

Providing Global Public Goods under Uncertainty[†]

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Abstract

We study how uncertainty and risk aversion affect international agreements to supply global public goods. We consider a benchmark model with homogeneous countries and linear payoffs. When countries directly contribute to a public good, uncertainty tends to lower signatories' efforts but may increase participation. Despite risk aversion, uncertainty may improve welfare. In contrast, when countries try to reduce a global public bad, uncertainty tends to increase signatories' efforts and decrease participation. In that case, an ex-ante reduction of uncertainty may have a large positive multiplier effect on welfare.

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1 Introduction

In this paper, we study how uncertainty and risk aversion affect international agreements to supply global public goods. When a public good is truly global, as with climate, disease eradication, or research on energy, the effort of one country benefits to all.¹ The problem of free-riding tends to be exacerbated by the international dimension. The principle of national sovereignty means that countries cannot be coerced into joining an agreement to supply the public good. Countries staying out of the agreement free-ride on those within. Given the importance of this issue, it is critical to understand the determinants of international cooperation and how it could be facilitated.

Uncertainty is a key feature of most global public goods. Consider, for instance, research on nuclear fusion. Many countries have agreed to fund coordinated work on an experimental fusion reactor.² Yet, the chances of success and ultimate social benefits of the project are very uncertain. Uncertainty, of course, also plays a central role in the science, economics and politics of climate change. The link between actions (CO₂ emissions) and consequences (damages due to climate change) is subject to multiple layers of uncertainty. This has greatly hindered the establishment of effective international cooperation to address “the biggest market failure the world has seen”, see Stern (2008).

Under risk aversion, uncertainty itself adds significant costs. Indeed, this partly explains the magnitude of the welfare loss estimates obtained in the Stern review, see Stern (2007). More generally, economists working on integrated assessment models usually assume risk aversion and this has first-order effects on the outcomes, see e.g. Nordhaus (1994), Heal and Kriström (2002).

¹Barrett (2008) presents a unifying analysis of the various types and properties of global public goods.

²This project, ITER, is currently supported by the European Union, China, India, Japan, South Korea, Russia and the United States, see the project’s website (www.iter.org) for more information.

In these models, an ex-ante reduction of uncertainty has straightforward positive effects. While this property is well-grounded in situations with a unique decision-maker, its validity in a strategic setting is less clear. How does uncertainty affect the emergence of international cooperation aimed at supplying a global public good? Does an ex-ante reduction of uncertainty always help cooperation, or could it have negative effects? Through what channels does uncertainty affect the incentives to join or quit an international coalition? How does uncertainty affect welfare in a second-best context characterized by partial, endogenous cooperation? We seek to provide formal answers to these questions.

To do so, we extend the model of treaty formation with linear payoffs discussed in Barrett (2003). This model is simple and well-understood. It provides a natural benchmark to incorporate and analyze new features.³ Countries face a n -player prisoner's dilemma and can join a coalition trying to take collective action. We consider two formulations of the model: one where countries contribute to a global public good, such as R&D, and one where countries seek to reduce the production of a global public bad like CO₂ emissions. In both formulations, we introduce two new assumptions. First, the benefits from collective action are subject to uncertainty.⁴ This notably captures uncertainty on pollution damages. Second, countries have risk averse preferences.⁵ We study how uncertainty and risk aversion affect the outcomes of the treaty formation game and especially effort, participation and welfare in equilibrium.

We find strong effects. Surprisingly, the two formulations yield radically different outcomes. Uncertainty tends to increase participation and improve welfare in the public good formulation.

³And, indeed, many authors have built on this model, see e.g. Barrett (2001), Kolstad (2007), Rubio & Ulph (2007), Ulph (2004).

⁴We also study an extension of the baseline model where the costs of action are subject to uncertainty.

⁵Economists often view countries as risk neutral due to their size and to the possibility to pool independent risks across individuals, see Arrow and Lund (1970). This view, however, is not appropriate to think about climate change and most other global issues. Climate risks are highly correlated within communities and potential damages are large, involving a sizable portion of the economies even in conservative estimates.

In contrast, it tends to reduce participation and welfare in the public bad one. This difference in outcomes arises from the different consequences of free-riding in the two models. In either case, countries who do not sign the treaty exert no effort. In the first formulation, it means that they do not contribute to the public good. Signatories' payoffs thus only depend on their own effort, and the *variability* of these payoffs increases with effort. Welfare is more uncertain for higher levels of signatories' contribution. Because of risk aversion, uncertainty then tends to decrease the contribution of signatory countries at any level of participation. Cooperation is not fixed, however. Stability of the treaty requires signatories to reach a critical mass. A treaty is stable at the precise level of participation such that an additional country signing has little effect while a country leaving has a strong impact. Because of the decreased contribution schedule, this threshold level is shifted to the right. Thus, while effort tends to decrease at given participation levels, participation itself tends to increase in equilibrium.

The mechanism is similar but leads to opposite outcomes in the public bad formulation. Countries who free-ride pollute as usual. This now generates an important background risk faced by signatory countries. As a consequence, the variability of signatories' payoffs becomes *negatively* related to signatories' effort. Signatories tend to increase their effort under uncertainty at any level of participation to shield themselves against this risk. Again, participation is not fixed and the critical mass of countries necessary to obtain a stable treaty is now shifted to the left. In the public bad formulation, participation tends to decrease under uncertainty.

In addition, we find that multiple equilibria may emerge in some circumstances.⁶ A stable treaty with low action and low participation may coexist with one with high action and high participation. Multiplicity, however, appears more likely in the public good formulation.

⁶This complements earlier findings by Ulph (2004) and Kolstad (2007) who show the emergence of multiple stable treaties in a context with learning and risk neutrality.

Finally, we study the robustness of our analysis to uncertainty on the cost of collective action. We find that our main results extend to a setting with uncertain costs and that some new, and somewhat subtle, differences arise between outcomes in the two formulations.

Thus, our analysis uncovers important strategic effects associated to an ex-ante reduction of uncertainty on international agreements. When trying to provide a public good, reducing uncertainty may hurt the establishment of international cooperation. When trying to reduce a public bad, reducing uncertainty may in contrast have a positive multiplier effect due to an increase in participation.

Our analysis contributes to two literatures. First, there is an active research agenda studying international environmental agreements, see Barrett (2003). Uncertainty is mostly ignored in this literature, however, with the exception of a few papers studying the effect of learning under risk neutrality.⁷ These papers usually contrast different timings for the resolution of uncertainty. For instance, Kolstad (2007) shows in a static framework that it may be better to negotiate after uncertainty is resolved rather than before. In contrast, Ulph (2004) finds that, in a model with two periods, the positive effects of learning may not be robust to the introduction of renegotiation between periods.⁸ We do not look at learning here. Rather, we relax the assumption of risk neutrality. We provide the first analysis of the effect of risk aversion on international environmental agreements. To our knowledge, the finding that the two formulations yield radically different outcomes is the first of its kind in this literature.⁹

Second, a growing literature examines the effect of uncertainty and risk aversion in strategic settings.¹⁰ Especially, Bramoullé and Treich (2009) look at global pollution under damage

⁷See Kolstad (2007, 2005), Ulph (2004), Ulph and Maddison (1997), Na and Shi (1998).

⁸Also, it may be easier to reach an agreement before the “veil of uncertainty” has been pierced. Kolstad (2005) and Na and Shi (1998) provide some supporting arguments, under risk neutrality.

⁹In all existing models, the two formulations are equivalent.

¹⁰See e.g. Gradstein et al. (1992), Sandler and Sterbenz (1990) on the exploitation of a renewable resource, Eso and White (2004) on auctions, and White (2008) on bargaining.

uncertainty. They assume that countries do not cooperate and find that uncertainty may decrease emissions and increase welfare in equilibrium. In contrast, we endogenize cooperation in this paper. In circumstances where participation is not affected, we indeed find similar results.¹¹ However, cooperation levels are usually affected and in ways leading to a reversal of the effect of uncertainty. Thus, reducing uncertainty before international negotiations take place could have a large, positive multiplier effect.

Our analysis is also related to the literature on discrete public goods and uncertainty. In this literature, the public good is binary and provided only if private contributions exceed a required threshold level. When this level is certain the first best outcome can be attained in equilibrium, as shown early on by Palfrey and Rosenthal (1984). Several authors have studied how uncertainty on the threshold contribution level affects the analysis. Nitzan and Romano (1990) find that uncertainty often leads to inefficiency and underprovision of the public good, see also Suleiman (1997). In contrast, McBride (2006) finds that, in some circumstances, an increase in uncertainty can increase equilibrium contributions and welfare.¹² As here, these studies emphasize the strategic effects of uncertainty in a context of public good provision. However, our analysis differs from these studies in key respects. In our setup, the public good is continuous and not subject to an exogenous contribution threshold.¹³ Rather, a threshold level of *participation* emerges endogenously from stability conditions in the treaty game. Moreover, while this threshold level depends on uncertainty, it is itself not uncertain. It depends in a non-stochastic way on the level of risk affecting the economic parameters. Finally, the distinction between the public good

¹¹This may happen, for instance, following a small enough increase in uncertainty.

¹²The intuition is that when uncertainty is greater, there is more variation in possible values of the threshold contribution level, which may increase the equilibrium probability that an agent is pivotal and his incentives to contribute.

¹³This notably means that the continuous public good is severely under-provided under certainty, in contrast to what happens in the discrete case.

and the public bad formulations plays a central role here, but is absent from these studies.¹⁴

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 analyzes the case where countries directly contribute to a global public good. Section 4 looks at the reduction of a global public bad. Section 5 extends the analysis to uncertain costs and Section 6 concludes.

2 The Model

In this section, we introduce uncertainty and risk aversion to the model of Barrett (2003). We study two different formulations of the model. While under risk neutrality these two formulations are generally equivalent, we will show that they lead to remarkably different conclusions under risk aversion. Consider n identical countries who face a problem of collective action. In the public good formulation, each country i exerts some costly effort q_i which benefits to all other countries. Individual payoff is equal to $b(\sum_{j=1}^n q_j) - cq_i$ where b represents the marginal benefit from overall effort and c the marginal cost from individual effort. Because payoff is linear, we assume that there is a maximal level of effort q_{\max} . This public good formulation captures situations where countries may directly contribute to a public good, such as R&D.

In contrast in the public bad formulation, countries seek to reduce the emissions of a global pollutant. Country i emits a level $e_i \in [0, q_{\max}]$ where q_{\max} represents business as usual emissions. Individual payoff is equal to $ce_i - b(\sum_{j=1}^n e_j)$. Effort q_i represents the level of reduction of emissions: $q_i = q_{\max} - e_i$.¹⁵ The parameter c now represents the private marginal benefit from pollution while b captures the marginal social damage. In fact, they have the same interpretation as in the public good formulation since abating one unit in one country costs c and increases the

¹⁴Also, we look at risk averse countries while the literature on discrete public goods has, so far, focused on risk neutral agents.

¹⁵Expressed in terms of efforts, individual payoff is now equal to: $b(\sum_{j=1}^n q_j) - cq_i - (nb - c)q_{\max}$.

payoff of all countries by b .

Our main assumption is that the marginal social benefit b is subject to uncertainty. For simplicity, we suppose that b can take two values.¹⁶ Marginal benefit b is equal to b_L with probability p and to $b_H > b_L$ with probability $1 - p$. It is useful to introduce the expected marginal benefit $\bar{b} = pb_L + (1 - p)b_H$. In contrast, uncertainty does not affect the marginal cost of effort c in our baseline model.¹⁷ We study how our analysis is affected by cost uncertainty in section 5.

