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Essais en Théorie d'Intermédiation Financière

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Résumé

L'intermédiation financière assure la transformation de l'épargne accumulée dans l'économie en fonds disponibles pour le financement des projets. L'efficience de cette fonction de transformation d'actifs est soumise à un large éventail de facteurs de risque. Le risque de défaut des entrepreneur-emprunteurs potentiels et la volatilité des fonds d'épargne collectés sont les deux principales sources d'incertitude. Afin de surmonter les inefficiences causées par une information incomplète sur le risque de défaut des emprunteurs, les intermédiaires financiers pourraient vouloir adopter un design institutionnel permettant soit de réduire les incitations adverses des emprunteurs ou de dépister les projets à faible risque parmi ceux faisant l'objet d'un financement potentiel. Outre les frictions de nature informationnelle affectant le marché de crédits et le design institutionnel qu'ils peuvent adopter, les intermédiaires financiers sont également invités, par le biais d'une gestion active de leur bilan, d'optimiser la fonction de transformation d'actifs dans la perspective de la maximisation des avoirs des actionnaires. La fixation de la marge des taux d'intérêt, en fonction de la volatilité des dépôts et la qualité de crédit des actifs, est au centre d'une telle démarche.

Le premier essai fournit une investigation de l'impact de la mutualité comme étant un design institutionnel des intermédiaires financiers sur la capacité des ces derniers à faire face au comportement opportuniste des emprunteurs en termes de prise de risque. Un modèle de risque moral est développé dans lequel il est démontré que l'efficience de l'allocation de ressources achevée par les mutuels de crédit est intiment liée à leur schéma de partage du surplus commun. L'analyse met l'accent sur l'impact de l'interaction stratégique existent entre les membre-emprunteurs affiliés due à la propriété commune du surplus d'intermédiation généré par l'institution. Le modèle permet également d'apprécier les effets en termes de bien-être social liés à la combinaison de la

mutualité et l'intermédiation financière dans un contexte d'une surveillance coûteuse des emprunteurs. Les tests empiriques effectués à partir des données disponibles sur les unions de crédit américaines supportent de manière significative les prédictions théoriques du modèle.

Dans le même cadre de l'étude du design institutionnel des intermédiaires financiers, le second essai s'intéresse à l'émergence des formes hybrides d'intermédiation financière et le profile risque/collatéral des entrepreneur-emprunteurs qu'elles permet d'attirer dans un marché de crédit sujet à des frictions de nature informationnelle. Un modèle de sélection adverse est monté dans lequel il est démontré comment des formes hybrides de bailleurs de fonds institutionnels, tels que les banques mutuelles et les sociétés de capital-risque, pourraient émerger. L'analyse révèle que le rôle principal des banques mutuelles est de fournir de crédit aux entrepreneurs à faible risque de défaut, tandis que le capital-risque constitue un design organisationnel approprié pour les intermédiaires financiers voulant cibler les entrepreneurs à faible collatéral. Le modèle indique que la coexistence de ces instances de la finance hybride avec la forme standard de la finance incarnée par les banques est expliquée par des mécanicismes d'auto-sélection et de rationnement de crédit empêchant une certaine population d'entrepreneurs d'être financés par les banques.

Le dernier essai se préoccupe des déterminants de la politique de la marge des taux d'intérêt des banques. Un modèle de gestion de bilan bancaire est construit dans lequel les banques contrôlent de manière proactive leur exposition aux principales sources de risque d'intermédiation : le risque de défaut affectant le portefeuille de prêts et la fragilité des dépôts. L'analyse montre qu'une détérioration de la qualité des actifs ou une liquidité accrue des dépôts pousse les banques à hausser leur marge de taux d'intérêts. Toutefois, il est démontré que l'impact de la compétition dans l'industrie bancaire ne présente pas un effet prévisible. Pour les décideurs économiques, ceci limite de manière sérieuse le contenu informationnel des marges de taux d'intérêt observées en termes de pouvoir de marché des banques. Par ailleurs, le modèle offre des applications prometteuses dans des champs actuels d'intérêt reliés aux questions de la stabilité des systèmes financiers et la réglementation de capital des banques.

Summary

Financial intermediation ensures the transformation of savings into loanable funds. The economic efficiency of this asset transformation activity is subject to a wide range of risk factors. The credit risk of potential entrepreneur-borrowers and the volatility of collected saving funds are the main sources of uncertainty. To overcome inefficiencies due to the incomplete information available on the credit risk of borrowers, financial intermediaries would adopt an institutional design permitting either to reduce the adverse risk-taking incentives of borrowers or to screening low-risk projects. Beyond the informational frictions affecting the credit market and the institutional design they adopt, financial intermediaries are allowed, by the mean of an active asset-liability management, to optimize the asset transformation activity in the perspective of shareholders' value maximization. Setting the interest rate margin in function of both the volatility of deposits and the credit risk of assets is at the hart of such as policy.

The first thesis essay investigates whether mutuality as an institutional design of financial intermediaries would help to reduce inefficiencies arising from the adverse incentives of borrowers for undertaking high-risk projects. A moral hazard model is developed in which it is shown that the efficiency of the resources allocation achieved by credit mutuals is directly affected by the common surplus' sharing rule they are adopting. The analysis stresses the impact of the strategic interaction between affiliated borrower-members due to the joint ownership of the intermediation surplus. The model also permits to assess the welfare effects of combining mutuality and financial intermediation in a costly monitoring environment. Empirical tests conducted using the American credit unions data significantly support the theoretical predictions of the model.

In the same vein of the study of the institutional design of financial intermediaries, the second essay provides an account for the emergence of hybrid forms of financial intermediation and the risk/collateral profile of entrepreneur-borrowers they permit to attract in a credit market subject to asymmetric information frictions. An adverse selection model is proposed in which it is shown how hybrid forms of lenders using costly screening devices, such as mutual banks and venture capitalists, would emerge. The analysis reveals that the main function of mutual banking is to provide finance to low-risk entrepreneurs, while venture capital finance emerges as an attractive design for financial intermediaries providing finance to low-collateral entrepreneurs. The model shows that the coexistence of these instances of hybrid finance with standard bank finance is explained by both self-selection and credit rationing mechanisms that prevent some population of entrepreneurs to be served by standard banks.

The last thesis essay focuses on the determinants of the interest rate margin policy of banks. An asset-liability management model is built in which banks are allowed to actively manage their exposures to the main sources of intermediation risk: the credit risk of loans portfolio and the fragility of deposits. The analysis reveals that both a deterioration of the credit quality of assets and an enhanced liquidity of deposits pushes banks to increase their interest rate margins. The impact of interbank competition, however, is shown to be unpredictable making interest rate margins useless for policy makers to evaluate the market power of banks. Moreover, the model exhibits promising applications to recent issues related to the stability of financial systems and the regulation of banks capital adequacy.

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Mohamed Ridha Mahfoudhi

Table des matières

Contents

Essay 2:
Adverse Selection in Credit Market and the Emergence of Hybrid Finance

Essay 3:
Deposits Fragility, Others' Money Rents and Competition:
An Asset-Liability Management Model for the Bank Interest Rate Margin

Essay 1:

The Stick and the Carrot as Cooperation Device: A Model of Moral Hazard in Credit Mutuals with a Test

Abstract

Mutual credit institutions are a popular form of financial intermediaries in developed countries and now are becoming a key actor in developing economies. The theory of the firm, however, fails to account for the sustainability and the success of these institutions. This paper provides a line of arguments based on the theory of moral hazard in teams that help to assess the economic function served by credit mutuals. We develop an incentive model in which we show how the design of the strategic interaction between affiliated members is important to exploring this issue and test the model predictions using U.S. credit unions data.

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I. Introduction

Mutual credit institutions represent an intriguing form of financial intermediaries that have long captured the interest of economists. While the theory of the firm predicts severe agency problems between the owner-members and managers in these institutions (e.g., Rasmusen 1988), their sustainability and remarkable growth during the last century mitigate the importance given to this agency relationship. Interested in financial intermediation in developing countries, the literature of development economics offers alternative arguments based on the incentives and welfare theories that permit to appreciate the economic function served by mutual credit institutions. Indeed, studies examining credit cooperatives (Banerjee et al., 1994; Besley and Coate, 1995) and informal institutions such as *rotating savings and credit associations* (Besley et al., 1993, 1994) suggest that both the interaction between members and the cooperation device implemented are determinant for the efficiency of resource allocation achieved by these institutions.

In this paper, we study mutual credit institutions, which we term *credit mutuals*, such as *credit unions* and *savings & loans* operating in Canada and the United States.[1] We address the following main issue: How do the arrangements used by credit mutuals, distinguishing them from conventional stock banks, affect the efficiency of the resource allocation they achieve? We answer this question in the perspective of assessing the community-oriented economic function of credit mutuals: the internalization of deposits-taking and lending services, and the sharing of the common intermediation surplus that results.

We develop an incentive model in which the credit mutual (Principal) provides loan financing to members (Agents) having adverse incentives to undertake risky projects that do not maximize

[1] The term *mutuals* is generally used to designate groups of agents sustaining a production function cooperatively and operating under the economic principle of common surplus's sharing. From the point of view of the theory of the firm, although the owner-members of mutuals could be viewed as firms' shareholders, mutuals are different from common-stock companies essentially because of their governance model characterized by the one-man/one-vote rule (Fama and Jensen, 1983). We find mutuals in most of the financial intermediation's sectors such as credit allocation and insurance. The key feature distinguishing credit mutuals from other forms of mutuals operating in the real economy's sectors such as agricultural co-ops is that the owner-members are at the same time the suppliers and consumers of the products and services offered (Taylor, 1971).

the common intermediation surplus to be shared. We examine how mutuality devices influence the equilibrium outcome in this moral-hazard problem. By mutuality (or cooperation) device we essentially mean the sharing rule of the credit mutual's surplus. In fact, we examine two distinct sharing rule designs that are commonly used by real-world institutions. The first, based on bonuses, stipulates that members' compensations take the form of a fraction of interests paid on loans or received on deposits. The second design is more conservative and defines members' compensations as dividends on the subscribed capital shares.

The analysis reveals that a strategic interaction exists between the borrowers' choices of projects due to the key provision of mutuality, i.e., the sharing of the common surplus. This makes the moral hazard problem faced by credit mutuals similar to that occurring in teams (Holmström, 1982; Rasmusen, 1987).[2] We thus use insights from the theory of teams to investigate the efficiency of the equilibrium outcomes achieved by the two sharing rules considered in our analysis. In particular, we pay attention to whether the sharing rule adopted by the credit mutual encourages free-riding behavior among borrower-members. We also ask whether the members' compensation arrangements such as bonuses and dividends used by credit mutuals help to improve efficiency. The results we get are discriminating and, overall, imply that the bonus-based sharing rule is better from the point of view of equilibrium efficiency than that based on dividends. Moreover, we find that the heterogeneity in the borrower-members' population may reinforce the relative efficiency of the bonus-based sharing rule.

Furthermore, we extend analysis by allowing the credit mutual to implement a costly monitoring function. We ask how the mutuality device affects the ability of credit mutuals to perform the monitoring function efficiently. Since monitoring helps lenders to enhance efficiency in imperfect credit markets, the answer will be useful to assess the welfare effect of combining mutuality and

[2] Teams can be defined as groups of agents in which the individual action of each agent does not influence his own utility only, but also those of the rest of the group' members. In contrast to the Principal$-n$ Agents relationship, teams do not necessarily require the presence of a Principal. Rather, the interdependence of agents' actions is the key feature of teams. Labor-owned firms, partnerships, work groups in decentralized firms, university departments, and theater groups are different examples of teams.

financial intermediation in a costly monitoring environment.

Monitoring performed by nonmarket credit institutions has recently attracted the interest of economists. During the last decade, many (governmental and non governmental) credit programs inspired by the Grameen Bank of Bangladesh were implemented in numerous developing countries. The overwhelming success of the Grameen Bank's program is due to the use of group-based credit allocation according to which each member benefiting from the credit program monitors the other members of her or his group. The idea behind is that community-based monitoring, the *peer monitoring* mechanism (Stiglitz, 1990; Besley and Coate, 1995), generates a sort of social collateral and reputational effects that prevent opportunistic behavior. At first glance, the continuation of the credit program, and thus the access to the financing resources, will encourage members to monitor each other. Even though the thesis of peer monitoring is plausible in itself, it implicitly conjectures the social cohesion of the community of members, and most of its expected benefits are attainable only under this hypothesis. Indeed, Arnott and Stiglitz (1991) show that without a perfect observability of efforts among members, peer monitoring cannot mitigate moral hazard in the insurance market. In the context of credit cooperatives, Banerjee et al. (1994) find that without enforcement constraints such as an unlimited liability rule, members will not have incentives to monitor each other. Furthermore, in conformity with the hypothesis of social cohesion, Pitt and Khandker (1998) report that peer monitoring-based credit programs in Bangladesh are exclusively implemented in *small* rural communities.

Although credit mutuals are based on the same notions of cooperation as nonmarket institutions introduced in developing countries, their institutional and social environments is not appropriate for implementing functional peer monitoring as suggested by the studies cited above. Hence, answering the question of whether credit mutuals are able to perform conventional (or direct) monitoring efficiently is helpful to answering whether peer monitoring is indispensable to enhancing the efficiency of resource allocation achieved by credit mutuals. This issue is also of interest from the perspective of the moral-hazard theory, since it consists of asking how the sharing rule design can substitute for mechanism design in teams where the latter fails to be functional.

In the last part of the paper we confront the theoretical model with U.S. credit union data. The panel contains around 4,900 institutions observed over the period of 1994−1999. Observed variables permit us to test only for the incentive effects of the two sharing rule designs studied. The empirical analysis of the monitoring function, however, requires information more refined than that available. Nevertheless, the relatively long time-horizon for the data and the clarity of variables reported have enabled us to evaluate the testable predictions of the theoretical model accurately enough. The tests we implement permit us to discriminate between the incentive schemes of the two sharing rules designs examined in the paper (i.e., the bonus-based and dividend-based sharing rules).

The rest of the paper is organized as follows. Section II presents the model and the institutional rules governing credit mutuals. Section III characterizes the equilibrium in the projects selection game and discusses its efficiency properties. Section IV assesses the efficiency and welfare effect of endowing credit mutuals with a costly monitoring function. Section V discusses the robustness of the model. Section VI contains the empirical analysis. Section VII concludes. Proofs are regrouped in the Appendix.

II. The Model

The model we develop is inspired by the salient features of credit mutuals operating in North America such as credit unions and savings & loans. We describe their institutional environment and the contractual arrangements they employ that differentiate them from stock banks.

Consider a credit mutual offering both deposit and lending services to its client-members. The economy lasts for two dates− 0 and 1. There are two members, i and j, each endowed with a project idea that requires an initial investment l_h $(h = i, j)$ at date 0. Without loss of generality, projects are entirely financed through loans that are supplied by the credit mutual at the gross interest rate R_L. For simplicity, members other than i and j subscribe to deposit shares the credit mutual issues at date 0 to finance loans. Deposits promise a gross interest rate R_D $(1 < R_D < R_L)$

at date 1, and their total amount, which we denote by D, exactly balances the total loans amount, so that the budget condition of the credit mutual at date 0 is $D = l_i + l_j$. For simplicity, we suppose that loans are entirely funded upon deposits. Our analysis could be easily extended to allow loans to be financed by a mix of deposits and the capital of the credit mutual. However, this would not add much intuition, since we focus on the behavior of borrower-members in terms of risk-taking rather than on the balance-sheet risk management policy of the credit mutual.

A. *Investment Projects*

At date 1, each project returns with probability p, a strictly positive payoff z if it succeeds, and nothing otherwise. Each project has a distinct distribution of success. We assume that by choosing a particular technology or management style at date 0, each borrower-member h ($h = i,\ j$) is *ex ante* able to select a target level for the likelihood, p, of the success of his project. Once the desired level for p is chosen at date 0, the date 1-project payoff is randomly generated by nature. The selection of the likelihood p_h of project success by each borrower-member h is a nontrivial choice problem. The set of risk-return opportunities in the economy stipulates that: (a) high levels of p are matched with low payoffs z, and vice versa; (b) the payoff z drops at an increasing marginal return to scale as the risk taken decreases. Formally, we assume the following.

Assumption 1. *The payoff function* $z(p_h) : (0, 1] \longrightarrow \mathbb{R}_+$ *verifies* $z',\ z'' < 0$ *for each borrower-member* $h = i,\ j$.

Given that the profitability of projects affects the borrower-members' choice of risk, making them reluctant to choose low-risk projects, the set of viable projects for each borrower-member h only includes those satisfying the inequality $p_h \leq \bar{p}_h$ where $\bar{p}_h = z^{-1}(R_L l_h) < 1$.

B. *The Sharing Rule's Design*

Since borrower-members' projects are risky, the date 1-recovery value of loans is uncertain. The mutual is exposed to the default risk of borrowers and, hence, may default on the repayment of the promised amount of capitalized deposits $R_D D$. As in the context of stock banks, depositor-members are the prior claimants if the credit mutual defaults (i.e., the usual absolute priority rule protecting depositors applies). Otherwise, the resulting intermediation surplus (i.e., the strictly positive margin between the recovery value of loans and the amount of capitalized deposit shares) is immediately transferred to members. This is the key feature distinguishing credit mutuals from stock banks.[3]

Let x_h denotes the risky payoff promised by the borrower-member h's project ($h = i,\ j$). As assumed in the model, $x_h = z(p_h) \geq R_L l_h$ with probability p_h and zero otherwise. Therefore, the date 1-recovery value v_h of the borrower-member h's loan to be received by the credit mutual is,

$$v_h = \min\left[x_h,\ R_L l_h\right], \tag{1}$$

Hence, the distribution of the total recovery value of loans, revealed at date 1, is:

$$\begin{array}{lll} \text{state 1:} & R_L(l_i + l_j) & \text{with probability} \quad p_i p_j, \\ \text{state 2:} & R_L l_i & \text{with probability} \quad p_i(1-p_j), \\ \text{state 3:} & R_L l_j & \text{with probability} \quad (1-p_i)p_j, \\ \text{state 4:} & 0 & \text{with probability} \quad (1-p_i)(1-p_j). \end{array}$$

It then follows that the date 1-surplus of the credit mutual, given the absolute priority of depositors, is

$$\pi = \begin{cases} R_L(l_i + l_j) - R_D D = (R_L - R_D)D > 0 & \text{state 1,} \\ \max\left[0,\ R_L l_i - R_D D\right] & \text{state 2,} \\ \max\left[0,\ R_L l_j - R_D D\right] & \text{state 3,} \\ 0 & \text{state 4.} \end{cases} \tag{2}$$

Remark that we allow the mutual's surplus at states 2 and 3 to take a strictly positive value or zero depending on the numerical values of the model parameters. This permits us to keep the model as general as possible. Since we suppose that there are only two borrower-members, it can

[3] As in the context of stock banks, depositors are assumed here to be the prior claimants by virtue of the deposits-insurance mechanism. Hence, they are not expected to play an active role in the model.

be supposed that, for not largely dispersed loans amounts and reasonable levels of interest rates, the credit mutual's surplus at these two intermediate states of nature will be likely equal to zero. However, one can imagine that the borrower j in our model represents what could be the whole community of borrower-members other than the borrower i in a generalized n-borrower setup. Moreover, we will demonstrate later in the paper that our model is robust whenever we allow for more than two borrower-members.

We are now ready to introduce the two distinct designs of the surplus sharing rule widely implemented by credit mutuals in real life.

B.1 *The Bonus-Based Sharing Rule*

The bonuses received by members under the *bonus-based sharing rule* (BBSR) take the form of refunded interests for borrower-members and supplementary interests for depositor-members. This constitutes what we call the *bonus provision*. The eligibility of members to receive bonuses under the BBSR is *ex post* contingent to their contributions to the mutual's surplus. Thus, only borrower-members who do not default on their loans are eligible to receive bonuses: this is the *penalty provision*.

Note that because depositor-members do not borrow funds in our model, they receive bonuses out of the mutual's surplus whenever it is strictly positive. In the real world, this dichotomy between depositors and borrowers does not exist, since members are often depositor and borrower agents at the same time. In this regard, it is important to note that even in the absence of this dichotomy, the penalty provision still applies to determine whether borrower-members, even those who also subscribed to deposit shares, are eligible to receive bonuses on loans. In fact, bonuses apply separately to deposits and loans accounts rather than to the members holding these accounts. This makes clear that the dichotomy we impose ensures that our theoretical model entirely reflects the mechanisms governing the BBSR implemented by credit mutuals.

Let b_h denote the bonus that the borrower-member $h = i, j$ receives at date 1 upon the distribution of the credit mutual's surplus. Given the penalty provision, we have

$$\begin{aligned}
&\text{state 1:} \quad b_i = \alpha(R_L - 1)l_i > 0, \quad && b_j = \alpha(R_L - 1)l_j > 0,\\
&\text{state 2:} \quad b_i = \hat{\alpha}_i(R_L - 1)l_i \geq 0, \quad && b_j = 0,\\
&\text{state 3:} \quad b_i = 0, && b_j = \hat{\alpha}_j(R_L - 1)l_j \geq 0,\\
&\text{state 4:} \quad b_i = 0, && b_j = 0,
\end{aligned}$$

which in turn yields,

$$b_h = \begin{cases} \alpha(R_L - 1)l_h > 0 & \text{with probability} \quad p_h p_{-h}, \\ \hat{\alpha}_h(R_L - 1)l_h \geq 0 & \text{with probability} \quad p_h(1 - p_{-h}), \\ 0 & \text{with probability} \quad (1 - p_h), \end{cases} \tag{3}$$

for any $(h, -h) \in \{(i, j), (j, i)\}$; where for each $h = i,\ j$, $\alpha > 0$ represents the bonus percentage rate used for setting eligible borrowers' compensations, and $\hat{\alpha}_h = \alpha$ if the parameters $(R_L, R_D, (l_h/D))$ are such that $R_L l_h - R_D D > 0$ and zero otherwise.

B.2 *The Dividend-Based Sharing Rule*

Under the *dividend-based sharing rule* (DBSR), the credit mutual's surplus is transferred to members in the form of dividends on capital shares. For credit mutuals, capital shares represent a sort of membership cost. The payment of dividends to members depends on whether the mutual's surplus is strictly positive. Thus, in contrast to the BBSR, borrower-members receive dividends as long as the credit mutual does not default on its obligations.

Define d_h as the dividend the borrower-member $h = i,\ j$ receives at date 1. Therefore, we have,

$$d_h = \beta_h \pi, \tag{4}$$

where β_h denote the fraction of the capital held by the borrower-member h. Note that under a one-share/one-member rule, this fraction simply reduces to $1/N$, where N is the total number of members.

III. Equilibrium Projects and Efficiency

In this section, we investigate the projects selection problem of borrower-members. We examine the properties of equilibrium projects and the impact of the sharing rule's design on the equilibrium outcome.

A. *Projects Selection*

Given Eq. (1), the residual payoff to be received by each borrower-member $h = i,\ j$ after the reimbursement of his loan is given by,

$$e_h = \max\left[0,\ x_h - R_L l_h\right], \tag{5}$$

Hence, given (2)−(5), the terminal wealth of borrower-member h, which we denote by W_h for any likelihood p_h of the project's success, is distributed as follows:

Under BBSR:

$$\begin{aligned} W_h &= (e_h + b_h) \\ &= \begin{cases} z\left(p_h\right) - l_h\left[R_L - \alpha(R_L - 1)\right] & \text{with probability} \quad p_h p_{-h}, \\ z\left(p_h\right) - l_h\left[R_L - \hat{\alpha}_h(R_L - 1)\right] & \text{with probability} \quad p_h(1 - p_{-h}), \\ 0 & \text{with probability} \quad (1 - p_h), \end{cases} \end{aligned} \tag{6}$$

Under DBSR:

$$\begin{aligned} W_h &= (e_h + d_h) \\ &= \begin{cases} z\left(p_h\right) - R_L l_h + \beta_h(R_L - R_D)D & \text{with probability} \quad p_h p_{-h}, \\ z\left(p_h\right) - R_L l_h + \beta_h \max\left[0,\ R_L l_h - R_D D\right] & \text{with probability} \quad p_h(1 - p_{-h}), \\ \beta_h \max\left[0,\ R_L l_{-h} - R_D D\right] & \text{with probability} \quad (1 - p_h)p_{-h}, \\ 0 & \text{with probability} \quad (1 - p_h)(1 - p_{-h}), \end{cases} \end{aligned} \tag{7}$$

for any $(h, -h) \in \{(i, j), (j, i)\}$. It is worthwhile to note that the terminal wealth of a *similar* borrower-client of a stock bank, which reduces to the project's residual return, e_h, can be derived through our model by simply substituting the bonus (dividend) rate α (β) by zero under the BBSR (DBSR) of the credit mutual.

The two borrowers are risk-averse decision makers, with preferences toward risk described by a strictly increasing and concave utility function U. Without loss of generality, let $U(0) = 0$. At date 0, once loan funds are in place, each borrower-member proceeds to the selection of his optimal project. The program of each borrower h consists of selecting the optimal likelihood of project's success that maximizes his expected utility function $E[U(W_h)]$ subject to $p_h \leq \bar{p}_h$.

In contrast to the context of a stock bank, the terminal wealth of each borrower-member of the credit mutual depends not only on his own choice of risk, but also on the risk taken by the other borrower (see Eqs. (6) and (7)). The mechanism is that the risk taken by each borrower influences the mutual's surplus, and, thus, affects the terminal wealth of his *neighbor*. In addition to the idiosyncratic risks of their own projects, borrower-members, therefore, bear a common uncertainty, i.e., the mutual's surplus. In a rational anticipations setup, this results in a strategic interaction between the borrowers' optimal projects, implying that the projects selection problem consists of a strategic game between borrower-members.

In this projects selection game, each borrower's strategy corresponds to a given level of the likelihood of project success. Further, the payoff function $z(.)$ and rationality are common knowledge for borrower-members.

The credit mutual cannot observe the projects' payoffs revealed at date 1, and thus borrower-members' compensations cannot be set in function of the likelihoods of projects' success chosen at date 0. Rather, only the repayment capacity at date 1 of borrower-members is received by the credit mutual as a verifiable but imperfect signal of the (hidden) riskiness of projects undertaken.

Banerjee et *al.* (1994) examined a similar moral-hazard problem between a credit mutual and a borrower-member. Our model, however, differs from the latter by allowing for more than one borrower-member. Each borrower's choice of project influences the terminal wealth of the other borrower by affecting the credit mutual's surplus to be shared between members. The strategic interaction resulting from the common uncertainty faced by the borrower-members (i.e., the credit mutual's surplus) makes the incentive problem we analyze similar to that occurring in teams. Issues related to moral hazard in teams were examined by Holmström (1982), Mookherjee (1984), and Rasmusen (1987).[4]

B. *Equilibrium Projects under the BBSR*

Lemma 1. *Under the BBSR, the Nash best-replies of borrower-members in the projects selection game are increasing correspondences, thus implying no free-riding.*

Proof. See the Appendix.

In the language of game theory, Lemma 1 means that the projects selection game is *supermodular.*[5] More intuitively, this result tells us that the risk-taking strategies of borrower-members are "strategic complements", in the sense that each borrower-member would increase the likelihood of success of his project (i.e., take less risk) if he anticipates that the other borrower is doing so as well. In other words, borrowers have no incentives to free-ride each other.[6] This differs from the general context of teams, where it has been shown that the incentive of members to free-ride each others' efforts is the main source of deficiency.[7] Therefore, Lemma 1 raises the following question: Why do borrower-members not free-ride each other? In fact, although the projects' outcomes are completely observable, one can see that each borrower is able to free-ride the other borrower's action by increasing the risk of his own project. This is because the credit mutual cannot verify the likelihoods of success of the initially chosen projects, but only the projects' outcomes realized

[4] Because the aggregated output in teams results from the whole efforts or actions of members, the question of free-riding is critical when analyzing moral-hazard problems. As shown by Holmström (1982) and Mookherjee (1984), the sharing rule of the aggregated output critically influences the outcome of the incentive problem in teams.

[5] Moreover, the supermodularity of the projects selection game ensures that the set of Nash equilibria is nonempty. For a more detailed discussion of the properties of supermodular games and their applications, see Vives (1999).

[6] In fact, free-riding should be reflected by decreasing best-reply correspondences. This is because in such a case, each borrower has an incentive to take more risks when he anticipates that his *neighbor* (the other borrower) is doing the contrary.

