Practical Considerations in Establishing the Statistical Reliability of Geomechanical Data

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ABSTRACT

In an underground mining operation, the design of safe excavations can be influenced by the quality and quantity of collected geomechanical data. Data collection is the first step in mine design, and a sufficient level of confidence in the input data should be reached depending on the project stage and the design requirements (e.g. temporary and non-entry vs. permanent and entry excavations). This paper compares two statistical analysis methods for quantifying the level of confidence in the intact rock properties obtained through a series of laboratory tests. The laboratory testing database of an underground hard rock mine was used to highlight the variations in the two methods. The impact of the two methods, from an engineering perspective, was illustrated with an example using the Kirsch analytical solution. This investigation demonstrated that the selection of the appropriate analysis method should be guided by the project requirements.

KEYWORDS

Confidence interval, Data confidence, Laboratory rock testing, Geomechanical data reliability, Geomechanical data collection, Small-sampling theory

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1 INTRODUCTION

Although geomechanical data collection is recognised as the first step in geomechanical design, the reality is that there are considerable variations in how these data are analysed and used. A fundamental goal of gap analysis of geomechanical data is to establish if adequate data are collected and the confidence in these data. This would dictate whether it is necessary to undertake additional data collection campaigns. In this context there are two approaches, both based on small-sampling theory that can be employed. The first approach requires a decision on what precision index is required a priori. This is directly related to the specifics of a project. It is then possible to identify the level of confidence that meets this objective and to determine, if necessary, the additional number of data required to reach higher levels of confidence. The second approach selects a defined confidence level in the interval estimate (i.e. the lower and upper confidence limits on the true mean) for a given rock property. This second approach looks at the precision index to determine if this is acceptable for the project under consideration. This is a precursor to further data collection.

Even if these approaches appear very similar, there are significant differences on how the two problems are tackled from a statistical and engineering perspective. This paper compares both approaches using historical data from the Raglan underground hard rock mine. This particular mine site undertook several data collection campaigns and comprehensive laboratory testing.

2 MINE SITE AND DOMAINS

Raglan Mine, a Glencore Company, is located in Quebec, Canada, north of the 55th parallel (Nunavik). The property is about 70 km long and consists of a series of high-grade sulphide deposits (nickel, copper and platinum-group elements). Ore zones are generally found in intrusive or extrusive mafic–ultramafic rocks. The footwall is composed of gabbro rocks, while ultramafic rocks, volcanic rocks and metasediments characterize the hanging wall. Argillaceous sediments can be present in the vicinity of the mineralized zones. A typical section view of the geology is presented in Fig. 1. Table 1 presents a description of the Raglan Formation identifying the rock types and rock units (domains) investigated in this paper.

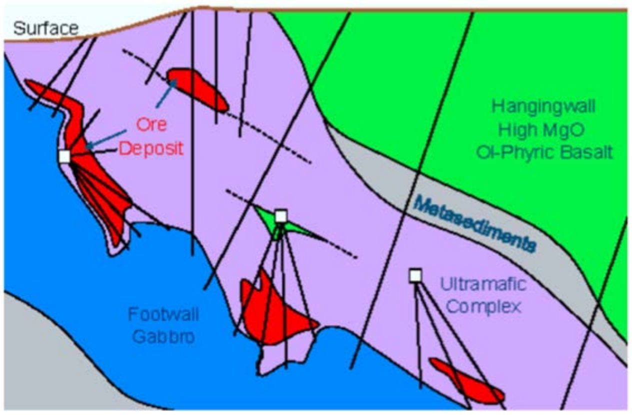

Figure 1. Typical geological cross-section of existing Raglan Mine workings. (Glencore 2012)

Table 1. Rock types and rock units in the Raglan formation

Rock type	Rock unit	ID
Ultramafic (undivided) (10)	Peridotite	10b
	Olivine Pyroxenite	10c
	Pyroxenite	10d
Metasediments (4)	Argillite	4a
	Hornsfeld	4f
	Siltstone	4g
Volcanics (undivided) (6)	Massive flows	6a
	Pillowed flows	6b
	Agglomerate Brecchia	6e
	Komatiitic Basalt	6h
Gabbro (undivided) (9)	Normal Gabbro	9a
	Leucogabbro	9b
	Mafic Gabbro	9c
Ore	Ore	Ore

The various mining projects at Raglan are shown in Fig. 2. Raglan Mine (Phase I) began in 1997 and current operations consist of Mine Katinniq, Mine 2, Mine Kikialik and Mine Qakimajurq. Phase I operations are expected to gradually cease after 2020. In order to extend the life of Raglan Mine, the Sivumut Project was launched (Phase II). The Phase II of Raglan consists of two new underground mining project: Mining Project 8H (PM8H in Fig. 2) and Mining Project 14 (PM14 in Fig. 2).

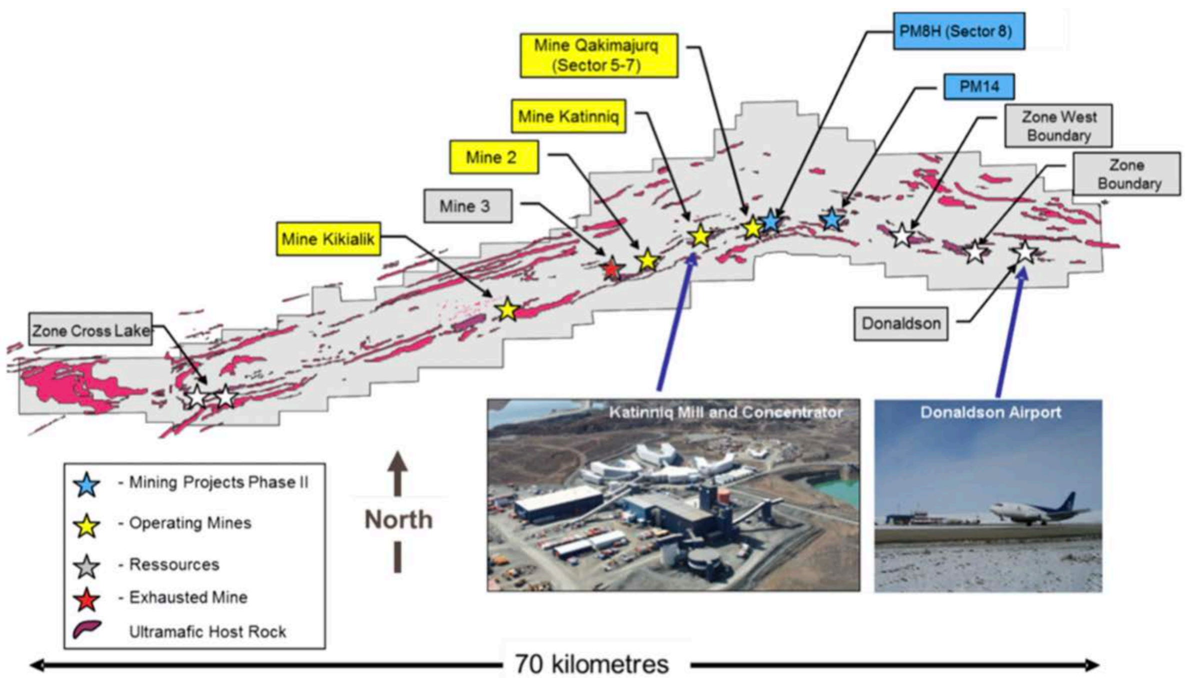

Figure 2. Raglan mine operations (Phase I) and developing projects. (Glencore 2018)

3 LABORATORY TESTING

The compilation of the laboratory testing database includes data from Qakimajurq Mine, PM8H, PM14 and Donaldson (potential mining project). The most recent laboratory test results database (i.e. the early 2018 database) includes uniaxial compressive strength UCS (σ_c), tensile strength (σ_t), density (ρ), Young's modulus (E) and Poisson's ratio (v) properties. The rock sample density was evaluated for UCS, tensile strength and triaxial laboratory tests and the elastic properties (Young's modulus and Poisson's ratio) were assessed from instrumented uniaxial compressive strength (UCS) laboratory tests. All testing was conducted at external laboratories that complied with the ISRM suggested methods (ISRM 2007). The compiled number of laboratory tests, per mining project, is presented in Table 2. Due to the location and distinctions between the rock unit properties, the mining projects are characterized separately.

