Optimization of Ship-Pack in a Two-Echelon Distribution System

by

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Abstract

The traditional Economic Order Quantity (EOQ) model ignores the physical limitations of distribution practices. Very often distribution centers (DC) have to deliver merchandize in manufacturer-specified packages, which can impose restrictions on the application of the economic order quantity. These manufacturer-specified packages, or ship-packs, include cases (e.g., cartons containing 24 or 48 units), inners (packages of 6 or 8 units) and eaches (individual units). For each Stock Keeping Unit (SKU), a retailer decides which of these ship-pack options to use when replenishing its retail stores. Working with a major US retailer, we have developed a cost model to help determine the optimum warehouse ship-pack. Besides recommending the most economical ship-pack, the model is also capable of identifying candidates for warehouse dual-slotting, i.e., two *picking modules* for the same SKU that carry two different pack sizes. We find that SKUs whose sales volumes vary greatly over time will benefit more from dual-slotting. Finally, we extend our model to investigate the ideal case configuration for a particular SKU, that is, the ideal size for an inner package.

Thesis Supervisor: Stephen C. Graves
Abraham J. Siegel Professor of Management Science

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Chapter 1

Introduction

1.1 Project Motivation

Much effort has been spent on optimizing inventory levels in a two-echelon distribution system. However, one important factor is often largely ignored: the choice of pack size that is to be shipped from the distribution center (DC) to the retail stores for a particular item. Our research is motivated by such a real problem of choosing the right ship-pack quantity for a major US-based retailer. The ship-pack quantity can typically be one of three choices: an "each" or individual unit, an "inner" (a packaged set of eaches, on the order of 6 to 8 units), or a case (e.g., a box of 24 units). The DC will incur a greater handling cost when it replenishes with eaches or inners rather than full cases for two reasons. First, warehouse associates need to spend time cutting open boxes and putting them in the appropriate locations. Second, each replenishment order from the store entails more work picking the packages. However, replenishing with cases could pose many problems for stores as well as DCs. First, the store inventory cost may increase since the order amount has to be a multiple of the case quantity, which could result in more store inventory. Second, stores with limited shelf space will have to put units that do not fit onto the shelves in the backroom, and this practice in turn incurs extra labor cost. Handling individual units at store level also increases the chances of pilferage and damage. Finally, the DC sees larger demand variability when stores are replenished in cases, and as a consequence, the DC has to carry more safety stock. Thus, it is of both the DC's and the stores' interest to find the optimal ship-pack that balances the DC handling cost, the store handling cost and inventory-related costs at both the DC and the stores. This paper addresses this problem by working with a major US retailer.

Currently, the major US retailer, which we refer to as Beta due to confidentiality, uses an Excel model to determine the optimal ship-pack, based on an analysis of the cost at an "average" store, namely a store with the average demand rate. This analysis ignores the wide variation in demand rates across the retail stores served by a DC. We believe this model can be improved by looking at specific demands from individual stores. Thus, our goal is to develop a comprehensive model that incorporates all essential cost components and generates the most economical warehouse ship-pack.

1.2 Two-Echelon Distribution System and the (R, s, S) Policy

We consider the two-echelon distribution system with a central DC and multiple non-identical retailers, as shown in Figure 1-1. Each retailer is replenished based on a pre-determined schedule that is subject to change depending on various factors such as sales promotion. Nonetheless, the schedule is relatively fixed for a considerable duration and the period between two planned replenishments for a particular store is the review period, R.

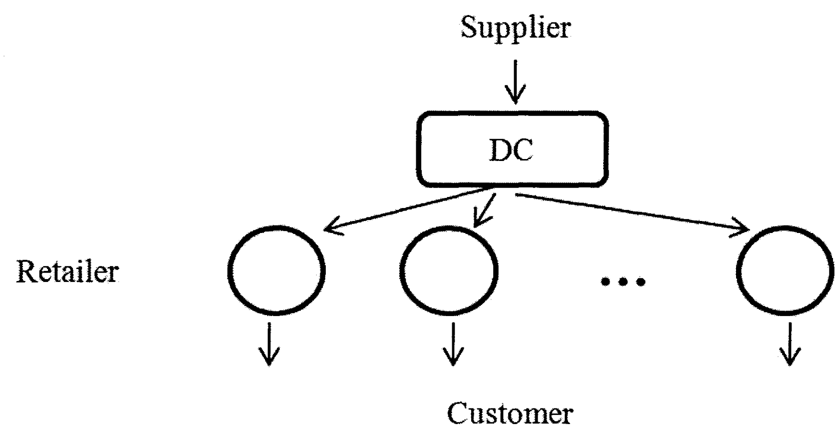

Figure 1-1: Two-echelon distribution system with single warehouse and multiple retailers

At the end of each review period R, the inventory control system checks the Inventory Position (IP) of all Stock Keeping Units (SKUs) at the store. If IP $\leq s$ for an SKU, then an order will be placed for that SKU to bring its inventory level to at least S. We also term s the Re-Order Point (ROP) and S the Order-Up-To-Level (OUTL).

1.3 Company Background and Inventory Policies

Beta is a major retail company with over 1,500 stores in the United States and several hundreds more overseas. It carries approximately 12,000 SKUs.

The SKUs at each store are replenished either from the DC or directly from the vendor (or supplier) by a flow-through policy. Under the flow-through policy, goods from the vendor are received at the DC and then directly sent to respective picking locations, from which store orders are fulfilled. Thus, the stores receive virtually everything from the DC. In this project, we focus on the distribution process between the DC and the retail stores, i.e., the two-echelon distribution system within Beta.

Each store is replenished on a regular weekly schedule. Low volume stores are replenished once a week on a fixed day; higher volume stores are replenished two to five times a week, also on fixed days. Beta follows the (R, s, S) inventory control policy described in the preceding section. Every night, specialized software collects and analyzes customer sales data from its retail stores. Based on these data, on-hand inventory at each store is updated. Prior to the replenishment of each store, a DC-to-store replenishment order is placed for each SKU that is below the store ROP; the replenishment is set to the minimum number of ship-packs that increase the stock to equal or exceed the OUTL.

1.4 Literature Review

1.4.1 Economic Order Quantity

The economic order quantity (EOQ) problem is a century-old research topic that traces its root to a 1913 article by Ford Whitman Harris in *Factory: The Magazine of Management* (Erlenkotter, 1990). Today, the EOQ formula has become a pervasive textbook formula which every supply chain student has to learn. Traditional EOQ model assumes instant and infinite availability of products, deterministic and constant demand, constant fixed order cost and no shortages allowed (Hopp and Spearman, 1996). Three basic components are incorporated in the model: a fixed order or setup cost, a holding cost and a variable order or unit production cost. Later variations of the EOQ model have relaxed some of the assumptions. The Economic Production Lot size (EPL) model assumes a finite and fixed production rate; the Wagner-Whitin model relaxes the assumption on constant demand rate; and a variant of EOQ allows shortages and considers a back-order cost.

1.4.2 Impact of the Pack Size and the Minimum Order Quantity (minOQ)

Although a great deal of academic literature exists on the EOQ model and its variants, very few studies have been done relating to pack size restrictions. Wagner (2002) acknowledges that the pack size could affect the order quantity in the real. Silver et al. (1998) suggest a simple way of dealing with the pack sizes based on the form of the total cost curve in classical EOQ model. Since the total cost curve is convex, the best integral multiple of the pack sizes must be one of the two possible values surrounding the optimal continuous Q. However, a critical factor is ignored in the classical EOQ model: the handling cost of dealing with different case packs (including the individual unit which is essentially a case pack of one) both in the DC and the stores.

Van Zelst et al. (2006) recognize shelf stacking process as the largest driver of the store operational cost. Moreover, the paper also demonstrates that the case pack size is the most important driver for stacking efficiency and concludes that increasing the case pack size could increase the stacking efficiency. However, Broekmeulen et al. (2007) later develop a regression model to show that high case pack sizes tend to cause shelf space shortages. Ordering behaviors from store managers are also significantly affected by the case pack size. The larger the case pack size for an SKU is, the more the store managers tend to deviate from system generated orders (van Donselaar et al. 2006). Thus, it is difficult to decide the best case pack size even at the store level.

Besides analysis that focuses on the impact of the case pack size on the retail level, some papers have extended such studies onto the DC level. A few papers show that pack size constraints could cause bullwhip effect in the supply chain system, which consequently increase total system cost (Geary et al. 2006, Lee et al., 1997a). This is in line with our modeling that larger ship-pack size induces larger demand variances at the DC level. Yan et al. (2009) address the problem of whether large case packs should be split prior to the retail level. They consider a two-echelon supply chain with a single distributor and multiple retailers under periodic review inventory system. Assuming retail demand from an equicorrelated multivariate Poisson distribution, Yan et al. designed a factorial experiment with eight parameters including the number of retailers, the average retailer demand, heterogeneity of the retailer demand, the spatial correlation between retailer demands, the delivery pack size, the inventory safety factor, the review period at the retailer level and the critical protection period at the distributor level. Each parameter has three values that represent low, medium and high levels respectively. It is worth noting that the three pack sizes experimented are 1, 6 and 24, since these three pack sizes are also the most common among Beta's SKUs. Through simulation and ANOVA analysis, they find that of the eight parameters, the pack size has the most significant effect on amplifying demand variance up the supply chain, and it is also one of the most significant factors that result in larger stock-on-hand and back-orders at retailer level. Thus, the recommendation is to split packs at the distributor level. However, the paper ends on a cautionary note that soft costs such as breakage, pilferage and increased labor costs should be considered by management before any decision is made. It also suggests future research to include such financial implications, which is what this project does.

1.4.3 Current Practices at Beta

Currently Beta employs a decision tool built into MS Excel to determine the optimal warehouse ship-pack. It first calculates the economic order quantity using the traditional EOQ formula $\sqrt{\frac{2AD}{h}}$ (Hopp and Spearman, 1996), where D is the average annual demand in units per store, A is the fixed order cost and h is the inventory carrying cost. Then, for each of the three possible ship-packs (eaches, inners and cases), the EOQ is rounded up to a multiple of the ship-pack quantity. For each ship pack, the number of orders per year is then expressed as $\frac{D}{EOQ_{sp}}$. The order quantity has an sp subscript because it is dependent on the ship-pack sp. Currently, Beta categorizes eaches and inners as both being break-packs and does not differentiate the DC picking and replenishment costs based on whether the ship-pack is break-pack BP or $case$. The model assumes that the picking cost for each BP is on a per order basis while the picking cost for each case is on a per case basis. The DC replenishment cost is always on a per case basis. The DC handling cost includes the picking cost and replenishment cost, therefore, the detailed formula for each of the three possible ship-packs is as follows:

$$DC\ Handling\ Cost_{each} =$$

$$\frac{D}{EOQ_{each}} * picking\ cost_{BP} per\ order + \frac{D}{case\ quantity} * replen\ cost_{BP}\ per\ case$$

$$DC\ Handling\ Cost_{inner} =$$

$$\frac{D}{EOQ_{inner}} * picking\ cost_{BP} per\ order + \frac{D}{case\ quantity} * replen\ cost_{BP}\ per\ case$$

$$DC\ Handling\ Cost_{case} =$$

$$\frac{D}{case\ quantity} * (picking\ cost_{case} per\ case + replen\ cost_{case}\ per\ case)$$

Finally, the impact on inventory costs of changing from each to inner and from each to case is calculated by looking at how much more inventory is held, based on different assumed EOQ's. The final decision is made based on the after tax handling cost and the inventory impact cost for the three ship-packs. The one with the least cost is the chosen ship-pack.

The good point about Beta's current ship-pack tool is its simplicity and ease of use. However, it ignores the differences between retail stores as it uses average store demand. This omission could lead to a sub-optimal result because a few unusually high-volume stores could distort the calculation. Moreover, the tool omits the extra handling cost at the stores that is attributable to different ship-packs. Last but not least, the potential saving from the reduced number of store orders is not considered by the ship-pack tool.

1.5 Organization

In Chapter 2 we will first describe the existing operations in Beta, then present in detail the development of the warehouse ship-pack cost model in accordance with current practices and finally introduce how we implement the model. Chapter 3 contains illustrative results that show the capability of the ship-pack model, including ship-pack change recommendations and the corresponding cost savings if Beta were to follow the recommended changes. In Chapter 4 we also extend our project from finding the optimal ship pack to searching for an ideal case configuration. Finally Chapter 5 concludes the thesis and suggests future research directions.

Chapter 2

Warehouse Ship-Pack Cost Model

2.1 Existing Operations

We made a few site visits to gain first-hand experience of the ship-pack's role in the distributor-retailer system. First we visited one of Beta's DCs in the northeastern region, and then we made two trips to a medium-volume store in the greater Boston area.

