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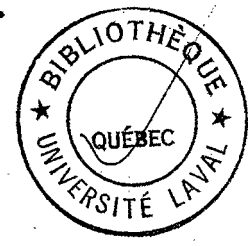
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THOMISM AND MATHEMATICAL PHYSICS

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CHAPTER ONE

THE PROBLEM OF MATHEMATICAL PHYSICS.

1. A Symbol of Progress.

"On the second floor of the Hall of Science at the Century of Progress Exposition, held at Chicago in the summers of 1933 and 1934, reaching up into the great tower of the building was a smaller tower designed to symbolize the interrelations and interdependence of the physical sciences. The huge base on which the remaining sciences were supported and uplifted was assigned to mathematics. Astronomy, physics, chemistry, the medical sciences, geology, geography, engineering, architecture, the industrial arts -- all had their roots in the science by whose methods and attainments they have learned and continue to learn to express themselves." (1)

The milling throngs that crowded the pavillions of Chicago's Exposition found a great many things to make their visit rewarding. For there, under a great variety of forms, were the concrete and tangible results of a century of amazing scientific and technological progress which had gone to almost incredible lengths in penetrating into the inner secrets of Nature and in controlling its hidden forces. But for those who were interested not merely in things, but in their meanings, the tower of the sciences resting

upon the base of mathematics was the most significant object in the whole Exposition. For it was the symbol of a human triumph that was the source from which had come all the other remarkable achievements on display -- a source so fruitful that it reached beyond the limitations of these particular achievements, and would ever continue to reach beyond the even more remarkable accomplishments that would come from it in the future. More than that, it was the symbol of something that was far too great to be put on display: the amazing theoretical attainments of Einstein, Planck, Bohr, Heisenberg, Schrodinger, Dirac, and De Broglie -- to mention only a few of the names which have made modern physics great.

But there were even more far-reaching implications in this symbolism. For it was a revelation of what has happened to the human intellect in modern times. And here we have in mind, not merely a question of scientific methodology, but something far deeper. In this symbolism could be found an indication of the precise direction in which the mind of man has progressed in the modern era. For in so far as the speculative intellect is concerned, modern progress has not been a progress in wisdom, but in sciences; and not in science in the full and perfect sense of the term in which it was understood by the Greeks and the Medievalists -- the sense in which it signifies an intellectual triumph over the obscurity of matter to the extent of laying hold of

the objective logos of nature with clarity and certitude -- but in that dialectical type of knowledge into which science necessarily issues as it pursues its development in the direction of increasing concretion in matter. And in so far as the practical intellect is concerned, modern progress has not been a progress in prudence, but in art; and, once again, not in the higher form of art, the art of imitation or fine art, in which the darkness of matter is transfused by the light of the mind, but in technological art, in which the intellect is bent upon the exploitation of matter, and at best achieves only a kind of compromise with it. And as this development has gone on, not only has dialectical science tended to dispute the hegemony of wisdom in the speculative order, and technological art that of prudence in the practical order, but science and art have been drawn closer and closer, and united in a new and strange intimacy.

Obviously, the matrix of this distinctive intellectual growth, so characteristic of our times, is something highly complex, and it would be a naive oversimplification to attribute it to any one factor. Nevertheless, we feel that the source which has contributed most to it, and given it its strongest impetus, and dictated its precise direction has been the erection of the tower of the sciences upon the base of mathematics: the interpretation of the physical world in the light of the world of mathematics.

For the moment we shall not attempt to establish this point. It has been suggested here merely to orientate properly the problem we are undertaking to discuss, and further development of it now would take us too far afield and make it necessary to anticipate much of what is to follow. But perhaps it would not be irrelevant to quote a passage from one of the greatest contemporary mathematical physicists, in which what we have been saying finds at least a general confirmation. In the introduction to his Electrons, Protons, Neutrons, and Cosmic Rays, Professor Millikan points out that it is only through the application of mathematics to the physical world that the secrets of nature can be effectively laid bare, and the road thrown open to man's control over nature through technological art:

For it usually happens that when nature's inner workings have once been laid bare, man sooner or later finds a way to put his brains inside the machine and to drive it whither he wills. Every increase in man's knowledge of the way in which nature works must, in the long run, increase by just so much man's ability to control nature and to turn her hidden forces to his own account. . . .

In this presentation I shall not shun the discussion of exact quantitative experiments, for it is only upon such a basis, as Pythagoras asserted more than two thousand years ago, that any real scientific treatment of physical phenomena is possible. Indeed, from the point of view of that ancient philosopher, the problem of all natural philosophy is to drive out qualitative conceptions and to replace them by quantitative relations. And this point of view has been emphasized by the farseeing throughout all the history of physics clear down to the present. One of the greatest of modern physicists, Lord Kelvin, writes: "When you can measure what you are speaking about and express it in numbers, you know something about it, when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind. It may be the beginning of knowledge, but you have scarcely

in your thought advanced to the stage of a science." (2)

Perhaps enough has been said to suggest that there is hardly a more important or more pressing task confronting contemporary philosophy, nor one which promises greater intellectual fruitfulness, than the analysis of the significance of the symbolism of the scientific tower resting upon the base of mathematics, the attempt to unfold one by one its manifold implications in their proper focus. Such is the purpose of this study. We shall not attempt to unravel completely the whole complicated maze of epistemological problems that have arisen out of mathematical physics, and particularly out of its more recent development. The state of this development is still too fluid perhaps to make any attempt of that kind feasible. We shall content ourselves with an analysis of the basic significance of the interpretation of nature in terms of mathematics.

It would be interesting to know how many of the hundreds of thousands of visitors at the Chicago Exposition found the tower within the tower worthy of special interest, and how many grasped the profound

meaning of its symbolism. Prima facie, it would undoubtedly seem preposterous to suggest that no one among those who had reaped the fruits of modern progress, or even among those whose genius had been immediately responsible for its great achievements, could understand this symbolism quite so well as some who lived centuries before the Century of Progress began. Yet it does not seem necessary, or even possible to rule out such a supposition in a priori fashion. And if this supposition could be proved to be true, it would provide striking evidence that not everything that has happened in the century of progress has been progress. In any case, it is important to understand that modern progress has not been abiogenetic. The mathematical interpretation of nature is indeed characteristic of the modern mind, but not in the sense that it was first discovered or created in recent times. Like most modern things it has its roots deep in the past. This has already been suggested in the passage just quoted from Millikan, and it will be one of the main purposes of this essay to show how important these roots are. But for the present it is necessary to examine its historical background only in a summary way, so that our problem will be thrown into proper focus.

2. Historical Perspective.

Not a few historians have considered the Renaissance as the origin of the physico-mathematical method in science and have generally accorded to Galileo or to Descartes the honor of being its creator. But history is there to contradict the historians, and Pierre Duhem, among others, has shown with what remarkable clarity of outline the so-called modern scientific method had already been conceived in ancient times. We shall have occasion later to show that this is true of all the major elements in this scientific method, but for the moment we are interested only in the application of mathematics to physics. It is true, of course, that only in modern times have the far-reaching possibilities and remarkable fruitfulness of this application been fully realized -- realized both conceptually and practically. That is why Duhem himself could write: "Créée au XVII^e siècle, la physique mathématique a prouvé qu'elle était la saine méthode physique par les progrès prodigieux et incessants qu'elle a faits dans l'étude de la nature." (3) It is also true that the modern developments of mathematical physics have brought to light, or thrown into sharper outline, certain new epistemological aspects of the general physico-mathematical method. And it is probably these new aspects that have led Sir James Jeans to declare: The fact that the mathematical picture fits nature must, I think, be conceded to be

a new discovery of science, embodying new knowledge of nature such as could not have been predicted by any sort of general argument." (4) But these new aspects do not change the essence of the method. And it is this essence which has its roots in the past. It is, moreover, this essence which has the deepest and most interesting philosophical implications. That is why we must, if we would see things in their proper perspective, try to situate our problem in its historical context.

Already among the ancient Greeks the physico-mathematical method was clearly conceived, and actually put to considerable use. In this connection the name of Archimedes comes readily to mind, for it was through him that this method achieved its fullest fruitfulness in ancient times, and actually led to the definite and clear cut formulation of the sciences of mechanics and hydrostatics. But Archimedes was not the inventor of the method. Long before his time, the Greek astronomers, such as Eudoxus of Cnidos, had united mathematics and physics by attempting to "save the phenomena" through deduction drawn from geometrical hypotheses. (5) In the same way mathematics had been applied successfully in other sciences, such as optics. But since the purpose of this historical sketch is to orientate a philosophical problem, we are interested less in those who actually

applied mathematics to nature, than in those who in some reflective way attempted to bring to light the philosophical significance of this application. And in this connection it has become customary to designate two Greek philosophers as the ones who in ancient times grasped more clearly than any others the meaning of the mathematical interpretation of nature and the reach of its possibilities. They are Pythagoras and Plato.

The basic doctrine of the Pythagoreans is well known. The ultimate reality of things was for them essentially mathematical; the structure of the universe was based on numbers and their relations. Aristotle characterizes their position in the following terms:

Contemporaneously with these philosophers and before them, the so-called Pythagoreans, who were the first to take up mathematics, not only advanced this study, but also having been brought up in it they thought its principles were the principles of all things. Since of these principles numbers are by nature the first, and in numbers they seemed to see many resemblances to the things that exist and come into being -- more than in fire and earth and water (such and such a modification of numbers being justice, another being soul and reason, another being opportunity -- and similarly almost all other things being numerically expressible; since, again, they saw that the modifications and the ratios of the musical scales were expressible) in numbers; -- since, then all other things seemed in their whole nature to be modelled on numbers, and numbers seemed to be the first things in the whole of nature, they supposed the elements of numbers to be the elements of all things, and the whole heaven to be a musical scale and a number. And all the properties of numbers and scales which they could show to agree with the attributes and parts and the whole arrangement of the heavens, they collected and fitted into their scheme; and if there was a gap anywhere, they readily made additions so as to make their whole theory coherent. (6)

For the Pythagoreans the divine One was a mathematical god; he was the supreme number, and the source and cause of all the numbers that constituted the universe. (7)

All this seems to be a distant anticipation of the conclusions that one of the greatest contemporary mathematical physicists has arrived at as the result of his many years of work in the field and of his philosophical reflections upon its meaning.

"Our contention," writes Sir James Jeans, "is that the universe now appears to be mathematical in a sense different from any which Kant contemplated or possibly could have contemplated -- in brief, the mathematics enters the universe from above rather than from below." (8) "From the intrinsic evidence of his creation, the Great Architect of the universe now begins to appear as a pure mathematician." (9) More and

more modern scientists are looking back to Pythagoras as to the one who first conceived the vision that they are laboring to realize. Whitehead, for example, has this to say:

So today when Einstein, and his followers proclaim that physical facts, such as gravitation, are to be construed as exhibitions of local peculiarities of spatio-temporal properties, they are following the pure Pythagorean tradition. Truly, Pythagoras in founding European philosophy and European mathematics, endowed them with the luckiest of lucky guesses -- or, was it a flash of divine genius, penetrating to the inmost nature of things. . . Finally, our last reflection must be, that we have in the end come back to a version of the doctrine of old Pythagoras, from whom mathematics and mathematical physics, took their rise. (10)

Ernst Cassirer also sees in Pythagoras the progenitor of modern science:

In the times of Pythagoras and the first Pythagoreans Greek philosophy had discovered a new language, the language of numbers. This discovery marked the natal hour of our modern conception of science. . .

The Pythagorean thinkers were the first to conceive number as an all-embracing, a really universal element. It use is no longer confined within the limits of a special field of investigation. It extends over the whole realm of being. When Pythagoras made his first great discovery, when he found the dependence of the pitch of sound on the length of the vibrating chords, it was not the fact itself but the interpretation of the fact which became decisive for the future orientation of philosophical and mathematical thought. Pythagoras could not think of this discovery as an isolated phenomenon. One of the most profound mysteries, the mystery of beauty, seemed to be disclosed here. To the Greek mind beauty always had an entirely objective meaning. Beauty is truth; it is a fundamental character of reality. If the beauty which we feel in the harmony of sounds is reducible to a simple numerical ratio it is number that reveals to us the fundamental structure of the cosmic order. "Number," says one of the Pythagorean texts, "is the guide and master of human thought. Without its power everything would remain obscure and confused." We would not live in a world of truth, but in a world of deception and illusion. In number, and in number alone, we find an intelligible universe. . .

In this general methodological ideal we find no antagonism between classical and modern physics. Quantum mechanics is in a sense the true renaissance, the renovation and confirmation of the classical Pythagorean ideal. (11)

But Pythagoras is not the only one among the ancient Greeks to whom modern scientists and philosophers of sciences are looking back for inspiration. In the question of the mathematical interpretation of nature he is made to share his honors with Plato:

An intense belief that a knowledge of mathematical relations would prove the key to unlock the mysteries of the relatedness within Nature was ever at the back of Plato's cosmological speculations. . .

His own speculations as to the course of nature are all founded upon the conjectural application of some mathematical construction. . .

Plato's mathematical speculations have been treated as sheer mysticism by scholars who follow the literary traditions of the Italian Renaissance. In truth, they are the products of genius brooding on the future of intellect exploring a world of mystery. (12)

The Platonic doctrine on the question of mathematical physics is considerably more difficult to define than the Pythagorean. For in the time that had elapsed between Pythagoras and Plato the development of the philosophical mind had gone a long way; it had gone far enough to reach a high degree of complexity, but not far enough to reduce this complexity to the clarity of an accurately defined and well articulated system. Historians have presented the position of Plato in a way which makes it appear extremely paradoxical. On the one hand, it is often identified with that of Pythagoras. It is in this way that it is presented by Emile Meyerson: "Pour Platon, le fin fond de la nature, ce que nous appelons actuellement, d'un terme kantien, la chose en soi, est mathématique et n'est que mathématique. (13) Tout le réel se compose uniquement de figures de géométrie." Since mathematics is in a sense the most perfect form of rationality for the human mind, it would seem to follow that for Plato nature was in itself something perfectly rational. And Meyerson seems to accept in substance this inexorable consequence, for he writes: "Platon. . . croyait fermement à l'explicabilité de l'univers. . . Pour lui, en effet, la régularité de la nature, sa légalité, n'était précisément qu'un corollaire de cette

(14)
rationalité.

On the other hand, nature would seem to have been in a sense completely irrational for Plato, for he held that no true science (episteme) of it was possible. About the material universe man could have only opinion (doxa).⁽¹⁵⁾ And it has been customary to draw a sharp contrast between the irrationality of the universe of Plato and the rationality of the universe of Aristotle, who made a science of nature possible by incarnating, so to speak, the Platonic ideas in the world of sense. The paradox could scarcely be more incisive; on the one hand the transparent intelligibility of mathematics, the most rational of all the sciences; on the other an unintelligibility so complete as to preclude the possibility of any true science.

We are evidently faced here with the traditional problem of the conflict between the rationality and the irrationality of the cosmos which had been so acute for the philosophers who had preceded Plato, especially Heraclitus and Parmenides. In a sense it is this conflict that is at the bottom of the problem we are undertaking to solve. But we feel that in so far as Plato himself is concerned the paradox has been rendered more acute than it actually is by the more or less arbitrary oversimplifications of historians.

In the first place, though it is true that Plato borrowed heavily from the Pythagoreans, his position cannot be

identified with theirs. The impact upon the Platonic physics of other systems, especially that of Heraclitus, was too strong to allow such an identification. (16) For Plato the mathematical world was not realized as such in the world of sense; the ideal mathematical forms were not given in nature, but merely suggested by it, in so far as nature in some more or less obscure way participated in them. The world of mathematics was not simply immanent in the physical world, but to some extent transcendent from it. Yet it was not so far removed from it as the world of pure ideas. It occupied, in fact, a kind of intermediary position between the ideas and the world of changing things. That is why the mathematical forms were realized in nature more easily and more perfectly than the other ideas. But at the same time this realization came from without.

The following passage of Aristotle brings out the difference between the position of Plato and that of the Pythagoreans:

But he agreed with the Pythagoreans in saying that the One is substance and not a predicate of something else; and in saying that the Numbers are the causes of the reality of other things he agreed with them; but positing a dyad and constructing the infinite out of great and small, instead of treating the infinite as one, is peculiar to him; and so is his view that the Numbers exist apart from sensible things, while they say that the things themselves are Numbers, and do not place the objects of mathematics between Forms and sensible things. His divergence from the Pythagoreans in

making the One and the Numbers separate from things, and his introduction of the Forms, were due to his inquiries in the region of definitions (for the earlier thinkers had no tincture of dialectic). . . .(17)

It is clear from this text that the reason why Plato separated the mathematical forms from the physical world was that the absolute, universal, and necessary definitions characteristic of mathematics could not be realized as such in the essentially mutable world of sense. Nevertheless, physical reality in some way participated in these mathematical forms, and it seems that for Plato our knowledge of nature could approximate to the true scientific knowledge that is characteristic of the intelligible world in so far as it could take on the form of precise measurement and mathematical formulation. In the Philebus (18) for example, he distinguishes between the arts "which have a greater participation in true scientific knowledge and those which have less." And to illustrate his point he says, "If we took away the numbering and measuring and weighing from all the arts, what would be left in each case would be called a poor thing..."

Ernst Cassirer has characterized the position of Plato in the following terms:

It is rooted in Plato's interpretation of mathematics, which is for him the 'mediator' between the ideas and the things of sense. The transformation of empirical connections into ideal ones cannot take place without this middle term. The first and necessary step throughout is

to transform the sensuous indefinite, which as such cannot be grasped and enclosed in fixed limits, into something that is quantitatively definite, that can be mastered by measure and number. It is especially the later Platonic dialogues, as for example the Philebus, which most clearly developed this postulate. The chaos of sense perception must be confined in strict limits, by applying the pure concepts of quantity, before it can become an object of knowledge. We cannot rest with the indefinite 'more' or 'less', with the 'stronger' or 'weaker', which we think we discern in sensation, but we must strive throughout for exact measurement of being and process. In this measurement, being is grasped and explained (cf. Philebus, 16, 24f) Thus we stand before a new ideal of knowledge, one which Plato himself recognized as in immediate harmony with his teleological thought, and combining with it a unified view. Being is a cosmos, a purposively ordered whole, only in so far as its structure is characterized by strict mathematical laws. The mathematical order is at once the condition and the basis of the existence of reality; it is the numerical determinateness of the universe that secures its inner self preservation. (19)

Plato's doctrine here, as in so many questions, is far from being easily definable. But perhaps enough has been said to show that his position can be identified with that of Pythagoras only by considerable oversimplification. On the other hand, it is perhaps an even greater oversimplification to draw the contrast between him and Aristotle so incisively that the Peripatetic world appears as something completely rational and the Platonic world as something completely irrational. We shall point out later what a large part the paralogon played in the system of Aristotle. It was precisely because of the irrationality he saw in the cosmos that he conceived of mathematical physics as a scientia media, an intermediary science

in which it was necessary to reach out beyond the realm of physics to that of mathematics in order to rationalize nature. Paradoxical as it may appear, the Aristotelian cosmos is at once both less rational and more rational than the Platonic, and the solution of this antinomy lies in a distinction between two types of rationality. We consider this distinction to be of capital importance; it will, in fact, be one of the keys for the solution of our whole problem.

The first type of rationality is that proper to the physical world itself. It is a rationality that arises out of the existence of foci of intelligibility in the obscure mass of materiality, of rallying points of intellectual stability in the flux of contingency. Because the mind can discover and disengage these intelligible forms, in a confused way at least, a science of nature in the strict sense of the word, in the sense of episteme, is possible. It would seem that Plato never arrived at the realization of this possibility, and it remained for Aristotle to found the philosophy of nature. From this point of view, the Platonic cosmos was irrational; it was the Heraclitean cosmos of change and obscurity. Of it the mind could not have true episteme, but only doxa.

The second type of rationality is the mathematical rationality of which we have already spoken. From this point

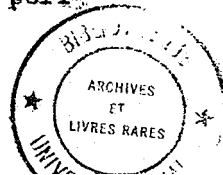
of view the Platonic world was extremely rational. For even though in the scheme of Plato nature was not composed intrinsically of mathematical forms, and the process of mathematization came in some way from without, nevertheless nature was profoundly mathematical in the sense of being highly amenable, perhaps indefinitely amenable, to this process of mathematization. Professor A. E. Taylor sums up Plato's doctrine on this point in the following terms:

The identification of the forms ($\xi\iota\delta\eta$) with numbers means that the "manifold" of nature is only accessible to scientific knowledge in so far as we can correlate its variety with definite numerical functions of "arguments". The "arguments" have then themselves to be correlated with numerical functions of "arguments" of higher degree. If this process could be carried through without remainder, the sensible world would be finally resolved into combinations of numbers, and so into the transparently intelligible. This would be the complete "rationalization" of nature. The process cannot in fact be completed, because nature is always a "becoming", always unfinished; in other words, because there is real contingency. But our business in science is always to carry the process one step further. We can never completely arithmetise nature, but it is our duty to continue steadily arithmetising her. "And still beyond the sea there is more sea"; but the mariner is never to arrest his vessel. The "surd" never quite "comes out", but we can carry the evaluation a "place" further, and we must. If we will not, we become "ageometretes". (20)

Plato seems to have considered this mathematization as the revelation of a logos that was proper to nature. That is why in his system mathematical rationality could supplant physical rationality, and his mathematical interpretation of nature become a philosophy of nature. From this

point of view, Aristotle's attribution of mathematicism to the ~~Platonists~~ would seem to apply to Plato himself: "Mathematics has been turned by our present day thinkers (21) into the whole of philosophy".

Aristotle's discovery of the physical rationality of nature did not make him lose sight of two important facts. The first fact was that this rationality is only partial, indeed extremely meager. He too recognized a doxa of nature along with the episteme he had discovered. As we have already suggested, and as we shall explain more fully later, it is only as long as the mind remains in generalities that it is able to lay hold of an objective logos of nature with certitude; and as it follows its natural development towards fuller concretion, this certitude very quickly fades into a dialectical knowledge that is similar to the Platonic doxa. The second fact was that Aristotle also recognized the part played by mathematical rationality in the study of nature. Indeed, one of the main objectives of this study is to show with what clarity and precision he recognized it. But we shall not take time out now in this brief historical sketch to set forth his position on this point. For besides the general fact that all that is to follow will be an explanation and development of it, we intend later in this chapter to give special attention to the question of the relevance of peri-



pateticism in the problem of mathematical physics. Let it suffice for the moment to have pointed out why the Aristotelian cosmos was at once both more rational and less rational than the Platonic. The universe of Plato seems to have been completely rational from the mathematical point of view, at least in the sense of being indefinitely amenable to mathematization. It was at the same time completely irrational from the purely physical point of view. The universe of Aristotle was at once partially rational and partially irrational from both points of view.

Another interesting paradox emerges from a comparison of the positions of Plato and Aristotle. In the doctrine of Plato the mathematical world is closer to the physical world and at the same time farther away from it than in the doctrine of Aristotle. It is closer to it for the reasons just indicated: for Plato the physical world is indefinitely amenable to mathematization, and this mathematization is a revelation of a logos that is proper to nature; for Aristotle only one aspect of nature is susceptible of the application of mathematics, and even with regard to this one aspect, the application always remains essentially extrinsic in the sense of providing only a substitute rationality.

The mathematical world is at the same time farther

away from the physical world in the position of Plato than in that of Aristotle. In separating the mathematical world from the physical world with which it was identified in the doctrine of the Pythagoreans, Plato gave to it an ontological existence that was independent of the material cosmos. Aristotle also separated the mathematical world from the physical world, but in doing so he gave it only a conceptual existence. For him mathematical forms are abstracted by the mind from the quantitative determinations of the material cosmos. As such they can exist only in the mind. In so far as ontological existence can be attributed to them at all, this existence must be found in the material cosmos. (22) But they can have this existence only at the expense of being robbed of the specific state of abstraction that is proper to them, and that is why, in themselves, they always remain essentially extrinsic to nature. Since, then, the mathematical forms of Aristotle have no ontological existence apart from sensible things and always have an essential physical reference they are closer to the physical world than those of Plato. But since the abstraction that is proper to them makes it impossible for their properties to be attributed to the things of nature, they are at the same time farther away from the physical world.

It is clear, then, why Aristotle was justified in

claiming that the Platonists had turned mathematics into the whole of philosophy. For because of the closeness of the mathematical world to the physical world in the doctrine of Plato, his physics was a kind of mathematical physics. On the other hand, because of the ontological existence attributed to the mathematical world, his mathematics took on a metaphysical character, and to that extent his metaphysics was a kind of mathematical metaphysics. That is why so much of his speculation about reality, whether physical or metaphysical, is involved in mathematics. And that is why on the face of things his system might appear as the best philosophical explanation of the mathematical interpretation of nature. But we feel that a deeper analysis will reveal that this is not true. For his mathematical physics is far from being the mathematical physics of modern science. Strange as it may seem, the very proximity of his mathematical world to the physical world prevents his doctrine from being the true explanation of modern mathematical physics. On the other hand, the very fact that he invested the mathematical world with an ontological existence of its own drew mathematics out of its proper sphere and away from its proper function, and got it involved in intellectual situations alien to its true character and to the role it plays in modern science.

The following lines of Professor Strong are extremely

pertinent here:

To substitute mathematical objects for the "fiction" of Forms makes ideal and mathematical number the same and destroys the distinction by which mathematical number is valid no matter what metaphysical theory of the universe is advanced; "for they state hypotheses peculiar to themselves and not to those of mathematics." (Aristotle: Met.XIII, 1086 a 9)

The hypotheses in respect to the metaphysical status of number are peculiar to metaphysics and not to mathematics. To make ideal and mathematical number the same is a verbalism, a figurative way of speech disguising the fact that the ideal number is not the mathematician's science nor the use of mathematics in dealing with physical phenomena. Optics, music, and astronomy are open to mathematical treatment or involve a mathematical element. Their subject-matter is mathematically formulable, because objects can be designated by number and can present quantitative aspects. Further to posit mathematical objects and relations as having substantial existence not only does not advance mathematical science, but also results in a confusion of mathematical procedures and properties with the first principles of being. . .

Plato, if we may judge from Aristotle's account proposes a scientific myth. Aristotle would object to identifying mathematics, the demonstrative science, with the conjectural theories of existential number; at least he objects to supposing that "ideal" mathematical number is, in fact, what mathematics is before going to the length of paying it metaphysical compliments.

If we suppose that God is a geometer who geometrizes continually, we have carried mathematical certainty to the throne of metaphysical or theological certainty. It will thence be delivered back to us in the creation of things, by figure and number. It will enter into knowledge, since the soul itself will be a number. What actually returns in the philosopher's account is the discretion and classification of Intelligences, Ideas, the soul, and the existences which make up the world after the patterns, paradigms exemplars, divine or seminal numbers in the mind of God. The procedures without which there is no demonstrative science do not come back from this journey. Numbers and figures are valued in respect to their reality and this depends upon their status in respect to God and not to mathematical use. In the face of such a transformation, arithmetic and geometry are pro-paedeutic to theological arithmetic, ancillary sciences for a kind of superscience in which they become metaphores and analogues. (23)

As has already been noted, it is being frequently urged by contemporary philosophers of science that the doctrine of Plato and the Platonic tradition are the metaphysical forebear of modern mathematical physics.

"In modern times," writes Cassirer, "mathematical physics first seeks to prove its claims by going back from the philosophy of Aristotle to that of Plato."⁽²⁴⁾ This

claim might mean several things. In the first place, it might mean that historically it was the platonic tradition that actually gave birth to modern mathematical physics, that it provided the metaphysical basis and the intellectual impetus which brought about its origin and development. It is in this way that the claim is understood by many modern critics, and Professor Burtt, among others, has gone to some lengths in his Metaphysical Foundations of Modern Physical Science to give it substance.⁽²⁵⁾

We do not think that the claim, understood in this sense, has as much importance as might first appear. For history is not logic; nor, generally speaking, is its development shaped by per-se determined causes. There is consequently no reason why a philosophical system which is wholly inadequate to explain the true meaning of mathematical physics might not have been the actual historical impetus which brought about the origin of modern physical science.

Yet it is interesting to note that an accurate and detailed study of this question recently undertaken by Professor Strong has made the claim that the Platonic tradition sired modern science appear extremely dubious. Strong undertook this study with the intention of consolidating the opinion of Burtt, but all the evidence that emerged from a close examination of the work of the scientists of the early-modern period forced him to arrive at the opposite conclusion. In his Procedures and Metaphysics he writes:

A Pythagorean-Platonic (or Neo-Platonic) conception of mathematics is regarded by some present-day critics as the realistic and rationalistic doctrine of a mathematical structure of nature. This may mean that we are today (in the light of contemporary Platonic scholarship) in a position to establish critically analogies between Plato's writings and prominent characteristics of modern science and philosophy. If, however, it is asserted that the early-modern mathematical investigators based their science upon metaphysical foundations, Platonic or otherwise, the weight of evidence gleaned from a survey of some of the Italian scientists is opposed to such an assertion. The historical problem should here be disentangled from modern critical exposition. By such exposition, it can be maintained that a Pythagorean-Platonic metaphysics is compatible with the mathematical treatment of nature. In the light of historical evidence, however, we may question whether the Platonism of the fifteenth and sixteenth centuries had at that time the role and significance which philosophers now critically assign to it in connection with modern science. The assertion that the Platonic metaphysics laid the foundations for the mathematical science of Galileo is at odds with the positive evidence already presented. Furthermore, it appears highly questionable when the tradition of Platonism is examined. The Neo-Platonic doctrines of Ficino, Giovanni

Pico, and Reuchlin, and of the mathematical writers -- Zamberti, Domenico and Dee -- express metamathematical doctrines carried over from Proclus and his predecessors with additional cabalistic embroideries. If this archaic tradition is characteristic, we are in a position to recall the objections and difficulties raised against Nicomachus, Theon, and Proclus. The main intention of this chapter is to expose the definitely archaic character of the Platonizing tradition of metamathematics preserved in several mathematical writers -- archaic that is, in the sense of its ineptness and nonconnection with the scientific work of the period in which it is reinvoked...

The Neo-Pythagoreans and Neo-Platonists were impressed with the mathematical disciplines, particularly arithmetic. Mathematics is taken over and given a cosmological significance, but the doctrines presented, the metamathematics of Platonizing thinkers, are foreign to the method and use of mathematics. The role attributed to number satisfied the assertions of metaphysics, but these assertions could not be applied or substantiated by either the logic or the practice of the mathematician. The metamathematicians assume a being and function for mathematical objects superior to the subject-matter and procedures of the science proper and assume that this metaphysical status is more real and important. Mathematics and mathematical science could not and were not expected to substantiate the assertion that one could by mathematics mount to a knowledge of a superior realm of being; yet a propaedeutic value was supposed to lie in this initiative capacity of mathematical study. The converse of this assertion is equally unsubstantiated, namely, that he who knows the mysteries of ontological and cosmological number-forms is able to penetrate into the inner significance of natural things. This is not a hypothesis for mathematical procedure. The basic supposition is the notion that natural things are the created copies of a creating form, inferior effects in an individual of a superior, unitary cause. Thus, although the metamathematicians employed a number-symbolism, the symbolism stood for forms and efficacies not mathematically conceived. . . .

It is a sobering reflection to consider how long the Pythagorean arithmology and its constitution in the Neo-Platonic system persisted in claims unsubstantiated in fact. Demands of logical and doctrinal consistency were satisfied so far as the purpose and end of the metaphysician were concerned. To suit a metaphysical purpose, mathematics was thrown into a status and assigned a role divorced from mathematical conception and meaningless for procedure. The

metaphysical and of cosmological status and divine residence was assumed to be the goal for which mathematics was preparatory as an intellectual purification; and since the One is causal of the many and the archetypal number-form is the unity of the individual, created thing, the use of mathematics is supposed to depend upon the constitution of natural things by the metamathematical patterns. Modern mathematical-physical science established its method and achieved its results in spite of, rather than because of, this kind of metamathematical tradition. Had the early-modern mathematical investigators in general, rather than by exception, taken the philosophical tradition seriously, history might have seen more mixtures of metaphysics and science similar to Kepler's, without, perhaps, the saving conditions that brought Kepler's metaphysical predispositions to a scientific issue. (26)

But the modern critics' insistence upon the relevance of the doctrine of Plato for modern science might also be taken to mean that among all philosophical systems, or at least among those which have come down to us from antiquity, this doctrine provides the most adequate explanation of the true meaning of mathematical physics. Understood in this sense, the claim is of extreme importance. And it is the purpose of this study to dispute its validity. But in doing so we have no intention to minimize the genius of Plato or his contributions to the philosophy of science. In his doctrine the philosophical mind made a great advance towards providing the true explanation of the mathematical interpretation of nature. The concept of the world of mathematics as occupying a kind of intermediary position between the physical world and the world of pure ideas was a significant contribution. Even more significant was the corollary that naturally flowed from it;

the mathematization of the cosmos is in some sense imposed upon nature from without. Moreover, there are a number of striking analogies between prominent features of modern science and points of Platonic doctrine. The view now generally accepted by the best scientists and philosophers that experimental science can never give more than probable knowledge would seem to be a confirmation of the Platonic doxa. The increasingly evident fact that modern science is essentially constructed of idealizations, that is to say of ideal forms and limit cases which are not given in nature but merely suggested by it, that scientific laws are not discovered in the objective universe but imposed by the mind in its attempt to rationalize experience would seem to be reminiscent of the Platonic doctrine of the relation between ideas and physical reality. Out of this mathematization and rationalization of experience through the process of idealization has come the ever increasing use of hypothesis, which played such an essential role in the method of Plato. ⁽²⁷⁾ And there would seem to be something kindred to Platonism in the a priori character of the modern scientific world which is made up so largely of constructs of the mind. All of these points are significant, but we do not feel that they suffice to constitute the doctrine of Plato as an adequate philosophy of science.

Continuing now our historical sketch, we find that in the middle ages the problem of the mathematical inter-

pretation of nature received comparatively little attention, though, as we shall see, its true nature was far from being ignored by the Thomistic school. Grosseteste at Oxford seems to have had considerable interest in the possibilities of mathematical physics. We are told that he tried to reduce all the sciences of nature to the one universal science of optics, that he considered mathematical principles as the key to all knowledge of the physical universe, and consequently tried to explain natural phenomena in terms of geometrical lines, figures and angles. This same interest is found in Roger Bacon, who in this, as in so many ways, anticipated the so-called modern mind. Bacon held that the book of nature is written in the language of geometry, and that mathematics is "the alphabet of all philosophy." How accurately he had conceived the mathematico-observational method of modern physics may be gathered from the following lines:

It is true that mathematics possesses useful experience with regard to its own problems of figure and number, which apply to all the sciences and experience itself, for no science can be known without mathematics. But if we wish to have complete and thoroughly verified knowledge, we must proceed by the methods of experimental science. (28)

With the dawn of the early modern period a new, spontaneous enthusiasm for mathematics began to make itself manifest. And this gravitation of the mind towards mathematical science soon became all of a piece with the general pattern of Renaissance philosophy, which was so profoundly

humanistic. For, as we shall explain later on, mathematics is the most "human" of all the sciences, in the sense that it has the greatest connaturality with the human intellect. It is also the science in which the mind can in some way imitate the a priori and creative character of divine knowledge, and as a consequence it offers to the mind a great measure of autonomy. That is why it was almost inevitable that there should be a natural gravitation towards mathematics in the period of humanism in which the intellect of man tended to become the measure of all things and to that extent necessarily divine, and in which there was such a universal vindication of the complete autonomy of the mind. "Through Copernicus', Kepler's and Galileo's great discoveries," writes Dilthey, "and through the accompanying theory of constructing nature by means of mathematical elements given a priori was thus founded the sovereign consciousness of the autonomy of the human intellect and of its power over nature; a doctrine which became the prevailing conviction of the most advanced minds."⁽²⁹⁾

This gravitation towards mathematics is already found in the doctrine of Cardinal Nicholas of Cusa, in whom were burgeoning practically all the trends which were subsequently to give direction to the development of the modern mind. He held that "knowledge is always measurement",⁽³⁰⁾ that "number is the first model of things in the mind of the Creator",⁽³¹⁾ and that "there is nothing certain in our

knowledge except mathematics." (32) From these principles he derived the idea of a universal mathematical structure and determination of reality, or a reality whose spiritual core and origin is revealed in its being the subject of universal laws, laws of number and magnitude." (33)

In the early modern period the one who grasped most clearly the significance of mathematics for the study of nature was undoubtedly Leonardo da Vinci. For Leonardo science was genuine only in the measure in which it was mathematical. "No human investigation can call itself true science unless it proceeds through mathematical demonstrations." "There is no certainty in sciences where one of the mathematical sciences cannot be applied, or which are not in relations with these mathematics." (34) "Oh, students, study mathematics, and do not build without a foundation." This enthusiasm for mathematics did not, however, lead him to believe that nature itself is mathematical; he attributed to the mathematical world only conceptual existence: e tuta mentale. And he was insistent upon combining observation with mathematical speculation. "Those sciences are vain and full of errors which are not born from experiment, the mother of all certainty, and which do not end with one clear experiment." (35) That all this was not pure theory in the mind of Leonardo is well known. His important contributions to the development of

mechanics, hydraulics, and optics were an impressive confirmation of his belief in the fruitfulness of the mathematico-observational method.

This method was taken up by Kepler and applied with great success to problems of astronomy. "Astronomy is subordinate to the genus of Mathematical discipline and uses Geometry and Arithmetic as two wings: through them, it considers quantities and figures of mundane bodies and movements, and enumerates times, and in this way prepares its own demonstrations: and it brings all speculations into use or practice." (36)

We have already remarked that there is no conclusive evidence to show that Platonic philosophy provided a foundation for the scientific work of any of the early-modern scientists. It might seem, however, that a case could be built up for Kepler. For his writings are saturated with a deep conviction that the cosmos is made up of hidden mathematical harmonies, a conviction that seems impregnated with the quasi mystical attitude of the Pythagoreans and Neo-Platonists, which attached a recondite religious significance to the mathematical character of reality. "Geometry," he writes, "was the form of creation and entered into man with the image of God" (37)

There can be no doubt that a great deal of philosophical reflection distinctively Neo-Platonic in tone accompanied the scientific work of Kepler, but it remains extremely questionable to what extent, if any, the former provided a foundation

for the latter, or exercised any true causal influence
(38)
upon it.

In the work of Galileo the mathematico-observational method became a well-defined scientific procedure. In his famous experiment of rolling a ball down an inclined plane at the tower of Pisa and of describing the phenomenon in terms of a mathematical equation, modern scientific method was clearly crystallized. And he pointed out the fundamental principle of this method when he wrote: "To be placed on the title-page of my collected works: Here it will be perceived from innumerable examples what is the use of mathematics for judgments in the natural sciences and how impossible it is to philosophize correctly without the guidance of Geometry, as the wise maxim of
(39)
Plato has it." "Philosophy is written in that great book which ever lies before our eyes -- I mean the universe -- but we cannot understand it if we do not first learn the language and grasp the symbols, in which it is written. This book is written in the mathematical language, and the symbols are triangles, circles, and other geometrical figures, without whose help it is impossible to comprehend a single word of it; without which one wanders in vain through a
(40)
dark labyrinth."

All scientific method involves selection, and it was

inevitable that the growing consciousness of the fruitfulness of mathematics in the explanation of natural phenomena should result in an increasing concentration of attention upon the quantitative aspects of nature. But scientific methods all too easily tend to become tyrannical, and what begins as a mere selection for the purpose of explaining phenomena often issues into an explaining away of the elements left out of the selection. Galileo was probably the first in modern times to call into question the existence of the non-quantitative aspects of reality. Kepler seems to have supposed that the non-mathematical properties of nature were in some way less real, but he did not deny their objective existence. This denial is found explicitly in Galileo, for whom the qualitative properties of nature had existence as such only in the faculties of man.

I feel myself impelled by necessity, as soon as I conceive a piece of matter or corporal substance, of conceiving that in its own nature it is bounded and figured by such and such a figure, that in relation to others it is large or small, that it is in this or that place, in this or that time, that it is in motion or remains at rest, that it touches or does not touch another body, that it is single, few or many; in short by no imagination can a body be separated from such conditions. But that it must be white or red, bitter or sweet, sounding or mute, of a pleasant or unpleasant odour, I do not perceive my mind forced to acknowledge it accompanied by such conditions; so if the sense were not the escorts perhaps the reason or the imagination by itself would never have arrived at them. Hence I think that those tastes, odours, colours, etc. on the side of the object in which they seem to exist, are nothing else but mere names, but hold their residence solely

in the sensitive body; so that if the animal were removed, every such quality would be abolished and annihilated. (41)

This quantification of nature found its full realization in the philosophy of Rene Descartes.

It has been customary to consider Descartes as the philosopher of modern mathematical physics. Meyerson writes: "C'est Descartes, incertestablement, qui a été le véritable législateur de la science moderne." (42)

This opinion is shared by Maritain:

...il (Descartes) a eu la claire vue intellectuelle du constitutif propre et des droits de la science physico-mathématique du monde, avec toutes ses exigences et, si je puis dire, sa férocité de discipline originale, d'habitus irréductible. IL mérite vraiment, à ce point de vue, d'être regardé comme le fondateur de la science moderne, non qu'il l'ait créée de toutes pièces, mais parce que c'est lui qui l'a tirée à la lumière du plain jour et établie à son compte dans la république de la pensée. (43)

We believe that this passage is filled with errors and ambiguities. It will eventually become clear, we hope, that Descartes' intellectual view of the "constitutif propre" of mathematical physics was extremely confused and profoundly erroneous. As a consequence he could have no just notion of its rights and exigencies. As a matter of fact, the extent to which he exaggerated them was nothing less than monstrous. Since mathematical physics is, as we shall see, an intermediary science, and since it is, in fact, not a science in the strict and formal sense

of the word, but dialectics, nothing could be more false than to apply to it the terms "discipline originale" and "habitus irréductible". Much could be said, moreover, in criticism of the expression "république de la pensée" for taken as it stands it could easily lead to a false notion of the independence of the sciences, but this is not the place to develop such a criticism.

We do not believe that Descartes deserves to be called the founder of modern science. Nevertheless, his doctrine had an extremely important historical influence upon the development of mathematical physics and for that reason it merits considerable attention.

For Descartes the mathematization of nature was not a mere scientific method; it was a world vision. The story of how that vision came to him on that winter's night at Newburg on the Danube is one of the best known events in the history of philosophy. It had been preceded by another great discovery which was to play an all important part in the fruitful development of mathematical physics -- the discovery of Analytical Geometry. Having succeeded in reducing geometry to arithmetic and algebra, in spite of the fact that the aristotelians had always insisted on their formal distinction, the next step was to reduce physics completely to mathe-

matics. It was a tremendous step, but Descartes did not hesitate to take it. In actual fact he went much farther than this and reduced the whole of philosophy to mathematics in the sense that his universal method was the geometrical method of beginning with a clear and distinct intuition and proceeding by means of deduction. All this lay behind the "Cogito." That is why his whole philosophy may be considered a kind of mathematicism. But we are not interested in this aspect of Cartesianism here.

The vision of which we have spoken is summed up in the epitaph written by his closest friend, Chanut: "In his winter furlough comparing the mysteries of nature with the laws of mathematics he dared hope that the secrets of both could be unlocked with the same key." And he has himself described this vision for us in the following terms:

As I considered the matter carefully it gradually came to light that all those matters only are referred to mathematics in which order and measurement are investigated, and that it makes no difference whether it be in numbers, figures, stars, sounds, or any other object that the question of measurement arises. I saw consequently that there must be some general science to explain that element as a whole which gives rise to problems about order and measurement, restricted as these are to no special subject matter. This, I perceived, was called universal mathematics.

Such a science should contain the primary rudiments of human reason, and its province ought to extend to

the eliciting of true results in every subject. To speak freely, I am convinced that it is a more powerful instrument of knowledge than any other that has been bequeathed to us by human agency, as being the source of all others. (44)

Having once laid down this principle, Descartes did not hesitate to follow its consequences to the very end. "My whole physics," he wrote to his friend Merseenne, "is nothing but geometry." (45) "I accept no principles in physics which are not at the same time accepted in mathematics." And he goes on to explain:

Nam plane profiteor, me nullam aliam rerum corporearum materiam agnoscere, quam illam omnimode divisibilem, figurabilem et mobilem quam Goemetrae quantitatem vocant et pro objecto suarum demonstrationum assumunt; ac nihil plane in ipsa considerare, praeter istas divisiones, figuras et motus; nihilque de ipsis ut verum admittere, quod non ex communibus illis notionibus de quarum veritate non possumus dubitare, tam evidentur, deducatur, ut pro mathematica demonstratione sit habendum. Et quia sic omnia Naturae phaenomena possunt explicari, ut in sequentibus apparebit, nulla alia Physicae principia puto esse admittenda, nec alia etiam optanda." (46)

The immediate and necessary consequence of the transformation of physics into mathematics was the identification of the nature of bodies with extension, of matter with quantity. What is matter, asks Descartes in the Principia. And his answer is that "its nature consists neither in hardness, nor in weight, nor in heat, nor in any other qualities, but only in extension in length, breadth, and depth, which the geometricians call quantity." "Those

who distinguish between material substance and extension or quantity, either have no real idea corresponding to the name of substance, or else have a confused idea of material substance."⁽⁴⁷⁾

Motion had traditionally been the main stumbling block for those who had tried to mathematicize nature. Aristotle's criticism of the Pythagoreans and Platonists had been that mathematization means the exclusion of movement, and he who is ignorant of movement cannot understand nature. And Saint Thomas had said: "Ex mathematicis non potest aliquid efficaciter de motu concludi."⁽⁴⁸⁾ This problem proved no obstacle to Descartes. He was convinced that even movement could be mathematicized, not in the sense in which it would be mathematicized later by the calculus of Newton and Leibniz, but in a sense far more radical. Descartes thought that motion was in its very essence mathematical, that in the last analysis it could be reduced to the displacement of a point on a plane. And this seemed so evident to him, and the nature of motion seemed so immediately clear that he scorned the definition of Aristotle whose profundity appeared to him to be nothing but the obscuration of something essentially simple and transparent.

Some modern philosophers find in this difference

in the concept of motion the best expression of the difference between the ancient and the modern mind. Thus, M. Brunschvicg believes that in the modern concept of motion "une forme de l'intelligence apparait, qui remplace une autre forme de l'intelligence, avec qui elle est sans aucun rapport." (50) Whatever may be thought of this view, it is certain that in this difference between the obscurity of the Aristotelian definition of motion and the clarity of Cartesian motion we have a striking symbol of the vast change wrought by Descartes in the history of philosophy. Reality, which for the Greeks and the medievalists had always been something profoundly complex, suddenly became transparently clear. This is a very significant point.

But in a particular way, we find in this question of motion the sharpest contrast between Aristotelian and Cartesian physics. In fact, a more incisive antinomy could hardly be imagined. For Aristotle movement was a becoming; for Descartes it was a state; For Aristotle it was a process; for Descartes it was a relation. For Aristotle it was self-evident that because of the principle of inertia the cessation of a body in motion demanded a cause. We shall return to this antinomy in the course of our analysis.

With these two clear intuitions of matter and motion as points of departure, Descartes set out to deduce the whole cosmos even to its smallest detail. He felt confident that with matter and motion alone he could construct the world. In commenting upon this attempt of Descartes, Duhem writes:

Ainsi, dans tout l'univers, est répandue une matière unique, homogène, incompressible et indilatable dont nous ne connaissons rien sinon qu'elle est étendue; cette matière est divisible en parties de diverses figures, et ces parties peuvent se mouvoir les unes par rapport aux autres; telles sont les seules propriétés véritables de ce qui forme les corps; à ces propriétés doivent se ramener toutes les apparentes qualités qui affectent nos sens. L'objet de la Physique cartésienne est d'expliquer comment se fait cette réduction.

Qu'est-ce que la gravité? L'effet produit sur les corps par des tourbillons de matière subtile. Qu'est-ce qu'un corps chaud? Un corps composé de petites parties qui se remuent séparément l'une de l'autre d'un mouvement très prompt et très violent. Qu'est-ce que la lumière? Une pression exercée sur l'éther par le mouvement des corps en flammes et transmise instantanément aux plus grandes distances. Toutes les qualités des corps, sans aucune omission, se trouvent expliquées par une théorie où l'on ne considère que l'entendu géométrique, les figures qu'on y peut tracer et les divers mouvements dont ces figures sont susceptibles. 'L'univers est une machine en laquelle il n'y a rien du tout à considérer que les figures et les mouvements de ses parties.' Ainsi la Science entière de la nature matérielle est réduite à une sorte d'Arithmétique universelle d'où la catégorie de la qualité est radicalement bannie." (51)

When he had finished his task, Descartes stopped to contemplate it with pride and satisfaction, and he declared that nothing was lacking, that his work was perfect.

One of the last paragraphs in the Principia has as its title: "That there is no phenomenon that is not included in what has been explained in this treatise." (52) It was no slight claim on the part of Descartes to pretend to have a direct intuition of the inner essence of physical reality and to be able to embrace all its phenomena in a type of knowledge that was clear and exhaustive.

The proclamation of Descartes as the founder or legislator of modern mathematical physics is susceptible of a variety of interpretations. It may, in the first place, be taken to mean that his philosophical system affords the truest explanation of the meaning of physico-mathematical knowledge. We believe that any claim of this kind is far from being justified, but it would be premature to embark upon a discussion of this point here. It may also be taken to mean that he formulated with accuracy and clarity the method that has been responsible for the development of modern physics. We do not think that even this claim is admissible. Cartesian physics as a system was extremely short-lived. This in itself is not necessarily a condemnation of cartesian method, for it is possible for a thinker to work out a true scientific method, and yet in spite of it be led into numerous errors in the order of application, and this faulty application may be due to circumstances beyond con-

trol. But in the case of Descartes the errors were for the most part because of his method rather than in spite of it. His physics is a tissue of arbitrary assumptions precisely because he refused to recognize the inductive character of physical science. Modern science is constituted essentially of both a priori and a posteriori elements, and Descartes was as blind to the latter as Francis Bacon was to the former.

Nevertheless there is something to be said for Descartes. His discovery of analytical geometry provided an extremely useful instrument for the mathematization of nature, even though he failed to recognize the true nature of his own creation. But more than that, his ambition of a completely mathematicized physics bequeathed to physicists a dialectical goal towards which they would never cease to strive; to bring all the phenomena of nature under the control of number. That is why it may be said that in the philosophy of Descartes the mathematical interpretation of nature seemed to have received its official charter. From then on there was never any question of the road that physics would follow in its development.

Added to the general inspiration given to mathematical physics by cartesian philosophy, was the tremendous impetus coming from the new discoveries in mathematics;

No picture however generalized of the achievements of scientific thought in this century can omit the advance in mathematics. Here as elsewhere the genius of the epoch made itself evident. Three great Frenchmen, Descartes, Desargues, Pascal, initiated the modern period in geometry. Another Frenchman, Fermat, laid the foundations of modern analysis, and all but perfected the methods of the differential calculus. Newton and Leibniz, between them, actually did create the differential calculus as a practical method of mathematical reasoning. When the century ended, mathematics as an instrument for application to physical problems was well established in something of its modern proficiency. (53)

As a result of the philosophical influence that stemmed from Descartes and of the discovery of more powerful mathematical instruments, the role of mathematics in physics continued to grow with ever increasing fruitfulness. There were a few reactionary attempts made, particularly in Germany by Goethe, Schelling and Hegel, but they had no lasting success, and left behind them no positive trace in science.

In the physics of Newton the mathematical interpretation of nature seemed to have reached its crowning achievement. "The outstanding fact that colors every other belief in this age of the Newtonian world," writes Randall, (54) "was the success of the mathematical interpretation of nature." The part that mathematics played in the work of Newton himself is aptly expressed by the title he chose for his classical work, The Mathematical Principles of Natural Philosophy, and by the brief interpretation he gave of its significance in

the preface:

We offer this work as mathematical principles of philosophy . . . By the propositions mathematically demonstrated in the first book, we then derive from the celestial phenomena the forces of gravity with which bodies tend to the sun and the several planets. Then, from these forces, by other propositions which are also mathematical, we deduce the motions of the planets, the comets, the moon, and the sea...(55)

Although throughout his work Newton acted as though in nature there were a possibility of infinite determination, it may be doubted perhaps just what significance he attached to this methodological principle. "To Newton, at any rate," says J.W.N. Sullivan, "the attempt to describe nature mathematically was an adventure that might or might not be successful." (56) And Dingle writes:

In the matter of fitting observations into a mathematical framework, Newton was both more and less thoroughgoing than Galileo. He himself enlarged the framework considerably, so that while to Galileo mathematics was mainly geometry, to Newton geometry occupied only a subordinate place. Thus he was able to conduct a mathematical treatment of the phenomena of colour which Galileo had relegated to the rank of a subjective quality. On the other hand, he did not regard the whole of external Nature as necessarily mathematical in character, although he hoped it might prove to be so. (57)

It would be too long and tedious to trace the subsequent development of mathematical physics in full detail. Much could evidently be said about Leibniz whose doctrine, in so far as it related to the physical universe, was, in the last analysis, a kind of mathematicism. Much could be said in

particular about Kant, whose *Transcendental Aesthetics* deals with the question of pure mathematics, and whose *Transcendental Analytic* is an explanation of the mathematical science of nature. One of the greatest contemporary philosophers of physical science, Sir Arthur Eddington, has this to say about the doctrine of Kant:

If it were necessary to choose a leader from among the older philosophers, there can be no doubt that our choice would be Kant. We do not accept the Kantian label; but, as a matter of acknowledgment, it is right to say that Kant anticipated to a remarkable extent the ideas to which we are now being impelled by the modern developments of physics. (58)

We shall not stop to evaluate this statement now, nor to discuss in detail the relation of mathematical physics to the philosophy of Kant. This we hope to do in chapter XIII. By that time we shall be in a position to see how many large concessions must be made to Kantianism if we are to understand the true nature of physico-mathematical knowledge. For the present let it suffice to point out that Kant considered Newtonian physics as the only genuine type of science, and that there is a sense in which it is true to say that he made it the foundation of his whole elaborate philosophical system. From the following lines it is evident that for him the physical world can be known scientifically only through mathematics:

Les suppositions de la géométrie ne sont pas des déterminations d'une simple création de notre fantaisie poétique, ne pouvant ainsi être rapportées avec certitude

à des objets réels, mais elles sont nécessairement valables pour l'espace, et par suite pour tout ce qui peut se rencontrer dans l'espace, parce que l'espace n'est pas autre chose que la forme de tous les phénomènes extérieurs sous laquelle des objets des sens peuvent nous être donnés. La sensibilité sur la forme de laquelle se fonde la géométrie, est ce dont dépend la possibilité des phénomènes extérieurs; ceux-ci ne peuvent donc jamais renfermer autre chose que ce que la géométrie leur prescrit. (39)

For Kant space and time which are the a priori forms that determine all our scientific knowledge of the material world are reducible to the abstract concepts of continuous and discrete quantity. In his First Metaphysical Principles of the science of Nature he writes: "In every particular theory of nature the only thing that is scientific in the strict sense of the word is the quantity of mathematics it contains." (60)

The progress of physics in recent years, particularly since the advent of the theory of relativity, the quantum theory and wave-mechanics, has resulted in a mathematization of nature never dreamed of by even the most enthusiastic of the classical physicists. (61) In one sense at least, the mathematical element seems to be supplanting more and more the purely physical. An obvious example of this is the way in which the problem of gravitation, which in classical physics was a question of dynamics involving the motion of force, has in Einsteinian physics been reduced to a problem of pure geometry. Moreover, in the comparison

with classical physics, the conceptual mathematical implements now being used are of a much more abstract nature, and are taken from what is sometimes known as "pure mathematics". Sir James Jeans sees in this application of "pure mathematics" to the physical universe a new epistemological phenomenon which constitutes a major difference between contemporary and classical mathematical physics. (62)

On the other hand, paradoxical as it may seem, Relativity and Quantum physics are at the same time less mathematical and more physical than classical physics. Cartesian and Newtonian physics were in many ways extremely simplistic. They attempted to impose upon the physical universe absolute quantitative determinations such as they may be conceived of by a mathematician who does not have to worry about concrete physical processes of observation and concrete physical procedures of measurement. Einstein brought to light the vast difference between a pure mathematician and a mathematical physicist by showing how much is involved in the concrete procedures of observation and measurement. As a result, science has been brought closer to the objective physical universe. Moreover, contemporary physics has become less mathematical and more physical in the sense that it has come to realize more clearly that nature overflows any geometrical frame that we may attempt to impose

upon it, that there is a greater irrational element in nature than was suspected before. However, underneath this revolutionary character of contemporary physics there is, of course, a fundamental continuity with the past, as (63) we shall try to make clear later on.

One of the characteristic features of recent physics which is of particular interest to us is its self-consciousness. Classical physics was self-conscious but it was, so to speak, the naive self-consciousness of adolescence. In recent years physical science has begun to achieve the self-consciousness of maturity, which consists chiefly in a detached self-criticism. All of the greatest contemporary mathematical physicists, those who have contributed most to the advancement of science, such as Einstein, Planck, De Broglie, Weyl, Dirac, Heisenberg, Schrödinger, Eddington and Jeans, have felt the need of doing some serious reflective thinking about the nature of their science. This thinking is of unequal philosophical value, to be sure, but out of it has come a wealth of helpful insights into the nature of physical science. At this point we can do no more than select from these contributions a few typical observations on the general nature of mathematical physics. These will be sufficient to situate our problem accurately in its contemporary context, and that is all that interests us for the moment.

But before indicating the characteristic positions taken by some of the more recent mathematical physicists as to the general nature of their science, perhaps it would be worth while to consider here a highly significant passage of one of the most outstanding of nineteenth century biologists, Claude Bernard. Bernard was one of those who made the greatest contributions to the growth of the critical view of science, and his observations on the general character of natural science are of the greatest value:

The absolute principle of the experimental sciences is a necessary and conscious determinism in the conditions of the phenomena. It is of such a sort that a natural phenomenon, whatever it is, being given, the experimenter can never admit that there is a variation in the expression of this phenomenon, unless at the same time there be the intervention of new conditions in its manifestation; moreover, he has an a priori certitude that these variations are determined by rigorous and mathematical connections. Experience simply shows us the form of the phenomena; but the connection of the phenomenon to a determined cause is necessary and independent of experience, and it is necessarily mathematically absolute. We thus see that the principle of the criterion of the experimental sciences is in reality identical with that of the mathematical sciences, since in each of them this principle is expressed by a necessary and absolute relation of things. However, in the experimental sciences these connections are surrounded by numerous, complex, and infinitely varied phenomena, which hide the connections from our view. By the aid of experience we analyze, we dissociate the phenomena, in order to reduce them to relations and conditions that are more simple. We wish in this way to seize the form of scientific truth, that is to say, to find the law which should give us the key to all the variations of the phenomena. This experimental analysis is the only means that we have for searching out the truths in the experimental sciences; and the absolute determinism of the phenomena, of

which we have an a priori consciousness, is the sole criterion or the sole principle which directs and supports us. In spite of our efforts, we are still very far from this absolute truth; and it is probable, especially in the biological sciences that we shall never see it in its nudity. (64)

When scientists speak of the general question of determinism in nature, it is sometimes difficult to know whether they are talking of determinism as a methodological principle or as a physical principle. In fact the two are often enough confused in the mind of the scientists themselves. Determinism is, of course, legitimate and necessary as a methodological principle. Without it there could be no science. But it is evident from the passage just quoted that for Bernard determinism is not merely a method existing in the mind of the scientist and in the process through which he studies nature, but a reality existing in nature itself. In the physical universe is objectively realized the infinite rigor of the mathematical world. This view of Bernard seems to have been the generally accepted opinion of the classical physicists, though among them there was this difference that while for some the infinite determination of nature could be arrived at by science, at least theoretically, for others it was an objective limit towards which science must ever move. The ever increasing success of the application of mathematics to nature tends almost inevitably to lead scientists to some position of this kind, for

as Professor Bridgman has pointed out:

...it is a result of every day experience that as we refine the accuracy of our physical measurements the quantitative statements of geometry are verified within an ever decreasing margin of error. From this arises that view of the nature of mathematics which apparently is more commonly held; namely that if we could eliminate the imperfections of our measurements, the relations of mathematics would be exactly verified. Abstract mathematical principles are supposed to be active in nature, controlling natural phenomena, as Pythagoras long ago tried to express with his harmony of the spheres and the mystic relation of numbers. (65)

And although Heisenberg's principle of uncertainty, which expresses the high degree of indeterminism recently discovered by scientists on the level of microscopic phenomena, has thrown wide open the whole problem of the determination of nature, there are still many scientists who hold that this indeterminism is purely subjective and that it gives no reason for doubting the objective existence of a mathematical determination in the universe.

In the annals of modern science there is no greater name than that of Albert Einstein, and consequently his opinion on the nature of mathematical physics is of the utmost interest. Of the many important statements he has made on the subject the following is perhaps the most significant for us and the most relevant to our present purpose.

On the contrary, the scientists of those times were for the most part convinced that the basic concepts and laws of physics were not in a logical sense free inventions of the human mind, but rather that they were derivable by abstraction, i.e. by a logical process, from experiments.

It was the general theory of Relativity which showed in a convincing manner the incorrectness of this view. For this theory revealed that it was possible for us, using basic principles very far removed from those of Newton, to do justice to the entire range of the data of experience in a manner even more complete and satisfactory than was possible with Newton's principles. But quite apart from the question of comparative merits, the fictitious character of the principles is made quite obvious by the fact that it is possible to exhibit two essentially different bases, each of which in its consequences leads to a large measure of agreement with experience. This indicates that any attempt logically to derive the basic concepts and laws of mechanics from the ultimate data of experience is doomed to failure. If then, it is the case that the axiomatic basis of theoretical physics cannot be an inference from experience, but must be free invention, have we any right to hope that we shall find the correct way? Still more -- does this correct approach exist at all, save in our imagination? Have we any right to hope that experience will guide us aright, when there are theories (like classical mechanics) which agree with experience to a very great extent, even without comprehending the subjects in its depths? To this I answer with complete assurance, that in my opinion there is the correct path, and, moreover, that it is in our power to find it. Our experience up to date justifies us in feeling sure that in Nature is actualized the idea of mathematical simplicity. It is my conviction that pure mathematical construction enables us to discover the concepts and laws connecting them which give us the key to the understanding of the phenomena of Nature. Experience can of course guide us in our choice of serviceable mathematical concepts; it cannot possibly be the source from which they are derived; experience of course remains the sole criterion of the serviceability of a mathematical construction for physics but the truly creative principle resides in mathematics. In a certain sense, therefore, I hold it to be true that pure thought is competent to comprehend the real, as the ancients dreamed.(66)

This passage is so lucid and precise that it scarcely needs a commentary. The important point to be drawn from it is that although the mathematical concepts and principles used in physics are not derived directly from nature, but come

from the productive activity of the mind, nevertheless there exists in the cosmos a basic mathematical structure and through the progress of science the mathematical construction of the mind can ultimately be brought into exact conformity with it.

Allusion has already been made to the views of Sir James Jeans on the significance of the application of mathematics to nature. For Jeans recent developments in physics have produced a new and highly significant epistemological phenomenon: the successful application of "pure mathematics" to the physical universe. In classical physics the use of mathematics had been large and fruitful, but the mathematics used was something that had been previously drawn from nature; it was not "pure mathematics" deriving solely from the creative activity of the intellect. "By 'pure mathematics' is meant those departments of mathematics which are creations of pure thought, or reason operating solely within her own sphere, as contrasted with 'applied mathematics' which reasons about the external world, after first taking some supposed property of the external
(67)
as its raw material." It is this "pure mathematics" which is now used in Relativity and Quantum physics. And the great mystery is that nature seems to conform to these free creations of pure thought:

We could not of course draw any conclusion from this if the concepts of pure mathematics which we find to be inherent in the structure of the universe were merely part of, or had been introduced through, the concepts of applied mathematics which we used to discover the workings of the universe. It would prove nothing if nature had merely been found to act in accordance with the concepts of applied mathematics; these concepts were specially and deliberately designed by man to fit the workings of nature. Thus it may still be objected that even our pure mathematics does not in actual fact represent a creation of our own minds so much as an effort, based on forgotten or subconscious memories, to understand the workings of nature. If so, it is not surprising that nature should be found to work according to the laws of pure mathematics. It cannot of course be denied that some of the concepts with which the pure mathematician works are taken direct from his experience of nature. An obvious instance is the concept of quantity, but this is so fundamental that it is hard to imagine any scheme of nature from which it was entirely excluded. Other concepts borrow at least something from experience; for instance multidimensional geometry, which clearly originated out of the experience of the three dimensions of space. If, however, the more intricate concepts of pure mathematics have been transplanted from the workings of nature, they must have been buried very deep indeed in our sub-conscious minds. This very controversial possibility is one which cannot be entirely dismissed, but it is exceedingly hard to believe that such intricate concepts as a finite curved space and an expanding space can have entered into pure mathematics through any worth of unconscious or sub-conscious experience of the workings of the actual universe. In any event, it can hardly be disputed that nature and our conscious mathematical minds work according to the same laws. She does not model her behaviour, so to speak, on that forced on us by our whims and passions, or on that of our muscles and joints, but on that of our thinking minds. This remains true whether our minds impress their laws on nature, or she impresses her laws on us, and provides a sufficient justification for thinking of the universe as being of mathematical design. Lapsing back again into the crudely anthropomorphic language we have already used, we may say that we have already considered with disfavour the possibility of the universe having been planned by a biologist or an engineer; from the intrinsic evidence of his creation, the Great

Architect of the Universe now begins to appear as a pure mathematician. (68)

It is to be noted that for Jeans the mathematical interpretation of nature gives exhaustive knowledge of it, for he says: "The final truth about a phenomenon resides in the mathematical description of it; so long as there is no imperfection in this, our knowledge of the phenomenon (69) is complete."

If we were to stop at this point and look back over the historical sketch we have been giving, we would find this one central thought running through the various opinions discussed: the fundamental reason why mathematics can be applied to nature is that nature is ultimately mathematical, that in the physical universe there is realized a basic mathematical structure; mathematical physics simply means that in the last analysis mathematics and physics are in some sense identified. Most of the authors we have mentioned would subscribe to the opinion of Juvet: "Sans préciser davantage notre pensée, nous dirons que le monde physique n'est qu'un reflet ou une section du monde mathématique." (70)

But at the present time a large number of authors are advancing an opinion which on the surface at least seems to be directly opposed to the position just stated. For many

modern philosophers of science, mathematics is nothing but formal logic, and the part that it plays in physics has no other significance than the part that logic plays in all the sciences. Vassily Pavlov has summed up this position in the following terms:

It were well, then, to introduce briefly the claim that mathematics at bottom is only logic. To many this claim has been demonstrated for all time in the work of Frege, Peano, Bertrand Russell, A.N. Whitehead, and others, who developed the subject of "symbolic" or "mathematical" logic. Mathematics and formal logic have been declared to be identical. Both have been pictured as vast systems of so-called "tautologies", substitutions, identities, possessing novelty only in a psychological sense. The entire system of mathematics (or logic) is said to be contained in its postulate sets, which are nothing but the "rules of the game", a game conventional to the core, possibly derived from reality but nastily indifferent to it. In short, there has occurred an apotheosis of the rules, the rules without the game.

Many of us are very uncomfortable over the sharp separation which has occurred between the rules of the game and the game itself. Every application of mathematics-logic to nature, then, seems to us a promise of a happy reunion. We return to nature only that which belonged to it in the first place. The mystery, if any, lies in the original separation, rather than in the application.(71)

Taken as it is presented here, this opinion means that mathematics is used in physics merely as an instrument that remains extrinsic to the essence of the science in which it is employed, just as logic is a mere instrument that remains essentially extrinsic to the inner constitution of the sciences which employ it. But it must be noted that not all the authors who teach that mathematics is only a tool in phy-

sics necessarily hold that it is a purely extrinsic instrument. For, as we shall explain presently, it is possible to hold that the mathematics employed in physics constitutes an essential part of the object of physical science and still consider it as purely instrumental in the sense that the whole purpose of physical science is to know the physical universe and not the mathematical world, and consequently the whole raison d'être of the use of mathematics is to enable the mind to come into closer contact with the objective cosmos. Perhaps it is in this light that we must interpret the opinion of Dirac:

From the mathematical side the approach to the new theories presents no difficulties, as the mathematics required (at any rate that which is required for the development of physics up to the present) is not essentially different from what has been current for a considerable time. Mathematics is the tool specially suited for dealing with abstract concepts of any kind and there is no limit to its power in this field. For this reason a book on the new physics, if not purely descriptive of experimental work, must be essentially mathematical. All the same the mathematics is only a tool and one should learn to hold the physical ideas in one's mind without reference to the mathematical form. (72)

It seems quite probable that it is also in this light that the position of Sir Arthur Eddington must be understood. Contrary to the opinion of Jeans, he holds that the physical universe is not mathematical, and that if mathematics enters into physical science it is only because the mind has introduced it from without. Nor can the role of mathematics be

reduced to a question of mere symbolism. Mathematics is able to get a grip on the cosmos because physical reality can by processes of measurement be transformed into series of measure-numbers, and the relation between these measure-numbers can be built up into a mathematical system principally through the instrumentality of the theory of Groups. In The Philosophy of Physical Science he has this to say:

Theoretical physics to-day is highly mathematical. Where does the mathematics come from? I cannot accept Jean's view that mathematical conceptions appear in physics because it deals with a universe created by a Pure Mathematician; my opinion of pure mathematicians, though respectful, is not so exalted as that. An unbiased consideration of human experience as a whole does not suggest that either the experience itself or the truth revealed in it is of such a nature as to resolve itself spontaneously into mathematical conceptions. The mathematics, is not there till we put it there. The question to be discussed in this chapter is, At what point does the mathematician contrive to get a grip on material which intrinsically does not of itself render a subject mathematical. If in a public lecture I use the common abbreviation No. for a number, nobody protests; but if I abbreviate it as N, it will be reported that "at this point the lecturer deviated into higher mathematics". Disregarding such prejudices, we must recognize that the allocation of symbols A, B, C, ... to various entities or qualities is merely an abbreviated nomenclature which involves no mathematical conceptions. (73)

And he goes on to explain how the Theory of Groups is employed in transforming physical science into a mathematical system. (74)

There is still another opinion which in the mind of many of the authors who advance it may not represent anything substantially different from the position of those who hold that

mathematics is nothing more than a logical tool, but which if taken literally amounts to something quite different. It is the view that the role played by mathematics in physics, is that of a universal and extremely convenient language. In so far as it is used in physics, mathematics is just a code, a kind of symbolic language, a sort of esperanto of science. (75) "Mathematics," says Herzfeld, "is only a tool, a short-hand way of expression, but cannot add anything to the physical concept, although it might occasionally suggest a physical law because its mathematical expression might be particularly simple."

For some who hold this opinion, the role of mathematics in physics is reduced to that of a stenographic method; and just as short-hand is a mere substitute for long-hand, and everything it expresses can be expressed with equal fulness and accuracy, though not with equal convenience, by the ordinary mode of writing, so everything contained in a world geometry could, strictly speaking, be expressed in purely "physical language." For others the symbolism of number has advantages over the symbolism of ordinary language which reach far beyond mere convenience, and which are the source of the fruitfulness of the application of mathematics to physics. For the symbolism of ordinary language can represent reality only in a dispersed and isolated way, whereas the symbolism

of number is essentially a relational symbolism and that is why it is able to represent the structure of the universe and thus open up its secrets. Perhaps the clearest expression of this opinion is found in Ernest Cassirer:

The symbols of language themselves have no definite systematic order. Every single linguistic term has a special "area of meaning". It is, as Gardiner says, "a beam of light, illumining first this portion and then that portion of the field within which the thing, or rather the complex concatenation of things signified by a sentence lies." But all these different beams of light do not have a common focus. They are dispersed and isolated. In the "synthesis of the manifold" every new word makes a new start, This state of affairs is completely changed as soon as we enter into the realm of number. We cannot speak of single or isolated numbers. The essence of number is always relative, not absolute. A single number is only a single place in a general systematic order. It has no being of its own, no self-contained reality. Its meaning is defined by the position it occupies in the whole numerical system... We conceive it as a new and powerful symbolism which, for all scientific purposes, is infinitely superior to the symbolism of speech. For what we find here are no longer detached words but terms that proceed according to one and the same fundamental plan and that, therefore, show us a clear and definite structural law. (76)

This view, which at first glance, at least, seems to reduce the role of mathematics in physics to a question of language, differs from the opinion of those who identify

mathematics with formal logic to the extent that language differs from logic, though perhaps the distance between logic and mathematical language would not be so great as that between logic and ordinary language, it might be argued that in the measure in which mathematics would be considered a universal language it would be lifted out of the materiality of individuality and brought closer to the universal laws of thought. At first sight, this position would seem to be at the other extreme from the opinion which sees the mathematical world realized in the physical world, but perhaps if we looked deeper we might find ourselves in the presence of a case where extremes meet, for, if mathematical language is but a substitute for "physical language" might not the reason be that the mathematical world and the physical world are really one?

3. Relevance of Thomism

In undertaking to establish the significance of Thomism (77) for the problem of mathematical physics we are not insensible to the fact that such an undertaking calls for an apologia. For historians almost without exception have represented the rise and development of modern physics as something completely antithetical to the whole structure of peripatetic philosophy. Speaking of Galileo Bertrand Russell says: "His few facts sufficed to destroy the whole vast system of supposed knowledge handed down from Aristotle, as even the palest morning sun suffices to extinguish the stars." (78) And Professor Burt writes:

But now, of course, the question which Copernicus has thus easily answered carries with it a tremendous metaphysical assumption. Nor were people slow to see it and bring it to the forefront of discussion. Is it legitimate to take any other point of reference in astronomy than the earth? Mathematicians who were themselves subject to all the influences working in Copernicus' mind, would, so he hoped, be apt to say yes. But of course the whole Aristotelian and empirical philosophy of the age rose up and said no. For the question went pretty deep, it meant not only, is the astronomical realm fundamentally geometrical, which almost any one would grant, but is the universe as a whole, including our earth, fundamentally mathematical in its structure? Just because this shift of the point of reference gives a simpler geometrical expression for facts, is it legitimate to make it? To admit this point is to overthrow the whole Aristotelian physics and cosmology (79)

We are dealing here not merely with those who hold it as an indisputable methodological principle that enlightenment first dawned upon the world at the time of the Renaissance. Such as these we could afford to ignore. But there are many others who while they have a sincere admiration for all that Greek and medieval culture has to offer us in the way of art, of metaphysics, and of morals, nevertheless believe that if there is one field in which both Aristotle and the Medievalists are completely barren, it is the field of science. Most of these might be willing enough to concede to Professor Whitehead that scholastic logic and theology prepared the soil in which modern science took its roots, (80) but this could scarcely serve as a sufficient basis to constitute Thomism as a significant philosophy of science.

Among contemporary philosophers of science few have won for themselves wider recognition and a greater name than Emile Meyerson, particularly in questions of the relation between modern science and its historical background. Yet if there is one theme which runs through all of Meyerson's voluminous works it is that peripateticism has absolutely nothing to offer to science. In Identité et Réalité he writes: "Le retour au péripatétisme, préconisé avec tant de force et de savoir par Duhem nous paraît impossible.

Il ne nous semble pas, en effet, que la pure doctrine
d'Aristote ait été une doctrine véritablement scientifique." (81)

And again in Du Cheminement de la Pensée, he says: "La
science péripatétiqué, assurément, a péri et, quoi qu'en
pensent certains partisans extrêmes du retour au moyen âge,
péri totalement et irrémédiablement. Il est aussi impos-
sible de la maintenir en face du triomphe de la physique mo-
derne qu'il l'est de la concilier, fût-ce même partiellement
avec celle-ci." (82)

In recent years, a few historians have, indeed,
come to recognize the eminence of the scientific spirit and
method of Aristotle, and the worthwhile significance of the
accomplishments which were the fruit of that spirit and me-
thod; but the tributes of these few are entirely restricted
to the field of biological science. That these tributes are
merited is evident to anyone who has ever taken the pains to
read the physical treatises of Aristotle, but they leave un-
solved the question in which we are directly interested.
In fact some have seen in the intense devotion of the Sta-
girite to research in the field of biology an argument against
the contention we have set out to substantiate. Dopp, for
example, writes:

Il est arrivé qu'Aristote s'est senti peu de goût pour les
mathématiques, ne s'est point consacré à ces sciences qui
les utilisaient, mais s'est donné surtout à des recherches

d'histoire naturelle et de biologie, lesquelles consistaient essentiellement en descriptions ou en analyses de qualités ou d'activités plus ou moins discontinues, donc qualitatives. . .

Cette doctrine avait en somme pour portée de libérer le physicien à l'égard de la pensée mathématique. Elle pèsera sur toute la tradition philosophique du Moyen Age et, par certaines de ses conséquences, sur la philosophie moderne jusqu'à nos jours. (83)

The view is now being advanced by more than one philosopher of science that there is a direct connection between Aristotle's predominant interest in biological sciences and the type of logic he evolved, and that Aristotelian logic is not only of little use for the development of mathematical physics, but in some sense an obstacle to it. For biology is essentially qualitative and classificatory, that is to say, it attempts to classify living beings in a schema of genera and species that is based upon qualitative characteristics. And that explains, we are told, why Aristotelian logic is essentially classificatory, and not relational like modern mathematical logic. Professor Whitehead has laid considerable emphasis on this point:

In a sense, Plato and Pythagoras stand nearer to modern physical science than does Aristotle. The two former were mathematicians, whereas Aristotle was the son of a doctor, though of course he was not thereby ignorant of mathematics. The practical counsel to be derived from Pythagoras is to measure, and thus to express quality in terms of numerically determined quantity. But the biological sciences then and till our own time, have been overwhelmingly classificatory. Accordingly, Aristotle by his Logic throws the emphasis on classification. The popularity of Aris-

totelian Logic retarded the advance of physical science throughout the Middle Ages. If only the schoolmen had measured instead of classifying, how much they might have learnt. (84)

Professor Etienne Gilson, who is considered by many to be one of the most eminent modern champions of Thomism, has gone far beyond either Dopp or Whitehead by claiming that peripateticism has been utterly sterile in the realm of physics because Aristotle attempted to biologize the whole of physical reality, that he actually made physical bodies into so many animals. In his essay, "Concerning Christian Philosophy" (85) we find the following devastating criticism:

...We are bound to condemn the scientific sterility of the Middle Ages for those very reasons which to-day make us condemn the philosophic sterility of "scientism". Aristotle also had exaggerated the scope of one science and the value of its method, to the detriment of the others; and in a sense he was less excusable than Descartes, for in this he came into open contradiction with the requirements of his own method, whereas Descartes was only carrying his through. And yet, philosophically, Aristotle's was the less dangerous error, for it was an error of fact, and left the question of principle untouched; to biologize the inorganic as he and the medieval philosophers did, was to condemn oneself to ignorance about those sciences of the inorganic world whose present popularity comes chiefly from the inexhaustible fertility which they display in things practical; but to mathematize knowledge entirely, and on principle, was to set strange limits to physics and chemistry, and to make impossible biology, metaphysics, and consequently moral theory. . . Aristotle's error lay in not being true to his principle of a science of the real for every order of the real, and the error of medieval philosophy lay in following him in this. Committing the opposite mistake to that of Descartes, Aristotle set up the biological method as a physical method. It is generally admitted that the only positive kinds of knowledge

in which Aristotelianism achieved any progress are those which treat of the morphology and the functions of living beings. The fact is that Aristotle was before everything a naturalist just as Descartes was before everything a mathematician; so much so indeed that instead of reducing the organic to the inorganic like Descartes, Aristotle claimed to include the inorganic in the organic. Struck by the dominance of form in the living being, he made it not only a principle of the explanation of the phenomena of life, but even extended it from living beings to mobile beings in general. Hence the famous theory of substantial forms, the elimination of which was to be the first care of Descartes. For a scholastic philosopher, as a matter of fact, physical bodies are endowed with forms from which they derive their movement and their properties; and just as the soul is a certain species of form -- that of a living being -- so is form a certain genus of soul -- the genus which includes both the forms of inorganic beings and the forms of souls of organized beings.

This explains the relative sterility of the scholastic philosophy in the order of physics and even chemistry, as well as the inadequacy of Cartesianism in the order of the natural sciences. If there is in the living being anything other than pure mechanism, Descartes is foredoomed to miss it; but if there is not in physical reality that which defines the living being as such, then the scholastic philosophy will not only fail to find it there, but will never discover even what is there. Nevertheless it wasted its time in looking for what was not there; and as it was convinced that all the operations of inorganic bodies are explained by forms, it strove with all its might against those who claimed to see there something else, and clung to that impossible position until, in losing it, it lost itself. Three centuries spent in classing what must be measured, as to-day some persist in measuring what must be classed, produced only a kind of pseudo-physics, as dangerous to the future of science as to that of the philosophy which imagined itself bound to it; scholasticism was unable to extract from its own principles the physics which could and should have flowed from it. . . . Formae naturales sunt actuosae et quasi vivae, said the Scholastics: between the Cartesian artificialism which makes animals into so many machines, and the Aristotelian vitalism which makes physical bodies into so many animals, there must be room for a mechanism in physics and a vitalism in biology. (86)

To this criticism Gilson appends the following inte-

resting footnote:

It is clear that Aristotle's error, less serious than that of Descartes from the point of view of philosophy, was more serious from the point of view of science. To extend, like Descartes, a more general science to the less general sciences, leaves it possible to reach in these last what they have in common with the first; hence a mechanization, always possible though always partial, of biology; but to turn the method of a more particular science back upon a more general science amounts to leaving the more general without an object. Now, in missing the real objects of physics and chemistry, Aristotle missed at the same time all that bio-chemistry teaches us concerning biological facts -- which, although it is neither the whole nor the most important part, is possibly the part which is most useful. And this, as well as being a serious gap in his theory, is the thing that human utilitarianism will never forgive him. (87)

It is to be noted that these lines are written by an historian who does not cite so much as one text to substantiate his criticism. Moreover, the only thing that presents the semblance of a reason for the assertions made is that Aristotle extended his doctrine of substantial form to inorganic as well as organic bodies, "and just as the soul is a certain species of form -- that of a living being -- so is form a certain genus of soul -- the genus which includes both the forms of inorganic beings and the forms or souls of organized beings." The sophistry of this argument is so obvious that it does not have to be pointed out.

Gilson holds that peripatetic sterility in the realm of physics derives from the fact that Aristotle failed to recognize or at least to follow the principles that were inherent

in his doctrine, but he admits that these principles could provide a fruitful philosophy of science. This, however, has been denied by M. Augustin Mansion, who in a long article entitled "La Physique Aristotélicienne et la Philosophie," (88) has tried to show not only why nothing of any consequence for mathematical physics is found in the doctrine of Aristotle, but even why it was theoretically impossible for it to be found therein. According to Mansion, mathematical physics could find no proper place in the doctrine of Aristotle because by an unfortunate and highly arbitrary division of the sciences he created an abyss between physics and mathematics by placing them in formally different degrees of abstraction. Having once made this fatal blunder, he could not but be embarrassed by the actual existence of certain physical sciences already to some extent mathematicized, such as astronomy, optics, etc., and recognizing the utter impossibility of finding a special place for them in the schema he had conceived a priori, he was forced to class them among the mathematical sciences, while at the same time attempting to save the situation in some fashion by pointing out that they were "more physical" than pure mathematics. In this way he removed these sciences from the realm of physics proper. This, added to the fact that Aristotle had a personal aversion for mathematical speculation, explains why peripateticism is completely barren from the

point of view of mathematical physics.

Voilà donc écartées de l'oeuvre d'Aristote, avant tout physicien et Naturaliste, -- quand il n'est pas logicien et métaphysicien, -- les sciences mathématiques proprement dites. Mais il est allé plus loin, et, cette fois, il a, de façon expresse, fait appel à ses principes, pour alléger son programme de certaines sciences auxquelles on ne peut guère dénier le caractère de sciences physiques. Ce sont celles précisément qui, de son temps, se trouvaient être les plus avancées et qui avaient pris déjà la forme qui leur fait reconnaître la qualité de sciences au sens moderne du mot: astronomie, optique, harmonique ou acoustique, mécanique. La supériorité caractéristique de ces disciplines, comparées à d'autres encore moins développées, provenait du fait que le côté quantitatif des phénomènes envisagés était non seulement reconnu et décrit en termes généraux, mais était étudié en détail, par l'application poussée aussi loin que possible. Dès lors, il fallait une compétence suffisante en mathématiques pour aborder ces branches de savoir, qui par le fait même étaient devenues l'apanage des mathématiciens. Aussi Aristote les classe-t-il sans hésitations parmi les *μαθηματικά* -- les sciences mathématiques, -- tout en leur attribuant un caractère "plus physique" qu'aux mathématiques pures (Physic, B.2, 194 a 7 - 12)

On touche du doigt ici les conséquences de la doctrine des deux premiers degrés d'abstraction, en même temps que de l'éloignement qu'éprouvait Aristote pour la spéculation mathématique. Les sciences ou branches de la physique déjà mathématisées auraient dû constituer pour lui le type le plus achevé des sciences physiques particulières, à condition, bien entendu, d'assigner à chacune d'elles l'étude complète des phénomènes d'un domaine bien délimité, celui de l'astronomie ou de la mécanique par exemple...

On voit donc comment, en écartant de la physique, pour les assigner au domaine mathématique, les sciences mentionnées à l'instant, Aristote a manqué l'occasion de traiter à fond sur des cas concrets parfaitement adaptés, le problème de la différence entre une étude philosophique et une étude purement scientifique de telle ou telle portion du monde matériel. Ses vues sur le degré d'abstraction de l'objet mathématique en sont responsables pour une part; mais, d'un autre côté, une fois admises, elles eussent aussi bien postulé une astronomie ou une mécanique complète, à la fois mathématique et physique, en effet, de l'aveu même du Stagirite, les entités mathématiques sont *ἐξ ἀδιαιρέτων*; ce sont des abstraits ou des extraits d'un ensemble plus complexe, qui constitue précisément l'objet physique. Donc elles en font partie et pour étudier ce dernier objet de

façon intégrale, le physicien lui-même n'en peut négliger l'aspect quantitatif jusque dans ses dernières déterminations.

Nous savons donc pourquoi, -- touchant la question de fait, -- nous ne trouvons pas et nous ne pouvons pas trouver, dans l'oeuvre d'Aristote, des exposés ou des traités ressortissant au domaine physique et répondant à des sciences particulières assez avancées pour avoir revêtu une forme mathématique quelque peu développée. (89)

Some authors have sought for a source of this barrenness in the Aristotelian doctrine on sensible knowledge which establishes an absolute identity between the sensible and the physical, thus precluding the possibility of a physical science that would be based not on the sensible qualities of nature, but upon its quantitative relations. Speaking of the physico-mathematical sciences in relation to the system of Aristotle, Salman writes:

Elles ne dérivent pas en effet normalement de la théorie des degrés d'abstraction, mais sont des données de fait, assez gênantes d'ailleurs, que le théoricien intègre comme il le peut dans une synthèse qui ne les prévoyait pas. Pour les auteurs scolastiques il n'y avait donc qu'une physique unique, homogène et uniforme, qui expliquait tout, depuis le Premier Moteur jusqu'à la salure des mers, et le régime des vents. Et ces conceptions épistémologiques étaient fondées sur une doctrine délibérée de la connaissance sensible, qui identifiait résolument le physique et le sensible. (90)

Salman makes much of this Scholastic identification between the physical and the sensible. He finds in it a reason to reject not only that part of Scholastic natural doctrine which corresponds to modern physics, but even the whole philosophy of nature:

Les scolastiques croyaient déboucher de plain-pied dans le réel, en percevoir d'emblée et par les sens l'organisation intime. Gratifiés d'une donnée immédiate et parfaitement simple, ils pouvaient édifier une scientia naturalis unique et homogène qui épuisait la connaissance de l'univers sensible. Les modernes sont moins bien partagés. Ils savent qu'il leur faut transgresser la zone du sensible, qui est physiquement impure avant de retrouver un monde matériel vraiment objectif; ce n'est qu'ensuite, lorsqu'une pénible reconstruction leur aura rendu des données authentiquement physiques, qu'ils pourront songer à en faire la philosophie. La "Philosophie de la Nature" si éventuellement elle se reconstitue, sera l'analogue de la philosophia naturalis médiévale; tandis que la science physique moderne, malgré ses ressemblances superficielles avec l'ancienne, est d'un type épistémologique radicalement nouveau, dont il serait naïf de chercher la formule chez les auteurs du moyen âge.

