{-k\Delta t}} \tilde{L}_t^T + e^{-kT} E_t \left[V_{t+2T}^\infty \middle| \tilde{L}_{t+T}^T > \tilde{L}_t^T \right] \right) \\ &+ \; \Pr\left(\tilde{L}_{t+T}^T \leq \tilde{L}_t^T\right) e^{-kT} E_t \left[V_{t+T}^\infty \middle| \tilde{L}_{t+T}^T \leq \tilde{L}_t^T \right].\end{aligned}$$

Since $\tilde{L}_{t+T}^T \leq \tilde{L}_t^T$ if and only if $R_{t+T} \leq R_t$, the last expression can be rewritten as

$$\begin{aligned}\frac{\left(1-e^{-kT}\right)\tilde{L}_t^T}{1-e^{-k\Delta t}} \; &+ \; \Pr\left(R_{t+T} > R_t\right) e^{-kT} \left(\frac{1-e^{-kT}}{1-e^{-k\Delta t}} \tilde{L}_t^T + e^{-kT} E_t \left[V_{t+2T}^\infty \left| R_{t+T} > R_t \right.\right] \right) \\ &+ \; \Pr\left(R_{t+T} \leq R_t\right) e^{-kT} E_t \left[V_{t+T}^\infty \left| R_{t+T} \leq R_t \right.\right].\end{aligned}$$

Note that

$$E_t \left[V_{t+2T}^\infty \left| R_{t+T} > R_t \right.\right] \; = \; E_t \left[\frac{R_{t+2T}}{1-e^{(\mu-k)\Delta t}} \middle| R_{t+T} > R_t \right] \; = \; \frac{e^{\mu T}}{1-e^{(\mu-k)\Delta t}} E_t \left[R_{t+T} \middle| R_{t+T} > R_t \right]$$

and

$$E_t \left[V_{t+T}^\infty \left| R_{t+T} \leq R_t \right.\right] \; = \; \frac{1}{1-e^{(\mu-k)\Delta t}} E_t \left[R_{t+T} \middle| R_{t+T} \leq R_t \right].$$

[4] In this letter, transaction costs (broker fees, searching costs, moving costs...) have not been considered. However, one can add the transaction costs δ in the moving decision by comparing $\tilde{L}_{t+T}^T + \delta$ with $\tilde{L}_t^T$.

As shown in appendix, $E_{t+j}\left[R_{t+j+T}\right| R_{t+j+T} \leq R_{t+j}] = \Omega_d R_{t+j}$ for any $j \geq 0$, where Ω_d is a constant, and $E_{t+j}\left[R_{t+j+T}\right| R_{t+j+T} > R_{t+j}] = \Omega_u R_{t+j}$ for any $j \geq 0$, where Ω_u is a constant. This gives

$$E_t\left[V_{t+2T}^{\infty}\,|R_{t+T} > R_t\right] \;=\; e^{\mu T}\Omega_u V_t^{\infty} \quad \text{and} \quad E_t\left[V_{t+T}^{\infty}\,|R_{t+T} \leq R_t\right] \;=\; \Omega_d V_t^{\infty}.$$

Let $\theta = \Pr\left(R_{t+T} \leq R_t\right)$. The present value of an infinite sequence of T-year once-renewable contracts is then equal to

$$\frac{\left(1-e^{-kT}\right)\tilde{L}_t^T}{1-e^{-k\Delta t}} \;+\; (1-\theta)e^{-kT}\left(\frac{1-e^{-kT}}{1-e^{-k\Delta t}}\tilde{L}_t^T \;+\; e^{(\mu-k)T}\Omega_u V_t^{\infty}\right) \;+\; \theta e^{-kT}\Omega_d V_t^{\infty},$$

which can be written as

$$\frac{\left(1-e^{-2kT}\right)\tilde{L}_t^T}{1-e^{-k\Delta t}} \;-\; \theta e^{-kT}\frac{1-e^{-kT}}{1-e^{-k\Delta t}}\tilde{L}_t^T \;+\; (1-\theta)e^{(\mu-2k)T}\Omega_u V_t^{\infty} \;+\; \theta e^{-kT}\Omega_d V_t^{\infty}.$$

Since $(1-\theta)\Omega_u = e^{\mu T} - \theta\Omega_d$, we can write

$$\frac{\left(1-e^{-2kT}\right)\tilde{L}_t^T}{1-e^{-k\Delta t}} \;-\; \theta e^{-kT}\frac{1-e^{-kT}}{1-e^{-k\Delta t}}\tilde{L}_t^T \;+\; e^{2(\mu-k)T}V_t^{\infty} \;+\; \theta\Omega_d e^{-kT}\left(1-e^{(\mu-k)T}\right)V_t^{\infty}. \tag{5}$$

for the value of an infinite sequence of T-year renewable contracts.

