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Multiple Marketing Mechanisms and the Performance of Multi-Unit Demand Sequential Auctions: The Case of Quebec Hog Market

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From ALLAH to ALLAH

Résumé

Le but de cette thèse est d'étudier théoriquement et empiriquement l'encan du porc au Québec. Nous avons analysé les allocations, le nombre d'équilibres, les tendances des prix et l'efficacité dans les enchères séquentielles de second-prix ainsi que l'impact du nombre d'enchérisseurs, des fusions et de l'ajout de mécanismes de commercialisation sur la performance de ces enchères.

Dans la première note, nous démontrons que des allocations symétriques et tendances constantes de prix sont possibles que sous des conditions contraignantes. Ainsi, les allocations asymétriques sont les résultats d'équilibres les plus probables dans ce cadre. Dans la deuxième note, on montre que le résultat d'unicité d'équilibre dans les enchères avec deux enchérisseurs n'est pas valide pour le cas de trois enchérisseurs et quatre objets.

Le troisième papier montre que le revenu du vendeur augmente avec le nombre de d'enchérisseurs symétriques, mais ce n'est pas nécessairement le cas lorsque les enchérisseurs sont asymétriques. Notre analyse empirique montre une diminution du revenu du vendeur lorsque des enchérisseurs additionnels de l'Ontario étaient invités à participer. Cependant, le modèle avec correction du biais de sélection de Heckman suggère que cette baisse serait plus élevée en l'absence de soumissionnaires de l'Ontario.

Le quatrième papier montre que, même en l'absence de synergies post-fusion, la fusion peut simultanément accroître le revenu du vendeur et améliorer l'efficacité des enchères. L'utilisation d'un test de changement structurel endogène affirme que la fusion a eu un effet anticoncurrentiel sur les prix reçus par les producteurs de porcs du Québec. En outre, nous soulignons que la coexistence du mécanisme de pré-attribution et les enchères peut faire augmenter ou diminuer le revenu du vendeur et changer la tendance des prix et l'efficacité, dépendamment de qui gagne les objets en pré-attribution.

Abstract

The purpose of this thesis is to investigate theoretically and empirically the performance of the Quebec hog auction. Several issues are analyzed such as equilibrium uniqueness, price trends and efficiency in sequential second-price auctions as well as how the performance of these auctions is impacted by the addition of bidders, bidder mergers and the introduction of concurrent marketing mechanisms.

The first two notes analyze allocations, number of equilibria and price trends in sequential second-price auction games under complete information. We find that symmetric allocations and constant price trends are supported by rather stringent conditions in the first note. Thus, unique asymmetric allocations are the most common equilibrium outcomes in this setting. In the second note, we show that the result about the existence of a unique Nash perfect equilibrium in two-bidder auctions is not robust in higher dimensional auctions.

The third paper shows that the seller's revenue increases with the number of symmetric bidders but this is not necessarily the case when bidders are asymmetric. Our empirical evidence finds that the seller's revenue significantly decreases with the number of invited bidders from Ontario. However, the model with Heckman's selection bias correction suggests that this decrease would be higher in the absence of bidders from Ontario.

The fourth paper shows that even in the absence of post-merger synergies, mergers may simultaneously increase the seller's revenue and improve efficiency in sequential second-price auctions. Using an endogenous structural change test, we find that the merger has an anti-competitive effect on prices received by Quebec hog producers. A pre-attribution scheme used concurrently with the auction may increase or decrease the seller's revenue from the auction and change the price trend and efficiency, depending on how pre-attributed objects are allocated.

Remerciements

Dans le processus de rédaction de cette thèse, j'ai énormément bénéficié du soutien, des conseils et de l'amitié de plusieurs personnes. S'il n'avait pas été pour eux, cette thèse n'existerait probablement pas.

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Je voudrais aussi exprimer ma gratitude aux enseignants, chercheurs et le personnel administratif du Centre de recherche en économie de l'agroalimentaire (CRÉA) de l'Université Laval. Vous m'avez fait beaucoup de place!

Cette thèse est consacrée à mes parents, à mon fils Ahmed, à mes frères et sœurs, pour le soutien et l'amour qu'ils m'ont toujours donné. Inconditionnellement, ils m'ont encouragé à suivre mes propres décisions, même si cela signifie avoir à vivre les uns des autres. Mon père aurait été fier de ce travail, qui est aussi dédié à sa mémoire. Ma mère m'a toujours infusé son enthousiasme, qui a été précieux dans les moments les plus difficiles de cette thèse. Mon père m'a appris à aborder les choses en allant à la racine et ne prendre rien pour acquis.

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In the process of writing up this thesis, I have benefited from the support, advice and friendship of many people. Had it not been for them, this thesis would most probably not exist.

My major supervisor, the Professor Bruno Larue, deserves an important place in the large list of people I am indebted to. His role as a supervisor has been invaluable. Not only has he helped me to develop my economic intuitions and express them in rigorous and intelligible terms; he has also demonstrated his continuous support towards my work and academic career, without which I would have lacked the enjoyment and confidence that have spired me on during these years. The Professor Bruno Larue, has been, and will continue to be, my academic role model.

I would also like to express my gratitude to the teachers, researchers and the administrative staff of the Centre for Research in the Economics of Agri-food (CRÉA) at the University Laval. You made me a lot of place!

This thesis is dedicated to my parents, to my son Ahmed, to my brothers and sisters, for the support and love they have always given me. Unconditionally, they have encouraged me to follow my own decisions even if this has meant having to live apart from each other. My father would have felt proud of this work, which is also dedicated to his memory. My mother has always infused me her enthusiasm, which was invaluable in the more difficult moments of this thesis. From my father I had learned to approach things by going to the roots and not taking anything for granted.

Mohamed Jeddy

Avant-propos

Les chapitres de la présente thèse sont constitués d'une note publiée, et une autre note et des papiers soumis ou à soumettre à des revues scientifiques. Dans ces travaux, je suis le principal auteur.

Le deuxième chapitre est composé de deux notes. La première est publiée à *Economics Bulletin* ayant comme co-auteurs mon directeur de thèse le professeur Bruno Larue et le professeur Jean-Philippe Gervais. La deuxième note a été soumis pour évaluation à une revue scientifique avec comité de lecture ayant comme co-auteur mon directeur de thèse le professeur Bruno Larue.

Le troisième chapitre est un article réalisé avec mon directeur de thèse le professeur Bruno Larue. Il est en cours de finalisation pour être soumis à une revue scientifique avec comité de lecture.

Le quatrième chapitre est un article ayant comme co-auteur mon directeur de thèse le professeur Bruno Larue. Il a été soumis pour évaluation à une revue scientifique avec comité de lecture.

Preface

The chapters of this dissertation are notes and papers that are either published, submitted to peer-reviewed academic journals, or are being prepared to be submitted to journals. In all these works, I am the principal author.

The first note in Chapter 2 was co-authored with my major professor, Bruno Larue. It has been published in *Economics Bulletin*. The second note was co-authored with my major professor, Bruno Larue. It has been submitted to a peer-reviewed journal.

The paper in Chapter 3 was co-authored with my major professor, Bruno Larue. It will be submitted to a peer-reviewed journal as soon as it goes through some final editing.

The paper in Chapter 4 was co-authored with my major professor, Bruno Larue. It has been submitted to a peer-reviewed journal.

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1. Introduction

The hog industry is particularly important in Quebec's agri-food sector. It accounted for 12% of Quebec's agricultural receipts in 2009 and only the dairy sector could claim to be more important with 28% of agricultural receipts.[1] Quebec's hog exports are rather insignificant as most of the production is processed in Quebec. As such, Quebec is quite different from Ontario and Manitoba which have specialized in exports of piglets that end up being "finished" and slaughtered in the United States. Quebec's pork exports, which account for half of the domestic production, are sold to 75 countries. The main destinations in 2009 were the United States (25.2%) and Japan (18.7%), Hong Kong (10.3%), South Korea (6.6%), Russia (5.9%), Philippines (5.5%), Australia (5.3%), and other countries (22.5%).[2] Historically, the United States has been the main destination, but the steady appreciation of the Canadian dollar with respect to the U.S. dollar has made the U.S. market less profitable for Quebec pork processors. The main reason why Quebec hogs are all processed in Quebec and that Quebec exports so much pork has to do with the marketing arrangements between hog producers and processors and the presence of a generous revenue insurance program.

A glance at the Organisation for Economic Co-operation and Development (OECD) producer support estimates reveal that hog production is not highly subsidized in Canada (i.e., 8% of the value of pig meat production in comparison to 52% for milk production). The main programme helping hog producers in Quebec is the *Assurance Stabilisation des Revenus Agricoles* (ASRA)[3] which is funded by producers (33%) and the provincial and federal governments (66%). Since 2010, larger farms pay 50% of the premium. The ASRA program

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[1] Source : http://www.mapaq.gouv.qc.ca/fr/md/statistiques/Pages/production.aspx (consulted on May 03, 2011).

[2] Source: http://www.laterre.ca/elevage/le-japon-nest-plus-le-principal-client-du-porc-que/ (consulted on May 03, 2011).

[3] The law supporting the programme is described at: http://www2.publicationsduquebec.gouv.qc.ca/dynamicSearch/telecharge.php?type=2&file=%2F%2FA_31%2FA31.htm (consulted on February 24, 2011).

guarantees a minimum price to producers. For processors, this is a major advantage because supply is not affected by decreases in the market price below the guaranteed price (Larue et al., 2004). As a result, hog supply has been more stable and more predictable in Quebec than in other provinces.

The marketing of hogs in Quebec has experienced several changes over the last 20 years. The *Fédération des Producteurs de Porc du Québec* (FPPQ) was granted permission to create a marketing board in 1981, but the FPPQ's first important realization was the implementation of a daily electronic auction in 1989. Between March of 1989 and January of 1994, all of the hogs produced in Quebec were sold through an electronic auction. In 1994, the FPPQ and the processors decided to change the marketing system. They opted for a hybrid system in which part of the hogs are pre-attributed (i.e., to be delivered to slaughterhouses at a negotiated price) while the remainder was sold through the daily electronic auction. In January of 2000, the hog marketing system entered a new phase in which pre-attributions became less important, as the percentage of hogs marketed through that mechanism dropped from 72% to 60%. The share of the hog supply sold on the auction was also reduced from 28% to 25%. The novelty was the introduction of a third mechanism, accounting for 15% of the average weekly number of hogs slaughtered during the last three months. This new mechanism was an auction of contracts for fixed supplies to be delivered over a month. Another important event that impacted on the industry is the merger of the two biggest processors, Olymel and Groupe Brochu, in October of 2004. Following the demand of the FPPQ, the auction mechanism was suspended by the *Régie des marchés agricoles du Québec* (RMAQ) from October 2006 to April 2007 and officially stopped on February 13, 2009. The reason was the high difference between the US reference price and the average auction price (more than $45 per 100kg at the end of 2008). After more than a year and one half of deep negotiations, Quebec hog producers and processors agreed on a new hog marketing agreement on September 7, 2009. Buyers commit to pay a predetermined reference price for all hogs purchased. Following the agreement, the FPPQ decides the allocation of hogs to each buyer according to some fixed rules and priorities (packer-owned hogs, specialty hogs and commodity hogs).[4]

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[4] For further details, see Gervais and Lambert (2010).

The electronic auction played an important role in hog marketing in Quebec. Gervais and Lambert (2010) contend that the new system lacks the flexibility that the electronic auction provided in transmitting positive and negative shocks impacting on the margins of hog processors. The electronic auction is no longer in use and its performance was criticized by the FPPQ when auction prices fell below the US reference price. An analysis of the Quebec hog auction remains most relevant because it remains to be ascertained whether the criticisms of the auction were well founded. Furthermore, as argued by Engelbregth-Wiggans and Kahn (1999), little is known about livestock auctions even though they have been in use for millenniums all around the world.

We develop simple theoretical models of the multi-unit sequential auctions under complete information with asymmetric bidders having multi-unit demands in keeping with the structure of the Quebec hog processing sector. The seller is presumably poorly informed while each bidder is completely informed, not only about the item(s) being auctioned or pre-attributed, but also about his own valuation and that of his competitors. The number of bidders was small (seven) as only slaughterhouses located in Quebec were allowed to bid[5]. Quebec hog processors sell to the same domestic and foreign firms and have known fixed capacities and technologies. They sell to a few large distributors/retailers on the domestic market and face competition from many foreign firms on export markets. Accordingly, they should possess declining marginal valuations for hogs and ought to have precise estimates of what these valuations are. As such, we contend that it is reasonable to use a complete information framework. Our justification is very close to the one invoked by Bernheim and Whinston (1986) regarding the bidding for construction contracts.[6] In their case as in ours, the bidders have reliable information about each other's costs, capacity and market opportunities.

Hog processing in Quebec is dominated by a processing firm, Olymel, which was created in March of 1991 from the merger of Olympia and Turcotte&Turmel. At that time, Olymel

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[5] There have been exceptions when the FPPQ has invited an outside bidder from Ontario to participate. This has not occurred often, but we should be able to see it from the data. The idea was to send a signal to the regular bidders that the FPPQ was expecting more aggressive bidding.

[6] For more justifications, see Gale and Stegeman (2001, p. 77).

reportedly controlled 75-80 percent of Quebec's hogs slaughter business (Larue et al., 2000). The second and third largest firms at the time were *Le Groupe Brochu* and *Le Groupe Breton* with respective shares of 14.4% and 10%. The other four firms involved in slaughtering had shares varying between 3 and 6 percent. Over the next ten years, Olymel saw its market share erode to about 50-60%. On the 13th of October in 2004, *Groupe Brochu* merged its Supraliment pork and poultry processing operations with Olymel, to create a larger company better suited to compete nationally and internationally. The presence of a dominant firm suggests large asymmetries in bidders' valuations. The history of Olymel also motivates investigation of about the impact of mergers on the auction's performance, including efficiency and the seller's revenue.

As pointed out earlier, there was a time when all hogs were sold through the auction, but prices tended to be lower and this is why additional mechanisms were created. A glance at price data suggests that these mechanisms have improved the performance of the auction from the seller's point of view (FPPQ). We use our models to shed some light on this empirical observation. Accordingly, we hope to make a contribution to the rather thin literature about the complementarity between marketing mechanisms (see Salmon and Wilson (2008) for a rare and recent example) as most studies tend to compare different individual marketing mechanisms (e.g., Wang, 1993; Bulow and Klemperer, 1996 and Hailu and Schilizzi, 2004). Our theoretical models generate propositions whose validity can be assessed empirically. We are especially fortunate that the FPPQ has graciously accepted to grant us access to part of their auction data. This is indeed the first time that such data is being analyzed for academic purposes.

The rest of this doctoral thesis is structured as follows. Chapter 2 deals with the allocations, price trends and equilibria in multi-unit sequential auctions with symmetric or asymmetric bidders. First, we analyze auctions involving two or more bidders with similar decreasing marginal valuations. We generalize insights from Krishna (1999) by showing that asymmetric allocations arise except in specific conditions about the decline of valuations. The conditions supporting a symmetric allocation are increasingly restrictive as the number of object increases. Second, we show that the result on the existence of a unique Nash perfect equilibrium (as in Krishna, 1993; Katzman, 1999; and Gale and Stegeman, 2001) is not robust in higher dimensional auctions.

Chapter 3 discusses an important issue about the variation of the seller's revenue when the number of bidders is increased. We show that the seller's revenue increases with the number of symmetric bidders but this is not necessarily the case when bidders are asymmetric. Our empirical evidence finds that the seller's revenue significantly decreases with the number of invited bidders from Ontario. However, the model with Heckman's selection bias correction suggests that this decrease would be higher in the absence of bidders from Ontario.

Chapter 4 analyzes the effects of mergers on the seller's revenue, price trend and efficiency in sequential auctions under complete information with asymmetric bidders. First, we provide the conditions when bidders are strategic and when a merger can take place. Second, we show that mergers may simultaneously increase the seller's revenue and improve efficiency. Third, we show that having a marketing mechanism working alongside the auction can increase or decrease the average auction price. We use weekly data about Quebec's daily hog auction to ascertain the effects of a merger and of changes in the weights of concurrent marketing mechanisms on daily auction prices. Our empirical analysis relies on an endogenous structural change test which detected three breaks corresponding to: i) the introduction of a new concurrent mechanism, ii) a joint-venture partnership of the two largest hog processing firms and iii) an announcement by Canada's Competition Bureau authorizing the full merger of the same two firms.

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2. Allocations, Price Trends and Equilibria in Multi-Unit Sequential Auctions

Résumé

Dans la première partie de ce chapitre, nous démontrons que des allocations symétriques et tendances constantes de prix ne sont possibles que sous des conditions contraignantes. Ainsi, les allocations asymétriques sont les résultats d'équilibres les plus probables dans ce cadre. Dans la deuxième partie, on montre que le résultat d'unicité d'équilibre dans les enchères avec deux enchérisseurs n'est pas valide pour le cas de trois enchérisseurs et quatre objets.

Abstract

We analyze allocations, number of equilibria and price trends in sequential second-price auction games under complete information. We find that symmetric allocations and constant price trends are supported by rather stringent conditions in the first note. Thus, unique asymmetric allocations are the most common equilibrium outcomes in this setting. In the second note, we show that the result about the existence of a unique Nash perfect equilibrium in two-bidder auctions is not robust in higher dimensional auctions.

2.1. Allocations and Price Trends in Sequential Auctions under Complete Information with Symmetric Bidders

Abstract. We analyze sequential second-price auctions under complete information involving two or more bidders with similar decreasing marginal valuations. Krishna (1999) designed a 2-bidder numerical example to show the existence of two symmetric equilibria characterized by an asymmetric allocation and weakly declining prices. We generalize Krishna's insights by showing that symmetric (asymmetric) allocations imply constant (weakly declining) price patterns and we derive the necessary conditions supporting symmetric allocations. The conditions become increasingly restrictive as the number of objects increases.

2.1.1. Introduction

Bernheim and Whinston (1986) have argued that the complete information assumption is appropriate for the analysis of frequently-held auctions involving the same bidders. In such settings, the bidders know each other's valuations, but the seller is poorly informed. It is easy to construct a second-price auction under complete information involving two symmetric bidders with declining valuations that will support two symmetric equilibria[7] characterized by a constant price pattern. Consider the outcome tree of the game illustrated in Figure 2.1. Arrows denote the allocation in each subgame and prices are given next to the paths. At each node, the bidders' gross payoffs are put in parenthesis. Each unit could go to either bidder A (left branch) or B (right branch). The equilibrium outcome is solved by backward induction and bids reflect the opportunity cost of not winning. Bidders' valuation

[7] The equilibrium is said to be symmetric when bidders are symmetric and use the same strategy while the equilibrium allocation could be either symmetric or asymmetric.

of the first and second objects are θ_1 and θ_2. Bidder *i* has gross payoffs of $\theta_1+\theta_2$, θ_1 and 0 from winning both objects, one object and nothing. Provided bidder A won the first object, he would bid his gross payoff differential θ_2 for the second object. Conditional on bidder A having won the first object, bidder B would have a gross payoff differential of θ_1-0 and would win the second object at price $p_2^B=\theta_2$. Conditional on the first object being won by bidder B, it is easy to see that bidder A would win the second object by bidding θ_1 and paying $p_2^A=\theta_2$. Moving up the tree, the payoffs at the two nodes account for allocations and prices derived for the second object: $(\theta_1,\theta_1-\theta_2)$ vs $(\theta_1-\theta_2,\theta_1)$. Thus both bidders end up bidding θ_2, knowing that if they lose the first object they will get the second at the same price.

However, as for the Heckscher-Ohlin model in the trade literature, the results of this 2x2 auction are not robust when the number of objects *n* or the number of bidders increases.[8] In an example of a four-object auction involving two bidders with symmetric valuations, Krishna (1999) uncovered two symmetric equilibria characterized by an asymmetric allocation and declining prices.[9]

The analysis of sequential auctions under complete information with symmetric bidders has been largely ignored in the literature and it is the purpose of this note to shed more light on such auctions. We show that when the number of objects is even, but greater or equal to 4, symmetric allocations and a constant price trend arise under specific conditions about bidders' valuations. Otherwise the allocations are uneven and prices are declining with possibly flat segments. When the number of objects is uneven, allocations are asymmetric and prices are declining.

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[8] When the two bidders have asymmetric valuations, Katzman (1999) has shown that the equilibrium is unique, possibly inefficient and that the price pattern may be constant or declining.

[9] Katzman (1999) and Gale and Stegeman (2001) analyzed cases with asymmetric valuations.

2.1.2. The Model

The auction is a sequential second-price one involving two completely informed bidders with identical decreasing marginal valuations: $\theta_1 > \theta_2 > ... > \theta_{n-1} > \theta_n$.[10] Part of the 4-object version of the game is illustrated in Figure 2.2. As it is explained about figure 2.1, the equilibrium outcome is solved by backward induction. In this instance, a symmetric allocation with bidders A and B getting two objects each can be achieved through six equilibria provided valuations decrease at a decreasing rate, $\theta_2 - \theta_3 > \theta_3 - \theta_4$: {A,B,A,B}, {B,A,B,A}, {A,A,B,B}, {B,B,A,A}, {A,B,B,A} and {B,A,A,B}. Equilibrium prices are constant and the seller's revenue is $R = 4\theta_3$.

If the two bidders had symmetric valuations such that $\theta_2' - \theta_3' < \theta_3' - \theta_4'$, prices would weakly decline $p = \{3\theta_3' - \theta_2' - \theta_4', \theta_2' + \theta_4' - \theta_3', \theta_2' + \theta_4' - \theta_3', \theta_4'\}$, one player would get 1 object and the other would get 3 and $R' = \theta_2' + \theta_3' + 2\theta_4' \lesseqgtr R$.[11] Symmetric allocations are also possible in higher-dimensional games. We show that the condition just derived for the *n*=4 case is a special case of a more general set of conditions.

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[10] The case of endogenous valuations is analyzed by Krishna (1999). In her two-object auction, a snowball effect arises because bidders use the object as inputs and compete on the "output" market. The bidder who won the first object has a higher valuation for the second object because that second object would secure a monopoly position. In our case, we treat valuations as exogenous. This could be rationalized by the existence of alternative marketing mechanisms preventing monopoly outcomes. For example, the daily hog auction in the province of Quebec involves a small number of bidders. However, they get a large share of their hog supply through a pre-attribution/formula pricing mechanism based on historical market shares.

[11] Consider the following examples with valuations adding up to the same total such that the seller's revenue from selling the 4 objects as a block would be the same: $\theta = \{10, 9, 6, 5\}$, $\theta' = \{10, 9, 6.7, 4.3\}$, $\theta'' = \{10, 9, 8, 3\}$ and $\theta''' = \{10, 8, 7, 5\}$. When the objects are sold sequentially, the first set of valuations produces a symmetric allocation, identical prices *p* = 6 and revenue *R* = 24. For the asymmetric allocations with weakly declining prices, we have $R' = 24.3, R'' = 23, R''' = 25$.

Proposition 1. *Consider two bidders {A,B} having similar strictly declining marginal valuations and let k=n/2 where n is an even number of successive second-price auctions with* $n \geq 4$. *There are multiple symmetric equilibria with a constant price pattern or weakly declining pattern generating identical payoffs for the two bidders. The bidders get the same number of objects k if and only if the price pattern is constant which requires* $\sum_{m=1}^{k}\theta_m - k\theta_{k+1} > \sum_{m=1}^{k-p}\theta_m - (k-p)\theta_{k+p+1}\ \forall p = 1,\ldots,k-1$.

Proof. Intuitively, bidder A must be indifferent between his allocation and that of bidder B, whether the allocations are symmetric or asymmetric. Under a symmetric allocation derived through backward induction, let us assume that bidder A has won *k* objects and bidder B has won $0 \leq j \leq k-1$ objects. When *j=k*-1, one object remains to be auctioned. Bidder A bids his valuation for the n^{th} and last object and this is the price that bidder B will pay given that his valuation is higher: $p^{B}_{k+j+1} = \theta_{k+1} < \theta_k$. Thus, at the (*n*-1)th auction, bidders know that if they lose the object they will win the last one and gain $\theta_k - \theta_{k+1}$. Because bidders must be indifferent between winning and losing the (*n*-1)th object, prices for the (*n*-1)th and n^{th} objects must be the same. This is just like the 2x2 auction in Figure 2.1.

Consider now $j = k-2$. Bidder B knows that bidder A has used up his first *k* valuations. Bidder B can win the last two objects by bidding in excess of θ_{k+1} and gain $\theta_{k-1} - \theta_{k+1} + \theta_k - \theta_{k+1}$ for these last two objects, or win one object and gain $\theta_{k-1} - \theta_{k+2}$ or win none and gain nothing. The latter option is dominated because valuations are strictly declining. If bidder B is to win the last two objects, it must be that: $\theta_k - \theta_{k+1} > \theta_{k+1} - \theta_{k+2}$ or valuations must decrease at a decreasing rate at the k^{th} valuation. This is the condition required to have symmetric allocations for the 4-object auction in Figure 2.2. If it is not met, an asymmetric allocation emerges and prices must decline.

In this 4 object-auction, if bidder B is to win only one object, his maximum payoff is achieved by having bidder A get the first three objects. Hence, $\pi^B = \theta_1 - \theta_4$ which must equal $\sum_{m=1}^{3}(\theta_m - p_m)$. Clearly the average price on the first three objects must be above θ_4. Furthermore, if one of the first three objects was to be sold below θ_4, bidder B would prefer getting this object instead of the fourth object. But bidder A would prefer bidder B's payoff and so a price below θ_4 cannot be observed. Therefore, prices must be weakly declining. Consider now the case *j*=0 (i.e., bidder A has won the first *k* objects and *k* others remain to be auctioned). A symmetric allocation requires that bidder B wins the last *k* objects and that both bidders get the same payoff. This requires that $\sum_{m=1}^{k}\theta_m - k\theta_{k+1} > \sum_{m=1}^{k-p}\theta_m - (k-p)\theta_{k+p+1}\ \forall p = 1,\ldots,k-1$. **QED**

The number of conditions increases with *n* because the symmetric allocation is pitted against a larger number of potential asymmetric allocations. Furthermore, the conditions supporting a symmetric allocation become increasingly stringent when the number of objects increases. For $k=3$ $(n=6)$, it must be that $\sum_{m=1}^{3}\theta_m - 3\theta_4 > \max\left(\sum_{m=1}^{2}\theta_m - 2\theta_5, \theta_1 - \theta_6\right)$. These inequality restrictions can be rearranged as: $Min(\theta_3 + 2\theta_5, \theta_2 + \theta_3 + \theta_6) \geq 3\theta_4$. Clearly the differences between the first three valuations and the fourth one must be large compared to the differences between the 4th and the 5th and 6th. For $k=5$ $(n=10)$, one of the necessary conditions is $\theta_5 - \theta_6 \geq 4(\theta_6 - \theta_7)$. Clearly $(\theta_k - \theta_{k+1}) - (\theta_{k+1} - \theta_{k+2})$ must increase significantly as the number of objects increases if a symmetric allocation is to be observed.

Proposition 2. *When n, the number of successive second-price auctions with two bidders {A,B} having similar declining marginal valuations, is uneven, the allocation is asymmetric and the price pattern is always declining with possibly flat segments.*

Proof. As for an asymmetric allocation when the number of objects is even in proposition 1, prices must be weakly declining because of payoff symmetry. Consider an auction with *n* = 3 and bidder A winning 2 objects and bidder B winning only one. Bidder B "waits" to win the last object for a payoff of $\pi^B = \theta_1 - \theta_3$. Bidder A must be indifferent between winning the first two objects or taking bidder B's place as the winner of a single object and vice versa. Furthermore, when the second object is put for sale, bidder B must be indifferent between his payoffs from waiting for the third object or getting the second object. Bidder A knows that and the price for the second and third objects is the same: θ_3 which explains the flat segment. Therefore, payoff symmetry requires that the price sequence be: $p = \{\theta_2, \theta_3, \theta_3\}$. Because players A and B can be interchanged, there are two symmetric equilibria with the same weakly declining price pattern.

QED

Figure 2.3 illustrates the results of proposition 1 and 2 via a few examples. The first example illustrates the case for 4 objects with declining valuations equal to $\{10,7,5,4\}$ for each bidder. The condition in proposition 1 is met and the equilibrium is characterized by a constant price. The 5-object example with bidders' valuations equal to $\{20,15,12,10,2\}$ generates weakly declining prices : $p = \{16,8,8,8,3\}$. A similar outcome also emerges with our 6-object example with bidders' valuations equal to $\{20,15,12,10,7,6\}$. Even though bidders have symmetric valuations, they can safely exploit rapid declines in valuations through asymmetric allocations. For the same reason, a symmetric (inefficient) allocation can arise when bidders have asymmetric valuations as shown by Katzman (1999).

Our analysis can be generalized for cases involving more than 2 bidders. In the 3-bidder case with *n* a multiple of 3, the symmetric allocation entails having bidders A,B,C winning $k \equiv n/3$ objects at a constant price $p = \theta_{k+1}$. When the game is at a point where *n*-3 objects have been sold such that bidders A,B,C have $\{k,k,k-3\}$ objects, then bidder C must decide whether it is best to get the last three objects or to get only one and letting the other bidders get one as well: $\sum_{i=k-2}^{k} \theta_i - 3\theta_{k+1} \geq \theta_{k-2} - \theta_{k+2}$. This is a necessary, but not sufficient condition. However, if $n = 9$, we are comparing allocations $\{3,3,3\}$ and $\{4,4,1\}$

and our necessary condition for a symmetric allocation is $\theta_2+\theta_3+\theta_5>3\theta_4$. Other asymmetric allocations, $\{5,2,2\}$ and $\{7,1,1\}$ impose additional conditions, namely: $\sum_{i=1}^{3}\theta_i-3\theta_4>\max\left(\sum_{i=1}^{2}\theta_i-2\theta_6,\theta_1-\theta_8\right)$ or $Min(\theta_3+2\theta_6,\theta_2+\theta_3+\theta_8)>3\theta_4$. The drop in valuation between the kth and (k+1)th objects must be large, as shown for 2-bidder cases.

