Automatic drainage pattern recognition in river networks

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Automatic Drainage Pattern Recognition in River Networks

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In both GIS and terrain analysis, drainage systems are important components. Owing to local topography and subsurface geology, a drainage system achieves a particular drainage pattern based on the form and texture of its network of stream channels and tributaries. Although research has been done on the description of drainage patterns in geography and hydrology, automatic drainage pattern recognition in river networks is not well developed. This paper introduces a new method for automatic classification of drainage systems in different patterns. The method applies to river networks and the terrain model is not required in the process. A series of geometric indicators describing each pattern are introduced. Network classification is based on fuzzy set theory. For each pattern, the level of membership of the network is given by the different indicator values. The method was implemented and experimental results are presented and discussed.

Keywords: river network; drainage pattern; terrain analysis; fuzzy logic

1 Introduction

A natural drainage system is the pattern formed by streams, rivers and lakes in a drainage basin. The drainage system is an important landscape feature for terrain analysis within Geographic Information System (GIS) since it provides a fundamental hydrological and morphological partition of river basins into fluvial areas and hillslopes. In a drainage system, a stream or a river is a natural watercourse, flowing towards an ocean, a lake, or another river. Apart from a few cases where a river simply flows into the ground or dries up completely before reaching another body of water, rivers always connect together to form networks, achieving a particular drainage pattern. The river pattern describes the morphological structure of a river network at the river basin scale and is different from the channel pattern which describes the river morphology at the river channel scale (Leopold and Wolman, 1957). There are several types of drainage patterns. They are commonly classified as dendritic, parallel, trellis, rectangular, radial, centripetal and reticulate patterns (Ritter, 2003). Dendritic patterns, also named tree-like patterns, can usually be found where there is no strong geological control (Charlton, 2008). Parallel, trellis and rectangular drainage patterns develop in areas with strong regional slopes but have their own specific characteristics. Streams radiating from a high central area form a pattern of radial drainage while streams forming a centripetal one gather in low-lying land. Reticulate drainage patterns are usually found on floodplains and deltas where rivers often interlace with each other (Simon and Gerald, 2004).

In GIS, the drainage system can be digitized manually or extracted from the Digital Elevation Model (DEM) by computing the flow direction and accumulation on the terrain (Mark, 1984; Tarboton et al., 1991, 1997; Vogt et al., 2003; Nardi et al., 2008; Florinsky, 2009; Ortega and Rueda, 2010) and is represented as a river network where each tributary stream is defined by a polyline connected to its main stream. Although semantic information can be added at the river level, no semantic information is computed and stored at the network level. Inside a network, different patterns can be observed and related to other geographical factors. In a drainage basin, a number of factors such as topography, soil type, bedrock type, climate and vegetation cover influence input, output and transport of sediment and water (Charlton, 2008). These factors also influence the nature of the pattern of water bodies (Twidale, 2004). As a consequence, to a certain extent, a drainage pattern can reflect the geographical characteristics of a river network. In structural geology, drainage patterns not only offer clues to geological structure, but also help to decode regional geological chronology (Hills, 1963). Moreover, drainage patterns are useful in the search for minerals (e.g. Binks and Hooper, 1984; de Wit, 1999). At present, much research has been done on the description of drainage patterns in geography and hydrology (e.g. Howard, 1967; Lambert, 1998; Twidale, 2004; Pidwirny, 2006). However, automatic drainage pattern recognition in river networks is not well developed.

In this paper, a method for drainage pattern extraction and classification from a river network is introduced. A drainage pattern can be identified for a whole network corresponding to a catchment area or for any sub-catchment. Classification relies on different geometric indicators such as the junction angle between streams and the shape of a catchment. A network belongs to a pattern if its indicators fall into some sets of values. Providing crisp sets as threshold values or intervals is not reliable. Therefore fuzzy sets are defined and thus pattern classification depends on the degree of membership of the network for each pattern.

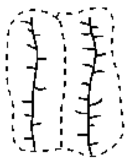
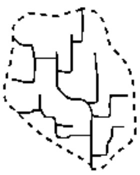
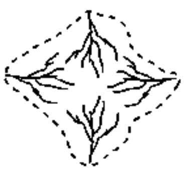
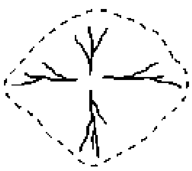
The DEM is not required for pattern identification so that the method does not depend on a specific terrain representation, e.g. grid, Triangular Irregular Networks (TIN) or contours, and does not suffer from inconsistencies that may occur between the network and the terrain (Chen et al., 2007). In GIS, such classification can be useful for terrain analysis as it can help provide a qualitative description of the terrain, or it can help with generalisation as the process may be adapted to the type of network. At present, many researchers have started to pay attention to geographical features of river networks during the process of generalisation (Ai et al., 2007; Buttenfield et al., 2010; Stanislawski, 2009, 2011). Considering drainage pattern as a geographical factor in river network generalisation helps to retain geographical features of the networks.

The paper is organised as follows. Section 2 briefly reviews related works in drainage pattern recognition and its applications. Geometric indicators characterising each drainage pattern are described in section 3. Section 4 presents a new methodology for pattern classification based on fuzzy logic. In section 5, the method is tested on a river network and results are discussed. Finally, conclusions and research perspectives are presented.

2 Related work

Drainage patterns are classified on the basis of their form and texture according to the terrain slope and structure. Their shape or pattern develops in response to the local topography and subsurface geology. There are 7 types of drainage network patterns as follows (see Table. 1).

Table 1. Drainage network patterns (diagrams modified from Ritter, 2006)

Name	Schematic Diagram	Description	Geometric and Topologic Characteristic
Dendritic		Dendritic pattern is the most common form of river system. In a dendritic river system, there are many contributing streams (analogous to the twigs of a tree), which join together and are the tributaries of a main river (Lambert, 1998).	-Tributaries joining at acute angle
Parallel		Parallel patterns form where there is a pronounced slope to the surface. Tributary streams tend to stretch out in a parallel-like fashion following the slope of the surface (Ritter, 2006).	- Parallel-like - Elongated catchment - Long straight tributaries - Tributaries joining at small acute angle
Trellis		In a trellis pattern, as the river flows along a strike valley, smaller tributaries feed into it from the steep slopes on the sides of mountains. These tributaries enter the main river at approximately 90 degree angles, causing a trellis-like appearance of the river system (Ritter, 2006).	- Short straight tributaries - Tributaries joining at almost right angle
Rectangular		The rectangular pattern is found in regions that have undergone faulting. Movements of the surface due to faulting offset the direction of the stream. As a result, the tributary streams make sharp bends and enter the main stream at high angles (Ritter, 2006).	- Tributary bends - Tributaries joining at almost right angle
Reticulate		Reticulate drainage patterns usually occur on floodplains and deltas where rivers often interlace with each other forming a net (Simon and Gerald, 2004).	- Tributaries cross together forming a cycle
Radial		The radial pattern develops around a central peak or dome. This pattern is common to such conically shaped features as volcanoes. The tributary streams flow from the top downward to the bottom around a mountain (Ritter, 2006).	
Centripetal		The centripetal pattern is just the opposite of the radial as streams flow toward a central depression. During wetter portions of the year, these streams feed ephemeral lakes, which evaporate away during dry periods (Ritter, 2006).	