[7] See Holmström (1982, p. 326).

at date 1. Therefore, the observability of individual project's outcome alone cannot explain in full the absence of the free-riding problem. The answer consists of the incentive effect of the sharing rule. Under the BBSR each borrower is compensated upon his own project's outcome. This implies that the borrowers' compensations (bonuses) are fully separable because of the separability of the projects' outcomes. Consequently, each borrower cannot extract wealth from his neighbor's action. The separability of individual outputs (i.e., projects' outcomes here) plays a key role. In the general theory of teams, Holmström (1982) shows that a linear sharing rule, combined with inseparable individual outputs, inevitably leads to free-riding.[8] However, he also finds that the Nash equilibrium efforts of team's members yield an efficient common outcome (Pareto optimal outcome) whenever a penalty provision is implemented.

To gain more intuition of the role of the penalty provision under the BBSR, we note that although borrower-members receive fully separable compensations upon their separable individual outputs, they still initially bear a common uncertainty due to sharing of the credit mutual's surplus. Given the fact the mutual's surplus is randomly (exogenously) generated by nature at date 1, one can expect that an *ex post* free-riding problem (*ex post* extraction of rents) may persist. That is, even though a borrower cannot extract wealth systematically from his neighbor's action, he can benefit *ex post* from the occurrence of a favorable state of nature. The role of the penalty provision is critical here, since it diffuses an *ex ante* threat effect that discourages borrowers whom initially anticipating to benefit from this *ex post* free-riding mechanism. It is worthwhile to note that although borrowers are exposed to individual penalties because of the separability of the projects' outcomes, a common penalty could be implemented elsewhere.[9]

[8] A *linear sharing rule* allows members to receive a nonzero fraction of the aggregated output independently from the level of that output, and hence independently from the actions they have undertaken.

[9] Holmström (1982) shows that in the context of not separable individual outputs, a common penalty indexed on the aggregated output serves to police team members effectively. In fact, the effectiveness of the penalty provision does not rely on the separability of individual outputs, but on the credibility of the threat it generates. The required condition for a credible threat effect consists of bringing a party, a Principal, who will assume the residual outcome when punishment applies, which does not permit agents to renegotiate or to cancel penalties once applicable. Indeed, self-imposed penalties are not sustainable, since they do not achieve a perfect equilibrium.

While there is no free-riding under BBSR, different behavioral pattern acting in opposite directions do occur. Indeed, Lemma 1 means that each borrower-member would increase his project's risk if he anticipates that the other borrower-member is doing so, and vice versa. Borrowers have, therefore, an incentive to imitate each other. This herding phenomenon is due to the strategic complementarity between the borrowers' choices of projects. Even though our model supposes identical borrowers' preferences toward risk, one can intuitively expect that the intensity of this herding behavior must decrease as the levels of borrowers' risk aversions become largely dispersed. Interestingly, the credit mutual can use this endogenous mechanism of herding to enhance efficiency. The next result shows how.

Proposition 1. *Increasing the bonus rate α makes the Nash equilibrium point shifting upward near the first-best solution, i.e., the corner point $(\bar{p}_i, \bar{p}_j)$.*

Proof. See the Appendix.

The BBSR generates an incentive effect reflected by the negative equilibrium relationship between the borrowers' choices of risk level and promised bonuses. The rationale is straightforward. The bonus received at date 1 compensates for a fraction of the loss in term of project's payoff that each borrower *ex ante* bears when reducing risk. The strategic complementarity between borrower-members' choices of projects amplifies this individual trade-off between bonus and project payoff opportunities, thus, yielding a positive externality effect. This is because an increase of each borrower' strategy (likelihood of project's success) due to an increase of the bonus rate leads the other borrower to increase his own strategy in response.

Combining Lemma 1 and Proposition 1, we conclude that both the penalty and the bonus provisions are complementary. While the first generates a credible threat discouraging free-riding, the second reinforces this effect by providing borrowers with incentives to select low-risk projects.

C. *Equilibrium Projects under the DBSR*

Lemma 2. *Under the DBSR, a borrower-member h ($h = i, j$) is a free-rider whenever l_{-h}/D exceeds some critical level. This critical level is increasing in the absolute risk aversion of h.*

Proof. See the Appendix.

This lemma states that as the ratio $l_{-h}/D = (1 - \frac{l_h}{D})$ rises, as the borrower-member h has an incentive to select riskier projects if he anticipates that his neighbor, the other borrower-member $-h$, is selecting a low-risk project (i.e., decreasing Nash best-reply correspondence). That is, borrower-members would have incentives to free-ride each other under the DBSR. The rationale is that for a high ratio (l_{-h}/D) the utility of the strictly positive dividend received by borrower h while his own project fails (see the distribution of borrowers' wealth under the DBSR given by Eq. (7)) becomes high enough, thus creating scope for free-riding. In our two-person game, this result means that *small* borrowers, i.e., those with a relatively low ratio (l_h/D), will tend to be free-riders more than other types of borrowers.

The assertion that the risk of failing to free-riding decreases with the level of risk aversion of borrower-members is very intuitive and agrees with the Rasmusen's (1987) result on moral-hazard in teams. Contrary to Holmström (1982) who assumes risk-neutrality, Rasmusen shows that budget-balanced sharing rules achieving an efficient outcome in the sense of Pareto optimality may exist whenever we allow team members to exhibit a high enough risk-aversion. In other words, the incentive for free-riding critically depends on the degree of the agents' risk aversion. In our model's context the idea is very simple. Since free-riding consists of strategic risk-taking, its optimality in the sense of the best-response concept must depend on how much borrower-members are risk-averse.

Overall, Lemma 2 reveals that, in contrast to BBSR, the DBSR design may lead to a free-riding problem. This is mainly due to the fact that DBSR constitutes a linear sharing rule, since it does not involve a penalty provision. Borrower-members under DBSR are, therefore, not exposed to any threat, which creates a room for opportunistic behavior. In this regard, Lemma 2 suggests that under the DBSR, the homogeneity in the borrower-members' population (the amounts of borrowing transactions as well as members' preferences toward risk are the criteria that determine homogeneity) is crucial for the efficiency of the equilibrium outcome.

IV. Monitoring

As we have shown, the BBSR allows credit mutuals to enhance efficiency better than the alternative DBSR design. In this section, we investigate the following question: Are credit mutuals implementing BBSR able to perform conventional monitoring efficiently? By conventional monitoring we mean the monitoring function delegated to managers and analysts, as in stock banks, in contrast to peer monitoring done by members themselves.

We assume that the monitoring function is costly but has an effective impact on the borrowers' choice of projects. We model this as follows: At time 0, the credit mutual's manager selects the amount to spend on monitoring borrower-members before they select their projects. The available monitoring technology allows manager to influence only the borrowers' strategies of risk taking, so that selected projects in equilibrium under monitoring are less risky than those without monitoring. In other words, the manager does not have the capacity to force borrowers to undertake the projects he desires. By doing so, we avoid the extreme situation where the moral-hazard problem is fully overcome by monitoring. Accordingly, the projects that the borrower-members select are still given by the Nash equilibrium of the projects selection game described in the first part of the model. The incremental feature here is that each borrower's choice is not affected by the other borrower's choice only (i.e., the strategic interaction), but also by the monitoring exercised by the credit mutual's manager. Namely, the borrower h's ($h = i,\ j$) problem of project selection is,

$$\max_{p_h} E\left[U\left(W_h\left(p_h(m_h;\alpha);p_{-h};\alpha\right)\right)\right],$$

subject to $p_h \leq \bar{p}_h$, where m_h is the cost inflicted on the mutual when monitoring borrower h. This cost indicates the intensity or effort of monitoring chosen by the manager. Further, we shall assume the following.

Assumption 2. *For each borrower-member $h = i,\ j$, we have,*

$$\frac{\partial p_h}{\partial m_h} \equiv g\left(m_h\right) \geq 0. \tag{8}$$

This means that for any $m_h > 0$, the project the borrower h undertakes in equilibrium subject to monitoring is less risky than the one he *would have* selected in equilibrium without monitoring, i.e., $p_h^*(m_h;\alpha) \geq p_h^*(0;\alpha)$.

Given the borrowers' choice spaces, monitoring must be effective, so as to keep the credit mutual's policy (α, R_L) fixed, $m_h \leq \bar{m}_h$ respects $p_h \leq \bar{p}_h$ for each $h = i,\ j$. Moreover, for a credit mutual (α, R_L), the monitoring problem of a risk-neutral manager maximizing the expected surplus from credit allocation net of promised bonuses is,

$$\max_{\{m_h\}_{h=i,j}} \sum_{h=i,j} E\left[v_h\left(p_h(m_h;\alpha); p_{-h}\left(m_{-h};\alpha\right)\right) - b_h\right] - m_h,$$

subject to $m_h \leq \bar{m}_h$, recognizing that the likelihoods p_h and p_{-h} are determined from the Nash equilibrium of the projects selection game.

We start by examining the effect of the mutual's bonus rule on the optimal monitoring plan. This is answered by the following.

Proposition 2. *The optimal cost m_h^* inflicted to the mutual from monitoring each borrower $h = i$, j, is decreasing with the bonus rate α.*

Proof. See the Appendix.

The idea here is that increasing the promised bonus reduces the borrowers' appetite for risk-taking (see Proposition 1) and hence lowers the intensity of monitoring required. Interestingly, this means that promising bonuses can be viewed as a partial substitute for monitoring. This is because paying out one dollar of bonus at date 1 permits the credit mutual to save a given amount of monitoring fees at date 0.

We now investigate the relative efficiency of the monitoring function of credit mutuals. We use stock bank as a benchmark monitor. Consider a stock bank endowed by the monitoring technology

$g_{SB}(.)$. Recall that any stock bank can be treated as a credit mutual implementing a zero bonus rate (i.e., $\alpha = 0$) in our model. Let $m^*_{h,SB}$ be the optimal cost the stock bank will choose to spend on monitoring borrower $h(h = i,\ j)$ and define $p^*_{h,SB}$ as the project undertaken by h in equilibrium when financed by the stock bank, i.e.,

$$p^*_{h,SB} = \underset{p_h \leq \bar{p}_h}{\arg\max}\ E\left[U\left(W_h\left(p_h(m^*_{h,SB};0);p_{-h};0\right)\right)\right].$$

Consider now a credit mutual endowed by the monitoring technology $g_{SB}(.)$ (i.e., the same of the stock bank) and paying a bonus rate $\alpha > 0$. Therefore, given Proposition 2, there exists a (constrained) monitoring cost $m^c_{h,M}$ that the credit mutual has to spend when financing borrower h leading to the same equilibrium project $p^*_{h,SB}$ financed by the stock bank such that,

$$m^c_{h,M} = m^*_{h,SB} - t_h(\alpha), \tag{9}$$

where $t_h(\alpha) \geq 0$ is increasing in α.

The point here is that by employing the incentive effect of BBSR, the credit mutual is able to achieve less costly monitoring than a similar stock bank whenever the projects financed by the two institutions are *ex ante* constrained to exhibit equivalent risks. Thus, a synergy occurs under the BBSR between monitoring and the incentive effect due to mutuality.

Since the efficiency of credit mutuals' monitoring is sensitive to the promised bonuses to borrowers, a question arises: Are the monitoring fees $t_h(\alpha)$ that the credit mutual saves budget-balanced by the bonuses to be paid out to the borrowers? The following result shows that *ex ante* the answer is not affirmative. Define *welfare* as the sum of the borrower terminal wealth and the return to the lender (bank or mutual) net of the monitoring cost and the payment of bonus. Hence, we have,

Corollary 1. *Consider a credit mutual endowed by the monitoring technology $g_{SB}(.)$. Let the credit mutual be constrained to spend $m^c_{h,M}$ leading to the same equilibrium project $p^*_{h,SB}$ as financed by the stock bank. The credit mutual's BBSR ($\alpha > 0$), therefore, achieves a welfare improvement that Pareto dominates the policy of zero bonus of the stock bank.*

Proof. See the Appendix.

The rationale is related to the well-known idea from the theory of financial securities (Arrow, 1964; Ross, 1976) of the Pareto dominance of state-contingent claims. Indeed, while the bonus paid out at date 1 is contingent on the project' success, monitoring fees, in contrast, are unconditionally expended at date 0.

As shown here, credit mutuals may dispose of sophisticated contractual arrangements, enabling them to perform conventional monitoring more efficiently than stock banks. It is worthwhile to note that stock banks benefit from advantages that credit mutuals do not, however. For example, Rasmusen (1988) shows that stock banks are endowed with a management function that is more efficient than that of credit mutuals. This is because of the absence of takeover threats and executive compensation plans in the context of mutuals. Further, Hart and Moore (1998) compare optimal investment made by cooperative firms with that achieved by common-stock firms. They find that mutuality is less efficient in competitive industries than joint-stock ownership. Indeed, the following proposition is similar to the Hart and Moore' result in the sense that it shows that mutuality in not uniformly welfare-improving. It mitigates the previous corollary by assessing the sensitivity of total welfare to the bonus instrument.

Proposition 3. *In (unrestricted) equilibrium, the total welfare from the credit allocation achieved by the credit mutual is not increasing with the bonus rate α everywhere.*

Proof. See the Appendix.

There is a region for which the marginal welfare gain in terms of monitoring fees' economies does not cover the welfare loss in terms of a more conservative economic profitability, both caused by an incremental increase of bonus (as indicated by Proposition 1, increasing the bonus rate α will result in a higher likelihood of success p, but this may lower the expected project payoff $pz(p)$ since $z' < 0$). Hence, the economic function of stock banks would be the financing of more aggressive and thus more profitable projects than those financed by credit mutuals.

V. Model Robustness

A. *Generalized n-Borrower Game*

For simplicity, we have so far assumed that there are only two borrower-members. In real-world situations, however, credit mutuals deal with a large number of borrowers $n \geq 2$. Below we generalize the model by allowing for a finite number of borrowers where the probability of default of the credit mutual depends on the recovery value V of the n borrower-members' loans. In this setting, $V = \sum_{h=1,\dots n} v_h = \sum_h \min[x_h,\ R_L l_h]$ is randomly generated by the entire set of the n financed projects' outcomes following a multinomial probability distribution; the credit mutual's surplus is $\pi = \max[0,\ V - R_D D]$.

Therefore, recalling that $U(0)$ is normalized to zero, the expected utility of each borrower-member $i \in \{1, 2, \dots n\}$ under the BBSR is given by

$$
\begin{aligned}
E[U(W_i)] \;&=\; \Pr[V > R_D D;\ x_i = z_i]\ U(z_i - l_i(R_L - \alpha(R_L - 1))) \\
&\quad + \Pr[V < R_D D;\ x_i = z_i]\ U(z_i - R_L l_i)\,. \qquad (10)
\end{aligned}
$$

Since we have that,

$$
\begin{aligned}
\Pr[V > R_D D;\ x_i = z_i] \;&=\; p_i \Pr\left[\sum_{h \neq i} v_h > R_D D - R_L l_i\right] \\
&=\; p_i p_j \Pr\left[\sum_{h \neq i,\, j} v_h > R_D D - R_L(l_i + l_j)\right] \qquad (11) \\
&\quad + p_i(1 - p_j) \Pr\left[\sum_{h \neq i,\, j} v_h > R_D D - R_L l_i\right],
\end{aligned}
$$

for any $j \neq i$, Eq. (10) can be rewritten as follows:

$$
\begin{aligned}
&E[U(W_i)] = \\
&\quad p_i[p_j(1 - F_{i,j}) + (1 - p_j)(1 - G_{i,j})]\ U(z_i - l_i(R_L - \alpha(R_L - 1))) \qquad (12) \\
&+ p_i[p_j F_{i,j} + (1 - p_j) G_{i,j}]\ U(z_i - R_L l_i)\,,
\end{aligned}
$$

where,

$$
F_{i,j} \equiv \Pr\left[\sum_{h \neq i,\, j} v_h < R_D D - R_L(l_i + l_j)\right],
$$

$$G_{i,j} \equiv \Pr\left[\sum_{h \neq i,\, j} v_h < R_D D - R_L l_i\right].$$

Since $F_{i,j} < G_{i,j}$ for any $i,\ j \in \{1, 2, ...n\}$ $(i \neq j)$, it is easy to check that the projects selection game remains supermodular, which ensures that Lemma 1 still holds for any integer $n \geq 2$. Then, one can show that the risk-return relationship, $z' < 0$, is sufficient to claim the incentive effect of mutuality (i.e., Proposition 1) in this generalized n-borrower game.

Furthermore, given Eqs. (4) and (11), it can be easily demonstrated that Lemma 2 still holds in this generalized n-borrower game. Indeed, we need only to remark that the expected utility of each borrower-member $i \in \{1, 2, ...n\}$ under the DBSR can be expressed as follows:

$$\begin{aligned} E\left[U\left(W_i\right)\right] = & \\ & p_i p_j \left(1 - F_{i,j}\right)\ U\left(z_i - R_L l_i + \beta_i \pi_{ijF}\right) \\ & + p_i \left(1 - p_j\right)\left(1 - G_{i,j}\right)\ U\left(z_i - R_L l_i + \beta_i \pi_{ijG}\right) \\ & + p_i \left[p_j F_{i,j} + \left(1 - p_j\right) G_{i,j}\right]\ U\left(z_i - R_L l_i\right) \\ & + \left(1 - p_i\right)\left[p_j \left(1 - F_{i,j}\right) U\left(\beta_i \pi_{jF}\right) + \left(1 - p_j\right)\left(1 - G_{i,j}\right) U\left(\beta_i \pi_{jG}\right)\right], \end{aligned} \tag{13}$$

for any $j \neq i$, with π_{ijF}, π_{ijG}, π_{jF} and π_{jG} being (stictly positive) realizable values of the credit mutual's surplus; $F_{i,j}$ and $G_{i,j}$ are defined above.

B. *Repeated Agency*

Although the one-shot game on which we have focused in the above theoretical analysis captures the salient features of the agency relationship between the credit mutual and its borrower-members, this relationship may also involve a dynamic dimension. There are two different hypotheses that help assess the impact of allowing for this dynamic dimension.

The first hypothesis, examined by Banerjee et *al.* (1994), attributes the sustainability of credit mutuals to the long-term relationship of these institutions with their members. In the context of our model, this would suggest that extending the moral-hazard problem to the repeated setting will likely attenuate the difference between the equilibrium outcomes achieved by the two sharing rules' designs (if ever the static equilibrium for a given case's parameters implies a free-riding problem under the DBSR). Even though it is intuitive to expect such a phenomenon, this hypothesis lacks precision and may need arguments or considerations besides those explicitly modeled in this paper.

The second hypothesis, in contrast to the first, yields more precise predictions by exploring the long-term interaction between the borrower-members. In the context of work teams, Che and Yoo (2001) find that because of the long-term interaction between team workers, the Principal is better off implementing a joint performance evaluation than evaluating the performances of workers relatively to that of their peers. The authors argue that under a joint performance evaluation, opportunistic agents (who decide to shirk) will be punished more severely in the repeated setting than in the benchmark case of static game because of the subsequent shirking of their partners. Borrower-members' compensations (bonuses or dividends) in our model are jointly determined in function of the credit mutual's surplus. Overall, the Che and Yoo' results would suggest that the long-term interaction between members will help mutuals to enhance efficiency. More precisely, when separately analyzing the studied sharing rules' designs (BBSR and DBSR), we can conclude the following. First, since borrower-members under the BBSR do not have incentives to free-ride in the one-shot game, repetition will preserve this property of the static equilibrium because of the (spot) punishment provision embedded in the BBSR. Second, if ever the characteristics of the borrowers' population (e.g., the distribution of the ratio $(\frac{l_i}{D})$ and the borrowers' risk aversions) are such that the static equilibrium implies a free-riding problem under the DBSR, the threat of being punished by peers in the long-term (alway receiving zero dividends) will provide borrower-members with implicit incentives to undertake moderate-risk projects. Hence, the potential imperfections of the DBSR in the static setting could be counter-balanced by the implicit incentive effect of the long-run interaction.

VI. Empirical Analysis

In this section we test some of the propositions on the empirical data. Unfortunately, the lack of detailed information on individual loan contracts limits our empirical analysis to work at the level of credit mutuals. Nevertheless, the relatively long time-horizon for the data and the clarity of variables reported have enabled us to conduct empirical analyses that are detailed enough to evaluate the main theoretical predictions of the model. In particular, we test for the relative efficiency of the credit mutual's designs: BBSR and DBSR. The tests we implement, indeed, permit to discriminate between the incentive schemes of these two distinct sharing rules designs.

A. *The Data*

We use data from the National Credit Unions Administration of the United States. The data contains a wide range of financial information collected from a large number of U.S. credit unions (CUs) semiannually. More specifically, we have observed more than 7,000 CUs in a common calendar of 12 semesters, from June 30, 1994 to December 31, 1999.

Before presenting our sample we have to report one salient feature in the data: Observed CUs do not implement the sharing rules, BBSR and DBSR, in an exclusive fashion. Rather, practically all CUs use the DBSR regularly, but combine, on a few occasions, both dividends and bonuses during some exercise periods. For CUs combining these two distinct compensation schemes, interest revenues refunded to borrower-members (bonuses) are reported in the Income Statement as a deduction from the interest income account. Independently of whether a CU pays bonuses or not, dividends distributed to all members are accounted for in the Income Statement as an interest expense item.

Our sampling method consists of selecting only CUs for which observations are available for more than two time periods, starting from the first calendar period. The panel sample we obtain contains 4,993 CUs observed over 12 consecutive time periods (semesters). This yields 59,916 observations, with only 3,747 observations missing.

As reported in Table 1, our sample presents two types of CUs in terms of the use of BBSR and DBSR. The first type paid bonuses for at least one time period. Since all CUs distribute dividends regularly, this type of CUs can also be defined as a CU that combines DBSR and BBSR at least once. We note, however, that most of these CUs paid bonuses more than once. The second type is CUs that never use bonuses, but distribute dividends regularly.

The fact that few of CUs observed in data distribute bonuses is striking. While the theoretical model tells us that credit mutuals are better with the BBSR, however, it cannot help us to determine what pushes them to implement a given sharing rule rather than the other. Many factors would be behind this wide use of the DBSR. Indeed, most American CUs hold member funds as "deposit shares". These shares are considered as capital shares and are compensated with dividends. The ratio of capital to assets for all CUs over all time periods is about 80.7% in our sample. Interestingly, in opposition to the American CUs, the largest network of credit mutuals in Canada, *Mouvement Desjardins*, uses the BBSR in an exclusive *way*. We suspect therefore the institutional background as well as tax considerations to be the main factors at play.

B. *Testable Implications*

Since our model supposes a one-shot game, borrower-members are supposed to select their projects simultaneously without the possibility of repetition. In real-world situations, however, we observe overlapping generations of investment projects as well as repeated project selection games. In addition, the historical performances of CUs are common knowledge for borrower-members, even for new entrant-members. In such a framework, a learning process will likely take place, thus enabling borrower-members to formulate their prior beliefs from the current set of observables, which in turn leads to the two following nulls.

Hypothesis 1. (Incentive effect of BBSR) *For CUs implementing the BBSR, the rate of failure on loans is decreasing in the bonus rate.*

The theory presented in this paper reveals that the surplus sharing rule involving bonus and punishment provisions (BBSR) disciplines borrower-members effectively. It is also shown that the threat effect generated by the punishment provision prevents credit mutuals from facing a free-riding problem (see Lemma 1), thus allowing them to overcome the moral hazard problem efficiently through the incentive effect of the bonus provision (see Proposition 1). Hence, CUs implementing BBSR should exhibit a negative equilibrium relationship between 'bad' loans and the bonus rate as stated in Hypothesis 1.

Hypothesis 2. (Incentive effect of DBSR) *For CUs implementing the DBSR only, no significant relationship may exist between the rate of failure on loans and the dividend rate.*

With regard to the DBSR design, the theoretical model suggests that the equilibrium outcome crucially depends on the homogeneity of the borrower-members' population (see Lemma 2). Since we do not observe borrower-members individually in our CUs data, the incentives scheme of the DBSR is expected to vary across observations.

According to the theoretical analysis, there is a potential risk of observing free-riding under the DBSR. Borrowers endowed with the relatively riskiest investment opportunities would therefore choose to be affiliated with credit mutuals distributing dividends rather than those implementing the BBSR. This potential adverse selection problem would emerge in our empirical data. There are interesting econometric tools recently developed in the empirical literature on contracts that permit to deal with the simultaneous presence of moral hazard and adverse selection patterns in data. Abbring, Chiappori and Pinquet (2003) and Abbring, Chiappori, Heckman and Pinquet (2003) provide a treatment of this issue in the context of insurance market. To perform the tests recommended by the cited authors, one needs in our context to observe the loans contracts individually. Unfortunately, we only observe here the aggregated performances of credit mutuals, which constrain us to conduct reduced-form tests. This kind of empirical test does not allow to controlling for adverse selection patterns potentially present in data. Nevertheless, there are two reasons pushing us to expect a moderated presence of adverse selection patterns in data.[10] First,

[10] It is important to understand that the adverse selection problem we are talking about here is

as we have stated above, most of CUs observed in data adopt the DBSR. This makes the impact of a potential simultaneous presence of moral hazard and adverse selection patterns less severe than what could be expected if CUs would have frequently implemented the BBSR as much as the DBSR. Second, because of geographical and business relationship considerations, the extent to which borrowers are able to shift form one credit mutual to another depending on the sharing rule adopted is expected to be reasonably low.

C. *Empirical Tests and Results*

To capture the incentive effects we aim to study, we select two basic explanatory variables from the data. The first variable, the *BonusRate*, is calculated as the periodic interests refunded to borrower-members (i.e., bonuses) divided by the periodic amount of gross interest income (i.e., before bonuses). The second, the *DividendRate*, is obtained by dividing the periodic amount of dividends transferred to the entire population of members by the total balance-sheet value of capital shares. Finally, as an indicator of the surplus sharing rule implemented, we use a shift variable Z that takes the value of 1 when the CU pays bonuses to borrower-members and 0 otherwise. Note that since CUs distribute dividends regularly but pays bonuses only occasionally, the dummy Z permits us to isolate CUs that have implemented the BBSR.

Furthermore, we define the dependent variable Y as the rate of failure on loans obtained after dividing the number of delinquent loans by the total number of outstanding loans.[11] To allow our endogenous variable to be sensitive to the exogenous economic shocks, we regress Y on the credit spread (CS) observed on the U.S. corporate bond market during the time calendar considered. This (time-varying) variable is defined as the spread between the yield return on long-term risky

that associated with the sharing rule and not the one related to the loans' interest rates largely analyzed in the adverse selection literature.

[11] According to the theoretical model (see Lemma 2), an average failure rate weighted by the amounts of loans would be a better measure of the riskiness of financed projects. Unfortunately, we do not observe the amounts and performances of individual loans, but only the aggregated performances of the credit mutuals. We have therefore to deal with the equally-weighted failure rate available in data.

bonds rated Baa by Moody's and the rate of return of the three-month Treasury bills. The choice of the credit spread is motivated by its documented power to forecast changes in the business cycle (Harvey 1988; Chen 1991). As control variables, we use the total assets A of the CU and a variable $BBias$ measuring the dominance of borrower-members in the membership. The variable $BBias$ is defined as the number of loans over the total number of members. One should expect a positive relation between this variable and the dependent variable. Table 2 summarizes the descriptive statistics of the sample.

First, we consider a simplified empirical model that tests for the theoretical predictions formulated in the previous subsection. More specifically, the model we implement is

$$\begin{aligned} Y_{i,t} = \beta_0 + \beta_1 CS_t + \beta_2 \log A_{i,t} + \beta_3 BBias_{i,t} \\ + \beta_4 BR_{i,t-1} + \beta_5 DR_{i,t-1} + \upsilon_i + \tau_t + e_{i,t}, \end{aligned} \tag{14}$$

where $BR_{i,t-1}$ is defined as the product of the dummy $Z_{i,t-1}$ and the variable $BonusRate_{i,t-1}$, while $DR_{i,t-1}$ is constructed as the product of the dummy $(1-Z_{i,t-1})$ and the variable $DividendRate_{i,t-1}$. Given the fact that bonuses are strictly positive only for CUs with $Z = 1$, the variable BR simply reduces to the original explanatory variable $BonusRate$. However, since all CUs distribute dividends, the variable DR takes the value of the $DividendRate$ only if the CU has not paid bonuses to borrower-members during the time period considered. Otherwise, DR is equal to zero in spite of the fact the variable $DividendRate$ is strictly positive. Hence, the variables BR and DR capture the incentives schemes of BBSR and DBSR, respectively. Further, υ and τ are the individual and time effects, respectively, arising from the panel nature of the data.

