An Exact Solution Approach for Multi-objective Location-Transportation Problem for Disaster Response

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Abstract

This paper considers a three-objective location-transportation problem for disaster response. The location problem aims at determining the number, the position and the mission of required humanitarian aid distribution centers (HADC) within the disaster region. The transportation problem deals with the distribution of aid from HADCs to demand points. Three conflicting objectives are considered. The first objective minimizes the total transportation duration of needed products from the distribution centers to the demand points. The second objective minimizes the number of agents (first-aiders) needed to open and operate the selected distribution centers. The third objective minimizes the non-covered demand for all demand points within the affected area. We propose an epsilon-constraint method for this problem and prove that it generates the exact Pareto front. The proposed algorithm can be applied to any three-objective optimization problem provided that the problem involves at least two integer and conflicting objectives. The results obtained in our experimental study show that the computing time required by the proposed method may be large for some instances. A heuristic version of our algorithm yielded, however, good approximation of the Pareto front in relatively short computing times.

Keywords: Emergency response, location-transportation problems, multi-objective combinatorial optimization, exact method, epsilon-constraint method.

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1. Introduction

Recent years have seen a significant growth in human life losses and material damages caused by anthropogenic and natural disasters such as earthquakes, flooding, tsunamis and terrorist attacks. In its December 2011 news release, the reinsurer Swiss Re reported that "2011 will be the year with the highest catastrophe-related economic losses in history, at USD 350 billion". This has prompted researchers in different fields to intensively address the problems of emergency management. Emergency management is commonly divided into four main phases: mitigation, preparedness, response and recovery (Altay and Green, 2006; Haddow et al., 2008). The *mitigation* and *preparedness* phases are pre-crisis. They aim to define the necessary measures to reduce, mitigate or prevent the impacts of disasters and to develop action plans that will be implemented upon the occurrence of a disaster. When the crisis occurs, the phases of *response* and *recovery* take place. The response phase, or intervention, is the mobilization and deployment of emergency services within the affected area in order to protect people and reduce the human and material damages. The recovery phase defines the measures leading to the return to normal, that is, a standard of living of the same quality as it was before the disaster occurs.

This paper focuses on the logistics aspect of the response phase and more precisely on two important related problems: location and transportation. The location problem aims at designing a network for distributing humanitarian aid (e.g., water, food, medical goods and survival equipment). It mainly consists in determining the number, the position and the mission of required *humanitarian aid distribution centers* (HADC) within the disaster region. The transportation problem deals with the distribution of humanitarian aid from HADCs to demand points. When both problems are solved simultaneously, we speak about a location-transportation problem.

Decision-making in the context of humanitarian aid distribution requires careful trade-offs between a number of conflicting objectives. Opening many HADCs would allow reducing transportation duration (which is composed of transportation time, docking, loading and unloading time of vehicles) of needed products from the distribution centers to the demand points. However, opening many HADCs would also require considerable human and material resources to operate them, which may be not appreciated and sometimes even impossible. In practice, nobody wants to bring more people (e.g., drivers, policemen, technicians) into the disaster zone than necessary because more people would require more food and water and would increase the need for coordination, as well as the potential risk to these people's lives. On the other hand, opening a too small number of HADCs may result in insufficient capacities to meet people demand and thus leading to excessive non-covered (unsatisfied) demand.

In disaster situations, it makes sense thinking that each particular house or building within the affected region could require relief or humanitarian aid, thus becoming a potential demand point. In a severe crisis that affects a large area, the number of demand points and the number of types of products and services required may be very high. The amount of information to be managed is huge, and it would be impractical (if not impossible) to consider the fine details in designing and deploying humanitarian aid distribution networks. To cope with these difficulties, demand points are aggregated into demand zones and needed products are grouped into generic humanitarian functions (Rekik et al., 2013).

Based on this particular context, our paper deals with a multi-objective emergency location-transportation problem (MOELTP). Three objectives are considered. The first objective is to minimize the total transportation duration of needed products. The second objective is to minimize the number of agents (first-aiders) needed to open and operate the selected distribution centers. The third objective minimizes the non-covered demand for all demand points.

We consider the MOELTP in a static and deterministic environment since the decisions to be made are taken immediately after the disaster (i.e., a few hours later) with the information available at that moment. We assume that requests for products are estimated by homeland security organizations, or their experts, based upon their experience and their evaluation of the disaster's seriousness. Estimated transportation times are considered to take into account routing access difficulty of the region (Yuan & Wang, 2009) and the infrastructures condition (Minciardi et al., 2007).

We propose an epsilon-constraint (ϵ-constraint) method and prove that it generates the exact Pareto front of the multi-objective location-transportation problem addressed. We observe, through a computational experiment, that this exact method sometimes requires large computing times. Thus, we propose an approximate method that imposes a number of stopping criteria on the exact method. Our experimental study proves that the approximate method reduces solution time, when compared to the exact method, while generating a relatively good approximation of the Pareto front.

The remainder of the paper is organized as follows. A literature review focusing on emergency logistics is presented in Section 2. Section 3 defines the multi-objective location-transportation problem considered and proposes a mathematical formulation for it. Section 4 recalls some key concepts of multi-objective optimization problems and briefly summarizes the main solution approaches proposed for them. In Section 5, we describe the solution method we propose and prove that it generates the exact Pareto front of the multi-objective location-transportation problem considered. Section 6 presents the problem tests generated for our experimental study and explains how the exact algorithm of Section 5 can be slightly modified to generate approximate solutions. It also reports and compares the results obtained by both the exact and the approximate solution approaches. Section 7 summarizes our findings and opens on future research avenues.

2. Literature review

Altay and Green (2006) report that, although emergency management problems fit perfectly into the discipline of operations research and management science (OR/MS), the research conducted by the OR/MS community on the subject is still limited especially in the response phase. However, during the past five years, the literature related to emergency logistics has greatly expanded treating both the location and the distribution problems either separately or simultaneously. Balcik et al. (2010) and de la Torre et al. (2012) showed that disaster relief presents unique logistics challenges and is a good example of research and practice integration. In the following, we briefly review recent works on location and distribution problems in emergency situations. A more complete literature review on the optimization models proposed for emergency logistic problems is presented in Caunhye et al. (2012).

Özdamar et al. (2004) addressed the problem of planning vehicle routes to collect and deliver products in disaster areas. To handle the dynamic aspect of supply and demand, the authors proposed to divide the planning horizon into a finite number of intervals and solve the problem for each time interval, taking into account the system state. Jia et al. (2007) propose a maximal covering location model to determine facilities location in response to large-scale emergencies. The objective of their model is to maximize the population coverage. Yi and Kumar (2007) proposed an ant colony meta-heuristic for relief distribution. The proposed approach decomposes the original emergency logistics problem into two sequential phases of decision making: a vehicle route construction and a multi-commodity dispatch. Sheu (2007a) proposed an approach to plan aid distribution that includes three phases: 1) forecasting the demand of the affected regions, 2) grouping the affected areas based on the estimated severity of the damage, and 3) determining the priorities for aid distribution to affected areas. Balcik et al. (2008) studied delivery of relief supplies from local distribution centers to beneficiaries affected by disasters. They minimized the sum of transportation costs and penalty costs for unsatisfied and late-satisfied demands for two types of relief supplies. Berkoune et al. (2012) studied a complex multi-vehicle, multi-depot and multi-product transportation problem in disaster response operations. They proposed a mathematical model and a genetic algorithm to solve realistic sized instances. The genetic algorithm has been useful not only to produce solutions of good quality but also to generate many alternative solutions to the emergency managers. Naji-Azimi et al. (2012) studied the case where people are required to get to some satellite distribution centers in order to get the survival goods, provided that these centers are not too far from their domiciles. These satellite distribution centers are supplied from a central depot using a heterogeneous and capacitated fleet of vehicles. They modeled this situation as a generalization of the covering tour problem and introduce the idea of split delivery. Rawls and Turnquist (2011, 2012) proposed an allocation model to optimize pre-event planning for meeting short-term demands for emergency supplies. They consider uncertainty about what demands will have to be met and where those demands will occur. Murali et al. (2012) considered a facility location problem to determine the points in a city where medicine should be handed out to the population. They consider locating capacitated facilities in order to maximize coverage, taking into account a distance-dependent coverage function.