2.1.3. Conclusion

We analyze sequential second-price auctions under complete information when bidders have identical decreasing marginal valuations over *n* objects ($\theta_1>...>\theta_n$). We show that a symmetric (asymmetric) allocation with each bidder getting *k* objects is characterized by constant (weakly declining) prices. Generally, symmetric allocations require that valuations be such that $\theta_k-\theta_{k+1}$ be larger than $\theta_{k+1}-\theta_{k+2}$. The decreases in valuations from the k+1th object must be increasingly small relative to the decrease in valuation between the kth and (k+1)th objects as the number of objects auctioned increases, thus making asymmetric allocations more likely when the number of objects is large.

2.1.4. References

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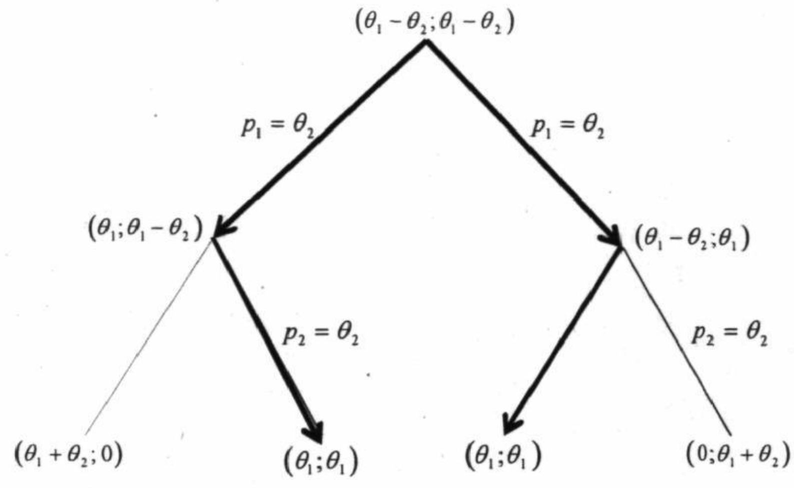

Figure 2.1. The complete information two-bidder two-object second-price auction with symmetric valuations.

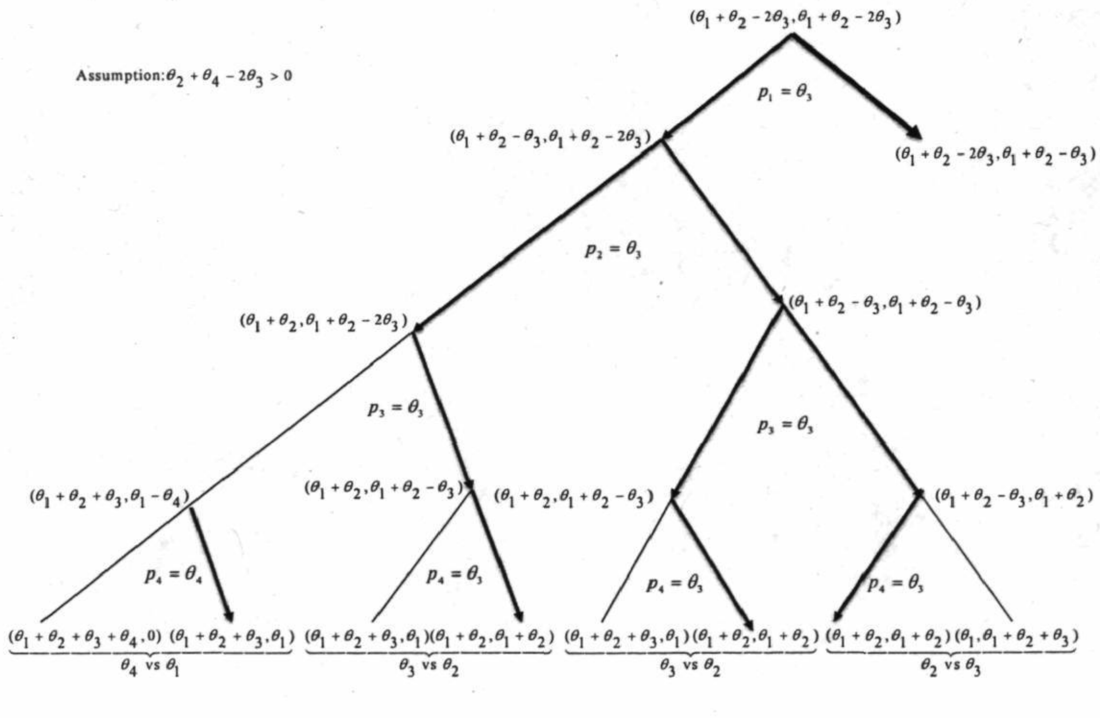

Figure 2.2. A 2-bidder 4-object auction with symmetric allocations.

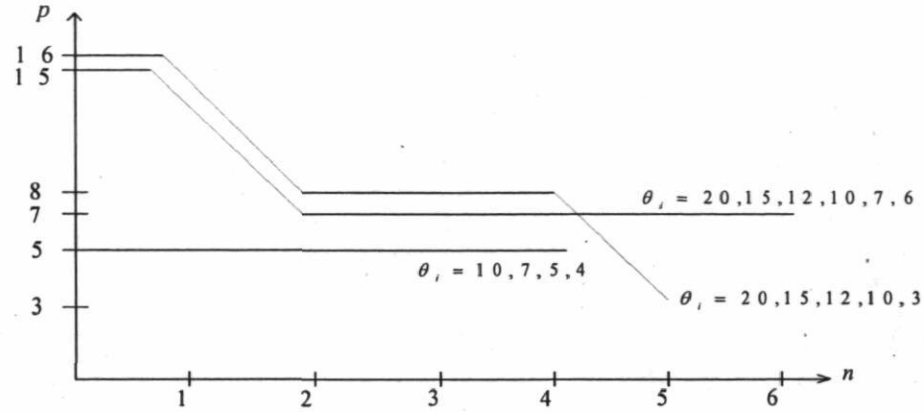

Figure 2.3. Examples of price patterns when bidders are symmetric.

2.2. Multiplicity of Equilibria in Multi-unit Demand Sequential Auctions under Complete Information

Abstract. We show that the result about the existence of a unique Nash perfect equilibrium in two-bidder multi-unit sequential second-price auctions under complete information (as in Krishna, 1993; Katzman, 1999; and Gale and Stegeman, 2001) is not robust in higher dimensional auctions. Using an example featuring three bidders competing for four objects, we found multiple equilibria characterized by different vectors of prices and allocations.

2.2.1. Introduction

In many real world auctions, the seller is poorly informed while each bidder is well informed, not only about the item(s) being auctioned and hence his own valuation, but also about his competitors' valuations. Bernheim and Whinston (1986) use the example of a few firms relying on a common technology and routinely bidding on construction contracts to justify the complete information assumption. Gale and Stegeman (2001, p.75) argue that the assumption is justified in cases where two well informed sellers bid sequentially for contracts, like for waste disposal, consulting services and military hardware. Electronic livestock auctions are also good examples. The Quebec hog auction has been in operation every day except on weekends between 1989 and 2008 as a small number of meat processors (seven) were bidding on fixed-size lots of hogs scoring 100 on a quality index.[12]

[12] When hogs were delivered to a plant, a quality grid was used to make price adjustments for hogs scoring below or above 100. Therefore, quality issues were internalized. Furthermore, it is not heroic to assume that each meat processor knows the production capacity, cost structure and market opportunities of other meat processors.

The analysis of multi-unit demand sequential auction under complete information with more than two asymmetric bidders has been largely ignored in the literature possibly because of a presumption that results for two bidders could be generalized to the *k*-bidder case. Important contributions on multi-unit auctions under complete information by Krishna (1993), Katzman (1999), Gale and Stegeman (2001) and Rodriguez (2009) demonstrate the existence of a unique Nash perfect equilibrium when there are two bidders. Jeddy, Larue and Gervais (2010) analyzed price trends and allocations when *k* bidders have identical decreasing valuations. They found that symmetric allocations and constant price trends are supported by rather stringent conditions. Thus, unique asymmetric allocations are the most common equilibrium outcomes in this setting.

As is common in the auction literature (e.g., Engelbrecht-Wiggans, 1999), we rely on a numerical example for a sequential second-price auction involving three bidders and four objects to show that equilibrium uniqueness charactering 2-bidder auctions under complete information, does not hold generally.

2.2.2. The model and Discussion

Consider a sequence of four second-price auctions where three individual bidders have diminishing marginal valuations such that: $V_1^j > V_2^j > V_3^j > V_4^j \; \forall j = A, B, C$ where V_i^j is the i^{th} valuation of bidder *j*. They compete for four homogenous objects under complete information. The seller is non-strategic and sets a reserve price equal to zero.

The strategic behaviour of bidders in second-price multi-unit sequential auctions under complete information is well-documented in the literature (e.g., Krishna, 1993, 1999; Katzman, 1999; Gale and Stegeman, 2001 and Jeddy et al., 2010). Each bidder is assumed to follow the weakly dominant strategy of sincere bidding in the last and 4^{th} round. For $k < 4$, it is a weakly dominant strategy for each bidder to place a bid in the k^{th} round that would make him indifferent between winning and losing the k^{th} round, considering the contingent outcomes from the $(k+1)^{th}$ to the 4^{th} rounds. The existence of equilibria in these games under complete information is obvious. It can be easily shown that a unique equilibrium exists for the case of three bidders and two objects. Intuitively, the uniqueness property holds for cases with more than two bidders because some bidders are nonstrategic and the auctions end up being equivalent to auctions with fewer bidders. However, we will show that uniqueness may not hold for three-bidder four-object auctions.

Example. Let us assume that: $V^A = \{20, 15, 14, 12\}$, $V^B = \{18, 13, 10, 5\}$ and $V^C = \{17, 11, 9, 3\}$.

Consider the parts of the outcome tree of the game illustrated in figures 2.4-2.6. Figure 2.4 (2.5) [2.6] shows the possible allocations for the last three objects provided the first object is allocated to bidder A (B) [C]. Arrows denote the allocation in each subgame and prices are given next to the paths. At each node, the bidders' gross payoffs are put in parentheses. Each unit could go either to bidder A (left branch), to bidder B (middle branch) or to bidder C (right branch). The equilibrium outcome is solved by backward induction and bids reflect the opportunity cost of not winning. The outcome tree, unlike the extensive form, features gross payoffs at every node which are obtained through subgame replacement. At nodes associated to the j^{th} object, gross payoffs are defined as the sum of valuations for objects won along the given path minus the sum of prices for objects that would be won among the last n-j+1 objects. For the last object, gross payoffs are simply the sum of the valuations.

Starting at the bottom of the subgame tree of figure 2.4, we can see that the vector of gross payoffs when bidder A wins all four objects is (61; 0; 0), which is simply the sum of the valuations for the objects won by the bidders. Provided the first three objects are won by bidder A, the fourth object may be won by bidder A, bidder B or bidder C. In these cases, the vectors of gross payoffs are (61; 0; 0); (49; 18; 0) and (49; 0; 17) respectively. It follows that at node N_{13}, the fourth object is worth at most 12 (i.e., 61-49 = 12) for bidder A, at most 18 (i.e., 18 - 0 = 18) for bidder B and at most 17 (i.e., 17-0 = 17) for bidder C. If the game were to reach node N_{13}, the fourth object would be won by bidder B at price 17. Therefore, the gross payoff at node N_{13} is (49; 18-p_4; 0) = (49; 1; 0).

The same reasoning could be used at nodes 14-39 and nodes 4-12 (see figures 2.4-2.6). It is easy to verify that if the game reaches N_4-N_{12} that the vectors of gross payoffs are respectively (35; 4; 3); (22; 18; 4); (22; 5; 17); (22; 18; 4-€); (9; 31; 6); (9; 18; 17); (22; 5; 17); (9; 18; 17) and (7; 5; 28). Particularly, at node N_7, it is a dominant strategy for bidder B to bid 13 to prevent the other bidders from acquiring the third object. Bidder A is willing to pay as much as 15 to counter bidder B and only 13 to counter bidder C while bidder C would pay 14 to prevent bidder A from winning and would pay 15 to prevent bidder B from winning. Taking into account bidder B's strategy, we can then analyze the subgame between bidders A and C with the normal representation illustrated in table 2.1. There are multiple equilibria but bidder C wins the third object and pays $p_3 = 13 + \varepsilon$ such that $0 < \varepsilon \leq 1$. The vector of net payoffs is (22; 18; 4-Є).

At node N_1, bidder B or bidder C wins the second object and pays 13 while at node N_3, bidder A or bidder B wins the object and pays 13. The vectors of net payoffs are respectively (22; 5; 4) and (9; 5; 17). We must determine the gross payoffs at N_1, N_2 and N_3 to determine who gets the first object and to solve the game from N_0. At node N_2, bidder A or bidder B wins the second object and pays 13. If bidder A wins the object then the vector of net payoffs is (9; 18; 4-Є) but if bidder B wins it, the vector of net payoffs is (9; 18; 6). In the former case, bidder C is better off to bid more than 13 and wins the first object at node N_0. Bidder C is willing to pay as much as 13 to counter bidder A and 13 + Є to counter bidder B. It is a dominant strategy for bidders A and B to bid 13 to prevent the other bidders from acquiring the object. If bidder C bids 13, his payoff will be (12-Є)/3 and if he bids more than 13, his payoff will be 4. Therefore, bidder C wins the first object and the entire game has at least three equilibrium allocations given by: E_1 = (C, B, A, A) with vector of prices (13; 13; 13; 13); E_2 = (C, B, B, A) with vector of prices (13; 13; 13; 11) and E_3 = (C, A, B, A) with vector of prices (13; 13; 13; 13). All three equilibria are characterized by a single vector of net payoffs: (9; 5; 4), and E_1 and E_3 characterize the same efficient allocation. Starting at the second object, figure 2.6 illustrates the three equilibrium paths.

However, if bidder B wins the second object and pays 13 at node N_2 (i.e., the vector of net payoffs is (9, 18, 6)), bidder C is better off bidding less than 13 at node N_0. Bidder C is willing to pay as much as 13 to counter bidder A but only 11 to counter bidder B. It is a dominant strategy for bidders A and B to bid 13. If bidder C bids less than 13, his payoff will be 5 because his payoff is 4 if bidder A wins and it is 6 if bidder B wins and, bidders A and B win with probability ½. If bidder C bids 13, his payoff will be 14/3 and if he bids more than 13, his payoff will be 4. Therefore, bidder C bids less than 13 and either bidder A or B wins the first object and pays 13. Consequently, the entire game has three other potential equilibrium paths given by: E_4 = (A, B, C, A) with vector of prices (13; 13; 13; 13); E_5 = (A, C, B, A) with vector of prices (13; 13; 13; 13) and E_6 = (B, B, C, A) with vector of prices (13; 13; 11; 11). The vector of net payoffs is (9; 5; 4) in E_4 and E_5 the same as for E_1-E_3 while it is (9; 5; 6) in E_6. Bidder C hopes that bidder B will win the first object neither bidder B nor bidder A has an incentive to coordinate on E_6.

In summary, we found five equilibrium paths with a single vector of bidders' net payoffs (9; 5; 4). Four of these have the same vector of prices (13; 13; 13; 13) and the same allocation. As such, these paths define the same equilibrium. The fifth equilibrium path yielding net payoffs (9; 5; 4) is inefficient and produce weakly declining prices (13; 13; 13; 11). There exists a sixth equilibrium path with vector of prices (13; 13; 11; 11) that leads to a different vector of bidders' net payoffs (9; 5; 6). Thus, we found three distinct equilibrium allocations with different price vectors.

Proposition. *In multi-unit demand second-price sequential auction under complete information, there can be more than one pure strategy Nash perfect equilibrium.*

Thus, the uniqueness property in Katzman (1999) 2-bidder and 2-object sequential auction and in Gale and Stegeman (2001) 2-bidder and *n* object auctions does not generalize.[13] However, as in Katzman (1999), an inefficient equilibrium can emerge. Given that in our example, an efficient allocation can also emerge, the inefficient allocation must entail a declining price pattern.

We could have used an example with more than three bidders and four objects to show multiplicity of equilibria. The intuition is that some bidders may be strategic in some subgames and nonstrategic in others because of asymmetric valuations. Thus, price vectors may be different along different equilibrium paths.

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[13] Cai et al. (2007) show that a pure strategic symmetric equilibrium does not exist in sequential auctions in which all bids are revealed after each auction and bidders have single-unit demand.

2.2.3. Conclusion

We analyze multi-unit demand sequential second-price auction under complete information with asymmetric bidders. We rely on a three bidder – four object example to show that the result about equilibrium uniqueness in the case of two bidders and *n* objects (e.g., Gale and Stegeman, 2001) is not robust. The implication is that different allocations may be observed in frequently repeated auctions involving the same bidders even if their valuations do not change. Casual empirical evidence from the Quebec daily hog auctions between February 1st of 2006 and August 31th of 2006 supports this hypothesis. The coefficient of variation for U.S. hog price over this period is 0.09. Given that the Canadian and US markets are highly integrated, the US price is a proxy for the variability of the market. The relative stability of the market over this short period suggests that processors' valuations probably did not change much. Yet, the coefficient of variation of the Herfindahl index, which captures changes in the allocations on the auction, is 0.23. This evidence does not constitute a formal test, but it is consistent with multiplicity of equilibrium allocations in the daily sequential auctions.

2.2.4. References

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Table 2.1. The payoff matrix for the second object at node N_7 given bidder B's dominant strategy is to bid 13, $\forall\varepsilon$ and ε' such that $.5 < \varepsilon \le 1$ and $.5 < \varepsilon' \le 1$.

		Bidder C		
Bidder A		14	14.5	$14+\varepsilon'$
	13	(22; 18; 4)	(22; 18; 4)	(22; 18; 4)
	13.5	(22; 18; 3.5)	(22; 18; 3.5)	(22; 18; 3.5)
	14	(21.5; 18; 3)	(22; 18; 3)	(22; 18; 3)
	14.5	(21; 18; 3)	(21.25; 18; 2.75)	(22; 18; 2.5)
	$14+\varepsilon$	(21; 18; 3)	(20.5; 18; 3)	$(21-\varepsilon'; 18; 3)\ if\ \varepsilon > \varepsilon'$ $(21; 18; 3-\varepsilon)\ if\ \varepsilon' > \varepsilon$ $(21.5-\frac{\varepsilon'}{2}; 18; 3-\frac{\varepsilon}{2})\ if\ \varepsilon = \varepsilon'$

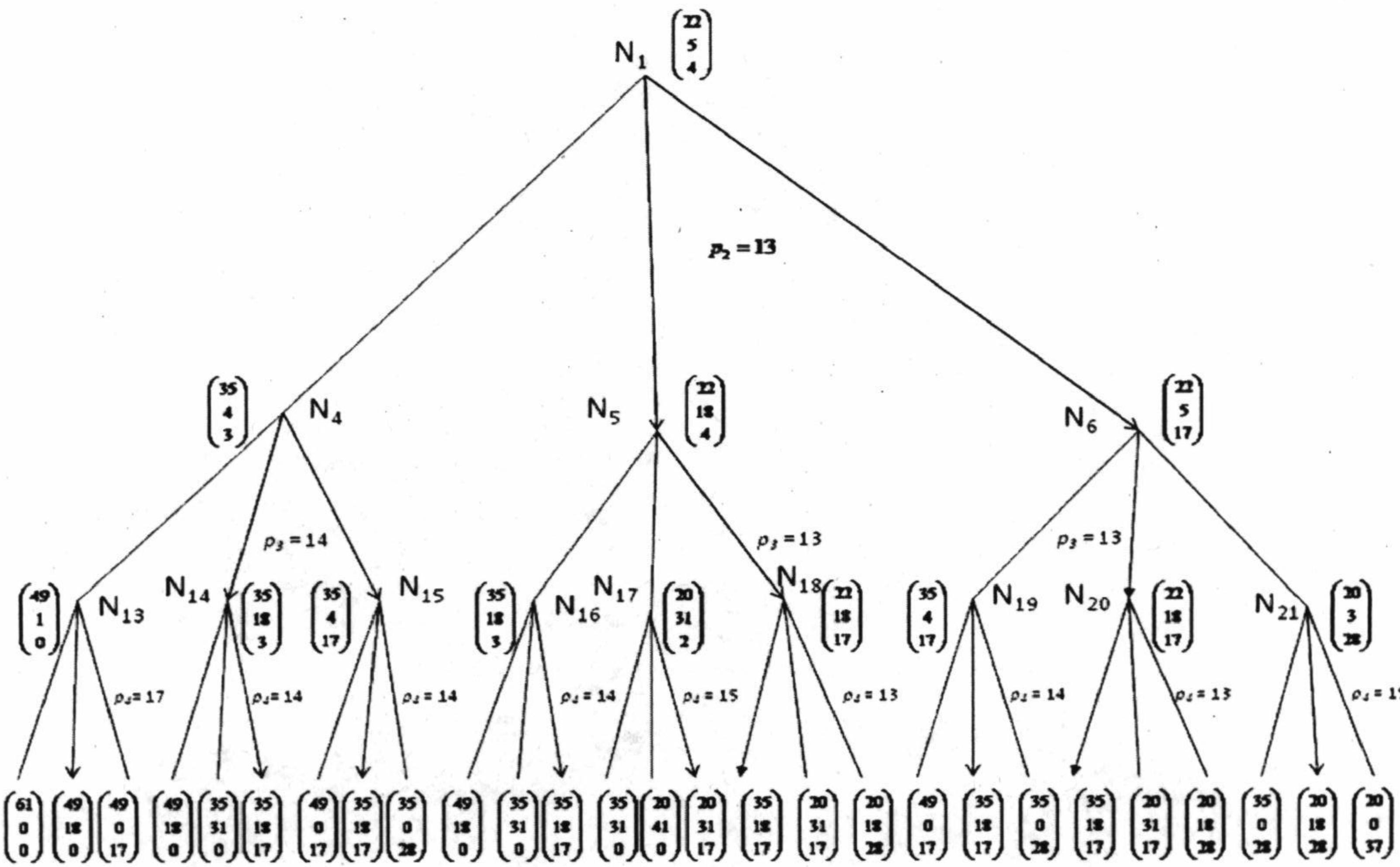

Figure 2.4. The outcome tree at node N_1.

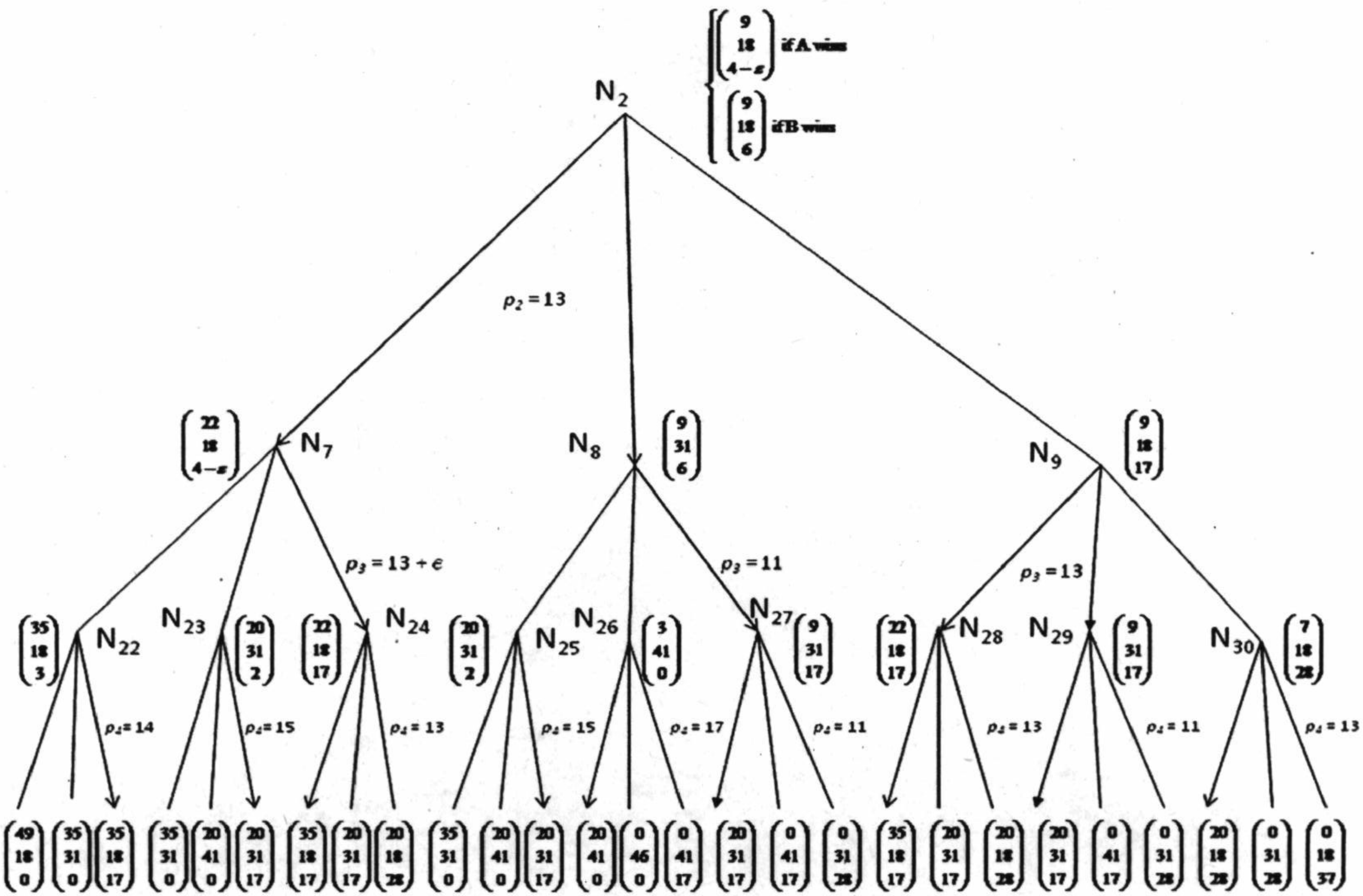

Figure 2.5. The outcome tree at node N_2.

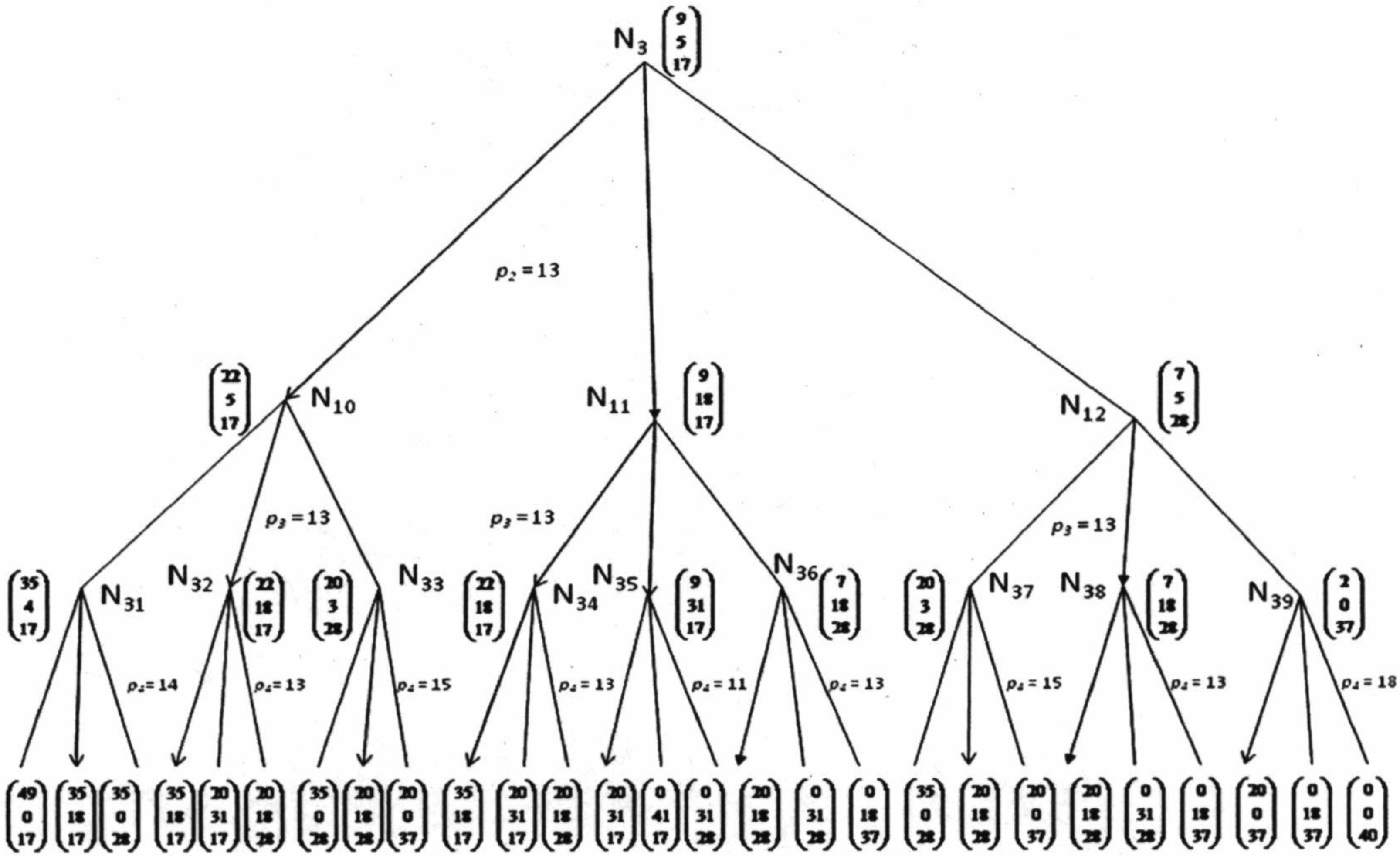

Figure 2.6. The outcome tree at node N_3.