On peut mesurer du même coup la portée véritable de la physique scolastique, et ses possibilités d'adaptation. Il est manifestement futile, en effet, de multiplier les "objets formels", dont les nuances plus subtiles devraient remplacer les vues insuffisamment différenciées des anciens. Car, pour user de ce langage scolastique, c'est l'"objet matériel" lui-même qui se dérobe. Ces qualités sensibles, sur lesquelles repose toute la construction médiévale, n'ont point la portée ontologique qu'on leur accordait. Elles n'existent pas dans les corps de la nature, mais seulement dans la perception de qui les connaît. La Physique ancienne n'est donc pas seulement erronée dans telle ou telle de ses conclusions, elle est atteinte, dès son point de départ, d'un subjectivisme radical dont se ressent profondément le système dans son ensemble. Plusieurs de ses thèses essentielles conservent sans doute une valeur permanente, et seront peut-être sauvées. Mais elles ne pourront revivre qu'après de nouvelles démonstrations fondées sur de nouvelles données, exprimées surtout dans un langage et avec une technique conceptuelle inspirés du réel physique et non par la vaine imagerie du sensible. Le seul parti raisonnable dès lors est de renoncer définitivement aux rapprochements superficiels et de reprendre l'élaboration d'une philosophie naturelle sur les bases toutes nouvelles que nous imposent une connaissance plus nuancée du monde physique et de son difficile accès. (90 a)

Other arguments of this kind could be easily adduced.

One of the most telling consists in this that for Aristotle

Physics is the study of mobile being (*ens mobile*), and everything it considers must be studied in the light of mobility; yet the Aristotelians have always taught that mathematics necessarily excludes motion. As we have already pointed out, Aristotle himself used this argument against the mathematization of nature taught by the Pythagoreans and the Platonists and St. Thomas stated explicitly: "*ex mathematicis non potest aliquid efficaciter de motu concludi.*" It would seem impossible, then, for a science to exist which would be at once physical and mathematical.

Montaigne once said of Aristotle that he had an "oar in every water and meddled with all things." However, the arguments we have just considered seem cogent enough to force the conclusion upon us that there was one expanse of water in which the Aristotelian oar never dipped: that of mathematical physics.

These are serious charges. They question the competence of Thomism in the whole realm of thought where philosophy comes to grips with science and with the multitudinous epistemological problems which have arisen out of its modern development. They go far deeper than even those who proffer them may suspect. In a sense they touch Thomism at its heart. For if there is one thing upon which Thomism prides itself, it

is its preeminence in that part of philosophy that is truly wisdom. Now it pertains to wisdom not only to have a critical knowledge of its own nature, but also to have that same critical knowledge of all the other sciences and of all their manifold interrelations. If Thomism cannot find within itself the principles which will be able to open up the inner meaning of mathematical physics and to situate it accurately in the whole epistemological scheme, it must renounce its claim to the possession of integral wisdom.

We do not propose to answer here all the charges indicated above. The whole study we are undertaking will be an answer to them. Yet it seems necessary at this point to purify the atmosphere of irrelevant considerations so that the real issue will be thrown into sharper focus.

In the first place, it must be pointed out that in seeking to establish the significance of Thomism as a philosophy of science we hold no brief for the decadent Scholasticism which first felt the impact of the rise of modern science and which has persisted in so many ways down to our own day. It is a sign of a singular lack of discernment on the part of historians to confuse true Thomism with this grotesque caricature. Galileo, who has traditionally been held up as the direct antithesis of all that peripateticism

stands for, realized the necessity of distinguishing between them. In his Lettere Intorno Alle Macchie Solari he says: "Nec sum ignarus, quam haec opinio sit inimica philosophiae Aristotelicae: sectae magis quam principi est diversa. Da mihi redivivum Aristotelem." (91)

This does not mean that the advancement of physical science has not resulted in the liquidation of a good many of the theories proposed by Aristotle in his treatises which deal with nature in its concretion. But only those who are utterly ignorant of the meaning of experimental science can find in this a reason to condemn him. In dealing with nature in its concretion error is normal. As we pointed out in considering the philosophy of Descartes, it is important, when one wishes to evaluate the work of a thinker of the past, to distinguish between the errors for which his system and method are intrinsically responsible, and those over which he had no control. The historians who are so eloquent in ridiculing the physics of Aristotle fail to realize that the only goal that experimental science can attain is, in the last analysis, to "save the phenomena", and that the physics of Aristotle saved the phenomena that were known in his time just as accurately and as perfectly as the theory of Relativity saves the phenomena that are known today. And we may well wonder how much of Einstein's work will be still standing after as many

thousands of years have passed over it as have elapsed since the time of Aristotle.

We think that the following passage of Charles Singer is extremely discerning:

Against Aristotle it has been urged that he obstructed the progress of astronomy by not identifying terrestrial and celestial mechanics, and by laying down the principle that celestial motions were regulated by peculiar laws. He placed the heavens beyond the possibility of experimental research, and at the same time impeded the progress of mechanics by his assumption of a distinction between "natural" and "unnatural" motion. On the other hand, we should remember that Aristotle gave an interest to the study of Nature by his provision of a positive and tangible scheme. It seems unfair to bring his own greatness as a charge against him. All our conceptions of the material world -- "scientific theories" as we call them -- are but temporary devices to be abandoned when occasion demands. That the scheme propounded by Aristotle lasted more than two thousand years is evidence of its symmetry and beauty and of the greatness of the mind that wrought it. That it received no effective criticism is no fault of Aristotle's, but is evidence of what dwarfs the men who followed him were by comparison with him. (92)

It is significant that the first one to call into question Aristotle's theory of the heavens seems to have been Thomas Aquinas, who considered Aristotle's doctrine as a mere opinion. (93)

It is clear, then, that in attempting to establish the relevance of Thomism for mathematical physics, we are not seeking to revive outmoded physical theories. Nor are we presuming to maintain that Aristotle or any of the Medievalists were great mathematical physicists. The point is that

Aristotle was something greater than a mathematical physicist: he was a great philosopher. Unquestionably, a full and exact knowledge of mathematical physics is indispensable for any philosopher who attempts to come to grips with the highly specific and concrete epistemological problems that arise out of the advanced development of physical science. But this knowledge is not necessary in order to discover the key which will open up a clear and precise view of the true nature of mathematical physics and its relations to all the other sciences. We believe that Aristotle discovered that key. We believe that that key is necessary today if we are to find our way out of the epistemological maze into which the progress of science has led us.

It may readily be admitted that from a purely material point of view Aristotle had very little to say about mathematical physics. The few passages in which he touches upon the subject are almost swallowed up in the great bulk of his writings. But that point of view is entirely irrelevant. Moreover, there are other reasons to explain this phenomenon other than the purely extrinsic reasons which delight so many historians. It has often been maintained that Aristotle knew very little mathematics, and that he had a

particular aversion for mathematical speculation. Gil-son, for example, tells us that if Aristotle did not get very far with scientific enquiry in terms of quantity and measurement, "it may be simply because of his ignorance of mathematics, of which he seems to have known only simple proportion. It is possible that this fact had a considerable influence on the general trend of his labours." (94)

This is also the opinion of Mansion, as we have seen. Gil-son gives us neither reasons nor references to support his assertion. And all that Mansion has to offer is an allusion to a text in the twelfth book of the Metaphysics where Aristotle, speaking of the movements of the heavenly bodies, writes:

That the movements are more numerous than the bodies that are moved is evident to those who have given even moderate attention to the matter; for each of the planets has more than one movement. But as to the actual number of these movements, we now -- to give some notion of the subject -- quote what some of the mathematicians say, that our thought may have some definite number to grasp; but, for the rest, we must partly investigate for ourselves, partly learn from other investigators, and if those who study this subject form an opinion contrary to what we have now stated, we must esteem both parties indeed, but follow the more accurate. (95)

Of this text Mansion says: "témoin la confession à peine voilée qu'il en a fait au XII^e livre de la Métaphysique à propos des astronomes, traités comme des spécialistes, devant la compétence duquel il s'incline sans vouloir discuter ni leur titre ni leurs hypothèses." (96) Even a casual reading

of the text of Aristotle reveals the utter gratuity of Mansion's inference. No one who is at all acquainted with the writings of Aristotle is unaware of the fact that it is customary for him to introduce a question by considering what authorities in the field have had to say about it, and that he always has respect for the opinions of these authorities unless his own reasoning has produced evidence to induce him to differ from them. In this case, it is evident from the text and context in question that he is interested merely in arriving at some probable opinion about the number of the movements of the heavenly bodies so that the mind will be able to fix itself upon a definite number. (97)

And since the opinions of Eudoxus and Callippus seem probable to him he accepts them.

As a matter of fact, scholars are now coming to recognize that Aristotle's knowledge of mathematics was far advanced for his day. "It was knowledge, rather than ignorance of the mathematics of his time," writes F.S.C. Northrop, "which supported Aristotle in the formulation of his logic." (98)

Aristotle's polemic against the mathematicism of the platonists was not a polemic against the existence of mathematical science, as some seem to think, but against the ontological existence of mathematical entities. By dissipating

the confusion of mathematics with both physics and metaphysics that was characteristic of the doctrine of the platonists, Aristotle established its true epistemological status. He thus freed it of all the associations which tended to draw it away from its proper function, and made of it a more apt instrument for the use of scientists. Professor Strong has brought out this point with remarkable clarity, and we cannot refrain from quoting the following passage in spite of its length:

Critics can criticize Aristotle for his refusal to accept the doctrine of Form as metaphysical number, but certainly not upon the ground that he failed to consider the meaning of mathematics. Rather, one may say, it was because Aristotle refused to confuse mathematical science with metaphysical principles, and because he insisted upon the operational character and physical reference of mathematics that he refused to identify mathematical number with ideal number existing in a separate realm of reality. This means that Aristotle did not advocate the formulation of a metaphysics in mathematical terms and relations and saw such a metaphysics as a confusion of the notion of mathematics with ontological realities. Hence Aristotle held no doctrine of the universe framed in mathematical universals of relation, for he regarded the ratios and proportions of mathematics as constituting no class of existences-in-themselves. They are relational only of entities of a mathematical character in arithmetic, geometry, or some more physical science such as mechanics.

The Physica, De Caelo, and Problemata reveal passages in which he used mathematics in connection with physical problems. This is of course not equivalent to saying that the basic principles of Aristotle's physical science were mathematical. Aristotle recognized mathematics as a self-contained science and as an instrument in the physical sciences. So far as he mainly directed his own treatment of nature to the problem of growth where mathematical formulation was not relevant, so far we may say that his in-

terest and approach were directed to other than the quantitative aspects and concepts of nature. It is characteristic of Aristotle's approach to his predecessors that he regards them as men striving for the theoretical view. His analyses of his predecessors are thus a source of knowledge with respect to their "metaphysics." His own inquiry ends in a position opposed to the views of Democritus and Plato. The opposition, in accordance with the view presented in the foregoing analysis, is not to mathematics or to the use of mathematics in natural science, but to the rôle which number and mathematical objects are supposed to have as ontological existences. To insist upon the distinction between the mathematician's subject-matter and the substantial and ideal number attributed to Plato, does not involve a rejection of mathematics proper. It does involve a rejection of theories about the "real" existence of number-forms. Those who assume that a mathematical metaphysics is fundamentally important in a regulative and interpretative rôle to the development of mechanics and mathematical physics charge Aristotle, upon the basis of his different conclusion in metaphysics, with having obstructed the progress that would supposedly have followed from his acceptance of the Platonic theories of existential number. So far as Plato and the Academy were actually engaged in mathematical work, the argument appears to carry weight. Nevertheless, the assumption that metaphysics is important in respect to subject-matter and procedure must first be established before Aristotle can be held responsible for obstructing the development of mathematical science. (99)

It is clear, then, that there must be other reasons besides a lack of knowledge of mathematics to explain why Aristotle, having once discovered the true principles of mathematical physics, did not devote himself to their development. In the first place, in order for any substantial progress to be made in the application of mathematics to nature two kinds of instruments are essential; conceptual mathematical instruments, and physical instruments of exact experiment and mea-

surement. Without these only extremely meager progress can be made, and Aristotle lacked both. It was only after the Renaissance that the necessary physical instruments were invented, and the conceptual instruments which were to prove so fruitful, such as analytical geometry and the calculus, were discovered. The development of mathematical physics depends completely upon these instruments, and, as Meyerson has pointed out, "si les mathématiques accomplissaient à l'heure actuelle un progrès comparable, ne fût-ce que dans une certaine mesure, à celui qui a été effectué par la création du calcul infinitésimal, la physique à son tour ferait, presque immédiatement, un bond en avant immense."⁽¹⁰⁰⁾

Another possible explanation of why Aristotle failed to give more attention to the exploitation of the fruitful principles he had discovered may be that he was far from realizing the vast extent of the applicability of his own principles. But before considering this possibility it is necessary to examine the major texts in which these principles are laid down.

There are two capital texts in which Aristotle deals explicitly with the nature of mathematical physics. These will constitute the seed out of which our whole study will grow:

The first of these two texts is found in the Posterior Analytics. This whole work is devoted to a discussion of the principles that are common to all the sciences, In chapter thirteen of the first book Aristotle explains how knowledge of the fact (scientia quia) differs from knowledge of the reasoned fact (scientia propter quid). After showing how they differ within the same science, he goes on to explain how they differ when they are found in different sciences: and in making this explanation he brings in the question of the subalternation of the sciences which we consider the key to the whole problem of mathematical physics.

But there is another way too in which the fact and the reasoned fact differ, and that is when they are investigated respectively by different sciences. This occurs in the case of problems related to one another as sub-alternate and superior, as when optical problems are sub-alternated to geometry, mechanical problems to stereometry, harmonic problems to arithmetica, the data of observation to astronomy. Some of these sciences are almost synonymous, e.g. mathematical and nautical astronomy, mathematical and acoustical harmonics. Here it is the business of the empirical observers to know the fact, of the mathematicians to know the reason for the fact. For the latter are in possession of the demonstrations giving the causes, and are often ignorant of the simple fact: just as those who know universals are often ignorant of some of its particular instances through lack of observation. Such are all the sciences which, though differing by their essence, use forms. For the mathematical sciences have to do with forms; they are not concerned with a subject, since, even though geometrical properties are predicable of a subject, it is not as predicable of a subject that they consider them. As optics is related to geometry, so another science is related to optics, namely the theory of the rainbow. Here it pertains to the physician to know the fact, but

to the optician to know the reason for the fact, either qua optician or qua mathematician. Many sciences, though not subalternated, are mutually related in a similar way, e.g. medicine and geometry: it is the business of the student of medicine to know that circular wounds heal more slowly, but it pertains to the geometer to know the reason why. (101)

The second important text is found in chapter two of the second book of the Physics. Since some historians have failed to see why this passage should be in this particular place and have preferred to seek for some extrinsic reason to explain its presence here, it is worthwhile to point out its connection with the context. After having discussed in book one the problem of the principles of nature, Aristotle takes up in book two the principles of the science of nature. The general principles common to all science had already been considered in the Posteriora Analytica. But each science has its own proper method, and consequently it was necessary for Aristotle after having determined upon the principles of nature to discuss the method to be used in the investigation of nature. It was necessary to consider the causes according to which demonstration may be had in natural science. Now it happens that the natural scientist in seeking for the cause of natural phenomena often turns to mathematics for light. Aristotle had to explain the significance of this recourse to mathematics. In other words, after having discussed in the Posterior Analytics the general principles governing the subal-

ternation of one science to another, he now applies these principles to the subalternation of physics to mathematics.

Having determined the different ways in which the term "nature" is used, we must now consider how the mathematician differs from the physicist. For physical bodies contain surfaces and volumes, lines and points, and these are the object of the mathematician. Moreover, astronomy is either different from physics or a part of it. For it seems strange that it should pertain to the physicist to know the nature of the sun or the moon, but not to know any of their accidents, especially since writers on physics obviously do discuss their shape also and whether the earth and the world are spherical or not. Now the mathematician, though he treats of these things, nevertheless does not treat of them as the limits of a physical body; nor does he consider the accidents precisely as accidents of such bodies. That is why he abstracts them; for in thought they are abstractable from motion, and it makes no difference, nor is any falsity involved if he so abstracts them. The holders of the theory of Forms are unaware of this. For they abstract physical things, even though these are less abstractable than mathematical things. This becomes plain if one tries to state in each of the two cases the definitions of the things and of their attributes. 'Odd' and 'even', 'straight' and 'curved', and likewise 'number', 'line', and 'figure', do not involve motion; not so 'flesh' and 'bone' and 'man' -- these are defined like 'snub nose', not like 'curved'. Similar evidence is supplied by the sciences which are more physical than mathematical, such as optica, harmonics, and astronomy. These are in a way the converse of geometry, for while geometry investigates physical lines but not qua physical, optica investigates mathematical lines, but qua physical, not qua mathematical. (102)

The central idea that emerges from these two texts is that mathematical physics is a hybrid science in which physics is subalternated to mathematics. It is, to use the technical Thomistic expression, a scientia media, an intermediary science between physics and mathematics; it involves a kind of neutric hylemorphism in which the material element is drawn from

physics and the formal element from mathematics. The purpose of this study is to analyze the unique type of knowledge that is born of this union. As we have already indicated, it is not our intention to attempt to come to grips with all the complicated epistemological problems which have evolved out of the development of mathematical physics. Rather we have in mind to take this one idea of a scientia media and explore all of its implications. But we hope to draw out these implications far enough to make it clear that in this one idea is found the central key which will open up the meaning of all the other problems encountered in physics.

Before undertaking the detailed analysis of these texts several general considerations are in order. In the first place, for the purpose of indicating the direction that this analysis will follow, it is helpful to try to orientate the position of Aristotle in relation to the other positions outlined earlier in this chapter. As we have already suggested, most of these opinions can be reduced to two categories: the role of mathematics in physics is either considered to be that of a pure instrument (whether logical or merely linguistic,) that is employed by the scientist in order to work more effectively upon his sole direct object which is nature; or it is considered to be that of the direct object of the science it-

self in the sense that the mathematical world is identified with or realized in the physical world. Now the position of Aristotle is located squarely between these two extreme positions.

In the first place, the role of mathematics in physics is essentially instrumental in the sense that the whole raison d'être of its introduction into physics is to enable the mind to get to know the physical universe better. The goal at which the whole of mathematical physics aims is not to know the mathematical world (for that is already known) but the physical world. Mathematics is employed as a means to that end.

On the other hand, mathematics is much more than a mere tool in physics, that is to say, it does not remain extrinsic to the science; on the contrary it enters intrinsically into its very constitution. And it enters into it intrinsically not merely in the sense of providing the principles from which physics may draw conclusions concerning its own proper object which in itself remains untouched by mathematics, but in the sense of entering into the very object of the science. For, as we shall see in chapter three, the type of subalternation found in mathematical physics is not merely subalternation according to principles, such as is found in the dependence of theology upon the science of the blessed, but subalternation according to

the object. This means that the formal object of mathematical physics is constituted by a combination of both a mathematical and a physical element.

But the nature of this combination must be rightly understood. It does not mean that mathematical physics studies as such the quantitative determinations found in nature from the point of view that is proper to them. Such a study is possible, but it will be either pure physics (if the quantitative determinations are considered in relation to mobility) or metaphysics (if the nature of quantity and its properties are considered). Mathematical physics studies the quantitative determinations found in nature, not just in the light of their ontological status, but in the light of the status that is proper to mathematical abstraction. For example, when the physicist says that light is propagated in a straight line, the line he is talking about is neither a mere physical, sensible line, such as is found in nature, nor is it merely a mathematical line; it is a combination of the two: the sensible line is considered in the light of a mathematical line.

In this way mathematics enters into the very essence of the object of physics, but it does so in such a fashion that the mathematical world is not identified with the physical world. It retains the extrinsic character that is proper to it.

And this is extremely important. For only by remaining extrinsic can it fulfill its essentially functional and instrumental role, by retaining all the pliancy and inexhaustible virtuosity that is proper to mathematical abstraction.

This brings us to a delicate point that must be touched upon before proceeding further in our analysis. It would seem that for Aristotle and the medieval Thomists the combination between the mathematical and the physical element in the object of mathematical physics was in a sense more intimate than it is possible to admit today. Because of a lack of refinement in their means of observation, they seem to have held that there are quantitative determinations in nature which come sufficiently close to the absolute state of perfection that they enjoy in the mathematical world to allow for a true scientific (103) handling of them in terms of mathematics. The heavenly bodies, for example, were for them perfect spheres, and consequently there was sufficient conformity between them and mathematical spheres to allow the mathematical properties of sphericity to be applied to them directly and adequately. This does not mean, of course, that mathematical entities were realized as such in the physical universe, for that would involve a confusion of mathematics and physics, and Aristotle and St. Thomas go to great lengths in inveighing against those who proposed such a confusion. (104) But it does mean that some phy-

sical entities possessed a determination which was in close enough conformity with the perfect determination of mathematical entities for mathematics to give an adequate explanation of them. That is why Aristotle and St. Thomas could look upon the combination of mathematics and physics as giving rise to a science in the strict sense of the term.

It would seem that this particular aspect of their doctrine is open to modification. Because of our more highly refined instruments of research, we are not longer inclined to believe that such a conformity exists between physical and mathematical entities. As a consequence, the mathematical interpretation of nature is never more than an extrinsic approach to nature. And that is why from this point of view mathematical physics cannot be considered a science in the strict Aristotelian sense of the term, but a species of dialectics.

There is another closely related point that must be underscored here in order to establish accurately the connection between Thomistic doctrine and modern mathematical physics. When Aristotle and the medieval Thomists speak of mathematics they understand it in the sense in which it was generally understood until recent years -- that is to say, as a science which deals with quantitative relations that are capable of realization

in the sensible world though not in the state of abstraction that is proper to them -- "oportet salvari principia mathematica in omnibus naturalibus, ut dicitur III Caeli et Mundi." As is well known, modern mathematics is no longer restricted to these limits. It now embraces a great range of conceptual constructions which reach far beyond these quantitative relations. Now it is bootless to dispute about names, but it is extremely important to keep in mind what they are meant to signify. And in so far as our problem is concerned, it is necessary to recognize the fact that from the point of view of Thomistic terminology, the part of modern mathematics which does not deal with quantitative relations abstracted from the sensible world is not mathematics, but a tissue of dialectical constructions. Now these dialectical constructions have been employed with great success in the recent developments of physics. The obvious example which immediately suggests itself is the use of non-Euclidian geometry in the theory of relativity. Does this mean that the Thomistic doctrine of scientia media has no relevance for recent mathematical physics. We do not believe that such a conclusion is legitimate. For the application of the dialectical constructions of modern mathematics to nature follows the same general pattern as the application of mathematics in the restricted sense in which it was understood by Aristotle and the Medievalists, and is go-

verned by many of the same general principles. Nevertheless it is necessary to keep in mind that in so far as these conceptual constructions are employed, mathematical physics is dialectical in a sense never envisaged by Aristotle and St. Thomas, that is to say, although their notion of dialectics is applicable, they never envisaged this application.

In connection with this question of the meaning of the term "mathematical" it will be helpful to determine here what breadth of meaning the phrase "mathematical physics" will have throughout this study. This is a double problem, involving the range of applicability of both the term "mathematical" and the term "physics." In so far as the first aspect of the question is concerned, it is to be noted that some authors restrict the phrase "mathematical physics" to those parts of physics which have attained the highest degree of mathematization. Professor Lenzen, for example, divides physics into experimental physics, theoretical physics, ideal theoretical physics, and mathematical physics. (105) The Thomistic acceptance of the phrase is much broader. It includes any part of physics in which a mathematical element is introduced to determine the object in such a way that new significant truths result which would

not arise without this determination.

The second question which must be determined is the meaning of the term "physics". A reading of the texts of Aristotle cited above raises a problem about the range of applicability which the principles laid down in them had for Aristotle and the medieval Thomists. The examples given in these passages are restricted to a very few especially privileged cases in which the presupposition of all mathematization, namely, order and regularity, is found in a particularly high degree - whether it be the geometrical order that is found in astronomy, for example, or the arithmetical order that is found in music. It would seem that the examples given are more than examples, that they are an exhaustive indication of the fields in which physics had to some extent been subalternated to mathematics. Did Aristotle or the medieval Thomists look beyond these fields? Did they conceive the possibility of a universal interpretation of nature in terms of mathematics? It seems quite possible that they did not. It is probable that the honor of this discovery must be accorded to the scientists and philosophers of the Renaissance. But this admission in no way compromises the objective applicability of these principles, nor their real fecundity.

Mathematics is almost synonymous with determination, and as a consequence nature is refractory to mathematization to the extent in which it participates in some form of indetermination. That is why it is necessary to understand the ways in which nature is subject to indetermination if we are to see the extent to which mathematics may be applied to nature. Now there are two types of indetermination: passive indetermination which is an imperfection arising out of the potentiality of matter, and active indetermination which is a perfection deriving from the actuality of form. Passive indetermination is found in all beings which have any share in potentiality; active indetermination is found in its fullness only in the liberty proper to spiritual beings, but it is also found anticipated to a greater or lesser degree in the spontaneity of all living things.

Now in Aristotle's and the medieval Thomists' concept of the cosmos, the heavenly bodies occupied a very privileged position. Though mobile, they were incorruptible, and they consequently occupied a position between the metaphysical realm of immobile beings and
(106)
the terrestrial world of corruptible beings. Though

inanimate they were in a sense more perfect than the living beings of the earth, even than man, in that they were subject to no intrinsic corruption, but only to the extrinsic mobility of local motion. (107) They were thus free of both the passive indetermination that is proper to corruptible things, and the active indetermination that is found in living beings. That is why for the ancients they constituted the part of nature that was most highly amenable to mathematization. It would be difficult to say just what possibilities of mathematization Aristotle and St. Thomas saw in the terrestrial world of corruptible things in which both passive and active indetermination play such a large part. But at least this much can be said: they would readily grant the possibility of a mathematical interpretation of the corruptible world to the extent in which definite regularity and order could be discovered in its phenomena. (108)

But whatever Aristotle or Saint Thomas may have thought about the extent to which nature may be mathematized, there is no doubt that their principles are applicable to the whole range of mathematization which modern physics has achieved. And that is all that is of any real importance. This universal applicability of Thomistic

principles is so true that in this study we shall, when speaking of mathematical physics, take the term "physics" in its primitive Aristotelian meaning in which it is coterminous with the whole of nature. In this sense it includes not only chemistry but even biology and psychology. As we shall see, according to Thomistic principles of the unity and distinction of the sciences, all of the sciences which deal with nature, whether it be inanimate, animate, or even psychic nature, constitute one indivisible science. In recent years there has been an attempt made by many Thomists to depart from this doctrine, but we shall point out in Chapter Two the error involved in this attempt. That mathematics has been successfully and fruitfully applied to all of these different fields of study is well known. And all of these applications (and whatever new applications the future may discover) constitute the *scientia media* of which Aristotle and Saint Thomas speak. (109)

Not all fields in the study of nature are equally amenable to mathematization. This is evident a posteriori from the history of science. It is even more evident a priori. For the objective basis of mathematization is, as we shall see, the homogeneous exteriority found in

nature. In the measure, then, in which the object of a certain branch of natural doctrine has to do with homogeneous exteriority and in the measure in which it excludes heterogeneous interiority, to that extent mathematization is possible. The field in which this condition is found in its highest degree is, of course physics, in the modern sense of the term. And that explains not only why mathematization is possible to such a large extent in physics, but also why it is necessary. For, to the extent in which heterogeneous interiority is excluded, physical rationality loses ground. That is why, if scientific investigation in the realm of physics is to advance at all, it must proceed in the light of mathematical rationality.

For experimental scientists, physics realizes the ideal type of science. And it is perfectly legitimate and natural for them to make every effort to bring the other branches of natural doctrine into as close conformity with physics as possible. As we shall see later, homogeneity is from one point of view more knowable than heterogeneity, and as Aristotle and St. Thomas point out, it is natural for the intellect to reduce the less knowable to the more knowable. But there is no doubt that

(110)
this conformity will never be complete. Mathematics is not competent to treat adequately of all natural being. For the subject of mathematics is quantity, which is the order of the parts of the substance in which it inheres. But the parts in question are always material parts, and hence must not be confused with the form of the substance. This confusion would lead to a denial of what is best in natural things.

In other words, in the measure in which beings are ontologically more perfect, they lend themselves less to mathematical interpretation. For a being is perfect in proportion to the extent that its form emerges above the potentiality of matter, that is to say, triumphs over the potentiality of matter. Now, in the structure of material being, while quantity follows upon matter, quality follows upon form. That is why as we ascend the scale of material being qualitative determinations assume an ever increasing importance. This is particularly true of living beings. For the formal principle of life is form, and if a thing is living it is because its form has emerged to a sufficient extent above the potentiality of matter. That is why qualities and classification play such an important role in biology. Moreover, in living beings we find not only the

passive indetermination common to all material things, but also the active indetermination of their vital spontaneity. This double indetermination will always provide great resistance to mathematization.

All this amounts to saying that as we ascend the scale of being heterogeneous interiority constantly increases. Within the cosmos it finds its fullest realization in man, the most perfect cosmic being. And we are referring here not merely to the psychic side of man, but also to the somatic part of his make-up. Of all the bodies in the universe, the body of man has the greatest heterogeneous interiority; it is the farthest removed from the Cartesian body, which is the ideal of an autonomous and self-sufficient physics. It is this heterogeneity of living beings that makes it possible for us to have a valid science of biology without mathematization—a science of classification.

It is interesting to note here in passing that whereas for physical science (in the modern sense of the term) heterogeneity is an irrational element, for philosophy it is homogeneity that is in some sense irrational. Here we are touching upon an important point to which we shall

return in chapter nine: the difference in the measurement that is proper to each science. For every science, even metaphysics, is in a way based upon measurement, but in each science there is a vast difference in the measure which provides the norm in relation to which everything that falls within its object is determined.

The important point to be borne in mind for the present is that in spite of the great heterogeneity found in nature, all natural things are spatio-temporal beings and consequently subject to a common measure. In discussing the problem of Indeterminism, Professor DeKoninck has emphasized this point:

Qu'on ne croie pas échapper à cette conséquence en disant que l'animal et la plante sont hétérogènes et rebelles à une mesure homogène. Ne peut-on mesurer leur durée par une même horloge? Cependant, puisque l'existence est proportionnelle à l'essence -- quantum unicuique inest de forma, tantum inest ei de virtute essendi -- la durée des êtres cosmiques est aussi de plus en plus simple, de moins en moins temporelle; il existe ainsi toute une hiérarchie de durées cosmiques. Mais cette hétérogénéité ontologique n'empêche pas le temps physique, que l'on définit par la description de son procédé de mesure, d'enlacer tous les êtres spatio-temporels par ce qu'ils ont d'homogène entre eux au point de vue durée. Cette commune mesure est fondée sur le genre commun de corporéité dans lequel conviennent tous les êtres naturels. Le temps physique n'atteint que leur bas-fond, et encore n'y touche-t-il que du dehors. L'homogénéité est fondement de toute mesure quantitative; ce genre physique

commun explique suffisamment l'unité spécifique du temps expérimental et pourquoi l'hétérogénéité des durées échappe aux prises d'une métrique calquée sur l'extériorité homogène. La science expérimentale débouche là où tous les êtres se touchent et se confondent: l'échelle graduée sur la balance n'indique aucune différence entre 150 livres d'homme et 150 livres de briques. Si maintenant le temps physique touchait les êtres dans leur fond ontologique et spécifique, si ce temps épuisait le réel, ne fût-ce qu'au point de vue de la durée, les différents degrés d'êtres ne seraient que des épiphénomènes de complexité matérielle croissante. Même si les choses sont plus que du dehors, cela n'empêche pas que la mesure de leur extériorité homogène soit commune et vraie. Ces deux perspectives ne sont point contraires, elles se complètent l'une l'autre. Sans connaître la complexité expérimentale d'une chose on ne peut saisir la richesse de son unité ontologique. (111)

The same author has elsewhere summed up the question at issue:

La biologie expérimentale est une science exacte. Les sciences expérimentales peuvent être appelées exactes dans la mesure où elles nous permettent de faire des prédictions. C'est en ce sens que la physique peut être dite la plus exacte des sciences expérimentales. En astronomie on peut prédire des éclipses qui n'auront lieu que dans plusieurs siècles, à une fraction de seconde près. La science expérimentale est essentiellement métrique. Elle ne peut définir les propriétés que par la description de leur procédé de mesure. Aucune loi expérimentale -- relation algébrique entre des nombres-mesures -- n'est absolument rigoureuse. Cependant, dans l'ensemble, les lois strictement physiques sont plus rigoureuses que les lois biologiques. Nulle raison de s'en étonner. Nous venons de dire qu'il y a dans les êtres vivants une spontanéité toujours croissante qui dans

l'homme aboutit à une véritable liberté. Il est absolument impossible à un physicien de prédire d'avance quel mouvement de bras je ferai dans les cinq minutes à venir, si j'y prête attention. Il peut mesurer le mouvement que je fais quand je le fais. Mais de cette mesure il ne peut pas déduire le mouvement suivant. Chaque mouvement que j'effectue librement est quelque chose d'absolument nouveau dans le monde. Dès lors on peut dire que plus un être vivant est parfait, plus il échappe à la rigueur métrique. Plus il est concentré au-dessus de l'espace-temps, plus il échappe aux prises de la science expérimentale. Ainsi, de toutes les sciences expérimentales, la psychologie expérimentale est la plus imparfaite, la plus inadéquate, bien qu'elle étudie la plus haute forme d'organisation naturelle.

En philosophie, c'est le contraire qui est vrai. Plus nous nous éloignons de l'homme pour descendre l'échelle des vivants, plus leur vie devient obscure. Ainsi, la vie des plantes est plus obscure pour nous que la vie animale. Nous reviendrons là-dessus. Il suffit de remarquer pour le moment qu'il existera une certaine complémentarité compensatrice entre ces deux ordres de connaissance si profondément distincts. Et par cette complémentarité compensatrice, je n'entends pas qu'à un certain point ces deux ordres de connaissance se fusionnent l'un dans l'autre. Non, ils ne sont jamais plus éloignés l'un de l'autre qu'au point où ils se touchent: comme des points sur une droite non euclidienne qui sont infiniment proches, mais aussi infiniment éloignés." (112)

In chemistry we already find an element which is refractory to complete mathematization. For the part that
(113)
qualitative diversity plays in chemistry is essential.

And even though history has made short shrift of Comte's rejection of the possibility of the mathematization of

(114)
chemistry, as it has of many another Comtian theory, it is safe to conclude that in this science there will always remain a margin impenetrable to complete mathematization.

In biology this margin will always be immeasurably larger than in chemistry, for the reasons indicated above. Nevertheless, the attempts already made towards mathematization in this field have been surprisingly fruitful, and there is no way of laying down any well defined limits beyond which this mathematization may not go. As Whyte has pointed out, "if the laws of life were independent of the physical laws, life could neither exist within the physical universe nor discover its laws." (115) And just as it is the duty of every scientist to proceed in practice as though there were no limit to the determination coming from per se causality, that is to say, as though there were no chance in nature, so it is the duty of the biologist to act as though there were no limit to mathematization in biology, even though he may realize that the immanence that is characteristic of life will always remain superior to pure corporality, and thus to some extent escape measurability.

It does not fall within the scope of this study

to discuss in detail the various ways in which mathematics
(116)
have been applied to biology. But the work already
carried on in biomathematics by such men as D'Arcy Thompson,
W.R. Thompson, Janisch, A.J. Lotka, Vito Volterra, and R.A.
Fisher, for example, has been sufficient to demonstrate
how promising this line of research in biology is. To cite
only a few typical examples, mathematics have been applied
successfully and fruitfully to problems of organic structure,
(117)
laws of growth, laws of reproduction, etc.

Of particular interest are the attempts being made to relate
biological phenomena with the discoveries of modern physics.
In this connection the experiments carried on by Timofeef-
Ressovsky, Zimmer and Delbruck on the relation between genes
and molecules, and those carried on by Stanley on the
relation between virus individuals and molecules seem
especially suggestive. Moreover, recent experimentation
on the biological effects of radiation seem to indicate
some promise of the general usefulness of an atomic-
physical and quantum-physical interpretation of fundamental
life processes. And it is interesting to note that Bohr
has lent the great weight of his name to the belief that
the new physics will ultimately have profound repercussions
upon biological science. There can be no doubt that by

abandoning the mechanism of the nineteenth century in favor of the analysis of phenomena in terms of constituent functional relationships, physics has immeasurably increased its significance for biology, and opened up in the latter science great possibilities of mathematization.

As we have already suggested, experimental psychology is of all the fields of natural doctrine the least congenial to mathematical interpretation. Yet even here the application of mathematics has been large and fruitful. (118) The use of mathematical formulations in the intelligence tests of Binet and his followers is well known. (119)

The Weber-Fechner law for the intensity of sensation, the logarithmic laws governing rote memory and forgetting, the Spearman factorial analyses of mental abilities are only a few of the results of the application of mathematics to experimental psychology. And what we have said of biology applies here as well: there is no way of laying down definite limits beyond which this mathematization may not go.

4. Some Implications of the Problem

In the beginning of this essay we alluded to

the importance of the philosophical study of the nature of mathematical physics. Perhaps it would be well, before bringing this chapter to a close, to try to round out our introductory considerations by indicating briefly some of the major issues involved in the study we are undertaking.

In the first place, this study is of vital importance for physical science itself. There was a time when philosophy was hermetically sealed off from science. Even when scientists did not feel it necessary to be inimical to philosophy, they thought that they could remain completely aloof from it. That time has passed.

"It is a well-founded historical generalization," says Whitehead in a somewhat different context, "that the last thing to be discovered in any science is what the science is really about. Men go on groping for centuries, guided merely by a dim instinct and a puzzled curiosity, till at last 'some great truth is loosened.'" (120)

Great truths have been loosened in modern physics and they have made us realize that in order to carry on the progress of science it is necessary to find out what science is really about. We have already pointed out

how all of the greatest contemporary physicists have been forced by the very needs of their science to invade the realm of philosophy. This is a highly significant phenomenon. It means that science is beginning to recognize a need for wisdom. In this connection Heisenberg writes:

Many of the abstractions that are characteristic of modern theoretical physics are to be found discussed in the philosophy of past centuries. At that time these abstractions could be disregarded as mere mental exercises by those scientists whose only concern was with reality, but today we are compelled by the refinements of experimental art to consider them seriously. (121)

Of the many great physicists who have felt the need of turning to philosophy, no one has contributed more to scientific epistemology than Sir Arthur Eddington. In his Philosophy of Physical Science Eddington discusses the significance of the need that science has of philosophy:

It is however, important to recognize that about twenty five years ago the invasion of philosophy by physics assumed a different character. Up till then traffic with philosophy had been a luxury for those scientists whose dispositions happened to turn that way. I can find no indication that the scientific researches of Pearson and Poincare were in any way inspired or guided by their particular philosophical outlook. They had no opportunity to put their philosophy into practice. Conversely, their philosophical conclusions were the outcome of general scientific training, and

were not to any extent dependent on familiarity with recondite investigations and theories. To advance science and to philosophise on science were essentially distinct activities. In the new movement scientific epistemology is much more intimately associated with science. For developing the modern theories of matter and radiation a definite epistemological outlook has become a necessity; and it is the direct source of the most far-reaching scientific advances.

We have discovered that it is actually an aid in the search for knowledge to understand the nature of the knowledge which we seek.

Theoretical physicists, through the inescapable demands of their own subject, have been forced to become epistemologists, just as pure mathematicians have been forced to become logicians. The invasion of the epistemological branch of philosophy by physics is exactly parallel to the invasion of the logical branch of philosophy by mathematics. Pure mathematicians, having learnt by experience that the obvious is difficult to prove -- and not always true -- found it necessary to delve into the foundations of their own processes of reasoning; in so doing they developed a powerful technique which has been welcomed for the advancement of logic generally. A similar pressure of necessity has caused physicists to enter into epistemology, rather against their will. Most of us, as plain men of science, begin with an aversion to the philosophic type of inquiry into the nature of things. Whether we are persuaded that the nature of physical objects is obvious to common sense, or whether we are persuaded that it is inscrutable beyond human understanding, we are inclined to dismiss the inquiry as unpractical and futile. But modern physics has not been able to maintain this aloofness. There can be little doubt that its advances, though applying primarily to the restricted field of scientific epistemology, have a wider bearing, and offer an effective contribution to the philosophical outlook as a whole.

Formally we may still recognize a distinction between science, as treating the content of knowledge, and scientific epistemology, as treating the nature of knowledge of the physical universe. But it is no longer a practical partition; and to conform to the present situation scientific epistemology should be included in science. We do not dispute that it must also be included in philosophy. It is a field in which philosophy and physics overlap. (122)

Scientists are becoming increasingly conscious of the fact that what they get to know of reality is inextricably bound up with the way they get to know it, and that as a consequence they cannot be sure of what they know except by studying the way in which they get to know it. To use the happy expression of Leon Brunschvicg, they are no longer satisfied with giving an artificial communique of their victories over reality, as was their wont in the past; they are finding it necessary to give an account of their battles.

But philosophy has as much to draw from scientific epistemology as physics has -- and more. For the philosopher few undertakings are more rewarding than the study of the mystery of knowledge. And of all the different types of knowledge none presents greater epistemological complexity than mathematical physics.

In physico-mathematical knowledge there are implications that are deep and far-reaching. A false view of its nature leads inevitably to a false view of the nature of human knowledge in general or to a false view of the nature of reality, or to both. It would be interesting to point out the connection between modern physical science and the many modern theories of knowledge, but that would take us too far afield. We have already alluded in a general way to this connection in Cartesianism and Kantianism, and this must suffice for the moment.

Because the true nature of physico-mathematical knowledge has been generally misunderstood, it has been almost universally substituted since the time of the Renaissance for the philosophy of nature. And the results have been disastrous for both philosophy and physics. Out of this substitution has arisen the great historical misunderstanding of the relation between Aristotelian and modern physics.

Looking back at the physics of Aristotle through the eyes of modern mathematical physics, and not taking the trouble to find out what Aristotle was actually talking about, scientists and philosophers of science have become a prey

to the fallacy of ignoratio elenchi. They have not suspected that when Aristotle was talking about motion his approach to the question was something entirely different from that of Descartes. If this study should accomplish no other purpose than to help to clear up this unfortunate misunderstanding, our efforts will be more than justified.

But even when mathematical physics has not been substituted for the philosophy of nature, the failure to grasp its true epistemological character has led to abortive and extremely unhappy attempts to integrate it directly with philosophy. These attempts have been numerous both inside and outside Scholastic circles. Before the true relation between philosophy and science can be worked out, an immense epistemological task of purification and clarification of notions must be undertaken. It is hoped that this study will contribute something to the furtherance of this task.

As we have said, the consequences of a false view of the nature of mathematical physics are far-reaching. It would be easy to show for example how it leads (and de facto has led) to a deterministic view of

the whole of nature. In this connection Boutroux writes:

Telle est la racine du déterminisme moderne.
Nous croyons que tout est déterminé nécessairement, parce que nous croyons que tout, en réalité, est mathématique. Cette croyance est le ressort, manifeste ou inaperçu, de l'investigation scientifique. (123)

But the implications are even deeper than this.

In the course of history the human mind has often been turned on the dialela of materialism and idealism. It is significant that a false notion of the nature of mathematical physics leads to both of these diametrically opposed extremes.

The reason for this derives from the peculiar character of mathematical science. As we shall see there is something necessarily material about mathematics in the sense that it deals with quantity, which, while it abstracts from sensible matter does not abstract from intelligible matter, and even intelligible matter implies homogeneity. In so far as mathematics has reference to reality, that reality can be nothing but material. Hence any possible real mathematical order is necessarily material. That is why universal mathematicism can lead and has led to materialism. On the other hand, mathematics is the most

abstract of all the sciences, in a sense even more abstract than metaphysics. For mathematical entities are considered by the mathematician in their very state of abstraction, and as a consequence they are indifferent to reality. Moreover, these mathematical entities in their abstract state are prior to the sensible reality to which we apply them. That is why universal mathematicism can lead and has lead to idealism.

During the years when mechanism held complete sway over mathematical physics the tendency of mathematicism was towards materialism. In recent years, however, since the breakdown of classical physics, the tendency has largely been towards idealism. Professor Joad has described the dialectic by which mathematicism leads to idealism;

But if the entities of which the universe is on a naively realistic view supposed to consist; substance and space-time, turn out to be mathematical, that is completely resolvable into mathematical formulae, and if to be mathematical is to be mental, more will be implied by the various statements asserting the mathematical nature of things than that the universe is describable in terms of mathematics; it will be implied that the universe somehow is mathematics. And, since mathematics is thought, to be mathematical will also be to be mathematical thought. (124)

Of all the modern mathematical physicists who have been drawn towards idealism, Sir James Jeans is perhaps the most outstanding example:

The terrestrial pure mathematician does not concern himself with material substance, but with pure thought. His creations are not only created by thought but consist of thought, just as the creations of the engineer consist of engines. And the concepts which now prove to be fundamental to our understanding of nature ... seem to my mind to be structures of pure thought, incapable of realisation in any sense which would properly be described as material ... The universe cannot admit of material representation, and the reason, I think is that it has become a mere mental concept.(125)

And elsewhere he writes:

Broadly speaking, the two conjectures are those of the idealist and realist -- or, if we prefer, the mentalist and materialist-- view of nature. So far the pendulum shows no signs of swinging back, and the law and order which we find in the universe are most easily described-- and also, I think, most easily explained -- in the language of idealism. Thus, subject to the reservations already mentioned, we may say that our present-day science is favourable to idealism. In brief, idealism has always maintained that, as the beginning of the road by which we explore nature is mental, the chances are that the end also will be mental. To this present-day science adds that, at the farthest point she has so far reached, much, and possibly all, that was not mental has disappeared and nothing new has come in that is not mental. Yet who shall say what we may

find awaiting us round the next corner? (126)

We must try to see whether it is necessary
to choose between materialism and idealism.

CHAPTER TWO

THE SPECIFICATION OF THE SCIENCES

1. The Problem

The expressions "mathematical physics" and "physico-mathematical science" immediately suggest an epistemological dualism which implies both a distinction and a union. And the crux of our whole problem lies in analyzing accurately the nature of that distinction and that union. In the present chapter we shall endeavour to lay bare the basic principles which determine the distinction between mathematics and physics; in chapter three we shall consider the principles which govern the union of the two. And the principles laid down in these two chapters will serve as the foundation upon which the entire superstructure of the chapters which are to follow will be built; they will guide and shape the whole subsequent analysis.

Our first concern, then, is to see how physics and mathematics are distinguished from each other. The mere recognition of the dualism implied in the expression

"mathematical physics" does not of itself predetermine the solution of our problem. For a dualism may be only nominal; it may be only the superficial expression of a basic identity. As a matter of fact, the dictionary of modern science is filled with expressions which suggest epistemological dualism: bio-chemistry, astro-physics, etc. And the very creation of these apparently hybrid sciences seems to have come from a recognition of a basic identity between the branches of knowledge joined together. As science progresses, this basic identity seems to be growing increasingly evident. Barrier after barrier between the sciences is being broken down; there is steady progress towards epistemological homogeneity. And on the face of things this seems to hold for mathematical physics as well as for the other hybrid sciences. Recent developments seem to be wearing pretty thin the traditional distinction between physics and mathematics. The most abstract conceptions of pure mathematics are being "incarnated" in the physical universe; the most concrete elements of the physical universe are finding a mathematical explanation. And perhaps few would hesitate to deny that there is a greater dichotomy between mathematics and physics than between biology and chemistry.

Our problem, then, is to try to discover how deep this dichotomy is between physics and mathematics. It is a problem which has innumerable ramifications, and which cannot be dealt with adequately in isolation from its epistemological context. In order to get at the nature of the distinction between physics and mathematics we must see how they fit into the whole epistemological scheme of things. In other words, we are faced with the question of a classification of the sciences . And we must explore this general question at least to the extent in which it is necessary to throw light upon the specific problem we have in hand.

It has often been remarked that the human mind has an instinctive tendency towards monism. It is an extremely significant tendency, and one which reveals the inner nature of the intellect. The history of philosophy has been a constant manifestation of this tendency under a great variety of forms. There have for example been countless attempts at some kind of ontological monism. But this is not the aspect of the tendency in which we are interested here; we are concerned with what might be called epistemological monism: the attempt to reduce

all human knowledge to one homogeneous type; the failure to recognize the radical heterogeneity of the ways in which the human mind enters into contact with reality. It would hardly be an exaggeration to say that one of the greatest intellectual evils of modern times has been this persistent attempt to homogenize knowledge. It is an evil which has had far reaching consequences, notably in the field of education. But these consequences are not particularly relevant here.

In this connection, positivism and scientism readily come to mind. But even philosophical circles which have rejected positivism and scientism (including the majority of modern scholastic circles) have been affected by this evil in a number of ways. Typical examples are: the identification of speculative and practical knowledge; the identification of metaphysics and the philosophy of nature; the identification of dialectical knowledge and true scientific knowledge, and the identification of mathematical and physical knowledge. This last example is obviously the one which affects us most directly. But all the others have definite repercussions upon our problem as we shall eventually see. It is worth while

pointing out here that the unification of knowledge has historically been associated with mathematicism. And the reason is that in no science can this tendency be carried so far as in mathematics.

Now it is extremely significant to note that homogeneity is at once the source of unity and the source of multiplicity -- infinite multiplicity. That is why the melting down of human knowledge to one standard type has almost inevitably resulted in the breaking up of the sciences into almost innumerable branches. One has only to study the classification of the sciences attempted by Bacon, Comte, Spencer, Bain, Karl Pearson and Huxley, to (1) mention only a few, in order to see how highly arbitrary the distinctions between the sciences must necessarily be if all knowledge is of one homogeneous type. And because these distinctions are arbitrary, the advancement of science has made short shrift of many of them. That is why some have come to the conclusion that all distinctions between sciences are purely capricious. And in this connection the following lines of Max Planck are significant:

Looked at correctly, science is a self-contained unity; it is divided into various branches, but this division has no natural foundation and is

due simply to the limitations of the human mind which compel us to adopt a division of labour. Actually there is a continuous chain from physics and chemistry to biology and anthropology and thence to the social and intellectual sciences; a chain which cannot be broken at any point save capriciously. (2)

In the sixteenth century two contemporary philosophers wrote on the question we are discussing. The one represented the birth of a new philosophical movement; the other represented the end of an old philosophical tradition that was passing away. The first was Rene Descartes, and the second John of Saint Thomas. Descartes was the principal source of what Maritain has justly called "the radical levelling of the things of the spirit" (3) that is so characteristic of modern times. In his famous page in the Regulae on the unity of knowledge, modern epistemological monism received its first explicit formulation. And the source of this formulation was the mathematization of nature, about which we spoke in Chapter (4) One. Around the time that Descartes wrote this page in Regulae, John of St. Thomas wrote an article on the unity and distinction of the sciences at the end of his (5) Ars Logica. - an article which summed up and synthesized with admirable clarity and precision all of the fundamental Thomistic principles governing the classification of the

sciences. Though it must be admitted that in his philosophical writings he neglected the order of concretion, and that he seemed completely unaware of the great scientific discoveries that were going on around him, no one ever achieved a better exposition of the fundamental notions of science and the principles which determine the unity and distinction of the sciences. It is principally to him that we shall look for a guide in our discussion of the present question. At the same time it must be noted that he merely synthesized principles already found in Aristotle and St. Thomas; he in no way changed or added to these principles, as some have maintained.

But before embarking upon this discussion it is important to point out that there are two fundamentally distinct aspects to the question of epistemological pluralism. For the problem may be considered either from the point of view of the plurality of formally distinct objects that the mind lays hold of in reality, or from the point of view of the plurality of the means of knowing employed by the mind, namely the intelligible species. In other words there are two distinct problems of the One and the Many. (6) Because we are engaged here with human knowledge,

both aspects enter into our problem. But it is important to keep in mind that a plurality on the part of the objects does not necessarily imply a plurality on the part of the means of knowing. In fact, in proportion as an intelligence is more perfect, the plurality of its means of knowing decreases while the distinctness with which it knows objective reality increases. The divine intelligence sees the whole of reality exhaustively in its ultimate distinction in the one intelligible species which is His essence. At the other extreme of the scale of intelligences, the human mind needs as many intelligible species as there are natures to be known. If the human intellect were in a state of perfection, the problem of the distinction of the sciences would be easily solved: there would be as many species of science as there are species of things. Saint Thomas explains that in the infused knowledge of Christ there were as many species of science as there were species of things known by Him. (7)

But because of the imperfection of the human intellect, it is necessary for it to know a plurality of objects which in themselves are specifically distinct in the light of a common scientific species. This commonness, however, is something quite different from the commonness of the intelligible species possessed by the higher intelligences which enables them to grasp reality in its distinction.

It is a commonness of potentiality which hides rather than reveals the distinction of reality.

In connection with the question of epistemological monism mentioned above it seems necessary to point out here that if the monistic tendency consists merely in an attempt to reduce the plurality of the means of knowing, as is done in the method of limits, it is a legitimate and laudable thing. It is reprehensible, however, when it consists in a reduction on the part of the objects.

These remarks should suffice to show that the question of the distinctions and specification of the sciences is an extremely complicated thing, which depends essentially upon the nature of the intellect in question. For God, for example, there is no speculative science distinct from His one science which is wisdom, since He necessarily must see all reality in terms of Himself, the First Cause. This does not mean, of course, that He fails to grasp the ratio mobilitatis, for example, which, as we shall see presently, is the formal ratio of all natural things, but He sees it sub ratione Deitatis.

For all created intelligences there is a distinction of speculative sciences even though all of them

must remain essentially subordinated to wisdom. And the nature of this distinction depends upon the nature of the intelligence in question. That is why there is a plurality of sciences peculiar to the human intellect which, unlike the angelic intellect whose knowledge is prior to things in so far as it is derived from the species divinae rerum ⁽⁸⁾ factivae, is dependent upon things for its knowledge.

This dependence, plus the fact that its object is necessarily material things, make human knowledge essentially abstractive. And that is why the plurality of the human speculative sciences is determined by abstraction. No other principle of division is possible.

But before we come to the question of how the speculative sciences are distinguished by the different degrees of abstraction, it is necessary to go back further in our analysis of the heterogeneity of knowledge. For reasons which will become apparent later, particularly in Chapter IV, we must begin with the primordial distinction between speculative and practical knowledge.

2. Speculative and Practical Knowledge

The implications of this distinction are manifold,

and it would take us too far afield to consider even the more important ones. We shall content ourselves with a summary consideration of those implications which have a particular relevance for the understanding of mathematical physics. (9)

Briefly, then, speculative and practical knowledge differ by their end. (10) The end of speculative knowledge is truth; the end of practical knowledge is an operation, that is, a work to be done or made. (11) When we say that the end of practical knowledge is an operation, or a work to be done or made, we mean an operation or a work that is outside the intellect. (12) For as Saint Thomas points out, an operation may be either exterior or interior to the intellect. In the latter case the operation is a mere contemplation of truth, and in this speculative knowledge consists. Moreover, within the intellect there may be a kind of opus consisting in an ordering and a construction. In this case we have an art, but only a speculative art, and not a practical art, for the opus remains interior to the mind. Both logic and mathematics are arts of this kind. This distinction between speculative and practical art is of some importance, since both of them have a vital

part to play in the construction of mathematical physics.

The object of all practical knowledge, then,
(13)
is something outside the limits of the intellect. It is,
in fact, primarily and essentially the object of an
appetite, for the intellect can have practical knowledge
only because it submits itself in some way to an appetite
(even though practical knowledge in itself does not consist
in a mere extrinsic submission). Hence it follows that
practical knowledge has as its object the good as good
(bonum ut bonum), and not the good as true (bonum ut verum)
which is the object of speculative knowledge. That is why
in order to have true practical knowledge it is not
sufficient that the object be in itself an operabile, i.e.
something that in itself is "makeable"; it is necessary
that this object be considered precisely in ordine ad
(14)
operationem, or per modum operandi. Now whereas the
object of speculative knowledge is something within the
intellect, and that of practical knowledge is something out-
side the intellect, if we consider the principles of these
two types of knowledge, the situation is exactly the
reverse (at least in so far as human knowledge is concerned).
The principles of speculative knowledge are in things, and

the movement is from things to the mind; the principles of practical knowledge are in the mind and the direction is from mind to things. That is why St. Thomas writes: "Practicus intellectus est de his quorum principia sunt in nobis, non quomodocumque, sed in quantum sunt per (15) nos operabilia."

Consequently, the mind is the measure of the things of which it has practical knowledge, whereas it is measured by the things of which it has speculative knowledge, as St. Thomas explains in the following passage:

Res aliter comparatur ad intellectum practicum, aliter ad speculativum. Intellectus enim practicus causat res, unde est mensuratio rerum quae per ipsum fiunt; sed intellectus speculativus, quia accipit a rebus, est quodammodo motus ab ipsis rebus; et ita res mesurant ipsum. Ex quo patet quod res naturales, ex quibus intellectus noster scientiam accipit, mesurant intellectum nostrum, ut dicitur X Metaphys. (com.9): sed sunt mesuratae ab intellectu divino; in quo sunt omnia creata, sicut omnia artificata in intellectu artificis. Sic ergo intellectus divinus est mesurans non mesuratus; res autem naturales, mesurans et mesurata; sed intellectus noster est mesuratus, non mesurans quidem res naturales sed artificiales tantum. (16)

Now there is an analytical connection and a direct proportion between the operabilitas (the "makeableness") of a thing and its degree of immateriality. Here

it must be noted immediately that we are taking the term "immateriality" in its broadest significance, in the sense in which it is opposed to any kind of potentiality, and hence to any form of contingency. The first condition required for a thing to be the object of practical knowledge is that its essence be not identified with its existence. For the practical knowledge is knowledge of things to be brought into existence. That is why God is the only being who cannot be the object of practical knowledge (except in the sense that He is attainable by intelligent creatures through practical knowledge).⁽¹⁷⁾ As John of St. Thomas points out,⁽¹⁸⁾ the speculative abstracts in some way from the existential (ab exercitio existendi), whereas the practical considers its object in its existential state (ut stat sub exercitio existendi). Yet it would be highly ambiguous to say, as some authors have done, that speculative knowledge has to do with the essential order, and practical knowledge with the existential order. For there is an operabilitas in the essential order as well as in the existential order. All beings which have potency in their essence, i.e. matter in the strict sense of the terms have an intrinsic ontological plasticity, a "formability" which pure forms do not have. In all material creatures,

"formability" touches the very substance. In their very essence is found the reason for their intrinsic physical contingency.

Viewing the hierarchy of being dialectically, we may say that in the measure in which we get farther and farther from pure immateriality in which the essence is identified with existence, in the measure in which we get deeper and deeper into materiality, the closer we approach to pure operabilitas and hence the greater becomes the scope of practical knowledge. We are getting deeper and deeper into contingency and hence farther and farther away from the necessary, which is the object of speculative knowledge. In this dialectical process we start with the Being of which only speculative knowledge is possible, and we tend towards a limit which would be an object that would be purely practical. This object does not exist, nor can it exist, but there is something like it in moral knowledge. Saint Thomas points out that the study of morals is not
(19)
for the contemplation of truth.

It should be pointed out, perhaps, that we are considering this descending scale from the point of view of natures, for if other points of view were introduced,

such as the large place that fortune plays in human life, and the immense amount of contingency involved in the supernatural order, what we have just said might be open to modification. Perhaps some might be tempted to take exception to the last paragraph on the score that the ultimate elements might very well prove to be few in number and highly determined in their constitution. But even if this should prove to be true what we have said would still hold. For elements are by their very nature for the whole, and from this point of view they would possess indefinite malleability and "formability" and serviceability because of the fact that everything in material creation would be made out of them.

Now all this has an extremely important bearing upon the nature of physics. For the object of physics is down very far in the scale we have been considering. This is particularly true of that part of physics which is far advanced towards concretion. And the farther physics advances the deeper it gets into materiality. That is why the things with which physics deals are principally operabilia, more operabilia than speculabilia. And as physics progresses, the things with which it deals become less and less amenable to speculative knowledge and more

and more amenable to practical knowledge.

Moreover, in order to possess fully the speculative knowledge of which these things are capable, it is necessary to have practical knowledge of them. For even though speculative knowledge always remains something distinct from practical knowledge, in order to have perfect speculative knowledge of things that are in their very nature operabilia, it is necessary to have practical knowledge of them. And the more things are operabilia in their very nature, the greater becomes the necessity of having practical knowledge of them in order to possess with any kind of adequacy the speculative knowledge that it is possible to have of them.

Now the difficulty is that this practical knowledge is not open to us. For we cannot make natures. We can only imitate them by making artificial things. Natures are, in fact, essentially "rationes artis divinae," as Saint Thomas points out in the second book of the Physics. In other words, art is essentially an extrinsic principle, and it is only in divine art that this extrinsic principle can be the cause of the intrinsic principle. The reason is that whereas all created art presupposes a subject, divine art does not, and as a consequence it can reach the very

first principle of the things it makes.

But even though man cannot have a practical knowledge of natures which alone would make it possible for him to have perfect speculative knowledge of them, he can have practical knowledge in relation to natures, and by means of it acquire a more perfect speculative knowledge of them. As a matter of fact, in order for man to have a profound speculative knowledge of natural things in their concretion it is necessary for him to have recourse to an immense amount of practical knowledge. He must operate upon nature with instruments devised by himself. And the deeper he plunges into concretion the more highly complex and subtle must these instruments and operations become. In this way practical knowledge becomes more and more an implement of speculative knowledge. Man must construct before he can contemplate, and it is precisely because of the weakness of his speculative knowledge that he must have recourse to practical knowledge.

Not only must physical construction enter into physics in an increasingly large measure as it advances, but mental construction as well. In theory-building, which still falls within the genus of art, though it be a

speculative art, the scientist makes, as it were, an ersatz logos which can never do more than connote objective nature. Moreover, in order to rationalize nature the physicist is forced to borrow heavily from mathematics which is also a speculative art.

Thus in a number of ways construction enters into the object of physics -- enters into it so profoundly that it becomes impossible to distinguish between what is derived from nature and what comes from art. All this is necessary but it constitutes a danger. For it is all too easy for man to come to look upon nature as a mere malleable matter to be worked upon and used. Moreover, the knowledge we acquire by having recourse to this construction makes possible such extensive mastery over nature that the practical power that is derived from this knowledge all too easily becomes confused with the purely speculative knowledge of nature which is the basis of the practical knowledge. In other words there is the danger of confusing the speculative knowledge we have of natural things with the knowledge of what we can do with them, or at least of subordinating the speculative knowledge of nature to the practical knowledge we are able to have in relation to it, in somewhat the same way as is found in

the case of the artist who is concerned with the nature of the material he uses only to the extent to which that is necessary for the achievement of his work of art. Then the practical knowledge is no longer the instrument of the speculative knowledge, but just the contrary. And even when the confusion between speculative and practical knowledge, or the perversion of the right order that should exist between them does not occur, there is at least the danger that the abundant use that we can make of nature might lead us to cease to wonder at nature, and without this wonderment, as Aristotle has pointed out, speculative knowledge cannot thrive.

That the tendencies we have just mentioned have been prevalent in modern times is all too evident. Already in Descartes we find the following:

Mais sitôt que j'ai eu acquis quelques notions générales touchant la physique, et que, commençant à les éprouver en diverses difficultés particulières, j'ai remarqué jusques où elles peuvent conduire et combien elles diffèrent des principes dont on s'est servi jusqu'à présent, j'ai cru que je ne pouvais les tenir cachées sans pécher grandement contre la loi qui nous oblige à procurer autant qu'il est en nous le bien général de tous les hommes: car elles m'ont fait voir qu'il est possible de parvenir à des connaissances qui soient fort utiles à la vie;

et dans les écoles, on en peut trouver une pratique, par laquelle, connaissant la force et les actions du feu, de l'eau, de l'air, des astres, des cieux et de tous les autres corps qui nous environnent, aussi distinctement que nous connaissons les divers métiers de nos artisans, nous les pourrions employer en même façon à tous les usages auxquels ils sont propres, et ainsi nous rendre comme maîtres et possesseurs de la nature. (22)

These tendencies have continued to grow since the time of Descartes, and today it is not rare to find even in the writings of those who have otherwise made valuable contributions to the philosophy of science passages in which the important distinction between speculative and practical knowledge seems to have faded to a large extent. The following lines of F.C.S. Schiller are fairly typical:

The mental attitude which entertains hypotheses.. feels free... to rearrange the world at least in thought, to play with it, and with itself. For hypothesis is a sort of game with reality, akin to fancy and make-believe, fiction and poetry ... It is by this hypothesis - building habit that science touches poetry on the one side, and action on the other; for it is akin to both. The play of fancy and the constructive use of the imagination reveal the creativeness of human intelligence; by their use the scientist becomes a "maker" like the poet... Yet on the other side, this hypothetical attitude mediates between thought and action, and helps to break down the superficial distinction between the theoretic and the practical. It drives the scientist out of the

purely receptive attitude, and makes him a doer. For to entertain a hypothesis is to hold a mental content hypothetically, and this is to hold it experimentally, which, again is to operate on it and to manipulate it. (23)

From many points of view it is in Marxism that the tendencies of which we have been speaking have found their fullest expression. Marx' eleventh thesis on Feuerbach states that "the philosophers have only interpreted the world differently; the point is to change it." At the heart of Marxism is a revolt against the humble state of being measured by things that is characteristic of speculative knowledge and a desire to become their measure through practical knowledge. There is a seeking to transform nature completely, to reconstruct it to one's own image and likeness, to subject it entirely to the exigencies of one's life of praxis. In his Introduction to Dialectical Materialism, Edward Conze, a faithful disciple of Marx, has this to say:

Dialectical materialism is surrounded by the glamour of being something specially strange, mysterious and startling. To the extent to which this new method of thinking becomes better known, the charm of the unknown will vanish. It will be seen that it is not a nice piece of decoration, but a very prosaic and practical tool. It has more the functions

of an axe than of a Chinese vase...
Not the mere understanding, but an increased
control of the world, is the ultimate purpose
of scientific method.(24)

But all this is an anticipation of what is to
come in subsequent chapters. Consequently, we must leave
this point, and having seen the nature of the distinction
between speculative and practical knowledge, we must pass
on now to a consideration of the hierarchy of speculative
science. This will bring us directly to the central point
around which the whole of the present discussion is re-
volving; the nature of the distinction between physics
and mathematics.

3. The Hierarchy of Speculative Science.

Science, writes Professor Urban, "is the most
(25)
ambiguous concept in the modern world." In order to
avoid confusion it seems necessary to point out immediately
that at the beginning of this discussion and until further
notice we shall take the term "science" in its strict
Aristotelian sense. Both Aristotle and Saint Thomas
sometimes use the expression "scientific knowledge" in a
(26)
fairly loose fashion. Thus, in the Posterior Analytics
"quaelibet certitudinalis cognitio" is called scientific

(27)
knowledge. In the Summa St. Thomas sometimes uses the word "scire" in terms of knowledge of particular contingent facts. But outside of a few exceptions of this kind, "science" in the peripatetic tradition has consistently meant a knowledge that is universal and necessary, a knowledge that has been arrived at by demonstration, and a knowledge that has been fixed and determined in an intellectual habitus. (28)

Now, in coming to grips with the problem of the distinction and classification of the sciences, it is extremely important to discover the true criteria by which one type of scientific knowledge is distinguished from another. One cannot select these criteria in an arbitrary fashion, for, as we have already pointed out, this inevitably leads to epistemological confusion. What, then, will reveal to us the true criteria of an objective and necessary classification. Obviously, the nature of knowledge itself.

Knowledge is essentially objective, for, in Thomistic terminology, to know is to be the thing known in its very "otherness." But human knowledge, because of its limitations, is never completely objective under every aspect. Potentiality always involves some kind of

subjectivity, and the intrinsic potentiality of man's nature necessarily limits the objectivity of his knowledge. quidquid recipitur ad modum recipientis recipitur; hence if the knowing faculty is very imperfect, the objectivity of its knowledge, however true it may be, must necessarily be very imperfect. It would seem to follow from this that the segmentation of scientific knowledge into specifically distinct types must be based on something which is fundamentally objective, but which has, at the same time, a subjective determination.

As we have already remarked, if human knowledge were in a state of perfection the problem of the distinction of the sciences would be simple, since there would be as many species of science as there are species of things. But because man is incapable of grasping things perfectly, it is necessary for him to know a plurality of objects which in themselves are specifically distinct in the light of a common scientific species. Now in order to grasp clearly the nature of this common scientific species we must introduce here the distinction between "thing" and "object". By "thing" we understand what is commonly known as the material object of knowledge, i.e. that which is known, the res in se,

considered purely in its entitative status. By "object" we understand what is commonly known as the formal object of knowledge, i.e. the particular determination or formality by which the cognitive power lays hold of the "thing". For a thing can become the object of knowledge only in so far as it is orientated to a cognitive power in a certain determined way. Thus, an eye can perceive a wall only because the wall is orientated to the eye by means of its color. From what has already been said about the nature of human knowledge it must be evident that the specification of scientific knowledge must come from reality, (29) not however in so far as reality is a "thing", but in so far it (30) is constituted as a scientific object.

Consequently, whenever St. Thomas uses such expressions as "scientiae secantur quemadmodum et res," (31) he understands "res" in the sense of formal object; for in the text just cited he immediately adds: "nam omnes habitus distinguuntur per objecta, ex quibus speciem habent."

In relation to the formal object, Cajetan introduces a further distinction which will be extremely useful for us, not only for our present purpose, but also for the final explicit formulation of the nature of physico-

mathematical knowledge which we shall attempt in Chapter
XI (32). He points out that there are two kinds of formal
object; one is the formality existing in the thing itself
which directly terminates the act of cognition, and by
means of which the "thing" is made apprehensible by the
cognitive power; the other is a formality which actualizes
the first formality. The concrete example usually given
to illustrate this distinction is borrowed from the realm
of sense cognition: in visual cognition there are two
formalities: the color existing in the wall, and the light
which plays upon the wall and actualizes its color. By
transposing this example to the realm of intellectual
cognition we discover that the second formality is a kind
of objective spiritual light (33) which manifests and
actualizes a determined formality existing in the thing,
which in turn renders the thing intelligible by constituting
it as an object. The first of these two formalities is
known in Thomistic terminology as the "objectum formale quod"
or the "ratio formalis quae", or the "ratio formalis objecti
ut res." The second is known as the "objectum formale quo,"
or the "ratio formalis sub qua," or the "ratio formalis
objecti ut objectum." This distinction may appear extremely
subtle, but Cajetan rightly insists upon its necessity:

Necessitas autem, qualitas et distinctio harum rationum sumenda est ex distinctione duorum generum, in quibus oportet locare obiectum scientiae. Oportet enim quod formaliter sit talis res, taliter scibilis. Et ideo oportet quod habeat et rationem formalem constituentem formaliter ipsam in tali esse reali, et rationem formalem constituentem formaliter ipsam in tali esse scibili. (34)

Now, from what has been said thus far it should be evident that the point of departure of the whole question of the specific distinction of the sciences must be an attempt to discover in the entire realm covered by scientific knowledge specifically distinct "rationes formales sub quibus," For, as we have just seen, it is the "ratio formalis sub qua" that actualizes the ratio formalis quae . In other words, what we are trying to decide is whether or not there are specifically distinct ways in which reality is scientifically knowable, and it is precisely ratio formalis sub qua which constitutes reality as scientifically knowable, i.e. in esse scibili . But where shall we turn to discover the specifically distinct rationes formales sub quibus by which one science will be distinguished from another? Once again our answer will be found in the nature of knowledge in general, and the nature of scientific knowledge in particular. (35)

(36)

The root of all knowledge is immateriality.

This immateriality is required first of all on the part of the knower which, in order to be open to other forms besides its own, must enjoy a certain independence of the restrictions of matter which is essentially a subject and hence entirely closed in upon itself. It is also required on the part of the thing known, for a thing can be known only in the measure in which it is constituted as an object, that is to say in the measure in which it is lifted out of the state of being a pure subject. When sufficient immateriality is not possessed by the object in the state in which it is found in nature, the knower must operate upon it and lift it to the state of immateriality required.

Because of this dependence of knowledge upon immateriality, if in the realm of speculative knowledge different levels of immateriality are discernible, there will be a stratification of the sciences corresponding to these different levels. Moreover, necessity pertains to the essence of science, for no truly scientific knowledge is possible of things in their contingency. (37) Consequently there will be as many different sciences as there are different types of necessity; that is to say, the sciences will be distinguished by the specifically different levels

according to which the scientific object can be lifted out of the flux of contingency. Hence St. Thomas concludes: "Et ideo secundum ordinem remotionis et a materia et a (38) motu scientiae speculativae distinguuntur." But the sciences will not be specified by the degree of immateriality and necessity of the object considered in its entitative state in such a way that the species of science will correspond to the degrees of being. If this were the case, the specification would be coming from the material object, which as we have seen, is impossible. It is the degree of immateriality and necessity arising out of the way in which the object is known by the intellect that is the principle of specification.

Now the means by which the intellect lifts its object out of the opacity of matter and the flux of change is called abstraction. Hence it will be the specifically different degrees of abstraction that will give us the rationes formales sub quibus we are looking for, and these in turn will actualize in the object different rationes formales quae. But before pursuing the discussion of the diverse degrees of abstraction, it is necessary to point out that we are concerned here not with total but with formal abstraction. This distinction is of capital im-

portance for the philosophy of science, and no one has probed its profound implications with greater acuteness than Cajetan. (39) Since all positive abstraction involves some kind of separation, the basis of this dual abstraction is a dual composition: the composition of matter and form, and the composition of a universal whole and its subjective parts. Abstraction is called formal when it consists in disengaging a form from the matter in which it is concretized; it is called total when it consists in laying hold of a universal whole apart from the subjective parts in which it is distributed. When a mathematician abstracts a certain quantitative concept, such as the notion of line, from the sensible matter in which it is concretized in the real world, he is practising formal abstraction. For "line" stands in relation to "sensible" as form to matter. When, however, one abstracts the concept of animal from its subjective parts, man and brute, to consider it apart, he is using total abstraction.

In order to avoid confusion it is necessary to point out that when we say that formal abstraction consists in abstracting a formal element from its material concretion it is never a question of abstracting the substantial form from the matter to which it is united, for as St. Thomas

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points out, the interdependence existing between a substantial form and its corresponding matter is such that one cannot be understood without the other. Thus, the student of nature never abstracts the substantial form from its matter; he merely prescind from the contingent materiality proper to individuals. This point is of extreme importance for a proper appreciation of the nature of physics, and it is usually misunderstood by scholastic writers. Similarly, the mathematician does not abstract the substantial form, but the accidental form of quantity. The metaphysician lays hold of substantial form only in so far as it is a co-principle of material being.

There is a world of difference between the two intellectual processes involved in formal and total abstraction. In the first case the separation results in a double concept each of which is complete by itself. The notion of line does not involve the notion of sensible matter, nor does the notion of sensible matter necessarily involve the notion of line. Hence each can be perfectly conceived in separation from the other. But in total abstraction only one complete concept results: the idea of animal is conceivable without the notion of either man or brute; but neither man nor brute is intelligible without

the notion of animal. Because formal abstraction reveals a form that is purified of the potentiality of its material concretion, it gives rise to greater objective intelligibility. In fact, this greater intelligibility is the very reason for the separability of the form. The notion of line, for example, is much more intelligible in its state of abstraction from sensible matter, than in its state of concretion. And let it be noted in passing that here we are touching upon the pivotal point of the whole problem of mathematical physics. In fact, we have given here the fundamental reason why physics in its development must necessarily become mathematical physics. But lest any confusion arise, it is necessary to emphasize the fact that we have been speaking here of greater objective intelligibility. Because of the imperfect condition of our intellect, greater objective intelligibility (intelligibilitas natura, or, intelligibilitas in se) does not necessarily mean greater intelligibility for us. In fact, there is ordinarily an inverse proportion between the two, as we shall have occasion to point out in chapter IV. We say "ordinarily," because mathematical science presents a unique case which we shall study in chapter VI. And this unique case will have an extremely important role to play in

the solution of our problem.

From the point of view of actuality, the movement of total abstraction is the reverse of that of formal abstraction. For, in ascending from brute and man to animal, and from there to higher genera in the Porphyrian tree, we are moving from what is more determined and more actual, and hence more intelligible objectively, to what is more confused, more potential, and hence less intelligible objectively. For the mind can abstract a universal whole from the subjective parts of which it is predicable only by retreating from the actuality and determination of these subjective parts into a state of greater potentiality. But it happens that in moving from what is less intelligible to what is more intelligible objectively we arrive at what is more intelligible for our imperfect intellects. The only kind of mind that can be realized in a being composed of matter and form is one which must acquire its knowledge through experience, and which must, therefore, begin with pure noetic potentiality, a tabula rasa, and move on gradually to greater and greater noetic actuality. That is why things are more intelligible for us in the potential and confused state of their

universality, than in the state of concretion. It is much easier for us to understand what a living being is than to understand what a cow is. We shall discuss this important point in considerable detail in Chapter IV, but it was necessary to bring it out here because, as we shall see in a few moments, a number of modern Thomists, while admitting in the abstract the distinctions we have laid down, have allowed themselves to arrive at erroneous conclusions about the nature of science because of a confusion between the two kinds of intelligibility we have just mentioned.

It should be clear from what has been said why the degrees of abstraction which specify the sciences are necessarily degrees of formal abstraction. Total abstraction is, in fact, common to all the sciences, since scientific knowledge deals necessarily with universals. But before leaving the question of abstraction in general to discuss the degrees of formal abstraction, there is another distinction to be made which will be of considerable usefulness for us as our analysis proceeds. We have in mind the distinction between positive and negative abstraction. In discussing total and formal abstraction we have been dealing with positive abstraction. Negative abstraction is

something quite different, and since its use in experimental science is extensive, it will be necessary to describe its nature briefly.

There are two distinct types of negative abstraction. The first type is called negative because it does not achieve a noetical separation in the strict sense of the word. Just as a sense can pick out a certain quality existing in an object, e.g. the color, and leave aside all the other qualities coexisting with it, so the mind when confronted by a plurality of formalities can concentrate its attention on one of them to the neglect of all the others with which it is connected. In thus concentrating its attention on one formality, the mind does not lift this formality out of its context, set it forth by itself, and consider it formally as separated. Hence the term at which it arrives remains tied to the context from which it has been abstracted. That is why this type of abstraction does not achieve even one complete and independent concept, and in this it differs from both formal and total abstraction, as is evident from what was said above. Negative abstraction is like total abstraction in that it arrives at a common notion, but it differs from it in that this common notion is not considered in relation to its inferiors. It is

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like formal abstraction in that it lays hold of a certain formality, but it differs from it in that the separation is only negative, and consequently it does not consider the formality formally as separated.

The second type of negative abstraction is that
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used in logic. It gives rise to an object which, though related to something in nature, does not have being in nature, but only in the mind. Positive abstraction always gives rise to an object that has being in reality, even though, as in the case of mathematical abstraction, the mind separates it from something to which it must be united if it is to have its being in reality. It is of great importance to distinguish carefully between mathematical abstraction and this second type of negative abstraction. In mathematical abstraction the mind does not supply the object but merely the conditions of the object, whereas in negative abstraction the mind supplies the very object. This type of negative abstraction plays an important role in experimental science because of its dialectical character and because of the extensive use of mental constructs. The first type mentioned above is employed in the formation of the universals that are characteristic of experimental science, for since the universals are merely hypothetical they cannot be the

result of positive abstraction.

We are now in a position to pursue our discussion of the degrees of formal abstraction. They are brought out with admirable clarity by Saint Thomas in his Commentary on the De Trinitate of Boethius, (43) and we scarcely need to do more than paraphrase his treatment of them. There are three kinds of matter, and consequently three distinct levels in the process by which the mind lifts its scientific object out of the potentiality in which it is concretized. First there is individual matter, i.e. the matter which sets individual things off from each other with all the particular individualizing characteristics proper to each. As long as these individualizing characteristics are retained no science is possible, for: omne individuum ineffabile. The reason is that a thing is intelligible only to the extent to which it is in act. Matter is being in potency and everything that is dependent upon it essentially and inseparable from it is not intelligible in act. Hence it is from the knower that intelligibility in act must come. Consequently the first step in the process of scientific abstraction is to slough off these particular characteristics and by so doing arrive at a specific intelligible essence. This first step is called physical abstraction, and it is used by all the

disciplines which study nature. The second kind of matter is known as common sensible matter. By sensible matter is meant matter that is apprehensible by the senses, and hence something that involves material qualities. This common sensible matter remains untouched by the first degree of abstraction, for the biologist, for example, studies flesh and blood, even though he does not study this particular flesh and blood, the flesh and blood of Socrates, for example. ⁽⁴⁴⁾ The second step in scientific abstraction consists in disengaging an intelligible form from this sensible matter. This is known as mathematical abstraction, for it is the abstraction employed by the mathematical sciences. In spite of its high degree of abstraction, mathematics does not succeed in freeing itself of all matter, for whatever is quantitative is necessarily material. But the matter which it retains though apprehensible by the intellect is no longer apprehensible by the senses, since all sensible qualities have been refined away. Hence it is known as intelligible matter. The last step in our intellectual purification succeeds in freeing the scientific object of this last vestige of matter and in setting it forth in its pure intelligibility. This is known as metaphysical abstraction.

There is another and more profound way of presenting these three degrees of abstraction: Some scientific objects depend upon sensible matter both for their being and for their "being known", that is to say, both for their objective existence outside the mind and for their subjective existence in the mind. As a consequence, they can neither exist nor be conceived or defined except in terms of sensible matter. These constitute the objects of the disciplines which study nature. All of the natural sciences study the material cosmos in its state of concretion in sensible matter. And they study it precisely from the point of view of sensible matter; that is to say, all the definitions of natural science are in terms of sensible matter. One may be tempted to contest this statement, since sensible matter means, as we have said, sensible qualities invested in matter, and physics seems to prescind from all qualitative determinations and to study the universe only in terms of the category of quantity. The answer to this objection is, of course, that modern physics is mathematical physics, and consequently not a pure natural science. Other scientific objects depend upon sensible matter for their being, but not for their "being known." That is to say, in order for them to exist outside the mind

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in the world of reality they must be invested in sensible matter. But they are conceived and defined independently of it. The notions of line, triangle, number three, etc. contain no sensible matter, nor are they ever defined in terms of it; yet if they are to exist at all in the objective world, they must be concretized in it. These form the objects of the mathematical sciences. Still other scientific objects depend upon sensible matter neither for their being, nor for their "being known". Not only are they conceived and defined independently of all matter, but they can exist in objective reality independently of all matter, either because they necessarily do not exist in matter, as for example God and the Angels, or because they do not necessarily exist in matter, as the concepts of substance, quality act and potency, etc. Here we have the
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objects of metaphysical science.

St. Thomas points out that this threefold division is exhaustive. For the only other possible case that might be imagined would be that of objects that would be independent of sensible matter in their objective existence, but dependent upon it for their subjective existence in the mind. Though completely immaterial in their being, they would have to be materialized in order to be

conceived and defined by the intellect. The inadmissibility of such a case is evident, since it implies that the intellect is essentially material and supposes the primacy of matter. Moreover such a process of materialization would be just the opposite of abstraction.

It is necessary to point out here in passing something that will be of considerable significance for us later. Even a casual consideration of the three degrees of abstraction brings to light the fact that there is something peculiar about the type of abstraction used in the mathematical sciences. In it alone do we find a deep dichotomy between the way things exist in reality and the way they are conceived by the mind. In both physical and metaphysical abstraction there is a correspondence between the way things exist objectively and the way they are conceived and defined by the mind. This correspondence is lacking in the second degree of abstraction. In order for mathematical objects to exist at all outside the intellect they must be immersed in sensible matter, whereas inside the intellect they are conceived and defined in complete independence of it. Hence in this case abstraction involves a separation that is not found in the other degrees. Later on in our analysis this dichotomy will throw a great deal of light upon the nature of mathe-

mathematical physics.

This threefold level of formal abstraction provides us with the specifically different rationes formales sub quibus that we set out to find. We have three different grades of immateriality, three different ways of abstracting and defining the scientific object. In metaphysics everything is defined without relation to matter of any kind. In Mathematics everything is defined in terms of intelligible matter alone. In the study of nature everything is defined in terms of sensible matter. Now these three rationes formales sub quibus in turn actualize and light up three specifically distinct rationes formales quae: being in metaphysics; quantity in mathematics; mobility in the study of nature. The first of these three objects is not of any special interest for our problem. We shall remit the question of the second object to Chapter VI where we shall discuss in some detail the nature of mathematical science. Since we are particularly concerned with physics, the scientific object which has the greatest interest for us is the one that is born of the first degree of abstraction. Thomists have traditionally insisted that the proper object of the study of nature is ens mobile: mobile being. For those who approach the question for the first time it is not

immediately evident perhaps why this should be so. There are a number of other ways of expressing the object studied by natural science which would seem to suggest themselves more spontaneously than "mobile being;" such as: "natural body", "natural substance", "sensible being", "physical body", "natural being", etc. In fact some modern Thomists have seen fit to substitute "sensible being" for the traditional "mobile being".⁽⁴⁷⁾ This question has been studied with great profundity and acuteness by Cajetan⁽⁴⁸⁾ and John of St. Thomas,⁽⁴⁹⁾ and though it would be too long and tedious to summarize all of their arguments, there are a few points which must be insisted upon with special emphasis. The reason why mobilitas is taken as the formal object of the study of nature is that better than any other point of view that might be selected, it opens up the inner essence of natural things. In other words, it is only in terms of mobility that nature can be truly explained. The history of philosophy gives us an extrinsic reason for this: from Heraclitus down to Bergson and Whitehead, the problem of mobility has been the crucial point in the study of nature. In his Commentary on the Physics, St. Thomas suggests an intrinsic reason:

De his vero quae dependent a materia non solum secundum esse sed etiam secundum rationem est Naturalis, quae Physica dicitur. Et quia omne quod habet materiam mobile est, consequens est quod ens mobile sit subiectum naturalis philosophiae. Naturalis enim philosophia de naturalibus est; naturalia autem sunt quorum principium est natura; natura autem est principium motus et quietis in eo in quo est; de his igitur quae habent in se principium motus, est scientia naturalis.(50)

The expression "sensible being" which some modern Thomists have attempted to substitute does not bring out the true objective formality in terms of which nature must be studied. For, things in nature are not sensible for the separated substances, but only for us. Hence "sensible" does not directly explain what things are in themselves, but only how they are known by us. Of course, every mobile being is at the same time a sensible being, for there is an analytical connection between motion and sensible matter in that both of them involve material potency. But sensibility does not explain the objective nature of things; it merely explains how we know them. Mobility, on the other hand, is something objective. Even the separated substances know natural things as mobile beings, not, indeed, as we do, merely in terms of the general formality of mobility, but in terms of the specific type of mobility proper to each ontological species.

And just as no other word may be substituted for "mobile", so no other expression can adequately take the place of "being": not "substance", for that would exclude the consideration of accidents; not "body", for as St. Thomas points out, (51) it pertains to the science of physics to prove that all mobile beings are bodies, and no science proves its own subject. John of St. Thomas clearly indicates the positive reason why the expression must be "mobile being:" Motion is not defined in relation to substance or body, but in relation to being, for it is: "actus entis in potentia in quantum huiusmodi":

Fundamentum huius conclusionis sumitur ex his, quae paulo ante sunt insinuata, quia videlicet propria et adaequata passio, quam physicus demonstrat de suo subiecto, est motus. Motus autem non definitur explicando ordinem ad corpus vel substantiam, sed ad ens mobile; definitur enim, quod est "actus entis in potentia", ut patet in hoc tertio libro. Ergo formalis ratio subiecti physici non explicat rationem corporis. Nam quod formaliter est subiectum scientiae, explicatur etiam in formali definitione propriae passionis tamquam id, ad quod passio dicit habitudinem. Ergo cum non explicetur in definitione motus ratio corporis, sed ratio entis in potentia, non pertinet ad formalem rationem subiecti corpus, licet in re verum sit, quod non sit mobile motu physico nisi id, quod est corpus. Sed tamen sub formalitate, qua pertinet ad Physicam, ita se habet, quodsi per impossibile daretur aliquod mobile, quod non esset corpus, adhuc tractaretur a Physica, sicut si per impossibile daretur aliquod coloratum extensum, quod non esset corpus, adhuc videretur ab oculo. (52)

It is extremely important to insist upon the unity and indivisibility of the object of the study of nature. The composition found in the expression "ens mobile" is only verbal. It does not imply a composition of two objective formalities, the formality of being and the formality of mobility. Mobile being does not mean "being" with the addition of a specific difference, "mobile". If this were true, philosophy of nature would be a part of metaphysics or at least a science subalternated to it.