Although location-routing problems (LRP) have been intensively studied in the literature in the context of business logistics (See Min et al. (1998) for an exhaustive literature review on LRP), a few papers addressed simultaneously the location and the transportation problems in emergency contexts. Dessouky et al. (2006), for example, propose two models for solving facility location and vehicle routing problems in the context of a response to a large-scale emergency. The first model minimizes the total distance between the demand points and the selected facilities. The second model minimizes the unmet demands over all the demand points. Yi and Özdamar (2007) propose an integrated location-distribution model for coordinating logistics support and evacuation operations in disaster response activities in a multi-period planning horizon. The proposed model aims to coordinate the transportation of commodities from major supply centers to distribution centers in affected areas and the transport of wounded people from affected areas to temporary and permanent emergency units. The goal is to minimize the delay in the arrival of commodities at aid centers and in the provision of healthcare for the injured. When compared to classical business LRPs, location-routing problem in emergency contexts have particular characteristics. For example, emergency LRPs focus on people safety and minimum delivery times rather than monetary costs. Sheu (2007b), Kovács and Spens (2007) and Balcik et al. (2010) identified a number of challenging issues proper to emergency logistics problems which may not be addressed as easily as in business logistics.

In their review on business LRPs, Min et al. (1998) pointed out that although the multi-objective nature of LRP problems is well recognized, a few papers (five in their study) addressed multi-objective LRPs. Since then, still a relatively small number of papers addressed such challenging problems (Averbakh and Berman, 2002; Cappanera et al., 2004). In the same trend, a few papers consider multi-objective emergency logistics problems. Tzeng et al. (2007) proposed a humanitarian aid distribution model that uses multi-objective programming. Three objectives are considered: minimizing costs, minimizing travel time and maximizing the satisfaction of demand points (or minimizing unsatisfied demand). The authors handle the dynamic data by considering a multi-period model in which most of parameters and variables are time-related. The goal of the model is to determine the transfer (i.e., distribution) centers to be opened and the quantities of products to be transported from collection points to transfer points and from transfer points to the final demand points. A fuzzy multi-objective programming method is used to solve the problem. Chern et al. (2009) proposed a multi-step heuristic to solve a transportation problem. The solution is evaluated over three objectives to be minimized: delay, flow time and the total cost of transportation set-up. Nolz et al. (2010) studied a multi-objective covering tour problem for distributing aids to a population. They considered three criteria : a minisum facility criterion, a tour length criterion and minmax routing criterion. They proposed a ϵ-constraint method and a memetic solution approach. Vitoriano et al. (2011) proposed a flow model for humanitarian aid distribution. They used a goal programming model to deal with eight criteria which correspond to cost, time, equity, priority, reliability (two criteria) and security (two criteria).

The problem we consider in this paper is quite similar to that addressed by Tzeng et al. (2007). However, while the work of Tzeng et al. (2007) focuses on the multi-period aspects, we mainly concentrate on a detailed modeling of the location-transportation problem. We also consider different objective functions and propose a different approach to solve the problem.

3. The multi-objective emergency location-transportation problem

This section describes the problem and presents a mathematical model to formulate it.

3.1. Problem description

In the following, the set of demand points and required products are denoted, respectively, I and J. The request for each demand point for each product is denoted by d_{ij}.

To decide on the location of HADCs, it is assumed that a set of candidate sites already exists. This set is denoted by L in the following. In disaster situations, one wants to ensure that every demand point is accessible (can be covered) from at least one HADC in a time less than or equal to a *maximum covering time*, denoted π. We denote t_{il} the time needed to travel from site $l \in L$ to demand point $i \in I$, which takes into account the state of roads (e.g., broken, damaged, intact). We also define for each demand point $i \in I$, a subset L_i of potential sites that are within the maximum covering time, i.e., $L_i = \{l \in L: t_{il} \leq \pi\}$.

We assume that we know in advance the number of people needed to make a candidate site operate as a HADC. This number is denoted $N_l, \forall l \in L.$ Moreover, each potential site has a global and a per product capacity that fixes the maximum quantity that can be stored within the site. The global capacity of a site l is denoted S_l and its capacity for product j is denoted s_{lj}.

In addition, it is assumed that at each potential site l, there are m_l vehicle types, h=1 … m_l, and u_{hl} vehicles of each type h. Since all potential sites may not be equally equipped for receiving a particular vehicle type, different docking times, τ_{lh}, are considered, one for each vehicle type h and the corresponding site l.

The vehicles available at candidate sites have different characteristics. Indeed, some vehicles may have certain handling equipment that makes them more efficient at manipulating some products. The time needed for loading and unloading one unit (for example, a pallet) of product j into a vehicle of type h is defined as α_{jh}, where $\alpha_{jh} = \infty$ if product j cannot be loaded into a type-h vehicle. There are also some restrictions on the total weight and the total volume associated with vehicles. These restrictions depend on the vehicle type used. Formally, a loaded vehicle of type h must not weigh more than Q_h weight units nor have a volume over V_h volume units. To determine the total weight (the total volume) corresponding to a given vehicle's load, the weight w_j in weight units (the volume v_j in volume units) of each product j is assumed to be known with certainty.

Finally, a maximum daily work time, D_h (in time units) for each vehicle type h is imposed. A given vehicle can perform as many trips as needed during a day as long as the corresponding work time limit is respected. As requested quantities are generally large in terms of vehicle capacity (in weight and/or volume), each vehicle trip is assumed to visit only one demand point at a time. In other words, only back and forth trips are considered. Obviously, a demand point may be visited many times. However, because of the maximum daily work time, the number of trips performed to delivery point i by a specific vehicle will be limited to a maximum value r. In our experimental study, we set r = 2.

3.2. Mathematical model

The definitions of parameters and decision variables used in the proposed mathematical model are summarized as follows:

Parameters

I	Set of demand points; $I = \{1, \dots, n\}$
J	Set of products; $J = \{1, \dots, p\}$
L	Set of candidate sites; $L = \{1, \dots, u\}$
d_{ij}	Demand of point i for product j
S_l	Capacity of site l for all products
s_{lj}	Capacity of site l for product j
N_l	Number of required agents at site l
π	Maximum covering time
t_{il}	Time needed to travel from demand point i to site l
L_i	Set of candidates sites $l \in L$ which can cover demand point i within the maximum covering time τ, $L_i = \{l \in L,\ t_{il} \leq \pi\}$
m_l	Number of vehicle types available at site l (h=1,…, m_l)
u_{hl}	Number of vehicles of type h available at site l (k=1,…, u_{hl})
τ_{lh}	Docking time for a vehicle of type h at site l
Q_h	Weight capacity of a vehicle of type h
V_h	Volume capacity of a vehicle of type h
α_{jh}	Time of loading and unloading one unit of product j into a vehicle of type h
D_h	Maximum daily work time for a vehicle of type h
w_j	Weight of one unit of product j
v_j	Volume of one unit of product j
r	Maximum number of trips performed by a vehicle

Decision variables

y_l	Equal to 1 if a $HADC$ is open at site l, 0 otherwise.

x_{ilhkv} Equal to 1 if demand point i is visited from $HADC\ l$ with the k^{th} vehicle of type h on its v^{th} trip to i.

Q_{ijlhkv} Quantity of product j delivered to point i from $HADC\ l$ with the k^{th} vehicle of type h on its v^{th} trip to i.

p_{jl} Quantity of product j provided at site l.

The MOELTP can be modeled as follows:

$$Min\ f_1 = \sum_{i=1}^{n}\sum_{l=1}^{u}\sum_{h=1}^{m_l}\sum_{k=1}^{u_{hl}}\sum_{v=1}^{r}\left((2t_{il}+\tau_{lh})x_{ilhkv} + \sum_{j=1}^{p}\alpha_{jh}Q_{ijlhkv}\right) \tag{1}$$

$$Min\ f_{2=}\sum_{l=1}^{u} N_l y_l \tag{2}$$

$$Min\ f_3 = \sum_{i=1}^{n}\sum_{j=1}^{p}\left(d_{ij} - \sum_{l=1}^{u}\sum_{h=1}^{m_l}\sum_{k=1}^{u_{hl}}\sum_{v=1}^{r} Q_{ijlhkv}\right) \tag{3}$$