According to the description of drainage patterns in Table 1, each drainage pattern has its own characteristics. Howard (1967) pointed out that dendritic patterns appear in horizontal sediments or uniformly resistant crystalline rocks with a gentle regional slope at present or at time of drainage inception. In Schumm et al. (2000), parallel drainage systems have moderate to steep slopes and appear in areas of elongated landforms. Trellis patterns usually exist in dipping or folded sedimentary, volcanic, or low grade sedimentary rocks. A rectangular pattern is with joints and faults at right angles, in which streams and divides lack regional continuity.

Some experimental works have been done concerning morphological dependencies of river channel patterns, such as straight, meandering and braid patterns. Schumm and Kahn (1972) determined an experimental relationship between slope and sinuosity for a fluvial channel, which can show threshold changes between pattern types. Here, sinuosity is the ratio of channel length to valley length. Results show that braided patterns appear on steep low-sinuosity channels. Schumm (1977) improved his model and pointed out that pattern adjustments, measured as sinuosity variations, are closely related to the type, size, and amount of sediment load. Although these works (e.g. Knighton, 1998; Lewin, 2001) about morphological dependencies apply to river channel patterns rather than river networks, some of the above relationships will be considered in this paper.

In GIS, ordering schemes based on the hierarchical structure of a river network have been developed. The structure is built by assigning an order number to each tributary. Ordering starts by assigning order 1 to branchless tributaries. The order of a stream is always higher than the order of its tributaries so that the highest order is assigned to the segment connected to the outlet. In this procedure, the most relevant ordering schemes are the Horton-Strahler scheme based on (Horton, 1945) and modified by Strahler (1957), and the Shreve scheme (Shreve, 1966). The Horton-Strahler scheme can be computed recursively (Gleyzer et al., 2004). In order to support GIS-based hydrological analyses, much research on coding drainage networks has been done (e.g. Verdin and Verdin, 1999; Fürst and Hörhan, 2009; Li et al., 2010).

At present, much research has been done on the definition, classification and description of drainage patterns in geography and hydrology. Many scholars work on predicting river channel patterns from in-channel characteristics, such as slope and discharge, but not drainage patterns. Touya (2007) and Jiang et al. (2009) both acknowledge drainage patterns as an important factor in river network generalisation. In order to preserve the main hydrographical properties, Jiang et al. (2009) obtained a simple representation of river networks by keeping the same drainage pattern after a selection operation but they did not go further to explain how patterns are recognised or preserved. Indeed, drainage pattern is recognised as an important element in GIS but its classification has not yet been considered. Therefore, this paper studies the geometric and topologic characteristics of each type of drainage pattern to allow river network classification.

3 Characterisation of drainage patterns

Based on the description of different types of drainage patterns, each pattern has its own geographical characteristics, which can be reflected in some quantifiable variable related to some topological and geometrical aspects. Therefore, each pattern can be characterised by a combination of different variables. First, terms describing river networks are defined. Second, the characteristics of each pattern are summarized from previous works. Finally, a set of indicators formalising the characteristics of each pattern and defining criteria for classification are introduced.

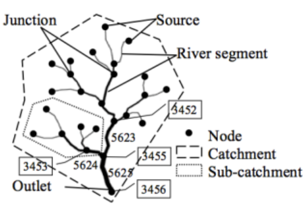
3.1 Definitions

A river network is composed of several connected river segments stored as line entities in GIS. The end points of the river segments are the nodes. There are three types of node: the junction node connecting river segments, the source node corresponding to river springs and the outlet towards where the flow goes. There is always one outlet in a river network except in reticulate drainages. A river network is located in a catchment, also called drainage basin. The catchment controlled by a tributary flowing into a main stream is called a sub-catchment. All these features are illustrated in Figure 1(a). In Figure 1(b), the topological structure of the river network used in this paper is illustrated including a node list and a river segment list. The connectivity of the river segments are set by FNode and TNode pointing to NID in node list. The “FNode” and “TNode” are also the “from” and “to” node respectively, which can show the flow direction of a river segment. From the node list, the river segments connecting to a node can be found by ConnectedRS pointing to RID in the river segment list.

3.2 Drainage pattern characteristics

In section 2, seven types of drainage pattern were introduced. Among them, the first five patterns are characterised by different geometric indicators measured on each segment of a network or describing the shape of the drainage while radial and centripetal patterns depend on the spatial organisation of a group of networks. This work focuses on the description of individual patterns, and addresses the identification of the first five patterns based on geometric characteristics identified inside a network. Based on the description of drainage patterns in Table 1, the authors propose a list of characteristics for each of them, which are also shown in the last column of Table 1.

Non-reticulate river networks are represented by a hierarchical graph and are characterised by geometric parameters related to the length and angle measured in the network. The reticulate pattern is a specific pattern because rivers intersect and cross together like a net. Due to that, a river network would form a cycle instead of a tree. Therefore, reticulate networks are identified first. They are taken out of the graph and replaced by nodes. The remaining part forms a hierarchical network with the outlet as the root which can be characterised by one of the four remaining patterns. Recognition and removal of reticulate networks is discussed in the next section while identification of other patterns based on geometric indicators is introduced in section 4.



(a) Features in river network (modified from Li, 2007)

Node list

NID	ConnectedRS	Shape
3456	{5625}	Point
3455	{5623, 5624, 5625}	Point
...	...	

River segment list

RID	FNode	TNode	Order	Shape
5623	3452	3455	4	Polyline
5624	3453	3455	3	Polyline
5625	3455	3456	4	Polyline
...	...	...	...	

(b) River network data structure

Figure 1. Features in river network and the data structure

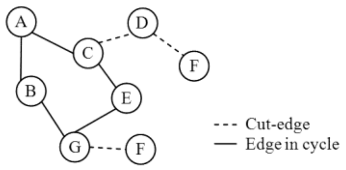


Figure 2. Cut-edges in an undirected graph

3.3 Reticulate pattern recognition

In graph theory, a cut-edge (also known as a bridge) is an edge whose removal produces a graph with more components than the original (Bondy and Murty, 2008). Equivalently, an edge is a bridge if, and only if, it is not contained in any cycle. Figure 2 illustrates the cut-edges in an undirected graph, where the dashed line is a cut-edge and the solid line is an edge contained in a cycle.

Considering the river network as a graph by setting river segments as edges and nodes in river network as nodes in graph, all cut-edges are found using a bridge-finding algorithm (Tarjan, 1974). Edges which are not identified as cut-edges are components of cycles and form reticulate patterns.

3.4 Geometric indicators

In this section, some geometric quantitative indicators are defined to recognise dendritic, parallel, trellis and rectangular patterns. From the geometric characteristics of drainage patterns in Table 1, the most important variable is the angle formed by a tributary with its main stream at a junction node. The average junction angle of all angles in a catchment is one quantitative indicator. In order to distinguish rectangular pattern, the shape of a tributary is also needed. In this pattern, tributary streams make sharp bends almost to a right angle. The amount of bending of tributaries can be estimated by the sinuosity of the river segments. Another difference between parallel and trellis pattern is length: the tributaries in parallel pattern are long relative to trellis. The average length ratio of tributaries to the main stream is the third indicator. The fourth indicator is the catchment elongation used to identify parallel patterns in an elongated basin. The catchment elongation is characterised by the ratio of long edge to short edge of the Minimum Bounding Rectangle (MBR) of the catchment. If the catchment is elongated, this ratio is large.