3. More Bidders May not Increase the Seller's Revenue: The Case of Quebec Sequential Hog Auctions

Résumé

Ce chapitre montre que le revenu du vendeur augmente avec le nombre de d'enchérisseurs symétriques, mais ce n'est pas nécessairement le cas lorsque les enchérisseurs sont asymétriques. Notre analyse empirique montre une diminution du revenu du vendeur lorsque des enchérisseurs additionnels de l'Ontario étaient invités à participer. Cependant, le modèle avec correction du biais de sélection de Heckman suggère que cette baisse serait plus élevée en l'absence de soumissionnaires de l'Ontario.

Abstract

This chapter shows that the seller's revenue increases with the number of symmetric bidders but this is not necessarily the case when bidders are asymmetric. Our empirical evidence finds that the seller's revenue significantly decreases with the number of invited bidders from Ontario. However, the model with Heckman's selection bias correction suggests that this decrease would be higher in the absence of bidders from Ontario.

3.1. Introduction

In the auction literature, it is well known that increasing the number of bidders in a unit-demand auction with independent private values increases the seller's expected revenue (e.g., Krishna, 2002, p. 19 and Pinkse and Tan, 2005). Ooi et al. (2006) examine the price formation process in land auctions with a small number of bidders. Intuitively, the probability of winning for each bidder decreases when the number of bidders increases and as a result bidders respond by increasing their bids. The empirical evidence supports this relation. Krishna (2002, p. 219) shows that the expected revenue is increasing with the number of bidders in sequential second-price auctions with stochastically equivalent bidders such that their valuations are uniformly distributed on the same interval. Similarly, Mishra et al. (2005) show that in sequential auctions with synergies and asymmetric bidders, realized prices are increasing in the number of bidders. The same result is obtained by Kittsteiner et al. (2004) who consider sequential unit-demand auctions with independent private values where bidders' valuations for objects are decreasing with rounds.

The expected seller's revenue may also be decreasing in the number of bidders. In single-unit auction mechanisms, Matthews (1984) showed that in first-price auctions with common values and winner's curse, the more bidders there are, the lower their bids because the impact of the increasing winner's curse gets stronger as the number of bidders increases.[14] In first-price auction with symmetric and affiliated private values, some bidders may decrease their bids because they realize that competition is weaker than they expected (Menicucci, 2009). Furthermore, in multi-unit demand auctions, Elmaghraby (2005) analyzed two successive second-price procurement auctions in the presence of bidders with asymmetric capacity production and economies of scale. Global bidders are able to supply both units while small bidders can supply only one unit due to capacity constraints. She showed that under some circumstances, more bidders does not translate into lower procurement cost. The buyer should exclude local bidders and invite more global bidders to participate because the latter always inflate their bids in the presence of small bidders. In

[14] This result is also obtained in an almost common values auction for a single object or multiple units (Bulow and Klemperer, 2002).

two successive second-price procurement auctions involving bidders with asymmetric capacities (i.e., small bidders are only able to bid for the second object), Rong and Zhi-xue (2007) showed that inviting small bidders can reduce the average expected procurement cost, the increase in the number of small bidders may also increase procurement cost because large suppliers have incentive to inflate their bids in the first round which depend on their expected payoff in the second round.

Under complete information, the seller's revenue may decrease and even be zero in Vickrey-Clarke-Groves (VCG) auctions when the number of bidders is increased (e.g., Ausubel and Milgrom, 2002; Milgrom, 2004). In unit demand auction, the VCG auction is similar to a second-price sealed bid auction. However, in multi-unit demand auctions under complete information, the VCG auction mechanism is different from sequential second-price auctions for two reasons. First, VCG auctions are combinatorial auctions. Each bidder submits his sealed bids for all of the objects and payments are determined so as to allow each bidder a payoff equal to his opportunity cost for the units won. Sequential second-price auctions are sequences of unit demand auctions. In each sequence, bidders submit their willingness to pay and prices are the highest losing bids. Second, in VCG auctions, the outcome is always efficient, while in sequential second-price auctions, this is not so (Katzman, 1999).

Although the literature on multi-unit demand sequential auctions under complete information (e.g., Katzman, 1999; Gale and Stegeman, 2001; Rodriguez, 2009 and Jeddy, Larue and Gervais, 2010 and 2011) is growing, the analysis of the effect of the number of bidders on the seller's revenue in such auctions has not been undertaken. We fill this void by developing a simple model of multi-unit sequential auctions under complete information with three bidders and three objects. The seller is presumably poorly informed while each bidder is completely informed, not only about the item(s) being auctioned, but also about his own valuations and that of his competitors (e.g., Bernheim and Whinston, 1986; Gale and Stegeman, 2001 and Jeddy et al. 2011). We show that in the case where bidders are symmetric the seller's revenue is always increasing with the number of bidders. Bidders with identical declining valuations and symmetric strategies can be interchanged and the increased competition between them increases prices because they tend to report their valuations more truthfully. However, when they are asymmetric and depending on the level of the highest valuation of the additional strategic bidder, the seller's revenue may not increase with the number of bidders. From Katzman's (1999) 2-object auctions, we know that prices depend on the three or the four highest valuations, depending on cases. As it was shown in Jeddy et al. (2011), when only two objects are sold, a third bidder matters only if he is strategic (i.e., if his highest valuation is among the top four). In 3-object 3-bidder auctions, a third bidder is strategic only if his highest valuation is among the top six valuations. In this case, the addition of a third bidder can influence prices but not necessarily in the expected positive way. The competition between bidders is increased if the highest valuation of the additional bidder is among the first three highest valuations. In this case, bidders bid aggressively and equilibrium prices increase. However, if the new bidder's highest valuation is between the fourth and sixth ranked valuations and if one of the other two bidders has the three highest valuations, this bidder may elect to win the second and third objects and let one of his rivals win the first object to generate lower prices. The empirical pertinence of these theoretical insights is assessed using data from the Quebec daily hog auction.[15] The Quebec daily hog auction was used alongside other marketing mechanisms. The percentage of Quebec's hog supply marketed through the daily auction varied over time and was one of the elements of the contract negotiated between the FPPQ, the union representative hog producers, and handful of processors.

Our sample contains data of Quebec' daily hog auction prices in February, May and September of each year between 1995 and 2006, U.S. daily hog prices, identity of winning bidders, the predetermined daily quantity of hogs to be sold on the daily auction and the presence of outside bidders. Our empirical evidence shows that the equilibrium average price was significantly decreasing with the number of invited bidders. However, the model with Heckman's selection bias correction suggests that this decrease would be higher in the absence of bidders from Ontario.

This paper is structured as follows. The next section and section III deal with our formal results in the cases where bidders are symmetric and asymmetry, respectively. Section IV

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[15] We are especially fortunate that the FPPQ has graciously accepted to grant us access to part of its auction data. This is indeed the first time that such data is being analyzed for academic purposes.

tests these results by using empirical evidence from the Quebec hog auctions. Some conclusions are contained in section V. The last section provides the appendices of the proofs.

3.2. Sequential Auctions with Symmetric Bidders

The analysis of sequential auctions under complete information with symmetric bidders has been largely ignored in the literature. Krishna (1999) analyzes sequential auctions with two symmetric bidders and endogenous valuations. When additional scarce resources become available and sold sequentially, the author shows that convexity of payoffs in the final stage of the auction game results from one bidder snowballing into a dominant firm. Particularly, she designed a 2-bidder numerical example to show the existence of two symmetric equilibria characterized by an asymmetric allocation and weakly declining prices. Jeddy et al. (2010) generalized these insights by showing that asymmetric allocations arise in auctions with completely informed bidders with identical and declining valuations except under specific conditions about the decline of valuations. The authors gave the necessary conditions supporting symmetric allocations that become increasingly restrictive as the number of objects increases. In this section, we examine the effect of an increase in the number of bidders on the seller's revenue. Our analysis begins with two symmetric bidders playing three second-price auctions under complete information. Then, we introduce a third bidder and determine how the seller's revenue may be affected. Two cases emerge depending on whether this additional bidder has similar or different valuations compared with the two initial symmetric bidders.

The auction is a sequential second-price one involving two completely informed bidders with identical decreasing marginal valuations. Formally, we have: $H = H_1 = H_2 > M = M_1 = M_2 > L = L_1 = L_2$, where H_i, M_i and L_i is the bidder *i*'s valuation for the first, second and third object won, respectively. The additional bidder is throughout denoted by bidder #3. Each bidder is assumed to follow the weakly dominant strategy of sincere bidding in the third round. It is a weekly dominant strategy for each bidder to place a bid equal to the second (resp. first) round price that would make them indifferent between winning and losing the second (resp. first) round. In figure 3.1, arrows denote the allocation in each subgame and prices are given next to the paths. At each node, the bidders' gross payoffs are put in parentheses. Each unit could go either to bidder #1 (left branch), to bidder #2 (middle branch) or to bidder #3 (right branch). The equilibrium outcome is solved by backward induction and bids reflect the opportunity cost of not winning. The outcome tree, unlike the extensive form, features gross payoffs at every node which are obtained through subgame replacement. At nodes associated to the j^{th} object, gross payoffs are defined as the sum of valuations for objects won along the given path minus the sum of prices for objects that would be won among the last *n-j*+1 objects. For the last object, gross payoffs are the sum of the valuations.

Figures 3.1 and 3.2 depict examples of the auction outcome tree for cases with two and three symmetric bidders, respectively. The bidders' valuations are given by: $H = 20 > M = 12 > L = 5$. The seller's revenue increases with the addition of bidder #3, from 22 to 36.

Starting at the bottom of the tree of figure 3.1, we can see that the vector of gross payoffs when bidder #1 wins all three objects is (37; 0), which is simply the sum of the valuations for the objects won by the bidders. Provided the first two objects are won by bidder #1, the third object may be won by bidder # 1 or bidder #2 and the vectors of gross payoffs are (37; 0) and (32; 20), respectively. It follows that at node 2, the third object is worth at most 5 (37 – 32 = 5) for bidder #1 and at most 20 (20 - 0 = 20) for bidder #2. If the game were to reach node 3, the third object would be won by either bidder #1 or bidder #2 at price 12. Therefore, the gross payoff at this node is (32– p_3; 20) = (20; 20). The gross payoffs are (32; 15) and (20; 20) at node 1. It follows that the second object is worth at most 12 for bidder #1 and at most 5 for bidder #2. Thus, bidder #1 wins the second object and pays 5. Because bidders are symmetric, at node N_0 the first object would be won by either bidder #1 or #2 at price 12. Therefore the seller's revenue is equal to 22.

Starting at the bottom of the tree of figure 3.2, we can see that the vector of gross payoffs when bidder #1 wins all three objects is (37; 0; 0), which is simply the sum of the valuations for the objects won by the bidders. Provided the first two objects are won by bidder #1, the third object may be won by bidder # 1, bidder #2 or bidder #3. In these cases, the vectors of gross payoffs are (37; 0; 0); (32; 20; 0) and (32; 0; 20), respectively. It follows that at node 2, the third object is worth at most 5 (37 – 32 = 5) for bidder #1, at most 20 (20 - 0 = 20) for bidder #2 and at most 20 (20 - 0 = 20) for bidder #3. If the game were to reach node 2, the third object would be won by bidder #1 at price 20. Therefore, the gross payoff at node 2 is (32; 20 – p_3; 20 – p_3) = (32; 0; 0). The third object would be won by either bidder #2 or #3 at price p_3 = 20. The same reasoning could be used at nodes 3 and 4. It is easy to verify that if the game reaches these nodes the last object would be won by bidder #3 at node 3 and by bidder #2 at node 4 at the same price 12. The vectors of gross payoffs at these nodes are respectively (32; 0; 0); (20; 20; 8); and (20; 8; 20). At N_0, it is a dominant strategy for each bidder to pay up to 12 to prevent the other bidders from acquiring the first object. Therefore, by symmetry the first object is won by one of the bidders at price 12. Consequently, the seller's revenue is equal to 36.

The presence of an additional bidder whose valuations are similar to the initial bidders gives incentives to each bidder to bid more aggressively. This result is consistent with the classical result of sequential second-price auctions under incomplete information with stochastically equivalent bidders such that their valuations are uniformly distributed on the same interval and the expected revenue is increasing with the number of bidders (e.g., Krishna, 2002, p. 219).

The following proposition summarizes the results.

PROPOSITION 1. *Consider a three successive second-price auctions under complete information. If the benchmark auction involves two identical bidders, then the seller's revenue cannot fall if a third bidder is introduced.*

PROOF. See appendix 1.

3.3. Sequential Auctions with Asymmetric Bidders

In the literature of multi-unit demand auctions, the seller's revenue increases with the number of bidders (e.g., Krishna 2002, p. 219). In this section, we show that this is not the case.

In complete information second-price auctions with n objects and k bidders such that $n < k$ (resp. $k \leq n$), Jeddy et al. (2011) show that a bidder i is strategic if and only if its highest valuation is among the (n+2) (resp. (2n)) highest valuations. These results are useful because if the additional bidder is not strategic, the seller's revenue will not be affected by the presence or absence of this bidder. If he is strategic he can win one object or more objects or none and the seller's revenue may decrease with the number of bidders.

Figure 3.3 illustrates the outcome tree of the auction game played between bidder #1 and bidder #2 while figure 3.4 depicts the outcome tree of the game with the presence of an additional bidder #3 such that the bidders' valuations are ranked as follows:

$$H_1 = 21 > M_1 = 20 > L_1 = 15 > H_3 = 12.7 > H_2 = 12.6 > M_2 = 12.5 > M_3 = 10 > L_3 = 5 > L_2 = 3.95$$

We find that the seller's revenue decreases, from $R(2)$ = 38.85 to $R(3)$ = 38.1. In this game, bidder #3 does not win a single object, but his participation decreases the seller's revenue.

Starting at the bottom of the tree of figure 3.4, we can see that the vector of gross payoffs when bidder #1 wins all three objects is (56; 0; 0), which is simply the sum of the valuations for the objects won by the bidders. Provided the first two objects are won by bidder #1, the third object may be won by bidder # 1, bidder #2 or bidder #3. In these cases, the vectors of gross payoffs are (56; 0; 0); (41; 12.6; 0) and (41; 0; 12.7) respectively. It follows that at node 4, the third object is worth at most 15 (56 – 41 = 15) for bidder #1, at most 12.6 (12.6 - 0 = 12.6) for bidder #2 and at most 7 (12.7 - 0 = 12.7) for bidder #3. If the game were to reach node 4, the third object would be won by bidder #1 at price 12.7. Therefore, the gross payoff at node 4 is (56 - p_3; 0; 0) = (43.3; 0; 0). The same reasoning could be used at nodes 5-12. It is easy to verify that if the game reaches N_5-N_{12} that the last object would be won by bidder #1 at prices equal to 12.7; 12.6; 12.7; 12.7; 10; 12.6; 10 and 12.6, respectively. The vectors of gross payoffs at these nodes are respectively (28.3; 12.6; 0); (28.4; 0; 12.7); (28.3; 12.6; 0); (8.3; 25.1; 0); (11; 12.6; 12.7); (28.4; 0; 12.7); (11; 12.6; 12.7) and (8.4; 0; 22.7). It follows that the vectors of gross payoffs at nodes 1-3 are respectively (30.6; 0; 0); (15.6; 12.6; 0) and (15.8; 0; 12.7). At N_0, it is a dominant strategy for bidder #2 (#3) to pay up to 12.6 (12.7) to prevent the other bidders from acquiring the first object. Bidder #1 is willing to pay as much as 15 to counter bidder #2 and only 14.8 to counter bidder #3. Given that the bid of bidder #2 is the minimum bid, the game is played between bidder #1 and bidder #3. Thus, bidder #1 wins the first object and pays 12.7 (the bid of bidder #3).

From Katzman's (1999) 2-object auctions, we know that prices depend on the three or the four highest valuations, depending on cases. Intuitively, with only two objects to be sold, a third bidder matters, or is pivotal, only if at least one of his valuations is among the top four. In 3-object auctions, if one or more valuations of a third bidder make up the top six, then the addition of a 3rd bidder will likely influence prices and the seller's revenue. Otherwise, this additional bidder is nonstrategic. Proposition 2 provides necessary conditions about the increase and decrease of the seller's revenue when a third bidder is introduced. Particularly, this proposition deals with the case where the introduction of a third bidder decreases the seller's revenue even when the third bidder does not win a single object. This situation is similar to contestable markets which are characterized by "hit and run" competition (Baumol et al., 1982).

PROPOSITION 2. *In three sequential second-price auctions with two asymmetric bidders, the addition of a third bidder*

1) *increases the seller's revenue if the highest valuation of bidder #3 is among the top three.*
2) *increases or decreases the seller's revenue if the highest valuation of bidder #3 is between the fourth and sixth highest valuations.*

PROOF. See appendix 2.

The seller's revenue always increases with the addition of a new bidder when the highest valuation of the additional bidder is among the three highest valuations. As shown in cases 2a and 2b of appendix 2, the bidder who has the three highest valuations may exploit his rivals' sufficiently low valuations for the second and third objects by choosing to win these objects and letting his rivals compete for the first object (*allocation* effect). In equilibrium, the asymmetry between bidders makes each bid being composed by the sum of two opposing effects—the *competitive* effect and the *allocation* effect. The competitive effect emerges when the introduction of the third bidder increases the competitiveness between all bidders and then at least the equilibrium price of the first object increases. There are situations where the allocation effect dominates as the number of bidders increases. In our case, independently on if the additional bidder wins (proposition 2) or not (proposition 3), the allocation effect may dominate such that the sum of prices is decreased after the addition of the third bidder. Indeed, in the 2-bidder auction game, the declining price trend implies a price of the first object greater than the second and the third prices which they are in general equal (allocation effect). In the 3-bidder auction game, the constant price trend may lead to an equilibrium price greater or lesser than the price of the second and the third object price in the 2-bidder auction. In the latter case, it is evident that the seller's revenue decreases with the introduction of the third bidder while in the former this is so if the price of the first object is sufficiently higher in the 2-bidder auction game. However, in the case where this price is not higher enough, the seller's revenue increases with the introduction of the third bidder. This situation is an example of competitive effect dominance.

Figure 3.5 illustrates the outcome tree of an auction game example played between bidder #1 and bidder #2 while figure 3.6 depicts the outcome tree of the game with the presence of an additional bidder #3 such that: $H_1 = 20 > M_1 = 17 > L_1 = 16.5 > H_3 = 15 > H_2 = 14.3 > M_3 = 14.2 > L_3 = 14.1 > M_2 = 13.5 > L_2 = 9.9$. In this case, bidder #3 wins an object and the seller's revenue decreases, from $R(2) = 43$ to $R(3) = 42.9$.

PROPOSITION 3. *In three sequential second-price auctions with asymmetric bidders, under complete information, the addition of a third bidder can increase or decrease the seller's revenue even if the third bidder wins an object.*

PROOF. See appendix 3.

This proposition confirms that our analysis of the additional bidder effect on three sequential second-price auctions under complete information with two asymmetric bidders is sufficient to know how the increase in the number of bidders affects the seller's revenue because bidder #3 can win and the seller's revenue decreases.

As it is mentioned in the introduction, this result contrasts the presumption that the seller's revenue must increase with the number of bidders which finds support in several studies pertaining to both unit and multi-unit auctions (e.g., Klemperer, 1999 and Krishna, 2002). Under complete information, an exception is the VCG auction where the sellers' revenue is non-monotonic and objects are non substitutes.

The lesson we can learn from this result is that in an auction design perspective, the seller must be conscious about the failure he can have when entry of new bidders is allowed. In Quebec hog auctions, the *Fédération des Producteurs de Porc du Québec* (FPPQ) has on rare occasions invited an outside bidder from Ontario to participate. The intent of the FPPQ was to send a signal to regular bidders that the FPPQ was expecting more aggressive bidding. However, as our proposition 2 and 3 show, this may be counterproductive.

3.4. Empirical Evidence from the Quebec Hog Auctions

In the Quebec hog market, the additional bidder is an invited bidder from Ontario who has already used his highest valuations to buy a greater part of his desired quantity of hogs in Ontario. Thus, he seemingly participates in the Quebec hog auctions with low valuations. The number of local bidders is few (seven) who compete in the same domestic and foreign output markets as well as on the sequential auctions every day except on weekends. They sell to a few large distributors/retailers on the domestic market and face competition from many foreign firms on export markets. Accordingly, they should possess declining marginal valuations for hogs and ought to have precise estimates of what these valuations are. As such, we contend that it is reasonable to use a complete information framework. Our justification is very close to the one invoked by Bernheim and Whinston (1986) and Jeddy et al. (2011). Bidders may have reliable information about each other's costs, capacity and market opportunities.

Therefore, data of this market is useful to test our theoretical results about how the increase of the number of bidders can affect the seller's revenue. As our propositions 2 and 3 show, when the FPPQ invites an outside bidder from Ontario to participate may increase or decrease the seller's revenue. Our sample contains data from Quebec hog auction prices in February, May and September of each year between 1995 and 2006, U.S. daily hog prices, identity of winning bidders, identity of bidders who bid minimum prices, the available daily quantity of hogs sold at auctions and the identity of non local bidders.

We first consider the following regression equation (I):

$$\begin{aligned} aucprice_i &= \beta_1 + \beta_2 usprice_i + \beta_3 aucquantity_i + \beta_4 Dinvited_i + \beta_6 DMay + \beta_5 DAugust \\ &+ Dyear1995 + Dyear1996 + Dyear1998 + Dyear2000 + Dyear2002 + Dyear2003 + \\ &Dyear2005 + Dyear2006 + \varepsilon_i ; i = 1,...,444. \end{aligned}$$

where $aucprice_t$ is the average auction price at day i (i = 1,...,444); β_1 is the intercept term; $usprice_i$ is the U.S. daily price at day i; $aucquantity_i$ is the daily quantity available at day i; *Dinvited* is the dummy variable which equal to 1 if there is an invited bidder and 0 otherwise; *DMay* is the dummy variable which is equal to 1 if the month is May and 0 otherwise; *DAugust* is the dummy variable which is equal to 1 if the month is August and 0 otherwise and $Dyear_s$ is the dummy variable which is equal to 1 if the year is s and 0 otherwise. Note that, some years from our sample are omitted because of the absence of invited bidders in these years. Table 2.1 summarizes the results.

From this table, we see that the coefficient of the dummy variable of the number of invited bidders is significantly negative (-12.720). However, this does not mean that the equilibrium average price or the seller's revenue decreases with the number of invited bidders. We do not know if in the absence of invited bidders, this price decreases more or not. Hence, the results will tend to be biased (sample selection bias). We use the two stage estimation method of Heckman (1979) to correct the bias.[16] We model the program

[16] For more details, see for example (Greene, 2003, p. 780).

participation as follows: $Dinvited_i^* = w_i^{'}\gamma + u_i$ such that $Dinvited_i = 1$ if $Dinvited_i^* > 0$ and 0 otherwise. We have: $\text{Prob}(Dinvited_i = 1 \mid w_i) = \Phi(w_i^{'}\gamma)$ and $\text{Prob}(Dinvited_i = 0 \mid w_i) = 1 - \Phi(w_i^{'}\gamma)$

and, (u_i, ε_i) □ bivariate normal $[0, 0, 1, \sigma_\varepsilon, \rho]$.

First, we estimate the probit equation by the maximum likelihood to obtain estimates of γ. The dependent variable is *Dinvited* and the independent variables, *X*, are the same as in the equation (I) and $aucprice_{i-1}$; $aucprice_{i-2}$; $aucprice_{i-3}$; $aucprice_{i-4}$ and $aucprice_{i-5}$. For each observation in the selected sample, we compute $\hat{\lambda}_i = \varphi(w_i^{'}\hat{\gamma}) / \Phi(w_i^{'}\hat{\gamma})$ and $\hat{\delta}_i = \hat{\lambda}_i(\hat{\lambda}_i - w_i^{'}\hat{\gamma})$.

$$E(y_i / X, Dinvited_i = 1, W) = x'_i \beta + \delta + E(\varepsilon_i / X, Dinvited_i = 1, W)$$

$$= x'_i \beta + \underbrace{\delta}_{ET=9.02} + \underbrace{\rho\sigma_\varepsilon \frac{\varphi(w'_i \gamma)}{\Phi(w'_i \gamma)}}_{ES=-18.92}$$

$$E(y_i / X, Dinvited_i = 0, W) = x'_i \beta - \rho\sigma_\varepsilon \frac{\varphi(w'_i \gamma)}{(1-\Phi(w'_i \gamma))}$$

Second, we estimate equation (I) where $\hat{\lambda}$ is included as an independent variable. We obtain. Table 2.2 reports the results. Since the coefficient of invmills variable (inverse Mills ratio) is positively significant ($-\rho\sigma_\varepsilon$ = 9.02) then in the same period the price decreases by 18.92 from its habitual level. Therefore, we invite Ontarian players to avoid the decrease of the average auction price that would be higher in their absence.

Despite our theoretical result about the increase of the seller's revenue with the number of symmetric and sometimes asymmetric bidders in sequential second-price auctions under complete information, the FPPQ must be careful in inviting an outside bidder from Ontario to participate.

3.5. Conclusion

The theoretical contribution of this paper is an important result that can be added to the literature about how the seller's revenue may increase or decrease with the number of bidders in multi-unit demand auctions. Both cases about symmetry and asymmetry between bidders are discussed. In the case where bidders are symmetric, the seller's revenue increases with the number of bidders. However, we have shown that the magnitude of asymmetry between bidders' marginal valuations is crucial to know if the seller is better off in inviting new bidders to increase his expected revenue. Our first empirical model shows that in Quebec daily hog auctions, the seller's revenue decreases significantly by the introduction of invited bidders from Ontario. However, the model with Heckman's selection bias correction suggests that this decrease would be higher in the absence of bidders from Ontario.

3.6. References

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Table 3.1. The variation of average daily auction price with the number bidders.

	Model 1	
	coefficient	P-value
usprice	.3411	0
quantity	-.0037	0
Dinvited	-12.720	0
DMay	-8.007	0
DAugust	-18.14	0
Dyear1995	26.356	0
Dyear1998	34.66	0
Dyear2000	35.47	0
Dyear2002	36.38	0
Dyear2003	13.35	0
Dyear2005	29.43	0
Constant	112.1	0

Table 3.2. The Probit model and the regression with Heckman's selection bias correction.

	Probit Model		Regression Model	
	coefficient	P-value	coefficient	P-value
usprice	.01	.310	.33	0
quantity	-.00004	.685	-.00331	0
DMay	.	.	-28.46	0
DAugust	5.24	0	-38.25	0
Dyear1995	.97	.202	28.67	0
Dyear1998	1.74	.001	38.84	0
Dyear2000	.56	.474	34.92	0
Dyear2002	1.73	.001	41.07	0
Dyear2003	.227	.662	13.23	0
Dyear2005	.321	.625	28.63	0
invmills	.	.	-10.23	0
x1	-.027	.001	.	.
D_invited	.	.	9.023	.074
Constant	-4.63	.013	109.98	0

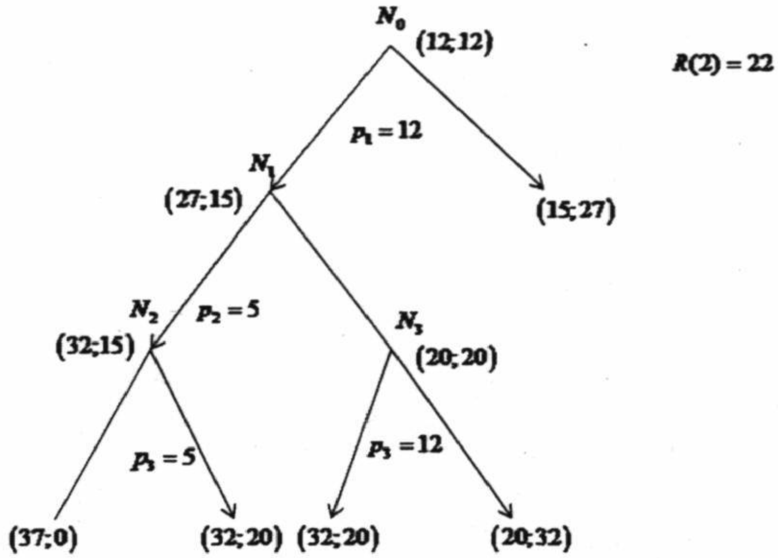

Figure 3.1. The 2-symmetric bidders game with three objects

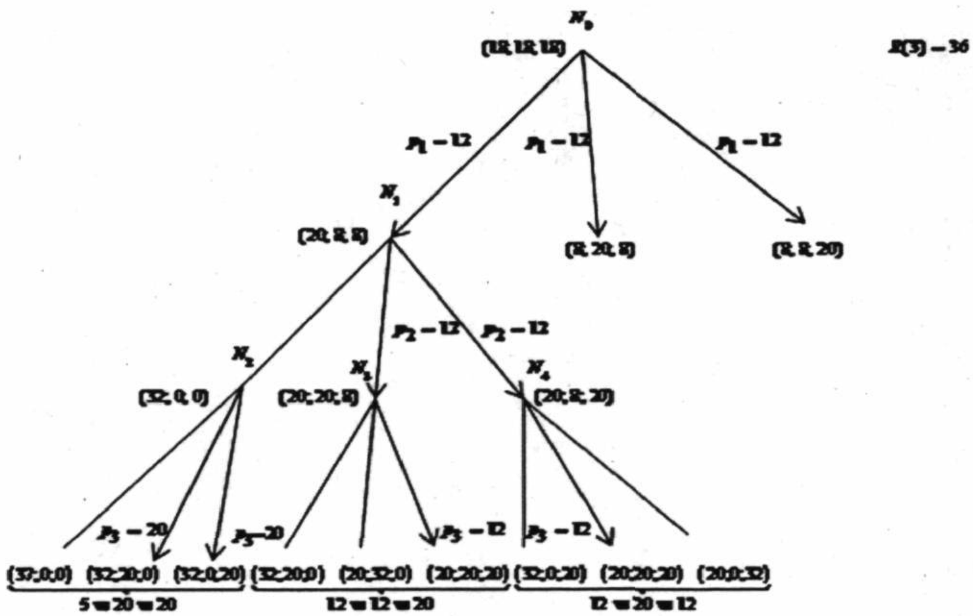

Figure 3.2. Example 1 of 3-symmetric bidders game with three objects.