Subject to

$$\sum_{\substack{l=1\\ l\in L_i}}^{u}\sum_{h=1}^{m_l}\sum_{k=1}^{u_{hl}}\sum_{v=1}^{r} Q_{ijlhkv} \le d_{ij} \qquad i = 1, \dots, n\, ; j = 1, \dots, p \tag{4}$$

$$\sum_{i=1}^{n}\sum_{h=1}^{m_l}\sum_{k=1}^{u_{hl}}\sum_{v=1}^{r} Q_{ijlhkv} - p_{jl} \le 0 \qquad j = 1, \dots, p\, ;\ \ l = 1, \dots, u \tag{5}$$

$$\sum_{i=1}^{n}\sum_{v=1}^{r}\left((2t_{il}+\tau_{lh})x_{ilhkv} + \sum_{j=1}^{p}\alpha_{jh}Q_{ijlhkv}\right) \le D_h\, y_l \qquad l = 1, \dots, u\, ; h = 1, \dots, m_l\, ; k = 1, \dots, u_{hl} \tag{6}$$

$$\sum_{j=1}^{p} w_j Q_{ijlhkv} \le Q_h x_{ilhkv} \qquad i = 1, \dots, n\, ; l \in L_i\, ; h = 1, \dots, m_l\, ; k = 1, \dots, u_{hl}; v = 1, \dots, r \tag{7}$$

$$\sum_{j=1}^{p} v_j Q_{ijlhkv} \leq V_h x_{ilhkv} \qquad i = 1, \dots, n\,; l \in L_i\,; h = 1, \dots, m_l\,; \; k = 1, \dots, u_{hl}; v = 1, \dots, r \quad (8)$$

$$\sum_{j=1}^{p} p_{jl} \leq S_l\, y_l \qquad l = 1, \dots, u \quad (9)$$

$$p_{jl} \leq s_{lj} \qquad j = 1, \dots, p\,; \; l = 1, \dots, u \quad (10)$$

$$y_l \in \{0,1\} \qquad l = 1, \dots, u \quad (11)$$

$$x_{ilhkv} \in \{0,1\} \qquad i = 1, \dots, n\,; l \in L_i\,; h = 1, \dots, m_l; \; k = 1, \dots, u_{hl}; v = 1, \dots, r \quad (12)$$

$$Q_{ijlhkv} \geq 0 \qquad i = 1, \dots, n\,; l \in L_i\,; h = 1, \dots, m_l\,; \; k = 1, \dots, u_{hl}; v = 1, \dots, r \quad (13)$$

$$p_{jl} \geq 0 \qquad j = 1, \dots, p\,; \; l = 1, \dots, u \quad (14)$$

The three objectives are given by equations (1)-(3). The objective function (1) minimizes the sum of all transportation durations. In fact, the duration of the v^{th} trip of the k^{th} vehicle of type h from site l to demand point i is given by $\left(2t_{il}x_{ilhkv} + \tau_{hl}x_{ilhkv} + \sum_{j=1}^{p} \alpha_{jh}Q_{ijlhkv}\right)$, where the first part ($2t_{il}$) represents the back and forth travel times, the second part (τ_{hl}) is the docking time, and the last part ($\sum_{j=1}^{p} \alpha_{jh} Q_{ijlhkv}$) is the loading and unloading time of all the products delivered from site l to point i. Objective function (2) minimizes the number of agents needed to operate the opened HADCs. The third objective function (3) minimizes the total uncovered demand. Constraints (4) ensure that the quantity of product j delivered for each demand point i does not exceed its demand. Constraints (5) ensure that the total quantity of a given product j delivered from a HADC l does not exceed the quantity of product j available in this HADC. Constraints (6) express the maximum daily work time restrictions associated with each vehicle k of type h located at a HADC l. These constraints also prohibit trips from unopened sites. Constraints (7) and (8) impose the vehicle capacity constraints for each trip, in terms of weight (Q_h) and volume (V_h). Constraints (9) and (10), respectively, insure that the global, respectively, the per product, capacity of each HADC is satisfied. Constraints (11) - (14) express the nature of decision variables used in the model.

One should notice that model (1) – (14) presents some symmetries especially in situations where many vehicles of the same type are present in the same HADC. Indeed, when examining x and Q variables, one could switch the k^{th} vehicle with any vehicle of the same type and obtain the same solution. Such symmetry is inherent to fleet composition and routing models (Hoff et al. 2010) and may complicate the resolution process when solving the problem to optimality.

4. Multi-objective optimization problems

This section reviews the main concepts and definitions related to multi-objective optimization problems (MOP). In general, a MOP, in a minimization case, is formulated as:

$$Min\ f(x) = (f_1(x), f_2(x), \dots, f_m(x))$$
$$\text{s.t} \quad x \in D;$$

where m ($m \geq 2$) is the number of objectives, $x = (x_1, x_2, \dots, x_n)$ is the vector representing the decision variables, and D is the set of feasible solutions. A MOP usually has not a unique optimal solution, but a set of solutions known as the Pareto-optimal set. Each Pareto optimal solution represents a compromise between different objectives, and the components of the corresponding vector of objectives cannot be all simultaneously improved. Comparing two solutions in multi-objective optimization is more complex than in the single-objective optimization case. Two concepts are indeed of a great importance in multi-objective optimization: Pareto dominance and Pareto optimality. In a minimization case, Pareto dominance and Pareto optimality are defined as follows.

Definition 1 (Pareto dominance): A given vector $u \in D$ dominates a vector $v \in D$ in the Pareto sense, if and only if u is partially less than v ($u \prec v$), i.e.

$$\begin{cases} f_i(u) \leq f_i(v) & \text{for all } i \in \{1, \dots, m\} \\ f_j(u) < f_j(v) & \text{for at least one } j \in \{1, \dots, m\} \end{cases}$$

Definition 2 (Pareto optimal solution): A solution $u \in D$ is a Pareto-optimal solution, if and only if there is no $v \in D$ such that v dominates u. Pareto-optimal solutions are also called efficient or non-dominated solutions.

Definition 3 (Pareto optimal set): The Pareto optimal set or the efficient set is defined as $P = \{x \in D: x \text{ is a Pareto optimal solution in } D\}$.

Definition 4 (Pareto front): The Pareto front is defined as $P\mathcal{F} = \{f(x) \ : \ x \in P\}$, where P is the Pareto optimal set.

Different solution approaches, either exact or heuristic, have been proposed in the literature to solve multi-objective optimization problems (MOP). Exact approaches compute the entire Pareto front whereas heuristic methods search for good solutions that are relatively close to Pareto-optimal solutions but with no guarantee of their Pareto-optimality. Ehrgott and Gandibleux (2002) reviewed available solution methodology, both exact and heuristic. Talbi (2009) presents the main issues related with the design and implementation of multi-objective metaheuristics. An in-deep presentation of multi-criteria optimization can be found in Ehrgott (2005).

A multitude of approaches exist for MOPs and scalarization methods are among the most popular. The weighted sum method reduces a MOP to a mono-objective problem by considering a weighted sum of all objective functions. The performance of this method strongly depends on the choice of the weighting coefficients. Its main drawback is that it does not generate all Pareto optimal solutions for non-convex objective spaces like those of MOP (Das and Dennis 2007).

Unlike weighted sum methods, the well-known ϵ-constraint method is another scalarization technique which works well for non-convex MOPs. This method has been widely used in the literature and consists in solving a sequence of constrained mono-objective problems. In Chankong and Haimes (1983), it is proven that for general MOP, the exact Pareto front can be found by the ϵ-constraint method, as long as we know how to modify the parameters of the method. Recently, Bérubé et al. (2009) consider a bi-objective traveling salesman problem with profits and propose an ϵ-constraint method to generate the exact Pareto front for the problem.

In the next section, we propose a solution approach for MOELTP based on the ϵ-constraint method and prove that it generates the exact Pareto front. Although the proposed approach is presented for MOELTP, it can apply to other three-objective optimization problems presenting some characteristics that will be discussed at the end of Section 5.

5. Exact solution method for MOELTP

The ϵ-constraint method solves a set of constrained single-objective problems $P^k(\epsilon)$, $\epsilon = (\epsilon_1, \dots, \epsilon_{k-1}, \epsilon_{k+1}, \dots, \epsilon_m)$ obtained by choosing one objective f_k as the only objective to optimize and incorporating inequality constraints for the remaining objectives of the form $f_i \leq \epsilon_i, i = 1, \dots, k-1, k = 1, \dots, m$. The set of problems $P^k(\epsilon)$ is obtained by assigning different values to the components of the ϵ-vector.

In our context, the MOELTP is a three-objective combinatorial optimization problem for which we proposed the mathematical formulation (1)-(14) (see Section 3.2). The three objectives f_1, f_2, f_3, defined by equations (1)-(3), consist in minimizing, respectively, the total vehicles trip duration (including travelling time as well as products' loading and unloading times), the number of agents needed to operates the opened HADCs and the total uncovered demand. In the proposed ϵ-constraint-based solution approach, we choose as objective function to optimize, the one taking non-integer value, that is objective f_1. As will be shown in the proof of Theorem 1, this choice is necessary to yield the exact Pareto front. Although objective f_3 may also take non-integer value in theory, we make the assumption in the following that both demand and Q variables take integer values yielding thus an objective function f_3 with only integer values as well. Such an assumption is realistic in emergency contexts since products are generally packaged in boxes and transported in pallets.