Table 2. Number of laboratory tests for the Qakimajurq, PM14, PM8H and Donaldson mining projects

Rock properties	Mining project			
	Qakimajurq	PM14	PM8H	Donaldson
Uniaxial compressive strength (σ_c)	115	131	67	154
Tensile strength (σ_t)	89	109	50	80
Density (ρ)	235	295	163	165
Young's modulus (E)	115	77	40	80
Poisson's ratio (v)	115	75	40	78
Total	669	687	360	557

The best fit between the normal or lognormal distribution, was assigned to the rock properties based on the distribution fit on the histograms, the probability plots and the goodness of fit testing (Mathwave 2017). It was possible to evaluate the compatibility of the samples with the theoretical normal or lognormal probability distribution functions. When both distributions were compatible with the geomechanical data samples, the best fit between the normal or lognormal probability distribution functions was assigned. The goodness of fit tests agreed well with the majority of the analysed rock

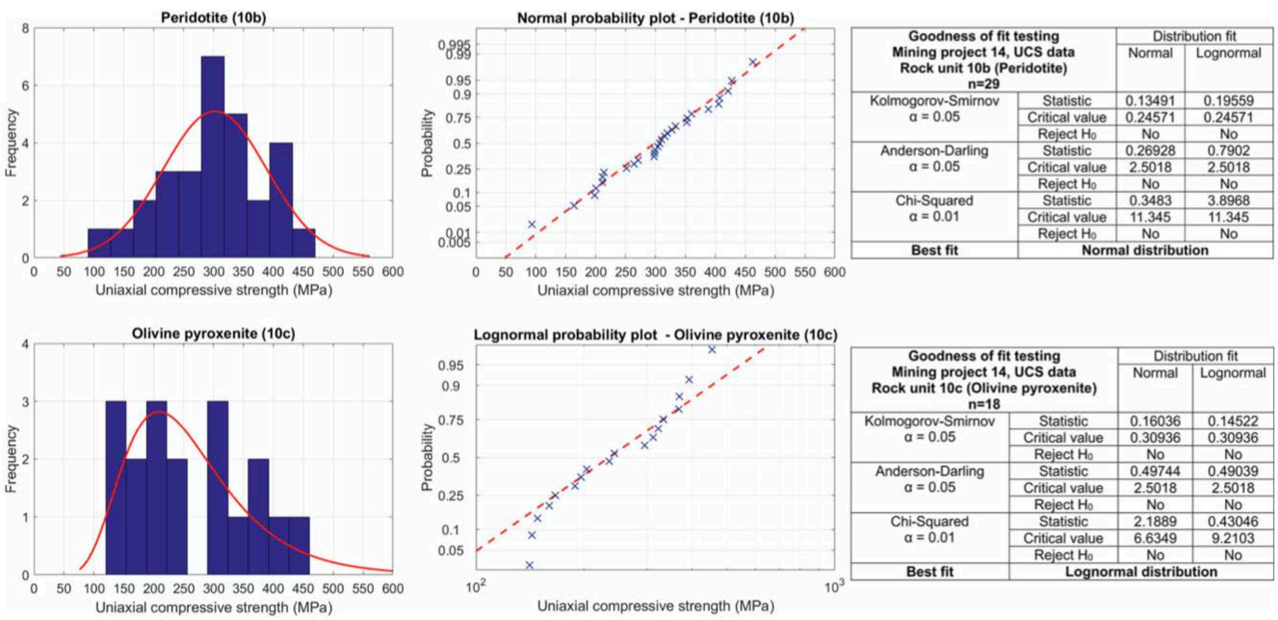

Figure 3. Example of UCS data distribution fit for rock units 10b (normal distribution) and 10c (lognormal distribution)

properties. An example of normal and lognormal data distributions, and the associated goodness of fit test results, is given in Fig. 3 for UCS data from the peridotite (10b) and olivine pyroxenite (10c) rock units collected for mining project PM14. The data distribution was determined qualitatively based on the similarity of the distribution fit on the histograms (e.g. symmetrical for the normal distribution and rightly skewed for the lognormal distribution) and the linearity of data on the normal or lognormal probability plots. A quantitative assessment of the best distribution fit was obtained with the acceptance of the null hypothesis (H_0 = the data follow the specified distribution) from goodness of fit. For the goodness of fit testing, the distribution parameters were estimated with the method of moments (Mathwave 2017).

4 QUANTIFYING THE CONFIDENCE LEVEL IN THE GEOMECHANICAL PROPERTIES OF ROC

The confidence level in the geomechanical rock properties was evaluated using two statistical methods based on small-sampling theory employing the confidence interval approach. This is an interval estimate, defined by upper and lower limits, of a population parameter (e.g. the true mean) that potentially includes the true population parameter with a stated confidence level. The confidence level is the probability that the value of a population parameter falls within the specified limits of the confidence interval. For example, a 95% confidence level implies that 95% of the confidence intervals would include the true population parameter. The small-sampling theory is a widely used statistical method to determine the confidence interval for the true mean when the number of specimens is smaller than 40 (Montgomery and Runger 2003) and has several applications in rock engineering (Fillion and Hadjigeorgiou 2017; Gill et al. 2005; Grenon et al. 2015; Ruffolo and Shakoor 2009). In practice, due to cost constraints, it is sometimes difficult to justify the requirement for additional data collection. Consequently, the geomechanical properties of rock are routinely estimated based on less than 40 laboratory tests on a given rock type or rock unit. A prerequisite of the use of small-sampling theory is that the data are normally distributed. This is justified by histograms and probability plots that are representative of a normal distribution, and by the results of the goodness of fit testing that corroborate the hypothesis that the data follow the normal distribution at a selected significance level (α). The 100(1-α) % confidence interval on the true mean of a normal distribution is obtained using Eq. 1 (Hines et al. 2003):

$$\bar{X} - t_{\frac{\alpha}{2},n-1}\frac{s}{\sqrt{n}} \leq \mu \leq \bar{X} + t_{\frac{\alpha}{2},n-1}\frac{s}{\sqrt{n}} \tag{1}$$

where $\bar{X}$ = the arithmetic sample mean; s = the sample standard deviation, μ = the population arithmetic mean, $t_{\frac{\alpha}{2},n-1}$= the confidence coefficient obtained from the Student t distribution for a two-side confidence on μ with n - 1 degrees of freedom, α = a confidence parameter, n = the number of elements in the sample.

In some cases, a lognormal distribution may be more representative of the laboratory test results for geomechanical rock properties. There have been several studies in the literature on constructing the confidence interval on a lognormal mean (Zhou and Gao 1997). The methods are evaluated based on the coverage error, the interval width and the relative bias. Olsson (2005) presented a modified version of the Cox method to improve the coverage for small-sample sizes. To use the modified version of the Cox method, the lognormally distributed variable *X* must first be transformed to its natural logarithm ($Y = ln(X)$) to obtain a normal distribution of data.

The 100(1 - α) % confidence interval for the lognormal mean obtained from the modified version of the Cox method is presented in Eq. 2.

$$\bar{Y} + \frac{S_Y^2}{2} - t_{\frac{\alpha}{2},n-1}\sqrt{\frac{S_Y^2}{n} + \frac{S_Y^4}{2(n-1)}} \leq \ln(\mu_X) \leq \bar{Y} + \frac{S_Y^2}{2} + t_{\frac{\alpha}{2},n-1}\sqrt{\frac{S_Y^2}{n} + \frac{S_Y^4}{2(n-1)}} \tag{2}$$

where $\bar{Y}$ = the arithmetic sample mean of the normally distributed variable $Y = ln(X)$ and X is the lognormally distributed variable, S_Y^2 = the variance of variable $Y = ln(X)$ and S_Y is the standard deviation of variable Y, $t_{\frac{\alpha}{2},n-1}$ = the confidence coefficient obtained from the Student t distribution for a two-sided confidence on μ_X and with n-1 degrees of freedom, α = a confidence parameter, n = the number of elements in the sample, μ_X = the mean of the lognormally distributed variable *X*.

The confidence limits (i.e. the lower and upper bounds of the confidence interval) of the lognormally distributed variable *X* are obtained by calculating the inverse of the natural logarithm (i.e. base e) of the confidence interval limits obtained from Eq. 2.

The two statistical methods used in this paper to evaluate the confidence level in the mechanical rock properties were developed from the theoretical concepts presented in Eq. 1 for normally distributed data and in Eq. 2 for lognormally distributed data.

The first method, described in detail by Fillion and Hadjigeorgiou (2017), is based on the confidence interval (*CI*) calculated for a desired precision index on the estimation. The precision index is the ratio of the upper and lower limits of the confidence interval and controls the length of the confidence interval, i.e. a lower precision index implies a wider confidence interval. It is desirable to obtain a confidence interval that is small enough for decision-making purposes and that also provides an adequate or desirable confidence level. One way to achieve this is by choosing the sample size (n) to be large enough to give a CI of specified length (precision) for a prescribed confidence level (Montgomery and Runger 2003). Figure 4 illustrates the confidence interval method. It shows that, by fixing the precision index to p = 1.35 (i.e. the relative error on the true mean E_r = 15%), the length of the confidence interval is the same regardless of the confidence level. For a fixed p, additional data must be collected to increase the confidence level (e.g. from CI=60%toCI=80%).

The second method, presented by Grenon et al. (2015), links target levels of data confidence ($TLDC$) to the relative error obtained for a fixed confidence level of 95%. The maximal relative error (E_r) is defined by the half length of the confidence interval divided by the sample mean. This was originally applicable to normally distributed data. It was modified for this work to lognormally distributed data. Figure 5 illustrates the $TLDC$ method. This figure shows that for a fixed CI = 95%, the length of the confidence interval is reduced as the relative error (E_r) on the true mean is reduced. Additional data must be collected to reduce E_r (e.g. from 40% to 10%) in order to increase the $TLDC$ (e.g. from 60% to 90%). The two methods are detailed in Sects. 4.1 and 4.2.