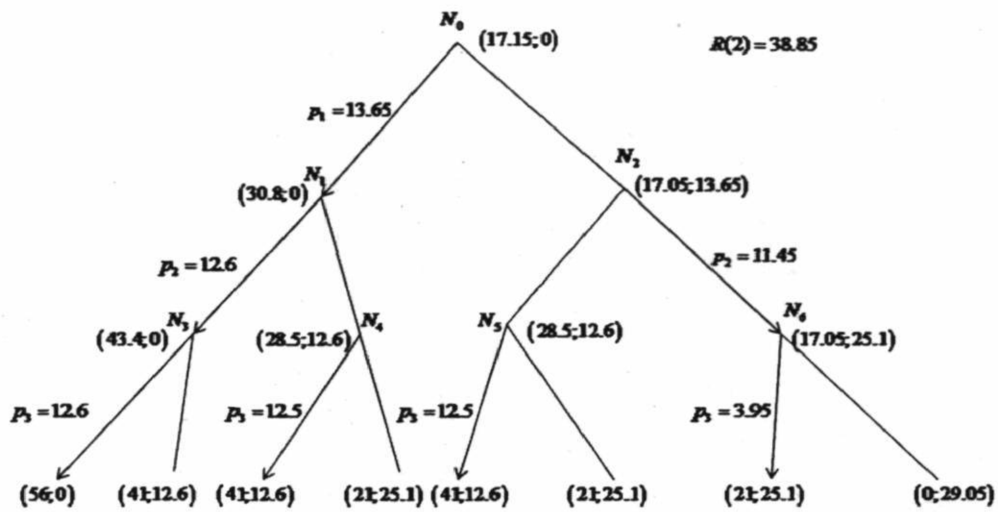

Figure 3.3. The 2-asymmetric bidders game with three objects.

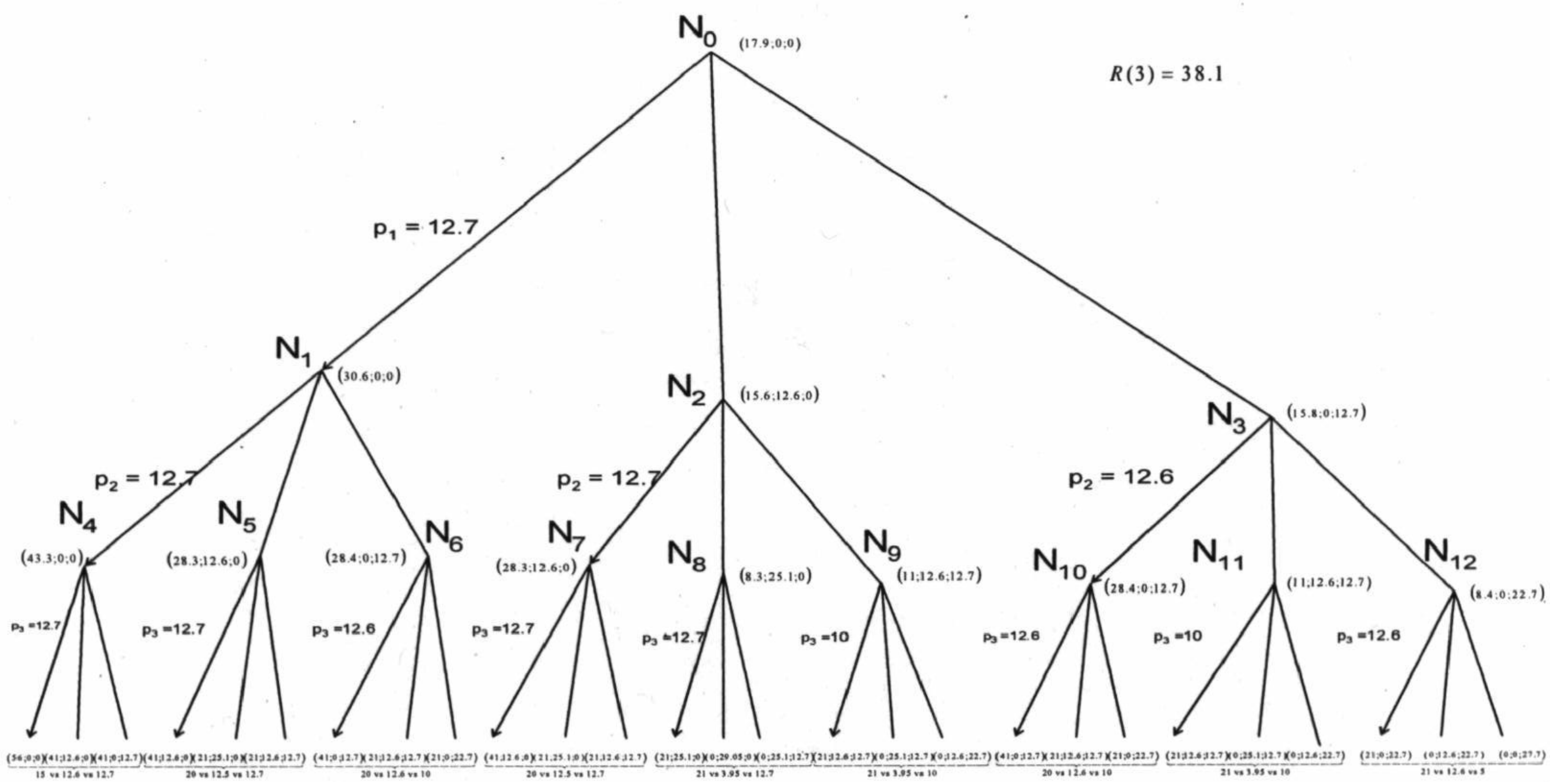

Figure 3.4. The seller's revenue decreases with the number of bidders.

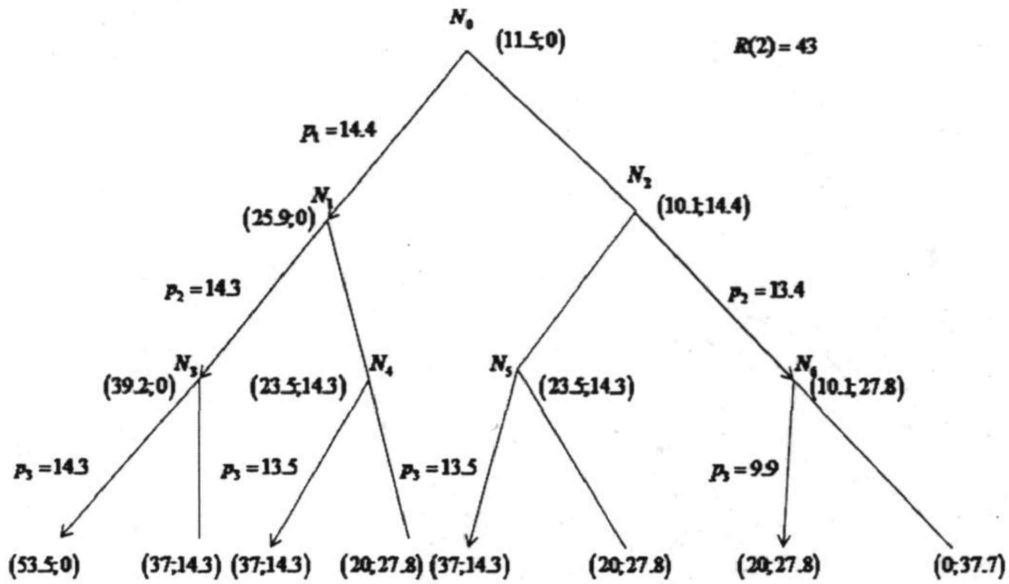

Figure 3.5. The 2-asymmetric bidders game with three objects.

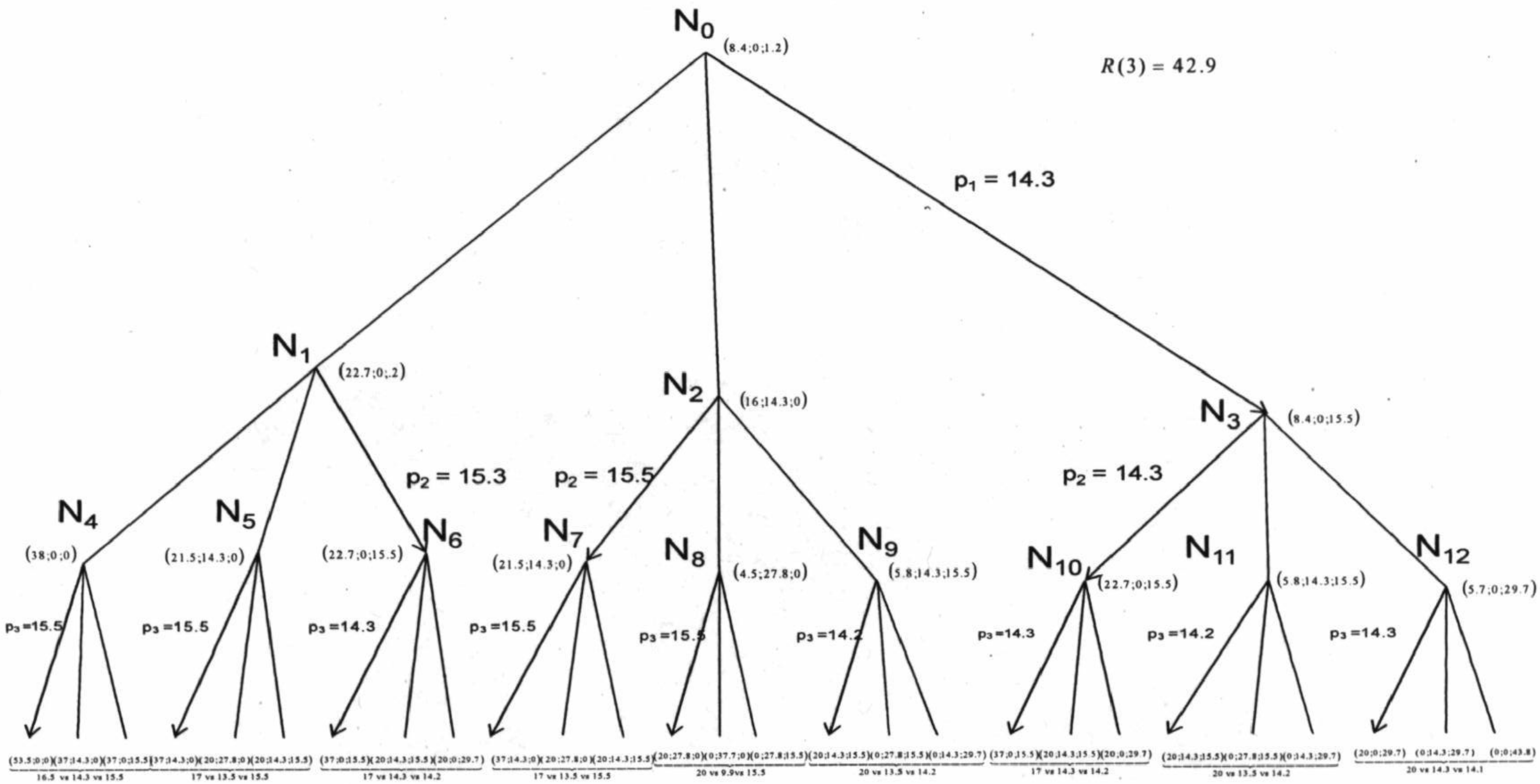

Figure 3.6. The additional bidder wins an object and the seller's revenue decreases.

3.7. Appendices

Appendix 1. There are four possible cases to be discussed because the additional bidder may or may not be identical to the other two. With two symmetric bidders and three objects such that: $H = H_1 = H_2 > M = M_1 = M_2 > L = L_1 = L_2$, the seller's revenue is equal to $R(2) = 2L + M$. The participation of bidder #3 in this auction shifts the seller's revenue to $R(3) = 3M$ in all cases except when the third bidder has no strategic significance ($H_3 \leq L$). Then, *R*(3) = *R*(2) because the auctions are equivalent. Figure 3.A.1 illustrates the 2-bidder 3-object auction. Because payoffs must be symmetric and allocations cannot, prices cannot be constant. Each bidder gets an object at price *L* for a gain of *H* – *L*. The last object is sold at price *M* and both bidders are indifferent between winning and losing.

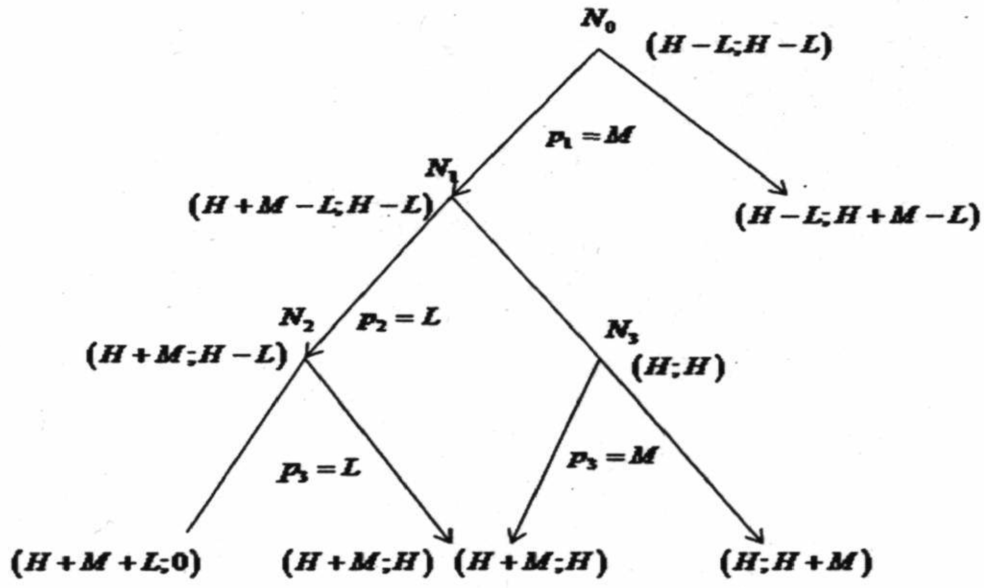

Figure 3.A.1. The 2-symmetric bidders game with three objects.

Appendix 2. Before proving proposition 2, it is necessary to show the corresponding results when only bidders #1 and #2 are active. The following lemma describes these results. The corresponding outcome trees are illustrated in figures 3.A2-3.A9. Lemma 1 describes the outcomes of 2-bidder auctions for different rankings of bidders' valuations. These outcomes can then be compared to outcomes of auctions with an additional bidder. Katzman (1999) analyzes four situations of bidders' valuations rankings where three situations are equivalent in the sense that they are characterized by constant price pattern and efficient allocation while in the fourth situation, where bidders are largely asymmetric, the price pattern is declining and the allocation is inefficient. For this reason, in this lemma, we focus on the investigation of two important cases of bidders' asymmetries, low asymmetry (assertion (i)) and high asymmetry (assertion (ii)). Moreover, our empirical evidence deals with these kinds of asymmetries. The high asymmetry exists between the dominant processor, Olymel, and its rivals while the low asymmetry is between processors rather than Olymel.

LEMMA 1. *In three successive second-price auctions with two asymmetric bidders, bidders #1and #2, and under complete information, we have:*

i) If $H_1 > H_2 > M_2 > M_1 > L_1 > L_2$ *then :*

a) *If* $L_1 + M_2 - 2M_1 > 0$ *then bidder #2 wins the first and the second objects and pays* $p_1 = 2M_1 - L_1 - M_2 + L_2$ *and* $p_2 = L_2$ *while bidder #1 wins the third object and pays* $p_3 = L_2$. *Then, the seller's revenue is* $R(2) = 3L_2 + 2M_1 - L_1 - M_2$.

b) *If* $L_1 + M_2 - 2M_1 < 0$ *then bidder #2 wins the first and the second objects and pays* $p_1 = p_2 = L_2$ *while bidder #1 wins the third object and pays* $p_3 = L_2$. *Then, the seller's revenue is* $R(2) = 3L_2$.

ii) If $H_1 > M_1 > L_1 > H_2 > M_2 > L_2$ *then we have:*

a) If $L_1 + M_2 - 2H_2 > 0$; $L_2 + M_1 - 2M_2 < 0$ *and* $L_1 + 2L_2 - 3H_2 - 2M_2 + 2M_1 < 0$ *then bidder #2 wins the first and the second objects and pays* $p_1 = L_1 + M_1 + L_2 - 2H_2$ *and* $p_2 = L_2 + M_1 - M_2$ *while bidder #1 wins the the third object and pays* $p_3 = L_2$. *Then, the seller's revenue is* $R(2) = 3L_2 + L_1 + 2M_1 - M_2 - 2H_2$.

b) If $L_1 + M_2 - 2H_2 > 0$; $L_2 + M_1 - 2M_2 < 0$ *and* $L_1 + 2L_2 - 3H_2 - 2M_2 + 2M_1 > 0$ *then bidder #1 wins all the objects and pays* $p_1 = H_2 + 2M_2 - L_2 - M_1$ *and* $p_2 = p_3 = H_2$. *Then, the seller's revenue is* $R(2) = 3H_2 + 2M_2 - L_2 - M_1$.

c) If $L_1 + M_2 - 2H_2 > 0$; $L_2 + M_1 - 2M_2 > 0$ *and* $L_1 + 2M_2 - 3H_2 > 0$ *then bidder #1 wins all the objects and pays* $p_1 = p_2 = p_3 = H_2$. *Then, the seller's revenue is* $R(2) = 3H_2$.

d) If $L_1 + M_2 - 2H_2 > 0\,; L_2 + M_1 - 2M_2 > 0$ *and* $L_1 + 2M_2 - 3H_2 < 0$ *then bidder #2 wins the first object and pays* $p_1 = L_1 + 2M_2 - 2H_2$ *while bidder #1 wins the second and the third objects and pays* $p_2 = p_3 = M_2$*. Then, the seller's revenue is* $R(2) = L_1 + 4M_2 - 2H_2$.

e) If $L_1 + M_2 - 2H_2 < 0\,; L_2 + M_1 - 2M_2 < 0$ *then bidder #2 wins the first and the second objects and pays* $p_1 = p_2 = L_2 + M_1 - M_2$ *while bidder #1 wins the third object and pays* $p_3 = L_2$*. Then, the seller's revenue is* $R(2) = 3L_2 + 2M_1 - 2M_2$.

f) If $L_1 + M_2 - 2H_2 < 0$ *and* $L_2 + M_1 - 2M_2 < 0$ *then bidder #2 wins the first object and pays* $p_1 = M_2$ *while bidder #1 wins the second and the third objects and pays* $p_2 = p_3 = M_2$*. Then, the seller's revenue is* $R(2) = 3M_2$.

PROOF OF LEMMA 1

i) Let us assume that : $H_1 > H_2 > M_2 > M_1 > L_1 > L_2$

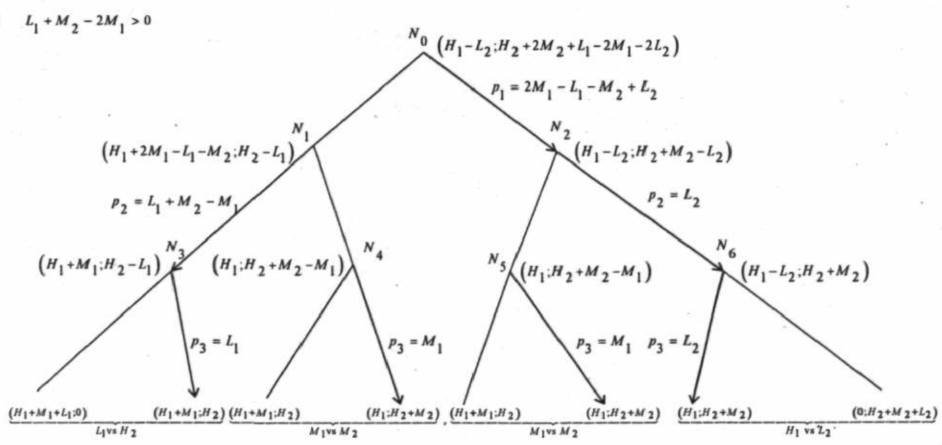

Figure 3.A.2.

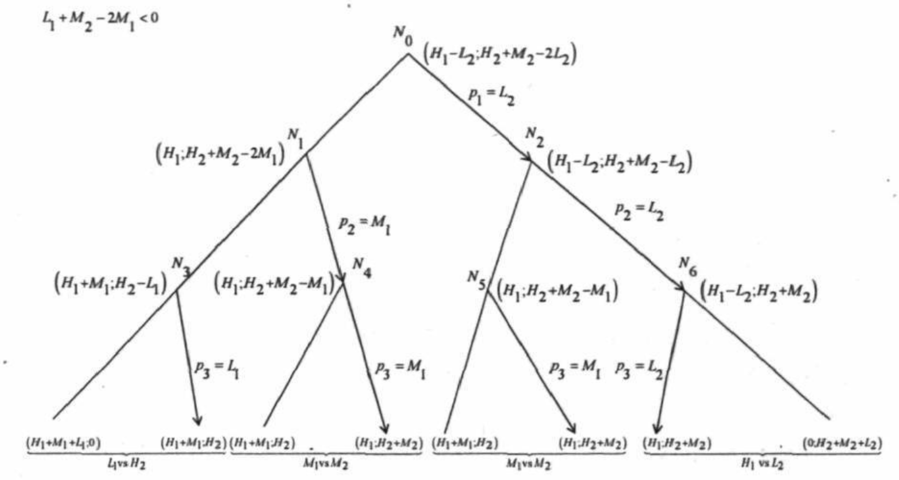

Figure 3.A.3.

ii) Assume that : $H_1 > M_1 > L_1 > H_2 > M_2 > L_2$

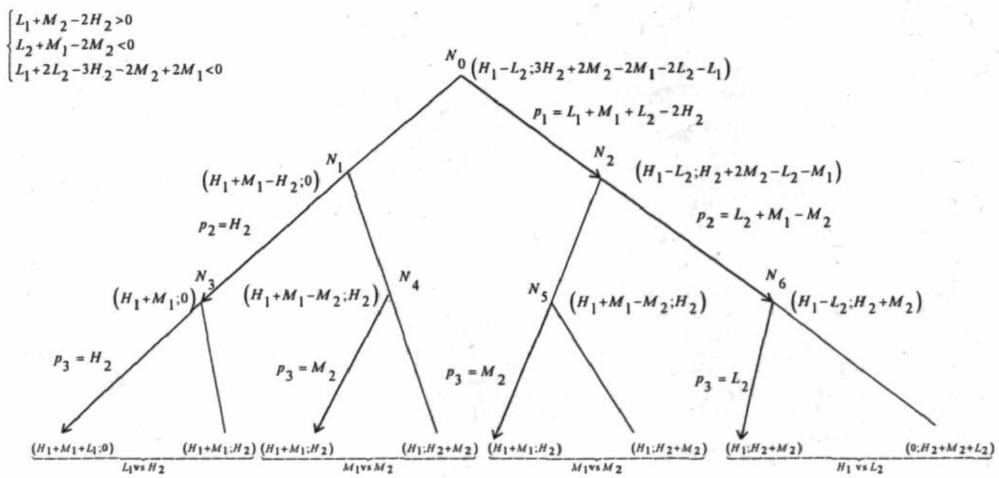

Figure 3.A.4.

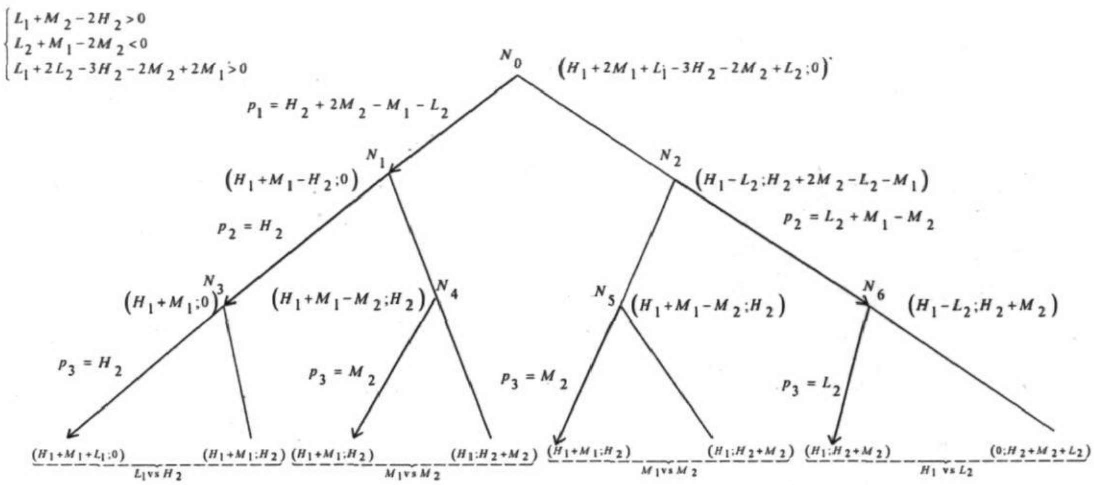

Figure 3.A.5.

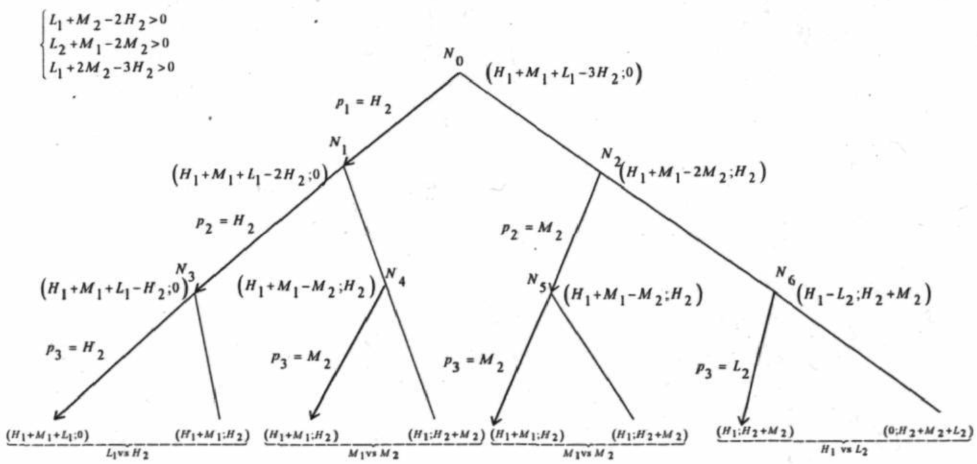

Figure 3.A.6.

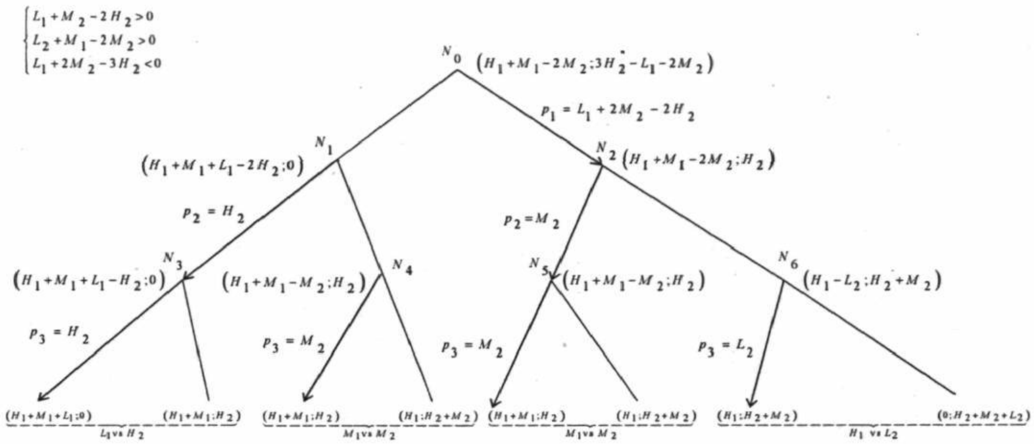

Figure 3.A.7.

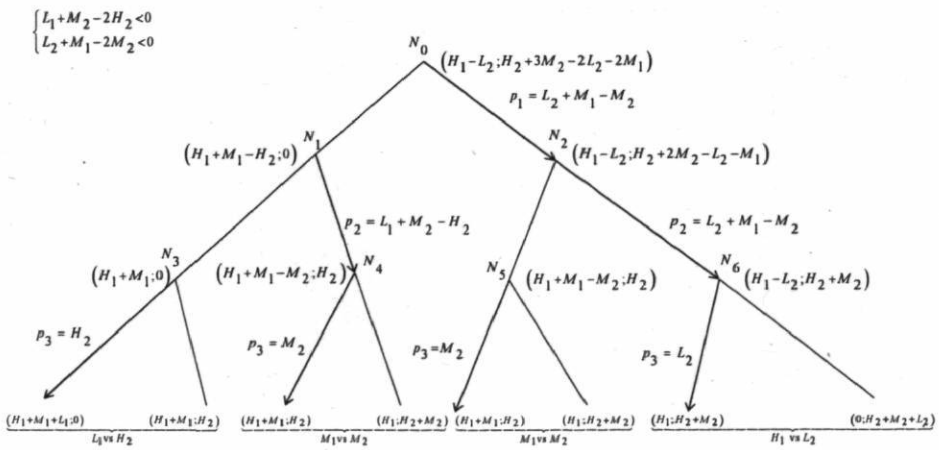

Figure 3.A.8.