Preferences of countries towards risk are identical, and represented by a strictly increasing and concave Von-Neumann Morgenstern utility function U . Thus, country i seeks to maximize his expected utility

$$EU(q_i, q_{-i}) = pU[b_L(\sum_{j=1}^n q_j) - cq_i] + (1 - p)U[b_H(\sum_{j=1}^n q_j) - cq_i]$$

in the public good formulation, and

$$EU(e_i, e_{-i}) = pU[ce_i - b_L(\sum_{j=1}^n e_j)] + (1 - p)U[ce_i - b_H(\sum_{j=1}^n e_j)]$$

in the public bad one. Since countries are ex-ante identical, we define social welfare W as the sum of the expected utilities of all countries. As in the model with certainty, we consider two restrictions on the parameters. First, $b_H < c$, so that playing $q_i = 0$ or $e_i = q_{\max}$ is a strictly dominant strategy for all countries in the game without treaty. Second, $nb_L > c$, so that the first-best outcome requires $q_i = q_{\max}$ or $e_i = 0$ for all i . Without treaty, there is severe under-provision

¹⁶Some of our results are valid for any distribution of the parameter, see the discussion in conclusion.

¹⁷In a climate change context, this assumption reflects the fact that scientific uncertainty has little effect on private abatement costs, but is a main source of uncertainty regarding the future damages from greenhouse gas emissions.

of the public good.

To study the formation of an international environmental agreement, we adopt the approach pioneered by Barrett (1994). Countries have to decide whether to join an international agreement before contributing to the global public good. More precisely, the game with treaty unfolds in three stages:

Stage 1 Simultaneously and independently, all countries decide to sign or not to sign the agreement.

In what follows, we denote by k , the number of signatories.

Stage 2 Signatories choose their effort in order to maximize their *collective* payoff.

Stage 3 Non-signatories independently choose the effort maximizing their individual payoff.

A key feature of our approach is that uncertainty is resolved *after* stage 3. Therefore, countries are uncertain about the state of the world when deciding whether to sign the agreement. Our main objective is to study how uncertainty and risk aversion affects this decision, and hence the existence and properties of a stable treaty. We make use of the usual notion of stability.

Definition 1. *A treaty is stable when signatories have no incentive to quit and non-signatories have no incentive to join.*

Conditions for stability can be formally summarized with a unique function Δ . Given symmetry and concavity, non-signatories will play identical actions, and this holds for signatories as well. Therefore, we denote by $q^s(k)$ and $q^{ns}(k)$ the optimal levels of effort exerted respectively by a signatory and a non-signatory country when the number of signatories is equal to k , and by $U^s(k)$ and $U^{ns}(k)$ their expected utilities. We introduce $\Delta(k) = U^s(k) - U^{ns}(k - 1)$ which represents the net benefit to a signatory of staying in the agreement. The fact that a non-signatory country does not want to join at k^* is equivalent to the fact that a signatory country wants to quit at

$k^* + 1$. Thus, a treaty is stable at $k^* \in [2, n - 1]$ if and only if $\Delta(k^*) \geq 0$ and $\Delta(k^* + 1) \leq 0$.¹⁸

While a treaty with no action and $q^s = 0$ is always stable, we restrict attention in what follows to treaties with strictly positive effort.

When there is no uncertainty, $b_L = b_H = \bar{b}$ and the analysis of Barrett (2003) applies. Both formulations yield the same outcomes. Non-signatories always play $q^{ns} = 0$ or $e^{ns} = q_{\max}$. The collective payoff of signatories is $k(k\bar{b} - c)q^s$ in the public good formulation and $k(k\bar{b} - c)q^s - k(nb - c)q_{\max}$ in the public bad one. In either case, signatories play $q^s = 0$ if $k < c/\bar{b}$ and $q^s = q_{\max}$ if $k > c/\bar{b}$. Signatories must reach a critical mass before action becomes worthwhile. Linearity of the payoff function induces a bang-bang solution. A treaty is stable if and only if $c/\bar{b} \leq k^* \leq c/\bar{b} + 1$. This usually pins down a unique number. Social welfare in the stable treaty is strictly greater than without treaty. Interestingly, the analysis under certainty also covers the case where countries are risk neutral. This justifies the introduction of risk aversion, which we assume in the remainder of the paper.

We next analyze the treaty game under uncertainty. Since $b_L < b_H < c$, a non-signatory country always plays $q^{ns} = 0$ or $e^{ns} = q_{\max}$ no matter the number or the actions of signatories. This solves Stage 3. We study the two formulations, in turn, in what follows. In each case, we first study Stage 2 and the actions of signatories. This study is critical to understand when a treaty is stable. The analysis under certainty illustrates how the shape of q^s determines stability. Under certainty, q^s is a step function and k^* is the first integer situated just above the step. On the one hand, $q^s(k^*) = q_{\max}$ and $q^s(k^* - 1) = 0$. If a signatory quits, the drop in collective action is very large, hence a signatory does not want to quit. On the other hand, $q^s(k^* + 1) = q^s(k^*)$. If a non-signatory joins, the other signatories do not change what they do, which gives no incentives

¹⁸A treaty with n signatories is stable if and only if $\Delta(n) \geq 0$. In that case, all countries are signatories, and the condition on non-signatory countries disappears.

for non-signatories to join. The general intuition carries over more generally. A signatory does not want to quit if he expects that his departure would induce a high drop in the actions of the remaining signatories. In contrast, a non-signatory does not want to join if he anticipates his decision to yield little increase, or even a decrease, in overall effort. We then make use of our study of q^s to analyze Stage 1, and the existence and properties of stable treaties in both formulations.

3 Providing a global public good

3.1 Signatories' actions

In this section, we study the actions of signatories $q^s(k)$ when countries contribute to a global public good. We have three results. First, we find that the bang-bang property which characterizes q^s under certainty partially disappears under uncertainty. Effort may take intermediate values in situations where action is desirable in one state of the world but not in the other. In that case, signatories trade-off the benefit of action in one state against its cost in the other. Second, we find that uncertainty unambiguously lowers q^s . Holding k constant, an increase in uncertainty always leads to a (weak) decrease in signatories' efforts. And third, we show that an increase in risk aversion has a similar effect and also lowers signatories' actions.

Recall, signatories jointly decide how to maximize their collective welfare, defined as the sum of their expected utilities. It means that q^s is the solution of the following program:

$$\max_{0 \leq q \leq q_{\max}} EU[(kb - c)q] \tag{1}$$

We first describe when the solution is at a corner. Observe that the objective function is strictly

concave. Marginal expected utility is equal to $E(kb - c)U'[(kb - c)q]$. At $q = 0$, this reduces to $(k\bar{b} - c)U'(0)$. Since $U' > 0$, $q^s = 0$ when $k \leq c/\bar{b}$. This notably covers the case where $k \leq c/b_H$, and signatories prefer to exert no effort in both states of the world. In contrast, when $k \geq c/b_L$, the marginal expected utility is always strictly positive. Signatories prefer to play q_{\max} in both states of the world. Thus, $q^s = q_{\max}$ when $k \geq c/b_L$. When k lies between c/\bar{b} and c/b_L , $kb_L - c < 0$ while $kb_H - c > 0$, and the solution may be interior. The first-order condition of program (1) can be written as follows:

$$p(c - kb_L)U'[(kb_L - c)q^s] = (1 - p)(kb_H - c)U'[(kb_H - c)q^s] \quad (2)$$

This condition says that marginal utilities are equal in both states of the world. It expresses a trade-off between the positive marginal utility from action when $b = b_H$ and the negative marginal utility when $b = b_L$. This equation always has a solution $q > 0$ if $U'(-\infty) = +\infty$ or $U'(+\infty) = 0$. Then, q^s is equal to the solution of this first-order condition if this solution is lower than q_{\max} , and to q_{\max} otherwise. In any case, $q^s(k)$ is continuous over $[0, n]$ when k is allowed to take real values. In summary,

Proposition 1. *Effort q^s exerted by a signatory country when k countries have signed the treaty is such that:*

(1) $q^s(k) = 0$ if $k \leq c/\bar{b}$ and $q^s(k) = q_{\max}$ if $k \geq c/b_L$.

(2) If $c/\bar{b} < k < c/b_L$, q^s is equal to the smaller of q_{\max} and of the solution to equation (2).

Signatories' effort under uncertainty is always lower than or equal to effort under certainty.

Observe that when $b_L = \bar{b}$, there is no uncertainty and Proposition (1) reduces to the result described in the previous section.

Given that q^s varies from 0 to q^{\max} , a natural question is whether q^s is necessarily increasing

over $[c/\bar{b}, c/b_L]$. Does more participation always lead to a higher effort per signatory? It turns out that the answer is negative. Even with standard preferences, q^s may be decreasing over some range of participation levels. We explore this issue in more detail in the Appendix. We show that if the level of absolute risk aversion is decreasing and decreases sufficiently rapidly, then q^s is indeed non decreasing. However, monotonicity of q^s in general is ambiguous and we provide counterexamples below.

We next study the effect of an *increase in uncertainty* on $q^s(k)$. Proposition (1) shows that $q^s(k)$ is lower under uncertainty than under certainty. Our next result shows that this property holds more generally for any increase in uncertainty in the sense of Rothschild and Stiglitz (1974). Let b and b' be two binary distributions of marginal benefit values with the same expected value $\bar{b} = \bar{b}'$. Here, b' is more risky than b if and only if $b'_L \leq b_L$ and $b'_H \geq b_H$.

Proposition 2. *Suppose that b' is more risky than b . Then $\forall k \in [0, n]$, $q^s(k, b') \leq q^s(k, b)$. An increase in uncertainty always reduces signatories' actions.*

The intuition behind this result can be seen by looking at the expected value and the variance of ex-post payoffs π for signatories. Here, $E(\pi) = (k\bar{b} - c)q^s$ and $Var(\pi) = p(1 - p)(b_H - b_L)^2 k^2 (q^s)^2$. When $k \geq c/\bar{b}$, an increase in q increases the expected payoff and also increases its variance. One is desirable, but the other is not under risk aversion. The optimal choice of q results from a trade-off between these two motives.¹⁹ Then, holding q and k constant, an increase in uncertainty leads to an increase in the payoff's variance while leaving its expected value unchanged. The marginal effect of a decrease in q on the variance is greater when uncertainty is greater, hence signatories' action is lower.

This effect is confirmed by looking at an increase in risk aversion. Recall that utility function

¹⁹In contrast, when $k < c/\bar{b}$, an increase in q decreases the expected value and increases the variance, and there is no trade-off. This provides another explanation for the fact that $q^s = 0$.

V represents more risk averse preferences than U if there is an increasing and concave function Φ such that $V = \Phi(U)$. In our context, the effect of risk aversion is similar to the effect of uncertainty.

Proposition 3. *Suppose that countries with utility V are more risk averse than countries with utility U . Then $\forall k \in [0, n]$, $q^s(k, V) \leq q^s(k, U)$. An increase in risk aversion always reduces signatories' efforts.*

For a given participation level, uncertainty and risk aversion tend to reduce signatories' actions. Signatories exert less effort in order to diminish the variability of their payoffs. While this effect may seem to be purely negative (lower level of collective action), we will see in the next section that it can have a positive indirect consequence. Lower action at all participation levels may increase the participation level that is sustainable in equilibrium.

To conclude on signatories' actions, we look at specific utility functions. We are able to obtain closed-form expressions in three important cases. These expressions are useful for applications and numerical simulations, and also allow us to obtain more precise information on the shape and monotonicity of q^s . Corresponding expressions can easily be derived for the public bad formulation. Denote by $\tilde{q}^s(k)$ the solution to equation (2).²⁰

Quadratic utility. Suppose that $U(\pi) = \pi - \lambda\pi^2$. We impose that $(nb_H - c)q_{max} \leq 1/(2\lambda)$ to ensure that utility is strictly increasing over the strategy space. We have:

$$\tilde{q}^s(k) = \frac{1}{2\lambda} \frac{k\bar{b} - c}{p(kb_L - c)^2 + (1-p)(kb_H - c)^2}$$

We show in the Appendix that q^s is non decreasing in k when U is quadratic.