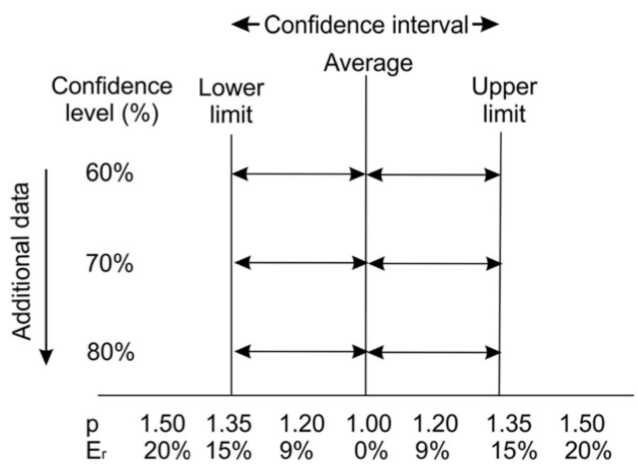

Figure 4. Example of the confidence intervals obtained by fixing the precision index to p = 1.35

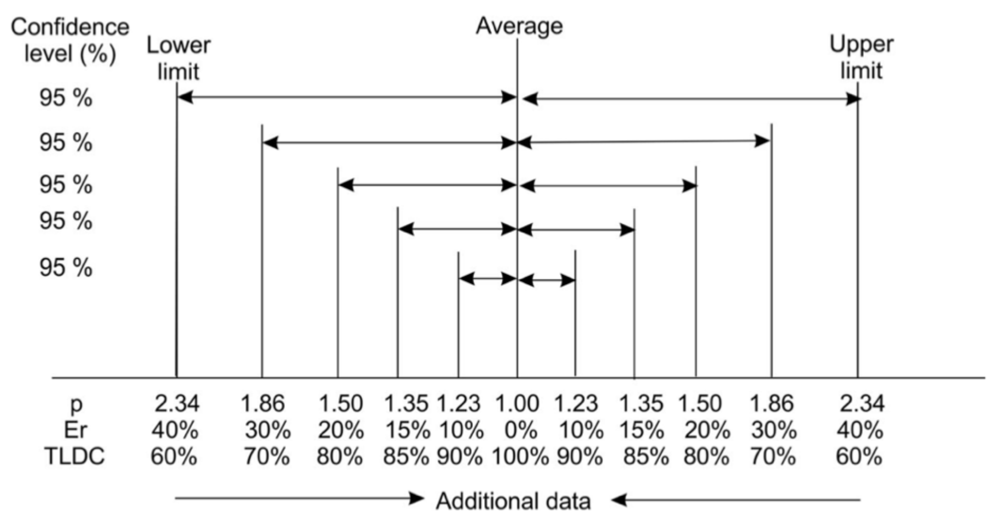

Figure 5. Example of the confidence intervals obtained by fixing CI = 95%

4.1 Confidence Interval (CI)

Gill et al. (2005) were the first to suggest a method to determine the confidence interval characterizing the mean value of normally distributed rock properties for a pre-determined precision index. Selecting a precision index implies choosing the maximum relative error on the determination of the true mean and controls the length of the confidence interval. Fillion and Hadjigeorgiou (2017) modified and extended the approach by Gill et al. (2005) to be used with lognormal data distributions. The confidence level on the interval for the true mean (CI) is obtained from the confidence coefficient, $t_{\frac{\alpha}{2},n-1}$, where α is obtained from the Student's *t* probability density function. The confidence level is calculated using Eq. 3.

$$CI(\%) = 100(1-\alpha) \tag{3}$$

For normally distributed data, the confidence coefficient is obtained from Eq. 4 that incorporates the desired precision index (*p*), i.e. the ratio of the upper and lower limits of the confidence interval. The precision index (*p*) is linked to the relative error on the true mean, i.e. $E_R = \frac{p-1}{p+1}$, and a higher p implies a higher relative error.

$$t_{\frac{\alpha}{2},n-1\_normal} = \left(\frac{p-1}{p+1}\right)\frac{\bar{X}}{s}\sqrt{n} \tag{4}$$

For lognormally distributed data, the confidence coefficient is obtained from Eq. 5, which includes the desired precision index (p_Y) on the confidence interval on $ln\ (\mu_X)$.

$$t_{\frac{\alpha}{2},n-1\_lognormal} = \sqrt{\frac{2n(n-1)}{(nS_Y^2+2n-2)}}\cdot\frac{(p_Y-1)}{(p_Y+1)} \tag{5}$$

Because of the multiplicative effect of the error, the initially selected p_Y, i.e. the precision index on the transformed variable $Y = ln(X)$, must be smaller than the targeted precision index (p) on the back transformed confidence interval (i.e. the interval for the lognormally distributed variable X). p_Y is selected by trial and error to ensure the targeted p for the back transformed confidence interval is reached.

4.2 Target Levels of Data Confidence $TLDC$

In previous work, Grenon et al. (2015) proposed linking target levels of data confidence ($TLDC$ (%)) to the relative maximal error (E_r) on the mean of the investigated geomechanical property, as shown in Eq. 6. This implies that the $TLDC$ is further increased as E_r is reduced. In this previous work, a fixed confidence level of 95% in the interval estimate is suggested. The justification to fix the confidence level at 95% is that a high confidence level in the interval estimate for the true mean should be targeted for all project stages, i.e. from the early conceptual stage to the more mature operational stage. Accordingly, a large confidence interval can be accepted at the early stages of a mining project and this interval should reduce as the project advances towards construction and operation (Grenon et al. 2015).

$$TLDC(\%) = 100(1-E_r) \tag{6}$$

The maximal relative error (E_r) is obtained by dividing the half length of the confidence interval of a 100(1 - α) % confidence level by the sample mean.

For normally distributed data, the confidence interval is symmetric and E_r can be reduced to Eq. 7 (Grenon et al. 2015).

$$E_{r\_normal} = \frac{t_{\frac{\alpha}{2},n-1}\frac{s}{\sqrt{n}}}{\bar{X}} \tag{7}$$

The confidence coefficient, $t_{\frac{\alpha}{2},n-1}$, is obtained for a 95% (α = 0.05) confidence interval on the true mean (μ).

In the present investigation, the approach by Grenon et al. (2015) was modified to accommodate data distributions using Eq. 2. For lognormally distributed data, the confidence interval is asymmetric and E_r is calculated using Eq. 8. This represents the half length of the confidence interval on μ_X, obtained with the inverse of the natural logarithm of the confidence limits and divided by the sample mean of the lognormally distributed variable $E[X] = e^{\bar{Y}+\frac{S_Y^2}{2}}$.

$$E_{r\_lognormal} = \frac{e^{\left(\bar{Y}+\frac{S_Y^2}{2}+t_{\frac{\alpha}{2},n-1}\sqrt{\frac{S_Y^2}{n}+\frac{S_Y^4}{2(n-1)}}\right)} - e^{\left(\bar{Y}+\frac{S_Y^2}{2}-t_{\frac{\alpha}{2},n-1}\sqrt{\frac{S_Y^2}{n}+\frac{S_Y^4}{2(n-1)}}\right)}}{2E[X]} \tag{8}$$

The confidence coefficient, $t_{\frac{\alpha}{2},n-1}$, is obtained for a 95% (α = 0.05) confidence interval on the true mean (μ_X).

4.3 Additional Number of Tests Required to Reach Higher Confidence Levels

It has been demonstrated that in certain cases the minimum number of specimens proposed in rock engineering best practice, e.g. the ISRM suggested methods (ISRM 2007), can be inadequate (Gill et al. 2005; Ruffolo and Shakoor 2009; Fillion and Hadjigeorgiou 2013). The ISRM suggested methods state that the minimum number of specimens to test should be at least five for UCS, Young's modulus and Poisson's ratio, three for density and ten for tensile strength. In fact it is not possible to determine a priori the number of required tests for a given rock type, and for a targeted precision index and confidence interval. When a number of laboratory test results become available, and the confidence level in the existing data is considered too low for the purposes of the work desired, it is possible to evaluate the number of additional laboratory tests required to reach a higher level of confidence.

According to small-sampling theory, the minimum number of specimens in a group (minimum sample size) can be expressed in terms of the precision index p, $t_{\frac{\alpha}{2},n-1}$Student's statistic and the coefficient of variation $cv = \frac{s}{\bar{X}}$. The minimum sample size n for normally distributed data is defined by Eq. 9.

$$n\_normal = \left[\left(\frac{p+1}{p-1}\right)t_{\frac{\alpha}{2},n-1}cv\right]^2 \tag{9}$$

It is not straightforward to solve Eq. 9 because the confidence coefficient, $t_{\frac{\alpha}{2},n-1}$, is a function of the sample size (n). Consequently, the required n must be determined through trial and error or by using a solver for nonlinear equations. Because the coefficient of variation can be different with additional data, i.e. cv is function of n, and n is a function of the confidence coefficient, iterations must be performed to solve Eq. 9.

For lognormally distributed data, the equation for the minimum number of specimens to reach a predetermined confidence level is derived from Cox's modified equation (Eq. 2) (Olsson 2005) to determine the confidence interval on $ln(\mu_X)$ and from the precision index (p_Y). The detailed steps are presented by Fillion and Hadjigeorgiou (2017). The minimum sample size (n) for lognormally distributed data is defined by Eq. 10.

$$n\_lognormal = \left[\frac{S_Y^2 n}{2(n-1)}+1\right] \cdot \left[\left(\frac{p_Y+1}{p_Y-1}\right)\cdot cv_Y \cdot \frac{t_{\frac{\alpha}{2},n-1}}{1+\frac{S_Y^2}{2\bar{Y}}}\right]^2 \tag{10}$$

In practice, it is not straightforward to solve Eq. 10 for the minimum number of specimens because the standard deviation (S_Y), the average ($\bar{Y}$) and the coefficient of variation (cv_Y) of variable $Y = ln(X)$, as well as the confidence coefficient, $t_{\frac{\alpha}{2},n-1}$, are also functions of the number of specimens (n). Consequently the required n must be determined through trial and error or using a solver for nonlinear equations.

Regardless of the data distribution (i.e. normal or lognormal), additional laboratory testing is required for each performed iteration to determine the minimum number of specimens to test until the targeted precision index and confidence level are reached.

