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A Practical Contribution to Quantitative Accelerated Testing Of Multi-Failure Mode Products Under Multiple Stresses

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Résumé

La mise en place d'un programme de tests accélérés (AT) est accompagnée de plusieurs préoccupations et incertitudes quant à l'estimation de la fiabilité qui peut causer un écart par rapport au service réel. Cette thèse vise à présenter les outils nécessaires et auxiliaires antérieurs aux tests, ainsi qu'à proposer des approches techniques et des analyses pour la mise en œuvre de tests accélérés pour l'estimation de la fiabilité, la comparaison de produits, l'identification des modes de défaillances critiques ainsi que la vérification de l'amélioration de la fiabilité (après modification de la conception). Tout programme de tests accélérés doit faire l'objet d'une investigation économique, de même que la similitude entre tests et modes de défaillances doit être vérifiée. L'existence de variables aléatoires dans le service en utilisant le profil et le temps de défaillance dans les tests accélérés sont les causes de l'incertitude pour estimer la fiabilité qui doit être résolu numériquement. La plupart des programmes de tests de dégradation accélérés ont été mis en œuvre à des fins qualitatives et d'analyse de comparaison, de sorte que le concept de tests de dégradation accélérés doivent être étendus et généralisés au cas de produits sujets à de multiples modes de défaillance, avec ou sans modes de défaillance dépendants. Si des échantillons, neufs ou usagés, d'un produit sont disponibles, la méthode de vieillissement partielle est proposée afin de diminuer considérablement le temps de test.

Abstract

The implementation of an accelerated testing (AT) program accompanies several concerns and uncertainties to estimate reliability that might cause a deviation from real service. This thesis aims at presenting necessary and auxiliary tools prior to the test, and proposing practical methods and analyses to overcome such uncertainties for reliability estimation, comparison of products, recognition of critical failure modes, and verification of reliability growth (after design modification).

Every accelerated testing program must present a suitable accelerating variable and verify similarity of failure modes in test and service. Any information from field including service usage profile and field failure data could be useful for an accurate quantitative analysis of reliability. Furthermore, for economic considerations, test samples should be selected so that they could provide a satisfactory level of confidence. There is not a classical approach to identify the above-mentioned items, so they are investigated through available related literature in order to be classified as a reference to preliminary tools to conduct accelerated testing. Furthermore, required parameters and their measurement to transfer from an accelerated catastrophic testing to an accelerated degradation testing are introduced.

The existence of random variables in service and test are the causes of uncertainty to estimate reliability. To overcome the problem, virtual sample method as a numerical technique, is proposed to replace every probability diagram with its representative values. For multi-failure mode products, the results obtained by AT could be as complementary data for incomplete service results to assess the critical failure mode. The proximity of test and service results is also proposed to be estimated by multinomial and ranking methods.

The concept of accelerated degradation testing is extended for multi-failure mode products (either with or without dependent failure modes) under some stresses for quantitative purposes, and the system of equations has been solved for each virtual sample. The concept is employed to estimate failure times of field samples returned from service due to a degrading failure mode. If used and new samples of a product are available, partial aging method is proposed to considerably decrease the time of an unknown aging process.

Preface

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To my dear wife and son

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Chapter 1

General Introduction

1.1 Introduction

The most realistic estimation of reliability or lifetime is best achieved by field failure data, because they reflect the failures in service. For a newly designed product, these data are not available. On the other hand, field failure data for the old version of design might be invalid for the new version. The utilization of field failure data (of the old version) for the new version must be studied according to the changes that have been made to product components. In addition, reported failures after warranty period might be incomplete for reliability analysis. Such issues motivate manufacturers and testers to conduct accelerated testing to estimate lifetimes and reliability of their products.

Accelerated testing consists of a variety of test methods for shortening the life of products or hastening the degradation of their performance. The aim of such testing is to quickly obtain data which, properly modeled and analyzed, yield desired information on product life or performance under normal use ([69]). An AT deals with theoretical considerations (including mathematical and statistical backgrounds of reliability) and technical concepts of products (including their failure modes, the economic and business considerations), so the identification of these elements by the manufacturer is necessary to obtain more realistic results from the AT.

The test time must relate to its equivalent service time in order to obtain a relationship between their results. The problem is that the failure times obtained from the test are stacked in a narrow band of time in comparison to service lifetimes which are distributed in a wider band because of variety of uses by customers, so the usage of products in service must be identified, and considered in the relationship.

To estimate failure time, some samples of the product must be selected to pass the aging process. The estimation of sample size is the economic aspect of AT to fulfill an acceptable confidence level. The calculation of lifetime or reliability by testing very few samples could lead to inaccuracy of test results. On the other hand, employing a lot of samples for test leads to increase the cost. The estimation of sample size could be a challenging task if the product is highly reliable so the failures of its samples are not attainable. Furthermore, for multi-failure mode products, to achieve every failure mode, more samples are required to pass the test. The utilization of service samples for test (if there are any) could considerably decrease the cost of AT. The destructive nature of a test (in the AT) results in preparing more samples for testing. In addition, in design stage, there might be a cost constraint for manufacturer to produce required prototypes for testing. As a result, manufacturers must recognize different approaches to decrease the sample size and the geometrical size of test samples.

Continuous monitoring of the failure for a sample (to measure the exact failure time) during an AT might be time-consuming. As a solution, the interval monitoring is suggested by literature to capture the failure time in an interval. This brings the concern of censored time which is not eligible for an accurate statistical analysis.

If the product is expected to be highly reliable, the failures of its samples might be unattainable, so statistical analysis of lifetime might be impossible because the lack of any failure times. For degrading failure modes, all the above-mentioned problems could be more critical because the lack of any interruptions in the operation, so the manufacturer must present a clear definition of these failure modes by relating their failure times and physical properties.

Severe environmental and/or operative stresses on samples of the product might result in another failure mode(s) than the actual ones in service. On the other hand, the frequencies of failures for each failure mode in test and service must be in consistent. Otherwise, the accelerated testing will be invalid.

In a multi-failure mode product, the failure (or degradation of physical property) of one failure mode can affect the failure (or degradation of physical property) of another one. The identification of dependent failure modes in a product is a technical issue that has to be done prior to the test. Then, the failure time of the dependent failure mode must be related to the operation of the other one. This leads the manufacturer to recognize the interaction of components inside the product.

For a complex product (having many components and failure modes) under multiple stresses, the manufacturer might be unable to use statistical analysis to obtain a mathematical formula to estimate reliability, so a numerical method is needed to transfer the problem from statistical analysis to a simple mathematical calculation.

The thesis is presented in the form of a collection of scientific contributions submitted for publication. The following contributions are submitted for publication, and they are placed in chapters 2-4:

- Contribution 1: S. Hossein Mohammadian and Daoud Aït-Kadi. An Integrated Approach to Implement Accelerated Testing. Submitted in March 2009 to *Relia-*

bility Engineering and System Safety.

- Contribution 2: S. Hossein Mohammadian and Daoud Aït-Kadi. A Contribution to Accelerated Catastrophic Testing. Submitted in February 2009 to *Reliability Engineering and System Safety.*

- Contribution 3: S. Hossein Mohammadian, Daoud Aït-Kadi, and François Routhier. Quantitative Accelerated Degradation Testing: Practical Approaches. Submitted in February 2009 to *Reliability engineering and system safety.*

This chapter aims to present the motivation and problems of the thesis, to provide an overview of related literature, and to summarize the methodologies that are applied in subsequent chapters. The rest of this chapter is organized as follows. Section 2 classifies the problems of the thesis. Section 3 presents the main objectives of the thesis. Section 4 reviews related literature to accelerated testing. Section 5 describes methodologies and assumptions. Section 6 presents conclusions.

1.2 Classification of Problems

There is not a unique classical approach to identify required tools, methods, and analyses to implement an accelerated testing. In order to provide a user-friendly reference to engineering applications, all problems dealt in the thesis are categorized into three groups, and for each group, its problems are itemized for quick citation.

1.2.1 The Problems of Service-Related Data

For catastrophic failure modes, the problem concerns the estimation of failure time, censored time, acceleration factor, service reliability, and number of failures. For each used sample returned from service because of a degrading failure mode, relating its service age to a product physical property can lead to assess its lifetime which is a challenging task. The above problems are listed in detail as follows:

- Problem 1.1. There is no unique standard format for exploiting field failure data in accelerated testing, and available formats might be different from one manufacturer to another. Furthermore, the variety of data types in samples/customers' portfolios can be the source of confusion for any service estimation.

- problem 1.2. Failure times of samples obtained from an aging process are stacked in a narrow band of time in comparison to service lifetimes which are distributed in a wider band because of variety of uses by customers.
- Problem 1.3. An AF-based aging process would not be applicable in accelerated testing without the identification of acceleration factor, which should be estimated by field failure data.
- Problem 1.4. Time-based analysis of catastrophic failure modes might be insufficient to identify critical and non-relevant failure modes in a product, because the statistical analysis of incomplete data is unable to estimate the reliability.
- Problem 1.5. For a degrading failure mode, the lack of any interruptions in the operation might cause a difficulty in defining its failure time. For this case, there might be a contrast between the customers' decisions to report failures, and the manufacturer's criterion to accept or refuse the failures.
- Problem 1.6. In general, there is no known technical relationship between reliability of the old and new versions of design, so any test and service related data for the old version of design might be useless for the new version.
- Problem 1.7. For highly reliable products, the number of reported failures may be insufficient for estimating reliability. For multi-failure mode products, this problem can prevent individual analysis of every failure mode due to the shortage of reported failures, so the comparison of failure modes cannot be achieved.
- Problem 1.8. A direct estimation of usage profile for a product by service data is usually inaccessible.

1.2.2 The Problems of Test Time and Cost

Prior to carrying out any tests, the accelerated testing program must economically be studied for its time spending and cost. The problem of test samples concerns the economic aspect of accelerated testing, and it is characterized by the number of samples (sample size), sample geometry and the total number of aging processes (or the total number of conducting performance test). Accelerating variable is any arrangement to shorten test time. The application of employing these variables in AT could result in the concern of test integrity (the consistency of failure modes and their occurrences in test and service).

- Problem 2.1. Estimating reliability by testing very few samples could lead to inaccuracy. So a minimum number of samples is required to fail for statistical analysis. If the product is highly reliable, no failure is expected to be achieved. Thus, statistical analysis is impossible.

- Problem 2.2. The above problem for a multi-failure mode product is more critical because many samples are required to assure achieving every failure mode.

- Problem 2.3. In an ADT, its related performance test (to obtain the performance factor) may be destructive, and consequently costly.

- Problem 2.4. After a modification to design, there might be a cost constraint to manufacture the samples of the new version of the product for AT. There might also be a limitation in time to dispatch samples to the market.

- Problem 2.5. The utilization of new samples for testing might be expensive. As a substitution, used samples could considerably decrease test expense if their residual lifetimes could be related to the equivalent test time.

- Problem 2.6. In an ADT, if many service samples are returned from service, the implementation of the aging process and performance test on every sample might be time-consuming.

- Problem 2.7. Stimulation of failure mechanism by any modifications to the stresses and samples might raise the concern of irrelevant failure modes and test integrity.

1.2.3 Related Problems to the AT Methods

The selection of a proper accelerated testing method depends on the ability of recognition the product that could be highly or lowly reliable, and single or multi-failure mode. In addition, both catastrophic and degrading failure modes might be under study. The aging process must be defined based on the availability of field failure data and usage profile. Sample size and test samples have to be identified based on their availabilities and any limitations to test time and expense. Finally, the employed analytical method has to be verified for its ability to solve the complexity of the problem.

- Problem 3.1. Most ADTs are intended for qualitative or comparative analysis (rather than reliability analysis), so the concept of ADT needs to be developed for reliability analysis, especially for the products under multiple stresses in different levels.

- Problem 3.2. The problem of multi-failure mode products may be more complicated if any dependent failure modes are available in the product.
- Problem 3.3. Compared to ACT, the results of an ADT are more meaningful, so in order to transfer an ACT to its corresponding ADT, required tools and knowledge need to be identified.
- Problem 3.4. The results obtained from an unknown aging process cannot be used to predict product lifetime.
- Problem 3.5. For complex usage profiles, regarding the existence of many random variables (usage frequency and failure time), analytical methods are usually unable to estimate reliability.
- Problem 3.6. For catastrophic failure modes, a comparative analysis might be needed for the available products on the market, or different failure modes (in a multi-failure mode product).
- Problem 3.7. The comparison of test and service results is crucial to their conformity, especially if the time-based data of service are unavailable.
- Problem 3.8. For highly reliable products, there might be a lot of right censored times for ACT, and less degradation of physical properties for ADT, which are not sufficient for reliability analysis.

The above problems are discussed in the chapters 2-4. As a quick reference, Table 1.1 summarizes the section numbers which are related to each mentioned problem.

1.3 Objectives

The objectives of the thesis are summarized as follows:

- To present practical approaches to decrease the cost and time of AT, including any arrangement to select test samples, aging process and measurement of physical properties.
- To classify technical approaches to obtain accelerating variables, and to verify the conformity of test and service failure modes.
- To propose a numerical method to solve the complexity of random variables for both ACT and ADT.

Table 1.1: Related sections to each problem

Classification of problems		Referred section							
Catastrophic and degrading field failure data	Prob. 1.1	2.6.1	4.3.1	4.3.2					
	Prob. 1.2	2.6.1	3.3.1	3.3.2	3.5.2				
	Prob. 1.3	2.4	2.4.1	2.6.1	2.6.2	3.5.2	4.3.1	4.4.1	4.5
	Prob. 1.4	2.6.1	3.4.3						
	Prob. 1.5	2.6.2	4.3.1	4.3.2					
	Prob. 1.6	3.5.2	3.5.3						
	Prob. 1.7	2.6.1	3.4.1	4.3.1					
	Prob. 1.8	2.2.1	2.2.2	3.3.1	3.3.2				
Test time, expense, and integrity of AT	Prob. 2.1	2.4.3	2.5	2.3.1	3.5.3	4.5			
	Prob. 2.2	2.3.1	3.5.3	4.4.1					
	Prob. 2.3	2.3.1	4.2.2	4.2.3					
	Prob. 2.4	3.5.3							
	Prob. 2.5	2.5	2.6.1	2.6.2	3.3.1	4.2.2	4.3.1	4.5	
	Prob. 2.6	2.6.1	4.3.1	4.3.2	4.5				
	Prob. 2.7	2.4.1	2.4.2	2.4.3	2.7	3.4.3			
Accelerated testing methods and analyses	Prob. 3.1	2.3.1	2.3.2	4.2.1	4.2.2	4.2.3	4.4.1		
	Prob. 3.2	4.4.2							
	Prob. 3.3	2.3.1	2.3.2						
	Prob. 3.4	4.5							
	Prob. 3.5	2.2.2	3.3.1	3.3.2	4.4.1				
	Prob. 3.6	3.4.1	3.4.2						
	Prob. 3.7	3.4.3							
	Prob. 3.8	2.4.1	2.4.3	2.5	3.2	3.4.1	3.5.3	4.2.3	4.5

- For comparative analysis, related indicative factors to compare the test and service results are intended to be defined if time-based field data are not available. Furthermore, the comparison of different failure modes in a product to identify the most probable and irrelevant failure modes is also aimed.

- To state the concept of ADT under s single stress by relating failure time of the product to one of its properties, and to extend it for multi-failure mode products under multiple stresses. It is also intended to be generalized for dependent failure modes.

- To present required approaches for reliability analysis of field data for both catastrophic and degrading failure modes.

- To propose a practical approach to estimate reliability and acceleration factor of a product, while the related aging process is unknown.

1.4 Literature Review

1.4.1 The Concepts of Accelerated Testing

Theoretical aspects of accelerated testing such as probability and life-stress models, data analysis and maximum likelihood estimation have been presented and classified by Nelson [69] to provide a statistical comprehensive reference for AT. For practical purposes, there have been a few efforts to categorize general methods (e.g. [98]).

Meeker et al. [62] presented the concept of ADT as degradation of product property based on a known degradation model. The unit-less damage has been defined as the proximity of sample to its inoperable state (e.g. [38, 55, 83]). Various damage models have been presented according to their applications and products (e.g. [38, 102, 29]). The most popular model is the linear damage model which is known as Miner's rule (e.g. [89, 9, 88]). The application of damage rate which is the derivative of damage function versus time has geometrically been explained by Jung et al. [42]. Every ACT might be converted into corresponding ADT by relating the catastrophic failure mode to a suitable parameter of the product expressing its degradation. For example, Minh et al. [63] simulated the failure of concrete by measuring the size of the created crack.

A few articles have been devoted to study the complexity, difficulties and limitations of conducting an accelerated testing (e.g. [61]). Clark et al. [20] technically discussed limitations in stress levels in order to use fewer samples to implement an accelerated testing.

The application of life-stress models in accelerated testing is to extrapolate failure results at the service level of a stress from the results obtained at its high levels. Based on the stresses to be stimulated, such models are presented and classified by many papers (e.g. [69, 28, 4, 17, 92, 70]). Among the models, Arrhenius model is the most well-known and applicable one if temperature is the stress [49, 48, 3, 34]. For non-thermal stresses (especially for mechanical stresses), the inverse power law has been employed [49, 100, 107, 73, 9]. Wu et al. [100] also used this model to estimate the acceleration factor and required test time for known model parameter. The linear relationship between failure cycles (in logarithm scale) and corresponding levels of stress (S-N diagram) are estimated for materials under different mechanical stresses by fatigue test [41, 16, 22]. For sandwich beams with different failure modes, Harte et al. [36] obtained such diagrams for every failure mode in order to recognize the most likely one to happen.

Life-stress models are also formulated to relate failure times under a combination of thermal and non-thermal stresses. In order to estimate the acceleration factor of an aging process in high level of temperature and UV irradiation, Koo and Kim [49] carried out two different types of aging process. Firstly, the samples were exposed to high levels of temperature while UV irradiation was kept constant at its normal level, and the acceleration factor of the first aging process was estimated based on the Arrhenius model. The second aging process was carried out in high levels of UV irradiation while temperature was kept constant at a specified high level, and the inverse power law was employed to estimate the acceleration factor of the second aging process. Then, the total acceleration factor of the aging process in high levels of temperature and UV irradiation was estimated as multiplication of the above-mentioned acceleration factors.

1.4.2 Qualitative Accelerated Testing

Qualitative ACT

Qualitative accelerated catastrophic testing is applied to detect latent failure modes and to verify their elimination after design modification if no field data are available. Murphy and Baxter [66] assessed usage profile of water heaters including temperature, voltage and their levels. Then, accelerated testing was carried out in two stages: before modification in order to detect latent failure modes, and after modification in order to verify the positive effects of the modification to eliminate or moderate detected failure modes. If no estimation of usage profile is available, highly accelerated life testing (HALT) could be used in high levels of environmental stresses and their combinations while the product operates in normal levels of operative stresses (e.g. [1, 11]). The stress level in HALT continuously increases and decreases in each cycle, and once it reaches at the highest/lowest level, it must be kept constant over a period of time which is called dwell time (e.g. [8]).

Qualitative ADT

A qualitative accelerated degradation testing can be used for many applications. It may be applied for a simple observation of product to explain the degradation of its physical properties over the aging process (e.g. [85, 6]). The more accurate estimation of these degradations could be obtained from natural (outdoor) weathering test [35, 76]. Fekete and Lengyel [30] proposed a combination of accelerated and natural tests to study the effect of natural exposure on coatings. In order to verify the conformity of ADT results, a comparison could be made between the results of natural and accelerated degradation testing (e.g. [76, 54, 86, 47, 10, 24, 57]).

Comparative analysis is widely used in ADT to study the ability of following items to degrade physical properties:

- Stress levels of a single stress on one property (e.g. [105]) or some properties (e.g. [39, 33, 5]).
- Stresses ([12, 77, 5]).
- Environmental conditions (e.g. [30, 2]).
- Materials (e.g. [46, 30, 19, 63, 23, 81, 44]).
- A known aging process on some products [68].

1.4.3 Analysis of Service Data

The most accurate estimation of usage profile of a product (including stresses, their levels and usage frequencies) results from real service. Mohammadian et al. [64] assessed the operative hours of lathe machines in service, and made their probability diagrams in order to provide their usage frequencies. Such information had already been achieved by technicians during maintenance activities and recorded in customers' portfolios. The usage profile has also been estimated by Kim et al. [45] for a train bridge by presenting some assumptions to specify actual weights of trains which are not known due to the uncertainty in the number of passengers and their weights. Burgess et al. [14] statistically investigated the gradient of bicycle roads in Great Britain in order to be related to stress levels on bicycles.

Deflorian et al. [24] presented the (average) levels of some meteorological stresses like temperature, radiation and humidity for each month of the year. The probability of stress levels has also been expressed over a year [13] for each stress.

In general, stress level is a random variable, and it can randomly change within an interval. In order to overcome such difficulties and uncertainty, Tang and Zhao [91] and Palin-Luc et al. [56] suggested a method to simply simulate a wide range of a random variable by considering the maximum level of stress during its operation. This modification removes unimportant levels of the stress to shorten test time.

If none of the above approaches is available, a preliminary test could be used to estimate usage profile. Wu et al. [100] carried out preliminary tests on various vehicles including helicopters and ground vehicles to assess transmitted levels of vibration to the interior of the vehicles.

Field failure data for catastrophic failure modes and the measurement of physical properties of used samples returned from service due to a degrading failure mode, are two sources of service data for reliability estimation [84]. Field failure data are compared with corresponding test results in order to estimate the acceleration factor of the aging process (e.g. [74]). In the literature, field failure data are usually employed for time-based analysis of reliability, maintenance and availability (e.g. [67, 21]). They are also used for number-based purposes such as comparison of failure modes (e.g. [96, 103, 97, 104]), failure mechanisms and components. Used samples can also be analyzed for root cause analysis in order to obtain the main causes of failures (e.g. [82]).

For complex CNC machines, Guangwen et al. [109] counted the number of failures for each component to obtain its relative failure probability (occurrence probability). Such information could also result from known failure modes and effects analysis (FMEA) for different failure modes (e.g. [60, 68, 37, 93, 95, 75]).

After-warranty field data are usually incomplete. Oh and Bai [72] modified Likelihood function in order to be applicable for analyzing field failure data of warranty and after-warranty periods. For complex machines, the desire of customers to report failures near the end of the warranty period could cause a deviation from actual time of failure, so the accuracy of statistical analysis may be in doubt (e.g. [78]).

For a degrading failure mode, Marahleh et al. [58] employed the linear damage model for relating the failure time of a used sample to its physical property. Degradation diagram of the failure mode could result in a correlation between the times of service and test while both have same effect on degradation [33]. Drew et al. [26] took some specimens from field samples in service in order to confirm their acceptable level of quality. If many properties are available to be investigated, Cox approach could be used to analyze and compare the effects of the aging process on required properties (e.g. [50]).

1.4.4 Auxiliary Tools

For a newly designed product, the finite element method (FEM) could be used to predict the locations of hot spots which are some local elements under high mechanical stress. The hot spot(s) are used to verify and validate the failure location obtained by accelerated testing (e.g. [89, 94, 99]). The ability of software to perform dynamic analysis of stress/strain could yield the distribution of stress level in hot spots (obtained by static analysis) over time in order to assess maximum stress and the most crucial hot spot (e.g. [43, 16]). If life-stress material diagram is available, an estimation of failure time could be made (e.g. [89]). The comparison of the results obtained by individual finite element analysis for all stresses could help recognize the most suitable stress to degrade samples in shorter time (e.g. [101]).

In general, international standards do not present a whole AT program. Depending on the required application and product in ADT, the standards might suggest the most suitable property to explain degradation (e.g. [5, 10]) and corresponding test to measure its values over the aging process (e.g. [5, 105]). They could also be used to identify normal and accelerated levels of environmental stresses like temperature and radiation, to identify usage profile, and to suggest the required accelerated testing (e.g. [5, 105, 10]).