We estimate this model using both fixed effects (FE) and random effects (RE). The FE procedure uses the "de-meaning" approach (Hsiao, 1986). The FE was implemented to control for individuals only, and an ANOVA analysis was performed on the residuals to check whether residual time effects remained. The F-test yielded a value of 32.2 with a significance of 0.001 or less. Thus, series were de-meaned for both individual and time effects. We also found that an OLS with de-meaned observations −for both effects− presents a Durbin-Watson (DW) statistic of 0.38, suggesting a high serial correlation of residuals. Accordingly, we adopted the procedure suggested by Hsiao (1986, p. 55) under which errors are specified as following a first-order autoregressive process, and the model was estimated by means of a maximum likelihood grid search procedure.[12] Under this error specification the DW statistic jumped to 1.997, which is within the acceptable range. When outliers were eliminated, however, the DW once again fell below the acceptable level. The DW statistic returned to the acceptable range only when the lagged values of the dependent variable $Y_{i,t-1}$ were included in the regression. The presence of lagged values of the dependent variable suggests that we are in the presence of a *dynamic data process*, where serial correlation can be expected to be high. Hence, neither the FE or the RE estimation procedure would yield efficient estimates. Based on Hsiao (1986) and Woolridge (2002, p. 283), we shifted to a differentiated model specification recommended for this type of situation.

Furthermore, model (14) assumes that the surplus sharing rule is exogenously set by CUs. To generalize our test, we implement a two-stage estimation. As Laffont and Matoussi (1995), who employ the same procedure to test for the incentive effects of sharecropping contracts, we first regress the variable BR (defined as noted below Eq. (14)) against a set of instruments using a Probit procedure,[13]

$$BR_{i,t} = \gamma_0 + \gamma_1 \log IntInc_{i,t} + \gamma_2 \log Cap_{i,t} + \gamma_3 \log NM_{i,t} + \mu_{i,t}, \tag{15}$$

where $IntInc$, Cap and NM are the gross interest income (before bonuses), the total amount of capital shares, and the current number of members, respectively. Then, at a second stage, we perform the panel estimation by the first-difference method to evaluate the model (14) augmented by the residue of the regression. The regression (15) permits us to examine the factors that would explain the CUs' decision of distributing bonuses. While the interest income earned during the time period captures the financial capacity of the CU to refund interests paid to borrower-members, the variables Cap and NM indicate its accumulated wealth and maturity. Due to a consistent absence of

[12] Estimates based on the Hildreth-Lu grid search procedure yielded essentially the same results.

[13] In contrast to Laffont and Matoussi (1995), who use a single cross-section observation, we use a panel. This leads to differences in the actual statistical procedure used, although the approach remains the same.

significance of $\log NM$ we eliminate it from the actual regressions. There are eleven cross-sectional regressions from which we extract the residues (one for each semester beginning from December 1994). To assess the capacity of these variables to account for bonus payments, we estimated the following using all panel observations,

$$
\begin{aligned}
BR_{i,t} = & \underset{(12.73)}{0.0012} \log IntInc_{i,t} - \underset{(-3.45)}{0.001} \log Cap_{i,t} + \mu_{i,t}, \\
& F = 23.67 \qquad N = 56,169
\end{aligned}
\tag{16}
$$

which suggests a reasonable specification for the decision of CUs of paying out bonuses to borrower-members.

The coefficient for the residue μ of the eleven Probit estimations incorporated in (14) was highly significant with a t-statistic of -3.03. The same coefficient becomes non significantly different from zero after extracting the residue μ from the regression of $BR_{i,t}$ against the lagged variables $\log IntInc_{i,t-1}$ and $\log Cap_{i,t-1}$ (for each of the eleven semesters) in the Probit model. This suggests that (14) should be estimated using a 2SLS procedure. In doing so, the same variables employed in Eq. (15), excepting $\log NM$, are used as instruments. Tests of autocorrelation of the preliminary runs' residues yielded an autocorrelation coefficient of -0.297, sufficiently close to the -0.5 that would be expected from a random process not to be alarming, but still significantly different from -0.5 in a two-sided t-test (-57.96). In addition, the DW statistic in those regressions was 2.38, still within the permissible range but very close to the upper bound. Hence, we adopt an autocorrelation and heteroschedasticity robust estimation technique based on GMM and use the Newey-West approach for computing the covariance matrix with the Bartlett window and one lag. After eliminating outliers with residues exceeding three standard errors, the GMM based 2SLS regression with $IntInc$ and Cap as instruments yields the following:

$$
\begin{aligned}
\Delta Y_{i,t} = & \underset{(0.53)}{0.006} \Delta CS_t - \underset{(-3.57)}{0.005} \Delta \log A_{i,t} + \underset{(4.14)}{0.001} \Delta BBias_{i,t} - \underset{(-2.93)}{0.231} \Delta BR_{i,t} - \underset{(-6.76)}{0.041} \Delta DR_{i,t} + \varepsilon_{i,t}, \\
& \text{Adj. } R^2 = 0.02 \qquad \text{DW} = 2.40 \qquad J(3) = \underset{(0.13)}{5.49} \qquad N = 45,528
\end{aligned}
\tag{17}
$$

We note that the Hansen J-statistic has a *p-value* of 0.13, which is in excess of the acceptable significance standards. This provides further support for the choice of instruments, since we cannot reject the hypothesis that they are adequate in terms of their correlation with both the disturbance term ε and the dependent variable.

Regarding the control variables introduced in the model, we note that the level of CU's assets significantly affects the dependent variable. In particular, we observe that large CUs benefit from a size effect enhancing loans' performances from which small CUs do not benefit. The intuition behind this is that the efficiency of the credit allocation performed by CUs is positively correlated with their experience and thus with their size. Remark also that the credit spread's changes (ΔCS) has not a significant explanation power of changes occurring on the rate of failure on loans. This can be explained by the fact that loans allocated by CUs often serve to finance small business, while the credit spread observed on the bonds market captures economic shocks that specifically affect larger and more sophisticated business. Furthermore, as expected, empirical data suggest that the dominance of borrower-members in the total population served by the CU has a significant and negative impact on the loans' performances (see the coefficient associated with $\Delta BBias$).

With respect to the variables of interest, we note that the empirical results significantly support the existence of a positive incentive effect of the BBSR as predicted by the first null. However, hypothesis 2, according to which the DBSR has no significant incentive effect, is rejected. Concerning the BBSR, the incentive scheme documented here is in conformity with the theoretical model's predictions (i.e., Lemma 1 and Proposition 1), according to which increasing the bonus rate enhances efficiency. Again, based on the theoretical model, the incentives scheme of the DBSR reflected by the empirical results in (17) suggests that CUs observed in our sample do not suffer from a free-riding problem. The factors leading to this result, as discussed in the theoretical part of the paper, would be the homogeneity in the borrower-members' populations served by CUs (see the discussion of Lemma 2) as well as the long-term interaction effect discouraging free-riding behavior. Unfortunately, the data available do not permit us to discriminate between these two effects.

Last, but not the less important, the empirical results in (17) are clearly discriminating. The BBSR is better from the point of view of equilibrium efficiency than the DBSR. Indeed, we observe that the coefficient for ΔBR is about six times as big as the coefficient for ΔDR, suggesting that the dependent variable (bad loans' rate) is much more sensitive to bonuses than to the same amount of dividends. This provides a qualified empirical support for the relative efficiency of the BBSR as predicted by the theoretical model.

VII. Conclusions

In this paper we examined the ability of credit mutuals to overcome the moral-hazard problem arising from the adverse incentives of their borrower-members to undertake projects that are not socially optimal. We constructed a simple moral-hazard model in which we investigated the efficiency of the common surplus' sharing rule adopted by these credit institutions. Our analysis stressed the impact of the strategic interaction between members on the outcome of this incentive problem. The model provides some answers on whether cooperation devices implemented by mutual credit institutions can help to overcome imperfections that affect credit markets in a general fashion. It also permits to assess the welfare effects of combining mutuality and financial intermediation in a costly monitoring environment. In this regard, the results obtained are helpful to appreciate the capacity of mutual credit institutions to perform a conventional (direct) monitoring function efficiently. Our model, however, does not capture the agency problem occuring between management and members and assumes that monitoring plans chosen by managers are common surplus-maximizers. Further research allowing for a potential management distortion might be useful to discriminate between the alternative hypotheses on the efficiency of mutual credit institutions.

By using a panel data of American credit unions we implemented an empirical model to test the main theoretical results derived in the formal part of the paper. After controlling for factors beyond those explicitly modeled and effectuating the required checks, we found that the theoretical predictions were significantly supported by the empirical results. The loans recovery performances of American credit unions are sensitive to the surplus sharing rule adopted, which confirms the presence of behavioral patterns in data consistent with the strategic interactions between members predicted by theory.

Appendix: Proofs

First, let us define for each borrower-member $h = i,\ j$:

Under the BBSR:

$$W_h^u \equiv z\left(p_h\right) - l_h\left[R_L - \alpha(R_L - 1)\right],$$

$$W_h^d \equiv z\left(p_h\right) - l_h\left[R_L - \hat{\alpha}_h(R_L - 1)\right],$$

Under the DBSR:

$$W_h^{uu} \equiv z\left(p_h\right) - R_L l_h + \beta_h(R_L - R_D)D,$$

$$W_h^{ud} \equiv z\left(p_h\right) - R_L l_h + \beta_h \max\left[0,\ R_L l_h - R_D D\right],$$

$$W_h^{du} \equiv \beta_h \max\left[0,\ R_L l_{-h} - R_D D\right].$$

Claim 1. *The expected utility function of each borrower-member $h = i, j$ is concave in the likelihood of the success of his own project.*

Proof. Given Eqs. (6) and (7) we have,

Under the BBSR:

$$\begin{aligned}\frac{d^2 E\left[U\left(W_h\right)\right]}{(dp_h)^2} &= 2\left[p_{-h} z'\left(p_h\right) U'(W_h^u) + (1 - p_{-h}) z'\left(p_h\right) U'(W_h^d)\right] \\ &\quad + p_h p_{-h}\left[z''\left(p_h\right) U'(W_h^u) + \left(z'\left(p_h\right)\right)^2 U''(W_h^u)\right] \\ &\quad + p_h\left(1 - p_{-h}\right)\left[z''\left(p_h\right) U'(W_h^d) + \left(z'\left(p_h\right)\right)^2 U''(W_h^d)\right],\end{aligned} \tag{18}$$

Under the DBSR:

$$\begin{aligned}\frac{d^2 E\left[U\left(W_h\right)\right]}{(dp_h)^2} &= 2\left[p_{-h} z'\left(p_h\right) U'(W_h^{uu}) + (1 - p_{-h}) z'\left(p_h\right) U'(W_h^{ud})\right] \\ &\quad + p_h p_{-h}\left[z''\left(p_h\right) U'(W_h^{uu}) + \left(z'\left(p_h\right)\right)^2 U''(W_h^{uu})\right] \\ &\quad + p_h\left(1 - p_{-h}\right)\left[z''\left(p_h\right) U'(W_h^{ud}) + \left(z'\left(p_h\right)\right)^2 U''(W_h^{ud})\right].\end{aligned} \tag{19}$$

Providing Assumption 1, it is easy to check that terms in the brackets on the right-hand side of both (18) and (19) are non-positive. Q.E.D.

Proof of Lemma 1. Providing the facts that $z' < 0$ and $W_h^u \geq W_h^d$, the cross-derivative of the expected utility function of each borrower-member $h = i,\ j$ under BBSR verifies,

$$\frac{\partial^2 E\left[U\left(W_h\right)\right]}{\partial p_h \partial p_{-h}} = \left[U(W_h^u) - U(W_h^d)\right] + p_h z'\left(p_h\right)\left[U'(W_h^u) - U'(W_h^d)\right] \geq 0,$$

for any increasing and concave utility function U (increasing differences). Hence, given Claim 1, the optimal likelihood of project success for each borrower h is increasing in the other borrower $-h$'s likelihood of project success. Q.E.D.

Proof of Proposition 1. The proof consists of examining the sensitivity of the Nash equilibrium projects to changes of the bonus rate α. Indeed, considering the individual maximization problem of each borrower-member $h = i,\ j$, the cross-derivative below

$$\begin{aligned}
&\partial^2 E\left[U\left(W_h\right)\right] / \partial p_h \partial \alpha = \\
&\qquad \left(dW_h^u / d\alpha\right)\left[p_{-h} U'(W_h^u) + p_h p_{-h} z'\left(p_h\right) U''(W_h^u)\right] \\
&\quad + \left(dW_h^d / d\alpha\right)\left[\left(1 - p_{-h}\right) U'(W_h^d) + p_h\left(1 - p_{-h}\right) z'\left(p_h\right) U''(W_h^d)\right],
\end{aligned}$$

is positive given Assumption 1 and the fact that $dW_h^u/d\alpha = (R_L - 1)l_h > 0$ and $dW_h^d/d\alpha = (R_L - 1)l_h > 0$ if $R_L l_h - R_D D > 0$ and zero otherwise. Providing Claim 1, we have $\partial p_h^* / \partial \alpha > 0$. From Lemma 1, the strategic interaction (indirect) effect $\left(dp_h^*/dp_{-h}^*\right)\left(\partial p_{-h}^*/\partial \alpha\right) > 0$. Hence, according to the envelope theorem, $dp_h^*/d\alpha > 0$ for each borrower-member $h = i,\ j$. Q.E.D.

Proof of Lemma 2. Under the DBSR, we have for each borrower-member $h = i,\ j$,

$$\begin{aligned}
\frac{\partial^2 E\left[U\left(W_h\right)\right]}{\partial p_h \partial p_{-h}} \quad = \quad & \left[U(W_h^{uu}) - U(W_h^{ud})\right] + p_h z'\left(p_h\right)\left[U'(W_h^{uu}) - U'(W_h^{ud})\right] \\
& - U(W_h^{du}).
\end{aligned} \tag{20}$$

Since $z' < 0$, $U'(.)$ is a decreasing function and $W_h^{uu} \geq W_h^{ud}$, the two first terms on the right-hand side of Eq. (20) are positive. As a result, the sign of this cross-derivative is undeterminate, since the quantity $\left[U(W_h^{uu}) - U(W_h^{ud}) - U(W_h^{du})\right]$ can be either positive or negative depending on the values taken by W_h^{uu}, W_h^{ud}, and W_h^{du}. Indeed, although $W_h^{uu} - W_h^{ud} - W_h^{du} \geq \beta R_D D > 0$, the concavity of the utility function U leads us to observe the following mechanism: as the ratio $l_{-h}/D = l_{-h}/(l_h + l_{-h})$ increases, as l_{-h} is much higher than l_h, which lowers the quantity $\left[U(W_h^{uu}) - U(W_h^{ud}) - U(W_h^{du})\right]$ below zero. This leads the cross-derivative above to take a negative value. Further, observe that the lower the absolute risk aversion of borrower-member h, the lower is the absolute value of $\left[U'(W_h^{uu}) - U'(W_h^{ud})\right]$. This implies that lowering the absolute risk aversion makes the cross-derivative in Eq. (20) decreases more rapidly when the ratio (l_{-h}/D) is increased. Q.E.D.

Proof of Proposition 2. Let us first define the credit mutual's net surplus,

$$
\begin{aligned}
&F(m_i, m_j, \alpha) \\
&\quad = \textstyle\sum_{h=i,j} E\left[v_h\left(\hat{p}_h(m_h;\alpha); \hat{p}_{-h}\left(m_{-h};\alpha\right)\right) - b_h\right] - m_h \\
&\quad = \textstyle\sum_{h=i,j} \hat{p}_h(m_h;\alpha)\left[A_h(\alpha)\hat{p}_{-h}\left(m_{-h};\alpha\right) + A_h(\hat{\alpha}_h)\left(1 - \hat{p}_{-h}\left(m_{-h};\alpha\right)\right)\right] \\
&\qquad - \textstyle\sum_{h=i,j} m_h,
\end{aligned}
$$

where $(\hat{p}_h(m_h;\alpha),\ \hat{p}_{-h}\left(m_{-h};\alpha\right))$ denotes the pair of Nash equilibrium projects to be selected by borrower-members and $A_h(y) \equiv l_h\left[(1-y)R_L + y\right]$ for any $y \in \Re$ ($(h,-h) \in \{(i,j),(j,i)\}$). Therefore, the optimal monitoring plan $\{m_i^*,\ m_j^*\}$ follows from,

$$
\frac{\partial F}{\partial m_h} = \frac{\partial \hat{p}_h}{\partial m_h}\left[\left(A_h(\alpha) + A_{-h}(\alpha) - A_{-h}(\hat{\alpha}_{-h})\right)\hat{p}_{-h} + A_h(\hat{\alpha}_h)\left(1 - \hat{p}_{-h}\right)\right] - 1,
$$

for each $h = i,\ j$, which, added to the fact that $\partial\hat{p}_h/\partial m_h \geq 0$ (Assumption 2) and Proposition 1, yields

$$
\frac{\partial^2 F}{\partial m_h \partial m_{-h}} = \frac{\partial \hat{p}_h}{\partial m_h}\frac{\partial \hat{p}_{-h}}{\partial m_{-h}}\left[\left(A_h(\alpha) - A_h(\hat{\alpha}_h)\right) + \left(A_{-h}(\alpha) - A_{-h}(\hat{\alpha}_{-h})\right)\right] \leq 0,
$$

$$
\begin{aligned}
\partial^2 F/\partial m_h \partial\alpha = &\left(\partial\hat{p}_h/\partial m_h\right)\ \hat{p}_{-h}\ \tfrac{d}{d\alpha}\left(A_h(\alpha) + A_{-h}(\alpha) - A_{-h}(\hat{\alpha}_{-h})\right) \\
&+ \left(\partial\hat{p}_h/\partial m_h\right)\ \left(1 - \hat{p}_{-h}\right)\ dA_h(\hat{\alpha}_h)/d\alpha \\
&+ \left(\partial\hat{p}_h/\partial m_h\right)\left(\partial\hat{p}_{-h}/\partial\alpha\right)\times \\
&\quad \left[\left(A_h(\alpha) - A_h(\hat{\alpha}_h)\right) + \left(A_{-h}(\alpha) - A_{-h}(\hat{\alpha}_{-h})\right)\right] \\
\leq\ & 0.
\end{aligned}
$$

This implies that the credit mutual's net surplus $F(m_i, m_j, \alpha)$ is a submodular function. Hence, by making use of the Topkis's theorem, both m_i^* and m_j^* are decreasing in α, as claimed by Proposition

2 (for more details concerning the use of the Topkis's theorem in comparing equilibria, see Milgrom and Roberts, 1996).

Proof of Corollary 1. Let Π_h denote the welfare generated from financing the borrower h's project, which we define as the sum of the borrower h's terminal wealth and the return to the lender, net of the monitoring cost and the payment of the bonus. Namely,

$$\Pi_h = W_h + [v_h - b_h - m_h], \quad \text{for each } h = i,\ j.$$

It is easy to check that independently from the considered lender (stock bank or credit mutual) we have:

$$\Pi_h = \begin{cases} z(p_h) - m_h & \text{with probability} \quad p_h, \\ -m_h & \text{with probability} \quad (1-p_h), \end{cases}$$

which implies,

$$E[\Pi_h] = p_h\ z(p_h) - m_h.$$

In an economy where the credit mutual is constrained to spend $m^c_{h,M}$ leading to the same equilibrium project $p^*_{h,SB}$ financed by the stock bank, the expected welfares associated with both the stock bank and the credit mutual are, respectively,

$$E[\Pi_{h,SB}] = p^*_{h,SB}\ z\left(p^*_{h,SB}\right) - m^*_{h,SB},$$

$$E[\Pi_{h,M}] = p^*_{h,SB}\ z\left(p^*_{h,SB}\right) - m^c_{h,M},$$

implying that,

$$E[\Pi_{h,M}] - E[\Pi_{h,SB}] = m^*_{h,SB} - m^c_{h,M} \equiv t_h(\alpha) \geq 0.$$

This establishes the welfare improvement due to the credit mutual's bonus policy.

Proof of Proposition 3. Consider the credit mutual (α, R_L) and let the equilibrium be characterized by the credit mutual's optimal monitoring plan $\{m^*_i, m^*_j\}$ and the Nash projects $(\hat{p}_i(m^*_i;\alpha)$, $\hat{p}_j(m^*_j;\alpha))$. Therefore, the expected welfare achieved at equilibrium is,

$$E[\Pi_h] = \hat{p}_h(m^*_h;\alpha)\ z\left(\hat{p}_h(m^*_h;\alpha)\right) - m^*_h, \quad \text{for each } h = i,\ j.$$

Differentiating with respect to the bonus rate, we get

$$\frac{dE\left[\Pi_h\right]}{d\alpha} = \frac{\partial \hat{p}_h(m_h^*;\alpha)}{\partial \alpha}\left[z\left(\hat{p}_h(m_h^*;\alpha)\right) + \hat{p}_h(m_h^*;\alpha)z'\left(\hat{p}_h(m_h^*;\alpha)\right)\right] - \frac{dm_h^*}{d\alpha}.$$

Given Propositions 1 and 2 and $z' < 0$, it is clear that $E\left[\Pi_h\right]$ is not a monotonic function of α. There is a region for the Nash equilibrium point $(\hat{p}_i(m_i^*;\alpha), \hat{p}_j(m_j^*;\alpha))$, so that raising the bonus rate α will decrease the expected welfare $E\left[\Pi_h\right]$. Q.E.D.

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Table 1: The use of the BBSR and DBSR by U.S. credit unions

This table shows the number of credit unions in our sample that have distributed bonuses (the second colon) to their members as well as the number of those that have paid only dividends (the third colon), and this during the time calendar June 1994–December 1999. Data is available from the National Credit Unions Administration of the United States.

Semester	Number of CUs paying bonuses	Number of CUs paying dividends only
June 1994	112	4,881
December 1994	193	4,800
June 1995	94	4,899
December 1995	205	4,788
June 1996	94	4,899
December 1996	221	4,772
June 1997	102	4,891
December 1997	240	4,753
June 1998	109	4,884
December 1998	198	4,795
June 1999	98	4,895
December 1999	184	4,809

Table 2: Descriptive statistics of the sample

This table summarizes the descriptive statistics of the basic variables used in our empirical tests. The variables 'Total Shares' and 'Interests Refunded' are defined as the book value of capital shares of the credit union and the amount of bonuses paid out to borrower-members, respectively. The variable 'Delqquency Rate of Loans' is the dependent variable of the empirical model, and is obtained by dividing the number of delinquent loans by the total number of loans. The variable 'BBias' is defined as the number of loans over the total number of members, and measures the intensity at which the decisions taken by the credit union are influenced by the voting-power of borrower-members. The 'Bonus Rate' is obtained as the ratio of interests refunded (i.e., 'Interests Refunded') to the gross interest income (before bonuses). The 'Dividend Rate' is obtained by dividing the amount of dividends transferred to the entire population of members by the value of capital shares (i.e., 'Total Shares'). Data is available from the National Credit Unions Administration of the United States.

Variable	Mean	Std-Deviation	Minimum	Maximum
Total Assets*	33 441 692	171 602 306	14 492	11 241 755 360
Total Shares*	29 194 311	139 745 130	2 487	9 075 375 019
Interest on Loans*	1 309 655	7 306 808	11	675 699 160
Dividends on Shares*	812 958	4 461 979	0	356 089 060
Interests Refunded*	1 718	27 372	0	2 370 458
Number of Loans	3 681	19 008	1	1 134 817
Number of Current Members	6 811	28 891	1	1 901 023
Number of Delinquent Loans	45	135	0	9 705
Delqquency Rate of Loans	0.03233	0.05065	0.0000	1.00000
BBias	0.00185	0.01038	0.0000	0.95366
Bonus Rate	0.00197	0.01600	0.0000	0.96860
Dividend Rate	0.02649	0.01217	0.0000	0.45034
Number of observation: 56,169				

(*) : in dollars

Essay 2:

Adverse Selection in Credit Market and the Emergence of Hybrid Finance

Abstract

An adverse selection model is developed in which it is shown how hybrid forms of lenders using costly screening devices, such as mutual banks and venture capitalists, would emerge. The analysis reveals that the main function of mutual banking is to provide finance to low-risk entrepreneurs, while venture capital finance emerges as an attractive design for financial intermediaries providing finance to low-collateral entrepreneurs. The model shows that the coexistence of these instances of hybrid finance with standard bank finance is explained by both self-selection and credit rationing mechanisms that prevent some population of entrepreneurs to be served by standard banks.

JEL classification: D82; G21; G24

Keywords: Financial intermediation; Credit market; Adverse selection; Screening; Credit rationing

I. Introduction

The lending industry as observed presently in most of developed financial systems regroups a wide range of credit institutions. Banks, venture capital companies, mutual banks like savings & loans and credit unions, conglomerates and many others intermediaries all share the same economic function: supplying credit to entrepreneurs. Beyond this main similarity, however, we lack arguments justifying the coexistence of all theses instances of financial intermediation over the same credit market as well as the important differences existing between them. Specifically, the following questions need to be answered:

(1) What explains the emergence of credit institutions that differentiated themselves from the standard model of banks? During the last century, many financial intermediaries other than banks have emerged. For example, the beginning of the last century was characterized by the creation of a large number of *savings & loans* in Europe, particularly in rural communities. The same phenomenon was thereafter observed in North-America by the creation of *credit unions*, particularly in labor and agricultural communities. During the late 1970s and early 1980s, we have also assisted to the emergence of new forms of nonbank finance like venture capital and conglomerates. The institutional background of all these *new* intermediaries is now considered as complex and very specific.

(2) What sort of projects or entrepreneurs does each institution finances? When looking at the type of clients served by different credit institutions, one can easily remark significant differences across the risk/return of the financed projects and the profile of entrepreneurs undertaking them. Many other criteria, such as the size of projects, the institutional affiliation and geographic location of entrepreneurs as well as their proximity to the lender could also indicate a large dispersion of the pool of clients.

(3) How can we explain the coexistence of different forms of financial intermediation? Since their emergence, nonbank intermediaries have grown fast and succeeded in preserving their

specificities without being engaged in an intensive competition with banks already established. Presently, all of the different credit institutions, including banks and nonbank intermediaries, *seem to* have substantial market shares that will ensure their long-term growth.

In this paper, I explore the informational aspect of credit markets in the perspective of examining all of these issues. I develop a unified adverse selection model that accounts for the emergence of different forms of nonbank finance, which I call *hybrid finance* in opposition to bank finance I qualify as *standard finance*. In particular, my model provides a line of imperfect information-based arguments giving rationales to the emergence of the two most popular and important forms of hybrid finance, mutual banking and venture capital finance, as well as to their continuing coexistence with banks.

In credit markets affected by the adverse selection, lenders as well as entrepreneurs would seek to attenuate the negative effects of this market friction by trading credit contracts that are adapted to their profiles. The model develops this idea by exploring explicit screening technologies that capture real-world credit contracts offered by nonbank intermediaries, such as mutual banks and venture capitalists. Broadly speaking, the model shows that by using more screening devices than the interest rate, the hybrid finance supplied by nonbank intermediaries is advantageous for the two following reasons:

(i) Attenuates the severity of adverse selection for lenders;

(ii) Allows entrepreneurs to get a specialized finance adapted to their risk/collateral profiles.

In the first part of the paper (sections II and III), I consider the economic framework of Stiglitz and Weiss (1981). I start by examining standard credit contracts offered by banks, such those analyzed by Stiglitz and Weiss. Thereafter, I investigate a class of hybrid credit contracts, which I call *interest rate-based hybrid contracts*, that stipulates the payment of a contingent compensation flowing from lender to entrepreneur in addition to the flow already involved in the standard contract. By taking market equilibrium prevailing under the standard credit contract (i.e., bank finance) as a benchmark, the analysis shows that this form of hybrid finance essentially serves low-risk entrepreneurs that, by self-selection mechanism, will drop out of the credit market in the benchmark equilibrium. Furthermore, the analysis reveals that alternative forms of this interest rate-based hybrid finance would emerge, depending on whether lender decides to offer the hybrid credit contract alone or the menu composed of both standard and hybrid credit contracts. Therefore, equilibrium in this setting is shown to be either pooling or separating depending on the supply strategy of lender (i.e., offering the hybrid contract only versus offering the menu of hybrid and standard contracts). Since they promise to entrepreneurs the opportunity to jointly share the intermediation surplus (profit) of the lender, the hybrid credit contracts studied exhibit the same profile as the participating contracts employed by mutual banks. The model in this sense accounts for the emergence of mutual banking as a form of hybrid finance that essentially serves low-risk entrepreneurs. In addition, it predicts a set of different equilibriums that correspond to lending industries with various specialization degrees of lenders, which gives rationale to the continuing coexistence of both standard banks (i.e., stock banks) and mutual banks.

The second part of the paper (section IV) extends the basic framework of Stiglitz and Weiss (1981) to allow for heterogeneous collateral endowments. In this generalized framework, entrepreneurs exhibit a risk-collateral profile that determines their demand for the credit contracts offered by lenders. In contrast to the basic framework of Stiglitz and Weiss where credit rationing affects entrepreneurs uniformly, analysis reveals that in the presence of heterogeneous collateral endowments, separating equilibrium under standard finance (i.e., bank finance) makes poor-collateral entrepreneurs suffering from credit rationing more severely than others. Starting from this observation, I investigate a new form of hybrid finance, which I term by *collateral-based hybrid finance*, stipulating the payment of a contingent compensation by poor-collateral entrepreneurs to lenders in addition to the flow already involved in the standard credit contract. In particular, the hybrid credit contract I explore constitutes a mixture of debt and equity claims, in which the lender conserves the position of prior claimant in the bad states of nature, while she benefits from the success of the project in the good states. This form of hybrid finance resembles venture capital finance and permits lenders to compensate the lack of guarantee while financing poor-collateral entrepreneurs.