Hence, the constrained single-objective problem $P(\epsilon_2, \epsilon_3)$ considered in the proposed ϵ-constraint-based solution approach is defined by:

$P(\epsilon_2, \epsilon_3)$ Min $f_1(X)$

s.t

$$\begin{cases} f_2(X) \le \epsilon_2 \\ f_3(X) \le \epsilon_3 \\ X = (x, y, Q, p) \in D \end{cases}$$

where $X = (x, y, Q, p)$ denotes the set of variables defined in the mathematical model (1)-(14) (Section 3.2); and D is the feasible region defined by equations (4)-(14).

The proposed approach consists in defining a sequence of ϵ-constraint problems based on a gradual variation of parameters ϵ_2 and ϵ_3. We will prove that the optimal solutions of some of these ϵ-constraint problems are Pareto optimal. We will also prove that the identified Pareto optimal solutions generate the exact Pareto front of MOELTP. Algorithm 1 hereafter describes the way parameters ϵ_2 and ϵ_3 are fixed to yield the exact Pareto front. In this algorithm, an optimal solution to a problem $P(\epsilon_2, \epsilon_3)$ is denoted by $opt(f_1, \epsilon_2, \epsilon_3)$ and $f_i^- = \max_{x \in D} f_i(x)$; for $i = 2,3$. We also define F_2 as the set of the different values, denoted f_2^t, that can be taken by objective f_2; these values being sorted in an increasing order. Observe that set F_2 is finite since the set of candidate sites L is finite and objective function f_2 takes only integer values. Set F_2 can be simply determined by enumerating the different combinations of potential sites ($l \in L$) and computing the corresponding total number of required agents (N_l).

The main steps of Algorithm 1 are as follows. We assign to parameter ϵ_2 defining the inequality constraint $f_2(X) \le \epsilon_2$ the different integer values that can be taken by objective f_2, starting from the minimum value (f_2^1) and ending with the maximum value ($f_2^{|F_2|}$). For each value f_2^t of ϵ_2, we assign to parameter ϵ_3 defining the inequality constraint $f_3(X) \le \epsilon_3$ the maximum value that could be taken by f_3 (f_3^-). The value of parameter ϵ_3 is decremented by one as long as the resulting problem $P(\epsilon_2 = f_2^t, \epsilon_3)$ is feasible. If for a given value of ϵ_3, problem $P(\epsilon_2 = f_2^t, \epsilon_3)$ is infeasible, the value of ϵ_2 is incremented to f_2^{t+1} and the process is reiterated. A pair $(\epsilon_2^*, \epsilon_3^*)$ for which the resulting problem $P(\epsilon_2^*, \epsilon_3^*)$ is feasible is solved to optimality and yields an optimal solution $X^* = opt(f_1, \epsilon_2^*, \epsilon_3^*)$. Solution X^* is Pareto optimal if $f_2(X^*) = \epsilon_2^*$ and there is no Pareto optimal solution X' already identified in the previous iterations such that $f_1(X^*) = f_1(X')$ and $f_3(X^*) = f_3(X')$.

Algorithm 1. ϵ-constraint method for MOELTP

S $:= \emptyset$
for $t = 1$ **to** $t = |F_2|$ **do**
    $\epsilon_3 = f_3^-$ **and** $\epsilon_2 = f_2^t$
    while $P(\epsilon_2, \epsilon_3)$ has a feasible solution **do**
        $\epsilon_2^* \coloneqq \epsilon_2$; $\epsilon_3^* \coloneqq \epsilon_3$
        $X^* \coloneqq opt(f_1, \epsilon_2^*, \epsilon_3^*)$
        if $f_2(X^*) = \epsilon_2^*$ **and** $\forall X' \in S,\ f_1(X^*) \neq f_1(X')$ or $f_3(X^*) \neq f_3(X')$ **then**
            S $:=$ S $\cup \{X^*\}$
        end if
        $\epsilon_3 \coloneqq \epsilon_3 - 1$
    end while
end for

Theorem 1. The set S of solutions produced by Algorithm 1 generates the exact Pareto front of the three-objective emergency logistics problem MOELTP.

Proof:

The proof is divided in two parts. In the first part, we prove that each solution in S is Pareto optimal. In the second part, we prove that $f(S) = \{ f(X) : X \in S\}$ gives the exact Pareto front.

Part 1: Each solution in *S* is Pareto optimal

Consider $X^* \in S$ with $X^* = opt(f_1, \epsilon_2^*, \epsilon_3^*)$. Let X be a feasible solution in D that is different from X^*. The cases for which X could dominate X^* are:

(1) $f_3(X) = f_3(X^*)$ and
(2) $f_3(X) < f_3(X^*)$

Notice that since $X^* \in S$, $f_2(X^*) = \epsilon_2^* = f_2^{t^*}$ for a given $t^* \in \{1, \dots, |F_2|\}$.

Case 1: $f_3(X) = f_3(X^*)$
Two cases must be discussed: the case where $f_2(X) = f_2(X^*)$ and the case where $f_2(X) < f_2(X^*)$.

- $f_2(X) = f_2(X^*)$:
  One can easily prove that X is a feasible solution for $P(\epsilon_2^*, \epsilon_3^*)$ ($X \in D$, $f_2(X) = f_2(X^*) \leq \epsilon_2^*$ and $f_3(X) = f_3(X^*) \leq \epsilon_3^*$). Since X^* is an optimal solution of $P(\epsilon_2^*, \epsilon_3^*)$ then $f_1(X) \geq f_1(X^*)$. Hence, either $f_1(X) = f_1(X^*)$ and in this case X and X^* yield the same objective vector ($f_1(X) = f_1(X^*)$, $f_2(X) = f_2(X^*)$ and $f_3(X) = f_3(X^*)$) ; or $f_1(X) > f_1(X^*)$ and in this case X^* dominates X. Notice that the case where X and X^* yield the same objective vector is the case where problem $P(\epsilon_2^*, \epsilon_3^*)$ has multiple optimal solutions.
- $f_2(X) < f_2(X^*)$:
  We show in the following that in this case $f_1(X) > f_1(X^*)$.

X is a feasible solution for $P(\epsilon_2^*, \epsilon_3^*)$ $(X \in D,\ f_2(X) < f_2(X^*) \leq \epsilon_2^*$ and $f_3(X) = f_3(X^*) \leq \epsilon_3^*)$. Since $X^*$is an optimal solution of $P(\epsilon_2^*, \epsilon_3^*)$ then $f_1(X) \geq f_1(X^*)$. Hence, either $f_1(X) > f_1(X^*)$ and in this case X cannot dominate X^* or $f_1(X) = f_1(X^*)$. Next, we prove by contradiction, that $f_1(X)$ cannot be equal to$f_1(X^*)$.

$f_2(X) < f_2(X^*)$ implies that there exist a $t \in \{1, \dots, |F_2|\}$ such that $f_2(X) = f_2^t$ and $t < t^*$. Consider problem $P(\epsilon_2 = f_2^t, \epsilon_3^*)$. Clearly, X is a feasible solution for $P(\epsilon_2 = f_2^t, \epsilon_3^*)$ $(X \in D$ and $f_3(X) \leq f_3(X^*) \leq \epsilon_3^*)$. Thus $P(\epsilon_2 = f_2^t, \epsilon_3^*)$ is feasible. Since $\epsilon_2 = f_2^t < \epsilon_2^* = f_2^{t^*}$, then according to Algorithm 1, problem $P(\epsilon_2 = f_2^t, \epsilon_3^*)$ is solved before problem $P(\epsilon_2^*, \epsilon_3^*)$. If $f_1(X) = f_1(X^*)$, X is also an optimal solution for problem $P(\epsilon_2^*, \epsilon_3^*)$. Problem $P(\epsilon_2^*, \epsilon_3^*)$ is a relaxation of $P(\epsilon_2 = f_2^t, \epsilon_3^*)$ (since $f_2^t = f_2(X) \leq f_2(X^*) \leq \varepsilon_2^*$) and its optimal solution X is feasible for $P(\epsilon_2 = f_2^t, \epsilon_3^*)$. Thus X is an optimal solution for $P(\epsilon_2 = f_2^t, \epsilon_3^*)$.