CARA utility. Suppose that $U(\pi) = -e^{-A\pi}$ where A denotes the level of absolute risk aversion.

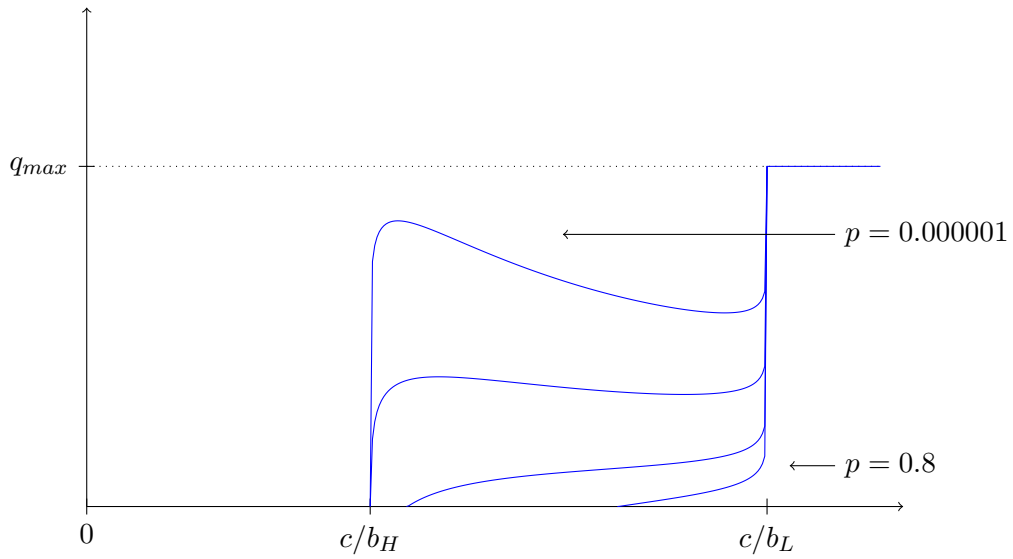
²⁰As shown in Proposition 1, $q^s(k) = \min(\tilde{q}^s(k), q_{max})$.

We obtain:

$$\tilde{q}^s(k) = \frac{1}{Ak(b_H - b_L)} \ln \left[\frac{(1-p)(kb_H - c)}{p(c - kb_L)} \right]$$

We ask whether q^s is increasing in k and find that two cases appear. Either q^s is increasing, or it is first increasing, then decreasing, and again increasing over $[c/\bar{b}, c/b_L]$. Also, q^s is more likely to be increasing when p is higher. Figure 1 illustrates.

Figure 1 : Effort with uncertainty, CARA function



$$n = 100, c = 15, b_H = 12, b_L = 5, A = 1, p = \{0.000001, 0.001, 0.2, 0.8\}, q_{max} = 3.5$$

CRRA utility. Suppose that $U(\pi) = \frac{1}{1-\gamma}(\pi_0 + \pi)^{1-\gamma}$ if $\gamma \neq 1$, and $\ln(\pi_0 + \pi)$ if $\gamma = 1$.

Introducing a baseline payoff π_0 is necessary to ensure that ex-post payoffs are always positive.

We obtain:

$$\tilde{q}^s(k) = \pi_0 \frac{(1-p)^{1/\gamma}(kb_H - c)^{1/\gamma} - p^{1/\gamma}(c - kb_L)^{1/\gamma}}{p^{1/\gamma}(c - kb_L)^{1/\gamma}(kb_H - c) + (1-p)^{1/\gamma}(kb_H - c)^{1/\gamma}(c - kb_L)}$$

Especially, when $\gamma = 1$ this reduces to $\tilde{q}^s = \pi_0 \left[\frac{1-p}{p(c - kb_L)} - \frac{p}{(1-p)(kb_H - c)} \right]$, which is clearly increasing in k . This may not necessarily be the case, however, when $\gamma \neq 1$.

3.2 Stable treaties

In this section, we study the properties of stable treaties. We first show that a stable treaty always exists and that participation in any stable treaty is always higher than under certainty. We then characterize participation in the best stable treaty when q_{\max} is large enough. We finally discuss the impact of uncertainty on welfare and illustrate these effects through numerical simulations.

We first show that a stable treaty with positive action always exists.

Proposition 4. *In the public good formulation, a stable treaty always exists. Participation in any stable treaty is such that $k^* \in [c/\bar{b}, c/b_L + 1[$. Participation under uncertainty is always greater than or equal to participation under certainty.*

Our proof relies on the study of the function Δ . Recall, $\Delta(k) = U^s(k) - U^{ns}(k - 1)$. On the one hand, $U^s = U^{ns} = 0$ on $[0, c/\bar{b}]$. This means that $\Delta(k) = 0$ if $k \in [1, c/\bar{b}]$ and $\Delta(k) > 0$ if $k \in]c/\bar{b}, c/\bar{b} + 1]$. On the other hand, $q^s = q_{\max}$ on $[c/b_L, n]$. This implies that $\Delta(k) < 0$ on $[c/b_L + 1, n]$. If we define k^* as the greatest integer such that $\Delta(k^*) > 0$, we have: $\Delta(k^*) > 0$, $\Delta(k^* + 1) \leq 0$, and $k^* > c/\bar{b}$, which means that $q^s(k^*) > 0$.

If the number of signatory countries lies just below c/\bar{b} , an additional country has a strictly positive incentive to join. If he does not join, signatories do not exert any effort. If he joins, all signatories start exerting positive effort. The number of signatories then increases until an additional country does not want to join, at which point the treaty is stable.

Clearly, in any stable treaty with positive action, $c/\bar{b} \leq k^* < c/b_L + 1$. We next derive two further results on this participation level. First, if for some integer k , $q^s(k) < q^s(k - 1)$, we have $U^s(k) < U^{ns}(k - 1)$, hence $\Delta(k) < 0$. Applying the previous argument shows that there exists a stable treaty with $k^* \leq k$. Second, we show that an equilibrium around c/b_L is generally guaranteed if q_{\max} is large enough. More precisely, let $[\pi]$ denote the smallest integer greater than

or equal to π .

Proposition 5. *Suppose that either $\lim_{\pi \rightarrow -\infty} U'(\pi) = +\infty$ or $\lim_{\pi \rightarrow +\infty} U'(\pi) = 0$, and that either c/b_L is not an integer or $\lim_{\pi \rightarrow +\infty} U(\pi) = +\infty$. Then, there exists $\bar{q} > 0$ such that if $q_{\max} \geq \bar{q}$, a treaty with $k^* = \lceil c/b_L \rceil$ and $q^s = q_{\max}$ is stable.*

The intuition behind this result lies in the fact that $q^s(k)$ becomes very steep when k gets close to c/b_L . Recall that at c/b_L , action becomes desirable in both states of the world, hence, given linearity of the payoffs, should be as high as possible. If q_{\max} is large, the drop in collective action if one country quits is large, which gives an incentive for signatories to stay in the treaty.²¹

How does uncertainty affect welfare in a stable treaty? In this model, uncertainty affects welfare through three different channels. First, holding q^s and k fixed, it has a direct negative effect resulting from risk aversion. Second, Proposition 2 shows that, holding k fixed, it has an indirect negative effect through its reduction of $q^s(k)$. Finally, there is a third countervailing effect. Proposition 4 shows that uncertainty has a positive impact on participation k^* . This third effect may overcome the the first two and lead to an overall increase in welfare. Note that when $q^s = q_{\max}$, the second effect disappears but the effect of uncertainty on welfare is still ambiguous. In this case, eliminating uncertainty lowers the incentives to participate. In the end, *an ex-ante reduction of uncertainty may hurt the prospects for international cooperation on the provision of a global public good.*

We illustrate these effects by reporting results from numerical simulations in Figures 2 - 4. We consider a utility function with constant relative risk aversion $U(\pi) = \frac{1}{1-\gamma}(\pi_0 + \pi)^{1-\gamma}$. We look at the effect of an increase in uncertainty on stable treaties through a decrease in b_L , holding b_H and \bar{b} constant. Parameters are set at the following values: $n = 100$, $\gamma = 0.5$, $\pi_0 = 10^3$, $c = 1000$,

²¹Either technical condition stated in the Proposition ensures that an arbitrarily large drop in collective action indeed translates into an incentive not to quit. The result may not hold, however, if c/b_L is an integer and U is bounded.

$\bar{b} = 991$, $b_H = 992$. Figure 2 represents signatories' effort in stable treaties, Figure 3 the level of participation and Figure 4 welfare.

Observe first that multiple equilibria emerge for $b_L \leq 100$. For any value of b_L , here, the equilibrium identified in Proposition 5 is always stable. When uncertainty is not too high, this is the unique stable treaty. It is characterized by high participation *and* high effort. When uncertainty is very high, however, a second treaty becomes stable.²² This second treaty yield much worse outcomes: lower participation, effort and welfare. Thus, if countries are stuck in this bad equilibrium, a decrease in uncertainty could initially lead to strong increase in participation and effort if it allowed countries to leave the the multiplicity domain.

Next, focus on the properties of the good treaty. Figure 3 illustrates how participation increases when uncertainty increases. This shows that Proposition 4 extends here to increases in uncertainty. Figure 4 illustrates both the direct negative effect of uncertainty and the indirect positive effect due to an increase in participation. When the decrease in b_L is relatively low, k^* does not change and welfare decreases as a consequence of the direct effect. In contrast, when b_L decreases below some threshold values, an additional country signs the treaty and welfare increases discontinuously. Overall, the positive effect clearly dominates and welfare may be much higher at high levels of uncertainty than under certainty.

4 Reducing a global public bad

4.1 Signatories' actions

We next turn to the analysis of the public bad formulation. To better differentiate the two models, we focus on emissions in what follows. As in the previous section, we first analyze how signatories'

²²Extensive simulations, not reported here, show that equilibrium multiplicity is indeed a robust phenomenon in the public good formulation. Multiplicity tends to be more prevalent for higher levels of risk and risk aversion.

emissions depend on participation. We then use this understanding to study the properties of stable treaties.

When countries try to reduce a global public bad, optimal signatories' emissions solve the following program:

$$\max_{0 \leq e \leq q_{\max}} EU[(c - kb)e - (n - k)bq_{\max}] \quad (3)$$

The study of this program, in Appendix, yields the following result.

Proposition 6. *In the public bad formulation, the emissions e^s of a signatory country when k countries have signed the treaty is such that:*

(1) $e^s(k) = q_{\max}$ if $k \leq c/b_H$ and $e^s(k) = 0$ if $k \geq c/\bar{b}$.

(2) If $c/b_H < k < c/\bar{b}$, e^s is equal to the solution of the first order condition of program (3) if this solutions lies within $[0, q_{\max}]$.

Signatories' effort under uncertainty is always higher than or equal to effort under certainty.

Comparing Propositions 6 and 1, we see similarities as well as key differences between the two formulations. The solution still lies at a corner except maybe when action is desirable in one state of the world but not in the other. However, the precise ranges where effort may take intermediate values differ. Effort becomes maximal at different threshold participation levels: c/\bar{b} in the public bad formulation, c/b_L in the public good one. A striking consequence is that uncertainty has opposite effects on signatories' efforts in both formulations. Uncertainty increases effort in the public bad formulation but lowers it in the public good one. We will see below that this also induces opposite effects on participation.

The main reason behind this difference lies in the impact of non-signatories' actions. In both cases, non-signatories do not exert any effort. When countries contribute to a public good, the baseline payoff for signatories is simply equal to zero. In contrast, in a context of global pollution,

no effort yields pollution damages. Signatories thus face a baseline uncertain loss caused by non-signatories' emissions. As is well-known, the presence of a background risk may deeply affect decisions of risk averse agents, see Gollier (2001).

Our next results confirm this reversal in qualitative outcomes. We look at the impacts of an increase in uncertainty and an increase in risk aversion. We provide the counterparts to Propositions 2 and 3 for the public bad formulation.