It also permits to this class of entrepreneurs to get loans when standard finance (i.e., bank finance) is rationed. These results accounts for the emergence of venture capital finance as an attractive design of lenders serving poor-collateral entrepreneurs. The model also gives rationale for the coexistence of banks and venture capital finance, as well as for the contracting practice used by venture capitalists.

This paper contributes to the strand of literature in financial intermediation theory focusing on the emergence and the contractual devices of nonbank intermediaries. A closely related paper is Bond (2004) developing a costly state verification model that accounts for the specificities of conglomerates and trade credit as well as the continuing coexistence of the credit institutions ensured by these non-financial specialists with banking. Other related papers are Nöldeke and Schmidt (1998), Repullo and Suarez (1999), Schmidt (2003) and Ueda (2004) focusing on the role of venture capitalists and the contractual devises they employ distinguishing them from banks. In the same vein, but less linked to our credit market-oriented topic, papers such as Boyd, Prescott and Smith (1988) and Smith and Stutzer (1995) provide an account for the endogenous formation of mutual insurers.

From the methodological point of view, the present paper is related to the literature on adverse selection, credit rationing and screening in credit markets. Beyond the fact that the model developed in section II borrows insights from Stiglitz and Weiss (1981), collateral and credit rationing related issues I briefly explore in section IV are similar to those studied more in details by Bester (1985) and Besanko and Thakor (1987). Further, even tough the issues they examine are quite different, Wang and Williamson (1998) and my paper, overall, share the same focus on the impact of costly screening devices on credit market equilibrium.

The remainder of the paper is organized as follows. Section II presents the analysis framework. Section III examines interest rate-based hybrid finance. Section IV extends the basic framework described in section II and then explores collateral-based hybrid finance. In both sections III and IV, a discussion of the organizational design of hybrid finance is also provided. Section V concludes.

Proofs are regrouped in the Appendix.

II. Framework

The economic environment I consider is similar to that described in the adverse selection model of Stiglitz and Weiss (1981). Economy lasts for two periods and there is a continuum of entrepreneurs, each endowed with an opportunity of investment in a given project. All projects require the same investment cost of one dollar. In period 0, entrepreneurs decide to whether or not invest in projects. In period 1, projects (if they were undertaken initially) return nonnegative payoffs, which are randomly (exogenously) generated by nature. Each project θ is characterized by the ex ante probability distribution $F(z,\theta)$ of its date 1-payoff z defined over the support $[0,\infty)$. The parameter $\theta \in \left[\theta^{-},\theta^{+}\right]$ indicates the riskiness of the project. It captures the fact that different projects have different distributions of return. Nature maps each project θ to each entrepreneur in period 0, so that we can associate to the continuum of projects' risk-type θ the distribution function $P(\theta)$ reflecting the dispersion of riskiness over the entrepreneurs' population.

As Stiglitz and Weiss (1981), I assume that greater values of θ are associated with higher levels of risk of projects in the sense of mean preserving spread. More formally, I consider the following.

Assumption 1. *For any $a \geq 0$ and $\theta_1 > \theta_2$, we have*

$$\int_0^{\infty} zdF(z,\theta_1) = \int_0^{\infty} zdF(z,\theta_2), \tag{1}$$

$$\int_0^{a} F(z,\theta_1)dz \geq \int_0^{a} F(z,\theta_2)dz. \tag{2}$$

Entrepreneurs are initially endowed with a physical asset with a fixed value $C < 1$, which is assumed identical for all entrepreneurs. Technology in this economy is such that entrepreneurs cannot put their physical asset into the production function of the projects they would like to undertake. In addition, because projects are indivisible and $C < 1$, entrepreneurs cannot sell their physical asset to undertake a partial investment. However, they can offer this asset as collateral to a lender and obtain a loan. As entrepreneurs themselves, the lender observes the value C of collateral and also knows the impact of risk on the project payoff described by $F(z,\theta)$ and Assumption 1. In contrast, the project's riskiness θ is a private information to entrepreneur, and thus lender does not observe this attribute. Hence, in equilibrium, the lender would never accept to finance entrepreneurs refusing to transfer their physical asset as collateral, so that the access to loan financing for entrepreneurs (and thus the feasibility of investment) is conditional upon the supply of collateral C.

In exchange of a loan financing, lender will charge to entrepreneurs a gross interest rate of R remunerating her hired capital of 1\$. That is, all loans have the same nominal value of 1\$ equalizing the investment cost of projects. Moreover, since $C < 1$ and $F(1,\theta) > 0$ for any θ, supplying loan financing is a risky activity for lender who, in consequence, would never charge a gross interest rate R lower than 1. As in Stiglitz and Weiss (1981), the lender observes the distribution $P(\theta)$. She will offer "take it or leave it" credit contracts to entrepreneurs, specifying the lender's claim to the project return as well as the interest rate R to be paid. In particular, the *standard credit contract* $(\pi(.), R)$ we shall use as a benchmark in our analysis, stipulates that lender gets the capitalized nominal of $R \times 1\$ = R\$$ in period 1 whenever the value of collateral C summed by the payoff z exceeds this promised amount. Otherwise, there is default, and the contract gives lender the ownership of both the collateral C and the project payoff $z \in [0, R-C)$ in such a state.[1] That is, the net return to an entrepreneur accessing to the credit contract $(\pi(.), R)$ is,

$$\rho(R) = \max[z - R, \; -C],$$

while the return to the lender offering the credit contract $(\pi(.), R)$ is given by,

$$\pi(R) = \min[z + C, \; R],$$

with, of course, the budget-balancing condition, $\pi(R) + \rho(R) = z$, is satisfied.

[1] For the purpose of illustrating the standard credit contract $(\pi(.), R)$, we shall consider here the interest rate R as given. In the next section, I examine the optimized credit contract $(\pi(.), R)$ where the interest rate R is endogenously set.

Observe that collateral is treated as an exogenous endowment in the model, so that lenders are collateral-takers. This accords with the basic framework of Stiglitz and Weiss (1981) that would seem more realistic than the Bester's (1985) model where lenders employ both interest rate and collateral as screening devices. In our framework, indeed, collateral cannot be used by low-risk entrepreneurs to signal their type to the lender in order to benefit from a low interest rate. Collateral only reduces the risk the lender bears by attenuating the date 1-loss caused by defaulting projects (i.e., those yielding $z < R$). However, this effect is homogenous for entrepreneurs with different (ex ante) probabilities of default, and thus cannot be optimized by our collateral-taker lender. Furthermore, as in Stiglitz and Weiss (1981), we will focus on debt contracts only. Our aim is to give rationales for the emergence of hybrid forms of finance. Therefore, in order to keep the model within the aimed scope, we do not investigate other standard forms of finance such as pure equity contracts.

III. Interest Rate-Based Hybrid Finance

In this section, I examine a family of screening credit contracts that, by the mean of interest rate-based contingencies, permit the lender to refine the pool of entrepreneurs applying for loans in function of their risk-type.

A. *The Standard Credit Contract*

I start by showing how the standard credit contract described in the previous section acts as a screening device. The results presented in this subsection directly follow from the adverse selection model of Stiglitz and Weiss (1981).

Lemma 1. *For any standard contract $(\pi(.), R)$ announced by lender, there is $\hat{\theta} \in [\theta^-, \theta^+]$ increasing in R, so that an entrepreneur exhibiting a risk-type θ will apply for a loan if and only if $\theta \geq \hat{\theta}$.* (Theorems 1 and 2, Stiglitz and Weiss, 1981).

Proof. Given the fact that the return $\rho(R)$ is a convex function of z, the expected return,

$$\bar{\rho}(R,\theta) = \int_0^{\infty} \rho(R)\, dF(z,\theta), \tag{3}$$

is non-decreasing in θ. Therefore, there is a cutoff level of risk $\hat{\theta}$, such that $\bar{\rho}(R,\hat{\theta}) = 0$. Since $\bar{\rho}(R,\theta)$ is decreasing in R, $\hat{\theta}$ is increasing in R. Q.E.D.

This indicates that for any standard contract $(\pi(.), R)$ offered by the lender, only entrepreneurs endowed with projects riskier than $\hat{\theta}$ will apply for loans, while the other entrepreneurs endowed with safer projects (i.e., $\theta < \hat{\theta}$) will drop out of the market.

Now, we would like to know whether credit market equilibrium in the presence of adverse selection may lead to credit rationing. Theorem 5 of Stiglitz and Weiss (1981) provides the answer.

Proposition 1. *There is a nonempty set of economies in which the demand and the supply functions of funds are such that competitive equilibrium maximizing the lenders' profits entails credit rationing.*
Proof. See the Appendix.

Henceforth, $(\pi(.), \hat{R})$ refers to the optimal standard contract that maximizes the lender's aggregated return. I will focus in the subsequent analysis on the group of entrepreneurs that leave the credit market after the lender announces her optimal standard contract $(\pi(.), \hat{R})$, i.e., entrepreneurs exhibiting a risk-type θ lower than the cutoff level $\hat{\theta}(\hat{R})$. I call them *interest rate-constrained entrepreneurs* (IRCEs). I study credit contracts permitting the finance of this class of entrepreneurs. In doing so, I consider economies in which $\hat{\theta}(\hat{R}) > \theta^{-}$, so that the group of entrepreneurs considered as IRCEs is non empty. I take equilibrium prevailing under the optimal standard contract $(\pi(.), \hat{R})$ as a benchmark.

B. *The Pooling-Hybrid Credit Contract*

Now, I examine the "take it or leave it" credit contract $(\pi(.), \phi(.), R, \alpha)$, which I call *hybrid credit contract* in opposition to the standard credit contract investigated above., promising to entrepreneur the return,

$$\begin{aligned}\rho_h(R,\alpha) &= \rho(R)+\phi(R,\alpha)\\ &= \max[z-R,\ -C]+\phi(R,\alpha),\end{aligned}$$

while pays off to the lender,

$$\begin{aligned}\pi_h(R,\alpha) &= \pi(R)-\phi(R,\alpha)\\ &= \min[z+C,\ R]-\phi(R,\alpha),\end{aligned}$$

where $\phi(R,\alpha)=\alpha(R-1)>0$ if $z\geq R$ and $\phi(R,\alpha)=0$ otherwise; with $\alpha\in(0,1)$. It is worthwhile to note that all we need for our results to hold is to assume $\phi(R,\alpha)>0$ if $z\geq R$ and $\phi(R,\alpha)=0$ otherwise. But it is intuitive to consider the contingent compensation flow $\phi(R,\alpha)$ as a bonus that takes the form of interests refunded to the entrepreneur whenever the project succeeds (i.e., whenever the project performs without the consumption of collateral). Remark that the hybrid credit contract defined above is essentially a debt contract, since the contingent compensation flow $\phi(R,\alpha)$ only impacts the amount of interests received by the lender. Indeed, both the *residual claimancy* of the entrepreneur and the *prior claimancy* of the lender, characterizing debt contracts, are still conserved.

The following lemma describes the participation constraint of entrepreneurs to the hybrid credit contract $(\pi(.),\phi(.),\hat{R},\alpha)$.

Lemma 2. *For any optimal standard contract $(\pi(.),\hat{R})$ that would have been announced by lender, there exists $\dot{\theta}\in[\theta^{-},\hat{\theta})$ function of $(\hat{R},\alpha)$ and decreasing in α, so that IRCEs exhibiting a risk-type $\theta\in[\dot{\theta},\hat{\theta})$ and all of non-IRCEs will apply for the hybrid credit contract $(\pi(.),\phi(.),\hat{R},\alpha)$.*

Proof. Observe that given any pair $(\hat{R},\alpha)$, the expected value of the contingent compensation $\phi(\hat{R},\alpha)$,

$$\bar{\phi}(\hat{R},\alpha,\theta)=\int_0^{\infty}\phi(\hat{R},\alpha)dF(z,\theta)=\alpha(\hat{R}-1)[1-F(\hat{R},\theta)], \tag{4}$$

is positive for any risk-type θ considered. Since the utility from applying for the standard contract $(\pi(.),\hat{R})$ for any $\theta<\hat{\theta}$ is negative by Lemma 1, there exists for any $\alpha\in(0,1)$ a cutoff level of risk-type $\dot{\theta} \in [\theta^-, \hat{\theta})$ function of $(\hat{R}, \alpha)$, so that $\bar{\phi}(\hat{R}, \alpha, \dot{\theta}) = -\bar{\rho}(\hat{R}, \dot{\theta}) > 0$. Because $\bar{\phi}(\hat{R}, \alpha, \theta)$ is increasing in α and $\bar{\rho}(\hat{R}, \theta)$ is increasing function of θ but negative over $[\theta^-, \hat{\theta})$, the cutoff level of risk $\dot{\theta}$ is decreasing in α. Further, since $\bar{\phi}(\hat{R}, \alpha, \theta)$ is decreasing in θ (this follows from the definition of the payoff z-function $\phi(.)$ and Assumption 1, the utility from applying for the hybrid contract $(\pi(.), \phi(.), \hat{R}, \alpha)$ given by,

$$U_h = \bar{\rho}(\hat{R}, \theta) + \bar{\phi}(\hat{R}, \alpha, \theta), \tag{5}$$

is positive for any IRCE exhibiting a risk-type $\theta \in [\dot{\theta}, \hat{\theta})$, while is strictly negative for any IRCE endowed with $\theta < \dot{\theta}$.

Now concerning non-IRCEs, it is sufficient to see that,

$$U_h = \bar{\rho}(\hat{R}, \theta) + \bar{\phi}(\hat{R}, \alpha, \theta) > \bar{\rho}(\hat{R}, \theta) \geq 0, \tag{6}$$

for any $\theta \geq \hat{\theta}$. Figure 1 illustrates this lemma. Q.E.D.

Of course, the hybrid contract $(\pi(.), \phi(.), R, \alpha)$ is not optimally set for $R = \hat{R}$ and an arbitrary compensation rate α despite the fact the standard contract $(\pi(.), \hat{R})$ is optimal by construction. The optimal hybrid contract, indeed, must specify the pair (R^*, α^*) maximizing the lender's aggregated return. In fact, this lemma only means that given the (optimal) standard contract $(\pi(.), \hat{R})$, the lender is able to attract a subpopulation of IRCEs in addition to non-IRCEs that *would have* applied for the standard contract, by offering the contract $(\pi(.), \phi(.), \hat{R}, \alpha)$, and this for any arbitrary chosen $\alpha \in (0, 1)$. It also says that there exists a group of IRCEs (the safest ones) whom always will be kept out of any market deal.

Our aim in the following is to answer whether the lender is better-off with offering the hybrid contract instead of the standard contract. This occurs whenever the optimal hybrid contract $(\pi(.), \phi(.), R^*, \alpha^*)$ achieves an equilibrium aggregated return higher than that the lender earns by offering the optimal standard contract $(\pi(.), \hat{R})$. The proof strategy I follow consists of checking whether the suboptimal hybrid contract $(\pi(.), \phi(.), \hat{R}, \hat{\alpha})$, optimizing the compensation rate α for $R = \hat{R}$, may dominate the optimal standard contract $(\pi(.), \hat{R})$. Then, a positive answer will be sufficient to accomplish the task.

Let's give the expected lender's return aggregated over the population of entrepreneurs applying for the credit contract offered. Because the standard contract excludes all of IRCEs, the aggregated return to the lender offering $(\pi(.), \hat{R})$ is,

$$\Pi(\hat{R}) := \arg\max_{\{R\}} \Pi(R) = \int_{\hat{\theta}}^{\theta^{+}} \bar{\pi}(\hat{R}, \theta) dP(\theta), \tag{7}$$

where $\bar{\pi}(\hat{R}, \theta)$ is the expected return $\pi(\hat{R})$ to the lender from financing the entrepreneur θ. Further, because the hybrid contract pools all of non-IRCEs and a subgroup of IRCEs (see Lemma 2), the aggregated return to the lender supplying the contract $(\pi(.), \phi(.), \hat{R}, \alpha)$ is given by,

$$\Pi_h(\hat{R}, \alpha) = \int_{\dot{\theta}}^{\theta^{+}} [\bar{\pi}(\hat{R}, \theta) - \bar{\phi}(\hat{R}, \alpha, \theta)] dP(\theta). \tag{8}$$

where $\bar{\phi}(\hat{R}, \alpha, \theta)$ is the expected value of the contingent compensation $\phi(\hat{R}, \alpha)$ promised to the entrepreneur θ. Recall here that $\hat{\theta}$ and $\dot{\theta}$ are functions of $\hat{R}$ and $(\hat{R}, \alpha)$, respectively. Hence, taking the difference between these two aggregated returns yields that $\Pi_h(\hat{R}, \alpha)$ exceeds $\Pi(\hat{R})$ only if,

$$I(\dot{\theta}, \hat{\theta}) := \int_{\dot{\theta}}^{\hat{\theta}} [\bar{\pi}(\hat{R}, \theta) - \bar{\phi}(\hat{R}, \alpha, \theta)] dP(\theta) > \int_{\hat{\theta}}^{\theta^{+}} \bar{\phi}(\hat{R}, \alpha, \theta) dP(\theta) := J(\hat{\theta}, \theta^{+}). \tag{9}$$

This means that lender will find interesting to offer the hybrid contract $(\pi(.), \phi(.), \hat{R}, \alpha)$ whenever the expected aggregated surplus (net of compensations) from financing IRCEs, $I(\dot{\theta}, \hat{\theta})$, exceeds enough the expected aggregated compensations, $J(\hat{\theta}, \theta^{+})$, that will go to non-IRCEs.

Proposition 2. *The lender is either indifferent between the optimal standard contract* $(\pi(.), \hat{R})$ *and the optimal hybrid credit contract* $(\pi(.), \phi(.), R^{*}, \alpha^{*})$ *(in such a case,* $\alpha^{*} = 0$ *and* $R^{*} = \hat{R}$*), or better-off with* $(\pi(.), \phi(.), R^{*}, \alpha^{*})$ *and in such a case* $\alpha^{*} > 0$ *and* $R^{*} \leq \hat{R}$.

Proof. See the Appendix.

It is easy to see that because the hybrid contract nests the standard contract, the lender offering the hybrid contract is always ensured to achieve at least the same aggregated return expected from the standard contract. The most important result the proposition above reveals, indeed, is that whenever the hybrid contract strictly dominates the standard contract, the equilibrium interest rate observed in such a case will be lower than which the lender would have charged under the standard contract. The rationale is that if lender finds more profitable to serve IRCEs by offering the hybrid contract, compensations promised to entrepreneurs cannot permit her to charge an interest rate higher than $\hat{R}$ while still being *profitable* (in the sense of reducing adverse selection effect) to justify the supply of hybrid finance in such a situation. An immediate consequence of this equilibrium condition is that the pooling-hybrid credit contract to be strictly preferred by the lender, a group of entrepreneurs qualified as IRCEs under the (optimal) standard credit contract must be served, or at least be eligible in the sense of self-selection to apply for loans.

C. *The Separating-Hybrid Credit Contracts*

I begin by investigating the case where the lender offers the menu of contracts composed of the standard contract $(\pi(.), R)$ and the hybrid contract $(\pi(.), \phi(.), R', \alpha)$. The hybrid contract here has exactly the same features than the pooling-hybrid contract studied previously. The incremental innovation, however, is that the lender will offer both of the standard and the hybrid contract with the possibility of charging different interest rates across the two contracts.

The following lemma describes the participation and the incentive compatibility constraints of entrepreneurs. Without loss of generality, I consider the optimal standard contract $(\pi(.), \hat{R})$ as a benchmark by studying the menu composed of $(\pi(.), \hat{R})$ and $(\pi(.), \phi(.), R', \alpha)$.

Lemma 3. *For any optimal standard contract $(\pi(.), \hat{R})$ that would have been announced by lender, the menu of contracts composed of the standard contract $(\pi(.), \hat{R})$ and the hybrid contract $(\pi(.), \phi(.), R', \alpha)$ permits lender to serve the entrepreneurs' population so that: a) there exist $\dot{\theta}$ and $\ddot{\theta}$ both functions of (R', α); b) there exists $\bar{R} > \hat{R}$ where for any $R' \in (\hat{R}, \bar{R})$, there is a compensation rate $\hat{\alpha} \in (0,1)$ function of $(R', \hat{R})$ verifying $\ddot{\theta}(R', \alpha) = \hat{\theta}(\hat{R})$; where we have:*

1) If $R' \in (\hat{R}, \bar{R})$ and $\alpha \in (\hat{\alpha}, 1)$, therefore $\dot{\theta} < \hat{\theta} < \ddot{\theta}$, and hence:

i) each entrepreneur exhibiting a risk-type $\theta > \ddot{\theta}$ will apply for the standard contract $(\pi(.), \hat{R})$;

ii) each entrepreneur exhibiting a risk-type $\theta \in [\dot{\theta}, \ddot{\theta}]$ will apply for the hybrid contract $(\pi(.), \phi(.), R', \alpha)$;

iii) each entrepreneur exhibiting a risk-type $\theta < \dot{\theta}$ will not apply for any contract;

2) If $R' \in (\hat{R}, \bar{R})$ but $\alpha \in (0, \hat{\alpha}]$, or $R' \geq \bar{R}$ and $\alpha \in (0,1)$; therefore $\ddot{\theta} < \hat{\theta} < \dot{\theta}$, and then the market of hybrid finance vanishes: all of IRCEs (i.e., those exhibiting $\theta < \hat{\theta}$) will drop out of the credit market while all of non-IRCEs (i.e., those exhibiting $\theta \geq \hat{\theta}$) will apply for the standard contract $(\pi(.), \hat{R})$;

3) If $R' \leq \hat{R}$, therefore for any $\alpha \in (0,1)$, we have $\dot{\theta} < \hat{\theta}$ and all of entrepreneurs exhibiting $\theta < \dot{\theta}$ will drop out of the credit market while all of entrepreneurs exhibiting $\theta \geq \dot{\theta}$ will apply for the hybrid contract $(\pi(.), \phi(.), R', \alpha)$;

with $\dot{\theta}$ ($\ddot{\theta}$) is defined as the critical level of risk-type of an entrepreneur indifferent between the hybrid contract and nothing else (the hybrid contract and the standard contract).

Proof. The statement provided by the lemma enounces all of the details we need, so that the proof immediately follows from: i) the fact that for any R' and $\alpha \in (0,1)$, the entrepreneur θ's utility from applying for the hybrid contract $(\pi(.), \phi(.), R', \alpha)$ given by $[\bar{\rho}(R', \theta) + \bar{\phi}(R', \alpha, \theta)]$ is increasing in θ, and this for the simple reason that $\bar{\rho}(R', \theta)$ is increasing in θ and $\bar{\phi}(R', \alpha, \theta) \geq 0$ for any θ; ii) the fact that for any $R_1 < R_2$, we have $\bar{\rho}(R_1, \theta) > \bar{\rho}(R_2, \theta)$ for any θ; iii) the given definitions of $\dot{\theta}$, $\ddot{\theta}$ and $\hat{\alpha}$; iv) Lemma 1 and the given definition of $\hat{\theta}$; and finally, v) the observation that for any admissible compensation rate verifying $\alpha < 1$, we are always able to find an upper bound $\bar{R}$ for the interest rate R', so that we have $\ddot{\theta} < \hat{\theta} < \dot{\theta}$ for any $R' \geq \bar{R}$.

To gain more intuition, Figure 2 illustrates the different outcomes we may confront when the lender offers the menu composed of the standard contract $(\pi(.), \hat{R})$ and the hybrid contract $(\pi(.), \phi(.), R', \alpha)$. Q.E.D.

This lemma holds for any other menu composed of $(\pi(.), R)$ and $(\pi(.), \phi(.), R', \alpha)$, excepting that in general, $\hat{\theta}$ defined by Lemma 1 will refer to the cutoff level of risk-type corresponding to the standard contract $(\pi(.), R)$ (i.e., $\hat{\theta} = \hat{\theta}(R)$).

This result shows that by charging an interest rate $R' > \hat{R}$ for entrepreneurs applying for the hybrid contract, the lender will deter the riskiest non-IRCEs from applying for the hybrid contract, originally supplied for IRCEs. To continue to operate, however, this effect cannot be pushed at its desired maximum level (i.e., the situation where only IRCEs apply for the hybrid contract), since there is always a group of non-IRCEs (the safest ones) whom will apply for the hybrid contract. Lemma 3 also tells us that there are extreme situations (scenarios (2) and (3) described by Lemma 3) in which the market for one of the two contracts offered vanishes, since no entrepreneur will be eligible (in terms of the individual rationality constraint or incentive compatible constraint) to apply for that contract. Overall, Lemma 3 shows how both the interest and compensation rates act as screening devices. Based on this lemma, the following proposition examines whether or not the supply of both standard and hybrid finance is attractive for the lender.

Proposition 3. *The lender is either indifferent between the optimal standard contract* $(\pi(.), \hat{R})$ *and the optimized menu composed of the standard contract* $(\pi(.), R^*)$ *and the hybrid credit contract* $(\pi(.), \phi(.), R'^*, \alpha^*)$*, or better-off with that menu of contracts and in such a case* $\alpha^* > 0$ *and* $R'^* > \hat{R}$.

Proof. See the Appendix.

This proposition essentially means that the only way the lender is able to screening (even partially) entrepreneurs in equilibrium, IRCEs versus non-IRCEs, is to index the hybrid credit contract by the highest interest rate. The rationale is that by benefiting from the promised compensation, IRCEs (the riskiest ones as indicated by Lemma 3) will find interesting (in the sense of self-selection) to apply for the hybrid contract in spite of the fact that by doing so, they will pay for an interest rate higher than at which they dropped out of the market under the benchmark equilibrium. This is because the expected compensation for these *low-risk* entrepreneurs is sufficient enough to cover the differential interests of $R'^* - \hat{R}$, so that they find the hybrid contract attracting. The same mechanism, however, acts in the opposite direction for the riskiest non-IRCEs, since the high probability of default makes the expected compensation less important for these *high-risk* entrepreneurs to cover the same differential interests of $R'^* - \hat{R}$.

Since the main motivation of supplying the hybrid credit contract consists of attracting IRCEs, this equilibrium outcome (i.e., $R'^* > \hat{R}$) may look undesirable although it effectively permits the financing of these entrepreneurs (at least, a subpopulation of IRCEs, since the extremely safest IRCEs will be always kept out of any market deal). Hence, we would like to know whether there is an alternative form of hybrid finance that ensures the financing of IRCEs without leading to such an outcome. As the next analysis shows, the answer is affirmative.

Let's consider the menu of contracts composed of the standard contract $(\pi(.), R)$ and the hybrid contract $(\pi(.), \phi(.), R, \alpha, k)$; both charging the same interest rate. The last contract promises to the entrepreneur and the lender the same date 1-returns, $\rho_h(R, \alpha)$ and $\pi_h(R, \alpha)$, respectively, than the pooling-hybrid contract $(\pi(.), \phi(.), R, \alpha)$. Additionally, as used notations indicate, the contingent compensation $\phi(R, \alpha)$ involved in this new hybrid contract is identically the same offered by the hybrid contract analyzed previously. The incremental innovation, however, is that to access to the hybrid contract $(\pi(.), \phi(.), R, \alpha, k)$, the entrepreneur must leave at date 0 a part $k \in (0, C)$ of collateral C to the lender in order to benefit from the promised date 1-contingent compensation $\phi(R, \alpha)$. That is, in contrast to the previous cases where whenever the project succeeds (i.e., $z \geq R$), the collateral C is put back to entrepreneur, the lender here will return back to entrepreneur at date 1 the amount of $C - k$ only and keeps for herself the amount k independently from the loan's status. Therefore, k can be viewed as a sunk cost of hybrid finance the entrepreneur bears. Summarizing, $(\pi(.), \phi(.), R, \alpha, k)$ returns to entrepreneur the net return,

$$\rho_h(R, \alpha, k) = \max[z - R, \ -(C - k)] + \phi(R, \alpha) - k,$$

while pays off to the lender,

$$\pi_h(R, \alpha) = \min[z + (C - k), \ R] - \phi(R, \alpha) + k.$$

Remark that with this hybrid contract, the lender appears as a taker of lower collateral comparatively to the previous contracts. This, however, is not true since the amount k (as the remaining part of collateral $C - k$) can be used by lender to attenuate her loss if the project fails as if she disposes of the same integral amount of C as collateral. The net advantage with this contract is that lender now will not return back the fraction k to entrepreneur, but only the residual capital of $C - k$.