X is an optimal solution for a problem $P(\epsilon_2 = f_2^t, \epsilon_3^*)$ such that $f_2(X) = \epsilon_2$. If $X \in S$, then there is no $X' \in S$ such that $f_1(X') = f_1(X)$ and $f_3(X') = f_3(X)$. However $X^* \in S$ and $f_1(X^*) = f_1(X)$ and $f_3(X^*) = f_3(X)$. Thus X cannot be in S. X is an optimal solution for a problem $P(\epsilon_2 = f_2^t, \epsilon_3^*)$ such that $f_2(X) = \epsilon_2 = f_2^t < \epsilon_2^* = f_2^{t^*}$and $X \notin S$. Thus, there exist a solution $X' \in S$ such that $f_1(X') = f_1(X)$ and $f_3(X') = f_3(X)$ with $f_2(X') < f_2(X)$. This X' is in fact an optimal solution for a problem $P(\epsilon_2' = f_2(X'), \epsilon_3^*)$ that is solved before problem $P(\epsilon_2^*, \epsilon_3^*)$ (since $\epsilon_2' = f_2(X') < f_2(X) < f_2(X^*) = \epsilon_2^*$). Solution $X' \in S$ is such that $f_1(X') = f_1(X) = f_1(X^*)$ and $f_3(X') = f_3(X) = f_3(X^*)$. Which is in contradiction with the fact that $X^* \in S$. Hence $f_1(X) \neq f_1(X^*)$.

Case 2: $f_3(X) < f_3(X^*)$

In this case, X would dominate X^* if and only if $f_1(X) \leq f_1(X^*)$ and $f_2(X) \leq f_2(X^*)$. We prove in the following that if $f_3(X) < f_3(X^*)$ and $f_2(X) \leq f_2(X^*)$ then necessarily$f_1(X) > f_1(X^*)$.

If $f_3(X) < f_3(X^*)$ and $f_2(X) \leq f_2(X^*)$ then X is a feasible solution for $P(\epsilon_2^*, \epsilon_3^*)$ $(X \in D,\ f_2(X) \leq f_2(X^*) \leq \epsilon_2^*$ and $f_3(X) < f_3(X^*) \leq \epsilon_3^*)$. Since $X^*$is an optimal solution of $P(\epsilon_2^*, \epsilon_3^*)$ then $f_1(X) \geq f_1(X^*)$. Hence, either $f_1(X) > f_1(X^*)$ and in this case X cannot dominate X^* or $f_1(X) = f_1(X^*)$.

In the following, we prove by contradiction, that $f_1(X)$ cannot be equal to $f_1(X^*)$. The basic idea is to prove that if $f_1(X) = f_1(X^*)$ then X^* cannot be an optimal solution for $P(\epsilon_2^*, \epsilon_3^*)$.

Recall that solutions X and X^* are feasible solutions for model (1)-(14) and could thus be represented by tuples $(x_{ilhkv}, y_l, Q_{ijlhkv}, p_{jl})$ and $({x^*}_{ilhkv}, y_l^*, {Q^*}_{ijlhkv}, p_{jl}^*)$, respectively.

If $f_1(X) = f_1(X^*)$ then $\sum_{i,l,h,kv}((2t_{il} + \tau_{lh})x_{ilhkv} + \sum_j \alpha_{jh} Q_{ijlhkv}) = \sum_{i,l,h,kv}((2t_{il} + \tau_{lh}){x^*}_{ilhkv} + \sum_j \alpha_{jh} {Q^*}_{ijlhkv})$.

On another hand, $f_3(X) < f_3(X^*)$ implies that $\sum_{i,j,l,h,k,v} Q_{ijlhkv} > \sum_{i,j,l,h,k,v} {Q^*}_{ijlhkv}$.

Consider a vector X'=$({x'}_{ilhkv}, y_l', {Q'}_{ijlhkv}, p'_{jl})$ that is deduced from X as follows:

- $y' = y$,
- $x' = x$,
- $p' = p$,
- $Q' = Q$ except for one component *(i',j',l',h',k',v')* such that $Q_{ijlhkv} \geq 2$ for which the quantity delivered to demand point *i'* is decreased by one unit for X' with respect to X. Consequently, $\sum_{i,j,l,h,k,v} {Q'}_{ijlhkv} = \sum_{i,j,l,h,k,v} Q_{ijlhkv} - 1 < \sum_{i,j,l,h,k,v} Q_{ijlhkv}$.

One can verify that $X' \in D$. Moreover, $f_2(X') \leq f_2(X) \leq f_2(X^*) \leq \epsilon_2^*$. Since, $\sum_{i,j,l,h,k,v} {Q'}_{ijlhkv} = \sum_{i,j,l,h,k,v} Q_{ijlhkv} - 1$; $\sum_{i,j,l,h,k,v} Q_{ijlhkv} > \sum_{i,j,l,h,k,v} {Q^*}_{ijlhkv}$ and variables Q take integer values (by assumption), then $\sum_{i,j,l,h,k,v} {Q'}_{ijlhkv} \geq$

$\sum_{i,j,l,h,k,v} Q^{*}{}_{ijlhkv}$. Thus, $f_3(X') \leq f_3(X^*) \leq \epsilon_3^*$. Hence X' is a feasible solution for $P(\epsilon_2^*, \epsilon_3^*)$. Furthermore, $f_1(X') = \sum_{i,l,h,kv}((2t_{il} + \tau_{lh})x'_{ilhkv} + \sum_j \alpha_{jh} Q'_{ijlhkv}) = \sum_{i,l,h,kv}\left((2t_{il} + \tau_{lh})x_{ilhkv} + \sum_j \alpha_{jh} Q'_{ijlhkv}\right) < f_1(X) = f_1(X^*)$. Which contradicts the fact that X^* is an optimal solution for $P(\epsilon_2^*, \epsilon_3^*)$.

Notice that if all non-null components of vector Q in X are such that $Q_{ijlhkv} = 1$ and if each HADC l delivers only one demand point i, then one cannot construct a solution X' from X with the same HADC (i.e., such that $y' = y$). In this case, X' will be defined as previously except that for one HADC $l \in L$ such that $y_l = 1$ (chosen arbitrarily), we fix the corresponding $y'_l = 0$. The same conclusions proved previously still apply in this case since $f_2(X') \leq f_2(X)$, $\sum_{i,j,l,h,k,v} Q'_{ijlhkv} = \sum_{i,j,l,h,k,v} Q_{ijlhkv} - 1$ and $\sum_{i,l,h,kv}((2t_{il} + \tau_{lh})x'_{ilhkv}) < \sum_{i,l,h,kv}((2t_{il} + \tau_{lh})x_{ilhkv})$ (one trip is missing in X' with respect to X).

In both cases ($f_3(X) = f_3(X^*)$ and $f_3(X) < f_3(X^*)$), we prove that X does not dominate X^*. Hence, $\forall X^* \in S$, $\nexists\, X \in D \setminus \{x^*\}$ such that X dominates X^*.

Let PF denote the exact Pareto front of MOELTP. In the following, we prove that $f(S) = \{f(X): X \in S\}$ is the exact Pareto front. Part 1 of the proof permits us to assert that $f(S) \subseteq PF$. In part 2, we prove that $PF \subseteq f(S)$.

<u>Part 2: $f(S)$ is the exact Pareto front</u>

One needs to prove that a vector $X \in D\backslash S$ will be either dominated by a solution $X^* \in S$ or will yield an objective vector $f(X)$ that is equal to $f(X^*)$ for $X^* \in S$.
Let $X^0 \in D\backslash S$, $\epsilon_2^0 = f_2(X^0)$ and $\epsilon_3^0 = f_3(X^0)$.
Clearly, X^0 is a feasible solution for $P(\epsilon_2^0, \epsilon_3^0)$. Let X' be an optimal solution for $P(\epsilon_2^0, \epsilon_3^0)$. Thus, $f_2(X') \leq \epsilon_2^0 = f_2(X^0)$, $f_3(X') \leq \epsilon_3^0 = f_3(X^0)$ and $f_1(X') \leq f_1(X^0)$. That is, X' dominates X^0.

- If $X' \in S$, the proof is done.
- If $X' \notin S$, then according to Algorithm 1, either $f_2(X') \neq \epsilon_2^0 (= f_2(X^0))$ or $f_2(X') = \epsilon_2^0$ and there exist already $X^* \in S$ such that $f_1(X^*) = f_1(X')$ and $f_3(X^*) = f_3(X')$.
    - If $f_2(X') \neq f_2(X^0)$ then one can easily prove that X' is an optimal solution for $P(f_2(X'), \epsilon_3^0)$ (the same proof established in part 1). Since $X' \notin S$, then there is necessarily $X^* \in S$ such that $f_2(X^*) < f_2(X')(\leq f_2(X^0))$, $f_1(X^*) = f_1(X')(\leq f_1(X^0))$, and $f_3(X^*) = f_3(X')(\leq f_3(X^0))$.
    - If $f_2(X') = f_2(X^0)$. Since $X' \notin S$, then there is necessarily $X^* \in S$ such that $f_2(X^*) < f_2(X') \leq f_2(X^0)$, $f_1(X^*) = f_1(X') \leq f_1(X^0)$ and $f_3(X^*) = f_3(X') \leq f_3(X^0)$.