2.1.1 DC operations

We visited the DC that is closest to MIT in terms of geographical distance. This particular DC carries 80% of the total SKUs and serves about 280 stores in the region. Thirty percent of the SKUs are flow-through items, which do not go to the reserve area. However, they still go through the same replenishment and picking processes, so we do not need to make exceptions for these items.

Every day, the DC receives pallets of products from suppliers according to a pre-fixed schedule. The items are first unloaded from the containers at the receiving docks and temporarily put aside in the staging area, where sorting may be done for mixed pallets. A mixed pallet contains more than one SKU. Then the product is moved from the staging area to the Pick-and-Drop (PND) area. Depending on whether the product is flow-through or not, it will next be put away either in the reserve area (non-flow-through items) or in its respective picking modules (flow-through items). The non-flow-through product in the reserve area is used to replenish the picking modules, as the inventory in the picking module is depleted. This process is termed the DC replenishment process, namely the process that transfers the items from the reserve area (or PND for flow-through item) to the picking modules. There are two types of picking modules: break-pack (BP) and full case (FC). BP modules contain broken cases in which individual consumer units (eaches) or inner packs are accessible and therefore easy for warehouse associates to grab, whereas FC modules store the unopened cases. Based on the destinations, DC replenishment can be BP replenishment or FC replenishment. Since most SKUs are packed in cases when they are received into the DC, additional work is required for BP replenishment, e.g. cutting open and emptying the cases. Currently, Beta estimates its BP replenishment at a per-case cost that is 4.5 times greater than that for its FC replenishment.

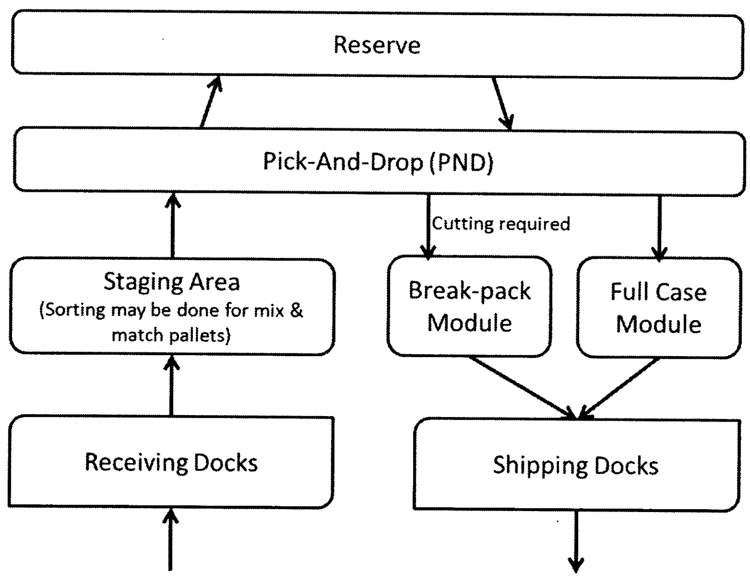

Figure 2-1: Warehouse operations; for illustration purpose only, not drawn to scale

Another DC operation essential to our ship-pack analysis is the picking activities. Different picking behaviors are observed in the two picking modules. When a DC-to-store order (i.e., a store replenishment order) is placed by the warehouse management system, picking labels are generated for full case selection if the ship-pack is cases; if the ship-pack is a BP, then picking is done using traditional pick lists. For an FC picking task, warehouse associates will take out the cases corresponding to the picking labels destined to a store, and place them on the conveyor belt, which connects the modules with the shipping docks. For a BP picking task, an associate will affix the barcode from the pick list to a tote and then follow the pick list to fill the tote manually. Each tote is dedicated to a single customer (store). Flashlight indicators are used to facilitate the picking process. When a tote is full, it is sent to the shipping docks by the conveyor belt. According to the warehouse manager, these associates know their products and their areas, so the accuracy is high enough for us to ignore the effect of picking errors.

The DC has a state-of-the-art system that sorts cases and totes at the end of the conveyor belt by their destinations. Then Beta's shipping fleet will dispatch these orders to their respective destination stores. The lead time ranges from one day to three days depending on the geographical location and the sales volume of the store.

2.1.2 Store operations

To understand how the ship-pack quantity affects the store operation cost, we have made two visits to a medium-volume store in the greater Boston area. One was to observe the receiving process and the other was to see the stacking process. Although there is no Standing Operating Procedure (SOP) for store receiving, sorting and putting away, these processes are fairly similar across stores.

Stores usually receive the delivery from the DC in the afternoon, and it takes about half an hour to unload the pallets from the truck. Then one associate is responsible for sorting the products according to their locations in the store, e.g., the aisle number. The next morning, two or three associates will come in and take the sorted items, ideally separated by aisles, to their respective destinations and put them onto the shelves. A major complication arises when there are more units than can fit onto the shelves. The store we visited has an area called the "middle/top section," which is the space in the upper part of the sales shelves. More specifically, the middle section is the highest level inside the shelves, and the top section is above (or atop) the shelves. When the associate cannot put everything onto the regular sales shelves, he/she will determine whether these extra units go to the middle or top section, usually based on the number of units left and the availability of these sections.

It is not difficult to see that the choice of ship-pack affects the number of units per store replenishment for a particular SKU. Generally, the larger the ship-pack size is, the larger the order quantity, and therefore the more likely it is to result in more units going to the mid/top section. A large pack size does not only incur extra labor cost, it also increases inventory cost at the store level. The fact that the inventory carrying cost (ICC) is higher at retail stores than at the DC makes it undesirable to stock more inventory than necessary at the stores. However, the positive side of having a larger ship-pack is the ease of handling at the DC and the potential savings in the fixed order cost due to less frequent replenishment orders.

Fortunately, Beta is able to provide an estimate of the store receiving cost as well as the extra handling cost on a per item basis when items cannot be fit onto the shelf. We will use these cost estimates in our ship-pack model. As will be seen, we will also need an estimate of the shelf capacity for each SKU at each store; the company liaison suggests approximating this with a linear function of the OUTL, which is what we do.

2.2 Model Overview

Our goal is to develop a cost model that captures all the essential cost components affected by the ship-pack size for an SKU in the two-echelon distribution system. With such a cost model, we can determine the total system cost for an SKU for each choice of ship-pack, i.e., eaches, inners and cases. Then the ship-pack with the lowest total cost is the optimal financial decision. More specifically, since we consider a single DC and multiple non-identical retailers, we will model the expected cost for an SKU for a store as a function of the ship-pack quantity, then sum up the costs for all stores plus an additional DC inventory cost to obtain the total system cost.

In general, there are three categories of costs to consider based on our observation and analysis of the existing operations: the DC handling cost, the store handling cost and inventory-related costs.

DC Handling Cost: As introduced earlier, the picking and replenishing activities at the DC differ substantially for FCs and BPs (including eaches and inners). Thus, the DC handling cost is comprised of the picking cost and replenishment cost.

Store Handling Cost: At the store level, if an item does not fit onto the shelf during the regular shelf-stacking process, then it has to go to the middle/top section or the backroom. Either way extra labor work is needed to retrieve that item and put it back onto the shelf at a later time. As a result, the store handling cost includes the normal receiving cost and any extra handling cost.

Inventory-related Costs: The constraint of a large ship-pack size (SPQ) could induce the bullwhip effect in the supply chain, as shown by Yan et al. (2006). Intuitively, a larger SPQ will make the replenishment orders from the stores less frequent and larger. Consequently, the total demand seen by the DC (equal to the sum of the demand processes from the stores) will be more variable; thus we expect the DC to need more safety stock. Therefore, higher inventory costs will be incurred at the DC (the upper echelon in the supply chain system). In addition, because a larger ship-pack tends to increase the size of the store replenishment order, average store inventory may increase. However, a positive effect of large pack size is the decrease in the number of store replenishment orders, and therefore reduces the fixed order cost.

In summary, for an (SKU k, store i) pair, the expected cost consists of six cost components: the DC replenishment cost of SKU k attributed to store i, the DC picking cost of SKU k attributed to store i, the normal receiving cost at store i, the extra handling cost at store i if there are units that do not fit onto the shelf, the average inventory cost of SKU k at store i, and the fixed order cost of SKU k for store i. The total system cost for SKU k includes the summation of all the expected (SKU k, store i) costs across all stores together with the expected DC inventory cost of SKU k.

Before we explain how we model each cost component, let us first introduce the notation and assumptions used in the model.

2.3 Model Notation and Assumptions

2.3.1 Notation

We consider store-specific demands from individual retailers in our model. Beta has the projected weekly sales forecasts for all stores available in the central database. Thus, we define the following terms with the units in the parenthesis:

$d_{i,k,t}$ = forecast of demand of SKU k at store i in week t (units)

c_k = unit cost of SKU k ($ per unit)

K = fixed order cost ($ per order)

$\tilde{n}^{sp}_{i,k,t}$ = expected number of ship-pack sp to be shipped from DC to store i for SKU k in week t

$\tilde{Q}^{sp}_{i,k,t}$ = expected order quantity for store i, SKU k in week t in ship-pack sp (units)

$SPQ_{sp,k}$ = the pack size for ship-pack sp of SKU k (units per pack)

$replen_{sp}$ = cost of replenishing ship-pack sp at DC ($ per case)

$pick_{sp}$ = cost of picking ship-pack sp at DC ($ per line or per case), depending on sp, where sp = case, inner, each. A line corresponds to a store replenishment order. This will be explained in more detail in the next section 2.3.2 Assumptions.

$OUTL_{i,k,t}$ = order-up-to-level for store i, SKU k in week t (units)

$OUTLdays_{i,k}$ = order-up-to-level for store i, SKU k (in days of demand)

$MaxShelf_{i,k,t}$ = the shelf capacity for store i, SKU k in week t (units)

$IP_{i,k}$ = a random variable to denote the inventory position of SKU k at store i when the store replenishes the SKU (units)

$ROP_{i,k}$ = re-order point of SKU k for store i (units)[1]

HC = normal receiving cost at store ($ per unit)

$ExtraHC$ = extra handling cost at store ($ per unit)

[1] The $ROP_{i,k}$ is not defined for week t because it is not subject to change over time according to Beta's practice.

$ExtraUnits$ = expected number of extra units that need to go to mid/top section or the backroom at store (units)

ICC_{st} = store inventory carrying cost (% per year)

ICC_{dc} = DC inventory carrying cost (% per year)

$D_{dc,k}(t, t + L)$ = random variable to denote the demand at the DC over time interval (*t*, *t*+*L*) where *L* is the replenishment lead time for the DC for SKU *k*

$D_{system,k}(t, t + L)$ = random variable to denote the demand for the system over time interval (*t*, *t*+*L*), i.e., the total demand across all stores served by the DC in our two-echelon distribution system

z = the safety factor used in the DC

2.3.2 Assumptions

ASSUMPTION 1. The fixed order cost *K* is assumed to be constant regardless of SKUs or stores. However, our model can be easily modified to account for different fixed order costs if necessary.

ASSUMPTION 2. We distinguish the relevant costs for the two types of a BP, i.e., inners and eaches. Thus, there are three possible values for both the DC replenishment cost and the picking cost, instead of two in the current practice at Beta. When *sp* = inner or each, $pick_{sp}$ is on a per line basis. A line refers to a physical aisle where products are stored in the BP form. When we say per line basis, we assume the picking cost is independent of the number of ship-packs a warehouse associate takes from the picking area but is dependent on the number of orders, because the physical act of going to the aisle and locating the desired item constitutes the major portion of the cost for one store replenishment order, while taking one item or two does not matter much. This assumption works well when the number of ship-packs picked, be it eaches or inners, is relatively small. When the number of ship-packs (eaches or inners) is large, cases will clearly be a superior choice. Thus, we believe this assumption is good enough. When *sp* =

case, the picking cost is proportional to the number of cases picked. Moreover, we also assume the cost of inner replenishment per case is between the each and case replenishment costs per case basis.

ASSUMPTION 3. At the suggestion of our Beta liaison manager, we assume that the following relationship between $MaxShelf_{i,k,t}$ and $OUTL_{i,k,t}$ holds:

$$MaxShelf_{i,k,t} = 1.25 \times OUTL_{i,k,t}$$

It seems counter-intuitive that the shelf capacity changes with time. However, one way of interpreting a changing shelf capacity is that stores will allocate more space for a particular SKU when demand increases, for example, when a sales promotion is on. Our model can also be easily modified to accommodate a fixed time-independent $MaxShelf_{i,k}$ for store i and SKU k.