(1) *Junction angle*. The angle at a junction is a useful parameter that can be used in flow direction and main stream inference (Serres and Roy, 1990; Paiva and Egenhofer, 2000). In general, a tributary joins into a main stream (Figure 3a), or two tributaries gather together forming a new stream (Figure 3b). In this situation, the angle is easy to obtain. However, it is further complicated when several tributaries (more than three river segments join at a junction) flow into a main stream at the same place (Figure 3c and 3d).

In the case of three river segments joining at one junction, the angle is formed by the two upper river segments (e.g. river segments r1 and r2 in Figure 3a and 3b). In another case, where more than three river segments connect to a junction, the most important thing is to find the main stream in the upper river segments to measure the angles between the tributaries and the main stream. In order to get the main stream, the stream order is considered in the first place. The upper river segment with the highest order is the main stream. If there are two or more upper river segments with the same highest order, Rusak and Caster (1990) set two rules to determine the main stream: ① It has the same direction as the lower river (without consideration of any other geographic conditions); ② It has the longest length if several streams have a similar direction as the lower river.

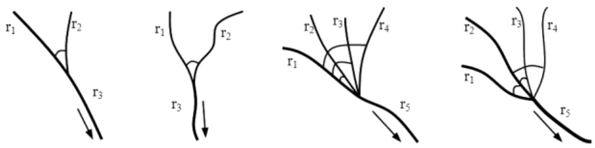


Figure 3. Different cases of river segments joining at a junction. The arrow refers to the flow direction. River segments in bolder line are with a higher stream order.

For example, in Figure 3c, r1 is the main stream because it has the highest order of all four upper river segments r1, r2, r3 and r4. Angles are formed by r1 with r2, r3 and r4 respectively. In Figure 3d, r1 and r2 have the same order, but r2 is the main stream because it has the same direction from the junction with lower river r5. Three angles are computed in the average, which are formed by r2 with r1, r3 and r4 respectively.

For a tributary joining a main stream at junction P_1, supposing points P_2 and P_3 are the "from" nodes of the upper stream and the tributary, the junction angle $\angle P_2P_1P_3$ can be computed by law of cosines:

$$\angle P_2P_1P_3 = \arccos\left(\frac{a^2+b^2-c^2}{2ab}\right) \tag{1}$$

where a is the distance between P_1 and P_2, b is the distance between P_1 and P_3, and c is the distance between P_2 and P_3.

The first parameter is the junction angle between the tributaries and the main stream. The parameter is given by the average value α of angles measured at all junctions. The dendritic pattern only requires that junction angles are acute, which can be translated by $\alpha < 90°$. Parallel patterns are characterised by angles more acute than in dendritic patterns, therefore $\alpha \ll 90°$. For trellis and rectangular patterns, tributaries join at a right angle and $\alpha \approx 90°$.

(2) *Sinuosity*. Schumm (1977) set the sinuosity variable of a stream as the ratio of the channel length to the valley length to quantify how much a river or stream meanders. In GIS, a stream is stored as a polyline. Then, the sinuosity can be approximately calculated as polyline length divided by length between end points. Supposing a river segment is composed of N points P_i with P_1 and P_N the end points, the sinuosity ratio SI is

$$SI = \sum_{i=1}^{N-1} Dis(P_i, P_{i+1}) \Big/ Dis(P_1, P_N) \tag{2}$$

where *Dis*() is the distance between two points.

A perfect straight stream would have a sinuosity ratio of 1; the higher this ratio is above 1, the more the stream meanders. If the sinuosity ratio is equal to or is greater than 1.5, the stream is considered to be meandering (Ritter, 2003). In both trellis and rectangular patterns, tributaries connect to the main stream at right angles. However, in trellis, tributaries are straight, while in rectangular pattern, most tributaries have sharp bends. A tributary is considered to have sharp bends if it has a high sinuosity. The indicator that is chosen is not the overall sinuosity of the network as a rectangular drainage can contain straight and sinuous streams which may yield a relatively low sinuosity value. Instead, the number of bended tributaries is considered. A parallel drainage or a trellis shall have very few bended tributaries in comparison to a rectangular drainage. A second indicator, the percentage of bended tributaries β is used. This parameter is calculated as the number of bended tributaries divided by the total number of tributaries, where a bended tributary has the sinuosity ratio ≥ 1.5. A rectangular pattern should yield a high value of β while, in trellis and parallel, β should tend towards 0.

(3) *Length ratio*. Long tributaries in a parallel pattern and short tributaries in a trellis pattern are relative conceptions in geography. The river absolute length cannot be used to distinguish different drainage patterns directly. This paper takes the length ratio between the tributaries and the main stream as an indicator. Here, the main stream is not only a river segment straight connected to the tributary. It is composed of several segments connected together with the same direction and same order. This is illustrated in Figure 4.

The parameter average length ratio γ is used to distinguish parallel and trellis patterns. In parallel patterns, tributaries have long length so $\gamma > 1$; otherwise, $\gamma << 1$ indicates that most tributaries are shorter than the main stream, as expected in a trellis.

(4) *Catchment elongation*. The exact location of the catchment area is usually computed from the DEM which is not available. Approximations can be obtained from the river network such as the convex hull, the axis-aligned bounding box (AABB) or the oriented minimum bounding rectangle (MBR) (Figure 5). In this paper, the objective is to estimate whether the catchment is elongated or not. The MBR of the river network is considered as it follows the orientation of the network. The breadth of the river network is given by the length of the MBR side that forms the largest angle with the main stream. The length of the other side which is roughly aligned with the main stream corresponds to the depth. The elongation is defined by the ratio between its depth and breadth. For example, in Figure 5, the depth is smaller than the breadth. The catchment area is not elongated, so that the drainage cannot be considered as parallel. Parallel and trellis patterns form in elongated catchments and are therefore characterised by a high elongation δ.

Geometric characteristics of different patterns presented in Table 1 are only defined qualitatively. In order to identify patterns based on these characteristics, statistical measures are obtained from the network and compared with threshold values. The different indicators are summarised in Table 2. They are expressed by qualitative predicates and are translated into geometric indicators. These indicators can be directly implemented and measured on a river network. Values associated with each pattern are vague as they represent qualitative properties, and classification into one pattern depends on several of these values.

Setting crisp threshold values defining the acuteness of an angle or the breadth of a catchment is an empirical task which relies on the user's judgment and expertise and which does not reflect the inherent vagueness of drainage patterns. Furthermore, they do not provide a robust enough classification. Too restrictive threshold values will leave many networks unclassified while too loose values will end up in networks that may belong to different patterns. Therefore, assertion of each predicate is not defined by crisp sets of values but by fuzzy sets and the membership to a set is based on fuzzy logic (Zadeh, 1965).

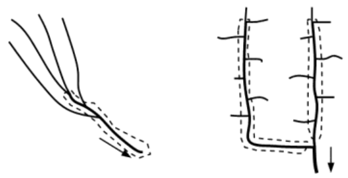


Figure 4. Main streams calculated in length ratio, where the arrow refers to the flow direction, and the river segments in dashed boxes are main streams.