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Both Cajetan and John of St. Thomas lay considerable stress upon this point, and we shall see its importance in a few moments.

The assigning of "mobile being" as the object of the science of nature gives rise to a difficulty, the solution of which will enable us to penetrate more deeply into the nature of physical science. We said above that science is possible only in so far as its object is lifted above the flux of change, for science is about necessary and not contingent things. The etymological root of the word episteme means firmness and stability. Consequently a science of mobile being would seem to be a contradiction in terms.

... de permutante, idest de eo quod movetur... nihil verum dicitur in quantum mutatur. Quod enim mutatur de albedine in nigredinem, non est album nec nigrum in quantum mutatur. Et ideo si natura rerum sensibilium semper permutatur, et omnino, idest quantum ad omnia, ita quod nihil in ea est fixum, non est aliquid determinata verum dicere de ipsa. (55)

In raising this question we are touching upon one of the most persistent antinomies in the whole history of philosophy. Ever since the time of the ancient Greeks philosophers have sought to reconcile the fluidity of nature, clearly revealed by the senses, with the necessity of science. In the doctrines of Heraclitus, Parmenides, Plato, and their followers philosophy and nature were in some measure in constant conflict. Sometimes it was philosophy that suffered from the conflict, and at other times it was nature. It took the genius of Aristotle to reconcile the two and to give birth to a philosophy of nature. (56) It is true that natural things are in a constant state of flux, of generation and corruption. Yet in the midst of this fluidity of phenomena there is in nature a permanent, general structure which the mind can lay hold of through the process of abstraction described above.

Contingentia dupliciter cognosci possunt. Uno

modo secundum rationes universales; alio modo secundum quod in particulari. Universales quidem igitur rationes contingentium immutabiles sunt, et secundum hoc de his demonstrationes dantur et ad scientias demonstrativas pertinet eorum cognitio. Non enim scientia naturalis solum est de rebus necessariis et incorruptibilibus, sed etiam de rebus corruptibilibus et contingentibus. Unde patet quod contingentia sic considerata ad eandem partem animae intellectivae pertinent ad quam et necessaria, quam Philosophus vocat hic scientificum. (57)

It is not necessary to transcend nature in order to find immutable types. Basic regulations in the stream of phenomena reveal the fact that there are immutable types immanent in nature itself. It is only in their individual composite existence, not in their universal essences that the things of nature are fluid. As Aristotle points out in the eighth book of the Metaphysics, it is only an individual house that is brought into existence, not the nature of house in general. In like manner when an individual man dies, the definition of man does not perish. "Etsi enim ista sensibilia corruptibilia sint in particulari, (58) in universali tamen quamdam sempiternitatem habent." It is in this way that definitions of natural things are possible, and wherever definitions are possible, science is possible. These definitions give the universal essences that are concretized in nature, shorn of their individual matter

(materia signata, in Thomistic terminology) but not of common sensible matter (materia non signata). Hence as we have already pointed out, it is not a question of abstracting a substantial form from its corresponding matter, for a form thus abstracted would have no meaning. Now as St. Thomas points out, ⁽⁵⁹⁾ these abstract essences can be considered in two ways: first, in their abstract state in which they exist in the mind alone, and in this way they are without matter (individual) and motion; secondly, in relation to the mobile material things outside the mind from which they have been abstracted, and in this way they are the medium by which physical reality is known, for things are known by means of their form. Thus it is possible to have a science of mobile being.

Nevertheless, it is important to point out that the mobility of the things which form the object of the science of nature has profound repercussions upon the necessity of that science itself. It is only at the price of retreating into broad generalities that the study of nature can enjoy true necessity. Once it begins to press its object more closely as it is bound to attempt to do, the necessity starts to fade. That is why as the study of nature follows its natural course towards greater

concretion, true scientific knowledge (episteme) peters out into opinionative knowledge (doxa). We shall call this opinionative knowledge dialectical knowledge, for reasons which will become apparent later.

In connection with the type of necessity found in the study of nature the following lines of St. Thomas are significant:

Modus autem demonstrationis est diversus; quia quaedam demonstrant magis necessarie, sicut mathematicae scientiae, quaedam 'vero infirmius', idest non de necessitate; sicut scientiae naturales, in quibus multae demonstrationes sumuntur ex his quae non semper insunt, sed frequenter. (60)

Almost instinctively the "doxa" will attempt to erect itself into an "episteme"; the "modus infirmior" demonstrandi will reach out for support to a more sure type of demonstration, the science of nature will seek to rid itself of the mobility to which it is a prey. And that is why physics will inevitably become mathematical.

And now we are in a position to see how the degrees of formal abstraction give us three levels of immobility as well as three levels of immateriality. The science of nature has to do with objects which in their

existence in reality are mobile, and which in their existence in the mind are from one point of view mobile and from another immobile: mobile in the sense that they are conceived of as mobile; immobile in the sense that they are conceived in an immobile way by virtue of a retreat into universality. Mathematical science deals with objects which have mobility in their objective existence, but absolute immobility in intellect. Metaphysical science considers objects which are absolutely immobile in both their objective and subjective existence.

In order to round out our consideration of the hierarchy of speculative science it is important to see the connection this hierarchy has with both an objective stratification in the structure of physical reality, and a subjective stratification in the cognitive powers. Physical reality is constructed in such a way that in it substance has a natural priority over the accidents which inhere in it and determine it. But even among the accidents quantity has a natural priority over the sensible qualities. Quantity is, in fact, the first accident; of all the accidents it is the closest to substance, for it is quantity which orders the parts of material substance and gives it actual extension. It is only because of this

extension that the other accidents can inhere in the substance. For example, a body can be determined by a certain color only because there is an extended surface that can receive this color. Hence sensible qualities are not rooted directly in the substance, but in the quantity. Only through it are they rooted in the substance. Because of its closeness to the substance, quantity possesses a source of intelligibility which the other accidents do not have. But at the same time it must be pointed out that from another point of view it has less intelligibility than the sensible qualities, for these latter follow upon the substantial form, whereas quantity follows upon the matter. We shall return to this paradox later, for it has an important part to play in the solution of our problem.

We find, then, in the structure of physical reality a definite stratification: substance, quantity, sensible qualities. It is possible for the mind to consider the essential determinations of reality independently of any relation to its quantitative and qualitative determinations. It is likewise possible for the mind to consider reality in terms of its quantitative determinations without any relation to its qualitative relations. But the reverse of this process is not possible. It is impossible,

for example, to conceive of quantity without substance, for quantity is precisely the order of the parts of the substance, and order cannot be conceived of without the parts. At first glance this point may seem to be in conflict with what was said above about the nature of formal abstraction. It was pointed out that total and formal abstraction differ in that the latter results in two independent concepts. And we added by way of example that just as the concept of quantity is independent of sensible matter, so the concept of sensible matter is independent of quantity. But from what has just been said it would seem that the concept of sensible matter cannot be independent of the concept of quantity. The solution of this apparent conflict lies in this that there are two kinds of quantity: abstract, mathematical quantity, and concrete quantity. The notion of sensible matter is independent of the former, though not of the latter.

This distinction between abstract and concrete quantity is of great importance for the question of mathematical physics. Since it is possible to lay hold of the concrete quantitative determinations existing in nature by a kind of negative abstraction the road is open to a confusion between this way of considering quantity and

the way it is considered in mathematics which is the fruit of a special type of formal abstraction. As a matter of fact, some authors have fallen into this confusion, as we shall point out in a few moments. The consequences of this confusion are disastrous. For if mathematical physics consisted merely in a study of the concrete quantitative determinations existing in nature by means of negative abstraction, it would not be a hybrid science, but a pure physical science. The mind would not travel out beyond the physical world to a realm apart, to return to the physical world later with a rationality fundamentally alien to it, yet in a mysterious way capable of being applied to it. The mind would remain enclosed within the physical world. This would change the whole epistemological character of mathematical physics.

Now the relation between this stratification and the hierarchy of speculative science does not consist in this that natural science studies the sensible qualities alone, mathematics the concrete quantity as it is found in nature, and metaphysics the substance of reality without any consideration of the accidents. All three of these statements are false. Rather, the connection between the two hierarchies must be expressed in this way: because of

the logical priority existing in the objective structure of the universe, it is possible for the mind in its attempt to lay hold of reality scientifically to take three specifically distinct steps: first to prescind only from the individual characteristics and to consider reality in terms of all its concrete determinations, including the qualitative determinations of sensible matter; secondly to prescind from all sensible qualities and to consider reality in terms of its quantitative determinations alone (but here it must be noted again that it is not concrete quantity that is being considered, but abstract quantity, for concrete quantity is precisely quantity concretized in sensible matter -- here we have a key to the paradox just mentioned about the greater and lesser degree of intelligibility possessed by quantity); thirdly, to prescind from all matter and to consider being as such.

The hierarchy of speculative science also has an essential connection with a hierarchy of cognitive powers. All knowledge begins in the external senses, but not all knowledge terminates there. Likewise all the sciences considered from the point of view of their origin have some kind of relation to the external senses, but considered from the point of view of their term, some

sciences are independent of the external senses, and bear an essential relation to some other cognitive power. For example, our knowledge of God depends upon the external senses for its origin, since the only way we can get to know God is through the material things in the world about us. But it does not terminate there, that is to say, in our conclusions about the nature of God we do not judge that He is like the sensible things in the material cosmos.

This is the basis of St. Thomas' doctrine that natural science terminates in the external senses, mathematical science in the imagination, and metaphysical science in the intellect alone. The reason why natural science must terminate as well as originate in the external senses is clear: all of its conceptions and definitions are necessarily in terms of sensible matter. As St. Thomas puts it, "qui sensum negligit in naturalibus incidit in
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errorem." Hence all of its judgements must be verifiable in sensible experience. It is to be noted that we say "verifiable" and not "verified" in sensible experience, for as we shall see later, it is only that part of natural doctrine which is purely dialectical that must necessarily be verified in sensible experience. We shall discuss this question of the relation between the study of nature and

sense experience in Chapter IV.

The connection between mathematics and the imagination is not so immediately evident perhaps. Since we have the intention of considering this problem in some detail in Chapter VI we shall content ourselves here with merely indicating the basis of the connection. In the first place it is fairly clear that mathematical science does not terminate in the external senses. It is independent of sensible matter in its conceptions and definitions. No mathematician has ever seen in the world of sense a straight line, a perfect circle, or a line touching a sphere at only one point. (64) But that does not affect his science in any way. Yet, while independent of sensible matter, the mathematician still retains intelligible matter, and it is because of this intelligible matter that his science must terminate in the imagination. For intelligible matter signifies homogeneous exteriority, that is to say, the multiplication of the same form through either continuous or discrete homogeneity. This exteriority and multiplicity demands some kind of individuation, and it is precisely the imagination that provides this individuation which in physical things is provided by the matter. Of itself, the intellect has to do with pure form, separated from matter.

Hence if the intellect alone functioned in mathematics we could not have the notion of homogeneous multiplicity. At first glance this may seem to give rise to a serious difficulty. For it is certain that God knows mathematics, and yet He is without imagination. The difficulty vanishes, however, when we take into account the vast difference between the human and the divine intellects. Man's knowledge is posterior to things and his intellect is dependent upon them and measured by them. All of his mathematical notions are drawn from concrete material things. Consequently, when they are lifted out of concrete matter, there must be something to substitute for the individuation which this matter provides. But God's knowledge is prior to things, and His intellect is not measured by them. That is why He does not have need of imagination to provide for individuation.

The connection between metaphysical science and intellect is quite clear. We may arrive at the notion of immaterial things by means of material things presented to us by the external senses and the imagination. But in the end we do not judge that immaterial things are like material things.

This point of Thomistic doctrine must be rightly understood if confusion is to be avoided. Even though only the study of nature terminates in the senses in the way in which we have explained, all science of reality must retain an essential connection with the deliverances of the senses if it is to have any validity. That is to say, it must be able to be resolved back to the sense experiences from which it took its rise. For abstraction does not consist in burning bridges behind one. And this is true even of metaphysics, as St. Thomas explains in the following lines:

Sed quia primum principium nostrae cognitionis est sensus oportet ad sensum quodammodo resolvere omnia de quibus judicamus; unde Philosophus dicit in III Caeli et Mundi quod complimentum artis et naturae est res sensibilis visibilis ex qua debemus de aliis judicare; et similiter dicit in VI Ethicorum (cap.VIII in fin.) quod sensus sunt extremi sicut intellectus principiorum; extrema appellans illa in quae fit resolutio judicantis.(65)

Taken in this sense, the principle of logical positivism that nothing has meaning except in the measure in which it is capable of verification in sense experience is quite acceptable, and is actually realized fully in metaphysics, in spite of the violent opposition to metaphysics on the part of the logical positivists.

Our discussion of the specification of the sciences would not be adequate if we did not touch at least briefly upon another point which emerges from a reading of the passages in which St. Thomas treats the problem. John of St. Thomas (66) calls our attention to the fact that there are a number of texts in which Aquinas seems to use other criteria for the distinction and specification of speculative science than the one upon which we have based our entire discussion, namely the three degrees of formal abstraction. Sometimes he founds the distinction upon a difference in the type of medium used by a science in its demonstrations. (67) In other places he appeals to a difference in the type of definition employed in the science for definitions are the principles of scientific demonstration. (68) With admirable clarity John of St. Thomas goes on to show how all of these different points of view are reducible to the same thing. In doing so he is merely making explicit what is found in St. Thomas himself, for in his Commentary on the Metaphysics (69) the coincidence of the three points of view is already clearly indicated. Since scientific knowledge is precisely knowledge arrived at by demonstration, it is clear if there are specifically different sciences there will be specifically different types of media used in the demonstrations by which

they arrive at their conclusions. Now these media are the premises employed in the scientific syllogism. These premises in turn are necessarily definitions, and hence a specific difference of media reduces itself to a specific difference of definition. But a specifically different type of definition can be had only by means of a specifically different type of formal abstraction. Since immateriality is the source of intelligibility, a specifically distinct level of immateriality is at the root of the specifically distinct ways the mind has of rendering reality intelligible, i.e. of laying hold of its essence, of setting forth its "quod quid est." But to set forth the quod quid est of a thing is to define. Hence the source of the unity and distinction of the sciences is the specific types of immateriality. These types of immateriality result in different types of definition. And this difference in definition gives rise to a specific difference in the principles and media used in scientific demonstrations. The difference in immateriality or intelligible light found in the principles are communicated by means of the demonstration to the scientific conclusions.

In introducing this question of the distinction of the speculative sciences, we said that we would adopt as

our guide the treatment of the problem given by John of St. Thomas. At the same time we noted that this treatment is merely a summary of the doctrine of St. Thomas and Aristotle, and that it in no way adds to it or modifies it in any respect. Perhaps the numerous references of St. Thomas and Aristotle adduced in our discussion of the question suffice to establish the truth of this assertion. But because the issue is of some moment for our study, and because some contemporary Thomists have thrown doubt upon it, we consider it worth while to stop for a moment to consider the problem explicitly.

It has been maintained that the doctrine of the three degrees of formal abstraction taught by Cajetan and John of St. Thomas is not found in St. Thomas himself. Aquinas, we are told, taught that only mathematical abstraction is formal abstraction and that the study of nature employs nothing but total abstraction. Certain texts of the Angelic Doctor seem at first sight to give substance to this claim. In the second article of the fifth question of his Commentary on the De Trinitate of Boethius, he seems to say that only in mathematical science do we have the abstraction of a form from matter. And speaking of the kind of abstraction found in the study of nature, he adds:

"Ideo praedicta abstractio non dicitur formae a materia absolute, sed universalis a particulari." In the next article, he explains that there are three different kinds of intellectual operation found in the three ^pseculative sciences and that the one that is proper to the study of nature is had "secundum oppositionem universalis a particulari, et haec competit etiam physicae, et est communis omnibus scientiis, quia in omni scientia praetermittitur quod est per accidens, et accipitur quod est per se."

It is obvious that these texts must be interpreted in the light of St. Thomas' general doctrine. And in the first place it must be noted that if there is no formal abstraction of any kind in the study of nature, it cannot be a science, for without formal abstraction it cannot have a ratio formalis. Consequently, to hold that St. Thomas and Aristotle in no way associated formal abstraction with the study of nature is equivalent to saying that for them natural doctrine was not a true science -- which is patently absurd. Moreover, there is a special reason why St. Thomas associates total abstraction with the study of nature, for it is only in the things of nature that there are individuals which are not species, and consequently it is only in natural

doctrine that it is necessary to begin by abstracting from individuals in order to get at the object of science.

Many of those who deny formal abstraction to the study of nature admit it for metaphysics. This admission should lead them to recognize the fact that when St. Thomas says that formal abstraction is found only in mathematical science he is taking the term in a very special sense. As a matter of fact it is only mathematics which considers forms that are separated from the sensible matter ^{which} ~~to~~ they must be united if they are to exist. In other words, there is formal abstraction in all of the three species of speculative science, but over and above this there is in mathematics a particular kind of formal abstraction. The proper nature of this type of abstraction will be analyzed in detail in Chapter VI. When St. Thomas seems to restrict formal abstraction to mathematics he warns us how this should be interpreted for he says: "...praedicta abstractio non dicitur formae a materia absolute." It is true that in the essences which constitute the object of the study of nature there is common matter as well as form, but it is illegitimate to use this as a foundation for a denial of formal abstraction in natural doctrine, for St. Thomas points out in innumerable places that even material essences

can be considered as forms in relation to the individual matter from which they have been abstracted.

And now we feel that enough has been said to bring out the central point about which this whole Chapter revolves: the basic principles which govern the distinction between physics and mathematics. Subsequent chapters will provide an elaboration of these principles. But perhaps it would be well at this point, in order to give sharper outline to the distinction, to consider briefly some observations made by a contemporary Scholastic on the Aristotelian doctrine of physical and mathematical abstraction in so far as it applies to the problem of mathematical physics. In an article to which we have already made reference Professor Mansion of Louvain has this to say:

Notons enfin que les déterminations quantitatives ne sont pas plus indépendantes de l'expérience concrète et de la réalité existante que les autres attributs, — d'ordre qualitatif — appartenant au monde des corps. Elles présentent seulement cet avantage que, isolées par l'abstraction, elles se prêtent mieux, — merveilleusement mieux, à une élaboration conceptuelle ultérieure: cette élaboration, œuvre de raison tout à fait remarquable, a donné naissance, en effet, à des disciplines indépendantes, construites suivant une rigueur logique inégale. Si l'on voulait soumettre à

un traitement semblable un concept tel que celui de chaleur, j'entends le concept répondant de façon immédiate dans l'abstrait à notre sensation de chaud, nos spéculations s'arrêteraient court avant^{qu'elles} arrivées fort loin. Cette notion, en effet, paraît réfractaire à toute analyse un peu poussée; elle est inapte à entrer telle quelle dans une systématisation plus développée, où seraient déterminés ses rapports avec des objets connexes, tels que le froid, etc. Ce n'est pourtant pas que nous ayons affaire ici à un concept abstrait à un moindre degré, que la notion de nombre par exemple; mais simplement que nous sommes en présence d'un concept de contenu différent, moins accessible à notre intelligence humaine dans ses conditions actuelles.(71)

This passage is filled with ambiguities and contradictions. In the first place, it must be admitted, of course, that there is a sense in which the initial statement (that the quantitative determinations of nature are no more independent of concrete experience and of existing reality than the sensible qualities) is true. It is obvious that we get to know these quantitative determinations only by grasping them in their state of concretion through concrete experience. It is likewise obvious that they are directly given in existing reality along with the qualitative determinations.

In this sense Mansion is justified in remarking:

Toutes (les notes caractéristiques de l'objet physique et celles de l'objet mathématique) font partie originairement d'un même complexe sensible, objet d'une perception globale, et dans lequel on retrouve les déterminations quantitatives au même titre que les autres."(72)

But at the same time there is a sense in which it is true to say that they are more independent of concrete experience and existing reality than the qualitative determinations. (73) Because of the hierarchical structure of physical reality, we get to know the quantitative determinations only by means of the qualitative determinations. This does not involve a process of illation, of course. It merely means that all the proper objects of the senses are qualitative determinations, and that it is only through them that this quantitative determinations can be grasped at all. Moreover, even though these quantitative determinations never exist objectively except in the state of concretion with sensible matter, they are, as we have seen, conceptually independent of this sensible matter in the sense that quantity is the first accident and the subject of all the other accident. That is why they can be lifted out of it and elaborated into a world apart -- a world of knowledge which does not have to terminate in the world of existing reality as presented by concrete experience, but merely in the intuitive imagination.

Does not all this involve an independence of both concrete experience and existing reality in which the qualitative determinations have no share? Does not Mansion himself admit this independence when he states that once isolated by abstraction these quantitative determinations can be elaborated into "des disciplines independantes"? Nor is there any force in Mansion's argument when he claims that Aristotle contradicts himself by postulating a special degree of abstraction for mathematics and at the same time admitting that mathematical beings are $\tau\alpha \acute{\epsilon}\gamma\lambda\alpha\iota\upsilon\sigma\tau\epsilon\omega\varsigma$, that is to say, extracted from the ensemble perceptible to the senses, which constitutes the physical object. (74) How else could mathematical beings have a special degree of abstraction except by being abstracted from the physical objects presented by the senses?

After pointing out that the quantitative determinations in their state of abstractive isolation lend themselves readily to a remarkable conceptual elaboration, Mansion goes on to say that this does not argue to a higher degree of abstraction. This statement seems to ignore completely the nature of formal abstraction which, as we have pointed out, is based precisely upon greater objective intelligibility. Moreover, to attempt

to establish a parallel between the way the concept of heat is abstracted from the other sensible qualities, and the way the concept of straight line is abstracted from sensible matter is to vitiate the whole Thomistic doctrine of abstraction. For the process of singling out the quality of heat from among the other sensible qualities is not necessarily positive abstraction at all, to say nothing of formal abstraction. Actually it is merely a kind of negative abstraction in which the mind fixes its attention on one point while neglecting everything else that is connected with it. And even if it were positive abstraction, there would still be a vast difference between it and the type of abstraction proper to mathematics. Enough has been said to show that quantity is in se more "abstractable" than the sensible qualities. The former can be conceived without the latter, but not vice versa. We can get at the quod quid est of a straight line, for example, and define it, but it is impossible to give a proper definition of heat or any of the sensible qualities. Perhaps we should mention here something that will be discussed in a later context: it is possible for the student of nature to consider quantitative determinations of the cosmos, but in his consideration they will always be united with sensible qualities and

connected with mobility; it also pertains to the metaphysician to study quantity, but only in so far as it is a principle of being. Both of these ways of considering the quantity of nature are vastly different from the way it is considered by the mathematician in the second degree of abstraction. The central error of this whole section of Mansion's essay seems to be a confusion between the way of grasping quantity that is proper to the mathematician and the other ways in which it may be laid hold of by the mind. This is evident in the following lines:

En s'en tenant à ce point de vue, on serait donc autorisé à affirmer qu'il y a moyen d'abstraire et d'isoler — par la pensée seule, bien entendu, — tel groupe particulier de qualités sensibles, appartenant à l'objet physique global, — le chaud et le froid, par exemple, — aussi bien que l'ensemble des déterminations quantitatives. On aurait ainsi un objet plus abstrait, parce que plus simple, que si l'on retenait tous les groupes de qualités sensibles analogues: on n'aurait pas pour autant un dégré d'abstraction caractéristique, mais une même abstraction poussée un peu plus loin, dans un certain sens, choisi d'ailleurs de façon arbitraire.(75)

Arising out of this initial confusion is the confusion between the concrete quantitative determinations as they exist in nature and the abstract quantity that is the object of mathematics. Professor Mansion seems to

hold that the object of mathematics is what is known in Thomistic terminology as the common sensibles, and in modern terminology as the primary qualities. That is why he objects so strongly to Aristotle's distinction between sensible and intelligible matter:

Or on est forcé de constater ici dans cet emploi des mots 'intelligible' et 'sensible', un abus de langage d'autant plus grave, qu'il paraît couvrir une confusion dans la pensée et constituer ainsi le point de départ d'une erreur formelle ... Cet objet (mathématique) est, à l'origine et fondamentalement, perceptible par les sens, tout autant que l'objet physique, et de manière aussi directe.(76)

It is this same reason that leads him to write:

Il y a plus, et cette particularité ne manque pas de saveur: le mouvement d'après lui est caractéristique de l'objet physique: l'objet mathématique en fait abstraction. Or le mouvement est aussi rangé parmi les sensibles communs, mais, en outre, c'est par la perception du mouvement, que nous avons celle de tous les autres, notamment les déterminations quantitatives, que retient seule le mathématicien. (De Anima T, l.425, al§ -19)(77)

The basis of these difficulties vanishes when one points out that Aristotle never held that the common sensibles constitute the object of mathematics. As for the question of movement, it is sufficient to remark that it falls under the common sensibles only indirectly, because of the

extension of space covered by the movement. Movement in itself, i.e. the act of being in potency in so far as it is in potency, is not a common sensible. The student of nature considers it, not as a common sensible, but in its intrinsic nature.

And thus St. Thomas writes:

Motus secundum naturam suam non pertinet ad genus quantitatis, sed participat aliquid de natura quantitatis aliunde, secundum quod divisio motus sumitur ex divisione spatii vel ex divisione mobilis: et ideo considerare motum non pertinet ad mathematicum, sed tamen principia mathematica ad motum applicare possunt: et ideo secundum hoc quod principia quantitatis ad motum applicantur, naturalis considerare debet de divisione, et continui, et motus, ut patet in VI Physicorum. Et in scientiis mediis inter mathematicam et naturalem tractatur de mensuris motuum, sicut in scientiis de sphaera mota, et in astrologia. (78)

The last remark of Mansion quoted above has no particular relevance, for in the place indicated in the De Anima Aristotle merely states that sensibles are perceived only through an immutation of the sense.

We have devoted considerable attention to these difficulties proposed by Professor Mansion not only because they serve as an excellent back-drop against which to bring

out in clearer focus the fundamental notions we have been laboring to formulate in this chapter, but also because if left unsolved they inevitably give rise to an entirely faulty view of Thomistic philosophy of science in general, and of mathematical physics in particular. As a matter of fact, they have led Professor Mansion to the fundamentally erroneous view of mathematical physics already pointed out earlier in this chapter -- that of considering it not as an interpretation of physical nature in terms of higher science, but merely as a study of the concrete quantitative determinations existing in the cosmos.

He Writes:

Car, remarquons-le bien, s'il est question ici de science ou de physique mathématisée, ce n'est pas qu'on ait substitué, dans l'objet d'expérience brut, à des attributs qualitatifs, apparaissent comme tels dans la sensation, des entités géométriques ou purement mathématiques; ces sciences ne sont encore mathématisées que parce qu'on a fait entrer dans la construction scientifique du phénomène la mesure exacte de ce qui est déjà donné comme quantitatif ou quantifié dans l'objet d'expérience lui-même. La part d'hypothèses géométriques qui s'y ajoute, par exemple en astronomie, pour importante qu'elle soit dans la construction systématique de la science, n'a qu'un rôle secondaire et simplement instrumental dans la détermination des lois quantitatives -- de forme mathématique -- régissant les phénomènes étudiés. Et de plus, à ce stade de l'évolution des sciences, les hypothèses utilisées ne sont, par ailleurs, pas encore hétérogènes au donné empirique, dont on cherche à formuler les lois.

(79)

We shall analyze the falsity of this position later.

In the difficulties enumerated above Professor Mansion finds the reason why, according to him, Aristotle cut himself off from the study of mathematics and of mathematical physics. From them he draws his conclusion that in Aristotelianism no true science of mathematical physics is even theoretically possible. We have referred to this conclusion in Chapter I and perhaps enough has already been said to call its validity into question.

4. Ultimate Specification

The above sketch of the hierarchy of speculative science will serve to draw a clear cut line of demarcation between physics and mathematics and at the same time to localize both of these sciences in the general field of knowledge. But it is extremely important for a true understanding of the nature of mathematical physics to press this question of epistemological pluralism a bit further. The three degrees of formal abstraction provide us with the basic structure of speculative science. But it may be asked whether they give us the absolutely ultimate specification of the sciences. Is it not conceivable that

in the general framework provided by a certain degree of abstraction a plurality of more specific formalities might be discovered which would serve as the basis for a sharper and more ultimate specification of the sciences? In this case, the degrees of abstraction would be a genus containing within it a number of scientific species. To the question posed in this general fashion the Thomists have traditionally given an affirmative answer. And John of St. Thomas provides (80) us with the reason. Because abstraction is a kind of process or movement, there are in it two points to be considered: the point of departure and the terminal point. This point of departure is the materiality that is sloughed off; and corresponding to the three types of matter there are three levels of abstraction. The terminal point is the particular grade of immateriality, the specific spiritual mode, the special type of intelligibility that an object is brought to when it is once cut free of a certain level of materiality. It is not the mere leaving behind of a certain general type of materiality that gives us the ultimate specific difference of the sciences, but the particular mode of intelligibility that is arrived at. For it is possible within one and the same degree of abstraction to have an intrinsic differentiation consisting in a greater or lesser

approach to immateriality. In other words, once the mind has performed the initial abstraction which gets rid of a certain general level of materiality, it may have the freedom to move to different points of terminal abstraction. Thus all of mathematics has the same general degree of abstraction: the leaving behind of sensible matter. Yet Thomists agree that within this degree of abstraction two specifically distinct sciences are found: geometry, which deals with continuous quantity, and arithmetic which deals with discrete quantity. All of the other branches of mathematics are either further elaborations, or appendages, or combinations or dialectical superstructures of these two fundamental sciences. The reason why they are specifically distinct is that arithmetic achieves a closer approach to immateriality than geometry. This can be brought out both by a proof and by a sign. The proof consists in this that continuous quantity has more subjectivity and more potentiality than discrete quantity. Continuous quantity is, in fact, principally matter, whereas number is principally form. In continuous quantity there is a subject which has infinite potentiality for division. It is true that number can be added to ad infinitum, but this potential infinity lies outside the number that is being added to, whereas in

the case of continuous quantity the infinite potentiality is within. Number is something definite and determined. Continuous quantity is something intrinsically indetermined.

Aristotle brings out the distinction between arithmetic and geometry in the Posterior Analytics:

A science such as arithmetic, which is not a science of properties qua inhering in a substratum, is more exact than and prior to a science like harmonics, which is a science of properties inhering in a substratum; and similarly a science like arithmetic, which is constituted of fewer basic elements; is more exact than and prior to geometry, which requires additional elements. What I mean by 'additional elements' is this: a unit is substance without position, while a point is substance with position; the latter contains an additional element. (81)

It is clear that the distinction laid down here by Aristotle is based upon the greater immateriality of arithmetic. In fact, as St. Thomas explains in his commentary on this passage, the contrast brought out by Aristotle between geometry and arithmetic is a contrast between matter and form: "alii autem duo modi accipiuntur secundum quod forma est certior materia, utpote quia forma est principium cognoscendi materiam." (82)

A sign of the more abstract character of arithmetic is found in the fact that it is far less dependent

upon the imagination than geometry. We can imagine any kind of a thing as a phantasm for number, as long as there is homogeneous plurality; but not any kind of thing represents a circle, for example. Another sign consists in this that by extension number can be applied to spiritual beings, whereas continuous quantity cannot.

Geometry still has something of the qualitative clinging to it, even if it be only a question of quantitative quality, such as figure. Speaking of this distinction between geometry and arithmetic, Duhem writes:

Parmi les sciences, l'arithmétique seule, avec l'algèbre, son prolongement, est pure de toute notion empruntée à la catégorie de la qualité; seule, elle est conforme à l'idéal que Descartes propose à la science entière de la nature. Dès la géométrie, l'esprit se heurte à l'élément qualitatif, car cette science demeure 'si astreinte à la considération des figures qu'elle ne peut exercer l'entendement sans fatiguer beaucoup l'imagination.' -- 'Le scrupule que faisaient les anciens d'user des termes de l'arithmétique en la géométrie, qui ne pouvait procéder que de ce qu'ils ne voyaient pas assez clairement leur rapport, causait beaucoup d'obscurité et d'embarras dans la façon dont ils s'expliquaient.' Cette obscurité, cet embarras, disparaîtront si l'on chasse de la géométrie la notion qualitative de forme, de figure, pour n'y conserver que la notion quantitative de distance, que les équations qui relient les unes aux autres les distances mutuelles des divers points étudiés. (83)

John of St. Thomas makes the following clear out distinction between the two:

Sed Mathematica considerat proportionem et mensuras, quae in quantitate discreta et continua ita variantur, quod ad diversa principia reducuntur et ad diversas abstractionem et modum definiendi, quia mensuratio per magnitudinem nullo modo convenit cum modo mensurandi penes numerationem. Haec enim abstractiori modo procedit, quia magnitudo mensurat per modum continentis, ut locus, numerus per intellectum numerando. (84)

(85)

In the Ars Logica he points out that geometry not only has greater dependence upon place but also upon time. It is not too clear just what this dependence upon time consists in, but in all probability he is referring to the generation of the figures in geometry.

A further indication of the greater materiality of geometry is found in the fact that some modern authors erroneously believe that, at least in certain aspects, it is more truly a physical science than a pure mathematical science. (86)

Telle était déjà l'idée de Gauss. 'Nous devons admettre humblement, écrivait-il à l'astronome Bessel, que, le nombre est uniquement le produit de notre esprit, l'espace, même au point de vue de notre esprit, constitue une réalité à laquelle nous ne pouvons a priori dicter complètement ses lois'. Dedekind, dans la préface de son fameux opuscule sur la nature du nombre a vivement insisté sur cette idée de l'autonomie de l'arithmétique à l'égard du réel. Le nombre est 'une émanation immédiate des lois pures de la pensée' et 'entièrement'

indépendant des concepts de temps et d'espace'; les nombres sont 'des créations libres de l'esprit humain, ils servent de moyen pour saisir plus aisément et avec plus de précision la diversité des choses' (Was sind und was sollen die Zahlen? 52^e ed., Brunswick 1923, p.111 ...

"Mais Locke, déjà, jugeait que 'le nombre est la plus simple et la plus universelle de toutes nos idées" (Essai Philosophique, II, Ch. XVI, no. 1), et Hume considérait la géométrie comme moins assurée que l'arithmétique et l'algèbre au point de vue de la valeur apodictique de ses affirmations. (Psychologie, tr. Renouvier et Fillon, Paris 1878, p.98).

Similarly within the general scientific framework which leaves all matter out of consideration, Thomists distinguish three specifically distinct sciences: metaphysics, logic, and supernatural theology, and once again the distinction is based upon different modes of immateriality. Supernatural theology is distinguished from the other two in that it enjoys the highest grade of immateriality that any speculative science can have — that provided by the light of revelation. Logic is distinguished from metaphysics in that its abstraction is purely negative, that is to say, since the object of logic is not anything real, it has only a negative immateriality.

Thus far all Thomists are in agreement. But when the question is raised about the possibility of a

plurality of sciences within the first degree of abstraction, the issue becomes highly controversial. The problem is whether the study of nature is specifically one, or only generically one. In its concrete form it reduces itself to the problem of the kind of distinction existing between philosophy of nature and experimental science. Since this question is of considerable importance for our purpose, we must endeavour to give it a rather exact analysis.

Speaking in a general way, we may say that until recent years Thomists recognized no formal distinction between the philosophy of nature and what has come to be known as "science." -- at least no distinction of such a nature as to give rise to two specific sciences. And this is of considerable significance, for if there is anything that the medieval Thomists took pains to do it was to introduce formal distinctions wherever there was any basis for them. This was particularly true in the realm of knowledge. (87) But some modern Thomists, notably M. Maritain, while recognizing the absence of any formal distinction between the philosophy of nature and "science" in the writings of Aristotle and the medieval Thomists, believe that this was a serious error on their

part -- an error due to "intellectual precipitation" and
an unwarranted "optimism".⁽⁸⁸⁾ They have consequently seen
fit to reject this point of Thomistic doctrine, and have
gone to great pains to elaborate an epistemological theory
which attempts to set off the philosophy of nature and
experimental science as two formally distinct sciences.⁽⁸⁹⁾
While commending the motive behind this elaboration --
that of attempting to integrate Thomistic philosophy with
modern achievements, we feel that it has resulted in a
theory that is in conflict with basic Thomistic epistemo-
logical principles. We must try to see why this is so,
and why these principles must be retained if modern ex-
perimental science is to have its true explanation.

In order to set the question in a clearer light,
it will be necessary to make several distinctions. In the
first place, it is evident that there is a specific
difference between philosophy of nature and mathematical
physics. For as we have already suggested, mathematical
physics does not fall completely under the first degree
of abstraction. It is a hybrid science whose formal
element is borrowed from the second degree of abstraction.
Hence it is formally distinct from science that is of a
purely physical character. The whole question at issue is

whether there can exist a plurality of specifically distinct sciences which fall completely within the first degree of abstraction. In the second place, we do not deny that there is a profound epistemological difference between philosophy of nature and experimental science. In fact, we shall lay considerable stress upon this difference in Chapters IV and V. But, it is not a question of a difference between two specifically distinct sciences of nature, in the strict sense in which science signifies universal and necessary judgments. Rather, it is a distinction between a science of nature in the strict sense (philosophy of nature) and a purely dialectical continuation of that science (experimental science). We shall try to make it clear later that experimental science is not science in the strict sense just defined. None of its judgments are universal and necessary; they never go beyond a greater or lesser degree of probability. Only the facts of science have certainty. And we shall see that the greatest of modern scientists and philosophers of science are in agreement on this point. In other words, the reasoning used in experimental science proceeds from hypothetical premises to probable conclusions. It is for this reason that we shall call this type of knowledge

dialectical knowledge. And in the future when we speak of experimental science it must be understood that we are taking the term science in the broad sense in which it signifies purely dialectical knowledge. The ambiguity of the word easily gives rise to confusion, and lest some may suspect that it is merely this ambiguity that is at the basis of the difference between Maritain's position and ours, we shall quote the following lines from Yves Simon, who is recognized as the most authentic interpreter of M. Maritain. In explaining Maritain's philosophy of the sciences he writes:

Whenever the mind seizes an essence, a ratio entis, albeit in the blind way proper to the perinoetical intellection, a genuinely scientific treatment remains possible. Any universal and necessary form of being, however obscure may be the way it is grasped, constitutes a matter to which the mind can apply the principles of scientific thought, that is, causal and explanatory schemes.(90)

Because of the essentially dialectical character of all experimental science, it is evident that there is no possibility of a plurality of specifically distinct sciences in the strict sense of the word within the first degree of abstraction. But we do not intend to argue from this point of view here. Rather, we have in mind

to approach the problem from an entirely different angle. Our position is that even if experimental science were science in the strict sense of the term it would not be formally distinct from philosophy of nature, but united with it to form one indivisible science of nature. On the other hand, if mathematical physics were science in the strict sense of the term, it would be formally distinct from the science of nature.

We can best settle the issue by first considering it in a positive way before taking up the arguments of M. Maritain and his followers. John of Saint Thomas whose doctrine M. Maritain generally professes to follow, has written a special article to show that a plurality of sciences in the first degree of abstraction is incompatible with basic Thomistic epistemological principles. (91) The clarity of the article is admirable, and we can do no better than to summarize its content. The study of nature covers a broad field; it includes a number of branches which extend to a great variety of things. Yet a close consideration of this study reveals the fact that all of these branches must of necessity fall under one indivisible science. For (prescinding from the difference between dialectical and truly scientific knowledge, which John of

St. Thomas does not consider) the only fundamental difference between these various parts of natural doctrine is the difference between generality and concreteness. This difference cannot constitute a formal distinction between sciences. For, as St. Thomas points out on (92) innumerable occasions every science necessarily begins with generalities and progresses to greater and greater concreteness. We have already indicated the reason for this: the human mind begins with potency and moves on slowly to greater actuality. And on these innumerable occasions St. Thomas makes it very clear that the various branches of natural doctrine do not constitute a variety of sciences but only a difference of greater or lesser concretion. John of St. Thomas wisely points out that if the difference between generality and concretion were sufficient to constitute a plurality of sciences, it would be impossible for a specifically distinct science to exist. For, every science that might be set up would necessarily move from some level of generality to greater concreteness.

Consequently, every science which deals with a certain genus necessarily deals with all the species which fall under that genus. Not only do these species not have

the full liberty of specifically distinct sciences, they do not even have the restricted liberty of a subalternated science, because the difference which they add to the generic study is not accidental and extrinsic, but intrinsic and essential. (93) As we pointed out above, sciences are distinguished by the essentially different principles which they employ, for each science has principles that are proper to it. Each science presses on towards its goal in the light of these proper principles, and consequently as it moves from generality towards greater concretion it cannot suddenly change its principles at a certain point along the way. It is true that from a purely material point of view new principles may be added. In this sense each new natural species that the student of nature discovers in experience becomes a new scientific principle for him and the source of new truths. But it is obvious that in this context we are taking scientific principles from the formal point of view which is determined by the modus definiendi that is characteristic of them. In this sense, the principles of a science cannot change. No matter how many new species the student of nature may discover they must all be defined in terms of sensible matter and considered from the point of view of

the ratio mobilitatis. It is evident that if the advent of new principles from the material point of view were sufficient to give origin to new sciences, there would be as many sciences as there are natural elements or species.

Just three things can happen to a science as it moves from generality to greater concretion. First, it may retain its character of strict science all the way, and then no profound epistemological change takes place at any point. This is what happens in the case of geometry, which begins with axioms and postulates of great generality, and which in pursuing its ambition to derive all the implications latent in these axioms and postulates, remains a strict science throughout. Secondly, it may at a certain point lose its character as a strict science and issue into dialectical knowledge. In this case the dialectical knowledge is a necessary continuation of the science as it moves towards concretion. It uses the same principles, but not in such a way as to arrive at strict demonstrations. Obviously this does not give rise to a plurality of sciences. Thirdly, it may call in the help of an outside science in such a way that the two constitute a scientia media. In this last case we

have the only way in which other principles besides the ones that science started out with can be introduced. These three cases are exhaustive. We do not see how any other possibility can be adduced. Let us apply these general considerations to our specific problem of the study of nature.

This study begins with the consideration of mobile being in its broadest generalities: what is motion in general, what are the constituents of all mobile beings, etc. These generalities form the subject matter of the eight books of the Physics. From this point the study moves gradually towards greater concretion, and the other natural treatises are devoted to following out this movement. We do not see how at any point new principles can be suddenly introduced to transform the science into a different science, unless they be brought in ab extrinseco. But if they are brought in ab extrinseco, they necessarily give rise to an intermediary science. This is what actually happens in the study of nature when mathematics is applied. But in this case we have a hybrid science composed of elements from two degrees of abstraction; we do not have a plurality of sciences in the first degree of abstraction.

It is true that as the study of nature progresses it eventually issues into a purely dialectical type of knowledge. But this does not give us a new science. If that dialectical knowledge could be suddenly transformed into strictly scientific knowledge it would merely constitute a continuation of the one science of nature in its movement towards concretion.

The obvious objection at this point is: what about mathematics in which you have two specifically distinct sciences within the same degree of abstraction. And the answer is not difficult to find: There is no science of quantity as such, as there is a science of mobile being as such. In other words a general science of mathematics does not exist, nor can it exist. If it did, geometry and arithmetic would not be specifically distinct, for as we pointed out above, the science which deals with the genus deals also with the species that fall under it. In other words, mathematics is not the study of quantity from the point of view of its essence; nor are geometry and arithmetic studies of continuous and discrete quantity from the point of view of their essence. The study of quantity and its species from the point of view of essence is distinctly a meta-

physical consideration. For it pertains to metaphysics to explore the nature of all the categories from the point of view of their essence, i.e. in so far as they are principles of being. This includes even the categories that involve matter. Nor is this a contradiction of what was said above about metaphysics prescinding from all matter, for metaphysics considers and defines these categories not from the point of view of their materiality but in so far as they are principles of being. This explains why St. Thomas can say: "De quolibet enim ente in quantum est ens, proprium est metaphysici considerare." (94)

And later in the same lectio he writes:

Licet ad considerationem primae philosophiae pertineant ea quae sunt separata secundum esse et rationem a materia et motu, non tamen solum ea; sed etiam de sensibilibus, in quantum sunt entia, philosophus perscrutatur. (95)

And so he concludes: "Geometria accipit quid est magnitudo a philosopho primo." (96)

The case of the study of nature is entirely different from that of mathematics. And it will sharpen the issue to present it in the form of a disjunction. Either there is a specific science of mobile being as such, or there is not. If there is not a special science, then

under what science does the study of mobile being fall? Certainly not metaphysics, for mobile being is not a category or a principle of being, as quantity is. On the other hand, if there is a science of mobile being as such, then everything that falls under the formality of mobility from the broadest generality to the ultimate concretion will pertain to the same science. One cannot begin the study of mobile being in its generalities and then somewhere along the road to concretion suddenly shift to other principles. A particular, concrete type of movement is a contraction of movement in general. But continuous quantity is not a contraction of discrete quantity or vice versa. In this case there is something entirely new.

This clarification of the difference between mathematics and the study of nature will help to bring out the ambiguity in the following statement of Maritain:

.. la différence entre la philosophie de la nature et les sciences des phénomènes, soit empiricostriques soit empirico-schématiques, apparaît comme beaucoup plus accusée que la différence entre l'arithmétique et la géométrie, lesquelles étaient pour les scolastiques deux sciences spécifiquement distinctes. (97)

Several distinctions are necessary here. There is a greater distinction between the philosophy of nature and experimental science in the sense that the former is strictly scientific knowledge, while the latter is only dialectical; whereas in both geometry and arithmetic there is strictly scientific knowledge. On the other hand, however, there is a greater difference between geometry and arithmetic in the sense that they are two formally distinct sciences, each possessing its own proper principles. Of course in the case of the sciences which Maritain calls empiriometric there is a deeper dichotomy separating them from philosophy of nature because of the fact that they constitute a hybrid science.

As a confirmation of his position, Maritain writes: "Jean de Saint-Thomas distingue ainsi la philosophie naturelle et la médecine." (98) It seems almost incredible that this argument should be adduced, especially since the word "ainsi" refers directly to the lines immediately preceding wherein Maritain explains his distinction between philosophy and experimental science. For John of Saint Thomas, while admitting a distinction between medicine and philosophy of nature (which in his terminology included the entire study of mobile being)

explicitly and in so many words rejects this distinction as an argument for a plurality of sciences of mobile being. And the reason for this rejection ultimately boils down to this that medicine and the study of nature are formally distinct because medicine is not a speculative science like the study of nature but a practical science. For while they both have the same material object: body, they have a distinct formal object in that natural doctrine considers bodies as mobile and medicine considers them as curable. Even though the act of curing takes place by means of motion, medicine does not consider its object in terms of the formality of motion, but in terms of curability.

St. Thomas brings this point out with great precision in his Commentary on the De Trinitate:

Quaeris enim corpus sanabile sit corpus naturale, non tamen est subjectum medicinae, prout est sanabile a natura, sed prout est sanabile per artes... Et sic relinquatur quod physica secundum se, et secundum omnes partes eius est speculativa, quaevis alicuae operativae subalternentur ei. (99)

It is precisely because medicine is a practical science that John of St. Thomas writes "magis concretive procedit (100) magisque ad singularia et praxim accedit. And while

experimental science actually proceeds in a more concrete way than philosophy of nature, and comes closer to singulars, no parity can be established between it and medicine, because even though as experimental science progresses it takes on more and more the character of practical knowledge, as we shall see, it remains essentially a speculative science. It is difficult to see how a distinction between a speculative and a practical science can afford any argument to prove the existence of a plurality of speculative sciences in the first degree of abstraction.

But it is time now to consider briefly the
(101)
positive arguments of M. Maritain. The basis of his distinction between philosophy of nature and experimental science seems to consist in this: The object of the study
(102)
of nature is sensible being -- ens sensibile. This object presents a dualistic or bipolar character, and it is this dualism or bipolarity which gives rise to two vastly diverse ways of studying nature. For it is possible to study sensible being in such a way that the emphasis is placed upon "being", and when this is done you have philosophy of nature. It is likewise possible to study sensible being in such a way that the emphasis is put upon "sensible", and then you are in the realm of experimental science. Out of this difference of accentuation arise

two diverse conceptual schemes, two diverse modes of definition. The philosopher of nature defines his concepts in terms of intelligible being, the experimental scientist in terms of sense phenomena. The one employs dianoetical intellection, which consists in penetrating to the essence of things. The other uses perinoetical intellection which consists in grasping the essence only in a blind and remote way in the phenomenal regularities themselves. The one resolves its concepts in an ascending analysis which goes up to intelligible being. The other resolves its concepts in a descending analysis which goes down to the sensible, the phenomenal. Hence the one moves from the visible to the invisible. The other from the visible to the visible.

Professor Simon, with his usual clarity, has attempted to give an exact and concrete explanation of Maritain's ascending and descending analysis:

Let us try a rigorous ascertainment of the meaning of a word found in both philosophical and in positive contexts. The example chosen may be very simple. To the question what does the word man mean? the answer will be 'rational animal'; now, none of the elements of this definition presents a character of irreducible clarity. Take one of them, for instance, animal. What does the word mean? A correct definition would be: "a living body endowed with sense knowledge?",

and these are so many terms which badly need clarification. Take one of them, for instance, 'living.' I would say that a body is a living one when it moves itself, when it is the active origin of its own development. If we go any step farther, we go beyond the limits of physical thought. In order to render the idea of life clearer, we would have to define it as self-actuation. The concept of self-actuation does not imply any reference to the proper principles of corruptible and observable things: it is a metaphysical concept. Its elements are identity and causality. Identity is the first property of being. Causality can be analysed into potency and act. Identity, potency and act are so many concepts directly reducible to that of being, which is, in an absolute sense, the first and most intelligible of all concepts. We have reached the ultimate term of the analysis, the notion which neither needs to be nor can be defined and which does not admit of any beyond

For the zoologist, man is a mammal of the order of Primates. How would he define such a term as mammal? A vertebrate characterized by the presence of special glands secreting a liquid called milk. How is milk defined? In terms of color, taste, average density, biological function, chemical components, etc. Here the ultimate and undefinable element is some sense datum; it is the object of an intuition for which no logical construction can be substituted and upon which all the logical constructions of the science of nature finally rest.(103)

We fail to find in all this the slightest basis for a duality of sciences in the study of nature. There are two main differences between the definition of the philosopher of nature and that of the experimental (104) scientist. Both of them, far from constituting a

a specific distinction between sciences, absolutely exclude the possibility of such a distinction. In the first place, the definition of the philosopher is strictly scientific, whereas that of the zoologist is purely dialectical. Obviously, if the definitions of experimental science are purely dialectical, it cannot be a specifically distinct science, for the simple reason that it isn't a science. The second difference between the two definitions is one of generality and concreteness. Whereas the philosopher of nature deals in broad generalities the experimental scientist is far advanced along the road to concretion. In this sense the former is far less immersed in the directly observable than the latter. If this is what M. Maritain means by saying that the one moves from the visible to the invisible, while the other goes from visible to the visible, he is correct; but besides being an extremely ambiguous and confusing way of explaining the situation, it provides no foundation for a specific distinction between sciences.

Because the experimental scientist is deeply immersed in concrete materiality, it is only natural that he will clarify his definitions in terms of concrete,

material observable things. If we asked St. Thomas to clarify his material definition of a house: "a structure made of stones, cement, and wood" ⁽¹⁰⁵⁾ he would undoubtedly do so in terms of material observable things.

It should now be fairly clear that the difference in materiality between philosophy of nature and experimental science upon which M. Maritain seems to base his specific distinction is not one that derives from formal abstraction which alone can specify the sciences, but merely from total abstraction, since it is a question of a difference between generality and concreteness. This difference, far from constituting a duality of sciences, absolutely excludes the possibility of such a duality, for we have already seen that the more particular must pertain to the same science as the more general.

But it may be objected: if the main difference between the definition of the philosopher and that of the experimental scientist consists in a question of generality and concreteness, why should it not be possible for the experimental scientist to clarify his definition by re-treating into higher levels of generality and thus rejoin the philosopher, and why should it not be possible for the

philosopher to push ahead into concretion and rejoin the experimental scientist. Our answer is that not only is such a thing possible, but in a certain sense absolutely necessary. Let us try to see why this is so.

In the first place, it must be noted that the ascending analysis attributed to the philosopher of nature is nothing but an ascent of the Porphyrian tree, a retreat into potentiality, that is to say into generalities that become increasingly more vague and more empty. The philosopher of nature may, indeed, make this ascent, provided he does so in terms of mobility. But it is important for him to realize that while this ascent is leading him in the direction of that which more knowable quoad nos, it is leading him farther and farther away from that which is more knowable in se. In other words, by the very fact that he is practicing total abstraction he is achieving greater intelligibility quoad nos only at the expense of sacrificing intelligibility in se. Now philosophy does not consist merely in giving terms that are more knowable for us, but in manifesting the natures of things as perfectly as possible. It consists in getting at what is more knowable in se and not merely what is more knowable quoad nos. Definitions are

supposed to manifest things to us and this manifestation does not come from a retreat into notions that become increasingly more vague and empty. The only way in which a philosopher can truly philosophize is, not by retreating backward into potentiality, but by pressing forward into fuller actuality. In no other way can he succeed in bringing to light the proper natures of things. That is why, as we noted above, St. Thomas in all of his pro^oemia to the natural works of Aristotle, keeps insisting that the philosopher of nature must constantly move forward into fuller concretion.

With these remarks in mind let us return to the passage quoted above from Mr. Simon. In the first place, it must be noted that Mr. Simon has chosen his examples with care, for apart from the fact (over which we shall not linger) that he has made the philosopher explain the generic part of his definition, and the zoologist the specific part of his definition, he has, in selecting the example of rational animal, chosen a very privileged case. (106)
As he himself suggests man is the only natural species for which it is possible to give a strictly scientific definition. From this point of view it provides a kind of terminus for the natural philosopher's quest to get

at the proper natures of things. This is far from saying, however, that his movement towards concretion has come to an end in so far as the nature of man is concerned. For both "animal" and "rational" are rather vague notions which must be explored and concretized. Having determined that man is a rational animal, the student of nature is forced to attempt to find out, for example, what precise structure of body is proper to rational animality. And this attempt will very speedily bring him to the definition given by the zoologist. But in order to bring out the issue clearly let us use another example.

Let us ask the philosopher of nature to tell us what a horse is. And while we await the answer let us recall a remark of Professor Simon: philosophy of nature "does not reach its end until it is able to answer the question 'What is the thing under consideration?' " (107) Where will the philosopher turn to tell us what a horse is? Will he turn upwards in his ascending analysis? If so, we are justified in becoming impatient and calling him back, for he is not telling us what a horse is; he is merely telling us what all animals in general are. Is it not evident that in order to answer the question "what is a horse" he must move in exactly the opposite direction?

It is useless to retreat into logical potentiality; he must push forward along the road to concretion into greater actuality. It may be that he will never be able to give us a perfect answer, but if he is true to his science that will not keep him from an endless striving to get at least a partial answer. M. Maritain seems to admit the necessity of this movement towards concretion in every science, for he writes: "Toute science allant d'ailleurs dans cet ordre vers la plus grande détermination, exigeant que l'objet soit serré, pour ainsi dire, dans une notion propre, et non pas enveloppé dans une notion commune plus ou moins flottante."
(108)

We know what reply this objection would
(109)
receive:

the philosopher of nature must not attempt to answer such questions. He must practice the spirit of poverty; he must not be guilty of the exaggerated optimism and philosophical imperialism of the ancient Thomists. He must leave questions of that kind to the experimental scientist who with his special science completes the philosopher's study of nature. And why? Because philosophy of nature is wisdom within the order of physical reality. Or "toute sagesse est magnanime, ne s'embarrasse pas du détail matériel

des choses, pauvre donc en ce sens, et libre, comme les vrais magnanimes; et cette sagesse-là est obligée à la pauvreté; elle doit se résigner à connaître, elle doit s'honorer de connaître le réel par des moyens pauvres, sans prétendre épuiser le détail des phénomènes, compter (110) les cailloux du torrent." We fail to see the force of this argument. Strange magnanimity this, the renunciation of the knowledge of things in their proper specificity. Far from being a property of wisdom, such magnanimity is directly opposed to its true nature. And if human wisdom cannot succeed in reaching things in their proper specificity, it is not because it is wisdom but because it is human and therefore extremely imperfect. But precisely because it is wisdom it must ever strive towards the knowledge of specific natures. These last lines of Maritain are rather hard on St. Thomas. For let us recall that he has already told us that the doctrine of the ancient Thomists (St. Thomas included) which held that the philosopher of nature should push forward into concretion was a grave error. If then the reason why the philosopher of nature must abstain from concrete questions is that he is obliged to do so by the very fact that philosophy of nature is wisdom, the con-

clusion is inevitable: St. Thomas was unaware of the true nature of wisdom. We prefer to believe that his ideas on the nature of wisdom were more exact than those of M. Maritain.

We admit that there is a sense in which it is true to say that the philosopher of nature is brought up short before such concrete questions. But the reason is not that he runs into another science, but that he runs out of science. But there is no reason why he should not prolong his study of nature dialectically even when he is unable to do so scientifically. And when this is done the philosopher and the zoologist inevitably meet.

If there were any valid reason why the philosopher of nature should remain in his generalities and feel satisfied with his ascending analyses, it would have to be because in this way he could ~~derive~~ the greatest illumination concerning nature and obtain the deepest insights into physical reality. But this would necessarily mean that the generalities would contain all their inferiors actually and distinctly, and that what is more knowable for us would be at the same time more knowable secundum se. Not a few modern scholastics, with their false air of profundity in dealing with these

vague generalities which considered from the point of view of the proper natures of things that constitute the goal of the science of nature, provide the most empty and superficial knowledge it is possible to have of the cosmos, seem to hold such a view, at least implicitly. And to hold a view of this kind is to fall into the error of the Platonists who wanted to reach the terminus of science merely by division. Plato's attempt to arrive at the notion of angler through a mere process of division beginning with the general notion of art is well known. (111)

In the last analysis this kind of philosophy of nature is nothing but Hegelianism. Karl Marx's explanation of Hegel on this point is extremely illuminating:

Quand, à partir des pommes, des poires, des fraises, des amandes réelles, je forme la représentation générale: fruit, quand je vais plus loin et que je me figure que ma représentation abstraite: le Fruit, obtenue à partir des fruits réels, est une essence qui existe en dehors de moi, est même l'essence véritable de la poire, de la pomme, je déclare -- en termes spéculatifs -- que le Fruit est la "substance" de la poire, de la pomme, de l'amande, etc. Je dis donc que l'essentiel de la poire, de la pomme ce n'est pas d'être poire ou pomme. L'essentiel de ces choses n'est pas leur être réel, tombant sous les sens, mais l'essence de ma représentation: le Fruit. Je déclare donc que la pomme, la poire, l'amande, etc. sont de simples modes -- modi -- du Fruit. Mon entendement fini, soutenu par les sens, distingue sans doute une pomme d'une poire, et une poire d'une amande, mais ma

Raison spéculative déclare que cette distinction sensible est inéssentielle et indifférente. Elle voit dans la pomme la même chose que dans la poire, et dans la poire la même chose que dans l'amande, à savoir le Fruit. Les fruits réels particuliers ne sont plus que des apparences du fruit, dont la véritable essence est la substance, le fruit ... Le Fruit n'est pas une essence sans vie, sans caractères distinctifs, sans mouvement, mais une essence vivante, distincte en soi, en mouvement. Le caractère distinct des fruits profanes ne relève aucunement de mon entendement sensible, mais du Fruit lui-même, de la Raison spéculative. Les fruits profanes distincts sont des manifestations vivantes, distinctes, du Fruit unique, ils sont des cristallisations qu'élabore le Fruit lui-même. Par exemple, dans la pomme, le Fruit se donne une apparence de pomme, dans la poire une apparence de poire. On ne doit donc plus dire, comme du point de vue de la substance: la poire est le fruit, la pomme est le fruit;, l'amande est le fruit, mais bien plutôt: le Fruit se présente comme pomme, comme poire, comme amande, et les différences qui séparent les unes des autres la pomme, la poire, l'amande sont les différences mêmes du Fruit et elles font des fruits particuliers des chaînons différents dans le processus vital du fruit. Le Fruit n'est donc plus une unité sans contenu, sans distinctions, il est l'unité en tant que généralité, que "totalité" des Fruits, qui forment une succession, le fruit se présente comme une existence plus développée, plus complètement exprimée, jusqu'à ce qu'il soit enfin "le résumé" de tous les fruits en même temps que leur unité vivante.(112)

We have quoted this long passage because it characterizes so well the attitude of many modern scholastics who seem to look upon the general as the very substance of things and the specific as a mere phenomenal mode which is of little interest for the philosopher who

must concentrate his attention upon the profound essences of things. We believe that the doctrine of Maritain tends to encourage this attitude. It does so in many ways: by insisting upon ascending analyses and neglecting the movement towards concretion; by describing experimental science as something which merely deals with phenomenal details, without explaining that it is precisely through experimental science that we are constantly carried closer and closer to the proper natures of things which constitute the goal of the whole study of nature, closer and closer to the most profound knowledge that it is possible to have of the cosmos -- to the kind of knowledge that God has of nature; etc. Maritain does, indeed, point out the poorness of the knowledge provided by philosophy of nature, but he does so in such a way as to make it appear that the riches which it renounces are hardly worth having. He compares the knowledge that experimental science gives with counting the stones in a stream. St Thomas had already taken care of this counting of stones when in explaining the opening lines of Aristotle's Physics where we are told that in the study of nature the mind must move in the direction of concretion by progressing from universals to singulars, he wrote:

Hic autem singularia dicit non ipsa individua, sed species; quae sunt notiores secundum naturam, utpote perfectiores existentes et distinctam cognitionem habentes; genera vero sunt prius nota quoad nos, utpote habentia cognitionem in potentia et confusam.(113)

The same point is brought out by Saint Thomas in the Prooemium of his Commentary on the Libri Meteorologicorum:

Unde manifestum est quod complementum scientiae requirit quod non sistatur in communibus, sed procedatur usque ad species; individua enim non cadunt sub consideratione artis; non enim eorum est intellectus, sed sensus.

But there is even a greater danger in Maritain's doctrine that the one just mentioned. We believe that it tends to lead to a confusion between philosophy of nature and metaphysics, in spite of Maritain's explicit efforts (114) to keep the two distinct. The difficulty here arises from the initial error of seeing in the object mobile being a dual or bipolar character which gives rise to two formalities. Earlier in this chapter we have rejected this error and pointed out that the great Thomists have traditionally insisted that the dualism in the expression "mobile being" is purely verbal, that it signifies one indivisible formality. Having created his two formalities, Maritain goes on to say that the object of philosophy of nature is mobile being or sensible being considered

precisely in so far as it is being. Now, as we saw above, St. Thomas in his Commentary on the Sixth Book of the Metaphysics repeatedly insists upon the fact that no other science can deal with any particular type of being precisely in so far as it is being except metaphysics. And he says explicitly that this is true of sensible being: "etiam de sensibilibus, in quantum sunt entia, Philosophus perscrutatur." (115) And the difficulty is only augmented when one constantly runs across such misleading statements as the following: "... il faut dire que l'objet propre de la philosophie de la nature ... n'est constitué que par le transcendantal être en tant que déterminé et particularisé au monde corporel, mobile et sensible." (116) "En réalité elle (la philosophie de la nature) considère les choses corporelles et mobiles au point de vue du transcendantal être imbibé en elles." (117)

And even if philosophy of nature could in this position save itself from identification with metaphysics, it would at best have the appearance of an intermediary science subalternated to metaphysics. We do not accuse M. Maritain of holding this view, but it is interesting to note that more than one author who have followed in his wake have explicitly arrived at this conclusion. (118)

And a greater epistemological perversion could hardly
(119)
be imagined.

But let us return to the definitions of the philosopher and the zoologist. From the foregoing it should now be clear why the philosopher of nature must move forward towards concretion and join the zoologist. But the question now suggests itself: can this meeting be brought about by having the zoologist move backwards as well as by having the philosopher move forwards? Once again the answer must be in the affirmative. If we ask a zoologist what a vertebrate is, he will probably answer: an animal with a spinal column. By seeking for an explanation of "animal" we can make the same ascent in the Porphyrian tree made by the philosopher. But one will immediately be tempted to object: granted that such an ascent is possible, why is it that it is never made by the experimental scientist? Why is it that, as Simon
(120)
points out, such a way of explaining terms would ordinarily move a zoologist to laughter? The reasons are not far to seek. Modern experimental scientists have chosen to ignore completely the higher levels of generality in the science of nature, and to begin their study with purely experimental propositions. Experimental propositions

are concrete and dialectical. The reason why the subject and the predicate are united is concrete experience alone. Hence it is only natural that when asked to explain the terms in such definitions they should turn to concrete experience. While it is not necessary for them to know philosophy of nature in order to become expert experimental scientists, such a knowledge would enable them to understand the meaning of their science and the proper significance of the terms and propositions they employ. A zoologist with a knowledge of philosophy of nature would have no difficulty in making an ascending analysis of his terms and thus rejoin the definition of the philosopher of nature. And in connection with the question why the zoologist ordinarily makes a descending rather than an ascending analysis perhaps this last remark should be made: experimental scientists have understood far better than scholastic philosophers of nature that the proper movement of the study of nature is forward into actuality, rather than backward into potentiality.

Before leaving this criticism of the doctrine of Maritain, we should like to put it to a final test. We are told that dialectical intellection is characteristic

of the science of nature which employs ascending analyses, while perinoetical intellection is proper to the science which employs descending analyses. Let us take the example of a definition of man in terms of the tongue and the hands. Now while most definitions in terms of the concrete structure of the body are purely synthetic and hence dialectical, as in the case of the definition of man as a mammal, it seems that the definition in terms of the tongue and the hands is analytic, for there is a necessary connection between rational animality (which implies an animal that possesses both a speculative and a practical intellect) and these two organs. If then one were to attempt to resolve the concepts contained in this type of definition in which direction would he turn? Would he not be led to explain himself in terms of concrete, material observable things? We are consequently faced with this question: what kind of intellection do we find in the proposition just mentioned? Is it dianoetical? If so, why do we have a descending rather than an ascending analysis? Is it perinoetical? If so, how explain that we have an analytic proposition, for in all analytic propositions the essence is opened up and does not remain covered over.

5. Natural Doctrine and Practical Knowledge

At this point it is necessary to introduce a problem which arises out of a text of Aristotle. The solution of this problem will serve to clarify our conception of the nature of natural doctrine and of its relations to the other branches of knowledge. The text we have in mind is found in the first chapter of the first book of the De Partibus Animalium. It is a text to which comparatively little attention has been given by the commentators of Aristotle; yet it is pregnant with profound implications. In spite of the fact that in all the other passages in his writings where he considers the nature of natural doctrine he classes it among the speculative sciences, in this particular text he seems to set it in opposition to the speculative sciences.

The causes concerned in the generation of the works of nature are, as we see, more than one. There is the final cause and there is the motor cause. Now we must decide which of these two causes comes first, which second. Plainly, however, that cause is the first which we call the final one. For this is the Reason, and the Reason forms the starting-point, alike in the works of art and in the works of nature. For consider how the physician or how the builder sets about his work. He starts by forming for himself a definite picture, in the one case perceptible

to the mind, in the other to sense, of his end -- the physician of health, the builder of a house -- and this he holds forward as the reason and explanation of each subsequent step that he takes, and of his acting in this or that way as the case may be. Now in the works of nature the good end and the final cause is still more dominant than in works of art such as these, nor is necessity a factor with the same significance in them all; though almost all writers, while they try to refer their origin to this cause, do so without distinguishing the various senses in which the term necessity is used. For there is absolute necessity manifested in eternal phenomena; and there is hypothetical necessity, manifested in everything that is generated by nature as in everything that is produced by art, be it a house or what it may. For if a house or other such final object is to be realized, it is necessary that such and such material shall exist; and it is necessary that first this and then that shall be produced, and first this and then that set in motion, and so on in continuous succession, until the end and final result is reached, for the sake of which each prior thing is produced and exists. As with these productions of art, so also is it with the productions of nature. The mode of necessity, however, and the mode of ratiocination are different in natural science from what they are in the-oretical sciences; of which we have spoken elsewhere. For in the latter the starting-point is that which is; in the former that which is to be. For it is that which is yet to be -- health, let us say, or a man -- that, owing to its being of such and such characters, necessitates the pre-existence or previous production of this and that antecedent; and not this or that antecedent which, because it exists or has been generated, makes it necessary that health or a man is in, or shall come into, existence. (121)

We have italicized the lines in this passage to which we wish to call particular attention. There can be no doubt that in these lines physics is distinguished from speculative science. And after all that was said above about the place it occupies in the first degree of formal abstraction which distinguishes the speculative sciences, this presents us with a problem that must be solved. Two possible interpretations of the passage just cited suggest themselves: natural doctrine is distinguished from the speculative sciences either because it is essentially a practical science, and consequently not speculative at all, or because though essentially a speculative science, it has some characteristics in common with practical knowledge and in some measure falls short of the perfection of speculative knowledge. After all that has been said thus far it must be evident that only the second interpretation is acceptable. Natural doctrine must be essentially a speculative science, because in it knowledge is sought for its own sake.

As our analysis proceeds we hope to make it clear in how many ways natural doctrine comes close to practical knowledge, and we do not wish to anticipate these developments here. Yet it will be helpful, perhaps,

to set down in skeletal fashion some of the salient features of the striking resemblance between the study of nature and practical science.

In the passage cited above, Aristotle suggests the basic reason for this resemblance. Like all the characteristics of the study of nature, this resemblance derives from the fact that the object of this study is mobile being. Now mobile being means not only being that is, but being that becomes. And the study which deals with such a being precisely in terms of its mobility will deal with it not merely in its being but in its coming to be. And it is because all natural things are mobile beings that we find in nature something closely akin to ~~what~~ that is found in art and prudence; we find a becoming, a generation, a production, a movement towards an end. And whenever there is an end, it always acts as principle, as Aristotle points out in the text just cited: "in the former (the starting point is) that which is to be. While this characteristic of natural beings establishes a similarity between them and the things of art and prudence, it at the same time distinguishes them from mathematical and metaphysical things. For, as we have seen, the objects of both

mathematics and metaphysics are immobile. To this it might be objected that there is a kind of production in metaphysical beings, since angels produce a succession of actions. But because it is merely a question of actions, this production touches only the accidental order. In natural things, on the contrary, it touches the substantial order itself. Because of the matter and privation in the essence of these beings, there is in them an intrinsic plasticity that makes them substantially formable. They are not merely called into existence; their generation is the terminus of a lengthy process of composition and formation in which nature proceeds like art. In mathematics there is no formability. It is true that there is a kind of construction in mathematical science, but this does not involve movement or production in the true sense of the word. And that is why the only kind of art that is possible in mathematics is speculative art.

Now we are in a position to understand the profound distinction which Aristotle introduces here between the object of natural doctrine and the objects of the other speculative sciences. Since the objects of the other speculative sciences do not become, they

simply are. That is why Aristotle says that these sciences have to do merely with that which is. But mobile being becomes. And since all becoming, all movement gets its whole specification and determination from the terminus, the science which studies such a being will be engaged primarily not with that which is, but with that which will be, that is to say, the end, which is first in intention and last in execution. And this end is a good, and moves as a good. All this reveals the fundamental role that finality plays in the study of nature as in all practical science and explains why Aristotle insists so strongly upon finality in nature in the second book of the Physics.

It is because of this dependence upon the end that existence plays a part in the study of nature that it does not play in mathematics or metaphysics which deal with essences - - a part that is similar to the part it plays in practical science. For in the notion of end there are two aspects: end in the order of intention, i.e. end as a cause; and end in the order of execution, i.e. end as an effect. Now it is precisely existence which separates these two. And it is because of movement, becoming, that the two terms are united. The study of nature has to consider what goes on between these two terms. That is why

existence is so important for it. That is why it is not merely concerned with the quod quid est as mathematics and metaphysics are. And it is to be noted that the end involved in nature is the very form of natural things, and consequently it is due to becoming that the very object of the study of nature is constituted.

All this serves to bring out the striking resemblance between the study of nature and practical knowledge. But it also makes it clear that from this point of view natural doctrine can be called practical knowledge only by extrinsic denomination, that is to say, because of the nature of the things with which it deals.

What we have been saying enables us to understand the particular type of necessity that is found in the sciences of nature. Since, as we have pointed out, all science deals with necessity, the nature of the science is intrinsically determined by the kind of necessity that is proper to it. Now there are two kinds of necessity: absolute and hypothetical. As Aristotle explains at the end of the second book of the Physics,⁽¹²²⁾ things which have their necessity from a formal, material or efficient cause enjoy absolute

necessity. On the other hand, the necessity which derives from the final cause is only hypothetical. And hypothetical necessity consists in this: if a certain end is to be achieved, then such and such means are necessary. But it does not follow that given these means, the end will necessarily be achieved. For example, we may say that if a certain type of organism is to be generated, then the conjunction of a sperm and an ovum is necessary. But it does not follow from the fact of this conjunction that the organism will necessarily be for the end may fail to be achieved for ~~some~~ reason or other.

In order to understand this point clearly we must have recourse to a distinction made by Aristotle (123) in the second book of the Physics. The end that is found in natural things may be considered in two ways. It may first of all be considered as a principle of reasoning, and then it is taken as the cause from which we may demonstrate all the things that are necessary for the end to be realized. In this sense we can reason from the end to the means that are necessary for the end. But it may also be taken as a principle of action, that is to say as the cause moving the agent. In this sense it is impossible for demonstration to actually reach the end,

that is to say, we cannot reason from the fact that the means necessary for an end are given that the end is going to be realized.

In all of the speculative sciences besides the study of nature absolute necessity is found, but in natural doctrine there is only hypothetical necessity. Here we have another point in common with the realm of the practical. And so Aristotle concludes: "For there is absolute necessity, manifested in eternal phenomena; and there is hypothetical necessity, manifested in everything that is generated by nature as in everything that is produced by art, be it a house, or what it may." (124) Hence in natural science no true demonstration from prior causes is possible, for, from the point of view of prior causes, whatever happens, happens at best only for the most part - - ut in pluribus. (125)

Nature may in fact be characterized by what happens for the most part. And it is this that St. Thomas has in mind when in a text already quoted (126) he points out that the science of nature has a "modus infirmior demonstrandi" because "multae demonstrationes sumuntur ex his quae non semper insunt, sed frequenter."

This distinguishes it from the other speculative sciences whose demonstrations enjoy a greater necessity. At the same time it reveals the close similarity with practical knowledge, for as Aquinas points out in the same lectio, in the moral sciences the "principia sumuntur ex his (127) quae sunt ut in pluribus."

It is evident, then, that in natural science demonstration cannot arrive at the ipsum esse finis. For example, in the evolution of the cosmos, at no point was it possible to demonstrate with absolute necessity the (128) future existence of any particular natural species - - even though once the existence of a certain species is given in nature it can be the principle of what had to be in order for it to exist. In other words, natural things are not knowable except in the order of existence; that is to say, we cannot know them except by knowing them as existing. This creates a great difference between the science of nature and the other speculative sciences. We stand before the universe as before a work of art in the process of being made. We might have a general notion of what is to come about, but as long as we have no full share in the idea of the artist, we do not know just what is to come about or exactly how. Like practical

knowledge, therefore, the study of nature has a close and necessary relation with the existential order, and consequently with experience. This point will be developed at considerable length in Chapter IV, and in connection with it we shall discover another closely related reason why physics is associated with practical knowledge: it has to do with objects that are formed by divine art. This is not true in the same sense of metaphysics, for angels are not formed in the line of essence. In mathematics everything is analytical.

Besides being about things that are brought into existence by composition, natural doctrine must itself engage in composition. This is true not only in the construction of theories, but already in the gathering of the various subjects considered. The study of nature must be built up out of bits garnered from experience. And closely connected with this is another point of similarity with practical knowledge, namely its intimate relation with singulars. The student of nature cannot deal purely with universals. In fact, as he pursues his research in the direction of fuller concretion, it soon becomes impossible for him to rise successfully above the realm of singulars to true universality, and he is obliged to have

recourse to a kind of artificial and hypothetical construct that is fashioned by the mind. And in connection with the relation between natural doctrine and singulars it is worth while noting that in nature generation is always in the singular. In mathematics, on the contrary, it is possible to have a quasi universal generation, e.g. the generation of a line from a point. This makes it clear that the science of nature has somewhat the same character of singularity as moral science. In these two fields alone is it possible to have history.

As the student gets deeper into the realm of concrete singularity his science becomes conditioned by a constantly increasing multiplicity of elements. In this it becomes remarkably similar to moral science. And just as in the field of concrete human actions the multiplicity of elements is so great that action remains possible only because man can override this multiplicity by a deliberate act of the will, so in the parts of natural doctrine which are deeply immersed in concretion, experience is conditioned by such a multiplicity of elements that science becomes possible only because the scientist overrides this multiplicity by deliberate fiat.

All this makes it clear why physical science as it advances towards concretion soon issues into a purely dialectical extension. This happens both because of the materiality of natural things and because of man's way of knowing them. It is interesting to note that if we consider the whole range of natural doctrine from the highest generality to the ultimate concretion the part which has a truly scientific character is small indeed in comparison with the part whose character is merely dialectical. It is also interesting to point out that the passage of Aristotle which we used to introduce this problem is taken from a treatise which is already far along the road to concretion.

Now it is highly significant that no other speculative science has such a dialectical extension. Theology, metaphysics, logic, arithmetic and geometry can pursue their course in strictly scientific fashion. This does not mean, of course, that no probable factors enter into these studies. It means that in these sciences there are no sections whose whole structure is dialectical. Of all the speculative sciences this is characteristic of the study of nature alone.

But at the same time it is also characteristic of practical knowledge. In moral philosophy as soon as we leave the most general principles necessity likewise peters out into probability. (130) That is why St. Thomas often repeats that moral philosophy proceeds "figuraliter, idest verisimiliter." And the closer the moral philosopher draws to concretion, the less normative his science becomes. Nevertheless, the very nature of his science forces him to continue along this road, exploring the realms of sociology, economics, etc, always pressing forward towards greater concretion. Once again as in the study of nature, the part of the doctrine which enjoys strict scientific necessity is small indeed in comparison with the part which possesses only probability.

Our final point of comparison between natural doctrine and practical knowledge brings us back to something considered at the beginning of this chapter. We saw that as the scientist draws closer to the ultimate concretion, his attempts to lay bare the secrets of nature make it increasingly necessary for him to operate upon nature, to refashion it and reconstruct it. In this way physical science gradually takes on the aspects of an art. At the same time man's practical power over

nature increases. And not only does his power increase, but at the same time his ars cooperativa naturae, as in the cases of the arts of medicine and hybridization, for example, increases. And in this man knowingly and through his skilful action pursues a terminus that in itself is natural. (151)

These few ideas on the relation between the science of nature and practical knowledge must suffice for the moment. Later chapters will give them fuller embodiment. But it is worth while pointing out here what an important bearing all this has upon the problem of mathematical physics. For few things could seem more diametrically opposed than mathematics and practical knowledge. Yet it is to this cosmos, which in so many ways presents such striking resemblances to the object of practical knowledge, that mathematics is applied.

6. Specification and Method

From this general consideration of the specification of the sciences a conclusion must be immediately drawn which is of extreme importance for our purpose. It is this: the specification which sets off the various distinct sciences is neither arbitrary nor

fluid; it is something very objective and definite. As a consequence, each specifically distinct science has a special character of its own which the other sciences cannot share. Each science has its own particular questions and its own particular answers; it has principles that are peculiar to it; it has its own way of demonstrating; it has a unique method. (132)

Saint Thomas brings out this point in a general way in his Commentary on the De Trinitate when, after explaining the distinction between physics, mathematics and metaphysics, and pointing out how each of these sciences terminates in a different cognitive power, he concludes: "Et propter hoc peccant qui uniformiter in tribus (133) speculativae partibus procedere nituntur." As Maritain has remarked, these words should be written in letters of gold over the doors of every university. (134)

In his Commentary on the Posterior Analytics, Aquinas presses this point home with greater precision and greater insistence. In commenting on Chapter XII he (135) devotes a whole lectio to showing that each science has its own particular type of questions and answers and disputations. And he points out how this follows from the very specific character of the science. For,

as we have seen, the sciences are specified by the type of propositions they use as principles of their syllogisms.

But a scientific question and a scientific proposition are substantially the same, and differ only in the mode of expression. Since, therefore, each science has its own particular type of principles, it will necessarily have its own particular type of questions. And so

Aquinas concludes: "Non ergo quaelibet interrogatio est geometrica, vel medicinalis; et sic de aliis scientiis." (136)

Since an answer must be in the same genus as the question to which it replies, it follows that each science has its own type of answers. And consequently St. Thomas remarks:

"Non contingit de quolibet interrogato respondere: sed solum de his quae sunt secundum propriam scientiam." (137)

It likewise follows that each science has its own type of disputation, since disputations proceed by questions and answers. And in order to press this general point home with more precision he adds to this lectio another lectio in which he shows that each science has its own peculiar types of deception and ignorance. (138)

But of even greater significance for our purpose is his commentary on Chapter VII (139) wherein he proves that each science demonstrates by means of its own

proper principles, and that consequently the demonstrations of one science cannot be used to demonstrate something in another science. He writes:

In illis scientiis, quarum est diversum genus subiectum, sicut in arithmetica, quae est de numeris, et geometria, quae est de magnitudinibus, non contingit quod demonstratio, quae procedit ex principiis unius scientiae, puta arithmeticae, descendat ad subiecta alterius scientiae, sicut ad magnitudines, (quae sunt subiecta geometriae. (140)

And he goes on to give the reason: the principles and the conclusions of a scientific syllogism must be in the same genus, for the principles illuminate the conclusions; the latter are in fact precontained in the former.

This doctrine taken as it stands here immediately gives rise to serious epistemological difficulties. It seems to throw up rigid and insurmountable barriers between the sciences in such a way that one science cannot influence another, except perhaps in a very extrinsic fashion. And has not modern science given the lie to any doctrine that would establish barriers of this kind? Must we conclude that it is illegitimate to ask geometrical questions in terms of arithmetic or to seek to demonstrate geometrical propositions by means of arithmetical principles: If so, what about analytical geometry? And - - to come directly

to the issue with which we are concerned -- is it illegitimate to raise questions about physics in terms of mathematics or to arrive at conclusions about nature through mathematical demonstrations? If so, what about mathematical physics? There is not a modern scientist or philosopher of science who would not immediately reject any doctrine which would call into question the legitimacy of such procedures. And Emile Meyerson terms the doctrine taught by Aristotle in the chapter we have been considering: "si choquante pour le sentiment de l'homme moderne."
(141)

Fortunately, there is no reason to take scandal. All difficulties vanish when the Chapter is read in its entirety in the light of the commentary of St. Thomas, and in conjunction with the whole context, particularly Chapter XIII where Aristotle and St. Thomas consider the problem of the subalternation of the sciences. And this whole context must, of course, be integrated with their other writings which treat of this question, notably the passage from the second book of the Physics cited in Chapter I. This full and integral reading not only dispels all difficulties but it leaves us with a profound admiration for Aristotle and Aquinas whose analyses remain

accurate to this day.

In lectio 21 of the Posterior Analytics, after explaining that each science has its own particular questions, St. Thomas goes on to give an example taken (142) from geometry. In giving this example he brings in the case of the science of optics which is subalternated to geometry, and he points out that it is legitimate to ask geometrical questions in optics precisely because it is subalternated to geometry and to that extent integrated with it. And he concludes:

Et quod dictum est de geometria, intelligendum est de aliis scientiis: quia scilicet propositio, vel interrogatio dicitur proprie alicuius scientiae, ex qua demonstratur vel in ipsa scientia, vel in scientia ei subalternata.

In lectio 15, to the text cited above in which he says that arithmetical demonstrations cannot be employed in geometry he immediately appends this important qualification:

... nisi forte subiectum unius scientiae contineatur sub subiecto alterius, sicut si magnitudines contineantur sub numeris (quod quidem qualiter contingat, scilicet subiectum unius scientiae contineri sub subiecto alterius, posterius dicetur). Magnitudines enim sub numeris non continentur, nisi forte secundum quod magnitudines numeratae sunt. (143)

In this passage written centuries before Descartes St. Thomas

explicitly allows the possibility of a treatment of geometry in terms of arithmetic.

In giving the reason why demonstrations must be in the same genus, St. Thomas takes pains to explain and qualify his doctrine with great accuracy:

Quare manifestum est quod necesse est, aut esse simpliciter idem genus, circa quod sumuntur principia et conclusiones, et sic non est descensus, neque transitus de genere in genus: aut si debet demonstratio descendere ab uno genere in aliud, oportet esse unum genus sic, idest quodammodo. Aliter enim impossibile est quod demonstraretur aliqua conclusio ex aliquibus principiis, cum non sit idem genus vel simpliciter, vel secundum quid.

Sciendum est autem quod simpliciter idem genus accipitur, quando ex parte subiecti non sumitur aliqua differentia determinans, quae sit extranea a natura illius generis; sicut si quis per principia verificata de triangulo procedat ad demonstrandum aliquid circa isoscelem vel aliquam aliam speciem trianguli. Secundum quid autem est unum genus, quando assumitur circa subiectum aliqua differentia extranea a natura illius generis; sicut visuale est extraneum a genere lineae et sonus est extraneus a genere numeri...

Cum autem huic coniunxerimus quod diversae scientiae sint circa diversa genera subiecta; ex necessitate sequitur quod ex principiis unius scientiae non concludatur aliquid in alia scientia, quae non sit sub ea posita...

Et similiter, quod est unius scientiae non habet probare alia scientia, nisi forte una scientia sit sub altera; sicut se habet perspectiva ad geometriam, et consonantia vel harmonica, idest musica, ad arithmetica. (144)

A casual reading of these passages might give the impression that St. Thomas contradicts himself. First he denies the possibility of using the demonstrations of one science, such as arithmetic, in another science, such as geometry. In the next breath he seems to admit the possibility. There is no contradiction here. He is merely trying to insist upon the fact that in order to unite things correctly one must first distinguish them carefully, that union without accurate distinction can only result in confusion. He begins, therefore, by insisting upon the distinct character of the sciences, each of which has its own peculiar mode of demonstration. From this he concludes that per se, that is, absolutely speaking, the demonstrations of one science cannot be applied promiscuously to other sciences. Having laid down this basic principle he goes on to explain that under certain conditions one science may be brought to bear upon another, in the measure in which one can be to some extent integrated with the other through the process of subalternation. But in the union effected through this subalternation neither of the sciences loses its proper character. The union of mathematics and physics does not mean that physics is mathematics,

or that mathematics is physics. Saint Thomas is very careful to keep before our minds the fact that the demonstration of a geometrical proposition through arithmetical principles is a process that is essentially different from the demonstration of a geometrical proposition through geometrical principles. All too many modern scientists and philosophers of science have allowed themselves to lose sight of this fact. That is why their union is a confusion.

And now, having seen the principles which govern the distinction of the sciences, we must turn our attention to the problem of their subalternation.

CHAPTER THREE

THE SUBALTERNATION OF THE SCIENCES

1. The Species of Subalternation.

In this question of subalternation we are touching upon one of the most basic and pivotal notions in the philosophy of science. That is why it is imperative to handle it with as much incisiveness as possible. For the ancient Thomists subalternation had a rather well defined meaning. But unfortunately not all modern Thomists have kept its outlines clear and sharp, nor have they taken sufficient pains to keep distinct the various ways in which the general notion of subalternation may be applied. The question has been handled with considerable looseness and ambiguity, and the result has been confusion. Let us try to circumscribe the meaning of the word as closely as possible.

Subalternation is sometimes defined in terms of the application of one science to another, or the

dependence of one science on another, or the subordination of one science to another. Its notion involves all of these things, but they do not adequately explain its meaning. In the first place, not every case of the application of one science to another is a case of subalternation. For example, in philosophy of science there is a kind of application of metaphysics to experimental science. But this does not involve the subalternation of experimental science to metaphysics. The philosophy of science is a purely metaphysical study, for, as we pointed out in Chapter I, it pertains to wisdom to make a critique of the nature of all the sciences including itself. ⁽¹⁾ Secondly, subalternation is not coterminous with dependence. For example, theology, in so far as it makes use of philosophy, may in some sense be said to be dependent upon it. But it is not subalternated to it. ⁽²⁾ Thirdly, the notion of subordination is not sufficient to explain the meaning of subalternation. For, philosophy is subordinated to theology, but it is not subalternated to it. ⁽³⁾ Moreover, all practical science is in some way subordinated to speculative science, but this subordination does not necessarily involve subalternation. It is true that some practical sciences, such as medicine, agriculture, etc. are subalternated to the

science of nature, but that is because of the peculiar character of the relation that obtains between them, as we shall presently explain.