ASSUMPTION 4. We assume the inventory position (IP) of an SKU when a store makes a replenishment order is a random variable that follows a discrete uniform distribution with the lower bound being zero and the upper bound equal to the ROP. Thus, it is equally likely for IP to be any integer in [0, ROP]. We need this assumption in order to have a relatively simple way to determine the expected number of ship-packs $\tilde{n}_{i,k,t}^{sp}$.

ASSUMPTION 5. We assume demand over a week occurs at a constant rate. This assumption is very helpful in calculating the average store inventory.

ASSUMPTION 6. We assume the lead time for the store replenishment is zero. We only need this assumption for estimating the extra units that cannot fit on the shelf; this assumption results in an over-estimate of the extra units, as we ignore any store demand during the lead time.

ASSUMPTION 7. The transportation cost is assumed to be constant regardless of the ship-pack choices; with this assumption we do not need to include transportation costs in the model. This assumption is based on Beta's fixed schedule for the store deliveries and therefore a fixed cost for the overall transportation cost.

2.4 Model Formulation

2.4.1 Fixed Order Cost

The expected annual fixed order cost for SKU k and store i is as follows:

$$K \times (E(\text{number of orders per year}))$$

and

$$E(\text{ number of orders per year}) = 52 \text{ weeks/year} \times \frac{\text{weekly demand}}{\text{expected order quantity}}$$

The weekly demand is $d_{i,k,t}$. Dividing by the expected order quantity gives orders per week. We annualize this by multiplying by 52. Thus,

$$Fixed\ order\ cost = K \frac{52 \times d_{i,k,t}}{\tilde{Q}_{i,k,t}^{sp}}$$

To compute $\tilde{Q}_{i,k,t}^{sp}$, we need to first determine $\tilde{n}_{i,k,t}^{sp}$, since $\tilde{Q}_{i,k,t}^{sp} = \tilde{n}_{i,k,t}^{sp} \times SPQ_{sp,k}$. We estimate the expected number of ship-packs by the following expression:

$$\tilde{n}_{i,k,t}^{sp} = E(\left\lceil \frac{OUTL_{i,k,t} - IP_{i,k}}{SPQ_{sp,k}} \right\rceil)$$

$OUTL_{i,k,t}$ is calculated based on $OUTLdays_{i,k}$. More specifically, for store i, SKU k and week t,

$$OUTL_{i,k,t} = \frac{OUTLdays_{i,k}}{7} * d_{i,k,t}$$

Then if the store orders, then its inventory is at or below the ROP and it needs to order enough to bring its inventory position to $OUTL_{i,k,t}$. By Assumption 4, we assume that the $IP_{i,k}$ at the time of the order follows a discrete uniform distribution in the interval

$[0, ROP_{i,k}]$. In the above formula $\left\lceil\frac{OUTL_{i,k,t}-IP_{i,k}}{SPQ_{sp,k}}\right\rceil$ equals the number of ship-packs to bring the inventory position to the order up to level. We compute $\tilde{n}_{i,k,t}^{sp}$ by taking the expectation over the possible values for the inventory position at the time of order.

2.4.2 DC Replenishing Cost

The total replenishing cost attributed to store i is the replenishing cost per case multiplied by the total number of cases replenished at the DC attributed to store i.

$$\text{DC Replenishing Cost} = replen_{sp} \frac{52 \times d_{i,k,t}}{SPQ_{case}}$$

2.4.3 DC Picking Cost

When the ship-pack is each or inner, the picking cost is equal to the picking cost per line multiplied by the number of orders for the SKU for the store, whereas when the ship-pack is case, the picking cost is simply the picking cost per case times the total number of cases picked for the SKU for the store.

$$\text{DC Picking Cost} = pick_{sp} \times \begin{cases} \frac{52 \times d_{i,k,t}}{\tilde{Q}_{i,k,t}^{sp}}, & if\ sp = each\ or\ inner \\ \frac{52 \times d_{i,k,t}}{case}, & if\ sp = case \end{cases}$$

2.4.4 Store Receiving Cost

For every unit received at the store, normal store receiving cost is incurred. So the expected normal receiving cost is the normal handling cost multiplied by the annual demand.

$$\text{Store Receiving Cost} = HC \times 52 \times d_{i,k,t}$$

2.4.5 Store Extra Handling Cost

The store extra handling cost is equal to the extra handling cost per item times the expected number of extra units times the expected number of orders per year.

$$\text{Store Extra Handling Cost} = ExtraHC \times E(ExtraUnits) \times \frac{52 \times d_{i,k,t}}{\tilde{Q}^{sp}_{i,k,t}}$$

To determine the expected number of extra units that do not fit onto the shelf during regular shelf-stacking process, i.e., $E(ExtraUnits)$, we need to know the shelf space allocated for the SKU at the stores. The shelf space is estimated as described in the Assumption 3.

$$E(ExtraUnits) = E\left(\max\left(0, IP_{i,k} + Q^{sp}_{i,k,t} - MaxShelf_{i,t,k}\right)\right)$$

$$\Rightarrow$$

$$E(ExtraUnits) = E\left(\max\left(0, IP_{i,k} + \left\lceil\frac{OUTL_{i,k,t} - IP_{i,k}}{SPQ_{sp,k}}\right\rceil \times SPQ_{sp,k} - MaxShelf_{i,t,k}\right)\right)$$

Again $IP_{i,k}$ follows a discrete uniform distribution in the interval $[0, ROP]$.

2.4.6 Store Inventory Cost

Since we assume a constant demand rate, the store inventory level is illustrated in Figure 2-2. For simplicity, the subscripts are dropped in the figure. We use IP_1 and IP_2 to represent the inventory position at the two successive store replenishment orders. The shaded area is the store inventory. The expected store inventory is then shown in the following formula.

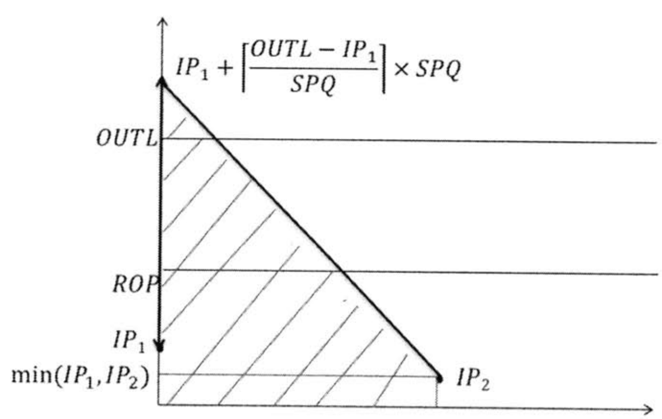

Figure 2-2: Store inventory illustration

$E(store\ inventory)$

$$= E(\min(IP_{i,k}^{1}, IP_{i,k}^{2}) + \frac{1}{2}\left(IP_{i,k}^{1} + \left\lceil\frac{OUTL_{i,k,t}-IP_{i,k}^{1}}{SPQ_{sp,k}}\right\rceil \times SPQ_{sp,k} - \min(IP_{i,k}^{1}, IP_{i,k}^{2})\right))$$

$$= E(\frac{1}{2}\min(IP_{i,k}^{1}, IP_{i,k}^{2}) + \frac{1}{2}\left(IP_{i,k}^{1} + \left\lceil\frac{OUTL_{i,k,t}-IP_{i,k}^{1}}{SPQ_{sp,k}}\right\rceil \times SPQ_{sp,k}\right))$$

Let $Q_{i,k,t}^{sp} = \left\lceil\frac{OUTL_{i,k,t}-IP_{i,k}^{1}}{SPQ_{sp,k}}\right\rceil \times SPQ_{sp,k}$ and $\tilde{Q}_{i,k,t}^{sp} = E(\left\lceil\frac{OUTL_{i,k,t}-IP_{i,k}^{1}}{SPQ_{sp,k}}\right\rceil \times SPQ_{sp,k})$ then

$$\text{E(Store Inventory Cost)} = ICC_{st} \times c_k \times E\left(\frac{1}{2}\min(IP_{i,k}^{1}, IP_{i,k}^{2}) + \frac{1}{2}(IP_{i,k}^{1} + Q_{i,k,t}^{sp})\right)$$

$= ICC_{st} \times c_k \times (\frac{1}{2}\tilde{Q}_{i,k,t}^{sp} + \frac{1}{4}ROP_{i,k} + \frac{2ROP_{i,k}^{3}+3ROP_{i,k}^{2}+ROP_{i,k}}{12(ROP_{i,k}+1)^{2}})$ (See Appendix for derivation.)

2.4.7 DC Inventory Cost

We model the safety stock needed by the DC as $z\sqrt{Var(D_{dc,k})}$. To find $Var(D_{dc,k})$, we use the following relationship, which follows from the inventory balance.

$$D_{dc,k}(t, t+L) = D_{system,k}(t, t+L) + \sum_{i=1}^{numStore} IP_{i,k}(t+L) - \sum_{i=1}^{numStore} IP_{i,k}(t)$$

In words, the demand seen by the DC over a time interval equals the demand at all of the stores over this interval plus any change in the inventory position at the stores between time t and time t+L.

To simplify the equation, let $IP(t) = \sum_{i=1}^{numStore} IP_{i,k}(t)$

We use the above equation to approximate the variability of demand over the lead time L at the DC. Namely, we develop the approximation from:

$$Var(D_{dc,k}) = Var(D_{system,k}) + Var(IP(t+L) - IP(t))$$

Here we assume the system demand over the lead time is independent of the change in IP for the stores. We think it is reasonable because L is likely to be much larger than the replenishment frequency at the stores. Since we can approximate $Var(D_{system})$ from their purchase projections, we assume it is known and given. This leaves us the job to compute the variance of the difference in IP. We take the following steps:

$$Var\big(IP(t+L) - IP(t)\big)$$

$$= Var\big(IP(t+L)\big) + Var\big(IP(t)\big) - 2Cov\big(IP(t+L), IP(t)\big)$$

$$\cong 2Var(IP(t))$$

Here we make another assumption, namely that $IP(t+L)$ is independent of $IP(t)$. The larger the lead time is, the better this assumption is. Moreover,

$$Var\big(IP(t)\big) = \sum Var\left(IP_{i,k}(t)\right) = \frac{\sum \tilde{Q}_{i,k,t}^{sp\ 2}}{12}$$

Here we assume that at each store the inventory position IP_i is uniformly distributed between $(x, x + Q_{i,k,t}^{sp})$ for some value of x; we don't need to specify x as it does not affect our model. We also assume that the inventory positions for any pair of stores are independent of each other.

Thus, we have 42

$$Var\big(D_{dc,k}\big) \cong Var\big(D_{system,k}\big) + 2\frac{\sum \tilde{Q}_{i,k,t}^{sp\ 2}}{12}$$

In summary, the DC inventory cost is given by the following equation.

$$DC\_InvCost_{\,k,t}(sp) = ICC_{dc} \times c_k \times z \times \sqrt{Var\big(D_{system,k}\big) + \frac{\sum \tilde{Q}_{i,k,t}^{sp\ 2}}{6}}$$

2.4.8 Total System Cost

The expected annual cost for an SKU for a store is the summation of the cost components described in 2.4.1 through 2.4.6:

$$Cost_{i,k,t}(sp) = K\frac{52\times d_{i,k,t}}{\tilde{Q}^{sp}_{i,k,t}}$$

$$+ replen_{sp}\frac{52\times d_{i,k,t}}{case}$$

$$+ pick_{sp} \times \begin{cases} \frac{52\times d_{i,k,t}}{\tilde{Q}^{sp}_{i,k,t}}, & if\ sp = each\ or\ inner \\ \frac{52\times d_{i,k,t}}{case}, & if\ sp = case \end{cases}$$

$$+ ICC_{st} \times c_k \times E\left(\frac{1}{2}\min\left(IP^1_{i,k}, IP^2_{i,k}\right) + \frac{1}{2}\left(IP^1_{i,k} + Q^{sp}_{i,k,t}\right)\right)$$

$$+ HC \times 52 \times d_{i,k,t}$$

$$+ ExtraHC \times E(ExtraUnits) \times \frac{52\times d_{i,k,t}}{\tilde{Q}^{sp}_{i,k,t}}$$

Where

$$\tilde{Q}^{sp}_{i,k,t} = \tilde{n}^{sp}_{i,k,t} \times SPQ, \text{ and } \tilde{n}^{sp}_{i,k,t} = E\left(\left\lceil\frac{OUTL_{i,k,t} - IP_{i,k}}{SPQ}\right\rceil\right),$$

$$E(ExtraUnits) = E\left(\max\left(0, IP_{i,k,t} + \tilde{Q}^{sp}_{i,k,t} - MaxShelf_{i,t,k}\right)\right)$$

Thus, the annualized total system cost in week t is the summation of the expected cost of all stores plus a DC inventory cost, i.e., $DC\_InvCost_{k,t}(sp) + \sum_{i=1}^{numStore} Cost_{i,k,t}(sp)$.