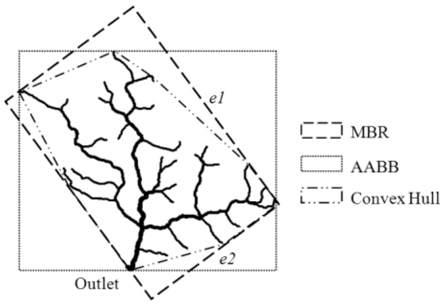


Figure 5. MBR of a river network. The edge e1 has a bigger angle with mainstream, *ratio = e2/e1 < 1*, it is not an elongated river basin.

4 Pattern classification based on fuzzy logic

A fuzzy set is a set whose membership is not defined by a binary value (an element belongs or not to a set) but by a value between 0 and 1 corresponding to different grades of membership. Fuzzy set theory allows approximated reasoning on values which are imprecise or incomplete. In this paper, fuzzy set theory is used to perform classification with predicates that cannot be asserted as true or false in all cases but require gradual assessment.

A total of eight predicates are extracted from indicators defined in Table 2. They are:

- α IS acute
- α IS very acute
- α IS right
- β IS bended
- γ IS long

- γ IS short
- δ IS broad
- δ IS elongated

Table 2. List of indicators

Drainage Pattern	Average Junction Angle (α)	Bended Tributaries Percentage (β)	Average Length Ratio (γ)	Catchment Elongation (δ)
Dendritic	Acute $\alpha < 90°$	-	-	Broad $\delta < 1$or $\delta \approx 1$
Parallel	Very acute $\alpha << 90°$	Not bended $\beta \to 0$	Long $\gamma \approx 1$or $\gamma > 1$	Elongated $\delta >> 1$
Trellis	Right angle $\alpha \approx 90°$	Not bended $\beta \to 0$	Short $\gamma << 1$	Elongated $\delta >> 1$
Rectangular	Right angle $\alpha \approx 90°$	Bended $\beta \to 100\%$	-	-

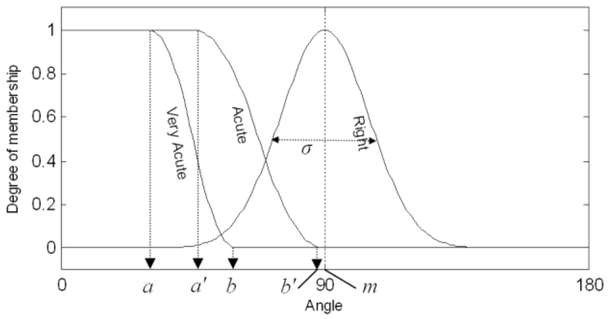


Figure 6. MFs for very acute, acute and right angle, input is the junction angle α

The degree of membership to each predicate is asserted by a membership function (MF). A membership function is a curve that defines how each element in fuzzy set is mapped to a membership degree between 0 and 1. Membership degrees of the first two predicates are defined by Z curves, i.e. asymmetrical polynomial curves open to the left (Figure 6) of the form $z(\alpha; a, b)$ where α is the junction angle and a and b locate the extremes of the sloped portion of the curve. The degree of membership is 1 if $\alpha < a$ and 0 if $\alpha > b$. If $a < \alpha < b$, the degree is decreasing. Obviously, a very acute angle should be smaller than an acute angle so that $a < a'$ and $b < b'$ in Figure 6. Membership functions can be non-zero for the same α value. That means that an angle may be considered as very acute, acute and right at different degrees. A Gaussian distribution curve $g(\alpha; \sigma, m)$ is used to define the degree of membership to the third predicate (Figure 6). The value m is the average angle on which the function is centred and is equal to 90°. Parameter σ controls the width of the curve; the larger it is, the broader the curve.

The degree of membership to the bend, long tributaries and elongated catchment predicates are estimated by S curves, i.e. asymmetrical polynomial curves open to the right, of the forms $s(\beta; a, b)$, $s(\gamma; a', b')$ and $s(\delta; a, b)$ (Figures 8a-8c). The smaller the input value, the smaller the degree of membership. Finally, the degree of membership to the short tributaries and broad catchment predicates are characterised by a Z curve where a small ratio has a high degree (Figure 7b and c).

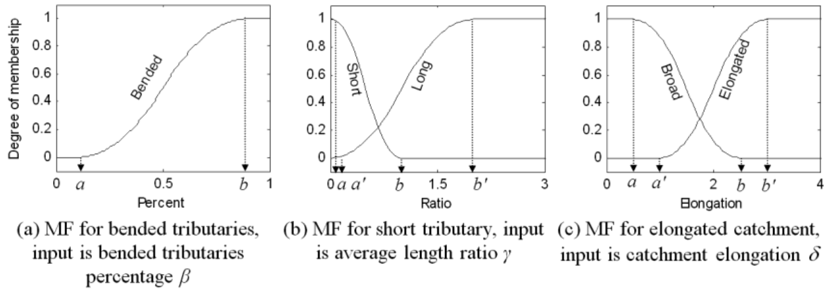


Figure 7. MFs for bended tributaries, short tributary and elongated catchment

Combining the predicates in more complex rules characterising each drainage pattern is done by using fuzzy Boolean operators AND, OR and NOT. In fuzzy logic, the truth of any statement is a matter of degree between 0 and 1. Zadeh (1965) suggested the minimum, maximum and complement methods for AND, OR and NOT operators respectively. For two fuzzy set values A and B within the range (0, 1), fuzzy logic operations are (Figure 8):

$$A\ AND\ B = min(A, B) \tag{3}$$

$$A\ OR\ B = max(A, B) \tag{4}$$

$$NOT\ A = 1 - A \tag{5}$$

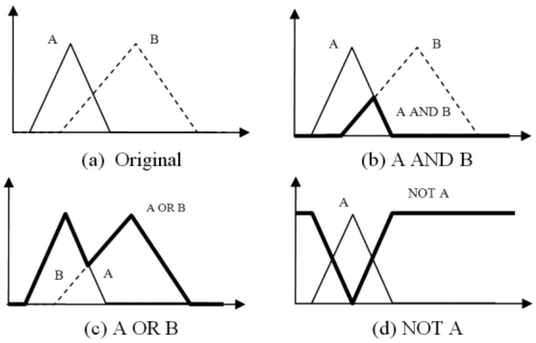


Figure 8. AND, OR and NOT operations

Each pattern of Table 2 is defined by the following IF-THEN rules based on fuzzy logic operations:

(1) IF (α IS acute) AND (δ IS broad) THEN pattern IS dendritic
(2) IF (α IS very acute) AND NOT (β IS bended) AND (γ IS long) AND (δ IS elongated) THEN pattern IS parallel
(3) IF (α IS right) AND NOT (β IS bended) AND (γ IS short) AND (δ IS elongated) THEN pattern IS trellis
(4) IF (α IS right) AND (β IS bended) THEN pattern IS rectangular

In fuzzy logic, there is no ELSE rule and all the rules should be evaluated. Therefore, each network is given a degree of membership for each pattern. To get a crisp decision, the maximum-method is used to defuzzify the set of singletons and the pattern with the maximum degree of membership is chosen.