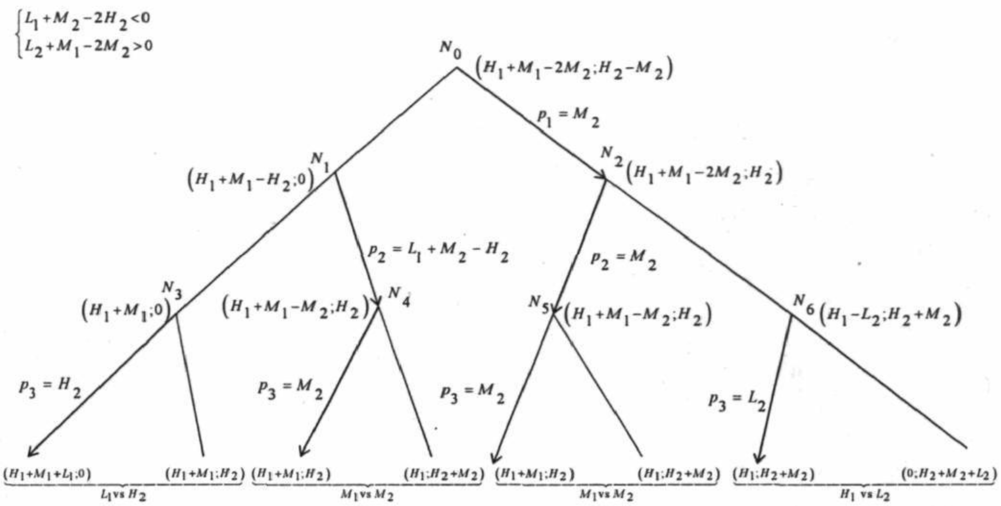

Figure 3.A.9.

PROOF OF PROPOSITION 2.

1. Let us assume that: $H_1 > H_2 > H_3 > M_2 > M_1 > L_1 > L_2 > M_3 > L_3$

We want to show that the seller's revenue always goes up when bidder #3's highest valuation is in the top three. Naturally, if H_3 was the first or the second highest valuation, the increase in the seller's revenue would be larger. In the auction game where $H_1 > H_2 > H_3 > M_2 > M_1 > M_3 > L_1 > L_2 > L_3$, the price trend is constant and prices are given by: $p_1 = p_2 = p_3 = M_1$. [17] Bidder #1 wins the first object while bidder #3 wins the second one and bidder #2 wins the last. The seller's revenue is $R(3) = 3M_1$. Figures 3.A.10-3.A13 illustrate the outcome tree of the game. Consequently from lemma 1-(i) we obtain:

a) If $L_1 + M_2 - 2M_1 > 0$ then $R(3) - R(2) = 3M_1 - (3L_2 + 2M_1 - L_1 - M_2) > 0$
b) If $L_1 + M_2 - 2M_1 < 0$ then $R(3) - R(2) = 3M_1 - 3L_2 > 0$.

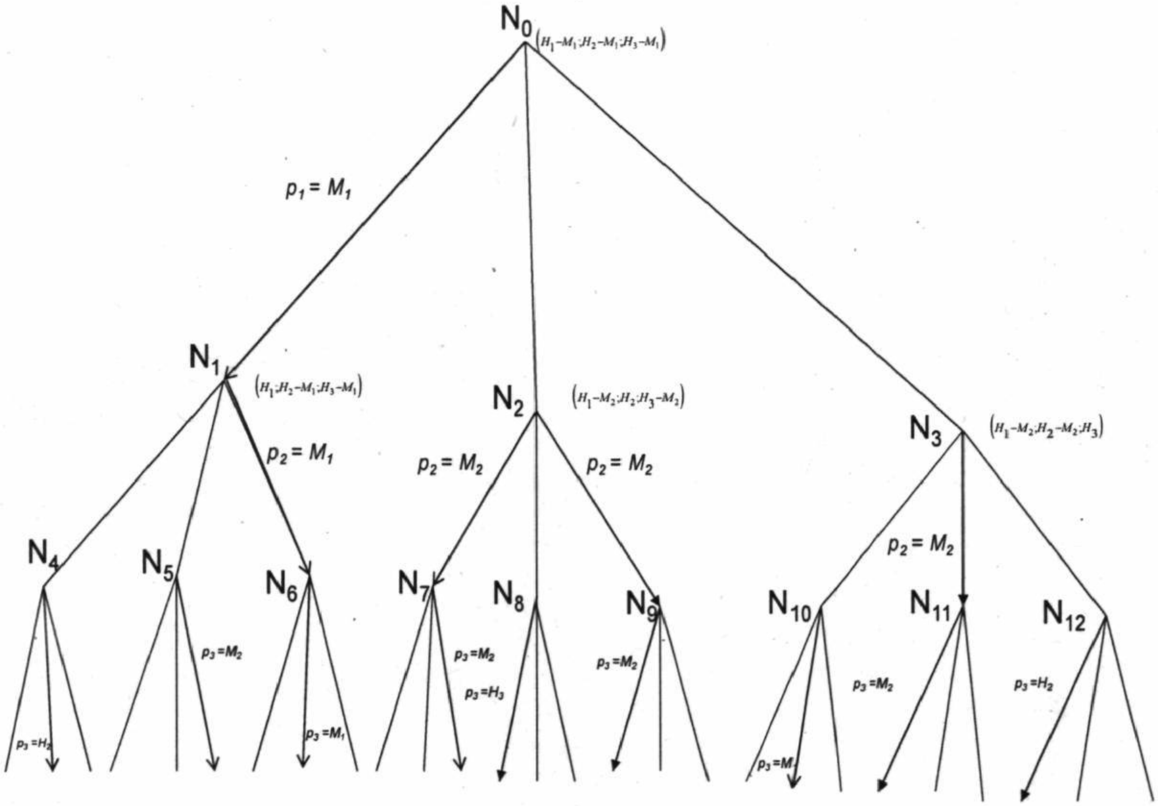

Figure 3.A.10. The outcome tree of the auction game in case (1) of proposition 2.

The subgames at nodes N_1, N_2 and N_3 are illustrated below.

[17] The same result is obtained when both the first and the second valuations of bidder #3 are strategic such that: $H_1 > H_2 > H_3 > M_2 > M_1 > M_3 > L_1 > L_2 > L_3$.

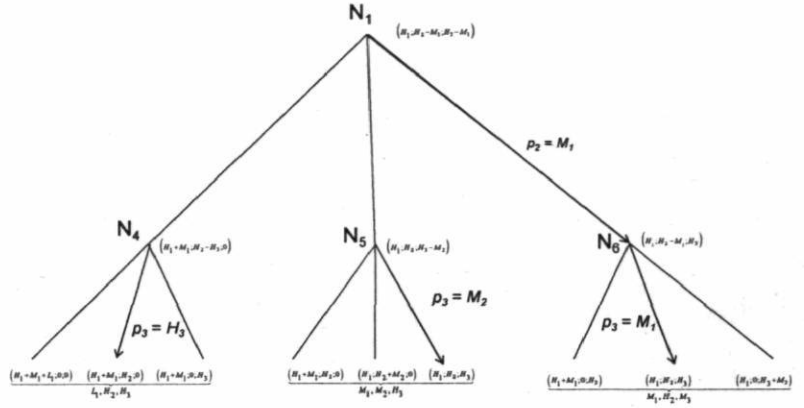

Figure 3.A.11. The subgame at node N_1.

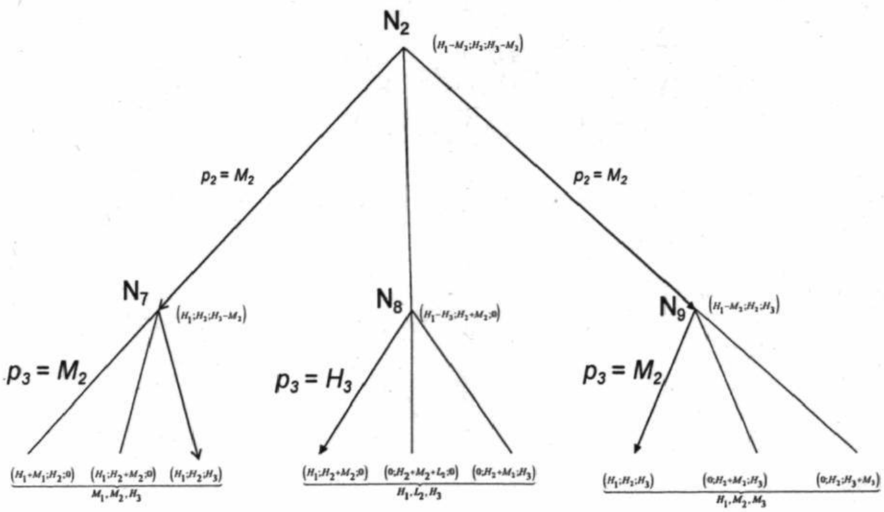

Figure 3.A.12. The subgame at node N_2.

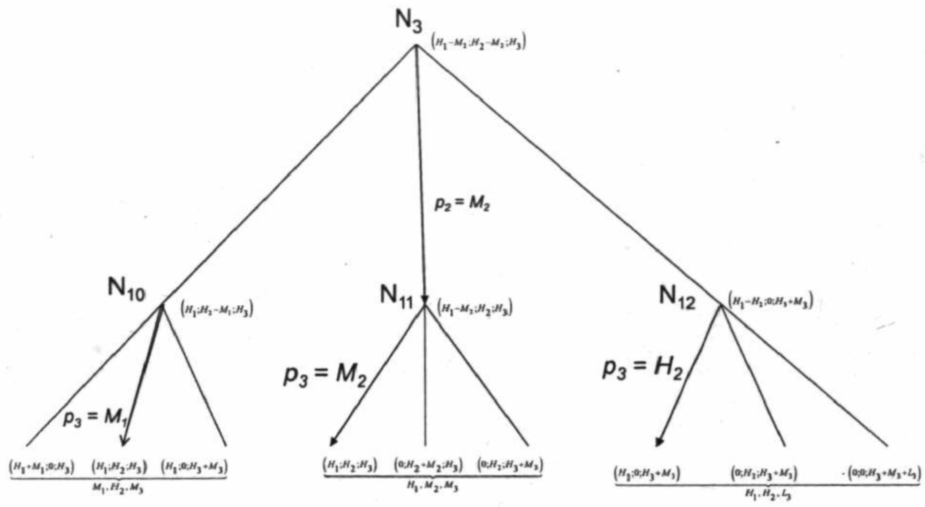

Figure 3.A.13. The subgame at node N_3.

This seller's revenue also increases when a third bidder is added for other cases of valuations' rankings if the highest valuation of the additional bidder is among the first three highest valuations and the competition between bidders is increased.

2. In what follows, it is sufficient to prove the result where the valuation of bidder #3 is the fourth and the sixth highest valuation because if the seller's revenue increases (resp. decreases) in both cases it is obvious that the seller's revenue increases (decrease) when bidder #3's highest valuation is in top five.

2. a. Let us assume that: $H_1 > M_1 > L_1 > H_3 > H_2 > M_2 > L_2 > M_3 > L_3$

The seller's revenue of this auction can be compared to the *R*(2) revenues for the six cases described by lemma 1-ii ($H_1 > M_1 > L_1 > H_2 > M_2 > L_2$). This case is interesting because H_3 is clearly strategic, unlike M_3 and L_3, as such, it is not surprising that the seller's revenue can increase with a third bidder. If $L_1 + H_2 - 2H_3 > 0$; $M_1 + M_2 - 2H_3 > 0$; $M_1 + M_2 - 2H_2 > 0$ and $L_1 + 2H_2 - 3H_3 > 0$, then bidder #1 wins all the objects and pays $p_1 = p_2 = p_3 = H_3$. The seller's revenue is $R(3) = 3H_3$. Figures 3.A.14-3.A17 depict the outcome trees. Therefore, from lemma 1-(ii-b) the seller's revenue may increase or decrease when bidder #3 is introduced because the difference between revenues $\Delta R = R(2) - R(3) = 3H_2 + 2M_2 - L_2 - M_1 - 3H_3$ may be negative or positive.

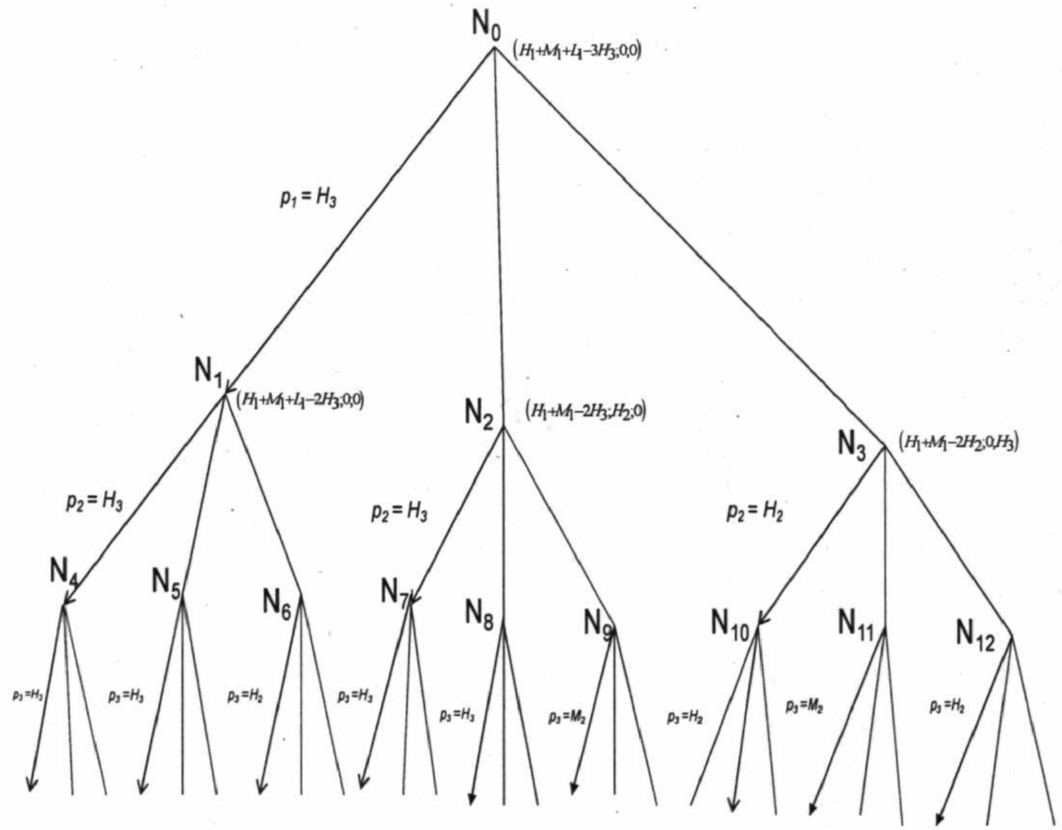

Figure 3.A.14. The outcome tree of the auction game in case (2.a) of proposition 2.

The subgames at nodes N_1, N_2 and N_3 are illustrated below.

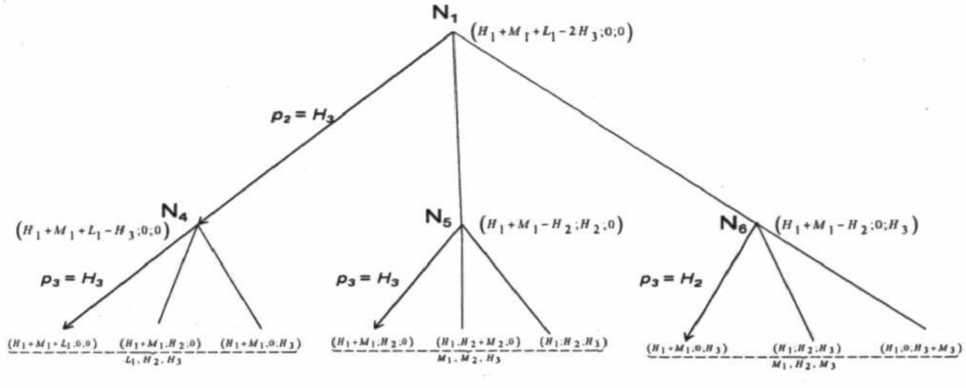

Figure 3.A.15. The subgame at node N_1.

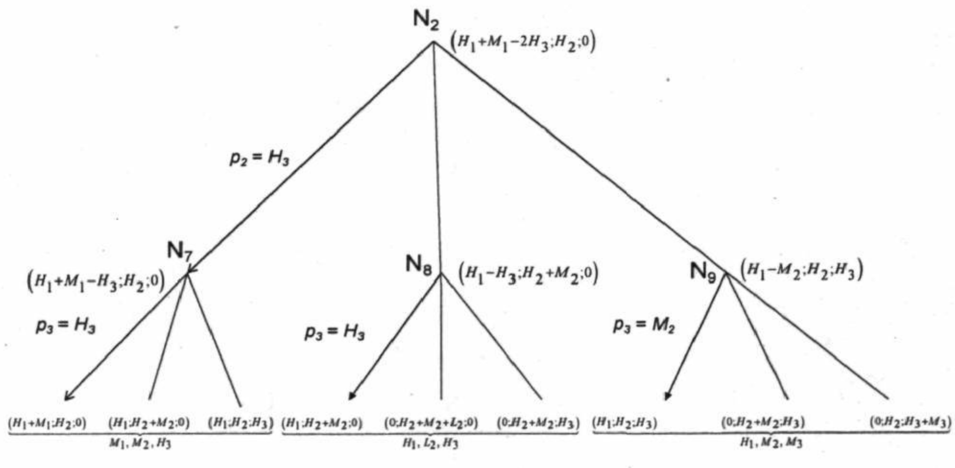

Figure 3.A.16. The subgame at node N_2.

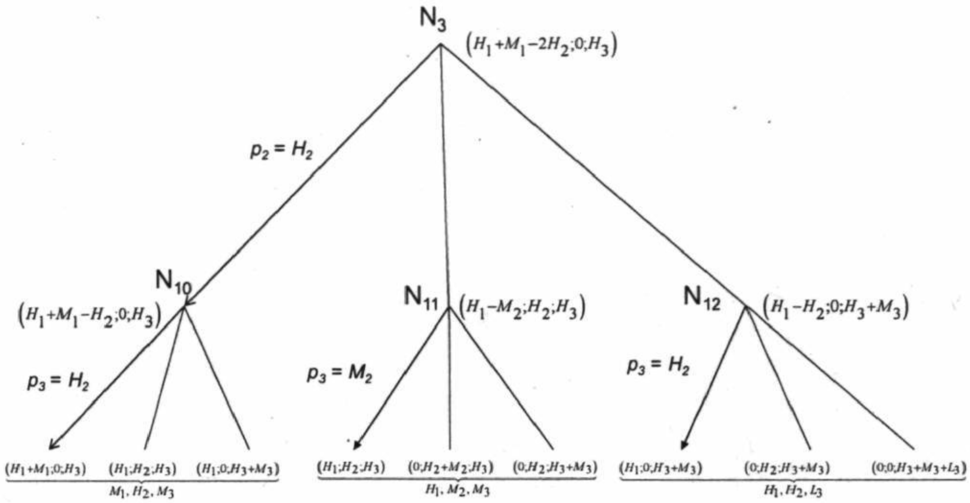

Figure 3.A.17. The subgame at node N_3.

Example 1. Let us assume that:

$H_1 = 21 > M_1 = 20 > L_1 = 15 > H_3 = 13 > H_2 = 12.1 > M_2 = 11.5 > L_2 = 2.5 > M_3 = 2 > L_3 = 1$

We have: $L_1 + M_2 - 2H_2 = 15 + 11.5 - 24.2 > 0$; $L_2 + M_1 - 2M_2 = 2.5 + 20 - 23 < 0$;

$L_1 + 2L_2 - 3H_2 - 2M_2 + 2M_1 = 15 + 5 - 36.3 - 23 + 40 > 0$;

$L_1 + H_2 - 2H_3 = 15 + 12.1 - 26 > 0$; $M_1 + M_2 - 2H_3 = 20 + 11.5 - 26 > 0$

$M_1 + M_2 - 2H_2 = 20 + 11.5 - 24.2 > 0$ and $L_1 + 2H_2 - 3H_3 = 15 + 24.2 - 39 > 0$

Therefore, $\Delta R = 3H_2 + 2M_2 - L_2 - M_1 - 3H_3 = -2.2 < 0$

Example 2. Let us assume that:

$H_1 = 21 > M_1 = 20 > L_1 = 15 > H_3 = 12.7 > H_2 = 12.6 > M_2 = 12.5 > L_2 = 3.95 > M_3 = 2 > L_3 = 1$
We have: $L_1 + M_2 - 2H_2 = 15 + 12.5 - 25.2 > 0$; $L_2 + M_1 - 2M_2 = 3.95 + 20 - 25 < 0$;

$L_1 + 2L_2 - 3H_2 - 2M_2 + 2M_1 = 15 + 7.9 - 37.8 - 25 + 40 > 0$;

$L_1 + H_2 - 2H_3 = 15 + 12.6 - 25.4 > 0$; $M_1 + M_2 - 2H_3 = 20 + 12.5 - 25.4 > 0$

$M_1 + M_2 - 2H_2 = 20 + 12.5 - 25.2 > 0$ and $L_1 + 2H_2 - 3H_3 = 15 + 25.2 - 38.1 > 0$

Therefore, $\Delta R = 3H_2 + 2M_2 - L_2 - M_1 - 3H_3 = 37.8 + 25 - 3.95 - 20 - 38.1 > 0$

2. b. Let us assume that: $H_1 > M_1 > L_1 > H_2 > M_2 > H_3 > M_3 > L_3 > L_2$. This is an interesting valuation ranking because bidder #3's highest valuation is only the sixth highest. If bidder #3 is introduced such that: $H_1 > M_1 > L_1 > H_2 > M_2 > H_3 > M_3 > L_3 > L_2$; $L_1 + M_2 - 2H_2 > 0$; $M_1 + H_3 - 2M_2 > 0$ and $L_1 + 2M_2 - 3H_2 > 0$ then bidder #1 wins all the objects and pays $p_1 = p_2 = p_3 = H_2$. The seller's revenue is $R(3) = 3H_2$. Figures 3.A.18-3.A21 depict the outcome tree. Therefore, from lemma 1-(a) and 1-(b) we obtain:

i) if $L_1 + 2L_2 - 3H_2 - 2M_2 + 2M_1 > 0$ then $R(2) = 3H_2 + 2M_2 - L_2 - M_1 > R(3) = 3H_2$.

ii) If: $L_1 + 2L_2 - 3H_2 - 2M_2 + 2M_1 < 0$ then $R(2) = L_1 + 3L_2 + 2M_1 - M_2 - 2H_2 < R(3) = 3H_2$

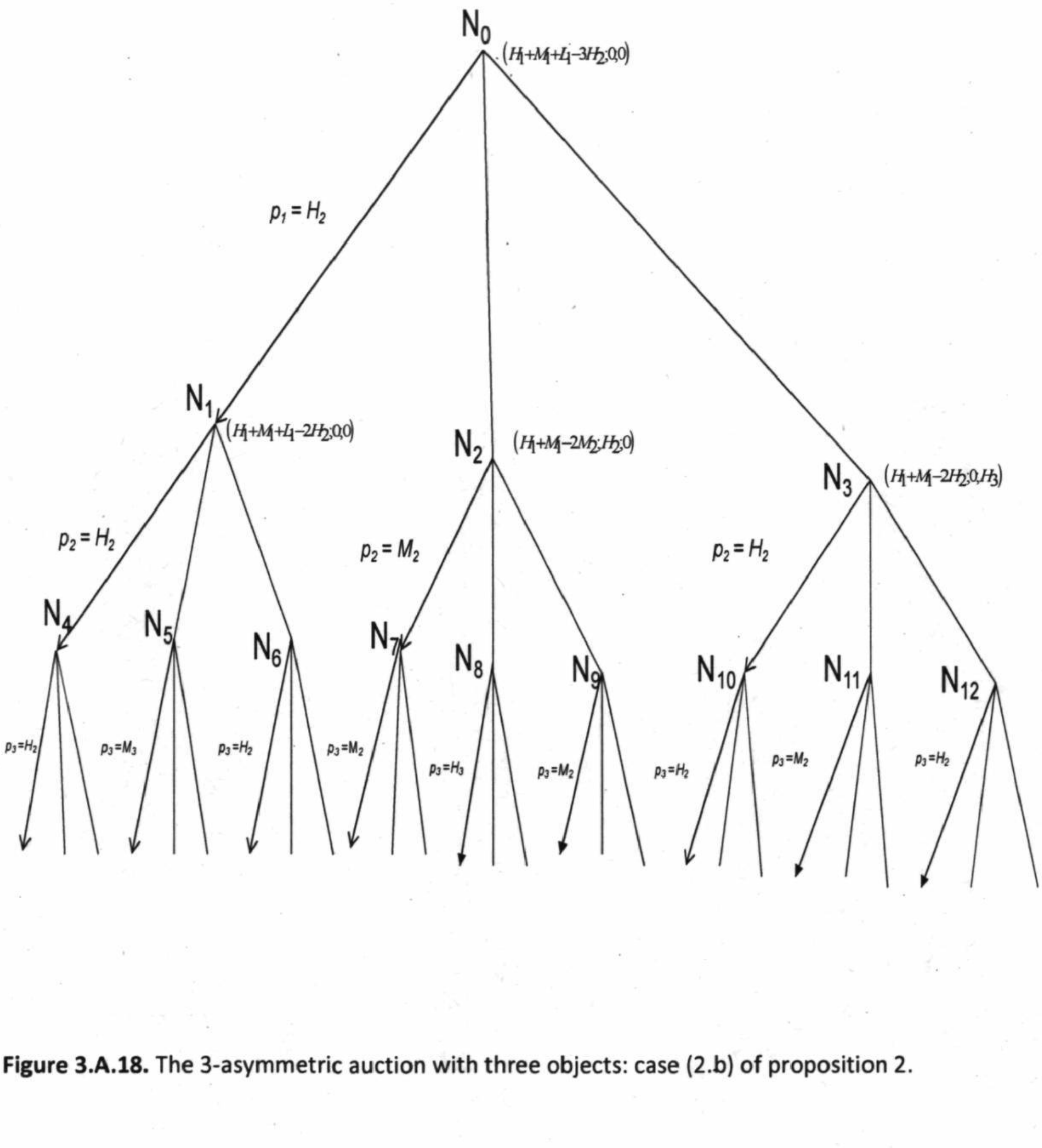

Figure 3.A.18. The 3-asymmetric auction with three objects: case (2.b) of proposition 2.

The subgames at nodes N_1, N_2 and N_3 are illustrated below.

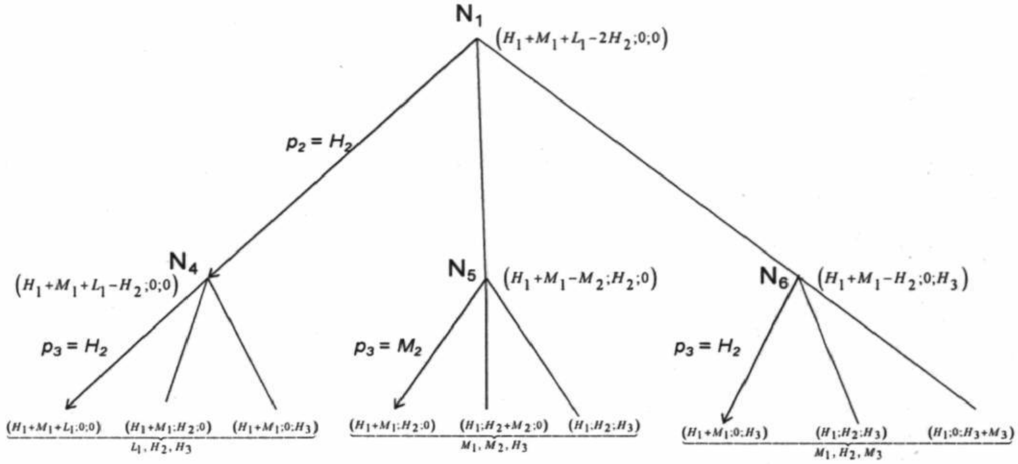

Figure 3.A.19. The subgame at node N_1.

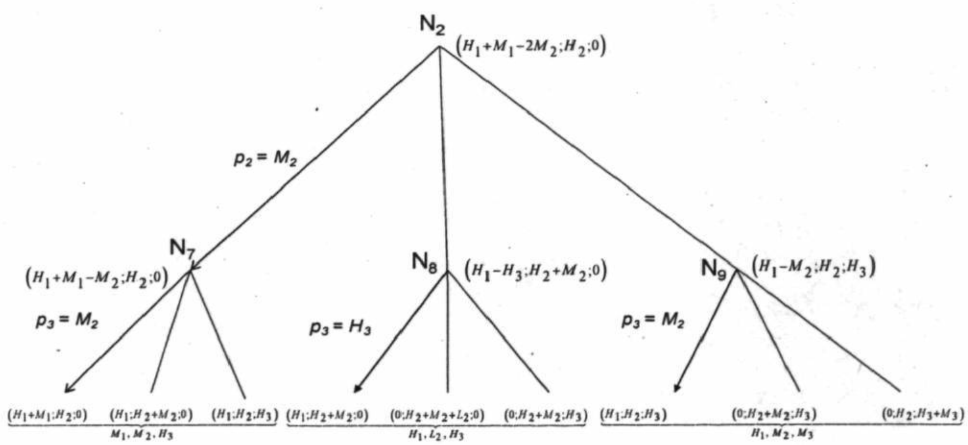

Figure 3.A.20. The subgame at node N_2.