With the objective function in place, we can then formulate the following minimization problem.

$$\text{Minimize } DC\_InvCost_{k,t}(sp) + \sum_{i=1}^{numStore} Cost_{i,k,t}(sp)$$

$$\text{s.t. } sp = \{each, inner, case\}$$

where *numStore* denotes the total number of stores the DC serves.

The optimal solution to the above problem is valid for only week t. When a multiple-week planning period is in question, we can extend the problem into the following form. Let $numWeek$ be the number of weeks in the planning period. We minimize the average total annual cost.

$$\text{Minimize } \frac{\sum_{t=1}^{numWeek}(DC\_InvCost_{k,t}\,(sp)+\sum_{i=1}^{numStore} Cost_{i,k,t}(sp))}{numWeek}$$

$$\text{s.t. } sp = \{each, inner, case\}$$

This time we will find the optimal ship-pack that is the most economic decision for the entire planning period from week 1 to week $numWeek$.

Complexity arises when we allow multiple ship-pack changes during the planning period and/or dual-slotting is practiced in the DC. For example, if we allow the warehouse to change its ship-pack each week, then the solution will be a vector of length $numWeek$, whose elements are the optimal ship-pack corresponding to each week. However, due to physical and practical constraints in the warehouse, it is more appropriate to limit the number of ship-pack changes throughout the planning period. The algorithm that finds such a solution will be discussed in more detail in the next section, Model Implementation.

2.5 Model Implementation

We use Visual Basic Applications (VBA) to implement the Warehouse Ship-pack Cost Model in the Microsoft Excel environment. There are two reasons for choosing VBA in Excel: first, Beta managers are proficient in Excel so it is very friendly to them in this sense; second, it is convenient to import data from an Access database using VBA.

The model is capable of running on multiple SKUs at one time and generating appropriate results according to the user's request. In a broad view, the core steps of the model include extracting appropriate data from Beta's central database, processing it accordingly and displaying it on an Excel worksheet. Figure 2-3 illustrates the main logic in a flow chart. It is important to use the right SQL queries to prepare the data in specific format for the model to interpret it correctly.

2.5.1 Single-slotting Algorithm

The second minimization problem in section 2.3 limits ship-pack change to only one time, in particular at the beginning of week one in a single-slotting warehouse. It is easy to solve, since the feasible set contains only three discrete choices. We can simply calculate the total annual cost for all three possible ship-packs and choose the *sp* with the least cost. To enhance its user-friendliness, the model will compare the calculated optimal ship-pack with current ship-pack, if available, and recommend whether a ship-pack change is needed.

It is also possible to relax the constraint to allow one ship-pack change in any week in the planning period. In this case, we not only need to search for the optimal ship-pack, but also when is the best time to change it. We use a straight-forward search for the timing with complexity $O(n)$. Suppose the number of weeks is n, then the ship-pack change can take place at the beginning of any week in these n weeks, i.e. there are a total of n possible changes. To find the optimal timing, our algorithm loops n times to calculate the most economical ship-pack pairs for each possible change opportunity and finally selects the one with the least cost.

The complexity increases exponentially with the increase in the number of changes allowed within the planning period. So our model restricts the ship-pack change to only one. Two alternatives are available to compensate for this limitation: first is to shorten the planning period and limit the ship-pack changes to one; second is to generate a vector of optimal ship-packs for each week and let the management decide. All three scenarios will be illustrated using a sample data in Chapter 3 Output Analysis.

2.5.2 Dual-slotting Algorithm

The huge differences in sales volumes of different stores for some SKUs prompted Beta managers to come up with the idea of dual-slotting in the warehouse. They are willing to invest the capital to set up two picking modules in the DC for the same SKU, one for ship-pack with larger quantity and the other smaller. Thus, we have built into our model the capability to determine the best two ship-packs the company should choose for dual-slotting. In fact, with three choices available, there are only three (three choose two) possible combinations. For each of the three combinations, the algorithm will determine for each store and each week the optimal ship-pack and calculate the corresponding total cost. Then the three values are compared and the best combination is selected. It is possible for certain SKUs that dual-slotting is unnecessary, i.e. one single ship-pack is optimal. In this case, two best ship-packs are still generated by the model but the percentages in units for these two ship-packs will be 100% and 0% respectively.

Figure 2-3 shows the overall logic.

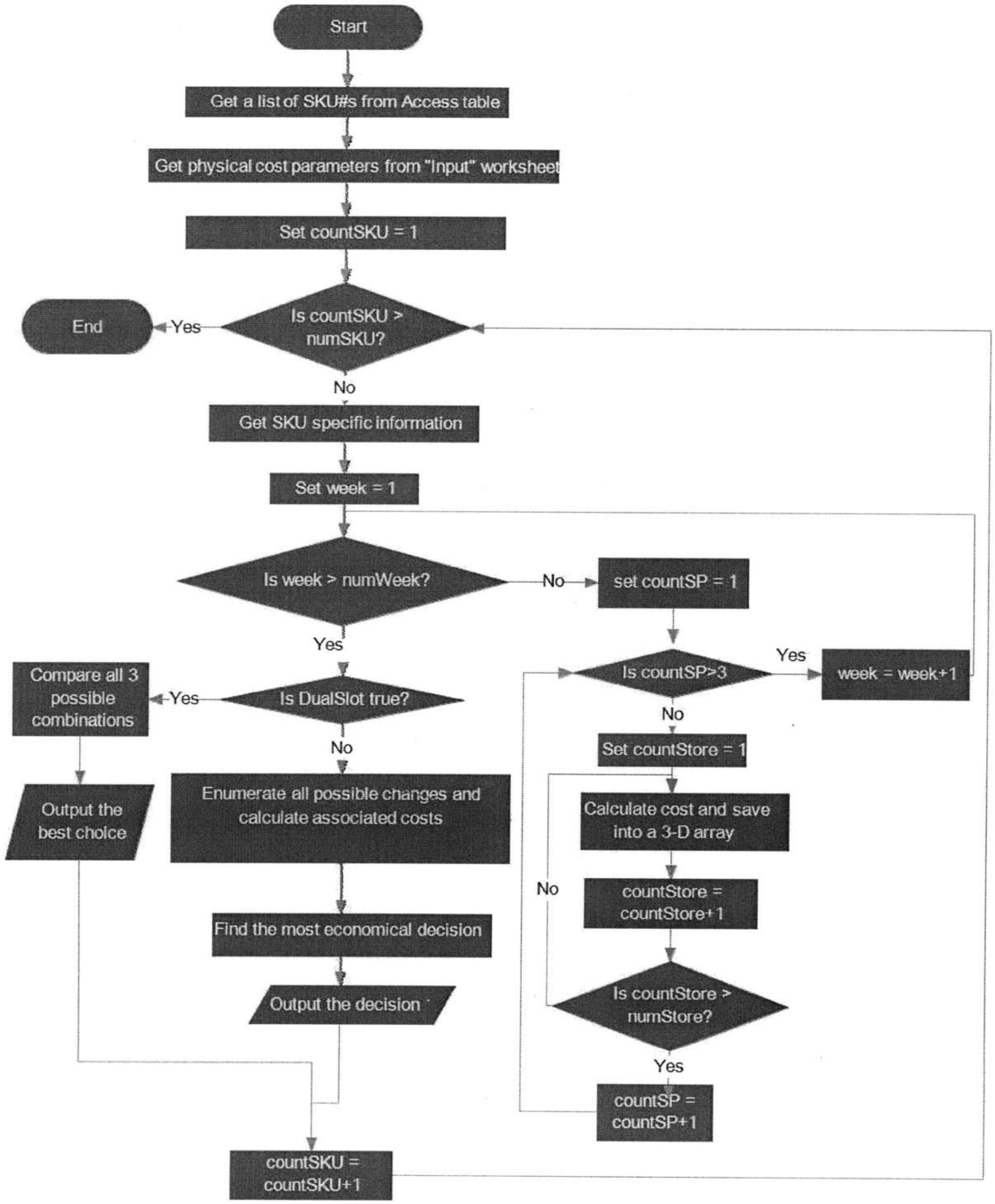

Figure 2-3: Model logic

Chapter 3
Model Output Analysis

3.1 Sample Data Description

3.1.1 General Description of the Sample Data

Beta provided us with the required cost parameters, including the DC and store handling rates, the fixed order cost and the inventory carrying cost etc. We also obtained a sample dataset for SKUs from three product families for the set of stores supplied by one DC. Before we show the model output, let us first have a look at what the data is. A number of interesting observations are made. These observations help us better understand the results, which will be presented in the next two sections 3.2 and 3.3.

The sample dataset contains a total of 529 SKUs, three of which have a case quantity of one. Beta terms such circumstance “case of 1”, and no ship-pack analysis is necessary. Thus, we are effectively dealing with 526 SKUs. Moreover, 369 out of the 526 SKUs have an inner quantity of one, meaning there is no inner pack for these SKUs.

3.1.2 Demand Projections of Selected SKUs

One of the complexities of determining an optimal ship-pack is the sales variance both across stores and throughout the planning period. The sample data includes 52 weeks of sales forecasts. We have identified three representative annual demand patterns. Figure 3-1 and 3-2 exhibit seasonal demands, with single and multiple peaks respectively, whereas Figure 3-3 shows relatively stable demand throughout the entire year. In each case we plot the total demand forecast for a single SKU, cumulated over all the stores that carry the SKU in the sample. In total, about one fifth of the total SKUs exhibit the single peak pattern; only 3% exhibits the double peak pattern; and the remaining 77% has relatively stable weekly sales forecasts.

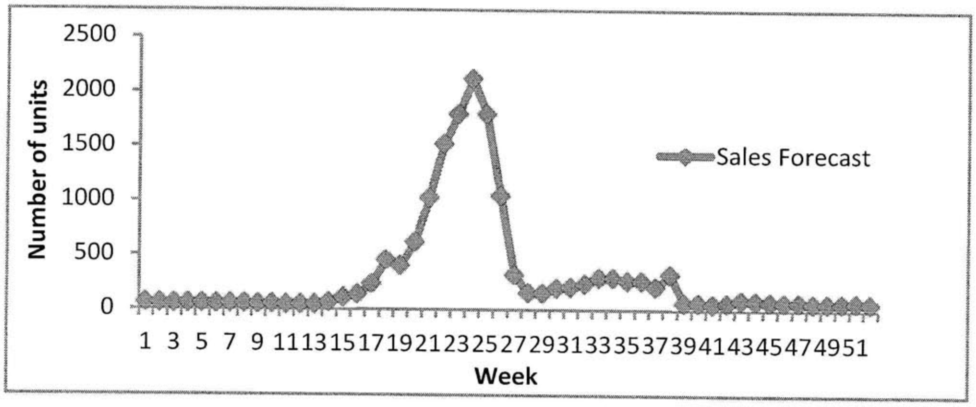

Figure 3-1: Weekly sales forecast for SKU 01

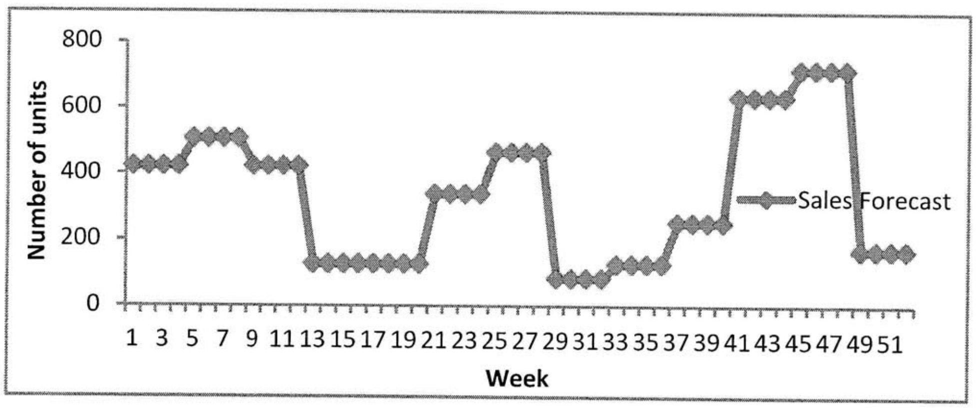

Figure 3-2: Weekly sales forecast for SKU 02

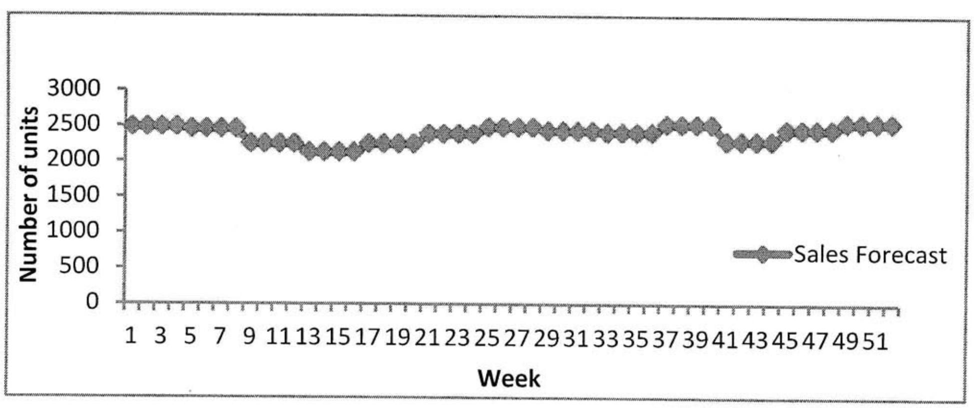

Figure 3-3: Weekly sales forecast for SKU 03

The variance across stores is even greater. The huge variance is not surprising because there are always low-, medium- and high-volume stores for any major retailer. A closer look reveals that the coefficient of variation (CV) of sales volume across stores for the given SKUs vary from 0.3 to 3.5. Figure 3-4 through 3-6 show the distribution of the annual demands by stores for SKU 01 to 03 respectively. All three graphs show that store annual demands vary greatly. In Figure 3-4, the number of low-volume stores is roughly equal to that of the mid-volume ones. There are two stores having extremely large annual demand such that the bar shows in the "more" column and a handful of stores with annual demand greater than 100 units. In Figure 3-5, about half of the stores are in the mid-volume range, one third are low-volume while the remaining one sixth are high-volume stores. Figure 3-6 shows a similar trend as Figure 3-5.