Figure 9 shows an example of fuzzy logic process to identify drainage patterns:

Step 1: input the indicators α, β, γ and δ of a river network;

Step 2: evaluate all rules according to fuzzy inputs by applying logic operations, and obtain the outputs of all rules;

Step 3: defuzzify the results and get the final output as the pattern with maximum degree.

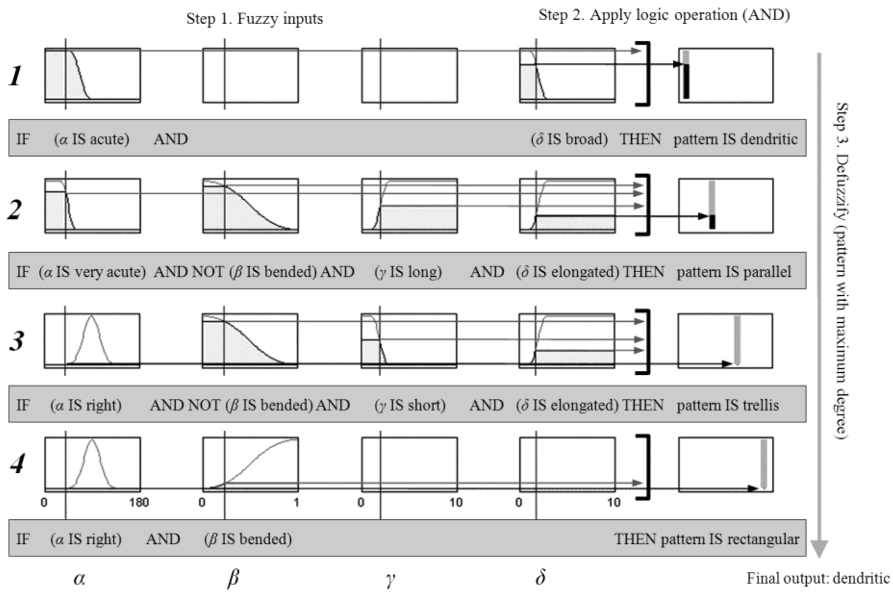


Figure 9. The fuzzy logic process for drainage pattern recognition

5 Results

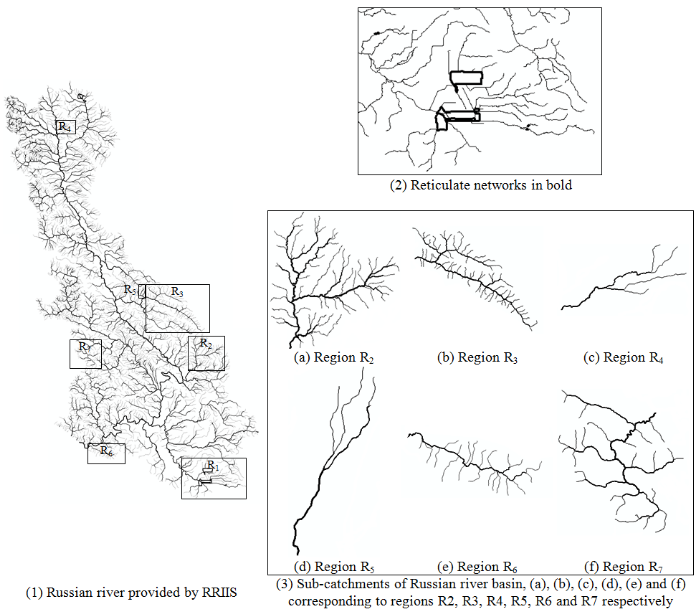


Figure 10. Experimental data

The method was implemented in C++ with the Boost Graph Library (BGL[1]) for cut-edge finding and indicator computation, and in MATLAB with Fuzzy Logic Toolbox for drainage pattern recognition. The experimental data is the Russian river shown in Figure 10(1), California at the scale of 1:24,000 stored in a Shapefile from the Russian

River Interactive Information System (RRIIS[2]). Original data set shown in Figure 10(1) is composed of 5699 river segments. The bolder the line, the greater the Horton-Strahler order. The highest order in the network is equal to 6.

5.1 *Data processing*

For the Russian river network, the Horton-Strahler order is already given; otherwise, it can be computed automatically from the network (Gleyzer et al., 2004). The data processing can be described as follows.

Input: a river network or a sub-network defined by a river stream and its tributaries.

Step 1: detect and remove reticulate networks;

Step 2: computation of indicators as defined in section 3.4;

Step 3: classification with fuzzy logic.

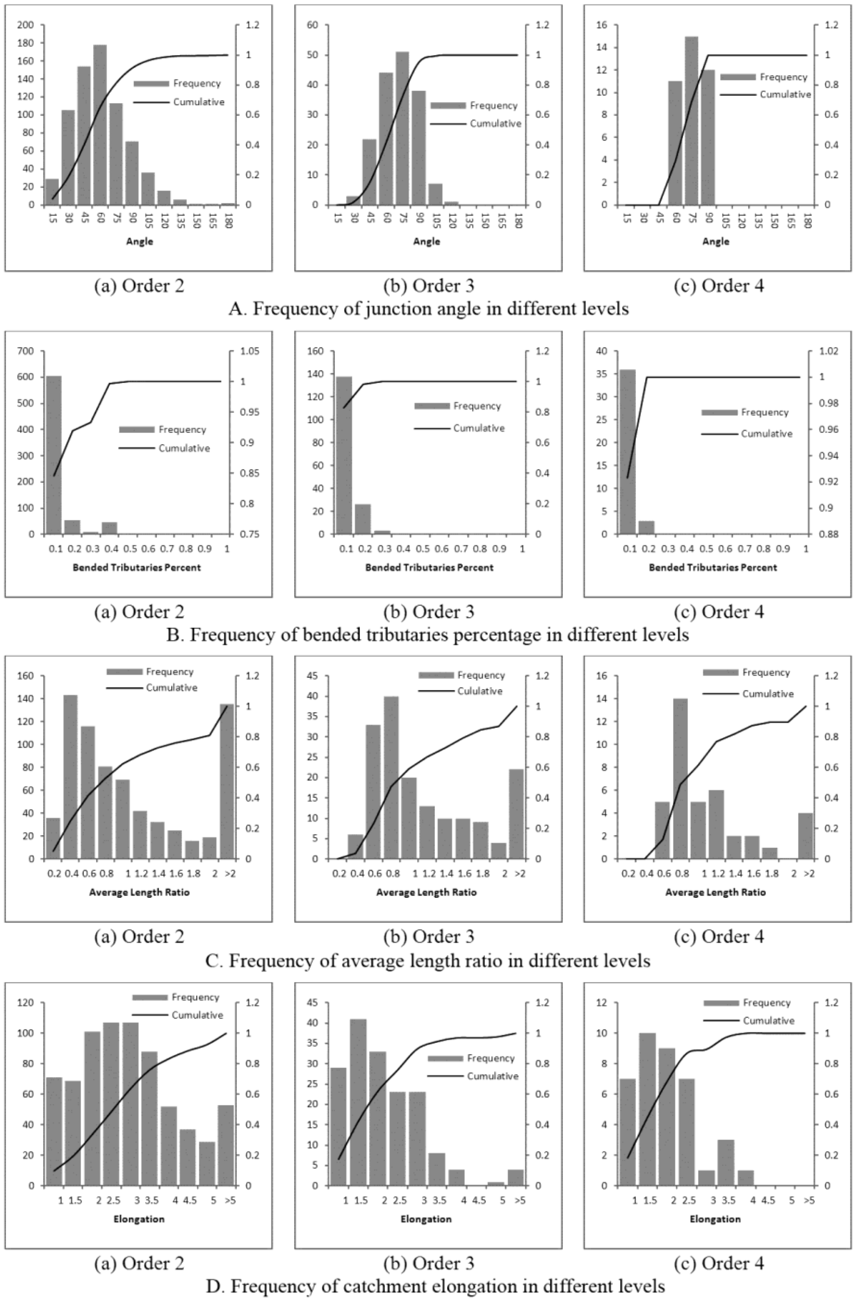
5.2 *Membership function parameter settings for Russian river*