In the literature, creating a deliberate defect in samples prior to the aging process has been used to stimulate their failure mechanism and to shorten test time. Such defects must address the failure mode under study. Dommarco et al. [25] studied the effects of different defects on a component for fatigue test of materials, and concluded that a combination of the defects could be applied in order to considerably shorten test time. For the quantitative purposes, the fatigue lifetimes of samples with and without initial damage were correlated by Burman and Zenkert [15]. Mechanical defects could also be analyzed by finite element analysis (e.g. [43]). If the lifetime of a product is potentially affected by many factors (such as design parameters and stresses), huge number of tests and samples are required in order to study the effect of each factor on failure time. In the literature, orthogonal arrays (e.g. [52, 51, 80]), design of experiments (e.g. [87]), and Latin square design method (e.g. [90]) have been used to specify a minimum number of tests required to achieve realistic and meaningful results. Then, such results should be analyzed by analysis of variance (ANoVA) to study the effect of the factors and their combinations on the product life.

For a problem including random variables, analytical approaches could present the most accurate results. Analytical methods are usually unable to solve the complexity introduced by many random variables, so such problems should preferably be solved by numerical analysis. For discrete variables, all possible scenarios must be considered. Nowak and Cho [71] and Choi et al.[18] identified every possible different failure scenario for bridges.

Reliability results presented by manufacturer can be useless for a retail user regarding its special application. Accordingly, an AT should be done by the retail user in order to achieve the required results for the application, so the physical property defined by the manufacturer might need to be modified to conform to the real application. Mohammadian et al. [65] modified the definition of battery capacity (by manufacturers) for valve regulated lead acid (VRLA) batteries according to their applications in motorized wheelchairs.

The idea of defining a minimum level of the physical property (critical level) being in the operable state has some applications in ADT. It could enable testers to recognize the effects of different stresses to degrade and fail samples (e.g. [7]). Such definition has also been applied by many others to estimate failure time (e.g. [49, 35, 108, 27]). For this purpose, the critical level might be extrapolated [49] or interpolated [32] from its measured values during the aging process.

For a complex product, its failure could be defined based on the degradation and failure of its components, the connection between its components, and the number of replacement. For series, parallel, standby and k-out-of-n products, such definition could be easily derived, whereas for other products, an exclusive definition is required. Fitzgerald et al. [31] defined the failure of a wheelchair based on the criticality of each component and the number of times it has been repaired or replaced. The failure of such products could technically be explained by chain of events [40].

1.5 Methodologies and Assumptions

In this thesis, failure modes are assumed to be independent, otherwise it will be mentioned. Every single-failure mode product is considered as binary-state with operable and inoperable states. At the beginning of its operation, it is considered to be in the operable state. There is no maintenance activity, i.e. once the state of the product changes from operable to inoperable, it cannot return to its previous state.

First Contribution

- Usage profile of a product is classified based on the number of levels for each stress, and it is identified by usage frequency of each level.
- An exclusive definition of performance factor is presented to relate physical properties of samples to their failure times, and it is measured by performance test. The values of performance factor decrease over the aging process.
- Different possible performance tests are classified based on related literature.
- Performance factor could be used to transfer a product from multi-state to binary-state and vice versa. For more information about reliability analysis of multi-state products refer to [59, 79, 106, 53].
- Failure mechanism is proposed to be stimulated by any changes in stresses, performance test and samples.
- Sample size is optimized by probability diagrams and confidence level. The physical concepts of products are suggested to be applied to transfer a destructive performance test to non-destructive one.
- Reliability for a product with a catastrophic failure mode is proposed to be analyzed by time-based and number-based data.
- To verify the conformity of the results obtained by accelerated testing and service, following approaches are proposed:
  - Considering usage profile.
  - Investigating the technical failure process.
  - Comparing test and service results.
  - Analysis of mechanical stress/strain.

Second Contribution

- Accelerated catastrophic testing is categorized based on the inspection strategy during the aging process.
- Virtual sample method has been presented for numerical analysis based on non-statistical reliability.
- For a single-failure mode product, test results are modified by related service results.

- Following indicative factors are defined to achieve obvious comparisons of time-based analysis:
  - Salvation factor to compare different products based on their reliabilities.
  - Longevity factor to compare different products based on their failure times.
  - Priority factor of each failure mode based on the first failure occurrence.
- For a multi-failure mode product, the following factors are defined to compare the number of failures in the test, and service occurrence probability for each failure mode based on the number-based data:
  - Conformity factor by multinomial distribution function.
  - Vicinity factor by ranking methods.
- In order to compare different products which might have high reliability, their allocated reliability and confidence levels are used.
- Acceleration factor is assumed to be a function of aging process (neither product nor design version), so a relationship is made based on the test and service results of old and new versions of design.

Third Contribution

- Performance factor is defined:
  - To relate physical property to failure time and lifetime.
  - To characterize an ADT.
  - To transfer an ACT to its corresponding ADT.
- ADT is categorized based on the number of performance test on each sample during the aging process.
- The performance factors of field samples are used to estimate service reliability.
- Regression analysis is used to interpolate and extrapolate performance factor.
- Critical performance is used for quantitative comparison of manufacturer and users' criteria to report failures for degrading failure modes.
- The unit-less damage (factor) is defined:
  - To specify the proximity of a sample to its inoperable state.

  - To show degradation diagrams of all physical properties (of a sample) in a unique coordinate system.

- The gradient of the damage factor is assumed to be a function of the damage factor (not time).

- The superposition principle is employed to study the effects of different stresses on failure modes either for independent or dependent failure modes.

- Virtual sample method has been presented for numerical analysis of non-statistical reliability.

- Partial aging method has been proposed to considerably decrease test time in an unknown aging process.

1.6 Conclusion

This chapter presented an overview of available problems of the thesis in order to categorize them for a user-friendly citation. Based on these problems and the thesis motivations, the objectives of the thesis are listed. Several articles have been overviewed, and the methodologies of each contribution are also listed.

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Chapter 2

An Integrated Approach to Implement Accelerated Testing

S. Hossein Mohammadian and Daoud Aït-Kadi

Abstract

The primary steps in conducting an accelerated testing are accompanied by several concerns about the economic considerations of time and cost and test validation. In this paper, a systematic study of related literature has been carried out to propose a comprehensive reference of the required theoretical and practical arrangements prior to the test. Usage profile of products in service is classified according to stress levels. In order to relate physical properties of products to their failure times, an exclusive definition of performance factor is presented, and a contribution to practical approaches to its measurement (performance test) is made. The concerns of applying accelerating variables as the main approach to shorten test time are explained, and the activities required to verify relevant failure modes are discussed. The sample size problem is studied based on the statistical considerations, confidence level, and destructive nature of performance test. Several formats of field failure data during the warranty period for catastrophic and degrading failure modes are analyzed, and unreported failures for after-warranty period are also discussed.

Key words: Accelerated testing; Accelerating variable; Test sample; Performance; Field failure data

2.1 Introduction

The implementation of an accelerated testing (AT) might be accompanied by several uncertainties, concerns and limitations about test time, cost and test validation that should be recognized prior to the test. For economic purposes of time and cost, the AT must include a suitable accelerating variable to shorten the test time and an estimation of sample size. In addition, relating physical properties of the product to its failure time could present more meaningful results about its degradation process. If field failure data are available, they could be used for test validation. Furthermore, the failure modes obtained from the test and the actual failure modes in service must be the same.

Accelerated testing is a practical/theoretical approach. There is huge number of literature dealing with statistical aspects of accelerated testing. Among them, Nelson [48] has been the pioneer to provide a comprehensive statistical reference. He also discussed several issues regarding test plans and listed a number of references to address related statistical problems [49]. The main concepts of accelerated testing have also been discussed by some literature to be applicable in real world testing (e.g. [8]). Life-stress models have been developed by many researchers (e.g. [18, 11]) in order to extrapolate service lifetime from the test results of AT in high levels of stresses.

For engineering purposes, a few efforts have been made to categorize AT methods (e.g. [19, 31]). A practical guide has been presented by Meeker and Escobar [43] to discuss the limitations and difficulties to conduct an accelerate testing. Lu et al. [39] discussed accelerated stress testing (AST) which is used to quickly fail products in order to detect failures.

The development in theoretical/statistical concepts of AT and the increasing number of practical applications in industry motivated us to propose an integrated approach in order to identify required activities and knowledge prior to implement any tests. Different classes of service usage profile including unique, multi, and continuous levels are introduced. Performance and damage are defined as the key factors to relate physics of the product to its failure time. The main causes of failure mode deviation are also discussed. Different formats of field failure data in the warranty period and afterwards are presented for catastrophic failure modes. For degrading failure modes, performance factor is used to estimate lifetimes of field samples returned from service. Sample size as the economic aspect of AT is proposed to be estimated by statistical analysis and confidence level.

The remainder of the paper is organized as follows. Section 2 presents different classes of usage profile in service. Section 3 introduces the concept of accelerated degradation testing. Section 4 describes accelerating variables to shorten test time. Section 5 discusses theoretical and practical approaches to estimate sample size. Section 6 deals with different formats of field failure data and their reliability analysis. Section 7 introduces technical aspects to verify relevant failure modes. Section 8 presents conclusions.

2.2 The Estimation of Usage Profile

The majority of failures for mechanical components are reported from service-related causes, mostly because of stresses [14]. Stresses are defined as operative (physical, chemical, electrical, etc.) and environmental forces applied on samples in service. Each stress is identified by its specifications, which are the level and usage frequency. The usage frequency refers to the ratio of the time in which samples of the product are under the stress to the total time in service, so its value is always less or equal than one. For example, the usage frequency of $q = 0.25$ means that the sample is under the stress for $(24)(0.25) = 6$ hours per day. Usage profile of a product in service is defined as the integration of stresses, their levels and usage frequencies.

The stress level and usage frequency are random variables and their values in service are uncertain. An assessment of the usage profile is necessary in order to select a proper usage profile for AT. In general, the estimation of usage profile for operative stresses is a complicated problem, but for many products under mechanical stresses, it could be assessed by a preliminary test for typical users (e.g. [10, 64]) or by theoretical concept of mechanical stresses or finite element method (e.g. [30]).

Low levels of stresses might be unable to activate the failure mechanism, so in order to compress time in the aging process; they should be eliminated from the usage profile. Luc et al. [40] and Tang and Zhao [59] proposed a method to select the maximum level of the stress and remove every other level from random cyclic profiles.

The specifications of environmental stresses such as temperature and UV irradiation may be assessed by available geographical data for the specified location over a period of time (e.g. [13]). In addition, international standards have been used in the literature to derive such information for their own laboratory tests (e.g. [4, 70, 7]).

2.2.1 Usage Profile Classes

For each stress, usage profile is categorized into three classes in order to be applicable to accelerated testing:

Unique Level Usage Profile

Each sample in the unique level class is considered under a constant level of the stress for its whole service life. This level might be different from sample to sample, but it must be constant for every sample (e.g. [36, 7, 71]).

Multi Level Usage Profile

The sample in a multi level class is considered under repeated cycles of the stress, so that each cycle includes different levels with different duration of time (according to the usage frequency of each level) in the cycle. This estimation has been used by Wu et al. [64] to define a cycle for a helicopter including different levels of vibration during several flying conditions such as warm-up, take-off, landing etc. The total effect of all stress levels on a sample in this class should be obtained from the superposition principle. Kim et al. [33] expressed the failure of a component under different stress levels by linear damage model (Miner's rule).

Continuous Level Usage Profile

The level of stress in a continuous level class repeatedly varies between a minimum and a maximum level (e.g. [10]). Multi and continuous levels are called cyclic levels, and their life unit could be expressed by cycle. The diagrams shown in Fig. 2.1 demonstrate the three mentioned classes to assess service usage profile.

2.2.2 Representative Values of a Probability Diagram

If many stresses and their levels are available in the usage profile, analytical methods are unable to present a definite solution, so numerical methods should be applied. Assume

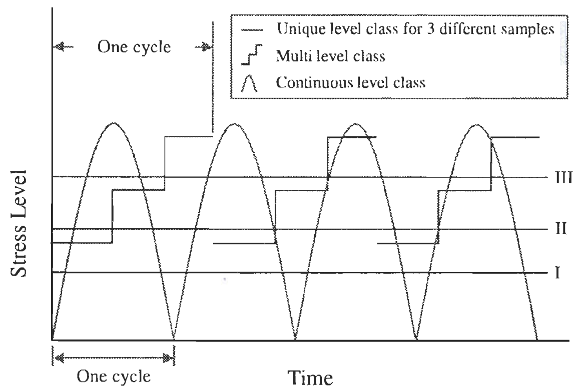


Figure 2.1: Unique, multi and continuous level classes of usage profile

that the probability diagram of the usage frequency for every stress level has already been identified. For numerical analysis, every diagram must be replaced with some representative values of the usage frequency as follows. If N values are required, the problem is to identify a set of N usage frequency values to be a reasonable representative of the diagram. Here, the approach is to split the cumulative interval $[0, 1]$ into N subintervals each has d_i ($i = 1, 2, ..., N$) length, so that from each subinterval, one and only one value must be taken as shown in Fig. 2.2. According to the multinomial distribution (2.13), the probability of this case is obtained as

$$Pr(N_1 = 1, N_2 = 1, ..., N_N = 1) = N!(1 - \sum_{i=1}^{N-1} d_i) \prod_{i=1}^{N-1} d_i$$

The maximum amount of Pr has to be estimated by solving the system of partial derivative equations as $\partial p/\partial d_i = 0$; $i = 1, 2, ..., N-1$. The system of equations simply results $d_1 = d_2 = ... = d_N = 1/N$. Then the middle points in the subintervals should be selected, so their corresponding usage frequency values are obtained from the cdf diagram as representative values.

2.3 Performance and Damage

Consider a product with a single degrading failure mode exposed to some stresses so that they can potentially degrade the product quality over time. In order to reveal and track such degradation, it should be characterized by a suitable property. In this paper, performance (factor) is defined as a suitable physical property that can be related to the failure mode. Performance must be a reasonable representative of the product degradation in long-term operation. It must also be measurable and sensitive

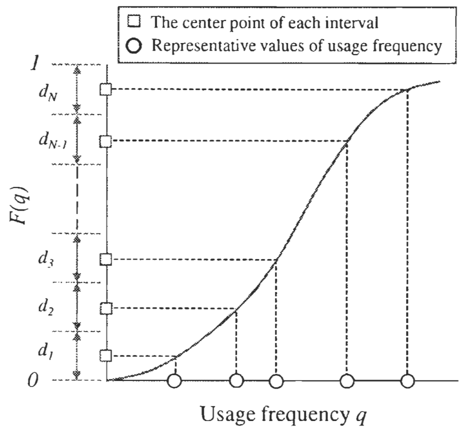


Figure 2.2: Obtaining N representative values of usage frequency

to the test time. Selecting the most suitable property as the performance factor is a challenging task which depends on the failure mode, meaningful results, test time and sample size. For each degrading failure mode in a multi-failure mode product, an exclusive performance factor must be selected. The performance diagram (performance versus time) must be a descending diagram over time. The performance factor might also be suggested by available international standards (e.g. [7, 4]).

The nominal performance P^N (of a performance factor) is defined as the target amount of the performance at the beginning of its operation in service, and it is used to define damage (factor) $D(t)$ in terms of performance $P(t)$ as [57, 38, 27]

$$D(t) = \frac{P^N - P(t)}{P^N - P^C} \tag{2.1}$$

where P^C is called critical performance which is defined as the minimum amount of performance being in the operable state. Damage factor is unit-less, and simply expresses how far the performance factor is from its nominal performance. It also specifies how close the performance factor is to the inoperable state. According to the above formula, the failure occurs if $D(t) = 1$. Afterward, the amount of damage tends to increase over 1. The damage diagram (damage versus time) is an ascending diagram over time.

The damage factor of a product is defined according to the damage factors of its physical properties, and the connection between the failure modes (e.g. [69]), which may be series, parallel, stand-by, etc. The diagrams in Fig. 2.3 show damage factors for series, parallel, stand-by and k-out-of-n products. For example, for series product, its damage factor is defined as its maximum damage factor, whereas for a parallel product, it's defined as its minimum damage factor. Once the damage factor of the product is identified, the failure time of the product could be defined at the time its damage reaches 1. For a complex product, the definition of its damage factor is not as simple as the above-mentioned products, and it may depend on many parameters like safety, criticality of each failure mode obtained from the failure modes and effects analysis (e.g. [26]) and the number of maintenance actions (e.g. [21]).

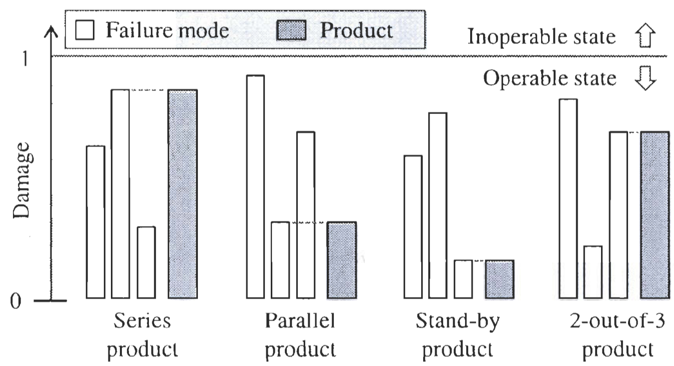


Figure 2.3: The definition of damage factor for series, parallel, standby, and k-out-of-n products according to their failure modes

2.3.1 Performance Test

Performance test (PT) aims to measure the amounts of performance factor for samples of the product during the aging process. Each performance factor must be related to its corresponding performance test. Regarding the nature of the test and required number of samples, this test is categorized into non-destructive and destructive tests. A sample under a non-destructive performance test can continue the rest of the aging process as shown in the left diagram of Fig. 2.4. The sample exposed to a destructive performance test is no longer acceptable to continue the aging process because its destructive nature causes a considerable defect in the sample, so the main conclusion of this test is to prepare more samples in parallel to pass the aging process as illustrated in the right diagram of the Fig. 2.4. Performance test is widely used by manufacturers and testers in different applications in industry. For some environmental stresses, international standards are used to identify the required performance test (e.g. [4, 70]).

Accelerated catastrophic testing (ACT) is a kind of AT on samples of a product with catastrophic failure modes, and aims at estimating failure times (or censored times) of the samples. The aging process is the only test on samples in an ACT. On the other hand, accelerated degradation testing (ADT) is a kind of AT on samples of a product with degrading failure modes, and aims at estimating their performance factors during the aging process. The samples under an ADT are expected to pass both the aging

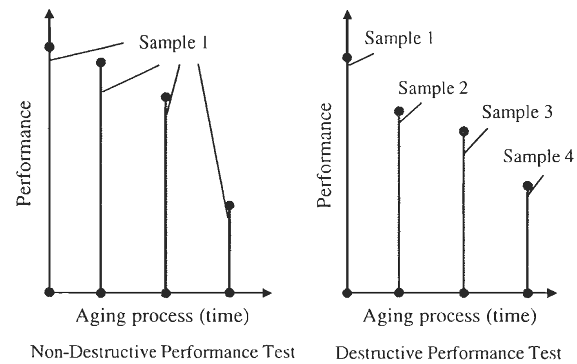


Figure 2.4: The samples required for non-destructive and destructive performance tests

process (main accelerated testing) and the performance test. Following approaches are employed to carry out performance test:

Measurement and Observation

The measurement and observation of most physical properties like physical volume and pressure [7], flow rate of an oil pump [22], steel remaining weight due to corrosion and wear [4, 22], remaining diameter due to the corrosion [4], non-corroded area of coatings [20], continuous difference in weight due to immersing in alkaline solution and water [32], transmittance of sunlight [52], gloss of coatings [9, 24], and capacity of batteries [5] are considered as the performance factors whose performance tests are non-destructive.

Ultimate Level of an Operative Stress

The ultimate strength is the static level of an operative stress in which catastrophic failure occurs. This level may be obtained by gradual or step increase in the stress level. The tensile strength test (e.g. [36, 70, 28, 32]) as the most popular mechanical destructive performance test, causes failing of samples by exposing them in high level of mechanical tensile force. In this case, the tensile force is considered as the performance factor. Tensile strength test might also be used to estimate elongation to failure as the performance factor (e.g. [4, 55]). A similar test may be performed by other types of mechanical forces (e.g. shear force [34]) or moments (bending and torsion).

Severe Environmental Stress Levels

In normal operative stresses, increasing the level of environmental stress can stimulate the failure mechanism. As the most popular approach, creep test (e.g. [42, 16]) subjects samples in relatively high temperature in constant mechanical stress. Note that in this approach, the performance is defined as the time required to fracture.

Fatigue Test

Fatigue test is usually used as the aging process in AT by increasing usage frequency of the stress in test. It could also be used as performance test by the following approaches:

- Exposing samples to the normal level of the stress in high usage frequency, like usual fatigue test.
- Exposing samples to high levels of the stress in high usage frequency, like low cycle fatigue testing (e.g. [4]).

For both approaches, performance factor is defined as the time (or cycles) to failure.

Shock and Impact

Any mechanical shock and impact could be used to obtain the robustness of samples. This approach is the quickest way to fail samples, and the number of shocks to failure should be considered as the performance factor. A study is needed to verify the relevancy of the failure mode obtained from shock and the expected failure in service.

2.3.2 Damage Diagram

There exists huge number of articles dealing with performance factors of products in test and in service to obtain their diagrams over time. For each performance factor, once its critical level is identified (e.g. [36, 24, 5]), the performance diagram could be easily converted to its corresponding damage diagram by the Eq. 2.1 and vice versa, as shown in Fig. 2.5. On the other hand, the diagrams of some physical properties are ascending (over time) like wear [12] and crack width [44]. Such diagrams could be converted to damage diagrams by making them unit-less.

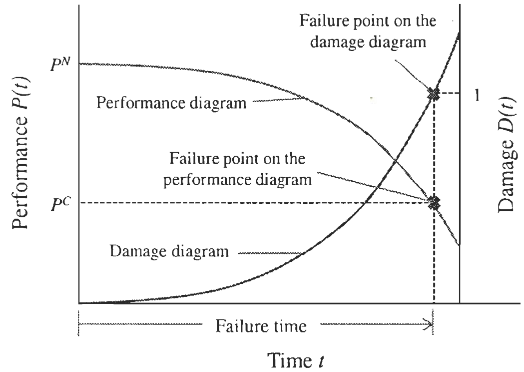


Figure 2.5: The comparison between damage and performance diagrams

2.4 Accelerating Variables

Accelerating variable is defined as any arrangements to shorten total test time of samples in accelerated testing. For this purpose, test conditions have to deviate from normal conditions in service. The reduction in time might be met by modifying stresses, performance test, physical characteristics of samples and any other parameters that can potentially speed up the degradation mechanism of the product.

The acceleration factor AF is defined as the ratio of service failure time (situation 1) to the failure time of the aging process (situation 2). Acceleration factor is a specification of the related aging process, i.e. its value depends on the accelerating variable. For an ACT, if test and field cdf diagrams are available, the value of AF could be estimated by comparing the service and testing times at a given reliability level as shown in the upper diagram of Fig. 2.6 in logarithm scales, and expressed as

$$AF = \frac{(\beta_{10})_1}{(\beta_{10})_2} = \frac{(\beta_{50})_1}{(\beta_{50})_2} \tag{2.2}$$

where β_{10} and β_{50} are the corresponding times for reliability levels of 90% and 50% respectively. Note that in the above formula, test and service diagrams are considered parallel.