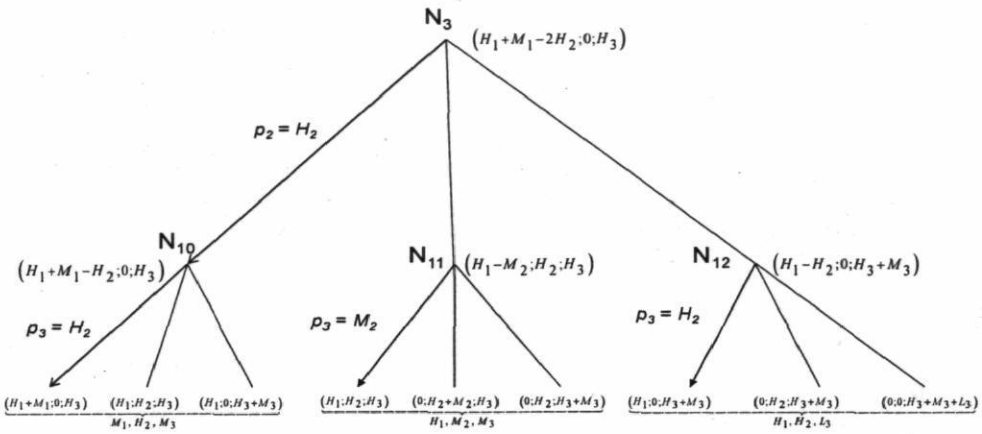

Figure 3.A.21. The subgame at node N_3.

Appendix 3.

PROOF OF PROPOSITION 3.

We show that even if bidder #3 wins an object the seller's revenue may decrease if the highest valuation of the additional bidder is the fourth or the fifth highest valuation. We first assume that: $H_1 > M_1 > L_1 > H_3 > H_2 > M_2 > L_2 > M_3 > L_3$. This case is interesting because it deals with the effect of only the highest valuation of the additional bidder as it is his sole strategic valuation. If bidder #3 is introduced such that: $L_1 + H_2 - 2H_3 < 0$ and $M_1 + M_2 - 2H_3 > 0$ then he wins the first object and pays $p_1 = H_2$ while bidder #1 wins the second and third objects and pays $p_2 = p_3 = H_2$. The seller's revenue is $R(3) = 3H_2$.

Figures 3.A.22-3.A.25 depict the outcome tree. Therefore, from lemma 1-(ii-b), if $L_1 + M_2 - 2H_2 > 0$; $L_2 + M_1 - 2M_2 < 0$ and $L_1 + 2L_2 - 3H_2 - 2M_2 + 2M_1 < 0$, the seller's revenue decreases ($R(2) = 3H_2 + 2M_2 - L_2 - M_1 > R(3) = 3H_2$) when bidder #3 is introduced such that: $L_1 + H_2 - 2H_3 < 0$ and $M_1 + M_2 - 2H_3 > 0$. On the other hand, we can easily show that if bidder #3 is introduced such that: $L_1 + H_2 - 2H_3 > 0$; $L_1 + 2H_2 - 3H_3 < 0$ and $M_1 + M_2 - 2H_3 > 0$ then he wins the first object and pays $p_1 = L_1 + 2H_2 - 2H_3$ while bidder #1 wins the second and third objects and pays $p_2 = p_3 = H_2$. The seller's revenue is $R(3) = 4H_2 + L_1 - 2H_3$. Therefore, from lemma 1-(ii-c), if $L_1 + M_2 - 2H_2 > 0$; $L_2 + M_1 - 2M_2 > 0$ and $L_1 + 2M_2 - 3H_2 > 0$, the seller's revenue increases ($R(2) = 3H_2 < R(3) = 4H_2 + L_1 - 2H_3$) when bidder #3 is introduced such that: $L_1 + H_2 - 2H_3 > 0$; $L_1 + 2H_2 - 3H_3 < 0$ and $M_1 + M_2 - 2H_3 > 0$.

Moreover, we can easily show that if $H_1 > M_1 > L_1 > H_2 > H_3 > M_2 > L_2 > M_3 > L_3$ then the seller's revenue can increase or decrease even if bidder #3 wins. However, if we have: $H_1 > M_1 > L_1 > H_2 > M_2 > H_3 > L_2 > M_3 > L_3$ then bidder #3 does not win a single object.

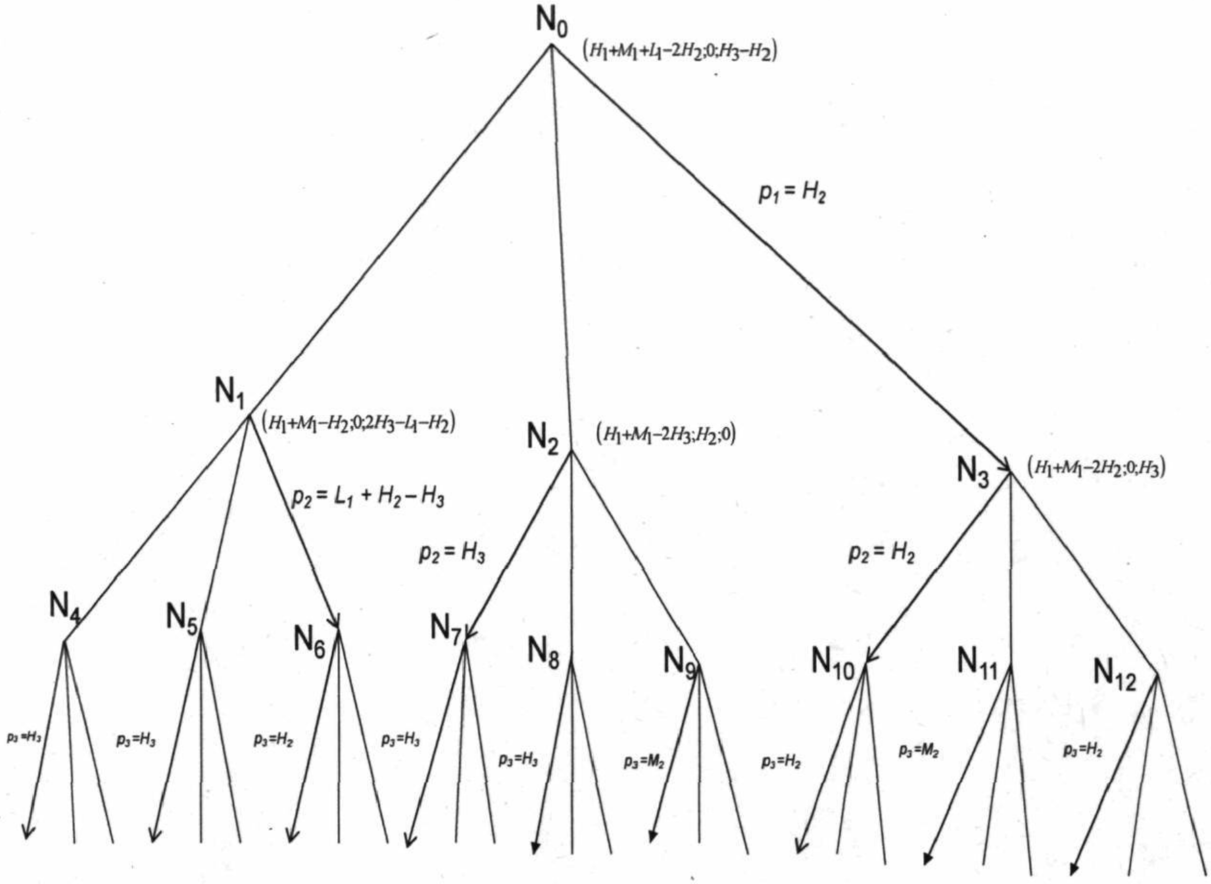

Figure 3.A.22. The 3-asymmetric auction with three objects: proposition 3.

The subgames at nodes N_1, N_2 and N_3 are illustrated below.

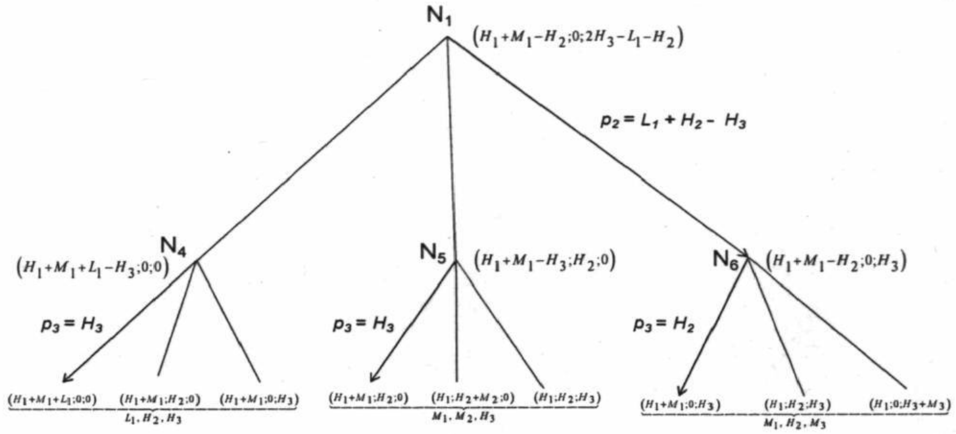

Figure 3.A.23. The subgame at node N_1.

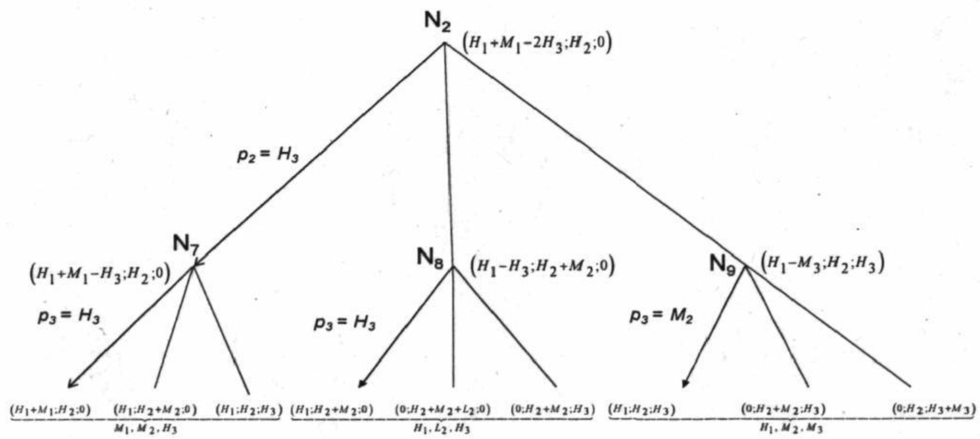

Figure 3.A.24. The subgame at node N_2.

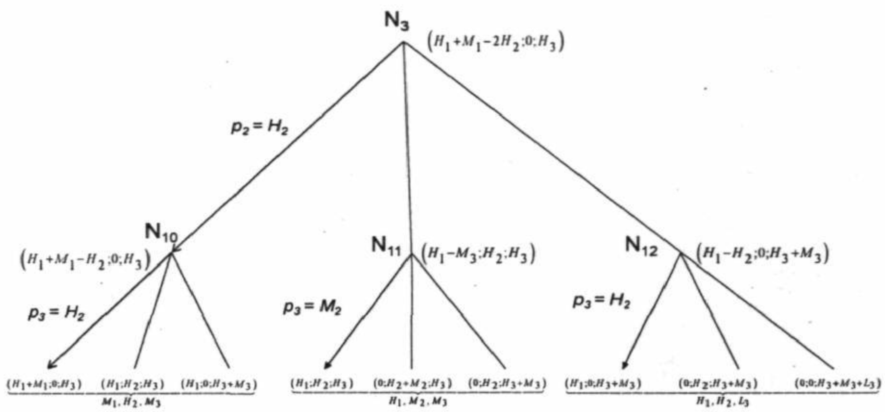

Figure 3.A.25. The subgame at node N_3.

4. Mergers, Concurrent Marketing Mechanisms and the Performance of Sequential Auctions

Résumé

Ce chapitre montre que, même en l'absence de synergies post-fusion, la fusion peut simultanément accroître le revenu du vendeur et améliorer l'efficacité des enchères. L'utilisation d'un test de changement structurel endogène affirme que la fusion a eu un effet anticoncurrentiel sur les prix reçus par les producteurs de porcs du Québec. En outre, nous soulignons que la coexistence du mécanisme de pré-attribution et les enchères peut faire augmenter ou diminuer le revenu du vendeur et changer la tendance des prix et l'efficacité, dépendamment de qui gagne les objets en pré-attribution.

Abstract

We analyze the effects of mergers on the seller's revenue, price trend and efficiency in sequential auctions under complete information with asymmetric bidders. First, we provide the conditions when bidders are strategic and when a merger can take place. Second, we show that mergers may simultaneously increase the seller's revenue and improve efficiency. Third, we show that having a marketing mechanism working alongside the auction can increase or decrease the average auction price. We use weekly data about Quebec's daily hog auction to ascertain the effects of a merger and of changes in the weights of concurrent marketing mechanisms on daily auction prices. Our empirical analysis relies on an endogenous structural change test which detected three breaks corresponding to: i) the introduction of a new concurrent mechanism, ii) a joint-venture partnership of the two largest hog processing firms and iii) an announcement by Canada's Competition Bureau authorizing the full merger of the same two firms.

4.1. Introduction

Mergers have inspired a vast and rich literature in industrial organisation. A perennial concern of competition regulations with mergers is their effects on prices and welfare. Farrell and Shapiro (1990) warned that mergers involving firms with combined pre-merger market shares exceeding 50% are likely to reduce welfare, but Heubeck et al. (2006) argue that such misgivings might be unwarranted even in the absence of direct cost efficiencies. The question about profitability of horizontal mergers has attracted much attention ever since the seminal paper of Salant et al. (1983). In one exemple (Salant et al., 1983 p.159), they showed that mergers between Cournot oligopolists producing a homogenous good with identical linear cost functions and facing a linear demand curve are unprofitable unless the merged firms account for at least 80% of the firms in the industry. The same result holds under Stackelberg competition (Daughety, 1990), but Deneckere and Davidson (1985) show that mergers are always profitable for the merging firms under Bertrand competition because prices are strategic complements.

The analysis of the impact of mergers on prices, allocation efficiency and the seller's revenue is also of great interest in the literature on auctions. Waehrer and Perry (2003) found that mergers increase the expected price in second-price procurement auctions in which the cost parameter of firms are drawn from the same type of distributions. Tschantz, Crooke and Froeb (2000) constructed a three-firm example to show that a merger between two identical bidders has a larger (smaller) price effect when the merging firms are larger (smaller) than the third firm. Thomas (2004) shows that mergers may decrease the expected price in one-shot procurement auctions. The above studies assume that bidders are incompletely informed and have unit-demands.

Mergers have not been analyzed under the complete information framework even though it is most suitable in the context of frequently repeated auctions involving the same bidders endowed with precise information about each other's costs, capacity and market opportunities (e.g., Bernheim and Whinston, 1986 and Gale and Stegeman, 2001). Our analysis fills this gap and contributes to the literature on multi-unit demand sequential auctions under complete information (e.g., Krishna, 1999, Katzman, 1999, Gale and

Stegeman, 2001, Rodriguez, 2009 and Jeddy et al., 2010) by considering higher dimensional auctions with several asymmetric bidders and several objects.

It is often assumed in single-unit private value auctions that the value of a merged firm is the maximum of its coalition member values which implies that the merged firm wins the auction that any of its pre-merger component pieces would have won (e.g., Baker, 1997). The allocation efficiency is maintained because the object is still won by the bidder with the highest valuation. Consequently, the amount extracted from the seller's revenue is totally transferred to the payoff of the merged bidder. We show that this outcome may also occur in a sequential multi-unit demand auction under complete information, but we also derive a new and interesting result pointing out that the higher payoff of the merged bidder need not be at the seller's expense. In such a case, if the pre-merger allocation is inefficient, a merger can produce an efficient allocation and increase the seller's revenue.

Most studies about marketing mechanisms tend to compare one mechanism against another, like two different types of auctions, or auctions versus contracts as in Bulow and Klemperer's (1996) classic paper. However, little is known about the performance of a given marketing mechanism when a different marketing mechanism is being used concurrently. The Quebec hog industry experimented with different mixes of marketing mechanisms.[18] Between 1989 and 1994, all of the hogs were sold through a daily electronic auction. Between 1994 and 2000, no less than 72% of the provincial hog supply was "formula-priced" in relation to a U.S. price, and "pre-attributed" to individual processors based on historical market shares. The remaining hogs were sold on the daily electronic auction. In 2000, a third

[18] We contend that Quebec's daily electronic hog auction can be analyzed in a complete information framework. The number of bidders (processors) was small (seven) and these bidders competed in the same domestic and foreign output markets with fixed production capacities. Furthermore, the Quebec federation of hog producers, known under the French acronym as FPPQ, has had the exclusive rights to market hogs in the province of Quebec and hence the opportunity to set a "fixed price take it or leave it" marketing mechanism. Instead, the FPPQ set up a daily electronic hog auction in 1989, which signals that it was poorly informed. To mitigate quality uncertainty, prices were determined for lots of virtual hogs scoring 100 on a quality index. Delivered hogs scoring higher or lower than 100 prompted automatic quality adjustments in price from an agreed upon grid of discount and premia negotiated on a regular basis between processors and the FPPQ.

mechanism was added as one-month supplies were being auctioned. Table 4.1 illustrates the relative importance of each marketing mechanism over time during what looks like a controlled experiment. We show that the presence of a pre-attribution scheme, as in the Quebec hog industry, may increase or decrease the average price generated by the sequential auction and may impact on price trends and allocation efficiency.

We use weekly data about Quebec's daily hog auction between 1996:1 and 2006:52 to ascertain the effects of a merger and of changes in the weights of concurrent marketing mechanisms on auction prices given that there are a large number of events that could have induced structural changes. We rely on a flexible endogenous structural change test which detected three break dates corresponding to: i) the introduction of a new mechanism (i.e., an auction of monthly supplies), ii) a joint-venture partnership of the two largest hog processing, and iii) an announcement by Canada's Competition Bureau authorizing the full merger of the same two firms. Comparing the prices predicted by the different regimes, we find a significant difference. Prices increase after the introduction of the third mechanism, but decrease after the partnership and eventual merger between the two largest processors.

The next section provides conditions when bidders can be strategic as well as when a merger can take place. Moreover, it presents the results about the effect of mergers on price trend, efficiency and seller's revenue in sequential auctions under complete information with asymmetric bidders. The third section focuses on how the presence of a concurrent pre-attribution mechanism impacts on the performance of the auction. The fourth section features an empirical analysis of the prices generated on the Quebec hog auction. Conclusions and policy implications are presented in the last section.

4.2. Mergers, Efficiency and Seller's Revenue

In this section, we derive necessary conditions for a merger to take place and investigate the implications of mergers on the seller's revenue, price trend and efficiency in sequential auctions. As in Krishna (1999), we begin with a simple example of a second-price sequential auction under complete information to present definitions and concepts to be used in our analysis of mergers.

Consider a sequence of two second-price auctions with three individual bidders (A, B and C) with diminishing marginal valuations. The seller is non-strategic and incompletely informed. We denote by V_j^i bidder *j*'s *i*th highest valuation. The valuations are ranked as follows: $V_A^1 = 10 > V_B^1 = 8 > V_A^2 = 7 > V_C^1 = 6 > V_B^2 = 5 > V_C^2 = 3$. The strategic behaviour of bidders in second-price multi-unit sequential auctions under complete information is well-documented in the literature (e.g., Krishna, 1993, 1999; Katzman, 1999; Gale and Stegeman, 2001 and Jeddy et al., 2010). Each bidder is assumed to follow the weakly dominant strategy of sincere bidding in the last round. It is a weakly dominant strategy for each bidder to place a bid equal to the first round price that would make them indifferent between winning and losing the first round object.

The outcome tree of the game is illustrated in Figure 4.1. Arrows denote the allocation in each subgame and prices are given next to the paths at the various nodes of the tree. At each node, the bidders' gross payoffs are put in parentheses. Each unit could be won by either bidder A (left branch), bidder B (middle branch) or bidder C (right branch). The equilibrium outcome is solved by backward induction and bids reflect the opportunity cost of not winning. The outcome tree, unlike the extensive form tree, features gross payoffs at every node which are obtained through subgame replacement. At nodes associated to the *j*th object, gross payoffs are defined as the sum of valuations for objects won along the given path minus the sum of prices for objects that would be won among the last *n*-*j*+1 objects. For the last object, gross payoffs are the sum of the valuations.

Starting at the bottom of the tree in Figure 4.1, we can see that the vector of gross payoffs when bidder A wins both objects is (17; 0; 0), which is simply the sum of the valuations for the objects won by the bidders. Provided the first object is won by bidder A, the second object may be won by bidder A, bidder B or bidder C. In these cases, the vectors of gross payoffs are (17; 0; 0); (10; 8; 0) and (10; 0; 6), respectively. It follows that at node N_1, the second object is worth at most 7 (17 – 10 = 7) for bidder A, at most 8 (8 - 0 = 8) for bidder B and at most 6 (6 - 0 = 6) for bidder C. If the game was to reach node N_1, the second object would be won by bidder B at price 7. Therefore, the gross payoff at this node is (10; 8– p_2; 0) = (10; 1; 0).

The same reasoning could be used at nodes N_2 and N_3. The vectors of gross payoffs at these nodes are respectively (4; 8; 0) and (2; 0; 6). At N_0, it is a dominant strategy for bidder C to bid up to 6 to prevent the other bidders from acquiring the first object. Because bidders A and B know that bidder C's bid will be 6, bidder A bids 6 and bidder B bids 7. Bidder B wins the first object and pays 6. Consequently, the seller's revenue is equal to $R^S = 6+6=12$ and the bidders' payoffs are given by: $\pi^A = (10-6)=4,\ \pi^B = (8-6)=2$ and $\pi^C = 0$. The price trend is constant, $p_1 = p_2 = 6$, and the allocation is efficient since the objects end up in the hands of the bidders with the highest valuations. The above example is interesting because even though bidder C has not won an object, his presence matters because equilibrium prices are equal to his highest valuation.

Definition 1: In auctions of *n* objects involving *k* bidders with valuations for the first object $V_A^1 > V_B^1 > \ldots > V_K^1$, a player *i* is strategic if its highest valuation V_i^1 is one of the two largest valuations in at least one the k^{n-1} nodes where the allocation of the last object is done in the outcome tree. The top two valuations in each of the k^{n-1} comparisons are said strategic, as they do impact on the allocation and price sequence determination. The lowest strategic valuation is called the residual strategic valuation.

Discussion: From the bottom nodes of the outcome tree, we are putting the valuation of each bidder for the last object at each of the k^{n-1} paths allocating the first *n*-1 objects. To be strategic, a player's highest valuation must be in the top two in at least one of the k^{n-1} comparisons. For example, in a 5-bidder 2-object auction where each bidder *i* has valuations $V_i^1 > V_i^2$ for the first and second objects, we have:

(i) $V_A^2 vs\, V_B^1 vs\, V_C^1\, vs\, V_D^1\, vs\, V_E^1$; (ii) $V_B^2\, vs\, V_A^1 vs\, V_C^1\, vs\, V_D^1\, vs\, V_E^1$; (iii) $V_C^2\, vs\, V_A^1 vs\, V_B^1\, vs\, V_D^1\, vs\, V_E^1$; (iv) $V_D^2\, vs\, V_A^1 vs\, V_B^1\, vs\, V_C^1\, vs\, V_E^1$ and (v) $V_E^2\, vs\, V_A^1 vs\, V_B^1\, vs\, V_C^1\, vs\, V_D^1$.

From $V_A^1 > V_B^1 > \ldots > V_K^1$ and from iii)-v), we know that V_A^1 and V_B^1 are strategic. From ii), we have that either V_A^1 and V_B^2 are strategic or either V_A^1 and V_C^1 are. Finally, from i) either V_A^2 and V_B^1 are strategic or V_B^1 and V_C^1 are. Intuitively, we know that bidder A is strategic because

V_A^1 is the highest valuation. However, the second highest valuation is either V_A^2 or V_B^1. From iii) we know that bidder B is strategic because $V_B^1 > \max\left(V_C^2, V_D^1\right) > V_E^1$. We know that bidders D and E cannot be strategic from i) to v). However, bidder C may or may not be strategic and V_C^1 may or may not be the residual strategic valuation.[19] The latter case occurs when $V_C^1 < \min\left(V_A^2, V_B^2\right)$. Clearly, increasing the number of objects tends to increase the number of strategic players. If 3 objects were auctioned, and that the valuation of player i for a third object is V_i^3, then bidder C could be strategic if the following condition holds: $V_C^1 > \min\left(V_A^3, V_B^3\right)$. This condition is less restrictive than $V_C^1 > \min\left(V_A^2, V_B^2\right)$ for the two-object case. The implication of the above definition is that the analysis of auctions can be simplified by taking into account only the strategic bidders.

Lemma 1: *The price of the last object is bounded from below by the residual valuation.*

Proof: The equilibrium path on the outcome tree auction may or may not include nodes involving the residual valuation. When it does not, the price of the last object is higher than the residual valuation. To see this, consider an auction with 5 bidders and 2 objects such that : $V_A^1 > V_A^2 > V_B^1 > V_C^1 > V_D^1 > V_E^1$. Bidder A has the two highest valuations and must decide whether his payoff is maximized with one or two objects, knowing that if he wins only one object, the other object will be won by bidder B. Thus, the parts of the outcome tree with the first object going to bidders C, D and E are irrelevant and we can focus on the branches for which the first object is attributed to bidder A or bidder B. When bidder A lets bidder B win the first object, the residual valuation is $\max\left(V_B^2, V_C^1\right)$. If $V_A^2 + \max\left(V_B^2, V_C^1\right) > 2V_B^1$ then $p_1 = p_2 = V_B^1 > V_B^2$ and bidder A wins both objects, but if $V_A^2 + \max\left(V_B^2, V_C^1\right) < 2V_B^1$, then bidder B (A) wins the first (second) object and $p_1 = V_A^2 + \max\left(V_B^2, V_C^1\right)$ $-V_B^1 > \max\left(V_B^2, V_C^1\right) = p_2$, which is similar to the 2x2 example in Katzman's (1999, p.81).

QED

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[19] From i) and ii), bidder C being strategic can be supported by the following sequences of valuations: $V_A^1 > V_B^1 > V_B^2 > V_C^1$, $V_A^1 > V_A^2 > V_B^1 > V_C^1$ and $V_A^1 > V_B^1 > V_C^1$.

Proposition 1: *In complete information second-price auctions with n objects and k bidders such that n<k, with bidders with declining valuations, $V_j^1 > V_j^2 ... > V_j^n$ where V_j^i is bidder j's i^{th} highest valuation, and without loss of generality, $V_A^1 > V_B^1 > ... > V_K^1$, then bidder j is strategic if and only if its highest valuation is among the (n+2) highest valuations.*

Proof: Going back to the proof of lemma 1 where bidder A has the *n* highest valuations and acts as a monopsonist, the residual valuation is the $(n+2)^{th}$ highest valuation, whether it belongs to bidder B or bidder C. Bidder C is strategic only when V_C^1 is the residual valuation. The result holds for other patterns of valuations. Let us derive the condition for bidder *K* to be strategic assuming that *n*=*k*-1 and that V_{K-1}^1 is the n^{th} highest valuation. This implies that objects are broadly allocated. For bidder *K* to be strategic, V_K^1 must be the residual strategic valuation or better. For this to happen it must the highest or second highest valuation when the last object is being allocated at one or more of the bottom k^{n-1} nodes in the outcome tree. Thus, V_K^1 matters at nodes where it compete with no more than one $V_j^1, j < k$. As a result, the other competing *k*-2 valuations must be the at most second highest valuations for the other *k*-2 bidders and it must be that $V_k^1 > V_l^i \; \forall l \neq \{k, j\}, 2 \leq i \leq n$. This allows for $V_k^1 \gtreqless V_j^2$. For example, in a 4-bidder 3-object auction with $V_A^1 > V_B^1 > V_C^1 > V_D^1$ and V_C^1 the 3rd highest valuation, V_D^1 is strategic if: i) $V_D^1 > \max\{V_A^2, V_B^2\}$ or ii) $V_D^1 > \max\{V_B^2, V_C^2\}$ or iii) $V_D^1 > \max\{V_A^2, V_C^2\}$. Note that i) is consistent with $V_C^1 > V_D^1 > \max\{V_A^2, V_B^2, V_C^2\}$ and $V_C^1 > V_C^2 > V_D^1 > \max\{V_A^2, V_B^2\}$. The same can be deduced from ii) and iii) and it follows that V_D^1 must be at most the $(n+2)^{th}$ highest valuation to be strategic. **QED**

Proposition 2: *In complete information second-price auctions with k bidders and n objects such that k ≤ n, with bidders with declining valuations, $V_j^1 > V_j^2 ... > V_j^n$ where V_j^i is bidder j's i^{th} highest valuation, and without loss of generality, $V_A^1 > V_B^1 > ... > V_K^1$, then bidder j is strategic if and only if its highest valuations is among the (2n) highest valuations.*

Proof: Let us assume, $V_A^1 > V_A^2 > ... > V_A^n > V_B^1 > V_C^1 >,,, V_K^1$ where bidder A can be likened to a monopsonist which must decide between buying only the last object, the last two, ..., the last *n*-1, or all *n* objects. If bidder A gets only the last object, bidder B gets between 1 and *n*-1 objects. When bidder B gets the remaining *n*-1 objects, then at the last node, it must be that: $V_A^1 > \max\left(V_B^n, V_C^1\right) > \min\left(V_B^n, V_C^1\right) > V_D^1 ... > V_K^1$. Bidder A wins the last object and V_1^C is the residual valuation if it is the $2n^{th}$ highest valuation. In such a case, bidder C is strategic even though it does not win any object. Bidder D can be strategic if bidder C wins at least one object. Consider the allocation with bidders A, B, C winning respectively 1, *n*-2, and 1 objects. At the last node of the equilibrium path in the outcome tree, we would have: $V_A^1 > \max\left(V_B^{n-1}, V_C^2, V_D^1, ..., V_K^1\right)$. Bidder D would be strategic if: $V_D^1 > \max\left(V_B^{n-1}, V_C^2\right)$. This could be supported by the following valuation ranking $V_A^1 >, ..., > V_A^n > V_B^1 >, ..., > V_B^{n-2} > V_C^1 > V_D^1$ in which case V_1^D is the $2n^{th}$ highest valuation. Similarly, V_K^1 can be strategic when bidder *K*-1 is allowed to win at least one object. Let us assume that the equilibrium allocation has bidder *K*-1 win *n*-*k*-2 objects and bidders A, B, ..., *K*-2 win one object each. Then at the last node, $V_A^1 > \max\left(V_B^2, ..., V_{K-2}^2, V_{K-1}^{n-k-1}, V_K^1\right)$. Bidder *K* is strategic if its first valuation is the maximum among the *k*-1 valuations in the parentheses. Given our assumptions, we must have: $V_A^1 >, ..., > V_A^n > V_B^1 > V_C^1, ..., > V_{K-2}^1 > V_{K-1}^1 > ... > V_{K-1}^{n-k-2} > V_K^1$. Thus, bidder *K* is strategic if its highest valuation is among the 2*n* highest valuation. **Q.E.D.**

The above results will be useful in our analysis of mergers, but they are interesting in their own right as they extend the litterature on auctions under complete information which tend to focus on auctions with small number of bidders and/or objects or symmetric bidders (Krishna, 1993, 1999; Katzman, 1999; Gale and Stegman and Jeddy et al. 2010).