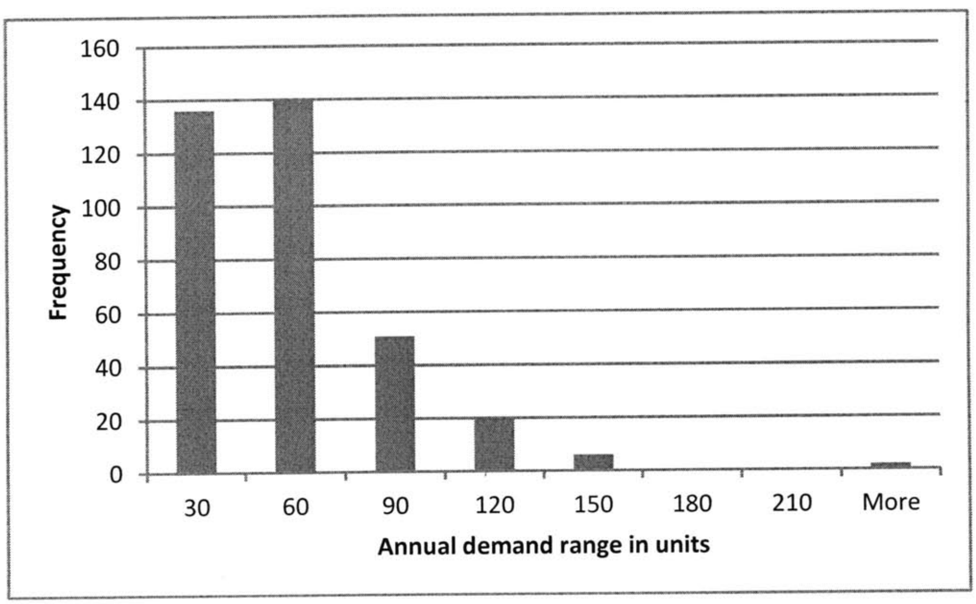

Figure 3-4: Distribution of the annual demand by stores for SKU 01

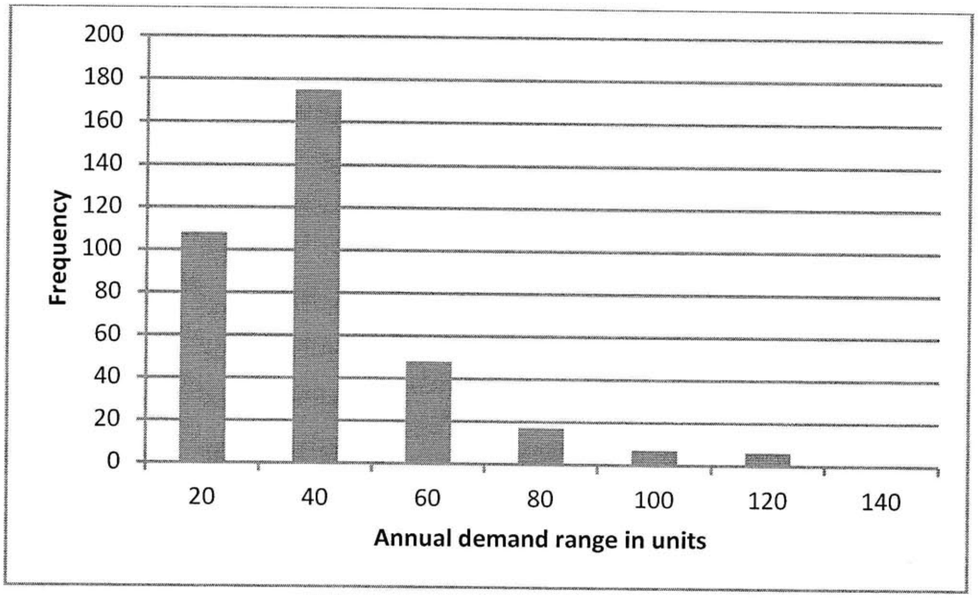

Figure 3-5: Distribution of the annual demand by stores for SKU 02

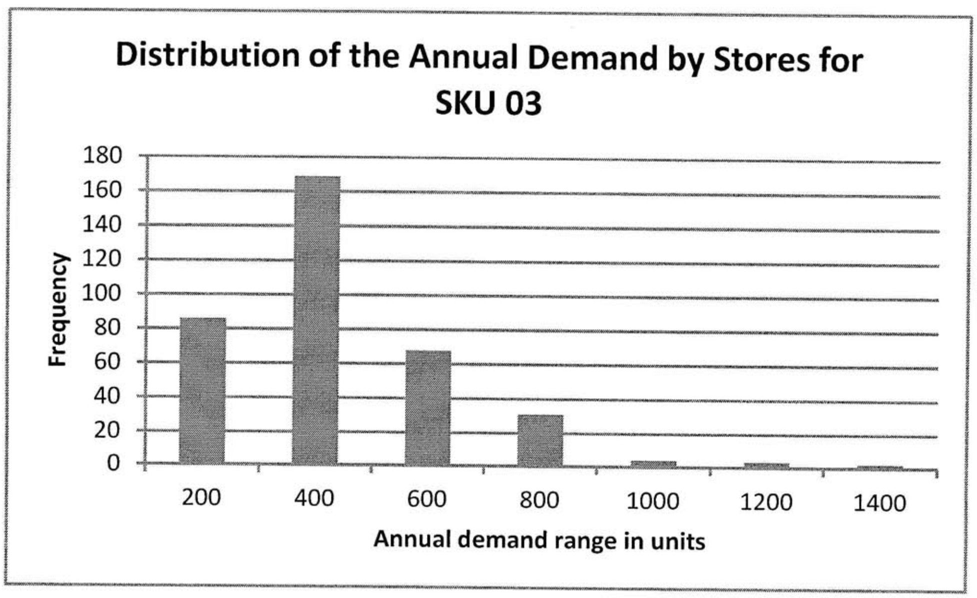

Figure 3-6: Distribution of the annual demand by stores for SKU 03

3.2 Single-slotting DC

This section describes the model output considering a DC with only single-slotting capability. To protect Beta we will not disclose any specific cost; instead, we will present the total cost savings as a percentage in the following three scenarios: restrict the ship-pack change to week one; restrict to one ship-pack change in any week in the planning horizon; and no restriction at all.

As mentioned earlier in Chapter 2 (2.2), we consider two types of break-packs (inners and eaches) and there is uncertainty about the replenishment cost for inners, relative to that for eaches and for cases. Thus, we perform a sensitivity analysis on this assumed cost. Since the given replenishment costs for eaches and cases are \$0.7789 and \$0.1716 respectively, we assume that the first cost is exactly in the middle of these two costs ($0.4753 = 0.7789 - \frac{0.7789-0.1716}{2}$), the second cost is nearer to \$0.7789 ($0.6271 = 0.7789 - \frac{0.7789-0.1716}{4}$), and the last cost is \$0.7789 itself. The results are shown in Table 3-1.

Table 3-1: Cost savings in percentage for three inner replenishment costs

Inner replenishment cost \$ per case	Percentage Saved		
	Restrict Change to Beginning	No Restriction on Timing	No Restriction
0.4753	0.32%	0.32%	0.34%
0.6271	0.28%	0.29%	0.30%
0.7789	0.35%	0.35%	0.37%

In Table 3-1 we report the total cost savings compared to the current ship-pack choices for a 52 week period for roughly 350 stores for 526 SKUs. From this table, we see that the savings percentages are not very sensitive to restrictions on ship-pack change. Basically one can save 0.02% more if no restrictions are enforced on when and how often to change the ship-pack, and almost zero improvement if we relax only the timing constraint. Since coordinating these changes for a single point in time is by far the most practical, we will concentrate our analysis on results when we limit ship-pack change to the beginning of the planning period from now on.

The ship-pack change recommendations corresponding to the cost savings in Table 3-1 "Restrict Change to Beginning" are listed in Table 3-2. In short, it shows that for 94% of the 526 SKUs, Beta is already operating with the optimal ship-pack, whereas only 6% of the SKUs, or slightly more than 30 SKUs, require some sort of action.

Table 3-2: Summary of ship-pack change recommendations for three inner replenishment costs

Inner replenishing cost $ per case	0.4753	0.6271	0.7789
Each to Each	383	387	391
Inner to Inner	102	98	88
Case to Case	10	10	10
subtotal	495 (94%)	495(94%)	489(93%)
Case to Each	3	3	3
Case to Inner	0	0	0
Each to Case	5	6	6
Each to Inner	15	10	6
Inner to Case	0	0	1
Inner to Each	8	12	21
subtotal	31(6%)	31(6%)	37(7%)
Total	526	526	526

A closer look at the results reveals that out of the 30-plus changes required, half of them provide 80% of the cost savings. As making a change to the ship-pack incurs a cost, this observation can be helpful in deciding how many and what changes should be done.

The three major categories of cost are also individually analyzed as shown in Figure 3-7. The bar shows the absolute savings for each cost component (exact figures omitted from chart) while the diamond dot shows the percentages. The DC handling cost is significantly reduced since there is a 2.9% reduction compared to only 0.08% and 0.03% percent savings.

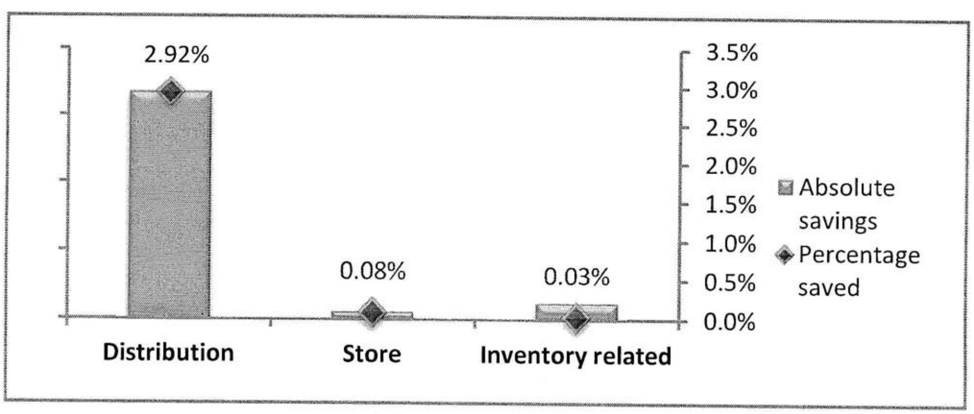

Figure 3-7: Savings breakdown for inner replenishment cost = $0.4753

If we were to allow a ship-pack change in any week, we find that the changes closely follow the demand pattern. We illustrate this in Figure 3-8 and 3-9 for two SKUs; the figures show both the weekly demand forecasts and the corresponding optimal ship-pack.