  In both cases, we prove that $\exists X^* \in S$ such that $f_1(X^*) \leq f_1(X^0)$; $f_2(X^*) \leq f_2(X^0)$. and $f_3(X^*) \leq f_3(X^0)$. The case for which X^* does not dominate X^0 is when $f_1(X^*) = f_1(X^0)$; $f_2(X^*) = f_2(X^0)$ and $f_3(X^*) = f_3(X^0)$. That is when problems $P(\epsilon_2, \epsilon_3)$ have multiple optimal solutions.□

Corollary 1. If problems $P(\epsilon_2, \epsilon_3)$ solved in Algorithm 1 have unique optimal solutions then the set S of solutions produced by Algorithm 1 defines the Pareto optimal set of the three-objective emergency logistics problem MOELTP.

*Proof***.**

The result is straightforwardly deduced from the proof of Theorem 1 (part 2). □

Based on the proof of Theorem 1, one can straightforwardly prove that Theorem 1 can be generalized to any three-objective optimization problem provided that the problem involves at least two objectives which take integer values (in our case f_2 and f_3) and a third objective that is conflicting with at least one of the first two ones (in our case f_1 and f_3). Formally, consider a three-objective minimization problem $Min\ f(x) = (f_1(x), f_2(x), f_3(x))$ s.t $x \in D$;
where $x = (x_1, x_2, \dots, x_n)$ is the vector representing the decision variables, and D is the set of feasible solutions. If:

(i) there exist two objective functions, for example and with no loss of generality f_2 and f_3, that take integer values, and
(ii) the third objective function, f_1, is conflicting with either f_2 or f_3 (for example f_3) . That is, for any $X, X' \in D$, if $f_3(X) < f_3(X')$ then necessarily $f_1(X) > f_1(X')$.

Then, Algorithm 1 generates the exact Pareto front for the considered MOP.

Moreover, Algorithm 1 can be slightly modified to act as an approximate method in case the exact approach requires a relatively long computing time. This can be done by imposing stopping criteria when solving problems $P(\epsilon_2, \epsilon_3)$. A stopping criterion could be for example a time limit or a non-null tolerance on the gap value.

6. Computational experiments

The proposed algorithm is implemented in VB.NET, and the MIP models are solved with CPLEX 12.2. Our experiments were performed on an IBM x3550 with an Intel Xeon E5420 running at 3.2 GHz with 4 Gig RAM.

6.1. Problem tests

In order to test the performance of the algorithm, we generate four sets of instances. Each set is defined by a pair $(|L|, |I|)$, where $|L|$ is the number of candidate sites and $|I|$ is the number of demand points. The four sets correspond to the following $(|L|, |I|)$ pairs: (3, 15), (4, 15), (4, 30) and (4, 45). For all sets, we consider a single product (family of products) and the loading time of a product depends on its compatibility with the vehicle used. Ten instances are generated for each set $(|L|, |I|)$. These instances are obtained by randomly generating a demand between a minimum and a maximum values for each demand point. We fix the maximum covering time to the same value (50 time units) for all sets.

To each distribution center is associated a number of available vehicles and an open cost which represents the number of required agents to open this center. We fixed for each vehicle type the weight and volume capacity, the maximum daily work time and the docking time at each of the distribution centers. In our problem, we considered two vehicle types and one vehicle of each type is available at each distribution center. Problems are available from the authors upon request.

6.2. Results for exact Pareto fronts

To generate the exact Pareto front for each instance, we let CPLEX run as long as needed to find an optimal solution for each problem $P(\epsilon_2, \epsilon_3)$. Table 1 displays the results obtained for all the 40 generated instances. Column $t_{\mathcal{F}}$ indicates the total computing time in seconds to find the exact Pareto front. Column $t_{\mathcal{F}}^{\epsilon}$ gives the computing time required on average for solving each problem $P(\epsilon_2, \epsilon_3)$. Column N_{ϵ} gives the number of ϵ-constraint problems solved and column $|\mathcal{F}|$ displays the size of the exact Pareto front. The $Average$ line refers to the average values of each column for the 10 instances of each set.

Table 1 shows that the proposed algorithm finds the exact Pareto front for all the instances. Computing time varies a lot from an instance set to another. The average time required to identify the Pareto front for instance set (3, 15) does not exceed 22 minutes (1 300 seconds) on average. This average time reaches 83 minutes (4 944 seconds) for instance set (4, 15) and exceeds 8 hours (16 535 seconds) for the larger problems of instance set (4, 45). This variation in computing times between the four instance sets is mostly due to the variation in the number of the ϵ-constraint problems solved for each set. In fact, as displayed in column N_ϵ, identifying the exact Pareto front for instances in set (3, 15) required solving 1 732 ϵ-constraint problems on average, whereas this number reaches 5 059 for instance set (4, 15), 7 144 for instance set (4,30) and 10 212 for instance set (4, 45). The computing time required for solving an ϵ-constraint problem in each set remains however small for all sets (between 0.64 and 8.23 seconds).

Table1. Results for exact Pareto fronts

$(\|L\|, \|I\|)$	N° inst.	$t_{\mathcal{F}}$	$t_{\mathcal{F}}^{\epsilon}$	N_{ϵ}	$\|\mathcal{F}\|$
(3,15)	1	1 114	0.66	1 664	766
	2	1 713	1.01	1 681	659
	3	1 323	0.72	1 814	828
	4	1 160	0.64	1 803	1 238
	5	1 254	0.73	1 714	1 025
	6	1 106	0.66	1 654	965
	7	1 824	0.94	1 932	902
	8	1 241	0.70	1 772	981
	9	1134	0.70	1 616	917
	10	1 128	0.67	1 677	912
	Average	**1 300**	**0.74**	**1 732.7**	**919.3**
(4,15)	1	4 300	0.85	5 056	1 199
	2	8 049	1.44	5 569	2 008
	3	5 175	1.00	5 173	1 234
	4	3 982	0.72	5 473	2 028
	5	4 282	0.81	5 242	2 399
	6	4 166	0.86	4 840	1 166
	7	6 278	1.26	4 980	1 238
	8	4 492	0.97	4 624	1 135
	9	4 743	1.03	4 583	1 109
	10	3 975	0.78	5 051	1 318
	Average	**4 944**	**0.97**	**5 059.1**	**1 483.4**
(4,30)	1	14 621	2.06	7 067	1 656
	2	6 784	1.13	5 984	1 463
	3	5 325	0.85	6 232	1 293
	4	37 437	4.80	7 784	1 938
	5	24 016	3.25	7 387	1 875
	6	28 168	3.21	8 761	4 279
	7	11 512	1.72	6 678	1 421
	8	15 527	1.77	8 763	2 567
	9	11 868	1.87	6 342	1 416
	10	10 094	1.56	6 447	1 426
	Average	**16 535**	**2.22**	**7 144.5**	**1 933.4**
(4,45)	1	29 041	2.85	10 189	2 275
	2	21 430	2.07	10 316	2 914
	3	38 445	3.56	10 789	3 030
	4	9 189	1.47	6 230	1 848
	5	78 542	8.23	9 535	1 925
	6	57 246	3.47	16 487	3 055
	7	17 135	1.42	12 043	3 161
	8	19 446	1.91	10 173	2 916
	9	19 761	2.09	9 413	2 950
	10	10 149	1.46	6 952	1 879
	Average	**30 038**	**2.85**	**10 212.7**	**2 595.3**

6.3. Results for approximate Pareto fronts

In order to reduce computing times, we modify Algorithm 1 and make it act as an approximate method by imposing a non-null tolerance on the relative gap (as computed by the CPLEX solver) when solving problems $P(\epsilon_2, \epsilon_3)$. The choice of this heuristic stopping criterion is motivated by the fact that we observed that for a large number of instances CPLEX spent a lot of time in proving solution optimality. Notice that the relative gap computed by CPLEX is the relative gap between the best upper bound (a feasible solution value) and the best lower bound (the linear relaxation value of the best node remaining). Hence, in Algorithm 1, X^* denotes a feasible (not necessarily optimal) solution of $P(\epsilon_2, \epsilon_3)$ with a relative gap that is lower than or equal to the tolerance value parameter considered and S is the approximate Pareto front. Notice that the condition of inclusion of a solution X^* in the approximate Pareto front S remains valid since it helps approximate more accurately the exact Pareto front. Indeed, if $f_2(X^*) = \epsilon_2^*$ and there exist $X' \in S$, such that $f_1(X^*) = f_1(X')$ and $f_3(X^*) = f_3(X')$ then X'dominates X^*; and should not be included in S.