For any given (optimal) standard credit contract $(\pi(.), \hat{R})$ considered as a benchmark, the following lemma shows how the sunk cost k introduced into the hybrid credit contract will act as a screening device.

Lemma 4. *For any optimal standard contract $(\pi(.), \hat{R})$ that would have been announced by lender, the menu of contracts composed of $(\pi(.), \hat{R})$ and the hybrid contract $(\pi(.), \phi(.), \hat{R}, \alpha, k)$ permits lender to serve the entrepreneurs' population so that there exists $\hat{k} \in (0, C]$, where we have:*

1) If $0 < k < \hat{k}$, therefore:

i) each entrepreneur exhibiting a risk-type $\theta > \tilde{\theta}$ will apply for the standard contract $(\pi(.), \hat{R})$;

ii) each entrepreneur exhibiting a risk-type $\theta \in [\ddot{\theta}, \tilde{\theta}]$ will apply for the hybrid contract $(\pi(.), \phi(.), \hat{R}, \alpha, k)$;

iii) each entrepreneur exhibiting a risk-type $\theta < \ddot{\theta}$ will not apply for any contract;

2) If $k \geq \hat{k}$, therefore the market of hybrid finance vanishes: all of IRCEs (i.e., those exhibiting $\theta < \hat{\theta}$) will drop out of the credit market while all of non-IRCEs (i.e., those exhibiting $\theta \geq \hat{\theta}$) will apply for the standard contract $(\pi(.), \hat{R})$;

with $\ddot{\theta} \in (\dot{\theta}, \hat{\theta})$ and $\tilde{\theta} \in (\hat{\theta}, \theta^{+})$ are both function of $(\hat{R}, \alpha, k)$; $\ddot{\theta}$ is increasing in k and $\tilde{\theta}$ is decreasing in k; and the cutoff levels of risk-type $\hat{\theta}$ and $\dot{\theta}$ are defined in Lemma 1 and 2, respectively.

Proof. Let us first define,

$$\bar{\rho}(R, k, \theta) := \int_0^{\infty} \max[z - R, \ -(C-k)]\, dF(z, \theta), \tag{10}$$

and let $U_h(k)$ be the utility for entrepreneur θ applying for the hybrid contract $(\pi(.), \phi(.), \hat{R}, \alpha, k)$ given by,

$$U_h(k) = \bar{\rho}(\hat{R}, k, \theta) + \bar{\phi}(\hat{R}, \alpha, \theta) - k, \tag{11}$$

with $U_h(0)$ representing the utility yielded by the pooling contract $(\pi(.), \phi(.), \hat{R}, \alpha)$.

Providing the fact that $\max[z - R, \ -(C-k)]$ is a convex function of z for any $k \in (0, C)$, the expected return $\bar{\rho}(R, k, \theta)$ is increasing in θ with,

$$0 < \bar{\rho}(R, k, \theta) - \bar{\rho}(R, \theta) < k, \tag{12}$$

for any θ and $k \in (0, C)$. When $R = \hat{R}$, the left-hand side of the inequality above immediately leads to $U_h(k) < U_h(0)$ for any θ and $k \in (0, C)$. This in turn implies that there exists $\ddot{\theta} \in (\dot{\theta}, \hat{\theta})$ for some k, function of $(\hat{R}, \alpha, k)$ and increasing in k, verifying

$$\bar{\rho}(\hat{R}, k, \ddot{\theta}) + \bar{\phi}(\hat{R}, \alpha, \ddot{\theta}) = k. \tag{13}$$

Consequently, for the same amount of k at which $\ddot{\theta}$ is defined, we have for IRCEs whom defined as entrepreneurs exhibiting a risk-type $\theta \in [\dot{\theta}, \hat{\theta})$ and having $\bar{\rho}(\hat{R}, \theta) < 0$ (i.e., negative utility from the standard contract):

i) $U_h(k) < 0 < U_h(0)$ for any $\theta \in (\dot{\theta}, \ddot{\theta})$;

ii) $0 \leq U_h(k) < U_h(0)$ for any $\theta \in [\ddot{\theta}, \hat{\theta})$.

Further, define $\hat{k}$ as the sunk cost satisfying,

$$\bar{\rho}(\hat{R}, \hat{k}, \hat{\theta}) + \bar{\phi}(\hat{R}, \alpha, \hat{\theta}) = \hat{k}. \tag{14}$$

Remark that since $\bar{\rho}(\hat{R}, \hat{\theta}) = 0$, inequality (12) implies that $\hat{k} \in (0, C]$. Hence, given the fact that $[\bar{\rho}(R, k, \theta) + \bar{\phi}(R, \alpha, \theta)]$ is increasing in θ (this since for $\bar{\rho}(R, k, \theta)$ is increasing in θ and $\bar{\phi}(R, \alpha, \theta) \geq 0$ for any θ), we find that for any IRCE $\theta \in [\dot{\theta}, \hat{\theta})$:

$$U_h(k) \leq 0 \text{ for any } k \geq \hat{k}.$$

This ends the proof of the participation constraint of IRCEs. Now, we examine the decision of choosing between the standard and the hybrid contracts for non-IRCEs whom exhibit a risk-type $\theta \geq \hat{\theta}$. For theses entrepreneurs, Proposition 1 shows that the standard contract $(\pi(.), \hat{R})$ yields a positive utility $\bar{\rho}(\hat{R}, \theta)$. Consequently, each non-IRCE will apply for the hybrid contract $(\pi(.), \phi(.), \hat{R}, \alpha, k)$ instead of the standard contract if and only if,

$$U_h(k) - \bar{\rho}(\hat{R}, \theta) = [\bar{\rho}(\hat{R}, k, \theta) - \bar{\rho}(\hat{R}, \theta)] + \bar{\phi}(\hat{R}, \alpha, \theta) - k > 0. \tag{15}$$

Given the fact that for any $R > 1$, in particular $\hat{R}$, inequality (12) holds and $\bar{\phi}(\hat{R}, \alpha, \theta)$ is positive but decreasing in θ, the non-IRCEs' incentive-compatible constraint above is satisfied for some $k \in (0, C)$ only for $\theta \in [\hat{\theta}, \bar{\theta})$ with $\bar{\theta}$ is a function of $(\hat{R}, \alpha, k)$ defined as the critical θ at which the incentive-compatible constraint is binding. Because $\bar{\phi}(\hat{R}, \alpha, \theta)$ is decreasing in θ, $\tilde{\theta}$ is decreasing in k. Finally, since $\tilde{\theta} \geq \hat{\theta}$, it is known form Proposition 1 that each non-IRCE exhibiting a risk-type $\theta > \tilde{\theta}$ will apply for the standard contract.

Figure 3 illustrates the different cases we may confront when lender offers the menu composed of the standard contract $(\pi(.), \hat{R})$ and the hybrid contract $(\pi(.), \phi(.), \hat{R}, \alpha, k)$. Q.E.D.

Here again the hybrid contract $(\pi(.), \phi(.), R, \alpha, k)$ is not optimally set for $R = \hat{R}$ and an arbitrary pair (α, k) despite of the fact the standard contract $(\pi(.), \hat{R})$ is optimal by construction. The lemma only shows that by introducing the sunk cost k into the hybrid contract, the lender will avoid to pay compensations to all of non-IRCEs by restricting the group of these entrepreneurs to only include the safest ones (those with a risk-type $\theta \in (\hat{\theta}, \tilde{\theta})$). As in the previous menu of contracts stipulating a free access to hybrid finance, this screening is costly, since it constrains the lender to abandon the opportunity of financing the safest IRCEs that *would have* been served by the pooling-hybrid contract.

It is clear form Lemmas 3 and 4 that the two menus of contracts studied in this subsection permit the lender to attract, if optimal (i.e., strictly preferred to supplying the standard contract only), populations of entrepreneurs identical in terms of riskiness. The following result immediately follows based on this observation.

Corollary 1. *For any optimized menu with no sunk cost provision, composed of the standard contract* $(\pi(.), R^*)$ *and the hybrid contract* $\left(\pi(.), \phi(.), R'^*, \alpha^*\right)$ *and strictly preferred to* $(\pi(.), \hat{R})$*, there exists a pair* $\left(\bar{\alpha}, \bar{k}\right)$*, such that the menu composed of* $(\pi(.), \hat{R})$ *and* $(\pi(.), \phi(.), \hat{R}, \bar{\alpha}, \bar{k})$ *achieves the same equilibrium aggregated return for the lender.*

Interestingly, the results derived in this subsection suggest that a specialization in the lending industry would take place. This occurs when *emerging* (or hybrid) lenders only offer hybrid credit contracts while others, say *old* (or standard) lenders, stay supplying the standard credit contract solely. In such a situation, credit market will be shared by the two types of lenders with some of clientele (those qualified as non-IRCEs with the standard credit contract) could be served by both emerging and old lenders. Whether these entrepreneurs eligible in the sense of self-selection to the more advantageous hybrid credit contract offered by emerging lenders, decide to go to old lenders depends on whether market equilibrium entails *hybrid-credit rationing* –an outcome that might be observed in equilibrium for the same known reasons than the *standard-credit rationing*. Indeed, after verification, the average return to the lender from offering both standard and hybrid contracts is still non monotonic function of the interest rate, which implies the possibility of observing credit rationing in equilibrium. This is true for the two menus of contracts studied in this subsection. The supply of hybrid finance, therefore, attenuates the adverse selection effect, but cannot completely offset this market friction.

D. *Institutional Design: Mutual Banking*

In contrast to standard banks (or stock banks) separating ownership from clientele, the governance model of mutual banks recognizes that both depositors and borrowers are the member-owners of the bank. Mutuality in fact implies the sharing of the intermediation surplus (the bank's profit) between members. The most observed scheme implemented by mutual banks stipulates the distribution of the intermediation surplus in the form of interest rebates on loans and interest bonus on deposits. While bonus serves to remunerate deposits shares uniformly, a commonly used provision, however, stipulates that only non-defaulting loan accounts are eligible to interest rebate treatment. This penalty provision is justified by the fact that defaulting loans lower the mutual bank's surplus and thus do not add wealth to the whole community of members.

As it has been shown in this section, hybrid credit contracts involving bonus arrangements have attractive features in credit market affected by the adverse selection problem. These credit contracts exhibit the same profile as the participating contracts used by mutual banks in practice. Financial intermediaries offering this form of hybrid finance closely resemble mutual banks. The model in this sense accounts for the mergence of mutual banking as an organizational design of financial intermediation permitting the finance of low-risk entrepreneurs. It is shown indeed that *low-risk* entrepreneurs (i.e., those qualified in the language of the model as IRCEs) will only be served by the mean of hybrid credit contracts involving bonus arrangements like those offered by mutual banks. In exchange of the promised bonus or interest rebate, these entrepreneurs signal their type by accepting to pay a higher interest rate or to spend a part of their endowment as a sunk cost in order to benefit from this form of hybrid finance. This ensures the existence of a separating equilibrium in which only low-risk entrepreneurs are served by mutual banks, while riskier entrepreneurs will get standard finance from stock banks. This model result provides a rationale for the continuing coexistence of both stock banks and mutual banks.

As the analysis of the two separating hybrid credit contracts suggests, there are different strategies by which mutual banks would implement this form of hybrid finance. The first consists of charging to IRCEs an interest rate higher than which required in the benchmark equilibrium prevailing under the standard credit contract. Although this strategy permits to finance IRCEs, it is in conflict with the function of mutual banks habitually aiming to offer advantageous rates to their members. In this regard, the model shows that an alternative form of hybrid finance imposing a sunk cost for applicants would achieve the same separating equilibrium without leading IRCEs to pay a higher interest rate. This model prediction is more consistent with real-world practice. Indeed, while mutual banks maintain the offer of competitive rates to their members, membership is often costly. Some mutuals impose to their new-affiliated members an entry cost, such as the subscription to capital or deposit shares. Other mutuals closer to the community like farm and rural credit mutuals need the human capital of their members to ensure the management of operations. Moreover, starting mutuals also require the investment of common goods like physical assets and land that are often supplied by members themselves.

Now, a question arises: *Why stock banks do not offer interest rate-based hybrid finance like mutual banks do?* There are two arguments suggesting a plausible answer. The first argument is closely related to the model. While it has been shown that hybrid credit contracts involving bonus arrangements permit the lender to attenuate the negative effect of adverse selection by attracting *low-risk* entrepreneurs (i.e., those qualified as IRCEs in the language of the model), it was not demonstrated that serving low-risk entrepreneurs only could be more profitable, however. Even if the lender's aggregated return from financing IRCEs is strictly positive, the model does not tell us that this return always exceed which the lender will earn from supplying loans to non-IRCEs. Indeed, this depends on the model parameters capturing the economy characteristics as well as the entrepreneurs' population. It can be though that in some economic environments, stock banks find more profitable to serve high-risk entrepreneurs only, more inclined to pay high interest rates, through the supply of standard finance. The model in this sense identifies the factors giving rationales for the emergence of mutual banking, but further research is needed to explore the endogenous formation of this form of financial intermediation.

The second argument relies to the organizational design issue in moral hazard theory. Although the model developed in this paper focuses on the adverse selection issue, the implementation of interest rate-based hybrid finance by lenders would have significant incentive effects. Fischer and Mahfoudhi (2005) show that among the surplus sharing rules adopted by mutual banks, those involving a bonus arrangement, like which supporting the screening technology examined in this paper, lead to a strategic interaction between the borrower-members similar to that occurring between teams' members. They find that this strategic interaction mechanism provides borrower-members with incentives for exploiting more conservative projects. This incentive effect essentially benefits to the community of depositor-members, more exposed to the risk of bank failure, but at the same time lower the profitability potential of mutual banks.

IV. Collateral-Based Hybrid Finance

Entrepreneurs are so far assumed to dispose of the same collateral. Now, I extend the analysis to a credit market in which entrepreneurs are endowed with heterogeneous collateral endowments. Without loss of generality, I assume that there are two collateral-types of entrepreneurs: type H endowed with the highest collateral, C_H, and type L having little collateral, $C_L \in (0, C_H)$. Inside each group of entrepreneurs (i.e., group H and group L), the marginal distribution of the risk-type θ is described by the same distribution function $P(\theta)$. That is, risk and collateral are exogenously and independently distributed in the economy. The lender is a collateral-taker, observes the collateral values C_H and C_L, but does not observe the collateral-type of entrepreneurs individually. Further, the lender knows that the percentage of collateral-type H (collateral-type L) in the whole population of entrepreneurs is equal to $q_H \in (0,1)$ ($q_L = 1 - q_H$).

A. *The Standard Credit Contracts*

Let's consider the menu of standard credit contracts $(\pi(.), C_H, R_H)$ and $(\pi(.), C_L, R_L)$. As the used notations indicate, these contracts are designated to collateral-type H and L entrepreneurs, respectively. They promise the same return functions ρ and π than the standard credit contract studied in the previous section, i.e., for each $i = H, L$,

$$\begin{aligned} \rho\left(C_i, R_i\right) &= \max\left[z - R_i,\ -C_i\right]. \\ \pi\left(C_i, R_i\right) &= \min\left[z + C_i,\ R_i\right], \end{aligned}$$

Comparatively to the basic framework where all of entrepreneurs are endowed with the same collateral, the existence of two collateral-types of entrepreneurs implies asymmetric participation constraints. Because the collateral-type is a private information to the entrepreneur and $C_H > C_L$, collateral-type H entrepreneurs would apply for the credit contract $(\pi(.), C_L, R_L)$ if they find more interesting to do so rather than to apply for $(\pi(.), C_H, R_H)$. Contrary to type H, collateral-type L entrepreneurs cannot apply for $(\pi(.), C_H, R_H)$ in any case because of their lack of collateral. The following lemma formalizes. For each $i = H, L$, define $\hat{\theta}_i$ as the cutoff level of risk-type associated with the contract $(\pi(.), C_i, R_i)$, i.e., the risk-type of the entrepreneur endowed with collateral C_i indifferent between applying for $(\pi(.), C_i, R_i)$ and nothing else. Therefore, we have the following.

Lemma 5. *For any standard contracts $(\pi(.), C_L, R_L)$ and $(\pi(.), C_H, R_H)$ announced by the lender:*

1) Collateral-type L entrepreneurs exhibiting a risk-type $\theta \geq \hat{\theta}_L$ ($\theta < \hat{\theta}_L$) will apply for $(\pi(.), C_L, R_L)$ (will not apply for any contract and thus drop out of the market);

2) There exist R_H^- and R_H^+ increasing functions of R_L and verifying $R_H^- < R_H^+ < R_L$, so that:

i) if $R_H \geq R_H^+$*, therefore, collateral-type* H *entrepreneurs exhibiting a risk-type* $\theta \geq \hat{\theta}_L$ ($\theta < \hat{\theta}_L$) *will apply for* $(\pi(.), C_L, R_L)$ *(will not apply for any contract and thus drop out of the market);*

ii) if $R_H^- \leq R_H < R_H^+$*, therefore* $\hat{\theta}_L < \hat{\theta}_H$ *and there exists* $\tilde{\theta}_H$ *increasing in* R_H *but decreasing in* R_L *and verifying* $\tilde{\theta}_H > \hat{\theta}_H > \hat{\theta}_L$*, so that each entrepreneur of collateral-type* H*:*

- exhibiting a risk-type $\theta \geq \tilde{\theta}_H$ *will apply for* $(\pi(.), C_H, R_H)$*;*

- exhibiting a risk-type $\theta \in [\hat{\theta}_L, \tilde{\theta}_H)$ *will apply for* $(\pi(.), C_L, R_L)$*;*

- exhibiting a risk-type $\theta < \hat{\theta}_L$ *will not apply for any contract and thus drop out of the market;*

iii) if $R_H < R_H^-$*, therefore* $\hat{\theta}_H < \hat{\theta}_L$*, and each collateral-type* H *entrepreneur exhibiting a risk-type* $\theta \geq \hat{\theta}_H$ ($\theta < \hat{\theta}_H$) *will apply for* $(\pi(.), C_H, R_H)$ *(will not apply for any contract and thus drop out of the market).*

Proof. The proof of the first part of the lemma is identical to that of Lemma 1. The second part of the lemma concerning the incentive compatibility constraint of collateral-type H entrepreneurs is more complicated. Define $\bar{\rho}(C_i, R_i, \theta)$ as the expected return $\rho(C_i, R_i)$ to an entrepreneur of risk-type θ applying for $(\pi(.), C_i, R_i)$. Remark that for each $i = H, L$, keeping θ fixed, $\bar{\rho}(C_i, R_i, \theta)$ is decreasing in both R_i and C_i. Therefore, because $C_H > C_L$, there exist R_H^- and R_H^+ increasing functions of R_L verifying $R_H^- < R_H^+ < R_L$, and,

$$\begin{aligned}
\bar{\rho}(C_H, R_H^+, \theta^+) &= \bar{\rho}(C_L, R_L, \theta^+), \\
\bar{\rho}(C_H, R_H^-, \hat{\theta}_L) &= \bar{\rho}(C_L, R_L, \hat{\theta}_L).
\end{aligned}$$

Hence,

i) for $R_H \geq R_H^+$, we have $\bar{\rho}(C_H, R_H, \theta) \leq \bar{\rho}(C_L, R_L, \theta)$for any θ;

ii) for $R_H^- \leq R_H < R_H^+$, we have $\bar{\rho}(C_H, R_H, \theta^+) > \bar{\rho}(C_L, R_L, \theta^+)$ and $\bar{\rho}(C_H, R_H, \hat{\theta}_L) < \bar{\rho}(C_L, R_L, \hat{\theta}_L) = 0$, implying that $\hat{\theta}_L < \hat{\theta}_H$, so that there exists $\tilde{\theta}_H \in (\hat{\theta}_H, \theta^+)$ increasing in R_H but decreasing in R_L and verifying $\bar{\rho}(C_H, R_H, \tilde{\theta}_H) = \bar{\rho}(C_L, R_L, \tilde{\theta}_H)$, which in turn implies the following:

$$\begin{aligned}
&\bar{\rho}(C_H, R_H, \theta) > \bar{\rho}(C_L, R_L, \theta) > 0, && \text{for any } \theta \geq \tilde{\theta}_H, \\
&\bar{\rho}(C_H, R_H, \theta) < \bar{\rho}(C_L, R_L, \theta) \text{ and } \bar{\rho}(C_L, R_L, \theta) > 0, && \text{for any } \theta \in [\hat{\theta}_L, \tilde{\theta}_H), \\
&\bar{\rho}(C_H, R_H, \theta) < \bar{\rho}(C_L, R_L, \theta) < 0, && \text{for any } \theta < \hat{\theta}_L;
\end{aligned}$$

iii) for $R_H < R_H^-$, we have $\bar{\rho}(C_H, R_H, \theta^+) > \bar{\rho}(C_L, R_L, \theta^+)$ and $\bar{\rho}(C_H, R_H, \hat{\theta}_L) > \bar{\rho}(C_L, R_L, \hat{\theta}_L) = 0$, implying that $\hat{\theta}_H < \hat{\theta}_L$, so that,

$$\bar{\rho}(C_H, R_H, \theta) > \bar{\rho}(C_L, R_L, \theta) \text{ and } \bar{\rho}(C_L, R_L, \theta) > 0, \quad \text{for any } \theta \geq \hat{\theta}_H,$$

$$\bar{\rho}(C_L, R_L, \theta) < \bar{\rho}(C_H, R_H, \theta) < 0, \quad \text{for any } \theta < \hat{\theta}_H.$$

Figure 4 illustrates all of these different outcomes. Q.E.D.

Now, let's introduce the following equilibrium concepts. Define $(\pi(.), C_H, \hat{R}_H)$ and $(\pi(.), C_L, \hat{R}_L)$ as the optimal standard contracts maximizing the lender's aggregated return.

Definition 1. *Equilibrium is said:*

1) separating if $\hat{R}_H < R_H^-(\hat{R}_L)$;

2) mixed (semi-pooling) if $R_H^-(\hat{R}_L) \leq \hat{R}_H < R_H^+(\hat{R}_L)$;

3) pooling if $\hat{R}_H \geq R_H^+(\hat{R}_L)$.

It remains to examine whether or not separating equilibria entail credit rationing. Since under the pooling equilibrium the *nature* of credit rationing is the same than which occurring where all of entrepreneurs are endowed with the same collateral (see Proposition 1), I will focus on non-pooling equilibria only. The next proposition provides the answer.

Proposition 4. *In any economy in which equilibrium is:*

1) separating, collateral-type L entrepreneurs will receive loans only if credit is not rationed to collateral-type H entrepreneurs;

2) mixed, entrepreneurs applying for $(\pi(.), C_L, \hat{R}_L)$ (i.e., all of collateral-type L entrepreneurs and some of collateral-type H entrepreneurs) will receive loans only if credit is not rationed to collateral-type H entrepreneurs applying for $(\pi(.), C_H, \hat{R}_H)$.

Proof. See the Appendix.

This result is similar to Theorem 13 of Stiglitz and Weiss (1981). It shows how credit rationing affects entrepreneurs differently in function of their collateral-type as well as the credit contract they apply for.

B. *The Hybrid Credit Contract*

The previous analysis (Proposition 4) reveals that in the presence of heterogeneous collateral endowments (unobservable by lender), non-pooling equilibria prevailing under the standard credit contracts makes low-collateral entrepreneurs (as well self-selected high-collateral entrepreneurs) suffering from credit rationing more severely than others. Starting from this observation, I investigate in the following a new form of hybrid finance designated to entrepreneurs posting the lowest collateral, which I term by *collateral-based hybrid finance*, that stipulates the payment of a contingent compensation flowing from entrepreneur to the lender in addition to the flow already involved in the standard credit contract. In particular, I examine the "take it or leave it" credit contract $(\pi(.), \psi(.), C_L, R_L, \beta)$ that promises to an entrepreneur posting the lowest collateral C_L the return,

$$\rho(C_L, R_L, \beta) = \rho(C_L, R_L) - \psi(R_L, \beta),$$

while pays off to the lender,

$$\pi(C_L, R_L, \beta) = \pi(C_L, R_L) + \psi(R_L, \beta),$$

where $\psi(R_L, \beta) = \beta(z - R_L)$ if $z \geq R_L$ with $\beta \in (0,1)$ and $\psi(R_L, \beta) = 0$ otherwise. Developing the cash flows promised to both parties of this hybrid contract, we have

$$\rho(C_L, R_L, \beta) = \begin{cases} (1-\beta)(z - R_L) & \text{if} \quad z \geq R_L, \\ z - R_L & \text{if} \quad R_L - C_L \leq z < R_L, \\ -C_L & \text{if} \quad z < R_L - C_L, \end{cases}$$

$$\pi(C_L, R_L, \beta) = \begin{cases} (1-\beta) R_L + \beta z & \text{if} \quad z \geq R_L, \\ R_L & \text{if} \quad R_L - C_L \leq z < R_L, \\ z + C_L & \text{if} \quad z < R_L - C_L. \end{cases}$$

Intuitively, this hybrid contract combines both debt and equity financing instruments. For the entrepreneur, the compensation flow $\psi(R_L, \beta)$ that goes to lender at the good states (i.e.,

$z \geq R_L$) can be viewed as a dividend payout. Indeed, the entrepreneur's return is identically the same as if the project was financed by the issue of new portion of equity with a marketable value equivalent to β percent of the project's value. For the lender, the compensation flow $\psi(R_L, \beta)$ can be interpreted as a bonus or an extra-payment remunerating her low-guaranteed loan financing. The main difference between this contract and the standard equity financing contract, however, is that the positions of lender and entrepreneur are asymmetric. This is because the debt component is still present in the contract which provides the lender with the status of a prior claimant at the bad states (i.e., $z < R_L$). To see this, observe that at the bad states, the lender's return is the same than which promised by a standard credit contract. The equity component, indeed, operates at the good states only, where the return to the lender is equivalent to the return of a portfolio composed of $(1-\beta)$ percent of debt and β percent of equity.

Lemma 6. *The participation and the incentive compatibility constraints of entrepreneurs confronting the menu composed of* $(\pi(.), C_H, R_H)$ *and* $(\pi(.), \psi(.), C_L, R_L, \beta)$ *are the same stated by Lemma 5 after replacing the applications for* $(\pi(.), C_L, R_L)$ *by those for* $(\pi(.), \psi(.), C_L, R_L, \beta)$. *In addition, the offer of the hybrid credit contract* $(\pi(.), \psi(.), C_L, R_L, \beta)$ *rather than the standard contract* $(\pi(.), C_L, R_L)$ *will result in: (i) a move of the cutoff level of risk-type* $\hat{\theta}_L$ *to a higher level* $\dot{\theta}_L > \hat{\theta}_L$ *increasing in* R_L *both and* β*; (ii) no change in the cutoff level of risk-type* $\hat{\theta}_H$*; and (iii) a downward move of the critical level of risk-type* $\tilde{\theta}_H$.

Proof. Define,

$$\bar{\psi}(R_L, \beta, \theta) = \int_0^\infty \psi(R_L, \beta) dF(z, \theta), \tag{16}$$

as the expected compensation flow $\psi(R_L, \beta)$ that goes to lender upon the financing of the entrepreneur of risk-type θ applying for the hybrid contract. The proof simply follows from Lemma 5, the fact that the utility form applying for the hybrid contact $(\pi(.), \psi(.), C_L, R_L, \beta)$ for an entrepreneur of risk-type θ is,

$$U_h = \bar{\rho}(C_L, R_L, \theta) - \bar{\psi}(R_L, \beta, \theta) < \bar{\rho}(C_L, R_L, \theta), \tag{17}$$

for any θ, and the observation that $\bar{\psi}(R_L, \beta, \theta)$ is increasing in β. Q.E.D.

This lemma shows that by displacing the cutoff levels of the entrepreneurs' risk-type, the introduction of the hybrid contract not only enables the lender to extract more rents from financing collateral-type L entrepreneurs, but also to shrink the pool of collateral-type H entrepreneurs that would like to post the lowest collateral of C_L (see Lemma 5). As the analysis below will reveal, the lender will exploit this screening effect of the hybrid contract-attached compensation rate β to optimize her aggregated return.

Define M_{sh} as the optimized menu composed of the standard contract $(\pi(.), C_H, R_H^*)$ and the hybrid contract $(\pi(.), \psi(.), C_L, R_L^*, \beta^*)$, and M_{ss} as the menu composed of the optimal standard contracts $(\pi(.), C_H, \hat{R}_H)$ and $(\pi(.), C_L, \hat{R}_L)$. Further, let's introduce the following equilibrium concept.

Definition 2. *Credit market equilibrium associated with the menu M_{sh} is said separating if R_H^*, R_L^*, and β^* are such that no collateral-type H entrepreneur applies for the hybrid contract.*

The next result shows how the introduction of the hybrid credit contract would be attractive for the lender.