Proposition 7. *Consider the public bad formulation and suppose that b' is more risky than b . Then $\forall k \in [0, n]$, $e^s(k, b') \leq e^s(k, b)$. An increase in uncertainty always increases signatories' effort.*

Proposition 8. *Consider the public bad formulation and suppose that agents with utility V are more risk averse than agents with utility U . Then $\forall k \in [0, n]$, $e^s(k, V) \leq e^s(k, U)$. An increase in risk aversion always increases signatories' efforts.*

Holding participation constant, signatories reduce their emissions and increase their efforts if uncertainty or risk aversion increases. These results are similar to results obtained in Bramoullé and Treich (2009). Indeed, they have a common explanation. The variance of the payoff is now equal to $p(1-p)(b_H - b_L)^2 k^2 e^2$ and is *increasing* in emissions. If uncertainty is higher, the payoff's variance is higher. To compensate, risk averse agents tend to decrease their emissions. Thus, exerting more effort reduces the payoff's variance in the public bad formulation but increases it in the public good one. This leads to opposite effects on signatories' actions hence, as we see next, on participation.

4.2 Stable treaties

We study stable treaties in a context of global pollution. We first show that the argument behind Proposition 4 extends. However, its conclusion is reversed.

Proposition 9. *In the public bad formulation, a stable treaty always exists. Participation in any stable treaty is such that $k^* \in c/b_H, c/\bar{b} + 1[$. Participation under uncertainty is always lower than or equal to participation under certainty.*

The reason for existence is essentially the same as under the public good formulation. Participation in a stable treaty must belong to the range where effort may take intermediate values. The different locations of these ranges lead to the different effects of uncertainty. The following result extends Proposition 5 to the public bad formulation.

Proposition 10. *Suppose that c/b_H is not an integer and that the level of absolute risk aversion of U is constant or decreasing. Then, there exists $\bar{q} > 0$ such that if $q_{\max} \geq \bar{q}$, there is a unique stable treaty with $k^* = [c/b_H]$ and $q^s = q_{\max}$.*

We emphasize two important differences between Proposition 10 and 5. On one hand, this result is more restrictive since we now need to make an assumption on the shape of preference towards risk. The reason relies, again, on the effect of non-signatories' actions. As q_{\max} increases, signatories face increasingly dire prospects. The background risk generated by non-signatories' emissions becomes worse. If risk aversion is constant or decreasing, this *increases* the incentives for signatories to exert effort.²³ On the other hand, this result is also sharper since we now get *uniqueness* of the stable treaty when business as usual emissions are high enough. This indicates that equilibrium multiplicity may generally be less frequent in the public bad formulation.²⁴

²³We suspect, but have not shown, that the result may not hold under increasing absolute risk aversion. Signatories would then tend to become *less* risk averse when faced with a worse baseline situation.

²⁴And indeed, multiplicity did not appear in the simulations we ran under the public bad formulation.

Finally, how does uncertainty affect welfare in a stable treaty? As in the public good formulation, uncertainty has three distinct effects on welfare. The first is similarly negative. Holding k and q^s fixed, uncertainty decreases welfare because of risk aversion. In contrast, the second effect has an opposite sign. Holding k fixed, uncertainty tends to increase signatories' efforts which improves welfare. However, the third effect also has an opposite sign. As shown in Proposition 9, uncertainty tends to decrease participation and this has a negative impact on welfare. In cases where $e = 0$ and effort is maximal, the second effect disappears and uncertainty unambiguously lowers welfare. This arises, for instance, in situations identified by Proposition 10. When this happens, *an ex-ante reduction of uncertainty has a positive "multiplier" effect in a context of global pollution*. This multiplier effect is purely strategic in nature. It captures the increased incentives to participate caused by the reduction of uncertainty.

As in the previous section, we run numerical simulations to illustrate, see Figures 5 - 7.²⁵ We still consider a utility function with constant relative risk aversion. Due to the different roles played by b_L and b_H in the two formulations, we now study the effect of an increase in uncertainty through an increase in b_H holding b_L and \bar{b} constant. Parameters are set at the following values: $n = 100$, $\gamma = 0.5$, $\pi_0 = 3.10^7$, $c = 1000$, $\bar{b} = 111$, $b_L = 90$.

Observe first that here the stable treaty is unique. As uncertainty increases, participation decreases. Figure 6 shows the existence of a threshold value for b_H such participation is equal to 10 below this value and drops to 9 above it. When $k^* = 10$, Figure 5 shows that effort is maximal. When it drops to $k^* = 9$, signatories' effort first drops discontinuously and then increases as uncertainty increases. The initial drop is consistent with the fact that, for given parameter values, effort tends to decrease if participation decreases. The following increase in effort illustrates Proposition 7: holding participation constant, effort increases as uncertainty

²⁵To facilitate comparison with the public good formulation, we look here at effort rather than emissions.

increases.

Next, look at welfare. As long as $k^* = 10$, q and k^* are constant and welfare decreases due to the direct negative effect of uncertainty. When k^* drops from 10 to 9, welfare also drops under the combined effect of the reduction in participation and the reduction in effort. When $k^* = 9$, we see a trade-off emerging between the direct negative effect and the indirect positive effect due to higher effort. Interestingly, the resulting outcome is non-monotonic. The positive effect first dominates. But the increase in effort occurs at a decreasing rate, and at some point, the indirect positive effect is overwhelmed by the direct negative one. Again, changes in participation have a crucial impact on welfare.

5 Uncertainty on the cost of effort

A restrictive assumption of our baseline model is that uncertainty only affects the benefits from action. In reality, the marginal cost of effort is often uncertain as well. For instance, when negotiating an international agreement on climate change, countries may not know exactly the benefits they will derive from their future CO₂ emissions. In international R&D projects, realized costs often differ substantially from their initial estimates. In this section, we study the robustness of our results to the assumption of certain costs. We relax this assumption, and ask whether uncertainty on costs tend to counteract or to amplify uncertainty on benefits. We find that our main results extend and that some new, and somewhat subtle, differences arise between outcomes in the two formulations. Throughout this section, we assume that marginal cost of effort c is equal to c_L with probability p' and to c_H with probability $1 - p'$, that cost and benefit are independent, and that $c_L > b_H$ and $nb_L > c_H$.

5.1 Providing a global public good

Suppose first that countries contribute to a global public good. Suppose that k signatories have signed the agreement, and look at how uncertainty on costs affects signatories' efforts. Signatories choose their action $q^s(k)$ to maximize $EU[(kb - c)q]$ where the expectation is now taken over all realizations of b and c . Marginal expected utility at $q = 0$ is equal to $(k\bar{b} - \bar{c})U'(0)$, hence $q^s(k) = 0$ if $k \leq \bar{c}/\bar{b}$. Among the four possible states of the world, the one which is least favorable to action is the one where benefits are lowest $b = b_L$ while costs are highest $c = c_H$. So action is desirable in all states of the world, and $q^s(k) = q_{\max}$, if $k \geq c_H/b_L$. Thus, if we compare a situation with certain cost \bar{c} to one with uncertain cost, the range of participation levels for which action may be interior expands from $]\bar{c}/\bar{b}, \bar{c}/b_L[$ to $]\bar{c}/\bar{b}, c_H/b_L[$. The first order condition of signatories' decision problem still trades-off the positive marginal utilities of action in some states of the world versus its negative marginal utilities in others. This condition is of course more complicated than with two states, and cannot be solved analytically for CARA or CRRA utility functions. However, a positive solution always exists if $U'(-\infty) = +\infty$ or $U'(+\infty) = 0$, as in Section 3.1.

In any case, signatories' action is necessarily (weakly) lower under uncertain costs when $k \in [\bar{c}/b_L, c_H/b_L]$. Uncertainty on costs tend to lower action at high participation levels. Does this property hold for all k ? More generally, does an increase in cost uncertainty always lowers effort? And does Proposition 2 still hold in a setting with uncertain costs? To gain some insight on these questions, we can look at the expected value and variance of the payoff. Here the expected payoff is equal to $E(\pi) = (k\bar{b} - \bar{c})q$ while its variance is $Var(\pi) = [p(1 - p)(b_H - b_L)^2k^2 + p'(1 - p')(c_H - c_L)^2]q^2$. An increase in risk on cost increases the variance of the payoff without affecting its expected value, which gives an incentive to signatories to reduce their effort. As it turns out, this intuition does *not* carry over to all risk averse utility functions. Effects related to moments of

order 3 and higher of the distribution of payoffs come into play. To obtain unambiguous results, we must impose some restrictions on the preferences, familiar from the theoretical literature on risk (Gollier (2001)). Introduce relative prudence, denoted by P_r , as:

$$P_r(\pi) = -\frac{u'''(\pi)}{u''(\pi)}\pi$$

Proposition 11. *Under uncertainty on benefits and costs, if $P_r \leq 2$, then any increase in uncertainty on b or c reduces signatories' actions at all participation levels.*

Especially, relative prudence is equal to zero, and Proposition 11 holds, if utility is quadratic. In contrast, Proposition 11 may not hold when relative prudence is not everywhere lower than 2. Thus, even though cost uncertainty often amplifies the effect of benefit uncertainty by further lowering signatories' effort, it could in some cases go in the opposite direction. In contrast, the effect of an increase in risk aversion is unchanged and still clear-cut. We show in Appendix that Proposition 3 directly extends to uncertain costs.

In turn, cost uncertainty affects stable treaties. We show in Appendix that our proof of existence extends. Participation k^* in any stable treaty now lies between \bar{c}/\bar{b} and c_H/b_L . More importantly, the argument behind Proposition 5 also extends. At $k = [c_H/b_L] - 1$, action is not desirable in at least one state of the world. When the interior solution exists, it is independent of q_{\max} . Then, as q_{\max} gets large, the drop in collective action when one signatory quits at $k = [c_H/b_L]$ becomes arbitrarily large. This guarantees that:

Proposition 12. *Consider uncertainty on benefits and costs. Suppose that either $\lim_{\pi \rightarrow -\infty} U'(\pi) = +\infty$ or $\lim_{\pi \rightarrow +\infty} U'(\pi) = 0$, and that either c_H/b_L is not an integer or $\lim_{\pi \rightarrow +\infty} U(\pi) = 0$. Then, there exists \bar{q} such that if $q_{\max} \geq \bar{q}$, a treaty with $k^* = [c_H/b_L]$ and $q^s = q_{\max}$ is stable.*

For the treaty identified in Proposition 12, uncertainty on costs plays a role similar to uncer-

tainty on benefits. It further increase participation, hence may further increase welfare despite its direct negative effect due to risk aversion. Interestingly, participation in this best possible treaty depends on the lowest value of benefit, but of the *highest* value of cost. The reason is that, taken together, these two values determine the worst possible state of the world and that in turn, this state determines the critical threshold level of participation.

5.2 Reducing a global public bad

Suppose next that countries try to reduce a global public bad. For signatories' actions, we show in Appendix that $q^s = 0$ if $k \leq c_L/b_H$, $q^s = q_{\max}$ if $k \geq \bar{c}/\bar{b}$ and q^s may be interior when $k \in]c_L/b_H, \bar{c}/\bar{b}[$. As in section 4.1, the critical state of the world is now the one which is *most* favorable to action, that is, when $b = b_H$ and $c = c_L$. As soon as participation raises above the threshold level for which action in this state is desirable, signatories' efforts may become positive. Since $c_L/b_H < \bar{c}/\bar{b}$, uncertainty on cost always (weakly) *increases* signatories' actions at low participation levels. As in the public good formulation, comparative statics with respect to risk are generally ambiguous. We obtain the following counterpart to Proposition 11. Introduce the level of absolute prudence, denoted by P , as:

$$P(\pi) = -\frac{u'''(\pi)}{u''(\pi)}$$

Proposition 13. *Under uncertainty on benefits and costs, if $P \leq 2/[nq_{\max}(c_H - b_L)]$, then any increase in uncertainty on b or c increases signatories' actions at all participation levels.*

The condition of Proposition 13 is actually more demanding than the condition of Proposition 11. The presence of a background risk implies that more restrictions have to be imposed on

preferences to obtain clear-cut comparative statics with respect to risk.²⁶ Interestingly, this background risk also affects the comparative statics with respect to risk aversion. If signatories become more risk averse, they do not necessarily increase their actions under uncertain costs.²⁷ This is related to standard results on the portfolio problem with a background risk, see Ch. 8.2 in Gollier (2001).²⁸ Especially, say that V is more risk averse than U in the sense of Ross (1981) if $V = \lambda U + f$ with $\lambda > 0$ and f decreasing and concave. Then, we show in Appendix that an increase in risk aversion in the sense of Ross always leads to an increase in the action of signatories.