Defining s_i as the number of strategic valuations associated with bidder *i*, the merger of firms produces two effects impacting on the allocation of objects and the price sequence. First, the new merged firm has weakly more strategic valuations than any of the firms involved in the merger had prior to the merger, and second, the merger weakly decreases the residual strategic valuation. The first effect tends to increase efficiency while the second tends to decrease it.

Proposition 3: *A merger will take place if and only if it decreases the residual strategic value of the auction.*

Proof: We have assumed that the individual firms' valuations are unaffected by mergers. As such, the mergers do not produce synergies in production and marketing activities. This simplifying assumption allows us to focus on the residual valuation.

Consider a 3-bidder 2-object auction and let bidders A and B merge. Without loss of generality, let $V_A^1 > V_B^1$. The merged firms will have valuations $\left\{V_A^1, V_B^1\right\}$ if $V_B^1 > V_A^2$ or $\left\{V_A^1, V_A^2\right\}$ if $V_B^1 < V_A^2$. In the latter case, one might think that bidder A would not have an incentive to merge because the merged firm ends up with its pre-merger valuations, but this is not necessarily the case. For example, consider the case $V_A^1 > V_A^2 > V_B^1 > V_B^2 > V_C^1 > V_C^2$. It is easy to verify that bidder A wins both object in the pre-merger equilibrium if $V_A^2 + V_B^2 > 2V_B^1$ and pays the residual strategic valuation V_B^1 for both objects. The equilibrium payoffs are $\left\{V_A^1 + V_A^2 - 2V_B^1, 0, 0\right\}$. If bidders A and B merge, the residual valuation is either V_C^1 or V_C^2 whether $V_A^2 + V_C^2 \gtrless 2V_C^1$. In the first case, the merged firm wins both objects, pays $p = \left(V_C^1, V_C^1\right)$ and ends up with a payoff of $V_A^1 + V_A^2 - 2V_C^1$ while in the second case it wins only one object with equilibrium prices and payoffs given by $p = \left(V_A^2 + V_C^2 - V_C^1, V_C^2\right)$ and $\left\{V_A^1 - V_C^2, 2V_C^1 - V_A^2 - V_C^2\right\}$. In both cases, post-merger equilibrium prices are lower than pre-merger prices[20] and when the residual valuation is at its lowest, the allocation is no longer efficient. It is easy to verify that mergers between bidders A and C and between B and C would produce the same prices as the pre-merger

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[20] If $V_A^2 + V_B^2 < 2V_B^1$ and $V_A^2 + V_C^2 < 2V_C^1$ the post merger price of the second object is lower than the pre merger price of the second object ($V_B^2 < V_C^2$) while the post merger price of the first object is lower than the pre merger price of the first object if and only if $V_B^2 - V_C^2 > V_B^1 - V_C^1$.

equilibrium because the strategic valuations remain the same with or without mergers. Thus mergers between A and C and between B and C would not be observed. **Q.E.D.**

Corollary: *A strategic firm has no incentives to merge with a non-strategic firm.*

Let us now consider a numerical example, based on the game described in Figure 4.1, to gain some insights as to how mergers can impact on the price trend, allocative efficiency and the seller's revenue. We will show that the seller's revenue may increase after the merger which is akin to what is known as a pro-competitive effect in the industrial organisation litterature. In what follows, we examine the equilibrium outcomes for three potential mergers A&C; B&C and A&B. It is initially assumed that valuations of bidders remain the same which would be the case if the merger could not create synergies between merging firms. We then discuss the implications of relaxing this assumption.

The pre-merger equilibrium is efficient. It is characterized by a constant price trend ($p_1 = p_2 = 6$) and payoffs for bidders A,B,C are $(4,2,0)$. Figure 4.2a illustrates the outcome tree when bidders A and C are merged. The valuations are ranked as follows: $V^1_{A\&C} = 10 > V^1_B = 8 > V^2_{A\&C} = 7 > V^2_B = 5$. As for the pre-merger equilibrium, the price trend is constant, but prices are lower: $p_1 = p_2 = 5$. Hence, the seller's revenue is $R^S = 10$ and the payoffs are given by: $\pi^{A\&C} = (10-5) = 5$ and $\pi^B = (8-5) = 3$. Since $\pi^{A\&C} > \pi^A + \pi^C$ the merger generates a net gain of 1 to be shared by A and C as per a pre-merger negotiation.

Figure 4.2b illustrates the outcome of an auction taking place after the merger between bidders A and B. The valuations are ranked as follows: $V^1_{A\&B} = 10 > V^2_{A\&B} = 8 > V^1_C = 7 > V^2_C = 5$. In contrast to the benchmark model, the price trend of this game is declining and prices are lower: $p_1 = 5$ and $p_2 = 3$. The seller's revenue falls to: $R^S = 8$ and the payoffs are given by: $\pi^{A\&B} = (10-3) = 7 > 6 = \pi^A + \pi^B$ and $\pi^C = (6-5) = 1 > 0$. It follows that bidder C also gains from the merger between bidders A and B. The allocation is inefficient.

Figure 4.2c illustrates the outcome of an auction taking place after the hypothetical merger between bidders B and C. The valuations are ranked as follows: $V_A^1 = 10 > V_{B\&C}^1 = 8 > V_A^2 = 7 > V_{B\&C}^2 = 6$. Prices are as in the benchmark case: $p_1 = p_2 = 6$. Hence, the seller's revenue is $R^S = 12$ and the payoffs are given by: $\pi^{B\&C} = (8-6) = 2$ and $\pi^A = (10-6) = 4$. The net gain of the merger is zero and as such this merger is less likely than the other two.

As already established by proposition 3 and its corollary, not all mergers are profitable. Mergers involving bidders with the highest valuations are more profitable because they tend to lower the residual valuation which in turn tends to induce inefficient allocations. The insight is similar to Katzman (1999)'s proposition 1. In our case, a merger makes it more likely that it will be profitable for the merging firms to "give up" one or more objects to get lower prices on the remaining objects. This inefficiency is caused by the elimination of competition among members of the merged firm as in Deneckere and Davidson (1985) and Mailath and Zemsky (1991). For regulators, pre and post merger market shares as indicators of competition could be misleading. Going back to our example, firm C does not get an object in the pre-merger equilibrium, but it wins one when firms A and B merge. The anti-competitive effect of the merger between bidders A and B is revealed by the changes in prices and seller's revenue. However, the seller's revenue need not always fall after a merger.

Proposition 4. *If the pre-merger allocation is inefficient, a merger can produce an efficient allocation and increase the sellers's revenue.*

Proof. See appendix A.

The above result is an illustration of second-best theory. Figure 4.3a illustrates the outcome tree of a game involving bidders A, B and C while figure 4.3b depicts the outcome tree of the game between merged firm A&B and firm C. The valuations are ranked as follows: $V_A^1 = 18 > V_A^2 = 17 > V_A^3 = 16.5 > V_B^1 = 15.39 > V_C^1 = 14.3 > V_B^2 = 14.2 > V_B^3 = 14.1 > V_C^2 = 13.5$ $> V_C^3 = 9.9$. It is easy to verify that the seller's post-merger revenue is slightly higher than the pre-merger benchmark: $R_{A\&B,C} = 43 > R_{A,B,C} = 42.92$. The merger improves upon a bad equilibrium in which the bidder with the three highest valuations, bidder A, acts as a dominant firm by giving up one object to bidder B to induce lower prices.[21] The merger between bidders A and B eliminates the competition between A and B and hence changes the incentives. The weakly higher prices under the merger generates higher gains for the merged firm relative to the gains of firms A and B prior to the merger because the gains under the merger are computed using the three highest valuations, V_A^1, V_A^2, V_A^3, as opposed to V_B^1, V_A^1, V_A^2 under the pre-merger equilibrium. Our result contrasts with the widely-held view that mergers are anti-competitive and lower welfare in the absence of post-merger synergies (e.g., Farrell and Shapiro, 1990).

In our analysis, we have assumed that a merger could not produce synergies between merged firms. In this case, the bidders' valuations are the same as in the absence of synergy. However, in the presence of synergies enhancing payoffs through cost savings, the post merger valuations of the merged firm become naturally higher. When the first valuations of the merged firm increase relatively more, the dominant firm/inefficiency effect alluded to earlier becomes more likely as the merged firm has more incentives to exploit the low valuations of its rivals by "giving up" objects to secure lower prices on subsequent objects. In contrast, when the last valuations increase relatively more, the merged firm will have incentive to get more objects.

4.3. Concurrent Marketing Mechanisms and the Performance of the Auction

Most studies about marketing mechanisms tend to compare one mechanism against another, as in Bulow and Klemperer (1996). The aim of this section is to analyze how the performance of sequential second-price auctions is impacted by the introduction of

[21] In our example, the allocation is inefficient and the price trend is declining. However, it is worth pointing out that sometimes the merger induces an efficient allocation even though the price trend is declining. In Katzman's (1999) analysis, a declining price trend was indicative of inefficiency. Thus, results derived from specific low-dimensional cases (i.e., *n*=*k*=2) may not generalize in higher dimensions.

concurrent mechanisms like the pre-attribution mechanism used in the marketing of hogs in Quebec. Between 1989 and 1994, all of the hogs produced in Quebec were sold through the daily electronic auction. Hog producers were critical of the performance of the auction and a pre-attribution mechanism was introduced to work alongside the daily auction. Hog processors were pre-attributed shares of the hog supply based on historical market shares. The price paid for these hogs was a US price minus a negotiated discount. As mentionned earlier, a third mechanism was introduced in 2000 and the relative importance of each mechanism changed over time.

Figure 4.4a illustrates the outcome tree of the benchmark model where valuations of bidders A and B are given by: $V_A^1 = 10 > V_B^1 = 8 > V_A^2 = 7 > V_B^2 = 6 > V_A^3 = 5 > V_B^3 = 3$. Bidder B wins the first two objects and pays $p_1 = 5$ and $p_2 = 4$ while bidder A wins the third object and pays $p_3 = 3$. The payoffs are $\pi^A = 7$ and $\pi^B = 5$. The price trend is declining and the allocation is inefficient. We are assuming that only one object is pre-attributed. Hence, there are two possible cases depending on who gets the pre-attributed object.

Figure 4.4b illustrates the outcome tree of the auction game when bidder A gets the pre-attributed object. Consequently, bidder A plays the auction game with valuations 7 and 5 because his highest valuation is used up on the pre-attributed object. The benchmark valuations of bidder B remains unchanged. In equilibrium, $p_2 = 5,\ p_3 = 5,\ \pi^S = p_{pre} + 10$, $\pi^A = (10 - p_{pre}) + (7 - 5) = 12 - p_{pre}$ and $\pi^B = 8 - 5 = 3$, where p_{pre} is the exogenously determined price for the pre-attributed object. In the benchmark case, prices were : $p_1 = 5, p_2 = 4$ and $p_3 = 3$. Thus the average auction price increases from 4 to 5. Prices on pre-attributed hogs were set in relation to a U.S. price, but prices over the 1979-2000 period reported in Larue et al. (2004, p.241) indicate that average daily auction prices in Quebec were systematically above (below) US prices after (before) the introduction of pre-attributions in 1994.

Figure 4.4c illustrates the outcome tree of the auction game when bidder B gets the pre-attributed object. Bidder B plays the auction game with valuations equal to 6 and 3 because his highest valuation is used up on the pre-attributed object. The valuations of bidder A are as in the benchmark case. Equilibrium prices and payoffs are $p_2 = 4, p_3 = 3, \pi^S = p_{pre} + 7, \pi^A = 7$ and $\pi^B = (8 - p_{pre}) + 2 = 10 - p_{pre}$. Thus, the average auction price decreases when the pre-attributed object is allocated to bidder B. This outcome happens because of the relatively low valuations of bidder B for the second and third objects. When the first object is pre-attributed to bidder B, bidder A is willing to let bidder B win the first object on the auction to get the last object at a very low price.

Proposition 5. *The introduction of a concurrent pre-attribution scheme may increase or decrease the average auction price and change the price trend and allocative efficiency of the auction.*

Proof: Consider an initial 2-bidder (A and B) 3-object second-price auction. The bidders' valuations are decreasing and ordered as: $V_A^1 > V_A^2 > V_A^3 > V_B^1 > V_B^2 > V_B^3$. This ordering supports several equilibrium allocations (bidder A winning 1,2 or all 3 objects), but it implies that the high-valuation bidder A always wins the last object. Let us define conditions C1, C2 respectively as $V_A^3 - V_B^1 > V_B^1 - V_B^2$, $V_A^3 - V_B^1 < V_B^1 - V_B^2$ and C3, C4 respectively as $V_A^2 - V_B^2 > V_B^2 - V_B^3$, $V_A^2 - V_B^2 < V_B^2 - V_B^3$. When C1 and C3 hold, two equilibrium allocations are possible. If C5 holds, $V_A^3 - V_B^1 > 2(V_B^1 - V_B^2)$, then bidder A wins all three objects and pays $p_{C1,C3,C5}^3 = (V_B^1, V_B^1, V_B^1)$. If C6 holds, $V_A^3 - V_B^1 < 2(V_B^1 - V_B^2)$, a declining price trend ensues, $p_{C1,C3,C6}^3 = (V_B^2 + (V_A^3 - V_B^1) - (V_B^1 - V_B^2), V_B^2, V_B^2)$, as bidder B wins the first object and bidder A wins the last two. Now, consider what happens when the first object is pre-attributed/sold to bidder B and the remaining two objects are auctioned. Two equilibria are possible. If C3 holds, then bidder A wins both objects at constant prices $p_{C3/B}^2 = (V_B^2, V_B^2)$. If C4 holds, bidder B wins the first object auctioned and bidder A wins the last one at a lower price as $p_{C4/B}^2 = (V_B^3 + V_A^2 - V_B^2, V_B^3)$. If the first object is pre-attributed/sold to bidder A, bidder A wins both objects when C1 holds and $p^2_{C1/A} = \left(V^1_B, V^1_B\right)$ while bidder B gets one and bidder A gets one when C2 holds. They then pay: $p^2_{C2/A} = \left(V^3_A - V^1_B + V^2_B, V^2_B\right)$. Comparing the average price of $p^3_{C1,C3,C5}$ with the average price of $p^2_{C3/B}$ and $p^2_{C1/A}$, we find that the average price is either lower or the same when one object is pre-attributed. If instead our benchmark is $p^3_{C1,C3,C6}$, it is easy to see that the average prices under $p^2_{C3/B}$ and $p^2_{C1/A}$ are respectively lower and higher than the average price when all three objects are auctioned. Under our assumptions, selling a pre-attributed object to the low valuation bidder lowers the average auction price. If two objects were pre-attributed, then equilibrium auction prices would be V^1_B, V^2_B and V^3_B if bidder A gets both pre-attributed objects, one or zero. The pre-attribution of two objects has an ambiguous effect on the average auction price. **QED**

Clearly, giving pre-attributed objects to bidders with rapidly decreasing valuations (like hog processors with low processing capacity) is not a good strategy for the seller, especially in the presence of a relatively large processor. The pre-attribution of hogs in Quebec was based on historical market shares. The data made available to us reveals a slowly declining market share for the largest processor over time which is consistent with the dominant firm-like behaviour.[22]

4.4. Empirical Analysis

In this section, we analyze prices generated by the Quebec daily hog auction. As hinted by our discussion about the evolution of the industry, there are a large number of events that could have induced structural changes, like the changes in the mix of marketing mechanisms displayed in Table 4.1. The merger of the two largest processors might also

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[22] This does not preclude other factors from impacting on market shares.

have caused structural changes in the prices generated by the daily auction. Because there is uncertainty regarding the number of structural changes, we must use an empirical approach that allows for several endogenous changes.

We use the flexible method developed by the seminal paper, Bai and Perron (2003), to determine endogenously the number of breaks and the dates at which they occured. Following several works (e.g., Ben Aissa and Jouini, 2003; Jouini and Boutahar, 2003 and Ben Aissa et al., 2004), the model and test statistics of the Bai-Perron recommended sequential procedure are briefly discussed below. A priori, we do not know which of the "events" we know about had a significant impact nor do we know whether bidders anticipated the events or responded with a delay to these events. Our data contains average weekly prices between 1996:1 and 2006:52. The data was provided by the FPPQ. We regressed the Quebec auction price on the U.S. reference price, total quantity of hogs available in the Quebec market, three dummy variables for seasonality and lagged dependent variables to account for marketing and biological production constraints. We consider the following multiple linear regression with *m* breaks (*m* + 1 regimes):

$$y_t = x'_t \beta + z'_t \delta_j + u_t \; ; \quad t = T_{j-1} + 1, \ldots, \; T_j \qquad (1)$$

for j = 1,..., *m*+1. Variable y_t is the observed dependent variable ("aucprice") at time *t*; $x_t(p \times 1)$ and $z_t(q \times 1)$ are vectors of covariates whose influence are respectively fixed and variable across regimes and β and δ_j (j = 1,..., *m*+1) are the corresponding vectors of coefficients; u_t is the disturbance at time *t*. The break points $(T_1, \ldots, \; T_m)$ are unknown such that $T_0 = 0$ and $T_{m+1} = T$. We set *p* = 1, the number of regressors x_t and *q* = 8, the number of regressors z_t, and *m* = 5 as the maximum number of breaks.[23] Each break date is asymptotically distinct and bounded from the boundaries of the sample. The estimation method is based on the least-squares principle proposed by Bai and Perron (1998). For each

[23] The test does not suffer from size distortions. There is no need to simulate critical values because the number of regressors is less than ten and the size of our sample is higher than 125 (see Prodan, 2008 for details).

m-partition $\left(T_1,\ldots,\ T_m\right)$ denoted $\left\{T_j\right\}$, the associated least-squares estimates of β and δ_j are obtained by minimizing the sum of squared residuals $\sum_{i=1}^{m+1}\sum_{t=T_{i-1}+1}^{T_i}\left(y_t - x_t'\beta - z_t'\delta_i\right)^2$. Let $\hat{\beta}\left(\left\{T_j\right\}\right)$ and $\hat{\delta}\left(\left\{T_j\right\}\right)$ denote the resulting estimates. Substituting them in the objective function and denoting the resulting sum of squared residuals as $S_T(T_1,\ldots,T_m)$, the estimated break points $\left(\hat{T}_1,\ldots,\ \hat{T}_m\right)$ are such that $\left(\hat{T}_1,\ldots,\ \hat{T}_m\right) = \arg\min_{(T_1,\ldots,T_m)} S_T(T_1,\ldots,T_m)$ where the minimization is taken over all partitions $(T_1,\ldots,T_m)$ such that $T_i - T_{i-1} \geq q$. Thus the break-point estimators are global minimizers of the objective function.

The sequential procedure consists of estimating the model with a small number of breaks that are thought to be necessary (or start with no break). It performs parameter constancy tests for every subsample (those obtained by cutting off at the estimated breaks), adding a break to a subsample associated with a rejection of the null hypothesis of no break using the $\sup F_T(l+1/l)$ test. This process is repeated increasing l sequentially until the test $\sup F_T(l+1/l)$ fails to reject the null hypothesis of no additional structural changes. As it was recommended by Bai and Perron (2003, 2006), a useful strategy is to first look at the UDmax or WDmax tests to see if at least a break is present.

Although the number of break dates can be determined by using the Bayesian Information criterion (BIC) suggested by Yao (1988) or the modified Schwarz criterion (LWZ) suggested by Liu et al. (1997), the sequential procedure is favoured because it directly addresses the presence of serial correlation in the errors and heterogeneous variances across segments (Bai and Perron, 1998). Bai and Perron (2006) compare the adequacy of different testing strategies in finite samples and in the presence of autocorrelation and/or heteroscedasticity. They show that even though the BIC works reasonably well in the absence of autocorrelation, sequential methods are still preferable. Several other studies have used the sequential procedure (e.g., see Jouini and Boutahar, 2003, Kerekes, 2007 and among others).

We conclude in favour of the presence of three breaks that correspond to the estimates found by the sequential procedure, using a null hypothesis of *m* breaks determined sequentially. These breaks are estimated at (2000:18); (2002:21) and (2005:20) with 95% non-overlapping confidence intervals given by [(2000:06); (2000:39)], [(2002:05); (2002:32)] and [(2005:11); (2005:22)], respectively.[24]

The introduction of the third mechanism in January of 2000 coincides with the first break identified by the sequential procedure. The second break corresponds to the date at which Olymel and Brochu, the two largest Quebec hog processors at the time, engaged in a partnership by purchasing Prince Foods, a processing firm specialized in bacon products. Olymel and Brochu submitted their merger proposol to Canada's Competition Bureau in October of 2004 and their merger was approved in April of 2005 which falls within the confidence interval of the third break. It was also announced on May of 2005 that three plants would close and that important capital investments would be made in three other plants.[25]

The identification of three breaks implies four regimes which are described in Table 4.2 and identified as R1, R2, R3 and R4. For each regime, we report coefficients along with their respective *p*-value. The parameter estimates associated with the U.S. price at date *t* in all four regimes are respectively 0.77, 0.55, 0.63 and 0.62, all with p-value close to zero. The changes in the coefficients suggest that the immediate impact of US hog prices on the Quebec auction price decreased substantially from the first to the second regime, but was not affected much when the third regime ended and the fourth one began.

We used matched pair tests, reported in table 4.2, to compare prices predicted over the same period by the current regime estimator and the previous' regime estimator. We found significantly higher prices after the introduction of the third mechanism, significantly

[24] The repartition procedure also used in Bai and Perron (2003) selects three break dates, two of which identical to the ones found by the sequential procedure: (2002:21) and (2005:20). However, the third date identified by the repartition procedure, (1999:14), end up being a full year before the introduction of the third mechansim in January of 2000 and hence is less plausible. The BIC procedure suggests a single break that is also identified by the other procedures: (2005:20).

[25] See http://www.ledevoir.com/economie/81238/olymel-supprime-366-emplois

decreased prices after the partnership and eventual merger between the two largest processors. Clearly, the smaller supply on the daily auction had a strong competitive effect offsetting the price-depressing effect of valuations transferred to the third mechanism. Moreover, Olymel's share had been declining and so had prices. This suggests that the dominant processor was giving up objects to get lower prices on the remaining ones.

4.5. Conclusion

In this paper, we have analyzed the impact of mergers and the introduction of concurrent marketing mechanisms on the performance of multi-unit sequential auctions under complete information with asymmetric bidders. Agricultural supply chains are characterized by high degrees of concentration at the processing and retail levels. In some cases, collective actions have led to the creation of producers-controlled marketing boards to counter the possible market power of processors and retailers. As shown in Table 4.1, the Quebec hog-pork sector has been experimenting with different combinations of marketing mechanisms in search of an ideal way of marketing hogs. Over the years, the relative importance of the electronic auction varied tremendously. Between 1989 and 1994, the electronic auction was the only mechanism in use, while starting in 2000, three different mechanisms were being used concurrently. Processing activities became even more concentrated when the two largest processors invested in a joint venture. We show that even in the absence of post-merger synergies, mergers can increase the seller's revenue and have pro-competition effects. This occurs when the pre-merger allocation is inefficient and the post-merger allocation is efficient. Such a peculiar result is new and specific to sequential auctions. Thus, whether a merger has pro-competition, anti-competition or no effects at all is an empirical question. The evidence produced through an endogeneous structural change test confirmed that the merger did have an impact, but an anti-competitive one on prices received by Quebec hog producers.

Finally, we have shown that a pre-attribution scheme used concurrently with the auction may increase or decrease the seller's revenue from the auction and change the price trend and efficiency, depending on how pre-attributed objects are allocated. Larue et al., (2004) had shown that long biological lags in hog production makes the supply very inelastic in the short run, thus making producers vulnerable to quasi-hold ups. In this context, a pre-attribution/price commitment scheme can improve the performance of auctions and this is what our empirical evidence confirms.

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Table 4.1. Evolution of hog supply shares sold on the three marketing mechanisms

	Shares of Hog Supply Allocated to Different Marketing Mechanisms		
Period	**% Pre-attribution**	**% Daily Auction**	**% Monthly Auction**
1996 :1 to 1997 :8	72%	28%	0%
1997 :9 to 1999 :8	76%	24%	0%
1999 :9 to 2000 :3	72%	28%	0%
2000 :4 to 2000 :52	60%	25%	15%
2001:1 to 2003 :53	55%	25%	20%
2004 :1 to 2006 :52	50%	25%	25%

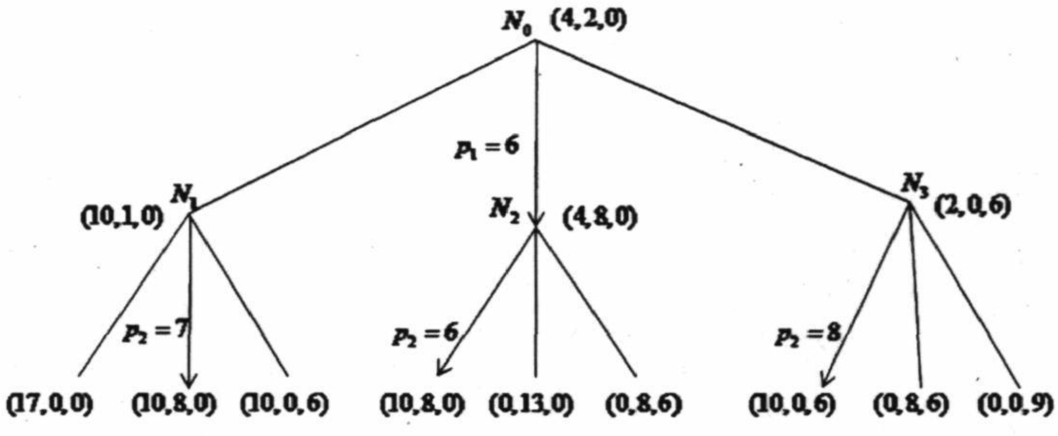

Figure 4.1. The outcome tree for the benchmark game. Bidder B wins the first object and bidder A wins the second. Prices are constant: $p_1 = p_2 = 6$.

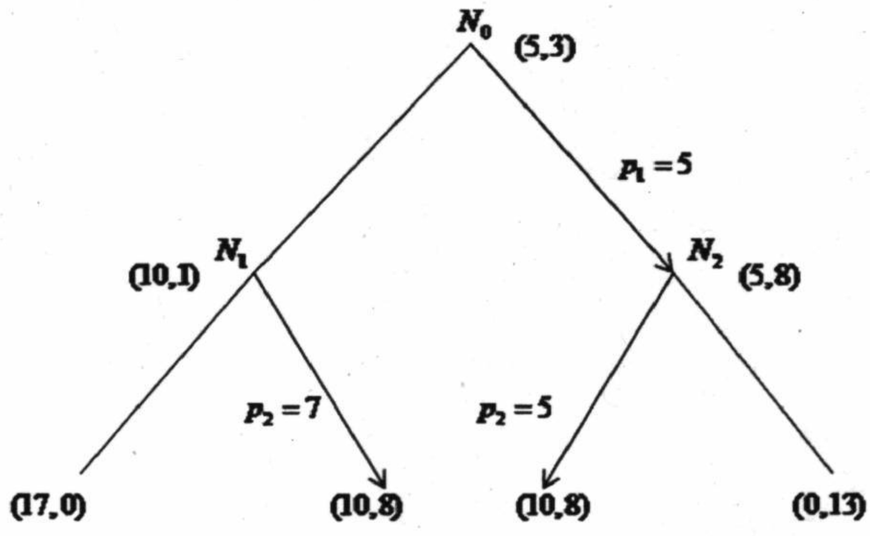

Figure 4.2a. The outcome tree for the merger between bidders A and C.

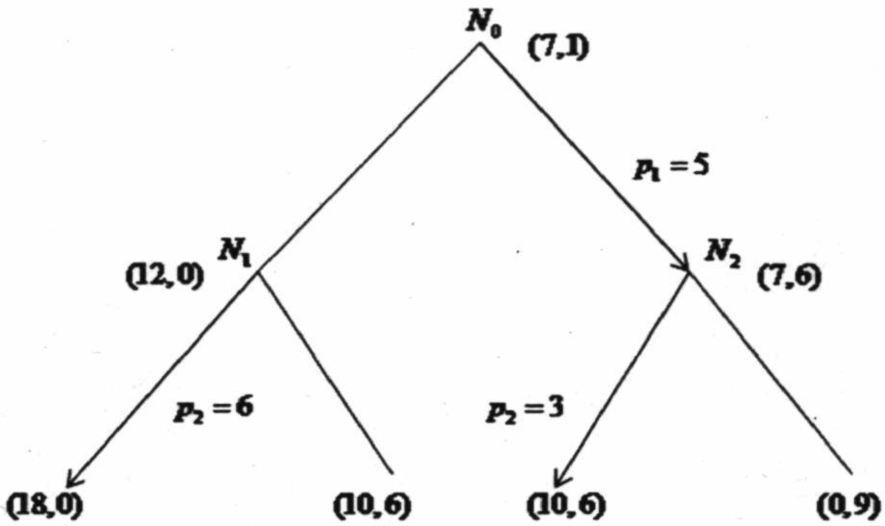

Figure 4.2b. The outcome tree for the merger between bidders A and B.