Proposition 5. *For any economy in which the menu of contracts M_{sh} leads to a separating equilibrium, the lender is strictly better-off with supplying the menu M_{sh} rather than M_{ss}, and this independently from the nature of equilibrium (separating or pooling) observed under M_{ss}.*

Proof. See the Appendix.

An immediate implication of this result is that upon the introduction of the hybrid contract, an economy in which equilibrium *would have* been pooling under the menu M_{ss} could achieve a separating equilibrium in which no collateral-type H entrepreneur will post the lowest collateral C_L to the lender. Another important implication is that for economies in which entrepreneurs exhibit different collateral-types, collateral-based hybrid finance targeting poor-collateral entrepreneurs will inevitably emerge if it leads to a separating equilibrium.

C. *Institutional Design: Venture Capital Finance*

The analysis presented above provides an account for the emergence of venture capital finance. Indeed, as venture capital finance does, the collateral-based hybrid finance studied permits: (i) the lender to earn rents from serving poor-collateral entrepreneurs by providing her the claim to share the project's good state-payoff with the financed entrepreneur, and (ii) entrepreneurs suffering from little collateral endowment, either self-selected or rationed, to get an adapted form of finance.

As the contracts used by real-world venture capitalists, the hybrid credit contract I analyzed constitutes a mixture of debt and equity claims. There are several theoretical models that have examined this feature. For example, Repullo and Suarez (1999) show that the return to venture capitalists could be approximated by a warrant like those issued by public firms to their stockholders. Nöldeke and Schmidt (1998) study a theoretical model in which the venture capitalist's right to cash flows is considered as an "option to own", i.e., a sort of option on the equity of the financed firm. Furthermore, because the lender has the right to consume a fixed fraction of the project's payoff in the good states of nature, the model also captures the convertible nature of securities employed by venture capital firms. In this regard, Schmidt (2003) shows that this convertibility property generates incentive effects that discipline contracting parties and enhance the efficiency of investment in ventures.

Interestingly, Proposition 5 shows that lenders are always better-off with offering collateral-based hybrid finance to poor-collateral entrepreneurs whenever this form of hybrid finance, when combined with standard finance, leads to a separating equilibrium. This means that a sufficient condition for the coexistence of both venture capital finance and bank finance is the existence of separating equilibrium in which banks (through standard credit contracts) will serve collateral-unconstrained entrepreneurs like established firms, while venture capitalists (through hybrid credit contracts) will supply credit to poor-collateral entrepreneurs like start-up firms. It is worthwhile to note that even if the attractiveness of collateral-based hybrid finance for lenders consists of the rents extracted from high-risk projects given the equity component embedded in the hybrid credit contract, the supply of this form of hybrid finance in my model is only motivated by the existence of poor-collateral type of entrepreneurs. This is consistent with the empirical findings of Sahlman (1990) showing that, in contrast to what could be subjectively expected, most of the projects financed by venture capital firms were moderately profitable, while only a few percentage of those projects were extremely successful.

Although the model provides a rationale for the coexistence of banks and venture capitalists, it cannot help us more to answer how competition for ventures would take place between these two intermediaries nor why poor-collateral entrepreneurs would decide to go to banks offering ventures rather than to apply for specialized venture finance. To address these issues, the model needs to be extended to account for the specialization skills of lenders and the impact of these skills on the choice of financer by the entrepreneurs. Nevertheless, Ueda (2004) provides a detailed response for this issue by showing that the capacity of specialized venture capitalists of evaluating projects more accurately than banks' divisions and to threaten to steal them from the entrepreneurs plays an important role in the choice of financer.

V. Conclusions

In this paper, I have developed an adverse election model of credit market equilibrium providing an account for the emergence of nonbank intermediaries and their continuing coexistence with banks. I have explored two credit contracts that share the use of costly screening technologies, explicitly formulated. In contrast to the standard credit contract supplied by banks, the two credit contracts (hybrid credit contracts in the language of the model) I investigated make use of more screening devices than the interest rate, and capture well the real-world contracts offered by nonbank intermediaries, such as mutual banks and venture capitalists. The analysis reveals that the supply of these contracts permits to achieve two main outcomes, giving rationales for the emergence of nonbank finance. First, it allows lenders to attenuate the negative effects of adverse selection and to enhance their profitability thanks to the screening effect it generate. Second, it permits entrepreneurs dropping out of the credit market under bank finance, either by self-selection or credit rationing mechanisms, to get loans.

This paper fills a gap in the financial intermediation theory by providing a unified model of both bank and nonbank finance that accounts for the contractual devices distinguishing each form of lending from the other. A subsequent research would address the endogenous formation of nonbank intermediaries and the risk profile of the securities they issue in order to complete our understanding of the institutional aspect of credit markets.

Appendix: Proofs

Proof of Proposition 1. Let us consider the average expected return to the lender (i.e., the mean return per capita) offering the standard contract $(\pi(.), R)$,

$$\bar{\pi}_{\text{avg}}(R) = \frac{1}{1 - P(\hat{\theta})} \int_{\hat{\theta}}^{\theta^+} \bar{\pi}(R, \theta) dP(\theta), \tag{18}$$

with $\bar{\pi}(R,\theta) = \int_0^\infty \pi(R) dF(z,\theta)$ is the expected return to the lender from financing the entrepreneur θ. Differentiating with respect to the interest rate R, we obtain,

$$\begin{aligned} \frac{d\bar{\pi}_{\text{avg}}(R)}{dR} = & \frac{p(\hat{\theta})\ d\hat{\theta}/dR}{(1 - P(\hat{\theta}))^2} \int_{\hat{\theta}}^{\theta^+} \bar{\pi}(R,\theta) dP(\theta) \\ & + \frac{1}{1 - P(\hat{\theta})} \left[\int_{\hat{\theta}}^{\theta^+} \frac{\partial \bar{\pi}(R,\theta)}{\partial R} dP(\theta) - \bar{\pi}(R,\hat{\theta}) \frac{d\hat{\theta}}{dR} p(\hat{\theta}) \right]. \end{aligned} \tag{19}$$

Since $d\hat{\theta}/dR > 0$ (see Lemma 1) and $d\bar{\pi}(R,\theta)/dR > 0$, the average return $\bar{\pi}_{\text{avg}}(R)$ is a nonmonotonic function of R. In competitive equilibrium, this, as shown by Stiglitz and Weiss (1981, Theorem 5), may lead to credit rationing as claimed. Q.E.D.

Proof of Proposition 2. Let's define $\Delta(\alpha) = \Pi_h(\hat{R},\alpha) - \Pi(\hat{R}) = \Gamma(\alpha) - \alpha\Phi(\alpha)$ with,

$$\begin{aligned} \Gamma(\alpha) \quad &:= \int_{\dot{\theta}(\hat{R},\alpha)}^{\hat{\theta}(\hat{R})} \bar{\pi}(\hat{R},\theta) dP(\theta), \\ \Phi(\alpha) \quad &:= \int_{\dot{\theta}(\hat{R},\alpha)}^{\theta^+} (\hat{R} - 1)[1 - F(\hat{R},\theta)] dP(\theta). \end{aligned}$$

Observe that $\Delta(\alpha) > 0$ for some $\alpha \in (0,1)$ requires the choice of a compensation rate α, such that $\Gamma(\alpha)/\Phi(\alpha) > \alpha$.

Because $\dot{\theta}(\hat{R},\alpha) = \hat{\theta}(\hat{R})$ for $\alpha = 0$, we have $\Gamma(\alpha = 0) = 0$ while $\Phi(\alpha = 0) > 0$. This added to the fact,

$$\begin{aligned} \Gamma'(\alpha) \quad &= \quad -\frac{\partial \dot{\theta}}{\partial \alpha} \bar{\pi}(\hat{R},\dot{\theta}) p(\dot{\theta}) > 0, \\ \Phi'(\alpha) \quad &= \quad -\frac{\partial \dot{\theta}}{\partial \alpha} (\hat{R} - 1)[1 - F(\hat{R},\dot{\theta})] p(\dot{\theta}) > 0, \end{aligned}$$

implies that for any $\alpha \in [0,1]$, $0 \leq \Gamma(\alpha)/\Phi(\alpha) \leq b$ with $0 < b \leq 1$. Resultantly, the non monotonic function $h(\alpha) = \Gamma(\alpha)/\Phi(\alpha)$ admits at least the origin ($\alpha = 0$) as a fixed point. The existence of any further strictly positive fixed point implies that given $\hat{R}$, there exists $\hat{\alpha} \in (0,1)$ for which lender always offer the hybrid contract $(\pi(.), \phi(.), \hat{R}, \hat{\alpha})$. This means that the statement in Proposition 2 holds for $(\pi(.), \phi(.), \hat{R}, \hat{\alpha})$. The proposition then follows by recalling that $(\pi(.), \phi(.), \hat{R}, \hat{\alpha})$ is a suboptimal hybrid contract.

Now, we show that we must have the inequality $R^* \leq \hat{R}$ satisfied in equilibrium. To prove this, assume that the hybrid contract dominates the standard contract so that for $\alpha^* > 0$, we have $\Pi_h(R^*, \alpha^*) > \Pi(\hat{R})$. Recall that at equilibria, the cutoff levels of risk-type $\hat{\theta}$ and $\dot{\theta}$ are functions of $\hat{R}$ and (R^*, α^*), respectively; that $\dot{\theta}$ is increasing in the interest rate while decreases with the compensation rate. Assume that $R^* > \hat{R}$. Therefore, we have two cases:

Case (I): $\alpha^* > 0$, so that $\dot{\theta}(R^*, \alpha^*) > \hat{\theta}(\hat{R})$. In such a situation, lender offering the dominating hybrid contract will only finance entrepreneurs qualified as non-IRCEs under the dominated standard contract. This implies that there exists $\bar{R}$ verifying $\hat{R} < \bar{R} < R^*$ and $\hat{\theta}(\bar{R}) = \dot{\theta}(R^*, \alpha^*)$, such that $\Pi(\bar{R}) = \Pi_h(R^*, \alpha^*)$. Since $(\pi(.), \hat{R})$ is the optimal standard contract, we have $\Pi(\hat{R}) \geq \Pi(\bar{R})$. This means that $\Pi(\hat{R}) \geq \Pi_h(R^*, \alpha^*)$, which is a contradiction.

Case (II): $\alpha^* > 0$, so that $\dot{\theta}(R^*, \alpha^*) < \hat{\theta}(\hat{R})$. In this case, there exists $\bar{R}$ verifying $\bar{R} < \hat{R} < R^*$ and $\hat{\theta}(\bar{R}) = \dot{\theta}(R^*, \alpha^*)$, so that $\Pi(\bar{R}) = \Pi_h(R^*, \alpha^*)$. Since $\Pi(\hat{R}) \geq \Pi(\bar{R})$, this leads to $\Pi(\hat{R}) \geq \Pi_h(R^*, \alpha^*)$, which is also a contradiction. Q.E.D.

Proof of Proposition 3. Observe that as indicated by scenario 3) in Lemma 3, lender is always able by the mean of market demand's mechanism to make the menu composed of $(\pi(.), \hat{R})$ and $(\pi(.), \phi(.), R', \alpha)$ reducing to the optimal pooling-hybrid contract. This constitutes the worst outcome lender would face. Hence, the proposition is immediate from Proposition 2. Finally, the equilibrium condition $R'^* > \hat{R}$ is justified from Lemma 3, elsewhere there will be a contradiction. Q.E.D.

Proof of Proposition 4. Given Lemma 5, the aggregated return $\Pi(R_L, R_H)$ to lender is equal to:

$$q_H \int_{\hat{\theta}_H}^{\theta^+} \bar{\pi}(C_H, R_H, \theta) dP(\theta) + q_L \int_{\hat{\theta}_L}^{\theta^+} \bar{\pi}(C_L, R_L, \theta) dP(\theta) := \Pi_1, \tag{20}$$

if $R_H < R_H^-$,

$$\int_{\hat{\theta}_L}^{\bar{\theta}_H} \bar{\pi}(C_L, R_L, \theta) dP(\theta) + q_H \int_{\bar{\theta}_H}^{\theta^+} \bar{\pi}(C_H, R_H, \theta) dP(\theta) + q_L \int_{\bar{\theta}_H}^{\theta^+} \bar{\pi}(C_L, R_L, \theta) dP(\theta) := \Pi_2, \quad (21)$$

if $R_H^- \leq R_H < R_H^+$, and

$$\int_{\hat{\theta}_L}^{\theta^+} \bar{\pi}(C_L, R_L, \theta) dP(\theta) := \Pi_3, \quad (22)$$

if $R_H \geq R_H^+$, where $\bar{\pi}(C_i, R_i, \theta)$ denotes the expected value of the return $\pi(C_i, R_i)$ to the lender offering the contract $(\pi(.), C_i, R_i)$ to an entrepreneur of risk-type θ $(i = H, L)$.

Observe that for equilibrium to be separating rather than pooling, we must have $\Pi_1 > \Pi_3$, which implies,

$$q_H \int_{\hat{\theta}_H}^{\theta^+} \bar{\pi}(C_H, R_H, \theta) dP(\theta) > (1 - q_L) \int_{\hat{\theta}_L}^{\theta^+} \bar{\pi}(C_L, R_L, \theta) dP(\theta). \quad (23)$$

Since $(1 - q_L) = q_H$, the inequality above reduces to,

$$\int_{\hat{\theta}_H}^{\theta^+} \bar{\pi}(C_H, R_H, \theta) dP(\theta) > \int_{\hat{\theta}_L}^{\theta^+} \bar{\pi}(C_L, R_L, \theta) dP(\theta). \quad (24)$$

By the same, for equilibrium to be mixed rather than pooling, we must have $\Pi_2 > \Pi_3$, implying that,

$$\int_{\bar{\theta}_H}^{\theta^+} \bar{\pi}(C_H, R_H, \theta) dP(\theta) > \int_{\bar{\theta}_H}^{\theta^+} \bar{\pi}(C_L, R_L, \theta) dP(\theta). \quad (25)$$

It thus follows that under both separating and mixed equilibria, collateral-type H entrepreneurs applying for $(\pi(.), C_H, \hat{R}_H)$ must be served first. This is because if assuming not, lender as indicated by (24) and (25) is able to increase her aggregated return by progressively substituting loans supplied to entrepreneurs applying for $(\pi(.), C_L, \hat{R}_L)$ with loans to collateral-type H entrepreneurs applying for $(\pi(.), C_H, \hat{R}_H)$ until serving all of them, which constitutes a contradiction with the optimality of $\Pi(\hat{R}_L, \hat{R}_H)$. Q.E.D.

Proof of Proposition 5. There are two cases.

Case (I): The menu of the standard contracts M_{ss} achieves a pooling equilibrium, so that the lenders' aggregated return is given by,

$$\int_{\hat{\theta}_L}^{\theta^+} \bar{\pi}(C_L, \hat{R}_L, \theta) dP(\theta) := \Pi(M_{ss}), \quad (26)$$

which can be rewritten as follows:

$$q_H \int_{\hat{\theta}_L}^{\theta^+} \bar{\pi}(C_L, \hat{R}_L, \theta) dP(\theta) + q_L \int_{\hat{\theta}_L}^{\theta^+} \bar{\pi}(C_L, \hat{R}_L, \theta) dP(\theta). \tag{27}$$

Remark that by offering the (non-optimized) menu composed of $(\pi(.), C_H, R_H)$ and the hybrid contract $(\pi(.), \psi(.), C_L, \hat{R}_L, \beta)$, the lender is able to realize an aggregated return equal to,

$$q_H \int_{\hat{\theta}_H}^{\theta^+} \bar{\pi}(C_H, R_H, \theta) dP(\theta) + q_L \int_{\dot{\theta}_L}^{\theta^+} [\bar{\pi}(C_L, \hat{R}_L, \theta) + \bar{\psi}(\hat{R}_L, \beta, \theta)] dP(\theta) := \Pi(\hat{R}_L, R_H, \beta). \tag{28}$$

Based on Lemma 5, setting $R_H = R_H^-(\hat{R}_L)$ we have $\hat{\theta}_H(R_H^-) = \hat{\theta}_L(\hat{R}_L)$ and $\bar{\rho}(C_H, R_H^-, \theta) \leq \bar{\rho}(C_L, \hat{R}_L, \theta)$ for any $\theta \geq \hat{\theta}_H(R_H^-)$, which given the fact that $\pi + \rho = z$ and Assumption 1 leads to $\bar{\pi}(C_H, R_H^-, \theta) \geq \bar{\pi}(C_L, \hat{R}_L, \theta)$ for any $\theta \geq \hat{\theta}_H(R_H^-)$ thus implying,

$$\int_{\hat{\theta}_H = \hat{\theta}_L}^{\theta^+} \bar{\pi}(C_H, R_H^-, \theta) dP(\theta) > \int_{\hat{\theta}_L}^{\theta^+} \bar{\pi}(C_L, \hat{R}_L, \theta) dP(\theta). \tag{29}$$

Since we can find at least $\tilde{\beta} \in (0, 1)$ so that,

$$\int_{\dot{\theta}_L}^{\theta^+} [\bar{\pi}(C_L, \hat{R}_L, \theta) + \bar{\psi}(\hat{R}_L, \tilde{\beta}, \theta)] dP(\theta) > \int_{\hat{\theta}_L}^{\theta^+} \bar{\pi}(C_L, \hat{R}_L, \theta) dP(\theta), \tag{30}$$

where $\dot{\theta}_L(\hat{R}_L, \tilde{\beta}) > \hat{\theta}_L(\hat{R}_L)$, the lender is able to realize an aggregated return $\Pi(\hat{R}_L, R_H^-, \tilde{\beta}) > \Pi(M_{ss})$. It hence follows from $\Pi(\hat{R}_L, R_H^-, \tilde{\beta}) \leq \Pi(M_{sh})$ that $\Pi(M_{ss}) < \Pi(M_{sh})$, with $\Pi(M_{sh})$ is the lender's aggregated return expected from the menu M_{sh}.

Case (II): The menu of standard contracts M_{ss} achieves a (fully) separating equilibrium where the lender's aggregated return is given by,

$$q_H \int_{\hat{\theta}_H}^{\theta^+} \bar{\pi}(C_H, \hat{R}_H, \theta) dP(\theta) + q_L \int_{\hat{\theta}_L}^{\theta^+} \bar{\pi}(C_L, \hat{R}_L, \theta) dP(\theta) := \Pi(M_{ss}). \tag{31}$$

Observe that by offering the (non-optimized) menu composed of $(\pi(.), C_H, \hat{R}_H)$ and the hybrid contract $(\pi(.), \psi(.), C_L, \hat{R}_L, \beta)$, the lender is able to realize an aggregated return equal to,

$$q_H \int_{\hat{\theta}_H}^{\theta^+} \bar{\pi}(C_H, \hat{R}_H, \theta) dP(\theta) + q_L \int_{\dot{\theta}_L}^{\theta^+} [\bar{\pi}(C_L, \hat{R}_L, \theta) + \bar{\psi}(\hat{R}_L, \beta, \theta)] dP(\theta) := \Pi(\hat{R}_L, \hat{R}_H, \beta). \tag{32}$$

As in Case (I), we can find at least $\tilde{\beta} \in (0, 1)$ verifying,

$$\int_{\dot{\theta}_L}^{\theta^+} [\bar{\pi}(C_L, \hat{R}_L, \theta) + \bar{\psi}(\hat{R}_L, \tilde{\beta}, \theta)] dP(\theta) > \int_{\hat{\theta}_L}^{\theta^+} \bar{\pi}(C_L, \hat{R}_L, \theta) dP(\theta), \tag{33}$$

where $\dot{\theta}_L(\hat{R}_L, \tilde{\beta}) > \hat{\theta}_L(\hat{R}_L)$, which implies that $\Pi(\hat{R}_L, \hat{R}_H, \tilde{\beta}) > \Pi(M_{ss})$. This combined with $\Pi(\hat{R}_L, \hat{R}_H, \tilde{\beta}) \leq \Pi(M_{sh})$ yield that $\Pi(M_{ss}) < \Pi(M_{sh})$.

Since the third case of mixed equilibrium is a mixuture of the two cases examined above, one can conclude that independently from the nature of the credit market equilibrium associated with the menu of the standard contracts M_{ss}, we have $\Pi(M_{ss}) < \Pi(M_{sh})$. Q.E.D.

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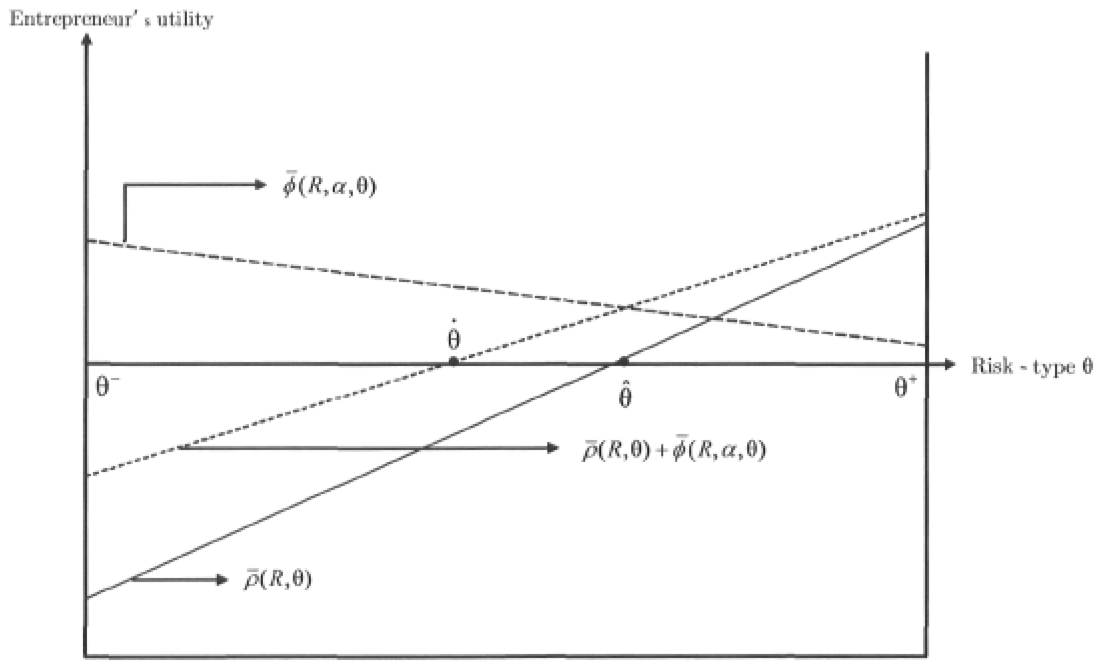

Figure 1. This figure shows how the introduction of the hybrid credit contract ($\pi(.)$,$\varphi(.)$,R,α) pushes the entrepreneur's utility upward thus making some of IRCEs applying for credit.

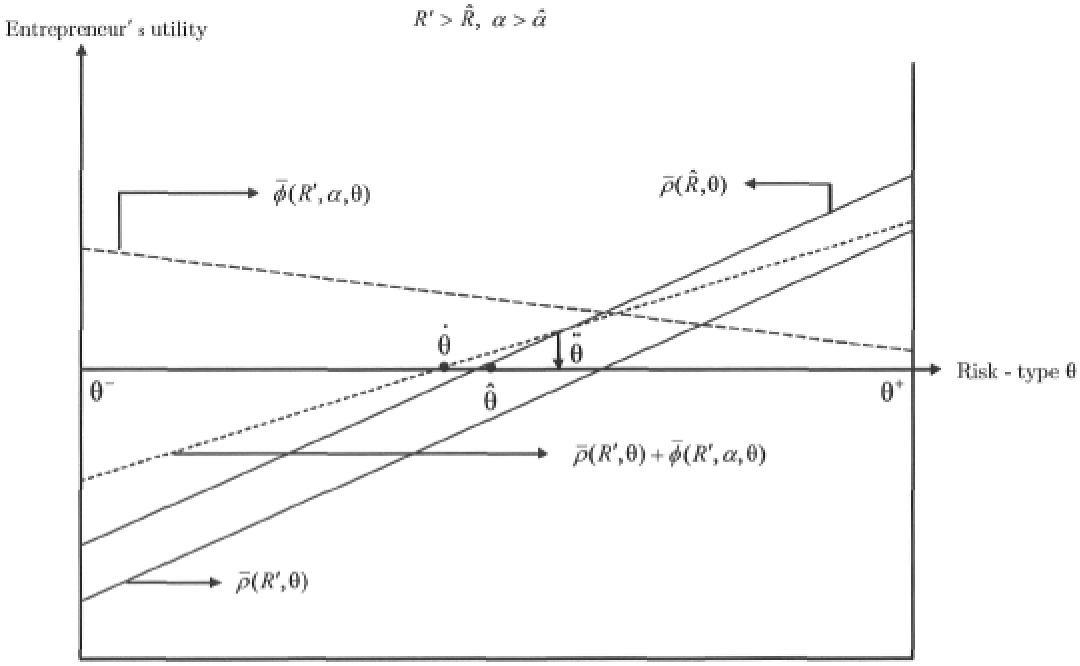

Figure 2 - a

Figure 2-a. This figure shows how the supply of the menu composed of the standard contract $(\pi(.), \hat{R})$ and the hybrid contract $(\pi(.), \varphi(.), R', \alpha)$, under the conditions required, permits the lender to screening IRCEs.

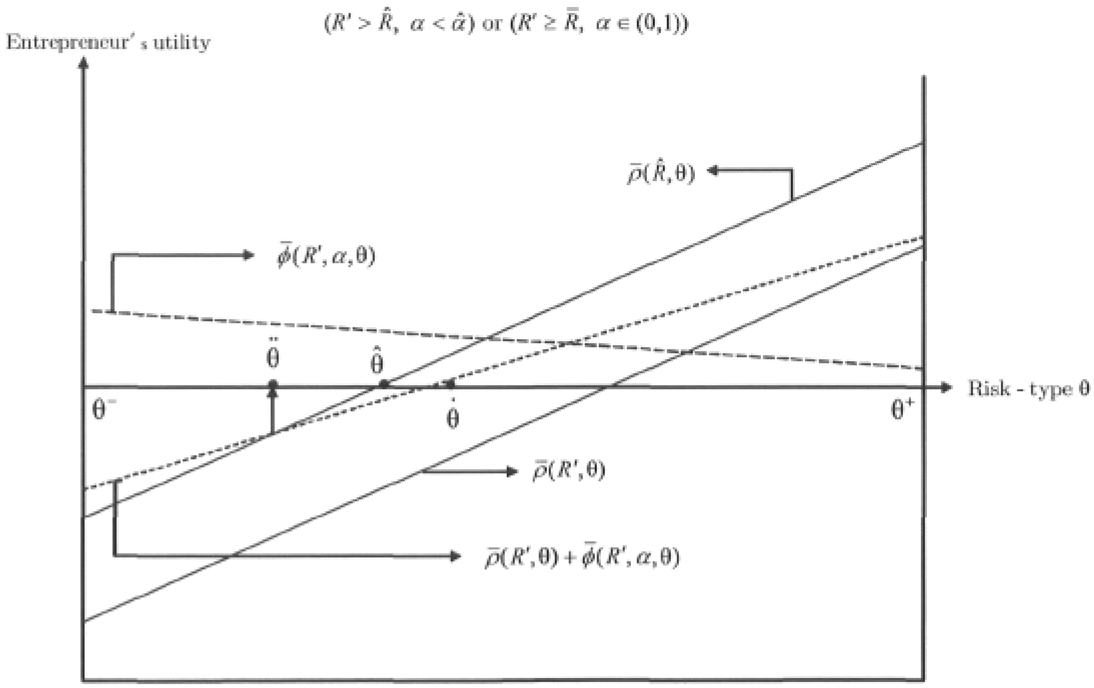

Figure 2 - b

Figure 2-b. This figure shows how the supply of the menu composed of the standard contract $(\pi(.), \hat{R})$ and the hybrid contract $(\pi(.), \varphi(.), R', \alpha)$, under the conditions specified, will achieve the same market outcome where hybrid finance is not allowed; that is where all of IRCEs drop out of the credit market.