For an ADT, acceleration factor is defined as the time to reach a certain level of damage ($D = 1$ or $D < 1$) if related damage diagrams are available. The times β_{10}

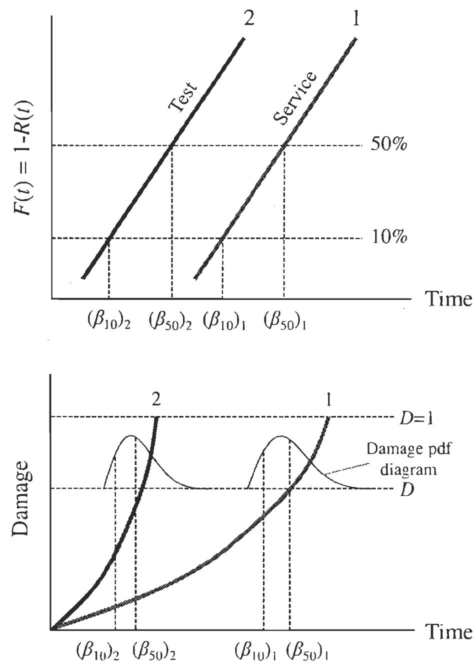


Figure 2.6: Service and test cdf diagrams for an ACT, and their damage diagrams for an ADT

and β_{50} in the damage pdf distribution functions located in the lower diagram of the Fig. 2.6 indicate that up to these times, only 10% and 50% of samples can reach the damage level of D, while damage levels of other samples are less than D. According to the stated definition of β_{10} and β_{50} for ADT, the acceleration factor is also defined by the Eq. 2.2. Note that, acceleration factor depends on both the level of probability (reliability) and the level of damage for the ADT.

2.4.1 Stress Accelerating Variables

The concept of this approach is based on shortening test time by increasing the usage frequency and/or the level of the stress for known usage profiles. If the usage frequency is considered as the accelerating variable while the stress level is kept constant at the normal level (like most fatigue tests), the corresponding aging process is called q-based aging process.

The acceleration factor of the stress level i (AF_i) is the ratio of the test usage frequency $q_{t,i}$ to the service usage frequency $q_{s,i}$ for this stress level:

$$AF_i = \frac{q_{t,i}}{q_{s,i}} \tag{2.3}$$

The problem of estimating acceleration factors of different levels of a stress is formulated by many literature in terms of stresses (e.g. [18, 11]). These formulations are called life-stress models. In engineering applications, many articles have focused on the Arrhenius model (as the most well-known life-stress model) when temperature is the stress (e.g. [36, 35, 2]) and the inverse power relationship for non-thermal stresses (e.g. [64, 71]). In addition, for most materials under mechanical stresses, life-stress diagrams are obtained from fatigue test.

Consider a single-failure mode product under K levels of a stress. The service damage factor of the product under the stress level i, i.e. $D_i(t)$, under the unique level usage profile is obtained from the linear cumulative damage model known as the Miner's law (e.g. [10])

$$D_i(t) = \frac{q_{s,i}}{q_{t,i}} \frac{t}{t_i} \tag{2.4}$$

where t_i is the failure time of the product under the stress level i with the test usage frequency $q_{t,i}$. The service damage factor of the product under multi level usage profile is also obtained from the superposition principle by the Miner's law (e.g. [10]) and the Eq. 2.4:

$$D(t) = \sum_{i=1}^{K} \frac{q_{s,i}}{q_{t,i}} \frac{t}{t_i} \tag{2.5}$$

For more information about cumulative damage models, the reader can refer to [66, 23].

For the multi level usage profiles, there are some different stress levels in the usage profile, so the AT must be conducted for each level individually. To avoid this, different stress levels must be replaced with an equivalent stress level. Then, the AT can only be conducted for the equivalent level. Then, the failure times of other stress levels must be estimated by the failure times of the equivalent stress level. An equivalent stress level with the usage frequency of $q_{s,e} = q_{s,1} + q_{s,2} + ... + q_{s,K}$ must be selected to acquire the same damage at any time (2.5)

$$\frac{q_{s,e}}{q_{t,e}} \frac{t}{t_e} = \sum_{i=1}^{K} \frac{q_{s,i}}{q_{t,i}} \frac{t}{t_i} \tag{2.6}$$

Consequently, t_e as the failure time of the equivalent stress level in test is estimated according to the above equation. Then, by knowing the life-stress model, the related stress level e (to the failure time t_e) can be estimated (e.g. [64]), and the problem is transferred to a unique AT under the equivalent stress level.

The possibility of substitution a new stress in an accelerated testing in order to speed up the failure mechanism might considerably decrease the time of testing. For this reason, a finite element analysis could be performed to compare stress distributions and hot spots of both stresses. Xie [65] studied the possibility of applying mechanical stresses instead of thermal cycling (which takes long time) in a creep test.

In order to prepare more severe conditions by stresses, samples might be exposed to step-by-step increasing stress level, each level once per sample whole life. The qualitative purpose of the step-by-step stress level in accelerated testing is to quickly fail samples to detect latent failure modes of the product. Murphy and Baxter [46] performed this approach by combining different levels of ambient air conditions and voltage to identify potential failure modes in a heat pump water heater. The stress level might also be increased continuously to detect latent failure modes as used in highly accelerated life testing (HALT) [1].

2.4.2 Performance Test Variables

Total test time of an accelerated degradation testing is the summation of the aging process, the performance test and the allocated time for installation and preparation for both tests. Selection of the optimum time intervals for performance test could considerably decrease the total test time. The suitable time intervals in an ADT must be obtained from the known performance models (linear, logarithm, exponential etc., which could be estimated by either field data or literature). Constant time intervals should be applied for the linear form of performance diagrams, whereas descending and ascending time intervals are more suitable for the downward and upward curvatures respectively as illustrated in Fig. 2.7.

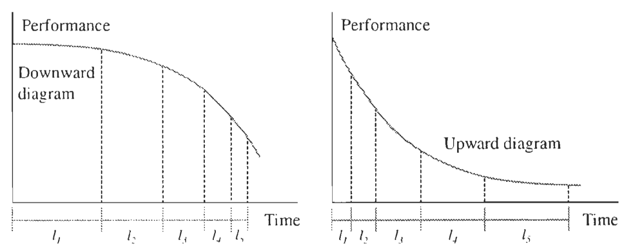


Figure 2.7: Descending and ascending time intervals for downward and upward performance diagrams respectively

Selection of a quick performance test can decreases the allocated time for whole test time. For example, Marahleh et al. [42] used the stress rupture test (SRT) instead of usual creep test to fail samples in shorter time.

The number of samples under the test, available instruments for the aging process (chambers), the number of equipment for the performance test, time intervals, and the duration of performance test must be considered in an ADT to draw a timetable for testing samples of the product as illustrated on Table 2.1 for four samples (A,B,C and D), three chambers for the aging process, one equipment for the performance test, and the ratio 1/3 for the duration of the performance test to the time of the aging process, for constant time intervals. To clarify the table, the consecutive tests (aging process in the chambers and PT) on the sample A have been highlighted.

Table 2.1: Time table for 4 samples (A,B,C and D), 3 chambers (for aging process) and one equipment (for performance test)

	Time interval	1	2	3	4	5	6	7	8	9	10	11	12	13	...
Chambers	1	A	A	A	D	D	D	C	C	C	B	B	B	A	...
	2		B	B	B	A	A	A	D	D	D	C	C	C	...
	3			C	C	C	B	B	B	A	A	A	D	D	...
	PT	B	C	D	A	B	C	D	A	B	C	D	A	B	...

2.4.3 Sample-Related Variables

From the geometrical point of view, smaller samples could stimulate failure mechanism under a common stress level. The failure time of the original product should be extrapolated from the failure times of the smaller samples (small in geometry) as shown in Fig. 2.8 by regression and extrapolation analysis. There are not classical mathematical formulas to relate the lifetime with the geometrical sizes of samples, but if the life-stress diagram of the material is available, the acceleration factor may be obtained from finite element analysis.

In literature, small specimens are used to result in rapid degradation (e.g. [4]), and changing the geometry of samples could enable a rapid exposure and consequently a quick aging (e.g. [34]). Any other approaches which indirectly stimulate the aging of samples could be considered as the artificial stimuli. For example, to meet catastrophic failure modes, an intentional small defect could accelerate the propagation process of crack (e.g. [15]). The form of the test equipment could also stimulate degradation mechanism (e.g. [12]). Adding foreign materials might help rapid aging, such as adding talc in the lubricant oil that leads to spoil lubrication [22].

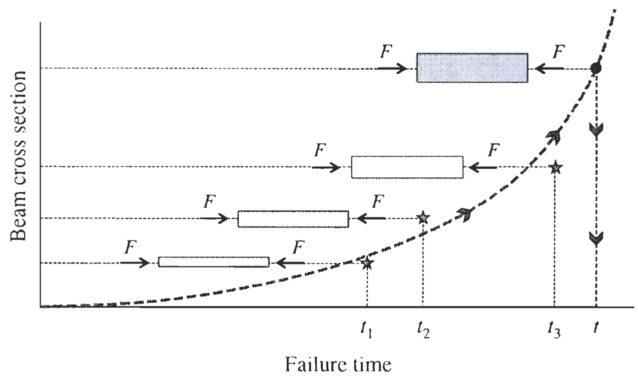


Figure 2.8: Life-cross section diagram for a beam under oscillatory compressive mechanical force

2.5 Test Samples

In accelerated testing, the problem of sample size refers to allocation a number of samples that have to pass the whole or some parts of the aging process and performance test. In fact, the problem concerns the economic view of accelerated testing to decrease the quantity of samples. The aim of exposing more than one sample under a common test conditions is to decrease the dependency of required results to only one sample.

If the main aims of an accelerated testing are to detect latent failure modes, and to validate their elimination from the product (like highly accelerated life testing-HALT), no quantitative analysis is needed, and the number of required samples depends on their availability (e.g. [46]).

Statistical Sample Size

A minimum number of failed samples is needed to enable statistical analysis of reliability. For probability diagrams, this number depends on the number of unknown parameters of the required function. For example, for a two parameter weibull function, there must be at least two failed samples to obtain its scale and shape parameters (e.g. [71]), whereas for the exponential life distribution function, at least one failed sample is required to obtain its only unknown parameter. The more samples to test, the more accurate the reliability could be estimated.

Minimum Confidence Level

The level of confidence has a direct relationship with the sample size, so that the minimal confidence level could be achieved by only one sample. In reliability analysis, sample size (S) to fulfill the allocated reliability (R) with the minimum requested confidence level (C) is estimated by the binomial distribution function in an inequality form as [62]

$$1 - C \geq \sum_{x=0}^{S_f} \binom{S}{x} R^{S-x}(1-R)^x \tag{2.7}$$

where S_f is the allowable number of failed samples. For zero failure test plan ($S_f = 0$), sample size is obtained as (see [68, 17])

$$S \geq \frac{ln(1-C)}{ln(R)} \tag{2.8}$$

To achieve a level of confidence in the test, the problem of validating a specified level of reliability, sample size and test time becomes a challenging task. If there are few samples available for testing (sample limitation), the time of testing must be extended (e.g. [51]). As a result, reliability should be decreased. If related probability diagram is available, this level of reliability could be estimated. Any limitations of test time (e.g. dispatching to the market) could result in increasing the number of samples and level of reliability. Such issues are shown in Fig. 2.9 which is drawn in logarithm scale. Note that both approaches must address the same confidence level.

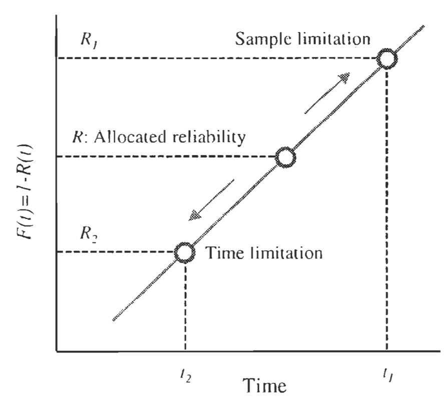


Figure 2.9: The estimation of reliability for time and sample limitations by cdf diagram

Consider a multi-failure mode product under an accelerated testing (at product level), and each component is expected to acquire a special level of reliability and confidence. In order to fulfill these requirements, the number of required samples should be calculated according to the Eq. 2.8 for each component. Because the test is at the product level, the ultimate number of samples required for testing should be the maximum number among the sample sizes obtained for the components. The more economic approach is to individually test every component (at component level) by exposing its samples to the same conditions inside the product.

Comparative Analysis

In accelerated testing, in order to compare different aspects in the product, more samples are required. Such comparison might be performed to study the effects of the stresses [36], their combinations [9] and their stress levels [36, 70, 71] on degradation of physical properties in an ADT. This comparison could also be carried out for different versions of design [47], materials [36, 12], products [21], manufacturing processes, and sample preparation [34] in order to recognize the most robust product. If the test has to be conducted in different levels of several stresses, orthogonal arrays (OA) could effectively decrease the number of the tests and sample size. Then, the analysis of variance (ANOVA) is used to study the effect of each stress on failure time (e.g. [37, 54]).

Sample Size for Performance Test

In ADT, every sample has to pass a series of aging process and performance test. If the corresponding performance test of the ADT is destructive, more samples must be prepared in parallel to continue the aging process. For multi-failure mode products, a number of non-destructive performance tests could be conducted on one sample to obtain its performance factors (e.g. [22]). In each case, if one destructive Performance test is available among them, it must be carried out as the last test (e.g. [4]).

A performance factor measured by a destructive performance test could be replaced with another property measured by a non-destructive test in order to avoid the failures of the test samples. For example, consider a rod with instant radius of $r(t)$ and known ultimate strength of S_u as illustrated in Fig. 2.10. The performance factor of the rod is defined as the ultimate tensile force measured by a destructive test. The rod is under corrosion, and its radius is assumed to be decreased over time in all directions. The ultimate tensile force could also be expressed as a function of the radius of the rod ($P(t) = \pi r^2(t) S_u$) which is obtained from a simple measurement.

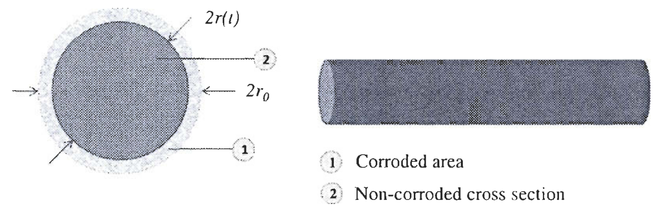


Figure 2.10: Corroded and non-corroded area of a beam, and the measurement of its diameters before and after corrosion

Economization In Materials

If the purpose of an accelerated testing on a component is to study the degradation of its material properties over test time, the economic way is to conduct the test at material level (rather than component level) by decomposition of material into small specimens (e.g. [34, 6]).

2.6 Field Failure Analysis

Field failure data are referred to as any types of information including failure and censored times (for catastrophic failure modes) and performance of field samples (for degrading failure modes) that might be used for reliability analysis. The majority of such information might be available in products/customers' portfolios, whereas performance of a sample returned from its own field, needs to be estimated by conducting a performance test. The analysis of field failure data can estimate reliability and/or failure occurrences of the failure modes in a multi-failure mode product. The comparison might be used to assess the most critical failure modes or to verify the conformity of the test and service results.

For reliability purposes, useful and useless information in field data might be mixed together that leads to considerable confusion in data analysis. Field data may suffer from any shortage of information, e.g. due to inexact reported time of failure (e.g. [53]) and unreported failures (e.g. [50]). Warranty considerations, failure types (catastrophic and degrading) and variety in failure modes, make field failure data analysis a challenging task.

To achieve meaningful results, each version of the product and each failure mode should be analyzed individually. If the sample version is not included in field data, the version might be assessed by comparing installation date of the sample with modification date (e.g. [45]). In the case of existing excessive number of field samples under study, a group of random samples should be selected so that they could cover a wide variety of applications and ages. If the aim is to estimate residual life of samples which are still operating in service, some specimens could be taken from the samples for performance test in order to estimate their residual lifetimes, or to validate their proper levels of performance (e.g. [16]).

For reliability purposes, if the sample is returned from service because of a catastrophic failure mode, its age is considered as its failure time, whereas for a degrading failure mode, failure time should be identified in terms of the performance factor of the sample.

2.6.1 Catastrophic Failure Modes

Warranty Period

Customers are eligible for some expense exemptions in the warranty period, so they are willing to report any failures to the manufacturer. According to this fact, the field failure data in this period of time are considered as reliable data to reflect the actual behaviour of products in service. If the product is under life warranty, or there is an obligation for the users to return their own failed samples to the manufacturer (according to a governmental or industrial rule) even after the warranty period (e.g. [61]), every field failure data is reliable.

For single-failure mode products, the sample i during the warranty period t may have one of the three following situations:

- It might have failed during this period of time $0 \leq t_{f,i} \leq t$, so $t_{f,i}$ is called its lifetime.
- It might survive from the warranty period $t_{r,i} = t$, so $t_{r,i}$ is called its right censored time.
- It is still in service operation during the warranty period $t_{r,i} < t$.

Note that the last two situations are both considered as right censored times. Let S_f and S_r the total numbers of failed samples and right censored samples respectively, so the likelihood function could be written as the Eq. 2.11 by considering $S_l = 0$.

Reliability analysis for a multi-failure mode product depends on failure definition of the product according to its failure modes. Furthermore, in order to obtain more meaningful results, each failure mode should be analyzed individually. If the product is series, failure time of the sample is considered as failure time for the first failure mode occurrence, and right censored time for every other failure mode. If the sample survives the warranty period, or it is still in operation during this period of time, the time is also considered as right censored time for every failure mode. The likelihood function (Eq. 2.11) should then be obtained for each failure mode to estimate its own failure probability diagram.

For other multi-failure mode products (such as parallel and stand-by), there might be a great deal of uncertainty about the occurrences of failure modes in the samples which are still in the operable state, so these products could only be analyzed at the product level (rather than failure mode level).

Interval Inspection

Depending on the warranty and maintenance policy, the product might be inspected in some time intervals. At the time of inspection, each failure mode may be in the inoperable state (left censored time) or in the operable state (right censored time). These data are not qualified for statistical reliability analysis, so non-statistical or high confidence reliability must be used.

As an illustrative example, consider 168 samples of a three-failure mode product. The product reliability is estimated and shown on Table 2.2. Non-statistical reliability and high confidence reliability ($C = 95\%$) for each failure mode are obtained according to the Eq. 2.14 and Eq. 2.15. Note that if the manufacturer decides to allocate a level of reliability to each failure mode, the most critical failure mode is the one having the least confidence level. As it is shown in the last column of the Table 2.2, failure mode 1 has the least confidence in comparison to others.

Occurrence Probability

Failure time is the key factor for reliability analysis of catastrophic failure modes, whereas these data might be unavailable in product profiles. Instead, the inspection of field returned samples could lead to recognize their failure modes over years. Occurrence probability of a failure mode is defined as the ratio of the number of samples returned

Table 2.2: Non-statistical reliability, high confidence reliability, and confidence level of a multi-failure mode product

Failure mode	S	S_l	Reliability estimation R_N	Reliability estimation R_C (C=95%)	Allocated reliability
1	168	4	97.6%	94.6%	R_C=95% / C=92.6%
2	168	7	95.8%	92.3%	R_C=92% / C=96.3%
3	168	10	94.0%	90.1%	R_C=90% / C=95.5%

due to that failure mode to the total number of failed samples. Occurrence probability might be demonstrated by a pie diagram (e.g. [56, 67]) or a bar diagram (e.g. [61]) to graphically compare failure modes. It is also used to compare with number of failures obtained by accelerated testing in order to make a correlation between test and service failure modes as demonstrated in Fig. 2.11.

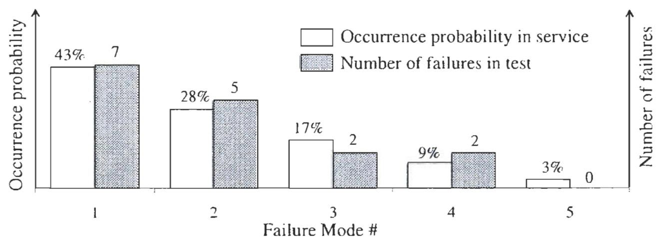


Figure 2.11: Comparison of the service occurrence probability and the number of failures obtained from test

After Warranty Field Data

After the expiration of the warranty period, there might be an uncertainty about unreported failures. The problem of unreported failures is not limited to only the lack of their failure times, but also the total number of unreported failures (and consequently, total number of failures) is unidentified, so reliability analysis faces a major uncertainty that even makes reported failures useless. Although there are not certain ways to obtain the total number of unreported failures after the warranty period, three approaches are recommended to have an approximate estimation of the number of unreported failures:

- The history of customers' portfolios in warranty period and afterwards could reveal the percentage of customers who are loyal to report their failures and use customer services.

- The cdf diagram during the warranty period could present an estimation of total number of failures after that. This method is especially useful for non-fixed warranty policy.
- Oh and Bai [50] modified the likelihood function (Eq. 2.11) based on the reported failures in warranty and after-warranty periods to estimate unknown parameters of pdf function.

2.6.2 Degrading Failures Modes

The lifetime for each field sample which is returned from service because of a probable degrading failure mode, should be specified in order to perform reliability analysis. As a simple assumption, the sample age could be considered as its failure time, so reliability analysis should be done by the above methods for catastrophic failure modes.

In reality, sample age is specified as its lifetime according to customer or technician's decision to return it back to the manufacturer, whereas the technical definition of lifetime should be realized by the manufacturer according to the level of performance. Accordingly, a performance test is needed to measure the performance level of each field sample.

After measuring the performance factor, each field sample is identified as a point in Cartesian coordinate system of performance-time. The technical definition of lifetime is presented as the time when its performance reaches the critical performance P^C. If the nominal level of performance is known (P^N), so for each field sample, a linear interpolation (or extrapolation) to the level of the critical performance as illustrated in Fig. 2.12 can identify its lifetime. Now, reliability analysis could be performed on these lifetimes by the methods presented for catastrophic failure modes.

In order to obtain more realistic results, the linear extrapolation method (Miner's rule) used in the above method should be replaced with a suitable performance diagram. This diagram could also be estimated by the least square estimation (rather than interpolation or extrapolation) of available data.

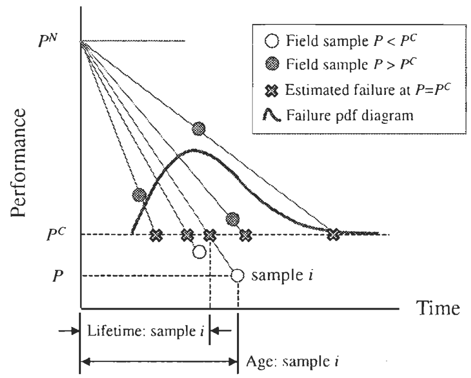


Figure 2.12: Failure time estimation of a field sample in terms of its age and performance factor

2.7 Relevant Failure Modes in AT

Applying accelerating variables to stimulate degradation mechanism of failure modes might bring the concern of non-relevant failure modes. For quantitative purposes, once the accelerated testing faces an irrelevant failure mode, the time must be considered as right censored time for every relevant failure mode. For qualitative reasons, i.e. to detect relevant failure modes, the irrelevant failure mode must be fixed by replacement to continue testing. There is no systematic approach to verify the relevant failure modes. Here, the problem is studied based on the available information and possible technical activities as follows.

Simple cases

The relevant failure mode can be confirmed:

- If there is only one possible failure mode in the product, i.e. the product is single-failure mode (ideal situation).
- If samples are under the same stresses and their levels as service. The main requirement of such assumption is to estimate service usage profile and considering probabilistic nature of the stress level and usage frequency.
- If relevant failure modes have already been identified.
- The similarity of failure modes in service and test is the most convenient approach to validate relevant failure modes. If the product is multi-failure mode, there

must be a consistency between the numbers of failure mode occurrences in test and service to validate the proximity of test and service results.

Technical Failure Process

The Manufacturers might be able to technically explain the failure process of their own products. This process might be presented as a chain of several events. For example, Jayatilleka and Okogbaa [29] explained that any failure in lubrication causes wearing in bush and increasing the gap between the bush and journal. Then, the pressure decreases and the surfaces will be in contact that results in increasing temperature and creating odours in the oil which causes damage to the bearing.