Drainage pattern classification depends on the definition of the membership functions. Based on the distribution of indicator values α, β, γ, δ among all network in Figure 11, membership functions used in this experiment are given in Table 3. For case I, the threshold values 30° and 60° are used to establish very acute MF. If the angle is smaller than 30°, it is definitely very acute angle while if it is greater than 60°, it cannot be very acute. Between 30° and 60°, the greater the angle is, the smaller the membership. Similarly, angles are considered acute under 45°, and their degree of membership decreases when the angle increases. The standard deviation for right angles was set to 10°. The closer to 90°, the higher the membership value. MFs of bended tributaries and elongated catchment are both S curves. The closer to 1 β is, the more tributaries bend.

Table 3. Specific membership functions

Case	I	II	III
Very acute angle	$z(\alpha;30°,60°)$	$z(\alpha;35°,65°)$	$z(\alpha;25°,55°)$
Acute angle	$z(\alpha;45°,90°)$	$z(\alpha;50°,95°)$	$z(\alpha;40°,85°)$
Right angle	$g(\alpha;10°,90°)$	$g(\alpha;12.5°,90°)$	$g(\alpha;7.5°,90°)$
Bended tributaries	$s(\beta;0,1)$	$s(\beta;0,0.9)$	$s(\beta;0.1,1)$
Long tributary	$s(\gamma;0,1)$	$s(\gamma;0,0.8)$	$s(\gamma;0.1,1.1)$
Short tributary	$z(\gamma;0,1)$	$z(\gamma;0.2,1.2)$	$z(\gamma;0,0.9)$
Broad catchment	$z(\delta;1,3)$	$z(\delta;1.25,3.25)$	$z(\delta;0.8,2.8)$
Elongated catchment	$s(\delta;1,3)$	$s(\delta;0.75,2.75)$	$s(\delta;1.2,3.2)$

Based on analyses of case studies, dendritic drainages had an elongation centred on 1 while trellis and parallel drainages had a much higher elongation (often greater than 2). Therefore, the elongated MF was set to $s(\delta;1,3)$ so that a network with an elongation up to 2 may still be considered as square. For the broad catchment, the MF is set opposite to elongated to $z(\delta;1,3)$. During the tests, short tributaries appeared to be a less relevant indicator than the elongation and the angle to characterise the networks. According to the rules given in Table 2, the MF for short tributaries should have a large support and thus is set to *z(γ;0,1)*, and the MF for long tributaries is set to *s(γ;0,1)* oppositely.



(a) Order 2 (b) Order 3 (c) Order 4

A. Frequency of junction angle in different levels

(a) Order 2 (b) Order 3 (c) Order 4

B. Frequency of bended tributaries percentage in different levels

(a) Order 2 (b) Order 3 (c) Order 4

C. Frequency of average length ratio in different levels

(a) Order 2 (b) Order 3 (c) Order 4

D. Frequency of catchment elongation in different levels

Figure 11. Frequency of indicators in different levels

In Table 3, two other cases are also presented. Membership functions in case II have a larger support than in case I, while in case III the support is smaller providing a stricter classification. As an example, an angle of 32° is definitely very acute in case II, but not in cases I and III. The degree of membership in case I would be higher than in case III though. These three cases were all tested in the experiment.

Sensitivity to parameter values was assessed by fixing all the parameters but one to values defined for case I. The free parameter was tested with values presented in cases II and III. Table 4 shows the results of sensitivity analysis and results are compared with case I classification. It appears that the model is mostly sensitive to *α* and *δ*. MFs of parameters *α* and *δ* for very acute angle and broad catchment have a strong influence on the classification results. On the opposite, the value of *β* has limited influence as few streams are bended.

Table 4. Number of drainage patterns with a parameter changing

Parameter	Test	Dendritic	Parallel	Trellis	Rectangular	Unrecognised	Changed
	Case I	405	339	130	18	28	-
α	VAA → $z(\alpha;35°,65°)$	382	370	122	18	28	31
	VAA → $z(\alpha;25°,55°)$	424	306	142	18	30	33
	AA → $z(\alpha;50°,95°)$	424	339	119	15	23	19
	AA → $z(\alpha;40°,85°)$	378	339	140	28	35	27
	RA → $g(\alpha;12.5°,90°)$	390	335	148	19	28	19
	RA → $g(\alpha;7.5°,90°)$	412	342	120	18	28	10
β	BT → $s(\beta;0,0.9)$	403	339	129	21	28	3
	BT → $s(\beta;0.1,1)$	407	339	131	9	34	9
γ	LT → $s(\gamma;0,0.8)$	401	343	130	18	28	4
	LT → $s(\gamma;0.1,1.1)$	409	335	130	18	28	4
	ST → $z(\gamma;0.2,1.2)$	401	339	136	17	27	6
	ST → $z(\gamma;0,0.9)$	406	341	123	19	31	7
δ	BC → $z(\delta;1.25,3.25)$	438	316	123	18	25	33
	BC → $z(\delta;0.8,2.8)$	364	361	143	20	32	41
	EC → $s(\delta;0.75,2.75)$	393	343	143	16	25	17
	EC → $s(\delta;1.2,3.2)$	420	331	116	21	32	22

(VAA, AA, RA, BT, LT, ST, BC and EC are short for very acute angle, acute angle, right angle, bended tributaries, long tributary, short tributary, broad catchment and elongated catchment respectively. "→" presents that the parameter apply another setting.)

5.3 *Case studies in Russian river*

5.3.1 *Reticulate pattern in Russian river*

The cut-edge finding algorithm is used to identify all river segment parts of a reticulate pattern. Figure 10(2) shows a part of the network from region R_1 of Figure 10(1) with its reticulate patterns.

5.3.2 *Dendritic, parallel, trellis and rectangular pattern in Russian river*

Results from several sub-catchments from the Russian river basin are provided for discussion with values of indicators as well as membership degrees for all four rules in Table 5. Selected sub-catchments are shown in Figure 10(3). Locations of sub-catchments in the whole river basin can be seen in Figure 10(1). Sub-catchments (a), (b), (c), (d), (e) and (f) have a highest Horton-Strahler order of 4, 4, 3, 3, 3 and 3 respectively.