We consider three tolerance values for this gap: 10%, 5% and 1%. For example, a tolerance of 5% implies that if the value of the relative gap falls below 5% during the branch-and-bound procedure of CPLEX, the optimization is stopped and the solution output by CPLEX is the one corresponding to the best upper bound.

Given that the approximation approach generates an approximate Pareto front (denoted hereafter $\mathcal{AF}$), one needs to evaluate the performance of the approximation approach with respect to the exact one. To this end, we consider two performance measures, used in Bérubé et al. (2009) and originally proposed by Cryzak and Jaszkiewicz (1998), as follows:

$$Dist1 = \frac{1}{|\mathcal{F}|} \sum_{z \in \mathcal{F}} \left\{ \min_{z' \in \mathcal{AF}} d(z, z') \right\}$$

$$Dist2 = \max_{z \in \mathcal{F}} \; \min_{z' \in \mathcal{AF}} d(z,\, z')$$

Where $\mathcal{F}$ denotes the exact Pareto front, $d(z, z') = \max \left\{ \max\left(0, \frac{z_i' - z_i}{z_i}\right) : \; i = 1,2,3 \right\}$,

and z_i denotes the value of the k^{th} objective ($k \; = 1, 2, 3$ in our case).

The first performance measure $Dist1$ gives information on the average distance from the exact Pareto front $\mathcal{F}$ to the approximate front $\mathcal{AF}$. The lower this value is, the better set $\mathcal{AF}$ approximates $\mathcal{F}$. The second performance measure $Dist2$ gives information on the worst case which corresponds to the largest distance between the two sets.

Tables 2, 3 and 4 display the results obtained for the tolerance gap values 10%, 5% and 1%, respectively. In each table, column $t_{\mathcal{F}}$ recalls the total computing time in seconds for finding the exact Pareto front (see Table 1). Column $t_{\mathcal{F}}^{\epsilon}$ gives the average computing time in second required to solve to optimality the ϵ-constraint problems for each instance. Column $t_{\mathcal{AF}}$ gives the total computing time in seconds for the approximate Pareto front and $t_{\mathcal{AF}}^{\epsilon}$ reports the average computing time needed to solve ϵ-constraint problems with the gap stopping criterion. As explained above, columns $Dist1$ and $Dist2$ give respectively the average and the maximum distances between the exact Pareto front $\mathcal{F}$ and the approximate Pareto front $\mathcal{AF}$. Finally, column $I(\%) = \frac{|\mathcal{F} \cap \mathcal{AF}|}{|\mathcal{F}|}$ is another performance measure which displays the percentage of elements belonging to both sets $\mathcal{F}$ and $\mathcal{AF}$. The $Average$ line refers to the average results for the 10 instances of each instance set.

Table2. Approximate Pareto front with tolerance=10%

$(\|L\|, \|I\|)$	N° inst.	$t_{\mathcal{F}}$	$t^{\epsilon}_{\mathcal{F}}$	$t_{\mathcal{AF}}$	$t^{\epsilon}_{\mathcal{AF}}$	Dist1 (%)	Dist2 (%)	I (%)
(3,15)	1	1 114	0.66	985	0.59	0.25	3.40	67.36
	2	1 713	1.01	894	0.53	0.54	5.11	44.61
	3	1 323	0.72	1 065	0.58	0.33	4.70	57.24
	4	1 160	0.64	1 081	0.60	0.29	3.70	61.14
	5	1 254	0.73	1 007	0.58	0.27	3.74	59.70
	6	1 106	0.66	931	0.56	0.32	4.67	58.13
	7	1 824	0.94	1 069	0.55	0.42	3.43	52.77
	8	1 241	0.70	927	0.52	0.36	9.05	60.65
	9	1 134	0.70	843	0.52	0.38	4.76	52.56
	10	1 128	0.67	965	0.57	0.27	5.38	68.85
Average		**1 300**	**0.74**	**977**	**0.56**	**0.34**	**4.79**	**58.30**
(4,15)	1	4 300	0.85	3 069	0.60	0.52	5.66	53.54
	2	8 049	1.44	3 380	0.60	0.43	7.47	55.82
	3	5 175	1.00	3 094	0.59	0.34	5.96	61.26
	4	3 982	0.72	3 312	0.60	0.48	7.25	49.06
	5	4 282	0.81	3 138	0.59	0.53	7.53	59.89
	6	4 166	0.86	2 908	0.60	0.61	7.10	53.00
	7	6 278	1.26	3 176	0.63	0.58	8.49	52.10
	8	4 492	0.97	2 858	0.61	0.61	7.34	49.07
	9	4 743	1.03	2 789	0.60	0.61	6.06	47.88
	10	3 975	0.78	3 094	0.61	0.49	8.18	55.15
Average		**4 944**	**0.97**	**3 082**	**0.60**	**0.52**	**7.10**	**53.67**
(4,30)	1	14 621	2.06	4 218	0.59	0.72	3.86	22.94
	2	6 784	1.13	3 280	0.54	0.62	6.00	31.10
	3	5 325	0.85	3 471	0.55	0.98	7.20	32.63
	4	37 437	4.80	4 571	0.58	0.75	5.65	21.15
	5	24 016	3.25	4 651	0.62	0.89	4.00	20.53
	6	28 168	3.21	5 636	0.64	0.77	4.54	26.10
	7	11 512	1.72	3 575	0.53	0.92	7.57	27.44
	8	15 527	1.77	5 403	0.61	0.53	4.01	30.30
	9	11 868	1.87	3 510	0.55	0.72	4.74	29.30
	10	10 094	1.56	3 417	0.53	0.87	7.00	28.33
Average		**16 535**	**2.22**	**4 173**	**0.57**	**0.77**	**5.45**	**26.98**
(4,45)	1	29 041	2.85	10 416	1.02	0.80	4.11	23,56
	2	21 430	2.07	11 183	1.08	0.76	4.90	21,98
	3	38 445	3.56	18 345	1.70	0.95	4.32	25,30
	4	9 189	1.47	8 010	1.28	0.90	3.82	33,75
	5	78 542	8.23	20 316	2.13	0.75	3.67	35,90
	6	57 246	3.47	8 310	0.50	0.78	2.75	20,30
	7	17 135	1.42	6 331	0.52	0.85	3.41	21,61
	8	19 446	1.91	5 430	0.53	0.67	5.02	26,45
	9	19 761	2.09	8 605	0.91	0.90	2.54	25,11
	10	10 149	1.46	8 354	1.20	0.77	2.87	18,30
Average		**30 038**	**2.85**	**10 530**	**1.08**	**0.81**	**3.74**	**25.22**

Table3. Approximate Pareto fronts with tolerance=5%

$(\lvert L\rvert, \lvert I\rvert)$	N° inst.	$t_{\mathcal{F}}$	$t_{\mathcal{F}}^{\epsilon}$	$t_{\mathcal{AF}}$	$t_{\mathcal{AF}}^{\epsilon}$	Dist1 (%)	Dist2 (%)	I (%)
(3,15)	1	1 114	0.66	1 082	0.65	0.12	1.78	79.37
	2	1 713	1.01	1 065	0.63	0.16	1.89	68.58
	3	1 323	0.72	1 209	0.66	0.09	2.11	77.77
	4	1 160	0.64	1 104	0.61	0.09	1.63	79.40
	5	1 254	0.73	1 157	0.67	0.10	1.94	79.02
	6	1 106	0.66	1 017	0.61	0.10	2.18	77.30
	7	1 824	0.94	1 240	0.64	0.13	2.35	71.61
	8	1 241	0.70	981	0.55	0.11	2.58	77.77
	9	1 134	0.70	891	0.55	0.19	2.72	64.77
	10	1 128	0.67	1 076	0.64	0.08	2.72	83.22
Average		**1 300**	**0.74**	**1 082**	**0.62**	**0.11**	**2.19**	**75.88**
(4,15)	1	4 300	0.85	3 382	0.66	0.20	3.44	67.88
	2	8 049	1.44	4 197	0.75	0.17	4.17	76.29
	3	5 175	1.00	3 389	0.65	0.11	2.31	76.01
	4	3 982	0.72	3 668	0.67	0.18	4.04	70.95
	5	4 282	0.81	3 967	0.75	0.15	3.77	80.07
	6	4 166	0.86	3 281	0.67	0.20	3.15	70.49
	7	8 628	1.73	3 958	0.79	0.17	3.91	71.97
	8	4 322	0.85	3 400	0.73	0.25	3.44	65.19
	9	4 081	0.89	3 336	0.72	0.25	3.07	67.26
	10	3 975	0.78	3 506	0.69	0.19	4.75	74.20
Average		**5 096**	**0.99**	**3 608**	**0.70**	**0.18**	**3.60**	**72.03**
(4,30)	1	14 621	2.06	5 583	0.79	0.24	1.61	41.00
	2	6 784	1.13	3 823	0.63	0.36	2.22	42.17
	3	5 325	0.85	4 020	0.64	0.26	2.51	55.37
	4	37 437	4.80	5 595	0.71	0.30	2.25	38.95
	5	24 016	3.25	6 886	0.93	0.26	1.83	38.93
	6	28 168	3.21	6 477	0.73	0.37	2.70	40.21
	7	11 512	1.72	4 160	0.62	0.42	3.60	38.49
	8	15 527	1.77	6 133	0.69	0.24	1.64	47.48
	9	11 868	1.87	4 150	0.65	0.35	2.09	42.01
	10	10 094	1.56	3 981	0.61	0.38	2.94	39.41
Average		**16 535**	**2.22**	**5 081**	**0.70**	**0.31**	**2.33**	**42.40**
(4,45)	1	29 041	2.85	15 230	1.49	0.20	2.40	33,21
	2	21 430	2.07	14 670	1.42	0.66	1.89	45.00
	3	38 445	3.56	24 896	2.45	0.45	3.05	18,03
	4	9 189	1.47	8 856	1.42	0.90	2.66	25,54
	5	78 542	8.23	47 798	5.01	0.75	2.12	15,30
	6	57 246	3.47	37 625	2.28	0.58	3.45	30,45
	7	17 135	1.42	12 956	1.07	0.55	3.00	22,40
	8	19 446	1.91	9 411	0.92	0.67	2.75	35,20
	9	19 761	2.09	10 097	1.07	0.90	3.50	36,02
	10	10 149	1.46	8 652	1.25	0.47	1.75	33,21
Average		**30 038**	**2.85**	**19 019**	**1.83**	**0.61**	**2.65**	**27.70**