Note that relevant failure modes are not only the ones under normal usage conditions. In service, there may be a considerable deviation from normal conditions that causes other relevant failure modes, so the recognition of these failure modes in high stress levels are necessary for manufacturers to detect and eliminate them from their products. As an example, the effect of higher oil temperature (than normal) on gears have been investigated by Hohn and Michaelis [25] to identify potential failure modes.

The Concepts of Mechanical Stress/Strain

For a newly designed product, the static finite element analysis can present the distribution of mechanical stresses in the product due to a particular load in order to obtain its hot spots (e.g. [58, 60, 63]), which are the locations under high mechanical stresses. The more accurate estimation is obtained by performing dynamic analysis in these hot spots over time in order to recognize crucial hot spots and their real maximum stress levels. If the life-stress model of the material is available, the lifetime of the product could also be estimated (e.g. [58]).

For complex products under mechanical loads, analytical and numerical methods are able to estimate the most probable failure modes. The truss in Fig. 2.13 is under an oscillatory force F. Probable failure modes of each member could be easily discovered. For example, the member BC, might fail due to only tensile yield failure. Then, the relevant failure modes must be identified according to physical properties (size, elasticity module, material etc.) of each member by considering the probabilistic nature of the load level.

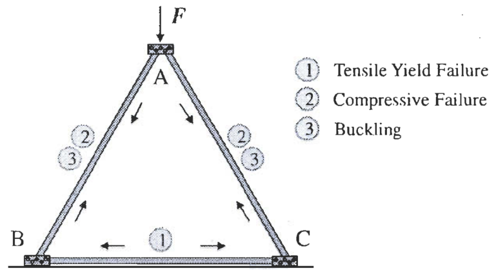
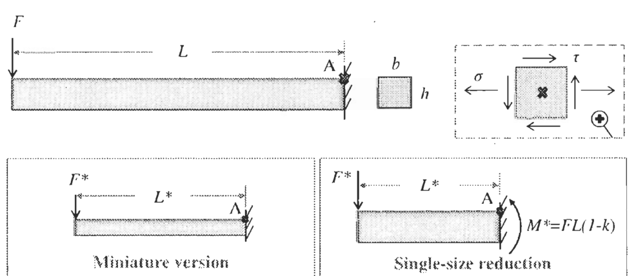


Figure 2.13: Probable failure modes in truss members

Any changes in geometrical size of samples must not cause new failure modes. For example, the excessive reduction in the beam diameter shown in the Fig. 2.8 under an oscillatory compressive force, may cause buckling which might be an irrelevant failure mode.

For the components under mechanical forces, a special attention is needed for any size reduction in order to keep the real stresses at their hot spots. Consider a rectangular cross section beam under a single force at the end as shown in Fig. 2.14. For the miniature version of order k (the same reduction ratio in every direction), the quantity of the mechanical force should be decreased by k^2 (not k) in order to prepare the same stress conditions ($\sigma = F/bh$ and $\tau = 6Fl/bh^2$) at the hot spot A. For single-size reduction version, a bending moment must be added at the point A while the mechanical force should be kept constant, to cause the same stress conditions.



Parameter	F^*/F	L^*/L	b^*/b	h^*/h
Coefficient	k^2	k	k	k

Parameter	F^*/F	L^*/L	b^*/b	h^*/h
Coefficient	1	k	1	1

Figure 2.14: Geometrical and load changes in miniature and single-size reduction versions to obtain the same mechanical stresses at the hot spot A

Relaxation Time

If a material is under an oscillatory stress, it must be relaxed (relaxation state) at the beginning of each cycle (rather than having a residual stress). The excessive increase of usage frequency does not allow some materials (like rubbers and polymers) to resume their operations from their relaxation states [38]. The main conclusion of this problem is to obtain longer failure time (deviation from real expected failure time) because of incomplete cycles of the stress as shown in Fig. 2.15.

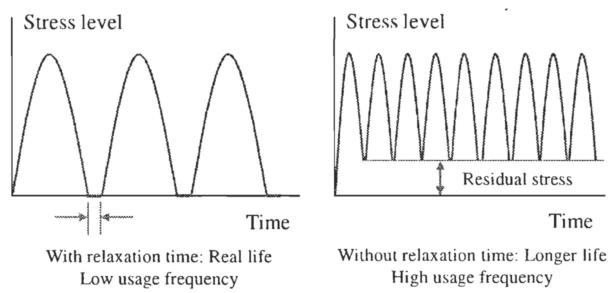


Figure 2.15: The effect of increasing the usage frequency to cause residual stress

2.8 Conclusion

In this paper, we presented an innovative study regarding the technical aspects of estimating usage profile and sample size, selecting suitable accelerating variables, analyzing field failure data, and verifying relevancy in accelerated testing. To implement an accelerated testing, these activities and knowledge should be recognized prior to any tests. In addition, it was emphasized the importance of defining the performance factor of products in order to dominate degradation mechanism.

Variety of accelerating variables related to stresses, performance test, and samples were introduced for stimulating degradation mechanism. We concluded that sample size is a function of many parameters like confidence level, allocated reliability, performance test and the level of test (system, component and material). This complexity led to an optimization problem.

Field failure data were analyzed during the warranty period for catastrophic failure modes. For degrading failure modes, we exploited performance factors to estimate lifetime. We recommended some approaches to deal with unreported failures after-warranty period and uncertainties about failure modes for multi-failure mode products. The practical contribution proposed in this paper to suggest required tools for AT, is expected to be a step forward to achieve a more systematic approach for AT.

2.9 Appendix: Theoretical Background

Continuous Probability Distribution Functions

Available discrete data of the random variable X, i.e. X_i $(i = 1, 2, ..., n)$, could be analyzed by a probability distribution function (pdf) in order to achieve a continuous formula for the probability $f(X)$ of the random variable $X \in [a, b]$. A pdf diagram is drawn in a two-dimensional Cartesian coordinate system, and each point in the diagram is denoted by a pair of $(X, f(X))$. Cumulative distribution function (cdf) at X is denoted by $F(X)$ and defined as

$$\begin{aligned} F(X) &= Pr[Y \leq X] \quad \text{for} \quad Y \in [a, b] \\ &= \int_a^X f(z)dz \end{aligned} \tag{2.9}$$

The cumulative distribution function is 0 at the point a and 1 at the point b, i.e. $F(a) = 0$ and $F(b) = 1$. A life distribution function such as weibull or exponential, in which the time is the random variable, is capable of estimating reliability at the given time T as

$$\begin{aligned} R(T) &= Pr[Y > T] \ \text{for} \ Y \in [0, \infty] \\ &= \int_T^\infty f(z)dz = 1 - F(T) \end{aligned} \tag{2.10}$$

The weibull and exponential cumulative life distribution functions are usually drawn in logarithm scales depending on the function. For example, the weibull function is written as

$$f(T) = \frac{\beta}{\alpha}(\frac{T}{\alpha})^{\beta-1}.e^{-(\frac{T}{\alpha})^\beta}$$

$$F(T) = 1 - e^{-(\frac{T}{\alpha})^\beta}$$

whereas in logarithm form, it could be expressed as

$$ln[-ln(1 - F(T))] = \beta.ln(T) - \beta.ln(\alpha)$$

that is written in the linear from of: $y = \beta x$. The same method can be used for the exponential function as

$$f(T) = \lambda.e^{-\lambda T}$$

$$F(T) = 1 - e^{-\lambda T}$$

$$ln[-ln(1 - F(T))] = ln(T) + ln(\lambda)$$

Maximum Likelihood Estimation

At the given time t, the sample i either in test or service, might be in one of the three situations as follows:

- It may fail at exactly this time, so $t_{f,i} = t$ is considered as its failure time.
- It may survive up to t, so $t_{r,i} = t$ is called its right censored time.
- It may have already failed at a time before t which is unknown, so $t_{l,i} = t$ is called its left censored time.

Let S_f, S_r, and S_l denote the total number of failed samples, right censored samples, and left censored samples respectively. The likelihood function is defined as [3]

$$L = \prod_{i=1}^{S_f} f(t_{f,i}).\prod_{j=1}^{S_r}[1-F(t_{r,j})].\prod_{k=1}^{S_l}F(t_{l,k}) \tag{2.11}$$

The maximum likelihood estimation aims at estimating unknown parameters of the pdf function $f(t)$ so that they could maximize L. As an example, the MATLAB statistical toolbox is capable of estimating unknown parameters of several pdf functions by the maximum likelihood estimation.

Binomial and Multinomial Distribution Functions

Consider S samples of a binary-state product exposed to the aging process of an accelerated catastrophic testing. The probability of failure for every sample at time t is known and equal to p. The probability of existing S_f samples in the inoperable state (and $S - S_f$ samples in the operable state) is obtained from the binomial distribution function

$$Pr(S_f) = \tbinom{S}{S_f}(1-p)^{S-S_f}(p)^{S_f} \tag{2.12}$$

Consider S service samples of a multi-state product with K states at service time t, so every sample is in one of the states at the time t. The probability of being in the state i is p_i ($p_1 + p_2 + ...p_K = 1$). The probability of existing s_1, s_2,..., s_K samples in the states 1, 2,..., and K respectively ($s_1 + s_2 + ... + s_K = S$), is obtained from the multinomial distribution function as [41]

$$Pr(s_1, s_2, ..., s_K) = S!\prod_{i=1}^{K}\frac{p_i^{s_i}}{s_i!} \tag{2.13}$$

Non-Statistical and High Confidence Reliability

At the given time t, if S_l samples (out of S samples) have already failed, the non-statistical reliability is defined as

$$R_N = \frac{S - S_l}{S} \tag{2.14}$$

The confidence level of such reliability is not high, i.e. if the test is repeated again; the possibility of obtaining the same level of reliability or higher is relatively low. High confidence reliability R_C at the time t with the confidence level C is estimated by solving the following implicit binomial distribution function

$$1 - C = \sum_{x=0}^{S_l} \binom{S}{x} . R_C^{S-x} . (1 - R_C)^x \tag{2.15}$$

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Chapter 3

A Contribution to Accelerated Catastrophic Testing

S. Hossein Mohammadian and Daoud Aït-Kadi

Abstract

Nowadays manufacturers' demands for the estimation of service reliability, the comparison of different brands of product on the market and the test validation, have increased considerably over the past few years. This paper presents the applications of accelerated testing for the products with catastrophic failure modes in order to achieve the abovementioned targets. Accelerated testing plans are categorized according to the inspection strategy for capturing failures. For a complicated usage profile (including many stresses and their levels), there might be several random variables (usage frequency and failure time) that result in complex integral equations for estimating statistical reliability. To solve this complexity, virtual sample method as a numerical approach is proposed to estimate non-statistical reliability. The method is demonstrated for two classes of usage profile by an illustrative example. Test results are harmonized with field failure data in order to identify a unique acceleration factor. The consistency between test and service results, are characterized by multinomial and ranking methods. The limitations of test time and manufactured samples for new version of design are also introduced, and the solutions to the problems are presented.

Key words: Accelerated catastrophic testing; Reliability; Virtual sample method; Comparative analysis

3.1 Introduction

Accelerated testing (AT) is a multi-purpose approach. Today, estimation of product reliability by AT is becoming a necessary activity for many manufacturers in order to plan their warranty policies, maintenance strategies and spare parts provisioning in industrial engineering. Accelerated testing might also be used to detect latent catastrophic failure modes, and to verify the elimination of the detected failure modes in new version of design. Reliability analysis before and after any modifications to design can be estimated by accelerated testing in order to ensure the positive effects of the modifications to improve the product reliability. For available products on the market to be purchased by a retail user, the comparison of their reliabilities by AT could be used to select the most suitable one for the required application. The available field failure data should be used to modify the results of the accelerated testing to be consistent with the service results. For a multi-failure mode product, the comparison of the occurrence probabilities for its failure modes leads to predict the most critical one.

Classically, accelerated testing was studied by Nelson [22] to introduce test plans, statistical models and their analytical methods. The estimated reliabilities at high and normal stress levels are related together by life-stress models, and they are classified according to stresses (e.g. [8, 4]). Among them, Arrhenius as the most well-known model when temperature is the stress (e.g. [14, 13, 1]) and the inverse power relationship for non-thermal stresses (e.g. [30, 34]), have widely been applied for practical problems. Furthermore, many efforts have been made to classify other practical AT methods (e.g. [9, 3, 29, 11]). Several limitations and difficulties of conducting AT methods are explained by Meeker and Escobar [18], but no solutions have been made to the problems. Utilization of AT methods for the above-mentioned applications might be accompanied by several uncertainties due to the random variables (usage profile and failure time), field failure data, and the complexity of product. Hence, analytical methods might be unable to estimate reliability.

Failure times obtained from an accelerated testing or lifetimes obtained from field failure data should be analyzed statistically to estimate product reliability (e.g. [34, 14]). Park et al. [24] exploited both data types (test and field) to obtain the acceleration factor. Such comparison is also proposed by Ranganathan et al. [10] to link between testing and service times while both having the same effect on the degradation of physical properties. In literature, acceleration factor is also proposed to be obtained by applying life-stress models like the Arrhenius model (e.g. [14]) and the inverse power relationship (e.g. [30]) if usage profile of the product is known. Once the acceleration factor is estimated, it could be used to obtain the service lifetime for the new version of design (e.g. [24]).

Any shortages in field-related data including inexact failure time (e.g. [26]), unreported failures (e.g. [23]), and the lack of design version in failed products (e.g. [19]) must be recognized prior to analysis. The failure data after the warranty period could be reliable if users are obliged to return failed samples to their own manufacturers due to a governmental or manufacturer rule (e.g. [28]). The time-based analysis of field failure data is sometimes impossible because of shortage in reported data. The problem could be more critical for a multi-failure mode product due to the uncertainty of reported failure modes.

If catastrophic field failure data are not accessible (e.g. for highly reliable products), the residual lifetime of field samples could be estimated by an accelerated catastrophic testing (ACT). Marahleh et al. [17] conducted an accelerated creep testing to fail field samples. The creep life of every sample was then used to estimate its residual lifetime. Such estimation was also made in terms of capacity of field-returned wheelchair batteries by Mohammadian et al. [20]. Because catastrophic failures of batteries were not accessible, a level of capacity was defined as the failing level, so the capacity of each battery was linearly extrapolated (or interpolated) to the failing level for estimating its lifetime. If there are not enough field samples for analysis, accelerated testing should be carried out on some specimens taken from the product in service [6]. For multi-failure mode products, field failure data could be presented as the number of failures for each failure mode rather than time-based data (e.g. [27, 31, 28]). The possibility of using field failure data to compare with accelerated testing results, needs to be recognized, firstly, to validate the test integrity of the AT and secondly to identify acceleration factor of the aging process.

Accelerated testing has been used by many people to carry out a qualitative comparison of different aspects in products. The work conducted by Knox and Cowling [12] compared the effects of applying different primers and surface conditions on long-term quality of adhesives. The shear force needed to fail the adhesion was considered as a factor to explain the quality of adhesives. Chou and Lamers [5] compared the degradation diagrams (wear) of different materials over time. Nagel et al. [21] statistically compared failure times of three different products (produced by different manufacturers) resulted from an accelerated testing in order to choose the most robust one. Comparison of the results obtained by accelerated testing in different stress levels shows that the results in severe levels are more likely to deviate from real service data (e.g. [33]). Potteau et al. [25] tested and analyzed the effects of different stresses on degradation of a product. Krivstove et al. [15] aged field samples to measure and compare degradation of their physical properties. For every comparative application, conducting a quantitative comparison could present more meaningful results.

The main aims of this paper are to propose numerical solutions for complex reliability analysis, and to validate the consistency between test and service results. For time-based failure data, test plans are divided into continuous, interval and single inspection methods according to related failure inspection strategy. Maximum likelihood estimation (MLE) and reliability validation methods are presented to estimate statistical and high confidence reliability respectively. For complicated problems including some random variables, there might be no analytical methods for reliability estimation. In this paper, the concept of virtual sample method as a numerical analysis is presented, and it is also validated for a known aging process including two types of random variable which are usage frequency and failure time. Salvation and longevity factors are defined based on reliability and lifetime of different products respectively, and they are used to select the most suitable product for the required application. Such selection is also suggested by considering the allocated reliability and confidence levels. For a single-failure mode product, the comparison between test and service results could lead to modify test probability diagram in order to be harmonized with the related service diagram. For a multi-failure mode product, the criticality of each failure mode is related to its priority factor which is defined based on the number of samples which is returned from service because of that failure mode. For such products, test and service results are compared to characterize the consistency of test results by defining conformity and vicinity factors. Detection of latent failure modes and confirmation of their elimination (after design modification) are discussed in qualitative ACT. In addition, the test and service results of old and new versions of design have been related by a unique acceleration factor, and any limitations of test time and manufactured samples are discussed.

The paper is organized as follows. Section 2 classifies ACT plans for single-failure mode products under a single stress based on the inspection strategies for time-based failure data. Section 3 presents virtual sample method and its applications in ACT for complex stresses. Section 4 deals with different uncertainties in comparative analysis of accelerated testing results. Section 5 presents design modification-related methods and their limitations and uncertainties. Section 6 discusses some concluding remarks.

3.2 Single-Stress ACT

The main aim of conducting a quantitative ACT on samples of a product with catastrophic failure mode(s) is to obtain their failure times and/or censored times of the samples. For this purpose, each sample must be exposed to the aging process for a duration of time t_a. For a number of test samples, complete data (e.g. [22]) refers to the ideal situation in which the exact failure time of every sample is obtained. Due to the limitation of time, complete data are not usually accessible.

The estimated failure and censored times are strongly influenced by the failure inspection strategy. So, accelerated catastrophic testing is divided into continuous, interval and single failure inspection methods as illustrated in Fig. 3.1.

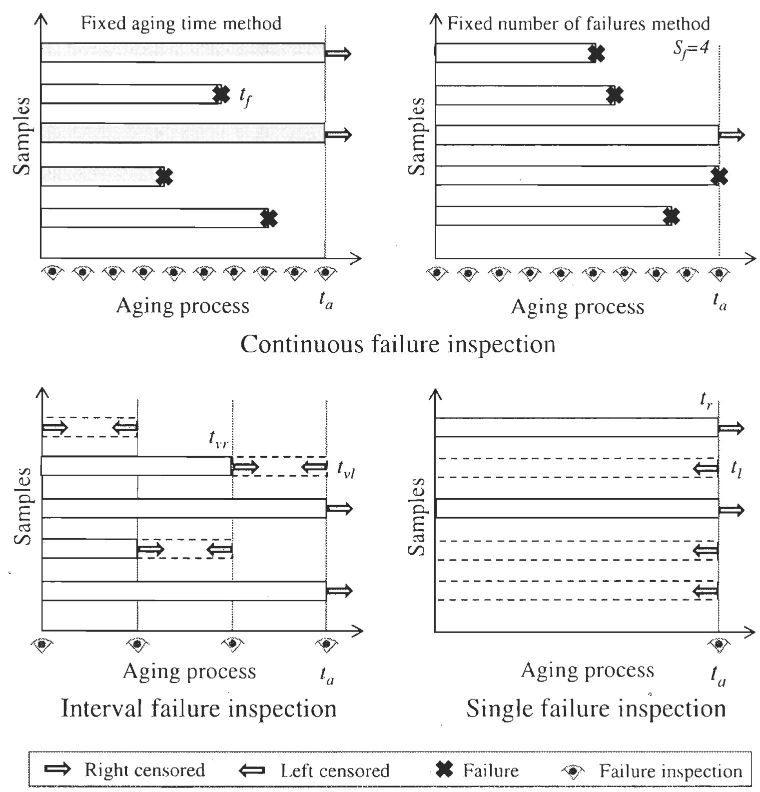


Figure 3.1: Accelerated catastrophic testing plans

The continuous failure inspection (usually time-consuming) is divided into fixed aging time and fixed number of failures methods enabling testers to detect the exact time of failure (failure time) for some samples. Although fixed number of failures method is more suitable to assure sufficient data for statistical analysis, but it is not recommended for high reliable products because of probable long failure time.

Any uncertainty in failure time is referred to as censored time which is divided into interval, left and right censored times. In the interval failure inspection method, probable failure time is recognized in a time interval (interval censored time) for each failed sample. In the single failure inspection method, all samples are inspected once (after the aging process), and this time is referred to as left censored time for each failed sample. For all the above methods, the time is considered as right censored time for every sample survived the aging process. Note that the dotted lines shown in the Fig. 3.1 in single and interval inspection methods emphasize that the exact time of failure is unknown, and it might have happened in any time along the dotted line. For all above methods except for the fixed number of failures method, the duration of time t_a has to be specified prior to the test.

For the continuous failure inspection methods, the maximum likelihood estimation (MLE) could be used to estimate unknown parameters of a specified probability distribution function (pdf). For the interval inspection method, it could be used if there are a few known failure times. Let S_r, S_l, and S_v denote the number of right censored samples, left censored samples and interval censored samples respectively, and S_f denote the number of samples whose exact failure times are identified. Likelihood function is expressed in terms of the pdf function f and its cumulative distribution function (cdf) F as [2]

$$L = \prod_{i=1}^{S_f} f(t_{f,i}).\prod_{i=1}^{S_r} [1 - F(t_{r,i})].\prod_{i=1}^{S_l} F(t_{l,i}).\prod_{i=1}^{S_v} [F(t_{vr,i}) - F(t_{vl,i})] \tag{3.1}$$

where $t_{f,i}$, $t_{r,i}$, $t_{l,i}$, $t_{vr,i}$, $t_{vl,i}$ are failure time, right censored time, left censored time, right and left times of the interval censored of the corresponding sample i respectively. The maximum likelihood estimation is based on estimating unknown parameters of the selected pdf function so that they could maximize L, i.e. the partial derivative of L with respect to every unknown parameter must be zero.

The above system includes complicated implicit equations in terms of unknown parameters, and there might be no analytical solution to explicitly extract the parameters, so it is recommended to use statistical software for this purpose. As a numerical solution, we used VB programming software and the MATLAB statistical toolbox by using trial and error method to find out the set of unknown parameters which maximizes the likelihood function L. For required applications in this paper, the two-parameter weibull function as the life distribution function and the Normal distribution function for any other applications like usage frequency; are employed.

For highly reliable products, the implementation of the above methods might present no failures (or a few failures) which are not sufficient for statistical analysis of reliability. Single inspection method can validate the allocated reliability level R at the time t_a with a specified minimum confidence level C by estimating the minimum number of samples S required to pass the aging process as [24, 21]

$$1 - C \geq \sum_{x=0}^{S_l} \binom{S}{x} R^{S-x}(1-R)^x \tag{3.2}$$

where S_l is the number of failures up to the time t_a. The single failure inspection method is called zero-failure test method if $S_l = 0$ and zero or one failure test method if $S_l = 1$ [32].

For a multi-failure mode product, in order to have control over each failure mode, to estimate its own probability diagram and to predict the most critical failure mode, the failure-related data should exclusively be estimated for every failure mode. For this purpose, failure time of each relevant (or non-relevant) failure mode that causes a stop in the aging process must be considered as right censored time for every other failure mode. For multi-failure mode products, many samples (and definitely long test time) might be needed to achieve probability diagram of every failure mode.

Up to now, the above testing methods are assumed to be done on new samples. The exploitation of both used (returned from service) and new samples in an accelerated testing presents more realistic results. In addition, used samples could be substituted in the case of any shortage of new samples. For this purpose, related aging process and the ages of used samples must be known. Furthermore, all used samples must have been returned from service because of another failure mode than the one under the study. Then, every used sample must pass the aging process for the duration of time t_a. For example, for an AF-based aging process (known acceleration factor), the equivalent test time $(t_a')_i$ for the used sample i is estimated as

$$(t_a')_i = t_a + \frac{(L_a)_i}{AF} \tag{3.3}$$

where $(L_a)_i$ is the age of the sample i in service.