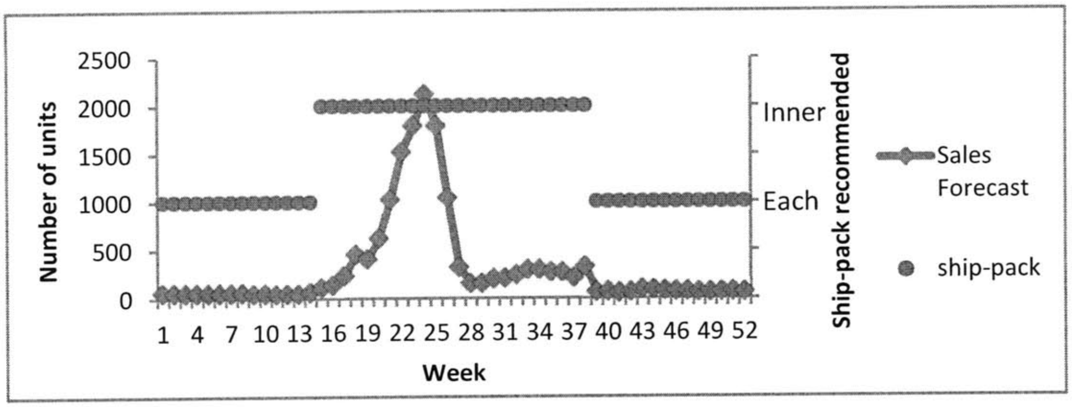

Figure 3-8: Weekly sales forecast of SKU 01and corresponding optimal ship-packs

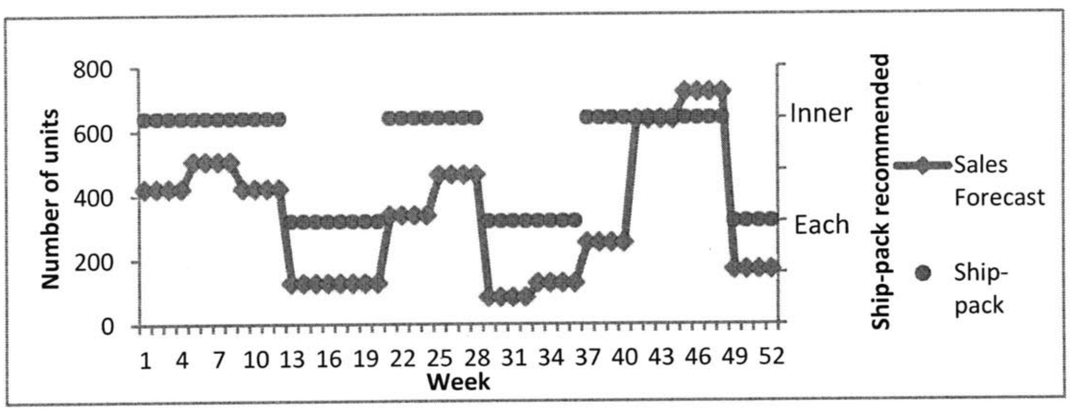

Figure 3-9: Weekly sales forecast of SKU 02 and corresponding optimal ship-packs

From Figure 3-8 and 3-9, we conclude that the optimal choice of ship-pack closely correlates to the demand, which is consistent with what Beta managers and we predicted.

3.3 Dual-slotting

Since Beta expressed interest in the possibility of dual-slotting in their warehouse, we also performed a series of experiments about dual-slotting on the sample dataset. Again we will present our results in relation to three inner replenishment costs. Table 3-3 shows the savings if every SKU has a chance to have two picking modules in the warehouse. In comparison with optimal single-slotting scenario, dual-slotting increase the percent savings from 0.3% to 0.5%.

Table 3-3: Cost savings in percentage for warehouse with dual-slotting

Inner replenishment cost $ per case	Percentage Saved
0.4753	0.51%
0.6271	0.49%
0.7789	0.58%

To achieve these savings, a total of 206 SKUs need dual-slotting. However, many of the SKUs only improve the cost by a few dollars or cents whereas a select few can save a lot, as the Pareto rule prevails here. Table 3-4 shows percentages in total savings when the number of dual-slots in the warehouse is pre-determined for each of the three inner replenishment costs. For example, if we allow only 30 SKUs to be dual-slotted in the DC, then a combination of dual-slotting and proper ship-pack changes can save 0.44% for Beta in the $0.4753 scenario.

Table 3-4: Savings in percentage with fixed number of dual-slots for three inner replenishment costs

	Percentage saved for different inner replenishment costs		
Number of dual slots	$0.4753	$0.6271	$0.7789
30	0.44%	0.42%	0.51%
50	0.47%	0.45%	0.54%
100	0.50%	0.48%	0.57%
All	0.51%	0.49%	0.58%

To understand the drivers behind the dual-slotting results in the DC, we compare the average coefficient of variation (CV) of the weekly sales forecasts for all SKUs and for the 30 most profitable dual-slotting SKUs, as shown in Table 3-5. The CV is a normalized measure of dispersion of a distribution and is defined as the ratio of the standard deviation to the mean. We first calculate the CV of the 52 weekly sales forecasts for each (SKU, store) pair. Then for each SKU, we take the weighted average of the CVs by the store demands. Finally, the average CV of all SKUs and that of the top 30 SKUs in savings are computed and compared. In all three scenarios, the average CVs of the top 30 SKUs in dual-slotting savings are significantly higher than that for all SKUs. Thus, we conclude that SKUs whose sales volumes vary greatly over time will benefit more from dual-slotting.

Table 3-5: Comparison of the average CV

	Average CV
All SKUs	0.2161
Top 30 SKUs in savings (\$0.4753)	0.4989
Top 30 SKUs in savings (\$0.6271)	0.4730
Top 30 SKUs in savings (\$0.7789)	0.5094

Currently no information is available on the capital investment of dual-slotting in the DC. If such information were known, then we could use it to determine which and how many SKUs should be dual-slotted.

Chapter 4

Optimal Case Configuration

4.1 Motivation

Finding an optimal case configuration is motivated by two factors. First, we observe there is a lack of inner packs for many SKUs and hence maybe an opportunity. Second, Beta might negotiate with vendors to modify their case configuration to include a more economical inner pack.

From the sample dataset, we noted that many SKUs have only a case pack of a large quantity of eaches, e.g. 24 units. This naturally made us ask what if there were an inner of 6 or 8 units for these SKUs. Furthermore we observed that many of the SKUs for which it was most economical to dual slot did not have an inner as a ship-pack option.

Table 4-1: Sample dual-slotting output ranked by cost savings for inner replenishment cost = $0.4753.

Savings Rank	Best Combination	Percentage of sales volume in units in Combination 1	Percentage of sales volume in units in Combination 2	Inner pack quantity
1	Each & Case	27.47%	72.53%	IP=1
2	Each & Case	54.94%	45.06%	IP=1
3	Each & Case	36.48%	63.52%	IP=1
4	Each & Case	16.26%	83.74%	IP=1
5	Each & Case	42.19%	57.81%	IP=1
6	Each & Case	58.70%	41.30%	IP=1
7	Each & Case	15.61%	84.39%	IP=1
8	Each & Case	15.59%	84.41%	IP=1
9	Each & Case	35.56%	64.44%	
10	Inner & Case	53.07%	46.93%	
11	Each & Case	16.49%	83.51%	
12	Each & Case	47.29%	52.71%	IP=1
13	Each & Case	63.14%	36.86%	IP=1
14	Each & Case	49.48%	50.52%	IP=1
15	Each & Case	16.14%	83.86%	IP=1

In Table 4-1 we report the best ship-pack combination for the 15 SKUs which yielded the greatest savings from dual slotting. In the last column we note those for which there was no inner ship-pack (i.e., IP = 1). We see that 12 of 15 have an inner quantity of one; these SKUs benefit the most from dual-slotting by having the option to ship either cases or eaches. But these SKU's might also benefit from having the option of an inner SPQ, which would bridge between the extremes of shipping eaches or cases.

Besides our research interest, Beta actually has its own case configuration analysis, which is a fairly elaborate process. It takes into account the dimension and weight constraints of both the full case and the break-pack at DC and retail stores, and then a financial decision and a logical decision will be recommended after a series of cost comparison. Our model will not consider the weight and physical dimension constraints; instead, it aims at determining the right pack quantity based just on the cost analysis.

4.2 Methodology

4.2.1 Practical Approaches

We modify our Warehouse Ship-Pack Cost Model into an Optimal Case Configuration Model. The new model determines the best ship pack size that the DC should replenish its stores. We assume a given case quantity and set it as the upper limit of the pack size, and the lower limit is naturally one, an each. Moreover, we also assume that the inner need be a divisor of the case quantity.

The steps of finding the optimal case configuration are described below.

1. Given a case quantity SPQ_{case}, we can find all the divisors, e.g. if the case quantity is 12, then the possible inner quantities are 2, 3, 4 and 6.
2. Based on the inner quantity, we approximate the inner replenishment cost by linear extrapolation between each replenishment cost and case replenishment cost. The exact formula used in the model is shown in Equation 4-1.

Equation: 4-1

$$replen_{inner} = replen_{each}\left(1 - \frac{SPQ_{inner} - 1}{SPQ_{case} - 1}\right) + replen_{case} * \frac{SPQ_{inner} - 1}{SPQ_{case} - 1}$$

3. We calculate the annual costs for all possible inner quantities; the smallest one is the preferred inner quantity.

The model is run and an example is shown in Figure 4-1. The SKU in question is the top saver in Table 4-1. The annual cost on the y-axis is omitted to protect the privacy of Beta.

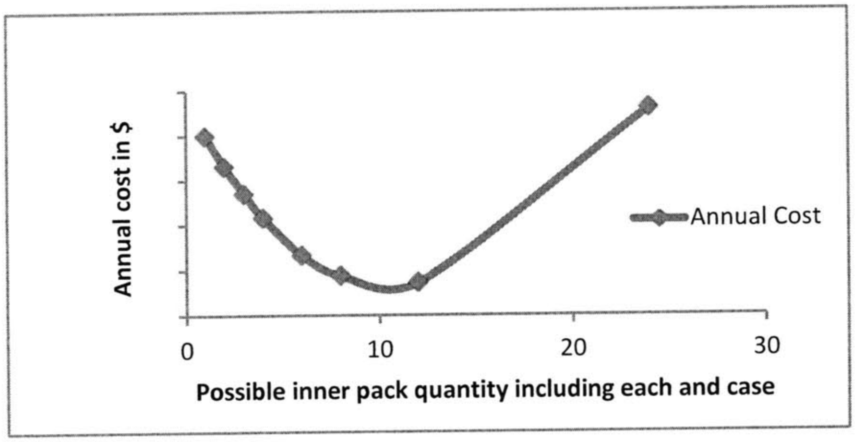

Figure 4-1: Annual cost over possible inner pack quantities for SKU 04

We obtain a nice (discrete) convex curve for this SKU. In this example, an inner pack of 12 is the optimal choice. In fact, it reduces the annual cost by 3.44% from the current single slotting ship-pack and 0.14% from the possible dual-slotting scenario.

Of course not every SKU has such a shape or potential improvement. Figure 4-2 shows a clear decreasing trend over the feasible range for another SKU. In this case, it may be better to increase the case pack size but we shall leave that to future research.

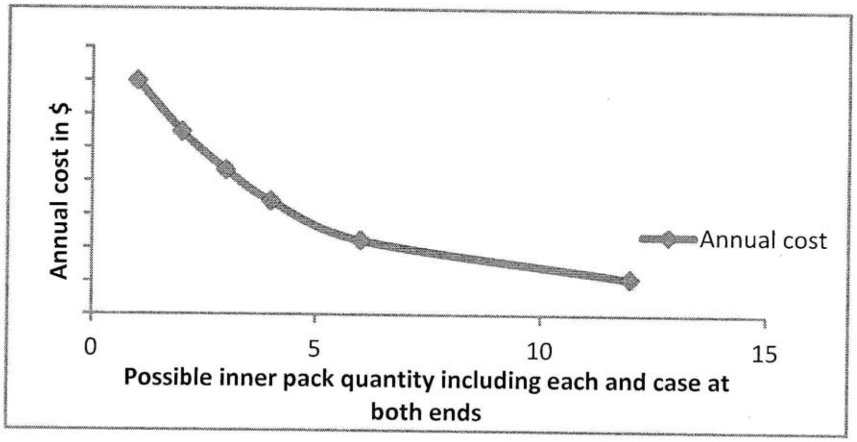

Figure 4-2: Annual cost over possible inner pack quantities for SKU 05

We run the Optimal Case Configuration Model for all the 526 SKUs and find that 171SKUs benefit from a more economical inner pack. By changing to the optimal inner packs, these 171 SKUs can reduce the total cost by 1.30%. For these 171 SKUs, the distribution of the optimal inner pack quantity (IPQ) against the current IPQ is shown In Figure 4-3. Clearly an IPQ of 2 or 4 is the most popular choice according to the results.