One of the difficulties encountered in the problem of subalternation arises out of the fact that the term is used in a variety of ways. Perhaps the best way to arrive at the positive meaning of the term is to begin by considering the different ways in which one science may be subalternated to another. John of St. Thomas (4) distinguishes three types of subalternation. One science may be subalternated to another either by reason of its end, or by reason of the principles it employs, or by reason of the subject it considers. Let us consider briefly each of these types.

Subalternation that derives from an end pursued is, as the very terms suggest, proper to the practical order; it is found in the practical sciences and in the arts. When the end of one science, though truly an end within its own order, is subordinated to the end of a higher science in such a way that it is controlled and directed by it, the first science is said to be subalternated to the second. Thus, for example, military science is subalternated to

political science. It is important to note that the first end must be truly an end within a certain order, for if it is only a means, if the higher science uses it merely as an instrument there is no real distinction of sciences and hence no subalternation. In this first type we are dealing with subalternation in a very broad and improper sense. For, subalternation implies the dependence of one science upon another with respect to the manifestation of truth, and very often when one science is subalternated to another by reason of its end there is no dependence of this kind, but rather dependence with respect to use, control, direction, and command, -- something akin to what is found in the interrelation of the virtues, as for example in the case of charity's command over temperance. And this follows from the very nature of the practical order, whose object is not the true as true, nor even the good as true, but the good as good. It is only in the speculative order that subalternation in the proper sense of the term is found, for the object of this order is always the true, and consequently subalternation in this order involves a manifestation of truth. We are particularly interested in the subalternation of the speculative sciences.

One speculative science may be subalternated to another in two ways: either by reason of its principles alone or by reason of its subject. The first case is had when a lower science borrows from a higher science the principles necessary to illuminate its own domain, and thus becomes dependent upon it. But in order to have subalternation of this kind in the full sense of the term the dependence must be necessary and essential, that is to say, the lower science must be lacking in per se evident principles within its own domain, and thus be forced to reach up to a higher science to have its principles made evident. This type of dependence is found in the subalternation of supernatural theology to the science of the blessed. Theology does not resolve its demonstrations into principles that are per se evident. For the theologian must accept his principles on faith. But these principles accepted by faith have their intrinsic evidence in a higher science -- the science of the blessed in heaven. It is in this higher science that they find their manifestation and their proof. That is why theology is essentially subalternated to the science of the blessed.

It is extremely important to insist upon the difference between this kind of dependence and the kind of

dependence that philosophy of nature and the other sciences have upon metaphysics. It is true that in some sense all of the sciences receive their principles from metaphysics, for as St. Thomas says, "ipsa (metaphysica) largitur principia omnibus aliis scientiis." (5) Nevertheless, the lower sciences do not depend upon metaphysics for the evidence of their principles. They are capable of resolving their demonstrations into per se evident principles which are proper to them. They do not have to turn to metaphysics to have the truth of their principles made manifest or proved. It is true that metaphysics explains the principles of the other sciences and defends them by a reduction ad impossibile, but it does not prove them in an a priori fashion. The principles of the other sciences come under the influence of those of metaphysics only in the sense that metaphysics is the most universal and the most basic of all the sciences. And even though it has become common for authors to state that the principles of philosophy of nature are contractions of the principles of metaphysics (e.g. that the principle of the composition of mobile being of matter and form is a contraction of the division of being into potency and act), we feel that such statements need qualification. For there is a world of

difference between the way in which the particular principles governing a certain type of motion are contractions of the general principles of motion, and the way in which the principles of the philosophy of nature are contractions of metaphysical principles. For, " as we saw in the two, in the latter case there is not merely a question of the application of the more general to the more specific; there is a question of two different orders. It is a serious error to confuse the two types of dependence described in these last two paragraphs.

It is true that the other sciences may sometimes use metaphysical principles in their demonstrations. It is likewise true that they may sometimes employ principles taken from the science of logic. But this amounts to no more than an occasional borrowing from these other sciences; it merely means the use of an extrinsic proof. All this explains why the dependence of the other sciences upon metaphysics and logic is not subalternation in the full sense of the word. And if the term subalternation is applied to this kind of dependence it should be made very clear that it is only a question of subalternation in a very partial and limited sense.

Now for our purpose, it is not subalternation by reason of the principles alone that is of particular interest, but subalternation by reason of the object. In this third type we have subalternation in the most perfect sense of the word. John of St. Thomas says: "Tertius^{modus}...
(7)
inducit propriissimam subalternationem." We must try to see why this is so.

This third species of subalternation arises when the object of one science falls under the object of another science. But as we pointed out in Chapter I in our discussion of the fifteenth lectio of the first book of the Posterior Analytics, one object may fall under another in two ways. First of all, it may merely be a question of a more specific object being contained in a more generic object, in the way in which, for example, animated mobile being falls under mobile being. In this case it is evident that there is no real distinction of science and hence no possibility of true subalternation. Every science explains its object by division as well as by definition, and consequently in order to have the formal distinction of science that is required for subalternation, it is not sufficient that one object add an essential specific difference to the other. And this explains why many of the apparently hybrid

sciences to which we alluded at the beginning of Chapter II (e.g. astro-physics, bio-chemistry, etc.) do not involve true subalternation, since it is merely a question of the union of two branches of the same science. There is, consequently, a world of difference between the hybrid character of these sciences and that of mathematical physics in which physics is truly subalternated to mathematics.

Because the subalternated science must be extrinsic to the subalternating science, the difference which the object of the one adds to the object of the other must be extrinsic and accidental. An example will make this point clear. Let us take the geometrical notion of "line". We may add to this notion in two ways. First of all, we may add the proper specific differences "straight" and "curved", and thus arrive the two specific objects, "straight line" and "curved line", both of which fall under the generic object, "line." By doing this we do not arrive at any new science, since the science which deals with a certain genus necessarily deals with all the proper species which fall under it. But it is also possible to add to the notion of line the extrinsic and accidental difference "visual", and thus arrive at a new object, "visual line".

This new notion is not a proper species of the generic geometrical notion of line. Hence it does not fall under the science of geometry in the sense of being a part of its object. In fact it constitutes a new science, the science of optics, known to the ancients as "perspectiva". This new science, while not falling under geometry in the sense of being a part of it, does come under it in some way, since the notion of line which is compounded with the notion of visual to constitute its object is borrowed from geometry. In other words, optics is subalternated to geometry by reason of its object.

Perhaps another simple example will clinch the point we are trying to make. We may add to the generic arithmetical notion of number the two proper essential differences "rational" and "irrational", and thus arrive at two numerical species, both of which pertain essentially to the object of arithmetic. But we may also add to the notion of number the extrinsic and accidental notion of sound and thus arrive at a compound object which constitutes a new science, distinct from arithmetic, but subalternated to it -- the science of music.

Now subalternation by reason of the object

always involves at the same time subalternation by reason of the principles. This should be fairly evident from the examples just cited. For, since the formal object of the subalternated science is constituted by the addition of an accidental difference to the object of the subalternating science, the subalternated science cannot treat its object and prove its properties except by having recourse to the conclusions of the subalternating science. But subalternation by reason of the principles does not always involve subalternation by reason of the object. The contrast between the way theology is subalternated to the science of the blessed and the way optics is subalternated to geometry brings this point out with sufficient clarity. As we saw, supernatural theology must reach up to the science of the blessed in order to find the evidence of its principles. Nevertheless, its object is not constituted by the addition of an accidental difference to the object of the science of the blessed. It is, in fact, the very same object viewed under two different lights: the light of faith on the one hand, and the light of vision on the other. But the difference between geometry and optics does not consist merely in two different ways of viewing the same object. In the first

case we have a simple notion that preexists from all sensible matter. In the second case we have a compound object made up of this simple notion plus an accidental element which involves sensible matter. There is an enormous difference between these two types of subalternation. In the first type, the subalternated science remains a simple science. In the second type, it becomes a complex science, a hybrid science, a scientia media, because its object is compounded of elements which involve two different levels of intelligibility.

The three fundamental types of subalternation just described are the only ones mentioned by John of St. Thomas in the article cited above. We may well wonder whether the list is exhaustive. For St. Thomas in his Commentary on the De Trinitate ⁽⁸⁾ gives us a case of subalternation which does not seem to fall under any of the three groups listed by his disciple. We are referring to the case already mentioned earlier in this chapter in which the practical sciences of medicine, agriculture, etc. are subalternated to the speculative science of nature. We pointed out that this subalternation does not arise merely from the subordination that all practical science has to speculative science, but from the special

character of the dependence which these few practical sciences have upon the science of nature. St. Thomas brings out the nature of this special relation with great clarity and precision:

Quamvis enim corpus sanabile sit corpus naturale, non tamen est subjectum medicinae, prout est sanabile a natura, sed prout est sanabile per artem. Sed quia in sanatione quae fit per artem, ars est ministra naturae, quia ex aliqua naturali virtute sanitas perficitur auxilio artis, inde est quod propter quid de operatione artis oportet accipere ex proprietatibus rerum naturalium. Et propter haec medicina subalternatur physicae, et eadem ratione alchimia, et scientia de agricultura, et omnia huiusmodi. Et sic relinquitur, quod physica secundum se, et secundum omnes partes eius est speculativa, quamvis aliquae operativae subalternentur.(9)

It does not seem possible to fit this type of subalternation directly into any of the three groups described above. It is not a case of subalternation by reason of the end, for we do not have one practical science subordinated to another practical science. Nor is it a question of subalternation by reason of the principles, for a practical science cannot receive its proper principles from a speculative science. Since the end of a practical science is not to know "why" but "how", it cannot receive a reason why or a propter quid from a speculative science. Finally, there is no possibility here

of subalternation by reason of the object, for elements from a practical science cannot be compounded with elements taken from a speculative science to constitute the object of a simple, unified science. As a matter of fact John of St. Thomas, after explaining the three types of subalternation, explicitly denies that medicine is subalternated to natural science: "Medicina (agit) de corpore sanabili, et tamen non subalternatur Philosophiae, quae agit de corpore." (10) From the context, however, it is evident that he is merely denying the possibility of subalternation by reason of the object. And even though the way in which medicine and agriculture are subalternated to natural science does not fit directly into any of the three groups listed by John of St. Thomas, it may be reduced to a case of the second group. For while it is true that a practical science cannot receive its principles from a speculative science, the principles of medicine and agriculture are completely determined by the principles of natural science because of the unique character of the relation existing between these sciences. Perhaps nowhere can the Aristotelian adage: Ars imitatur naturam be applied with such fullness as here. In fact, the imitation is so perfect that in a certain sense it

results in an identification, for in medicine and agriculture, the works of art must be at the same time works of nature.

It would seem that if the concept of subalternation is conceived as embracing all of the various cases we have described it can hardly have a strict unity. Nevertheless, there are two kinds of subalternation in which the concept is realized in its proper and strict sense, and in which it has a definite unity. We refer to subalternation by reason of the principles in which there is an essential relation of dependence between the subalternated science and the subalternating science, that is to say, when the former receives its proper principles from the latter, and to subalternation by reason of the object. When the ancient Thomists speak of subalternation, it is usually this strict and proper sense of the concept that they have in mind, and it is in this sense that we shall speak of it from now on.

And now, having reduced the notion to this definite meaning, it remains for us to explain in what its essence consists. But before pursuing this analysis it is worth while pausing at this point to remark that every effort should be made to maintain a clear cut

distinction between the various kinds of subalternation we have been describing. As we pointed out at the opening of this chapter, this has not always been done by modern Thomists. We are being told by more than one contemporary writer for example that philosophy of nature is a scientia media, born of a union of the first and third degrees of abstraction, or, even worse, arising out of the application of metaphysics to the data of empirical science. (11) And we consider it extremely misleading, unless all the necessary qualifications and distinctions are made, to insist, as some authors do, that in modern times mathematics has come to occupy the same position in relation to the experimental sciences that metaphysics held for the ancient Thomists.

2. The Essence of Subalternation

The intrinsic nature of subalternation follows from the intrinsic nature of science itself. Science is certain knowledge of things in their causes, and for the human intellect this means knowledge arrived at by a process of demonstration. Now knowledge that is arrived at by demonstration is never self-evident knowledge.

Conclusions do not have their evidence from themselves, but from something else, namely from the immediately evident principles from which they have been derived. That is why the intellectual virtue of science is essentially dependent upon another intellectual virtue, known as the intellectus principiorum, which is the habitus that enables the mind to grasp immediately the truth of self-evident principles. Now the essential difference between a subalternated science and a science that is not subalternated is that the habitus of the latter is in immediate continuity with the habitus principiorum, whereas the habitus of the former is only mediately in continuity with it, through the habitus of a higher science, known as the subalternating science. (12)

In other words, no science is a science in and by itself, but in and by its continuity with a superior habitus, for without this continuity its conclusions cannot have the certitude that is necessary for scientific knowledge. A science that is not subalternated is a science that is in direct continuity with the habitus principiorum from which it immediately derives the evidence of its conclusions. On the other hand, a subalternated science is one that is in direct continuity

with the habitus of a superior science, and only through this habitus is it in continuity with the habitus principiorum.

At this point it will be helpful to draw a contrast between the way supernatural theology is subalternated to the science of the blessed and the way other sciences are subalternated - - not because we are particularly interested in the subalternation of theology, but because the contrast will serve to accentuate the characteristic features that are found in the intermediary sciences in general and in mathematical physics in particular. In the subalternation found in all the other sciences besides theology, the proximate principles of the subalternated science are conclusions demonstrated by the subalternating science.

... scientia subalternata non utitur principiis aliarum scientiarum, sed conclusionibus: assumit enim principia quae probantur a scientia superiori tamquam conclusiones, non autem principiis superioris scientiae utitur resolvendo usque ad principia per se nota. (13)

When the subalternating science does not coexist in the same intellect along with the subalternated science, these conclusions are taken on faith. But this does not mean that in this case the principles of the subalternating

science are taken on faith. For the intellect which possesses the subalternated science may possess the principles of the subalternating science by means of the habitus principiorum, without possessing the habitus of the subalternating science itself. In this connection John of St.

Thomas writes:

... in scientiis naturalibus non potest verificari quod ipsa principia per se nota ipso lumine principiorum in superiori scientia, sint tantum credita, et non per se nota in inferiori: quia quod est per se notum lumine principiorum, omnibus est per se notum; et principia quanto sunt superiora, et ad scientiam superiorum pertinent, tanto sunt magis nota omnibus propter suam universalitatem. (14)

This only refers, of course, to principles that are self-evident, and not to the postulates which a science may take as its principles. In this kind of subalternation there are two points to be noticed about the proper principles of the subalternated science; first, they are not evident; secondly, they are mediate, that is to say, they are the fruit of demonstration from principles that are evident. These two points are not identical, for it is possible for principles not to be evident without their being mediate. And in this distinction we find a fundamental difference between the kind of subalternation we have just been considering and the kind that is found in supernatural theology.

The proper principles of theology are not evident; but not all of them are mediate, since some are as first reasons, and others are truths consequent upon these reasons.

(15)
Now as Cajetan points out, although both the element of inevidence and that of mediacy are ordinarily considered to pertain to the essence of subalternation in some way, the former pertains to it in a formal way, and the latter only in a material way. Hence, in order to have true subalternation it is not absolutely necessary that the proper principles of the subalternated be conclusions; it is sufficient that they be not evident. In fact, John of St. Thomas maintains that in theology's use of principles that are not conclusions there is a fuller kind of subalternation than that found in the natural sciences where all the proper principles of the subalternated science are necessarily conclusions. For, whereas in the latter case, as we pointed out above, at least the principles from which the conclusions are drawn are evident, in the former case the fundamental (16) principles are in no way evident.

But here it is important to distinguish between two kinds of continuity, which for want of better terms we shall call objective and subjective. When the continuity is considered from the point of view of the objects that

the science is about it is objective; when it is considered from the point of view of the scientist it is subjective. Another way of expressing the same idea is to say that objective continuity is the continuity that a science has by its very essence, while subjective continuity is the continuity that it has because of its actual state. When a subalternated science is in its perfect state there is subjective as well as objective continuity. But when it is in an imperfect state, subjective continuity may be lacking. And here it must be pointed out in passing that when Thomists raise the question about whether or not a certain subalternated science is in continuity with the subalternating science, it is to subjective continuity that they are referring, for, obviously, there can be no question about objective continuity since it is a necessary condition for the very possibility of subjective continuity. But perhaps the best way to explain this distinction is by means of an example. The science of optics necessarily has objective continuity with the science of geometry, that is to say, its proximate principles are geometrical conclusions, which in turn have their evidence from their continuity with self-evident principles. But from the point of view of the student of optics this continuity may

or may not exist. It exists if he is a mathematician as well as a student of optics. It does not exist if the geometrical conclusions which he applies to his particular matter are merely accepted by him on the authority of a mathematician without their intrinsic evidence being grasped. From this it follows that the habitus of the proximate principles of a subalternated science is per se the habitus of the subalternating science. Per accidens, however, it may be a matter of authority alone.

In this distinction of the two kinds of continuity we have the solution to a problem to which John of St. Thomas gives considerable attention. ⁽¹⁷⁾ The problem is this: when subjective continuity does not actually exist, is it possible for the subalternated science to be a true science? At first glance it would seem not. For scientific knowledge is necessarily certain knowledge. And how can knowledge be certain if it is reducible merely to principles which are held on authority and not to per se evident principles? Does not St. Thomas write: "quaecumque sciuntur proprie accepta scientia, cognoscuntur per relationem in prima principia, quae per se praesto sunt ⁽¹⁸⁾ intellectui."

As we have just said, the correct solution of this problem lies in the distinction between subjective and objective continuity. Even when subjective continuity is lacking, objective continuity is always there, and that is sufficient to insure the truly scientific character of the subalternated science. For objective continuity means that the proper principles of the subalternated science are de facto demonstrated in the subalternating science, and thus there is the essential connection between the subalternated science and self-evident principles which St. Thomas demands in the text just cited.

This problem has particular significance for the science of theology, which, in this life, is based completely on faith. But it also has relevance for the question in which we are interested. For we can imagine the hypothetical case of a student of nature who, though unacquainted with the pertinent mathematical demonstrations that are presupposed, might accept the mathematical conclusions he needs on authority and employ them in his interpretation of natural phenomena. The conclusions concerning nature that he would be able to arrive at by using the borrowed mathematical conclusions as principles would express objective truth, even though they could not be called

scientific truths on the part of the student himself.

From this we may conclude that a subalternated science is specifically the same scientific habitus whether there is subjective continuity with the subalternating science or not. For even when subjective continuity is lacking, the objective continuity establishes an essential relation between the subalternated and the subalternating science. It is this essential relation that determines the nature of the subalterned habitus. And this essential relation demands completion by subjective continuity. Hence, as long as subjective continuity is lacking the habitus of the subalternated science is in an imperfect state. But when it is acquired, no new habitus is born; the old habitus is merely brought to fullness and perfection. The following lines of St. Thomas throw light upon this subtle point:

... qui habet scientiam subalternatam, non perfecte attingit ad rationem sciendi, nisi in quantum eius cognitio continuatur quodammodo cum cognitione eius, qui habet scientiam subalternantem. Nihilominus tamen inferior sciens non dicitur de his, quae supponit, habere scientiam, sed de conclusionibus, quae ex principiis suppositis de necessitate concluduntur. (19)

At this point we must turn our attention to a highly significant passage of John of St. Thomas:

... non facit subalternationem simpliciter hoc quod est mutuari aliquod principium ab aliis scientiis, ad procedendum ex illo tamquam ex principio extraneo et mutuato. Ratio est, quia subalternatio propria et simpliciter, requirit quod aliqua scientia ex propriis principiis et intrinsecis non possit resolvere in principia per se nota; sed pro evidentia suorum principiorum necessario debeat recurrere ad aliquam aliam scientiam, quae talem evidentiam faciat. Si autem utitur principiis aliarum scientiarum tamquam extraneis et mutuatis, et in illis solum recurrit ad scientiam extraneam pro illorum evidentia: non manet subalternata intrinsece; quia quantum ad propria et intrinseca principia non accipit evidentiam ab alia scientia, sed solum quoad principia extranea. Et ex hoc judicanda est subalternatio propria et intrinseca: scilicet an inveniatur in principiis intrinsecis et propriis alicuius scientiae, an solum in externis et mutuatis;(20)

These lines have two obvious references. In the first place they refer to a point made by John of St. Thomas in the Cursus Philosophicus which we have discussed earlier in this chapter: an occasional and extrinsic borrowing of principles from other sciences, such as metaphysics and logic, does not constitute subalternation in the strict sense of the word. In the second place, they refer to the immediate context in which the author shows that theology cannot be subalternated to philosophy even though it uses philosophical principles in its demonstrations, for first of all it does not take them as its own proper principles,

and secondly it uses them only after having judged them in its own supernatural light and elevated them in some way to its own level, and thus the whole essence of the demonstration rests formally and ultimately upon the supernatural principle.

But it is not particularly because of these immediate references that we have introduced this passage here. Rather it is because some of the statements in it give rise to a problem which touches the very essence of the type of subalternation found in mathematical physics.

As this passage of John of St. Thomas suggests, the ancient Thomists do not seem to have considered what we shall call dialectical subalternation, that is to say, subalternation in which the subalternating science does not give to the subalternated science in an intrinsic and adequate way the evidence of the principles that are proper to the subalternated science - - one in which there is not realized a sufficiently perfect continuity between the two disciplines in question to permit the formation of a science in the strict sense of the term. Now this is the type of subalternation that is actually found in mathematical physics. And that is why we must develop this point a little further.

The medieval Thomists recognized the existence of mathematical physics, and they accurately analyzed its nature as an intermediary discipline that involves the fullest kind of subalternation -- subalternation by reason of the object. They carefully distinguished this type of subalternation from that found in theology where the principles alone are involved. Nevertheless, for them there was a fundamental parity between these two types of subalternation. Just as there was a perfect continuity between the principles of theology and those of the science of the blessed, so there was a perfect continuity between the principles of physics and those of mathematics -- at least sufficiently perfect to permit mathematical demonstrations to be applied adequately to physical phenomena.

We are referring here to a point already mentioned in Chapter I, where we explained that for Aristotle and the medieval Thomists mathematical physics could constitute a science in the strict sense of the term because physical entities realized a sufficiently perfect conformity with mathematical entities to allow for the former to be treated in terms of the latter in strictly scientific fashion. The reason why they held this view

was that they were without refined experimental instruments, and had to depend upon sense experience. Now rough sense experience is extremely illusive. It often gives the impression that things in nature have a perfection which as a matter of fact they lack. The sense of touch may convey the notion that a surface is perfectly flat; the sense of sight may give the impression that a physical sphere is a perfect sphere. Consequently, when there is nothing else to go on but this rough experience one is easily led to feel justified in positing the hypothesis that physical lines and figures reasonably approach mathematical perfection.

The refinement of our modern instruments has emphasized the gap between physical and mathematical entities. All of our measurements are only approximative. For this reason it now seems necessary to hold that mathematical physics is merely dialectics and not a strict science. That is why the subalternation involved in it is purely dialectical.

But perhaps we should immediately add that we are considering the question here merely from the point of view of the knowledge of which the human intellect is

capable in its present state. For we see no reason to exclude a priori the possibility of the existence in nature of entities whose perfection approaches mathematical perfection sufficiently to allow for their being treated in terms of mathematics in a strictly scientific way. We have no means at our disposal to make it possible for us to arrive at this perfection, but perhaps the knowledge of this perfection is possible for the angelic intelligences, or even for the human intelligence in a superior state. If this should be true, mathematics would be able to provide a strictly scientific propter quid for natural phenomena.

But perhaps what we have just said about the opinion of Aristotle and the medieval Thomists may give rise to a problem. For if they believed that there existed in nature entities whose perfection came reasonably close to mathematical perfection, why did not such entities fall directly under the object of the study of nature? Why was it necessary to study them in terms of mathematics and construct the theory of scientia media? Why was not the so-called science of mathematical physics nothing but physics? Does not this bring us back to something akin to the opinion of Professor Mansion criticized in the last

Chapter? The answer is that even if the conformity between physical and mathematical entities were perfect, physics would still have to be subalternated to mathematics. For the concrete quantitative determinations of nature, in so far as they remain attached to sensible qualities, are not susceptible of the conceptual elaboration of which mathematical quantity is capable. Quantity is by its very nature more abstract than the sensible qualities, and it has its own reasons prior to those of the sensible qualities, and this would necessarily lead to subalternation.

A few general remarks remain to be made in order to complete our consideration of the nature of subalternation. In the first place, it should be evident from what has already been said that a lower science must be subalternated to a higher science and not vice versa.

... quanto scientia aliqua abstractiora et simpliciora considerat, tanto eius principia sunt magis applicabilia aliis scientiis: unde principia mathematicae sunt applicabilia naturalibus, non autem e converso propter quod physica est ex suppositione mathematicae et non e converso, ut patet in III Coeli. (21)

A higher science may at times use the principles of lower science, but then the dependence is only material and not formal, for the higher science in that case interprets the principles of the lower in terms of its own superior light. (22)

In the Posterior Analytics, St. Thomas gives us an example in which a mathematical proposition is demonstrated in physics:

Sunt enim quaedam propositiones, quae non possunt probari nisi per principia alterius scientiae; et ideo oportet quod in illa scientia supponantur, licet probentur per principia alterius scientiae. Sicut a puncto ad punctum rectam lineam ducere, supponit geometria et probat naturalis; ostendens quod inter quaelibet duo puncta sit linea media. (23)

It should also be evident that the subalternated science and the subalternating science can coexist in the same subject, that is, in the same intellect. In fact, this coexistence is the normal case, for it is synonymous with the subjective continuity we spoke of above. One could not get very far in analytical geometry without possessing the science of arithmetic and algebra, nor in mathematical physics without a personal knowledge of mathematics. In the case of theology this coexistence or subjective continuity with the subalternating science is impossible in this life but it will be realized in the next, for after death, the habitus of theology will perdure, even though faith has disappeared.

The subalternated science and the subalternating science may also coexist in the same object. That this is true of the material object is obvious. It is also true

of the formal object (ratio formalis quae) but in that case there can be subalternation only by reason of the principles and not by reason of the object. And here we touch upon one of the fundamental differences between the two kinds of subalternation. Theology differs from the science of the blessed only by its ratio formalis sub qua: it studies God under a different light. But the ratio formalis quae, that is the ratio Deitatis is the same. But in the intermediary sciences, not only is the ratio formalis sub qua different (a different type of abstraction), but also the ratio formalis quae, for it is a compound object arising out of the addition of an extrinsic accidental difference to the object of the subalternating science. And in order to understand what this involves we must now analyse more closely the particular kind of subalternation found in the intermediary sciences.

3. Subalternation and Scientia Media

Let us begin our analysis by considering the conditions required in order for a scientia media to exist. We have already touched upon some of them.

In the first place, the object of the subalternated science must contract the object of the subalternating science and add something to it. This addition cannot be an essential, specific difference, for otherwise there will be no formal distinction of sciences. Neither can it be a property that flows essentially from the object of the subalternating science, for the same science which deals with a certain object deals with all the essential properties of it. Consequently, the addition must be an accidental difference which makes the matter of the subalternated science extrinsic to that of the subalternating science. But not any kind of accidental difference is sufficient to constitute a scientia media. For there are some accidental differences which are not the source of any special scientific properties, and as a consequence they are incapable of constituting a new science. For example, there is no scientific fecundity in the addition of the notions of "hot" or "cold" to the mathematical notion of "line". But there is great scientific fecundity in the addition of the notion of "visual", as the science of optics attests. In the same way, the addition of the notion of "visual" to the notion of number does not give rise to special scientific properties, while the addition of the notion of "sound"

does, as is evident in the science of music.

It is important to understand accurately the accidental character of the difference that is added to the object of the subalternating science. This accidental character must not be considered from the point of view of the two sciences themselves, in the sense of there being only an accidental difference between them. As a matter of fact, there is a specific and essential difference between the subalternating and subalternated sciences. Rather, it must be considered from the point of view of the being which constitutes the object of the sciences. In other words, to use scholastic terminology, the difference is accidental to the object, not in esse scibili, but in esse rei. But, as has already been suggested, not every accidental difference in esse rei is sufficient to constitute a mixed science. It must be a difference of such a nature that it gives rise to certain new scientific truths. And these truths must depend for their explanation upon the principles borrowed from the subalternating science.

In other words, the relation between the two elements that are combined to constitute the object of an

intermediary science must be a matter-form relation. The element taken from the superior science plays the role of form, and the element taken from the lower science plays the role of matter. For the subalternating science must illuminate, determine and inform the subalternated science.

This is what St. Thomas has in mind when he writes:

Scientiae mediae, de quibus dictum est, communicant cum naturali secundum id quod est materiale in earum consideratione, differunt autem secundum id quod in earum consideratione est formale. (24)
Subjectum inferioris scientiae comparatur ad subiectum superioris, sicut materiale ad formale. (25)

In every intermediary science we have an application of the object of a higher science to the object of a lower science. When, for example, in physics we speak of light being propagated in a straight line, the line in question is neither physical alone, nor mathematical alone. It cannot be purely physical, for it is conceived as being perfectly straight. Nor can it be purely mathematical, for it is the physical entity of light that is being propagated. Consequently, it must be both physical and mathematical at the same time.

But such a line does not exist as such in nature. It exists only in the mind. It does not however exist in the mind merely through a simple process of

abstraction. Rather it is born there through an act of composition on the part of the intellect. And it is extremely important to grasp the difference between the composite character of the notion of the physico-mathematical line, and the composite character of the notion of "rational animal", for example. In the latter case the composition is not created by the mind; it is merely discovered by it. That is why it comes into being through a simple process of abstraction. In the former case the composition is created by the mind. It is a priori in the Kantian sense of the term. This is an important point to keep in mind. It will be of vital importance when in Chapter XII we come to discuss how many concessions a realistic philosophy of mathematical physics must make to Kantianism. But lest confusion arise it must be pointed out immediately that even though created by the mind, the union between the two elements is not completely logical. They are brought together by the mind - - but for an objective reason.

Now this composite character of the object of the intermediary sciences gives rise to a serious difficulty for John of St. Thomas. (26) For an object that is constituted by the addition of an accidental difference

can have only an accidental unity, and it seems impossible to have a science that is essentially and specifically one if the object is only accidentally one: "de ente per accidens non datur scientia per se." It is impossible to have an essential definition of a being that is only accidentally one, since the definition gives the quod quid est, which is something strictly one, and being that is only accidentally one does not consist of a genus and its specific difference. But the unity of a science is determined by the unity of its definitions, since, as we saw in the last Chapter, definitions are the principles of every science.

Perhaps one might be tempted to think that this no longer constitutes a real problem, once we have granted that the intermediary sciences are not sciences in the strict sense of the word, but dialectics. We believe, however, that this would be an illegitimate inference. For though these sciences are dialectical they are not sophistical, and only sophistry deals with ens per accidens. Though they are not sciences in the strict sense of the word, they must proceed ad modum scientiae. Consequently, the problem is still relevant.

John of St. Thomas solves this problem by pointing out that a scientia media does not have as its object simply and directly the composite of the two elements considered as an accidental being. Rather it considers directly only one of the two elements -- not absolutely and by itself, but in so far as it connotes the other and is modified and informed by it. For example, the science of optics, as the very name implies, has as its direct object "the visual". However, it does not consider it independently by itself, but in so far as it is determined by certain mathematical properties. And thus it is possible to consider a certain object as being scientifically knowable per se, and as being the source of certain necessary scientific truths, even though in order to be the source of those truths it requires the accidental addition of an extraneous element. For there are a number of properties which do not flow from an object when it is merely considered absolutely by itself, but only when it is considered as determined, modified, and informed by a certain element, which, though accidental to it, is absolutely necessary in order for these properties to arise. For example, there are certain properties which flow from the notion of sound when it is considered not by

itself alone, but as determined by number. In other words, although the union between the two elements is accidental, the connotation is not accidental, since by means of it certain necessary properties are revealed. Perhaps a simple analogy will add clarity to this point. Paternity is something accidental to man in the sense that not all men are necessarily fathers. Nevertheless, a number of essential properties flow from the notion of man when it is considered precisely as connoting the notion of paternity, which do not arise when it is considered independently of this determination.

It must be noted here in passing that it is precisely because the mathematical element enters into the object of mathematical physics by way of mere connotation that the role of mathematics in physics is essentially functional and instrumental.

Now since the object of a mixed science is a composite of elements taken from different levels of intelligibility, the question arises whether the abstraction employed in it is dual, or specifically one. John of St. Thomas explains that it is only one, and that is a special intermediary abstraction that stands in between the two levels of intelligibility from which the elements

have been borrowed, and that participates in the nature of both.

Quod vero additur de Musica et aliis scientiis subalternis, respondetur in illis non esse duplicem abstractionem, sed unicam, quatenus principia superioris scientiae ex applicatione ad talem materiam redduntur minus abstracta et consequenter pertinentia ad diversam speciem in genere scibilis, et illa abstractio, quam induunt in tali materia, unica est, et ideo aliquid participant de utrisque, unica tamen abstractione, sicut medium unum existens dicitur participare ab extremis. (27)

The significance of the Thomistic doctrine of scientia media has not always been correctly understood. Thus, for example, Professor Salmon writes:

Quant aux scientiae mediae, dont on a d'ailleurs beaucoup exagéré l'importance théorique, il ne faut y voir qu'un simple accident historique. Quelques problèmes, plus faciles, avaient reçu des géomètres grecs des solutions fort précises, et dont le caractère mathématique était dès lors plus accusé. On a donc pu croire que la théorie des cordes vibrantes, la catoptrique, l'astronomie, se distinguaient de quelque manière des autres parties moins évoluées de la physique. La différence n'était cependant qu'apparente, comme on l'a souligné plus haut en faisant valoir des éléments mathématiques implicites des formules rudimentaires du langage commun. On remarquera d'ailleurs historiquement que ces sciences intermédiaires n'interviennent jamais directement dans la classification des sciences, mais sont seulement ajoutées dans les réponses aux objections. Elles ne dérivent pas en effet normalement de la théorie des degrés d'abstraction, mais sont des données de fait, assez gênantes d'ailleurs, que le théoricien intègre comme il le peut dans une synthèse qui ne les prévoyait pas. (28)

We fail to see any foundation for the objection that the intermediary sciences do not enter directly into the classification of the sciences. By the very fact that they are intermediary, they obviously could not be put directly into any one of the three general types of knowledge that are based on the degrees of abstraction. If this is what Professor Salman has in mind when he says that they do not derive normally from the theory of the degrees of abstraction, his observation is perfectly true. But then it is an observation that is utterly lacking in significance. On the other hand there is a sense in which it must be said that they derive essentially from the degrees of abstraction. For it is only by seeing these sciences precisely as intermediary sciences, that is, as combinations of two different levels of intelligibility which arise out of two distinct kinds of abstraction that we can understand their true nature. It is utterly impossible to grasp the meaning of these sciences except in relation to the degrees of abstraction. That is why it is completely false to say that they are mere "données de fait" which the philosopher must force arbitrarily into a synthesis which has no natural place for them. Nor did Aristotle or any of the great Thomists ever show any signs of the embarrassment of which Professor Salman speaks.

We feel that perhaps enough has already been said to show that the intermediary sciences were far from being "a simple historical accident," and that the difference between them and pure natural science is essential and not merely apparent. The further analysis which is to follow will add clarification and confirmation to these points. Mathematical physics is specifically distinct from pure natural science because it contains an essential element taken from the science of mathematics. And yet the introduction of this extrinsic element into experimental physics is necessary and not merely arbitrary. The ancient Thomists recognized clearly both of these points.

As for the remark that the theoretical importance of the intermediary sciences has been greatly exaggerated -- we feel that the contrary is the case. The great epistemological implications latent in this point of Thomistic doctrine and its relevance for modern physics have scarcely been recognized.

4. Scientia Media and Mathematical Physics.

To discover the special characteristics of mathematical physics as a scientia media we must turn to the two

pivotal texts of Aristotle and St. Thomas mentioned in Chapter I. As has already been explained, the text from the Posterior Analytics is introduced in connection with the discussion of the two types of demonstration: demonstratio quia, i.e. demonstration which arrives only at the existence of a fact without being able to give its proper reason and cause, and demonstratio propter quid, i.e. demonstration which gives the proper reason. After pointing out how these two types of demonstration differ in the same science, Aristotle and St. Thomas go on to explain how they differ in different sciences, and first of all in sciences which are subalternated one to the other. And they state that in this latter case it pertains to the subalternating science to know the propter quid, i.e. the proper reason, and to the subalternated science to know the quia, i.e. the simple fact. Both Cajetan and John of St. Thomas insist that in making this statement Aristotle was speaking of something that is special to the kind of subalternation found in mathematical physics and not something that is common to all types of subalternation.

In order to understand why this is so we must try to grasp the difference between a scientia propter quid and a scientia quia. A scientia propter quid is a science

that is explanatory in the strict sense of the word, that is to say a science that assigns the proper reason for things. It is knowledge that is arrived at by a propter quid demonstration, that is to say a demonstration which proves that a property belongs to a subject because of its very essence. A scientia quia is a science which arrives at the fact that certain things exist or happen in a certain way, but it cannot assign the proper reason for the fact. The demonstratio quia which gives rise to this type of science may be one of three kinds. In the first place, it may be an a priori demonstration, and then it consists in proving an effect by its cause. But in this case it is always a question of the remote and common cause. Secondly, it may be an a posteriori demonstration, which proves the cause by the effect; and this maybe either inductive such as is found in the study of nature, or deductive, such as is found in natural theology in the demonstration of God's existence. The last type of demonstratio quia is known as demonstratio a simultaneo; it is used in the demonstration of the existence of a thing by the existence of its correlative or of something that is distinct from it only by a distinctio rationis ratiocinantis.

Since we are dealing with the study of nature, we are interested in the type of scientia quia that arises from inductive a posteriori reasoning. But lest confusion arise, it must be pointed out that in mathematical physics, it is not the whole of physics (in the Aristotelian sense) that is subalternated to mathematics. The first part of natural doctrine that is known as philosophy of nature does not enter into subalternation. It can reduce its demonstrations to its own self-evident principles. It uses induction, to be sure, but a type of induction that arrives at analytic and not merely synthetic propositions. It is, therefore, a deductive as well as an inductive science. It is a scientia propter quid.

It is only the dialectical prolongation of philosophy of nature, known as experimental science, that is subalternated to mathematics. This part of natural doctrine uses a type of induction that arrives only at synthetic propositions. There result from this two important things to be noted about experimental science. First, it pertains to the type of knowledge known as scientia quia. It cannot arrive at a proper propter quid. The best it can do is to construct an imitation, a substitute propter quid by means of hypothesis. Secondly, it is not even a scientia quia

in the strict sense of the word, for it does not give certain knowledge, but only probability.

Now in these two characteristics we find two reasons why experimental science inevitably reaches out to mathematics. For science is certain knowledge of things in their causes. (31) And in order to have science in the full and perfect sense of the word, these causes must be the proper causes. That is why scientia quia is related to scientia propter quid, as an imperfect state of science to a perfect state. That is why all scientia quia aspires to scientia propter quid. Now experimental science is neither certain knowledge, nor is it knowledge of things in their proper causes. Hence it has a double reason for reaching out to a scientia propter quid, i.e. mathematics, in order to obtain for itself at least a substitute certitude and a substitute propter quid. That is why the subalternation of physics to mathematics is not an historical accident. It is the result of a necessary and inevitable scientific tendency. In this connection John of St. Thomas writes:

In illis scientiis subalternatis ipsi mathematicae, quae usque ad sensibilia excurrunt, pertinet scire scientia quia eo quod res sensibiles per inductionem attingunt et usque ad experientiam descendunt. Si autem illa eadem, quae per experientiam cognoscunt, velint scire propter quid, necessario debent uti principiis traditis a mathematica seu a scientia subalternante. (32)

In subsequent discussions we shall adduce fuller evidence to bring out the necessity of the subalternation of physics to mathematics, but perhaps enough has already been said to show how erroneous is the opinion of those modern scholastics who hold that the grounding of physics on mathematics is a great and fatal historical mistake. (33)

As John of St. Thomas points out, (34) when we say that the subalternating science of mathematics knows the cause, or the propter quid of the natural phenomena, this does not mean that it pertains to the subalternating science to know the conclusions of the subalternated science and to demonstrate them. This would mean that mathematics would descend to sensible matter, and in order to do this it would have to abandon its proper abstraction, and thus cease to be mathematics. The expression merely means, as (35) Cajetan explains that the subalternating science knows the propter quid in an abstract and general way, and it is the subalternated science which takes the general principles that are given to it and applies them to its own particular subject matter. This is what Aristotle and St. Thomas have in mind when they point out that the one who knows (36) the reason does not have to know the fact. It should be obvious from what has been said that when Aristotle and

St. Thomas say that the subalternated science knows only the quia, or the fact, this means by itself, independently of the subalternation to the higher science from which it receives its principles. For, by virtue of its subalternation the subalternated science is able to know the cause as well as the fact.

Just as it is possible to have subalternation in the strict sense of the word without the two sciences being related to each other in such a way that the one knows only the fact and the other the reason for the fact, so it is possible to have sciences related in this way without being subalternated to each other. Aristotle gives a simple example of this taken from the science of medicine. (37) A physician may learn from experience that circular wounds heal more slowly than other kinds of wounds; but it is geometry which gives the reason for this: the absence of angles. This, however, does not mean that medicine is subalternated to geometry.

Now St. Thomas makes it very clear that in mathematical physics we really apply abstract mathematical entities to the phenomena of nature.

Perspectiva applicat ad lineam visualem ea quae

demonstrantur a geometria circa lineam abstractam;
et harmonica, idest musica, applicat ad sonos ea
quae arithmeticus considerat circa proportiones
numerorum... Perspectiva accipit lineam abstractam
secundum quod est in consideratione mathematici,
et applicat eam ad materiam sensibile^m (38)

When a physicist speaks of light being propagated in a straight line his calculation proceeds from mathematical straightness. Of course, he is not properly concerned with the mathematical line, but with the physical line which connotes the mathematical line that is applied to it. It is extremely important to keep in mind that it is actually the abstract mathematical entity that is applied to nature.

This application is not merely the reverse of mathematical abstraction. It does not consist merely in fitting back into sensible matter what was lifted out of it by the second degree of formal abstraction. For, as we shall see in Chapter VI, the abstraction that is found in mathematics is different from that found in all the other sciences in this that we cannot go back to reality from the abstract notions and find them realized there. There is a world of difference between the abstract notion of man and the abstract notion of straight line. In the first case, we can find the notion of man realized in the concrete. In the second case, although we can find a line in nature, we cannot find a

perfectly straight line.

Although we cannot pass from the world of mathematics to the world of physical reality by a process of direct concretion, which would simply be the reverse of abstraction, we can do so by a process of extrinsic application. The fact that this is merely an application and not a direct realization shows that the mathematical interpretation of nature is necessarily a scientia media. It also shows that the propter quid which mathematics supplies to the study of nature always remains in some sense extrinsic to nature. This would be true even in the hypothetical case mentioned earlier in this Chapter in which a superior intelligence would find it possible to treat natural phenomena in terms of mathematics in a strictly scientific way.

For us the mathematical propter quid must also remain extrinsic to nature in the sense of its being dialectical. The inadequacy of all our measurements and the limitation of all our experience both with regard to space and time makes it necessary for us to operate within an extremely restricted frame where no phenomena can be sufficiently accounted for. Given this inadequacy of our

measurements and experiments and the uncertainty of our reasoning, the application of a mathematical proposition to a natural subject must be considered as something essentially tentative. The mind ever goes beyond the data of experience in this application, and in so far as this application inevitably outreaches what is conveyed to us by experience, the mind is out on its own, so to speak. As a consequence, the subject formally attained is never wholly divorced from the part played by reason itself. And to the extent in which there is in the subject something coming from reason alone, the subject itself must be called a dialectical entity.

It is clear, therefore, that in mathematical physics we can never arrive at anything more than a provisional and substitute propter quid. This is attested to by the history of physics. In Newtonian physics, for example, the propter quid for many natural phenomena was found in Euclidian geometry; in Einsteinian physics the propter quid for the same phenomena is found in non-Euclidian geometry.

As we have seen, physics reaches up to mathematics in an attempt to escape the dialectical status imposed upon

it by its lack of true universal necessity. But it is clear from what has just been said that, because mathematics cannot provide an explanation that will give universal necessity for the meaning of nature, physics does not succeed in escaping from its dialectical status by becoming subalternated to mathematics. In fact, it becomes doubly dialectical.

But for the present the important point is that physics, because of the opacity of the universe of matter, is forced to go out into a new world to find light, and having found it in the world of mathematics, it brings it back into the material world. As Cassirer has remarked, "that form of knowledge, whose task is to describe the real and lay bare its finest threads, begins by turning aside from this very reality and substituting for it the symbols of number and magnitude." (39) It is a strange light that we bring back from our excursion into the world of mathematics, for as we shall see, mathematical abstraction is in one sense richer and in another sense poorer than any other type of scientific abstraction. In this connection it is important to note the exact formality of the expressions used by St. Thomas in his discussion of

the application of mathematics to physics: "Huiusmodi scientiae utuntur speciebus idest formalibus principiis, quae accipiunt a mathematicis." This shows that the mathematical forms in physics are something essentially alien to the physical world, and that the role played by mathematics is from this point of view purely instrumental.

In mathematical physics, then, we take a mathematical line, for example, and apply it to the physical line. In other words we consider the latter as if it were a straight line. Mathematical physics is essentially a science of als ob. The line which we introduce into nature is the fruit of our own abstraction, and cannot exist as such in reality. We have here a kind of application of a priori forms, and consequently a kind of a priori knowledge. And once again it becomes evident how much Kantianism there is in mathematical physics.

In connection with this insistence that what is applied to nature is actually the abstract mathematical entity, we must consider for a moment a possible interpretation of mathematical physics which at first glance appears highly plausible, but which is fundamentally erroneous. We refer to an interpretation which would consider the so-called mathematical entities merely idealizations or limit

cases of physical entities. Experimental science deals constantly with idealizations and limit cases. When a physicist speaks of the laws of gases he has in mind a "perfect gas" which exists nowhere in nature. Does it not seem plausible that when he speaks of a "perfectly straight line" he is likewise speaking merely of an idealization of a sensible line, that is to say, a sensible line pushed to its limit case? If this interpretation were correct, mathematical physics would not be a scientia media, for just as the introduction of such idealizations and limit cases as "perfect gas", does not involve the application of a superior science, so neither would the idealization of a sensible line. This would bring us back to something similar to the doctrine of Professor Mansion discussed in the last chapter.

Such an interpretation cannot be admitted.

Idealizations and limit cases are not the product of formal abstraction, but merely of negative abstraction. It is possible, of course, to push certain physical entities to their limit case and thus arrive at something which superficially resembles mathematical entities. It is likewise possible to attempt to study nature in terms of these idealizations. However necessary negative abstraction of this kind may be, it remains something common, and does not account for the peculiar intelligibility provided by the application of the

positive abstraction of mathematics. The great rational elaborations of mathematical physics show that it is a specifically superior source of intelligibility that has been introduced into nature which of itself is less rational.

It is true that the basic relations between variable quantities out of ^{which} the mathematical physics is constructed are given implicitly in a concrete quantitative determinations of nature. But it is illegitimate to conclude from this, as Professor Renoirte seems to have done, that there is no subalternation of a lower to a higher (40) science involved. For mathematical physics is not a mere collection of concrete quantitative relations or of concrete measure -numbers. It is essentially a mathematical elaboration and interpretation of these initial data. And it is in this elaboration and interpretation that the subalternation consists.

After explaining that the subalternation of physics to mathematics consists in this that the former gets its propter quid, its cause and reason from the latter, Aristotle and St. Thomas go on to explain the particular (41) nature of this cause. Now the only propter quid which mathematics can give to the study of nature must be in the line of formal causality. For of all the four causes the

only type of cause that is found in mathematics is the formal cause. The mathematical world is a completely immobile world. In it there is no becoming and hence no subject, no agent, no purpose. It is a world of pure forms. And this gives us an insight into the peculiar nature of mathematical physics. If it were purely physics it would try to resolve things in terms of all the four causes. But because it is formally mathematical it can see things only in the light of formal causality. This is an extremely important point, and we shall return to develop it later. For the moment let it suffice to bear in mind that the cause which mathematics contributes to physics is in the general line of formal causality, and pertains in particular to the structural order.

Now since mathematical physics is an intermediary science between physics and mathematics, it is necessary to try to determine to what extent it participates in both of these sciences. Does it participate in both of them in equal measure, so to speak, or does none of the two predominate over the other? From what has been said up to this point one might easily be led to deduce conflicting answers to this question. For in discussing the structure of a mixed

science we stated that an accidental element taken from the object of the lower science is added to the object of the higher science. From this it would seem to follow that the most important element in the object of mathematical physics is the element taken from mathematics, and that the physical element is merely an accidental addition to it. On the other hand, when the question arose about the kind of unity found in the object of an intermediary science we said that the object which mathematical physics considers directly and per se is the physical element, and the mathematical element is brought into the consideration in a kind of oblique fashion by way of connotation.

If we look for the solution of this antinomy in the writings of Aristotle and St. Thomas, our difficulty is aggravated. For on the one hand, Aristotle seems to class the physico-mathematical sciences among the mathematical sciences. (42) Moreover, we read in Saint Thomas that these sciences are "magis affines mathematicis, quia in eorum consideratione id quod est physici, est quasi naturale; quod autem mathematici, quasi formale." (43) And John of St. Thomas says: "astrologus non agit de coelo et planetis, ut sunt entia mobilia, sed ut mensurabiles sunt eorum motus et secundum varios aspectus diversam

proportionem induunt, quod magis pertinet ad mathematicum
(44)
quam ad physicum." On the other hand we are told by
St. Thomas that these sciences are more physical than
mathematical: "Huiusmodi autem scientiae, licet sint
mediae inter scientiam naturalem et mathematicam, tamen
dicuntur hic a Philosopho esse magis naturales quam
mathematicae, quia unumquodque denominatur et speciem
habet a termino: unde, quia harum scientiarum consideratio
terminatur ad materiam naturalem, licet per principia
mathematica procedant, magis sunt naturales quam mathematicae." (45)

There is a text in the Summa which, together with
the commentary of Cajetan, throws light upon this apparent
paradox:

Quilibet habitus formaliter quidem respicit medium,
per quod aliquid cognoscitur; materialiter autem
id, quod per medium cognoscitur; et quia id quod
est formale, potius est, ideo illae scientiae quae
ex principiis mathematicis concludunt circa materiam
naturalem, magis cum mathematicis connumerantur,
utpote eis similiores, licet quantum ad materiam
magis convenient cum naturali; et propter hoc
dicitur in II Phys. quod sunt magis naturales. (46)

To this text Cajetan adds the following remarks:

In responsione ad tertium secundi articuli non
dicitur quod scientiae mediae sunt magis mathematicae
quam naturales: cum falsum sit, absolute loquendo:
quia simpliciter sunt scientiae naturales, utpote
non abstrahentes a materia sensibili; omnis enim
scientia non abstrahens a materia sensibili est
naturalis, ut patet VI Met. Sed dicitur quod

connumerantur magis cum mathematicis, utpote eis similiore. Et de connumeratione quidem liquet, quia cum geometria et arithmetica scientiae numerantur inter liberales artes. De similitudine autem in modo demonstrandi manifestum est, dum mensurando et quantificando conclusiones monstrantur. Verum quia medium utrumque sapit extremum; et scientiae istae ex parte formae ex mathematica veniunt et pendent, ex parte materiae physicae sunt; sermones Doctorum pie interpretandi sunt, si quando ad alterum extremum nimis declinant.

Perhaps a more sharply drawn distinction will serve to dispel all confusion on this point. From the point of view of its ratio formalis quae, mathematical physics is more physical than mathematical; from the point of view of its ratio formalis^{sub} qua, it is more mathematical than physical. The ratio formalis quae is the physical considered as connoting the mathematical and as determined and modified by it. Consequently the physical is considered directly, whereas the mathematical is brought in only indirectly and obliquely. The terminus or end of mathematical physics is the knowledge of nature. It is not the knowledge of the mathematical world that the mathematical physicist is striving for (that is already presupposed) but of the physical world. As we saw in Chapter II, mathematics does not terminate in sense experience, and the origin which it has in sense experience is only remote and pre-scientific. Mathematical

physics, on the other hand, both originates and terminates in sense experience, even though, due to the role played by mathematics, there are introduced between the origin and the terminus many elements which have no counterparts in sense experience. All this explains why we speak of mathematical physics and not of physical mathematics. And from this point of view, physico-mathematical science may be numbered among the physical sciences. As Cajetan points out in the passage just cited, mathematical physics does not abstract from sensible matter, and judged by this criterion it may be said to be a natural science.

Yet it would be erroneous to conclude that physico-mathematical science is formally identified with pure natural science. As a matter of fact, it is distinguished from it specifically both by its ratio formalis quae and its ratio formalis sub qua. For in so far as the ratio formalis quae is concerned, we have just seen that, while the physical is considered directly and primarily, it is nevertheless, considered only as connoting the mathematical and as modified by it. Now this connotation and modification introduces a profound change. As we pointed out in the last Chapter, the ratio formalis quae of all pure natural science is mobility. This, however, cannot be said

to be the ratio formalis quae of mathematical physics, for as we shall explain later on, the introduction of mathematics into physics destroys all true mobility by the very fact that there is no true becoming intrinsic to mathematics. Movement undoubtedly plays a large part in mathematical physics, but it is movement in the Cartesian sense, which is a state and a relation, and not a process or a becoming. Mathematical physics does not study the physical world as mobile, but as measurable. As John of St. Thomas says in a text already quoted, "Astrologus non agit de coelo et planetis ut sunt entia mobilia, sed ut mensurabiles sunt eorum motus et secundum varios aspectus diversam proportionem induunt, quod magis pertinet ad mathematicum quam ad physicum." (47)

Yet mathematical physics does not dispense completely with mobility. For there is an essential relation between its formal object and that of pure natural science. The extremely paradoxical character of mathematical physics has already been noted: in order to draw closer to the absolute world condition it draws away from it by going out into another world, that of mathematics. Applying this to the point under discussion, we may say that in order to understand the mobility of the cosmos it prescind from it by introducing mathematics. But the important point is that in prescinding

from it, it is tending towards a more perfect understanding of it. The limit of this tendency would be an identification of the formal object of mathematical physics with that of pure natural science. Even though this limit can never be reached, nevertheless there is in the state of tendency an essential relation between the two formal objects.

In mathematical physics there is a triple dialectical movement. First, there is the movement from the state of generality towards the ultimate concretion. Secondly, there is the movement from the state of probability towards the state of certitude. Both of these dialectical movements are common to all experimental science. And thirdly, there is the movement proper to mathematical physics - the one we have just explained. All of these three movements are intimately bound together.

Physico-mathematical science is distinguished from pure natural science not only by its ratio formalis quae, but also by its ratio formalis sub qua. In fact, from the point of view of this latter ratio it is closer to mathematics than to physics, just as from the point of view of the former it is closer to physics than to mathematics. Mathematical physics is formally mathematical. It gets its propter quid from mathematics, and since the propter quid

gives the reason and cause of the natural phenomena, it stands in relation to the latter as form to matter. All this means that mathematical physics proceeds under the light of mathematical evidence. This would seem to imply that the special type of abstraction which constitutes its ratio formalis sub qua, and which, as we saw above, stands in between mathematical and physical abstraction and shares in the character of both, is more mathematical than physical. Though principally mathematical it is not, however, specifically mathematical, since it is applied to a physical object in order to constitute a new subject and new principles proper to a science concerned with physical reality. In other words, though mathematical physics is formally mathematical, it is not specifically mathematical.

From what has just been said about the parts played by mathematics and physics, it should be clear that when we say that mathematical physics is formally mathematical and materially physical this does not mean that the formal object is mathematical and the material object is physical. For the objectum formale quod has to do with the physical world. Some modern scholastics seem to be confused on this point. (48) It should also be clear how completely Aristotle is misrepresented by Professor Mansion

when he writes:

On voit donc comment, en écartant de la physique, pour les assigner au domaine mathématique les sciences mentionnées à l'instant, Aristote a manqué l'occasion de traiter à fond sur des cas concrets parfaitement adaptés, le problème de la différence entre une étude philosophique et une étude purement scientifique de telle ou telle portion du monde matériel. (49)

Aristotle in no way removed the physico-mathematical sciences from the realm of physics. If he listed them among the mathematical sciences it was merely because they are formally mathematical. And he took pains to point out explicitly that while they are closer to mathematics from this point of view, they are at the same time more natural than mathematical. In his mind they were, of course, specifically distinct from pure natural science, but this did not remove them from the realm of physics, since their whole raison d'être was to get to know the physical universe.

At this point it is interesting to compare what has been said thus far about the nature of mathematical physics as a scientia media, formally mathematical and materially physical, with two passages from Albert Einstein, one of which has already been quoted. There is a remarkably close affinity between what the ancient Thomists taught about mathematical physics as formally mathematical and what

Einstein has to say in the following lines:

It is my conviction that pure mathematical construction enables us to discover the concepts and laws connecting them which give us the key to the understanding of the phenomena of Nature. Experience can of course guide us in our choice of serviceable mathematical concepts; it cannot possibly be the source from which they are derived; experience of course remains the sole criterion of the serviceability of a mathematical construction for physics, but the truly creative principle resides in mathematics.(50)

In the same way, the following passage seems an exact confirmation of the Thomistic doctrine that mathematical physics is materially physical:

Pure logical thought cannot give us any knowledge concerning the world of experience: all knowledge of reality begins in experience and ends in experience. The conclusions obtained by means of purely rational processes are, in so far as reality is concerned, entirely empty.(51)

We are now in a position to understand with greater exactness a point to which some attention was given in Chapter I. We refer to the question of whether or not the role of mathematics in mathematical physics is purely instrumental. It should be evident from what has been said that it cannot be purely instrumental in the sense of being a mere logical tool or a convenient language. For neither a logical tool nor a language enters into the very object of the science that employs them. They remain essentially extrinsic to that object. But in mathematical physics, an

element of mathematics enters into combination with a physical element to constitute the very object which specifies that science. And yet because it does not enter into it directly, but in an oblique fashion by way of connotation, and because as a consequence the objectum formale quod, that is, the thing that mathematical physics is trying to get to know, the thing that is the terminus and the end of the whole science, is something of the physical world, and not the mathematical world, we may say that in this sense the role of mathematics is purely functional. Mathematics is employed in physics only as a means to get to know the physical universe. As Professor Babin has pointed out, the physicist who loses sight of this purely functional character cannot fail to pervert his science:

Parce que la fin du savoir physico-mathématique est tout de même la nature sensible, le physicien mathématicien, à tendance mathématisante, pervertit sa science, quand il se désintéresse des choses naturelles elles-mêmes pour se complaire, comme dans un terme, dans l'ordre et la beauté de son objet formel, donc dans l'aggregatum ut sic, en tant que celui-ci est un composé accidentel et oeuvre de sa raison, et pure substitut de la nature. C'est un artiste égare' ou frustré, et qui se sert de la nature comme d'une matière ouvrable. Ce faisant, il érige en fin ce qui est moyen seulement, et préfère contempler l'oeuvre de sa raison plutôt que la nature, qui est l'oeuvre de l'intelligence divine. (52)

Emile Meyerson makes the following commentary on the pivotal text of the Posterior Analytics in which Aristotle explains his conception of mathematical physics as an intermediary science:

Il y a évidemment, dans ce dernier morceau une sorte de tendance panmathématique, laquelle n'a pas manqué d'embarrasser quelque peu les commentateurs dont certains même ont cru pouvoir observer que le Stagirite, transgressant les règles qu'il avait posées ailleurs, paraissait bien passer ici d'un genre à un autre. (Note: Cf. notamment la note de Barthélemy-St-Hilaire, Logique d'Aristote, t. III, Paris, 1842, p.85). Mais si l'on fait abstraction de ces passages, qui semblent plutôt un héritage provenant des philosophes de l'Académie, la pensée d'Aristote s'avère parfaitement orientée dans le même sens que celle de Bosanquet, tout en étant en quelque sorte plus extrême que celle-ci. (53)

It is extremely difficult to find any trace of a tendency towards panmathematicism in Aristotle's doctrine of mathematical physics. He never identified mathematics with physics. On the contrary, through his doctrine of subalternation, he kept them both distinct, while at the same time recognizing their intimate relation. He never held that the whole of physics could be subalternated to mathematics, to say nothing of the other sciences. Much less did he ever attempt to erect the mathematical interpretation of reality into a metaphysic². Nor have any of his great commentators - - those who have understood his

doctrine most correctly and given it most genuine and integral development - - ever manifested the slightest embarrassment over this text from the Posterior Analytics. On the contrary, they have considered it to be in perfect harmony with all of the epistemological principles of the Aristotelian synthesis.

There is no difficulty in admitting an influence of the Academy upon this particular point of Aristotle's doctrine. Aristotle himself would certainly be the last one to deny his great indebtedness to Plato. But it is not, as Meyerson suggests, a heterogeneous bit of doctrine that was accepted by a kind of strange concession to eclecticism. Rather it is something that has been purified of Platonist exaggerations and brought into perfect line with the whole body of Aristotelian epistemology. As for the charge that this text represents a transgression of rules laid down by Aristotle elsewhere -- we have already considered this point both in this Chapter and in the last part of Chapter II, and there is no need of reconsidering it here.

These remarks conclude our explanation of the basic principles underlying the Thomistic philosophy of

mathematical physics. The chapters which are to follow will be an elaboration of these. As we have seen, there are two pivotal points around which these principles revolve: the nature of the distinction between physics and mathematics, and the nature of scientia media. The next three Chapters will be a development of the first point, and the remaining Chapters a development of the second. The next two Chapters will be devoted to an analysis of the science of nature, and the one following them to an analysis of the science of mathematics. The study of scientia media will fall naturally in two parts: first we shall consider the way in which this intermediary science is constituted (Chapters VIII and IX), and secondly we shall analyze the nature of the physico-mathematical world which results from this mediation (Chapters X to XIII).

CHAPTER FOUR

COSMOS AND LOGOS

1. Movement Towards Concretion.

At the beginning of his Commentary on the De Coelo et Mundo, St. Thomas has this to say:

... Philosophus ostendit in scientiis esse processum ordinatum, prout proceditur a primis causis et principiis usque ad proximas causas, quae sunt elementa constituenta essentiam rei. Et hoc est rationabile: nam processus scientiarum est opus rationis, cuius proprium est ordinare; it unde in omni opere rationis ordo aliquis inventitur, secundum quem proceditur ab uno in aliud. Et hoc patet tam in ratione practica, cuius consideratio est circa ea quae nos facimus, quam in ratione speculativa, cuius consideratio est circa ea quae sunt aliunde facta.(1)

It is proverbial that the most characteristic property of wisdom is order: sapientis est ordinare. (2) And perhaps in no way does the profound wisdom of Aristotle and St. Thomas manifest itself with greater brilliance than by the order that is found in their writings. This order is sometimes left to impose itself upon the mind by its own clarity without explicit attention being called to it. At other

times, when there is special need of insisting upon the right order to be followed, an effort is made to explain and justify the order adopted. And nowhere in their writings do Aristotle and St. Thomas lay such particular stress upon the question of order as in their treatises on natural doctrine. It is the first problem discussed at the beginning of the eight books of the Physics, and time after time throughout the subsequent treatises it is brought back into focus, and the basic principles involved in it are reconsidered. ⁽³⁾ As we shall presently attempt to make clear, the history of philosophy, and the history of modern thought in particular, have shown that this emphatic insistence upon the correct order to be followed in the study of nature was far from being gratuitous.

But if this question is to be put into proper perspective, we must begin by recalling that there are two issues involved in the general problem of scientific order. First, there is the question of the right ordering of the different sciences among themselves, and this has been treated at some length in Chapter II. Secondly, there is the question of the right ordering of the different parts of the same science; this has been touched upon lightly in Chapter II, but we must now consider it in greater detail

in so far as it involves the study of nature.

St. Thomas brings out this double movement of the scientific mind in his Commentary on the De Sensu et
(4)
Sensato:

Et sicut diversa genera scientiarum distinguuntur secundum hoc quod res sunt diversimode a materia separabiles, ita etiam in singulis scientiis, et praecipue in scientia naturali, distinguuntur partes scientiae secundum diversum separationis et concretionis modum. Et quia universalis sunt magis a materia separata, ideo in scientia naturali ab universalibus ad minus universalis proceditur.

In other words, both the ordering of the different sciences and the ordering of the parts of the same science are determined by different degrees of mental separation, but in each case a distinct type of separation is involved. In the case of the ordering of the various sciences it is a question of a separation from materiality according to different levels of formal abstraction, and the natural movement of the mind is from the less abstract to the more abstract. In the case of the ordering of the different parts of the same science, it is a question of a separation from concreteness according to different levels of total abstraction, and the natural movement of the mind is from
(5)
the more abstract to the less abstract.

It is commonly supposed that progress in science means an increase in abstractness. As a matter of fact, it is just the contrary that is true. (6) This refers, of course, to the sciences whose object is to know existential reality. To get to know concrete reality better means to get to know it with greater concreteness. Mathematics, precisely because it is the science of the abstract qua abstract, can make progress by growing in abstractness, but in the study of nature and in metaphysics the movement must be towards fuller concretion. In metaphysics this movement is from the communia entis up through the realms of the created separated substance to Pure Act. In the study of nature the movement towards concretion carries the mind in some sense in the opposite direction - - into deeper immersion in matter.

Perhaps at first sight all this may seem to be in direct contradiction to the actual historical development of physics. Bertrand Russell has claimed that "in proportion as physics increases the scope and power of its methods, in that same proportion it robs its subject-matter of concreteness." (7) Surely relativity physics and quantum physics are immeasurably more abstract than anything that the past centuries have produced.

It cannot be denied that progress in modern physics has meant an increase in abstractness. But at the same time, it has also meant an increase in concreteness. Atomic physics, for example, in spite of its abstract constructions (or rather precisely because of them -- as we shall explain in a moment) has brought us into more intimate contact with concrete reality than we ever were before. There is nothing paradoxical in this double movement towards concreteness and abstractness. It merely reveals the fact that modern physics is not a pure physical science, but a scientia media in which physics a science of the concrete is subalternated to mathematics, a science of the abstract. (8)

In this Chapter we are concerned with the study of nature in so far as it prescind from subalternation to mathematics. That is why the movement that must claim our attention in a particular way is the one towards fuller concretion. Moreover, even in mathematical physics, the movement towards abstractness is secondary and purely functional, since its whole purpose is to assist the movement towards concretion. That is why it is of extreme importance to analyze the nature of this latter movement.

In the first Chapter of the first book of the

Physics, Aristotle writes:

The natural way of doing this is to start from the things which are more knowable and obvious to us and proceed towards those which are clearer and more knowable by nature; for the same things are not 'knowable relatively to us' and 'knowable' without qualification. So in the present inquiry we must follow this method and advance from what is more obscure by nature but clearer to us, towards what is more clear and more knowable by nature. Now what is to us plain and obvious at first is rather confused masses, the elements and principles of which become known to us later by analysis. Thus we must advance from generalities to particulars; for it is a whole that is best known to sense-perception, and a generality is a kind of whole, comprehending many things within it, like parts. (9)

It is clear from this capital text that for Aristotle the basic order to be followed in the study of nature is one which moves from the more confused to the more distinct, from the more universal to the more particular, from the more abstract to the more concrete. But he does not lay down this principle, which is to serve as the guiding light throughout his long researches into nature, without seeking to give it full justification. And St. Thomas, in his commentary on this passage, shows that this justification can be cast in the form of a simple syllogism:

Innatum est nobis ut procedamus cognoscendo ab iis quae sunt nobis magis nota, in ea quae sunt magis nota naturae; sed ea quae sunt nobis magis nota, sunt confusa, qualia sunt universalia; ergo oportet nos ab universalibus ad singularia procedere. (10)

Each of the propositions in this syllogism deserves attentive examination.

In the first place it is clear that in the pursuit of science we must start with those things which are most knowable for us, and gradually pass on to those things which are less knowable for us. This principle is so obvious that it does not need justification. But it so happens that there is an inverse proportion between the knowability that things have for us and the knowability that they have in se. And we do not have to seek very far to find the reason for this. For, since being and ontological truth are convertible, things are objectively knowable according to the measure of perfection of being which they possess. And since things have perfection of being to the extent in which they are in act, it follows that their objective knowability is determined by their degree of actuality. That is why, if our intellects were in the fullness of actuality, their order of knowing would coincide with the objective order of knowability. But it happens that they are far from possessing the fullness of actuality -- as far as it is possible for any intellect to be. As a matter of fact, they must begin the process of knowledge from noetic pure potency -- a tabula rasa -- and gradually move in the direction of fuller actuality.

And that is why the knowability of things for us is in inverse proportion to the knowability of things in se. In other words, the intellect must acquire knowledge, not in conformity with its act, but in conformity with its potency. If it were to acquire knowledge in conformity with its act, it would suffice for it to exist in order for it to have knowledge in act. Hence the first object of knowledge must be that which is most in conformity with the intellect's (11) state of potentiality.

In our discussion of the nature of abstraction in Chapter II we pointed out that one of the differences between formal and total abstraction emphasized by Cajetan consists in this that as we advance in formal abstraction we are moving from what is more knowable to us and less knowable in se to what is less knowable to us and more knowable in se, while an advance in total abstraction means a movement in the opposite direction. And this explains why in the ordering of the different sciences we must ascend the levels of formal abstraction and advance from the less abstract to the more abstract, whereas in the ordering of the different parts of the same science we must descend the levels of total abstraction and pass from the more abstract to the less abstract. In both cases we are moving from the more knowable

for us towards the more knowable in se, that is to say, from potentiality to actuality. In the first case it is a question of the potentiality of materiality; in the second case it is a question of the logical potentiality of universality.

And this brings us to an explanation of the minor of our syllogism. It is fairly obvious why the mind, if it is to follow its natural movement of passing from potentiality to actuality, must begin with the more general and gradually advance in the direction of the more particular. For universals contain their subjective parts only in a confused and indistinct way, that is to say, in potentiality. In other words, the universal stands in relation to the particular as indetermination to determination, and hence as potency to act.

In connection with the conclusion of the syllogism it is necessary to note that the expression "singularia" does not refer to individuals but to species. We have already brought out this point in our criticism of Maritain in Chapter II. And perhaps it is not superfluous to mention in passing that in this whole discussion Aristotle and St. Thomas are dealing only with intellectual knowledge, for obviously a

knowledge of particulars by the senses is a prerequisite for the formation of universals by the mind.

The terminus, then, towards which the whole study of nature must ever move is ultimate specific concretion. It does not aim to lose itself in the infinite potentiality of individual concretion -- de singulis non est scientia. It must begin with the consideration of mobile being in general and analyze its structure and properties; from there it must move towards the full and adequate determination of the unique mobility that is proper to each natural species. This is a goal that actually transcends the powers of the human mind, as we shall explain more fully a little later; but it provides a limit towards which natural science must ever tend if it is to be true to its own intrinsic nature.

The study of mobile being, therefore, is essentially a science that must ever remain in the state of mobility. For though from one point of view the generalities which constitute the first part of the science of nature are the most satisfying for the mind, since they are the truths that are most knowable for us, and, as we shall presently see, the truths about which we can have the greatest certitude, from another point of view they are

the least satisfying. For, by their very generality and vagueness, they give us only a superficial knowledge of nature; they provide only a kind of introduction to the study - material reality, in somewhat the same way as the communia entis in metaphysics provide only an introduction to the study of immaterial being. The true student of nature will never be satisfied with the superficiality of this introduction. He will want to come into more intimate contact with cosmic reality. And in order to achieve this, he will never cease his efforts to advance in the direction of fuller concretion. In his commentary on the Libri Meteorologicorum St. Thomas writes:

Sicut in rebus naturalibus nihil est perfectum dum est in potentia, sed solum tunc simpliciter perfectum est, quando est in ultimo actu; quando vero medio modo se habens fuerit inter puram potentiam et purum actum, tunc est quidem secundum quid perfectum, non tamen simpliciter; sic et circa scientiam accidit. Scientia autem quae habetur de re tantum in universali, non est scientia completa secundum ultimum actum, sed est medio modo se habens inter puram potentiam et ultimum actum. Nam aliquis sciens aliquid in universali, scit quidem aliquid eorum actu quae sunt in propria ratione eius: alia vero sciens in universali non scit actu, sed solum in potentia - - Unde manifestum est quod complementum scientiae requirit quod non sistatur in communibus, sed procedatur usque ad species.(12)

Aquinas points out elsewhere that natural forms have their
(13)
very being "in concretionem ad materiam." That is why one

can come into intimate contact with them only by delving deeper and deeper into matter.

Perhaps this last point will present a difficulty to the mind. For this delving into the depths of matter may seem to be leading us in the direction of greater objective unintelligibility, whereas we stated a few moments ago that the movement towards concretion means an advance towards things which are more intelligible in se. The solution of this difficulty is fairly simple: even though the things of nature because of their materiality are less intelligible in se than immaterial things, they are, nevertheless more intelligible in se in the state of concretion with matter than in the state of vague generality.

Having established the fact that natural science must move from generality to concretion we must now consider the problem of how this movement is carried out. This is a question of extreme importance, for it has to do with what is perhaps the most widely misunderstood point of the whole Thomistic philosophy of science.

It has become traditional among historians and philosophers of science to insist with great emphasis upon the completely antithetical character of the scientific

spirit of the Renaissance in comparison with the Aristotelianism that had dominated the preceding centuries. We are told (almost invariably without any attempt at proof) that Aristotle and his medieval followers had held that the whole of cosmic reality could be deduced a priori from a few general principles, and that it was only at the time of the Renaissance that the essential role played by experience and induction in the study of nature was first clearly recognized. This condemnation of Aristotelianism is so universal that it is found even among those who have won for themselves considerable reputations as historians of science. Emile Meyerson, for example, tells us in more than one place in his writings that, as Malbranche pointed out, Aristotle's natural science was not physics but logic, that it was, in fact, a panlogicism similar to that of Hegel. The following passage from De l'Explication dans les sciences is typical:

... elle (la théorie d'Aristote) présente également un essai de déduction globale de la nature. Comment s'opère effectivement cette déduction, par quel moyen à l'aide des concepts de matière et de forme les phénomènes se constituent, c'est ce que les manuels enseignent suffisamment pour que nous puissions nous abstenir de l'exposer ici. Contentons-nous de relever que la déduction domine le système entier. Tout doit se ramener au syllogisme, et Aristote ne connaît de démonstration scientifique que par le syllogisme, cette démonstration, comme l'a justement formulé Zeller, étant chez lui une conclusion résultant de prémisses qui sont elles-mêmes nécessaires. C'est au point que l'on a pu

dire que la science d'Aristote était, non pas une physique, mais une logique. C'est là en effet, l'impression qu'en reçoit un homme élevé à l'école de la science moderne. Mais il est clair que, pour le maître du péripatétisme, aussi bien que pour ses sectateurs de l'antiquité et du moyen âge, les deux se confondent puisque la nature ne peut être que logique C'est là un état d'esprit qui, sans doute, paraît fort éloigné du nôtre. Il n'est cependant pas impossible de lui trouver un parallèle à une époque très rapprochée de nous. Hegel, nous le verrons plus tard, a entrepris une tâche sinon identique à celle que se proposaient les Ioniens ou Aristote, du moins fort semblable, en ce sens que, tout en ne prétendant pas déduire la nature entière, il croyait cependant pouvoir recréer, par sa métaphysique, tout ce qu'il y avait en elle d'essentiel. (14)

Later in the same work Meyerson claims that Peripateticism was an even more extreme form of panlogicism than Hegelianism, since Hegel did not hold that the whole of natural science was deducible whereas Aristotle did. And he finds a reason for this difference in the fact that the great advances made in experimental science between the time of Aristotle and that of Hegel could not help but influence the latter, in spite of his "arrogance logique." Levelled (15) against the decadent Scholastics of the late middle ages, or against the modern writers of Scholastic manuals (to which, incidentally, Meyerson seems to have gone to find his "deduction globale") this accusation has some justification. But applied to Aristotle and St. Thomas it is nothing short of sheer calumny. We do not hesitate to

say that no system of philosophy is so diametrically opposed to Peripateticism as Hegelianism.

In the first place, it is extremely interesting and significant to note that in his commentary on the opening passages of the Physics which we have been trying to analyse, St. Thomas explicitly excludes the interpretation of Aristotle which has become current among modern historians and philosophers of science. This interpretation had already been proposed as far back as the time of Averroes. According to Averroes, when Aristotle speaks of the movement from generalities to particularities he has in mind a process of deduction or demonstration whereby the latter are drawn from the former, in which they are already precontained as parts in a composite whole. St. Thomas' refutation of this interpretation is precise and to the point:

Sciendum autem quod Commentator aliter exponit. Dicit enim quod ibi, Innata autem est etc., vult ostendere Philosophus modum demonstrationis huius scientiae, quia scilicet demonstrat per effectus et posteriora secundum naturam: ut sic quod ibi dicitur, intelligatur de processu in demonstrando, et non in determinando. Cum autem dicit, Sunt autem nobis etc., intendit manifestare, secundum eum, quae sunt magis nota quoad nos et minus nota secundum naturam, scilicet composita simplicibus, intelligens composita per confusa. Ultimo autem concludit quod procedendum est ab univesalioribus ad minus universalia, quasi quoddam corollarium. Unde patet quod eius expositio non est conveniens,

quia non coniungit totum ad unam intentionem; et quia hic non intendit Philosophus ostendere modum demonstrationis huius scientiae, hoc enim faciet in secundo libro secundum ordinem determinandi; iterum quia confusa non debent exponi composita, sed indistincta; non enim posset concludi aliquid ex universalibus, cum gēnera non componantur ex speciebus. (16)

The last lines of this passage which we have italicized are extremely important. They show that for St. Thomas absolutely nothing can be deduced from the generalities with which the study of nature begins. But in order to come to understand this point as clearly as possible, it is necessary to analyze the nature of the universality that is found in the first part of natural doctrine.

(17)

According to St. Thomas there are two kinds of universality - - universality by predication and universality by causality. As the name implies, universality by predication arises from the possibility which a universal notion has of being predicated of a number of inferiors. It consists, therefore, in pure generality, and as a consequence, the greater universality of this type a notion possesses, the emptier, the more confused, the more indetermined it is. Because of this indetermination, notions and principles which have mere universality of predication cannot be sources of deduction; their emptiness renders

them barren. Universality of causality, on the other hand, arises from the capacity of producing a number of effects. Increase in universality of this kind means an increase in richness and fullness of being; it means an increase in fecundity, since the effects actually derive from the principle which possesses this universality as from a source.

The notion which possesses the greatest universality of predication is obviously the general and confused notion of being. On the other hand, the principle which possesses the greatest universality of causality is the Subsistent Being, or God. That is why no greater error could be made than to confuse these two kinds of universality. And in this connection Professor DeKoninck writes:

Il me semble que l'idéalisme de Hegel est la philosophie la plus universellement opposée à la nôtre. Cet idéalisme nous est plus distant que le matérialisme; il est, à parler absolument, plus matérialiste que le matérialisme; il accorde, en effet, au premier connu, à l'être prédicat le plus universel, le plus confus, le plus indéterminé, le plus pauvre, le plus inévident en soi, la place qui, dans notre philosophie, revient à Dieu. La position de Hegel est dès lors inférieure, même à celle de David de Dinant, 'qui stultissime posuit Deum esse materiam primam'. (Ia, q. 3, a. 8, c.) Car son principe en soi premier a plus raison de matière que la matière physique. (18)

Now the generalities with which the study of nature begins possess only universality of predication.

From this point of view they are the emptiest, the most indetermined, the most confused, the most superficial of all the truths that can be learned about the cosmos. That is why they cannot be sources of deduction.

There are some scientific first principles which have not only universality of predication, but at the same time something which may be compared with universality of causality. These are found in mathematics, and that is why from a few primary axioms and postulates a whole geometry can be rigorously deduced. There is a world of difference between the principles from which mathematics takes its start and the generalities which constitute the beginning of the science of nature. Mathematics can progress by sheer deduction; the science of nature cannot. Yet deduction is something for which the mind instinctively reaches out, since through it man can become prior to things and in some sense the cause of them. And that is one of the reasons why it is inevitable for the science of nature to be subalternated to mathematics so that nature may be transformed to some extent at least into a deductive system.

But for the moment we are interested only in the way in which the study of nature advances from generalities

to fuller concretion. Enough has been said to show that this cannot be accomplished by means of deduction. That leaves us with only one alternative: experience and induction. It is important to come to see that the potentiality native to the intellect not only demands that we begin with generalities, but also that in attempting to escape from these generalities we take every step in complete dependence upon the data of experience. And thus we are brought to a consideration of the part that induction and experience play in the Thomistic philosophy of science. This consideration will serve to clear up not only the historical misunderstanding mentioned above, but also another misunderstanding closely associated with it: the often reiterated accusation that the generalities with which Aristotle and St. Thomas proposed to begin the study of nature were nothing but abortive and ill-founded hypotheses. (19)

2. Thomism and Experience.

We know of no better way of introducing this question than by quoting a text of Aristotle which the historians of science have consistently ignored:

Of things constituted by nature some are ungenerated, imperishable, and eternal, while others are subject to generation and decay. The former are excellent beyond compare and divine, but less accessible to knowledge. The evidence that might throw light on them, and on the problems which we long to solve respecting them, is furnished but scantily by sensation; whereas respecting perishable plants and animals we have abundant information, living as we do in their midst, and ample data may be collected concerning all their various kinds, if only we are willing to take sufficient pains . . . Having already treated of the celestial world, as far as our conjectures could reach, we proceed to treat of animals, without omitting, to the best of our ability, any member of the kingdom, however ignoble. For if some have no graces to charm the sense, yet even these, by disclosing to intellectual perception the artistic spirit that designed them, give immense pleasure to all who can trace links of causation, and are inclined to philosophy. Indeed it would be strange if mimic representations of them were attractive, because they disclose the mimetic skill of the painter or sculptor, and the original realities themselves were not more interesting, to those at any rate who have eyes to discern the reasons that determined their formation. We therefore must not recoil with childish aversion from the examination of the humbler animals. Every realm of nature is marvellous: and as Heraclitus, when the strangers who came to visit him found him warming himself at the furnace in the kitchen and hesitated to go in, is reported to have bidden them not to be afraid to enter, as even in that kitchen divinities were present, so we should venture on the study of every kind of animal without distaste; for each and all will reveal to us something natural and something beautiful. . . If any person thinks the examination of the rest of the animal kingdom an unworthy task, he must hold in like dis-esteem the study of man. For no one can look at the primordia of the human frame - - blood, flesh, bones, vessels, and the like - - without much repugnance. (20)

We feel that this text brings into clear light the spirit of research and the respect for concrete facts which animated Aristotle's study of nature. Nor must it be looked upon as an exceptional and isolated passage that demands some ingenuity in order to be reconciled with the actual practice and the epistemological principles of the Stagirite. For other texts of like character could easily be adduced, as for example the one found in the first book of De Generation et Corruptione, where he points out that the main obstacle to the study of nature is insufficiency of experience and that only those who live in great intimacy with natural phenomena can succeed in such a study. (21) As far as actual practice is concerned, one has only to read the natural treatises that are far advanced in the direction of concretion, as for example, the Historia Animalium and the De Partibus Animalium, to see to what extremes he pushed the experimental method. It is said that Alexander the Great had thousands of men engaged in research in every part of the world that was then known in order to assist Aristotle in the writing of his (22) Historia Animalium. It is true that most of this experimental research is restricted to the field of biology, but sufficient reasons have already been brought forward in

Chapter I to explain why this is so.

But the most important point in this discussion is to show that this experimental method follows logically and inevitably from Peripatetic epistemological principles. And in order to do this we must return to what we saw in Chapter II about the intrinsic nature of physical science.

In discussing the distinction of the sciences we explained that natural doctrine differs from all the other sciences by the fact that it does not abstract from sensible matter, and that as a consequence all of its definitions must be formulated in terms of sensible matter. Propositions which prescind from sensible matter can have nothing more than a dialectical meaning in physics. We pointed out that St. Thomas drew from this the principle that unlike mathematics and metaphysics, physics must not only begin in sense experience, it must also terminate in it. Scientific conclusions have no meaning in natural doctrine unless they are verifiable in sense experience. And that is why Aquinas could write: "qui sensum negligit in naturalibus incidit in errorem. Et haec sunt naturalia quae sunt concreta cum materia sensibili. (23) It is only experience that can provide us with natural definitions.

All this evidently ties up with the Peripatetic doctrine of hylemorphism. Natural forms, which are the object of natural science, have their very being "in concretione ad materiam." And this refers not merely to their existence, but to their very essence. It is extremely important to keep in mind that a material form is not a quiddity. It is not knowable in itself and by itself independently of matter -- just as matter is not knowable independently of form. Even God does not know material forms except in relation to matter, since independently of matter a natural form is nothing. As a consequence, the perfection of our knowledge of these forms depends upon the intimacy of our contact with sensible matter. And that is why every true Thomist will unhesitatingly subscribe to the principle formulated by Eddington: "Every item of physical knowledge must therefore be an assertion of what has been or would be the result of carrying out a specified observational procedure."⁽²⁴⁾

There are many reasons why the whole study of nature is completely dependent upon experience, but in some respects the most profound reason is the one hinted at by Aristotle in the passage quoted above from the De Partibus Animalium: the material universe is a work

of art. And it is impossible to understand the role played by experience in the Thomistic philosophy of science except by coming to see the precise way in which art enters into the structure of the cosmos.

Towards the end of the long analysis of the meaning of nature carried on in the second book of the Physics, St. Thomas arrives at his well-known definition: "Natura nihil est aliud quam ratio cuiusdam artis scilicet divinae, indita rebus, qua ipsae res moventur ad finem
(25)
determinatum." A nature is something essentially rational; it is a divine logos. And this applies even to the purely material principle out of which cosmic
(26)
reality is constructed. The whole purpose of the study of nature is to come to know these divine logoi in their ultimate specific concretion.

Now at first glance, all this may seem to add up to an argument against complete dependence upon experience rather than one for it. For to say that the cosmos is constructed out of divine logoi might seem to indicate that it is a perfectly logical and perfectly rational system, and that it therefore lends itself more to deduction than to induction. As a consequence Meyerson

might seem to be justified in writing: "La science d'Aristote était non pas une physique, mais une logique . . . Mais il est clair que, pour le maître du péripatétisme, aussi bien que pour ses sectateurs de l'antiquité et du moyen âge, les deux se confondent puisque la nature ne peut être que logique." ⁽²⁷⁾ Moreover, the immaterial universe is also a work of divine art, and yet the science which deals with it is not completely dependent upon experience.

As a matter of fact, however, there is a vast difference between the art which has formed the immaterial universe and that which has formed the material universe. For in the cosmos there is a plasticity and a malleability that is utterly foreign to a universe that is free of matter. And it is in this plasticity and malleability that the complete dependence on experience has its root.

Immaterial forms are fashioned by divine art, but only with respect to their existence. This does not mean that their essence is in no way formed by God; it merely means that this formation consists only in bringing the form into existence. Because of their simplicity, immaterial forms have no plasticity intrinsic to their very essence, and consequently within this realm of essence the art that

produces them cannot compose. Material forms, on the other hand, are fashioned by divine art, not only with respect to their existence, but also with respect to their essence. The very fact that they are not pure forms, that in their very essence there is a principle of indetermination that is susceptible of an infinite variety of determinations, gives them an intrinsic malleability that leaves free scope for composition. This principle of indetermination, this source of plasticity, is obviously prime matter, which is in potency to all forms. And all this brings us back to something we saw in Chapter II in connection with the similarity between the study of nature and practical knowledge: as we descend the hierarchy of beings the operabilitas of things increases.

But perhaps we can give clearer outline to this point by having recourse to a rather crude illustration, drawn from the realm of mathematics. Between any two given numbers in the series of integral numbers there is only a finite multiplicity of numbers. And the numbers in this multiplicity are already predetermined. In order to actualize them a simple process of designation is sufficient. But between any two points in a continuum there is an infinity of points, and these points are not

predetermined. In order to actualize a certain magnitude a simple process of election is not sufficient. There is required a previous process of determination by which the magnitude in question is carved out, so to speak, of the potentiality of the continuum.