3.3 Virtual Sample Method

The usage profile, failure time and physical properties of products are the main random variables in accelerated testing, although the latter, i.e. the physical properties are usually studied in accelerated degradation testing (ADT). The complexity of random variables in accelerated testing might make analytical methods unable to estimate reliability, especially in the case of existence many random variables. Therefore, a proper numerical method is needed for reliability analysis.

The usage profile of a stress might consist of two types of random variables: its levels and usage frequencies. The usage frequency is referred to as the ratio of the time in which the sample is under the stress to the total time in service. It is concluded that the value of usage frequency is always less or equal than one. If there are more than one stress level in the usage profile, to increase the accuracy of reliability estimation, usage frequency and failure time must individually be estimated for each stress level.

The virtual sample method presented here, as a numerical method to estimate reliability, is based on Monte-Carlo method by taking some representative values of random variable from each identified probability diagram. The process of selecting such values is described in Appendix. Accordingly, each probability diagram is introduced by its representative values instead of its mathematical formula. Then, a number of virtual samples must be defined each including some representative values of random variables according to the problem. Non-statistical reliability of the virtual samples is considered as an estimation of reliability of the product in service.

To demonstrate the application of the virtual sample method, service reliability of a product is estimated by its usage profile and failure times resulted from accelerated testing as follows. The product is under low levels of usage frequency in service, so accelerated testing is conducted by usage frequency as the accelerating variable. The main requirement is to identify the usage profile of the product in service including stresses, their levels and usage frequencies. Here, for the simplicity reason, the product is assumed to be single-failure mode under M levels of a single stress in service. Probability diagram of usage frequency has to be identified for each stress level. In order to estimate failure probability diagram of the product, for each above identified stress level, accelerated testing must individually be conducted on some new samples. The level of the stress must be kept constant during the non-stop aging process. If many levels of the stress are identified in the usage profile, in order to economize on samples and time, some of the levels (the lowest, a few middle and the highest levels) have to be chosen for testing, and their testing results must be used to predict related results for other levels by an interpolation method or a life-stress model (if it has already been identified).

The usage frequency probability diagrams (predicted by usage profile) and the failure probability diagrams (estimated by accelerated testing) must individually be replaced with some of their representative values as $(q_s)_i$ and $(t_f)_i$ respectively for the stress level i. For the stress level i, the number of representative values for usage frequency and failure time are denoted by $(N_q)_i$ and $(N_f)_i$ respectively. Here, the virtual sample method is individually applied for two classes of usage profile.

3.3.1 Unique Level Usage Profile

Every sample in this class is considered under a unique level of the only stress for its whole service life. The stress level for every other sample might be different, but it must also be unique for that sample. The usage profile of personal bicycles could be placed in this class if user weight is considered as stress. Each bicycle is intended to be used by a specified person, i.e. it is under a unique level of weight for its whole lifetime.

The number of virtual samples under the stress level i is obtained from $S_i = (N_q)_i.(N_f)_i$, and each virtual sample is identified by its stress level i, usage frequency $(q_s)_i$ and failure time $(t_f)_i$ resulted from the ACT. The total number of virtual samples is obtained by

$$S = \sum_{i=1}^{M} (N_q)_i.(N_f)_i \tag{3.4}$$

The user ratio of the stress level i is denoted by UR_i and is defined as the ratio of the service samples which are utilized in this stress level, to the total number of samples in service, and obviously $UR_1 + UR_2 + ... + UR_M = 1$. The user ratio of each stress level must be specified in the usage profile. For every stress level, there must be a direct relationship between its sample size S_i and its user ratio UR_i, i.e.

$$\frac{S_i}{UR_i} = \text{Constant}, \text{ for } i = 1, 2, ..., M \tag{3.5}$$

The service lifetime of a virtual sample under the stress level i is estimated as

$$t_i = \frac{q_i}{(q_s)_i}(t_f)_i \tag{3.6}$$

where t_i is the service lifetime for a virtual sample under the stress level i, and they are used to estimate service reliability of the product at this stress level, and q_i is the usage frequency of the accelerated testing at the stress level i. At the time t_s, non-statistical service reliability of the product under the stress level i is estimated as

$$R_i(t_s) = \frac{S_i - (S_f)_i}{S_i} \tag{3.7}$$

where $(S_f)_i$ is the number of those virtual samples whose estimated service lifetimes are less than the given time t_s. The lifetimes of all virtual samples, i.e. $t = \{t_i | i = 1, 2, ..., M\}$ are used to estimate non-statistical service reliability of the product as

$$R(t_s) = \frac{S - S_f}{S} \tag{3.8}$$

where $S_f = (S_f)_1 + (S_f)_2 + ... + (S_f)_M$.

Table 3.1 presents the parameters of Normal and weibull probability diagrams for usage frequency and failure time respectively for a product under three stress levels in the unique level usage profile. The user ratio and the number of representative values for each pdf function are also included on the Table. Note that the Eq. 3.5 has already been validated by selecting suitable numbers of representative values of failure time. Non-statistical service reliability is estimated based on the Eq. 3.7 and Eq. 3.8. The accuracy of estimated reliability could be improved by employing more virtual samples. Service reliability is estimated as 90.3% for 300 virtual samples as shown on the Table 3.1, and 88.9% for over 100,000 virtual samples (according to our calculations). For the data presented on the Table, representative values and three typical virtual samples are shown in Fig. 3.2.

Table 3.1: The estimation of service reliability for the unique level usage profile

Stress level i	UR_i	Usage frequency			Failure time				S_i	R_i %
		μ	σ	$(N_q)_i$	α (h)	β	q_i	$(N_f)_i$		
1	0.30	0.25	0.08	10	3,230	7.0	1	9	90	85.6
2	0.50	0.15	0.05	10	2,150	8.2	1	15	150	92.0
3	0.20	0.10	0.03	10	1,480	6.8	1	6	60	93.3
								Overall	300	90.3

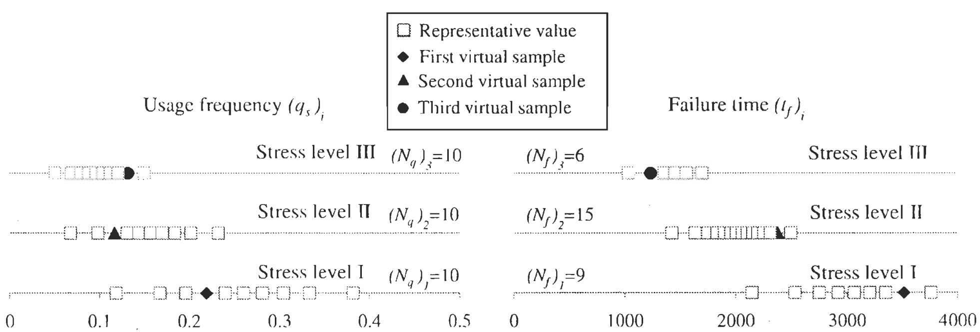


Figure 3.2: The representative values and three typical virtual samples for the unique level usage profile

3.3.2 Multi Level Usage Profile

In this class, every sample in service is assumed to be exposed to all stress levels with their own usage frequencies. In other words, each sample experiences every stress level during its lifetime. The bicycles for public uses, e.g. for renting, which are used by different people in variety of weights (as the stress) could be placed in this class.

A virtual sample is identified by its own usage frequency (a value from $(q_s)_i$) and failure time (a value from $(t_f)_i$) at the stress level i for $i = 1, 2, ..., M$. The total number of virtual samples is calculated as

$$S = \prod_{i=1}^{M} (N_q)_i.(N_f)_i \tag{3.9}$$

Note that, for each virtual sample; its total usage frequency must not exceed 1. The lifetime t is estimated according to Miner's rule (e.g. [89, 9, 88]) as

$$t = (\sum_{i=1}^{M} \frac{(q_s)_i}{q_i} \frac{1}{(t_f)_i})^{-1} \tag{3.10}$$

All these lifetimes t are used to estimate non-statistical service reliability of the product at the time t_s by the Eq. 3.8.

Table 3.2 shows the probability diagrams of usage frequency and failure time of a multi level usage profile. Note that the usage frequency of the first stress level is considered constant. Non-statistical service reliability is estimated as 83.5% for 100,000 virtual samples (as shown on the Table) and 82.2% for more than 100 million virtual samples (according to our calculations). The representative values and three virtual samples are also shown in Fig. 3.3.

Table 3.2: The estimation of service reliability for the multi level usage profile

Stress level	Usage frequency			Failure time				S	R (%)
	μ	σ	$(N_q)_i$	α (h)	β	q_i	$(N_f)_i$		
I	0.02	_	1	3,230	7.0	1	10		
II	0.10	0.03	10	2,150	8.2	1	10	100,000	83.5
III	0.05	0.015	10	1,480	6.8	1	10		

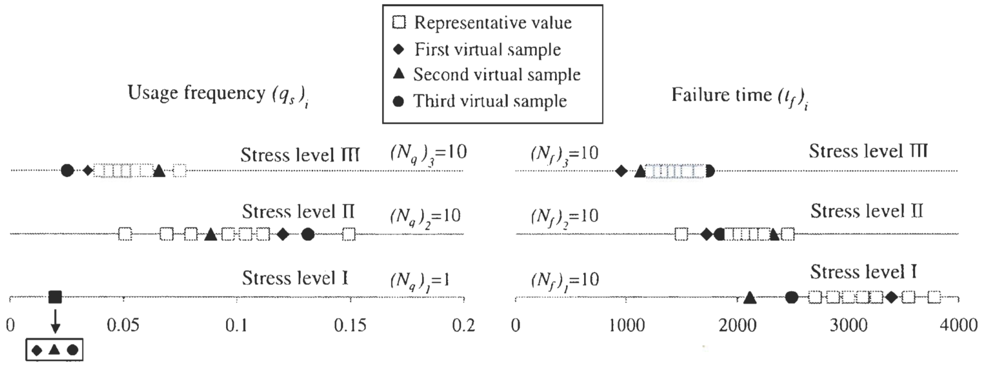


Figure 3.3: The representative values and three typical virtual samples for the multi level usage profile

3.4 Comparative Analysis of ACT Results

3.4.1 Comparison of Products

Accelerated testing might be used to compare different aspects in design, material and manufacturing process in order to select the most suitable one for required application. In fact, there is a competition between above issues in order to get more attention from customers and manufacturers. Such comparison might also aim at comparing different products available on the market. In this case, in order to avoid extra tests, available field failure data of a product should be used to compare with test results of other products if acceleration factor of the aging process has already been identified.

Any comparison must be based on an indicative parameter which could be reliability, lifetime, confidence level etc. The selection of an appropriate indicative parameter could depend on warranty policy, maintenance strategy, customer expectation or required application the product is expected to be used.

Comparison of Reliability

Let test probability diagrams of T different products be identified either from accelerated testing or field failure data. If test reliability is considered as the indicative parameter, at the test time t_t (which is assumed to be known by required service time and known acceleration factor), the product i should be selected as the most reliable product if

$$R_i(t_t) > R_j(t_t) \text{ for } j = 1, 2, ..., T \text{ and } j \neq i \tag{3.11}$$

In order to make the above relationship more obvious, salvation factor of the product i (SF_i) is defined as the probability of its survival up to the time t_t while others have already failed in $[0, t_t]$. Based on the definition of probability distribution function $f(t)$, cumulative distribution function $F(t) = \int_0^t f(z)dz$, and reliability $R(t) = 1 - F(t)$, the salvation factor can be estimated as

$$\begin{aligned} SF_i(t_t) &= R_i(t_t) \prod_{j=1, j\neq i}^{T} \int_0^{t_t} f_j(z)dz \\ &= \frac{R_i(t_t)}{F_i(t_t)} \prod_{j=1}^{T} F_j(t_t) \end{aligned} \tag{3.12}$$

The salvation factor could be normalized as

$$SF_i^*(t_t) = \frac{SF_i(t_t)}{\sum_{j=1}^{T} SF_j(t_t)} \tag{3.13}$$

where the accent * denotes a normalized factor.

Comparison of Lifetime (Failure Time

The time to reach an allocated reliability R (in percentage) for every product could be used as indicative parameter in order to select the product with longest lifetime. According to the definition, the product i has to be selected if

$$(\beta_{100-R})_i > (\beta_{100-R})_j \text{ for } j = 1, 2, ..., T \text{ and } j \neq i \tag{3.14}$$

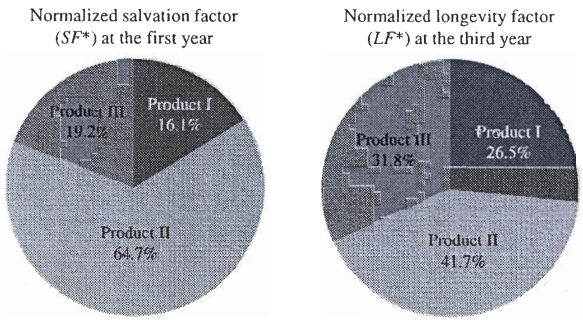
where the time $(\beta_{100-R})_i$ indicates that up to this time, $100 - R$ percent of samples (of the product i) have already failed. If $R = 50\%$, the comparison is based on the median life. In addition, the mean time could also be defined as the indicative parameter.

For the product i, the probability of having the longest life (while others fail in $[0, t_t]$) is defined as the longevity factor $LF_i(t_t)$, and it is mathematically expressed as

$$LF_i(t_t) = \int_0^{t_t} \frac{f_i(z)}{F_i(z)} \prod_{j=1}^{T} F_j(z)dz + SF_i(t_t) \tag{3.15}$$

Note that the last term, i.e. $SF_i(t_t)$ indicates that the failure time of the product i is greater than t_t while others have failed in $[0, t_t]$.

The diagrams in Fig. 3.4 show the comparison of three products based on their salvation and longevity factors with known test weibull parameters α and β. The acceleration factor of the aging process is 10. Product II is the most reliable one by considering reliability or salvation factor as the indicative parameter for the first year in service ($t_t = 36.5$). This product should also be selected if the longevity factor is considered as the indicative parameter up to the third year.



	Prod.	α	β	Reliability at t_t=36.5 days
	I	195	2.2	97.5
	II	178	3.2	99.4
	III	145	2.8	97.9

Figure 3.4: The comparison of three products according to their salvation and longevity factors

Comparison of Allocated Reliability and Confidence Level

For highly reliable products, accelerated testing might be unable to present sufficient failure data for statistical analysis, and accordingly, related probability diagrams could not be obtained. In this case, a number of samples from each product (the number of samples might be different from product to product) must pass a common aging process up to the test time t_t. To decrease the cost of the ACT, failure inspection is done once at the end of the aging process, so right censored and left censored times will be the only data available for each product.

The first approach to compare different products is to allocate a common reliability level R for every product and to set the confidence level as the indicative parameter obtained by the binomial distribution as

$$C_i = 1 - \sum_{j=0}^{S_{l,i}} R^{S_i - j}(1 - R)^j \tag{3.16}$$

where S_i and $S_{l,i}$ are the number of samples and the number of failed samples (left censored samples) for the product i, and C_i is its confidence level. The product with the highest level of confidence must be selected as the most suitable product, although its reliability level is allocated at the same level as others.

The second approach is to allocate a common level of confidence C for every product and then to set reliability as the indicative parameter. For the product i, its reliability R_i has to be obtained by solving the following implicit equation (see the Eq. 3.16)

$$C = 1 - \sum_{j=0}^{S_{l,i}} R_i^{S_i - j}(1 - R_i)^j \tag{3.17}$$

Then, the product with highest reliability must be selected as the most reliable product.

As an illustrative example, Table 3.3 shows the number of samples and failed samples for three products. The data for the first product are derived from service. The acceleration factor of the aging process is estimated as 5, so for the first year in service, the aging process must take $t_t = 365/5 = 73$ days. The seventh and eighth columns show the results of the first and the second approaches to compare products with common reliability of 90% and common confidence level of 90% respectively. Note that the most suitable product suggested by the first approach (product III) is different from the most reliable one estimated by the second approach (product I).

Table 3.3: The comparison of the allocated reliability and confidence level for three products according to the field failure data for the product I and the results of the accelerated testing for the products II and III

Prod. i	Source of data	Time t_t or t_s	AF	Sample size S	Failed samples S_l	Confidence level for R=90%	Reliability for C=90%
I	field	365	-	156	12	79.3%	88.8%
II	test	73	5	38	2	74.6%	86.6%
III	test	73	5	30	1	81.6%	87.6%

3.4.2 Comparison of Failure Modes

For a product including N failure modes, predicting the most critical one is necessary in order to moderate or remove it from the product. The criticality (as expressed in failure modes and effects analysis) of the failure mode i depends on both its severity s_i and its occurrence probability OP_i which is the probability of returning samples from service because of the failure mode i up to the service time t_s, and obviously $OP_1 + OP_2 + ... + OP_N = 1$. The failure mode i in service is the most critical one if

$$s_i.OP_i > s_j.OP_j \;\; for \;\; j = 1, 2, ..., N \text{ and } j \neq i \tag{3.18}$$

Consider probability diagram of each failure mode be identified by accelerated testing. Priority factor of the failure mode i up to the time t_t is denoted by PF_i and is defined as the probability of happening the failure mode i before every other failure mode in $[0, t_t]$. Based on the definition of probability distribution function $f(t)$, and reliability $R(t) = 1 - \int_0^t f(z)dz$, the priority factor can be estimated as

$$PF_i = \int_0^{t_t} \frac{f_i(z)}{R_i(z)} \prod_{j=1}^{N} R_j(z)dz \tag{3.19}$$

The most critical failure mode in the test is the one having the highest value of $s_i.PF_i$.

The comparison of failure modes leads to detect the non-important failure mode in the product if there is any. The failure mode i is defined as non-important, if $s_i.OP_i$ or $s_i.PF_i$ is relatively much less than the corresponding values for other failure modes. As an illustrative example, three failure modes of a product have been investigated by accelerated testing as shown in Fig. 3.5. The failure mode I is the most critical one, whereas the probability of occurring the second failure mode and its low severity make it as a non-important failure mode.

Failure mode	I	II	III
α (hours)	145	185	170
β	8.9	9.1	7.1
Severity s	10	1	8
PF^* (%)	57.7	5.8	36.5

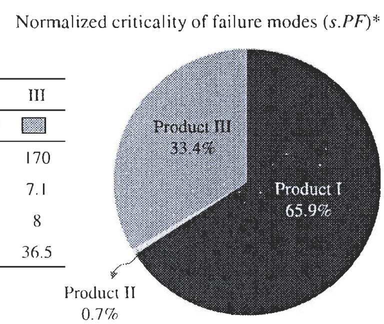


Figure 3.5: The most critical and non-important failure modes for the first 100 hours in test ($t_t = 100$ hours)

3.4.3 Comparison of Field and Test Results

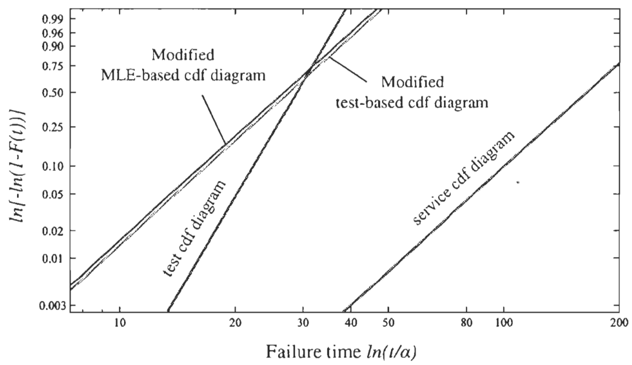
Single-Failure Mode Products

The comparison of test and service failure-related data for a single-failure mode product, aims at modifying test results in order to be consistent with field results. In general, the obtained service and test cdf diagrams for a particular product are not necessarily parallel in logarithm scales, e.g. the shape parameter of the test weibull cdf diagram is greater than the one for service ($\beta_t > \beta_s$). The main reason of this contrast is about the variety of stresses; their levels and usage frequencies in service that create a wide range of lifetime. For the weibull functions, these conditions decrease the shape parameter of the service cdf diagram.

Here, two modification methods are presented to modify the test cdf diagram in order to make it parallel to the field cdf diagram for the weibull functions, although the methods could be extended for other distribution functions. Such modifications allow manufacturers to estimate a common acceleration factor. In both methods, the modified shape parameter of the tests must be set to the corresponding parameter for service. In test-based method, scale parameter of the modified diagram must be kept constant at α_t. In MLE-based method, parameter α has to be estimated by maximum likelihood estimation (MLE). As an illustrative example, the service and test cdf diagrams are estimated for complete data of 6 samples in service and 4 samples in test respectively as shown in Fig. 3.6. The modified test diagrams are also illustrated based on the above modification methods. Note that there is no considerable difference between both modified test diagrams.

Multi-Failure Mode Products

In multi-failure mode products, the individual comparison of test and service probability diagrams for every failure mode cannot ensure the consistency between test and service results. The test failure-related data of a failure mode might completely be consistent with its service related data, while for other failure modes, there might be no such consistency. In addition, the individual comparison might lack of accuracy due to the existence of many right censored times. Instead of time-based analysis, a unique comparison of all failure modes should be done based on the number of failures in test and occurrence probability in service. Here the suggested methods aim at characterizing the proximity of test and field results to validate accelerated testing.



cdf Weibull diagram	α	β	Failure time data					
Service	179.7	3.85	98	116	156	167	199	237
Test	30.3	7.27	25	26	28	35		
Modified test-based	30.3	3.85						
Modified MLE-based	29.3	3.85						

Figure 3.6: Test-based and MLE-based methods

Consider S new samples of a series product with N failure modes subjected to an accelerated testing. The number of failed samples due to the failure mode i after the test is a random variable and is shown by S_i, and obviously $S_1 + S_2 + ... + S_N = S$ if every sample fails. The total number of failure mode combination $(S_1, S_2, ..., S_N)$ depends on the number of samples and the number of failure modes. On the other hand, occurrence probability of each failure mode is assumed to be known. Here, three methods are presented to validate accelerated testing:

- Multinomial Method: The probability of each failure mode combination could be estimated based on occurrence probability by multinomial distribution function as (e.g. [16])

$$Pr(S_1, S_2, ..., S_N) = S! \prod_{j=1}^{N} \frac{OP_j^{S_j}}{S_j!} \tag{3.20}$$

The highest $Pr(S_1, S_2, ..., S_N)$ means the closest results of test and field data, because the amounts of OP_j/S_j for $j = 1, 2, ..., N$ are close to each other and their multiplication will be high. In order to characterize the test and service proximity, all probable failure mode combinations must be listed in descending order in terms of their probabilities Pr obtained by the Eq. 3.20. The first combination in the list (having the probability of Pr_1) is the most likely failure mode combination in the test. If the actual failure mode combination in the AT is placed in the ranking i (having the probability of Pr_i), its conformity factor

CF_i is defined as

$$CF_i = 1 - \sum_{j=1}^{i} Pr_j \tag{3.21}$$

Test and service results are considered close together for high levels of the conformity factor.

- Integer Ranking Method: In this method, the failure modes must be placed in the test and service ranks as follows. In the test rank, they must be sorted in descending order in terms of their numbers of failures. In the service rank, they have to be sorted according to their occurrence probabilities in descending order. The ranking number (for both test and service ranks) for each failure mode has to be an integer value between 1 to N (including 1 and N). Let $r_{t,i}$ and $r_{s,i}$ the rankings of the failure mode i in the test and in the service ranks respectively ($r_{t,i} \neq r_{t,j}$ and $r_{s,i} \neq r_{s,j}$ for $i, j = 1, 2, ..., N$ and $i \neq j$). For each failure mode, the difference between its test and service rankings reveals a deviation in test result compared with service. Accordingly, the total deviation factor (DF) is defined as

$$DF = \sum_{i=1}^{N} (r_{t,i} - r_{s,i})^2 \tag{3.22}$$

The most deviant case occurs when $r_{t,i} = N - r_{s,i} + 1$ for $i = 1, 2, ..., N$, and the test results are completely in contrast to the service-related results. In this case, the highest possible deviation factor DF_{max} is obtained from the Eq. 3.22 as

$$DF_{max} = \sum_{i=1}^{N} (N - 2r_{s,i} + 1)^2 = \frac{N}{3}(N^2 - 1)$$

Note that $\sum_{i=1}^{N} r_{s,i} = N/2(N+1)$, and $\sum_{i=1}^{N} N^2 = N^3/3 + N^2/2 + N/6$. The vicinity factor is defined as the proximity of test and field results as

$$VF = \frac{DF_{max} - DF}{DF_{max}} \tag{3.23}$$

The vicinity factor is 1 for ideal testing results, and 0 for the most deviant case presented above.