5 CONFIDENCE LEVEL IN GEOMECHANICAL DATA FOR UNDERGROUND MINING PROJECTS

The geomechanical design and development of an underground mine is a complex process. This requires a sufficient degree of confidence in the geomechanical data used for the design of the underground excavations. The level of knowledge in the rock properties should arguably improve during the design process from the early conceptual stage to the late operation stage. Read and Stacey (2009) suggested guidelines for the targeted levels of confidence in the different components of a geotechnical model, as a function of the different stages of a mining project. Those guidelines were developed as part of the large open pit mines project (LOP) (Read and Stacey 2009). For underground openings at the operation stage, Grenon et al. (2015) made the distinction between temporary and permanent underground excavations. A higher level of confidence is suggested for permanent excavations that will be accessible to mine personnel for a long period of time, as opposed to temporary excavations. Cepuritis and Villaescusa (2012) presented a reliability-based approach to open stope span design in underground mining. Different levels of data reliability are suggested for different project stages with a distinction, at the operations stage, between early to mid-life and mature operations. Table 3 shows the suggested degrees of confidence in the rock properties as a function of the project stages. As shown in Table 3, the degrees of confidence in the mechanical rock properties for underground mining projects are similar and a confidence level of 85% is suggested for mature or permanent excavations at the operations stage (Grenon et al. 2015; Cepuritis and Villaescusa 2012).

The existing guidelines have some limitations. As outlined, the reporting systems that define levels of confidence or reliability in the data is qualitative (or subjective). Furthermore, there is no universally accepted process to quantify the required number of laboratory tests to determine the rock properties with a targeted confidence.

This paper provides a quantitative assessment of the confidence in geomechanical data collected for an underground mining project, using the two statistical methods presented in Sect. 4. The results for mining project 14 (PM14) are presented in this paper. This mining project was selected for the higher number of laboratory tests available in a variety of domains.

A fixed precision index (p) of 1.35 or less was targeted with the use of the Fillion and Hadjigeorgiou (2017) method. In reality, there are no established guidelines for determining an acceptable precision index. For example, Gill et al. (2005) suggested that for permanent mine openings, p should be $\leq$ 1.35 while keeping the confidence interval at 95%. This condition ensures a maximum relative error on the true mean of approximately 15%. However, an equally valid case could have been made that a different precision index could have been employed to determine CI. A fixed confidence interval of 95% was targeted with the use of the Grenon et al. (2015) method. A 95% confidence interval implies that the true mean will lie in the calculated confidence interval 19 times out of 20. It cannot be certain that the interval contains the true unknown population mean, but the interval is constructed so there is a higher degree of confidence that it does. The results for UCS, tensile strength, density, Young's modulus and Poisson's ratio data are respectively presented in Sects. 5.1–5.5.

Table 3. Suggested degrees of confidence in the rock properties for the different stages of a mining project

Project level status	Conceptual (%)	Pre-Feasibility (%)	Feasibility (%)	Design and construction (%)	Operations early to mid-life or temporary (%)	Operations mature or permanent (%)
Read and Stacey (2009)	> 30	40–65	60–75	70–80	> 80	
Grenon et al. (2015)	> 30	40–65	60–75	70–80	80–85	> 85
Cepuritis and Villaescusa (2012)	< 50	50–60	60–70	70	80	85

5.1 Uniaxial Compressive Strength

Uniaxial compressive strength data were available for a total of 11 domains for PM14 at Raglan mine. The confidence level in the uniaxial compressive strength data was calculated for 9 geotechnical domains present in Mining Project PM14. It was not possible to calculate the confidence for 2 domains because the data distribution could not be assigned due to the limited amount of UCS data. Table 4 presents the lower and upper limits of the confidence intervals, the corresponding confidence level, and the minimum number of specimens that should be tested ($Nmin$) to reach 85% confidence, for mature operations or permanent excavations at the operations stage, based on the recommendations in Table 3. For comparison purposes, the length of the confidence intervals constructed with the two methods and the associated levels of confidence are illustrated in Fig. 6.

Table 4. Confidence intervals and minimum number of specimens required to reach an 85% confidence level, obtained with the CI and $TLDC$ methods for PM14 UCS data

Rock unit	n	Data distribution	Sample average (MPa)	cv	Fillion and Hadjigeorgiou (2017)				Grenon et al. (2015)				
					Confidence interval for p = 1.35 (E_r = 15%)			*Nmin* for *CI* 85%	Confidence interval for *CI* = 95%		E_r (%)	*TLDC* (%)	*Nmin* for *TLDC* 85%
					Lower (MPa)	Upper (MPa)	*CI* (%)		Lower (MPa)	Upper (MPa)			
10b	29	Normal	302.4	0.29	257.3	347.4	99.1	–	269.6	335.2	10.9	89.1	–
10c	18	Lognormal	259.4	0.40	224.6	302.6	86.7	–	213.7	318.1	20.0	80.0	19
4a	6	Normal	211.6	0.16	180.1	243.1	93.3	–	176.9	246.3	16.4	83.6	7
4f	2	N/A											
6a	12	Normal	239.4	0.17	203.7	275.0	98.9	–	213.8	264.9	10.7	89.3	–
6b	5	Normal	168.8	0.22	143.6	193.9	79.3	6	122.3	215.2	27.5	72.5	8
6e	5	Normal	132.2	0.15	112.5	151.9	91.2	–	107.9	156.5	18.4	81.6	6
6 h	4	N/A											
9a	28	Normal	210.6	0.33	179.2	241.9	97.7	–	183.9	237.3	12.7	87.3	–
9b	9	Normal	171.3	0.49	145.8	196.9	61.1	15	106.8	235.9	37.7	62.3	26
Ore	13	Normal	258.2	0.37	219.8	296.7	82.3	14	199.9	316.6	22.6	77.4	17

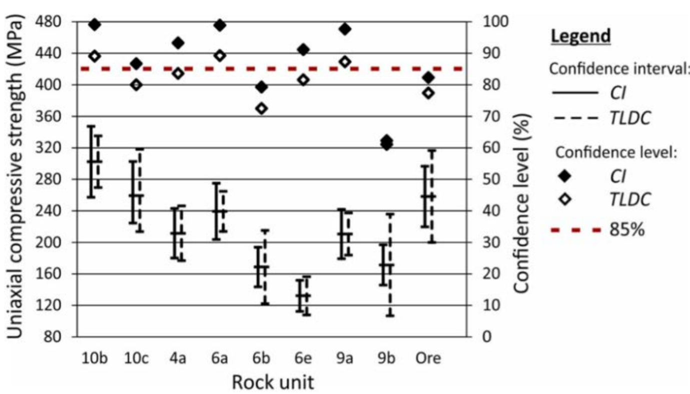

Figure 6. Confidence intervals (whisker plots) and associated levels of confidence obtained with the CI and $TLDC$ methods (black and white diamond symbols on top) for PM14 UCS data

The confidence intervals are represented by the whisker plots in Fig. 6, with straight lines extending from the minimum to the maximum limits of the confidence intervals. The middle line is the average value, which is the same for a specific rock unit using both methods. The values for the average and confidence limits correspond to the left vertical axis. The confidence levels obtained using both methods are illustrated with the black (CI method) and white ($TLDC$ method) diamond symbols at the top of Fig. 6. The values for the level of confidence can be obtained from the right axis of the chart. The red dashed line represents the 85% confidence level suggested for the design of a mature or permanent excavation at the operation stage. The 85% confidence level is given as an indicator of domains that potentially require additional data collection.

The results show that generally wider confidence intervals and lower confidence levels are obtained with the $TLDC$ method. The results in the $Nmin$ column of Table 4 show that the $TLDC$ method requires more specimens to reach the same precision (e.g. E_r = 15% which implies p = 1.35) and confidence than the CI method. Finally, additional laboratory testing would be recommended to reach an 85% confidence level for 5 domains (i.e. rock units 4f, 6b, 6 h, 9b and Ore) using the CI method. Based on this analysis additional testing would be required for 8 domains (i.e. rock units 10c, 4a, 4f, 6b, 6e, 6 h, 9b and Ore) if the $TLDC$ method is employed.

Table 5. Confidence intervals and minimum number of specimens required to reach an 85% confidence level, obtained with the CI and $TLDC$ methods for PM14 tensile strength data

Rock unit	*n*	Data distribution	Sample average (MPa)	*cv*	Fillion and Hadjigeorgiou (2017)				Grenon et al. (2015)				
					Confidence interval for p = 1.35 (E_r = 15%)			*Nmin* for *CI* 85%	Confidence interval for *CI* = 95%		E_r (%)	*TLDC* (%)	*Nmin* for *TLDC* 85%
					Lower (MPa)	Upper (MPa)	*CI* (%)		Lower (MPa)	Upper (MPa)			
10b	17	Normal	20.9	0.33	17.8	24.0	91.8	–	17.4	24.5	17.0	83.0	18
10c	22	Normal	21.8	0.16	18.6	25.1	100.0	–	20.3	23.3	6.9	93.1	–
4a	6	Normal	19.9	0.33	16.9	22.9	68.8	8	13.1	26.7	34.1	65.9	13
4f	2	N/A											
6a	12	Normal	24.0	0.07	20.4	27.5	100.0	–	22.9	25.0	4.4	95.6	–
6b	4	N/A											
6e	4	N/A											
9a	21	Normal	19.1	0.26	16.3	22.0	98.3	–	16.9	21.4	11.9	88.1	–
9b	8	Normal	18.1	0.27	15.4	20.8	83.6	9	14.0	22.2	22.7	77.3	10
Ore	11	Normal	17.7	0.27	15.1	20.4	90.5	–	14.5	20.9	18.0	82.0	12