Finally, consider stable treaties. Existence still holds, but Proposition 10 does not extend as directly as Proposition 5. We show the following result.

Proposition 14. *Consider uncertainty on benefits and costs. Suppose that the level of absolute risk aversion of U is constant or decreasing. There exists \bar{q} such that if $q_{\max} \geq \bar{q}$, then $\forall k \geq \bar{c}/b_H$, $q^s(k) = q_{\max}$ and participation k^* in any stable treaty satisfies $[c_L/b_H] \leq k^* \leq [\bar{c}/b_H]$. We may have $q^s(k^*) \neq q_{\max}$.*

So if q_{\max} is large enough, adding uncertainty on cost to uncertainty on benefits decreases participation further. However, and in contrast to what happens with in the public good formulation, it may not decrease all the way towards the threshold level $[c_L/b_H]$. To illustrate why, consider CARA utility functions. We show in Appendix that if $k^* \in]c_L/b_H, \bar{c}/b_H[$ (and only if), signatories' emissions tend to the following limit as q_{\max} tends to infinity:

$$e^s(k) = \frac{1}{A(c_H - c_L)} \ln \left[\frac{1 - p'}{p'} \right] \frac{c_H - kb_H}{kb_H - c_L}$$

²⁶In both Propositions 11 and 13, the sufficient conditions depend on the magnitude of the increase in risk. While in the first, it is captured by the index of relative prudence, in the second, it appears directly in the bound (see the Appendix for details).

²⁷So while Proposition 3 directly extends to uncertain costs, this is not the case for Proposition 8.

²⁸Most results assume independence between risks, hence cannot be applied here.

This follows from two observations. First, as q_{\max} increases, the background payoff in the bad situation $-(n-k)b_H q_{\max}$ becomes increasingly worse than the background payoff in the better situation $-(n-k)b_L q_{\max}$. What happens in the bad state dominates, and signatories act as if the bad state were certain. Second, income effects are absent under CARA, which implies that optimal emissions when $b = b_H$ do not depend on q_{\max} . So in that range of participation levels, the emission's limit is independent of q_{\max} . This means that if $k = \lceil c_L/b_H \rceil - 1$ the effect of one signatory joining becomes larger and larger. Yet, at $k = \lceil c_L/b_H \rceil$, an additional signatory may still have an incentive to join, since this may further decrease emissions, and the treaty may not be stable. However, when $k > \bar{c}/b_H$, signatories' emissions tend to zero, which means that $k^* \leq \lceil \bar{c}/b_H \rceil$ hence that participation is lower than under certain costs if q_{\max} is high enough.

6 Conclusion

We conclude with a discussion of some limitations of our analysis and of promising directions for future research. In this paper, we introduce uncertainty and risk aversion to a simple model of international agreement to supply a global public good. We find that uncertainty and risk aversion significantly affect the analysis of treaty formation. They yield qualitative changes as well as significant quantitative changes on the outcomes of the game. This complexity provides some justification, ex-post, for the study of a simple benchmark model. It also raises the question of the robustness of our results. Five features of the model especially deserve attention: the fact that the risk is binary, linearity of the payoffs, homogeneity of the agents, the static dimension, and the simplicity of the policy framework.

Many of our results would directly extend to an arbitrary risk. For instance, the argument behind Propositions 1 and 6 hold in general. Similarly, our existence results Propositions 4 and

9 are valid for any risk. In contrast, signing the effects of risk and risk aversion in general may be more complicated. As is well-known from risk theory, comparative statics may be ambiguous and may involve conditions based on the third and higher derivatives of the utility function, see Gollier (2001). Section 5 already illustrates some of these complexities in a setup with uncertain costs of action.

Linearity of the payoffs is a critical assumption. Under certainty, non-linear payoffs may lead to very different equilibria, see Barrett (2003). An interest of the model with linear payoffs, however, is that it neatly captures the idea of critical mass. Collective action becomes worthwhile only when enough countries have joined in and once this threshold is reached, there is little benefit from an additional signatory. As such, studies of models with linear payoffs may be useful to understand what happens more generally. With any payoff function, stability captures a form of local critical mass. A treaty is stable when the drop in collective action if one country is high enough and when the addition of one signatory has little effect. Thus, we conjecture that our results may extend, under conditions to be determined, to models with non-linear payoffs. As soon as uncertainty lowers signatories' actions, corresponding threshold levels may increase which may improve participation in equilibrium. In contrast, if uncertainty increases signatories' actions the threshold levels may decrease and participation may drop.

Under heterogeneity, anonymity is lost. The idea of local critical mass is still relevant, but the identity of the signatories and non-signatories now matters. To understand the effects of heterogeneity under uncertainty, however, we would first need to understand its effects under certainty.²⁹ Despite some recent advances in that direction (see, e.g., Barrett (2001) and McGinty (2007)), a general analysis of international agreements under heterogeneity is still lacking. If anything, we

²⁹Note that by continuity, we expect our results to hold under heterogeneity, as long as the extent of heterogeneity is not too high.

expect the effects of uncertainty and risk aversion to be exacerbated under heterogeneity, given that countries may differ in the risk they face and in their degree of risk aversion.

Our model is static. However, the intertemporal dimension clearly matters for many global public goods, climate change being a prime example. Several non trivial issues arise in a dynamic framework. First, the agreement may have to be renegotiated over time and the set of signatories may evolve. Second, agreements have to address the intertemporal dimension. They may, for instance, specify temporal *sequences* of action. Third, the timing (and conditions) of future renegotiations may itself be an important part of the negotiations. We note that the study of these issues *under certainty* has barely started.³⁰ Thus, as with heterogeneity, a general understanding of the dynamics of international agreements under certainty is still lacking. Uncertainty would have interesting and complicated dynamic effects. Especially, uncertainty generally evolves over time thanks to learning, and the learning process itself may be affected by signatories' and non signatories' actions.

Finally, our current policy framework is extremely simplified, both within and between countries. Within countries, national considerations such as elections and political constraints could very well affect countries' ability to contribute to the global public good, and hence international negotiations. And countries often interact with each other through a variety of channels such as trade, health, and foreign aid, and these other dimensions can have deep impacts on negotiations on a specific global public good.³¹ It would be interesting to introduce uncertainty to a model of international agreement incorporating some of these considerations.

Overall, there is much research to be done to better understand the determinants of interna-

³⁰See Rubio and Ulph (2007) for one of the first truly dynamic analysis of international environmental agreements in the presence of a stock pollutant.

³¹See e.g.Barrett (2003, Ch. 12), and the references therein, on linkage between different issues in international negotiations.

tional agreements in more realistic settings. In this paper, we have shown that uncertainty and risk aversion have strong strategic effects in a simple benchmark model. Given the importance of uncertainty in reality, we think that such strategic effects will also play a major role in the analysis of more realistic models incorporating arbitrary risks, general payoffs, heterogeneity, dynamics, and richer policy considerations.

APPENDIX

Proof of claim p.11. Denote by $\tilde{q}^s(k)$ the solution to condition (2). Rewriting the condition yields: $pU'_L(kb_L - c) + (1-p)U'_H(kb_H - c) = 0$ where the indices L and H stand for the function arguments $(kb_L - c)q$ and $(kb_H - c)q$. Derivating with respect to k yields:

$$\begin{aligned} & pU'_L b_L + p(kb_L - c)U''_L[(kb_L - c)\frac{\partial \tilde{q}^s}{\partial k}(k) + b_L^s \tilde{q}^s(k)] \\ & + (1-p)U'_H b_H + (1-p)(kb_H - c)U''_H \left[(kb_H - c)\frac{\partial \tilde{q}^s}{\partial k}(k) + b_H \tilde{q}^s(k) \right] = 0 \end{aligned}$$

Rewriting using $A = -U''/U'$ shows that $\tilde{q}^s(k)$ is weakly increasing iff:

$$pU'_L [-b_L + (kb_L - c)A_L b_L \tilde{q}^s(k)] + (1-p)U'_H [-b_H + (kb_H - c)A_H b_H \tilde{q}^s(k)] \leq 0$$

From the f.o.c., we have $\frac{U'_L}{U'_H} = \frac{(1-p)(kb_H - c)}{p(c - kb_L)}$. Substituting, we obtain that $\tilde{q}^s(k)$ is weakly increasing iff:

$$\tilde{q}^s(k) (A_L b_L - A_H b_H) \geq \frac{-c(b_H - b_L)}{(kb_H - c)(c - kb_L)}$$

If $A(\pi_H) \leq (b_L/b_H)A(\pi_L)$, then this condition is clearly satisfied since the LHS is non-negative and the RHS is negative. This means that the level of absolute risk aversion at π_H is sufficiently lower than the level of absolute risk aversion at $\pi_L \leq \pi_H$.

Proof of Proposition 2. Consider q^s as a function of b_L and b_H , holding \bar{b} constant . We have $p = (b_H - \bar{b})/(b_H - b_L)$. Substituting, condition (2) becomes: $U'_L(b_H - \bar{b})(kb_L - c) + U'_H(\bar{b} - b_L)(kb_H - c) = 0$. Derivating this last expression with respect to b_H , we see that:

$$U'_L(c - kb_L) \geq k(\bar{b} - b_L)U'_H \implies \frac{\partial q^s}{\partial b_H}(k, b_H) \leq 0$$

The left-hand side holds for $k \in [c/\bar{b}, c/b_L]$. Similarly, derivating with respect to b_L , we have:

$$U'_H(kb_H - c) \leq U'_L(b_H - \bar{b})k \implies \frac{\partial q^s}{\partial b_L}(k, b_H) \geq 0$$

where the left-hand side also holds when $k \in [c/\bar{b}, c/b_L]$. Thus, holding b_H constant, q^s decreases if b_L decreases and holding b_L constant, q^s decreases if b_H increases. This means that $q^s(b'_L, b'_H) \leq q^s(b_L, b'_H) \leq q^s(b_L, b_H)$.

Proof of Proposition 3. Denote by $F(q) = EV[(kb - c)q]$ the objective function of the signatories when agents have utility V . Since $V = \Phi(U)$, we have:

$$F'(q) = p\Phi'(U[(kb_L - c)q])U'[(kb_L - c)q] + (1 - p)\Phi'(U[(kb_H - c)q])U'[(kb_H - c)q].$$

$$\text{At } q = q^s(U), pU'[(kb_L - c)q] + (1 - p)U'[(kb_H - c)q] = 0, \text{ and}$$

$F'(q^s(U)) = (1 - p)U'[(kb_H - c)q](\Phi'(U[(kb_H - c)q]) - \Phi'(U[(kb_L - c)q]))$. Since $(kb_H - c)q > (kb_L - c)q$, U is increasing, and Φ' is decreasing, we have $F'(q^s(U)) \leq 0$. Since F is concave, F' is decreasing, and $F'(q^s(V)) = 0$, we have: $q^s(V) \leq q^s(U)$.

Computations for specific utilities.

Quadratic utility : Consider $q^s(k) = \frac{1}{2\lambda} \frac{k\bar{b} - c}{p(kb_L - c)^2 + (1-p)(kb_H - c)^2}$. Introduce $\bar{b}^2 = pb_L^2 + (1-p)b_H^2$.