In somewhat the same way, we may say that between any two given angelic species in the hierarchy of the separated substances only a finite number of species is possible. This is not a limitation of God's power to imitate His essence in immaterial forms since just as there is no superior limit to the series of integral numbers so there is no superior limit to the hierarchy of separated substances which God can create. But between any two given material species, no matter how close they may be to each other, an infinite number of other species is possible. Immaterial forms, like integral numbers, are predetermined; their actualization consists in a simple process of election by which existence is given to them. But material forms are not predetermined; if they were, prime matter would not be pure potentiality -- there would be a latitatio formarum. That is why previous to the process of election by which existence is given to them there must be a process of composition by which their very essence is formed. In other

words, the production of immaterial forms merely consists in giving existence to essences already ^{de}pretermined in the divine exemplary ideas; there is no composition in these exemplary ideas themselves. But in the case of material beings there is composition in the very exemplary ideas according to which they are produced.

In the mathematical world nothing is formed in the true sense of the word; nothing depends upon art in the sense of depending upon free determination, for in mathematics all things are analytical. And if mathematics is called an art, it is only on the sense of its being a speculative art, like logic. In the metaphysical world there is formation by art in the sense of dependence upon free determination, but only with respect to existence. But in the physical world there is formation both in the realm of existence and of essence. The material universe is essentially plastic.

That is why there is no way of arriving at a more profound view of the cosmos than by seeing it as a work of art. In spite of his tendency to look upon the universe as essentially mathematical, Sir James Jeans touched upon this truth when he wrote: "To my mind, the

laws which nature obeys are less suggestive of those which a machine obeys in its motion than those which a musician obeys in writing a fugue, or a poet in composing a sonnet." (28)

But in order to understand just how completely and essentially the cosmos is a work of art it is necessary to recall that because of its transcendental freedom, divine art is not tied down to the vias determinatas that are characteristic of human art. In this respect divine art is similar to prudence which proceeds per vias determinandas. Divine art can dominate contingency in a way that completely transcends human art; it can order it with infinite finesse. In fact, divine art shines nowhere with greater brilliance than in the realm of indeterminism and chance. And in the Thomistic view of things, the physical universe is essentially immersed in contingency, simply because it is essentially material. That is why the divine logos that is found everywhere in the cosmos is not the perfectly analytical rationality that is found in the mathematical world, nor the type of rationality that is found in the metaphysical world. It is essentially an artistic logos -- ratio artis divinae -- which orders contingency without destroying it. And a greater calumny could hardly be levelled against the Thomistic view of the cosmos than to

say that in it physics and logic coincide since the universe is a perfectly logical system. One has only to read the remarkable passages written by Aristotle, St. Thomas and Cajetan on the part that contingency and chance play in the universe to appreciate the falsity of this charge. The Peripatetic and the Spinozistic (29) universes are completely antipodean.

All this helps us to understand the part that experience plays in natural science. For as we saw in Chapter II, in the study of nature we stand before the universe as before a work of art. There is no way of telling a priori what an artist is going to do. One has to wait to see ~~what~~ he actually accomplishes. Nor is it possible to deduce from the first general outlines the particular details that will eventually enrich the composition. The only way in which a priori knowledge may be had of a work of art is for the artist to reveal what he intends to do. Something of this nature has actually occurred in the case of the angels, into whose intellects God infused the intelligible species of all the things which were to come from His creative art. But for us whose knowledge is posterior to things, the only way in which we can get to know nature is by ex-

perience. It is true that given the subject of a certain work of art some vague generalities may immediately be known about it. Given, for example, the fact that an architect is going to build a house, there are some general things common to all structures which serve as shelters that we can immediately know about it. These do not depend upon the free disposition of the artist. But as soon as we wish to come down to particularities we become dependent upon the free will of the artist. For there is an infinity of ways of making houses. In somewhat the same way, given the idea of a material universe, there are some things that we can immediately know about it. We can know for example that man must exist in it, since man is the raison d'être of the whole universe. But there is an infinity of ways in which the material universe in its evolution may prepare for the final production of man. From the beginning the cosmos has been in a continual process of formation and artistic composition. That is why there is a great deal of truth in Plato's idea of the demiurge which constantly works the world. And the only way to discover the actual line of species that has led up to man is by natural history, as St. Augustine has pointed out. This brings us back to the profound significance of the "erit" in the passage

of Aristotle quoted in connection with the question of the relation between physics and practical knowledge. Natural things are not knowable except in the order of existence. The only way to get to know them is by knowing them as existing, that is to say by experience. As we remarked in Chapter II, the study of nature, because of its likeness to practical knowledge, must be built up out of bits garnered from experience. This constitutes a great difference between the science of nature and the other sciences.

There is, then, great wisdom in Aristotle's remark that it is noble to soil one's hands in experiments because by so doing one gets to know the art of Him who made all things. There is all the difference in the world between a "naturalist" and a peripatetic. The former merely delves deeper and deeper into the obscurity of matter. His knowledge is something like the cognitio nocturna of the fallen angels, because it is not referred to God. But whereas the end of his study is night, the end of the study of the peripatetic is light -- the light of divine intelligence, for the deeper he delves into matter the closer he is coming to divine art, since he is getting into more intimate contact with things in their plasticity. The farther advanced science gets towards concretion, the more it gets into the

realm where divine art composes more than anywhere else.

That is why every true Thomist has a profound respect for experience. For it takes the place of the infusion of the angelic species; it gives a share in the scientia visionis of God. And the farther advanced the student of nature gets in experience, the more his knowledge becomes like that of the angels which depends directly upon the divine species - - the more he participates in the scientia visionis of God. And in this connection it is interesting to note that if the term of this increase in experience could be realized, if the ultimate concretion could be reached, there would be a complete destruction of experience, for there would be perfect a priori knowledge. This is just one instance of a very significant truth which we shall examine in some detail in Chapter XI, namely that if the term of the tendency of experimental science could be reached there would be a contradiction. "L'esprit humain est absurde par ce qu'il cherche; il est grand par ce qu'il trouve."
(31)

The conclusion that this discussion imposes upon us is that every part of the study of nature is dependent upon experience, but not in the same degree. The gener-

alities with which this study begins are not a priori hypotheses, as so many critics of Peripateticism are inclined to think. They are truths that are drawn from experience. But precisely because they are so general and superficial, and because they are the truths that are the most proportionate to our minds, they do not demand a great deal of experience; it is comparatively easy for the mind to disengage them from the world of sense. In order to arrive at the general nature of motion, for example, one simple experience with any kind of motion, such as the fall of a leaf, the movement of a finger, or a change of color in the sky is sufficient, for everything that can be known about the general nature of motion is contained perfectly and completely in any one of these examples. But in order to get at the nature of the special type of mobility that is proper to a particular natural species it is necessary to have recourse to long and complex experimental research. In other words, as we advance towards concretion, the dependence of the mind upon experience increases. And it is perhaps the relative simplicity of the experience that is required for the generalities which mark the beginning of the study of nature, and the comparative ease with which the mind disengages them that have led to the erroneous opinion that they are

nothing but abortive, hastily formed and ill-founded generalizations.
(32)

But perhaps at this point one might be tempted to object: did not Aristotle frequently have recourse to hypotheses that were not fully founded in reality? Assuredly - and so has every other scientist worthy of the name who has really understood the nature of science, from Thales to Einstein. And this applies even to Newton, in spite of his well-known dictum: hypotheses non fingo. Newton merely failed to grasp the full significance of the method which he put to such good advantage. Hypotheses, as we shall bring out presently, are of the very essence of the study of nature. And to admit that Aristotle had recourse to them is simply to say that while on the one hand he had no part with the apriorism of Descartes who spurned sense experience and wished to deduce more geometrico even such specific elements in nature as "the heavens, the stars, the earth, and on the earth: water, iron and minerals," (33) on the other hand he was far from falling into the naive empiricism of Francis Bacon. Although both Descartes and Bacon are counted among the principal founders of modern science, it is certain that modern science has sprung neither from the rejection of experience of the one, nor the

rejection of hypothesis of the other, but from a union of experience and hypothesis, such as is found in the doctrine of Aristotle.

But were not the hypotheses of Aristotle hastily formed? The answer is yes and no. For in a sense all good scientific hypotheses are hastily formed. Of their very nature they must anticipate reality; they must reach beyond the actual deliverances of experience. From this point of view a scientist who is too cautious is a poor scientist. It is true that as we look back now from the vantage point of many centuries of scientific progress some of the hypotheses of Aristotle look extremely precipitant. But, as we suggested in Chapter I, is it so certain that when as many centuries of progress have passed over the hypotheses of Einstein they will not appear just as precipitant as the Aristotelian hypotheses look to us today? The following well-known passage of Poincaré is extremely relevant here:

Chaque siècle se moquait du précédent, l'accusation d'avoir généralisé trop vite et trop naïvement. Descartes avait pitié des Ioniens; Descartes à son tour nous fait sourire; sans aucun doute nos fils riront de nous quelque jour. Mais alors ne pouvons-nous aller tout de suite jusqu'au bout? N'est-ce pas le moyen d'échapper à ces railleries que nous prévoyons? Ne pouvons-nous nous contenter de l'expérience toute nue?

Non, cela est impossible; ce serait méconnaître complètement le véritable caractère de la science. Le savant doit ordonner; on fait la science avec des faits comme une maison avec des pierres; mais une accumulation de faits n'est pas plus une science qu'un tas de pierres n'est une maison."(34)

In connection with this question of hypothesis one often encounters the charge that the Peripatetics were notoriously guilty of arbitrarily and artificially forcing facts to fit into preconceived theoretical frames. We do not believe that this charge is justified. For, in the first place, it is something that was explicitly and strenuously combatted by Aristotle. In the second book of the De Coelo, for example, he writes:

In fact their (the Pythagoreans') explanation of the observations is not consistent with the observations. And the reason is that their ultimate principles are wrongly assumed: they had certain predetermined views, and were resolved to bring everything into line with them.. But they, owing to their love for their principles, fall into the attitude of men who undertake the defence of a position in argument. In the confidence that the principles are true they are ready to accept any consequence of their application. As though some principles did not require to be judged from their results, and particularly from their final issue. And that issue, which in the case of productive knowledge is the product, in the knowledge of nature is the unimpeachable evidence of the senses as to each fact. (35)

Moreover, a number of cases could be cited in

which the great respect they had for sense experience led them to formulate points of doctrine that could only with some difficulty be harmonized with their fundamental principles. An example which immediately suggests itself is that of the doctrine of incorruptible matter. Because sense experience revealed no other changes in the heavenly bodies except local motion, they were led to the doctrine that these bodies were intrinsically incorruptible, and that consequently the prime matter which entered into their composition was different from that found in terrestrial bodies. We do not say that it is impossible to reconcile this with the pure indetermination of prime matter. In fact even today, after science has shown that the celestial bodies are susceptible of the same intrinsic changes as terrestrial bodies, and made up of the same stuff, we do not think it possible to prove apodictically that incorruptible matter cannot exist somewhere in the cosmos. Yet this reconciliation demands considerable ingenuity, and if the peripatetics had had less respect for sense experience it would have been a good deal easier to arrive a priori at the conclusion that the celestial bodies were capable of intrinsic mutations.

Another example of this kind is found in the

doctrine of spontaneous generation. This doctrine was formulated because sense experience revealed the generation of living beings out of putrefying matter, and at the time there were no adequate means for detecting the fact that eggs had previously been laid in the decaying mass. Here again we have a doctrine which was adopted in order to save sense experience even though it could only with considerable difficulty be reconciled with the basic principle of the essential difference between living and non living matter.

One of the most common objections brought to bear against peripatetics is that they failed to recognize the hypothetical character of their hypotheses, that they consistently mistook them for certain principles. In order to assess the justice of this charge we must consider a few texts. Speaking of the theory of the incorruptibility of the matter of celestial bodies, Aristotle remarks:

The mere evidence of the senses is enough to convince us of this, at least with human certainty. For in the whole range of time past, so far as our inherited records reach, no change appears to have taken place either in the whole scheme of the outermost heaven or in any of its proper parts.(36)

Commenting on this text, St. Thomas has the following to say:

Secundum signum ponit ibi; Accidit autem hoc et per signum etc.: quod quidem accipitur ab experientia longi temporis. Et dicit quod id quod probatum est per rationem et per communem opinionem, accidit, idest consequitur sufficienter; non quidem simpliciter, sed sicut potest dici per comparationem ad humanam fidem, idest quantum homines possunt testificare de his quae parvo tempore et a remotis viderunt . . . Nec tamen hoc est necessarium, sed probabile. Quanto enim aliquid est diuturnius, tanto maius tempus requiritur ad hoc quod eius mutatio deprehendatur; sicut transmutatio hominis non deprehenditur in duobus vel tribus annis, in quibus deprehenditur transmutatio canis, vel alicuius alterius animalis brevioram vitam habentis. Posset igitur aliquis dicere quod, etsi caelum sit naturaliter corruptibile, est tamen tam diuturnum quod totum tempus cuius memoria potest haberi non sufficiet ad deprehendendam eius transmutationem. (37)

In the second book of the same work, Aristotle writes:

Duabus autem dubitationibus entibus, de quibus merito utique quilibet dubitabit, tentandum dicere quod videtur; dignum esse reputantes promptitudinem magis imputari verecundiae quam audaciae, si quis, propter philosophiam stare, et parvas sufficientias diligit, de quibus maximas habemus dubitationes. (38)

St. Thomas' commentary on this passage is extremely enlightening:

Dicit ergo primo quod, cum circa stellas sint duae dubitationes de quibus rationabiliter quilibet potest dubitare, tentare debemus dicere circa istas dubitationes id quod nobis videtur; ita scilicet quod nos reputemus dignum esse quod promptitudo hominis considerantis huiusmodi quaestiones magis debeat imputari verecundiae, idest honestati vel modestiae, quam audaciae, idest praesumptioni; si tamen ille qui huiusmodi dubitationes considerat, diligit etiam parvas sufficientias, i.e. parum sufficientes rationes, ad inveniendum de illis

rebus, de quibus habemus maximas dubitationes; et hoc propter desiderium quod quis habet ad philosophiam, ut scilicet eius principia stent, idest firma permaneant...

Illorum (Eudoxi, Aristotelis, et Ptolemai) tamen suppositiones quas adinvenerunt, non est necessarium esse veras: licet enim, talibus suppositionibus factis, apparentia salvarentur, non tamen oportet dicere has suppositiones esse veras; quia forte secundum aliquem alium modum, nondum ab hominibus comprehensum, apparentia circa stellas salvantur. Aristoteles, tamen, utitur huiusmodi suppositionibus quantum ad qualitatem motuum, tamquam veris.(39)

Another very significant text is found in the Summa:

Dicendum quod ad aliquam rem dupliciter inducitur ratio. Uno modo ad probandum sufficienter aliquam radicem; sicut in scientia naturali inducitur ratio sufficiens ad probandum quod motus coeli semper sit uniformis velocitatis. Alio modo inducitur ratio, quae non sufficienter probet radicem, sed quae radici iam positae ostendat congruere consequentes effectus; sicut in astrologia ponitur ratio excentricorum et eipycolorum ex hoc quod, hac positione facta, possunt salvari apparentia sensibilia circa motus coelestes: non tamen ratio haec est sufficienter probans, quia etiam forte alia positione facta salvari possent.(40)

We believe that these texts, which were completely ignored by historians until several of them were
(41)
brought to light by Pierre Duhem, establish beyond a doubt the fact that Aristotle and Saint Thomas were acquainted with the hypothetical method employed by modern science.

It would be interesting to examine each of them

in detail. But for our purpose a summary conclusion will suffice. We believe that they make it abundantly clear that the peripatetics had accurate knowledge of the hypothetical method that has become the very soul of modern science. The fact that in individual cases they may have erroneously believed that they had apodictic arguments in favour of certain propositions when such arguments did not exist, does not in any way invalidate this claim. As is evident from these texts, the position of Aristotle in this matter is less unambiguous than that of St. Thomas. But there is ample reason for believing that even the former had great diffidence about the truth of the theories he proposed, that he attributed to them the certitude that is necessary for working hypotheses, that he posited them as if they were true in order to save the phenomena. But whatever may be thought about the position of Aristotle, there can be no doubt about that of Aquinas. In the passages just cited from him there is an accurate description of the hypothetical method used in modern science.

It is not without interest to note that the theories to which St. Thomas attributed only probability were precisely those upon which rested the whole doctrine of the structure of the heavenly spheres, which has seemed

so utterly naive to modern critics. What these modern critics fail to realize is that this doctrine saved the phenomena that were known at that time just as successfully as the theories of classical physics saved the phenomena that were known during the seventeenth and eighteenth centuries - - just as successfully as the theories of Einstein save the phenomena that are known today. It is extremely significant that nowhere do we find in the writing of those who are credited with being the founding fathers of modern science, such as Copernicus, Kepler and Galileo, anything that comes so close to a description of the true method of science as that found in the writings of Aquinas. It is true that Copernicus in his Commentariolus de Hypothesibus Motuum Caelestium seems to posit his fundamental principles as mere postulates: "si nobis aliquae petitiones . . . concedentur." But later in his De Revolutionibus Caelestibus Libri Sex his attitude is far less reserved. In his introduction to this latter book, Osiander brought out with great accuracy the true scientific method: "Neque enim necesse est eas hypotheses esse veras, imo, ne verisimiles quidem; sed sufficit hoc unum, si calculum observationibus congruentem exhibeant." But Kepler would have no part with such a doctrine: "Je n'hesite pas

a déclarer que tout ce que Copernic a amassé a posteriori et prouvé par l'observation, tout cela pourrait, sans nulle entrave, être démontré a priori, au moyen d'axiomes géométriques, au point de ravir le témoignage d'Aristote, s'il vivait." (42) Galileo distinguished between the point of view of astronomy in which the hypotheses have no other sanction except conformity with experience, and that of philosophy of nature which bears upon the objective nature of things. (43) But if we are to believe Duhem this was a purely theoretical distinction formulated to avoid the censures of ecclesiastical authority, and Galileo accorded full certitude to all of his theories. In any case there can be no doubt that throughout the reign of classical physics full certitude was universally attributed to doctrines which were in reality only hypothetical. And if today the hypothetical character of sciences has become generally recognized, it is undoubtedly due in large measure to the rude awakening occasioned by the downfall of Newtonian physics. St. Thomas did not need such an awakening. In spite of the fact that the physical theories he held saved the phenomena known at the time as successfully as modern theories save the phenomena known now, he was sagacious enough to recognize their hypothetical character.

But even more important than the consideration of the texts cited above is the consideration of the certitude that the propositions of experimental science enjoy de jure in the Peripatetic philosophy of science. And this requires an analysis of the relation between certitude and experience in the study of nature. Before embarking upon this analysis, however, at least passing attention must be paid to one last objection that is frequently proposed against the position we have been maintaining with regard to the importance of the role of experience in the Thomistic philosophy of science. It is this: if according to Thomism experience plays such an indispensable role in the study of nature, and particularly in that part of it which is to some degree advanced in the direction of concretion, why is it that St. Thomas and the medieval schoolmen were so notoriously remiss in the actual practice of experimentation. We do not hesitate to grant the premises upon which this objection is based. Aristotle was, as we have already pointed out, a great experimenter. But St. Thomas and the medievalists, with a few notable exceptions, such as St. Albert the Great, were not. There was, however, a reason for this. The medievalists were primarily theologians. This does not mean that there were not at the same time great philosophers, nor that theology

dictated to their philosophy in the manner usually described by historians. It merely means that their interest in philosophy was concentrated chiefly upon the problems that had a bearing upon theology and upon the problems that had the greatest significance for human life. They were moreover primarily interested in science in the full and perfect sense of the word, that is to say, science in which there is certitude, and as we shall see in a few moments, experimental science does not give true certitude.

Whatever may have been the actual practice of St. Thomas and his followers, the only important point is that in principle according to the Thomistic philosophy of science, the student of nature must, if he is to realize his purpose, be carried constantly forward toward fuller concretion, and this advance demands an ever increasing dependence upon experience. Here we run across a remarkably striking paradox. Auguste Comte, the father of Positivism, denied the necessity and validity of extended experimentation. He rejected, for example, what he called the abuse of ex-
(44)
tended microscopic research. Nowhere do we find anything of this sort in the doctrine of Aristotle or St. Thomas, which, if we are to believe critics, was so thoroughly anti-positivistic. On the contrary, the very principles of this

doctrine demand unceasing experimentation and recourse to the most refined instruments of research available. It may readily be admitted that neither Aristotle nor St. Thomas ever anticipated the perfectibility of our means of observation and experimentation that modern progress has revealed, and that as a consequence some of the positions assumed by them were far more provisory than they suspected. But the fact remains that their conception of natural science demands a conformity with observation which must constantly increase both in breadth and in depth.

3. Experience and Certitude.

Let us begin our analysis of this problem by considering the following text of Aristotle:

The science which is knowledge at once of the fact and of the reasoned fact, not of the fact by itself without the reasoned fact, is the more exact and the prior science. A science such as arithmetic, which is not a science of properties qua inhering in a substratum, is more exact than and prior to a science like harmonics, which is a science of properties inhering in a substratum; and similarly a science like arithmetic, which is constituted of fewer basic elements, is more exact than and prior to geometry, which requires additional elements. What I mean by 'additional elements' is this: a unit is substance without position, while a point is substance with position; the latter contains an additional element. (45)

In this passage Aristotle brings out the three basic principles which determine the relative certitude found in the sciences. Although in writing this passage he did not have explicitly in mind the point which is of interest to us here, we may apply these principles to our purpose, which is to show that in the measure in which the study of nature becomes increasingly dependent upon experience, its certitude decreases.

The first principle laid down by Aristotle is this: a science which not only gives us facts (the quia) but also the reasons for the facts (the propter quid) is more certain than a science which provides only the facts without the reason for them. Now as increasing experience carries us forward towards fuller concretion, the abundance of facts continually grows, but at the same time it becomes constantly more difficult to disengage the propter quid to explain these facts. And the reason for this is fairly evident: the more we advance, the more we approach things under the aspect in which they depend completely upon the practical knowledge of God, and scientia visionis, which involves something that is outside the realm of knowledge, namely the divine free will. (46) It is precisely because it eventually becomes impossible to discover a

proper propter quid in the parts of natural doctrine that are advanced towards concretion that it becomes necessary to reach up to mathematics to find a substitute propter quid through a process of subalternation. That is another way of saying that as we emerge from the part of the study of nature that is most conformable to our minds it ~~becomes~~ necessary to substitute the science that of all the sciences is most in harmony with the human intellect. (47)

The second principle of Aristotle consists in this that a science which deals with a subject is less certain than a science which does not. In his commentary on this passage, St. Thomas explains what Aristotle means by the term "subject": "Et accipitur hic subjectum pro materia sensibili;... incertitudo causatur propter transmutabilitatem materiae sensibilis; unde quanto magis acceditur ad eam, tanto scientia est minus certa." (48) Now just as a science which deals with sensible matter is less certain than one that does not, so that part of the study of nature which experience has carried deeply into concretion is less certain than that part which is not so completely immersed in concrete sensible matter.

In his third principle Aristotle states that a

science which has to do with fewer elements is more certain than one in which the elements are more numerous. This has a direct application to our problem. For increasing experience carries the study of nature forward from generality to greater specificity, in such a way that the proper distinctions of things gradually emerge. This is why the farther the study advances the greater becomes the need for more particular and consequently more numerous principles. For the proper differences of the natural species cannot be deduced from each other, as we have already pointed out. Hence the necessity of as many principles as there are natural species to be known. It may be said that the number of principles in experimental science tends towards infinity. Each natural species is a primary datum and the source of a number of original propositions. And the multitude of possible natural species is infinite. It is true that the theories of evolution will attempt to reduce this great variety to a basic unity, but these theories presuppose experience with the original variety and must succeed in leading back to it.

From all this it follows that there is an inverse proportion between the dependence of natural science upon experience and the degree of certitude that is possible

in it. That is why the prudent student of nature will commit himself less categorically and with greater reserve and with more abundant qualifications the more he advances towards concretion. As Aristotle points out, "since the truth seems to be like the proverbial door, which no one can fail to hit, in this respect it must be easy, but the fact that we can have a whole truth and not the particular part we aim at shows the difficulty of it." (49) And it is for this reason that the universal propositions advanced in the more concrete parts of natural doctrine do not enjoy true certitude. Nor is it any cause for wonder that in a science which deals with mobile being, certitude so quickly fades into mere probability. (50) But it is necessary to try to analyze this question more accurately. In the general propositions which the mind first disengages from its experience with cosmic reality, perfect certitude is possible, for in such propositions an analytical relation exists between subject and predicate, For example, there is an analytical relation between substantial mobility ~~and substantial mobility~~ and substantial composition of matter and form. In propositions of this kind the mind not only grasps the quia, but also the propter quid. That is why the parts of natural doctrine which are made up substanti-

ally of propositions of this kind, i.e. the Physics and the De Anima constitute true scientific knowledge in the strict sense of the word. In this case there is direct correspondence between the clarity which these propositions have for us and their certitude, in contrast to what is found in theology whose principles though extremely obscure for us have greater certitude than principles which have greater clarity for us.

But as natural science advances towards concretion and dependence upon experience increases, analytical relations become less and less apparent. Propositions become more and more experimental. There ultimately comes a point (and it is very quickly reached) at which the propositions are purely experimental, that is to say, they merely formulate what experience presents to the senses. From that point forward no true scientific knowledge in the strict sense of the word is possible. The propositions give only the quia and not the propter quid. In other words they are not analytic, but purely synthetic; It is true, as we shall try to bring out presently, that the mind will not rest satisfied with this pure synthesis. It will try to triumph over it by the projection of its own subjective logos by the creation of a "propter quid", in such a way

that in a sense it will be able to arrive at synthetic a priori judgments. But in the last analysis the propositions remain synthetic and never become analytic. At this juncture we have arrived at the frontiers of philosophy and experimental science.

John of St. Thomas has brought out this point with considerable precision:

Non est idem propositio per se nota quod intuitiva sive per experientiam sensuum nota, quia quod sensu cognoscitur, non cognoscitur ut propositio, sed ut simplex objectum apprehensum, neque ex sola explicatione terminorum innotescit, sed quia experientia externa attingitur. Et sic nivem esse albam, licet in sensu sit per experientiam notum, in intellectu tamen non est propositio nota ex terminis per se connexis, sed potius in materia contingenti. (51)

Even though all experience that has ever been had with snow has presented it as white, this experience does not prove that it is contradictory for snow not to be white. It remains possible, of course, that there is some incompatibility between the essence of snow and any other color, and further experience will render this possibility increasingly probable. But of itself experience will never transform this probability into certitude. Nor does it do any good to have recourse to the principle that what happens ut in pluribus comes from nature. For though this principle

is unquestionably valid, it does not settle the problem about what nature is involved. In other words, the regularity of the whiteness of snow is obviously a sign that it is coming from nature. But is it coming from the nature of the snow? Perhaps it derives from some atmospheric condition or complexity of conditions proper to our planet. There are so many natures involved in even such a relatively simple process as the production of snow that it remains impossible to trace the regularity back to its source. It becomes apparent, then, that the proposition "snow is white" is not universal and necessary at the same time. In so far as it is proposed as necessary, it is not universal, but restricted to the snow that has been met thus far in experience. In so far as it is proposed as universal it is not necessary. As a consequence, it cannot be a scientific proposition which must be both universal and necessary. Hence it is evident that the universalization that is effected in experimental science is purely functional. That is to say, when propositions are universalized without evidence, there must be a functional reason for doing so. In other words, when we act "as if" this does not mean essentially that in so doing we may be right, but rather that in so doing we may get somewhere.

It is clear, then, that the propositions of experimental science remain completely tied down to experience. It can never truly abstract from experience because experience is never complete. This means that they can never effectively rise above the realm of singularity. In this sense all experimental science is essentially nominalistic. That is why experimental science must ever remain in a state of becoming. And we mean by this something over and above the progress that is characteristic of all human science. We mean that the very genesis of the concepts employed in experimental science is never terminated. There must be a constant recourse on the part of the intellect to sense experience which is immersed in contingency and the flux of time. And this flux and contingency will ever remain refractory to complete abstraction. It will always be possible that further experience may change to a greater or less degree of concepts already formed, or at least the relations between them suggested by previous experience. That is why, as Professor DeKoninck has pointed out, ⁽⁵²⁾ history pertains to the very essence of experimental science, whereas the disciplines that are sciences in the strict sense of the word are only accidentally implicated in history. And in this connection it is interesting to

note that even if per impossibile the cosmos were the perfectly rational system that the historians have wished upon Peripateticism, it could never be known as such by the methods that are proper to experimental science. Its necessary structure would only be a dialectical limit which experimental science could constantly approach without reaching.

All this discussion about the part played by experience in the study of nature leads inevitably to the problem of induction over which logicians have labored so much, especially since the time of Hume. We believe that much of this labor has been futile because a few basic distinctions have been neglected. And perhaps the best way to embark upon this question is by citing the following significant text of John of St. Thomas:

Omnis nostra speculatio dependet ab inductione sicut dependet a sensu et experientia; unde si propositiones universales alicuius scientiæ non sunt ita abstractæ et communes quod ex quocumque individuo manifestari possit ipsarum veritas, sed ex plurium numeratione et experientia pendeat, sicut scientiæ naturales, non sunt ita certæ sicut aliæ scientiæ abstractiores et communiores, ut metaphysica et mathematicæ, quorum principia in uno individuo habent totam certitudinem ut; quodlibet est vel non est.(53)

When John of St. Thomas says that all of our

speculation depends upon induction just as it depends upon the senses and experience, he is evidently taking the term in a rather broad sense, in a sense in which it is coterminous with any deliverance of sense experience to the intellect. But under this generic notion it is possible to distinguish three types of induction. In the first place, induction may be understood to mean the abstraction of universal concepts from singular objects. Taken in this sense, it is found in all of the sciences and in all intellectual activity.

Secondly, it may signify the arrival at analytic propositions from sense experience, and here it must be noted that the term "analytic propositions" is not taken in the superficial sense in which it is understood by Kant. It means all propositions in which the predicate is for any reason necessarily (and therefore universally) connected with the subject. Since all sciences in the strict sense of the word must begin with necessary principles, and since all of our knowledge is drawn from sense experience, this type of induction is found in all of the disciplines which are truly sciences, that is to say in mathematics, in metaphysics, and in philosophy of nature. The way in which this induction takes place is not in every respect the same for all the sciences. Mathematics presents an especially particular

case about which much has been written in recent years. It is not to our purpose to embark upon this question here, and it is sufficient to point out that even mathematical principles, in spite of their intuitive and a priori character are originally drawn from sense experience, even though they are not found there in the state of abstraction and perfection that is characteristic of the mathematical world. (54) In metaphysics principles applicable to the whole range of being can be drawn from sense experience for they are realized in sensible being not because it is sensible, but because it is being. In philosophy of nature analytic principles governing mobile being are disengaged from experience, and unlike metaphysical propositions, are enunciated in terms of sensible matter. And in all of these cases the passing from the singularity and contingency of experience to the universality and necessity of analytic principles is not logically invalid, simply because the basis of the universality and necessity is not the fact that the subject and predicate are united in experience, but the fact that the mind can see that the predicate pertains to the very nature of the subject. For example, the principle that the whole is greater than any of its parts is drawn from experience in which concrete

wholes are presented as greater than concrete parts, but the universality and necessity of the principle is founded on the analytical nexus which the mind discovers between the subject and the predicate.

Perhaps the passage quoted above from John of St. Thomas may give rise to doubt about the possibility of such analytic principles in philosophy of nature, for at first glance he may seem to restrict them to metaphysics and mathematics. A more careful reading of the text, however, suggests another interpretation. In comparison with all of the propositions found in natural science, the number of truly analytical propositions is almost infinitesimally small, and that is why synthetic propositions may be considered as characteristic of the study of nature. Moreover, even the few analytical propositions that are found in philosophy of nature, though fully certain in themselves, are less certain in comparison with metaphysical and mathematical principles because of the materiality involved in them.

The third type of induction is the one that is of special interest for us. It is the type that is characteristic of experimental science, and it takes the form of an illation in which the mind progresses from a

multiplicity of singular experiences to a judgment which is proposed as universal, but which can never be anything more than tentatively universal because the nexus of the judgment is based merely upon repeated experience and not upon the apprehension of a necessary connection between subject and predicate. Such propositions, as we have already pointed out, can never be anything more than probable. It is true that as the experiences are multiplied the probability may in some cases increase to the extent of reaching practical certitude, but it can never reach the infinite limit of theoretical certitude. It is our contention that experimental science is made up completely of this probable knowledge, and that as a consequence it is not science in the strict sense of the word. But lest misunderstanding arise, it must be noted immediately that this probability refers only to universal propositions and there is no intention of calling into question the certitude of facts established by experimental science. The whole point is that science is constituted essentially of universal propositions and not of singular facts.

The type of induction we have just described is known as ascending induction. There is also a corresponding descending process in which the mind passes from a

universal proposition to singulars. This descending induction is often confused with deduction. There is, however, a vast difference between the two, for like ascending induction, descending induction lacks a true middle term. (55) This descending induction is also used extensively in experimental science. For since the universal proposition arrived at by ascending induction is only tentative it must be continually submitted to further experience for verification, and it is by a process of descending induction that this submission takes place. It remains true, of course, that deduction plays an important role in physics, but that is principally because of the introduction of mathematics which is a true deductive science.

The most important point which emerges from this discussion is the clear cut distinction between the second and third types of induction. Most of the difficulty that has arisen about the nature of induction has resulted from a confusion of these two. Until fairly recently it was customary to identify the third type with the second in the sense that the induction of experimental science was believed to give absolute certitude. Until the downfall of classical physics, nothing seemed more certain than Newtonian science. But since this downfall occurred it has

become customary to identify the second with the third and to extend the lack of certainty that is characteristic of experimental science to all science, ⁽⁵⁶⁾ and indeed to all human knowledge.

This distinction is important because upon it is based the distinction between philosophy and experimental science, as has already been suggested. The principles of the philosophy of nature are drawn from experience by induction, but because they are analytic, it is possible to infer from them conclusions that are certain. If the inference is good, the conclusions are necessarily true. These conclusions must indeed terminate in the senses in the way already explained in Chapter II. But this does not mean that they have to be submitted to sense experience for further verification -- since they are already necessarily true. In experimental science, on the other hand, the principles drawn from experience are only probable. Certain conclusions may be inferred from them, but even if the inference is good the conclusions are not necessarily true. That is why they must be submitted to observation and controlled by further experience. Experimental science is, consequently, doubly experimental -- both in its origin and in its terminus. Its principles are drawn from experience

and this "drawing" does not consist in a strict disengagement; the principles remain tied down to the actual experience already achieved. The conclusions of experimental science must be put back into experience again. Philosophy of nature on the other hand is experimental only in its origin and even here it transcends experience in the sense that the nexus of its propositions is not based upon experience. That is why, in opposition to the term "experimental" it may be called "rational".

And now, having arrived at this important distinction between philosophy and experimental science, we must pause to examine its nature in some detail.

4. Philosophy and Experimental Science.

It has become customary for modern writers to point out that in the writings of Aristotle no distinction between philosophy and experimental science is encountered. The inference that one is invited to draw from this observation is either that Aristotle was unacquainted with
(57)
experimental science or that he erred in failing to recognize that these two types of natural doctrine are formally

and specifically distinct sciences in the strict sense of
(58)
the word. Perhaps enough has already been said to show
that the basic structure of modern experimental science is
clearly and accurately outlined in the writings of Aristotle.
And in Chapter II we pointed out why Aristotle failed to
recognize the formal and specific distinction upon which so
much stress has been laid by some modern Thomists: such a
distinction neither exists nor can exist.

Does this mean that Aristotle recognized no
distinction between the two parts of natural doctrine that
have become known as philosophy of nature and experimental
science? In the first book of De Partibus Animalium we run
across the following passage: "It may, however, be asked, of
what mode of necessity are we speaking when we say this. For
it can be neither of those two modes which are set forth in
(59)
the philosophical treatises." These few lines make it
clear that Aristotle recognized a distinction between the
parts of natural doctrine that are advanced in the direction
of concretion and those which deal with generalities. To the
latter he applied the term "philosophical" and the evident
implication is that the former are in some sense not
philosophical. Yet later on in the same work he tells us
that it pertains to the philosopher to handle the subject of

(60)
this treatise. This seems at first glance to constitute a paradox. Yet we feel that a closer examination will reveal that these two texts implicitly suggest the correct solution of the problem of philosophy and science. They suggest both the precise way in which the two parts of natural doctrine are distinct and the way in which they must be kept united.

In the first place, let us recall that the term "philosophy" had for the ancients a much broader meaning than the one it now enjoys. It was, in fact, coterminous with all human science taken in the strict sense of the word (with the exception of theology for the medievalists). Consequently, when Aristotle says that the more abstract parts of natural doctrine are philosophical whereas the more concrete parts are not, he is simply saying that the former are strictly scientific and the latter are not. And this is precisely the conclusion to which our analyses have already led us. In Chapter II we demonstrated the impossibility of more than one true science in the first degree of abstraction. And earlier in this Chapter we saw that because of the type of induction employed by experimental science, it can never effectively rise above singularity to the point of achieving true universal and necessary propositions. We saw that whereas in philosophy of nature the

nexus of the propositions is strictly formal and analytic, in experimental science the nexus is material and synthetic. There are, of course, two types of material and synthetic nexus. There is, first of all, the completely material and synthetic nexus found in such propositions as: "this table is white." In this case we know that the nexus is merely material and synthetic because we have seen tables which are not white. But in the case of the propositions of experimental science we are not sure that the nexus is merely material and synthetic. In fact we tentatively arrive at something more than that. That is to say, there is a movement away from pure materiality and pure synthesis towards formality and analysis. Nevertheless this remains a purely dialectical limit that can never be reached. In other words, whereas in philosophy of nature we get at both the quia and the propter quid, in experimental science we get at only the quia. But we do not rest content with the mere quia. There is a constant striving towards the discovery of a propter quid. This is carried out by means of hypothesis. But the validity of every hypothesis depends upon an experimental confirmation, and this experimental confirmation gives us only an experimental proposition, and thus we have set out upon an infinite series of interplays between experience and hypothesis. All this

amounts to saying that all the concepts of experimental science ever remain incomplete and indefinitely open and perfectible. Because descending induction can never reach experience in such a way as to close the concept and make of it a true universal, experimental science though constantly striving towards formal abstraction never actually arrives at it nor at its certitude. The perfect certitude that experimental science seems to possess is, in the last analysis, nothing but an illusion deriving from the certitude that it is possible to have of a singular object or a group of singular objects.

Since, then, experimental science does not arrive at true formal abstraction, it cannot be a science in the strict sense of the word. And if it was experimental science that Locke had in mind when he said that natural philosophy is not capable of being made a science, (61) he was quite correct. As has already been stated, experimental science belongs to a type of knowledge which must be termed "dialectical." We shall devote the whole of Chapter V to an analysis of the meaning of this term, and for the moment it is sufficient to have pointed out in a general way the nature of experimental science in order to make evident the precise way in which it is

distinguished from philosophy of nature. It should be apparent from what has been said that the frontiers between philosophy and science are something definite and clear cut and not the nebulous thing that so much of the discussion of the question has made them. Just as soon the study of nature has arrived at the point at which the nexus of its propositions depend only upon experience, the frontiers between philosophy and experimental science have been reached. And it must be said in passing that if the reason why the term "experimental" is applied to science is not that the propositions are purely experimental, we know of no definite and absolute meaning that can be attributed to it.

At this juncture it is necessary to consider in some detail the distinction between philosophy and experimental science traditionally proposed by scholastic manuals: experimental science studies reality in terms of its proximate causes, whereas philosophy studies it in terms of its ultimate causes. We believe that in this distinction there is a extremely pernicious ambiguity that has confused the whole question of the relation between philosophy and science. For the expression "ultimate Cause" may be taken to mean two different things. It may,

first of all, mean the principles which enjoy true universality of causality, and not merely universality of predication. These causes can be arrived at as such and in an absolute fashion only by means of what are known as the proximate causes. Thus it is possible to demonstrate in the De Anima that man is the last end of all the natural species. But the knowledge that this gives us, though certain, is extremely obscure and confused. The theories of evolution are an attempt to dissipate this confusion and to arrive at this end in the order of concretion. And it is only by means of these theories that we can get at this ultimate cause which is man in a determined and absolute fashion.

But the expression "ultimate causes" may also be taken to mean the principles which have only universality of predication, that is to say, those encountered in the first part of natural doctrine. These causes may be called ultimate only in the sense that they are the farthest removed from what constitutes the essential and primary object of the study of nature -- the knowledge of things in their proper causes. They are not ultimate in the sense of being the terminus towards which the whole study of nature is orientated. In fact, they are at the opposite extreme.

That is to say, far from being the ultimate causes, they are the very first causes which the mind lays hold of in its initial contact with nature. Nor are they ultimate in the sense of being the most profound causes in the true sense of the word. For the most profound knowledge that one can have of nature is to know natural things in their proper causes, and the causes of which we are speaking are the most common that it is possible to discover. That is why from this point of view they provide us with the most superficial knowledge that it is possible to have of the cosmos. And it can be considered the most profound knowledge only by confusing the study of nature with the type of knowledge that is had in mathematics where the most known for us is also the most known in se. (62)

The following passage from the second book of the Physics brings out what Saint Thomas understood by profound cause:

... in naturalibus oportet semper supremam causam uniuscuiusque requirere, sicut contingit in artificialibus. Ut si quaeremus quare homo aedificat, respondetur, quia est aedificator; et similiter si quaeramus quare est aedificator, respondetur, quia habet artem aedificativam; et hic statur, quia haec est prima causa in hoc ordine. Et ideo oportet in rebus naturalibus procedere usque ad causam supremam. Et hoc ideo est, quia effectus nescitur nisi sciatur causa; unde si alicuius effectus causa sit etiam alterius causae effectus, sciri non poterit nisi causa eius sciatur; et sic quousque perveniatur ad primam causam. (63)

It is fairly clear from these lines that the most important cause -- the cause which constitutes the proper goal of science, the cause which gives us the most profound view of the nature of things, is not the remote cause, but the proper cause -- the cause which accounts for the ultimate concretion of the effect.

We believe that the majority of modern Scholastics have confused the two meanings of "ultimate cause" just defined. And this confusion has led to a good deal of unfortunate misunderstanding about the true character of the study of nature. From it has come that false air of profundity that so many Scholastics have assumed in dealing with things which in reality constitute the most indetermined and confused knowledge that it is possible to have of nature. From it, too, has come a view which when analyzed can hardly be distinguished from Hegelian idealism. We have in mind the notion that by means of the most general considerations possible one succeeds in grasping the very substance of things. Scholastic manuals give the impression that in the De Anima, for example, one grasps the very essence of the soul, and that the study of bees and birds and horses has to do only with accidental modalities of the substance of the brute animal. If this were true, the general would

be identified with substance, as in the doctrine of Hegel, and the species would be only a kind of phenomenal mode, or ulterior elaboration of the substance, which is not of interest to the philosopher whose task is to get at the profound essence of things. In other words, what is the most clear and the most knowable for us would be the essential substance of things, that is the most clear and knowable in se. Early in this Chapter we have seen that this is diametrically opposed to Aristotelian and Thomistic doctrine.

From this same confusion has arisen a false view of the order in which nature should be studied. Instead of following the traditional Aristotelian and Thomistic order which begins with generalities and moves on towards fuller concretion, in such a way that experimental science is a prolongation of philosophy of nature, most modern scholastics have made the philosophy of nature an extension of experimental science in such a way that the former in one fashion or another depends upon the latter. This dependence is often proposed as being absolute. Thus Fulton Sheen, for example, writes: "Under no consideration must it be thought the philosophy of nature does away with any experimental science. As a matter of fact, it would cease to exist without them?" (64) Such a position has led modern scholastics to

undertake such futile tasks as the demonstration of the doctrine of hylemorphism by means of physics and chemistry.

This view of the relation between science and philosophy is the one usually accepted among non-scholastic philosophers. Professor A.E. Taylor states the position in the following terms:

The work of the Philosophy of Nature and Mind only begins where that of the experimental sciences leaves off. Its data are not particular facts, as directly amassed by experiment and observation, but the hypotheses used by experimental science for the co-ordination and description of these facts. (65)

It is obvious that if this were the true relation between philosophy and science the former would be even more
(66)
dialectical than the latter.

In some quarters the anteriority of philosophy of nature to experimental science is recognized in one fashion or another, but then philosophy often becomes nothing but a highly theoretical vanguard of science born of hasty generalization which science gradually supplants by its constant progress. "The increasing independence of natural scientific branches from philosophy from Aristotle's time to the present," writes Pascual Jordan, "has simultaneously also emptied philosophy of its original

(67)
content and problems."

Some modern Thomists, while not making the dependence of experimental science upon philosophy complete and absolute, consider it nevertheless to be so essential that the constant progress of experimental science makes every treatise of the philosophy of nature extremely short lived. Thus Maritain says: "Je pense qu'un traité de philosophie de la nature, au maximum peut vivre une vie d'homme, cinquante ans, soixante-dix ans, si autem in potentatibus, octogenta anni - - et encore à condition d'être périodiquement remis à jour, à supposer qu'il ait des éditions successives; parce que ce traité de philosophie de la nature doit nécessairement avoir un contact intime avec les sciences des phénomènes, et ces sciences se renouvellent beaucoup plus rapidement que la philosophie." (68)

We cannot subscribe to such an opinion. We believe that a treatise of philosophy of nature, if it is good when first written, can live far beyond the life of a man. We believe that it can live forever without any substantial change.

In everything that is essential, the treatises of Aristotle and St. Thomas upon those parts of natural doctrine which are now known as philosophy of nature - - the eight books of the Physics and the three books of the De Anima - - are

just as alive today as when they were first written. All too many modern Thomists think that they have gone far in defending the perennial vitality of Thomism when they claim that although the writings of Aristotle And Aquinas on physical subjects are now obsolete, their metaphysics and moral philosophy remain eternally alive. It is safe to say that most of the Thomists who make such statements have never taken the trouble to give the Physics and the De Anima a close and intelligent reading, for such a reading would reveal that it is only in comparatively few and in extremely minor details that these treatises need revision. And the reason is simple: these treatises are essentially anterior to, and therefore independent of, experimental science. As we have already explained, in order to arrive at the general notion of the nature of motion Aristotle needed only the simple experience of the fall of a snow flake. The generic nature of motion was totally contained in this one instance and could be disengaged from it. If his analysis of this generic nature was correct, and we believe it was, then his definition of motion will ever remain unaffected by the innumerable highly complicated experiments subsequently made to determine the nature of motion in a more specific way.

And the same holds true of all the fundamental propositions
(69)
of the philosophy of nature.

This brings us to the consideration of an objection that has frequently been brought to bear against the view we have been upholding in relation to the question of philosophy and science. It has been formulated by Professor Alexander in the following terms:

Mr. Adler defines philosophy as a body of logical conclusions drawn from common sense observations, and science as a body of conclusions drawn from specific observations obtained by specific investigative methods. I agree with Mr. Adler's definition of science but not with his definition of philosophy. Mr. Adler reduces philosophy to reasoning about inadequate (common sense) observations, science representing at the same time reasoning about more adequate observations obtained by refined and improved methods of investigation. And yet, in order to save the medieval hegemony of philosophy, with a peculiar twist of reasoning, Mr. Adler tries to subordinate science - - that is to say conclusions drawn from improved observations - - to philosophy, which according to his own definition consists of conclusions from inadequate observations. If Adler's definition of philosophy is correct philosophy should be discarded in the proportion to which scientific knowledge progresses by the use of steadily improving special techniques of investigation. With this definition Adler himself speaks the death sentence of philosophy.(70)

Let us suppose that the term "philosophy" here refers to what Thomists understand by philosophy of nature, and that the expression "common sense observations" means the simple, ordinary observation that is the point of departure of the

first speculations of the mind about nature. It may readily be admitted that this common observation is completely inadequate for the solution of specific problems. But no one has ever claimed that it is adequate for such a purpose. Our position is that common observation is adequate for common, generic problems, and that only highly specialized observation is adequate for specific questions. The common observation from which is derived the generic notion of motion is completely inadequate for the solution of a very special problem concerning the respiratory tubes of a certain species of animal, let us say. But at the same time knowledge of the exact kind of motion found in a particular type of respiratory tubes is wholly unnecessary for a determination of the generic nature of motion.

Doctor Alexander's objection with regard to the subordination of the experimental sciences to philosophy recalls what was said in Chapter II in connection with our analysis of the twenty fifth lectio of St. Thomas' Commentary on the Posterior Analytics. This subordination does not mean subalternation in the strict sense of the word. From this point of view the experimental sciences are completely independent of philosophy. It can only mean a subordination arising from an order in which one

moves from the more generic to the more specific, that is to say a dependence of the more particular upon the more general. We feel that enough has already been said to make it clear that this dependence does not mean that the more general knowledge acquired in the philosophy of nature predetermines the solution of the more particular problems of the experimental sciences. Nevertheless, the anterior parts of natural doctrine have a definite influence upon the posterior parts. For the definitions arrived at in the philosophy of nature become methodological principles to guide the construction of hypotheses in the experimental sciences, to impose limits upon them, and to serve as criteria by which they may be criticized. Thus, for example, the definition of intellect in the De Anima becomes a methodological principle for experimental psychology. This role of philosophy of nature is not a restriction upon the experimental sciences. Rather it frees them from becoming enmeshed in false and useless hypotheses.

This discussion of the subordination of the experimental sciences to philosophy of nature suggests an important question: is it necessary, or at least helpful for experimental scientists to be acquainted with philosophy of nature? We know of no better answer to this

question than the one found in the following passage of Professor DeKoninck:

N'est-il pas vrai que les meilleurs physiciens modernes ignorent à peu près le tout des questions étudiées dans les premières parties de la philosophie de la nature? Seraient-ils meilleurs physiciens s'ils savaient la définition du mouvement, ou que la comparaison de mouvements d'espèce différente suppose une prédication d'identité et un mouvement dialectique de la pensée? A cela on peut répondre par la question: Le maçon serait-il meilleur maçon s'il était architecte? Les ouvrages des savants modernes sur les aspects 'plus philosophiques' de leur science, montrent suffisamment les désastres du maçon qui veut faire l'architecte en tant que maçon. Ils font violence à l'ordre qu'il nous faut suivre dans la connaissance si nous voulons en arriver à voir la partie dans son ordre au tout. Ils ont négligé les considérations logiquement antérieures à celle de leur propre sujet, négligence qui se fait sentir quand ils veulent sortir de celui-ci. Faire violence à l'ordre, ne fût-ce qu'à celui qui nous est imposé par la nature même de l'intelligence humaine, c'est faire violence à la sagesse, à la science de la nature en tant qu'elle est philosophique."(71)

The greatest mistake of the modern students of nature is that they have insisted on starting in midstream. The most fundamental and most basic questions have been ignored. Having started midway, and pursuing their progress into deeper concretion, they have thought that they could ultimately find the solution of the fundamental questions. But the progress of the study of nature does not move in a circle; it moves in a straight line. And one has only to consider the answers that scientists have brought forward to such fundamental questions

as: "what is life", to be convinced of this. Because the simple basic questions have been ignored, modern text books are filled with phrases and expressions which are utterly devoid of any definite meaning. They have much to say, for example, about "animal behavior" without ever having raised or solved the simple question: what is an animal in general. And all this brings home to us once again the utter futility of the efforts of modern scholastics to prove or disprove the doctrine of hylemorphism by means of chemistry and physics. The substantial composition of mobile being is a fundamental question that is anterior to, and therefore independent of all of the findings of modern experimental science.

The experimental sciences are, then, dependent in some way upon philosophy of nature. But from another point of view we may say that philosophy of nature is subordinated to the experimental sciences. For that which is less knowable in se is by nature subordinated to that which is more knowable in se. In other words the more abstract parts of natural doctrine are subordinated to the more concrete parts as potency is subordinated to act. In the concrete parts the abstract parts find their fulfillment. That is why the true philosopher of nature can never rest satisfied with the common,

general truths about nature, in spite of the fact that they alone provide him with scientific certitude. Such truths constitute only an introduction to the study of nature and are consequently completely orientated towards the more concrete parts which follow. The true philosopher of nature will never lose sight of that orientation, and he will be carried across the frontiers into the realm of experimental science. In so doing he will not be guilty of a naive optimism, or of a kind of "imperialism"; he will simply be obedient to the impetus of the dynamism that is intrinsic to the very study of nature. For the end towards which all the experimental sciences strive is at the same time the end towards which the philosophy of nature strives. And here we are touching upon the profound wisdom contained in the two texts from the De Partibus Animalium which seemed at first sight to constitute a paradox. On the one hand, the concrete parts of natural doctrine are distinguished from the more abstract parts by the fact that the latter are philosophical, that is to say truly scientific. But at the same time the philosopher of nature must study the concrete parts as well as the abstract parts, since the latter are a prolongation and a necessary fulfillment of the former. The following lines of Sir Arthur Eddington are relevant

here:

Not so very long ago the subject now called physics was known as 'natural philosophy'. The physicist is by origin a philosopher who has specialized in a particular direction. But he is not the only victim of specialization. By the breaking away of physics the main body of philosophy suffered an amputation.(72)

Perhaps we can sum up this discussion of the relation between philosophy of nature and the experimental sciences by drawing the following contrast between them. The former is of greater intrinsic importance than the latter for three reasons. First it provides us with the knowledge of nature that is most in conformity with the human intellect. It is significant that in modern times the mind in its dealings with nature has almost universally rejected the object that is most proportionate to it . But perhaps one might be tempted to object that experience shows that the experimental sciences are more easily accessible to a greater number than philosophy of nature. The answer to this objection has already been suggested earlier in this Chapter. In speaking of the relative "knowability" of the different parts of natural doctrine we have in mind only intellectual knowledge. In the measure in which sense knowledge enters into the discussion, it is evident that concrete singular sensible objects are the most easily

knowable. And in so far as the experimental sciences enjoy a close proximity to sensible singular objects they possess a facility that is not found in philosophy of nature. It must be noted, however, that in the measure in which physics is mathematicized it participates in the science that is the most proportionate to the human mind. We believe that these two facts explain the comparative accessibility of physics and the extreme attraction which it exercises over the mind.

Secondly, the philosophy of nature provides us with truly scientific knowledge. St. Thomas writes:

Illi qui sciunt causam et propter quid, scientiores sunt et sapientiores illis qui ⁱⁿignorant causam, sed solum sciunt quia. Experti autem sciunt quia, sed nesciunt propter quid. (73)

It remains possible to have scientific certitude as long as the mind remains in generalities. That is why the wiseman in the realm of nature must be humble. To reject certitude in these things is a kind of pride. Thirdly, the philosophy of nature has as its object the most noble thing existing in nature, the focal point of the whole of material creation - the spiritual soul of man.

On the other hand, the experimental sciences are more important than philosophy of nature in the sense that

they come closer to the realization of the goal of the whole study of nature -- the knowledge of things in their proper causes. (74) From this point of view they provide, as we noted in Chapter II, a type of knowledge that is closer to the knowledge that God has of the Cosmos than the knowledge found in philosophy of nature.

5. The Interrogation of Nature.

We have seen that nature may be defined in terms of a ratio indita ^o rebus. It is this intelligence, this logos realized in material things that makes the science of the cosmos possible. And the goal of this science is to capture this ratio in some partial way at least, to bring into contact with the ratio of man. We have seen that this becomes increasingly difficult as experience carries the mind forward into deeper concretization. Nature appears less and less rational, less and less homogeneous with the intellect. It continually throws up greater obstacles to the mind's attempt to disengage the objective logos from the materiality in which it is concretized. And there ultimately remains only one thing for the mind to do if it is to continue its task: to impose upon nature the rationality which it lacks, to extract the

objective logos of the cosmos by injecting its own subjective logos into it. This process of rationalization eventually terminates in the mathematization of nature, in which the most irrational of all the speculative sciences become subalternated to the most rational. The intellect finds, for example, that the visual line is not rational enough for it, so it substitutes the mathematical line. But even prior to the introduction of mathematics an extensive process of rationalization takes place. We must now try to analyze this process.

In the first place, it is important to recall that experimental knowledge is essentially imperfect, for it implies physical passivity. To have an experience means to become subject to something, and in the case of sense experience is always a question of becoming entitatively subject to material things which physically affect the sense organs. That is why man cannot be satisfied with purely experiential knowledge. By the very fact that knowledge is vital it is opposed to passivity, and by the fact that it is intentional it is opposed to the purely physical. (75) That is why the mind is impelled to go beyond experience, to anticipate it by searching for the

reason of what is presented in experience. The more the science of nature approaches concretion the more experience gets the upper hand, so to speak. The intellect cannot accept this state of affairs. It must try to rationalize experience and thus get the upper hand itself. For the intellect can never rest in pure givenness; it has, as Meyerson says, "une répugnance irrémédiable ... devant tout donné".⁽⁷⁶⁾ It cannot be content with a mere quia; it must search for the propter quid. It cannot remain imprisoned within singularity; it must strive to achieve universality. It cannot rest satisfied with purely synthetic judgments; it must find a way of making them a priori. And when nature does not provide what it seeks, it will reconstruct nature in such a way as to make it render what it wants, or at least in such a way as to allow the mind to give itself what it wants. All this explains why as soon as the propositions of the study of nature start to be purely experimental there begins a gigantic task of reconstruction of nature. And the greater the part that experience plays in this study, the greater must be the part that the mind plays. Science becomes a mixture of fact and fiction, and as fact increases so does fiction. As Duhem has remarked: "Le développement de la Physique provoque une lutte continuelle entre 'la nature qui ne se lasse pas de fournir' et la raison qui ne veut

(77)
pas 'se laisser de concevoir.'" We must now try to point out the most salient features of this rationalization of experience.

This is far from being an easy task. For not only do the objective and subjective logos ultimately become so inextricably fused that it is impossible to draw the line between them, but it is also impossible to find an absolute starting point for the introduction of the subjective logos, since the whole process is essentially circular. It might be suggested that the first step in the rationalization of experience consists in this that at the beginning of a scientific experiment the scientist makes a selection of the elements that are to enter into the experiment and places them in especially chosen conditions in such a way that the whole experiment is an artificially constructed process. It might further be suggested that the second step consists in an intellectual filtration and purification of the elements entering into the experiment in such a way that they become idealizations which have no exact counterparts in experience. There can be no doubt that experimental science deals with idealized entities of this kind, such as perfect gases movement without friction, absolutely rigid bodies, perfect levers, perfectly geometrical crystals, absolutely pure

(78)
metals, perfect fluidity, perfect elasticity, etc. And
all this represents a projection of thought into the cosmos.
But the nature of this projection must be rightly under-
stood. For at first glance it might seem that all that is
involved here is the substitution of limiting cases for the
brute phenomena that are directly perceptible. If this
were true, we could, as Cassirer has pointed out, "attempt
to do justice to this method by a simple extension of the
positivistic schema." (79) As a matter of fact, however,
the problem is much more complicated than that. And an
attempt to unravel it will immediately show that in the
process of rationalization there is a good deal prior to
the steps mentioned a moment ago.

This brings us to the central point of our present
discussion. And we know of no better way of coming to grips
with it than by considering a passage from Kant's Critique
(80)
of Pure Reason:

Mathematics and physics are two types of theoretical
knowledge which must determine a priori their object:
the first in an absolute way; the second at least in
part, and to the extent to which the other sources
of knowledge besides the reason allow it to do so.

After attempting to show that mathematics is a completely
a priori science and that it has made true progress only
since mathematicians have come to realize this, he goes on

to consider the a priori character of physics:

When Galileo rolled balls down an inclined plane with an acceleration determined and chosen by himself, when Torricelli attributed to the air a weight which he computed as equal to the weight of a known column of water, or when later Stahl transformed metals into lime, and the latter in turn into a metal, by separating and adding certain elements, then a new light dawned for all physicists. They understood that reason discovers only what it produces itself according to its own designs; it must take the lead with principles which determine its judgments according to constant laws, and force nature to respond to its questions, instead of leaving itself be conducted by nature as though by a string: for otherwise our observations made at random and without any plan traced beforehand would never lead to a necessary law, which the reason nevertheless looks for and demands. The reason must present itself before nature, holding in one hand its principles which alone are able to give the concordant phenomena the authority of law, and in the other hand it must hold the experiment such as it has planned according to the same principles. Reason demands to be informed not as a school boy, who is bound to speak only what pleases the teacher, but as a judge on his bench, who constrains the witnesses to answer the questions put to them. Physics, therefore, is indebted to the happy revolution which has been introduced into its method by this simple notion that it must seek for (and not imagine) in nature, in accordance with the ideas which the reason itself brings to it, what the reason ought to learn of nature, about which it can never learn anything simply by itself. It is thus that physics has been able to enter for the first time upon the sure road of science, after groping along for so many centuries.

The gist of this passage may be summed up by saying that according to Kant experimental physics owes its emancipation and its progress to the fact that it proceeds to a certain

extent in an a priori fashion by posing questions which anticipate experience and predetermine it.

This doctrine has in recent times been applied to biology by an ardent disciple of Kant, J. von Uexkull:

Natural science falls into two parts, doctrine and research. The doctrine consists of dogmatic assertions, which contain a definite statement concerning Nature. The forms these assertions take often suggest that they are based on the authority of Nature herself. This is a mistake, for Nature imparts no doctrines: she merely exhibits changes in her phenomena. We may so employ these changes that they appear as answers to our questions. If we are to get a right understanding of the position of science vis-a-vis of Nature, we must transform each of the statements into a question, and account to ourselves for the changes in natural phenomena which men of science have used for evidence for their answer. Investigation cannot proceed otherwise than by making a supposition (hypothesis) in its questions, a supposition in which the answer (thesis) is already implicit. The ultimate recognition of the answer and the setting up of a doctrine follow as soon as the investigator has discovered in Nature what he considers a sufficient number of phenomena that he can interpret as positive or negative on the lines of this hypothesis.

The sole authority for a doctrine is not Nature, but the investigator, who has himself answered his own question. (81)

We do not subscribe to all of the implications of the doctrine found in these two passages. Nevertheless, we believe that the central idea running through them is essentially correct. Kant was right in holding that if

experimental science is to have any significance it cannot rest satisfied with the purely synthetic character of experimental propositions. The mind must introduce an a priori element into them. And this introduction does not take place only after the process of experimentation has been accomplished. It is something that is effected during the process itself. The mind must anticipate experience and by this anticipation predetermine the experimental process. Kent was wrong in believing that Newtonian physics was definitive, and that as a consequence the a priori element introduced by the mind was something absolute and necessary. (82)

Let us examine each of these two points in turn.

We have already suggested that modern science is far from being an outgrowth of the naive empiricism of Francis Bacon whose ideal it was to have experimentation carried on without any preconceived ideas. In this connection Poincaré writes:

On dit souvent qu'il faut expérimenter sans idée préconçue. Cela n'est pas possible; non seulement ce serait rendre toute expérience stérile, mais on le voudrait qu'on ne le pourrait pas. Chacun porte en soi sa conception du monde dont il ne peut se défaire si aisément. (83)

Perhaps the first author in modern times to bring out with great clarity and emphasis the importance of pre-

conceived ideas in scientific experimentation was Claude Bernard. In his classic work, Introduction à l'étude de la médecine expérimentale, he says:

Il n'est pas possible d'instituer une expérience sans une idée préconçue; instituer une expérience... c'est poser une question; on ne conçoit jamais une question sans l'idée qui sollicite une réponse. Je considère donc, en principe absolu, que l'expérience doit toujours être instituée en vue d'une idée préconçue, peu importe que cette idée soit plus ou moins vague, plus ou moins bien définie... (C'est) l'idée qui constitue... le point de départ ou le primum movens de tout raisonnement scientifique, et c'est elle qui en est généralement le but, dans l'aspiration de l'esprit vers l'inconnu... Sans cela on ne pourrait qu'entasser des observations stériles. (84)

This opinion of Claude Bernard has become universally accepted among the best modern scientists and philosophers of science. Innumerable authorities besides the ones already cited could be brought forward to attest to this universal acceptance. (85) It has become increasingly clear that, as Meyerson says, " toute expérience n'est et ne peut être qu'une expérience de pensée." (86) And these authorities are unanimous in attributing the whole fecundity of experimental science to the projection of an a priori idea into experimentation. Without this projection experimentation could render only pure data without any unified significance. And these data could lead to nothing

beyond themselves. They would be utterly sterile, unable to carry the mind forward in any definite direction. It is from the a priori idea that science derives its essential dynamism.
(87)

But it is important to see in what precise way this projection of the a priori into experimentation is effected. The texts cited above have already suggested that it is brought about essentially by the way in which the experimenter interrogates nature. Every experiment is in fact a very definite question which the experimenter puts to nature. And the results of the experiment have no meaning except in so far as they are the answer to this definite question. That is why these results are already predetermined by the experimenter. The whole pattern of the experiment, the selection of the elements that are to enter into it, the structure of the instruments that are to be employed, the precise character of every action that carries the experiment forward -- all these are predetermined by the precise question that is in the mind of the experimenter. And this question has no meaning in relation to the very complicated theoretical background which forms its context. Max Planck has brought out this point with his usual clarity:

Therefore from the results that are given by experi-

mental measurement we must choose those which will have a practical bearing on the object of our inquiry, because each particular attempt at discovering reality in the physical universe represents a special form of a certain question which we put to nature. Now you cannot put a reasonable question unless you have a reasonable theory in the light of which it is asked. In other words, one must have some sort of theoretical hypothesis in one's mind and one must put it to the test of research measurements. This is why it often happens that a certain line of research has a meaning in the light of one theory but not in that of another. And very often the significance of a question changes when the theory in the light of which it is asked has already changed.(88)

But it is necessary to try and analyze more accurately the character of the questions that it is possible to put to nature in experimental sciences. There are in fact two conceivable ways in which a question may be posed. In the first place it is possible to ask a question which demands in an absolute fashion what the nature of a thing is, for example: "what is man?" Such a question can never be answered by either "yes" or "no". The answer must be "rational animal" or "featherless biped" or something similar. And the reason is that such a question does not contain an hypothesis. But there is another type of question which does contain an hypothesis, for example: "is the definition of man: featherless biped?" In this

case the hypothesis involved constitutes a suggestion to which one is forced to answer by either "yes" or "no". This suggestion is already in some sense a predetermination of the answer. And it is clear that in posing a question of ~~th~~is second type the mind is taking the initiative and anticipating nature.

Now it is only questions containing an implicit hypothesis that are used in experimental science. As Meyerson has remarked, "il est parfaitement impossible d'arracher à la nature ses secrets en l'interrogeant
(89)
directement." And because it becomes increasingly difficult to induce nature to yield up its secrets as progress is made towards fuller concretion, it is necessary that the questions posed by the scientist become increasingly artificial and hypothetical. Scientific method has often been compared to the methods employed in tracking down criminals. Now the criminal which is nature will never answer a direct question. And as a result the scientific detective never succeeds in pinning this criminal down in an absolute and definitive fashion. For there is this difference between nature and ordinary criminals that when for former answers "yes" it does not necessarily mean "yes" in an absolute way. That is to say, when the hypotheses

of the scientist's question is verified in experience, this does not mean that the hypothesis is necessarily true - "quia forte secundum aliquem alium modum apparentia salvantur". It does not follow from this, however, that von Uexkull is completely correct in maintaining that "the sole authority for a doctrine is not nature, but the investigator, who has himself answered his own question." For though it be true that nature's answers are to some extent predetermined by the questions formulated by the investigator, they are not completely determined thereby. It cannot be denied that nature has something to do with the answer, and that throughout the whole dialectical process of interrogation it remains the measure to which the scientist must ever seek to conform himself.

Even among those who readily admit that hypothesis plays a major role in experimental science the notion is often current that hypothesis is always something posterior to experimentation and merely superimposed upon it, in such a way that it remains a comparatively easy task to distinguish the factual elements deriving from experience from the hypothetical elements contributed by the mind. We feel that enough has already been said to show that this is false. Hypothesis must anticipate experience and pre-

determine it. And this predetermination is such that, in the more complicated experimental processes at least, it is impossible to distinguish sharply between the subjective and objective logos. The analysis which is to follow will serve to bring out this truth with greater evidence.

6. Operationalism

In order to come to understand more fully the way in which the subjective logos is projected into nature in the procedure of experimental science, it is necessary to examine closely the precise character of a scientific experiment. (90) During the reign of classical physics, it was generally believed that a scientific experiment was essentially a revelation of a property that existed as such in objective reality. It was taken for granted that the whole experimental procedure was merely a means by which the scientist was able to disengage a definite feature that was embedded in the absolute world condition. Contemporary physics has shown how naive this view was. In fact, we are touching here the very heart of the profound difference between Newtonian physics and Relativity and Quantum physics.

We have already laid considerable insistence upon the purely experimental character of the definitions that form the structure of experimental science. We have seen that experimental science never really succeeds in disengaging an essence, that it never really rises above the realm of singularity. As a consequence, the definitions of experimental science are merely formulations of what is presented by sense experience. All this is true even of propositions which derive from ordinary observation, that is to say, observation into which no element of control or artificial construction has been introduced.

But the true well spring of science, and particularly of physics, is not this ordinary observation. By the very fact that the scientist is unable to really disengage essences from it and thus rise to true universality and necessity, it appears as a frustration to the mind. For this reason the student of nature cannot rest satisfied with it. If nature will not yield up its secrets of its own accord, it must be forced to do so. That is why he finds it necessary to operate upon nature, to bring it under his guidance and control, to manipulate it in ways dictated by his preconceived ideas. All this is known as a scientific experiment.

An experiment has often been defined as controlled sense perception. But it should be clear from what has just been said that it is a good deal more than that. It is, in fact, a reconstruction of nature. Because the routes provided by nature are not sufficient to enable the scientist to arrive at his goal, it is necessary for him to construct an artificial detour. This detour carries him closer to his goal than he would have been able to get without it, but it does not do so in the way conceived by the classical physicists. For the detour is inseparable from the goal. And this brings us to an extremely significant paradox to which we shall return more than once in this study: scientific method carries us closer to nature only at the expense of carrying us farther away from it. (91)

And what happens to the scientific definitions in this process? The reconstruction of nature effected by the scientist enables the mind to penetrate more deeply into its meaning, but this penetration never arrives at a point at which the mind is able to rise above purely experimental propositions which are of the very essence of experimental science. In fact, as we have just suggested, from one point of view the very reconstruction makes it even less possible to escape from them. The mind remains bound

down to experience, bound down to a mere formulation of what is presented by experience. But now what is presented by experience has become something different. It is no longer something produced by nature, but rather something produced by the scientist himself in his operation upon nature. That is why the results of experiments have no meaning except in terms of the precise operations by which they are produced. They depend upon every element which enters into the experiment: upon what he does, the way in which he does it, all the concrete circumstances in which he operates, etc. And because it is impossible for him to know exactly what he is doing and all the circumstances of the operation, he is never able to rise above the sensible individual operation except by means of provisional and dialectical generalizations. All this amounts to saying that the definitions of experimental science derive their significance from the series of operations employed in the experiments which led to their formulation. That is to say, the only way to define physical quantities is by an enumeration of all the concrete operations by which these physical quantities have come to be known. And every attempt to analyze the meaning of the definitions of experimental science must necessarily end in the mere designation of a concrete series of operations performed with a concrete

(92)

set of instruments. There must be a reductio ad materiam sensibilem individualem. The more experimental science attempts to achieve the natural desire of the intellect to rise above the senses and the pure givenness of experience, the more is it obliged to fall back upon them.

In order to be convinced that all the definitions with which physical science deals are essentially operational one has only to open a book of physics and read the definitions of the fundamental quantities which constitute the science. Mass, force, temperature, electricity, magnetism, light, sound, energy, entropy, atomic and molecular properties, etc. -- all without exception are defined in terms of definite physical operations performed with definite physical instruments. And we must be constantly on guard against the natural tendency to hypostatize terms which designate no more than experimental processes. The way in which scientific progress forces physics to introduce progressive modifications into the definitions of its fundamental quantities should be a constant warning that these quantities are not real, ontological properties.

As we have suggested, the realization of the operational character of the definitions of experimental

science is the very core of the difference between classical and contemporary physics. One has only to read Einstein to be convinced of this. The relation to the central problem of the whole question of relativity - - that of simultaneity, he is constantly coming back to the query: what meaning can simultaneity have for me as a physicist? And his answer is always the same: a definition of simultaneity can have meaning for a physicist only if it designates a series of operations of measurement that can be realized in the concrete and that will make it possible to determine whether two events are simultaneous or not. Having posited this principle, it merely remains for him to show that every attempt to determine simultaneity by means of concrete operations involves a relation to a particular observer, and that consequently simultaneity cannot be considered by a physicist as an absolute property of two events themselves, but as something belonging to these events in so far as they stand in relation to a given observer, which relation is determined by velocity. We shall return to this question again in Chapter VIII. For the moment it is important to note that operational definitions maintain a vital union between experience and theory. No matter how far the experimenter and the theorist may go, each in his own direction, they will always

be sure of remaining in contact with each other as long as
(93)
their definitions are operational.

It is worth while pointing out in passing the similarity between this principle of operationalism and the fundamental thesis of logical empiricism: a proposition has meaning only if it states the means for its verification. This thesis is acceptable in so far as it applies to experimental science; the error of the logical empiricists is to have extended it to all knowledge.
(94)

This whole question of operationalism has been summed up by Sir Arthur Eddington in The Mathematical Theory of Relativity:

To find out any physical quantity we perform certain practical operations followed by calculations; the operations are called experiments or observations according as the conditions are more or less closely under our control. The physical quantity so discovered is primarily the result of the operations and calculations; it is, so to speak, a manufactured article - - manufactured by our operations. But the physicist is not generally content to believe that the quantity he arrives at is something whose nature is inseparable from the kind of operations which led to it: he has an idea that if he could become a god contemplating the external world, he would see his manufactured physical quantity forming a distinct feature of the picture. By finding that he can lay x unit measuring-rods in a line between two points, he has manufactured the quantity x which he calls the distance between the points; but he believes

that that distance x is something already existing in the picture of the world -- a gulf which would be apprehended by a superior intelligence as existing in itself without reference to the notion of operations with measuring rods...

Having regard to this distinction between physical quantities and world-conditions, we shall not define a physical quantity as though it were a feature in the world-picture which had to be sought out. A physical quantity is defined by the series of operations and calculations of which it is the result ... We do not need to ask the physicist what conception he attaches to 'length', we watch him measuring length, and form our definition according to the operations he performs.(95)

The epistemological implications of this principle of operationalism are far reaching. They may, perhaps, be summed up by saying that the physicist is never confronted with a pure object. The fundamental quantities, such as length, mass, energy, potential, etc. out of which the whole structure of physics is erected are not things or natures or properties or features of the absolute world condition. They are articles manufactured by the subject. They are synthetic products. They are not things of nature, but things fabricated in order to explain nature. As Professor Petit has remarked, "le faire est au coeur du
(96)
connaître expérimental." In other words, in the experimental sciences, speculative knowledge can reach out towards its object only by giving way in some measure to

practical knowledge.

All this, however, does not favor the idealistic position. For the operations which constitute a scientific experiment are physical, and they are performed upon objective physical nature. As a consequence, the results, while not purely objective, are not purely subjective. They are a composite of the objective and the subjective. But it is extremely important to recognize the part played by the subjective element. As we shall have occasion to point out in a future Chapter, it is only by acknowledging the role of the subjective in experimental science that we can become truly objective.

It should be clear from what has been said thus far about operational character of experimentation that the subjective enters into science in two ways. In the first place there is a mental intrusion through hypothesis and theory in the sense that all of the operations and the whole structure of the instruments employed are determined by some preconceived theory. Instruments are in fact nothing but materialized theories. This point has been developed in the last section of this Chapter. In the second place there is a physical intrusion in the sense

that the subject operates physically upon nature through physical operations carried on by physical instruments constructed of copper, and glass, and aluminum and silk, etc. This obviously results in a physical interaction between the object and the subject, which makes it impossible for the subject to get at the object in its pure state of objectivity. We intend to return to this question in our discussion of the limitations of measurement in Chapter VIII, but perhaps at this point it will be worthwhile to quote the following lines from Heisenberg, who has done so much to bring out the significance of this interaction:

Particularly characteristic of the discussions to follow is the interaction between observer and object; in classical physical theories it has always been assumed either that this interaction is negligibly small, or else that its effect can be eliminated from the result by calculations based on 'control' experiments. This assumption is not permissible in atomic physics; the interaction between observer and object causes uncontrollable and large changes in the system being observed, because of the discontinuous changes characteristic of atomic processes. The immediate consequence of this circumstance is that in general every experiment performed to determine some numerical quantity renders the knowledge of others illusory, since the uncontrollable perturbation of the observed system alters the values of previously determined quantities. If this perturbation be followed in its quantitative details, it appears that in many cases it is impossible to obtain an exact determination of the simultaneous values of two variables, but

rather that there is a lower limit to the accuracy with which they can be known. (97)

Until rather recently it was customary to contrast the method of introspection employed in experimental psychology with the methods used in the other experimental sciences by pointing out that in the case of introspection the intrusion of the subject makes it impossible to arrive at the object in its pure state of objectivity. And it was more or less taken for granted that this pure objectivity was attained in the other experimental sciences. Niels Bohr, however, has shown that this pure objectivity is a mere illusion and that throughout physics there is an intrusion of the subject comparable to that found in the method of introspection. One of the reasons why scientists become easily susceptible to this illusion is that, as Duhem (98) has brought out so fully and so accurately, they tend to substitute in their mind an idealized instrument, a kind of mathematical model for the actual physical instrument employed. For a copper wire of a certain breadth, for example, is substituted a geometrical circle without breadth; for a steel magnetic needle which has a definite magnitude and which is unable to move without friction is substituted an infinitely small horizontal magnetic axis which moves around a vertical axis without friction, etc. In fact there

is a tendency to go even beyond this; to dematerialize the instrument completely, to attribute to it the properties of a transsubjective cognitive faculty. And the reason for all this is clear: it pertains to the nature of the intellect qua intellect to know things independently of physical means.

Perhaps the most significant conclusion that can be drawn from this discussion of operationalism is that irrationality enters into experimental science in a way in which it does not enter into any other science. It is true that irrational elements enter into all the sciences in one way or another, but in all the other sciences these elements remain extrinsic to the formality of the concepts that are proper to these sciences. But because the very notions out of which experimental science is constructed remain inseparable from the physical, material operations by which they are formed, that is to say, because a mere series of physical operations plays the role that essences play in philosophical knowledge, there is a profound element of irrationality intrinsic to these notions. And it is all too easy to lose sight of this fact simply because of the operational clarity that these notions possess.

7. Laws and Theories.

But science is not made up merely of isolated notions. It is a highly coordinated and unified system. And this coordination and unification is brought about chiefly through the formulation of laws and theories. To this formulation we must now turn our attention. Since we shall have to return to this question later when we come to consider the mathematical transformation of physical science, we shall content ourselves here with a brief outline of the structure of physical laws and theories and with a summary discussion of their epistemological significance, in such a way that the central thought we have been pursuing, namely the projection of the subjective logos into nature, will be rounded out and fully crystallized.

Unity is a condition of intelligibility, for
(99)
pure diversity is essentially irrational. That is why the mind in its effort to rationalize nature cannot rest content with a mere collection or tabulation of phenomena. As we shall see in Chapter VIII, the process of measurement in physical experiment is already a unification, for measurement consists essentially in reducing a multiplicity to the unity of a standard. But this initial unification is not sufficient to satisfy the mind's desire for rationality. It has an innate aspiration to approach as closely as

possible to the higher forms of intellect which grasp an increasing plurality of things in a diminishing plurality of species. It instinctively tends to rise to a higher unity by establishing definite relations between the multiplicity of events which reveal themselves in experiment. That is why the development of science manifests two paradoxical tendencies. For on the one hand, we have seen that the movement towards concretion is a movement towards greater multiplicity, since it approaches things in their proper specific nature. This is a tendency towards a pluralistic universe. On the other hand, the mind instinctively seeks to reduce this multiplicity to an ever more perfect unity, and the terminus of this movement is a completely monistic universe. The amazing thing is that these two contrary movements, far from being irreconcilable, are actually cooperative. (100) The early part of this Chapter was devoted to a consideration of the movement towards pluralism. Now, before bringing this Chapter to a close we must discuss the tendency towards monism. This tendency is carried forward principally by means of laws and theories.

Now nature lends itself admirably to this tendency of the mind. For the events which present themselves in experience are not mere disparate phenomena. They reveal

themselves as belonging to a pattern. For nature is defined precisely in terms of those things which happen,
(101)
"ut in pluribus." This natural order and regularity makes it possible for the mind to establish legality among phenomena, and this is the first step in the movement of the mind towards a more perfect unification than that found in the reduction of phenomena to a standard.

But are physical laws a mere reflection of the order and regularity of nature? Classical physicists seem to have been persuaded that they are. All the best modern epistemologists, however, are agreed that this is very far from being the case. And we feel that enough has already been said to show why this is so.

For in the first place, it is clear from our discussion of the nature of the propositions of experimental science that the universality and necessity which are found in physical laws, and which are of the very essence of all law, can be nothing but a gift of mind to nature. Nor is this gift gratuitous. The mind bestows it only that it may be carried nearer to the goal towards which it is striving. That is why physical laws are essentially functional. That is why they must not be

looked upon as something fixed and static, as a finished reflection of an absolute order existing in nature.

But there is much more to the case than all that. For, as we have just seen, the quantities which form the stuff out of which physical laws are formulated are not objective entities. They are articles manufactured by the subject in his operations upon nature. (102) Into this manufacture has gone both hypotheses and physical action. That is why the resultant laws have no meaning except in terms of the projection of subjective logos that all this entails. Moreover, in the highly complex structure that is physical science, laws do not have a completely independent and absolute meaning in their own right. Their meaning is derived from their context, which is a closely woven pattern of mutually interdependent laws and theories. In this connection, Professor Campbell writes: "Nous remarquons d'abord que les termes ne sont pas habituellement des jugements simples et immédiats sur les sensations, mais des collections complexes de tels jugements. Dans la plupart des lois, ces collections sont telles que les lois ne sont vraies que si d'autres lois le sont. Elles en dépendent à la fois pour leur sens et pour leur vérité, Ce caractère de dépendance mutuelle est très important pour

(103)
nos recherches." The significance of laws also depends upon the particular theory into whose structure they are fitted, in such a way that if the theory changes the significance of the laws changes. Duhem writes: "selon que l'on adopte une théorie ou une autre, les mots même qui figurent dans l'énoncé d'une loi de physique changent de sens, en sorte que la loi peut être acceptée par un physicien qui admet telle théorie et rejetée par un autre physicien qui admet telle autre théorie." (104) The difference of meaning attached to the law of gravitation in Newtonian and In Einsteinian physics is a case in point.

It is evident, then, that there is a vast difference between the objective laws of reality and the laws of physical science. Eddington has brought out this difference in the following terms:

We are in danger of falling into a confusion regarding laws of nature -- a confusion between what they are and what we originally intended them to be. To avoid ambiguity I will discriminate (temporarily) between 'laws of nature' and 'laws of Nature'. Law of Nature will have the meaning that the term was originally intended to bear -- a law emanating from the world-principle outside us, which we often personify as Nature. Law of nature will mean as heretofore a regularity which we have found in our observational knowledge, irrespective of its source. In short a law of nature is whatever would be designated by that name in current physical practice.

It will be seen that a law of Nature is a law of the objective universe. But all recognized laws of nature are subjective. We have thus reached the verbal paradox that no known law of nature is a law of Nature. Effectively the terms have become mutually exclusive.

It is true that we have left an opening. A law of Nature is a law of nature if it would be (not necessarily if it already is) accepted as such in physics. This brings me to a further question. Have we any reason to believe that if a law of Nature -- a generalization about the objective world -- were to become known to us, it would be accepted by current physics as a law of nature? I think it would only be accepted if it conformed to the pattern of physical law that we are accustomed to, But this pattern is the pattern of subjective law. We shall try later to show by epistemological study how the pattern has grown out of the subjective aspect of physical knowledge. The pattern is the very hall-mark of subjectivity. Any expectation we may have formed that the objective laws of Nature, when they are discovered, will conform to the same pattern is quite unreasonable. (105)

In order to be convinced that physical laws are ideal constructions of the mind it is sufficient to analyze any one of them accurately. This analysis will reveal the utter impossibility of their being realized as such in nature. And this is true of even the most fundamental laws which have come to be considered as the principles of the whole structure of physical science. The principle of inertia is a case in point. (106) The verification of this law in nature would involve a contradiction. For in order to show that a moving body preserves its rectilinear and uniform motion unless influenced by another body, it would

be necessary to have only one body in existence - - and then all motion would be impossible, since bodies can move only in relation to one another. Moreover the exact verification of the principle would demand that the volume of the body
(107)
be reduced to zero. It is important to note that laws of this kind become conventions which serve to define the very concepts which are involved in them, in such a way that it becomes impossible for experience not to conform to them. If a moving body were to fail to preserve its rectilinear and uniform motion a scientist would never conclude that the law of inertia had been violated, but rather that some secret influence of which he was ignorant was being exercised upon the moving body. In like manner, the law which formulates the functional relation between the length of a piece of metal and its temperature is transformed into the definition of coefficient of linear expansion; the law which states the dependence of the stress in an elastic body upon the strain is transformed into a definition of elastic constant. First the law is established that light travels in a straight line, and then the path of light becomes the definition and the norm of a straight line. That is why Le Roy could write: "Les lois sont invérifiables, à prendre les choses en toute rigueur . . . , parce qu'elles constituent le critère même auquel on juge les apparences

et les méthodes qu'il faudrait utiliser pour les sou-
mettre à un examen dont la précision soit susceptible de
(108)
dépasser toute limite assignable."

It is necessary to conclude, then, that physical laws are not found -- they are made. They do not exist before they are formulated by the mind. This does not mean that they are purely fictitious. They have a basis in reality in the sense that they are suggested by experience. The law of inertia, for example, was formulated only after it had been suggested in countless ways by nature. Moreover, the term of the process which constructs physical laws is always the true, objective laws of nature. And that is something which those who insist upon the subjective character of scientific laws usually forget. Nevertheless, it remains true that only a suggestion of these laws is actually found in reality. That is why there is something essentially Platonic about them. That is why Kant was in this respect correct in making the mind the lawgiver of nature. For scientific laws come from reality only materially; formally they are from the mind. The essence of scientific knowledge is made up of a kind of noetic hylemorphism in which the matter presented by reality is formalized by the mind. In all of the laws of experi-

mental science, as Eddington writes: "the mind has by its selective power fitted the processes of Nature into a frame of law or a pattern largely of its own choosing; and in the discovery of this system of law the mind may be regarded as regaining from Nature that which the mind has put into Nature."
(109)

The establishment of legality among phenomena was for Comte the ultimate terminus of the scientific movement. But in this respect as in many others Comte failed to seize upon the true spirit that animates scientific endeavor. As Einstein and Infeld have pointed out, "la science n'est pas une collection de lois . . . Elle est une création de l'esprit humain au moyen d'idées et de concepts librement inventés. Les théories physiques essaient de former une image de la réalité et de la rattacher au vaste monde des impressions sensibles. Ainsi, nos constructions mentales se justifient seulement si, et de quelle façon, nos théories forment un tel lien."
(110)

Just as the mind's desire for rationality impells it to rise above the initial unification achieved in measurement to the higher unity of law which establishes a definite relation in the multiplicity of phenomena, so it likewise impells it to go further and arrive at a higher synthesis

which establishes relations in the multiplicity of laws. This higher synthesis is achieved by means of a physical theory. The kinetic theory of gases, for example, makes it possible to synthesize the laws of Mariotte, of Gay-Lussac, and of Avogadro. By means of this principle of gravitation Newton was able to synthesize the laws arrived at by Kepler and Galileo and the laws governing the tides.

Without theory the movement of the scientific mind would be essentially frustrated. For the two essential~~ly~~ properties of science are universality and necessity. By means of laws the mind is able to rise above the singularity of phenomena and arrive at a kind of universality. But this universality is lacking in necessity. That is to say, even when laws have been formulated there is nothing intrinsic to them which shows that could not have been otherwise. In other words, propositions which merely state an association between the values of one variable and the values of another variable are not logically necessary. For example, an increasing temperature is associated in a determined way with increasing volume but there is nothing in this law which shows that the reverse might not have been the case. The mind cannot rest satisfied with this contingency; it must strive to reduce

it to some kind of necessity by finding a reason which explains why increasing temperature is associated with increasing volume. This is accomplished by the construction of a theory which postulates the existence of unobserved entities whose hypothetical behavior will explain the observed phenomena. Thus physical theory becomes a substitute for the analytical character that the propositions of experimental science lack.