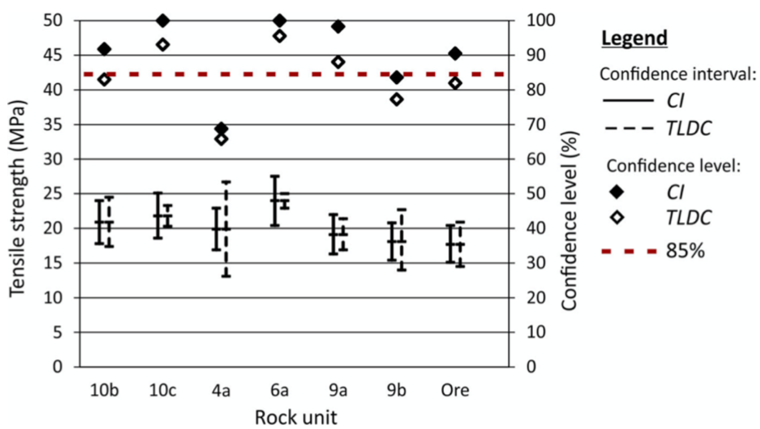

Figure 7. Confidence intervals (whisker plots) and associated levels of confidence obtained with the CI and $TLDC$ methods (black and white diamond symbols on top) for PM14 tensile strength data

5.2 Tensile Strength

Tensile strength data were available for a total of 10 domains for PM14 at Raglan mine. The confidence level was calculated for 7 domains. It was not possible to calculate the confidence for 3 domains because the data distribution could not be assigned due to the limited amount of tensile strength data. This highlights some of the practical issues in reviewing mechanical data. Table 5 and Fig. 7 present the confidence intervals and the associated levels of confidence obtained with the CI and $TLDC$ methods.

An analysis of the tensile strength data follow a similar trend to that obtained for the UCS data, i.e. generally wider confidence intervals, lower confidence levels and higher *Nmin* obtained with the $TLDC$ method. The results for rock unit 6a (Fig. 7) show that, when the coefficient of variation is low (i.e. 0.07 in that particular example), high precision and confidence can be obtained by applying the $TLDC$ method. This results in a narrow 95% confidence interval for rock unit 6a. Additional laboratory testing would be recommended to reach an 85% confidence for the remaining 5 domains (i.e. rock units 4a, 4f, 6b, 6e and9b) using the CI method, as opposed to 7 domains (i.e. rock units 10b, 4a, 4f, 6b, 6e, 9b and Ore) if the $TLDC$ method is employed.

Table 6. Confidence intervals and minimum number of specimens required to reach an 85% confidence level, obtained with the CI and $TLDC$ methods for PM14 density data

Rock unit	n	Data distribution	Sample average (t/m^3)	cv	Fillion and Hadjigeorgiou (2017) Confidence interval for p = 1.04 (E_r = 2%) Lower (t/m^3)	Upper (t/m^3)	CI (%)	Fillion and Hadjigeorgiou (2017) *Nmin* for *CI* 85%	Grenon et al. (2015) Confidence interval for CI = 95% Lower (t/m^3)	Upper (t/m^3)	E_r (%)	*TLDC* (%)	*Nmin* for *TLDC* 85%
10b	57	Lognormal	2.86	0.053	2.81	2.92	99.3	–	2.82	2.90	1.4	98.6	–
10c	55	Lognormal	2.81	0.024	2.76	2.87	100.0	–	2.79	2.83	0.7	99.3	–
4a	12	Normal	2.77	0.004	2.72	2.82	100.0	–	2.76	2.78	0.2	99.8	–
4f	4	N/A											
6a	32	Normal	2.98	0.023	2.92	3.04	100.0	–	2.96	3.01	0.8	99.2	–
6b	14	Normal	3.00	0.010	2.94	3.06	100.0	–	2.98	3.02	0.6	99.4	–
6e	12	Normal	2.97	0.012	2.91	3.03	100.0	–	2.95	2.99	0.7	99.3	–
6 h	6	Normal	3.06	0.039	*2.98*	*3.15*	*85.5*	–	2.94	3.19	4.1	95.9	–
9a	54	Normal	2.97	0.052	2.91	3.03	99.2	–	2.93	3.01	1.4	98.6	–
9b	17	Normal	2.99	0.029	2.94	3.05	98.6	–	2.95	3.04	1.5	98.5	–
Ore	32	Lognormal	3.10	0.034	3.04	3.16	99.7	–	3.06	3.13	1.2	98.8	–

Italics implies that the precision index is higher than the selected p = 1.04. For the CI method, when CI was too low, p was increased to reach 85% confidence

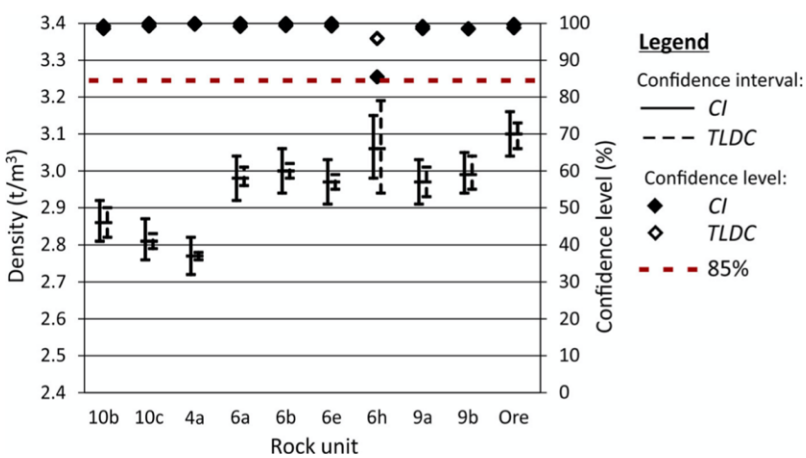

Figure 8. Confidence intervals (whisker plots) and associated levels of confidence obtained with the CI and $TLDC$ methods (black and white diamond symbols on top) for PM14 density data

5.3 Density

The density data were obtained for the specimens used in the UCS, tensile strength and triaxial laboratory tests. Density measurements were available for a total of 11 domains for PM14 at Raglan mine. The confidence level was calculated for 10 domains. It was not possible to calculate the confidence for rock unit 4f because the data distribution could not be assigned due to limited test results. Due to the naturally lower variability of the density data (i.e. density measurement generally about 2–3 t/m^3), the coefficient of variation (cv) of density laboratory tests is significantly lower than for other rock properties. To increase the precision on the estimation and reduce the length of the confidence interval, a precision index (p) of 1.04 instead of 1.35 was targeted using the CI method. This implies that the maximum E_r is about 2% and results in a smaller confidence interval. Table 6 and Fig. 8 show the confidence intervals and the associated levels of confidence obtained with the CI and $TLDC$ methods.

Table 7. Confidence intervals and minimum number of specimens required to reach an 85% confidence level, obtained with the CI and $TLDC$ methods for PM14 Young's modulus data

Rock unit	*n*	Data distribution	Sample average (GPa)	*cv*	Fillion and Hadjigeorgiou (2017)				Grenon et al. (2015)				
					Confidence interval for p = 1.35 (E_r = 15%)			*Nmin* for *CI* 85%	Confidence interval for *CI* = 95%		E_r (%)	*TLDC* (%)	*Nmin* for *TLDC* 85%
					Lower (GPa)	Upper (GPa)	*CI* (%)		Lower (GPa)	Upper (GPa)			
10b	16	Normal	83.49	0.08	71.05	95.92	100.0	–	79.80	87.18	4.4	95.6	–
10c	16	Normal	77.78	0.15	66.20	89.36	99.9	–	71.74	83.81	7.8	92.2	–
4f	2	N/A											
6a	8	Normal	83.78	0.16	71.30	96.25	96.5	–	72.44	95.11	13.5	86.5	–
6b	5	Normal	84.28	0.09	71.73	96.83	97.7	–	74.55	94.01	11.5	88.5	–
6e	5	Normal	71.08	0.21	60.49	81.66	80.8	6	52.31	89.84	26.4	73.6	7
9a	9	Lognormal	79.29	0.28	68.15	91.97	85.1	–	63.76	98.31	21.8	78.2	10
9b	5	Normal	95.02	0.07	80.87	109.17	99.0	–	86.46	103.58	9.0	91.0	–
Ore	11	Lognormal	66.56	0.12	57.33	77.36	99.8	–	61.58	72.03	7.8	92.2	–

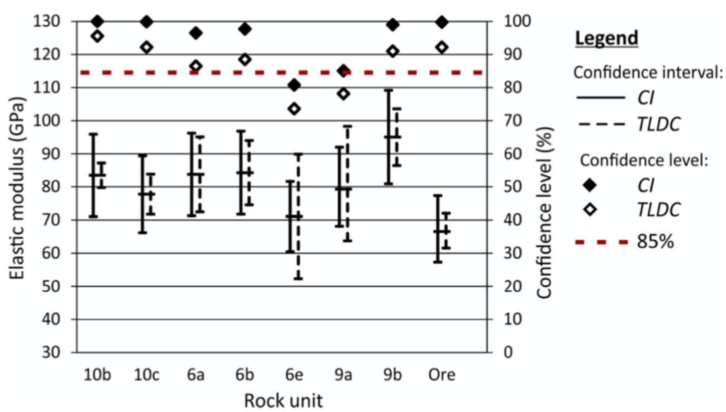

Figure 9. Confidence intervals (whisker plots) and associated levels of confidence obtained with the CI and $TLDC$ methods (black and white diamond symbols on top) for PM14 Young's modulus data

The results show that, contrary to more variable rock properties, the level of confidence in density data is generally high. This is partly due to the higher number of density test results (obtained during the testing of three other rock properties), and partly due to the naturally lower variability of density measurements, in comparison to other rock properties. For density data, the length of the calculated confidence interval is generally smaller with the $TLDC$ method, which implies a higher precision on the estimation. Furthermore, an issue with the pre-selection of *p* was observed with the CI method. For example, the selection of p = 1.04 for rock unit 6h resulted in a 72.3% CI of 3.00–3.12 t/m^3, which implies that additional density data should be collected for a mine at the operation stage. However, as shown in Table 6 for rock unit 6h, an 85% CI of 2.98–3.15 t/m^3 can be obtained by increasing *p* to 1.057 (i.e. E_r about 3%). This demonstrates that a significant difference in the confidence level can be obtained for a similar confidence interval by increasing p with only a small increment. Finally, additional laboratory testing is recommended for rock unit 4f to be able to identify the data distribution. The number of density test results is sufficient for all other geotechnical domains.