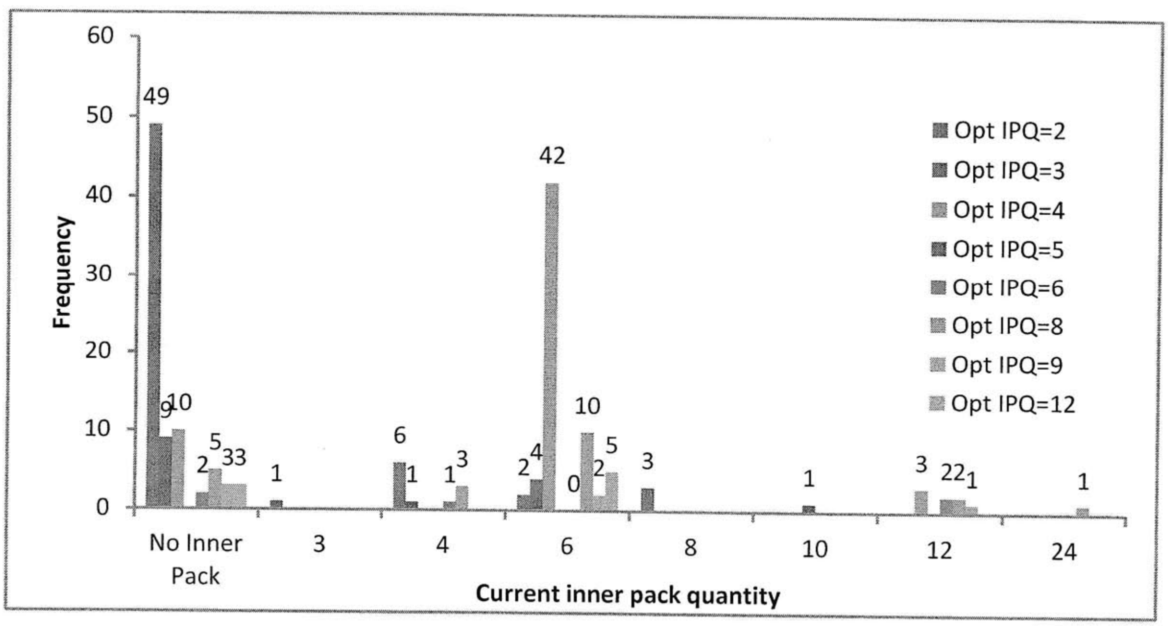

Figure 4-3: The distribution of optimal IPQ against current IPQ.

4.2.2 Theoretical Approaches

The convex shape in Figure 4-1 inspired a theoretical approach to this problem, since convex optimization is widely researched and many algorithms are readily available. Following the notation defined in section 2.2, we formulate the objective function as follows,

$$DC\_InvCost_{k,t}(SPQ_{inner}) + \sum_{i=1}^{numStore} Cost_{i,k,t}\,(SPQ_{inner})$$

$where$

$Cost_{i,k,t}(SPQ_{inner}) =$

$$K\frac{52\times d_{i,k,t}}{\tilde{Q}^{sp}_{i,k,t}}$$

$$+\,replen_{inner}\frac{52\times d_{i,k,t}}{SPQ_{case}}$$

$$+\,pick_{sp}\times\frac{52\times d_{i,k,t}}{\tilde{Q}^{sp}_{i,k,t}}$$

$$+\,ICC_{st}\times\ c_k\times(\frac{1}{2}\tilde{Q}^{sp}_{i,k,t}+\frac{1}{4}ROP_{i,k}+\frac{2ROP^3_{i,k}+3ROP^2_{i,k}+ROP_{i,k}}{12(ROP_{i,k}+1)^2})$$

$$+\,HC\times 52\times d_{i,k,t}$$

$$+\,ExtraHC\times\ E\big(\max(0, IP_{i,k,t}+Q^{sp}_{i,k,t}-MaxShelf_{i,k,t})\big)\times\frac{52\times d_{i,k,t}}{\tilde{Q}^{sp}_{i,k,t}}$$

and

$$DC_{InvCost_{k,t}}(SPQ_{inner}) = ICC_{dc,k}\times\ c_k\times\ z\times\sqrt{Var(D_{system,k})+2\sum\tilde{Q}^{sp\ 2}_{i,k,t}/12}$$

where $IP_{i,k}$ follows discrete uniform distribution.

We linearly approximate the expected order quantity $\tilde{Q}^{sp}_{i,k,t}$ as follows:

$$\tilde{Q}^{sp}_{i,k,t} = a_{k,1}\times OUTL_{i,k,t}+a_{k,2}\times ROP_{i,k}+a_{k,3}\times SPQ_{inner}$$

The linear approximation works very well empirically. We use the regression analysis to obtain the coefficients. To quantify the accuracy of our approximation, 30 SKUs are randomly selected and the regression statistics are reported in Table 4-2.

Table 4-2: Regression statistics of the sampled SKUs

	R^2	SE[2]	t-statistics				Coefficients			
			Intercept	$a_{k,1}$	$a_{k,2}$	$a_{k,3}$	Intercept	$a_{k,1}$	$a_{k,2}$	$a_{k,3}$
Min	0.95	0.08	0.15	0.98	0.31	48.89	-0.86	0.00	-0.68	0.50
Max	1.00	1.69	35.01	65535	212.27	828.48	0.10	1.04	0.34	0.99
Mean	0.98	0.47	7.68	4423.65	22.32	169.06	-0.36	0.65	-0.29	0.76
Median	0.99	0.38	3.79	10.07	4.35	132.93	-0.36	0.58	-0.27	0.86

From the table, the R^2 is extremely high with a minimum 0.95 and an average of 0.98. This fact implies a very good fit of the linear model. The standard errors are also small, with an average of 0.46 and a median 0.38. Since the median is smaller than the mean, the data is positively skewed with many smaller values and some large outliers, which further demonstrates that we can predict the expected order quantity within a narrow range. The t-statistics are also high enough support the use of the linear regression model.

To see how the approximation affects the total system cost, we choose three SKUs for comparison. The three SKUs are the SKU 04 and 05 in Section 4.2.1 and the SKU with the largest standard error in Table 4-2. We plot the annual cost curves for these three SKUs using the approximation technique mentioned earlier and set them against the curves calculated by using accurate expected order quantities. Figure 4-4 through 4-6 show such comparisons.

[2] SE stands for standard error.

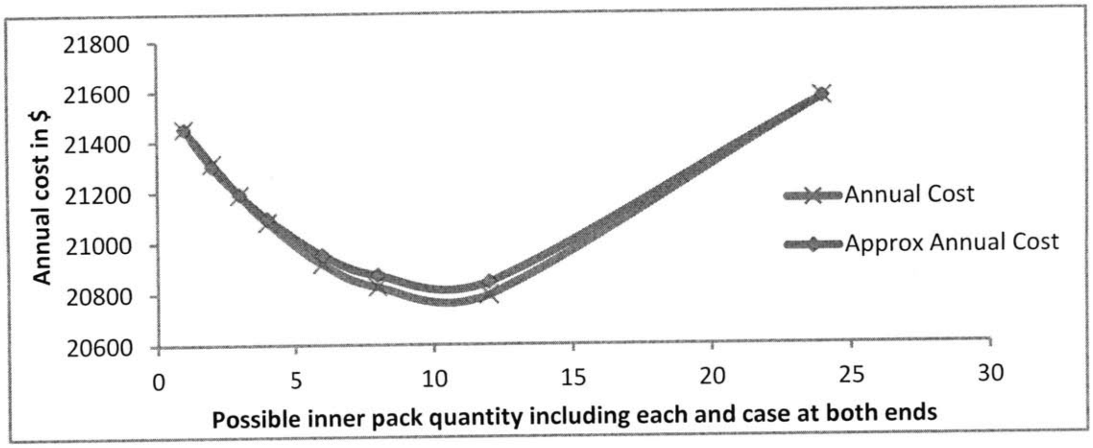

Figure 4-4: Actual annual cost vs. approximate annual cost for SKU 04

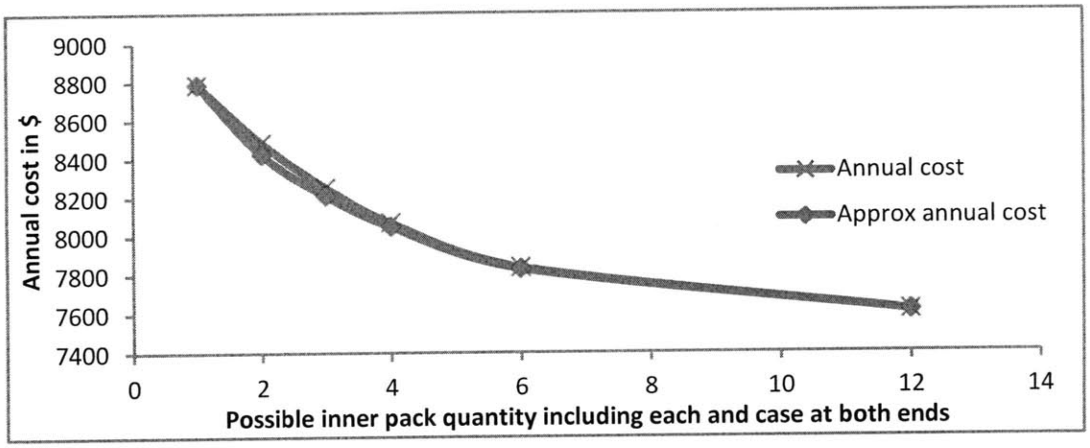

Figure 4-5: Actual annual cost vs. approximate annual cost for SKU 05

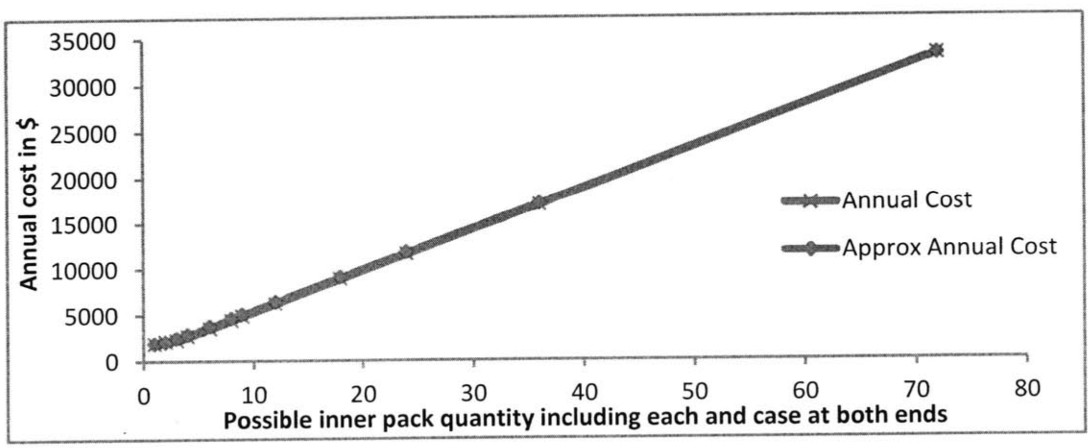

Figure 4-6: Actual annual cost vs. approximate annual cost for the SKU with the largest SE

For all three SKUs, the two curves have almost identical shapes and therefore we can use the linear approximation technique.

We also need to approximate the expected number of extra units that do not fit onto the shelves during the regular shelf-stacking process. The larger the SPQ_{inner} is, the more extra units we have. Thus, we use a simple fraction to model the $E(ExtraUnits)$, i.e.

$E(ExtraUnits) = b_{i,k}SPQ_{inner}$ where $b_{i,k,t}$is a constant less than 1 and it can be zero.

This approximation penalizes large ship-pack quantities, and it is exactly what we want to achieve in modeling the extra handling cost.