- Real Ranking Method: If there are only a very few failure modes, the integer ranking method is limited to a few values, and it is not able to present a precise estimation of the vicinity factor to validate related accelerated testing. In the real ranking method, like the integer ranking method, the failure modes must be sorted in descending order in terms of their numbers of failures in test rank and their occurrence probabilities in service rank as follows:

In the test rank, the failure modes j and k including the highest and the lowest number of failures are placed at the rankings 1 and N respectively, i.e. $r_{t,j} = 1$ and $r_{t,k} = N$. Then, the ranking of the failure mode i has to be obtained by the following implicit linear equation

$$\frac{r_{t,i} - r_{t,j}}{S_j - S_i} = \frac{r_{t,k} - r_{t,i}}{S_i - S_k} \tag{3.24}$$

In the service rank, a similar approach is used to estimate service ranking of the failure mode i based on its occurrence probabilities if the failure modes l and m have the highest and the lowest occurrence probabilities

$$\frac{r_{s,i} - r_{s,l}}{OP_l - OP_i} = \frac{r_{s,m} - r_{s,i}}{OP_i - OP_m} \tag{3.25}$$

Then, the deviation and vicinity factors must be obtained from the related equations presented in the integer ranking method (Eq. 3.22 and Eq. 3.23). In order to estimate DF_{max} in this method, the test ranking of each failure mode must be specified as the farthest possible ranking from its service ranking as

$$r_{t,i} = \begin{cases} 1, & \text{if } r_{s,i} \geqslant (1+N)/2 \\ N, & \text{if } r_{s,i} < (1+N)/2 \end{cases} \tag{3.26}$$

Table 3.4: The calculations of conformity factor by multinomial method for $S = 34$ test samples and $N = 6$ failure modes of the product

Ranking	Number of failures						P	CF
	1	2	3	4	5	6	(10^{-4})	%
	OP_1=0.21	OP_2=0.11	OP_3=0.24	OP_4=0.05	OP_5=0.30	OP_6=0.09		
1	7	4	8	1	11	3	4.737	99.95
2	7	4	9	1	10	3	4.632	99.91
...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...
71	6	5	8	1	11	3	3.474	97.22
72	6	4	9	1	10	4	3.474	97.22
...	...	...	...	...	...	...	...	...

As an illustrative example, for 34 samples of a six-failure mode product, occurrence probability and number of failures for each failure mode are shown on Table 3.4. The analysis of data is performed by a VB program for all three above methods. The failure mode combination $(6, 4, 9, 1, 10, 4)$ obtained from the ACT, has been placed in ranking 72 out of 575757 possible combinations in multinomial method, and its conformity factor is estimated as 97.22% that is relatively an excellent and acceptable level. According to our calculations, the worst failure mode combination is $(0, 0, 0, 34, 0, 0)$ with $CF = 0$, i.e. all samples fail because of the failure mode having the less occurrence probability. Table 3.5 shows the integer and real rankings of the above problem. The vicinity factors obtained from the integer ranking method (98.57) and real ranking method (98.54) are also in acceptable levels.

Table 3.5: The calculations of vicinity factor by integer and real ranking methods for $S = 34$ test samples and $N = 6$ failure modes of the product

Failure mode	OP_i	S_i	Integer ranking method DF_{max}=70		Real ranking method DF_{max}=106.76	
			Service	Test	Service	Test
1	0.21	6	3	3	2.80	3.22
2	0.11	4	4	4	4.80	4.33
3	0.24	9	2	2	2.20	1.56
4	0.05	1	6	6	6.00	6.00
5	0.30	10	1	1	1.00	1.00
6	0.09	4	5	4	5.20	4.33
Deviation factor (DF)			1		1.56	
Vicinity factor (VF) %			98.57		98.54	

3.5 Modifications to Design

In the design stage when the product is expected to be manufactured in near future, or after production and sale, due to the existence of failures in service as well as customer expectation, a modification to the present version of design might be necessary in order to improve product reliability and/or to remove (or moderate) its latent and observed failure modes that help the company to remain in intensive competition among other manufacturers. The implementation of accelerated testing on the new version of design aims at validating the elimination of failure modes, estimating its reliability or confirming a required reliability.

3.5.1 Qualitative ACT

The implementation of qualitative ACT on samples of a product in the present version of design is to identify product latent failure modes. These failure modes should be removed from the product by failure cause analysis and necessary modifications to design. The accelerated testing on the samples of the new version aims at confirming the elimination of the identified failure modes after the modifications. The identification of non-relevant failure modes is a technical issue of design and material, and it must be recognized by engineers, testers and technicians. Field failure data and available literature could also be used to verify the relevancy of the observed failure modes in the test with the actual ones in service. No statistical analyses are needed for detecting the failure modes and validating their elimination, so fewer samples are required for the test compared to quantitative ACT.

In order to meet the failures in shorter time, the level of each stress must be increased during the aging process. This augmentation might be step by step or continuous. In continuous augmentation, the product must operate at the highest level for a period of time which is called dwell time. There are not classical approaches to identify the highest applicable levels of stresses and to estimate the time required for the aging process. In comparison to quantitative accelerated testing methods, there are fewer limitations to increase stress levels and to combine stresses, so the time to failure considerably becomes shorter. The levels of environmental stresses such as temperature, vibration and humidity might increasingly be higher than the endurance limit of some components inside the product, so in order to avoid obvious non-relevant failure modes; they have to be isolated from exposing to high levels of these stresses. Qualitative accelerated methods are divided into single-modification and multi-modification methods. In both methods, once a non-relevant failure occurs, the failed component has to be replaced with a new one in order to continue the test.

In the single-modification method, the first stage of test is to detect latent failure modes of the product in the current version. Then related modifications to design and material must be done to remove the observed relevant failure modes at the first stage of test. At the second stage of the test, the manufactured samples (prototypes) in the new version must pass the same aging process in order to validate the positive effect of the modification to eliminate the failure modes. For both stages, in the case of a catastrophic failure in a component, it must be replaced with a new one in order to continue the test.

In the multi-modification method, a series of aging process (in order to detect failures), failure cause analysis, and modification must be performed on the product. The modification and/or failure cause analysis should be done whenever a relevant failure is observed in the product. Highly accelerated life testing (HALT) as a multi-modification method, has recently been developing for complex products, especially for electronic products. HALT chambers are capable of preparing a wide range of temperature, vibration, humidity etc., and their combinations in different geometric sizes. An economic evaluation is needed prior to perform HALT testing, because the chambers are relatively costly.

3.5.2 Acceleration Factor Method

The statistical test and service cdf diagrams of the new version of design might seem to be independent of the ones for the old version product because of existence new features in design. The acceleration factor method aims at relating the old and new versions of design to enable manufacturers to estimate related service results of the new version.

For the old version, the data obtained from service and the accelerated testing, must be statistically analyzed to estimate the field and test cdf diagrams. In this method, as the first assumption, the estimated field cdf diagram (of the old version) is considered as the base diagram so that every other cdf diagram must be parallel to this diagram. The parallel diagrams of the old version present a unique acceleration factor AF. As the second assumption, this factor (AF) is only considered as a function of the specified aging process (neither the product nor its design version), so the acceleration factor is the same for both the old and new versions of design.

After a modification to design, some samples (prototypes) must be manufactured in order to pass the same aging process to obtain the test cdf diagram of the new version. For the old version, let $t_{t,o}$ and $t_{s,o}$ the times to achieve an arbitrary level of reliability R in the test and service respectively. If $t_{t,n}$ is the test-related time for the new version, so its estimated service time, i.e. $t_{s,n}$ could be obtained by considering the same acceleration factor for both design versions as shown in Fig. 3.7

$$AF = \frac{t_{s,o}}{t_{t,o}} = \frac{t_{s,n}}{t_{t,n}} \tag{3.27}$$

3.5.3 Reliability Allocation Method

Although the acceleration factor method is an effective approach to estimate service reliability of the new version of the product, but the samples in this version might be strong enough to pass the aging process without any failures, so no useful statistical analysis could be obtained. On the other hand, the manufacturer might be interested to confirm an allocated reliability for the new version rather than estimating its reliability.

Reliability allocation method aims at confirming a level of reliability R with the

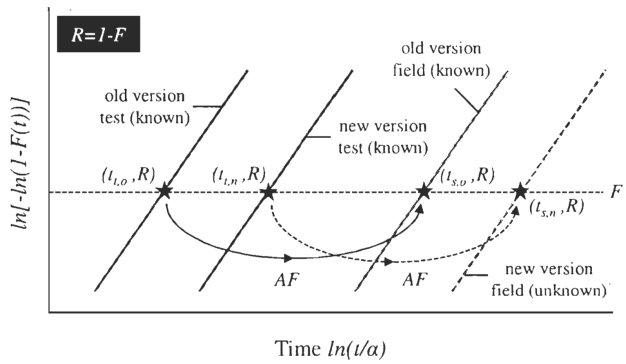


Figure 3.7: The estimation of field cdf weibull diagram for the new version of the product by acceleration factor method

minimum confidence level C at the time t_s in service. To use this method, the first stage of testing must be done like the acceleration factor method in order to estimate the acceleration factor of the aging process. The service and test cdf diagrams of the new version are parallel lines (to the cdf diagrams of the old version) that pass through the points (t_s, R) and $(t_t = t_s/AF, R)$ respectively in the coordinate system as illustrated in Fig. 3.8. So, the most distinction of this method is that, the cdf diagrams of new samples could be identified before testing the samples of new version. According to the binomial distribution function, the number of samples S to pass the aging process up to the time t_t is estimated as (see [32, 7])

$$S \geq \frac{ln(1-C)}{ln(R)} \tag{3.28}$$

The above equation is based on zero-failure test method, and it means that every sample must survive the aging process in order to confirm the allocated reliability R with the minimum confidence level C.

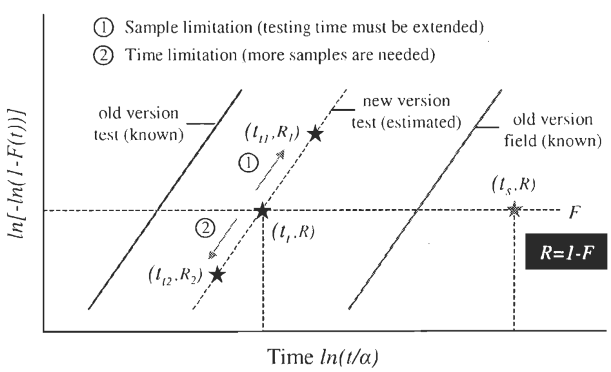


Figure 3.8: The allocated reliability methods for sample and time limitations

In real applications, there might be a technical or budget limitation to produce required number of samples (in new version) needed for accelerated testing (in the second stage). Here, the proposed method to compensate for the shortage of samples is to extend test time along the cdf test diagram in order to achieve a low level of reliability $R_1(< R)$ by keeping the same level of confidence as shown in the Fig. 3.8 (case 1). Then, the required samples S_1 should be estimated as $S_1 \geq ln(1-C)/ln(R_1)$, where $S_1 < S$.

On the other hand, there might be a constraint on test time of new samples due to market demand. Here, the proposed method is to prepare more samples in parallel to pass the aging process in shorter time. Such decrease in the test time results increasing reliability as illustrated in the Fig. 3.8 (case 2). The reliability $R_2(> R)$ estimated by the test cdf diagram is used to estimate number of samples needed for testing as $S_2 \geq ln(1-C)/ln(R_2)$, where $S_2 > S$.

3.6 Conclusion

ACT plans have been categorized based on their failure inspection strategies. If many random variables are available, virtual sample method is proposed to replace real test samples with virtual samples in order to estimate non-statistical reliability. This method has been validated for a single-failure mode product under different levels of a stress. In the case of existence many random variables, the number of representative values has to be assessed by the time needed for numerical analysis by computer. Test results obtained by accelerated testing have been modified based on service results. A variety of auxiliary factors is defined to enable manufacturers for comparative analysis of their products. The assumption of the unique acceleration factor for old and new version of design led to estimate service related results for the new version. Regarding the time and sample limitations, reliability level has been changed to compensate for these limitations.

3.7 Appendix: Representative Values

Regarding the complexity of analytical methods to analyze probability distribution functions, these diagrams should be replaced with some representative values so that they could be a reasonable representative of the diagram for numerical analysis. Consider a known continuous probability distribution function $f(X)$ and its cumulative distribution function $F(X)$ of the random variable $X \in [a,b]$. To take S representative values of the random variable X, the interval $[0,1]$ of the vertical axis of the cdf diagram must be divided into S unknown subintervals as d_k ; $k = 1,2,...,S$ $(d_1+d_2+...+d_S = 1)$. The probability of having one value in each subinterval $(s_1 = s_2 = ... = s_S = 1)$ is obtained from the multinomial distribution as [16]

$$Pr(d_1, d_2, ..., d_S) = S!(1 - \sum_{k=1}^{S-1} d_k)^1 \prod_{k=1}^{S-1} \frac{d_k^1}{1!}$$

The best set of d_k $(d_1, d_2, ..., d_S)$ must have the maximum value of probability in the above multinomial distribution, because it shows the closest sets of d_k and s_k for $k = 1,2,...,S$. Hence, the derivative of the probability with respect to every independent d_k ; $k = 1,2,...,S-1$ must be zero, i.e. $\partial Pr(d_1, d_2, ..., d_S)/\partial d_k = 0$; $k = 1,2,...,S-1$. Then, the length of each subinterval is simply estimated as $d_k = 1/S$ for $k = 1,2,...,S$. As a reasonable accuracy, the selected value in each subinterval should be in the middle of the subinterval as illustrated in Fig. 3.9. Then, its related random variable X_k has to be obtained by the cdf diagram.

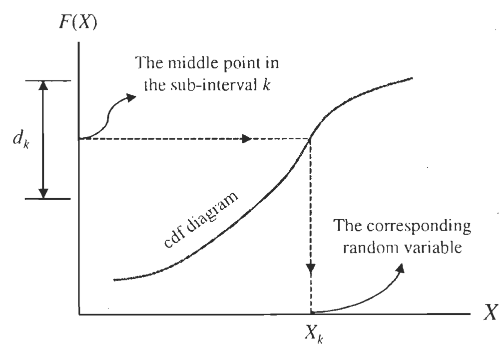


Figure 3.9: A typical representative value of the random variable X from a sub-interval from the cdf diagram

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Chapter 4

Quantitative Accelerated Degradation Testing: Practical Approaches

S. Hossein Mohammadian, Daoud Aït-Kadi and François Routhier

Abstract

The concept of single stress accelerated testing for a product with degrading failure modes needs to be developed for estimating the product reliability. This paper aims at proposing the technical and theoretical approaches required to conduct accelerated degradation testing for reliability analysis. For this purpose, each failure mode is related to a suitable physical property, which is called its performance factor. The failure mode occurs whenever its related performance factor reaches the critical level. The concept is also developed by the superposition principle for multiple stresses, and dependent failure modes. If some service samples (returned from service due to a degrading failure mode) of the product are available, this concept is used to estimate the product reliability. In this case, partial aging method is proposed to considerably shorten test time.
Key words: Accelerated degradation testing; Reliability; service samples; Partial aging method

4.1 Introduction

For a product with a degrading failure mode exposed to an accelerated degradation testing (ADT), there is an uncertainty in estimating its failure time because the lack of any obvious failures in operation. This problem could become more complicated for a multi-failure mode product under some stresses, especially in the case when there are interactions between its failure modes. On the other hand, carrying out an ADT might take long time to degrade samples. Furthermore, there might be no known relationship between the duration of the aging process and the service time. Accordingly, most of ADTs are mainly conducted for studying the degradations of physical properties over time rather than estimating reliability and lifetime. The lifetime of a sample reported as fail (due to a degrading failure mode) by its own user might be unsuitable for reliability estimation because of probable difference between manufacturer's criterion and user's decision to report the failure.

Accelerated degradation testing has widely been used by manufacturers and testers for qualitative explanation of a degradation process, and comparative analysis. The degradation process obtained for different design aspects could be used to predict the most robust case to tolerate service stresses. Knox and Cowling [17] compared the durability of two surface pre-treatments of adhesives. The effects of different stresses (e.g. [7]) and different stress levels (e.g. [31]) on degradations of physical properties of test samples can lead to a modification to design in order to remove (or moderate) their effects on the product.

Although natural and outdoor tests are time consuming and limited to only special levels of stresses (depending on the geographical situation), their results could be useful to make a comparison with corresponding results obtained from the ADT, especially if no field data are available. Such comparison might be used to recognize whether a physical property could be a reasonable representative of product degradation in service (e.g. [4]). One of the most probable reasons of deviation of an ADT (from natural test) is due to high stress levels (e.g. [27, 31]). For more realistic results, degradation of samples could be achieved by a combination of accelerated and natural test (e.g. [11]).

Conducting an ADT is a challenge of physical properties, stress levels and aging of samples [6]. In order to characterize an ADT, every failure mode has to be related to a suitable physical property (performance factor). Then, a critical level of the property has to be specified as failure criterion. The failure mode is considered in the inoperable state if its property is below the critical level (e.g. [18, 13, 3]). Such definition has been presented by Meeker et al. [23] as soft failure for degrading failure modes. This level could be specified based on customer expectation, the physics of the problem, interaction of failure modes, or safety considerations. It might also be suggested by related international and local standards (e.g. [1]).

Performance factors could be measured by any types of destructive and non destructive test. The estimation of yield stress [1, 21], cycles to failure in low cycle fatigue test [2] and tensile strength [16] are destructive, whereas the measurement of volume and pressure [4], flow rate of an oil pump [12], remaining weight of steel due to the corrosion and wear [1, 12], non-corroded area of coatings [11], and the capacity of batteries [3] are considered as non-destructive.

Service samples returned from service due to a degrading failure mode, could be used to estimate product reliability, either by a destructive test (e.g. [22]) or a non-destructive test (e.g. [25]). The estimation of failure time is achieved by relating residual lifetime of each sample with its performance factor. If field samples are still in operation (in service), some specimens could be taken for testing their materials (e.g. [8]). Krivtsov et al. [19] tested used and new car tyres of different brands in order to measure their physical properties. Then, the Cox survival regression model was employed to estimate the most sensitive properties to the aging process. Ranganathan et al. [12] measured different properties of used journal bearings in oil pumps. They also estimated the corresponding results for new journal bearings, and concluded that the service and test results of journal wear volume are consistent, so a correlation was made between test time (in terms of hours) and lifetime (in terms of kilometres).

The main aim of this paper is to present the practical approaches to characterize ADT for estimating product lifetime and reliability. Accelerated degradation testing is categorized into single and multi-aging process for a failure mode under a single-stress. For each category, sample-based and interval-based techniques are proposed to estimate failure time. Degradation diagrams of field samples returned from service due to a probable degrading failure mode claimed by users are estimated by the least square method. The contrast between manufacturer's criterion to accept a failure, and reported failures by users are characterized by relating their lifetimes and physical properties. A general formula for a failure mode of a multi-failure mode product under some stresses has been derived by the superposition principle. The formula is also extended for dependent failure modes. Partial aging method, as a new approach to estimate product reliability by unknown aging process, is introduced to considerably decrease required time for the corresponding ADT.

The paper is organized as follows. Section 2 classifies accelerated degradation testing for single-failure mode products under single stress. Section 3 introduces reliability analysis of samples returned from service due to degrading failure modes. Section 4 presents the general formula for a degrading failure mode of a multi-failure mode product under multiple stresses. Section 5 introduces partial aging method. Section 6 discusses some concluding remarks.

4.2 Single-Stress ADT

4.2.1 Definition

The primary requirement of an ADT is to identify the degrading failure mode of the product under study. Then, the failure mode has to be related to a suitable physical property, which is called performance (factor). For each performance factor, its nominal performance P^N is defined as the target amount of the performance at the beginning of its operation (initial performance). In general, initial performance might be different from sample to sample, i.e. it is a random variable, but in this paper, it is considered as constant and equal to the nominal performance for every sample.

Performance diagram (performance versus time) is assumed to be a continuous descending diagram over time. In the operable state, its value decreases from the nominal performance to critical performance P^C which is defined as the minimum allowable performance of the failure mode being in the operable state. Once the performance factor of a sample reaches P^C, i.e. $P(t) = P^C$, the state of the sample changes from operable to inoperable, so t is called sample failure time. After failure, the amount of performance continuously decreases.

During the aging process, performance should be measured at the end of each time interval by performance test (PT). Selecting the most suitable test as PT strongly depends on its related performance factor. If there are some options for PT, the non-destructive one should preferably be selected, because the destructive nature of a PT increases the sample size. Accordingly, each ADT consists of both aging process (as the main accelerated testing) and PT, whereas there is no performance test in an accelerated catastrophic testing (ACT).

Every ACT is limited to catastrophic failure modes, but ADT could be conducted for both degrading and catastrophic failure modes. There is not censored time data in an ADT, i.e. if the product is highly reliable and the failure of its catastrophic failure mode is not attainable, related ADT method should be used to extrapolate the performance factor to the critical performance.

Performance diagram is usually different from sample to sample, but every diagram might belong to a common performance model (if there is any). For example, the unknown parameters a and b in the quadratic performance model $P(t) = a - bt^2$ specifies different performance diagrams for different samples. For the sample i its performance values at time $t = 0$ and $t = t_a$ are denoted by P^N and P_i respectively, which are obtained from the PT, so

$$a = P^N \quad \text{and} \quad b = \frac{P^N - P_i}{t_a^2}$$

In this section, the product is considered to be single-failure mode under a single stress with a known performance factor and its related non-destructive PT. The values of nominal and critical performance must also be identified prior to the test. Related aging process is assumed to be known, i.e. its time (test time) can be related to service time somehow as follows. In a q-based aging process, samples are under normal service levels of stresses and degradation mechanism is stimulated by increasing the usage frequency of the stress (q) which is defined as the ratio of the time in which service samples are under this stress level to the total service time. In an AF-based aging process, acceleration factor of the aging process (the ratio of the service time to its equivalent test time to acquire the same level of degradation) is assumed to be known. In a stress-based aging process, the results of the test at high levels of stress could be used to estimate acceleration factor by known life-stress model (e.g. [10, 5]).

4.2.2 Single-Aging Process

In the single-aging process, each new sample must first pass the PT in order to obtain its initial performance (before the aging process). According to the above-mentioned assumption, the amount of initial performance must be the same and equal to the nominal performance for every new sample. Accordingly, the implementation of the PT in this stage aims at estimating the nominal performance which could be the mean value of measured initial performance values. If many samples are available for ADT, only a few of them are needed to pass the PT (at this stage) to save time.

Then, the sample has to pass the only aging process for the specified period of time t_a. Finally, it must again pass the PT to obtain its performance after the aging process. For the new sample i, P^N and P_i denote its performance factors before and after the aging process respectively. Performance test is conducted twice on each new sample in a single-aging process, so the performance model could simply be assumed to be linear (Miner's rule). Failure time of the sample is proposed to be estimated by sample-based or interval-based technique as follows.

In the sample-based technique, for the sample i, the time to failure $t_{f,i}$ is estimated as

$$t_{f,i} = t_a \frac{P^N - P^C}{P^N - P_i} \tag{4.1}$$

according to the linear performance model shown in Fig. 4.1. Then the estimated failure time values for S test samples, i.e. $t_{f,i}$, $i = 1, 2, ..., S$ are used for statistical analysis of reliability.