5.4 Young's Modulus

Young's modulus data were obtained from instrumented UCS laboratory tests for 9 domains for PM14 at Raglan mine. The confidence level was calculated for 8 domains because the data distribution could not be assigned to rock unit 4f due to the limited amount of Young's modulus data. Table 7 and Fig. 9 present the confidence intervals and the associated levels of confidence obtained with the CI and $TLDC$ methods.

Although the higher confidence is obtained using the CI method, the length of the confidence interval is generally smaller for the $TLDC$ method. This is due to the generally low variability of the measured Young's modulus data for this particular location. For more variable domains, such as rock units 6e and 9a, wider confidence intervals are obtained with the $TLDC$ method, and a greater number of specimens is required to reach 85% confidence compared to the CI method. Additional laboratory testing would be recommended to reach an 85% confidence for 2 domains (i.e. rock units 4f and 6e) using the CI method, as opposed to 3 domains (i.e. rock units 4f, 6e and 9a) if the $TLDC$ method is employed.

Table 8. Confidence intervals and minimum number of specimens required to reach an 85% confidence level, obtained with the CI and $TLDC$ methods for PM14 Poisson's ratio data

Rock unit	*n*	Data distribution	Sample average	*cv*	Fillion and Hadjigeorgiou (2017)				Grenon et al. (2015)				
					Confidence interval for p = 1.35 (E_r = 15%)			*Nmin* for *CI* 85%	Confidence interval for *CI* = 95%		E_r (%)	*TLDC* (%)	*Nmin* for *TLDC* 85%
					Lower	Upper	*CI* (%)		Lower	Upper			
10b	16	Lognormal	0.29	0.07	0.25	0.34	100.0	–	0.28	0.30	3.9	96.1	–
10c	16	Lognormal	0.29	0.10	0.25	0.34	100.0	–	0.28	0.31	5.4	94.6	–
4f	2	N/A											
6a	7	Normal	0.31	0.17	0.26	0.36	94.5	–	0.26	0.36	15.4	84.6	8
6b	4	N/A											
6e	5	Normal	0.26	0.16	0.22	0.30	89.0	–	0.21	0.31	20.2	79.8	6
9a	9	Lognormal	0.29	0.14	0.25	0.34	99.0	–	0.27	0.33	10.6	89.4	–
9b	5	Normal	0.30	0.12	0.22	0.30	95.2	–	0.22	0.30	14.7	85.3	–
Ore	11	Normal	0.29	0.07	0.25	0.33	100.0	–	0.28	0.30	4.7	95.3	–

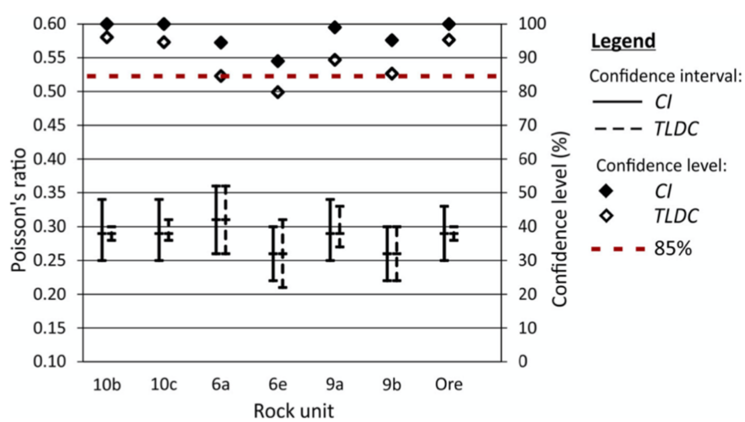

Figure 10. Confidence intervals (whisker plots) and associated levels of confidence obtained with the CI and $TLDC$ methods (black and white diamond symbols on top) for PM14 Poisson's ratio data

5.5 Poisson's Ratio

Poisson's ratio data were obtained from instrumented UCS laboratory tests. Poisson's ratio measurements were available for a total of 9 domains for PM14 at Raglan mine. The confidence level was calculated for 7 domains because the data distribution could not be assigned to rock units 4f and 6b due to the limited amount of Poisson's ratio results. Table 8 and Fig. 10 show the confidence intervals and the associated levels of confidence obtained using both statistical methods.

Similarly to the density data, the Poisson's ratio is naturally less variable than other rock properties with typical values between 0 and 0.5. Consequently, generally high confidence levels can be reached with limited Poisson's ratio measurements (5–16 for this particular case). For Poisson's ratio data, the length of the calculated confidence interval is generally smaller with the $TLDC$ method which implies a higher precision on the estimation. However, the confidence levels are smaller with the $TLDC$ method and a higher number of specimens ($Nmin$) must be tested to reach 85% confidence for domains with higher variability. For example, the confidence level is higher than 85% for rock units 6a and 6e if the CI method is used, but testing one additional specimen is recommended for those two domains if the $TLDC$ method is considered. Additional laboratory testing would be recommended to reach an 85% confidence for 2 domains (i.e. rock units 4f and 6b) using the CI method, as opposed to 4 domains (i.e. rock units 4f, 6a, 6b and 6e) if the $TLDC$ method is employed.

The results for rock unit 9b demonstrate the difference with both methods for the same CI = 95% and p = 1.35 (E_r = 15%). This implies that the length of the confidence interval is the same with both methods, i.e. 0.22–0.30 for this particular domain (Table 8 and Fig. 10). However, the confidence level is smaller with the $TLDC$ method because $TLDC$ (%) = 100–E_r with E_r calculated for a 95% CI. This illustrates why the $TLDC$ methods result in lower confidence levels than the CI method.

6 PRACTICAL CONSIDERATIONS

This section presents the practical considerations using two approaches based on small-sampling theory in evaluating the level of confidence in normally or lognormally distributed rock properties obtained through a series of laboratory tests (i.e. UCS, tensile strength, density, Young's modulus and Poisson's ratio). A practical example using the Kirsch analytical solution (Kirsch 1898) is presented to illustrate the implications of selecting one analysis method over the other. The two methods used are equally applicable to other rock properties following a normal or a lognormal data distribution. For other data distributions, those two methods cannot be used to determine the level of confidence. Other methods such as the modified Bayesian approach (Read 2013) could be employed to establish the reliability of laboratory test results when a data distribution other than normal or lognormal is identified. The use of other methods is outside the scope of this work.

For a statistical method to be valid, the sample must be representative of the population, i.e. it is desirable to select a random sample to avoid introducing a bias into the sample which may result in an over or underestimation of the parameter of interest. In reality, the sampling and testing methods can have a significant impact on the parameter estimate, the delineation of the geotechnical domains and the resulting confidence level in the rock properties (e.g. weaker samples not tested, test results excluded from the geotechnical database, variability underestimated for spatially correlated samples, borehole orientation bias, discrepancies in the rock type identification over time and with different personnel logging the core, high variability in the rock properties within a single geotechnical domain, etc.). The implications of the potential sources of sampling bias is outside the scope of this work. Nevertheless, statistical analyses can aid in identifying potential bias and may contribute to a better zoning of the geotechnical domains.

6.1 Differences Between the CI and the $TLDC$ Analysis Methods

The first approach, i.e. the CI method, requires that a precision index is targeted a priori, based on the specifics of the project. It is then possible to determine the level of confidence that meets this criteria and to decide if additional laboratory tests should be conducted to increase the confidence level. The second approach, i.e. the $TLDC$ method, suggests that a high confidence level of 95% in the interval estimate of the investigated rock property should be targeted, and the decision to collect additional data or not for the project under considerations is based on the associated precision index. Although these approaches appear very similar, the results presented in Sect. 5 for the different rock properties identified differences on how the two problems are addressed from a statistical and engineering perspective.

For both methods, it was shown that a higher number of laboratory tests does not necessarily imply a higher level of confidence. Distinct geotechnical domains have different levels of geological and structural complexity which results in different number of specimens to reach a targeted level of confidence. Furthermore, within a specific geotechnical domain, collecting additional data may result in a smaller level of confidence if the data variability was previously underestimated. Similarly, comparable coefficients of variation may lead to different confidence levels in the rock properties because the confidence level is function of various parameters, including the number of specimens in the sample. For example, for the tensile strength data samples of rock units 10b and 4a (Table 5), the coefficient of variation is equivalent (i.e. cv = 0.33), but the associated CI and $TLDC$ is significantly higher for domain 10b. This further demonstrates that a decision to test additional samples guided solely on cv values may be inadequate to assess the reliability of the collected geomechanical data. Both the CI and $TLDC$ methods can be employed in identifying gaps in data collection and in selecting drilling targets for additional data collection. Effectively, those two methods provide a quantitative assessment of the confidence level in geomechanical data based on a predetermined precision requirement. This quantitative measure aids in justifying the need for additional data collection in particular geotechnical domains in order to meet the precision and confidence criteria for a specific design. The main differences between the CI and the $TLDC$ methods are:

- For the same precision index p, the confidence level determined with the CI method is generally higher.