Derivating q^s with respect to k , we obtain:

$$\frac{\partial q^s(k)}{\partial k} = \frac{-k^2\bar{b}\bar{b}^2 + 2k\bar{b}^2c - c^2\bar{b}}{2\lambda(k^2\bar{b}^2 - 2ck\bar{b} + c^2)^2}.$$

Then, q^s is increasing iff $-k^2\bar{b}\bar{b}^2 + 2k\bar{b}^2c - c^2\bar{b} \geq 0$. In $k = c/\bar{b}$, this expression becomes $c^2(b^2 - \bar{b}^2)/\bar{b}$, which is always positive. In addition, we know that $q^s(c/b_L) > q_{max}$ from proposition 1. Since the previous condition is quadratic in k , there are two cases. Either q^s is increasing over $[c/\bar{b}, c/b_L]$, or q^s is first increasing and then decreasing, and $\max_{[c/\bar{b}, c/b_L]} q^s > q_{max}$. In either case, $\min(q^s, q_{max})$

is increasing over $[c/\bar{b}, c/b_L]$.

CARA utility : Let $q^s(k) = \frac{1}{Ak(b_H - b_L)} \ln \left[\frac{(1-p)(kb_H - c)}{p(c - kb_L)} \right]$. Introduce the three following auxiliary functions: $f(k) = \frac{(1-p)(kb_H - c)}{p(c - kb_L)}$, $g(k) = \frac{(kb_H - c)(c - kb_L)}{k}$, and $h(k) = \ln[f(k)]g(k)$. Derivating q^s with respect to k , we obtain:

$$\frac{\partial q^s}{\partial k} = \frac{1}{A(b_H - b_L)k} \left(\frac{-\ln(f)}{k} + \frac{c(\bar{b} - b_L)}{(1-p)(kb_H - c)(c - kb_L)} \right)$$

This means that q^s is increasing iff $\frac{c(\bar{b} - b_L)}{(1-p)(kb_H - c)(c - kb_L)} \geq \frac{\ln[f(k)]}{k}$, which is equivalent to $h(k) \leq c(b_H - b_L)$. We next study the properties of h . We have : $h' = \ln[f]g' + \frac{f'}{f}g$. Since $k \geq c/\bar{b}$, $f(k) \geq 1$ and $\ln[f(k)] \geq 0$. In addition, $f'(k) \geq 0$ and $g'(k) = -b_L b_H + c^2/k^2$. This means that , $h'(k) > 0$ if $k \in [0, c/\sqrt{b_L b_H}]$. Looking at the second derivative of h gives:

$$\frac{\partial^2 h}{\partial k^2}(k) = \ln[f(k)]g''(k) + \frac{c(b_H - b_L)}{k^2(c - kb_L)(kb_H - c)} [c(-k(b_H + b_L) + 2c)]$$

which is strictly negative for $k \geq c/\sqrt{b_L b_H}$. Therefore, h is increasing over $[0, c/\sqrt{b_L b_H}]$ and strictly concave over $[c/\sqrt{b_L b_H}, c/b_L]$. In addition, we know that q^s is increasing at c/\bar{b} and becomes arbitrarily large when k gets close to c/b_L . This implies that q^s is either increasing over $[c/\bar{b}, c/b_L]$, or increasing, decreasing, and increasing again. Besides, $\partial h/\partial p \leq 0$, hence the whole h function shifts downwards when p increases. Suppose that $p_1 > p_2$. If q^s is increasing for p_1 , then q^s increasing for p_2 . If q^s is increasing, decreasing, increasing for p_1 , then either q^s is increasing, or the interval on which it is decreasing is smaller.

Proof of Proposition 5. Let $[x]$ denote the smallest integer greater than or equal to x . Let $k^* = [c/b_L]$. Recall, $\Delta(k^* + 1) < 0$ since $q^s(k^*) = q^s(k^* + 1) = q_{\max}$. In addition, if a signatory quits at k^* , he obtains $(k^* - 1)bq^s(k^* - 1)$. Thus,

$$\begin{aligned}\Delta(k^*) &= p [U((k^*b_L - c)q_{max}) - U((k^* - 1)b_Lq^s(k^* - 1))] \\ &+ (1 - p) [U((k^*b_H - c)q_{max}) - U((k^* - 1)b_Hq^s(k^* - 1))]\end{aligned}$$

Since $q^s(k^* - 1) < c/b_L$, if q_{max} is large enough, $q^s(k^* - 1) < q_{max}$. Since $k^*b_H - c > 0$, if q_{max} is large enough, $(k^*b_H - c)q_{max} > (k^* - 1)b_Hq^s(k^* - 1)$. If $k^* \neq c/b_L$, we also have $k^*b_L - c > 0$, hence $(k^*b_L - c)q_{max} > (k^* - 1)b_Lq^s(k^* - 1)$ if q_{max} is large enough. In this case, since U increasing, $\Delta(k^*) > 0$. This result also holds if $k^* = c/b_L$, and $\lim_{\pi \rightarrow \infty} U(\pi) = +\infty$.

Proof of Proposition 6. If $k \leq c/b_H$, $c - kb$ is less than or equal to zero in both states of the world, and strictly less than zero in one, hence signatories' emissions are clearly maximal. Next, suppose that $k \geq c/\bar{b}$. The marginal expected utility of a signatory at $e = 0$ is equal to $E(c - kb)U'[-(n - k)bq_{max}]$. Observe that $c - kb$ is decreasing in b while $U'[-(n - k)bq_{max}]$ is weakly increasing in b by concavity of U . The covariance rule (Gollier 2001) implies that $cov[(c - kb)U'[-(n - k)bq_{max}]] \leq 0$, hence that $E(c - kb)U'[-(n - k)bq_{max}] \leq E(c - kb)EU'[-(n - k)bq_{max}]$. Finally, $E(c - kb) = c - k\bar{b} \leq 0$ and $EU'[-(n - k)bq_{max}] \geq 0$, which means that the marginal expected utility at 0 is less than or equal to zero. By concavity, $e^s = 0$. QED.

Proof of Proposition 7. Consider e^s as a function of b_L and b_H . We have $p = \frac{b_H - \bar{b}}{b_H - b_L}$. The f.o.c for the coalition's problem is $p(c - kb_L)U'_L + (1 - p)(c - kb_H)U'_H = 0$. Substituting with $p = \frac{b_H - \bar{b}}{b_H - b_L}$, it becomes:

$$(b_H - \bar{b})(c - kb_L)U'_L + (\bar{b} - b_L)(c - kb_H)U'_H = 0 \quad (4)$$

We now derive with respect to b_L , with $\frac{\partial \bar{b}}{\partial b_L} = 0$, we have:

$$\begin{aligned}\frac{\partial e}{\partial b_L}(k) [(b_H - \bar{b})(c - kb_L)^2U''_L + (\bar{b} - b_L)(c - kb_H)^2U''_H] &= (b_H - \bar{b})kU'_L \\ + (b_H - \bar{b})(c - kb_L)U''_L[ne_{max} - k(e_{max} - e(k))] &+ (c - kb_H)U'_H. \text{ Replacing } (c - kb_H)U'_H = -\frac{p}{1-p}(c -\end{aligned}$$

$kb_L)U'_L$, from the f.o.c., in the last equation leads to : $\frac{\partial e}{\partial b_L}(k) \geq 0$, for all $k \leq c/\bar{b}$. We now derivate (4) with respect to b_H , which gives us:

$$\begin{aligned} \frac{\partial e}{\partial b_L}(k) [(b_H - \bar{b})(c - kb_L)^2 U''_L + (\bar{b} - b_L)(c - kb_H)^2 U''_H] &= -(c - kb_L)U'_L \\ + (\bar{b} - b_L)(c - kb_H)U''_H[ne_{max} - k(e_{max} - e(k))] &+ k(\bar{b} - b_L)U'_H. \end{aligned}$$

Again, using the f.o.c., we have : $\frac{\partial e}{\partial b_H}(k) \leq 0$, for all $k \leq c/\bar{b}$. Since $\frac{\partial e}{\partial b_L}(k) \geq 0$ and $\frac{\partial e}{\partial b_H}(k) \leq 0$, for all $k \leq c/\bar{b}$, and both derivatives equal to zero for $k > c/\bar{b}$ (from proposition 6), this proves proposition 7

Proof of Proposition 8. Denote by $F(q) = EV[(c - kb)e - (n - k)be_{max}]$ the objective function of the signatories when agents have utility V . Since $V = \Phi(U)$, we have:

$$\begin{aligned} F'(q) &= p(c - kb_L)\phi'_L U'_L + (1 - p)(c - kb_H)\phi'_H U'_H. \end{aligned}$$

At $e = e^s(U)$, we have $p(c - kb_L)U'_L = -(1 - p)(c - kb_H)U'_H$, and $F'(q) = p(c - kb_L)U'_L[\phi'_L - \phi'_H]$ since $(c - kb_H)e < (c - kb_L)e$, U is increasing, and Φ' is decreasing, we have. $F'(e^s(U)) \geq 0$. Since F concave, F' decreasing, and $F'(e^s(V)) = 0$, we have: $e^s(V) \geq e^s(U)$.

Proof of Proposition 10. We will show that there is a $\bar{q} > 0$ such that if $q_{max} \geq \bar{q}$, then for any $k \geq c/b_H$, $e^s(k) = 0$. The first order conditions of program 3 can be written

$$p(c - kb_L)U'[(c - kb_L)e - (n - k)b_L q_{max}] = (1 - p)(kb_H - c)U'[(c - kb_H)e - (n - k)b_H q_{max}]$$

Denote by $\tilde{e}^s(k)$ its solution. When u is CARA, direct computations lead to:

$$\tilde{e}^s(k) = -\frac{1}{Ak(b_H - b_L)} \ln \left[\frac{(1 - p)(kb_H - c)}{p(c - kb_L)} \right] - \frac{n - k}{k} q_{max}$$

If q_{max} is large enough, this is clearly negative for any k which implies that $e^s(k) = 0$ as soon as $k \geq c/b_H$.

Next, take the derivative of the f.o.c. with respect to q_{\max} . Algebraic computations lead to:

$$\frac{\partial \tilde{e}^s}{\partial q_{\max}} = -\frac{n-k}{k} \frac{b_H A(\pi_H) - b_L A(\pi_L)}{(kb_H - c)A(\pi_H) + (c - kb_L)A(\pi_L)}$$

where $A(\pi) = -u''(\pi)/u'(\pi)$, $\pi_H = (c - kb_H)e - (n - k)b_H q_{\max}$ and similarly for π_L . Observe that $\pi_H - \pi_L \leq 0$ if $e \geq 0$. When u satisfies DARA, $\pi_H \leq \pi_L \Rightarrow A(\pi_H) \geq A(\pi_L)$, hence $\frac{\partial \tilde{e}^s}{\partial q_{\max}} \leq -\frac{n-k}{k^2} < 0$. This guarantees that if $\exists q_1$ such that $\tilde{e}^s(q_1) < 0$, then $\forall q_{\max} \geq q_1$, $\tilde{e}^s(q_{\max}) < 0$. Also, if $\exists q_2$ such that $\tilde{e}^s(q_2) > 0$, then \tilde{e}^s decreases in q_{\max} til it becomes negative for $q_{\max} \geq q_2$. In the end, $\exists \bar{q} > 0$ such that if $q_{\max} \geq \bar{q}$, then for any $k \geq c/b_H$, $e^s(k) = 0$. QED.

Proof of Proposition 11 We show that any increase in uncertainty reduces the level of action.