Table 5. Information of sub-catchments in Figure 10(3)

	Indicator values	Case	MF degree				Output
			rule 1 (p_1/p_2)*	rule 2 ($p_1/p_2/p_3/p_4$)*	rule 3 ($p_1/p_2/p_3/p_4$)*	rule 4 (p_1/p_2)*	
(a)	α=51.64° β=3.54% γ=0.74 δ=0.87	I	0.957 (0.957/1)	0 (0.155/0.998/ 0.865/0)	0 (0.001/0.998/ 0.135/0)	0.001 (0.001/0.003/)	dendritic
		II	0.997 (0.997/1)	0.007 (0.397/0.997/ 0.989/0.007)	0.007 (0.009/0.997/ 0.423/0.007)	0.003 (0.009/0.003)	dendritic
		III	0.866 (0.866/0.998)	0 (0.025/1/ 0.063/0)	0 (0/1/ 0.063/0)	0 (0/0)	dendritic
(b)	α=81.14° β=1.49% γ=0.74 δ=3.17	I	0 (0.078/0)	0 (0/0.999/ 0.865/1)	0.135 (0.675/0.999/ 0.135/1)	0 (0.675/0)	trellis
		II	0.003 (0.189/0.003)	0 (0/0.999/ 0.989/1)	0.423 (0.778/0.999/ 0.423/1)	0.001 (0.778/0.001)	trellis
		III	0 (0.015/0)	0 (0/1/ 0.741/0.999)	0.063 (0.498/1/ 0.063/0.999)	0 (0.498/0)	trellis
(c)	α=21.52° β=0 γ=1.28 δ=3.53	I	0 (1/0)	1 (1/1/1/1)	0 (0/1/0/1)	0 (0/0)	parallel
		II	0 (1/0)	1 (1/1/1/1)	0 (0/1/0/1)	0 (0/0)	parallel
		III	0 (1/0)	1 (1/1/1/1)	0 (0/1/0/1)	0 (0/0)	parallel
(d)	α=22.88° β=0 γ=1.55 δ=5.13	I	0 (1/0)	1 (1/1/1/1)	0 (0/1/0/1)	0 (0/0)	parallel
		II	0 (1/0)	1 (1/1/1/1)	0 (0/1/0/1)	0 (0/0)	parallel
		III	0 (1/0)	1 (1/1/1/1)	0 (0/1/0/1)	0 (0/0)	parallel
(e)	α=85.43° β=9.76% γ=0.62 δ=2.87	I	0.008 (0.021/0.008)	0 (0/0.981/ 0.711/0.992)	0.289 (0.901/0.981/ 0.289/0.992)	0.019 (0.901/0.019)	trellis
		II	0.072 (0.091/0.072)	0 (0/0.977/ 0.898/1)	0.647 (0.935/0.977/ 0.647/1)	0.024 (0.935/0.024)	trellis
		III	0 (0/0)	0 (0/1/ 0.539/0.946)	0.194 (0.831/1/ 0.194/0.946)	0 (0.831/0)	trellis
(f)	α=94.93° β=4.08% γ=0.33 δ=0.87	I	0 (0/1)	0 (0/0.997/ 0.218/0)	0 (0.886/0.997/ 0.782/0)	0.003 (0.886/0.003)	rectangular
		II	0 (0/1)	0 (0/0.996/ 0.340/0.007)	0.007 (0.925/0.996/ 0.966/0.007)	0.004 (0.925/0.004)	trellis
		III	0 (0/0.998)	0 (0/1/ 0.106/0)	0 (0.806/1/ 0.731/0)	0 (0.806/0)	-

* The content in the parenthesis is the degree of all predicates of each rule.

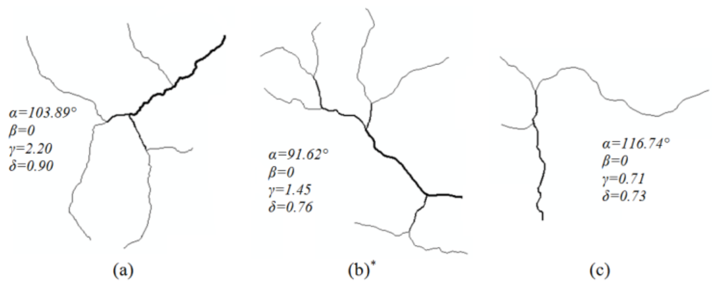


Figure 12. Some unclassified sub-networks in case I (*dendritic in case II)

Based on indicators, network (a) has acute junction angles and is broad. The MF value of the first rule is the highest in all cases (0.957/0.997/0.866), meaning that (a) is dendritic. From Table 5, (b) and (e) have a highest membership value for the third rule and other membership values are very small, so they are classified as trellis pattern. Networks (c) and (d) are definitely parallel, because they both have very acute junction angles (21.52° and 22.88°), elongated catchments (3.53 and 5.13 both bigger than 3) and long tributaries (1.28 and 1.55). Network (f) has average angle greater than 90°, so it is neither dendritic nor parallel. It is identified as rectangular in case I and trellis in case II, but cannot be recognised in case III. The membership value from rule 4, however, is only 0.003, which might be too small to consider (f) as rectangular. In case II, the membership value from rule 3 is 0.007, which is not large enough to consider (f) as trellis.

The quantitative indicators and fuzzy logic method introduced in this paper can characterise the drainage patterns of the river network. From the experiment, it is verified that a small variation of MFs has a limited impact on the classification result, but the MFs with a more tolerant setting can support more ambiguous situations. In addition, the maximum method for defuzzifying the fuzzy outputs may show some limitations in some cases. For example, sub-network (f) in Figure 10(3) has been output as a rectangular pattern as the final result in case I, but the degree of membership of the result is only 0.003 which is too farfetched to classify (f) as rectangular. In such a case, a solution may be not to use the maximum method but to obtain the final result by giving a threshold of the degree of membership.

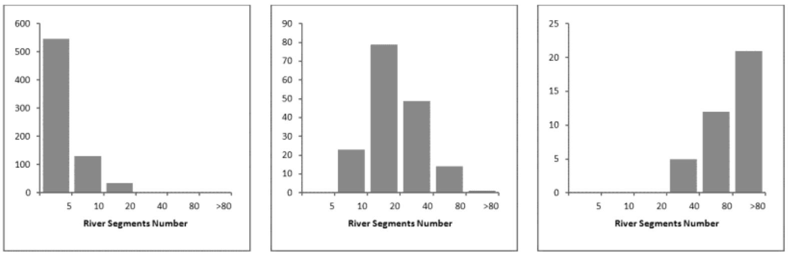
A few networks remained unclassified such as network (f) in case III of Figure 10(3), for which all membership degrees of all rules are 0. Some unclassified networks in case I are illustrated in Figure 12. Overall, unclassified networks are broad and have tributaries joining at obtuse angles. They also have few tributaries so that the indicators computed on an average may not be objective.

In Figure 12, sub-networks (a) and (b) have a junction angle (103.89°, 91.62°) close to a right angle, but catchments are not elongated (δ both smaller than 1). Moreover, neither of them have bended tributaries, and their tributaries are not short. So, (a) and (b) cannot be classified. The junction angle of (c) is bigger than 90°, so it cannot be dendritic and parallel. Also, (c) cannot be identified as trellis because its catchment is not elongated (δ = 0.73), neither can it be rectangular because there are no bended tributaries (β = 0). However, (b) can be classified as dendritic in case II due to the wide support on MF of acute angle, and the membership value of rule 1 is 0.011.