In other words, science, as we saw in Chapter II, is a knowledge of things in their causes arrived at by demonstration. But without theories experimental science is unable to discover the causes of the laws it formulates, nor can it deduce these laws. That is why it is only by having recourse to theories that the scientific mind can realize its ideal of rationalizing nature by making it deducible. We are touching here upon the central theme (111) which runs through the works of Meyerson. He has shown that the ultimate terminus towards which all science moves is the perfect rationalization of reality through deduction. The realization of this ideal would mean that the whole of nature could be deduced from one simple theory. And that would mean the destruction of nature, since it would involve the destruction of all heterogeneity. Thus the

full realization of the ideal of science of nature would mean its complete destruction. And this is just another example of a phenomenon which has already been noted and to which more attention will be given later: experimental science tends towards a contradiction. The realization of its ideal will ever remain a mere dialectical limit, for nature will never fail to reveal irrational elements to prevent its perfect deductibility.

To say that science tends towards monism while it moves towards pluralism is to say that it tends towards universality while it moves towards specific concretion. But it is important to note that the universality towards which it tends is not the same kind of universality from which it is escaping by its movement towards specific concretion. For as we have already pointed out, this latter universality is a mere universalitas in praedicando, which in no way lends itself to deduction. What science seeks to achieve in its construction of theories is a universality which will permit deduction. And that is why it instinctively reaches out to mathematics whose principles are not only universal in praedicando, but also in causando. And this explains why Descartes' attempt at the global deduction of nature by means of mathematics

was much more intelligent than Hegel's attempt to arrive at the same goal by means of logical categories.

It is in the construction of theories that the mind finds the fullest scope for the projection of its subjective logos into nature. For to a far greater extent than in the case of laws, physical theories are not so much a gift of nature to the mind as a gift of mind to nature. They are fictitious constructions freely chosen (112) by the subject. It is true that these constructions must be made to conform with reality. Nevertheless, this conformity is not a logical proof of the objective truth of the theory concerned, for : ex falso quodlibet. In other words, one cannot conclude to the truth of a theory from its perfect and constant verification in reality without falling into the logical fallacy of affirming the (113) consequent.

It is true that deduction from a theory can lead to the experimental discovery of a fact. For example, the law of gravitation as conceived by the theory of Relativity led to the discovery of the fact that in the neighborhood of ponderable bodies a path of light undergoes considerable deviation. This fact is true, but the truth does not derive

from the logical discourse which first suggested it. Rather, it derives formally from the experience by which it was actually discovered. And this brings us once again to the essential reason why experimental sciences are experimental: their truth is in experience only; the logical discourse is only an instrument, and even the conclusion of this discourse is only instrumental in the sense that it leads to or suggests an observation or experiment to be made. Consequently, hypotheses can be said to be "verified" only by extrinsic denomination. An infinity of theories can lead to the same conclusions. The laws of electrostatics, for example, can be "explained" successfully by a number of different theories, such as the theory of two electric fluids, or the theory of a single fluid, or the theory of discrete smallest charges, namely, electrons and protons. The corpuscular and undulatory theories of light, both of which have been successively "verified", are a classical example of the same thing.

The impression is fairly prevalent that physical theories are founded directly upon facts. This is, however, an inexact way of representing them. They are not founded directly upon experience, rather they seek to posit a point of departure from which experience may be arrived at, that

is to say, from which relations may be logically deduced which will be equivalent to those derived from experience.

It must not be thought, however, that theories may be constructed in a purely arbitrary fashion. There are certain criteria which must guide the mind in this construction. And the three most important of these criteria may be deduced from the foregoing analysis of the nature of physical theory. First, because every theory is an attempt to arrive at the most perfect unity possible, the one which has the greatest logical simplicity will be preferred to all others. Secondly, because every theory is an attempt to make nature deducible, the one which has the most perfect conformity with reality must be chosen. Thirdly, because the ideal of science is a merely dialectical limit towards which it must ever tend, that theory will be preferred which has the greatest fecundity, that is to say, which is most significantly suggestive of new experience. This last point means that a good theory is one which reaches beyond itself; if it does not give rise to new problems which it cannot adequately solve, it is not truly scientific. A good theory must not only solve problems; it must create them, for otherwise science will become static and sterile. (114)

The new experiments suggested by a theory will at once increase

the multiplicity of data and prepare for a higher synthetic
(115)
unity, that is to say, for a more perfect theory. That
is why a good theory must contain the seed of its own
destruction within its bosom. For a theory that explains
everything explains nothing. Newton's theory was good, not
only because it explained many things, but because it
brought to light things which it was unable to explain.
"Crises" are essential for the development of science,
and if contradictions did not continually arise it would
(116)
become stagnant. But it is significant that no matter
how many contradictions may arise in the face of one theory,
it is not abandoned until another theory is ready to take
its place. The mind will not descend from its plane of
rationality. All this amounts to saying that experimental
science develops through a constant intercation of objective
and subjective logos, and it is this interaction that we
must now attempt to analyze before bringing this Chapter
to a close.

8. Objective and Subjective Logos.

If there is any conclusion which emerges from
the preceding discussions it is that the evolution of science
is essentially a creative evolution. The mind does not merely

discover nature; it constructs it to its own image and likeness. And it is only by so doing that it is able to
(117)
discover it. But because this construction is never free from its relation to discovery, it is not a pure creation, but a re-creation. The mind can progress in production only by becoming increasingly dependent upon induction; it can perfect its construction, only by perfecting the instruction it receives from nature. (118) It can advance only by keeping up an incessant dialogue with reality. It cannot reason without experimenting, nor can it experiment without reasoning. This is not, however, a circle without any definite direction. For the reasoning is always orientated towards reality,

In other words, experimental science must be at once synthetic and a priori. And it is only by maintaining a proper balance between these two elements that the extremes of idealism or empiricism can be avoided. All this may be summed up by saying that experimental science is a mixture of science and art, and for this reason it is neither a science nor an art in the full sense of the word. And there is perhaps no better way of getting at its nature as a quasi science than by analyzing the way in which art enters into it.

Rousselot is correct in maintaining that in the epistemology of St. Thomas the sciences in the modern sense of the term are rather arts than sciences. (119) And it is highly significant that as the science of that part of reality which, as we saw above, cannot be defined in a more profound way than as a work of art, tends towards its perfection, its nature is transformed in such a way that there is no more penetrating way of knowing it perhaps, than by viewing it as an art. There is, in fact, a remarkable parallel between the way in which art enters into nature, and the way in which it enters into the experimental sciences of nature. As we pointed out earlier in this Chapter, all created reality is a work of art, but nowhere does divine art penetrate so deeply into reality than in the material cosmos. In the same way, art enters into all the sciences if for no other reason than that they all employ logic, but in no science does it penetrate so deeply as in the experimental natural sciences. And it is extremely important to see why this is so.

Logic reaches farther down into the structure of the sciences than might at first be supposed. It has to do even with the first operation of the mind. One might perhaps be tempted to doubt this statement. For logic has

to do with an ordering of thought, and since simple apprehension grasps things in an absolute fashion, it may be difficult to see how the mind can introduce order in relation to this first operation, as it does in the construction of propositions and syllogisms. Nevertheless, as John of St. Thomas says, "prima apprehensio absolute et
(120)
per se pertinet ad logicam." As is evident from The Categories of Aristotle, a certain distinguishing and ordering of terms is necessary prior to their construction into propositions. In this way, art surrounds the terms in all the sciences from the very beginning. But the vital point is that in all the other sciences besides the experimental sciences this art merely surrounds the terms - - it does not posit them. Only in the experimental sciences are the very terms themselves artefacta. The student of nature fabricates the very stuff out of which the whole universe of physical science is constructed. To use scholastic terminology, the objects are never a pure quod; they are always a mixture of a quod and a quo. The quod and the quo constitute an accidental unity and are considered ad modum unius.

This penetration of art into the very essence of experimental science is continued throughout its whole

structure. As we saw in our discussion of laws and theories, the form of experimental science proceeds not only from the object, but also from the subject. ⁽¹²¹⁾ The philosophical sciences, are constructed by means of art, but this art remains a purely extrinsic tool. It does not become a part of the structure itself. That is why these sciences are sciences in their own right independently of the logic they employ. But in the experimental sciences the art employed becomes an essential part of the scientific structure. That is why they are not sciences in their own right independently of the dialectics they use. They are dialectics. This point will be clarified in the next Chapter when we come to analyze the relation between experimental science and dialectics.

Another way in which art penetrates into the very essence of physics is found in its subalternation to mathematics, which is at once a science and a speculative art. How deep this penetration is may be seen by considering the intimate union existing between subalternating and subalternated sciences. The mind, which finds it necessary to reconstruct nature, discovers great scope for its artistic impulse in the vast constructibility of mathematics. In this connection attention must be called to a significant

text in the De Trinitate in which St. Thomas says: that logic, mathematics and mathematical physics "inter coeteras scientias artes dicuntur quia non solum habent cognitionem, sed opus aliquod, quod est immediate ipsius rationis, ut constructionem, syllogismum, et orationem formare, numerare, mensurare, melodias formare, cursus siderum (122) computare."

It is interesting and instructive to try to determine the nature of the art which enters into experimental science. A moment's reflection will reveal the extreme complexity of its character. For, in the first place, it is at once both speculative and practical. In so far as it involves the use of dialectics and mathematics, it is speculative; in so far as it involves a physical operation performed upon nature, it is practical. In the second place, it has characteristics which are proper both to fine art and to useful art. The fine arts are essentially arts of imitation. But as St. Thomas points (123) out, an imitation is not a mere similitude, that is to say a materially exact copy. It is the expression of an original by an intellect, and this means that the original has passed through an intellect, and in passing has acquired something of the order and light that are proper

to the intellect. And the purpose of all fine art, except religious art, is to make the original in some way better than it actually is. We believe that all this is true to some extent of experimental science. The physical universe constructed by the scientist is an imitation of the real world. It is not an exact copy or model of it. For the intellect has contributed much to this imitation. And in this imitation we make the world in some sense better than it really is. Our knowledge of material things is better than the things themselves; intelligence is the best thing in nature. The forms that are found in the mind are better than those found in reality.

But precisely because they are better they are worse. They are worse because experimental science is not a pure art but a science. That is why the whole purpose of these forms is to lead to the knowledge of the forms existing in nature. No matter how perfect the constructions of science may be, they are never anything more than mere scaffoldings. That is to say, the art that is found in experimental science is purely functional, and from this point of view it is utilitarian. Scaffolds are to some extent an imitation of the building against which they are erected for they must take on some of its general out-

lines at least. Nevertheless their most fundamental aspect does not consist in this but rather in the fact that they are built in order to reach the house.

The medieval schoolmen made a further distinction in the arts -- the distinction between those which cooperate with nature and those which do not. In the latter case there is a projection into matter of a form which is independent of the natural form that is native to the matter. In the former case there is an extrinsic assistance brought to bear to enable the natural form to achieve its end more fully. It would seem that the art which enters into experimental sciences participates in both of these categories. For in so far as it is purely functional, in so far as its purpose is to induce nature to yield up its logoi, it is an art cooperating with nature. But in so far as the projection of the subjective logoi is not a purely extrinsic assistance, as is true, for example, of the use of logic in the sciences; in so far as this projection results in the construction of a physical universe that is in a sense distinct from the absolute world condition, it shares in some way in the second category.

A number of recent authors have insisted upon the fact that modern scientific progress has meant a gradual

emancipation of science from the profound anthropomorphism that was characteristic of the views of nature current in (125) past centuries. And the truth of this can hardly be doubted. Yet if the foregoing discussion of the projection of the subjective logos into nature means anything at all, it must mean that from another point of view modern science is immeasurably more anthropomorphic than ancient science. For all art, as Bacon has remarked, is man added to nature. This is just another of the innumerable paradoxes that one constantly encounters in attempting to analyze the nature of experimental science: modern science is less anthropomorphic precisely because it is more anthropomorphic; in other words it is more objective precisely because it is more subjective. A specific example of this is found in the mathematization of nature. This mathematization is in a sense anthropomorphic for it consists in viewing nature in terms of the science that is most connatural to the human mind. And yet it is this mathematization that delivers us from the anthropomorphism which derives from the subjectivity of sense perceptions. Ernest Cassirer has brought out this paradox of modern science:

Physical thought strives to determine and to express in pure objectivity merely the natural

object, but thereby necessarily expresses itself, its own law and its own principle. Here is revealed again that 'anthropomorphism' of all our concepts of nature to which Goethe, in the wisdom of old age, loved to point, 'All philosophy of nature is still only anthropomorphism, i.e. . . . man, at unity with himself, imparts to everything that he is not, this unity, draws it into his unity, makes it one with himself... We can observe, measure, calculate, weigh, etc., nature as much as we will, it is still only our measure and weight, as man is the measure of all things. 'Only, after our preceding considerations, this 'anthropomorphism' itself is not to be understood in a limited psychological way but in a universal, critical and transcendental sense. Planck points out, as the characteristic of the evolution of the system of theoretical physics, a progressive emancipation from anthropomorphic elements, which has as its goal the greatest possible separation of the system of physics from the individual personality of the physicist. But into this 'objective' system, free from all the accidents of the individual standpoint and individual personality, there enter those universal conditions of system, on which depends the peculiarity of the physical way of formulating problems. The sensuous immediacy and particularity of the particular perceptual qualities are excluded, but this exclusion is possible only through the concepts of space and time, number and magnitude. In them physics determines the most general content of reality, since they specify the direction of physical thought as such, as it were the form of the original physical apperception. (126)

As Cassirer suggests, one of the fundamental differences between the anthropomorphism of past centuries and the anthropomorphism of modern science is that the former tended to be individualistic, whereas the latter tends to rise above the restrictions of individual sensuous

perceptions and of the interpretations proper to particular groups. There is some truth in Claude Bernard's remark, "Si l'art c'est moi, la science c'est nous." Yet of the art of which we have been speaking it may be said: "c'est nous." And the reason is that this art is at the same time a science.

All this explains the spell that mathematical physics has succeeded in putting upon the human intellect in modern times. For in it man can be at once both the homo sapiens and the homo faber.⁽¹²⁷⁾ The mind is allowed to indulge in unlimited speculation in the realm that is most connatural with it -- that of mathematics, and this speculation is inseparable from construction in which the intellect posits its own object. At the same time this speculation brings it closer to the object that is most proper to it -- the essence of ^{ori}material things. And this intimate knowledge of material things reveals the plasticity and malleability that is native to them and thus gives to the mind the power to refashion nature according to its own designs.

But this spell constitutes a great intellectual danger. For not only will man fall a prey to a kind of scientism which will make mathematical physics absorb his whole attention, in such a way that in the speculative intellect wisdom will be dethroned by science, and not by

science in the full sense of the word but by mere dialectical prolongation of science; and in the practical intellect, prudence will be dethroned by art, and not by highest form of art but by technological art - - not only will he fall a prey to this form of intellectual suicide, but because by nature he is more a being of action than of contemplation, more an artisan than a philosopher, he will be tempted to make all science a kind of art. That is to say, he will become so fascinated by the projection of his own subjective logos into nature that he will sever this projection from its complete orientation to the objective logos and make it an end in itself. Bergson has characterized this tendency in the following terms:

Nous ne dirions peut-être pas homo sapiens, mais homo faber. En définitive, l'intelligence envisagée dans ce qui en paraît être la démarche originelle, est la faculté de fabriquer des objets artificiels, en particulier des outils à faire des outils, et d'en varier indéfiniment la fabrication. "Son objet n'est pas . . . de nous révéler le fond des choses, mais de fournir le meilleur moyen d'agir sur elles." "quel est l'objet essentiel de la science? C'est d'accroître notre influence sur les choses.(128)

We have seen that experimental science is more a priori than the disciplines that are sciences in the strict sense of the word precisely because it is less a priori. That is to say, in the latter case the

connections of things are independent of the experience in which they are first recognized, and in this sense they are a priori. It is precisely because that is lacking in experimental science that a substitute a priori must be introduced. But this a priori of experimental science actually anticipates nature. The mind determines beforehand what is going to happen. And when experience confirms this anticipation the mind has in some way become the principle of experience. Experience does not manifest, it merely confirms the manifestation that the mind has made to itself. That is why the intellect in experimental science becomes the creator of the universe, as Professor Campbell has remarked:

Un Newton, un Faraday, un Maxwell, conçoivent une théorie, et la vie s'adapte pour toujours aux lois qu'ils ont prédites. Par la puissance de leur imagination, ils créent la structure durable du monde. Ils ne sont pas des créatures chétives, enchaînées par les lois du temps et des sens; ils sont les créatures qui enfantent ces lois; les vents et les flots leur obéissent. (129)

When this creative element is made an end in itself, the mind becomes utterly free, and the measure of all things. In this connection the following lines of Abel Rey are extremely pertinent:

The present era announces a new liberation, as profound perhaps as the two previous ones. It aims at these immutable, these mathematico-physical absolutes. There is no longer a tool

that serves the intellect, except the intellect itself in its inventive omnipotence. The universalization of the hypothetico-deductive method, in its broadest signification, is the logical illustration of it . . . It renews itself by changing, whenever necessary, even its very foundations. Logic, a collection of rational formulae, appears no longer as an architectural conception constructed once and for all into an unchangeable unity resting on an eternal foundation. Thought must constantly be ready to build on new foundations, or to modify the arrangement of the edifice, and consequently to complete, to adjust, and to renew its tools.(130)

This tendency has been extremely prolific and
(131)
extremely virulent in recent years. One of its results has been the instrumentalism of John Dewey. The following passage, which is typical of his thought shows how the creative element has been made the whole raison d'être of all scientific endeavor, how science has been transformed into art:

If Greek philosophy was correct in thinking of knowledge as contemplation rather than as a productive art, and if modern philosophy accepts this conclusion, then the only logical course is relative disparagement of all forms of production, since they are modes of practice which is by conception inferior to contemplation. The artistic is then secondary to the esthetic: 'creation', to 'taste,' and the scientific worker - as we significantly say - - is subordinate in rank and worth to the dilettante who enjoys the results of his labors. But if modern tendencies are justified in putting art and creation first, then the implications of this position should be avowed and carried through. It would then be seen that science is an art, that art is practice,

and that the only distinction worth drawing is not between practice and theory, but between those modes of practice that are not intelligent, not inherently and immediately enjoyable, and those which are full of enjoyed meanings. When this perception dawns, it will be a commonplace that art -- the mode of activity that is charged with meanings capable of immediately enjoyed possession -- is the complete culmination of nature, and that 'science' is properly a handmaiden that conducts natural events to this happy issue. Thus would disappear the separations that trouble present thinking: division of everything into nature and experience, or experience into practice and theory, art and science, of art into useful and fine, menial and free. (132)

Enough has been said to show that there is a sense in which the whole structure of experimental science is instrumental and functional, but as we shall point out in a few moments it is so primarily in relation to contemplation, to the apprehension of the objective logos of nature. Dewey segregates this instrumental and functional character and destroys its essential orientation.

But the tendency to exact the projection of the subjective logos has led man far beyond this form of instrumentalism. It has led him to conceive the mind as a kind of Platonic demiurge whose sole purpose is to work the world, to fashion it according to its own designs. Nature becomes merely a kind of matter for the art of man; it is viewed only in terms of its plasticity. Everything

in nature that does not yield itself up as malleable matter for the free play of human art is neglected or its existence is denied. All the proper distinctions which lift things out of pure plasticity and set them up as natures in their own right must be wiped out even at the expense of contradiction. Every determination in nature must give way before the constructive genius of man. Nature must no longer be defined as "ratio alicuius artis, scilicet divinae," but "ratio alicuius artis, scilicet humanae." Man substitutes himself for God.

We believe that this is the profound significance of the Marxist philosophy of nature and science, and in fact of the whole Marxist system. Marx writes: "La question de savoir si la pensée humaine peut comporter une vérité objective n'est pas une question théorique mais pratique. C'est dans la pratique que l'homme doit prouver la vérité de sa pensée, c'est-à-dire sa réalité, sa puissance, son en-de-ça." "Les philosophes n'ont fait qu'interpréter le monde de différentes manières. Or il s'agit de la transformer."
(133)

Bertrand Russell touched the core of Marxist philosophy when he wrote: "Roughly speaking, all matter, according to Marx, is to be thought of as we naturally

think of machinery: it has a raw material giving opportunity for action, but in its completed form it is a human product." (134) When man has succeeded in breaking down every determination which resists his creation of the cosmos, he will at last be able to "revolve ab⁽¹³⁵⁾out himself, his own true sun." Never before has there been let loose upon the world a more frightful philosophy, nor one that is more pregnant with fearful consequences.

From many points of view this doctrine is but the logical outcome of the general trend that modern thought has taken since the time of the Renaissance. In every order there has been a tendency to construct rather than to accept. And in the last analysis this revolt against mere givenness is nothing but a revolt against the finiteness of the human mind. As great an authority as Ernst Cassirer assures us that at the time of the Renaissance all the properties that the Deity had formerly claimed for itself were made the attributes of the human soul.

In so far as all this affects the philosophy of science, it is clear that the error of the moderns has been to divorce the projection of the subjective logos into nature from its essential orientation to the objective logos. The subject becomes the measure of the object only

in order that the object may in a more perfect way become
(136)
its measure. Kant was correct in pointing out that in
the construction of hypotheses we anticipate experience.
But even before we give our assent to an hypothesis we
have already admitted an objective criterion by which it
is measured, namely objective truth. For an hypothesis
must be likely, that is to say, have at least the appearance
of truth. We are not the ones who create this likeness
to truth. Moreover the only reason we posit an hypothesis
is to help us to know objective truth, and we submit it to
experience as to the determining measure of its worth. The
moderns see in the power to construct hypotheses a mani-
festation of the supreme excellence of man. Undoubtedly,
it is better to be able to construct hypotheses than to
have to remain in the state of pure passivity. But in the
last analysis the necessity of having recourse to hypothesis
in order to know nature springs from the extreme im-
perfection of the human intellect.

Yet the modern exaltation of the constructive
genius of man in experimen^tal science is but the exploitation
of a profound truth. For we have already noted that the
advancement of science means that man's knowledge of the
universe is becoming at the same time more objective and

more subjective. And it is interesting to note here in passing that something similar to this is found in Theology in which the more we get to know God the greater becomes our recourse to the via negationis which is in a sense getting us farther and farther away from Him. Now if the limit towards which experimental science tends could be reached man's knowledge of the universe would be completely objective, but at the same time the universe would be completely a projection of the subject. Man's speculative knowledge of nature would be one with his practical knowledge. Nature and art would be identified. In other words, man would be God. Surely there is profound wisdom in Dante's remark: "Si che vostr' arte a Dio quasi e nipote."

Perhaps to move
His laughter at their quaint opinions wide
Hereafter, when they come to model heaven
And calculate the stars: how they will wield
The might frame: how build, unbuild, contrive
To save appearances.

- - - Paradise Lost.

CHAPTER FIVE

EXPERIMENTAL SCIENCE AND DIALECTICS

1. The Problem

In the first book of the Topics Aristotle tells us that in seeking to discover the nature of an art it is advisable to begin by consulting those who are expert in that art. No one who attempts to follow this advice with respect to the nature of experimental science can fail to be struck by a remarkable unanimity in the opinions of those who in recent years have achieved the greatest renown as scientists. Experimental science is consistently described by them as a discourse in which from freely chosen suppositions certain conclusions are inferred. And in this hypothetical character attributed to experimental science two particular points are generally stressed: 1) it is, at least to some extent, a priori knowledge; 2) it never goes beyond probable knowledge.

In the foregoing pages some passages have already been cited which show that this represents the opinion

which the most eminent modern scientists have of their own art. Innumerable texts of similar character could easily be adduced from the writings of such experts as DeBroglie, Le Roy, Poincaré, Eddington, Planck, etc. etc. Perhaps the following lines of Sir Jeans will serve as a typical example:

We have seen that efforts to discover the true nature of reality are necessarily doomed to failure, so that if we are to progress further it must be by taking some other objective and utilizing some new philosophical principle of which we have not so far made use. Two such suggest themselves. The first is the principle of what Leibniz described as probable reasoning; we give up the quest for certain knowledge, and concentrate on that one of the various alternatives before us which seems to be most probably true. But how are we to decide which of the alternatives is most likely to be true? This question has been much discussed of late, particularly by H. Jeffreys. For our purpose it is sufficient to rely on what may be described as the simplicity postulate; this asserts that of the two alternatives the simpler is likely to be nearer to the truth . . .

In real science also a hypothesis can never be proved true. If it is negatived by future observations we shall know it is wrong, but if future observations confirm it we shall never be able to say it is right, since it will always be at the mercy of still further observations. A science which confines itself to correlating the phenomena can never learn anything about the reality underlying the phenomena, while a science which goes further than this, and introduces hypotheses about reality, can never acquire certain knowledge of a positive kind about reality; in whatever way we proceed, this is forever denied us. (1)

We cannot claim to have discerned more than a very faint glimmer of light at the best; perhaps it was wholly illusory, for certainly we had to strain our eyes very hard to see anything at all. So that our main contention can hardly be that the science of to-day has a pronouncement to make, perhaps it ought rather to be that science should leave off making pronouncements; the river of knowledge has too often turned back on itself.

Many would hold that, from the broad philosophical standpoint, the outstanding achievement of twentieth-century physics is not the theory of relativity with its welding together of space and time, or the theory of quanta with its present apparent negation of the laws of causation, or the dissection of the atom with the resultant discovery that things are not what they seem; it is the general recognition that we are not yet in contact with ultimate reality. (2)

This attitude, which Bertrand Russell character-
(3)
izes as "humble and stammering", is a far cry from the proud dogmatism of the classical physicists whose fundamental attitude towards experimental science had been summed up in Descartes' dictum that those who wish to find the true road in science must not occupy themselves with any object about which they cannot have certitude equal to that
(4)
found in the demonstrations of arithmetic and geometry.

If this new attitude is correct, then Jeans is surely right in suggesting that it represents a discovery of far greater import than the amazing discoveries of modern science itself. For the former means a growth in wisdom, whereas the latter means merely a growth in science. But

the full extent of this new attitude must be clearly recognized. The point is not that scientists have come to realize that modern experimental science knows nothing that is universal and necessary with absolute certitude, but rather that the nature of experimental science is such that it can never arrive at certain knowledge. In other words, the expression which Emil du Bois-Reymond made so famous must be applied to the very essence of experimental science: "Ignorabimus."

This new attitude raises a crucial problem for those who wish to establish the relevance of ancient epistemological schemes with modern science. In fact, the majority of contemporary writers both Scholastic and non-Scholastic seem to hold that this new attitude is incompatible with the epistemology of the ancient Peripatetics. The Scholastics see in this incompatibility a proof that the new attitude is false. The non-Scholastics see in it a proof that the old conception was only a provisional stage in the evolution towards modern thought. Both of these positions have consequences of great import. We believe that in the last analysis the first is a denial of experimental science and the second a denial of philosophy.

Sir Arthur Eddington has crystallized the issue

in the following terms:

In view of the closer contact which now exists between science and philosophy, I would like to raise one question which effects our cooperation. A feature of science is its progressive approach to truth. Is there anything corresponding to this in philosophy? Does philosophy recognize and give appropriate status to that which is not pure truth but is on the way to truth. . . It is essential that philosophers should recognize that in dealing with the scientific conception of the universe they are dealing with a slowly evolving scheme. I do not mean simply that they should use it with caution because of its lack of finality; my point is that a vehicle of progress is not furnished on the same lines as a mansion of residence. The scientific aim is necessarily somewhat different from the philosophic aim, and I am not willing to concede that it is a less worthy aim. (5)

Eddington's query: "Does philosophy recognize and give appropriate status to that which is not pure truth but is on the way to truth?" may be taken in two ways. In the first place, it may mean: does philosophy grant within its own realm a place to a vehicle of progress which is not furnished on the same lines as a mansion of residence? In the second place, it may mean: does the philosophy of science recognize the progressive approach to truth which for Eddington constitutes the very essence of experimental science, and does it admit its value and its meaning? Genuine Thomistic philosophy unhesitatingly gives an affirmative answer to both of these questions. And as we have already suggested, the explanation

of this answer must be sought for in the field of dialectics.

In so far as the first question is concerned it must be pointed out that Aristotle and St. Thomas in the most explicit fashion "recognize and give appropriate status to that which is not pure truth but is on the way to truth." And they do so not merely by granting this "vehicle of progress" an insignificant place within the realm of philosophy, but by admitting that it must make up the major portion of every philosophical treatise even of that which constitutes the very soul of all philosophy -- metaphysics. At the end of the first lesson of his Commentary on the Third Book of the Metaphysics Aquinas writes: "Dialecticam disputationem posuit quasi partes principia⁽⁶⁾ huius scientiae."

But it is evidently in the second sense that Eddington wishes his query to be understood. And here we come upon something quite different from the case just considered. Dialectics as a vehicle of progress must constitute the major portion of every philosophical treatise because the arrival at philosophical truth usually entails a long journey for the human mind. Nevertheless in philosophy there is an arrival, there is a mansion of residence furnished on different lines from the vehicle of progress, and the long journey is caused only by the

limitations of the human intellect. But in experimental science there is no arrival, there is no mansion of residence; one is committed to remain forever in the vehicle of progress. And the reason for the endless journey is not merely the limitations of the human mind, but the very nature of the object studied.

We must try to see why this is so. And our first concern will be to examine the nature of this vehicle of progress.

2. The Nature of Dialectics.

(7)

In his Commentary on the Posterior Analytics, St. Thomas brings out the difference between metaphysics, logic and dialectics:

Sciendum tamen est quod alia ratione dialectica est de communibus et logica et philosophia prima. Philosophia prima enim est de communibus, quia eius consideratio est circa ipsas res communes, scilicet circa ens et partes et passiones entis. Et quia circa omnia quae in rebus sunt habet negotiari ratio, logica autem est de operationibus rationis; logica etiam erit de his, quae communia sunt omnibus, idest de intentionibus rationis, quae ad omnes res se habent. Non autem ita, quod logica sit de ipsis rebus communibus, sicut de subiectis. Considerat enim logica, sicut subiecta, syllogismum, enunciationem, praedicatum, aut aliquid

huiusmodi. Pars autem logicae, quae demonstrativa est, etsi circa communes intentiones versetur docendo, tamen usus demonstrativae scientiae non est in procedendo ex his communibus, intentionibus ad aliquid ostendendum de rebus, quae sunt subiecta aliarum scientiarum. Sed 3
hoc dialectica facit, quia ex communibus intentionibus procedit arguendo dialecticus ad ea quae sunt aliarum scientiarum, sive sint propria sive communia, maxime tamen ad communia. Sicut argumentatur quod odium est in concupiscibili, in qua est amor, ex hoc quod contraria sunt circa idem. Est ergo dialectica de communibus non solum quia pertractat intentiones communes rationis, quod est commune toti logicae, sed etiam quia circa communia rerum argumentatur. Quaecumque autem scientia argumentatur circa communia rerum oportet quod augmentetur circa principia communia, quia veritas principiorum communium est manifesta ex cognitione terminorum communium, ut entis et non entis, totius et partis, et similium.

The term "dialectics" has come to possess a number of meanings, but its most fundamental meaning and the one to which all others can be reduced is indicated in this text:

dialectics consists in the application of an ens rationis to ens reale. That is to say, it is a process by which the intellect, starting from the modus intelligendi moves towards the modus rei. In other words, it is an attempt of the intellect to draw from mental constructs conclusions which regard reality.

This point is brought out with even greater clarity by St. Thomas when in his Commentary on the Fourth

Book of the Metaphysics he distinguishes between the dialectician and the philosopher:

Differunt autem ab invicem, Philosophus quidem a dialectico secundum potestatem. Nam majoris virtutis est consideratio philosophi quam consideratio dialectici. Philosophus enim de praedictis communibus procedit demonstrative. Et ideo eius est habere scientiam de praedictis, et est cognoscitivus eorum per certitudinem. Nam certa cognitio sive scientia est effectus demonstrationis. Dialecticus autem circa omnia praedicta procedit ex probabilibus; unde non facit scientiam, sed quamdam opinionem. Et hoc ideo est, quia ens est duplex: ens scilicet rationis et ens naturae. Ens autem rationis dicitur proprie de illis intentionibus, quas ratio adinvenit in rebus consideratis; sicut intentio generis, speciei et similium, quae quidem non inveniuntur in rerum natura, sed considerationem rationis consequuntur. Et huiusmodi, scilicet ens rationis, est proprie subiectum logicae. Huiusmodi autem intentiones intelligibiles, entibus naturae aequiparantur, eo quod omnia entia naturae sub consideratione rationis cadunt. Et ideo subiectum logicae ad omnia se extendit, de quibus ens naturae praedicatur. Unde concludit quod subiectum logicae aequiparatur subiecto philosophiae, quod est ens naturae. Philosophus igitur ex principiis ipsius procedit ad probandum ea quae sunt consideranda circa huiusmodi communia accidentia entis. Dialecticus autem procedit ad ea consideranda ex intentionibus rationis, quae sunt extranea a natura rerum. Et ideo dicitur, quod dialectica est tentativa, quia tentare proprium est ex principiis extraneis procedere. (8)

It is clear, then, that dialectics involves a process which begins with a construct and hence ab extrinsecis. That is why there is a movement in dialectics -- dialectica est tentativa; the mind attempts to pass from the extrinsic to

the intrinsic, from logical construction to reality. But as is evident from the two texts of St. Thomas just cited, there are more than one kind of construct from which the mind may attempt to reach reality. A close reading of these texts and of other passages in which Aristotle, Saint Thomas, and their medieval commentators discuss the nature of dialectics reveals that they recognized three distinct types of dialectical reasoning. The first type consists in reasoning from principles which are composed out of purely logical terms, that is to say, terms which signify second intentions. A good example of this is found in the seventh book of the Metaphysics in which the metaphysician employs a definition of substance which is not metaphysical but purely logical: substance is that of which everything is predicated and which is predicated of nothing. ⁽⁹⁾ The second type of dialectical reasoning is had when the principles employed are not proper to the science in which the reasoning takes place, but are common to several sciences. In this case the terms out of which the principles are constructed are not formed by the mind, but the principles themselves are, in the sense that their commonness is something that depends upon the intellect. It is only for the logician that angel and man are in the same genus, for

when things do not share in a natural genus, they can have only a logical genus in common. An example of this type of dialectical reasoning is suggested by Saint Thomas in the passage from the Posterior Analytics cited above: from the common principle that contraries are in the same category, one concludes that hatred pertains to the concupiscible appetite because it is the contrary of love. (10)

The third type of dialectics consists in reasoning from principles which are only probable but which are accepted as if they were certain. It might not be immediately apparent why principles of this kind can be considered logical, and how reasoning based on them can realize the property of dialectics insisted upon by Saint Thomas, namely that it be ex intentionibus, ex extraneis. The answer is this: syllogistic form necessarily requires universality, and when there is mere universality ut nunc, that is to say a universality that is not seen in things, but is supplied tentatively by the mind, there is obviously a formation by the mind. (11)

Whenever conclusions are drawn from any of these three types of principles they are purely dialectical. For conclusions must be considered formally in the light of the principles by which they are illuminated. This is

true even when only one of the premises is dialectical (in a way somewhat analogous to the case of reasoning which is formally theological even when only one of the premises is a datum of faith and the other is metaphysical). And in all reasoning of this kind the habitus employed is always the habitus of logic. That is why, if, as we shall try to show, experimental science is formally dialectics, it will be necessary to conclude that the habitus employed in it is the habitus of logic and not that of physical science. Nevertheless, it must be pointed out that while the use of dialectics in a certain matter pertains to a habitus other than the science of this matter, it is obviously necessary to have some exercise in the matter concerned in order to be able to use the dialectics. It is also worth while here calling attention to the fact that, speaking formally, the abstraction used in all types of dialectics is that of logic (i.e. a negative abstraction which falls reductive in the third degree of formal abstraction), even though the subject and predicate of the propositions may pertain to physics.

Now since all of these three types of dialectical reasoning are a functioning of a habitus that is extrinsic to the scientific habitus proper to the matter concerned,

they must from this point of view be distinguished from scientific reasoning. Yet from another point of view the first two types may be identified with scientific reasoning. For the essential property of scientific reasoning is that it is a strict demonstration, and it is evident that only the third type is lacking in demonstration.

Another way of bringing out this point is by saying that while all dialectics consists in an attempt to get at reality from a logical construct that is extrinsic to it, this construct may be extrinsic in two distinct ways. It may first of all be extrinsic from the point of view of truth, and then the reasoning is merely probable and does not give strict scientific certitude. Secondly, it may be extrinsic from the point of view of what is specifically proper to the reality concerned, and then the reasoning may give strict scientific certitude. Since a failure to grasp this important distinction may easily give rise to confusion about the way in which dialectics is employed in the study of nature, it is important to try to make it as explicit as possible. And we can best achieve this by considering the question in terms of definitions.

Definitions may be considered in two ways: either

merely as definitions, or as principles of reasoning. Taken by themselves, definitions are not propositions; they do not involve predication. Hence they cannot be either true or false, but only good or bad. Now definitions may be either intrinsic or extrinsic. They are intrinsic (or proper) when they define things in terms of what constitutes them intrinsically; they are extrinsic (or dialectical) when they define things in terms of something extrinsic to them. An apt example of this distinction is found in the two definitions of substance. The proper definition of substance is: that whose nature it is to exist in itself and not in another as in a subject. The dialectical definition is in terms of something extrinsic to substance, namely predication: substance is that of which everything is predicated and which is predicated of nothing. In this distinction we have the explanation of the contrast which Aristotle draws between the physician and the dialectician at the beginning of the De Anima:

Differenter autem definitur physicus et dialecticus unumquodque ipsorum: ut iram quid est. Hic quidem enim appetitum recontristationis, aut aliquid huiusmodi; ille autem fervorem sanguinis aut calidi circa cor. Horum autem alius quidem assignat materiam, alius vero speciem et rationem. Ratio quidem enim haec species rei. Necesse est autem hanc esse in materia huiusmodi. (12)

We have seen that since sensible matter pertains essentially to mobile beings, all physical definitions must be in terms of it. That is why any definition of the things of nature which does not include sensible matter, which attempts to define them in terms of the form alone, cannot be intrinsic and proper, since it does not touch cosmic reality in what constitutes its very being. It can be nothing but extrinsic and dialectical, for the forms of natural things can exist independently of sensible matter only in the mind; the very quod quid est of these forms demands matter.

Definitions however may not only be considered in themselves, but also in relation to the thing defined. In this sense they are virtual propositions and can become principles of syllogisms, as St. Thomas points out in the Posteriora Analytics: "Principium autem syllogismi dici potest non solum propositio, sed etiam definitio. Vel potest dici quod licet definitio in se not sit propositio in actu, est tamen in virtute propositio quia cognita definitione, (13) apparet definitionem de subjecto vere praedicari." Con- sidered in this way, definitions may be either scientific or dialectical. They are scientific if the connection with the thing defined is necessary, in other words if they are virtual propositions that are true. They are dialectical if

the connection is not necessary, in other words if they are
virtual propositions that are merely probable. (14) It is
clear, then, that definitions can be truly scientific and
at the same time dialectical in the first sense of the term.
It is likewise clear that they can be truly physical and
natural, and at the same time dialectical in the second
sense. Hence it is extremely important to keep distinct
these two ways in which the term "dialectics" is employed
by Aristotle and St. Thomas in relation to natural doctrine.

And now, having made these necessary distinctions
between the various meanings of dialectics, we must try to
see in what sense experimental science can be called
dialectical. From all that was said in the last Chapter it
should be evident that the most fundamental way in which
experimental science is dialectical consists in this that
in it the mind attempts to get at the truth about nature by
means of hypothetical and hence probable reasoning. Con-
sequently in this Chapter we shall concentrate upon the
meaning of dialectics in which it is opposed to what is
strictly scientific, that is to say, to what involves true
demonstration, and leave the consideration of other ways in
which physics is dialectical to future contexts. Taken in
this sense, dialectics is defined by Aristotle at the opening

of the first book of the Topics as: "methodus per quam possimus argumentari de omni proposito problemate ex probabilibus et ipsi disputationem sustinentes nihil dicamus repugnans." The central notion that must be analyzed in this definition is obviously that of probability.

There are two kinds of probability: real and dialectical. / The former belongs to objective reality independently of knowledge, and it arises from the indeterminism of nature. The existence of chance in nature means that there are some future events which are not completely predetermined in their causes. These events are not necessary, and hence are at best only probable. Only conjectural knowledge can be had of them. (15) Even the most perfect created intelligence is unable to foresee them with certitude. Of course a created intelligence can judge with certitude of the present probability of the future, and in this sense real probability can be the foundation of a true proposition. But the truth of the future event does not follow from the truth of the present probability. / Dialectical probability is not founded as real probability is upon an indetermination inherent in things, but upon an indetermination proper to the intellect which must move from potency to act. And it is with this type of probability that we

are concerned in the dialectics of experimental science.

Aristotle defines dialectical probability in the following terms: "Probabilia autem sunt, quae videntur omnibus vel plerisque vel sapientibus, atque his vel omnibus vel plerisque vel maxime notis et claris." ⁽¹⁶⁾ The important word in this definition is "videntur". Probability must be defined in terms of appearances. As Aristotle points out in the fourth book of the Topics, ⁽¹⁷⁾ the probable is not a species of being. It must not be defined in terms of being, but in terms of that which has the likeness of being -- that which appears to be. Just as being gives rise to truth in the mind, so the likeness of being gives rise to the likeness of truth. That is why in the Rhetoric Aristotle defines probability as that which is similar to ⁽¹⁸⁾ the truth. Probability means verisimilitude. In other words, just as truth is the adequation of the mind with what is, so probability is the adequation of the mind with what appears to be. And this explains why, as Aristotle ⁽¹⁹⁾ suggests in the Rhetoric, the same natural impetus which moves the mind to seek after truth and take delight in it, likewise moves the mind to seek after its likeness and take delight in it, even though this delight is not completely satisfying. In his commentary on the Topics Sylvester Maurus

writes:

Respondet Aristoteles Dialecticam distingui a Philosophia per hoc, quod licet dialecticus versetur circa res omnes et circa omnia problemata, sicut philosophus scientificus, adhuc differunt in modo considerandi. Philosophus enim non est contentus apparentia, sed examinat omnia secundum veritatem, ac quaerit, propria principia et proprias causas rerum; dialecticus e converso contentus est quadam apparentia veri et procedit ex communibus et probabilibus, quae causant solam opinionem. (20)

A first reading of Aristotle's definition cited above may make one wonder why in it he gives so much attention to the various kinds of knowers. But from what has just been said it should be clear that probability must necessarily be defined in terms of the knower and not in terms of the thing known. In other words, it is essentially related to appearances and hence to the apprehension of the knower and not to objective reality. (21)

The judgment which is the subject of the qualification "probable" is known as opinion. Just as a truly scientific judgment is necessarily true, so an opiniative judgment is necessarily probable. Opinion is opposed to certitude as indetermination to determination. And the indetermination that is proper to opinion is in the mind and not in things. (22) In other words, the object of

opinion considered formally as such exists only in the
(23)
apprehension, By the indetermination found in opinion
the mind is opposed to reality as logical being is opposed
to real being. In other words the mind interposes itself
so to speak between itself and reality. And the attempt
to arrive at reality from this state of indetermination will
be a dialectical process.

There was profound wisdom in the recognition by
the ancient Greeks of the fact that at least much of the
study of nature was merely doxa and not episteme in the
strict sense of the word. For a study which can never rise
above the appearances presented by experience except by
having recourse to hypotheses which are never more than
probable and whose sole purpose is to "save the phenomena",
can never rise above the state of opinion, can never become
a science in the strict sense of the word. In this
connection St. Thomas writes:

... ita et in processu rationis, qui non est
cum omnimoda certitudine, gradus aliquis
invenitur, secundum quod magis et minus ad
perfectam certitudinem acceditur. Per
huiusmodi enim processum, quandoque quidem
etsi non fiat scientia, fit tamen fides vel
opinio propter probabilitatem propositionum,
ex quibus proceditur: quia ratio totaliter
declinat in unam partem contradictionis,
licet cum formidine alterius, et ad hoc
ordinatur Topica, sive Dialectica. Nam
syllogismus dialecticus ex probabilibus est,
de quo agit Aristoteles in libro Topicorum.(24)

But before turning to consider the way in which the dialectics of probable reasoning is employed in experimental science, we must try to determine a bit more accurately its precise nature. It should be fairly evident from what has already been said that it pertains to what the schoolmen termed logica utens, as opposed to logica docens which merely gives the rules for the application of scientific principles that are already given and which does not enter into the very construction of these principles. But the term logica utens is employed in a variety of ways, and John of St. Thomas has brought out with great clarity the sense in which it must be understood here:

Tertius usus Logicae est ipsi specialissimus, quatenus praebet usum in aliis scientiis seu materiis probabiliter disputandi sine hoc, quod procedatur demonstrative et resolute usque ad prima principia. Et tunc proprie dicitur Logica utens, ut distinguitur a demonstrante et docente, eo quod demonstrans non praecise utitur discursu sistendo in eo sed pervenit resolvendo usque ad prima principia, quae discursu non probantur, sed sunt terminus discursus. Utens sitem discursu, sed non demonstrans, ita utitur et sistit in discursu, quod non pervenit ad terminum discursus, qui est resolutio usque ad prima principia, et hoc pertinet ad procesum disputativum seu tentativum, quando inquirendo, non autem resolvendo proceditur. Et ita vocatur probabilis processus, quia non cum certitudine ultimae resolutionis usque ad principia fit. Hic est actus Logicae utentis, et sic explicat illum D. Thomas opusc. 70, q.6. art.1 dicens

Logicam utentem esse, quae utitur discursu, sed non termino discursus, qui terminatur in principia per se nota, ubi cessat usus rationis discurrentis . . .

Logica utens tertio modo accepta solum versatur circa partem topicam et sophisticam, id est processu non resolutivo, sed probabili seu probativo et disputativo. Et si talis usus fiat in aliis scientiis ex principiis talium scientiarum disputando ex illis et non resolvendo, talis usus pertinet ad Logicam solum directive; si autem procedat ex principiis ipsius Logicae talis disputatio non resolutiva, non solum directive, sed elicitive erit a Logica, quasi actus secundarius et imperfectus . . .

Expressius autem hoc tradit D. Thomas opusc. 70 cit. q. 6, art. 1., ubi docet, 'quod aliquando dicitur processus rationalis ex termino, in quod sistitur procedendo. Ultimus autem terminus, ad quem rationis inquisitio perducere debet, est intellectus principiorum, in quae resolvendo iudicamus; quod quidem quando fit, dicitur demonstratio. Quando autem inquisitio rationis usque ad ultimum terminum non perducit, sed sistitur in ipsa inquisitione, quando scilicet quarenti adhuc manet via ad utrumlibet, sic rationalis processus distinguitur contra demonstrativum. Et hoc modo procedi potest rationabiliter in qualibet scientia, ut ex probabilibus paretur via ad necessarias conclusiones. Et hic est alius modus, quod Logica utitur in aliis scientiis, non ut est docens, sed ut utens. Sic D. Thomas . . .

Et si hoc faciat praebendo principia propria tali discursui et disputationi, elicitive totum illum discursum producet Logica, quia non solo praebet modum disputandi, sed etiam materiam seu principia. (25)

In order to understand that passage correctly it is necessary to recall the distinction made above between the two ways in which the extrinsic character of dialectics can be understood. When John of St. Thomas suggests that

the use of dialectics which he terms directivus does not provide the principles for the process of reasoning, but merely the modus disputandi he obviously has in mind the meaning of extrinsic in which it signifies something exterior to the matter that is specifically proper to the science involved, as in the case of the definition of anger in terms of form alone, or of substance in terms of predication. For if extrinsic were understood in the other sense, then even the dialectics of probable reasoning must be said to provide the principles.

In any case, it is in the use of logic which John of Saint Thomas calls directivus that we are now particularly interested. Later we shall have occasion to see that mathematical physics also involves a use of logic that is similar to what he terms elicitivus, in so far as an attempt is made to explain natural phenomena in terms of logical constructs.

It is clear that a study which remains within the dialectical discourse just described without ever being able to emerge from it can never be a science in the strict sense of the word. It is not a science in its own right, since it never achieves strict demonstration. Nor can it be considered

a logical science, since the logic involved is not logica docens but utens. The following passage from St. Thomas' Commentary on the Metaphysics is relevant here:

Licet autem dicatur, quod philosophia est scientia, non autem dialectica et sophistica, non tamen per hoc removetur quin dialectica et sophistica sint scientiae. Dialectica enim potest considerari secundum quod est docens, et secundum quod est utens. Secundum quidem quod est docens, habet considerationem de istis intentionibus, instituens modum quod per eas procedi possit ad conclusiones in singulis scientiis probabiliter ostendendas; et hoc demonstrative facit, et secundum hoc est scientia. Utens vero est secundum quod modo adiuncto utitur ad concludendum aliquid probabiliter in singulis; et sic recedit a modo scientiae. Et similiter dicendum est de sophistica; quia prout est docens tradit per necessarias et demonstrativas rationes modum arguendi apparenter. Secundum vero quod est utens deficit a processu verae argumentationis. Sed in parte logicae quae dicitur demonstrativa, solum doctrina pertinet ad logicam, usus vero ad philosophiam et ad alias particulares scientiae quae sunt de rebus naturae. Et hoc ideo, quia usus demonstrativae consistit in utendo principiis rerum, de quibus fit demonstratio, quae ad scientias regales pertinet, non utendo intentionibus logicis. Et sic apparet, quod quaedam partes logicae habent ipsam scientiam et doctrinam et usum, sicut dialectica tentativa et sophistica; quaedam autem doctrinam et non usum, sicut demonstrativa. (26)

From all that has been said thus far it follows that the meaning which the term "knowledge" has for us when applied to experimental science coincides exactly with the sense in which it is understood by Sir Arthur Eddington;

Some writers restrict the term 'knowledge' to things of which we are quite certain; others recognise knowledge of varying degrees of uncertainty. This is one of the common ambiguities of speech as to which no one is entitled to dictate, and an author can only state which usage he has himself chosen to follow. If 'to know' means 'to be quite certain of', the term is of little use to those who wish to be undogmatic. I therefore prefer the broader meaning; and my own usage will recognize uncertain knowledge. (27)

Enough has been said to show that if we wish to discover the principles which reveal the true nature of experimental science it is to the Topics especially that we must turn. And it is extremely significant that this part of logic has been almost completely neglected by modern scholastics. In fact, the teaching of logic has been almost exclusively limited to the Prior and Posterior Analytics. And we believe that there is a connection between the scholastics' neglect of dialectics and their neglect of movement towards concretion in the study of nature. This disregard for the importance of dialectics goes back as far as John of St. Thomas himself;

In secunda vero parte agemus de his quae pertinent ad materiam logicalem seu ad posterioristicam resolutionem, maxime in demonstratione, ad quam praecipue ordinatur. (28)
Quae enim pertinent ad partem topicam, quae agit de probabilitatibus, et quae pertinent ad libros Elenchorum qui agunt de parte sophistica, omittuntur in praesenti, quia non agunt de certa et perfecta

resolutione iudicii, et ideo solum libri Priorum
et Posteriorum vocantur analytici ab Aristotele. (29)

At the time that these lines were written the modern development of experimental science was already underway. Without realizing it, men like Galileo had already discovered in dialectics a potent intellectual instrument for the advancement of the study of nature in the direction of concretion. It remains for us to see just how this dialectical instrument is employed by experimental science.

3. Dialectics and Experimental Science.

As we have already explained, the propositions that are proper to experimental science are devoid of intrinsic and objective universality. But because the intellect cannot remain imprisoned in singularity, the scientist is lead to confer universality upon them ab extrinseco. In order to get at the reason for the regularity appearing in nature, the scientist is lead to act as if these propositions were universal. In so doing he is applying the principle laid down by Aristotle in the Topics :
"quaecumque in omnibus aut in plurimis apparent, sumenda
(30)
sunt quasi principia et probabiles theses." In this way

he uses the principle dici de omni in the sense in which it is employed in the Priora where it is not restricted to science in the strict sense of the term, but is common to both science and dialectics:

Ad quod sciendum est quod dici de omni, prout hic sumitur, addit supra dici de omni, prout sumitur in libro Priorum. Nam in libro Priorum accipitur dici de omni communiter, prout utitur eo et dialecticus et demonstrator. Et ideo non plus ponitur in definitione eius, quam quod praedicatum insit cuilibet eorum quae continentur sub subiecto. Hoc autem contingit vel ut nunc, et sic utitur quandoque dici de omni dialecticus; vel simpliciter et secundum omne tempus, et sic solum utitur demonstrator. (31)

We have already pointed out that these propositions which are posed by the scientist instead of being imposed upon him are purely functional. Their position must lead to something beyond themselves. They are instruments - - principles of research. In other words, they are dialectical. The mind uses them in order to get at reality.

But as we explained in the last Chapter, these universalized propositions do not satisfy the mind, for they do not "save the phenomena". That is to say, they merely state the connection between subject and predicate without giving the reason for it. Consequently, the mind is lead to reach out for the propter quid by constructing hypotheses which will give a provisional explanation of the

experimental propositions. In other words, purely experimental propositions contain an implicit problem, and in order to solve this problem we transform propositions into questions which anticipate experience. In connection with this use of hypothesis it is worth while pointing out, lest confusion arise, that the term "hypothesis" (suppositio) usually meant for Aristotle and St. Thomas something quite different from the sense in which it is now understood. It did not mean something that was lacking in certainty, and that as a consequence could not be demonstrated. On the contrary, it meant something that was absolutely certain, but that was accepted without demonstration either because of its self-evidence or because of its demonstration in another science, or at least because of its acceptance by the adversary or the disciple with whom he who used it had to deal. (32) It is clear, however, from the passages cited in the last Chapter from the De Coelo etc. with regard to the planetary systems that the ancients also recognized the use of hypothesis in the modern sense of the term. Taken in this sense it means, as we have already suggested, a proposition or a group of propositions posed by the mind in order to save sensible phenomena by offering a provisional explanation of the reason behind experimental propositions.

An hypothesis never goes beyond probability; it is, as someone has said "an educated guess" -- an anticipated solution of a problem. It is essentially the product of the creative imagination and of scientific construction. From hypotheses of this kind posited as premises, the mind seeks to deduce conclusions which square with sensible experience and thus explain it. It is clear that these hypotheses are purely dialectical: they are constructions by which the mind attempts to arrive at the nature of reality.

The scientist accepts what is similar to the truth as if it were the truth and uses it as a principle of research. In doing so he is following the natural appetite of the mind which as we saw above must seize upon what is similar to the truth when it cannot have the truth. The student of nature must multiply without end his conjectures and must fix attention more upon their operative, functional, instrumental value than upon anything else.

Les théories n'ont pas pour but de nous révéler la véritable nature des choses; . . . leur but unique est de coordonner les lois physiques que l'expérience nous fait connaître... Peu nous importe que l'éther existe réellement; c'est l'affaire des métaphysiciens; l'essentiel pour nous, c'est que tout se passe comme s'il existait, et que cette hypothèse est commode pour l'explication des phénomènes." (33)

Ever remaining within the realm of the conjecturable, the experimental scientist must carry on a methodical interrogation of nature which never has any final issue. The art which guides this methodical interrogation is dialectics.

The mind is therefore free in the construction of these hypotheses. We have already quoted several passages from Einstein which show that the premises of experimental science are free inventions, creations. This freedom is not absolute, to be sure, for the dialectics of experimental science must always be kept in tow, so to speak, by constant recourse to experience. Nevertheless there is liberty and creativity in this dialectics. The scientist is free to choose between contrary or contradictory hypotheses the one which seems to serve his purpose best at the moment. He is, for example, free to choose between the opposing corpuscular and wave theories of light the hypothesis which gives him the greater help in achieving his task. All this recalls what St. Thomas has to say about the dialectician:

Secundo, ibi: Dialectica etc., ponit differentiam inter dialecticam propositionem et demonstrativam, dicens quod cum propositio accipiat alteram partem enunciationis, dialectica indifferentem accipit quancumque earum. Habet enim viam ad utranque partem contradictionis, eo quod ex probabilibus procedit. Unde etiam et in proponendo accipit utramlibet partem contradictionis et quaerendo

proponit. Demonstrativa autem propositio accipit alteram partem determinate, quia nunquam habet demonstrator viam, nisi ad verum demonstrandum. Unde etiam semper proponendo accipit veram partem contradictionis. Propter hoc etiam non interrogat, sed sumit, qui demonstrat quasi notum. (34)

Because these hypotheses are never more than probable, experimental science must ever call into question not merely its conclusions but its very principles. And this characteristic of dialectics, as St. Thomas points out in his Commentary on the Posterior Analytics:

Sciendum tamen est quod interrogatio aliter est in scientiis demonstrativis et aliter est in dialectica. In dialectica enim non solum interrogatur de conclusione, sed etiam de praemissis: de quibus demonstrator non interrogat, sed ea sumit quasi per se nota, vel per talia principia probata: sed interrogat tantum de conclusione. Sed cum eam demonstraverit, utitur ea, ut propositione, ad aliam conclusionem demonstrandam. (35)

This brings out the difference in the way dialectics is employed by philosophy and by experimental science. In philosophy it is used merely as an instrument to search out principles which, when found, impose themselves upon the mind by their certitude. In experimental science, dialectics is employed not merely in the search for principles but in the very choice and positing of the principles. (36) This ties up with what we saw in chapter IV about the difference between the Thomistic and the Kantian meaning of a priori.

In all this we have the reason why experimental science is essentially variable and transitory -- a vehicle of progress and not a mansion of residence. And in this connection De Broglie writes:

Il ne faut pas s'étonner si souvent la découverte d'un ordre nouveau de phénomènes vient renverser comme un château de cartes nos plus belles théories, car la richesse de la nature dépasse toujours nos imaginations. Les savants sont bien hardis de vouloir reconstruire par la pensée des portions de l'univers: la grande merveille, c'est qu'ils y ont parfois réussi. (37)

As Dotterer has remarked, "the first principles of the sciences must be regarded as postulates; and there is a sense in which all science is founded on faith". (38) It was because Claude Bernard recognized the dialectical character of experimental science that he made doubt the great experimental principle: "The great experimental principle, therefore, is doubt, the philosophical doubt which leaves the mind its freedom and initiative, and from which come the most valuable qualities in an investigator in physiology and in medicine." (39)

Experimental science advances by a gradual rationalization of irrational elements; but this demands a continual reorganization of its rational system. Both the method employed and the corpus of doctrine achieved must ever

remain open to revision. The only way that experimental science has to develop is by a continual process of substitution. It can grow only by passing through crises
(40)
and revolutions.

It is clear, then, that all the propositions which make up the structure of experimental science are reducible to what St. Albert the Great calls "interrogatio consensus in probabile".

Sed dialectica propositio est interrogatio consensus in probabile, nec consensus requireretur si probari non deberet; manifeste autem falsum probari non potest, et manifeste verum non indiget probari, sed ad alterius alicuius assumitur probationem. In diffiniendo ergo propositionem dialecticam secundum potissimum suum statum dicimus, quod propositio dialectica est interrogatio probabilis, ita quod probabilis sit genitivi casus, hoc est, interrogatio de probabili, quod est materia propositionis dialecticæ. In probabili enim (quia ponitur in iudicio eius cui proponitur, utrum sic videatur vel non) oportet quaerere respondentis iudicium et consensum, antequam procedere possit opponens. Sic ergo dialectica propositio interrogatio est probabilis. Et hac ratione etiam Boetius in diffinitione syllogismi dicit, quod est oratio in qua quibusdam positis et concessis, respiciens ad propositionem syllogismi dialectici. Cuius causa est, quod probabile de se non habet sufficientem causam consequentiæ vel inferentiæ, et causam inferentiæ sufficientem accipit a concessione respondentis. Haec igitur est tota diffinitio propositionis dialecticæ.
(41)

Sir James Jeans has brought out the dialectical

character of the scientist's interrogation of nature:

Such an experiment, like every other, amounts in effect to asking a question of nature. This question can never be -- 'Is hypotheses A true?' but 'Is hypothesis A tenable?' Nature may answer our question by showing us a phenomenon which is inconsistent with our hypothesis or by showing us a phenomenon which is not inconsistent with our hypothesis. She can never show us a phenomenon which proves it; one phenomenon is enough to disprove a hypothesis but million million do not suffice to prove it. For this reason, the scientist can never claim to know anything for certain, except direct facts of observation. Beyond this, he can only proceed by building up hypotheses, each of which covers more phenomena than its predecessor, but each of which may have to give place to another hypothesis in due course. Strictly speaking, the time for replacing a hypothesis by a claim to certainty never arrives. (42)

(43)

As von Uexkull has pointed out, the art of interrogation of nature, the art of research is characteristic of the experimental scientist. We feel that enough has been said to show that this art is substantially the same as the "logica interrogativa", "tentativa", "inquisitiva", "inventiva" of the ancients, i.e. the dialectics of the (44)

Topics. And it is extremely significant to note the similarity between the following passage of von Uexkull and the lines quoted earlier in this Chapter from St. Thomas' Commentary on the Fourth book of the Metaphysics:

"dialecticis autem procedit ad ea consideranda ex intentionibus rationis, quae sunt extranea a natura rerum. Et ideo

dicitur, quod dialectica est tentativa, quia tentare proprium est ex principiis extraneis procedere."

In the present book I have endeavoured to frame the theoretical considerations concerning biology, in such a way that there can no longer be any doubt that, in their very nature, biological doctrines always remain unsolved problems.

In nature everything is certain; in science everything is problematical. Science can fulfill its purpose only if it be built up like a scaffolding against the wall of a house. Its purpose is to insure the workman a firm support everywhere, so that he may get to any point without losing a general survey of the whole. Accordingly, it is of the first importance that the structure of the scaffolding be built in such a way as to afford this comprehensive view; and it must never be forgotten that the scaffolding does not itself pertain to Nature, but is always something extraneous. (45)

The comparison of science with a scaffolding, which had already been employed by Goethe, is, as we suggested in the last chapter, very exact. It brings out the fact that experimental science is essentially a logical construction which the mind uses in an attempt to get at reality. As we shall point out in chapter XI, it is not a formal sign of nature, but purely an instrumental sign. Just as a scaffolding can be made to approach closer and closer to the form of the house and thus be brought to take on gradually a greater likeness to the house, so experimental science can approach ever closer and closer to nature and in so doing take on a greater likeness to

nature. But just as a scaffolding can never become the house and must ever remain an extrinsic construction, so science must ever remain an extrinsic construction of the mind. In fact, as we suggested in chapter IV the closer it gets to nature the more extrinsic it becomes, because of the fact that the subjective construction constantly increases. As we shall point out in chapter XI, there is a great deal of similarity between the dialectical approach of science to nature and the dialectical movement of a regularly inscribed polygon with constantly increasing sides towards a circle. Just as the multiplication of the sides of the polygon makes it more like a circle, and at the same time more of a polygon and hence more unlike a circle (which has only one "side") so the movement of science towards nature makes it at once more objective and more subjective.

A number of objections may suggest themselves in regard to this identification of experimental science with dialectics. In the first place, one may be tempted to ask: if experimental science is dialectics, in what sense can it be considered as a part of natural doctrine?

The answer is: experimental science is natural doctrine principally because of the limit towards which its dialectical movement is orientated, i.e. nature. In other words, it is natural doctrine not so much because of what it has achieved at any given stage of its development as because of what it is at all times attempting to achieve. To get back to the example used above -- the circle is the limit of the polygon only in so far as the latter is in a state of movement through the successive multiplication of its sides. If this movement should stop at any one given polygon, no matter how far advanced it may be in the series, the circle can no longer be considered as the limit. Similarly, natural doctrine, in so far as it is built upon hypothesis, must ever remain in a state of movement towards its limit which is nature, that is to say, the absolute world condition. No given stage of the development of experimental science can be considered natural doctrine in an absolute sense. To so consider it would be to identify a subjective construct with objective nature -- which would be comparable to identifying a polygon with a circle. Nevertheless, just as a given polygon that is far advanced in the series which tends

towards the circle is already in some way a revelation of the nature of the circle, so any given stage of the construction of experimental science is in some measure a revelation of objective nature. And just as a polygon of a million sides is closer to the circle than a polygon of ten sides, so modern physics knows nature better than the physics of the four elements. We shall return to examine these notions in fuller detail in chapter XI. For the moment, it is interesting to compare what has just been said with the following passage from von Uexkull:

A man may have assimilated the conclusions of natural science in the form of doctrine, and may know how to employ them in speculation, according to the rules of logic; but he still knows nothing whatsoever concerning Nature — or at any rate, infinitely less than does any peasant or gardener who is in daily intercourse with her. (46)

This statement, which at first sight appears to be an extreme exaggeration, can be accepted if viewed in the light of our foregoing remarks. In so far as experimental science is a subjective hypothetical construction, the scientist may be said to know nothing about nature in its purely objective condition. Nevertheless, because this subjective construction is in some measure a reflection of nature, von

Uexkull is correct in immediately qualifying his initial absolute statement. And there is a sense in which it is true to say that experimental scientists know infinitely less about nature than gardeners and peasants, who are, though in an extremely obscure way, in contact with objective nature. The actual vegetables with which gardeners deal are certainly not constructed according to the hypotheses of biology. This would suppose that biology had achieved a knowledge of the true essence of living things. "Scientific vegetables" are not edible.

A second objection to our identification of experimental science with dialectics might be that in innumerable places Aristotle and St. Thomas condemn the Platonists and the Pythagoreans for proceeding "logice sive
(47)
dialectice in naturalibus." An attentive examination of these texts, however, will immediately reveal that they do not condemn the use of dialectics as such in the study of reality. As a matter of fact, both of them have frequent recourse to it. What they do condemn is the abuse of dialectics, which consists in granting priority to principles over experience, when, as a matter of fact, the former should ever remain in complete dependence upon the

confirmation of the latter. Instead of rejecting principles in order to save appearances, the Platonists made it a practice of rejecting sensible appearances in order to save their preconceived principles. This is evident from the passage from the third book of the De Coelo quoted in the last chapter. In other words, the condemnations of Aristotle and St. Thomas are levelled against the logical error of confusing a formal consequence with an argument, which would make dialectics self-sufficient and independent in the study of nature.

A final objection which might be brought to bear against the identification of experimental science with the Aristotelian dialectics of the Topics is that the very definition which the Stagirite gives of the latter (48) seems to indicate that it is essentially a method of discussion with adversaries and that consequently it presupposes a dialogue. It is true that dialectics essentially involves a kind of dialogue, since, as we have seen, its principles are always "interrogationes probabiles." It may also be granted that in writing the Topics Aristotle had principally in mind the use of dialectics which involves a plurality of persons. But the dialogue of dialectics

tics does not necessarily suppose such a plurality. In dialectical reasoning one person can start with what seems probable to him and seek his own assent to it. Moreover, even without a plurality of persons there is always an adversary, namely the other part of the contradiction.

In this dialectical character of experimental science we find the basic reason why physics inevitably issues into mathematical physics. Not finding scientific certitude within its own realm, it attempts to acquire for itself a substitute certitude by reaching up to mathematics. From this point of view, Bertrand Russell is correct in saying that "physics is mathematical, not because we know so much about the physical world, but because we know so little."⁽⁴⁹⁾ What we have been saying in this chapter also brings to light the reason why mathematics in the modern sense of the term is a natural prolongation of the dialectics of experimental science. Dialectics bestows upon physics the hypothetico-deductive method which is so characteristic of modern mathematics. And in this connection it is extremely interesting to compare what we have said about the nature of dialectical reasoning from freely chosen hypotheses with Bertrand Russell's famous definition of mathe-

atics:

Pure mathematics consists entirely of assertions to the effect that, if such and such a proposition is true of anything then such and such another proposition is true of that thing. . . Thus mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true. (50)

This brings us to the task of analyzing the proper nature of mathematics.

CHAPTER SIX

THE NATURE OF MATHEMATICAL ABSTRACTION

1. Mathematical Abstraction.

History has played with the term "mathematics" in a way similar to that in which it has played with the term "science". We have seen that the latter term now has a meaning quite distinct from, and to a certain extent opposed to, the meaning it had for the ancients: it no longer signifies certain knowledge of things in their causes, but a purely dialectical type of knowledge that is lacking in certitude. In somewhat the same way, the meaning of the term "mathematics" has undergone a profound change. For the ancients it signified a strictly unified science specified by a definite formal object, namely quantity. But in recent years mathematics has been divorced from its essential relation to quantity and given a range that extends indefinitely beyond its confines.

In former days, it was supposed (and philosophers are still apt to suppose) that quantity was the fundamental notion of mathematics. But nowadays, quantity is banished altogether, except from one little corner of Geometry, while order more and more reigns supreme. The investigation of different kind of series and their relations is now a very large part of mathematics, and it has been found that this investigation can be conducted

without any reference to quantity, and, for the most part, without any reference to number. All types of series are capable of formal definition, and their properties can be deduced from the principles of symbolic logic by means of the Algebra of Relatives.
(1)

Mathematics is no longer a strictly unified science; it no longer has a definite formal object. And the result is that most of what is now considered mathematics is not mathematics in the original sense of the term; it is dialectics. In this chapter we shall try to analyze the nature of mathematics in the strict and formal sense of the term, in the sense
(2)
in which it was understood by the ancient Thomists.

One of the objections brought against the relevance of Peripateticism for the question of science is that it necessarily minimizes the importance of mathematics because of the fact that it considers quantity merely as one out of
(3)
ten predicaments. As a matter of fact, however, Peripatetics have always accorded to quantity a unique position among all the categories. For of all the nine accidents it is the one closest to substance. And it is the only one of the accidents that can be the subject of a special science. For all science deals with a subject manifesting itself through certain definable properties, and quantity is the only accident in

which there is found both subject and properties. This explains why quantity and the quantitative can constitute, in relation to knowledge, a closed universe apart from everything else:

Sciendum autem est quod quantitas inter alia accidentia propinquior est substantiae. Unde quidam quantitates esse substantias putant, scilicet lineam et numerum et superficiem et corpus. Nam sola quantitas habet divisionem in partes proprias post substantiam. Albedo enim non potest dividi, et per consequens nec intelligitur individuari nisi per subjectum. Et inde est quod in solo quantitatis genere aliqua significantur ut subjecta, alia ut passiones. (4)

But in order to get at the nature of this special science it is necessary to point out that it is not quite accurate to call mathematics the science of quantity. For the other two speculative sciences, metaphysics and philosophy of nature, also deal with quantity in some way. Metaphysics deals with it in so far as it is a principle of being -- one of the nine accidents. Philosophy of nature deals with it in so far as in nature there is mobility in the genus of quantity, which is characteristic of those mobile beings which have life. Consequently, in order to get at the intrinsic nature of mathematics, it will be necessary to consider the particular way in which it deals with the notion of quantity, it will be

necessary to analyze as accurately as possible the special nature of mathematical abstraction.

A number of things were said about the nature of this abstraction in Chapter II. Before pushing ahead in our analysis let us recapitulate briefly the points already laid down.

Mathematical abstraction is the second degree of formal abstraction. It stands midway between physical and metaphysical abstraction, and shares to some extent in both. Yet from another point of view, it is not midway between the first and third degrees of abstraction, in the sense of being in direct line with them. Rather it is out of line, off to one side, so to speak. And in this connection it is interesting to note that while the term "metaphysics" is an historical accident, it is an extremely happy accident in the sense that it characterizes quite accurately the nature of the science it has been chosen to designate. From this point of view it is highly significant that mathematics, though coming directly after physics in the degrees of abstraction, is not called metaphysics. Nor is metaphysics called metamathematics, though it comes immediately after

mathematics. And yet when physics begins to seek a substitute cause and reason to explain its facts, it is not to metaphysics that it naturally turns, but to mathematics. This is a paradox upon which we must endeavor to throw some light.

Mathematical abstraction prescind from all sensible matter, though not from intelligible matter. By sensible matter we understand matter with sensible qualities, and hence apprehensible by the senses. It is important to distinguish between mathematical quantity and the common sensibles. As we shall see there is a close connection between the two, but they are not identified, precisely because the common sensibles are sensible. A mathematical line, a number, etc. are by definition not sensible. (5) By intelligible matter we mean the substance considered as the subject of quantity, which is the order of the parts of the substance. This abstraction gives to mathematics an object which depends upon sensible matter for its being, but not for its "being known", that is, it is conceived by the mind and defined independently of all sensible matter, but in order for it to exist outside the mind, it must be realized in sensible matter.

As we pointed out in Chapter II, this profound dichotomy between subjective and objective existence is something peculiar to mathematical abstraction. It is found neither in physical abstraction, in which the object is dependent upon sensible matter both for its existence in the mind and its existence outside the mind, nor in metaphysical abstraction, in which the object is independent of sensible matter both for its existence in the mind and its existence outside the mind. We suggested that this dichotomy found in mathematical abstraction is extremely significant, and the time has now come to explore that significance.

We know of no better point of departure for this exploration than a consideration of a text of Saint Thomas which at first sight might appear somewhat confusing, but which actually contains the key to the nature of mathematical abstraction. As we noted in Chapter II, in the third article of the fifth question of the De Trinitate, Aquinas seems to restrict the expression "formal abstraction" to the type of abstraction found in the mathematical sciences. He points out in fact that there are two kinds of abstraction: the abstraction of a form from matter, and the abstraction of a universal from a particular. The former he considers to be proper to mathematics, while the latter is common to all

(6)
the sciences. We have already explained in a general way how this passage must be interpreted. But at this juncture it is necessary to analyze the mind of Aquinas with greater exactness, for by so doing we shall be able to lay bare the proper nature of mathematical abstraction.

In simple apprehension the intellect is able to separate certain things which in reality are not separated. It is in this way that the mind gets at the things which form the objects of the mathematical sciences. Objects such as line and number can be separated by the mind from the sensible matter with which in reality they are necessarily united. Now precisely because this union in reality is necessary, the separation effected by the mind in simple apprehension cannot be transposed to the second operation of the mind, the judgment. For the essence of the judgment is the copula, and this expresses existence, reality. That is why from the conception of a line separated from sensible matter we cannot pass on to the judgment: "the line exists without sensible matter." What about the judgment: "the line exists with sensible matter?" Such a judgment can be made, of course, but then we are no longer speaking about the separated line, the abstract line. There is, therefore, a kind of indifference in this abstraction.

On the one hand, it does not say that the line is with sensible matter. But on the other hand, it does not say that it exists separated from it.

This brings out the characteristic feature of mathematical abstraction, and explains what is meant by saying that quantity depends upon sensible matter in order to be, but not in order to be conceived. For on the one hand, in the case of the sensible qualities which enter intrinsically into the study of nature, there is no possibility of separation "secundum intellectum" since sensibility pertains to their very concept. Material substance, which is the object of the science of nature, even though as substance it is the first subject of all the determinations connected with it, cannot be conceived as material substance without mobility, and mobility necessarily involves quantity with sensible matter. On the other hand, while the objects with which metaphysics deals are separated "secundum intellectum", they are also separated "secundum esse", and that is why in metaphysics we can transpose the separation found in simple apprehension to the operation of judgment. "Considerare substantiam sine quantitate, magis pertinet ad genus separationis quam abstractionis . . .
(7)
Et haec competit scientiae divinae, sive metaphysicae."

All this helps us to see why St. Thomas is justified in calling the abstraction found in mathematics formal abstraction in a very special sense. In it alone there is a form lifted out of matter to which it is necessarily united in reality. And this enables us to grasp the difference between the formal abstraction characteristic of mathematics, and the "universalizing" abstraction found in the other sciences. For it follows from what has just been said that mathematical entities in one sense can and in another sense cannot be realized in nature. They may be said to be realized in nature in the sense that there are triangles, lines, etc. actually existing in the world of reality. But mathematical entities as such, that is, in their state of abstraction from sensible matter, cannot exist in reality. This point is important, for not only does it reveal the special nature of mathematical abstraction, but it also enables us to understand the true nature of mathematical physics. For as we have already pointed out, the application of mathematics to physics consists in the application of mathematical entities as such, that is, in their abstract state. It is not merely a question of finding in nature quantitative determinations as they exist in union with sensible matter.

But perhaps it is not sufficiently clear yet just how mathematical abstraction differs from the abstraction found in the other sciences. For all the sciences deal with abstractions, and abstract things as such, that is, in their state of abstraction cannot be realized in nature, even though they may be realized by the removal of this state. In what way, then, do mathematical entities differ from the abstract things with which the other sciences deal? There is a vast difference between mathematics and the other sciences. For, although all sciences deal with abstract things, only mathematics deals with abstract things as abstract. That is to say, the abstractions found in all the other sciences may be predicated directly of things existing in reality. Mathematical entities, on the other hand, can be predicated directly of nothing existing in reality, precisely because they are defined in a way in which they cannot exist, that is, as separated from sensible matter. In other words, the only difference between the abstract entities found in the other sciences and reality is that of universality and particularity. But in mathematics there is much more than this. Not only do universal mathematical entities not exist in reality, but even particular mathematical entities

do not exist. This point has been summed up with great exactness by Cajetan:

Cum ergo in littera dicitur quod mathematica non subsistant, non est interpretandum quod universalia mathematica universaliter sumpta non subsistant (hoc enim esset ridiculum pro ratione afferre); sed quod mathematica ut sic particulariter sumpta, non subsistant; seu, quod idem est, quod mathematica ut sic, non habent aliquod individuum existens in rerum natura. Et propterea neque sunt in universali, neque in particulari: ac per hoc bona esse non possunt. Quod de aliis rebus universaliter sumptis dici non potest. Et sic patet nullitas consequentiae ad oppositum factae: et quare singulariter dicatur de mathematicis quod non habent esse.(8)

This, then, is the essential difference between mathematical abstraction and the other types of scientific abstraction: In physical abstraction there is a kind of separation from matter through simple apprehension. But the only kind of matter from which separation is made is individual matter. All the matter pertaining to the essence of the thing abstracted is retained. And this explains two things. First it explains why the separation cannot be transposed to the operation of judgment, for only individuals exist, and things which have matter in their essence must have individual matter to exist. Secondly, it explains why we can, nevertheless, make a judgment which predicates the abstract essence of actually existing things,

for the predicate of a predication is a universal nature, and through physical abstraction nothing has been removed from the nature except individuation.

In metaphysical abstraction there is a separation from all matter, and this separation can be transposed to the operation of judgment, since there are beings existing without any matter. For the same reason, we can predicate metaphysical entities in their very state of separation or abstraction of actually existing things. As Cajetan points out: "Metaphysicabilia secundum propriam abstractionem sumpta subsistunt: quoniam habent in rerum natura individua abstrahentia ab omnia materia sensibili et intelligibili, ut patet de intelligentiis." ⁽⁹⁾ Metaphysical abstraction differs from physical abstraction in that in the latter the separation cannot be transposed to the operation of judgment, and though the abstract entities can be predicated of reality, they cannot be predicated in their very state of separation.

In mathematics there is something different from either of these two types of abstraction. Like physics and unlike metaphysics, mathematics deals with things which depend upon sensible matter for their existence outside the mind (in the sense explained above). Like meta-

physics and unlike physics, it deals with things which are independent of sensible matter for their conception and definition. Like the case of physics and that of metaphysics there is separation from matter. Like the case of physics and unlike that of metaphysics this separation cannot be transposed to the judgment. Unlike both the case of physics (because the separation now has to do with matter which pertains to the very essence of things abstracted in so far as those things are real) and that of metaphysics the things abstracted cannot be predicated of reality.

But even this does not bring out with complete clarity the distinctive character of mathematical abstraction. Following leads given us by Cajetan and John of St. Thomas we can push the question a little further:

(10)
Advertendum est ex Cajetano quod quantitas potest dupliciter abstrahi. Uno modo secundum abstractionem generis vel speciei ab individuis, remanente tota natura et quidditate quantitatis, sicut omnes aliae naturae quando in universali concipiuntur; et haec abstractio fit ab intellectu universalizante naturam; et hoc modo quantitas in abstracto consideratur a metaphysico et sic non amittit rationem perfectionis neque boni. Alio modo fit abstractio quantitatis denudando illam a sensibilitate, et fit per imaginationem: sicut imaginamur distantiam quantitatis in vacuo, lineas aut superficies in eo imaginantes; et talis abstractio non est universalis a particulari, sed solum quantitatis interminatae, seu imaginatae, a sensibili... (12)

We have already had the occasion to point out that it does not pertain to mathematics to consider the nature of quantity in itself, nor its ontological properties, nor even the nature and ontological properties of its two species: continuous and discrete. All this belongs to metaphysics. For quantity is a principle of being, one of the ten predicaments, and therefore comes under the object of metaphysics whose object is the being that is distributed through the ten categories. It is evident, then, that the mind is able to lay hold of quantity by another kind of abstraction than that found in mathematics. And it is clear from the passage just cited from John of St. Thomas that this abstraction is the kind we have been opposing to mathematical abstraction since the beginning of this discussion, that is, the universalizing abstraction, which considers quantity as a universal genus of being, apart from the real individuals in which it is realized. This abstraction lays hold of quantity in so far as it is a certain essence, a certain reality that exists ontologically. It considers quantity precisely in so far as it exists in reality as a principle of being, and not in so far as it is set off in a state in which it cannot exist in reality. It is to be noted that the metaphysical consideration of

quantity in some way abstracts from sensible matter (otherwise it would be a physical and not a metaphysical consideration). But it does not, like the mathematical consideration, explicitly separate it from sensibility, "denudando illam a sensibilitate," and explicitly set it off in a world apart from the real world. Rather, while not taking account of its sensible determinations, it considers it as it exists in reality along with the other accidents which constitute the structure of physical being. Mathematical abstraction, on the other hand, considers quantity not in so far as it is a principle of being, or a category of reality, or a certain form or essence, but from the point of view of the relations of order and measure that result when it is separated from all sensibility and set apart by itself.

It must be kept in mind that physical abstraction also lays hold of quantity in some way. For since quantity is the first accident, it is the matrix of all the sensible qualities, which consequently cannot be conceived of except in relation to it. All the mobility in the cosmos is inextricably bound up with quantitative determinations, and from this point of view quantity enters into the object of the study of nature. These quantitative determinations,

incidentally, form the basis of the mathematization of nature. But they are only the basis, for in mathematical physics they are considered from the point of view of the mathematician and not that of the physicist. Quantity is also studied by the philosopher of nature in a very particular way, in so far as in living mobile beings there is found a special kind of mobility pertaining to the genus of quantity.

It is obvious that this consideration of quantity is quite different from that of the mathematician.

Mathematica ex vi suae abstractionis et conceptus, excludunt a quantitate statum sensibilem, nec considerant quantitatem secundum illam realitatem qua potest cadere sub sensu, sed secundum extensionem imaginabilem praecise; quia, ut diximus, ad demonstrationes mathematicas sufficiunt lineae et figurae in imaginatione formatae, quantum ad id quod extensionis, proportionis vel continuitatis considerari potest; non vero quantum ad id quod sensibilitatis est in tali quantitate, seu in quantum ens naturale est. (13)

There would seem, then, to be three distinct ways in which quantity may be laid hold of scientifically by the mind.

First it may be considered explicitly in relation to sensible determinations, and in this way it is the object of the science of physics. Secondly, it may be considered as an ontological accident in so far as it exists in reality along with the sensible accidents - - abstracting

from them in some way, i.e. not explicitly as determined by them, and yet not explicitly as separated from them. In this way it is the object of the science of metaphysics. Finally, it may be considered as separated from all sensibility, set off in a state in which it cannot have actual reality, and contemplated precisely in terms of this abstract state. In this way it is the object of the sciences of mathematics.

All this makes it clear that mathematics not only deals with abstract things like the other sciences, but it deals with them precisely in so far as they are abstract, In this sense, Whitehead is justified in saying that "mathematics is the science of the most complete abstractions to which the human mind can attain. (14) The particular nature of the abstraction found in the mathematical sciences has not been generally recognized. Professor Lenzen, for example, writes: "The relational structure is a complex universal which may be exemplified in various instances, and hence the problem of the reality of mathematical objects is that of the reality of universals." (15) We hope that enough has been said to show that the problem of reality which results from the special kind of formal abstraction found in the mathematical sciences is something quite different from the

problem connected with the "universalizing" abstraction found in the other sciences.

This consideration of the abstract character of mathematics brings us to an interesting paradox. In a sense it is true to say that by the very fact that it is the most abstract of all the sciences, it is also the most concrete. What we mean by that is that in a sense the mathematical universal is the same as the mathematical particular. For mathematical particulars abstract from sensible matter just as the universal does. "Materia sensibilis non includitur in intellectu mathematicorum neque in universali, (16) neque in particulari." Nothing extrinsic is added to a mathematical particular to individuate it. A particular circle a or b may be considered the universal circle.

This truth has considerable importance for our problem of mathematical physics as may be gathered from the following passage of Ernst Cassirer. While not subscribing to everything contained in this passage we believe that it brings out effectively the point we are trying to make:

In his criticism of the logic of the Wolffian school, Lambert pointed out that it was the exclusive merit of mathematical 'general concepts' not to cancel the determinations of the special case, but in all strictness fully to retain them. When a mathematician makes his formula more

general, this means not only that he is to retain all the more special cases, but also to be able to deduce them from the universal formula. The possibility of deduction is not found in the case of the scholastic concepts, since these, according to the traditional formula, are formed by neglecting the particular, and hence the reproduction of the particular moments of the concept seems excluded. Thus abstraction is very easy for the 'philosopher', but on the other hand, the determination of the particular from the universal so much the more difficult; for in the process of abstraction he leaves behind all the particularities in such a way that he cannot recover them, much less reckon the transformations of which they are capable. This simple remark contains, in fact, the germ of a distinction of great consequence. The ideal of a scientific concept here appears in opposition to the schematic general presentation which is expressed by a mere word. The genuine concept does not disregard the peculiarities and particularities which it holds under it, but seeks to show the necessity of the occurrence and connection of just these particularities. What it gives is a universal rule for the connection of the particulars themselves. Thus we can proceed from a general mathematical formula, -- for example, from the formula of a curve of the second order, -- to the special geometrical forms of the circle, the ellipse etc., by considering a certain parameter which occurs in them and permitting it to vary through a continuous series of magnitudes. Here the more universal concept shows itself also the more rich in content; whoever has it can deduce from it all the mathematical relations which concern the special problems, while, on the other hand, he takes these problems not as isolated but as in continuous connection with each other, thus in their deeper systematic connections. The individual case is not excluded from consideration, but is fixed and retained as a perfectly determinate step in a general process of change. It is evident anew that the characteristic feature of the concept is not the 'universality' of a presentation, but the universal validity of a principle of serial order. We do not isolate any abstract part whatever from the

manifold before us, but we create from its members a definite relation by thinking of them as bound together by an inclusive law. And the further we proceed in this and the more firmly this connection according to laws is established, so much the clearer does the unambiguous determination of the particular stand forth. Thus, for example, the intuition of our Euclidian three-dimensional space only gains in clear comprehension when, in modern geometry, we ascend to the 'higher' forms of space; for in this way the total axiomatic structure of our space is first revealed in full distinctness. (17)

The mathematical universe is indeed a strange universe. Its abstract character gives it a high degree of intelligibility. And yet this intelligibility is extremely inadequate, for from the abstract mathematical entities we cannot arrive at actually existing things. The separation from matter gives it a perfection which the physical universe does not have. And yet, unlike the case of the separated substances, this removal of matter does not contribute to the perfection of natures. In fact, the separation from matter prevents mathematical entities from being natures. And yet, it is in the light of these entities that we shall try to understand the natures existing in the cosmos.

In order to add further precision to our notion of mathematical abstraction, it seems worth while, before leaving this question, to compare the way in which mathematical en-

tities are abstracted from the world of sensible matter and the way in which dialectical entities, such as the one discussed in the last chapter: the form of anger considered independently of the sensible matter to which it pertains, are abstracted. In both cases we have the abstraction of a form from the matter to which it belongs. But there is a vast difference in the way this abstraction takes place. In the case of the dialectical definition of anger, we have the form of a natural thing which is essentially inseparable from matter both for its being and for its "being known". Hence when it is set off by itself, it is in a purely logical state; it is a mere construction of the mind. Mathematical entities, on the other hand, are by their very nature separable from sensible matter secundum intellectum, even though they are not separable secundum esse. Consequently, when they are considered as separated, they are in their natural state; they are not dialectical. Anger as a pure form is ens logicum. A mathematical entity as a pure form is an ens naturae.

This brings us to the important question of the relation between mathematics and existence.

2. Mathematics and Existence

The question of the relation between mathematics and existence has been an acute philosophical problem ever since the time of the ancient Greeks. The analysis of the nature of mathematical abstraction has already thrown some light upon it. But the question demands closer attention. In fact, what we have seen thus far in a sense only serves to throw the problem into sharper focus. For if mathematical entities cannot exist as such in reality, must we not conclude that mathematics deals with entia rationis -- logical beings? John of St. Thomas has gone to great pains in the Cursus Theologicus (18) to settle this question. Let us consider briefly his solution.