Denote q^* before the increase in uncertainty. If q^* is not interior, the proof is obvious (see section 5.1). Then, assume that that k leads to an interior solution for q^* . We use the fact that a distribution x' is more risky than x iff there exist a white noise ε such that $x' = x + \varepsilon$. The first derivative of the problem of the coalition after the increase in risk, evaluated at q^* , is: $E_\varepsilon E_{b,c} U'(kb - c)q^* + \varepsilon q^*)(kb - c + \varepsilon)$. We can rewrite this last expression as $:E_\varepsilon E_{b,c} f(\varepsilon)$. If $f(\varepsilon)$ is concave, we have $:E_\varepsilon E_{b,c} f(\varepsilon) \leq E_{b,c} f(0) = 0$ (Jensen's inequality), so the optimal level of action has to be lower for higher risks (since U is strictly concave). We can compute $f''(\varepsilon) = U''' q^{*2}(kb - c + \varepsilon) + 2U'' q^*$, so $f''(\varepsilon) \leq 0$ iff $-\frac{U'''}{U''} q^*(kb - c + \varepsilon) \leq 2$.

Robustness results for the Public Good Model

The Effect of Risk Aversion We show that an increase in Risk Aversion reduces the level of action. Consider the first derivative of the coalition when preferences are represented by $V(x) = \phi(U(x))$, with $\phi' > 0$ and $\phi'' < 0$ (i.e. V more risk averse than U): $kE\phi'[U((kb - c)q)]U'[(kb - c)q](kb - c)$. Notice that, $(kb - c)\phi'(U[(kb - c)q]) \leq (kb - c)\phi'(U[0])$ for any values of k, b, c, q . Then, the first derivative is lower than $k\phi'(U[0])EU'[(kb - c)q^*(U)] = 0$ so the optimal

level of action is lower for more risk averse preferences.

Existence We show that there exist a stable treaty with positive action. Let k_1 be the first integer such that $q^*(k) > 0$. We know that $k_1 \geq \bar{c}/\bar{b}$. The expected utility for a non-signatory at $k_1 - 1$ is $EU^n(k_1 - 1) = EU[0]$, while the expected utility for a signatory at k_1 is $EU^s(k_1) = EU[(k_1 b - c)q^s]$. Then, at $k_1 - 1$ a non-signatory has a strict incentive to join the coalition. Now, let $k_2 = \lceil c_H/b_L \rceil$. The expected utility of a non-signatory at k_2 is $EU^n(k_2) = U[k_2 b q_{max}]$ while the expected utility of a signatory at $k_2 + 1$ is $EU^s(k_2 + 1) = U[k_2 b q_{max} + (b - c)q_{max}]$. Then, at k_2 , a non-signatory has no incentive to join the coalition. This shows that there exists a stable treaty $k^* \in [k_1, k_2]$. Since $EU'^s(0) > 0$ for $k \geq \bar{c}/\bar{b}$, we know that $q(k^*) > 0$.

Proof of Proposition 12 Define $k^* = \lceil c_H/b_L \rceil$. At k^* , we know that a non-signatory has no incentive to join the coalition. We show that if q_{max} is large enough, we have $q^*(k^* - 1) < q_{max}$. If $k^* - 1 < \bar{c}/\bar{b}$, the result is obvious, otherwise we show that the f.o.c holds. Notice that (at least) $(kb_L - c_H)$ is negative, and $(kb_H - c_L)$ is positive. Since the utility function is concave, the negative part of the first derivative is increasing in q in absolute value, while the positive part is decreasing in q . If $\lim_{x \rightarrow -\infty} U'(x) = \infty$ (or if $\lim_{x \rightarrow \infty} U'(x) = 0$) and q_{max} is large enough, there exists q^* such that the first derivative equals zero.

We now show that k^* is stable. The expected utility of a non-signatory at $k^* - 1$ is given by $EU^n(k^* - 1) = EU[(k^* - 1)bq^s(k^* - 1)]$ while the expected utility for a signatory at k^* is given by $EU^s(k^*) = EU[(k^* b - c)q_{max}]$. If c_H/b_L is not an integer, the payoff of a signatory at k^* is strictly increasing in q_{max} . If $\lim_{\pi \rightarrow \infty} U(\pi) = \infty$, there exist q_{max} such that $EU^s(k^*) > EU^n(k^* - 1)$. Then, if q_{max} is large enough, a signatory at k^* has no incentive to quit the coalition, since $q^s(k^* - 1)$ is strictly smaller than q_{max} and independent of q_{max} . Then k^* is stable.

Robustness results for the Public Bad Model

Level of emissions We show that $q(k) = 0$ if $k \leq c_L/b_H$, and $q(k) = q_{max}$ if $k \geq \bar{c}/\bar{b}$.

The first derivative for the coalition is $kEU'[(c-kb)e-(n-k) bq_{max}](c-kb)$, which is lower than $kEE\{U'[(c-kb)e-(n-k) bq_{max}](\bar{c}-kb)|b\}$, and lower than $kEU'[(c-kb)e-(n-k) bq_{max}](\bar{c}-k\bar{b})$ by the law of iterated expectations and the successive application of the covariance rule. Then, the level of emissions is null for $k > \bar{c}/\bar{b}$. Also, as in the public good model, we can show that the first derivative is strictly positive for $k \leq c_L/b_H$. Then, the level of emissions is maximal for $k \leq c_L/b_H$.

Risk Aversion in the sens of Ross (1981) Consider the maximization problem of the coalition when countries' preferences are represented by $v(x)$, more risk averse than $u(x)$ in the sens of Ross (1981). The first derivative can be written as a function of $u(x)$, i.e. $\lambda E(c-kb)u'[(c-kb)e-(n-k) bq_{max}] + Ef'[(c-kb)e-(n-k) bq_{max}](c-kb)$. By the covariance rule, we have that $E(c-kb)f'[(c-kb)e-(n-k) bq_{max}] \leq (\bar{c}-k\bar{b})Ef'[(c-kb)e-(n-k) bq_{max}] \leq 0$ since $k \leq \bar{c}/\bar{b}$. Then, we have that $\lambda E(c-kb)u'[x] + Ef'[x](c-kb) \leq \lambda E(c-kb)u'[x]$. Since the RHS is equal to zero at $e^*(u)$, an increase in risk-aversion in the sens of Ross (1981) leads to a lower level of emissions.

Existence

We show that there exists a stable level of participation k^* with positive action. Let k_1 be the first integer such that $e^s(k_1) < q_{max}$. Notice that $EU[(c-nb)q_{max}] < EU[(c-k_1b)e^s(k_1) - (n-k_1)bq_{max}]$ since $e^s(k_1) < q_{max}$ solves the problem of the coalition for $k = k_1$, and U is strictly increasing.

Now, let $k_2 = \lceil \bar{c}/\bar{b} \rceil$, we have that $EU[cq_{max} - (n-k_2)bq_{max}] < EU[-(n-k_2-1)bq_{max}]$. Since a country has a strict incentive to join the treaty at k_1 , and no incentive to join at k_2 , we know that there exist a stable treaty $k^* \in [k_1, k_2]$. Suppose that $e(k^*) = q_{max}$, then it means that there exists k_3 such that $e(k_3) < e(k_3 + 1)$. Then, at k_3 , no country has an incentive to join the coalition. There exists a stable treaty $\tilde{k} \in [k_1, k_3]$. Since $e(k_1) < q_{max}$, we know that there exists

some stable treaty with strictly positive action.

Proof of Proposition 13 We show that any increase in uncertainty increases the level of action.

Denote e^* the optimal level of emissions for a given level of uncertainty. If e^* is not interior, the solution is obvious. Then, assume that that k leads to an interior solution for e^* . The proof goes as the one of proposition 11.

First, we look at an increase in risk through c . The first derivative of the problem of the coalition after the increase in risk, evaluated at e^* , is: $E_\varepsilon E_{b,c} U'(c + \varepsilon - kb)q^*(c - kb + \varepsilon)$. We can rewrite this last expression as $:E_\varepsilon E_{b,c} f(\varepsilon)$. If $f(\varepsilon)$ is concave, we know that the optimal level of emissions has to be lower for higher risks (since U is strictly concave). We can compute $f''(\varepsilon) = U'''e^{*2}(c - kb + \varepsilon) + 2U''e^*$, so $f''(\varepsilon) \leq 0$ iff $-\frac{U'''}{U''}e^*(c - kb + \varepsilon) \leq 2$.

Next, we look at an increase in risk through b . Proceeding the same way, we find that $f''(\varepsilon) \leq 0$ iff $-\frac{U'''}{U''}(ke^* + (n - k)q_{max})(c - kb - k\varepsilon)/k \leq 2$

Denote c_H and b_L , the highest possible cost, and the lowest possible benefit *after* the increase in risk, then a sufficient for both conditions is: $-\frac{U'''}{U''} \leq \frac{2}{nq_{max}(c_H - b_L)}$.

Proof of Proposition 14 Suppose that U respects CARA or DARA. We show that if q_{max} is high enough, $q^s(k) = q_{max}$ for all $k \geq \bar{c}/b_H$, and that any stable treaty k^* respects $c_L/b_H \leq k \leq \bar{c}/b_H$.

First, define $\pi(e) = E_{b,c}u((c - kb)e - (n - k)bq_{max})$. Since $\pi'' < 0$, we have $e = 0 \Leftrightarrow \pi'(0) < 0$, where $\pi'(0) = E_b(\bar{c} - kb)u'(-(n - k)bq_{max})$, the expected marginal utility at $e = 0$.

Suppose first that u is CARA. Then $u'(x) = Ae^{-Ax}$ which yields $\pi'(0) = e^{A(n-k)b_Lq_{max}}[p(\bar{c} - kb_L) - (1 - p)(kb_H - \bar{c})e^{A(n-k)(b_H - b_L)q_{max}}]$. When q_{max} is large enough, $\pi'(0)$ has the same sign as $-(kb_H - \bar{c})$, hence negative if $k > \bar{c}/b_H$ and positive if $k < \bar{c}/b_H$. This shows that $e = 0$ if $k > \bar{c}/b_H$ and q_{max} is high enough.

Suppose next that u is DARA. We have $\pi'(0) = u'(-(n - k)b_Lq_{max})[p(\bar{c} - kb_L) - (1 - p)(kb_H - \bar{c})\frac{u'(-(n-k)b_Hq_{max})}{u'(-(n-k)b_Lq_{max})}]$.

Next, we know that $f(x) - f(y) = f'(z)(x - y)$ with $z \in [x, y]$ which means that $u'(-(n - k)b_H q_{\max}) - u'(-(n - k)b_L q_{\max}) = (n - k)(b_H - b_L)q_{\max}(-u'')(-(n - k)\hat{b}q_{\max})$ with $\hat{b} \in [b_L, b_H]$.

Thus, $\frac{u'(-(n-k)b_H q_{\max})}{u'(-(n-k)b_L q_{\max})} = 1 + (n - k)(b_H - b_L)q_{\max} \frac{(-u'')(-(n-k)\hat{b}q_{\max})}{u'(-(n-k)b_L q_{\max})}$.

Since u is DARA, prudence holds, $u''' > 0$, hence $(-u)''$ is decreasing. Since $-(n - k)\hat{b}q_{\max} \leq -(n - k)b_L q_{\max}$, we have $(-u'')(-(n - k)\hat{b}q_{\max}) \geq (-u'')(-(n - k)b_L q_{\max})$. Hence $\frac{(-u'')(-(n-k)\hat{b}q_{\max})}{u'(-(n-k)b_L q_{\max})} \geq A(-(n - k)b_L q_{\max}) \geq A(0)$.

Thus, $\frac{u'(-(n-k)b_H q_{\max})}{u'(-(n-k)b_L q_{\max})} \geq 1 + (n - k)(b_H - b_L)A_0 q_{\max}$, which tends to ∞ when q_{\max} tends to ∞ , which implies that $\pi'(0)$ also has the same sign as $-(kb_H - \bar{c})$, hence is negative if $k > \bar{c}/b_H$.

Remark on CARA utility functions For CARA utility functions, we show that, as q_{\max} tends to infinity, the solution with uncertainty on the cost and the benefit of action tends to the one with uncertainty on the cost, when $b = b_H$ with certainty iff $k \in]c_L/b_H, \bar{c}/b_H[$.