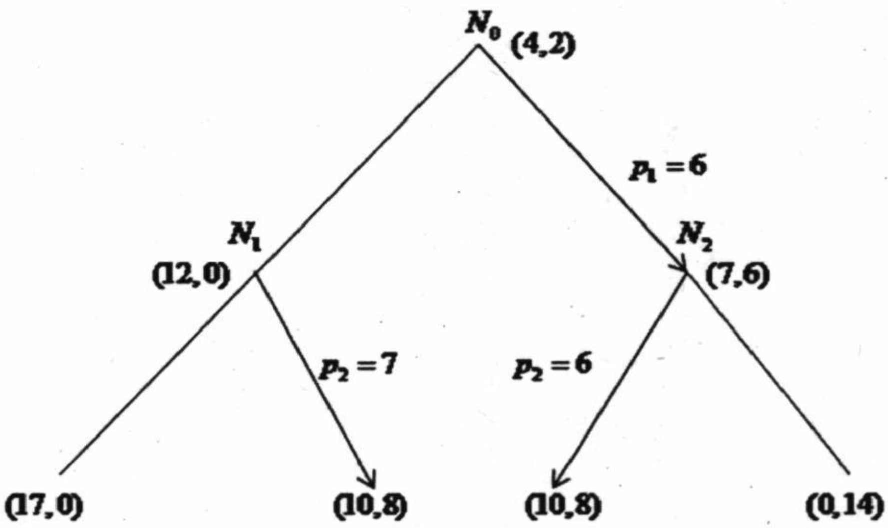

Figure 4.2c. The outcome tree for the merger between bidders B and C.

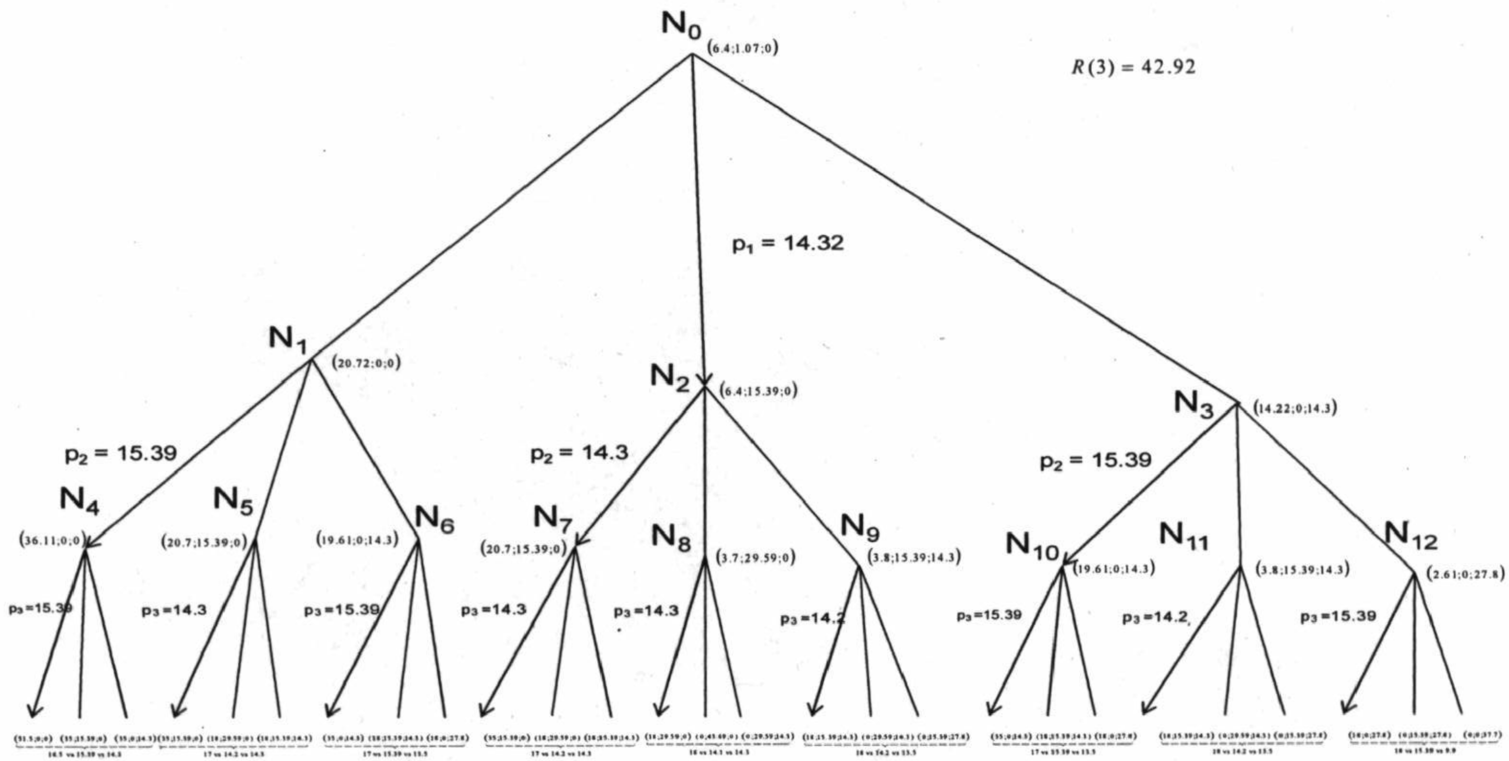

Figure 4.3a. The pre-merger auction game with three bidders and three objects.

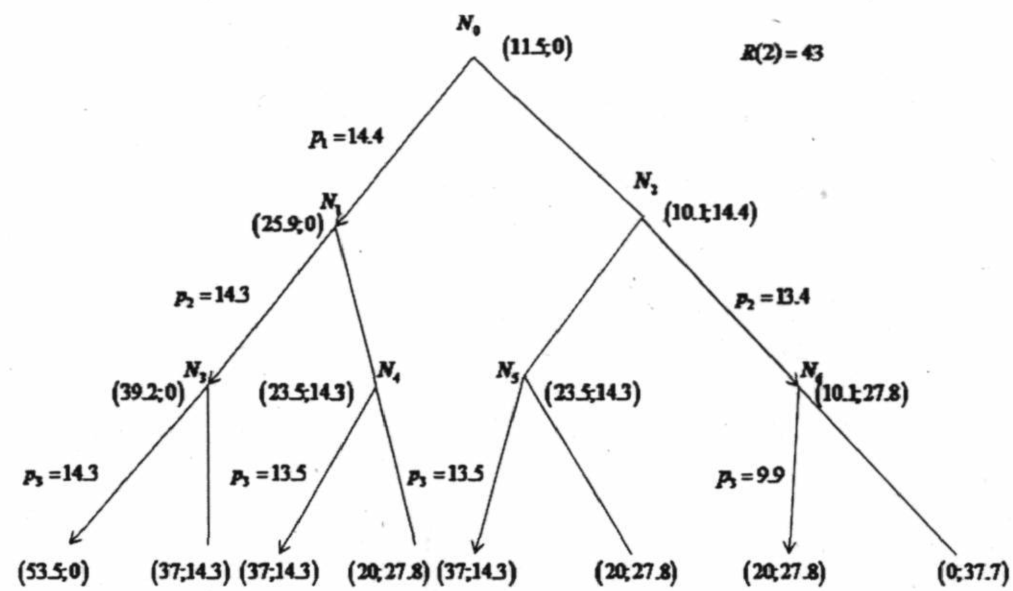

Figure 4.3b. The post-merger auction game with two bidders and three objects.

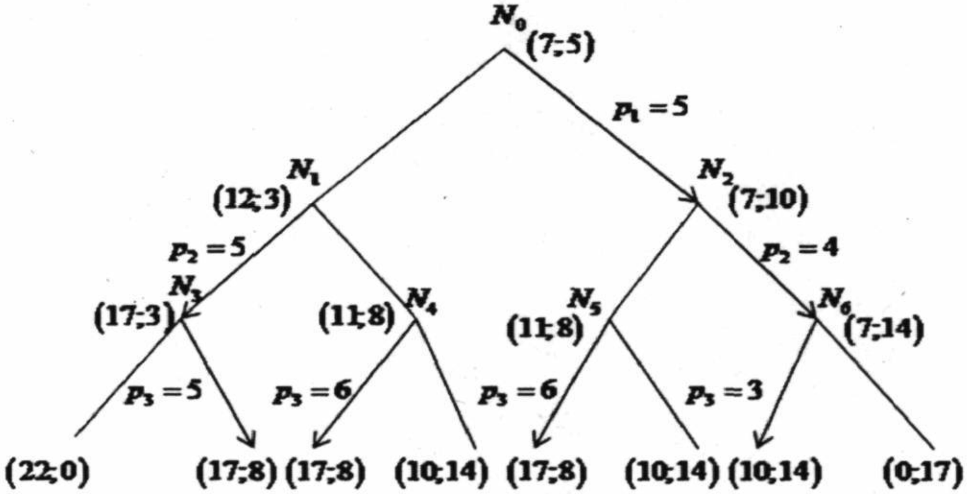

Figure 4.4a. The outcome tree of the benchmark case to analyze the introduction of a concurrent mechanism.

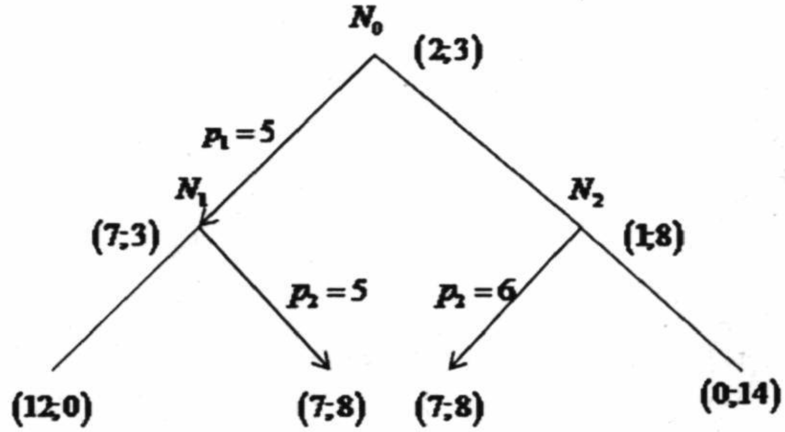

Figure 4.4b. The auction game when bidder A gets the pre-attributed object.

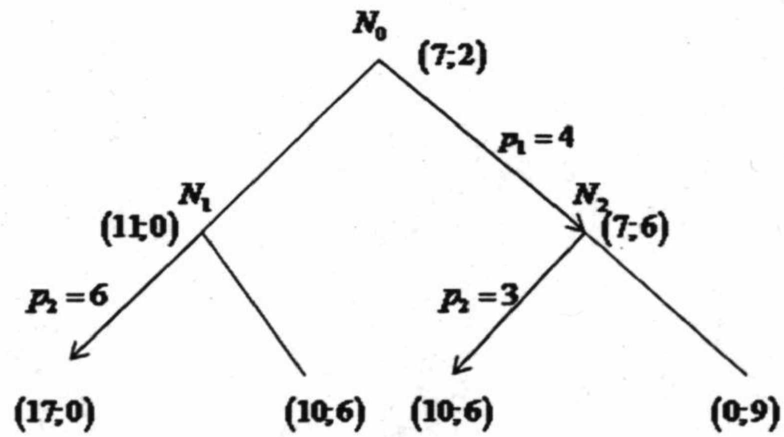

Figure 4.4c. The auction game when bidder B gets the pre-attributed object.

Table 4.2. Parameter estimates for each of the four identified regimes

	Regime 1		Regime 2		Regime 3		Regime 4	
observed mean price	151.38		171.31		151.9		135.28	
predicted mean price	-		$P^{R1} = 108.34$		$P^{R2} = 168.77$		$P^{R3} = 145.29$	
	-		$P^{R2} = 170.86$**		$P^{R3} = 152.11$**		$P^{R4} = 134.96$ **	
	Coefficient	P-Value	Coefficient	P-Value	Coefficient	P-Value	Coefficient	P-Value
aucprice(t-1)	0.357137	0	0.445065	0	0.553014	0	0.168963	0.026
aucprice(t-2)	-0.133176	0.004	-0.076974	0.331	-0.2427	0	0.103964	0.108
aucprice(t-37)	-0.005416	0.581	0.040761	0.062	-0.002429	0.874	0.118788	0
usprice(t)	0.766781	0	0.550153	0	0.630775	0	0.624878	0
qty(t)	0.000015	0.426	0.000029	0.315	0.000032	0.089	-0.000069	0.009
S1	4.004291	0	4.023262	0.003	6.974186	0	-3.190844	0.066
S2	4.064395	0	4.245774	0.002	5.545976	0	-2.088833	0.126
S3	0.753047	0.401	2.335926	0.115	0.179032	0.867	-0.343951	0.77
Constant	2.526223	0.347	2.526223	0.347	2.526223	0.347	2.526223	0.347

** indicates that predicted mean price P^{Rj+1} is statistically different from mean predicted price P^{Rj}.

4.7. Appendix

We assume that $V_A^1 > V_A^2 > V_A^3 > V_B^1 > V_C^1 > V_B^2 > V_B^3 > V_C^2 > V_C^3$ such that $V_A^3 + V_C^1 - 2V_B^1 > 0; V_A^2 + V_B^2 - 2V_B^1 > 0; V_A^2 + V_B^2 - 2V_C^1 > 0$ and $V_A^3 + 2V_C^1 - 3V_B^1 < 0$. The game is depicted by the outcome trees in Figure 4.A.1-4.A.4. Bidder B wins the first object and pays $p_1 = V_A^3 + 2V_C^1 - 2V_B^1$ while bidder A wins the other two objects and pays $p_2 = p_3 = V_C^1$. The outcome is inefficient and the seller's revenue is $R(3) = 4V_C^1 + V_A^3 - 2V_B^1$. Let us now consider the merger between bidder A and bidder B. The auction is a 2-bidder and 3-object game between the merged firm A&B and firm C such that: $V_{A\&B}^1 > V_{A\&B}^2 > V_{A\&B}^3 > V_C^1 > V_C^2 > V_C^3$; $V_{A\&B}^3 + V_C^2 - 2V_C^1 > 0$; $V_C^3 + V_{A\&B}^2 - 2V_C^2 < 0$ and $V_{A\&B}^3 + 2V_C^3 - 3V_C^1 - 2V_C^2 + 2V_{A\&B}^2 > 0$. Since the valuations of the merged firm is the maximum of its coalition member valuations, we have: $V_{A\&B}^1 = V_A^1; V_{A\&B}^2 = V_A^3$ and $V_{A\&B}^3 = V_A^3$. Figure 4.A.5 illustrates the outcome tree of this auction game. The merged firm wins all three objects and pays $p_1 = V_C^1 + 2V_C^2 - V_C^3 - V_A^2$ and $p_2 = p_3 = V_C^1$. The payoff of the merged firm is higher than the pre-merger payoffs of firm A and B: $\pi_{A\&B} = V_A^1 + V_A^2 + V_A^3 - 3V_C^1 - 2V_C^2 + V_C^3 + V_A^2 > \pi_A + \pi_B = (V_A^1 + V_A^2 - 2V_C^1) + (V_B^1 - V_C^1)$ and the merger is incentive compatible. The seller's revenue is given by: $R(2) = 3V_C^1 + 2V_C^2 - V_C^3 - V_A^2$. Therefore, $R(2) > R(3)$ if and only if $2V_B^1 + 2V_C^2 - V_A^2 - V_A^3 - V_C^1 - V_C^3 > 0$ (see p. 16 for a numerical example). The merger generates an efficient allocation and an increase in the seller's revenue. The payoff of bidder C is not affected by the merger as it remains zero. As such, the merger creates a Pareto improvement and is clearly pro-competition. **QED**

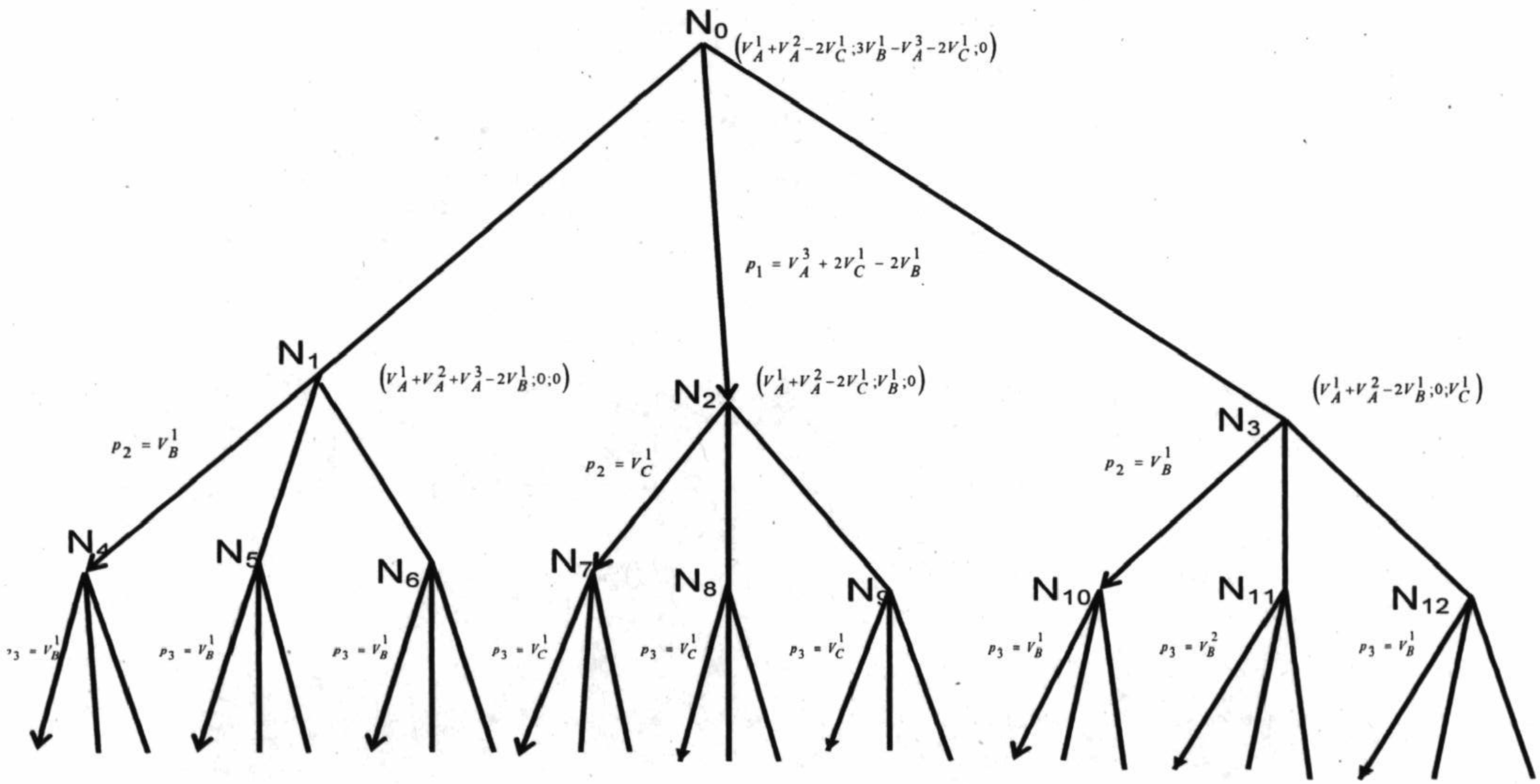

Figure 4.A.1. The pre-merger auction game with three bidders and three objects.

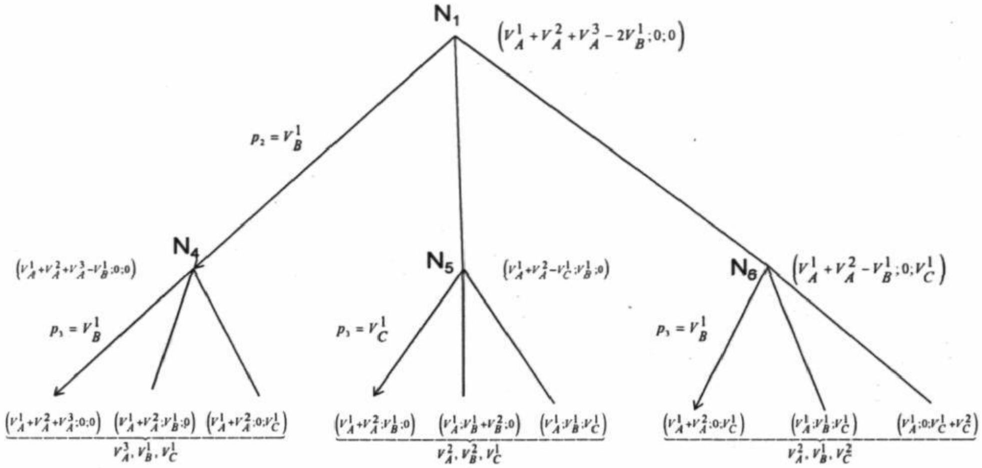

Figure 4.A.2 The outcome tree at node N1.

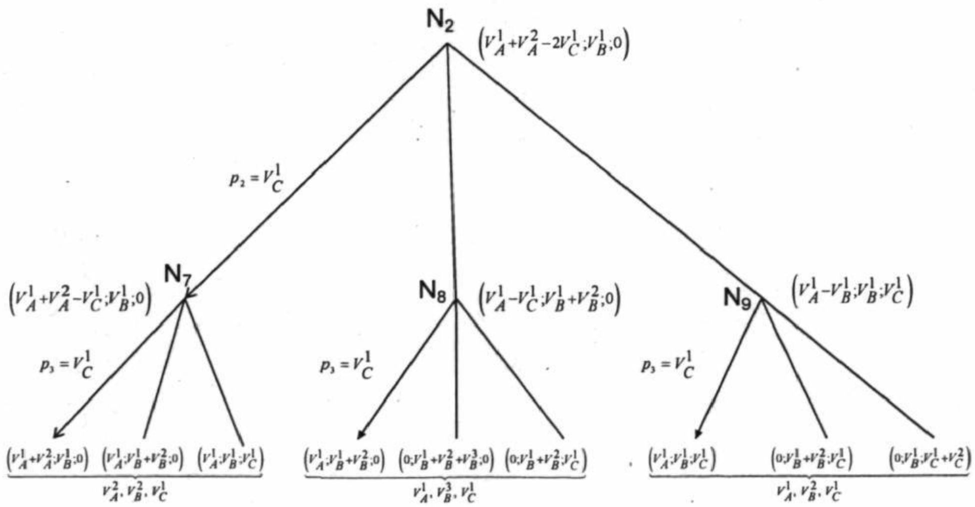

Figure 4.A.3 The outcome tree at node N2.

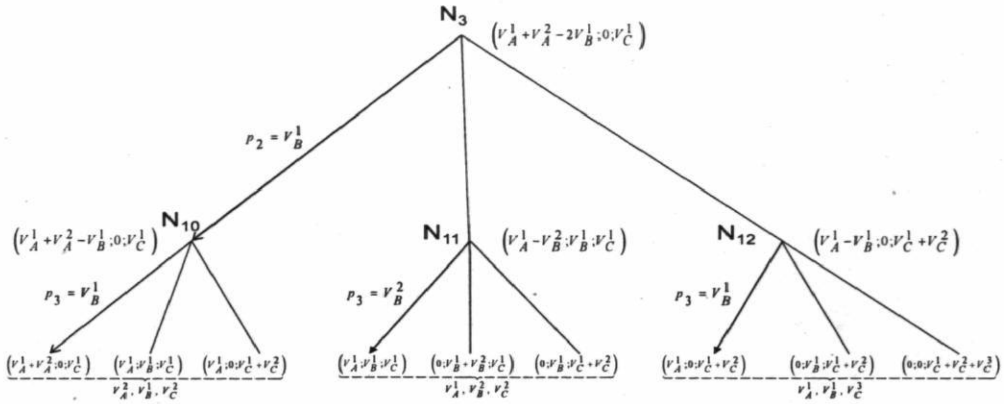

Figure 4.A.4 The outcome tree at node N4.

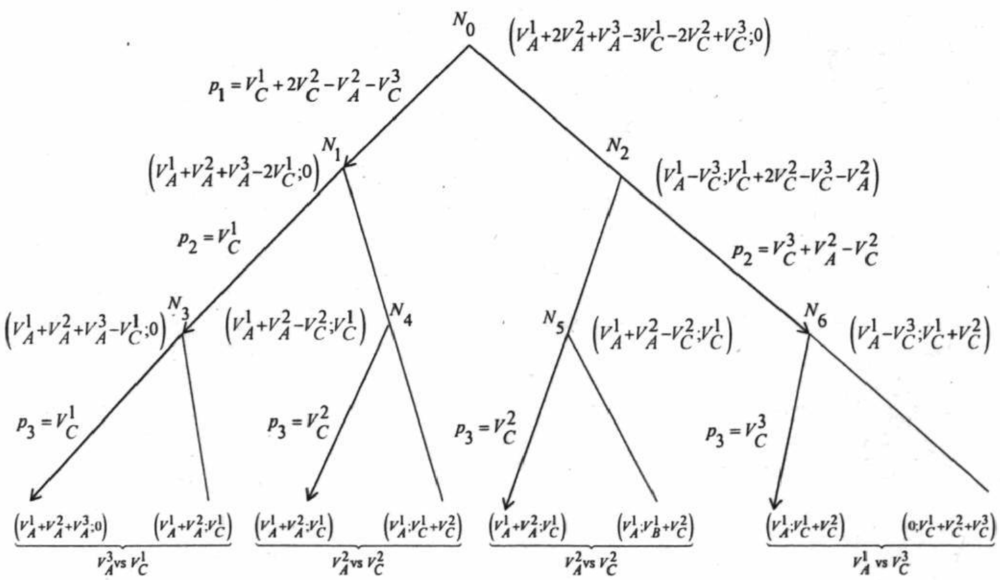

Figure 4.A.5. The postmerger auction game with two bidders and three objects.

5. Summary and Conclusion

Surprisingly little has been done on livestock auctions, despite their long history and ubiquity. As far as we know, our investigation is the first to theoretically and empirically analyze allocations, price trends and efficiency in sequential Quebec hog auctions, and to examine the performance of these auctions after mergers and the introduction of concurrent marketing mechanisms. Electronic livestock auctions are commonly used in countries, like the United Kingdom, Australia and Canada. The assumption of completely informed bidders is most realistic in the context of auctions that are repeated frequently involving the same bidders. Livestock auctions, especially the electronic kind in which bidders bid on homogenous virtual animals, are good examples.

The contribution of chapter 1 is twofold. First, it shows that asymmetric allocations of objects among completely informed and identical bidders in sequential auctions are to be regarded as a natural outcome, not as an exception. When allocations are asymmetric, prices are weakly declining (i.e., they fall but there could be flat segments) and payoffs are the same for all players, implying that bidders who win fewer objects get the ones sold at lower prices. In contrast, symmetric allocations which are possible under rather strict conditions on the bidders' valuations produce constant prices. The conditions required to support symmetric allocations become increasingly restrictive as the number of objects increase. The second note shows that the result about the existence of a unique Nash perfect equilibrium in two-bidder multi-unit sequential second-price auctions under complete information is not robust in higher dimensional auctions. Using an example of three-bidder four-object sequential second-price auction, we found equilibria characterized by different vectors of prices accompanied with either a single or different vector(s) of bidders' net payoffs.

Chapter 2 analyzes an important theoretical and empirical issue of the variation in the seller's revenue in the presence of additional bidders in sequential auctions under complete information. We theoretically show that the seller's revenue increases with the number of symmetric bidders, but this is not necessarily the case when bidders are asymmetric. Our empirical analysis of Quebec daily hog auctions between 1995 and 2006 shows that the seller's revenue significantly decreases with the number of invited bidders from Ontario.

Chapter 3 investigates the impact of mergers and the introduction of concurrent marketing mechanisms on the performance of multi-unit sequential auctions under complete information with asymmetric bidders. We show that even in the absence of post-merger synergies, mergers can increase the seller's revenue and have pro-competition effects. This occurs when the pre-merger allocation is inefficient and the post-merger allocation is efficient. Such a peculiar result is new and specific to sequential auctions. Thus, whether a merger has pro-competition, anti-competition or no effects at all is an empirical question. The evidence produced through an endogenous structural change test confirmed that the merger did have an impact, but an anti-competitive one on prices received by Quebec hog producers. Moreover, we point out that a pre-attribution scheme used concurrently with the auction may increase or decrease the seller's revenue from the auction and change the price trend and efficiency, depending on how pre-attributed objects are allocated. Larue et al., (2004) had shown that long biological lags in hog production makes the supply very inelastic in the short run, thus making producers vulnerable to quasi-hold ups. In this context, a pre-attribution/price commitment scheme can improve the performance of auctions and this is what our empirical evidence confirms.

The *electronic auction played an important role in hog marketing in Quebec.* Surely, an important question to be analyzed is whether the suspension of this mechanism and the creation of the new system improves the market conditions and increases the market price for producers. The elimination of the auction has reduced the flexibility of the marketing chain as the auction transmitted positive and negative shocks on the margins of hog processors to producers. Among several other important issues investigated in this thesis, we show how mergers may simultaneously increase the seller's revenue and improve efficiency in sequential second-price auctions. This is an interesting theoretical result because it confers an advantage to sequential second-price multi-unit demand auctions over other mechanisms in the context of mergers in auction games that are repeated frequently involving the same bidders.