Substituting the approximations into the original equation we obtain:

$$Cost_{i,k,t}(SPQ_{inner}) =$$

$$K\frac{52\times d_{i,k,t}}{a_{k,1}\times OUTL_{i,k,t}+a_{k,2}\times ROP_{i,k}+a_{k,3}\times SPQ_{inner}}$$

$$+\left(replen_{each}\left(1-\frac{SPQ_{inner}-1}{SPQ_{case}-1}\right)+replen_{case}*\frac{SPQ_{inner}-1}{SPQ_{case}-1}\right)\times\frac{52\times d_{i,k,t}}{SPQ_{case}}$$

$$+pick_{sp}\times\frac{52\times d_{i,k,t}}{a_{k,1}\times OUTL_{i,k,t}+a_{k,2}\times ROP_{i,k}+a_{k,3}\times SPQ_{inner}}$$

$$+ICC_{st}\times c_k\times(\frac{1}{4}ROP_{i,k}+\frac{2ROP_{i,k}^3+3ROP_{i,k}^2+ROP_{i,k}}{12(ROP_{i,k}+1)^2}+\frac{a_{k,1}\times OUTL_{i,k,t}+a_{k,2}\times ROP_{i,k}+a_{k,3}\times SPQ_{inner}}{2})$$

$$+HC\times 52\times d_{i,k,t}$$

$$+ExtraHC\times b_{i,k}\times SPQ_{inner}\times\frac{52\times d_{i,k,t}}{a_{k,1}\times OUTL_{i,k,t}+a_{k,2}\times ROP_{i,k}+a_{k,3}\times SPQ_{inner}}$$

$$DC\_InvCost_{k,t} = ICC_{dc}\times c_k\times z$$

$$\times\sqrt{Var(D_{system})+2\sum(a_{k,1}\times OUTL_{i,k,t}+a_{k,2}\times ROP_{i,k}+a_{k,3}\times SPQ_{inner})^2/12}$$

Let a constant $A_{i,k,t} = a_{k,1}\times OUTL_{i,k,t}+a_{k,2}\times ROP_{i,k}$

Consider the following objective function with N stores.

$$f = DC\_InvCost_{k,t}+\sum_{i=1}^{N}Cost_{i,k,t}$$

$$= ICC_{dc}\times c_k\times z\times\sqrt{Var(D_{system})+\frac{\sum_{i=1}^{N}(A_{i,k,t}+a_{k,3}\times SPQ_{inner})^2}{6}}$$

$$+\sum_{i=1}^{N}\{K\frac{52\times d_{i,k,t}}{A_{i,k,t}+a_{k,3}\times SPQ_{inner}}+\left(replen_{each}\left(1-\frac{SPQ_{inner}-1}{SPQ_{case}-1}\right)+replen_{case}*\frac{SPQ_{inner}-1}{SPQ_{case}-1}\right)\times\frac{52\times d_{i,k,t}}{SPQ_{case}}+pick_{sp}\times\frac{52\times d_{i,k,t}}{A_{i,k,t}+a_{k,3}\times SPQ_{inner}}+ICC_{st}\times c_k\times(\frac{1}{4}ROP_{i,k}+\frac{2ROP_{i,k}^3+3ROP_{i,k}^2+ROP_{i,k}}{12(ROP_{i,k}+1)^2}+\frac{A_{i,k,t}+a_{k,3}\times SPQ_{inner}}{2})+HC\times 52\times d_{i,k,t}+ExtraHC\times b_{i,k}\times SPQ_{inner}\times\frac{52\times d_{i,k,t}}{A_{i,k,t}+a_{k,3}\times SPQ_{inner}}\}$$

To investigate whether the function has convexity property, we need its second order derivative.

The first order derivative is as follows:

$$f'=ICC_{dc}\times c_k\times z\times\frac{\sum_{i=1}^{N}(A_{i,k,t}+a_{k,3}\times SPQ_{inner})a_{k,3}}{6\sqrt{Var(D_{system})+\frac{\sum_{i=1}^{N}(A_{i,k,t}+a_{k,3}\times SPQ_{inner})^2}{6}}}$$

$$+\sum_{i=1}^{N}\{-\frac{52\times K\times d_{i,k,t}\times a_{k,3}}{(A_{i,k,t}+a_{k,3}\times SPQ_{inner})^2}+\left(\frac{replen_{case}-replen_{each}}{SPQ_{case}-1}\right)\times\frac{52\times d_{i,k,t}}{SPQ_{case}}-\frac{pick_{sp}\times 52\times d_{i,k,t}\times a_{k,3}}{(A_{i,k,t}+a_{k,3}\times SPQ_{inner})^2}+ICC_{st}\times c_k\times\frac{a_{k,3}}{2}+ExtraHC\times b_{i,k}\times 52\times d_{i,k,t}\times(\frac{1}{A_{i,k,t}+a_{k,3}\times SPQ_{inner}}-\frac{a_{k,3}\times SPQ_{inner}}{(A_{i,k,t}+a_{k,3}\times SPQ_{inner})^2})\}$$

Here is the second order derivative:

$$f''=ICC_{dc}\times c_k\times z$$

$$\times\frac{6\left(Var(D_{system})+\frac{\sum_{i=1}^{N}(A_{i,k,t}+a_{k,3}\times SPQ_{inner})^2}{6}\right)\sum_{i=1}^{N}a_{k,3}^2-(\sum_{i=1}^{N}(A_{i,k,t}+a_{k,3}\times SPQ_{inner})a_{k,3})^2}{36\left(Var(D_{system})+\frac{\sum_{i=1}^{N}(A_{i,k,t}+a_{k,3}\times SPQ_{inner})^2}{6}\right)^{\frac{3}{2}}}$$

$$+\sum_{i=1}^{N}(\frac{52\times d_{i,k,t}\times a_{k,3}\times(2K+a_{k,3}\times pick_{sp}-2ExtraHC\times b_{i,k}\times A_{i,k,t}(A_{i,k,t}+a_{k,3}\times SPQ_{inner}))}{(A_{i,k,t}+a_{k,3}\times SPQ_{inner})^3})$$

The numerator can be rearranged into the following form.

$$6\left(Var(D_{system})+\frac{\sum_{i=1}^{N}(A_{i,k,t}+a_3\times SPQ_{inner})^2}{6}\right)\sum_{i=1}^{N}a_3^2-\left(\sum_{i=1}^{N}(a_3\times A_{i,k,t}+a_3^2\times SPQ_{inner})\right)^2$$

$= 6Var(D_{system}) \times N \times a_{k,3}^2 + (N-1)(a_{k,3}^2 \sum_{i=1}^{N} A_{i,k,t}^2 + N \times a_{k,3}^4 \times SPQ_{inner}^2) > 0$

Thus, $f'' > 0$ if $(A_{i,k,t} + a_{k,3} \times SPQ_{inner}) > 0$ and

$$2K + a_{k,3} \times pick_{sp} - 2ExtraHC \times b_{i,k} \times A_{i,k,t}(A_{i,k,t} + a_{k,3} \times SPQ_{inner}) > 0$$

Recall that $A_{i,k,t} + a_{k,3} \times SPQ_{inner}$ is the approximation for the expected order quantity, and it should be strictly positive as long as the SKU is selling. Thus, as long as $2\ K + a_{k,3} \times pick_{sp} - 2ExtraHC \times b_{i,k} \times A_{i,k,t}(A_{i,k,t} + a_{k,3} \times SPQ_{inner}) > 0$, the function is convex. Moreover, if the expected number of extra units is zero, the cost function is definitely convex because the term about the extra handling cost is eliminated from the original cost function.

Given that $K = 0.25, a_{k,3} = 0.76$ (the average value from the regression analysis), $pick_{sp} = 0.15, ExtraHC = 0.03$, the inequality is simplified to $0.614 - 0.06 \times b_{i,k} \times A_{i,k,t}(A_{i,k,t} + a_{k,3} \times SPQ_{inner}) > 0$. In other words, as long as $b_{i,k} \times A_{i,k,t} \times (A_{i,k,t} + a_{k,3} \times SPQ_{inner}) < 10$, the cost function is convex. Suppose $b_{i,k}$ is 0.5, which is an exaggerated approximation of the expected number of extra units, then as long as $A_{i,k,t}(A_{i,k,t} + a_{k,3} \times SPQ_{inner}) < 20$, the cost function is convex. An empirical analysis is done to find the values of $A_{i,k,t}(A_{i,k,t} + a_{k,3} \times SPQ_{inner})$ and the expected number of extra units. We find that about 91% of the time, it is safe to conclude that the cost function is convex. The convexity property is very useful to the inner pack size analysis because we can utilize the many algorithms available for convex optimization instead of using enumeration technique.

Chapter 5
Conclusion

In this collaborative project with Beta, we establish a cost model that can be used to optimize warehouse ship-pack in the two-echelon distribution system (distribution center - retail). The three major contributions of this model are the inclusion of store-specific demands, the inclusion of multiple weekly forecasts, and the consideration of extra-handling costs at store level for larger pack size. We start from the very basic objective of generating an optimal ship-pack for the DC given store-specific demands in a single week so that it takes the least cost to serve the stores in the region; then we move on to include multiple weeks because it was unrealistic to run the model every week; finally we add in the capability to connect it with the Microsoft Access database and the model can now be run until all SKUs in question are finished.

Besides generating the optimal ship-packs for the planning period, we provide information on how the optimal ship-pack alters with changing projected demands. This information can potentially be used to provide a demand threshold for ship-pack changes. Users can also enable the dual-slotting capability in the input control so that the model produces the best two ship-packs for the dual-slots in the warehouse.

Finally, utilizing the cost model we provide an algorithm to shed some light on the optimal case configuration that can further reduce the total system cost in the DC-to-store system.

The major assumptions in our model are constant and known demand rate and uniform distribution of the Inventory Position (IP) at stores at the time of order. We did try other popular distributions such as geometric and triangular, but we found the impact was minimal so we adhered to uniform distribution. One future research direction is to assume stochastic demand and generating the optimal ship-pack under such circumstances. Another interesting extension of the current model is to incorporate the capital investment of ship-pack changes as well as the capital investment of the dual-slotting. The inclusion of these cost parameters will make the cost model as a more comprehensive decision tool. If differentiated DC replenishment costs for different SKUs were known, more sensitivity analysis could also be undertaken.

Appendix

$$E\left(\frac{1}{2}\min\left(IP_{i,k}^{1}, IP_{i,k}^{2}\right) + \frac{1}{2}\left(IP_{i,k}^{1} + Q_{i,k,t}^{sp}\right)\right)$$

$$= E\left(\frac{1}{2}\min\left(IP_{i,k}^{1}, IP_{i,k}^{2}\right)\right) + E\left(\frac{1}{2}IP_{i,k}^{1}\right) + E\left(\frac{1}{2}Q_{i,k,t}^{sp}\right)$$

Clearly, $E\left(IP_{i,k}^{1}\right) = \frac{ROP_{i,k}}{4}$ and $E\left(\frac{1}{2}Q_{i,k,t}^{sp}\right) = \frac{1}{2}\tilde{Q}_{i,k,t}^{sp}$.

Since $IP_{i,k}^{1}$ and $IP_{i,k}^{2}$ both follow discrete uniform distribution in the interval $[0, ROP_{i,k}]$, we can derive a formula which involves only the $ROP_{i,k}$. We assume the two variables are independent, so there are all together $\left(ROP_{i,k} + 1\right) \times \left(ROP_{i,k} + 1\right)$ possible pairs for $\left(IP_{i,k}^{1}, IP_{i,k}^{2}\right)$. Out of the $\left(ROP_{i,k} + 1\right)^{2}$ pairs, only one pair will have the minimum value as $ROP_{i,k}$, i.e. when both $IP_{i,k}^{1}$ and $IP_{i,k}^{1}$ are equal to $ROP_{i,k}$, and there are three pairs with the minimum value as $\left(ROP_{i,k} - 1\right)$, five pairs with the minimum value as $\left(ROP_{i,k} - 2\right)$, and so on. In summary, there are $(2i + 1)$ pairs with the minimum value as $\left(ROP_{i,k} - i\right)$. Using the definition of the expectation, i.e., $E(x) = x_i Prob(x_i)$, we can calculate the minimum of $\left(IP_{i,k}^{1}, IP_{i,k}^{2}\right)$ as follows:

$$E\left(\frac{1}{2}\min\left(IP_{i,k}^{1}, IP_{i,k}^{2}\right)\right) = \frac{1}{2}\left(\frac{\sum_{i=0}^{ROP_{i,k}}(2i+1)\left(ROP_{i,k} - i\right)}{\left(ROP_{i,k}+1\right)^{2}}\right) = \frac{2ROP_{i,k}^{3} + 3ROP_{i,k}^{2} + ROP_{i,k}}{12\left(ROP_{i,k}+1\right)^{2}}$$

Thus,

$$E\left(\frac{1}{2}\min\left(IP_{i,k}^{1}, IP_{i,k}^{2}\right) + \frac{1}{2}\left(IP_{i,k}^{1} + Q_{i,k,t}^{sp}\right)\right)$$

$$= \frac{1}{2}\tilde{Q}_{i,k,t}^{sp} + \frac{1}{4}ROP_{i,k} + \frac{2ROP_{i,k}^{3} + 3ROP_{i,k}^{2} + ROP_{i,k}}{12\left(ROP_{i,k} + 1\right)^{2}}$$

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