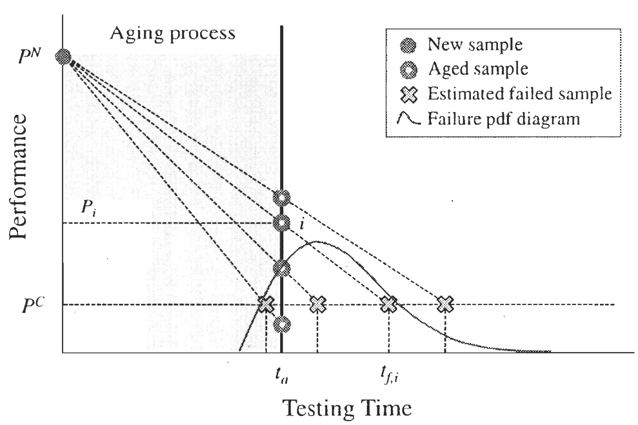

Figure 4.1: The estimation of failure time in single-aging process based on the linear performance model by sample-based technique

Note that the above technique is only valid for new samples. In order to obtain more realistic results, a number of new and used samples might be used together for ADT. For an AF-based aging process, this technique could be used for the service samples to obtain a correlation between test and service times. For the used sample i, let L_i denotes the age of the sample in service, so L_i/AF is its equivalent test time as illustrated in Fig. 4.2. According to the least square method, the best fitted line has to be drawn so that it passes through the point $(0, P^N)$ in performance-time coordinate system. The failure time of the used sample i is estimated as the line intersects the horizontal critical performance line.

For the interval based-technique, virtual sample approach is used as a numerical method to estimate reliability. For this purpose in general, a set of representative values must be taken from each available probability diagram of the random variable x according to Monte-Carlo method. The interval $[0, 1]$ of $F(x)$ (cumulative distribution

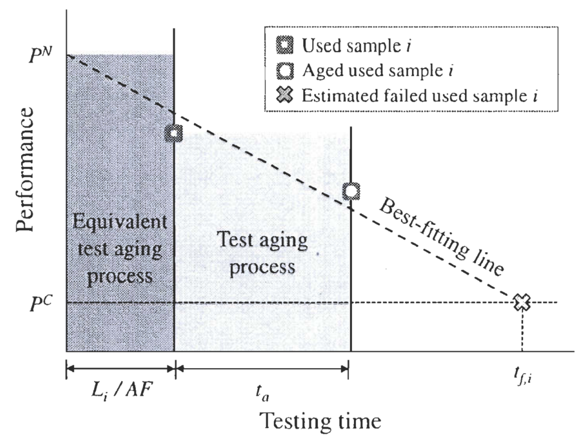

Figure 4.2: The estimation of failure time based on the linear performance model by sample-based technique for used samples

function) must equally be divided into N_x^* (required number of representative values) subintervals so that from each subinterval its middle point $F(x^*)$ should be selected, so its corresponding value of x^* is considered as one representative value. The integration of all such values makes a set of representative values of the probability diagram. According to the problem, a virtual sample must be defined based on such representative values. Then, its failure time has to be estimated by the applied performance model. If S^* virtual samples are defined, non-statistical reliability of the product at the given time t is estimated as

$$R(t) = \frac{S_r^*}{S^*} \tag{4.2}$$

where S_r^* is the number of those virtual samples whose estimated failure times are greater than t. In order to increase the accuracy of estimated reliability, high number of representative values (that results in high number of virtual samples) must be taken from each probability diagram. Note that if some probability diagrams exist in the problem, taking many representative values from each diagram might considerably increase the time needed for numerical analysis by computer.

To apply the above numerical approach for the interval-based technique, the performance values of all samples at the time t_a, i.e. P_i , $i = 1, 2, ..., S$ are used to estimate the performance probability diagram at the time t_a. Then, a number of representative values of performance (S^*) must be taken from the diagram and listed in descending order as P_i^* , $i = 1, 2, ..., S^*$. The performance value P_i^* ranked in position i must be devoted to the virtual sample i at the time t_a as illustrated in Fig. 4.3. Therefore, S^* virtual samples are defined. For the virtual sample i, its failure time $t_{f,i}^*$ is estimated by the Eq. 4.1 for $P_i = P_i^*$ and $t_{f,i} = t_{f,i}^*$. Then, the non-statistical reliability of the product is estimated by the Eq. 4.2.

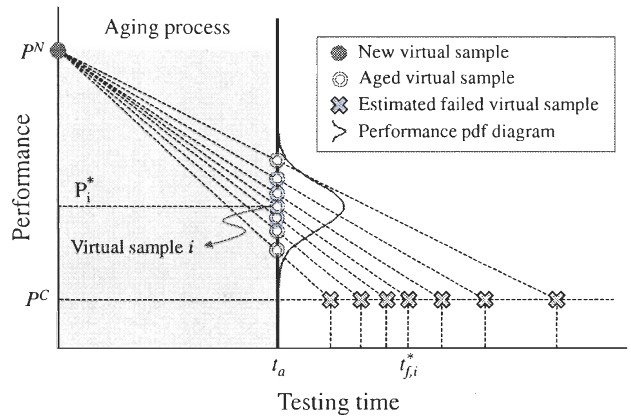

Figure 4.3: The estimation of failure time in the single-aging process based on the linear performance model by interval-based technique

4.2.3 Multi-Aging Process

In order to improve the accuracy of estimated failure time, the aging process must be divided into some time intervals, so a more realistic performance model (including more unknown parameters) could be obtained compared with single-aging process. Therefore, the whole aging process is divided into L time intervals ($L > 1$), and performance must be measured at the beginning of the test, and after each time interval (totally $L+1$ times for each sample). Like the single-aging process, the sample-based or the interval-based technique could be used to estimate failure time.

In the sample-based technique, for each new sample, according to its performance values obtained at the end of time intervals, the best fitted diagram has to be estimated by regression analysis. Then, its failure time is defined as the time the diagram intersects the horizontal critical performance line as shown in Fig. 4.4. All such estimated failure times must be used for statistical analysis of reliability. Because regression analysis is individually done for each sample, there might be no performance model, i.e. each sample has its own performance diagram which might inherently be different from others.

In the interval-based technique, at the end of each time interval, the measured performance must statistically be analyzed to estimate performance probability diagram at the time as shown in Fig. 4.5. Depending on required accuracy, an equal number of representative values (S^*) must be taken from every performance probability diagram, and such representative values for each diagram must be sorted in descending order. Accordingly, L sets of representative values in descending order are obtained. The virtual sample i is defined as the sample including every value of the sets listed in the

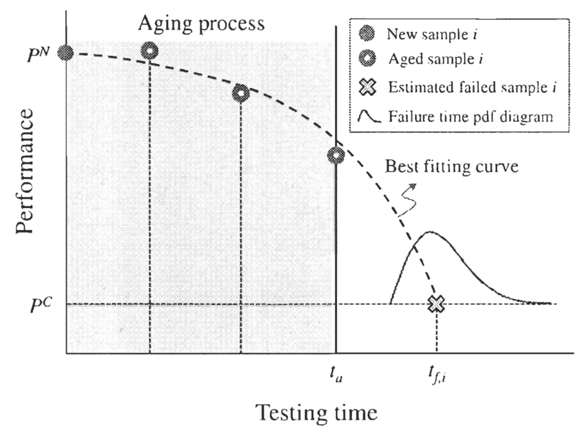

Figure 4.4: The estimation of failure time in the multi-aging process by sample-based technique and regression analysis

ranking i at its own time. Then, its performance diagram and failure time could be estimated by regression analysis. The virtual samples 1 and S^* respectively have the longest and the shortest failure times among virtual samples. Then, non-statistical reliability of the product should be estimated by the Eq. 4.2.

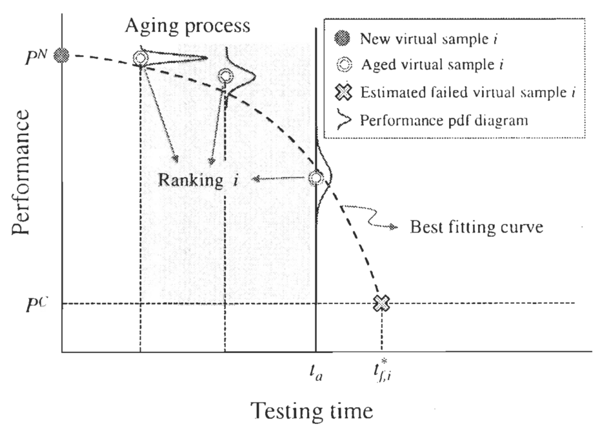

Figure 4.5: The estimation of failure time in the multi-aging process by interval-based technique and regression analysis

4.3 Degradation Analysis of Field Samples

Used samples of a product returned from service due to a degrading failure mode claimed by users and/or technicians, potentially possess valuable information about their degradation process in their own fields. Qualitative explanation of these samples (excellent, good, fair or poor) is not sufficient to predict their failure times for an exact reliability analysis. In addition, such explanation is not able to describe the available usual gap between manufacturer and user's criteria to report a failure.

Primary requirements for reliability analysis of the samples returned from their own fields are the identification of performance factor and its corresponding PT for the failure mode under study. In order to identify the frontier between operable and inoperable states, critical performance has to be specified. The implementation of the PT on a service sample (with its known age) reveals its location in performance-time coordinate system. Furthermore, in order to identify nominal performance, the PT must be conducted on some new samples, so their average performance values could be considered as the nominal performance.

In this section, in order to clarify different aspects in field failure analysis of degrading failure modes, the results of an empirical study on 12-volt valve regulated lead acid (VRLA) batteries used in 24-volt powered wheelchairs (2 batteries for each wheelchair) are presented. The failure nature of such batteries is degrading, and battery capacity, which is the time required to discharge a battery from fully charged to fully discharged, is selected as the performance factor (e.g. [9, 3]). Battery capacity is capable of fulfilling the main requirements of a performance factor as expressed by Mohammadian et al. [24]. It is measurable and sensitive to service time, and it continuously decreases over time. For our application in powered wheelchairs, the above definition has been re-defined as the time required to discharge the battery from $12.7V$ (not fully charged) to $12.0V$ (not fully discharged), because most powered wheelchair batteries are used in this range of voltage. Such modification represents a real connection between the performance factor of batteries and their required application. Critical performance is defined as 80% of the nominal capacity for such batteries by many manufacturers (e.g. [9, 29]). Hereafter, this appointment is referred to as manufacturer's criterion.

Performance test including both charging and discharging tests, are carried out in an approximate ambient temperature of 23 °C, whereas discharging current rate is kept constant at $6A$. Performance test has been conducted on 17 used batteries (claimed as failed by users) and 2 new batteries in order to obtain their capacities as shown in Fig. 4.6 by Mohammadian et al. [25]. Nominal performance (P^N) is calculated as the average capacity values of new batteries ($P^N = 366$ minutes), so critical performance is calculated as $P^C = (0.8)(366) = 292.8$ minutes according to the above-mentioned manufacturer's criterion. The age and performance of the used battery i are denoted by L_i and P_i respectively. The batteries below the horizontal critical performance line are considered as fail, so only 5 out of the 17 batteries are failed based on the manufacturer criterion.

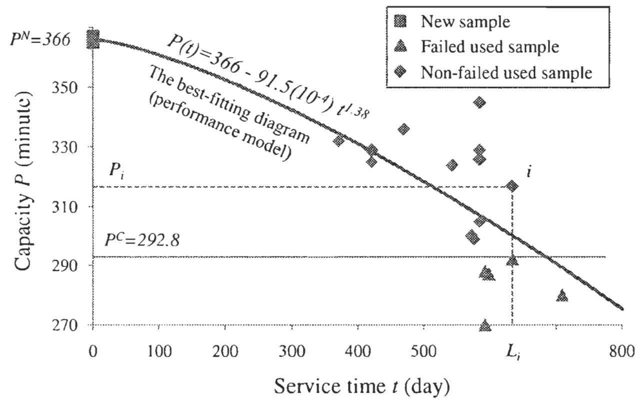

Figure 4.6: Performance-time coordinate system and the best fitted function estimated by the least square estimation

4.3.1 The Best Fitted Performance Model

The least square estimation is used to obtain the best fitted function (curve) among m mathematical functions as $T_j(t)$, $j = 1, 2, ..., m$. The square estimator of the j^{th} function (SE_j) is defined as

$$SE_j = \sum_{i=1}^{S} (\frac{T_j(L_i) - P_i}{L_i})^2 \tag{4.3}$$

where $T_j(L_i)$ is the estimated performance of the used sample i by the function j at the service time L_i, and S is the number of used samples. According to the least square method, the function j is the best fitted function if

$$SE_j < SE_k \ , \ k = 1, 2, ..., m \ , \ k \neq j \tag{4.4}$$

The least square method could also be used to estimate unknown parameters of a specified performance model. Then, all parameters must be kept constant at their own estimated values except one of them that has exclusively be obtained for each used sample based on its performance value at its age. As an illustrative example, we present the model $P(t) = P^N - at^b$ with two unknown parameters a and b for the wheelchair batteries. According to the least square estimation, the parameters a and b should be obtained so that they could minimize the square estimator SE. By a simple MATLAB program, the parameters a and b are estimated as $91.5(10^{-4})$ and 1.38 respectively as illustrated in the Fig. 4.6. The performance model $P(t) = P^N - at^{1.38}$ (by keeping b at its estimated value) is one of the best fitted models for the wheelchair batteries with the unknown parameter a. For the sample i, the unknown parameter a_i is obtained so that its performance sets to P_i at the service time L_i. Consequently, related unknown parameter and failure time of the sample i are obtained as

$$a_i = \frac{P^N - P_i}{L_i^{1.38}} \quad \text{and} \quad t_{f,i} = (\frac{P^N - P^C}{a_i})^{1/1.38}$$

Reliability of the wheelchair batteries during the first year in service is estimated by the weibull pdf diagram on the 17 used wheelchair batteries by some known performance models as illustrated in Fig. 4.7. It can be concluded that the selected performance model can considerably affect the estimated reliability of the product.

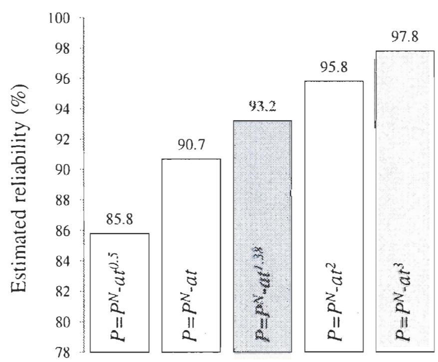

Figure 4.7: Comparison of reliability for different performance models

4.3.2 User-Based Criterion

For most catastrophic failure modes, the failure reported by a user is often acceptable by the manufacturer because the failure is obvious. For degrading failure modes, there might be a contrast between user and manufacturer's decisions to report a failure. Usually, critical performance is the base criterion to define failure from the manufacturer point of view. Such criterion is often in contrast to user expectation. In addition, the user satisfaction level might be different from one to another. For example, a sample of a product reported as failed by its user might still be in operable state according to the manufacturer's criterion. On the other hand, another sample might have already failed according to the manufacturer's criterion, but it is still satisfactorily used by its own user. Such contrasts motivate manufacturers and retail users (not real users) to specify the gap between manufacturer and users' criteria.

Returning sample from service due to a probable degrading failure mode is the result of customer and/or technician's decision to report it as a failed sample. The user-based criterion states that the age of a field-returned sample must be considered as its failure time regardless of the manufacturer's decision to accept or refuse the reported failure. So, for the used sample i, $t_{f,i} = L_i$. All these failure times are used for statistical analysis.

There are 17 used batteries belong to 12 different wheelchairs. To avoid entering extra failure time data, for each pair of batteries used in the same wheelchair, one failure time must be considered. Fig. 4.8 illustrates the weibull failure probability diagrams of batteries based on the best fitted performance model and the user-based criterion. The reliability is estimated as 97.4% and 93.2% at the first year based on the users and the manufacturer's criteria respectively, so fewer failed batteries are reported during the first year by users compared to the expected value estimated by the manufacturer's criterion. Surprisingly, more than 96% of batteries are returned between the first and the second years, whereas only 42.2% of them are expected to be returned during this period of time. The main reason of this major contrast might be due to users' perception that battery capacity considerably decreases after the first year.

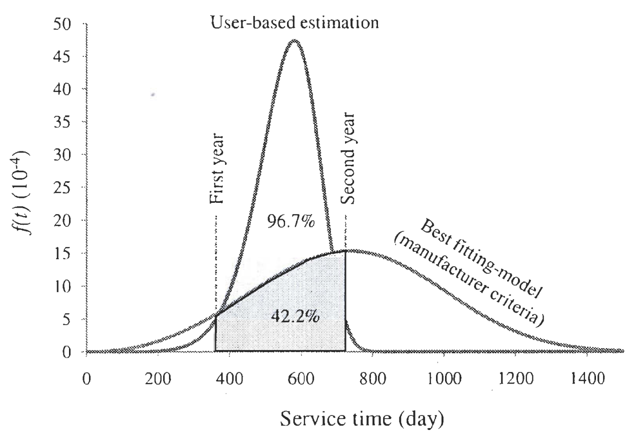

Figure 4.8: Weibull probability diagrams for batteries in 12 wheelchairs according to the manufacturer and user-based criteria

The user-based criterion could be stated in terms of performance values of samples. If each field-returned sample is accepted as a failed sample, then its performance must be considered as a critical performance. Hence, the critical performance is defined as a random variable introduced by field samples. The capacities of 17 field batteries are used to estimate the normal distribution diagram for critical performance as shown in Fig. 4.9. The average value of the random critical performance is estimated as 310.8 minutes, so there is a huge gap between critical performance defined by the manufacturer (292.8

min) and the one estimated by field samples (310.8). It is also concluded that almost near 20% of returned batteries are considered as failed by the manufacturer's criterion, and others are in the acceptable levels of capacity.

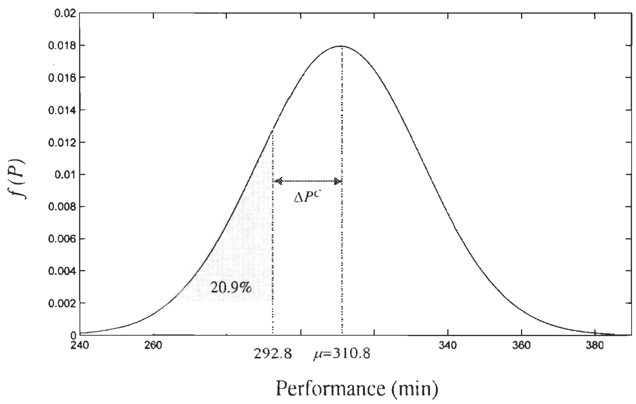

Figure 4.9: The normal distribution diagram for critical performance according to the user-based criterion

4.4 Generalized ADT

For a single-failure mode product under a single stress, accelerated degradation testing can estimate its performance model, failure times of its samples, and finally product reliability. Such problem could become complicated if the product is multi-failure mode, and its samples are under more than one stress in service. For each stress, an exclusive known aging process must be defined while all other stresses are inactive. Then, the effect of the stress must individually be estimated on every failure mode in order to estimate its performance model due to the stress. The existence of random variables inside performance models and usage profile make reliability estimation more complicated (or impossible) to be solved by analytical methods.

Unit-less damage (factor) at the time t is defined as [28, 20, 14]

$$D(t) = \frac{P^N - P(t)}{P^N - P^C} \tag{4.5}$$

where damage value is zero at the beginning of its operation and one at the failure time. After the failure, the amount of damage stands constant at one for catastrophic failure modes, whereas it tends to increase over one for degrading failure modes. Damage diagram (damage versus time) is assumed to be a continuous ascending diagram. Damage factor $D(t)$ and performance factor $P(t)$ of a failure mode are related together by the Eq. 4.5, so failure time could be estimated by using either performance or damage diagram.

For every physical property in a multi-failure mode product, performance-time coordinate system is scaled based on its physical unit. The performance diagram of other failure modes cannot be drawn in this coordinate system, because the coordinate system has already been scaled based on the first physical unit which might be different from the second one. The damage diagrams of all physical properties could be drawn together in a single coordinate system, because damage is a unit-less factor. Therefore, such illustration facilitates the observation and comparison of failure modes degradations.

For a failure mode under a specified stress, damage diagram could be different from sample to sample, but all diagrams might belong to a common damage model. According to the applications and stresses, variety of damage models is available [26, 30]. a Damage model depends on material, design, manufacturing process, applications and users. The most suitable damage model could be recognized by an apparent observation of available damage values in damage-time coordinate system. The best fitted model could also be detected by the least square method.

4.4.1 Independent Failure Modes

A failure mode is called independent if other failure modes cannot have any effects on its damage factor. Accelerated degradation testing on a product including N independent failure modes under M stresses must individually be performed for every stress. It is assumed that every combination of stresses in service levels is not able to activate another failure mechanism than the ones that have already been detected under the individual stresses, otherwise such combination must be considered as a new stress, and the new failure mode has to be included in the product.

Destructive nature of PT increases sample size and total time of accelerated testing. If the PT of every failure mode is non-destructive, for each stress, one set of test samples is needed for the ADT. An exclusive set of test samples is needed for any performance factor whose PT is destructive.

The mathematical function $G_{j,l}(t_a)$ denotes the test damage model of the failure mode j under the stress l (as the only stress while others are not active) in terms of the time of the aging process t_a. Such model obtained from ADT must be extended to its corresponding service model as follows. If the aging process is q-based, the time t_a must be replaced with $q_l t$, where q_l is the usage frequency of the stress l, and t is the service time. Service damage model is then expressed as

$$D_{j,l} = G_{j,l}(q_l.t) \tag{4.6}$$

The equivalent model for an AF-based aging process is expressed as

$$D_{j,l} = G_{j,l}(\frac{t}{AF_{j,l}}) \tag{4.7}$$

where $AF_{j,l}$ is the acceleration factor of the aging process.

The expression $dD_{j,l}(t)/dt$ is the time derivative of the damage model that is called damage rate (e.g. [15]), and it is assumed to be a function of D_j rather than the service time t. The damage rates of the q-based and AF-based aging processes are respectively obtained from the Eqs. 4.6 and 4.7:

$$\frac{dD_{j,l}(t)}{dt} = q_l \dot{G}_{j,l}(D_j) \tag{4.8}$$

$$\frac{dD_{j,l}(t)}{dt} = \frac{1}{AF_{j,l}} \dot{G}_{j,l}(D_j) \tag{4.9}$$

where $\dot{G}_{j,l}(D_j)$ is the time derivative of $G_{j,l}(t)$. It is concluded that for the failure mode j under the stress l, the constant acceleration factor in AF-based aging process and the random variable usage frequency in corresponding q-based aging process have an inverse relationship in above formulas. For instance, if the test damage model of the failure mode j under the stress l is obtained by an ADT as quadratic form $G_{j,l}(t_a) = ct_a^2$ where c is the model parameter, the service damage rates of the failure mode for q-based and AF-based aging processes are obtained as

$$\frac{dD_{j,l}(t)}{dt} = q_l(2\sqrt{c.D_j}) \tag{4.10}$$

$$\frac{dD_{j,l}(t)}{dt} = \frac{1}{AF_{j,l}}(2\sqrt{c.D_j}) \tag{4.11}$$

where damage rates are not dependent on time.

The damage rate $dD_{j,l}(t)/dt$ of the failure mode j is only the one caused by the stress l. According to the superposition principle, the total damage rate of this failure mode is equal to the summation of every damage rate, each caused by its own related stress. Therefore, the general differential equation for the failure mode j is expressed as

$$\frac{dD_j(t)}{dt} = \sum_{l=1}^{M} \frac{dD_{j,l}(t)}{dt} \tag{4.12}$$

where $D_j(t)$ is the (total) damage rate of the failure mode j. The above equation for all failure modes, i.e. $j = 1, 2, ..., N$ makes a system of N non-linear independent first order differential equations for the product.