By a logical being we understand: "ens habens esse objective in ratione, cui nullum esse correspondet in re." Consequently, if mathematical entities were logical beings it would be absolutely contradictory for them to exist in reality. Now, from what we have seen about the nature of mathematical abstraction it should be evident that we cannot say in absolute fashion that the real existence of mathematical entities always involves a contradiction. For there is a sense in which it is true to say that some mathematical entities may exist in reality, not indeed in their state of separation from sensible

matter. We say some mathematical entities, because there are obviously a good many mathematical entities, which are evidently mere logical beings, and whose real existence would necessarily involve a contradiction. An example such as the square root of minus one comes readily to mind. In fact, the whole point of John of St. Thomas' analysis is to show that mathematics, by the very nature of the abstraction it employs, remains indifferent to whether the entities it deals with are real or logical beings.

And he illustrates this point by having recourse to the example of predicamental relation. The essence of a relation consists in the ordering of one thing to another. But a relation may be of two kinds: it may be either real, that is, existing in reality, or it may be only logical, that is, created by the mind. A real relation is one of the nine accidental categories, and like all of the other accidental categories it has a real existence in the subject which it relates to something else. A logical relation does not have a real existence in the subject related, since it is the mind which creates the ordering. Now since the proper essence of relation which distinguishes it from all the other categories consists in the ordering of one thing to another, or in

Scholastic terminology, in the ratio ad, it is indifferent to either real existence (the ratio in) or purely logical existence. The ratio ad is common to both of these types of existence. In somewhat the same way mathematics is indifferent to whether the entities it deals with have real or only logical existence. In this way it differs from all the other sciences, and is a kind of medium between the science of nature and metaphysics on the one hand, and logic on the other. For both the science of nature and metaphysics deal necessarily with real beings. Logic deals necessarily with logical beings. Mathematics deals with either or both. It is true that entia rationis enter into both the science of nature and metaphysics, but their existence in these studies is purely functional, that is, the whole raison d'être of the construction of these entia rationis is to enable the philosopher of nature or the metaphysician to get to know reality; they do not constitute the object of these sciences, and are not considered for their own sake. In mathematics, however, the entia rationis are considered for their own sake. In this respect, mathematics is similar to logic. It differs from it, however, in that the entia rationis it considers are based on real beings which also constitute its object. In this sense Meyerson is justified in saying: "...chez le mathématicien,

réel et idée semblent en quelque sorte se confondre, on ne distingue pas immédiatement s'il traite de l'un ou de l'autre ...C'est là, encore un coup, la conséquence directe de l'accord de l'intellect et du concret dans la mathématique, et c'est ce qui fait de cet élément la vraie et unique 'substance intermédiaire,' dans le sens de Platon."⁽¹⁹⁾

As has already been suggested, this indifference on the part of mathematics to real or logical existence is something that arises out of the very nature of mathematical abstraction. As John of St. Thomas explains, it is precisely because mathematics considers quantity stripped of the definite determination and formation that it has in its state of union with sensibility that mathematical entities can be simple concepts capable of being realized in sensible matter, or concepts that have been elaborated by the mind into a state which cannot be realized in nature.

Mathematica ex vi suae abstractionis et conceptus, excludunt a quantitate statum sensibilem, nec considerant quantitatem secundum illam realitatem qui potest cadere sub sensu, sed secundum extensionem imaginabilem praecise; quia, ut diximus, ad demonstrationes mathematicas sufficiunt lineae et figurae in imaginatione formatae, quantum ad id quod est extensionis, proportionis vel continuitatis considerari potest: nonvero quantum ad id quod sensibilitatis est in tali quantitate, seu in quantum ens naturale est. Et sic apud Aver-

roem et alios antiquos considerabatur quantitas interminata et terminata: et illa interminata dicitur quae praecise extensionem considerat secundum quod praecise sequitur ad materiam, quantum ad id quod de extensione potentiali et formabili dicit; terminata vero quantitas est illa quae sub certa terminatione et formatione concipitur, et sic redditur sensibilis; . . . ita mathematica considerat quantitatem quantum ad id praecise quod habet de extensione interminata, et secundum id quod habet a materia: non secundum terminationem et modum quem habet a forma, ratione cuius redditur sensibilis. Quare quantitas mathematica habet conceptum positivum quantitatis interminatae; eo modo quo quantitas potest inveniri, sive imaginario, sive sensibiliter in ratione entis veri. Unde permissive se habet ad rationem entis realis et veri: neque positive includendo et considerando adaequate, neque positive excludendo per repugnantiam, realitatem ipsius quantitatis. Et in hoc differt a quantitate pure imaginaria, quae est ens rationis; haec enim repugnanter se habet ad quantitatem realem, quia ens rationis est. At vero quantitas mathematica non repugnanter se habet, sed indifferenter: quia neque bene potest facere suas demonstrationes in eis realibus, vel imaginariis; sicut si relatio consideretur secundum rationem ad praecise, nondum consideratur ut ens rationis; nec tamen ut determinate ens reale: sed indifferenter ad illud; quia non consideratur adaequata ratio eius ex omni parte quae requiritur ad realitatem, ad quam etiam requiritur ratio in: sed ex ea parte qua indifferens est ad realitatem, et solum explicat rationem ad. Sic quantitas consideratur a mathematico inadaequate, et sub ea ratione praecise extensionis interminatae: quae indifferenter se habet ad imaginariam et realem, et sic non excludit rationem entis, sed permittit: neque repugnanter se habet ad illud, sed indifferenter. Unde nec ens rationis est determinate, nec ens reale determinate: sed indifferenter et permissive se habet ad utrumque. Quod non solum contingit in ratione entis in communi, quae abstrahit ab ente reali, et rationis: sed etiam in relatione, quae abstrahit a reali, et rationis, secundum inadaequatum conceptum ad: et in quantitate

quae abstrahit ab imaginaria et sensibili, sub
inadaequato conceptu extensionis interminatae.
(20)

All this helps us to understand more accurately the meaning of the phrase to which we have already given some consideration: "mathematica dependent a materia secundum esse". The primary meaning is that while it doesn't pertain to the essence of a mathematical entity to be capable of realization, whenever it is capable, the realization always takes place in matter. But there is another important meaning which can also be attached to this phrase: in every mathematical entity, capable of realization or not, there is always an essential relation to matter. If prime matter were impossible, mathematics would also be impossible. Since prime matter is the principle of homogeneity, and since homogeneity is the fundamental postulate of all mathematics, there is obviously no possibility of mathematical science without an intrinsic reference to prime matter. But the important point is that while always intrinsically dependent upon matter, mathematical entities are not always necessarily capable of realization in matter, for the capability of realization does not enter into their intrinsic formality.

It is equally false to say that mathematical entities have this capability, or that they do not have it. In themselves they are indifferent.

But this may seem to involve us in a contradiction, or at least in a sophism. For in discussing the nature of mathematical abstraction we stated that mathematical entities as such are not capable of realization in nature, and now we seem to admit the possibility of their realization. The contradiction here is only apparent; both statements are correct, provided they be rightly understood. And it is precisely because the mathematical world is so strange that it gives rise to apparent contradictions of this kind. In the first place, it is obvious that abstract things are not capable of realization in their abstract state. In this sense not even the concepts arrived at by mere universalizing abstraction which lifts them out of individuation have such capability. But as we saw above, mathematical entities are incapable of realization in a deeper sense than this. For not only does mathematical abstraction lift them out of the accidental determinations of individuation, but it separates them from an element that pertains to their very essence if that essence is to be real. Mathematical entities are not

capable of realization, therefore, in the sense that they cannot exist in their state of separation from sensible matter. On the other hand there is a sense in which they are capable of realization, for there are actually existing lines and circles and a plurality of quantified things. These may be considered the realization of mathematical lines, circles and number. It is true that the realization is not perfect. Mathematical entities cease to be truly mathematical once they are realized. The realization robs them of the ideal purity and perfection they possess in their state of abstraction. The straight lines in nature are not perfectly straight, nor are natural circles perfectly circular. It would be a mistake to identify the mathematical zero with the philosophical concept of nothingness, or to confuse mathematical number with a plurality of natural beings. And all this results from the nature of mathematical abstraction which does not seize upon the ontological essence of the things it abstracts. On the other hand, the relation between mathematical lines and circles and the lines and circles existing in nature is not the same as that existing between logical beings and their foundation in reality. We cannot say that logical beings are realized in their objective foundation, as we can say

that mathematical lines and circles are realized in the lines and circles of nature.

All this makes it clear that mathematical being is a medium between possible being, arrived at by universalizing abstraction, and logical being. Possible being prescind only from the actual exercise of existence; it retains an intrinsic order to real existence. Mathematical being, by the very fact that it is indifferent to either real or logical existence, prescind not only from the actual exercise of existence, but also from any intrinsic order to existence; on the other hand, it does not absolutely exclude the possibility of actual existence. Logical being not only prescind from real existence; it positively excludes it.

The mathematical world is indeed a strange world. In it mind and nature, the real order and the ideal order are in some sense fused. On the one hand, mathematical being is not a pure creation of the mind; on the other hand it is not a pure discovery of the mind. For since mathematical abstraction never lays hold of quantity in its ontological essence, a mathematical entity is never a property of reality.

On the one hand, mathematical entities prescind not only from actual existence but from an intrinsic order to real existence. On the other hand, mathematical being has a necessary relation with the real, and the character of this relation is unique, for it never retains the ontological essence of the thing with which it is connected. Even the mathematical entities which are capable of realization in nature have an ideal character about them which they lose by this realization. Even those which are not capable of realization in nature are in one way or another elaborations of something that is capable of realization. At the basis of the whole mathematical structure is something found in reality: quantity taken by itself with its proper forms and specifications and relational structures. But right from the start the mind lays hold of this quantity in such a way as to establish its own priority and its own autonomy. For, as has been said repeatedly, it does not grasp its ontological nature; to do that would mean a complete submission of mind to ontological reality. Rather, it transforms quantity into a condition that is especially congenial to its own nature: it establishes it in an abstract state and deals with it precisely as abstract. By so doing the mind acquires for itself a freedom that is almost unlimited. Though dealing with

things originally connected with sense matter, it no longer has to be concerned with having its processes terminate in the external senses. There remains an intrinsic connection with the intuitive imagination, but as the mind exploits its freedom and pursues its process of intellectual elaboration this connection can be stretched to extreme limits of tenuity. And as the intellect takes fuller advantage of its liberty, it will tend more and more to impose its own nature upon the mathematical world. There will be an inevitable growth in spiritualization. The concreteness and potentiality of the continuum will tend to be absorbed by the greater abstractness and actuality of number. There will even be a reaching out beyond the confines of quantity itself to transcendental multitude and pure logical relations. And all this is perfectly legitimate, provided the intellect remains critically conscious of what it is doing. And in this intellectual movement, the mind is not bound down to dealing with real entities; it has at its disposal the vast possibilities of logical being. But in the last analysis it remains true that all logical mathematical beings are founded upon real mathematical beings, and that these real beings have by a process of mathematical abstraction been lifted out of actual experience with the real world. Thus the whole mathematical structure is rooted in

real quantity -- the same quantity which the philosopher grasps ontologically.

All this is extremely important for the problem of mathematical physics. As has already been suggested, mathematical physics does not mean the discovery of the mathematical world in the physical world. Nor does it imply the direct realization of the mathematical world in the physical world. Rather, it is a question of application. And by application we mean an intellectual interpretation of the cosmos which always remains in some sense extrinsic to the cosmos. This is true even when physics employs mathematical entities which are real beings and which are consequently capable of realization in the sense defined above. For, as we have already pointed out, when these entities are employed in physics they retain their mathematical character. In other words, they are applied to the physical world in their abstract state. It is the mathematically perfect straight line that the physicist has in mind when he tells us that light is propagated in a straight line.

If the use of mathematical entities which are real beings is always an extrinsic application, that is a fortiori true of the use of those entities which are merely logical

beings. And it is extremely significant to understand that by the very fact that it is a question of an extrinsic application, it is possible for logical mathematical beings to be more fruitful in the interpretation of the cosmos than real mathematical beings. As we have already pointed out, mathematical physics is essentially a doctrine of als ob. That is why a logical being may be able to "explain" better than a real being. And this point has a direct bearing upon the highly disputed question about whether the cosmos is Euclidian or non-Euclidian. We do not wish to attempt a solution of this question here. But there are a few things that must be pointed out as to the meaning this problem must have. First of all, to say that our cosmos is Euclidian cannot mean that Euclidian geometry as such, that is, in its ideal geometrisal state is realized in nature. Nor does it necessarily imply that Euclidian geometry is capable of "explaining" the cosmos with greater accuracy and fruitfulness than any other geometry. It can only mean that the mathematical entities which make up the structure of the Euclidian system are real beings and are capable of realization in nature in the sense explained. Moreover, this question cannot be solved by an appeal to the relative explanatory powers of the different geometries.

For it is possible for a Euclidian universe to be more rational for us when interpreted in terms of Riemannian geometry than when interpreted in terms of Euclidian geometry. That is why most of the arguments adduced by those who try to prove that the physical universe is non-Euclidian are inefficacious. The question is further complicated by the highly ambiguous meaning of "physical universe." But we do not wish to enter into the problem at this point.

In connection with this problem and with the general question of the relation between mathematics and existence, the oft-quoted remark of Sir James Jeans comes to mind: the cosmos was created by a pure mathematician. As we know, Jeans was lead to this conclusion because of the remarkable way in which modern physicists have been able to fit the most abstruse constructions of higher mathematics upon the material universe. But from what has just been said it is clear that this successful and fruitful application does not constitute a sufficient premise for such a conclusion. Moreover, it is worth while pointing out that there is a profound opposition between the concepts of a pure mathematician and a creator of a material universe. The pure mathematician is indeed a creator, but a creator

in the abstract speculative order. And the world he constructs is, as we have seen, not only cut off from concrete existence, but even from any intrinsic order to concrete existence. He deals with the abstract as abstract, and the whole movement of his science is in the opposite direction from any embodiment in the matter and motion which go to make up the substance of the material universe. In another work Jeans states: "Kronecker is quoted as saying that in arithmetic God made the integers and man made the rest; in the same spirit we may perhaps say that in physics God made the mathematics and man made the rest."⁽²¹⁾ Our analysis of the nature of mathematical abstraction has led us to a somewhat different conclusion, and while it would not be completely true, it would be much closer to the truth to say: in physics, man made the mathematics and God made the rest.

And now perhaps enough has been said to make it clear that mathematics and logic cannot be identified. The confusion between the two generally derives from a confused notion of the nature of logic. Nor are those who maintain this identity with such zeal always anxious to explain what they mean by logic. The science of logic is essentially a reflective science in the sense that its object is what

is known in scholastic terminology as "second intentions." That is to say, it considers what the mind knows in the other sciences, precisely as known by the mind. Mathematics is not a reflective but a direct science. It does not deal essentially with second intentions. It has as its object a proper realm of knowable "natures". That is why it cannot be identified with logic.

278 This discussion of the relation between mathematics and existence would not be complete unless at least passing mention were made of the question of whether mathematical beings have the property of goodness. The ancient Thomists paid considerable attention to this question. In fact it is principally in connection with it that they discussed the problem of the relation of mathematics to existence. And briefly their solution was this: precisely because mathematical being prescind not only from existence, but even from any intrinsic order to existence, it necessarily lacks the property of goodness. For the good is whatever can be the object of an appetite, and appetite has a necessary connection with the existential order. Or, to present the question in a slightly different fashion: because the mathematical world prescind from all order to existence, it is an immobile world of pure essences - - essences which

in no sense are natures. Consequently, in this world there is no becoming, no seeking for ends, no finality. And without finality there is no goodness. For the good is formally defined as: perfectivum alterius per modum finis.

In immobilibus non contingit aliquid esse per se bonum. Unde in mathematicis nihil per hanc causam probatur, neque est aliqua demonstratio. (22)

Mathematica non subsistunt separata secundum esse; quia si subsisterent, esset in eis bonum, scilicet ipsum esse ipsorum; sunt autem mathematica separata secundum rationem tantum, prout abstrahunt a motu et a materia; et sic abstrahunt a ratione finis, qui habet rationem moventis. (23)

This doctrine must be taken in the strictly formal sense in which it was understood by the ancient Thomists. It refers only to mathematical being considered intrinsically. For it is evident that extrinsically finality may enter into mathematics, and with it goodness. Mathematical being can be an end and a means to an end, and thus in both ways involve finality. In the first place, it can be an end in the speculative order in so far as there is truth in mathematics and truth is the good of the mind. But as John of (24) St. Thomas points out, this does not make mathematical beings intrinsically good, just as the knowledge of evil things may be a good for the mind without making the evil things good. Mathematics may be good as a means in relation

to the practical order, as is evident from the large part that mathematics plays in technology. It may also be good as a means in the purely speculative order. In this sense mathematics is a good for the physicist in so far as it becomes for him an instrument to open up the meaning of the universe. In fact, it is the goodness of a mathematical theory which primarily determines its acceptance or rejection by the physicist. For, as we shall see in Chapter XI, there is a sense in which it is true to say that theories are neither true nor false; they are only good or bad. From this point of view a scientist is essentially a pragmatist.

And this brings us to the question of whether or not there is truth in mathematics. Since the world of mathematics is a world of essences which constitute an object knowable by the mind, it is evident that there is truth in mathematics. But since this world of essences is separated off by itself without even an intrinsic order to existence, it is likewise evident that this truth is of
(25)
a very special sort. For the definition of truth as the conformity of the mind with existing reality cannot be characteristic of a world which is cut off from existing reality, and in which logical beings are accepted on equal

terms with real beings. The truth characteristic of such a world cannot consist essentially in a relation, one of whose terms is found in existence, but in a relation, both of whose terms are found within the realm of essence, or in other words, in intrinsic coherence. And that explains why mathematics is the most deductive of all the sciences. Free of any necessity of conforming to an objective order, it can follow out rigorously its own inner logic. It does not, like philosophy, have to keep in constant touch with experience. It affords the one chance that the mind has to triumph completely over mere givenness. It is worthwhile noting here that the coherence notion of truth is proper to the science of mathematics. Every other science, including logic, employs the conformity notion. From this point of view, mathematics is even more detached from the real than logic, although from another point of view, as we saw above, it is in closer relation to it. It is also worth while pointing out that the word "real" is often substituted for the word "true". For a mathematician whatever is mathematically true may be considered real. And this adds to the ambiguity of the question whether real space is Euclidian or non-Euclidian. The special meaning which truth has in mathematics is of

great importance for our problem. For a physicist by the very fact that he is a student of nature, must adhere in so far as he is able to the conformity notion of truth. What happens when these two notions of truth are brought together in mathematical physics we shall see later when we come to discuss the relation between the physico-mathematical world and the absolute world condition.

Though without goodness, mathematical beings possess beauty as well as truth. For as St. Thomas points out: "pulchrum proprie pertinet ad rationem causae formalis." (26)

And thus Aristotle writes:

The chief forms of beauty are order and symmetry and definiteness, which the mathematical sciences demonstrate in a special degree. And since these (e.g. order and definiteness) are obviously causes of many things, evidently these sciences must treat this sort of causative principle also. (i.e. the beautiful) as in some sense a cause. (27)

These remarks are not gratuitous, for the beauty of mathematics sometimes prevents the scientist from recognizing the essentially functional role that mathematics plays in physics. When that happens, the end of mathematical physics is made a means, and the means an end, and the scientist becomes, as Professor Babin has remarked, "un artiste égaré ou frustré."

This consideration of the nature of mathematical abstraction and of the detachment from existence that is consequent upon it helps us to understand the kind of causality that is found in the mathematical world. A world which is the result of formal abstraction in the strictest sense of the term, that is, an abstraction which detaches pure forms from the material embodiment in which they belong and sets them off by themselves, can be endowed with formal causality alone. In other words, in abstracting from matter, the mathematical world excludes material causality. Furthermore, the abstraction from matter involves abstraction from mobility, since mobility follows upon matter. Hence the mathematical world prescind from both efficient and final causality, which are, as it were, the two causal terms of mobility. Or, to put the matter in a slightly different way, in detaching itself from existence the mathematical world detaches itself from coming into existence, or becoming, and only formal causality can exist where there is no becoming, since the other three causes have an analytical relation with coming into existence.

This point is of supreme importance for a correct understanding of the nature of mathematical physics. For the scientist, by the very fact that he is a physicist,

must endeavor to know the cosmos in terms of all four causes. But by the fact that he is a mathematical physicist, and that he must interpret the cosmos in the light of mathematics which is the formal element in his study, whereas the physical is only material, he can see things only in terms of formal causality. What happens when these two tendencies meet we shall consider in some detail in Chapter IX.

The paradox of studying a universe in which efficient, final and material causes are essential in the light of a science which positively excludes all but formal causality is in the last analysis reducible to the paradox of introducing into a science whose object is essentially mobile being the principles of a science which absolutely excludes all mobility. We do not intend to consider this problem here, but perhaps it would be well at this point to eliminate a possible source of confusion. For it might be argued that there is mobility in the mathematical world, since the infinitesimal, vectorial and tensor calculus, for example, deal with the idea of variable quantities and the function concept. Thus we can speak of an infinitesimal as a quantity which approaches zero as its limit. Moreover, the inherent

constructibility of mathematical entities seems to involve motion, for we can speak of a surface being generated by a moving line.

There is indeed motion of a sort in the mathematical world. But it is merely dialectical and not real. It is a purely imaginary and instrumental thing, and does not involve becoming in the true sense of the word. Mathematical entities do not come into being; and they are neither the principle nor the terminus of becoming. We may have recourse to an imaginary movement in order to generate the figures, but that is due to the imperfection of our knowledge. The figures themselves do not originate that way.

Moreover, the exclusion of real motion from the mathematical world does not eliminate the possibility of an application of mathematics to real motion. For, as we have already pointed out, quantity is the primary accident and the matrix of all the others. And that is why all of the determinations of mobile being are endowed with a quantitative mode. This quantitative mode may be laid hold of, and treated mathematically. But we shall come back to this point later.

3. Mathematics and the Intuitive Imagination.

It is clear from the foregoing that, unlike physics, mathematics does not receive its subject from the external senses. It is true that mathematical entities are derived originally from sense experience. For example, we form our notion of a circle only after having experienced a concrete perceptible circular object such as a ball. But this sense experience has only a pre-scientific function. It is required by mathematics only as a presupposition, not as an intrinsic element in the science itself, as it is required by physics. Once derived from sense experience, mathematical notions by virtue of mathematical abstraction, become independent of sense experience. They are stripped of the experiential context in which they were discovered and invested with a new, idealized, non-sensible character. That is why mathematical judgments do not have to terminate in sense experience.

Recently a number of authors have called into question this detachment of mathematics from sense experience. For example, Professor Hogben whose popular book, Mathematics for the Million, is written from the point of view of

dialectical materialism even to the extent of being overt propaganda, says: "The statement $AB = CD$ does not mean 'the line AB is exactly equal to the line CD,' because no one knows how to make exactly equal lines with any actual compass or rule. Its correct translation is 'measure AB (28) to get the length of CD as accurately as you need it."

And as a refutation of the proposition that a straight line is the shortest distance between two points he cites the example of an experiment made on a shrimp whose directional movements are controlled by a certain organ connected with the nervous system. If this organ is filled with steel fillings, the shrimp swimming in a magnetic field will move in curves since the lines of force in the magnetic field are curved. Consequently for the shrimp a straight line (29) is not the shortest distance between two points. We do not consider it necessary to give an explicit refutation of this view of the nature of geometry. So much has already been said about the essential abstraction of mathematics from sensibility that it would be superfluous to labor the point any further. Nor does recourse to the etymology of the word geometry which signifies the science of surveying afford any rational basis for the advocates of "physical" geometry. In recent years the so-called "concrete" methods

of teaching geometry have become increasingly popular. Whatever we may think of these methods as a pedagogical device to gradually prepare the mind for the effort of mathematical abstraction it is evident that one does not really enter into the realm of geometry until this abstraction has been achieved.

Einstein's views on the nature of geometry are relevant here. In his book Geometry and Experience he divides geometry into two distinct branches. The first consists in purely formal knowledge based on axioms that are free creations of the human mind and made up of schematic concepts that are empty of all content. The second is called practical geometry; it is a natural science, and is in fact the most ancient of all the branches of physics. Taken as it stands, this opinion of Einstein is really a denial of the true nature of geometry. For his first branch of geometry seems to be nothing but dialectics, and if his second branch is identified with physical science, there is no place left for a specifically distinct and proper science of geometry.

Once again we do not feel it necessary to enter into a refutation of these views. They have been intro-

duced here to bring into focus the point to be discussed in this section, namely that while on the one hand mathematics is independent of sense experience and hence not to be identified with physical science, on the other hand it is not independent of all reference to sense, as dialectics may be.

Though detached from the external senses, mathematics has an essential connection with the internal sense of imagination. It is in the intuitive imagination that all the judgments of mathematics must terminate, either directly and immediately, or at least reductively. And this brings home to us once again the intermediary character of mathematics. Unlike physics and like metaphysics it is independent of external sense experience. But unlike metaphysics and like physics it still retains a terminal connection with sense life. Mathematics is at once both more free and less free than metaphysics. It is more free in that unlike metaphysics it not only does not have to terminate in sense experience, but its judgments do not have to correspond with anything that is given in objective reality. It is less free in that it has to terminate in the intuitive imagination. It is because of having abandoned this intrinsic connection with imaginative intuition that

modern mathematicians have arrived at the notion of mathematics as a science that is empty of any objective content, as a science that is in the last analysis identified with logic. It is evident that the true view of the nature of mathematics holds a middle course between the "concrete" notion of mathematics which seeks to establish an intrinsic connection between it and external sense experience, and the purely axiomatic notion which severs all connection with the internal sense. Both of these extreme views will evidently have repercussions upon our problem. By holding the first position one could be lead to believe that mathematical physics consists in discovering the mathematical world in the physical world. By holding the second one would be forced to conclude that mathematics provides the empty forms to which physics gives objective content, or that mathematics reveals the essential rules of the game which the scientist plays with the physical universe.

Mathematics and the imagination hold a parallel relation to external sense experience. Like mathematics, the imagination is dependent upon the external senses only as a presupposition. Once it has received its material from them, it can to some extent detach this material from the perceptual context from which it was drawn, that

is to say from the external physical conditions which embodied it originally; like mathematics, it can construct and reconstruct this material into new forms and patterns; it can create new entities only remotely connected with the material to which they owe their origin. And the reason why mathematics must retain some connection with the imagination is that though freed from the determinations of sensible qualities, it is not freed from all materiality and hence it must in some way remain bound up with a cognitive power related to materiality. Though prior to the whole sensible order by reason of its being the primary accident, quantity is nevertheless known to us only through sensible determinations, and hence even after it has been detached from sensible qualities there is still something of sense clinging to it. It is the imagination which, though a sense faculty and thus essentially distinct from the intellect, is nevertheless in the existential order bound up so inextricably with the workings of the intellect, which makes it possible for mathematics to retain its orientation towards sense, even though it is so far advanced in the order of intelligibility. The object of mathematics is never purely intelligible.

But this connection of mathematics with the

imaginative intuition must be rightly understood. In the first place, the intuitive schemes which the imagination presents are not in themselves the object of mathematics; they are only the sensible illustration of that object. Moreover, not all branches of mathematics are equally dependent upon these intuitive schemes. As has already been pointed out, arithmetic, because of its more abstract character, is more remotely connected with the imagination than geometry. For any kind of phantasm will serve to represent number, provided there is plurality; but only a very definite kind of phantasm will serve to represent a circle or a triangle. And as mathematics takes fuller advantage of its inherent liberty, and as it follows its natural tendency towards higher abstraction and spiritualization, the connection with the imagination becomes increasingly attenuated. It would be ridiculous to maintain that all mathematical entities must be capable of direct and perfect reconstruction in the imaginative intuition, and that in this sense all of the judgments of mathematics must terminate immediately in the imagination. Such an assertion would limit mathematics to an infinitesimal fraction of its actual range.

But it is impossible to have an adequate notion

of the orientation of mathematics towards the imagination without seeing the essential relation which the imagination has with intelligible matter, which enters intrinsically into mathematical abstraction. We have explained that mathematics, while prescinding from sensible matter, clings to intelligible matter. "Non possunt (mathematica) considerari sine intellectu substantiae quantitati subiectae; quod esset eas abstrahi a materia intelligibili communi." (30) By intelligible matter is understood the material substance as determined by quantity in so far as quantity is the order of its parts. Why it is called intelligible matter is explained by St. Thomas: "substantia enim remotis accidentibus non remanet nisi intellectu comprehensibilis, eo quod sensibiles potentiae non pertingunt usque ad substantiae comprehensionem. Et de his abstractis est mathematica." (31) Though this matter is rightly called intelligible, it has an intrinsic connection with the imagination, precisely because it is matter. For mathematical forms are not purely intelligible as metaphysical forms are. They are like natural forms in that they are in matter. "Sicut naturalia habent formam in materia, ita et mathematica." (32) And just as the presence of sensible matter in the object of the study of nature makes it necessary for

sense experience to enter into the understanding of this object, so the presence of intelligible matter in the object of mathematics makes it necessary for the imagination to play a part in mathematical intellection.

In his quae sunt per abstractionem, idest in mathematicis quorum ratio abstrahit a materia sensibili, rectum se habet sicut simum. Haec enim mathematica habent materiam, sicut et naturalia. Rectum enim mathematicum est, simum autem naturale. Ratio enim recti est cum continuo, sicut ratio simi cum naso. Continuum autem est materia intelligibilis, sicut simum materia sensibilis. Unde manifestum est, quod aliud est in mathematicis res et quod quid erat esse, ut rectum et recto esse; unde oportet quod alio cognoscat quod quid erat esse horum, et alio ipsa. . .

Unde sicut per naturalia ostenditur, quod intellectus, qui cognoscit quidditates naturalium, sit alius a sensu qui cognoscit ipsa naturalia singularia, ita ex mathematicis ostenditur quod intellectus qui cognoscit quod quid est ipsorum, sit aliud ab imaginative virtute, quae apprehendit ipsa mathematica. (33)

It is clear from this last quotation that intelligible matter plays the part of the material element in mathematical definitions.
(34)

The principal role played by the imagination in mathematics in connection with intelligible matter has already been pointed out in Chapter II. From what has been said about the nature of intelligible matter it is evident that it provides the homogeneous exteriority that is at the basis of the

whole mathematical structure. Now homogeneous exteriority means a multiplication of the same form - such a multiplication is impossible without individuation. And this individuation must take place in the imaginative intuition. For since mathematical entities are stripped of sensible qualities, the individuation cannot be effected by qualitative determinations grasped by the senses. On the other hand, the intellect of itself has to do with pure form separated from matter, and hence if it alone functioned in mathematics we could have no notion of homogeneous multiplicity. For things that are outside each other because of the form are formally different, hence heterogeneous. Speaking of Plato's doctrine of the intermediary position of mathematics, Aristotle says: "Further, besides sensible things and Forms he says there are the objects of mathematics, which occupy an intermediate position, differing from sensible things in being eternal and unchangeable, from Forms in that there are many alike, (35) while Form itself is in each case unique."

There remains just one last point of which passing mention must be made before we bring this discussion to a close. In his Commentary on the Posterior Analytics

St. Thomas explains that intelligible matter is ipsa
(36)
continuitas. Taken in its strictest sense, then, it
is essential only to geometry. Nevertheless, even
arithmetic must terminate in the imagination in some
way, in so far as number is caused by a division of the
continuum.

4. Mathematics and the Human Mind.

There are a number of reasons why physics inevitably reaches out to mathematics for illumination, and some of them have already been touched upon. But at this point we wish to call particular attention to one of the most significant causes of this natural gravitation: the profound congeniality existing between mathematical science and the human mind. Since the time of the Renaissance when mathematics commenced the phenomenal development which has brought it to its present high point of perfection, and when physics began to be increasingly quantified, the fact of this connaturality has been clearly recognized. Kepler is quoted as saying that our minds are so constructed that they can know nothing perfectly except quantities. "Just as the eye was made to see colours, and the ear to hear sounds,

so the human mind was made to understand, not whatever you
(37)
please, but quantity." And Descartes' insistence on the
close relation between the mind and mathematics is too well
known to need being mentioned. But while the fact of this
congeniality has become obvious, the reason for it has not
been so clearly recognized. It is significant that while
in comparison with modern developments mathematical science
and the quantification of physics were only in an incipient
state at the time of Aristotle and St. Thomas, both of these
philosophers not only grasped the fact of the intimate re-
lationship between the intellect and mathematics, but also
(38)
gave a clear and adequate explanation for it.

(39)
As Aristotle points out, difficulties which
stand in the way of the mind's perfect union with a scientific
object may come either from the mind or from the object. In
the case of metaphysics, the difficulties come from the weak-
ness of the human mind. For metaphysical objects because of
their complete separation from all matter are of all scientific
objects the most knowable in themselves. But in relation to
the human mind they are the least knowable. For their high
degree of immateriality keeps them from being within easy
reach of an intellect which is essentially united with matter

and which must derive all its knowledge from the material world through the medium of organic faculties. In relation to metaphysical objects, as Aristotle goes on to explain, the human mind is like the eye of the owl for which the light of day is too bright to see well, and which can see with greater clarity in the obscurity of night. And this explains why for Aristotle and St. Thomas metaphysical wisdom was something too divine to be possessed by man except in a very inadequate and precarious fashion, something rather loaned to man than actually given to him outright.

In the case of physics, on the other hand, the difficulties come from the object. For cosmic things, immersed as they are in matter and in the flux of mobility, are essentially obscure. It is true that by remaining in generalities the mind may triumph over this obscurity to some extent. But as it pursues its inevitable progress towards concretion, the light and certainty deriving from generality gradually fades. Now modern experimental physics is a stage in the study of the cosmos that is far advanced towards concretion. That is why its object is doubly obscure. It is obscure first of all because it is cosmic reality of matter and motion; it is obscure, secondly, because it attempts

to get at this cosmic reality in its concretion. In experimental physics the human intellect is caught in a kind of anguish. From a certain point of view, it is in a realm that is most proper to it. For since it is human, its proper object is the essence of material things; and since it is an intellect it is impelled to know them not just in a general way but in their proper specific concretion. And yet by following this instinct of its nature it inevitably becomes immersed in deeper and deeper obscurity.

Now mathematical science occupies a privileged position between these two extremes. On the one hand, since, it abstracts from matter and motion, its object is more intelligible in se than that of the science of nature. On the other hand, since it is not completely immaterial, since it always retains an essential connection with the imagination from which the human intellect derives all its concepts, it is more intelligible for us than that of metaphysics. "Sed mathematica sunt abstracta a materia, et tamen non sunt excedentia intellectum nostrum: et ideo in eis est requirenda
(40)
certissima ratio."

Another reason for the connaturality of mathematics

with the human mind is given by Aristotle and Saint
(41)
Thomas in the sixth book of the Ethics. The intellect
finds the science which deals with sensible things dif-
ficult because it demands a great deal of experience;
it finds the study of metaphysics difficult because it
transcends the imagination and is free of all reference
to sense. In between these two extremes stands mathematics,
"quae nec experientia indigent, nec imaginationem trans-
cendant." One of the signs of this connaturality is the
comparatively frequent occurrence of child prodigies in
mathematical science -- a phenomenon that is not found in
(42)
the other speculative sciences. (

This profound attraction which mathematics has for
the intellect can constitute a danger. For it is easy for
the mind to try in one way or another to reduce all knowledge
to mathematical knowledge, and to reject whatever does not
prove amenable to this reduction. Descartes, we know, fell a
prey to this tendency. As St. Thomas remarks, "quidam non
recipiunt quod eis dicitur, nisi dicatur eis per modum mathe-
(43)
maticum." It is true, as Aquinas goes on to explain, that
a similar monistic tendency is sometimes found with regard

to other types of knowledge. But the danger is more acute in connection with mathematics because of the connatural attraction of which we have been speaking. And that is why Aristotle and St. Thomas insist that the study of nature must not be reduced to a kind of mathematics:

Ostendit quod ille modus, qui est simpliciter optimus, non debet in omnibus quaeri; dicens quod 'acribologia' idest diligens et certa ratio, sicut est in mathematicis, non debet requiri in omnibus rebus, de quibus sunt scientiae; sed debet solum requiri in his, quae non habent materiam, Ea enim quae habent materiam, subjecta sunt motui et variationi; et ideo non potest in eis omnibus omnimoda certitudo haberi. Quaeritur enim in eis non quid semper sit, et ex necessitate; sed quid sit ut in pluribus."(44)

From all that has been said thus far it is clear that this passage does not intend to exclude the possibility of an application of mathematics to the study of nature. It is merely trying to point out that this application is not an identification.

But we have not yet fully explained the connatural attraction which mathematics exercises over the intellect. There is an innate tendency in the human mind to see one thing in another. This is the root of all scientific endeavor, whose purpose is to see things in their causes. And the source of this tendency we know: every intellect is a reflection

of the divine intellect which sees all things in their proper specification and in their ultimate concretion in the light of the one divine essence. And not only does every intellect seek to grasp one thing in another, it also seeks to construct otherness out of sameness. It strives to become like the divine intellect by constituting itself prior to things, by making itself the creator of its own object. Because the human intellect is human it will always in some measure be subjected to givenness; but because it is an intellect it will strive to triumph over this givenness by making itself the source of the things it knows, thus dominating its object completely. Now the unlimited constructibility of the mathematical world provides the fullest freedom for this tendency of the mind. In mathematics the intellect is able to construct its own object. From a point it is able to construct a line, from a line a plane, from a plane a solid, etc. And it is only after the construction of the subject that the properties of the subject become manifest. Thus the mind constructs the source of these properties. It does not as in the other sciences merely discover the properties and allow them to lead it to a knowledge of a given subject. In all the other sciences the sub-

ject is givenness there is obscurity.

Mathematical abstraction has this unique privilege that the most knowable in se is the most knowable for us. In the other two types of formal abstraction, the most knowable for us is the least knowable in se. Unlike the fundamental principles of the other speculative sciences, the principles of mathematics are at the same time universal in praedicando and universal in causando. And that is why the whole mathematical world is deducible from a few fundamental principles and postulates. And this explains why in some way mathematics is like wisdom, as Cournot has remarked: sophiae germana mathesis. For it is the property of wisdom to reveal all things in the light of an original source, and the perfect deductibility of mathematics enables the mind to see the whole mathematical world as flowing out of the original postulates. And since, as we explained above, mathematical particulars are abstract, and in some sense identified with universals, this process of mathematical wisdom is able^{to} reach even particulars. In a way, mathematics satisfies the mind's instinct for wisdom even better than metaphysics, for since in metaphysical abstraction the best known for us is the least known in se, the whole

metaphysical world can not be drawn out of the original principles. That is why after the mind has pursued its course from the original generalities up through the angelic universe to the divine being it must, in order to satisfy its quest for wisdom, complete its study by having recourse to a dialectical process by which the multiplicity of things are derived from the divine source.

In our introductory chapter we pointed out that Plato conceived the mathematical world as occupying a kind of intermediary position, and we suggested that this was an extremely profound and fruitful insight. There are, in fact, many ways in which mathematical being is truly a medium. Some of them have been touched upon and others could easily
(45)
be adduced. But here we wish to call attention to one particular aspect of this intermediary character of mathematics, for it will serve to throw light upon the point we are trying to develop.

Mathematical being is a medium between purely material and purely immaterial being, and it participates in the nature of both. In the first place, although it is distinct from material being because of the nature of mathe-

mathematical abstraction which frees it from sensible matter, it remains inseparable from it in the sense of always being linked to it by an intrinsic and essential bond. As a matter of fact, if the material world were impossible, the mathematical world would likewise be impossible. For it is only in a world of composed essences, in which formal oppositions are incomplete because of the common matrix of prime matter that the mathematical world can originate. It is this common matrix that provides the source of the homogeneity, and consequently of the univocal relations which are essential to mathematics. The mathematical world is a world of formality, but it is a strange formality, a kind of material formality, since it is immersed in homogeneity. It is something quite different from the heterogeneous formality of the world of separated substances. Because of the homogeneity and the common matrix found in the mathematical world there is a lack of the perfect unity and the pure distinctions found in the separated substances. But at the same time the homogeneity provides a substitute for this lack of unity by being the source of the relations out of which the mathematical world is constructed. On the other hand, the mathematical world is a world of formality even though this formality is not pure. And that is why

it transcends the world of contingency and obscurity, and becomes a world of rationality and necessity. This brings it close to the spiritual world and transpositions from one to the other become possible. It was indeed a profound intuition on the part of Plato to give to mathematics an intermediary position between the "Same" and (46) the "Other". By its very nature mathematics appears to us as a principle of reconciliation between reason and (47) material nature. And all this enables us to understand more clearly why the mathematization of the cosmos can lead, and often has led, to both materialism and idealism. It is only by understanding the true nature of mathematical abstraction and the intermediary character of the science that results from it that these two extremes can be avoided.

Now it is this intermediary character of mathematics that makes it the ideal instrument for physics. Because it is without matter secundum intelligi it participates in the immobile world of necessity and rationality; because it is with matter secundum esse it is applicable to cosmic reality. Hence it is the perfect instrument by which physics may be lifted out of its natural obscurity and contingency into the realm of perfect science, and even

into a state that is in some respects similar to wisdom. And while being a medium between the material and spiritual, it is at the same time a medium between the objective and the subjective, as we saw in our discussion of the relation it bears to existence. This adds immeasurably to its effectiveness as a scientific instrument. For it leaves the mind free to work out its own rational schemes, and yet it provides the possibility of these schemes being applied to cosmic reality. The following remark of Meyerson is extremely relevant here:

C'est que le mathématique, se détachant du reste du réel a l'air de pouvoir progresser sans faire appel à son comportement: c'est ce qui semble en faire la vraie 'matière intermédiaire' entre la pensée et le réel, et ce qui explique aussi l'attrait que le panmathématisme, en dépit du fruste irrémédiable de l'image de l'univers qu'il construit, exerce et exercera sans doute éternellement sur l'esprit humain.(48)

CHAPTER SEVEN

SCIENCE, SENSIBILITY, AND HOMOGENEITY

1. The Problem.

This Chapter marks a turning point in our study. In the last three chapters we have been concerned with a delineation of the salient characteristics of the two sciences whose union constitutes the intermediary science of mathematical physics. Whatever else this delineation has accomplished, it has certainly brought into clear relief the profound antithesis which lies between these two sciences: on the one hand, a science which sees everything in terms of mobility and sensible matter, a science of contingency and obscurity; on the other hand, a science which prescindess essentially from mobility and sensible matter, a science of necessity and rationality. A more radical antithesis could hardly be imagined than the one which exists between these two studies. And yet out of this antithesis must come a synthesis if mathematical physics is to exist. It is to the nature of this synthesis that we must now turn our attention. We shall devote three chapters to an analysis of how this synthesis is effected.

In the remaining chapters of our study we shall consider the results of this synthesis.

The general problem which immediately confronts us, then, is this: how does the mathematical world lay hold of the world of sensible phenomena and transform it into its own image and likeness? Anyone at all acquainted with science knows that the answer to this problem lies in the one word: measurement. But before we can come to an analysis of the process of measurement, a preliminary question imposes itself: what is there in nature itself which makes it amenable to this transformation through measurement into a system of mathematical symbolism? Measurement is the instrument of the mathematization of the cosmos! But there must be in the cosmos itself a basis for this mathematization.

Duhem has posed the question which confronts us here in the following terms:

Pour qu'une théorie physique se puisse présenter sous la forme d'un enchaînement de calculs algébriques, il faut que toutes les notions dont elle fait usage puissent être figurées par des nombres; nous sommes ainsi amenés à nous poser cette question: A quelle condition un attribut

physique peut-il être signifié par un symbole numérique?"(1)

And to this question he gives the following general answer:

Cette question posée, la première réponse qui se présente à l'esprit est la suivante: Pour qu'un attribut que nous rencontrons dans les corps puisse s'exprimer par un symbole numérique, il faut et il suffit, selon le langage d'Aristote, que cet attribut appartienne à la catégorie de la quantité et non pas à la catégorie de la qualité; il faut et il suffit, pour parler un langage plus volontiers accepté par le géomètre moderne, que cet attribut soit une grandeur. (2)

This general answer is fairly obvious, and was already implicit in what we saw in the last chapter about the nature of mathematics and the link which binds it to reality. But it is only a general answer, and it stands in need of a good deal of explication. And perhaps we can orientate ourselves towards a more definite solution by presenting the issue in the following terms: Since mathematical physics consists in the union of a sensible world with a world which prescind from sensibility, the suture which knits the two together must be along the lines of something which is at once connected with sensibility and independent of it, something which while not sensible in the fullest sense of the word, is nevertheless sensible in

a secondary sense. Presented in this way, the problem immediately calls to mind the Thomistic doctrine of proper sensibles and common sensibles, of which the latter are all reducible to quantity, even though in themselves they are not quantity, by the very fact that they are sensible. We believe that it is in this doctrine that the fundamental solution of our problem is to be found.

And we know of no better way of bringing the question into proper focus than by having recourse to the well-known adventure of Sir Arthur Eddington's elephant:

Let us then examine the kind of knowledge which is handled by exact science. If we search the examination papers in physics and natural philosophy for the more intelligible questions we may come across one beginning something like this: 'An elephant slides down a grassy hill-side...' The experienced candidate knows that he need not pay much attention to this; it is only put in to give an impression of realism. He reads on: 'The mass of the elephant is two tons.' Now we are getting down to business; the elephant fades out of the problem and a mass of two tons takes its place.... Let us pass on. 'The slope of the hill is 60° .' Now the hill-side fades out of the problem and an angle of 60° takes its place.... Similarly for the other data of the problem. The softly yielding turf on which the elephant slid is replaced by a coefficient of friction; which though perhaps not directly a pointer reading is of kindred nature... We have for example, an impression of bulkiness. To this there is presumably some direct counterpart in the external world, but that counter-

part must be of a nature beyond our apprehension, and science can make nothing of it. Bulkiness enters into exact science by yet another substitution; we replace it by a series of readings of a pair of calipers. Similarly the greyish-black appearance in our mental impression is replaced in exact science by the readings of a photometer for various wave-lengths of light. And so on until all the characteristics of the elephant are exhausted and it has become reduced to a schedule of measures. (3)

This remarkable passage brings out with great exactness the fact that it is through the instrumentality of various types of measurement that the cosmos is mathematized. But it also suggests what the basis of this mathematization is. For it is evident from the concrete example here given that when the mathematician seeks to lay hold of the material universe all the attributes of this universe which are known in Thomistic terminology as proper sensibles and in modern terminology as secondary qualities slip through his fingers. And no matter how many efforts he makes to recapture them, they continue to elude his grasp. With their passing, the very natures of the things he is dealing with vanish. The characteristic qualities of the hill-side, the greenness of the grass, the softness of the turf, etc. fade out of the picture of

the physicist -- and the hill-side fades with them. And the same is true of the elephant itself.

Yet it is clear that the exact scientist lays hold of something in the material universe, otherwise his science could in no sense be called physics. It is likewise clear that he lays hold of something which though in a sense independent of sensibility is at the same time essentially connected with it. He does not grasp the greyish-black colour of the elephant in its proper nature, yet the wave-lengths of light which register on his photometer are essentially connected with this greyish-black colour. And evidently the thing which he lays hold of can be approached through the avenues of more than one sense. For, a blind scientist can have a perfect knowledge of optics, a deaf scientist can be expertly proficient in acoustics, and if it were possible to live and have sentiency without the faculty of touch there would be nothing to preclude the possibility of the science of thermodynamics. This common character of the object with which exact science directly deals manifests its nature: it reveals the fact that it is intimately bound up with homogeneity. And all of these considerations lead us

to this conclusion: mathematical physics prescind from proper sensibles; its object falls within the domain of the common sensibles.

The views of modern scientists and philosophers of science confirm this conclusion, even though these views are not expressed in Thomistic terminology. Max Planck, for example, has this to say:

Now all physical experiences is based upon our sense perceptions, and accordingly the first and obvious system of classification was in accordance with our senses. Physics was divided into mechanics, acoustics, optics, and heat. These were treated as distinct subjects. In course of time, however, it was seen that there was a close connection between these various subjects, and that it was much easier to establish exact physical laws if the senses are ignored and attention is concentrated on the events outside the senses -- if, for example, the sound waves emanating from a sounding body are dealt with apart from the ear, and the rays of light emanating from a glowing body apart from the eye. This leads to a different classification of physics, certain parts of which are re-arranged, while the organs of sense recede into the background. According to this principle the heat rays emanating from a hot stove ceased to be the province of heat and were assigned to optics, where they were dealt with as though entirely similar to light waves. Admittedly such a re-arrangement, neglecting as it does the perceptions of the senses, contains an element of bias and arbitrariness. (5)

But this concentration upon primary qualities to

the exclusion of secondary qualities is by no means peculiar to modern science. A definite movement in that direction is discernible almost from the beginning of the systematic study of the cosmos. It is true, as Planck points out, that in the first stages of its development natural science identified the sensible and the physical. This was inevitable, since, as we have seen, pure natural science is a study of reality in terms of sensible matter. Physics took its origin when man began to observe and analyze perceptible properties and to express the results in descriptions. This enabled him to introduce order into his cognitions by means of classification. Regular recurrences in his sensory experiences (e.g. hot bodies become cold; a swinging object comes to rest etc.) made it possible for him to arrive at general laws based on qualitative uniformities. But the persistent attempt to perfect this rudimentary knowledge, to analyze these classifications and uniformities with greater exactness, and to render them more rational inevitably led to a dissolution of the relation of identity between the sensible and the physical, and a gradual abandonment of sensorial categories in the explanation of the physical world. In some cases this abandonment became not only methodological,

but philosophical. Already in Democritus and Lucretius we have an explicit denial of the ontological existence of what were later to be known as proper sensibles or secondary qualities. It is only by opinion or convention that they can be said to exist. At the time of the Renaissance this doctrine of the ancient atomists was revived by such men as Vives, Sanchez, and Campanella, and this revival, together with the astounding success of the new mathematical method in physics, had a profound influence on the epistemological views of subsequent scientists. As we saw in Chapter I, Kepler, while admitting the objectivity of the qualitative determinations of nature, maintained that they were somehow less real and fundamental than the quantitative determinations. Galileo went further than Kepler and made the secondary qualities subjective. For him the quantitative determinations of nature were absolute, objective, and immutable, and the object of true knowledge, whereas the qualitative determinations were relative, subjective, fluctuating and the source of mere opinion and illusion. Descartes' expulsion of qualitative determinations from both the physical and the geometrical world, and Newton's subsequent discovery of measurable correlates of colour in terms of differently refrangible rays (6)

provided both a theoretical and experimental foundation for this position. And it remained for Hobbes⁽⁷⁾ and Locke⁽⁸⁾ to lend the weight of their authority to make it the generally accepted philosophical and scientific view. In mechanism the divorce between the sensible and the physical was accepted as a fundamental dogma. And wherever mechanism was accepted as a philosophy, the denial of the ontological existence of the secondary qualities usually resulted.

Contemporary science has continued to maintain the divorce between the sensible and the physical. Max Planck sees the evolution of Physics as a progressive withdrawal from the world of sense:

But at the same moment the structure of this physical world consistently moved farther and farther away from the world of sense and lost its former anthropomorphic character. Still further, physical sensations have been progressively eliminated, as for example in physical optics, in which the human eye no longer plays any part at all. Thus the physical world has become progressively more and more abstract; purely formal mathematical operations play a growing part while qualitative differences tend to be explained more and more by means of quantitative differences. . .

As the view of the physical world is perfected, it simultaneously recedes from the world of sense: and this process is tantamount to an approach to the world of reality. (9)

The gap between the world of sense and the world of physics has become so wide that authors dispute whether "qualitative physics" might not be considered a contradiction in terms, or whether such qualitative propositions as "copper conducts electricity;" "the melting point of ice is lowered by pressure," can be called physical laws. (10)

Recent physics has introduced a new and significant aspect into this progressive recession from the world of sense. In classical physics, although the gap between the world of science and the world of external sensibility had already grown wide, there still remained a direct and immediate relation between the scientific world and the imagination. The scientific constructions of classical physics were susceptible of direct representation through concrete images. That is why mechanism was essentially a physics of models. Lord Kelvin's well-known remark that he had to be able to make a model of a thing before he could understand it is typical of classical physics. But in recent years science seems to have made a direct break not only with external sensibility, but even with the imagination. This break was first effected by the introduction of the theory of Relativity and the theory of Quanta. And more

recent developments have served to widen the gap immeasurably. The theories of Schrodinger and Dirac, for example, seem to be completely incapable of imaginative representation.

It is important to recognize the fact that this progressive withdrawal from the world of sense has sprung from a finality intrinsic to experimental science itself. It was not brought about by arbitrary, extrinsic influences. In particular, it did not grow out of any idealistic bias. When Galileo made the secondary qualities subjective, he understood subjective in the sense of intra-organic and not in the sense of psychic. They were for him the product of an interaction between an external object and a sense organ. Even Descartes, who might perhaps be suspected of a bias towards idealism, admitted the objective existence of a reality which caused the secondary qualities. (11) It is true that idealistic philosophers have seized upon this particular development of science as grist for their mill. But science cannot be held responsible for the interpretations and generalizations of philosophers.

And yet the directions in which science develops have great significance for philosophy. The particular development we have just sketched presents several important problems which we must try to solve if we are to understand the true nature of mathematical physics.

This should be evident from all that was said in Chapter II about the essential relation between physics and sensible matter. In some way physics seems to depend upon the senses for its very subject, ⁽¹²⁾ and yet as it develops it draws farther and farther away from the deliverances of the senses. What then is the precise relation between physical science and sensibility? Why has progress in science produced an ever widening gap between the sensible and the physical? In withdrawing from the world of sense, what is it that science is actually laying hold of in the cosmos? What is the nature and validity of the knowledge that results from this prescind^g from the determinations of the cosmos that are presented by the senses? Is Planck correct in stating that this withdrawal from the world of sense is tantamount to an approach to the world of reality? Has the progressive desensibilization of physical science demonstrated that the objective world is devoid of qualities or that qualities may

in some way be reduced to quantities? What is it that the intellect is attempting to achieve fundamentally in pursuing this progressive desensibilization? Does this development in any way favor idealism? These are some of the questions that demand our attention.

At the beginning of this chapter we suggested that the key to our general problem might be found in the Thomistic doctrine of proper and common sensibles. But the recent developments in physics to which we alluded above might seem to challenge this statement. For some authors see in this break with the imagination a demonstration of the illusory character of the common sensibles, just as they see in the previous withdrawal from external sensibility a demonstration of the illusory character of the proper sensibles:

Or on constaté sans peine que le discernement entre le sensible et le physique, si bien commencé jadis, n'avait pas été poussé aussi loin qu'il aurait pu, et que sans doute il aurait dû l'être. De quel droit affirme-t-on la valeur immédiatement physique des qualités premières et des autres données mathématiques perçues? La force, et l'inertie, sont des notions issues directement de l'expérience sensible. Et l'image, car c'est bien d'une représentation imaginative qu'il s'agit, l'image d'un corps à trois dimensions, dans l'espace euclidien, d'un corps qui se déplace sans se déformer et qui demeure im-pénétrable, dépend indubitablement des conditions

particulières de l'expérience sensorielle de l'homme. Notions anthropomorphiques donc, et qui ne sont pas moins liées à la structure particulière de notre sensibilité que ne l'était la couleur orangee ou le parfum de la violette. Il s'agit d'ailleurs de ce que les anciens appelaient des sensibles communs, qui ne sont jamais perçus qu'en liaison avec les sensibles propres; si donc ces derniers sont transposés du fait de la sensation, il est normal que les sensibles communs subissent le même sort."(15)

Perhaps the best way of coming to grips with these problems is by considering the relation between science and sensibility. But in order to understand this relation it will be necessary to recall a few fundamental notions about the nature of sense cognition.

2. The Nature of Sense Cognition.

Sensation is in many respects an anomalous thing. It represents the first confused awakening of matter to conscious life. It is at once an act of knowledge (which is defined in terms of immateriality) and an act of a material body. While on the one hand transcending pure corporeality, it remains immersed in it. By the fact that it is knowledge it involves a kind of immaterial trans-subjective union between subject and object. But because it is also an act of a material body, this union is bound up with a material sub-

jective union produced by a physical movement.

Now all knowledge is by its very nature objective, for to know is to become another thing in its very otherness. But not all knowledge is equally objective, for there is a direct proportion between the objectivity of knowledge and its perfection. Only divine knowledge is completely objective, for it alone is perfect. This does not mean that knowledge which is imperfect is subjective precisely in so far as it is knowledge. It merely means that its objectivity is conditioned by a certain measure of subjectivity.

Since sensation is the lowest form of knowledge, it is necessarily the most subjective. It is immersed in matter, and matter is by its very nature a subject and the farthest removed from the state of object. It is to be borne in mind that an object is an object not in so far as it acts physically upon a knower, but in so far as it specifies an act of knowing. As we have just suggested, sensation is dependent upon matter not only from the point of view of its object as the intellect is, but even in its own intrinsic nature. For the senses are not purely psychic powers; they are psychosomatic. Sensation is an actus coniuncti, and matter enters into it not merely as a necessary condition, but as a co-cause. That is why it cannot

possess the otherness necessary for pure objectivity for:
"intus existens prohibet extraneum." In the measure in
which cognitive powers must conform to their object in its
entitative state, they cannot conform to it in its objec-
tivity.

Professor DeKoninck has brought out with great
exactness the profoundly subjective character of sense cog-
nition:

Alors que l'intelligence est une faculté séparée
qui atteint les choses sans leurs conditions matérielles
individuanes, le sens reste, à tous les niveaux,
lié à ces conditions de la matière. Et cela est
le plus manifeste dans les sens externes. Ceux -
ci sont pour ainsi dire diffusés sur les choses
dans leur concrétion matérielle, et, par conséquent
dans ce qu'elles ont d'obscur en soi, sous ce rapport,
ils participent aux conditions mêmes de l'objet
dans ce qu'il comporte d'irréductiblement entitatif:
la sensation est liée à un organe corporel. On
le voit le mieux dans le toucher. L'organe de la
température a lui-même une température; il a lui-même
durété et mollesse; est étendu, et il est mesuré
par le temps; il a sa masse à lui; il se répand sur
l'objet étendu; il cède à l'objet dur, et il en épouse
la figure; ils'imprime dans l'objet qui l'enveloppe;
etc. Bien que les premiers philosophes se soient
trompés dans leur explication de la connaissance par
une similitude entitative qui serait requise de la
part du connaissant, ils ont néanmoins énoncé un prin-
cipe qui se vérifie du sens. Mais il s'y vérifie dans
la mesure où le sens s'éloigne de la pure objectivité.
La connaissance sensible est imparfaite parce qu'elle
demande cette immixtion de l'organe à la chose matérielle.
Le sens sera moins parfaitement l'autre dans la mesure
où il demande au préalable une assimilation entitative
dans laquelle le sens même est passif. Le toucher ne

peut sentir une température sans que l'organe ne prenne lui-même cette température. Cette passibilité, où nous sommes, pour ainsi dire, assimilés par une autre chose, est, comme telle, à l'extrême opposé de la connaissance: celle-ci est, en effet, une opération vitale; motus ab intrinseco. L'immixtion aux choses dans leurs conditions matérielles reste purement instrumentale. (14)

The subjectivity of sense cognition is so evident that it has become proverbial; *de gustibus et de coloribus non est disputandum*. The same subject may receive different sensations of the same object, as when, for example a person touches a piece of metal and a piece of wood in a cold room: though both are of the same temperature, the first will feel much colder than the second. The same subject may likewise receive the same sensation from different objects, as when one's hands have a different temperature and are brought into contact with bodies of different temperature.

Now we can best get at the exact nature of this subjectivity by having recourse to some fundamental principles laid down by St. Thomas. "*Nam sentire, quod etiam videtur esse operatio in sentiente, est extra naturam intellectualem, neque totaliter est remotum a genere actionum quae sunt ad extra.*" (15) Sensation is at the point in the universe where immanence first emerges from the transitive activity of material natures. It does not completely emerge

from it; it remains inextricably bound up with it. For in every act of sensation a physical, material interaction takes place between the material object and the material organ. Out of this interaction comes a "product" whose nature is determined both by the character of the stimuli which impinge upon the organ (and these are dependent upon the nature of the medium) and the character of the organ which receives them. It is this "mixture" of external stimuli (already a "mixture" arising out of the interaction between the distant object and the innumerable, indefinable elements which go to make up the medium) and the complex structure of the material organ which constitute the direct object of sensation. What is immediately sensed is not an absolute, distant object exactly as it exists in itself, but something intra-organic.

One of the most fundamental principles of cognition established by Aristotle and St. Thomas is that the sensible object in act is the same as the sense in act. There is a similar principle governing intellectual cognition: the intelligible object in act is identified with the intellect in act. But there is a vast difference between the significance of these two principles. For because of the material interaction of which we have been speaking, the

transition of the sensible object from the state of potency to that of act is not a pure actualization which leaves its intrinsic nature unchanged. The sensible object in act is physically different from the sensible object in potency. St. Thomas explains this point in the following significant passage:

Probat (Philosophus) quod supposuerat; scilicet quod unus et idem sit actus sensibilis et sentientis, sed ratione differant, ex his quae sunt ostensa in tertio Physicorum. Ibi enim ostensum est, quod tam motus quam actio vel passio sunt in eo quod agitur, id est in mobili et patiente. Manifestum est autem, quod auditus patitur a sono; unde necesse est, quod tam sonus secundum actum, qui dicitur sonatio, quam auditus secundum actum, qui dicitur auditio, sit in eo quod est secundum potentiam, scilicet in organo auditus. Et hoc ideo, quia actus activi et motivi fit in patiente, et non in agente et movente. Et ista est ratio, quare non est necessarium, quod omne movens moveatur. In quocumque enim est motus, illud movetur. Unde si motus et actio, quae est quidam motus esset in movente, sequeretur, quod movens moveretur. Et sicut dictum est in tertio physicorum, quod actio et passio sunt unus actus subiecto, sed different ratione, prout actio signatur ut ab agente, passio autem ut in patiente, ita supra dixit, quod idem est actus sensibilis et sentientis subiecto, sed non ratione. Actus igitur sonativi vel soni est sonatio, auditivi autem actus est auditio. Dupliciter enim dicitur auditus et sonus; scilicet secundum actum et secundum potentiam, Et quod de auditu et sono dictum est, eadem ratione se habet in aliis sensibus et sensibilibus. Sicut enim actio et passio est in patiente et non in agente, ut subiecto, sed solum ut in principio a quo, ita tam actus sensibilis quam actus sensitivi, est in sensitivo ut in subiecto. (16)

Sensation, then, is the result of a physical, material action which takes place within the material organ, and which produces there a material motion, and this involves a physical, material passio on the part of the organ which, paradoxically, is the source of both the objectivity and the subjectivity of sensation. It is the source of objectivity because it is the reception of an action coming from an external object; it is the source of subjectivity because it involves a physical change on the part of the instrument of sensation and a reaction which contributes to the constitution of the object immediately sensed. As St. Thomas points out, "non enim oportet quod actio agentis recipiatur in patiente secundum modum agentis, sed secundum modum patientis et recipientis" (17) On a number of occasions both Aristotle and St. Thomas state that sensation consists in a modification, an alteration of the sense organ; it is this alteration that is immediately sensed. "Sentire consistit in moveri et pati. Est enim sensus in actu quaedam alteratio: quod autem alteratur, patitur et movetur." (18)

Whitehead, then, is justified in remarking: "It is an evident fact of experience that our apprehensions of the external world depend absolutely on the occurrences within the human body We have to admit that the body is the

organism whose states regulate our cognisance of the world." (19)
By naively attributing absolute objectivity to our sense cognition we are, as Sir Arthur Eddington has remarked, "continually making the mistake of the man who on receiving a telegram, thinks that the handwriting is that of the sender." (20) And in the same context he points out that to attribute the taste we experience in eating an apple to the apple itself is something like saying that the pain we experience in a dental operation is in the dentist's drill. It is necessary then to recognize the enormous distance which separates us from the things that are the closest to us. The very physical proximity of sensible things is a sign of their distance in the order of knowledge.

It is important to note that this subjectivity of sense cognition in no way gives aid and comfort to the idealists, as some might be led to think. For, as we have already pointed out, the very source of the subjectivity is at the same time the guarantee of objectivity. That is why Aristotle, after pointing out that sensations are really nothing but "modifications of the perceiver" immediately adds: "but that the substrata which cause the sensation should not exist even apart from sensation is impossible. For sensation is surely not the sensation of

itself, but there is something beyond the sensation, which must be prior to the sensation; for that which moves is prior in nature to that which is moved." (21)

Moreover, to say that the qualities that are immediately sensed are intra-organic is not the same as saying that they are psychic. As a matter of fact, they are completely physical and independent of consciousness. (22) They are a part of the physical world, even though they do not exist in the place in which they are localized by the naive view. And the reason why they are where they are is determined by the very physical structure of the universe. Bertrand Russell brings out this point in Mysticism and Logic:

The view that sense-data are mental is derived, no doubt, in part from their physiological subjectivity, but in part also from a failure to distinguish between sense-data and 'sensations'. By a sensation I mean the fact consisting in the subject's awareness of the sense-datum. Thus a sensation is a complex of which the subject is a constituent and which therefore is mental. The sense-datum, on the other hand, stands over against the subject as that external object of which in sensation the subject is aware. It is true that the sense-datum is in many cases in the subject's body, but the subject's body is as distinct from the subject as tables and chairs are, and is in fact merely a part of the material world. So soon, therefore, as sense-data are clearly distinguished from sensations, and as their subjectivity is recognized to be physiologi-

cal not psychical, the chief obstacles in the way of regarding them as physical are removed. (23)

We have laid considerable emphasis upon the nature of sensation both because it is of great importance for the problem we are undertaking to solve, and also because the majority of modern Scholastic philosophers have presented sensation as though it possessed the same purity of objectivity as intellectual cognition. It is extremely important to realize that sense and intellectual knowledge differ generically and not merely specifically. From the point of view of objectivity there is a vast difference between sense and intellectual knowledge. Kant brings out this difference rather accurately when he writes: "Sensitive cogitata esse rerum repraesentationes, uti apparent, intellectuality autem, sicuti sunt." (24) The senses have to do with phenomena, with things as they appear and not as they are in themselves. Their object is not an essence - something absolute as it exists in se in the external world, but something essentially relative to the sense organ itself. It is true that when the intellect is brought to bear upon sense data there will be an instinctive attempt to assimilate them to the condition of intellectual objects, that is to lift the "uti apparent" to "sicuti sunt", and as we shall

point out presently, this is precisely what the intellect is trying to do in its mathematization of the sensible world, but the fact remains that in themselves the sense data are purely phenomenal. To lose sight of this and to project into the external world the sense data as sensed by us is tantamount to identifying the sensible in act with the sensible in potency. As we pointed out above, because of the material nature of the sense organ, there is a difference between the two, not only from the metaphysical point of view, but even from the physical and material point of view. We cannot say just how great this difference is. To do that it would be necessary for us to know actually the sensible in potency, which is a contradiction. Only the separated substances know actually the sensibilia in potentia, and, we may add, they know the sensibilia in actu in the only way in which they can be known: as sensed by material subjects, as existing within the organs of beings endowed with sense life. But even though we cannot say just how much a difference there is between the sensible in act and the sensible in potency we know that there is a difference. Things do not exist exactly as they are sensed by us. And we cannot insist too much upon the fact that we never sense the sensible in

potency, that is the separated object in its own absolute existence. Perhaps we can sum up this point succinctly in the following terms. On the one hand only the sensible in potency exists (i.e. outside the sense organ); on the other hand, only the sensible in act is known by us. Consequently, there is a real gap between the sensible and the physical (i.e. the extra-organic world). And the withdrawal of science from the sensible world is a clear recognition of this gap.

Paradoxical as it may seem, the attribution to sensation of the pure objectivity proper to intellectual knowledge comes closer to idealism than the clear recognition of the subjectivity that is characteristic of all sense operations. For in the last analysis this attribution consists in projecting into the external world something that is the product of the sentient subject. In other words, idealists identify the sensible in potency with the sensible in act; those who attribute pure objectivity to the senses identify the sensible in act with the sensible in potency. Ultimately, the two positions coincide. Aristotle and St. Thomas point out the consequences of this fatal identification:

Si omne apparens est verum, nec aliquid est verum nisi ex hoc ipso quod est apparens sensui, sequetur quod nihil est nisi in quantum sensibile est in actu.

Sed si solum sic aliquid est, scilicet in quantum est sensible, sequetur quod nihil sit si non erunt sensus. Et per consequens si non erunt animata vel animalia. Hoc autem est impossibile. Nam hoc potest esse verum quod sensibilia in quantum sensibilia non sunt, id est si accipiatur prout sunt ~~sensibilia~~ in actu, quod non sunt sine sensibus. Sunt enim sensibilia in actu secundum quod sunt in sensu. Et secundum hoc omne sensibile in actu est quaedam passio sentientis, quae non potest esse si sententia non sunt. Sed quod ipsa sensibilia quae faciunt hanc passionem in sensu non sint, hoc est impossibile. (25)

If the sensible in act and the sensible in potency are identified, either the objective world depends for its existence on sensation, or everything in the objective world is actually and constantly sensed, or nothing is sensed. This last consequence follows because in order for an object to be sensed there must be a physical mutation produced in the organ, and this mutation necessarily involves a transition from a potential to an actual state of sensibility. It is only by clearly distinguishing between the sensible in potency and the sensible in act that we can escape idealism and angelism.

And now a few notions relative to the object of sensation must be touched upon before we can consider the relation between science and sensibility. Aristotle and St. Thomas distinguish between objects that are sensible

per accidens and those that are sensible per se. Objects are said to be sensible per accidens when although they themselves are incapable of being sensed, they are connected with something that is the actual object of sensation. Thus, for example, substance cannot be actually sensed; nevertheless in so far as it is the substratum of the accidents that are sensed, it is said to be sensible per accidens. Objects that are sensible per se are those which are actually sensed in themselves. They are divided into two types: proper sensibles and common sensibles. It is this latter distinction that interests us particularly.

The proper sensibles are those which constitute the specific object of each individual external sense, and are consequently the exclusive property of only one sense, as, for example, color for the eye, sound for the ear, etc. The common sensibles are those which are the common property of more than one sense. There are five principal common sensibles: figure, motion, rest, number and magnitude; and to these are added three others: time, which is connected with motion and rest; position which is connected with external figure; and place, which is connected with magnitude.

These common sensibles comprise all of the predicaments except two. Action and passion are included under motion and rest; quantity comes in under number and magnitude; quality under figure; habitus is taken in by figure; situs has already been enumerated as one of the common sensibles; and ubi and quando are directly reducible to place and time. The only two predicaments not included are substance, which, as we saw is only a sensible per accidens, and relation, which cannot be sensed because it involves something that is proper to the intellect; an ordering of one thing to another. Hence, in so far as experimental science is based upon the common sensibles it will be incapable of attaining the substances of things or true predicamental relations. And yet quantity provides a substitute for both substance and predicamental relation. Because of the unique position which it occupies as the first accident and consequently the one closest to substance there is a quasi substantiality about it which, as we saw in the last Chapter, explains why it alone of all the accidents is capable of being the object of a special science. Because "in solo quantitatis genere, aliqua significantur ut subjects, alia ut passiones" quantity can constitute a world apart. And in this world mathematical order substitutes for

real predicamental relation.

Now perhaps the most important aspect of these common sensibles as far as we are concerned is that they
(26)
are all reducible to quantity. Number and magnitude are species of quantity; figure is a quality that is proper to quantity, since it consists in the termination of magnitude; motion (rest) and time are modes of quantity, "ex eo quod dividuntur secundum quantitatem ad divisionem
(27)
alicuius quantitatis"; and position and place, by being connected with figure and magnitude are reducible to quantity. The fundamental reason for this reductibility to quantity is that quantity by being the first accident is the matrix of all the others and hence contributes to them a quantitative mode. This common matrix on the part of the object is the foundation of the common sensibility on the part of the senses. The very homogeneity in which all of the common sensibles are rooted makes them common to several senses and prevents them from being proper to any one sense.

In connection with the proper sensibles a distinction must be made the importance of which will be apparent later. Among the external senses there is a

hierarchy in which sight occupies the highest place and touch the lowest. Of all the external senses sight is the most perfect because it is the most immaterial and the most objective. It is the sense which enables us to know the greatest number and the greatest variety of objects.

(28)
Of all the senses it is the most detached from its object.

Touch, on the other hand is the most material and the most subjective of all the sense faculties. It is the least detached; it has the weakest capacity for apprehending things in their distinctions. And yet it has a quality which makes it excel all the other external faculties.

Professor DeKoninck has analyzed with great accuracy and clarity this characteristic quality:

C'est pourtant le toucher qui nous enracine le plus directement et le plus sûrement dans les choses. Il est pour ainsi dire un prolongement en nous des choses telles qu'elles sont dans leur concrétion propre. Il coïncide le plus avec elles, dans l'espace et dans le temps; il revêt davantage leur condition. Pour cette raison, il est aussi, par excellence le sens de l'expérience et de l'intelligence. Au point de vue certitude, c'est le toucher qui l'emporte. Un signe en est que nous demandons de toucher les choses comme critère ultime. L'ouïe, et davantage encore la vue, à cause de leur proximité de l'imagination, peuvent être sujets d'illusion. Le toucher, au contraire est davantage soumis au choc des choses dans leur concrétion épaisse. Il est, d'après l'expression des anciens 'grossior' et 'crassior', mais cette grossièreté lui donne des avantages au point de vue de la sobre certitude. En tant qu'elle implique 'subir' la connaissance

expérimentale est essentiellement imparfaite, mais elle l'emporte chez nous en tant qu'elle est pour nous origine de toute connaissance, et principe de toute certitude: 'veritas principiorum quantumcumque per se nota, in nobis semper est reducibilis ad sensus ex quibus originatur, et eorum universalitas ex inductione facta per sensus dependet.' (Jean de S.Thomas, Curs. Theol., T. I. p.392b) C'est sous ce rapport qu'il répond le plus pleinement à la première exigence de l'intelligence. Il a par là une affinité à l'intelligence, qui se traduit même dans l'organe. 'Homo secundum tactum, multum differt in certitudine cognitionis ab aliis animalibus. Unde quia homo habet optimum tactum sequitur quod sit prudentissimum omnium aliorum animalium. Et in genere hominum ex sensu tactus accipimus, quod aliqui ingeniosi sunt, vel non ingeniosius non secundum aliquem alium sensum. qui enim habent duram carnem, et per consequens habent malum tactum, sunt inepti secundum mentem: qui vero sunt molles carne, et per consequens boni tactus, sunt bene apti mente. (In II de Anima, lect.19 nos.482 - 485) (29)

It is clear, then, that though from different points of view we may say that both sight and touch are at once the most objective and the most subjective sense faculties, the objectivity of touch has a very special significance for experimental science. In spite of its lack of distinction, it provides us with the greatest certitude, and in this it is like something that is found in the intellectual order: the most confused knowledge has the greatest certitude for us.

Now in so far as the sense of touch is the sense

of homogeneity, the sense which comes closest to the quantitative aspects of material objects, the sense that comes closest to pure corporeity and pure exteriority, it is the sense that is the most closely allied to mathematical physics. Modern science wants to reduce its sense experience with the universe to the minimum that is found in the sense of touch, and that means not merely to the generic sense of touch which includes perception of temperature, etc. but to pure tactility, that is to say to pure contact of point to point.

This brings us to the consideration of a final distinction that has a bearing upon our problem. -- the distinction between external and internal experience. External experience consists in the experience of the external senses of which we have been speaking. Internal experience consists in the experience had of one's own proper reality through the operations of the internal senses and the mind. Now all too often it seems to be taken for granted that the study of nature depends only upon external experience. This is far from being the case, especially when it is a question of the study of living nature. As a matter of fact it is true to say that in a certain sense the study of psychology is based principally upon internal

experience. We come to know what life is originally and primarily through our own proper experience of living. St. Thomas brings out this point in his Commentary on the De Anima of Aristotle: "Hoc enim quilibet experitur in seipso, quod scilicet habeat animam, et quod anima vivificet." (30)

This internal experience is so important that if one were to abstract completely from his own personal experience of living, he could not speak of life existing in anything. And it is important to insist upon the fact that this internal experience is not the flimsy and untrustworthy thing that many modern scientists attempt to make of it. On the contrary it enjoys the greatest certitude. In the text just cited, St. Thomas bases the eminent certitude which psychology possesses precisely upon the fact that life is known through internal experience. In comparison with the certitude which we have of our own life, our knowledge of the existence of life in other things, which depends upon external sensation, has only a greater or less degree of probability. It is precisely because psychology is based upon the experience we have of our own soul that the basic Aristotelian treatise on living nature is called De Anima. In it the soul is considered in quodam abstractione - - not in the sense that it is studied in complete abstraction from the sensible matter with which it is united, for then

it could not form a part of natural doctrine, but in the sense that it is considered to some degree in and by itself, And this dependence upon internal experience introduces a new factor into the ordering of the natural treatises about which we spoke in Chapter IV. Since the basic methodological principle is to begin with what is best known to us, the study of living nature must start with the soul as it is experienced by us, in quadam abstractione, and then pass on to things that are more intimately bound to matter. That is why De Sensu et Sensato comes after the De Anima. In the introduction to his Commentary on De Sensu et Sensato St. Thomas explains this ordering. (31) Vegetative life which is not attainable by direct internal experience is the most hidden form of life: "vita in plantis est occulta." (32)

But it would be a mistake to believe that internal experience enters only into the treatises on living nature. It is also used in the Physics. For example, in book three when Aristotle is looking for an illustration of motion, he has recourse to the example of a man building a house. One might be tempted to wonder why he deliberately chose the example of the becoming of an artefactum and not of a natural generation. But the illustration like all of the illustrations of Aristotle, is not without its profound

significance. For in the example of the building of a house we have a case of motion in which both external and internal experience enter. As a matter of fact, the striving of an agent for an end, which is so essential to the true concept of motion, is most clearly apprehended by us in our own internal experience. When this internal experience is completely set aside, it is all too easy to lose sight of the fact that motion involves the coming into being of a new actuality which is the end of an agent, and to look upon it as a pure degradation. As a matter of fact many modern scientists have come to look upon motion merely in terms of the second law of thermodynamics which states that the world is continually in a state of degradation, that is to say, continually losing actuality, and consequently destined ultimately to arrive at a state of thermodynamic equilibrium in which all of cosmic reality will be in a state of utter chaotic diffusion and formless homogeneity. In connection with this question of entropy which constitutes time's arrow for the scientists, it is interesting to note that in his commentary on Aristotle's treatise on time in the fourth book of the Physics St. Thomas teaches that if we abstract from the agent of motion and from its intention, time is a degrading factor: "mutatio est ad peiora ex natura sua." (33)

Mutation and time must be joined with the idea of an agent acting for a certain end in order to have the generation of a new actuality.

All this may appear to be an irrelevant digression, but as a matter of fact it is very a propos. For it serves to bring out the fact that the starting point of mathematical physics is diametrically opposed to that of philosophy of nature. Mathematical physics seeks to take its start from a minimum of experience. It excludes internal experience, and it reduces external experience to its very lowest form: pure corporeal contact. And out of this minimum of experience it seeks to construct the whole universe. Philosophy of nature on the other hand, has as its point of departure a maximum of experience. It employs not only the whole range of external experience, but also internal experience. And in connection with its dependence upon internal experience, it must be pointed out that this method of investigating problems is neither anthropomorphism nor subjectivism. On the contrary it enjoys a high degree of objectivity. For one's own internal states and experiences are as objective as anything in the universe.

This contrast between the points of departure of mathematical physics and the philosophy of nature brings

into relief a striking paradox. While from the point of view we have had in mind in this discussion philosophy of nature depends upon a maximum of experience and mathematical physics upon a minimum of experience, from the point of view from which we considered the problem of experience in Chapter IV the situation is completely reversed; a minimum serves as a starting point for philosophy, while a maximum is required for mathematical physics and all branches of experimental science. We may say, then, that because of a significant effort on the part of the intellect to shake itself loose from its dependence upon the senses, mathematical physics tends towards a minimum of experience. This tendency is seen first in the vast use of hypothesis by which the mind seeks to anticipate reality. It is carried forward by a reduction of sense experience to its lowest form: pure taction. But it is a tendency that can seek its end only by binding the intellect down to a maximum of experience.

But in order to become aware of all that is involved in this question it is not sufficient to consider the difference between the starting points of mathematical physics and philosophy of nature; we must also consider the terminal points at which they aim. Precisely because

philosophy of nature begins with a maximum of experience it has as its ultimate goal and as its most important object the noblest being existing in nature, the being which in some sense transcends nature, and yet is a part of it. The being which possesses the highest degree of heterogeneous interiority in the universe, ⁽³⁴⁾ the spiritual soul of man. On the other hand, precisely because mathematical physics begins with a minimum of experience, its ultimate goal must be to reduce the whole cosmos to pure homogeneous exteriority, to a state of pure otherness without any formal distinctions. As we shall have occasion to point out a little later, if mathematical physics could actually arrive at the goal towards which it is constantly striving, it would succeed in reducing the cosmos to a state of pure emptiness.

It should be obvious that this question is closely connected with the divergent forms of measurement employed in the philosophical sciences and in the experimental sciences, to which we alluded in Chapter I and which we shall consider in greater detail in Chapter IX. The method of mathematical physics has its many advantages and its rich returns, but when, as has often happened, the knowledge that it provides is proposed as the only valid

knowledge of nature, then we are asked to accept an epistemological monstrosity, an exaltation of the superficial, a radical form of nihilism.

3. Science and Sensibility.

We are now in a position to consider the problem of science and sensibility. From what was said above it is clear that it is especially in relation to the proper sensibles that the ever widening gap between science and the sensible world has occurred. We must now try to see what has created this gap. Perhaps enough has already been said to show that it is not an artificial and arbitrary creation, nor a fortuitous occurrence, but something that has come inevitably from the very nature of experimental science and the nature of sensibility.

The first cause of the withdrawal of science from the sensible world is obviously the subjectivity of sense cognition. Natural science is orientated completely towards the absolute world condition, and its whole inner finality urges it to draw ever closer to this goal. The inherent subjectivity of the ministrations of the senses

is a direct obstacle to this tendency. For the deliverances of the senses present an anthropomorphic world, a world that has been refashioned, to some extent at least, according to the structure of man's sense organs. They consequently present a relative world, a world of appearances. If science is to be true to its inner urge to strive for the absolute world condition, it must find a way to disanthropomorphize these deliverances; it must, as we have suggested, strive to transform the "uti apparent" of Kant to "sicuti sunt". And it does this by means of a double substitution: one on the part of the subject and one on the part of the object. On the part of the subject, it puts in the place of organic instruments of perception inorganic artificial instruments of measurement especially designed for the purpose in accordance with scientific theories. On the part of the object there is a corresponding substitution of quantitative for qualitative determinations. The scientific world that is built up by means of these artificial inorganic instruments of measurement will inevitably draw farther and farther from the sensible world that is built up by the organic instruments of perception. (35)

It is to be noted that the subjectivity of the senses is an individual subjectivity. The corresponding

sense of ten different subjects will not necessarily represent the same object in the same way. Ten different men, for example, may get ten different perceptions of the temperature of the same body of water. Now this is contrary to one of the ideals of science, which has come to be known in recent years as intersubjectivability. And science has found that by the double substitution mentioned above almost perfect intersubjectivability can be achieved. Norman Campbell has shown that the only exact judgments with regard to perceptions that are universally accepted are those that are based on quantitative determinations, and particularly those which have to do with the three categories of space, time and number. (36)

Another important reason for the withdrawal of science from the world of sense is that from the point of view from which experimental science approaches the cosmos, the proper sensibles are irrationals. And that for two reasons. In the first place, these proper sensibles cannot be defined. It is impossible to define heat; it is impossible to define a color or a sound. They are utterly incapable of analysis. They possess no inherent communicability. It is impossible to explain to a man born blind what red and blue are. And the reason for this is that the

proper sensibles are the primary and immediate data of sense cognition. Hence there are no prior notions in terms of which they may be defined; there are no more fundamental elements into which they may be analyzed.

Now it is different for the mind to rest satisfied with this state of affairs. It has an instinctive desire to define, to express to itself the quod quid est of things. That is why there have always been attempts to liberate the proper sensibles from the incommunicability that is native to them. The medieval Scholastics made attempts of this kind. For example, they defined white as disgregativum visus. But it is evident that such attempts can never yield strict definitions.

Similarly, the proper sensibles are indemonstrable. There are no prior principles in the sensible order from which they may be deduced. At the same time, they themselves are not principles of demonstration. Nothing can be deduced from them. However, it is only through them that the common sensibles can be perceived. That is why they may in a way be compared to what is known in the intellectual order as the supreme dignitates, which are necessary for every demonstration, but which are not in themselves the principles of any demonstration. Indefinable

incapable of analysis, indemonstrable, incapable of being a source of demonstration, the proper sensibles are merely given. Is it any wonder that science instinctively draws away from them?

The second source of their irrationality is very closely allied with the first: by the very fact that they are proper sensibles, they are irreducibly heterogeneous; they are isolated one from the other; they are not unified by a logical pattern. As we shall attempt to explain presently, not all types of heterogeneity are essentially and completely irrational. Nevertheless, in the measure in which heterogeneity is incapable of being reduced to some kind of unification it always presents an element of irrationality to the mind. Meyerson has laid considerable emphasis upon the isolation of the proper sensibles:

Il suffit en effet, de réfléchir à la nature de la qualité pour se rendre compte à quel point elle se prête difficilement aux tentatives consistant à relier, mentalement, le divers à l'identique, qui constituent l'essentiel de toute explication du réel. Car toute qualité nous apparaît comme quelque chose de complet en soi; non seulement le fait de son existence ne postule rien en dehors d'elle-même, mais elle est quelque chose d'intensif et ne paraît donc point susceptible de se combiner, de s'ajouter à autre chose. (37)

Material qualities lend themselves admirably to

descriptive knowledge, but they seem refractory to explanatory knowledge. They appear to be closer to sentiency, whereas quantity seems closer to rationality. Once again from this point of view, the proper sensibles are merely given, and this givenness is in direct opposition to the necessity that science seeks. Not being able to find this necessity in the realm of the proper sensibles, it will look for it elsewhere.

Another reason for the withdrawal of science from the sensible world arises from the extremely restricted nature of the senses. The crudity of our sense organs allow us to perceive only an infinitesimally small part of the cosmic occurrences. By the substitution of inorganic instruments of measurement for the organic instruments of perception the scope of science is increased immeasurably.

In general, then, we may say that we experience the outer world through small samples of it coming into contact with our sense-organs... Yet not all samples of the outer world affect our sense organs. Our ear-drums are affected by ten octaves, at most, out of the endless range of sounds which occur in nature; by far the greater number of air-vibrations make no effect on them. Our eyes are even more selective; speaking in terms of the undulatory theory of light, these are sensitive to only about one octave out of the almost infinite number which occur in nature
Science has of course provided us with methods of extending our senses both in respect

of quality and quantity. We can only see one octave of light, but it is easy to imagine light-vibrations some thirty octaves deeper than any our eyes can see. While philosophy is reflecting how different the world would appear to beings with eyes which could see these vibrations, science sets to work to devise such eyes -- they are our ordinary wireless sets. We also have means for studying vibrations far above any our eyes can see. Actually a range of vibrations extending over about 63 octaves can be detected and has been explored -- 63 times the range of the unaided eye. And even this limit is not one of the resources of science, but of what nature provides for us to see. In the same way, the spectroscope makes good the deficiency of our eyes for analyzing a beam of light into its constituent colours, and further enables us to measure the wave-length of each colour of light to a high degree of accuracy.

Science has extended the range and amplified the powers of our other senses in similar ways, in quality as well as in quantity. We cannot touch the sun to feel how hot it is, but our thermocouples estimate its temperature for us with great accuracy. We cannot taste or smell the sun, but our spectroscopes do both for us -- or at any rate give us a better acquaintance with the substance of the sun than any amount of smelling or tasting could do. We are entirely wanting in an electric sense, but our galvanometers and electroscopes make good the deficiency.(38)

As Hermann Weyl has pointed out, this crudity of the senses leads us to identify things which are physically distinct and thus runs counter to one of the most basic principles of science:

For the question forces itself upon us: why is

physics not content with this domain of perceived colors which has only two dimensions, what urges it to put oscillations of the ether or something similar in their place? After all, from our visual perceptions we know nothing about the oscillations of the ether; what we are given are precisely only these colors, the way we encounter them in our perception. Answer: To light rays which cause the same impression to the eye are in general distinct in all their remaining physical and chemical effects. If, for example, one illuminates one and the same colored surface with two lights which visually appear as the same white, the illuminated surface usually looks quite different in both cases. Red and green-blue together give white light, equally light brown together with violet. But the first light produces a dark hue on the photographic plate, the second a very light one. If one sends two lights which visually appear as the same white through one and the same prism, the intensity distribution in the spectrum arising behind the prism is different in both cases. Therefore physics cannot declare two lights which are perceptually alike to be really alike, or else it would be involved in a conflict with its dominating principle: equal causes under equal circumstances produce equal effects. Perceptual equality therefore appears to physics only as a somewhat accidental equality of the reactions which physically distinct agencies produce in the retina. The accidental equality of the reaction rests upon the particular nature of this receptive apparatus. (39)

In connection with this point it is not superfluous to add that the deliverances of the sense are extremely fluctuating and unstable. As Meyerson has remarked: "le retour de sensations véritablement identiques est excessivement rare." (40)

That is why science must look for a source of permanence which is so essential to its nature.