5.4 Drainage pattern recognition results and discussion

In the experiment, different catchment units lead to different results. According to the

Horton-Strahler order of its main stream, a catchment unit can belong to different orders from 2 to 4. Order 1 catchments were not considered as they correspond to single stream networks. On one hand, the smaller the order, the more catchment units. On the other hand, low order networks have fewer tributaries which may make average values less significant. Figure 13 shows the frequency of river segment numbers in a sub-catchment at different orders.



(a) Order 2 (b) Order 3 (c) Order 4

Figure 13. Frequency of river segment numbers in a catchment at different orders

Frequency distributions vary for different stream orders. From Figure 13, most of the river sub-catchments in order 2 are composed of fewer than 5 river segments, and no sub-catchment in order 4 has fewer than 20 river segments. The number of river segments in a sub-catchment would influence the indicators such as average angle and catchment elongation. The percentage of bended tributaries indicator is not related with the river segments number because it depends more on the shape of each single river segment. Table 6 shows the number of drainages at each order for each pattern by the MFs settings in Table 3. Reticulate networks are identified in a preliminary step.

Table 6. Number of drainage patterns

	Case	Dendritic	Parallel	Trellis	Rectangular	Unrecognised	Total	Reticulate
	I	253	320	107	9	25		
Order 2	II	253	329	111	5	16	714	
	III	247	300	114	10	43		
	I	119	19	18	8	3		
Order 3	II	119	21	19	7	1	167	12
	III	114	18	19	4	12		
	I	33	0	5	1	0		
Order 4	II	31	1	6	1	0	39	
	III	32	0	5	0	2		

In Table 6, although cases I, II and III have different sets of values of membership functions, there is little change in the number of drainages recognised for each pattern at a given order. This shows that the classification obtained with fuzzy logic is robust. Case II recognises the largest number of networks while case III recognises the smallest number. The result is expected as MF in case II have larger support. Therefore, changing the threshold values of membership functions can help to avoid unclassified networks to some extent. The proportion of dendritic drainages increases with the order while the proportion of parallel drainages decreases. This variation may be linked to some geomorphological properties of the terrain but may also be due to the fact that MFs "Acute" and "Very Acute" partially overlap and for some networks, both MFs are equal to 1. In that case, the catchment elongation becomes the main indicator. A river network at a higher order tends to form a more complex network with a larger number of rivers spreading in various directions and eventually to exhibit a broader catchment, hence a larger proportion of dendritic patterns at order 4. Indeed, in some cases, networks at order 4 can represent very large systems where a main stream goes through different types of terrain and follows different patterns. The number of trellis remains stable in proportion because drainages identified as trellis tend to form less complex networks with a smaller number of sub-networks and so remain elongated.

Statistics of average value of indicators for classified drainages is shown in Table 7. It can be noted that the junction angle of parallel drainages is close to 30°, indicating that most of the parallel drainages had a junction angle far below the limit. Junction angle of trellis is around 85° meaning that many streams do not join at right angles. Therefore, the membership function needs to be set with a rather large support. Trellis has a significantly smaller value γ than for other patterns for which it is twice as long as the main stream. This mostly relates to the way main streams are defined. Finally, it can be noted that the catchment elongation of parallel and trellis patterns is much larger than dendritic, which makes sense, but also that parallel drainages are on average more elongated than trellis.

Table 7. Statistics for each classified drainage

	Case	Average(α)	Average(β)	Average(γ)	Average(δ)
Dendritic	I	58.97°	4.27%	2.09	1.57
	II	59.50°	4.26%	2.08	1.56
	III	57.90°	4.31%	1.99	1.57
Parallel	I	34.82°	3.48%	2.49	3.84
	II	35.45°	3.48%	2.49	3.80
	III	33.52°	3.62%	2.61	3.85
Trellis	I	85.50°	3.29%	0.43	2.65
	II	87.09°	3.74%	0.44	2.53
	III	81.47°	3.14%	0.41	2.83
Rectangular	I	94.01°	18.81%	2.64	1.76
	II	95.46°	17.52%	3.31	1.81
	III	97.30°	26.07%	1.91	1.24

6 Conclusions

This paper aims to recognise the drainage pattern of a river network, which is an important geographic factor for drainage system analysis. Five types of pattern are classified: dendritic, parallel, trellis, rectangular and reticulate patterns. The method is based on geometric indicators, such as the junction angle, sinuosity, and catchment elongation, to classify the patterns automatically. Different patterns in a river network were identified separately and correspond to more or less complex networks with different Horton-Strahler orders. From the experiment in the Russian river network, the quantitative indicators and fuzzy logic method successfully classified the drainage pattern of a river network.

The advantage of this work is that proposed geometric indicators are easy to obtain and calculate. They can easily be implemented in a GIS and applied to a river network defined in a Shapefile or extracted from DEMs. However, in this last case, the quality of the classification may depend on the quality of the extracted network from the

DEM (Grimaldi et al., 2007, Nardi et al., 2008). As rules defining each pattern are vague and depend on a combination of indicators, classification made use of fuzzy logic to improve robustness of the result. Such classification and organisation can be useful for terrain analysis as it can help provide a qualitative description of the terrain or for generalisation as river selection can be adapted to the type of network.

Validation of the results is based on assessments done on case studies. Some networks still remained unclassified either because they could belong to several different patterns, or to none. They are usually networks at low order where there are not enough tributaries. Classification depends on the membership function definitions. Different definitions were tested and yielded consistent results. It appears that among the different indicators, the junction angle and the elongation are the most significant: parallel drainages are mainly characterised by very acute junction angles and elongated catchment, trellis are also elongated but with orthogonal junctions.

In the experimentation, it appeared that the proportion of drainages in each pattern varied with the order. One explanation may be that drainages at higher order tend to be broader and therefore more likely classified as dendritic. Further investigation is required in this direction, however, as large drainages may also be formed by clusters of drainages of different types as rivers go through different types of terrain. This evolution can be tracked by clustering adjacent drainages showing similar patterns into larger drainages that do not necessarily form a hierarchical structure as in this work.

The influence of scale shall also be studied. Results were discussed on a large scale model. However, the scale may affect the number of tributaries represented in the network and the computation of indicators. As the drainage system is often extracted from the terrain model, the accuracy of drainage pattern classification at different orders may be related to the resolution of the terrain model.

In terms of future research, for the short term, the first aspect for further work is the addition of other parameters for further pattern descriptions. On top of geometric indicators computed in a single network, other topologic indicators expressing relationships between networks can be considered. This would allow the study of the structure of the river network according to stream order and location inside the network. Drainage patterns such as radial and centripetal patterns have not been addressed in this paper. Their identification requires the characterisation of spatial relationships between networks rather than geometric indicators.

For the longer term, the drainage pattern can be considered for applications in terrain analysis and cartography. The drainage pattern can be used to analyse and correlate the drainage patterns with the catchment areas extracted from a terrain model. As the drainage pattern is related with the morphology of a terrain, it can be used to enrich the terrain model and characterise morphologic features. In cartography, river patterns provide information about the network structure and can be used in river tributaries selection for map generalisation (Touya, 2007). It may also be used in map updating to check the existence of inconsistencies between terrain elements and the streams and to correct the conflicts (Chen et al., 2007).

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1. http://www.boost.org/doc/libs/1_48_0/libs/graph/doc/index.html

2. http://www.rrwatershed.org

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