Let $\pi'(0) = e^{A(n-k)b_L q_{\max}} [p(\bar{c} - kb_L) - (1 - p)(kb_H - \bar{c})e^{A(n-k)(b_H - b_L)q_{\max}}]$. Since $kb_H - \bar{c} < 0$, $\pi'(0) > 0$ if q_{\max} is large enough. Then, either $\pi'(q_{\max}) > 0$, and $e = q_{\max}$ or there is an interior solution $e > 0$ solving $\pi'(e) = 0$.

In that case, $pe^{Ab_L(ke + (n-k)q_{\max})} E_c(c - kb_L)e^{-Ace} + (1 - p)e^{Ab_H(ke + (n-k)q_{\max})} E_c(c - kb_H)e^{-Ace} = 0$, so $e^{A(b_H - b_L)(ke + (n-k)q_{\max})} = -\frac{p}{1-p} \frac{E_c(c - kb_L)e^{-Ace}}{E_c(c - kb_H)e^{-Ace}} > 0$ implying that $E_c(c - kb_L)e^{-Ace} > 0$ and $E_c(c - kb_H)e^{-Ace} < 0$. Then, $E_c(c - kb_H)e^{-Ace} = -p'(kb_H - c_L)e^{-AcLe} + (1 - p')(c_H - kb_H)e^{-AcHe}$, where $kb_H - c_L > 0$ since $k > c_L/b_H$ and $c_H - kb_H > 0$ since $k < \bar{c}/b_H < c_H/b_H$.

Thus, $-p'(kb_H - c_L)e^{-AcLe} + (1 - p')(c_H - kb_H)e^{-AcHe} = M(e)e^{-A(b_H - b_L)(ke + (n-k)q_{\max})}$, where $M(e) = -\frac{p}{1-p} E_c(c - kb_L)e^{-Ace}$ is bounded. Therefore, the LHS tends to zero, which means that e tends to e^* which solves: $-p'(kb_H - c_L)e^{-AcLe} + (1 - p')(c_H - kb_H)e^{-AcHe} = 0$, which corresponds to the f.o.c. of the problem: $\max E_c u((c - kb_H)e - kb_H q_{\max})$. Finally, observe that $e^* > 0 \Leftrightarrow k < \bar{c}/b_H$

Figure 2: Effects of Uncertainty on Signatories' Effort (Public Good)

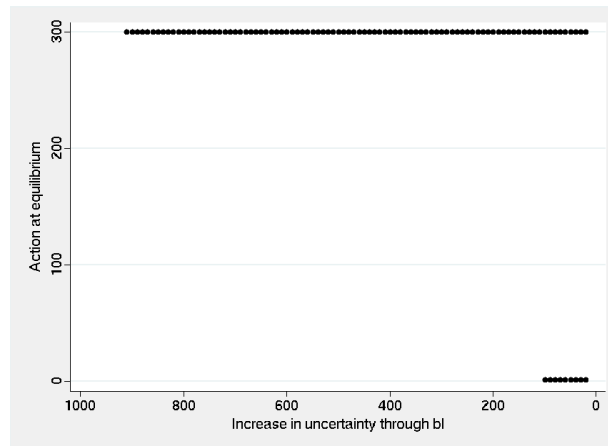


Figure 3: Effects of Uncertainty on Participation (Public Good)

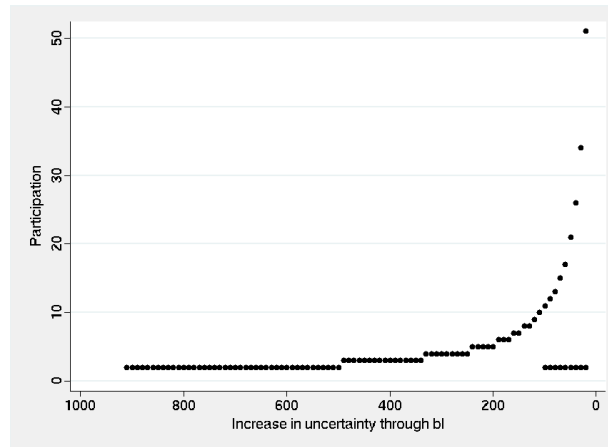


Figure 4: Effects of Uncertainty on Welfare (Public Good)

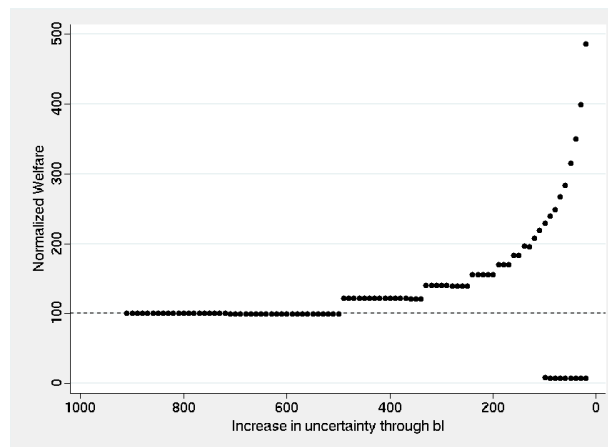


Figure 5: Effects of Uncertainty on Signatories' Effort (Public Bad)

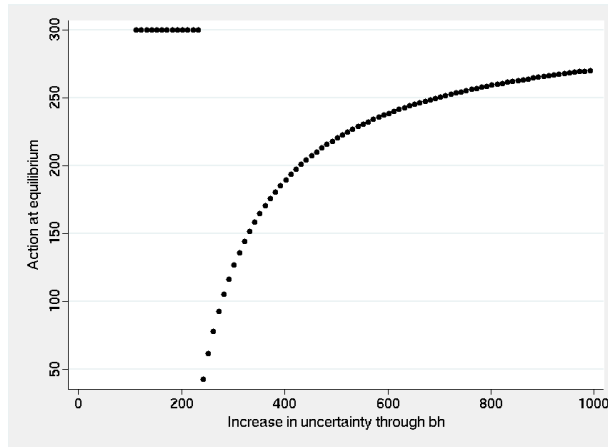


Figure 6: Effects of Uncertainty on Participation (Public Bad)

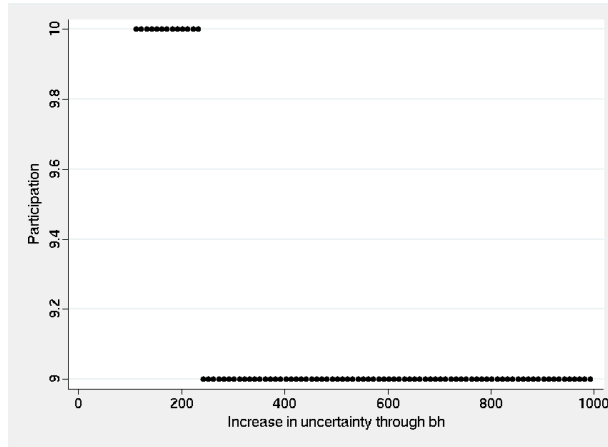
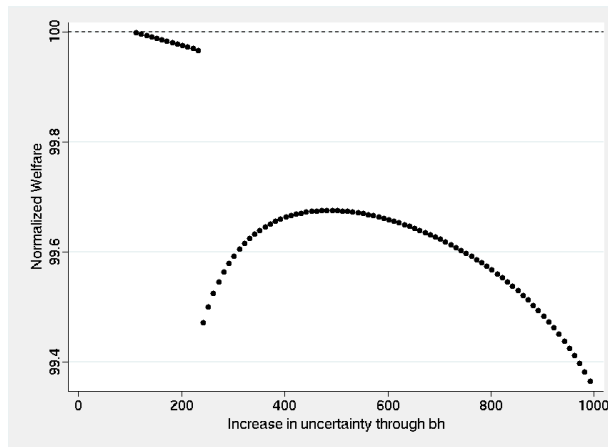


Figure 7: Effects of Uncertainty on Welfare (Public Bad)



REFERENCES

- Arrow, K. and R. Lind** (1970) "Uncertainty and the evaluation of public investment decisions", *American Economic Review*, 60, 364-78.
- Barrett, S.** (1994): "Self-Enforcing International Environmental Agreements," *Oxford Economic Papers*, 46, 878-894.
- Barrett, S.** (2001): "International cooperation for sale," *European Economic Review*, 45, 1835-1850.
- Barrett S.** (2003): *Environment & Statecraft*. Oxford: Oxford University Press.
- Barrett S.** (2008): *Why Cooperate? The Incentive to Supply Global Public Goods*. Oxford: Oxford University Press.
- Bramoullé, Y. and N. Treich.** (2009): "Can Uncertainty Alleviate the Commons Problem?," *Journal of the European Economic Association*, 7(5), 1042:1067
- Eso, P. and L. White.** (2004): "Precautionary Bidding in Auctions," *Econometrica*, 72, 77-92.
- Gollier C.** (2001): *The Economics of Risk and Time*. Cambridge: The MIT Press.
- Gradstein, M., Nitzan, S. and S. Slutsky.** (1992): "The Effect of Uncertainty on Interactive Behaviour," *The Economic Journal*, 102, 554-561.
- Heal, G. and B. Kriström.** (2002): "Uncertainty and Climate Change," *Environmental and Resource Economics*, 22, 3-39.
- Kolstad, C.D.** (2005): "Piercing the Veil of Uncertainty in Transboundary Pollution Agreements," *Environmental and Resource Economics*, 31, 21-34.
- Kolstad, C.D.** (2007): "Systematic uncertainty in self-enforcing international agreements," *Journal of Environmental Economics and Management*, 53, 68-79.
- McBride M.** (2006): "Discrete public good under threshold uncertainty," *Journal of Public Economics*, 90, 1181-1199.
- McGinty, M.** (2007): "International environmental agreements among asymmetric nations," *Oxford Economic Papers*, 59, 45-62.
- Na, S.-L. and H.S. Shin.** (1998): "International environmental agreements under uncertainty," *Oxford Economic Papers*, 50, 173-185.
- Nitzan S. and Romano R. E.** (1990): "Private provision of a discrete public good with uncertain cost," *Journal of Public Economics*, 42, 357-370.
- Nordhaus, W.** (1994): *Managing the Global Commons, The Economics of Climate Change*. Cambridge: the MIT Press.

Palfrey T, and Rosenthal H. (1984): "Participation and the provision of a discrete public goods: a strategic analysis," *Journal of Public Economics*, 24, 171-193.

Ross, S. A. (1981): "Some stronger measures of risk aversion in the small and in the large with applications," *Econometrica*, 3, 621-638.

Rothschild, M., and J. Stiglitz. (1970): "Increasing risk: I. A definition," *Journal of Economic Theory*, 2, 225-243.

Rubio, S., and A. Ulph. (2007): "An infinite horizon model of dynamic membership of international environmental agreements," *Journal of Environmental Economics and Management*, 54, 296-310.

Sandler, T. and F. P. Sterbenz. (1990): "Harvest Uncertainty and the Tragedy of the Commons," *Journal of Environmental Economics and Management*, 18, 155-167.

Stern, N. (2008): "The Economics of Climate Change", *American Economic Review Papers and Proceedings*, 98(2), 1-37.

Stern, N. (2007): *The Economics of Climate Change*, Cambridge University Press.

Suleiman, R. (1997): "Provision of step-level public goods under uncertainty: a theoretical analysis," *Rationality and Society*, 9, 163-187.

Ulph, A. (2004): "Stable International Environmental Agreements with a Stock Pollutant, Uncertainty and Learning," *The Journal of Risk and Uncertainty*, 29, 53-73.

Ulph, A. and D. Maddison (1997): "Uncertainty, learning and international environmental policy coordination," *Environmental and Resource Economics*, 9, 451-66.

White, L. (2008): "Prudence in bargaining: The effect of uncertainty on bargaining outcomes." *Games and Economic Behavior*, 62(1), 211-231