In general, there exists no analytical solution for the above system of equation regarding the existence of many random variables, so it should be solved numerically. Here, the virtual sample method is used by representative values of probability diagrams. For every stress, if its related aging process is q-based, a number of representative values must be taken from its usage frequency probability diagram. Representative values must also be taken from each failure probability diagram obtained by its related ADT. Then, such failure times should be used to estimate representative damage diagrams based on the known damage models as typically illustrated in Fig. 4.10 for the failure mode j under the stress l by linear damage model.

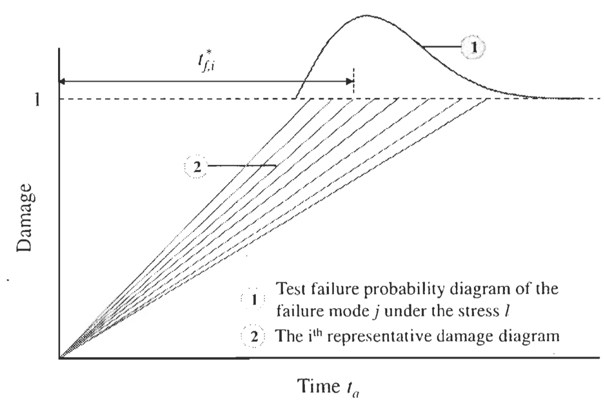

Figure 4.10: Obtaining representative damage diagrams (linear damage model $D = ct$) from the known failure probability diagram

For an AF-based aging process, each failure probability diagram must be modified based on its known service-related diagram in order to achieve a unique acceleration factor (e.g. same shape factor for both test and service weibull diagrams).

A virtual sample is identified by its damage diagrams (a damage diagram from each damage model) and usage frequencies (a value from each set of usage frequency representative values of a stress if its related aging process is q-based). The number of virtual samples is

$$S^* = \prod_{j=1}^{N} \prod_{l=1}^{M} S^*_{j,l} \cdot \prod_{l=1}^{M} S^*_l \tag{4.13}$$

where $S^*_{j,l}$ is the number of representative damage diagrams for the damage model of the failure mode j under the stress l, and S^*_l is the number of representative values of the usage frequency of the stress l if the related aging process is q-based. Hence, for each virtual sample, a system of equations (like Eq. 4.12) is obtained. The system can be solved by the finite difference approximation as

$$\frac{D_j^*(t+\Delta t) - D_j^*(t)}{\Delta t} = \sum_{l=1}^{M} \frac{dD_{j,l}^*(t)}{dt} \quad , \text{for } j = 1, 2, ..., N \tag{4.14}$$

where the accent * denotes a virtual sample. The time interval Δt specifies the precision and the speed of calculation. The step by step calculation leads to obtain the damage factor of each failure mode at any time. According to the overall assumption in this paper, the initial value of damage for each failure mode should be considered zero, i.e. $D_j^*(0) = 0$, $j = 1, 2, ..., N$, otherwise initial damage must be considered as a random variable, and its representative values must also be applied to the system of equation.

The failure mode j of a sample occurs whenever its damage D_j reaches 1, but it doesn't mean the sample fails. For this purpose, failure of the product has to be defined based on its failure modes and their connections which might be series, parallel, k-out-of-n, stand-by etc. According to the definition of the product, the damage factor of the product could be defined as its proximity to the inoperable state. For example, for series and parallel products, their damage factors could be defined as the highest and the lowest damage factors of their failure modes respectively. At any given time t, according to the number of virtual samples (S^*) and the number of survived virtual samples (S_r^*), non-statistical reliability of the product is estimated by the Eq. 4.2.

As an illustrative example, consider a product with two independent series failure modes under two stresses. The first ADT is an AF-based aging process that has no effect on the second failure mode, whereas the second ADT is a q-based aging process. The characteristics of both aging processes, scale and shape factors of each failure probability diagram, and service damage models are presented on Table 4.1.

Table 4.1: Characteristics of the aging processes, failure probability diagrams, and damage models

	First ADT (under the stress I) AF-based aging process					Second ADT (under the stress II) q-base aging process: μ=0.09, σ = 0.03		
Failure mode	α (day)	β	AF	β_s	Damage model	α (day)	β	Damage model
I	190	7.5	10	2.8	$G_{1,1}=at/AF$	120	8.1	$G_{1,2}=b\,(qt)^2$
II						65	7.5	$G_{2,2}=c\,(qt)$

For each virtual sample, the random variable q must be taken from related probability diagram of the usage frequency, and unknown parameters a, b and c have to be obtained from test failure probability diagrams. In other words, each set of (q, a, b, c)

specifies an exclusive virtual sample. For every virtual sample, its system of equations is written as (see the Eq. 4.10)

$$\frac{dD_1^*(t)}{dt} = \frac{a}{AF} + 2q\sqrt{bD_1^*}$$
$$\frac{dD_2^*(t)}{dt} = cq$$

Because the product is assumed to be series, the time of the first failure mode occurrence must be considered as the failure time of the virtual sample. Finite difference approximation has been used to solve the above system for each virtual sample in time step $\Delta t = 1$ day.

By a MATLAB code, reliability of the product at the first year is estimated and drawn in Fig. 4.11 in terms of the number of virtual samples, and it is estimated as 92.6% for large number of virtual samples.

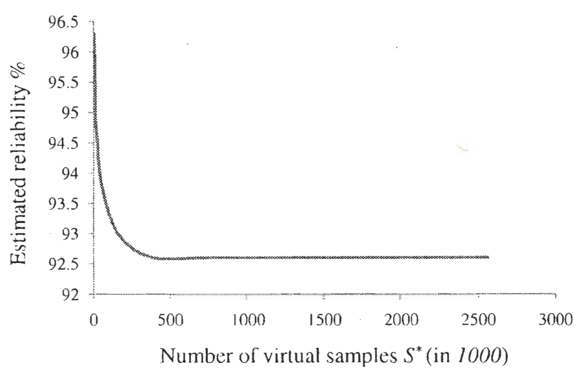

Figure 4.11: The estimated reliability in terms of the number of virtual samples by the finite difference approximation

Occurrence probability of a failure mode is defined as the probability of returning samples from service due to that failure mode (among failed samples). Virtual sample method is also able to estimate occurrence probability of each failure mode in order to predict the most probable failure mode and non-relevant failure mode (if there is any). The diagram in Fig. 4.12 shows the occurrence probabilities of the first and the second failure modes versus the number of virtual samples, and they are respectively estimated as 63.2% and 36.8%, i.e. the first failure mode is more likely to happen than the second one.

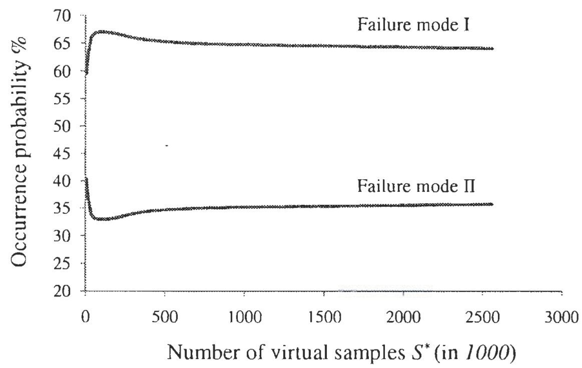

Figure 4.12: The estimated service occurrence probabilities of the failure modes in terms of the number of virtual samples

4.4.2 Dependent Failure Modes

The ability of ADT to estimate reliability for a product with dependent failure modes is one of ADT's most distinctive advantages over ACT. If the failure mode k can stimulate the degradation process of the failure mode j, then the failure modes j and k are called dependent and cause failure modes respectively. The general formula presented for independent failure modes (Eq. 4.12) is not valid for dependent failure modes, and it must be modified based on the physics of the problem and the type of the cause failure mode (catastrophic or degrading).

Catastrophic Cause Failure Mode

The catastrophic cause failure mode k during its operable state usually has no effect on the dependent failure mode j. The effect of the failure of the cause failure mode could be like a physical impact on the dependent failure mode so that it could suddenly increase the stress level afterwards, and consequently, change its damage diagram.

The system of beam and rope shown in Fig. 4.13 under the mechanical force F, has two failure modes which are the dependent failure mode j due to breaking the beam at point I, and the cause failure mode k due to the catastrophic failure of the rope. Once the rope fails at the time t_k, the stress at the point I suddenly increases so that it will result stimulating degradation of the beam at the point I. It is assumed that the physical impact on the failure mode j at the time t_k cannot activate degradation mechanism of other failure mode (than the one under study), otherwise the new failure mode has to be considered in the analysis.

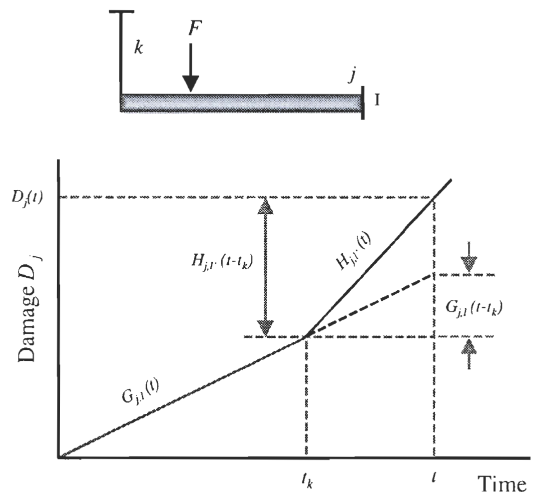

Figure 4.13: The effect of the catastrophic failure of the failure mode k on the failure mode j to increase its damage rate (linear model) for the system of beam and rope

In order to obtain the damage rate formula for the dependent failure mode j, ADT must individually be done on the system for both stress levels (stress level l before and stress level l' after the catastrophic failure) in order to obtain their damage models. Then, the general formula for the independent failure mode (Eq. 4.12) must be modified to be applicable for the dependent failure mode j as

$$\frac{dD_j(t)}{dt} = (1 - \phi^0(D_k))\,\dot{G}_{j,l}(D_j) + \phi^0(D_k)\,\dot{H}_{j,l'}(D_j) \tag{4.15}$$

where $\dot{G}_{j,l}(D_j)$ and $\dot{H}_{j,l'}(D_j)$ are damage rate models (Fig. 4.13) due to the stress l and l' respectively, and $\phi^0(D_k)$ is the zero order singularity function defined as

$$\phi^0(D_k) = \begin{cases} 0, & \text{if } D_k < 1 \\ 1, & \text{if } D_k \geq 1 \end{cases}$$

Degrading Cause Failure Mode

Regarding the physics of the product and the connection between its failure modes, the damage of the degrading failure mode k (not its failure) might continuously stimulate degradation mechanism of the failure mode j. In other words, the total damage rate of the failure mode j could be affected by the damage level of the failure mode k.

The original system of beam and spring shown in Fig. 4.14 includes the cause failure mode k due to continuous degradation of the spring constant (k) and the dependent failure mode j at point I. The supported mechanical force by the spring decreases over time because of degradation in the spring constant. Consequently, the level of mechanical stress at the point I continuously increases, and it will cause speeding up the damage rate of the failure mode j. The general formula for the failure mode j could be modified as

$$\frac{dD_j(t)}{dt} = A_{j,k}(D_k).\dot{G}_{j,l}(D_j) \tag{4.16}$$

where $A_{j,k}(D_k)$ is called dependency function of the failure mode j on the failure mode

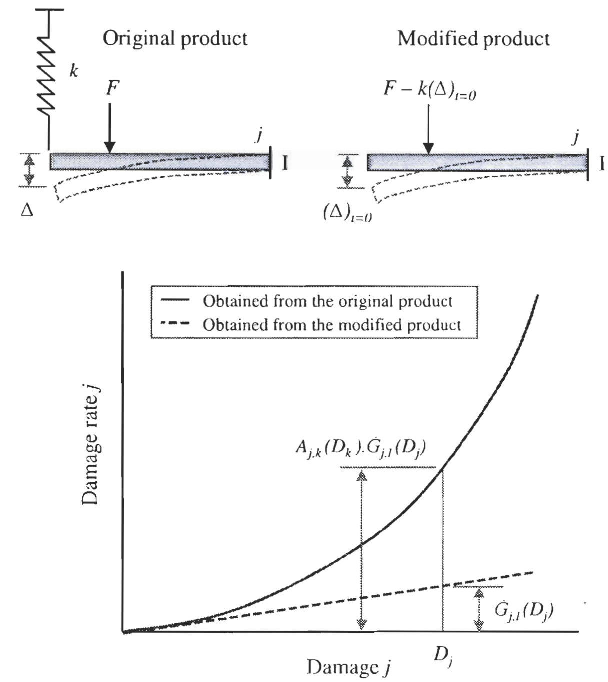

Figure 4.14: The continuous effect of the degrading failure mode k on the failure mode j to increase its damage rate for the original system of beam and spring

k, and it shows the tendency of the failure mode j to increase its damage level in comparison to the situation it is independent. Note that the function $\dot{G}_{j,l}(D_j)$ is the damage rate of the failure mode j without any effects from the failure mode k. For each level of D_k, $A_{j,k}(D_k)$ must be greater than one.

In order to estimate the dependency function, ADT must individually be conducted for the original and modified products (Fig. 4.14) as follows:

- ADT on the original product: The ADT test on the original product results in the damage rate diagram of the failure mode j by considering the effect of the failure mode k.

- ADT on the modified product: In order to remove the effect of the failure mode k, the original product must be modified so that the stress level on the failure mode j is kept constant and equal to its level at the beginning of its operation while damage of the failure mode k is zero.

The diagram in the Fig. 4.14 clearly shows the difference between damage rate diagrams obtained by the above ADTs. The dependency function should be estimated by dividing the first damage rate model by the second one. The diagrams also show the individual effects of the stress F and the failure mode k on the dependent failure mode j. This issue could be interesting for design engineers to decide necessary modifications to design in order to decrease the damage of the failure mode j.

4.5 Partial Aging Method

The concept of the partial aging method proposed here for a single-failure mode product is based on aging a few new and some available used samples (with their known ages) in a relatively shorter time than a regular accelerated degradation testing by an unknown aging process. If many used samples are available, some of them should be selected for testing so that they could cover a wide range of ages and applications. Critical performance must be identified prior to the test. In order to make an estimation of the time of a partial aging process, there should be an approximate prediction of the whole aging process (t_a), i.e. required time to fail a new sample.

To implement the method, each sample (new and used) must firstly pass the PT in order to estimate its damage before the aging process. Then the sample must pass a portion of the whole aging process for the time period of δt ($\delta t \ll t_a$). Finally the sample has to again pass the PT to estimate its damage after the partial aging process. Note that, the unknown aging process used in this method distinguishes it from the single-aging process.

Let $D_{i,1}$ and $D_{i,2}$ be the damages of the sample i before and after the partial aging process respectively, so the approximate damage rate of the sample at the damage level $D_{i,1}$ could be estimated as

$$\dot{D}_{i,1} = \frac{D_{i,2} - D_{i,1}}{\delta t} \tag{4.17}$$

The geometrical definition of damage rate for a sample is the gradient of its linear damage diagram (over the span of δt) as shown in Fig. 4.15 for the new sample j and the used sample k.

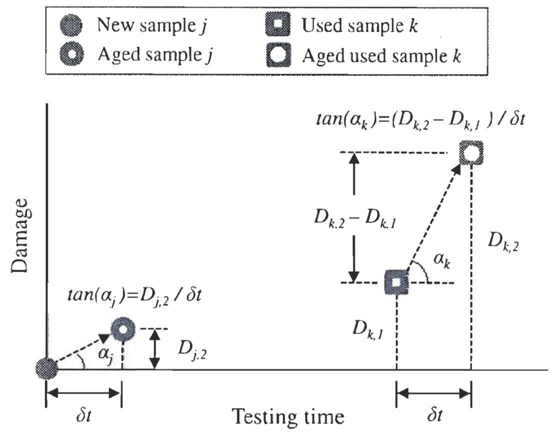

Figure 4.15: The damage rate of the new sample j and the used sample k in damage-time coordinate system

The main assumption in this method is that the damage rate of the product dD/dt is a function of the damage level D (not the age). Accordingly, for each sample, its damage before the aging process and its damage rate estimated by the Eq. 4.17 make a couple (a point) in the two dimensional coordinate system in which the damage rate is expressed in terms of damage. By the least square method, the best fitted function (curve) could be estimated as

$$\frac{dD}{dt} = g_a(D) \tag{4.18}$$

where the subscript a denotes aging process, and the mathematical function $g_a(D)$ is called damage rate model (DRM). Unlike performance and damage models, there is not any unknown parameter in DRM, because it represents an average function of damage rate. In order to obtain the damage rate diagram for the i^{th} used sample, i.e. $g_{a,i}(D)$, the difference between its damage rate (estimated by the Eq. 4.17) and the damage rate value obtained by DRM at the damage level $D_{i,1}$ specifies the deviation of $g_{a,i}(D)$ from DRM as illustrated in Fig. 4.16 and estimated by

$$g_{a,i}(D) = \frac{\dot{D}_{i,1}}{g_a(D_{i,1})} \, g_a(D) \tag{4.19}$$

The above differential equation could analytically (or numerically) be solved for the sample i in order to estimate its failure time, i.e. the time its damage reaches 1. Note that the initial value of damage (at the time $t = 0$) for every used sample is assumed to be zero.

The test damage rate diagram for the used sample i, i.e. $g_{a,i}(D)$ is used to estimate its service damage rate. For this purpose, the time in test t must be transferred to the

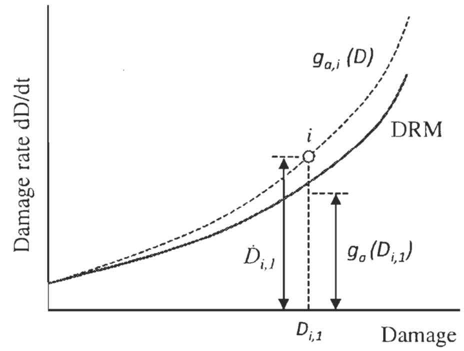

Figure 4.16: Obtaining damage rate diagram for the used sample i from the known damage rate model (DRM)

corresponding time in service t_s by defining the acceleration factor of the sample i as $AF_i = t_s/t$ which is unknown. Then (see the Eq. 4.9)

$$\frac{dD}{dt_s} = \frac{1}{AF_i} g_{a,i}(D) \tag{4.20}$$

The above differential equation should be solved so that service damage diagram (in terms of time t_s) must include the point $(L_i, D_{i,1})$ where L_i is the age of the sample i. Then, the unknown acceleration factor of the sample i, and consequently, its service damage could be obtained.

The damage factors of 7 used samples and a new sample before and after the partial aging process, are shown on Table 4.2. The approximate whole aging process time is considered as 1000 hours, so the partial aging process time has been selected as $\delta t = 100h$. Although the initial damage of used samples is considered zero, but this value for the only new sample is not zero. The weibull pdf is used to statistically analyze the test and service failure times. Scale parameters of the aging process and service are obtained as $\alpha_a = 884.9$ hours and $\alpha_s = 5798.1$ hours respectively, so the acceleration factor of the whole aging process is estimated as 6.55. In fact, the samples are aged for only 100 hours, so the acceleration factor of the partial aging is estimated as $5798.1/100 = 58.0$ which is much higher than the one for the whole aging process.

4.6 Conclusion

Accelerated degradation testing has been categorized based on the number of its aging process and performance test as single-aging and multi-aging processes. For both

Table 4.2: The partial aging method for 7 used samples and 1 new sample to estimate the acceleration factor of the whole aging process and the partial aging process

Samples		Damage		$\dot{D}_{i,1}$ (10^{-4}) $\delta t=100h$	$g_a(D)$	$\dot{D}_{i,1}/g(D_{i,1})$	$t_{f,i}$	AF_i	$t_{s,i}$
i	$L_i(h)$	$D_{i,1}$	$D_{i,2}$						
1	0	0.01	0.06	5	By least square method based on linear damage rate model: $g_a(D)=k.D+l$ $k=17.38$ (10^{-4}) $l=6.18$ (10^{-4})				
2	850	0.14	0.22	8		0.9288	829.0	4.13	3426.6
3	1870	0.13	0.24	11		1.3034	590.8	13.60	8031.7
4	2790	0.39	0.49	10		0.7718	997.8	5.05	5042.8
5	3450	0.69	0.81	12		0.6604	1166.0	3.67	4280.6
6	3790	0.53	0.75	22		1.4295	538.7	10.32	5558.4
7	4600	0.85	1.04	19		0.9069	849.1	5.94	5041.8
8	5110	0.86	1.11	25		1.1834	650.7	8.55	5563.2
					α=Scale factor of the Weibull function		$\alpha_a=884.9$		$\alpha_s=5798.1$

sample and interval-based techniques to estimate failure time, the multi-aging process is suggested in order to achieve more accurate estimation of reliability.

The accuracy of analyzing the results of field samples of a product strongly depends on its estimated performance model. The best fitted diagram (obtained by the least square estimation) has been proposed to be selected as the performance model. The gap between estimated reliability by the manufacturer's criterion and reported failures by users has been characterized by their estimated failure and performance probability diagrams.

A general formula is developed for failure modes of a multi-failure mode product under some stresses. Regarding the complexity of analytical methods to solve the system of differential equations due to the existence of random variables, virtual sample method has been introduced as a numerical technique to estimate non-statistical reliability. It has been concluded that the accuracy of virtual sample method has a direct relationship with selected number of virtual samples. The general formula has also been extended for a dependent failure mode. If new and used samples of the product are available, the partial aging method is proposed for an unknown aging process to considerably decrease required aging time.

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Chapter 5

General Conclusions

The first steps to conduct an accelerated testing include the identification of accelerating variables, test samples and usage profile. Each AT must be validated to address the same failure modes in test and service. Field data of a catastrophic failure mode are analyzed based on warranty and afterwards periods, and time-based or number-based analyses. The uncertainty for quantitative analysis in an ACT might be due to random variables, any types of shortage in time-based field data (especially for multi-failure mode products), the unknown aging process, and the unknown relationship between the test and service results of the old and new versions of design.

The existence of many random variables in the usage profile and failure time is the cause of complexity in an AT and disability of available analytical methods to solve the problem. The virtual sample method is proposed to estimate non-statistical reliability by replacing continuous probability diagrams with their representative values. Each virtual sample is defined according to its representative values. Then, non-statistical reliability is estimated. The virtual sample method is applied for unique and multi levels of usage profile of a q-based aging process. The selection of more virtual samples increases the accuracy of the estimated reliability.

For a single-failure mode product, test results have been modified based on related time-based service data. These data for a multi-failure mode product present the most probable and irrelevant failure modes. For these products, the conformity and vicinity factors can effectively demonstrate the proximity of test and service results.

A relationship is established between the service and test results of the old and new versions of design based on a unique acceleration factor. The limitations in test samples and test time for the new version are discussed so that the shortage of one of them could be compensated for the other one.

The concept of accelerated degradation testing for multiple stresses is developed by the superposition principle. The definition of unit-less damage factor is presented to show the degradation of different physical properties in one diagram. Damage rate of every failure mode is assumed to be the function of its damage level (not time). The system of equations for a multi-failure mode product is solved by the virtual sample and finite difference methods to estimate reliability. The concept of ADT has also been used to estimate failure time of a sample returned from service due to a degrading failure mode. This estimation leads to identify the difference between user's decision to report a failure, and manufacturer criterion to accept the failure. A major contrast has been obtained for the problem of wheelchair batteries.

The partial aging method is proposed for an unknown aging process while both new and used samples are available. For the presented illustrative example, the most distinction of the method to increase the acceleration factor has been confirmed.

Every previous qualitative ADT could be converted to quantitative ADT by defining its related critical and nominal performance factors as presented in the concept of ADT. Accordingly, non-statistical reliability of the product will be estimated.