Proposition 1 *Consider a world where tenants have two options regarding lease contracts: A fixed-rent $2T$-year contract, or a once-renewable T-year contract. Let $\tilde{L}_t^T$ and L_t^{2T} represent the renewable T-year rent and the fixed $2T$-year rent, respectively. Then the renewable rent can be expressed as*

$$\tilde{L}_t^T \;=\; L_t^{2T} \;+\; P_t,$$

where $P_t \geq 0$ represents the value of the option to sign another renewable contract with a better rate at time T.

The remaining of this section provides the proof of Proposition 1. In equilibrium, (3) and (5) must be equal and thus, writing (4) with $W = 2T$, replacing V_t^{2T} by $\frac{\left(1-e^{-2kT}\right)L_t^{2T}}{1-e^{-k\Delta t}}$, and replacing V_t^{∞} by $\frac{R_t}{1-e^{(\mu-k)\Delta t}}$, we have

$$\begin{aligned}
\frac{\left(1-e^{-2kT}\right)L_t^{2T}}{1-e^{-k\Delta t}} \;&=\; \frac{\left(1-e^{-2kT}\right)\tilde{L}_t^T}{1-e^{-k\Delta t}} \;-\; \theta e^{-kT}\frac{1-e^{-kT}}{1-e^{-k\Delta t}}\tilde{L}_t^T \;+\; \theta\Omega_d e^{-kT}\frac{\left(1-e^{(\mu-k)T}\right)R_t}{1-e^{(\mu-k)\Delta t}} \\
&=\; \frac{\left(1-e^{-2kT}\right)\tilde{L}_t^T}{1-e^{-k\Delta t}} \;-\; \theta e^{-kT}\frac{1-e^{-kT}}{1-e^{-k\Delta t}}\tilde{L}_t^T \;+\; \theta\Omega_d e^{-kT}\frac{\left(1-e^{-kT}\right)L_t^T}{1-e^{-k\Delta t}}.
\end{aligned}$$

Rearranging the last equation and using the identity $1-e^{-2kT}=\left(1-e^{-kT}\right)\left(1+e^{-kT}\right)$, we obtain

$$\tilde{L}_t^T \;=\; L_t^{2T} \;+\; \frac{\theta e^{-kT}}{1+e^{-kT}}\left(\tilde{L}_t^T-\Omega_d L_t^T\right) \;=\; L_t^{2T} \;+\; \frac{\Pr(R_{t+T}\leq R_t)}{1+e^{kT}}\left(\tilde{L}_t^T-E_t\left[L_{t+T}^T\middle| R_{t+T}\leq R_t\right]\right) \quad (6)$$

Define

$$P_t \;=\; \frac{\Pr(R_{t+T}\leq R_t)}{1+e^{kT}}\left(\tilde{L}_t^T-E_t\left[L_{t+T}^T\middle| R_{t+T}\leq R_t\right]\right), \quad (7)$$

which is the premium paid each period in the T-renewable contract over the fixed $2T$-year rent. P_t corresponds to the option-adjusted lease spread between the T-year renewable contract and the fixed $2T$-year contract. In the above equations, L_t^T would correspond to the monthly rent paid on a fixed T-year contract (an option that we have not considered available in this framework).

From (6), the T-year renewable rent is given by

$$\tilde{L}_t^T \;=\; \left(1-\frac{\theta}{1+e^{kT}}\right)^{-1}\left(L_t^{2T}-\frac{\theta\Omega_d L_t^T}{1+e^{kT}}\right).$$

Since $L_t^{2T}=\frac{1+e^{(\mu-k)T}}{1+e^{-kT}}L_t^T$, the last equation can be rearranged as

$$\tilde{L}_t^T \;=\; L_t^{2T}\left(1-\frac{\theta}{1+e^{kT}}\right)^{-1}\left(1-\frac{\theta\Omega_d(1-e^{-kT})}{(1+e^{kT})(1+e^{(\mu-k)T})}\right). \quad (8)$$

3 Application

Table 1 shows some numerical examples with varying volatility σ and two different expected growth rates in the market rents μ. We can see in this table that the value of the option to renew is increasing (and convex) with volatility, and that it is inversely related to the expected growth rate in the market rents μ.