<u>Table4</u>. Approximate Pareto fronts with tolerance=1%

$(\lvert L\rvert, \lvert I\rvert)$	N° inst.	$t_{\mathcal{F}}$	$t_{\mathcal{F}}^{\epsilon}$	$t_{\mathcal{AF}}$	$t_{\mathcal{AF}}^{\epsilon}$	Dist1 (%)	Dist2 (%)	I (%)
(3,15)	1	1 114	0.66	1 100	0.66	4.5 E^{-03}	0.58	98.30
	2	1 713	1.01	1 535	0.91	4.3 E^{-03}	0.46	97.87
	3	1 323	0.72	1 280	0.70	4.4 E^{-03}	0.75	98.18
	4	1 160	0.64	1 152	0.63	3.6 E^{-03}	0.71	98.54
	5	1 254	0.73	1 206	0.70	1.9 E^{-03}	0.40	98.73
	6	1 106	0.66	1 098	0.66	4.4 E^{-03}	0.77	98.34
	7	1 824	0.94	1 756	0.90	1.8 E^{-03}	0.58	99.00
	8	1 241	0.70	1 235	0.69	7.5 E^{-03}	0.89	97.75
	9	1 134	0.70	1 095	0.67	2.3 E^{-03}	0.42	98.14
	10	1 128	0.67	1 115	0.66	2.6 E^{-03}	0.49	98.90
Average		**1 300**	**0.74**	**1 257**	**0.71**	**3.7 E^{-03}**	**0.60**	**98.37**
(4,15)	1	4 300	0.85	4 221	0.83	9.6 E^{-03}	0.79	96.91
	2	8 049	1.44	7 299	1.31	5.6 E^{-03}	0.70	98.05
	3	5 175	1.00	4 702	0.9	4.4 E^{-03}	0.69	98.13
	4	3 982	0.72	3 975	0.72	2.8 E^{-03}	0.63	98.61
	5	4 282	0.81	4 141	0.79	5.3 E^{-03}	0.91	98.70
	6	4 166	0.86	4 104	0.84	5.2 E^{-03}	0.88	98.45
	7	8 628	1.73	8 236	1.65	0.05	0.70	79.48
	8	4 322	0.85	3 974	0.85	0.06	0.78	78.67
	9	4 081	0.89	3 991	0.87	0.07	1.19	78.99
	10	3 975	0.78	3 907	0.87	7.7 E^{-03}	0.71	97.11
Average		**5 096**	**0.99**	**4 855**	**0.96**	**0.02**	**0.79**	**92.31**
(4,30)	1	14 621	2.06	10 382	1.46	0.01	0.83	89.07
	2	6 784	1.13	5 919	0.98	0.02	0.74	89.13
	3	5 325	0.85	5 309	0.85	5.3 E^{-03}	0.46	97.13
	4	37 437	4.80	20 459	2.62	0.02	0.70	87.77
	5	24 016	3.25	14 452	1.95	0.02	0.65	87.46
	6	28 168	3.21	14 417	1.64	0.03	0.75	84.83
	7	11 512	1.72	8 384	1.25	0.02	0.62	85.99
	8	15 527	1.77	8 536	0.97	0.02	0.54	91.07
	9	11 868	1.87	7 985	1.25	0.01	0.52	89.33
	10	10 094	1.56	7 494	1.16	0.02	0.61	87.02
Average		**16 535**	**2.22**	**10 334**	**1.41**	**0.01**	**0.64**	**88.88**
(4,45)	1	29 041	2.85	22 007	2.18	0.19	0.79	75.25
	2	21 430	2.07	18 762	1.81	0.07	0.85	62.00
	3	38 445	3.56	26 240	2.58	0.21	0.76	42.58
	4	9 189	1.47	9 012	1.46	0.09	0.77	75.12
	5	78 542	8.23	62 359	6.54	0.05	0.85	70.50
	6	57 246	3.47	50 722	3.07	0.11	0.84	81.20
	7	17 135	1.42	15 120	1.25	0.09	0.72	43.05
	8	19 446	1.91	14 089	1.38	0.20	0.85	55.45
	9	19 761	2.09	11 210	1.19	0.19	0.75	79.00
	10	10 149	1.46	9 820	1.42	0.10	0.90	66.75
		30 038	**2.85**	**23 934**	**2.28**	**0.13**	**0.80**	**65.09**

One can see from Tables 2, 3 and 4 that computing times required for the approximate approach considerably decrease when the tolerance on the gap increases. On the counter part, the greater the value of the tolerance is the worse is the quality of the Pareto front. For example, when considering a tolerance of 10%, the approximate approach is 2.85 times faster than the exact approach for instance set (4, 45). In this case, the approximate algorithm locates only 25.2% of non-dominated points on average, the average distance between the approximate front and the exact front is of 0.81, which is relatively small. If we consider a small tolerance on the gap equal to 1%, the approximate algorithm found 98.4%, 92.3%, 88.9% and 65% of non-dominated points, on average, for the four sets of instances, respectively. However, the total computing time remains large on average reaching almost 6.6 hours (23 934 seconds) for the larger set. Finally, when considering an intermediate value for the tolerance, that is 5%, results are relatively balanced with regard to the quality of the approximate front and computing times. In this case, the approximate algorithm is 1.2 to 3.25 times faster than the exact algorithm and locates between 27.7% and 75.9% of the non-dominated points. The average distance between the approximate front and the exact front is however very small varying between 0.11% on average for the smallest instance set (3,15) and 0.61% on average for the largest instance set (4,45). The maximum distance remains also relatively small. It varies between 2.19 % and 2.65% on average for all the instance sets and does not exceed 4.75% for all the instances.

7. Conclusion

We proposed an epsilon-constraint method and prove that it generates the exact Pareto front of a complex three-objective location-transportation problem. The proposed algorithm can be applied to any three-objective optimization problem provided that the problem involves at least two integer and conflicting objectives. The results obtained in our experimental study show that computing time may be relatively large. A slight modification of our algorithm (as already proposed by other authors) yielded lower computing times without drastically deteriorating the quality of the solutions found. Although we are aware that the proposed exact algorithm may require large computing times for large instances, we believe that it consists in a good tool to evaluate the quality of other approximate algorithms for three-objective optimization problems.

Acknowledgements

This work was financed by individual grants [OPG 0172633 and OPG 0371655], by Collaborative Research and Development Grants [CG 095123] from the *Canadian Natural Sciences and Engineering Research Council* (NSERC), by DMR, a division of Fujitsu Consulting (Canada), and by *Partnerships for Research on Microelectronics, Photonics and Telecommunications* (PROMPT). This financial support is gratefully acknowledged. We would also like to thank the emergency managers of our industrial partner, Fujitsu Consulting (Canada), for providing us with relevant data and useful comments. The authors also gratefully acknowledge the insightful comments of an anonymous reviewer.

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