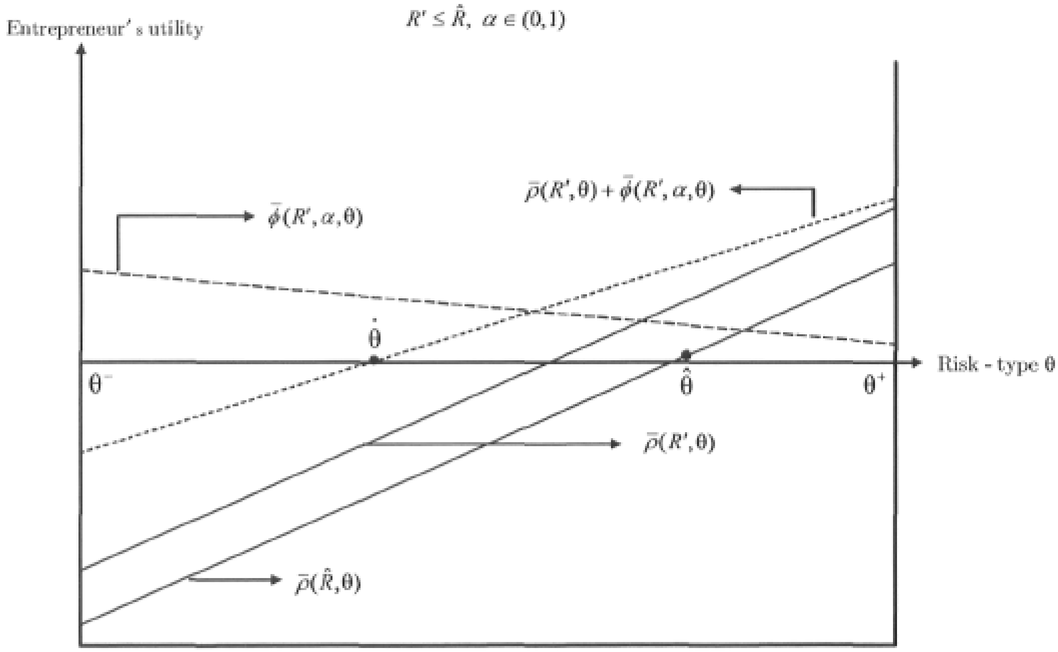

Figure 2-c

Figure 2-c. This figure shows how the supply of the menu composed of the standard contract $(\pi(.),\hat{R})$ and the hybrid contract $(\pi(.),\varphi(.),R',\alpha)$, under the conditions specified, will make all entrepreneurs leaving the standard contract.

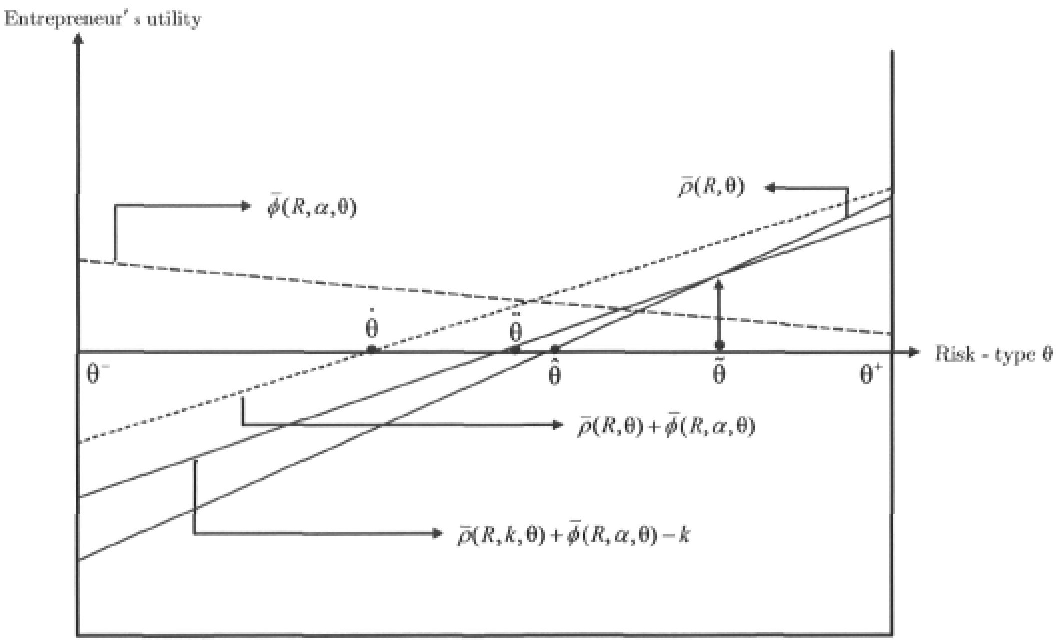

Figure 3. This figure shows how the supply of the menu composed of the standard contract $(\pi(.), \hat{R})$ and the hybrid contract involving a sunk cost component $(\pi(.), \varphi(.), \hat{R}, \alpha, k)$ permits the lender to screening IRCEs.

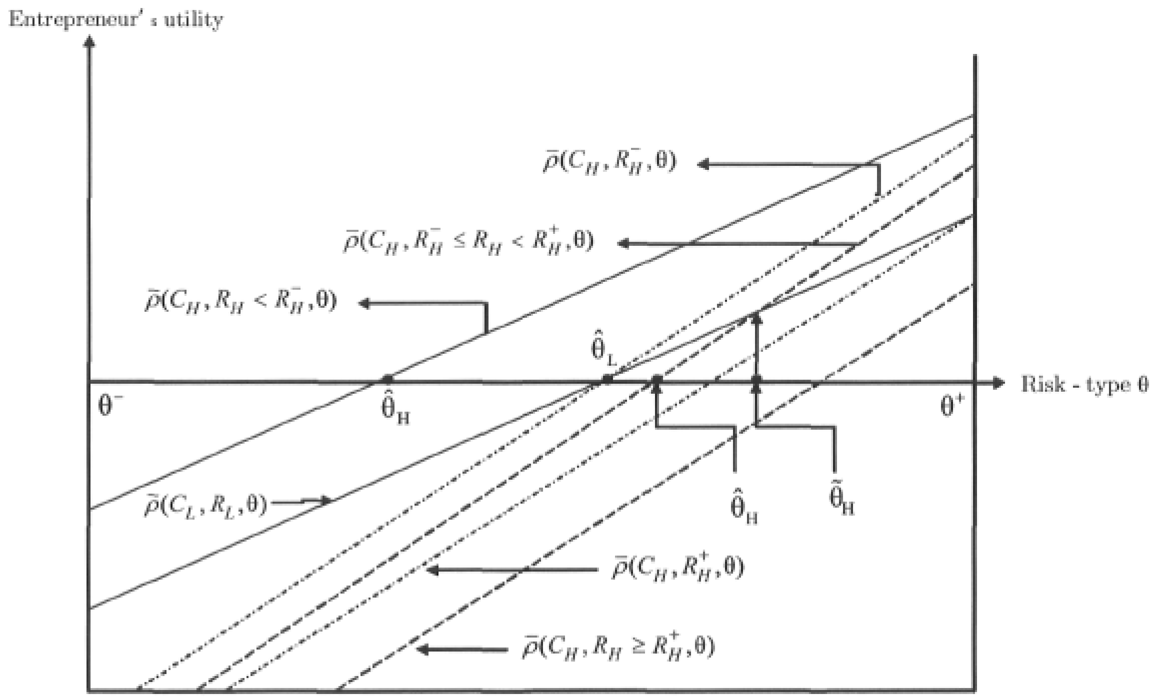

Figure 4. This figure illustrates the different outcomes the lender may face when offering the menu of contracts $(\pi(.), C_L, R_L)$ and $(\pi(.), C_H, R_H)$. We see that the level of R_H is determinant for the demand of credit contracts by entrepreneurs (both types *L* and *H*). The figure depicts all the different scenarios formally described in Lemma 5.

Essay 3:
Deposits Fragility, Others' Money Rents and Competition: An Asset-Liability Management Model of the Bank Interest Rate Margin

Abstract

The interest rate margin accounts for a large fraction of the banks' profitability. This paper develops an asset-liability management model of the determinants of this margin in which banks manage actively their exposures to the main sources of intermediation risk: the credit risk of loans and the fragility of deposits. The model reveals that both a deterioration of the credit quality of assets and an enhanced liquidity of deposits pushes banks to increase their interest rate margins. The impact of interbank competition, however, is shown to be unpredictable making interest rate margins useless to evaluate the market power of banks.

JEL classification: G20; G21; G29

Keywords: Bank interest rate margin; Liquidity risk; Interbank competition; Bank failure; Deposits insurance

I. Introduction

By intermediating savers and entrepreneurs, banks raise funds in the form of deposits, demandable in the short run, and transform them into illiquid and risky assets taking the form of loans maturing in the long run. To maximize shareholders' wealth, banks must make this basic function of financial intermediation risk-controlled as well as profitable. As for industrial firms, the maximization of the bank's profit is subject to a variety of risk factors and constraints. This task as accomplished by banks, commonly termed by asset-liability management, is more sophisticated than for industrial firms, however.[1]

This paper explores the determinants of the bank interest rate margin defined as the spread between the interest rate charged on loans and the interest rate paid on deposits. Empirically, the interest margin accounts for a large fraction of banks profits (for example, as reported by Wong 1997, this fraction was about 80 percent on average in the U.S. at the beginning of 1990s).[2] In competitive money markets, where the demand and the supply of loanable funds are interest-rate-elastic, the interest rate margin constitutes a meaningful indicator of the bank policy and how this policy is adjusted in response to both economic and institutional changes.

The determinants of the bank interest rate margin were first examined by Ho and Saunders (1981). Inspired from the theoretical models of bid-ask spread observed in the stock market, the authors consider the bank as a risk-averse "dealer" managing non-synchronic stochastic flows of demand and supply of loanable funds. Their main results reveal that the bank interest rate margin is sensitive to the management's risk-version and the volatility of interest rates. Allen (1988) extended the Ho and Saunders' model to allow for heterogeneous loans. Her analysis shows that because of the

[1] First, the asset transformation makes banks bearing the credit risk of borrowers. Second, because of the liquidity gap between loans and deposits, banks have to create liquidity. Third, banks are exposed to institutional/regulatory environments more constraining than the industrial firms would face. Deposits insurance, capital adequacy requirements and public policies of interbank competition are examples of institutional/regulatory constraints that influence the banks' decisions.

[2] In general, the term *interest margin* is used to qualify the margin between the interest revenue of the bank and her interest fees. For this reason, to indicate the interest rate spread rather than the dollar margin, I use the term *interest rate margin*.

cross-elasticities between the demands for (heterogeneous) loan products, banks may benefit from a portfolio effect (diversification effect) lowering the interest rate margins observed in equilibrium. Other than the dealership approach, Zarruk and Madura (1992) develop a firm-theoretical model in which the source of uncertainty is the credit risk of loans. The authors investigate the impact of deposits insurance and capital adequacy regulation on the bank's interest rate margin. They find that for banks exhibiting a non-increasing risk aversion, higher capital requirement or higher deposits insurance premium lead to higher interest rate margins. In the same vein, Wong (1997) proposes a generalization of the Zarruk and Madura's model by allowing for interest rate risk and different scenarios for the banker's risk aversion.

Although the issues these models focus on such as the risk preferences of the bank' management deserve attention from the theoretical point of view (e.g., see Hellwig, 2000), the factors ignored like the fragility of deposits,[3] the heterogeneity of bank's deposit products, the risk of bank failure and interbank competition are expected to affect the interest rate margin considerably.[4] But the main limit of these models motivating this paper is that they assign a passive role to banks in the determination of the interest rate margin. For example when dealing with the random arrival of loanable funds, banks in Ho and Saunders (1981) and Allen (1988) act as passive dealers without any ability of controlling their exposure to this basic intermediation risk. Similarly, by ignoring the asset-liability management dimension, Zarruk and Madura (1992) and Wong (1997) do not allow banks to play an active role in financial intermediation.

The goal of the paper is to provide a microeconomic account of bank behavior, consistent with the practice of banks management, permitting to explore the determinants of the interest rate margin in a complex environment of financial intermediation where banks manage their risk exposures actively. I develop an asset-liability management model of the bank interest rate margin in the

[3] An exception to be noted here is that the dealership models of Ho and Saunders (1981) and Allen (1988) allow for liquidity shocks by considering stochastic flows of loanable funds.

[4] The term "deposits fragility" I will frequently use in the rest of paper designates the demandability of deposits due to the allowance for deposits withdrawals. The term *fragile* expresses the idea that banks have to deal with a fragile liquidity structure when transforming deposits demandable in the short-term (demand deposits) into illiquid long-term assets.

presence of two sources of intermediation risk: the credit risk of loans and the fragility of deposits. The analysis is conducted in a realistic setup allowing for demand/term deposits mix, deposits insurance, heterogeneous assets, and the risk of bank failure (bankruptcy). A generalized market structure in which several banks compete for deposits-taking is also examined. The main contribution of the model is that it permits the investigation of some important issues the aforementioned models cannot explore because of their basic assumptions. The model also innovates by allowing for both deposits withdrawals and the risk of bank failure; stylized facts that are not captured by most of financial intermediation models.

In a simplified version of the model, the bank raises funds in the form of demandable deposits and immediately transforms them into a mix of: (i) liquid, non-interest-bearing assets; and, (ii) illiquid, interest-bearing assets. In the model, liquid assets are riskless (or in general, capital-preserving securities), since they pay off a certain return. In this sense, they are viewed as reserves the bank holds to meet deposits' withdrawals. In contrast, illiquid assets are loans serving to finance risky projects that promise uncertain returns, and thus are subject to credit risk. To capture the fragile liquidity structure caused by the asset transformation process, loans in the model cannot be liquidated before maturity. On the liability side, deposits are the primitive form of finance the bank uses to finance the lending activity. Furthermore, when the demand for liquidity exceeds reserves the bank issues nondeposit liability instruments on the capital market. Nondeposit liability allows the bank to avoid a forced premature (suboptimal) liquidation by providing her financing against the loans portfolio. Given this model of bank behavior, the optimal interest rate margin arises endogenously from maximizing the expected intermediation surplus of the bank.

In a more generalized setup, funds raised by the bank are a mix of demand and term deposits. Consistent with the empirical findings of McShane and Sharpe (1985), the model reveals that a lower fragility of deposits reflected by a lower demand deposits-to-total deposits ratio pushes banks to increase their interest rate margin. The main idea behind is that the fragility of deposits enables banks to benefit from others' money rents by transferring the asset's risk to nondeposit liability holders (bondholders). As a result, a lower fragility of deposits implies a lower liquidity creation, which makes the bank bearing asset's risk more than she *would have* assumed with fully demandable deposits. Because term deposits are closely exposed to the risk of bank's failure, this effect is amplified when introducing deposits insurance.

The last part of the analysis extends the model to allow for interbank competition. By analogy to industrial firms, viewing deposits as production inputs and loans as goods to be sold, the interest rate margin must reflect the market power of banks. The model, in contrast to the case of industrial firms, suggests that the intensity of competition between banks does not influence monotonically the equilibrium interest rate margin. This prediction is in line with related theoretical models such as Allen and Gale (2004) and Boyd and De Nicoló (2005) showing an ambiguous relationship between competition and the risk-taking behavior of banks. An immediate implication of these models and ours to the design of public policies of interbank competition is that the proposition of a negative tradeoff between competition and stability is not necessarily valid. For our concern, despite the fact that it is closely indicative of the effective exposure of banks to the industry's risk factors, the interest rate margin is not useful as much for assessing the banks' market power.

The remainder of the paper is organized as follows. Section II presents the model. Section III examines the determinants of the bank interest rate margin. Section IV discusses the consistency of the model results with the related empirical literature. Section V concludes. The proofs of propositions are relegated to the Appendix.

II. The Model

Consider an economy lasting for a time interval– date 0 to date T. Date 0 is the instantaneous planning period, and date T is the instantaneous consumption period. There are two classes of agents: savers and entrepreneurs. At date 0, each entrepreneur would like to exploit a project that requires some fixed amount of external finance. All projects end at date T. There is a bank with a function consisting of raising money from savers (depositors) at date 0, and immediately transforming it into loanable funds to entrepreneurs (borrowers). The pools of depositors and entrepreneurs remain unchanged over the time period $[0, T]$. That is, once money is raised from depositors at date 0, the bank does not collect new deposits. For the time being, the bank is assumed to benefit from a monopolistic position. This assumption is relaxed later in the paper by allowing for interbank competition.

A. *Demand and Supply of Funds*

The total supply of deposits is captured through an upward sloping inverse supply function $R_D(D)$, where R_D denotes the gross interest rate to be paid to deposits.[5] The symbol D refers to the total amount of deposits the bank raises. Further, define L as the total amount of loans. The gross interest rate the bank charges on loans follows from a downward sloping inverse demand function $R_L(L)$. The supply and demand functions of funds satisfy the following.

Assumption 1. (i) $R'_D > 0, R''_D \geq 0$; (ii) $R'_L < 0, R''_L \leq 0$; (iii) $R_L(0) > R_D(0) > 1$.

Note that condition (iii) ensures the existence of equilibrium in which the bank is able to intermediate the two groups of agents, the savers and entrepreneurs. At first glance, the bank faces a basic tradeoff when fixing its interest rate margin, the spread $R_L - R_D$. Raising more deposits implies supplying more loans, which given Assumption 1 will lead to a low interest rate margin, and vice versa. Throughout the rest of paper it will be shown that the economic environment in which the bank operates makes the optimal interest rate margin more complex to be consistently and simply predicted from this basic tradeoff.

B. *Deposits Fragility*

Deposits are fragile (demandable) in the sense that withdrawals may occur at any intermediate time $0 \leq t \leq T$. To capture this situation, let $w(t)$ be a (continuous) time- deterministic process starting from zero that represents the cumulated amount of withdrawals to occur up to the date

[5] All of the interest rates considered in the model apply to money flows to be exchanged at times zero and T. That is, if $T = 1$, interest rates will be interpreted as annual rates.

t, providing an initial deposit of 1 dollar at date 0. The matching of occurred withdrawals with depositors is not critical for our purpose, so that the process increment $dw(t)$ can be indexed on the whole amount of deposits without loss of generality. Consequently, the nominal value of remaining deposits at any time t is,

$$1 - \int_0^t dw(s),$$

for each one dollar deposited at date 0. The technical assumption we require here is $dw(t) > 0$ for any $0 \leq t \leq T$. This implies that the nominal value of remaining deposits is strictly decreasing in time because of the allowed withdrawals. Additionally, we need to impose that,

$$\int_0^T dw(t) = 1. \tag{1}$$

This means that deposits are running in full over the time period $[0, T]$. The assumption of a time-deterministic process $w(t)$ could be argued in the situation where the bank has acquired from previous exercise periods the ability to accurately predict future withdrawals. Elsewhere, the model can be easily extended to allow for stochastic withdrawals of deposits, but this would not add more intuition to our analysis.

C. *The Credit Risk of Loans*

The projects to be financed by the bank return uncertain payoffs at date T, such that for any entrepreneur-borrower the probability that the capitalized loan amount is not reimbursed in full is strictly positive. For our purpose, treating loans performances individually is not critical. The aggregated recovery value of loans is $\tilde{\varepsilon} L R_L(L)$ where $\tilde{\varepsilon} \in [0, 1]$ is a random variable generated by a continuous and an increasing probability distribution function $P(\tilde{\varepsilon})$, indicating the *ex ante* credit quality of loans. In other words, $(1 - \tilde{\varepsilon})$ represents the percentage of non-performing loans. Note here that the risk structure of loans does not depend on the size of the bank's loans portfolio. This would be the case where after exploiting the diversification potential of the faced pool of entrepreneur-borrowers, the bank will only bear the systematic risk of financed projects.

D. *Asset-Liability Management*

Because of the liquidity gap existing between demandable deposits and illiquid loans, a fraction of the bank's assets is held in the form of marketable securities. Although the institutional environment in certain countries does not stipulate reserve requirements anymore, banks usually obey to internal safety standards leading them to hold liquid assets, a sort of buffer reserves that help them to overcome deposits withdrawals. Without loss of generality, I assume that these reserves are composed of non-interest-bearing assets.

Define $S_\tau(D)$ as the amount of reserves balancing the deposits withdrawals capitalized at the gross interest rate R_D, to occur over the time interval $[0, \tau]$ with $0 < \tau < T$. Therefore, we have that,

$$S_\tau(D) = DR_D(D) \int_0^\tau \frac{t}{T} dw(t). \tag{2}$$

By disposing of the amount $S_\tau(D)$ of liquid assets at date 0, the bank is ensured that internal reserves will permit to overcome the demands for liquidity up to date τ without the need for external finance. Note here that reserves in the model could be either high or low depending on the safety standard τ of the bank. In this setup, the bank's budget-balancing constraint at time zero is

$$L + S_\tau(D) = D. \tag{3}$$

Summarizing, the sequence of events occurring instantaneously at date 0 is as follows: (i) The bank fixes the amount of total deposits D, and, thus, the gross interest rate R_D is revealed; (ii) Given the reserves standard τ, reserves $S_\tau(D)$ are allocated in function of D and R_D respecting (2); (iii) Given D and $S_\tau(D)$, the bank fixes the total amount of loans L to supply such as the budget equation (3) is satisfied, and the gross interest rate of R_L to charge to entrepreneurs will be revealed in consequence.

Once the time τ is reached, reserves $S_\tau(D)$ are consumed in full. To overcome the demand for liquidity, the bank proceeds to a *liability substitution* that consists of issuing nondeposit liability instruments. More precisely, the bank contracts a nondeposit financing at date τ permitting to acquire funds balancing the amount of future deposits withdrawals to occur from τ to date T,

$$(1-w(\tau))DR_D(D)\int_{\tau}^{T}\frac{t}{T}dw(t).$$

The reimbursement of this nondeposit liability is made upon the liquidation of the bank's loans portfolio at date T. The nondeposit liability holders (bondholders), therefore, are promised to receive at the maturity date T the amount,

$$B = R_B \times \left\{(1-w(\tau))DR_D(D)\int_{\tau}^{T}\frac{t}{T}dw(t)\right\}, \tag{4}$$

where R_B is the expected (gross) return required on nondeposit finance, which is exogenously fixed and, for convenience, normalized to one.[6] It is worthwhile to note that the loans securitization would be a concrete financing strategy that captures the liability substitution policy described in the model.

Providing the date 0-reserves and the nondeposit liability to be issued at the intermediate date τ, the bank is completely ensured to meet its contractual obligations towards depositors. This setup is useful for two reasons. First, it permits us to circumvent the problem of bank runs (Diamond and Dybvig, 1983). Second, it makes for instance deposits insurance irrelevant, since depositors are not exposed to the risk of bank's failure. Nevertheless, I examine later a generalized model in which deposits are risk-bearing claims and introduce deposits insurance.

In order to keep the model as simple as possible, the bank capital is ignored. Note however that incorporating bank capital is easily feasible. As in Zarruk and Madura (1992) and Wong (1997), one can consider a binding capital adequacy constraint such that the bank capital represents a given percentage of the amount of deposits. By doing so, the bank capital will simply displace the equilibrium outcome without affecting the model results qualitatively.

[6] Since the issue of nondeposit liabilities is operated through the liability substitution policy (i.e., the bank in the model cannot make use initially of this nondeposit finance), the bank cannot benefit from the rate differential $(R_D - R_B)$ at time zero. Hence, the simplifying assumption $R_B = 1$ should not affect the model results.

E. *The Optimal Interest Rate Margin*

In this model of bank behavior, the economic function of the bank consists of optimizing the deposits-taking activity subject to the market demand and supply of funds faced. Namely, the problem of fixing the amount of deposits D to raise for a risk-neutral bank given the limited-liability rule is,

$$\mathbf{Prg(a)}: \underset{\{D\}}{\text{maximizes}} \int_0^1 \max\left[0,\ \tilde{\varepsilon} L R_L(L) - B\right] dP(\tilde{\varepsilon}),$$

subject to Eq. (4),

$$\begin{aligned} L &= D \times \left\{ 1 - R_D(D) \int_0^{\tau} \frac{t}{T} dw(t) \right\}, \\ D &\geq 0. \end{aligned}$$

The interest rate margin arises endogenously from the model as the spread achieved in equilibrium between the interest rate charged on loans and the interest rate paid on deposits.

As one can see, increasing deposits will result in an increase of loanable funds, but will simultaneously cause an increase of nondeposit-liability. The equilibrium amount of deposits verifies the first-order condition,

$$E\left[\frac{d\tilde{\varepsilon} L R_L(L)}{dD} \middle| \tilde{\varepsilon} > \hat{\varepsilon} \right] = E\left[\frac{dB}{dD} \middle| \tilde{\varepsilon} > \hat{\varepsilon} \right], \qquad (5)$$

where $E[.]$ is the expectation operator associated with the probability function $P(\tilde{\varepsilon})$; $\hat{\varepsilon} = B/LR_L(L)$ is the critical level of $\tilde{\varepsilon}$ below which the bank defaults on his nondeposit- liability B.

Lemma 1. *For not excessively rate-elastic demand and supply of funds, the bank's deposits-taking optimization problem admits an interior solution for any standard τ of reserves.*

Proof. See the Appendix.

This simply means that by transforming a fraction of the loanable funds into safety reserves (or liquid assets), the bank needs market-power sufficient enough to be able to intermediate savers and entrepreneurs.[7] To provide more precision on the tolerated level for the rate-elasticity of the demand and supply of funds we need more explicit assumptions. However, simple numerical examples would give more intuition concerning this point. The next section examines how the credit quality of loans, the fragility of deposits and interbank competition affect the equilibrium interest rate margin.

III. The Determinants of the Bank Interest Rate Margin

A. *The Asset Quality*

The quality of banks' assets influences the stability and the systematic risk of the banking industry. Examining how banks set their deposit-taking and interest rate margin policies in function of the credit risk they bear constitutes a critical issue in this sense. The following proposition tells us how the credit quality of the bank's loans affects the equilibrium levels of both deposits and the interest rate margin.

Proposition 1. *For any standard τ of reserves, a mean-preserving increase of the credit risk of loans implies a decrease of the equilibrium amount of deposits and an increase of the interest rate margin.*

Proof. See the Appendix.

The rationale behind this result is due to the relationship between reserves and nondeposit liability in one side and the likelihood of bank failure in the other side. Remark that an increase of deposits, *ceteris paribus*, implies an increase of the amount of reserves. But because deposits are demandable, keeping the standard τ of reserves (or liquid assets) fixed means that marginal increases of deposits beyond some level will be counter-balanced by marginal increases of the nondeposit liability more than by incremental reserves, thus increasing the probability of bank's failure. Since

[7] Even though the bank is supposed here benefiting from a monopoly position, its market-power is reduced by the rate-elasticity of the demand and offer of funds. Similarly to industrial firms, the market-power of the bank can be measured by the *Lerner index* thanks to the demand and offer functions of funds.

the bank's solvency is subject to the credit quality of loans, this implies that the higher the default risk affecting the bank's loans, the higher is the marginal increase of the probability of bank failure caused by the marginal increase of the nondeposit liability.

Interestingly, this result shows how banks readjust their lending policy in function of changes occurring in the real economy. Because deposits raised by the bank reflect the funds available for lending, a decrease of deposits due to a lower credit quality of loans will cause a credit crunch in our model. This prediction is consistent with the observed behavior of banks in several OECD countries during the late 1980s and early 1990s. As reported by Hölmstrom and Tirole (1997), the higher interest rates observed during this period have reduced the cash flows and the corporate worth of firms, thus lowering the credit quality of borrowers operating in the real sectors. This has severely affected equity value in the banking industry, pushing banks to compress loans and to increase their interest rate margin.

Although deposits here are not exposed to the risk of bank's loans, they play a prominent role by acting as a channel for risk transfer from the bank's shareholders to capital market by the mean of the nondeposit liability. Indeed, collecting deposits and securitizing loans, which in the language of the model is called by the liability substitution, are both complementary and sequential stages of a risk transfer policy. In this regard, the risk of failure is crucial, since it disciplines banks by preventing them from proceeding to unlimited transfer of risk to nondeposit liability holders, thus permitting some trade-off to take place.

In spite of the fact that the impact of the credit quality of loans on the interest rate margin is essentially transmitted via the money market mechanisms captured through the demand and supply functions of funds, the proposition foreshadows the determinant impact of the demandability of deposits on the bank's interest rate margin. In the following subsection, I investigate this main issue by examining the sensitivity of the bank interest rate margin to the degree of deposits fragility.

B. *Demand Deposits versus Term Deposits*

Deposits are so far assumed demandable at any time over the period $[0, T]$. This means that the bank is dealing with an extremely fragile structure of liquidity. This assumption is now relaxed by allowing the bank to attract both demandable and non demandable deposits. I model this as follows: At date 0, the bank issues an aggregated amount of deposits D by fixing the gross interest rate R_D to be paid out to depositors. A fraction αD (with $0 < \alpha < 1$) of the total deposits, say *term deposits*, is maintained in full up to date T, while the remaining amount of deposits $(1-\alpha)D$, is demandable at any time, i.e., *demand deposits*. Aggregated withdrawals of demand deposits follow the same process $w(t)$ defined previously.