4 Conclusion

Lease options of many types are common in real world leasing agreements. Renewal (or break) options are the most common of these options and are therefore an important phenomenon. In this letter, we have derived a closed-form solution to the value of the renewable lease rate as well as the value of the option to renew using binomial model. In practice, the proposed model may be used to extract the information content in the embedded options. In particular, such a model can be used to predict future implied market rent volatility.

Volatility σ	T-year fixed $\left(L_t^T\right)$	$2T$-year fixed $\left(L_t^{2T}\right)$	T-year renewable $\left(\tilde{L}_t^T\right)$	Value of option to renew (P_t)
$\mu = 2\%$				
10%	104.86	109.69	113.14	3.45
15%	104.86	109.69	115.87	6.19
20%	104.86	109.69	118.73	9.03
25%	104.86	109.69	121.63	11.93
30%	104.86	109.69	124.57	14.88
$\mu = 4\%$				
10%	110.06	120.73	122.39	1.67
15%	110.06	120.73	124.70	3.98
20%	110.06	120.73	128.68	7.95
25%	110.06	120.73	131.83	11.10
30%	110.06	120.73	135.06	14.33

Table 1: Numerical examples with $R_t = 100$, $T = 5$, $\Delta t = 1/12$, $k = 5\%$.

This work opens the doors to many further researches and applications. In particular, extensions to more flexible lease options models are straightforward and empirical works on lease contracts may be done. Above all, application of this framework to design risk measurements for direct commercial office real estate assets may be considered.

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A Technical proof of Ω

Suppose that the market rent at time $t+j$ is equal to R_{t+j} and let $N = T/\Delta t$ denote the number of time steps during a time period T, N being divisible by 2. In what follows, $p_d = 1 - p_u$. Then

$$
\begin{aligned}
&E_{t+j}\left[R_{t+j+T} \middle| R_{t+j+T} \le R_{t+j}\right] \\
&= \frac{\binom{N}{N/2} p_u^{N/2} p_d^{N/2} u^{N/2} d^{N/2} R_{t+j} + \binom{N}{N/2-1} p_u^{N/2-1} p_d^{N/2+1} u^{N/2-1} d^{N/2+1} R_{t+j} + \ldots + \binom{N}{0} p_d^N d^N R_{t+j}}{\Pr\left(R_{t+j+T} \le R_{t+j}\right)} \\
&= \frac{\binom{N}{N/2} p_u^{N/2} p_d^{N/2} u^{N/2} d^{N/2} + \binom{N}{N/2-1} p_u^{N/2-1} p_d^{N/2+1} u^{N/2-1} d^{N/2+1} + \ldots + \binom{N}{0} p_d^N d^N}{\binom{N}{N/2} p_u^{N/2} p_d^{N/2} + \binom{N}{N/2-1} p_u^{N/2-1} p_d^{N/2+1} + \ldots + \binom{N}{0} p_d^N} R_{t+j} \\
&= \Omega_d R_{t+j}.
\end{aligned}
$$

Since $d = 1/u < 1$, $\Omega_d < 1$. Note that Ω_d is a constant regardless of j. Similarly, we have

$$E_{t+j}\left[R_{t+j+T} \middle| R_{t+j+T} > R_{t+j}\right] = \Omega_u R_{t+j},$$

where $\Omega_u > 1$ is a constant. Note that

$$E\left[R_{t+j+T}\right] = \Pr\left(R_{t+j+T} \le R_{t+j}\right) \Omega_d R_{t+j} + \Pr\left(R_{t+j+T} > R_{t+j}\right) \Omega_u R_{t+j} = e^{\mu T} R_{t+j}$$

and thus

$$\Pr\left(R_{t+j+T} \le R_{t+j}\right) \Omega_d + \Pr\left(R_{t+j+T} > R_{t+j}\right) \Omega_u = e^{\mu T}.$$

Note also that since L_t^T is a linear function of R_t, $\Omega_d L_{t+j}^T = E\left[L_{t+j+T}^T \middle| R_{t+j+T} \le R_{t+j}\right]$.