• As illustrated in Fig. 6, for rock properties that are generally more variable (e.g. UCS), the length of the confidence interval established with the CI method is generally smaller for the selected p = 1.35. Wider confidence intervals would be obtained with higher p.

• As illustrated in Figs. 8 and 10, for rock properties with a naturally low coefficient of variation (e.g. density, Poisson's ratio), the length of the confidence interval established with the $TLDC$ method is generally smaller. Smaller confidence intervals can be obtained with the CI method by reducing p, but this will also reduce the confidence level in the interval estimate.

• As shown in Tables 4, 5, 6, 7 and 8, the additional number of specimens required to reach higher confidence levels in the interval estimate for the rock properties is higher with the $TLDC$ method. This is due to the use of a fixed CI=95% to establish the lower and upper limits of the confidence interval.

Table 9. Advantages and limitations of the CI and $TLDC$ methods

Method	Advantages	Limitations
CI	More flexibility for the designer by allowing to target the required precision and confidence level	For a given *p*, *CI* can be quite low when few data are available or for data with high variability
	Higher confidence levels in the confidence interval (for the same dataset)	Pre-selecting a value for the precision index (*p*) can significantly influence the resulting *CI*
	For the same precision index *p*, a smaller number of specimens must be tested to reach higher confidence levels (for the same data set) which is beneficial economically	
TLDC	High level of confidence in the constructed confidence interval (*CI* fixed at 95%)	A higher number of additional specimens must be tested to reach higher confidence levels
	Generally provides high precision and high confidence levels for data with low variability	For data naturally more variable, the length of the resulting confidence interval can be quite large which makes it more difficult to select a design value

The results obtained from both the CI and $TLDC$ methods allowed the identification of advantages and limitations to those approaches, as summarized in Table 9. It is important to consider these advantages and limitations when selecting a method to determine the level of confidence in the laboratory test results. The selection of the analysis method depends on the project requirements.

A major advantage of the CI method is that it provides more flexibility to the designer, depending on the type of project. Contrary to the $TLDC$ method, which fixes th$e\ CI\ at\ 95\%, the\ CI\ m$ethod allows the targeting of different CI and p values. For example, there is less need to reach high CI values in domains that are not located in the vicinity of less critical mine excavations. Nevertheless, the CI value must be adequate if the specific domain is present in the back/walls of a temporary excavation, prior to the final excavation. It is further possible to increase p to reach higher confidence levels without having to collect additional data (e.g. rock unit 6h in Table 6). This implies a wider confidence interval. The impact of higher p on the selection of the design value and consequently on the project design, could be investigated through sensitivity analyses. This requires the use of a higher precision index on the confidence interval for the true mean to determine if the design requirements are met for the required precision.

A considerable benefit of using the $TLDC$ method is the high level of confidence of 95% in the interval estimate. This can be a critical criterion for entry excavations (i.e. personnel exposed) or permanent excavations such as the mine's shaft. Based on this approach, a higher number of additional specimens is required to increase the confidence in the rock properties values. This can be considered a relatively more conservative choice and may be more appropriate for critical excavations.

In underground mining projects, such practical site specific considerations have an impact on the number of data collected and can also influence the timing for data collection. For example, at Raglan mine, many drifts are excavated in the peridotite rock unit (10b) (Grenon et al. 2015) which explains the generally higher number of tests available for this specific domain in the mine's laboratory testing database.

6.2 Practical Example Using the Kirsch Analytical Solution

This section presents an example of the implications of quantifying the level of confidence in the geomechanical data used in mine design. Two approaches were employed to determine the level of confidence in rock unit 9a (Normal Gabbro) UCS data for two different stages of the PM14 mining project, i.e. a 2014 scoping study and a 2015 prefeasibility study. The results for the prefeasibility study represent the compilation of the 2014 and 2015 laboratory testing campaigns. The data distribution fit for the 2014 and 2015 project stages are presented in Fig. 11. The confidence intervals and the associated level of confidence for the two stages are presented in Table 10. The results show that, for this particular case, the additional data collection resulted in a different distribution best fit (i.e. lognormal at the 2014 scoping stage and normal at the 2014–2015 prefeasibility stage) and a higher level of confidence in the data at the later stage. Furthermore, as shown in Fig. 12, the confidence intervals obtained with both methods are similar at the 2014–2015 prefeasibility stage, but a wider confidence interval is obtained with the $TLDC$ method at the 2014 scoping stage. This can have significant implications in mine design.

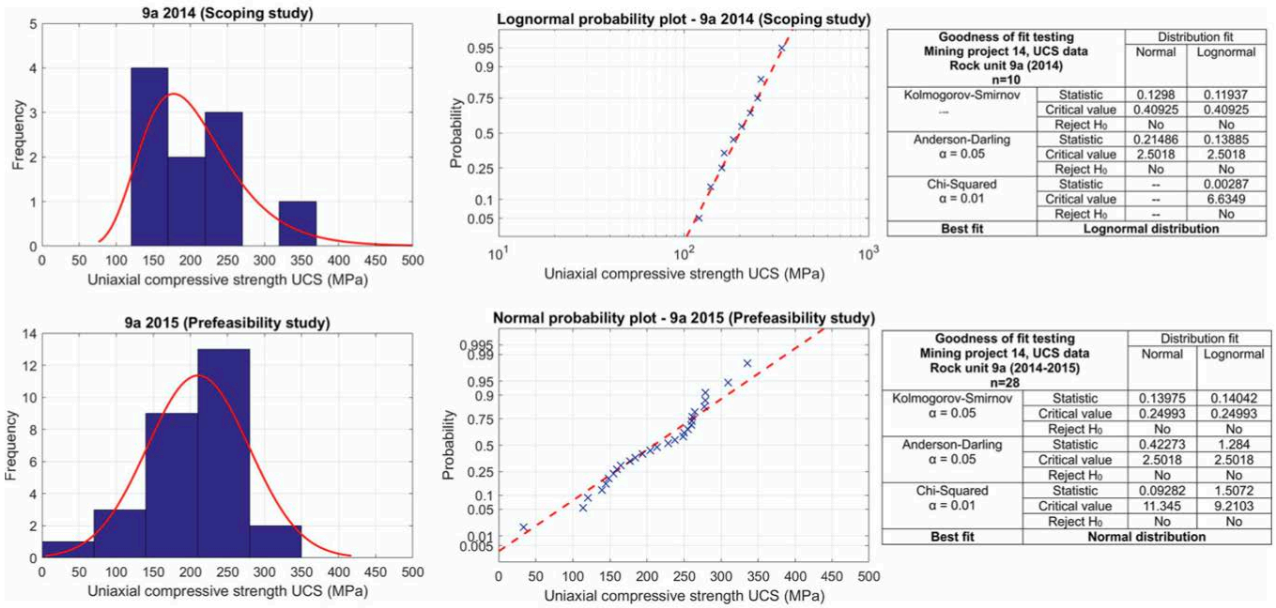

Figure 11. UCS data distribution fit for rock units 9a at the 2014 scoping stage (lognormal distribution) and at the 2015 prefeasibility stage (normal distribution)

Table 10. CI and $TLDC$ obtained for rock unit 9a UCS laboratory test results conducted for a scoping study (2014 database) and a prefeasibility study (compiled 2014–2015 database) undertaken for PM14 mining project

Rock unit	n	Data distribution	Sample average (MPa)	cv	Fillion and Hadjigeorgiou (2017)			Grenon et al. (2015)			
					Confidence interval for $p = 1.35$ ($E_r = 15\%$)			Confidence interval for $CI = 95\%$		E_r (%)	$TLDC$ (%)
					Lower (MPa)	Upper (MPa)	CI (%)	Lower (MPa)	Upper (MPa)		
9a 2014	10	Lognormal	205.0	0.32	176.8	238.7	82.6	163.2	258.6	23.2	76.8
9a 2014–2015	28	Normal	210.6	0.33	179.2	241.9	97.7	183.9	237.3	12.7	87.3

A simple example using the Kirsch analytical solution (Kirsch 1898) is presented to illustrate the potential consequences of the analysis method in the design process. The Kirsch solution was selected for this example because it is an analytical solution derived from the theory of elasticity. The use of a simple analytical solution is optimal for this example and helps illustrate the impact of data analysis.

The Kirsch solution is used to calculate the stresses around the circular excavation illustrated in Fig. 13. The stresses at the boundary of a circular opening (i.e. $r = a$) are given by Eq.11 to Eq.13:

$$\sigma_{rr} = 0 \tag{11}$$

$$\sigma_{\theta\theta} = p[(1+k) + 2(1-k)\cos(2\theta)] \tag{12}$$

$$\sigma_{r\theta} = 0 \tag{13}$$

where σ_{rr} = the radial stress and σ_{rr} = 0 because there is no internal pressure at the boundary, $\sigma_{\theta\theta}$ = the tangential stress, $\sigma_{r\theta}$ = the shear stress and $\sigma_{r\theta}$ = 0 at a traction-free boundary, p = the vertical stress, k = the stress ratio ($k = \sigma_h/p$, i.e. the ratio of the horizontal stress to the vertical stress), θ = angle from a reference direction in the polar coordinate system, r = distance from a reference point in the polar coordinate system, a = radius of the circular excavation.