The implications for the bank of confronting this mix of deposits supply are numerous. Indeed, reserves will only balance a fraction of the demand deposits. For term deposits, however, the bank needs no reserves. This will result, *ceteris paribus*, in a higher amount of funds available for loans comparatively to the previous case where all deposits are demandable.[8] The amount of reserves for a given safety standard τ is given by

$$S_\tau(D) = (1-\alpha)DR_D(D)\int_0^\tau \frac{t}{T}dw(t), \tag{6}$$

implying that the nondeposit liability balancing demand deposits withdrawals beyond date τ is,

$$B = (1-w(\tau))(1-\alpha)DR_D(D)\int_\tau^T \frac{t}{T}dw(t). \tag{7}$$

In contrast to the case of demand deposits, the suppliers of term deposits will be exposed to the risk of bank's failure in the absence of deposits insurance. By introducing a deposits-insurance system, however, the supply of term deposits will not be affected by the bank's risk which enables us to use the aggregated (inverse) supply of deposit funds $R_D(D)$ as it is assumed in Eqs. (6) and (7). Of course the risk of bank failure covered by deposits insurance is not the only factor that would explain potential scale differences between the supply of term deposits and that of demand

[8] Note here that there is no readily available argument leading us to expect that the bank's supply of loans will be higher than which prevailing in the previous case where deposits are fully demandable. In fact, the exact amount of loans supplied by the bank will be determined in equilibrium.

deposits. The opportunity cost of liquidity is also suspected to affect the supply of the two types of deposit differentlys. To simplify our analysis, however, we shall maintain our assumption according to which the supply functions of both the demand and term deposits are the same. In the light of our next results, a generalization of the model in this direction would not add more intuition to our analysis.

Let us assume that the bank pays a deposits-insurance premium at time 0. In real-world situations, reserves and liquid assets (marketable securities and short-term assets other than loans) held by banks are taken in account by the insurer (i.e., the deposits-insurance agency) when charging the insurance premium, since they can be immediately used by the bank to overcome withdrawals of demandable deposits, and, thus, avoids liquidity failure. In our model, this implies that the deposits-insurance premium is function of the amount of term deposits, since only term deposits are exposed to the risk of bank's failure. Let $h(\alpha D)$ denotes this function.

Assumption 2. *The deposits-insurance premium $h(.)$ is a strictly increasing and convex function of the amount of insured deposits.*

This assumption can be easily justified, since the deposits-insurance premium might be viewed as the value of a put option written on the bank's risky assets (i.e., the loans portfolio) with a strike price equal to the amount of insured deposits (Merton, 1977).

Under this generalized framework, the bank's optimization problem is given by,

$$\mathbf{Prg(b)}: \underset{\{D\}}{\text{maximizes}} \int_0^1 \max\left[0,\ \tilde{\varepsilon} L R_L(L) - \alpha D R_D(D) - B\right] dP(\tilde{\varepsilon}) - h(\alpha D),$$

subject to Eq. (7),

$$\begin{aligned} L &= D \times \left\{1 - (1-\alpha) R_D(D) \int_0^T \frac{t}{T} dw(t)\right\} - h(\alpha D), \\ h(\alpha D) &\geq 0, \\ D &\geq 0. \end{aligned}$$

Resolving this problem and examining the sensitivity of the equilibrium interest rate margin to the deposits mix of the bank yields the following result.

Proposition 2. *For any standard τ of reserves not excessively high, the interest rate margin is increasing with the deposits mix's parameter α.*

Proof. See the Appendix.

This means that the higher (lower) the weight of term (demand) deposits in the deposits financing mix, the higher is the bank's interest rate margin. This is due to two complementary effects. The first, *risk transfer effect*, is acting through demand deposits. Given Eq. (7), one can see that the nondeposit liability of the bank is increasing with the amount of demand deposits. As a result, the lower the weight of demand deposits in the deposits mix, the lower is the capacity of the bank to extract others' money rents by transferring the loans' risk to nondeposit liability holders.[9] The second effect, *deposits insurance effect*, is operating through term deposits. It implies that the higher the amount of term deposits, the higher is the deposits insurance premium the bank has to pay. Hence, combining these two effects yields that a high deposits mix's parameter α implies a high effective exposure of the bank to the loans' risk, thus, leading her to charge a high interest spread to borrowers. Figure 1 illustrates this mechanism. It shows for various values of the deposits mix's parameter α how these two effects influence the optimal interest rates margin via the equilibrium amounts of demand and term deposits.

This result is very important, since it does not only reveal how the equilibrium interest rate margin is sensitive to the intensity of deposits fragility, but also provides an economic interpretation of the interest rate margin itself. It challenges the proposition viewing the interest rate margin as a premium compensating the credit risk the bank bears when transforming riskless deposits into defaultable loans. Because of the asset-liability management, indeed, the interest rate margin should reflect the effective exposure of the bank to the asset's risk rather than the intrinsic risk of the bank's asset itself. To see the subtle difference between these two interpretations, one can simply

[9] Moreover, since demand deposits are demandable and term deposits are insured, the bank is unable to transfer the loans' risk to depositors.

combine Propositions 1 and 2 to deduce that, *ceteris paribus*, banks financing low-risk projects may charge an interest rate margin higher than that banks financing riskier projects would require. This interpretation of the interest rate margin is critical for empirical concerns, particularly when comparing banks exhibiting heterogeneous sizes, and, thus, different deposit-financing structures.[10]

C. *Interbank Competition*

A closely related extension of the model consists of allowing for interbank competition. In fact regulators mostly discourage monopolistic banking industries by granting bank charters to a number of banks. The entry to the banking sector, however, is subject to the economic value of bank charters themselves. For this reason, I allow for imperfect competition. Specifically, there are n banks competing for deposits-taking in a Cournot fashion.[11] Each bank i ($i = 1, ...n$) chooses the total amount of deposits D_i to raise, recognizing that the (inverse) supply function of deposits is given by $R_D(\Sigma_i D_i)$. The amount of loans to supply by bank i follows from the budget equation (3) and the (inverse) demand function $R_L(\Sigma_i L_i)$. Analysis is built in the framework allowing for mixed deposits and deposits insurance. In equilibrium, the set of market shares $\{(D_i)_{i=1,...n}\}$ follows from maximizing the expected net profit for each one of the n banks. Hence, for a common deposits-insurance agency $h(.)$ and n granted charters, the program of each bank i is,

$$\mathbf{Prg(c)} : \underset{\{D_i\}}{\text{maximizes}} \int_0^1 \max\left[0,\ \tilde{\varepsilon}_i L_i R_L\left(\sum_{i=1}^{n} L_i\right) - \alpha D R_D\left(\sum_{i=1}^{n} D_i\right) - B_i\right] dP_i(\tilde{\varepsilon}_i) - h(\alpha_i D_i),$$

[10] It is well documented that there is a significant divergence between the deposits-liabilities financing structures of small and large banks. In the context of US commercial banks, English and Nelson (1998) provide detailed statistics.

[11] It is quite intuitive to think that banks compete over market shares, since the hypothesis of differentiated deposit/loan products seems hard to defend.

subject to,

$$\begin{aligned}
L_i &= D_i \times \left\{1-(1-\alpha_i)R_D\left(\sum_{i=1}^{n} D_i\right)\int_0^{\tau_i}\frac{t}{T}dw(t)\right\} - h(\alpha_i D_i),\\
B_i &= (1-w_i(\tau_i))(1-\alpha_i)DR_D\left(\sum_{i=1}^{n} D_i\right)\int_{\tau_i}^{T}\frac{t}{T}dw(t),\\
h(\alpha_i D_i) &\geq 0,\\
D_i &\geq 0.
\end{aligned}$$

As in Boyd and De Nicoló (2005) analyzing banks stability, I shall simplify the analysis here to a symmetric Nash-Cournot equilibrium by setting $\alpha_i = \alpha, \tau_i = \tau$, and $P_i(.) = P(.)$ for each $i = 1, ...n$, so that the problem above reduces to the determination of the amount of deposits D to be raised by each bank $i = 1, ...n$.[12] Allowing for a more general framework of interbank competition that captures differentiated banking products would certainly increase the quality of the model predictions. However, such extension comes at the cost of loosing the analytic tractability of the model. Standard models of competition in industrial economics permit such analysis, since they deal with relatively simple production functions. In contrast here, the bank's objective function is quite sophisticated, which limits the model flexibility in this sense. Nevertheless, numerical resolution of the model remains an alternative approach for a further research.

In such a setup, we have the following result.

Proposition 3. *The interest rate margin at the Nash-Cournot equilibrium is a non-monotonic function of the interbank competition's intensity n.*

Proof. See the Appendix.

Similarly to industrial firms, by viewing deposits as production inputs and loans as goods to be sold, the interest rate margin must reflect the market power of banks.[13] Therefore, this proposition may appear surprising from the point of view of industrial economics, since an increasing

[12] The assumption of symmetric Nash-Cournot equilibrium substantially simplifies analytic development without making us loosing generality. By consulting the mathematical proof of the proposition provided in this subsection, one can easily deduce that the model prediction will remain qualitatively the same if we allow for asymmetric profiles of banks in competition.

[13] In the language of industrial organization models, the interest rate margin reflects the *mark-up* in the banking industry.

competition is expected to reduce the market power of firms. Related theoretical papers examining the relationship between competition and the risk-taking behavior of banks, however, have demonstrated a similar result. Allen and Gale (2004) and Boyd and De Nicoló (2005) show that the impact of interbank competition on the stability of banking industry is ambiguous.

Since the bank's objective function of the intermediation surplus's maximization differs sensibly from that of industrial firms because of the asset-liability management, we would like to know whether we can attribute this result to the fragility of deposits, the credit risk of loans or both of these two factors of risk. To accomplish this task, two particular equilibrium outcomes are examined. The first corresponds to the case where the totality of deposits attracted by banks are qualified as term deposits (i.e., this occurs where the parameter $\alpha = 1$). Without restricting the deposits mix's parameter α, the second equilibrium obtains where banks' loans serve to finance default-free projects. By investigating these two cases, it is revealed that taken individually, both deposits fragility and credit risk leads to a non monotonic relationship in equilibrium between the industry concentration and the interest rate margin. The same result also holds when combining these two polar cases. Nevertheless, as shown by the numerical simulations illustrated in Figure 2, the model predicts that beyond some critical level for the number of banks in competition, the effect of the deposits mix will completely disappear so that the equilibrium interest rate margin becomes monotonically decreasing with the parameter n. The simulations reported also suggest that the less the fragility of deposits (i.e., the higher the value of α), the lower is this critical intensity of competition beyond which such a monotonic behavior will be observed.

In the light of the last proposition, there is a potential gain of prudence from not treating stability and competition as acting in opposite directions from the point of view of welfare economics. As shown by Allen and Gale (2004), the proposition of a negative tradeoff is not necessarily valid when fixing optimal public polices of interbank competition. In our context, although the interest rate margin reflects the effective exposure of banks to the industry's risk factors, it does not reflect as much their market power. This means that observed interest rate margins would be useless for regulators in assessing the welfare effect of public policies of interbank competition and their impact on the stability of the banking industry.

IV. Related Empirical Work

The determinants of the bank interest rate margin were examined in several empirical studies. The existing empirical work, however, borrows insights from the theoretical model of Ho and Saunders (1981) based on the dealership approach. Nevertheless, the empirical findings reported are useful to evaluate the consistency of the theoretical results derived in this paper.

In an empirical study of US commercial banks, Angbazo (1997) finds that after controlling for key indicators of both on-balance and off-balance sheet activities, banks charging the highest interest rate margins are those having the highest exposure to the credit risk measured by the net charge-offs. This provides a clear support for Proposition 1 of the model.

Furthermore, it is documented from Angbazo (1997) and Saunders and Schumacher (2000) that banks bearing high implicit interest paid out in the form of services and extra payments to depositors charge high interest rate margins. The authors interpret this empirical result as consistent with the hypothesis according to which banks facing changes in the deposits-taking industry will finance implicit interest payments by increasing their explicit interest rate spread charged for borrowers. Alternatively, since implicit interest payments on deposits reflect the relative importance of demand deposits in the deposits mix of banks, the positive relationship between these fees and interest rate margins seems in conflict with Proposition 2 of the model. However, this interpretation is suggestive. A stronger and a more direct empirical evidence, which in conformity with Proposition2, is provided by an earlier study conducted by McShane and Sharpe (1985) examining the time series of Australian banks' interest rate margins over a period of twenty years. The authors find that the decline over time of the demand deposits-to-total deposits ratio, which exactly reflects the parameter $(1-\alpha)$ in our model, has caused a significant increase of the interest rate margin.

Saunders and Schumacher (2000) provide an empirical study investigating the relationship between competition and the interest rate margin. The authors examine this issue by comparing banks' interest rate margins using panel data of both US and European banks. After introducing a set of control variables in a first-step panel regression, the authors define the regression's intercept as the pure interest rate margin reflecting the country-specific structure of interbank competition. Their second-step regression then consists of explaining cross-country variations of the pure interest rate margin in function of country-specific market-structure dummies and the volatility of the money market-interest rates indicating the country-specific macro-economic conjuncture. Their results unambiguously reveal that the effect of the market structure on the interest rate margin is significant, but quite heterogeneous across countries. In particular, it appears after controlling for both heteroskedasticity and cross-correlation between residuals in data that the relative importance of pure interest rate margins estimated for the different countries of the sample cannot be consistently predicted from the concentration of the associated national banking industries (see Saunders and Schumacher, 2000, p. 828). This empirical evidence accords with Proposition 3 of the model predicting a non monotonic relationship between competition and the interest rate margin.

V. Conclusions

This paper has developed an asset-liability management model of the bank interest rate margin in the presence of two sources of intermediation risk: the credit risk of loans and the fragility of deposits. The impacts of the quality of assets, the heterogeneity of deposit products, and interbank competition on the interest rate margin are examined in a realistic setup of financial intermediation.

The model predicts the behavior of banks during periods of high interest rates where the deterioration of the credit quality of borrower-firms pushes banks to shrink loanable funds, which results in an increase of the banks' interest rate margin. Moreover, consistent with documented facts in the empirical literature, the model reveals that a lower fragility of deposits leads banks to increase their interest rate margin. The rationale behind is that the fragility of deposits enables banks to benefit from others' money rents by transferring the risk of assets to capital market. Hence, a lower fragility of deposits implies a lower liquidity creation, which in turn increases the exposure of banks to the asset's risk. Furthermore, in contrast with the context of industrial firms, the model shows that an intensification of the competition between banks does not affect the equilibrium interest rate margin in a monotonic fashion. Rather, in line with recent empirical and theoretical work, the model suggests an unpredictable relationship between the intensity of interbank competition and the interest rate margin. An important policy implication of this result is that observed interest rate margins are useless for regulators in assessing the effective market power of banks.

Finally, because of the realistic environment of financial intermediation it captures, the model would be attractive for addressing related issues, such as the risk-taking behavior of banks. Further extensions of the model, such as informational imperfections of the credit market and the risk aversion of the bank's management are also interesting for subsequent research.

Appendix: Proofs

To simplify notations, let us define

$$J(\tau) \equiv \int_0^{\tau} \frac{t}{T} dw(t), \tag{8}$$

$$K(\tau) \equiv \int_{\tau}^{T} \frac{t}{T} dw(t). \tag{9}$$

Proof of Lemma 1. First, remark that the first-order condition (5) issued from the bank's program Prg. (a) can be rewritten as follows,

$$\underbrace{\frac{\frac{dLR_L(L)}{dD}}{\frac{dB}{dD}}}_{\equiv\Phi(D)} = \underbrace{\frac{1-P(\hat{\varepsilon})}{E\left[\tilde{\varepsilon}\right|\tilde{\varepsilon} > \hat{\varepsilon}]}}_{\equiv\Gamma(\hat{\varepsilon})}, \tag{10}$$

where,

$$\begin{aligned} \frac{dLR_L(L)}{dD} &= \left[1 - J(\tau)\left(R_D(D) + DR'_D(D)\right)\right] \\ &\quad \times \left[R_L(L) + \left(D - DR_D(D)J(\tau)\right)R'_L(L)\right], \end{aligned} \tag{11}$$

$$\frac{dB}{dD} = \left(1 - w(\tau)\right)K(\tau) \times \left(R_D(D) + DR'_D(D)\right). \tag{12}$$

Since $\tilde{\varepsilon} \in [0,1]$, we have $\Gamma(\hat{\varepsilon}) > 1$ for any $0 < \hat{\varepsilon} < 1$. This means that the existence of an interior solution for the bank's deposits-taking program requires $\Phi(D) > 1$. By inspection, this is satisfied for any $0 < \tau < T$ whenever the demand and supply of funds, respecting Assumption 1, are not excessively rate-elastic.

Proof of Proposition 1. Based on Assumption 1, it is easy to check that the first-order derivative dB/dD is increasing in D. Moreover, for any not excessively rate-elastic demand and supply of funds respecting Assumption 1 and ensuring the existence of an interior solution, $dLR_L(L)/dD$ is decreasing in D for any $0 < \tau < T$. In sum, this implies that $\Phi(D)$ is decreasing in D: **(Claim 1)**.

Further, following the approach adopted in Sandmo (1971, p. 67), inspection reveals that a mean-preserving increase of the credit risk will result in an increase of the ratio $\Gamma(\hat{\varepsilon})$: **(Claim 2)**.

Hence combining Claims 1 and 2, a mean-preserving increase of the credit risk will push down the equilibrium amount of deposits.

The effect of an increase of the credit risk on the interest rate margin is transmitted through the functions of demand and supply of funds. Based on the expression for the amount of loans L given by the bank's program Prg. (a), observe that for a not excessively elastic supply function of deposits, a decrease of the amount of deposits at equilibrium will result in a decrease of the amount L of loanable funds. According to Assumption 1, this will lower the interest rate paid out to depositors and push up the interest rate charged to borrowers, which yields an increase of the bank's interest rate margin.

Proof of Proposition 2. Let's start by giving the first-order condition for the bank's deposits-taking problem Prg. (b):

$$\frac{dLR_L(L)}{dD}E\left[\tilde{\varepsilon}|\tilde{\varepsilon}>\hat{\varepsilon}\right]-\frac{dX}{dD}\left[1-P(\hat{\varepsilon})\right]-\frac{dh(\alpha D)}{dD}=0, \tag{13}$$

with,

$$\hat{\varepsilon} = \frac{X}{LR_L(L)}, \tag{14}$$

$$X \equiv \alpha DR_D(D)+B \tag{15}$$

$$= DR_D(D)\times\left[(1-\alpha)(1-w(\tau))K(\tau)+\alpha\right]. \tag{16}$$

I proceed in four steps.

1. Given Assumptions 1 and 2, as well as the facts that $h(\alpha D)\ll D$ and $0<(1-w(\tau))K(\tau)<1$ for any $0<\tau<T$, the derivatives

$$\begin{aligned}\frac{dLR_L(L)}{dD} &= \left[1-(1-\alpha)J(\tau)\left(R_D(D)+DR_D'(D)\right)-\alpha h'(\alpha D)\right]\\ &\quad\times\left[R_L(L)+\left[D-(1-\alpha)DR_D(D)J(\tau)-h(\alpha D)\right]R_L'(L)\right],\end{aligned} \tag{17}$$

$$\frac{dX}{dD}=\left[(1-\alpha)\left(1-w(\tau)\right)K(\tau)+\alpha\right]\times\left(R_D(D)+DR_D'(D)\right), \tag{18}$$

are increasing in α. Further, given Assumption 1 and Eqs. (8), (9), (17) and (18), it is easy to check that there exists $0 < \tau^* < T$, such that for any $\tau \leq \tau^*$, we have **(Claim 1)**:

$$\frac{d}{d\alpha}\left(\frac{dX}{dD}\right) > \frac{d}{d\alpha}\left(\frac{dLR_L(L)}{dD}\right) > 0. \tag{19}$$

2. Remark that because $h(\alpha D) \ll D$, the term

$$LR_L(L) = \{D \times [1 - (1-\alpha)J(\tau)R_D(D)] - h(\alpha D)\}\, R_L(L), \tag{20}$$

is increasing in α. Observe also that X is increasing in α, since $0 < (1 - w(\tau))K(\tau) < 1$ for any $0 < \tau < T$. But providing the fact that $(1 - w(\tau))K(\tau)$ is decreasing with τ, while $J(\tau)$ is increasing with the same parameter τ, one can easily see that given Eqs. (16) and (20), there exists $0 < \tau^{**} < T$, such that for any $\tau \leq \tau^{**}$, the default-trigger threshold $\hat{\varepsilon}$ is decreasing in α. Consequently, for any $\tau \leq \tau^{**}$, the probability of bank's solvency $[1 - P(\hat{\varepsilon})]$ is increasing in α: **(Claim 2)**.

3. By simple differential calculus,

$$\frac{d}{d\alpha}\left(\frac{dh(\alpha D)}{dD}\right) = h'(\alpha D) + (\alpha D)h''(\alpha D). \tag{21}$$

Given Assumption 2, the cross-derivative above is strictly positive: **(Claim 3)**.

4. Since $1 - P(\hat{\varepsilon}) = \Pr[\tilde{\varepsilon} > \hat{\varepsilon}] > E[\tilde{\varepsilon} | \tilde{\varepsilon} > \hat{\varepsilon}]$ for any $0 < \hat{\varepsilon} < 1$, combining Claims 1 and 2 implies that for any standard τ of reserves not excessively high (i.e., for any $\tau \leq \min(\tau^*, \tau^{**})$), the quantity

$$\frac{dLR_L(L)}{dD} E[\tilde{\varepsilon} | \tilde{\varepsilon} > \hat{\varepsilon}] - \frac{dX}{dD}[1 - P(\hat{\varepsilon})], \tag{22}$$

is decreasing in α. Given Claim 3, this means that for any standard τ of reserves not excessively high, the differentiated expected surplus of the bank expressed on the left-hand side of the first-order condition (13) is decreasing in α. Keeping the second-order condition satisfied, this means that increasing the deposits mix's parameter α will push down the amount of deposits to be raised by the bank in equilibrium. Hence, as established in the proof of Proposition 1, this decrease of the equilibrium amount of deposits will result in an increase of the bank's interest rate margin.Q.E.D.

Proof of Proposition 3. In the symmetric Nash-Cournot equilibrium, the amount of deposits D to be raised by each bank satisfies the first-order condition of the bank's problem Prg. (c) given by

$$\frac{dLR_L(nL)}{dD}E\left[\tilde{\varepsilon}\right|\tilde{\varepsilon}>\hat{\varepsilon}]-\frac{dY}{dD}\left[1-P(\hat{\varepsilon})\right]-\frac{dh(\alpha D)}{dD}=0, \tag{23}$$

with,

$$\begin{aligned}\hat{\varepsilon} &= \frac{Y}{LR_L(nL)}, &(24)\\ Y &\equiv \alpha DR_D(nD)+B &(25)\\ &= DR_D(nD)\times\left[(1-\alpha)(1-w(\tau))K(\tau)+\alpha\right]. &(26)\end{aligned}$$

In one side, based on Assumption 1, increasing the integer n will result in an increase of both $R_D(nD)$ and $R_D'(nD)$, and, hence, in an increase of both Y and,

$$\frac{dY}{dD}=\left[(1-\alpha)\left(1-w(\tau)\right)K(\tau)+\alpha\right]\times\left(R_D(nD)+nDR_D'(nD)\right). \tag{27}$$

In the other side, however, given the fact that,

$$L=D\times\left[1-(1-\alpha)J(\tau)R_D(nD)\right] \tag{28}$$

the interest rate charged on loans $R_L(nL)$ as well as the elasticity of the demand for loans $R_L'(nL)$ do not vary monotonically in response to an increase of the parameter n. This implies that increasing n will make both $\hat{\varepsilon}$ of and,

$$\frac{dLR_L(nL)}{dD}=\frac{dL}{dD}\left[R_L(nL)+nLR_L'(nL)\right] \tag{29}$$

varying non monotonically. This is sufficient to conclude that the equilibrium amount of deposits as well as the associated equilibrium interest rate margin is a nonmonotonic function of the integer n as claimed.

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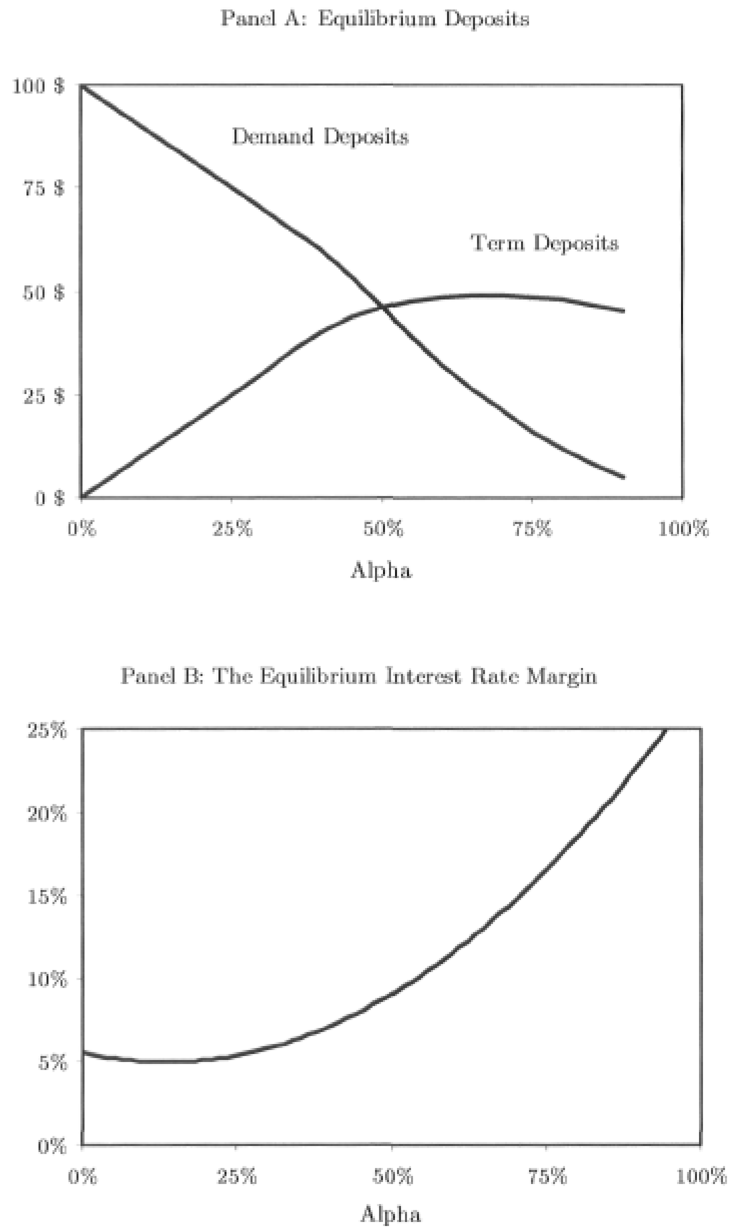

Figure 1. Equilibrium outcomes (demand deposits, term deposits and interest rate margin) for different values of the parameter α. The numerical results are obtained by assuming: (i) linear demand and supply functions of funds; (ii) a probability function $P(.)$ defined as a truncated lognormal distribution; and (iii) a deposits-insurance function $h(.)$ approximating the value of a put option written on the bank's loans portfolio with a strike price equal to the amount of term deposits.

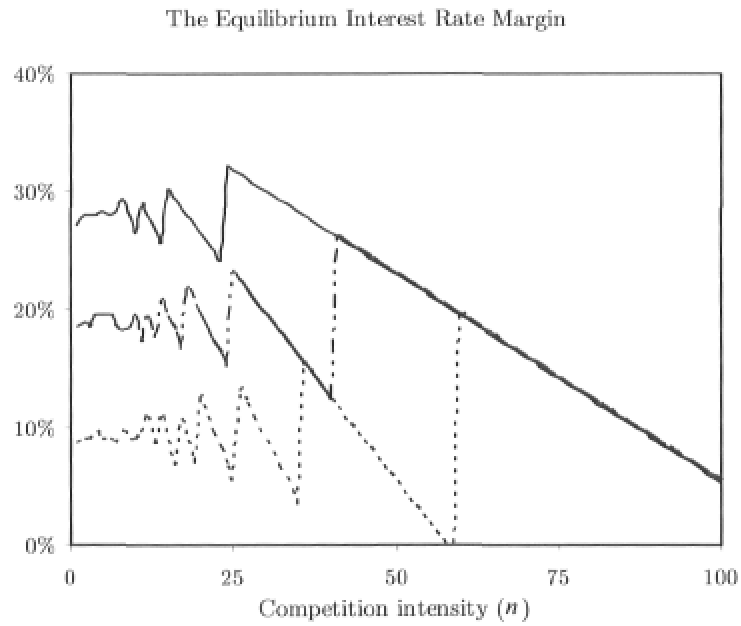

Figure 2. The equilibrium interest rate margin for different interbank competition intensities *n*. The numerical results are obtained by assuming the same primitive functions as in Figure 1. The parameter α takes the values of 1 (solid line), 3/4 (long-dashed line) and 1/2 (short-dashed line), respectively.