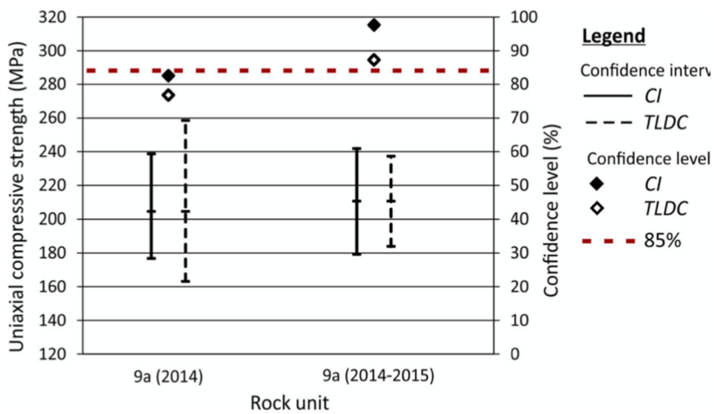

Figure 12. Confidence intervals (whiskers plots) and associated levels of confidence obtained with the CI and $TLDC$ methods (black and white diamond symbols on top) for PM14 UCS data collected to characterize rock unit 9a at two different project stages

In this example, the induced stress at the roof of a circular excavation (θ = 90°) in rock unit 9a (i.e. Normal Gabbro) is evaluated to determine if the compressive strength of the rock on the excavation boundary will be exceeded at depth z = 800 meters. Since the excavation has neither a support pressure nor an internal pressure applied to it, the local stresses at the boundary have $\sigma_3 = \sigma_{rr} = 0$ and $\sigma_1 = \sigma_{\theta\theta}$. The stress ratio is assumed to be k = 2. The density of rock unit 9a is estimated to be about 3.0 t/m^3 (i.e. unitweight γ = 29.4 kN/m^3). The vertical stress due to the weight of the overburden ($p = \gamma z$) is 23.5 MPa. By replacing the values in Eq. 12, the tangential stress ($\sigma_{\theta\theta}$) at the roof is approximately 118 MPa.

The confidence intervals for the mean uniaxial compressive strength obtained with the CI and $TLDC$ analysis methods for two project stages are given in Table 10. The calculated confidence intervals implies that the true mean can take any value in the specified range. To perform probabilistic analyses, the parameters (mean and standard deviation) of the probability distribution (normal or lognormal for this specific example) must be determined. Strength values are then generated randomly from the specific probability distribution. The lower bound of the confidence interval was selected as a design value for the stability analysis. Selecting another value (e.g. the sample average or the upper limit of the confidence interval) would result in a smaller probability of failure. The sample standard deviation of strength values for rock unit 9a at the scoping (2014) and the prefeasibility (2014–2015) stages are respectively 65.1 and 68.8 MPa. The probability that the compressive strength is exceeded for rock types at the roof of the excavation, i.e. the probability that the compressive strength will be less than 118 MPa, was calculated from the associated cumulative distribution function. The results are presented in Table 11. Figure 14 shows the data distributions obtained with the two methods with the simulation of 10,000 random UCS values at the 2014 scoping stage and at the 2014–2015 prefeasibility stage. The probability that the compressive strength will be less than 118 MPa is represented by the shaded area in Fig. 14.

In this example, for simplicity purposes, the focus was on the estimation of the level of confidence on the mean using a constant value for the sample standard deviation. In reality, the sample standard deviation is a variable parameter influenced by the number of specimens in the sample, as opposed to the population standard deviation which is a constant value. Confidence intervals for the true standard deviation can be constructed for normally distributed data, e.g. Hines et al. (2003), and for lognormally distributed data, e.g. Tang and Yeh (2016). For a predetermined confidence level, the precision index on the confidence interval for true standard deviation can be different than the precision index on the confidence interval for true mean. Tang and Yeh (2016) concluded that, typically, the number of specimens needed for the lognormal standard deviation confidence interval is larger than that for the lognormal mean confidence interval. The influence of the variability of the standard deviation estimate was outside the scope of this work.

For this particular example, the probability that the compressive strength is exceeded, i.e. P (strength \ 118 MPa), at the roof of a circular excavation is lower at the early 2014 stage (scoping study) compared to the 2015 prefeasibility study using both the CI and $TLDC$ methods. This suggests that, because of the smaller number of UCS data available at the 2014 project stage (i.e. 10 UCS values at the 2014 project stage vs. 28 at the 2015 project stage), the data variability was underestimated. With additional data collected in 2015 (i.e. 18 additional UCS laboratory tests conducted), a better understanding of the true variability can be achieved. This resulted in a slightly higher cv at the 2015 prefeasibility stage, but a complete change in the best data distribution fit (i.e. lognormal distribution at the 2014 project stage vs. normal distribution at the 2015 project stage). Underestimating the true data variability can have significant implications on the design. For example, the design can be accepted for a probability that the compressive strength is exceeded at the roof of the excavation ≤ 0.15. This acceptance criterion would be accepted at the 2014 project stage but would not be met at the 2015 prefeasibility stage. Other acceptance criteria could be selected based on the project requirements. The choice of an appropriate acceptance criterion is outside the scope of this work.

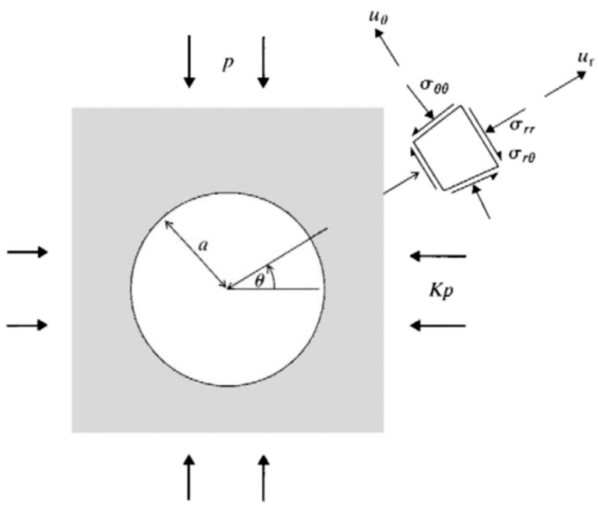

Figure 13. Stresses induced around a circular excavation. (Brady and Brown 2006)

Table 11. Practical example for the probability that the compressive strength is exceeded, i.e. P (strength \ 118 MPa), at the roof of a circular excavation using the CI and $TLDC$ analysis methods at two different stages of a mining project

Method	Project stage	Distribution	Average strength (MPa)	Standard deviation (MPa)	$P(UCS < 118$ MPa)
CI	2014	Lognormal	176.8 (5.175)	65.9 (0.313)	0.10
	2014–2015	Normal	179.2	68.8	0.19
TLDC	2014	Lognormal	163.2 (5.095)	65.9 (0.313)	0.15
	2014–2015	Normal	183.9	68.8	0.17

The results in brackets are for the log-transformed variable $Y = \ln(X)$

The results of the analysis, summarised in Table 10, illustrate that with additional collected data for the 2015 prefeasibility study, the level of confidence is significantly higher, i.e. 97.7% vs. 82.6% with the CI method and 87.3% vs. 76.8% with the $TLDC$ method. The relative error obtained with the $TLDC$ method is reduced significantly with additional data collected (i.e. 12.7% vs. 23.2%). This resulted in similar upper and lower limits for the confidence interval obtained with both methods and a similar UCS value selected for design purposes, i.e. 179 MPa with the CI method and 184 MPa with the $TLDC$ method (see Table 11). Since a similar design value is selected at the 2015 prefeasibility stage, the probability that the compressive strength is exceeded at the roof of the excavation is similar (i.e. 0.19 with the CI method vs. 0.17 with the $TLDC$ method).

This simple example illustrated the practical implications of selecting the CI or $TLDC$ method in establishing the level of confidence in laboratory test results on the resulting design output. The analysis of two different project stages (i.e. a 2014 scoping study and a 2015 prefeasibility study) further demonstrated the importance of reaching a sufficient level of confidence in the collected data to ensure a reliable design.

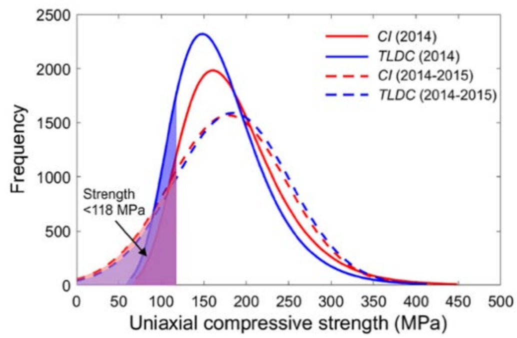

Figure 14. Data distributions obtained with the CI and $TLDC$ methods at the 2014 scoping stage and at the 2014–2015 prefeasibility stage

7 CONCLUSIONS

A major challenge in geotechnical analysis and design of underground mine excavations is the availability of sufficient and reliable data. Through the life of a mine, defining the scope of geotechnical data collection campaigns and the necessity for additional data collection is not a straightforward task. This requires knowledge of the level of confidence in the existing database to select targets for additional data collection. In this paper, two statistical approaches, based on small-sampling theory, were investigated to quantify the level of confidence in the rock properties obtained through a series of ISRM suggested methods for an underground mining project in Northern Canada. The level of confidence in UCS, tensile strength, density, Young's modulus and Poisson's ratio data was calculated for a specific mining project prior to the operation stage. The results identified the main differences, including the advantages and limitations, of the two methods. The results further illustrated the impact of choosing one analysis method over the other on the resulting confidence level, precision index and additional number of specimens required to reach a higher confidence level in the rock properties. This investigation demonstrated that the selection of the appropriate analysis method should be guided by the specifics of the project, i.e. the required confidence level and precision index.

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