Moreover, the qualitative determinations of nature permit of only general and loose propositions. In order to achieve accuracy, and in order to make its propositions capable of unambiguous confirmation or refutation, science must have recourse to quantitative determinations. For example, the statement: "fire causes water to boil," is not true unless a number of precise determinations be added with regard to temperature, pressure, respective masses of the water and fire, surface of radiation of the fire, etc. A certain arrangement of these conditions could actually keep water from boiling.

It seems necessary to add one final observation before we leave this question. The whole material universe is a mixture of qualitative and quantitative determinations. As we go up the scale of perfection in cosmic reality, the qualitative determinations assume an ascending importance, for they manifest the increasing triumph of form over matter. That is why they are so important in the biological sciences. But in inorganic matter it is the quantitative aspect that is in the ascendency. And that can perhaps be adduced as a further reason why physics as it progresses becomes more and more immersed in the quantitative.

And now, having considered the relation that exists between science and sensibility, we must try to see the way in which the mind triumphs over the limitations of the senses.

4. Science and Homogeneity.

In order to understand the part that homogeneity plays in science it is necessary to begin by making an important distinction between two types of heterogeneity. There is first of all a kind of heterogeneity which is found on the part of the object of knowledge and which we shall call "natural". This is the heterogeneity that exists between man and brute, between the numbers two and three, between the different angelic species, between the logically distinct rationes formales of the divine essence. This type of heterogeneity obviously springs from a difference of form (in the broad sense in which it signifies a ratio formalis). It is consequently a heterogeneity that is essentially rational. It has its source in intelligibility. And the more perfect an intellect is, the more perfectly does it grasp things in their proper and irreducible heterogeneity.

There is another type of heterogeneity that may be termed "noetic" because it is found not on the part of the object of knowledge but on the part of the intelligence itself. It consists in the multiplicity of media or concepts, or intelligible species which the intellect needs to employ in order to know reality. The more imperfect an intellect is, the greater is this multiplicity. This heterogeneity therefore is essentially irrational. It is a reflection of the original potentiality of the intellect. It is clear that perfect knowledge will consist in knowing natural heterogeneity in all the fullness and richness of its proper specific distinctness by means of absolute noetic homogeneity.

It is only in divine knowledge that this perfection of knowledge is found. In a unique intelligible species which is His essence God sees all the individual natures in existence, exhaustively and in their ultimate specific concretion. Here there is no possibility of any conflict between heterogeneity and homogeneity. In fact, it is only because God sees things in the one species which is Himself that he is able to grasp them in their absolute heterogeneity. But as we descend the scale of

beings, noetic heterogeneity gradually increases. The higher separated substances can know a large number of individual natures in their specific distinctness through a small number of intelligible species. In the lower separated substances a great multiplicity of media are required. And the limit of this process is found in the human intelligence which because it partakes of the diffusion of matter with which it is united, can know things in their distinctness only through a multiplicity of intelligible species equal to the multiplicity of ontological species.

In the intellect of man there is a profound conflict between homogeneity and heterogeneity. On the one hand, he is incapable of sharing in noetic homogeneity. He can, indeed, attempt to triumph over this limitation by having recourse to the dynamic method of limits, and this method is not without its fruitfulness. But it always remains only an attempt, since dialectical limits cannot be attained. On the other hand, natural heterogeneity, though something basically rational, will always present to him an irrational aspect in the measure in which it remains in its pure isolated givenness, in the measure in which it cannot be reduced to some kind of unification, to some type of homogeneity. It must be remembered that even though the

source of natural heterogeneity is fundamentally something rational, in so far as it is found in the material universe it also involves an irrational element in the sense that a plurality of really distinct forms is possible only because they are imperfect and limited.

The problem of the human intellect then, is to see the heterogeneity of nature in terms of some type of homogeneity. Here we are touching upon a conflict in the intellectual order of which there is something strangely analogous in the sensible order. We refer to the distinction pointed out above between the faculties of sight and touch. As we saw above the first is a faculty of heterogeneity in that, better than any other sense, it is capable of grasping things in the richness of their specific distinctness. The second is a faculty of homogeneity in that it has the least capacity for grasping distinctions and in that it seems to come into closest contact with the quantitative determinations of nature. It is also the most important sense faculty from the point of view of certitude, and this carries out the analogy still further, since, as we have seen, it is only by remaining in the homogeneity of generality that the mind is able to arrive at true scientific certitude in relation to the cosmos. The ideal towards which man will ever strive

will be a union of this distinctness and this certitude. In the sensible order this is possible, since sight and touch can be brought into a combined operation on the same object: "unless I see in his hands the print of the nails, and put my finger into his side, I will not believe." But in the intellectual order separate faculties of distinctness and certitude, or heterogeneity and homogeneity do not exist. Hence the mind will have to discover some other means of striving towards its ideal. Let us see how it goes about it.

There are two important ways by which man tries to triumph over the heterogeneity of reality through homogeneity. The first is by retreating into generality and consequently into logical potentiality. It is in this way that philosophy of nature studies the cosmos. By reducing the specific heterogeneity of the universe to the logical homogeneity of generalities, it is able to procure for itself a number of important advantages. It is able to get at the fundamental, common structure of the physical world, and to know it with certitude. It is able to view the cosmos in terms of unity and in terms of what is most knowable for it. But the price it has to pay for these advantages is great. For all the concrete richness of the

universe remains untouched. At the limit of this process of logical homogenization the universe would be reduced to the emptiest, most vague and most potential concept - - that of being, abstracted by mere total abstraction.

It is in order to get at the richness of nature that the mind starts its march towards concretion. But by advancing in this direction it soon gets involved in an intellectual crisis. For its gain from the point of view of heterogeneity means a loss from the point of view of homogeneity, and hence an increase in irrationality. And this increasing irrationality forces the mind to seek for some kind of homogeneity through which to triumph over it. But it will have to be a homogeneity that is quite different from the one from which it is emerging, i.e. one that will not lead it back into generalities, but will carry it forward into concretion. It will have to be a homogeneity that is not logical but ontological. It will have to be something which will afford at the same time both a unity to provide for what is lost by drawing away from generalities, and a distinctness to enable the mind to press forward towards concreteness. It will have to be something that

will make it possible for the mind to see nature in terms of what is most knowable for it (and thus make up for what is lost by drawing away from generalities) and at the same time in terms of what is most knowable in se (and thus make up for the deficiencies of purely generic knowledge.) And the mind finds a basis for what it is seeking for in a general substructure of cosmic reality, in a common matrix in which the heterogeneous determinations of the physical world are rooted. This, we believe, is the most fundamental significance of the mathematization of the universe. (41)

Now science gets at this homogeneous matrix by displacing its object from the realm of the proper sensibles to that of the common sensibles. And these common sensibles serve its purpose excellently by the very fact that, while they are not quantity in themselves, they are all reducible to quantity. Since they are sensibles, and hence not quantity specifically, the science which studies them is able to remain within the realm of physics. On the other hand, since they are all reducible to quantity, the mind is able to find the homogeneity it is seeking for, and physics becomes mathematical physics. Since quantity is the primary accident and the one closest to substance, all

the specific determinations of cosmic reality are rooted to it, and hence they all assume a quantitative mode. Because of the principle "quidquid recipitur ad modum recipientis recipitur," quantity necessarily modifies the qualities that are received into it. (42)

In order to understand the nature of these quantitative modes it must be noted that in the structure of physical reality, the qualitative and the quantitative determinations are not related to each other after the manner of two contiguous layers. Rather, there is an intimate, dynamic union between them. And this explains why the qualitative determinations can be "translated" into quantitative equivalents, why the colors and sounds and heat of the universe can become functions of the space, time, mass and other derivative relationships that exist between the various parts of nature. By getting at these quantitative modes, science is able to construct a physics that can be informed and rationalized by mathematics.

But at this point it must be noted that it is possible to study qualitative perfections in a quantitative way without having recourse to a physical quantitative mode. Intelligence, for example, is studied in experi-

mental psychology in terms of quantitative measurements based on an association between certain psychological reactions and a scale of numbers. Mathematical physics is primarily concerned not with an extrinsic and artificial correlation of this kind, but with an intrinsic correlation which springs from the very structure of physical reality. This intrinsic correlation is not a discovery of modern science; it was clearly recognized by the ancients, and (43) was the basis of their mathematical physics.

But in order to understand this point accurately it is necessary to introduce a distinction here, which will not only help us to clarify the present issue, but will also be useful for us in the next Chapter when we come to discuss the relation between science and measurement. We have in mind the distinction between predicamental and transcendental quantity. St. Thomas explains this distinction with great preciseness in the following passage:

Duplex est quantitas. Una scilicet, quae dicitur quantitas molis, vel quantitas dimensiva, quae in solis rebus corporalibus est. Unde in divinis personis locum non habet. Sed alia est quantitas virtutis, quae attenditur secundum perfectionem alicuius naturae, vel formae. Quae quidem quantitas designatur, secundum quod dicitur aliquid magis, vel minus calidum, in quantum est perfectius vel minus perfectum in tali caliditate. Huiusmodi autem quantitas virtualis attenditur primo quidem in radice, idest in ipsa perfectione formae, vel

naturae; et sic dicitur magnitudo specialis, sicut dicitur magnus calor propter suam intensionem, et perfectionem. Et ideo dicit August 8 de Trinit. cap. 18. quod in his quae non mole magna sunt, hoc est maius esse, quod est melius esse. Nam melius dicitur, quod perfectius est. Secundo autem attenditur quantitas virtualis in effectibus formae. Primus autem effectus formae est esse; nam omnis res habet esse secundum suam formam. Secundus autem effectus est operatio: nam homo agens agit per suam formam. Attenditur igitur quantitas virtualis et secundum esse, et secundum operationem. Secundum esse quidem, in quantum ea quae sunt perfectioris naturae, sunt maioris durationis. Secundum operationem vero, in quantum ea, quae sunt perfectioris naturae, sunt magis potentia ad agendum. (44)

The more or less of transcendental or virtual quantity is based on heterogeneity while that of predicamental or formal quantity is based on homogeneity. And it is interesting and helpful to view the latter as the dialectical limit towards which the former tends as the hierarchy of immaterial things descends towards the realm of corporeality. The difference of forms gradually diminishes and at the limit the definition of each part is the same as the definition of the whole. The diversity is no longer formal; it is purely material. In all material things both types of quantity are found together. The heterogeneity of the one is rooted in the homogeneity of the other and takes on its modes and determinations.

In Quaestiones Disputatae de Virtutibus in

Communi, St. Thomas explains that besides the magnitude which qualities and forms are said to possess per se, there is another magnitude that is attributed to them per accidens. It is this quantity per accidens that is of special significance for mathematical physics:

Omnibus qualitibus et formis est communis ratio magnitudinis quae dicta est, scilicet perfectio earum in subjecto. Aliquae tamen qualitates, praeter istam magnitudinem seu quantitatem quae competit eis per se, habent aliam magnitudinem vel quantitatem quae competit eis per accidens; et hoc dupliciter. Uno modo ratione subiecti; sicut albedo dicitur quanta per accidens, quia subiectum eius est quantum; unde augmentato subiecto, augmentatur albedo per accidens. Sed secundum hoc augmentum, non dicitur aliquid magis album, sed maior albedo, sicut et dicitur aliquid maius album . . . Alio modo quantitas et augmentum attribuitur alicui qualitati per accidens, ex parte obiecti in quod agit; et haec dicitur quantitas virtutis; quae magis dicitur propter quantitatem obiecti vel continentiam; sicut dicitur magnae virtutis qui magnum pondus potest ferre, vel qualitercumque potest magnam rem facere, sive magnitudine dimensionis, sive magnitudine perfectionis, vel secundum quantitatem discretam; sicut dicitur aliquis magnae virtutis qui potest multa facere . . . Sed considerandum est, quod eiusdem rationis est quod aliqua qualitas in aliquid magnum possit, et quod ipsa sit magna, sicut ex supra dictis patet; unde etiam magnitudo perfectionis potest dici magnitudo virtutis.(45)

It is clear from this passage that in so far as forms and qualities are found in corporeal beings they may

become quantitative per accidens in relation to predicamental quantity. And from the last line of St. Thomas just cited it is evident that there can be a direct relation between the transcendental per se quantity of these forms and qualities and the predicamentally quantitative modes which make them quantitative per accidens. This makes it possible for science to deal with the transcendental quantity of the specific perfections of reality in terms of predicamental quantity.

By fixing its attention upon the quantitative modes of the specific determinations of the cosmos, physics obtains for itself innumerable advantages. For, in the first place, nothing seems so real to common sense as quantity. As Spaiser has remarked, "c'est la quantité qui représente la réalité la plus solide . . . En un mot, le réalisme habituel est avant tout un réalisme de la quantité"
(46)

By adopting the quantitative method the mind enjoys an experience that is in some way similar to that of being able to reach out and touch and handle an object of sense. Whether or not along with this there is the advantage of being able to grasp things in their distinct-

ness in a way that would be similar to the perfection of the sense of sight is a question which we shall consider a little later. Moreover, nothing is capable of being so abstract and ideal as quantity. And this gives almost unlimited scope for the mind's desire for perfect rationality.

This reveals the profound significance of the homogenization of the cosmos. Because man is composed of both matter and spirit there are two fundamental tendencies in him: to draw everything from matter, and to draw everything from mind. The persistent recurrence of the extremes of materialism and idealism in the history of philosophy have been a constant manifestation of this. Now the quantitative homogenization of the cosmos makes it possible for man to realize both of these tendencies simultaneously. The mathematization of nature means something far deeper than an attempt to escape from the anthropomorphism involved in the subjectivity of sensibility. It is really an attempt on the part of the intellect to shake itself loose from the senses. This is in a way a natural movement, since intellect in its perfection is independent of sense. To construct the universe out of a minimum of experience is the next thing to positing the universe. To a certain extent the mind is successful in this attempt. But by an

ironical paradox this success involves a falling back upon something similar to the very lowest form of sense life -- pure taction. It is a conception of the universe in terms of the homogeneous exteriority of pure materiality.

All this explains why the goal towards which science is ever striving is to reconstruct the universe out of sameness. "The aim of the analysis employed in physics," writes Eddington, "is to resolve the universe into structural units which are precisely like one another." (47) The analysis of matter has gone far in this direction; it has succeeded in resolving cosmic reality into protons which are all alike and electrons which are all alike. And when nature seems to present an irreducible dualism in the heterogeneity existing between protons and electrons, the theory of relativity will attempt to dissolve this heterogeneity by suggesting that "they are actually similar units of structure, and the difference arises in their relations to the general distribution of matter which forms the universe." (48)

The end towards which physical science is aiming is to reconstruct the whole universe, i.e. to conceive the universe in terms of structural knowledge determined with exactness by mathematical formulae. Knowledge of this kind

prescinds completely from the nature of the units which constitute the structure. In their place are substituted manipulatable mathematical symbols, which while they serve as admirable instruments for knowledge of structure, at the same time blot out all that lies beneath the structure. Mathematics is especially competent to express patterns, but incompetent to reveal the proper natures of entities and operations. Through group-structure mathematics is able to lay hold of realities which in themselves are not directly susceptible of mathematical conceptions.

All this explains the increasingly important place of mathematics in physics, for it is only in mathematical form that purely structural knowledge can be adequately expressed. In particular it explains the central role played by the Theory of Groups.

This structural knowledge is at once extremely objective and extremely subjective. It is objective in the sense that by prescinding from the proper determinations of things, the knowledge of which involves so many subjective elements, it is able to constitute a type of knowledge that is exactly communicable to all minds. It is at the same time subjective in the sense that the essential

plasticity of the sameness out of which the structure is formed gives unlimited scope to the constructivity of the mind. In fact, this whole process must be looked upon as the mind's imposition of its engrained forms upon reality.

This is a point that has been stressed by Eddington:

Granting that the elementary units found in our analysis of the universe are precisely alike intrinsically, the question remains whether this is because we have to do with an objective universe built of such units, or whether it is because our form of thought is such as to recognise only systems of analysis which shall yield parts precisely like one another. Our previous discussion has committed us to the latter as the true explanation. We have claimed to be able to determine by a priori reasoning the properties of the elementary particles recognised in physics - - properties confirmed by observation. Accordingly we account for this a priori knowledge as purely subjective, revealing only the impress of the equipment through which we obtain knowledge of the universe and deducible from a study of the equipment. We now say more explicitly that it is the impress of our frame of thought on the knowledge forced into the frame . . . I want to show therefore that the concept of identical structural units expresses a very elementary and instinctive habit of thought, which has unconsciously directed the course of scientific development. Briefly, it is the habit of thought which regards variety always as a challenge to further analysis: so that the ultimate end-product of analysis can only be sameness. We keep on modifying our system of analysis until it is such as to yield the sameness which we insist on, rejecting earlier attempts (earlier physical theories) as insufficiently profound. The sameness of the ultimate entities of the physical universe is a foreseeable consequence of forcing our knowledge into this form of thought . . . I conclude there-

fore that our engrained form of thought is such that we shall not rest satisfied until we are able to represent all physical phenomena as an interplay of a vast number of structural units intrinsically alike. All the diversity of the phenomena will then be seen to correspond to different forms of relatedness of these units or, as we should say, different configurations. (49)

The foregoing analysis makes it clear that it is precisely through the source of homogeneity that the common matrix of quantity offers to the mind that it is possible for science to rationalize the cosmos. Much has been written on this point by modern philosophers of science. Professor Whitehead, for example has this to say in Process and Reality:

It is by reason of this disclosure of ultimate system that an intellectual comprehension of the physical universe is possible. There is a systematic framework permeating all relevant fact. By reference to this framework the variant, various, vagrant, evanescent details of the abundant world can have their mutual relations exhibited by their correlation to the common terms of a universal system. Sounds differ qualitatively among themselves, sounds differ qualitatively from colours, colours differ qualitatively from the rhythmic throbs of emotion and of pain; yet all alike are periodic and have their spatial relations and their wave-lengths. The discovery of the true relevance of the mathematical relations disclosed in presentational immediacy was the first step in the intellectual conquest of nature. (50)

But perhaps the author who deserves particular attention in relation to this question is Emile Meyerson,

for we are touching here upon the central theme which runs through all of his voluminous works. Meyerson has labored to show that the mind cannot understand reality except by reducing its diversity to some kind of identity, and that the identity in which it comes closest to realizing its ideal is that of undifferentiated spatiality. Unfortunately, there is usually a fairly thick penumbra surrounding his analyses because he fails to make a number of important and necessary distinctions. Like Parmenides and Anaxagoras, he confuses the noetic and the ontological problems of the one and the many; he does not seem to recognize the difference between what is more knowable for us and what is more knowable in se, between the rationality which things have for us and the rationality they have ontologically. From this arises a confusion between the different kinds of diversity and the different kinds of unity by which the mind seeks to triumph over the diversity, with regard to diversity, he fails to make the all important distinction between natural and noetic heterogeneity. And in his treatment of identity there is no attempt to distinguish clearly between the homogenization arising from the reduction of singularity to universality, from the coordination of laws in theories, from the relations of

causality, from the quantification of reality, and from the method of limits. It is especially important to keep this last type of unification distinct from all the others.

But in spite of these limitations, his fundamental tenets are quite correct. The following passage is a good expression of his central theme:

Ce à quoi la science tend de la manière la plus immédiate, c'est à établir un rapport logique entre les phénomènes, à les déduire les uns des autres. Mais cette tendance n'est au fond, qu'une conséquence, une expression particulière du postulat de la rationalité du réel: c'est, en quelque sorte, de la même monnaie de rationalité. Il n'est donc point étonnant qu'en l'accumulant nous finissions par reconstituer, au moins partiellement, le capital primitif, c'est-à-dire qu'à force de déduire les phénomènes les uns des autres, la science finisse par faire crouler les murailles qui en divisaient le domaine en parcelles distinctes, privées de communication les unes avec les autres. Cette opération, cela est de toute évidence, ne peut s'accomplir qu'en renonçant à ce qui est qualitatif, au profit de la quantité. En effet, tout ce qui est affecté d'un indice qualitatif devient, par la même, spécifique, isolé . . . Mais ce qui apparaît certain, c'est que l'éclosion de la notion de quantité dans l'ensemble des conceptions du sens commun, tout en étant favorisée par des constatations des expériences sur les phénomènes... est cependant surtout conditionnée par ce souci de l'explication, de la rationalisation, qui constitue le ressort fondamental de notre pensée tout entière. (51)

If the ideal of science could be adequately achieved, the entire universe would be reduced to an

immense tautology and would thus collapse and vanish completely. "La raison, en cherchant à expliquer, à rendre rationnelle la réalité extérieure, la fait disparaître finalement dans le tout indistinct de l'espace et du néant." (52) According to Meyerson, this collapse will not occur because the cosmos will ever remain propped up, so to speak, by irrational elements which are essentially refractory to the mind's process of homogenization. As we have already suggested, Meyerson fails to make it clear that from a more fundamental point of view these props are rational elements, in the sense that they derive from natural heterogeneity. It is because of them that our attempts at rationalization are kept from issuing into the utter irrationality of a purely homogeneous and amorphous universe which would correspond to the original irrationality of the human intellect in its state of tabula rasa. It is a striking and highly significant paradox that if our attempts at rationalization could succeed the universe would be rendered completely irrational.

Better than any one statement of Meyerson himself, the following passage of Prince Louis de Broglie sums up the essence of this doctrine:

Selon lui (Meyerson) dans la recherche scientifique

comme dans la vie quotidienne, notre raison ne croit vraiment avoir compris que si elle est parvenue à dégager dans la réalité mouvante du monde physique des identités, et des permanences. Ainsi s'explique en particulier la structure commune des théories physiques qui tentent de grouper des catégories de phénomènes par un réseau d'égalités, d'équations, cherchant toujours, autant que faire se peut, à éliminer la diversité et le changement réel et à montrer que le conséquent était en quelque sorte contenu dans l'antécédent. La réalisation complète de l'idéal poursuivi par la raison apparaît alors comme chimérique, puisqu'elle consisterait à resorber toute la diversité qualitative et toutes les variations progressives de l'univers physique en une identité et une permanence absolues. Mais si cette réalisation complète est impossible, la nature du monde physique se prête néanmoins à un succès partiel de nos tentatives de rationalisation. Il existe, en effet, dans le monde physique non seulement des objets qui persistent à peu près semblables à eux-mêmes dans le temps, mais des catégories d'objets assez semblables entre eux pour que nous puissions les identifier en les réunissant dans un concept commun. Ce sont ces 'fibres' de la réalité, comme dit M. Meyerson, que notre raison saisit dans l'expérience de la vie quotidienne pour constituer avec elles notre représentation habituelle du monde extérieur; ce sont ces fibres également et d'autres plus subtiles, révélées à notre connaissance par les méthodes raffinées de la recherche expérimentale, dont la raison du savant s'empare pour chercher à extraire de la réalité variée et mouvante la part d'identique et de permanent qu'elle renferme. Aussi, grâce à l'existence de ces fibres bien que l'idéal de la science soit en toute rigueur irréalisable, quelque science est possible: c'est la grande merveille. Cette situation se trouve résumée par une phrase de M. Paul Valéry, phrase sans doute inspirée par la lecture même des ouvrages de M. Meyerson: L'esprit humain est absurde par ce qu'il recherche; il est grand par ce qu'il trouve. Mais comme en définitive l'univers ne peut pas se réduire à une vaste tautologie, nous devons forcément nous heurter çà et là dans notre description scientifique de la nature à des éléments 'irrationnels' qui résistent à nos tentatives d'identification, l'effort

jamais lassé de la raison humaine s'acharnant
à circonscrire ces éléments et à en réduire le
domaine. (53)

It is clear, then, how the mind through the homogenization of the cosmos succeeds in triumphing over the irrationality that arises out of the pure givenness of the deliverances of the senses. Unlike the isolated perceptions of sense experience, the quantities with which mathematical physics deals lend themselves to the mind's desire for deduction: they can be both the conclusions and the principles of deduction. And to the highly integrative value of quantities which makes them derivable from each other is added the advantage of the wide scope of relational possibilities which arises from the extension of the quantitative system to include zero values, negative values, infinite values, etc.

But what is the price which the mind must pay for this triumph? From what has been said about the movement of science towards tautology, one might be led to suspect that the price is rather high, and to wonder what has actually been gained by abandoning the logical homogeneity of generality in which the specific distinctions of things are swallowed up. It might seem that the homogenization of experimental science is contrary to the very nature of that science,

which seeks to get at things in their specific natures and consequently in their heterogeneity. To put the question quite bluntly: does not the quantitative homogenization of the cosmos destroy the specific concretion of things and thus turn science back from its essential aim?

The answer is: yes and no. There is an essential difference between the logical homogeneity of generality and the ontological homogeneity of quantity. In the first case there is a complete renunciation of specific differences; in the second case the renunciation is only partial. For as we explained above, by locating its object in the realm of common sensibles, mathematical physics does not deal with pure quantity; it deals with the quantitative modes or, to use the expression of Meinong, the "quantified surrogates" of the specific determinations of nature. And because mathematics is not only a science of great generality, but also a science of great exactness, mathematical physics can, through a process of rigorous physical measurement, get at these specific determinations with far greater concrete precision than sensibility can. All of the qualitative aspects of nature have their quantitative modes and their variations involve quantitative mutations. And we pointed out above that there can be a direct correlation between the transcendental quantity

that is intrinsic to qualities and forms, and the predicamental quantity that is measured by physical processes. That is why the homogeneity of mathematical physics is not a complete renunciation of the heterogeneity of nature. From one point of view it is a means of knowing it better, and in this sense there is a distant resemblance here of the perfection of cognition found in the separated substances in which it is precisely through the homogeneity that the heterogeneity is known. And even though in its superstructures mathematical physics moves towards undifferentiated spatiality and tautology, it always starts out from, and must inevitably lead back to, the heterogeneity of nature. This makes it essentially different science based on logical homogeneity.

Thus the mind is able to enjoy an experience remotely analogous to the combination of sight and touch in sense experience. It is able to get at nature with something that resembles the certitude that is derived from touch, and with something that resembles the distinctness that comes from sight. But it is extremely important to recognize that in both cases it is a question of a mere substitute. Mathematical method affords a kind of exactness and certitude in dealing with nature, but from all that was said above about the essen-

tially dialectical character of experimental science it should be clear that it cannot provide true objective certitude. The same must be said of distinctness. For, with whatever extreme precision we get to know the quantified surrogates of the qualities and forms in nature, it is always with a substitution that we are dealing and never with the qualities and forms in their own proper, specific nature. Exact knowledge is not the same as specific knowledge. Moreover, a surrogate is always ambivalent; at the same time that it unites us with the object for which it substitutes, it separates us from it.

To attempt to get at the proper nature of the qualitative through purely quantitative methods is to accept one of the fundamental principles of Hegelian and Marxist dialectics: every quantity if sufficiently increased turns
(54)
into a quality.

That many have actually been led to identify the qualitative with the quantitative is well known. Spaier, for example, holds that our physical experiments succeed
(55)
in measuring quality directly. For him quantity is not something that exists objectively in the physical structure of reality, but a conceptual construction which results

(56)
from our process of measurement. But ordinarily this identification has been approached from the opposite direction by a sacrifice of quality to quantity. The evident dependence of the sense qualities upon the organic structure of the sense faculties, and the immense success of quantitative methods in science have led some to deny an objective status to all qualities and to conceive of the cosmos as a purely quantitative structure. Such a position is completely gratuitous. We have already shown that even though conditioned by the instruments of perception, the sensible qualities are not psychical, but physical and hence existing objectively in nature. And the fact that they do not exist in the distant object in exactly the same way as they are perceived, is no argument that the object is deprived of all qualitative determinations. (57) Moreover, the success of quantitative methods cannot be adduced as a demonstration of the non-existence of qualities without transforming a methodology into an ontology. (58)

As a matter of fact, the existence of an infinitely homogeneous reality is hardly conceivable. And even if it were a possibility, it could never be a source of knowledge. (59) It could not even be measured. For, as

Professor Thompson has remarked, "quantity, per se, in other words, pure undetermined quantity, is as unmeasurable as quality. It is measurable only when bounded, stamped, or permeated with quality. The quantitative picture of Nature, in spite of its satisfying accuracy is not self-supporting : it is executed in a framework of qualities, with which the savant must maintain contact." (60) It is worth while pointing out, moreover, that the numbers out of which the structure of mathematical physics is erected are concrete measure-numbers. This means that they involve something more than pure quantity. For even though they do not necessarily have a direct and immediate relation with our qualitatively different sensations or with the ontological qualities of reality, they are the results of qualitatively different processes of measurement.

All this enables us to see what is actually involved in the scientific homogenization of the cosmos. The barriers isolating the specific properties of nature are broken down; the pure givenness of these properties are mastered; nature is transformed into a deductive system; reality is rationalized; the most profound aspect of the cosmos: the order of the whole, is in a sense, revealed to the mind. At the same time contact is maintained with the specific properties through a process of correlation and substitution.

All this is a great achievement. But it is not without its price. For the determinant properties of things in their specific essences, the very inner natures of things have faded out of the picture. The hillside with its greenness and its softness of turf, the elephant in its own proper essence -- all of the things in Nature which seem to be of the greatest significance for the other sciences of reality, for all the arts, and for human life itself, have slipped through the fingers of the physicist and have left in their wake only a series of pointer readings. (61)

This raises the question of the relative rationality of the qualitative and the quantitative determinations of reality. It has often been stated, that the latter are more rational than the former. That there is a sense in which this is true is evident from all we have been saying. But perhaps one might be tempted to question this superior rationality on the score that quantity is said to follow upon matter which is the source of irrationality, whereas quality is said to follow upon the form. John of St. Thomas gives us the answer in the following terms:

Non est intelligendum, quod quantitas sequatur ad materiam nudam sine forma, cum constet sequi ad gradum corporeitatis qui praebetur a forma. Sed intelligitur sequi materiam, vel quia solum in-

venitur in rebus materialibus, qualitas autem sequitur actum, etiam si immaterialis sit, et sic proprium est qualitatis qualificare sicut et formae; tum etiam quia quantitas se habet in genere accidentium, sicut materia in genere substantiae, quia non est activa, sed medium receptivum aliorum accidentium et inter reliqua primum. (62)

Quantity has the great advantage of being the accident closest to substance. Material substance is a substance that can't contain itself, so to speak; it is dispersed, divided into parts; and quantity is the order of these parts. It is precisely because quantity consists in order that it can provide us with formal causality and not just with a kind of material causality, as one might be led to think because of the fact that it follows upon matter. Quantity is more abstractable than sensible qualities -- not, however, because the latter are qualities, but because they are sensible. Mathematical beings are more perfect than sensible beings from the point of view of exactitude and certitude. Their very homogeneity is the source of precision. Moreover, their very emptiness makes them more manipulatable by us. Finally, quantity provides the common matrix which, as we have just seen, is so necessary for the rationalization of the cosmos. For all of these reasons quantity has a source of rationality which the specific properties of reality do not possess. And it is a type of rationality that is par-

ticularly amenable to the methods of physical science.

On the other hand, the specific properties of reality are far more rational from another point of view. They reveal the proper natures of things. Consequently, it is in philosophical science that their rationality is particularly relevant. As we explained in Chapter I, the rationalities proper to physics and to philosophy are related to each other in inverse proportion. In the last analysis, it all comes down to a difference in the type of measurement proper to each science. In the following chapter we shall return to this point.

And now, having seen the way in which the mind triumphs over one of the sources of irrationality connected with sensible perceptions -- their isolation and pure givenness, we must turn our attention to the other element of irrationality about which we spoke earlier in this chapter -- the indefiniteness of proper sensibles. By the same processes which we have been describing science succeeds in mastering this second irrational element, it succeeds in defining the indefinable. Through its quantitative methods, physics is able to define heat and colour in terms of movement of molecules, light waves, etc. A non-scientific person with the faculty of sight cannot define what he means by redness, but a blind physicist can. And

the advantages of this definability are so obvious that they do not need to be mentioned.

But once again we must remain critically aware of what is actually involved in this defining of the indefinable. From what we have said about the impossibility of attaining the qualitative in its proper, specific nature by means of the quantitative it is obvious that the scientific definitions of heat, colour, etc. do not give us the quod quid est of these properties. There is a world of ambiguity in such expressions as "heat is a movement of molecules." All that they actually mean is: there is a correlation between the movement of molecules and heat. And science cannot even tell why there is such a correlation.

The scientist does not seek a derivative measure for qualities which are incapable of direct measurement in order to find what those qualities really are. The measure of an object, whether fundamental or derived, does not express what the object is; it expresses how the object, as an instance of a certain character, is related to another object chosen as a standard for that character or for a correlated character. (63)

The following lines of Dewey, in spite of their obvious instrumentalistic bias, bring out rather accurately the point we are trying to make:

The resolution of objects and nature as a whole into facts stated exclusively in terms of quan-

tities which may be handled in calculation, such as saying that red is such a number of changes while green is another, seems strange and puzzling only when we fail to appreciate what it signifies. In reality, it is a declaration that this is the effective way to think things; the effective mode in which to frame ideas of them, to formulate their meanings. The procedure does not vary in principle from that by which it is stated that an article is worth so many dollars and cents. The latter statement does not say that the article is literally or in its ultimate 'reality' so many dollars and cents; it says that for purpose of exchange that is the way to think of it, to judge it. It has many other meanings and these others are usually more important inherently. But with respect to trade, it is what it is worth, what it will sell for, and the price value put upon it expresses the relation it bears to other things in exchange... The formulation of ideas of experienced objects in terms of measured quantities, as these are established by an intentional art or technique, does not say that this is the way they must be thought, the only valid way of thinking them. It states that for the purpose of generalized, indefinitely extensive translation from one idea to another, this is the way to think them. . .

There is something both ridiculous and disconcerting in the way in which men have let themselves be imposed upon, so as to infer that scientific ways of thinking of objects give the inner reality of things, and that they put a mark of spuriousness upon all other ways of thinking of them, and of perceiving and enjoying them. It is ludicrous because these scientific conceptions, like other instruments, are handmade by man in pursuit of realization of a certain interest -- that of the maximum convertibility of every object of thought into any and every other. (64)

It is clear then that mathematical physics does

not succeed in actually defining the specific properties of nature, but merely something that is correlated with them. But even with regard to this correlation a further important qualification must be made. For, since scientific definitions are necessarily operational, the definitions of physics do not give us an absolute, objective, quantitative element that is in correlation with the specific properties; they necessarily involve the whole operational procedure by which this quantitative element has come to be known by us. This obviously removes them still further from a direct rendition of the quod quid est of the sensible properties. And in this connection it is necessary to point out that though the pointer readings which issue from our processes of measurement are not abstract but concrete numbers, they are not concrete in the sense that they directly correspond to certain sensations, but only in the sense that they are produced by concrete processes of measurement into which a multiplicity of concrete determinations have entered.

This brings us to another significant question. One of the important reasons given above for the adoption of quantitative methods in physics was the attempt to overcome the subjectivity and anthropomorphism of sensibility. We

pointed out how through a substitution of inorganic instruments of measurement for organic instruments of perception science has been able to triumph over the subjectivity of sense cognition. But just how complete is this triumph? Do our measuring instruments provide us with a perfectly objective rendition of reality? Until fairly recently, it was not uncommon for scientists to think so. (65) Yet a greater error could hardly be imagined. In the next Chapter when we come to analyze the process of measurement we shall try to show just how much subjectivity this process involves, and for the moment it will suffice to merely mention the more important sources of this subjectivity. In the first place, there is the mental operation involved in the conception and method of application of the measuring instrument: all instruments are constructed and applied in accordance with certain scientific theories, and hence participate in the subjectivity of these theories. In the second place, there is the physical operation involved in the actual process of measurement: the instruments of measurement enter intrinsically into the process of measurement in such a way that the results are not independent of them.

The measuring instruments are not merely pas-

sive recipients simply registering the rays impinging upon them: they play an active part in the event of measuring and exert a causal influence upon its result. The physical system under consideration forms a totality subject to law only if the process of measuring is treated as forming part of it. (66)

In principle a physical event is inseparable from the measuring instrument or the organ of sense that perceives it; and similarly a science cannot be separated in principle from the investigators who pursue it. (67)

In attempting to get away from a mixture of senses and objects we succeed only in arriving at a mixture of instruments and object.

While considering all the advantages that have accrued to science from the substitution of inorganic instruments of measurement for organic instruments of perception, it is important to realize that our senses are also instruments of measurement, and that from this point of view there is no essential difference between the two:

Perception is a kind of crude physical measurement. . . There is no essential distinction between scientific measures and the measures of the senses. In either case our acquaintance with the external world comes to us through material channels; the observer's body can be regarded as part of his laboratory equipment, and so far as we know, it obeys the same laws. We therefore group together perceptions and scientific measures, and in speaking of a 'particular observer' we include all his measuring appliances. (68)

The greater objectivity that comes to us by means of impersonal instruments differs from the objectivity that comes to us through the senses only by degree; there is no qualitative difference between the two cases. The sense of touch perceives differences of temperature, and it may be said that it is only by accident that one's finger is a poor thermometer. If it were possible to know the physiological state of the finger with great accuracy one could by means of it arrive at the degree of temperature with as great precision as that achieved by a thermometer. In general it must be kept in mind that in our perception of the common sensibles, even without the aid of impersonal instruments, we already have a comparison.

In connection with what was said above about the advantages of the homogenization of the universe deriving from the greatly extended range which measurement adds to our limited powers of perception, a reservation must also be made. For while it is true that there is much in nature which cannot be sensed but which can be measured, it is likewise true that there is a great deal which can be sensed and cannot be measured.

This analysis of the relation between science and

sensibility would not be complete if, before concluding, some attempt were not made to determine how closely the scientific world remains linked to the sense world. From one point of view the bond seems to have grown extremely tenuous. As has already been said, mathematical physics is based upon a minimum of experience. The only kind of sensibility that is directly required for the scientist to carry on his work is that which is necessary to recognize objects and instruments and to perceive the coincidence of a fixed line on a scale with another variable line. All that this demands is the ability to perceive a spatio-temporal exteriority that is qualitatively differentiated. It makes little difference just what the nature of this qualitative differentiation is, provided it affords a sufficient means for making necessary distinctions. In other words, science has come as close as possible to the lowest form of all sense experience -- the quantitative contact of pure taction. (69)

But it is important to keep in mind that in spite of its tenuity the bond between the scientific world and the senses remains essential:

What I mean is this: we rig up some delicate physical experiment with galvanometers, micrometers, etc., specially designed to eliminate the fallibility of human perceptions;

but in the end we must trust to our perceptions to tell us the result of the experiment. Even if the apparatus is self-recording we employ our senses to read the records. (70)

The desensitized processes of physics are not self-supporting. Independent of the whole background which they have in the sensible world they are meaningless. Moreover, it must not be forgotten that by the very fact that mathematical physics is physics, it must realize the reductio ad sensum mentioned in Chapter II, which is characteristic of every science of nature. It must both take its origin in the sense world and terminate in it. Planck explains this very clearly in The Universe in the Light of Modern Physics:

In my opinion, the teaching of mechanics will still have to begin with Newtonian force, just as optics begins with the sensation of colour, and thermodynamics with the sensation of warmth, despite the fact that a more precise basis is substituted later on. Again, it must not be forgotten that the significance of all physical concepts and propositions ultimately does depend on their relation to the human senses. This is indeed characteristic of the peculiar methods employed in physical research. If we wish to form concepts and hypotheses applicable to physics, we must begin by having recourse to our powers of imagination; and these depend upon our specific sensations, which are the only source of all our ideas. But to obtain physical laws we must abstract exhaustively from the images in-

roduced, and remove the definitions set up all irrelevant elements and all imagery which do not stand in a logical connection with the measurements obtained. Once we have formulated physical laws, and reached definite conclusions by mathematical processes, the results which we have obtained must be translated back into the language of the world of our senses if they are to be of any use to us. In a manner this method is circular; but it is essential, for the simplicity and universality of the laws of Physics are revealed only after all anthropomorphic additions have been eliminated.(71)

As physics progresses it inevitably becomes more abstract and more highly symbolic. But to even its most abstract symbolism there always remains attached a dictionary which links up the symbols with concrete entities. And these concrete entities ultimately lead back to the world of sense. Thus modern physics presents the paradox of an ever increasing detachment from the sense world, and at the same time an essential attachment to it. And this paradox is comprehensible only in terms of another paradox: modern physics is at the same time physics and not physics; that is to say, it is a hybrid science, an intermediary science. It is formally distinct from pure natural science, but at the same time it is a valid study of nature. Because it is formally mathematical it must in its development draw ever farther and farther away from the world of sense; but because it is terminative physical it must inevitably lead back to it.

This brings us to the final point that must be touched upon before we leave the general question which has formed the subject of this Chapter. In setting up the problem which has been occupying us we mentioned that some authors see in the recent developments of physics an abandonment of the common sensibles similar to the former abandonment of the proper sensibles and complementary to it. We do not believe that this is the correct interpretation of the newer scientific constructions. It is true that they are not susceptible of direct imaginative representation. But this does not mean that science has removed its object from the realm of the common sensibles as earlier it had removed it from the realm of the proper sensibles. It probably means several things. For one thing, in so far as these recent constructions have to do with the microcosmic world, it means that science is beginning to discover that phenomena on this microcosmic level may not be capable of direct representation in terms of phenomena on the macroscopic level, DeBroglie points this out in Matière et Lumière:

Plus nous descendons dans les structures infimes de la matière plus nous nous apercevons que les concepts forgés par notre esprit au cours de l'expérience quotidienne, et tout particulièrement ceux d'espace et de temps, deviennent impuissants à nous permettre de décrire les mondes nouveaux

où nous pénétrons. On dirait que le contour de nos concepts doit, si l'on peut s'exprimer ainsi, s'estomper progressivement pour leur permettre de s'appliquer encore un peu aux réalités des échelles subatomiques. (72)

But in general, the most fundamental significance of these developments seems to be that science, by using as its instruments mathematical entities, which, as we saw in the last Chapter, can stretch their connection with the imagination to the extremes of tenuity, has so intellectualized its subject as to place it outside of any immediate relation to the sensible. There is no reason why it should not do this, provided all of its intellectual constructions can be made to lead back ultimately to verification in the sensible world. In this way that can be said to "explain" the sensible world. But this does not mean that these constructions give us a direct and immediate revelation of things as they exist in the real world or that they prove the common sensibles to be illusory.

And now, having seen the basis for mathematization that exists in nature, we must see how science, by laying hold of this basis through the instrumentality of measurement, succeeds in transforming nature into a new world of symbolism. This Chapter has attempted to show that in mathe-

mathematical physics the mind's ambition is to transform the universe into a purely rational system in which multiplicity and differences will be constructed out of unity and sameness. It is in measurement that the mind finds a road towards its goal. For measurement consists in the repeated application to reality of the same unity - - a unity which the mind has determined.

CHAPTER EIGHT

AN ANALYSIS OF MEASUREMENT.

1. Science and Measurement.

This Chapter is in a sense the pivotal point of our whole study. For the central idea in mathematical physics is that of a scientia media involving a union of the physical and mathematical worlds, and it is precisely through measurement that these two worlds are brought into contact. This was already recognized by John of St. Thomas, for in speaking of the mathematical physics of his time, he writes: "Astrologus non agit de coelo et planetis, ut sunt entia mobilia, sed ut mensurabiles sunt eorum motus." (1) The reason why measurement is able to achieve this union between a science that is essentially experimental and one that prescind from experiment is that, while remaining a physical instrument of experiment, it is not an instrument which merely reveals physical phenomena; it both reveals them and transforms them into numerical values. "Ce qui distingue notre science," writes Bergson, "ce n'est ^{par} qu'elle

expérimente, mais qu'elle n'expérimente et plus généralement
ne travaille qu'en vue de mesurer." ⁽²⁾ It is significant that
the names of practically all of our modern experimental
apparatus end in "meter" whereas formerly they ended in
"scope".

In other words, there is something both physical
and mathematical about measurement. It is, as it were, a
transforming machine into which physical determinations enter
and from which numbers emerge. And even though the concrete
measure-numbers which issue from our pointer-readings are
not in themselves a mathematization of the physical in the
full sense of the word, they are the incoation of this mathe-
matization. They are the stuff out of which all the mathe-
matical elaborations of physical science evolve. Although
still directly linked with the physical, they already have
something of the idealization, the absolute character, the
necessity, etc. that belong to the mathematical world. And
just as the whole mathematical interpretation of nature
arises out of the physical through processes of measurement,
so it must ultimately lead back again to the physical through
processes of measurement. For no mathematical theory in
physics has any value if it cannot be verified in concrete
pointer-readings.

This explains why the whole progress of physical science is directly bound up with the refinement of measurement. (3) And, as Norman Campbell has pointed out, it is to the fact that it is a science of measurement that physics owes its ascendancy over the other natural sciences. (4) All this explains why nothing has any meaning in physical science except in terms of measurement. (5) For a physicist a thing is real only to the extent in which it is measurable and everything that falls outside the scope of measurement is irrational. To define a body by its physical properties means simply to enumerate the operational processes of measurement to which this body can be subjected, and to list the series of numbers which the instruments used in these processes render. For example, what meaning for a mathematical physicist can hydrogen have, with its various properties: colorless, of a certain density, liquifying at a certain temperature, etc.? It can have no meaning except the following: a body will be called hydrogen if when subjected to the instruments which define fluidity, viscosity, compressibility, temperature, refraction, etc., it produces a collection of pointer-readings which square with the numbers cited in the definition of hydrogen.

Among modern philosophers of science no one has

labored with greater zeal to make this point generally understood than Sir Arthur Eddington. (6) In connection with the adventure of elephant which we discussed in the last Chapter, he writes:

The whole subject-matter of exact science consists of pointer readings and similar indications. We cannot enter here into the definition of what are to be classed as similar indications. The observation of approximate coincidence of the pointer with a scale-division can generally be extended to include the observation of any kind of coincidence - or, as it is usually expressed in the language of the general relativity theory, an intersection of world-lines. The essential point is that, although we seem to have very definite conceptions of objects, in the external world, those conceptions do not enter into exact science and are not in any way confirmed by it. Before exact science can begin to handle the problem they must be replaced by quantities representing the results of physical measurement.

Perhaps you will object that although only the pointer readings enter into the actual calculation it would make nonsense of the problem to leave out all reference to anything else. The problem necessarily involves some kind of connecting background. It was not the pointer reading of the weighing-machine that slid down the hill: And yet from the point of view of exact science the thing that really did descend the hill can only be described as a bundle of pointer readings. (It should be remembered that the hill also has been replaced by pointer readings, and the sliding down is no longer an active adventure but a functional relation of space and time measures.) The word elephant calls up a certain association of mental impressions, but it is clear that mental impressions as such cannot be the subject handled in the physical problem . . .

The vocabulary of the physicist comprises a number of words such as length, angle, velocity, force, potential, current, etc., which we call "physical quantities." It is now recognized as essential that these should be defined according to the way

in which we actually recognize them when confronted with them, and not according to the metaphysical significance which we may have anticipated for them. In the old textbooks mass was defined as 'quantity of matter'; but when it came to an actual determination of mass, an experimental method was prescribed which had no bearing on this definition. The belief that the quantity determined by the accepted method of measurement represented the quantity of matter in the object was merely a pious opinion. At the present day there is no sense in which the quantity of matter in a pound of lead can be said to be equal to the quantity in a pound of sugar. Einstein's theory makes a clean sweep of these pious opinions, and insists that each physical quantity should be defined as the result of certain operations of measurement and calculation. You may if you like think of mass as something of inscrutable nature to which the pointer reading has a kind of relevance. But in physics at least there is nothing much to be gained by this mystification, because it is the pointer reading itself which is handled in exact science; and if you embed it in something of a more transcendental nature, you have only the extra trouble of digging it out again . . .

Whenever we state the properties of a body in terms of physical quantities we are imparting knowledge as to the response of various metrical indicators to its presence, and nothing more . . .

The recognition that our knowledge of the objects treated in physics consists solely of readings of pointers and other indicators transforms our view of the status of physical knowledge in a fundamental way. Until recently it was taken for granted that we had knowledge of a much more intimate kind of the entities of the external world. (?)

Perhaps a word of explanation should be immediately appended to this passage lest confusion arise. When we say that mathematical physics deals only with pointer readings, we do not mean that it begins and ends in numbers

alone. If this were the case it would be mathematics and not mathematical physics. The numbers it deals with are measure numbers. In other words, the experience which gives rise to these numbers has something more than a pre-scientific function as in mathematics. The physical process of measuring the quantitative determinations of nature is an integral part of mathematical physics. Consequently, even though the numbers dealt with do not represent things in the objective cosmos, as we shall see, they are always tied up with objective determinations of the physical universe out of which they have issued through measurement. In this sense there is a physical background in which they are embedded. Yet the mathematical physicist cannot get at this background in any other way than by measurement, and that is why as long as he remains true to the nature of his science this background will always elude him. Of course it is possible for him to go out beyond the limitations of his science and embed the measure-numbers in a background of his own choosing, but, as Eddington remarks, in so far as mathematical physics is concerned, there is nothing to be gained by doing so.

We shall return later to discuss the nature of knowledge which grasps reality only through measurement.

For the present we merely wish to emphasize the fact that this is the only type of knowledge that is had in mathematical physics. Of course, in actual practice, scientists never restrict themselves completely to measure-numbers. (8) As Poincaré has remarked, they cannot be denied the liberty of using metaphores any more than poets can. (9) But in the last analysis their grasp of the cosmos is restricted to metric knowledge. It is because this is not always recognized that much of the confusion about the meaning of modern science has arisen. This is particularly true of many of the abortive criticisms of the Theory of Relativity. Einstein's great merit is to have recognized clearly the complete dependence of mathematical physics upon measurement, and to have seen the implications and limitations of this dependence.

That mathematical physics is essentially a science of measurement is now becoming generally recognized. But what is not generally recognized is that every science of reality is essentially a science of measurement. This statement is, of course, ambiguous, for obviously the term "measurement" cannot be understood in the same sense in which we have been employing it in relation to physics. And yet it is not an equivocation; in both cases the term is used in

its strictly formal sense. And in order to understand accurately the part that measurement plays in physics it is extremely important to see how the other sciences, and particularly the philosophical sciences are related to measurement.

Taken in its general sense, measurement implies an effort on the part of the intellect to see a certain complexity in the light of a principle of simplicity. This principle is provided by a standard, and the attempt of the mind to reduce complexity to simplicity will be more or less successful in proportion to the degree of simplicity possessed by the standard. This explains why in physics there is a continual search for a minimum measure. But it is not only in physics that there is an attempt to see the complexity of reality in terms of the simplicity of a standard. This is found in the philosophical sciences as well, although the nature of the standard and hence the nature of the measurement is something quite different from what is found in physics.

St. Thomas defines measure as "that by which
(10)
the quantity of a thing is made known." But as we saw in the last Chapter, there are two kinds of quantity: predicamental and transcendental. The former consists in homogeneous

exteriority and the latter in interiority, that is, in perfection of being. Now whereas in physics it is predicamental quantity that is made known through measurement, in the philosophical sciences it is transcendental quantity. In both metaphysics and philosophy of nature it is the principal subject of the science which provides the ultimate principle of simplicity in relation to which every other subject in the science is measured. For, as John of St. Thomas remarks: "mensura importat perfectionem, cum semper accipiatur pro mensura id quod perfectissimum est in unoquoque genere; nec requiritur quod sit notificativum rei mensuratae, ut fundans imperfectam cognitionem; sed per modum alicuius magis simplicis et perfecti quo res mensurata magis ad unitatem et uniformitatem reducitur." (11)

In every order in which a relation of more or less is possible there is measure, and the "maxime tale" is always the measure of everything that is found in the order. (12) In metaphysics the principal subject which plays the part of the standard is God, known extrinscally, in so far as He is the cause of being. It is by comparison with God, Pure Act, that the transcendental quantity of all metaphysical beings is measured and their intrinsic perfection revealed. In philosophy of nature the principal

subject is man, and it is in relation to him that the transcendental quantity of all natural beings is determined. In this sense Protagoras was right in making man the measure of all things in the universe. Whereas from the point of view of the physicist man is the most complex being in the cosmos and the one that is the farthest removed from his standard of measurement and hence the one that is least amenable to his processes of measurement, from the point of view of the philosopher of nature he is the most simple being in the cosmos precisely because he possesses the highest degree of interiority. It is extremely significant that the measurements of physics and the philosophy of nature lead in opposite directions: the one determines things in relation to the simplicity of purely homogeneous exteriority, the other in terms of the simplicity of interiority. For physics interiority is irrational. That is why the experimental science which deals with man -- experimental psychology -- is the most irrational of all the experimental sciences. For the philosophy of nature it is homogeneous exteriority that is irrational and that explains why for the philosopher natural things become more obscure as one descends the scale of perfection. No one, perhaps, has handled this question with greater skill than Professor DeKoninck:

Toute science s'efforce de réduire le complexe au plus simple et de l'expliquer en fonction de lui. Mais il faut s'entendre sur la signification du terme 'simple'. La nature de la simplicité à laquelle on doit tout ramener différenciera profondément les savoirs. Or il est facile de montrer que ce que nous appelons simple en science expérimentale est tout opposé à ce que nous disons simple en philosophie. En science expérimentale une pierre est infiniment plus simple qu'une cellule; le va-et-vient d'un piston est beaucoup plus simple que le bond d'une panthère qui se jette sur sa proie; de tous les êtres qu'étudie la science expérimentale, l'homme est incontestablement le plus complexe. Or en philosophie c'est tout le contraire qui est vrai. L'animal est plus simple que la plante, et de tous les êtres qu'étudie la philosophie de la nature, c'est l'homme qui est le plus simple; de même qu'en métaphysique la mesure et la cause de tout être est la simplicité absolue de l'acte pur. En physique on mesure par la minima mensura - le temps par le temps atomique par exemple; en philosophie la mesure est toujours riche et compréhensive - - le temps est mesuré par l'événement, et tous les deux par l'éternité. En d'autres termes, la simplicité expérimentale est inversement proportionnelle à la simplicité ontologique. Le philosophe dira que le savant explique le supérieur par l'inférieur, le parfait par l'imparfait. Ainsi nous pouvons dire par avance que dans la mesure où une explication expérimentale de l'homme est possible, elle consistera à l'étudier dans la perspective de ce qui est expérimentalement plus simple que lui, non pas pour identifier entre eux le complexe et l'élémentaire, mais pour dériver l'un de l'autre. Il est donc tout naturel que le savant cherche à dériver l'homme de l'animal, celui-ci de la plante et à voir toute la hiérarchie des espèces naturelles s'ériger dans le sens d'une organisation toujours croissante et plus complexe. Le philosophe qui nie la possibilité même d'une théorie évolutionniste nie l'essence même de la méthode scientifique. S'il était logique il devrait nier aussi la valeur d'une mesure de longueur. (13)

This brings us back to what we saw in Chapter I in relation to the possible extent of the mathematization of nature.

But in order to understand the peculiar nature of the knowledge that is based completely on a measurement of things in terms of homogeneous exteriority we must try to analyze the nature of measurement.

2. The Nature of Measurement.

Measure, according to Aristotle and Saint Thomas,⁽¹⁴⁾ is that by which the quantity of a thing is made known. This definition immediately gives rise to a difficulty. For quantity may be known independently of any measure. In fact, homogeneous exteriority is an immediate datum of cognition, and consequently does not depend upon any medium such as a measure. Moreover, we have already pointed out that quantity is known and studied both by the philosopher of nature and the metaphysician, and in neither case does the knowledge of it involve measurement. This difficulty did not escape Aristotle and St. Thomas. For after laying down the fundamental definition just cited, they proceed to

qualify its meaning by adding the phrase: in quantum
quantitas. That is to say, measure is that by which the
quantity of a thing is made known precisely in so far as
it is quantity. At first glance this may not seem to help
matters, for is not quantity known as quantity independently
of measurement in the ways just mentioned?

St. Thomas throws light upon the question by
writing: "Addit autem (Philosophus) 'in quantum quantitas'
ut hoc referatur ad mensuram quantitatis. Nam proprietates
et alia accidentia quantitatis alio modo cognoscuntur." (15)
In other words, there are two fundamental aspects to
quantity. In the first place, in so far as it is one of the
nine accidents it is a certain essence and consequently can
be known in the same way that all the essences of reality
are known. In so far as it orders the parts of a material
substance by contributing to it homogeneous exteriority,
it is a primary and immediate datum of cognition. In so
far as it is involved in the mobility of the cosmos, it can
be studied by the philosopher of nature. In so far as it
is one of the principles of being it can be studied by the
metaphysician. In all of these cases it is a question of
"quidditative" knowledge, that is, knowledge that answers
the question: what is quantity? Now while this question

"what" can be asked of all the categories of reality, there is a special question that can be asked only of quantity - - "how much" (quantum). And it is knowledge which answers this question that is revealed by measure. Since, then, the question "how much" (quantum) is proper to quantity alone, Aristotle and St. Thomas are justified in saying that measure is that by which the quantity of a thing is made known; and they are speaking with strict formality when they add the phrase: in quantum quantitas.

It is extremely important to insist upon the precise nature of the knowledge of quantity that is given to us through measurement. It is not "quidditative" knowledge; it does not in any way answer the question: what is quantity. It merely tells us how much quantity there is. This knowledge is mediate and derivative, since it comes to us through the medium of measure. But a measure is a very special kind of cognitive medium. Unlike a sign, it does not substitute for the thing known, nor does it in any way manifest its nature. And the practical conclusion to be drawn from these considerations is that in so far as science is based upon measurement, not only does it not tell us the "whatness" of all the determinations of reality which fall outside the category of quantity, but it does not even tell us

the "whatness" of the quantitative determinations that are being dealt with. This point is frequently lost sight of.

But in order to understand more clearly the nature of this peculiar type of knowledge we must try to see just how quantity is revealed through measurement. A measure manifests the quantity of a thing not in any way whatsoever, but through the reduction of a certain type of complexity to simplicity, of indetermination to determination, of variability to uniformity -- in other words, of unintelligibility to intelligibility. When the determination of one thing manifests to us the determination of another thing, which without it would remain indetermined, we say that the first is the measure of the second. In this way the measure is a certification of the thing measured. From this it follows that there are two essential elements in measurement: a principle of perfection and uniformity and simplicity, which is the measure, and a process of reduction of the complex and variable to this principle. (16)

This second element obviously involves some kind of union between the measure and the thing measured. In order to understand the nature of measurement it will be necessary to analyze each of these two elements.

With regard to the first it is clear that in order for a thing to be a measure it must be one and indivisible, for in no other way can it be simple and determined. That is why in the tenth book of the Metaphysics St. Thomas begins his explanation of measurement by saying: "cum ratio unius sit indivisibile esse; id autem quod est aliquo modo indivisibile in quolibet genere sit mensura; maxime dicetur in hoc quod est esse primam mensuram cuiuslibet generis." (17) But it must be pointed out that the "one" is not as such a measure. That is to say, indivisibility of itself does not necessarily constitute a measure; it must be indivisibility in a certain given order. The transcendental One is not a measure because it is not in a definite genus. Moreover, it does not possess strict unity. (18)

Aristotle and St. Thomas make it clear why indivisibility is one of the essential qualities of a measure:

Assignat autem rationem quare mensuram oportet esse aliquid indivisibile; quia scilicet hoc est certa mensura, a qua non potest aliquid auferri vel addi. Et ideo unum est mensura certissima; quod unum quod est principium numeri, est omnino indivisibile, nullamque additionem aut subtractionem suscipiens manet unum. (19)

A measure is a certification of the thing measured. But it can be a certification of something else only to the extent in which it is fixed in certainty itself. And it can be fixed in certainty only by being fixed in indivisibility.

A thing can be a measure, then, only to the extent in which it is indivisible. But as St. Thomas goes on to explain:

Non similiter in omnibus invenitur indivisibile; sed quaedam sunt omnino indivisibilia, sicut unitas quae est principium numeri; quaedam vero non sunt omnino indivisibilia, sed indivisibilia secundum sensum, secundum quod voluit auctoritas instituentium tale aliquid pro mensura; sicut mensura pedalis, quae quidem indivisibilis est proportione, sed non natura.(20)

And elsewhere he writes: "Nec oportet, quod omnis mensura sit omnino infallibilis et certa, sed secundum quod est possible in genere suo". (21) In so far as the measurement of predicamental quantity is concerned it is only the one which is the principle of number that has absolute indivisibility. That is why it alone is the perfect measure. "Esse mensuram est propria ratio unius secundum quod est principium numeri." (22) And just as all of our notions of measurement are derived from predicamental quantity, (23) so within the realm of predicamental quantity itself all

our notions of measurement are derived from the measurement of discrete quantity:

Primo ostendit quod ratio mensurae primo invenitur in discreta quantitate, quae est numerus dicens, quod id quo primo cognoscitur quantitas 'est ipsum unum', idest unitas, quae est principium numeri. Nam unum in aliis speciebus quantitatis non est ipsum unum, sed aliquid cui accedit unum; sicut dicimus unam manum, aut unam magnitudinem. Unde sequitur, quod ipsum unum, quod est prima mensura, sit principium numeri secundum quod est numerus.. Hinc scilicet ex numero et uno quod est principium numeri, dicitur mensura in aliis quantitatibus, id scilicet quo primo cognoscitur unumquodque eorum. Et id quod est mensura cuiuslibet generis quantitatis, dicitur unum in illo genere. (24)

For us the "one" which is the principle of number is the model for every measure. It is that by which quantity is first made known to us: "id quo primo cognoscitur quantitas."

In the measurement of other kinds of predicamental quantity only quasi indivisibility is possible. It is impossible, for example, to have a length which will be a universal measure for all lengths as the one which is the principle of number is the universal measure for all numbers.

Hoc modo derivatur ratio mensurae a numero ad alias quantitates, quod sicut unum quod est mensura numeri est indivisibile, ita in omnibus aliis generibus quantitatis aliquod unum

indivisibile est mensura et principium. Sicut .
in mensuratione linearum utuntur homines quasi
indivisibile 'mensura pedali,' idest unius
pedis; ubique enim quaeritur pro mensura
aliquid indivisibile, quod est aliquid simple².(25)

And this quasi indivisibility is nothing but an imitation of
the true indivisibility that is found in the "one" which is
the principle of number. One inch, for example, is an
imitation, for it cannot be by itself an absolute measure.

Sed mensurae aliorum generum quantitatis imitantur
hoc unum, quod est indivisibile, accipiens aliquid
minimum pro mensura secundum quod possibile est.
Quia si acciperetur aliquid magnum utpote stadium
in longitudinibus, et talentum in ponderibus,
lateret, si aliquid modicum substraheretur vel
adderetur; et semper in majori mensura hoc magis
lateret quam in minori.

Et ideo omnes accipiunt hoc pro mensura tam in
humidis, ut est oleum et vinum, quam in siccis,
ut est granum et hordeum quam in ponderibus et
dimensionibus, quae significantur per grave et
magnitudinem; quod primo invenitur tale, ut ab
eo non possit aliquid auferri sensibile vel addi
quod lateat. Et tunc putant se cognoscere
quantitatem rei certitudinaliter, quando
cognoscunt per huiusmodi mensuram minimam.(26)

This attempt on the part of the measurement of
magnitude to imitate the measurement of multitude must be
considered in the light of what was said in Chapter II
about the difference between arithmetic and geometry. We
pointed out that the higher abstraction and superior intelli-
gibility of arithmetic was based upon the superior ration-
ality of number in comparison with magnitude. Number is in

fact more immaterial, more determined, more actual than continuous quantity. The continuum is something essentially obscure, indetermined and potential because of its intrinsic divisibility into infinity. As a result, the measurement of discrete quantity is something clear and absolute, while that of continuous quantity is always something obscure and relative. In ~~the~~ latter there is always a background
(27)
of irrationality.

But since measurement is always a rationalization in the sense that it manifests the quantity of the thing measured, the mind can never rest satisfied with this background of irrationality. That is why there will inevitably be a constant attempt to assimilate as much as possible the measurement of continuous quantity to that of discrete quantity. "Omnis mensuratio quae est in quantitatibus continuis aliquo modo derivatur a numero. Et ideo relationes quae sunt secundum quantitatem continuum etiam attribuuntur numero."
(28)

This process of assimilation will be at once both subjective and objective. In the first place, since a definite unit of measure is not given objectively for magnitude as it is for multitude, one must be constructed

by the mind, established by fiat.

Quaedam vero non sunt omnino indivisibilia, sed indivisibilia secundum sensum, secundum quod voluit auctoritas instituentium tale aliquid pro mensura. (29)

In gravitate ponderum accipitur ut unum indivisibile uncia, sive 'mna', idest quoddam minimum pondus; quod tamen non est simplex omnino, quia quodlibet pondus est divisibile in minora pondera, sed accipitur ut simplex per suppositionem. (30)

This point is of considerable importance for the philosophy of physical science. For the basic measurement in physics is that of magnitude. Though science employs a great variety of measurements, they are reducible in the last analysis to the measurement of length. It is clear, then, that the measurement out of which the whole structure of mathematical physics is erected, is not based on something absolute, something perfectly objective and given as such in nature, but upon a construction of the mind. Both the intellect and the will have to enter into the process of measurement to determine a standard and establish a unity that does not exist. Magnitude is lifted to a status of intelligibility that is not native to it. And all this obviously involves a separation of some sort from the real world. What is not by nature one and indivisible is considered by the mind as if it were. Once again, from this

point of view, mathematical physics is a science of als ob.

However, this construction is not purely subjective and arbitrary. In order to assimilate the measurement of magnitude to that of multitudo it is not sufficient to declare by fiat something indivisible that is by nature divisible; it is necessary that what is declared indivisible approach as closely as possible to that which is objectively indivisible. In other words, the less extension the standard chosen possesses, the more perfectly will it be able to serve as a measure. That is why science is always searching for the smallest possible measure -- the minima mensura. And this is true of ancient as well as of modern science:

Id quod est minimum in unoquoque genere, est mensura illius generis, sicut in melodia tonus, et in ponderibus uncia, et in numeris unitas; manifestum est autem quod minimus motus est qui est velocissimus, qui scilicet habet minimum de tempore, quod est mensura motus; omnium ergo motuum velocissimus est motus coeli. Et accipitur hic motus velocissimus, qui citius peragit cursum suum ex parte brevitatis temporis ... Unde ... attenditur secundum minimam magnitudinem. (31)

This choice of the speed of the movement of the heavens as the standard was based upon an hypothesis of ancient physics.

As Saint Thomas points out: "Ponit (Aristoteles) hanc

suppositionem quod motus coeli sit mensura omnium motuum." (32)

Today the standard has been changed and is now the speed of the movement of light. But whether the standard chosen be the speed of the movement of the heavens, or the speed of the propagation of light, or the wave-length of a red spectral line emitted by cadmium, the logical structure of the measurement of continuous quantity remains the same: it is always a question of a standard which is indivisible by fiat though not by nature, and which represents an attempt to come as closely as possible to the minima mensura.

It is clear, then, that there is something profoundly paradoxical about the measurement of continuous quantity. On the one hand, it is necessary for the scientist to search for the minima mensura, and the dialectical tendency towards certitude about which we spoke in Chapter V becomes in this field the search for an absolutely small measure. On the other hand, this infinitesimally small measure does not exist. "Sed in lineis non est invenire minimum secundum magnitudinem, ut sit scilicet aliqua linea minima; quia semper est dividere quamcumque lineam. Et similiter dicendum est de tempore." (33) An infinitesimally small measure would involve a contradiction, since it would consist in a continuum without extension. It is then a purely dialectical limit that can be approached indefinitely;

it is not a limit given in nature that can ultimately be arrived at. And this impossibility of arrival is not due to any lack of precision on our part; it is due to the very nature of continuous quantity. We must then be satisfied to accept the minimum measure that is possible for us to have -- "accipere aliquid minimum pro mensura secundum quod possibile est."

How is it possible for the mind in spite of this paradox to succeed in some way in assimilating the measure of magnitude to that of multitude? In order to answer this question it is necessary to recall that it is possible to know that two or more classes have the same number, without knowing what that number is. Thus, for example, if all the tickets to a certain theater have been sold, it is possible to know that there are as many people in the theater as there are seats without knowing in any definite way the number of the two classes involved. In the same way it is possible to know that two classes have different numbers, without knowing what these numbers are. Now something very similar is found in magnitude. By juxtaposing two rods x and y , I can discover that they are of equal or unequal length, even though I cannot say anything of the length of rod x or rod y taken separately.

If it happens that rod x can be placed twice along rod y , I can arrive at the formula: $y = 2x$. Yet once again, this does not reveal anything about the lengths of the two rods when they are taken separately. In other words, it is possible to arrive at the knowledge of continuous quantity (34) by establishing ratios. And since the structure of mathematical physics is based on the measurement of lengths, the knowledge that it gives us is reducible in the last analysis to a knowledge of ratios. When for example the wave-length of the line $H\alpha$ in the spectrum of atomic hydrogen is indicated by the measure-number 0.000065628, this does not reveal any absolute property; it merely tells us the ratio existing between the length of a wave of $H\alpha$ light to that of a centimeter, which is obviously an arbitrary standard. In like manner the whole of physics is built up out of ratios determined in relation to arbitrary standards. (35)

It is clear, then, how it is possible for the measurement of magnitude to imitate that of multitude. Just as I can know that two classes have the same number, so I can know that two rods have the same length. The two cases remain similar until I attempt to get at the meaning of the "same". In the case of multitude this meaning can be determined absolutely since it is based on cardinal number,

and consequently it is possible to escape from mere knowledge of proportion. In the case of magnitude, the meaning of the "same" cannot be determined absolutely; (36) it is impossible to escape from knowledge of proportion.

From all that has been said thus far about the nature of measurement of magnitude it follows that from the point of view of the physicist the standard of length has no length. Sir Arthur Eddington has brought out this point very forcefully in the Prologue to Space, Time, and Gravitation. (37) But lest confusion arise it is necessary to make several distinctions. The term "length" is in fact extremely ambiguous and is susceptible of a great variety of meanings. It may be taken to mean: 1) dimension as such (and this is its most proper meaning); 2) a line, that is to say, a finite length; (38) 3) the measured magnitude of a finite length; 4) a geometrical line; 5) the measured magnitude of a finite line; 6) a sensible line taken as a dimension; 7) a sensible line as a finite magnitude; 8) the measured magnitude of a sensible line. Now, if the term be taken in the senses indicated under numbers six and seven it is obvious that the standard of length is a length. If it were not it could not be a standard: "oportet mensuram homogeneam esse mensurato." But when a physicist speaks of length it

is particularly the sense indicated by number eight that he has in mind. Then it is a question of a magnitude that is expressible by a measure-number which answers the question "what is the length of this line? In this sense it is true to say that the standard of length has no length. In so far as it is a standard it can be defined only by designation and in no other way. The same is true of the measurement of time. Understood in this way, the theory of Relativity is correct in maintaining that if an object could move with the velocity of light it would be outside of time, for the speed of the propagation of light is taken as the fundamental standard of the measurement of time.

It is possible, of course, to define a certain designated standard in terms of another standard, but then ~~it is no longer~~ being defined qua standard, since another standard has been substituted. For example, we can define a meter in terms of a hundred centimeters, and this gives us the illusion that we can know how long the standard meter is. But obviously in this definition the standard is no longer the meter but the centimeter, and we are faced with the question: how long is a centimeter? There are just two ways by which one might attempt to answer this question: one is by saying that it is the hundredth part of

a meter, and this obviously involves a vicious circle; the other is by having recourse to a still smaller standard, and this involves a process ad infinitum; by the time we have come to the Ånström as the standard we are still as far from the answer to our question as we were in the beginning.

The infinity of the vicious circle and the indefinite process is a sign of what is at the bottom of this whole question: the inexhaustible potentiality of the continuum. And most of the difficulties that arise in connection with this problem have their origin precisely in this that we attempt to confer upon the continuum a degree of intelligibility that belongs only to discrete quantity. It is extremely important to keep in mind that the measures of continuous quantity are essentially inadequate and imitative. They do not do away with the inherent unintelligibility of the continuum, for they cannot change its nature. Measurement consists in the juxtaposition of an unknown with a scale. It is usually taken for granted that this scale is something definitely known by itself. As a matter of fact, it is not. And as a consequence, measurement, in the last analysis is merely the juxtaposition of an unknown with an unknown. But perhaps this whole question will become clearer later on when

we take up the distinction between intrinsic and extrinsic measure. In the meantime it is worth while noting that this point is obviously of extreme significance for the whole question of mathematical physics and particularly for the theory of Relativity.

Indivisibility is then the primary quality of the first element of measurement mentioned above: the principle of perfection and simplicity. But there is another extremely important quality that is closely allied to it: the measure must possess uniformity. In order for measurement to be able to reduce complexity to simplicity, indetermination to determination, and variability to invariability it is not sufficient that the measure be one and indivisible; it is also necessary that it be uniform. In no other way can it provide objective certification in respect to the thing measured. Consequently it is necessary to choose a standard that is controllable, precise, uniform and invariable.

Perfectio mensurae consistit in uniformitate et simplicitate, qua aliquid de se est notificativum alicuius quantitatis; hoc enim exigitur ad rationem mensurae ex parte suae perfectionis, eo quod perfectissimum in aliquo genere est mensura ceterorum. (39)

And obviously the uniformity required is uniformity with respect to the particular genus in which the measurement takes place.

Sola uniformitas seu regularitas, sumpta in abstracto, est communis ad omnem mensuram...Ergo oportet quod determinetur ratio talis mensurae essentialiter et intrinsece, per hoc quod sit uniformitas talis vel talis quantitatis, vel generis...Ergo pertinet ad ipsam essentialem rationem mensurae non solum habere uniformitatem, sed uniformitatem talis vel talis conditionis seu generis: ratione cuius sit apta et habilis mensura ad mensurandum talia mensurata. (40)

The perfection of a measure of length, for example, requires that it be uniform in the genus of length, in other words, that its length be objectively constant. Here we are touching upon one of the most important problems of measurement in so far as it effects mathematical physics -- the problem of the rigid rod. We shall have a great deal to say about this question later on, and at this point it will be sufficient to merely touch upon the fundamental issue. In every measure of continuous quantity there is from the point of view of uniformity and invariability an essential imperfection that parallels its imperfection from the point of view of indivisibility. For every measure of continuous quantity is an extended piece of matter which is an ens mobile and consequently subject to a continual state of flux. It is a part of an extremely complex and unstable cosmos. It is at every moment undergoing innumerable physical influences which necessarily produce changes in it. These physical influences cannot be eliminated completely without changing the nature of the material standard and without separating it completely from the cosmos. Of course, the changes produced can be controlled to some extent.

But in order to have perfect control, it would be necessary to know all of the laws of nature; it would be necessary to have an exhaustive knowledge of the cosmos. Once again it is evident that the perfectly uniform standard is only a dialectical limit that can be approached indefinitely, not a natural limit that is objectively capable of being reached. Once again the mind must step in and construct; it must provisionally declare to be uniform what is by nature lacking in uniformity. (41) And we may apply here what St. Thomas has to say about the fluidity of human law: "Mensura debet esse permanens, quantum est possibile; sed in rebus mutabilibus non potest esse aliquid omnino immutabiliter permanens." (42)

In connection with the first essential element we have been discussing -- the principle of perfection and simplicity there is one final point that must be touched upon. We have said that a measure is that by which the quantity of a thing is made known. But there are two ways in which one thing may manifest another. In the first place, a less perfect object may serve to manifest a more perfect object. It is in this way that creatures manifest their Creator, and this is in keeping with the limited nature of our human knowledge which in the order of generation progresses from

the less perfect to the more perfect. But it is also possible for a more perfect object to manifest a less perfect object, and it is obviously in this way that a measure manifests the thing measured, since in relation to the latter the former is always a principle of perfection.

Licet mensura de se ordinetur ad notificandam quantitatem formalem vel virtuales rei mensuratee, non tamen est de ratione mensurae quod notificet nobis quantitatem rei mensuratee modo imperfecto, seu juxta modum quo procedit nostra cognitio de imperfecto ad perfectum; sed requiritur quod ex ipsa ratione mensurandi notificet nobis mensuratum modo perfecto, seu procedendo a perfectiori ad minus perfectum seu minus nobis notificatum . . . Hoc enim modo mensura notificat, scilicet per modum perfectionis et simplicioris, quia perfectissimum in unoquoque genere est mensura ceterorum; unde per modum perfectionis, et non via generationis (sive processus de imperfecto ad perfectum), debet mensura notificare. (43)

Now it happens that in the type of measurement with which science is primarily concerned -- the measurement of length -- there is no objectively perfect standard, no absolutely perfect principle of simplicity, as is evident from all that has been said thus far. That is why science must ever remain in search of a more perfect standard to manifest the less perfect. And that is why its measurement will always remain imperfect and obscure.

And now, having analyzed the first essential element of measurement we must consider the second: the union between

the measure and the thing measured. In order for this union to be possible, it is obviously necessary that there be some kind of compatibility between the two. And this prerequisite condition is expressed in the fundamental Thomistic principle: "mensuram oportet esse homogeneam mensurato."

Mensura semper debet esse cognatum, scilicet eiusdem naturae vel mensurae cum mensurato: sicut mensura magnitudinis debet esse magnitudo; et non sufficiet quod conveniat in natura communi, sicut omnes magnitudines conveniunt; sed oportet esse convenientiam mensurae ad mensuratum in natura speciali secundum unumquodque, sic quod longitudinis sit longitudo mensura, latitudinis latitudo, vox vocis, et gravitas gravitatis, et unitatum unitas. (44)

But this immediately gives rise to several difficulties. In the first place, number is measured by the "one", which is not a number. Consequently, in this case the measure and the measured do not seem to be in the same genus. St. Thomas answers this difficulty in the paragraph which follows the one just cited: "Unde nihil aliud est dicere unitatem esse mensuram numeri, quam unitatem esse mensuram unitatum." (45)

In other words, even though the "one" is not a number, it belongs to the same genus in the sense of being the principle of number. Though not in itself discrete quantity it pertains to the order of discrete quantity in so far as it is its principle. A more serious difficulty arises

from the fact that God is said to be the measure of all beings, and eternity is said to be the measure of time; yet in neither case does it seem possible to apply the principle: "mensuram oportet esse homogeneam mensurato." St. Thomas suggests the solution for this difficulty in the Summa: "Mensura proxima est homogenea mensurato, non
(46)
autem mensura remota." In other words, in order to have measure in the strict sense of the word it is not necessary that the measure and the thing measured be in the same genus in the strict sense of the word. This is required only of the immediately proximate measure. For every other measure it is sufficient that they be in the same general category as for example in the case of time and eternity which belong to the category of duration, or even in the same universal order of being as in the case of God and
(47)
creatures. It is in the realm of magnitude that the principle which requires the measure and the thing measured to be homogeneous is most perfectly realized. For the measure of a length is not a point but another length. That is why St. Thomas in his commentary on the fifth book of the Metaphysics in speaking of the difference between number and mag-
(48)
nitude uses the phrase: "magnitudo sive mensura." Magni-
tude is, in fact, a measure, whereas number is not.

But this basic compatibility between the measure and the thing measured is only the prerequisite condition for the fulfillment of the second essential element in measurement. In order for the indetermination of the thing measured to be effectively reduced to determination some kind of union between the two is necessary. Now there are two ways in which a measure can be united with the object measured. In the first place, it can be united to it extrinsically by means of some kind of application. This application need not be physical; it may consist in a purely intellectual juxtaposition or comparison, as when, for example the transcendental quantity of creatures is measured by the Supreme Being. In physical science the application is in one way or another physical; but it does not have to be direct or immediate, otherwise it would be impossible to measure objects in motion and objects at a distance. Yet it must be pointed out in passing that physical measurement acquires certitude and objectivity to the extent in which the application becomes more direct and immediate. Now whenever a measure and an object measured are united, by means of an application the measurement is extrinsic.

But there is another and more intimate way in which a measure can be united with a measured object: by identification.

And when this type of union is realized the measurement is known as intrinsic.

This brings us to the distinction between extrinsic and intrinsic measure which is of considerable importance for an understanding of the nature of measurement. St. Thomas (49) touches upon this distinction in several places, but perhaps the clearest and fullest explanation of it is found in John of St. Thomas:

Oportet distinguere mensuram instrinsecam et extrinsecam. Extrinseca est quae mensurat aliquid extra se; et ideo per applicationem et continentiam illius dicitur mensurare, sicut duratio et motus coeli mensurat motus inferiores tamquam extrinseca mensura illorum, et ulna mensurat pannum, et libra pondus. Unde talis mensura terminat relationem realem sui mensurati. Intrinseca mensura est illa quae inest rei mensuratae; et ita non mensurat per applicationem, sed per informationem; unde habet perfectionem mensurae, licet non relationem realem et imperfectionem dependentiae qua mensuratum dependet a mensura; . . . et in unoquoque genere perfectissimum est mensura sui et ceterorum, sui quidem instrinseca, aliorum vero extrinseca. (50)

It is obvious that this distinction rests upon a difference in the kind of union existing between the measure and the object measured. Now just as a measure is more perfect to the extent in which its first essential element, that of simplicity and uniformity, is more perfectly realized, so likewise it is more perfect in proportion to the degree of intimacy found in the union with the measured object. This has

already been noted with regard to union by application in physical measurement: the certitude and objectivity of the measurement depends upon how direct and immediate the application is. But obviously union by identification is more perfect than any kind of union by application, no matter how direct or immediate it may be. That is why, speaking absolutely and objectively, intrinsic measure is more perfect than extrinsic measure. Thus John of St. Thomas writes:

Quanto perfectior est mensura, tanto perfectius coniungitur suo mensurato, illudque magis ad se trahit quantum possibile est. Et ita cum aeternitas sit mensura perfectissima, summe coniungitur suo proprio mensurato: ita quod habet identitatem cum illo. (51)

The difference, then, between extrinsic and intrinsic measure comes down to this that, whereas the former measures and manifests a certain object per applicationem, the latter measures and manifests per informationem. In the first case there is a real distinction between the measure and the thing measured; in the second case the distinction is only logical. That is the meaning of the principle "omnis mensura in suo genere seipsa mensuratur." In Thomistic terminology, an extrinsic measure measures its object ut quod, that is to say, per contactum rei ad rem. Intrinsic measure, on the other hand, measures its object ut quo, that is to say, it is the very form of the thing measured.

We are so accustomed to making measure coterminous with extrinsic measure that it is difficult to form a clear notion of intrinsic measure. And yet it is evident that the perfection, simplicity and uniformity of a thing can manifest another thing only by manifesting itself in some way. In this sense intrinsic measure is the very foundation of extrinsic measure. John of St. Thomas writes:

Quando mensura est intrinseca, idem quod est mensura intrinseca, est etiam forma: alioquin non esset mensura intrinseca, id est, per informationem mensurans; cum tamen necesse sit ponere aliquas mensuras intrinsecas, quia id quod est mensura in aliquo subjecto esse debet, et non mensuratur per aliquid extrinsecum, alioquin de illo inquiremus per quid mensuratur: et sic vel erit processus in infinitum, vel deveniemus ad aliquam mensuram, quae respectu sui subjecti sit forma et mensura respectu vero aliorum extra se sit mensura tantum. Nec tamen sub eadem formalitate est forma et mensura: sed est forma ut constituit formaliter; est autem mensura ut respicit quantitatem aliquam virtutalem vel formalem, uniformitate affectam, et sic mensuratam. (52)

In other words, by the very fact that a thing exists it has a certain perfection and simplicity, independently of any comparison with another object. Consequently, it possesses a measure intrinsic to itself. And since it is the form of a thing which makes it both be and be known, this intrinsic measure is the form which gives perfection, simplicity and uniformity to the thing it informs and by so doing manifests it. It is only because this perfection and uniformity is possessed in-

dependently of any comparison that there can be a basis for the comparison necessary for extrinsic measure.

Esse mensuram homogeneam mensurato potest intelligi vel ut quo vel ut quod, et respectu subiecti recipientis est homogenea ut quo, scilicet id, quo tale subjectum redditur homogeneum et uniforme alteri extrinseco, respectu cuius est homogeneum ut quod, si mensurat illud per applicationem et contactum rei ad rem. (53)

But the relation between intrinsic and extrinsic measure must be rightly understood. It is extremely important to keep in mind that the extrinsic measure does not reveal the intrinsic measure, as some might be tempted to think.

With regard to the nature of intrinsic measure, two important questions suggest themselves: first, does it manifest the quantity of the thing in the sense of answering the question "how much", secondly, is it something absolute? These questions are connected, but we shall consider them separately. With regard to the first, it is difficult at first glance to see how intrinsic measure manifests the "how much" of the quantity measured, since whenever we wish to find out how much quantity there is in a thing we inevitably have to fall back on extrinsic measure. On the other hand, we have defined measure in general as that by which the amount of quantity that a thing possesses is made known, and if this definition is valid it should apply to intrinsic measure. Perhaps the best way of

solving this problem is by considering the following passage of John of St. Thomas:

Aliud est considerare mensuram et mensuratum, ex parte rei cognitae, aliud ex modo et ex parte cognoscentis. Ex parte quidem rei cognitae, semper mensura est perfectior mensurato, et notificativa illius, atque explicativa confusionis eius via perfectionis et simplicitatis. At vero ex modo cognoscentis non semper mensura nostra, propter suam imperfectionem, attingit simplicitatem et uniformitatem rei mensurantis supra mensuratum: hoc tamen non tollit rationem mensurae ex parte ipsius rei cognitae, licet per accidens ob defectum cognoscentis non possit uti illa mensura ad cognoscendum per illam, tamquam per medium, rem mensuratam. (54)

Intrinsic measure does make the quantity of a thing known in the sense of manifesting the "how much," and therefore realizes the definition of measure. But this manifestation is dependent upon two factors. In the first place, it is dependent upon the nature of the subject to which the manifestation is being made. It is possible that an intrinsic measure may manifest the quantity of a thing in a clear and adequate way to a superior intellect but only in a vague and general way to an inferior intellect. In this case the inferior intellect will have recourse to extrinsic measure. This is true of the intrinsic measure of predicamental magnitude. The intrinsic measure of an isolated extended object manifests adequately the quantity of that object to the divine intellect. But to the

human intellect this manifestation is only vague and obscure. Before comparing one extended object with another we know the quantity of the first in a very loose and inadequate way. If we did not there would be no basis for comparison. To answer the question: how much quantity is there in an extended object, we can point to the object and say: that much. But the intrinsic measure does not give us any accurate and definite knowledge of the quantity. It does not give us the precision of knowledge that can be expressed in a measure-number. That is why recourse must be had to extrinsic measure.

In the second place, the manifestation deriving from intrinsic measure is dependant upon the nature of the object manifested. When the quantity of this object is something fixed and absolute, it can be manifested in a definite and absolute fashion. This is true of the transcendental quantity of immaterial things. But the extension of material objects is not fixed and absolute. For, as we have pointed out, all material objects are *entia mobilia* and are constantly in a state of flux. The extension of every material object is always in a state of becoming since it is forever undergoing the changes being produced in it by the innumerable physical influences to which it is subject. That is why even to the diving mind the intrinsic measure found in every material object cannot manifest the quan-

tity of that object as something fixed and definite. If it did, becoming would be identified with being.

And this brings us to the answer to the second question: is the intrinsic measure of material objects something absolute? The answer is yes and no. It is absolute in the sense of not possessing the relativity that is proper to extrinsic measure and that derives from the comparison of one object with another. It is not absolute in the sense of manifesting a quantity that is fixed and definite. The partisans of absolute dimensions in the cosmos consistently overlook this second point. (55) To their argument: omne ens est aliquid, must be appended the qualification: in quantum est ens. To the extent in which a thing is becoming it is not a being and hence is not absolute. And from this point of view it is likewise true to say that the standard of length has no fixed length. Through a progressive refinement of scientific processes, physics is constantly drawing closer to the absolute world condition. But in so far as the process of measurement is concerned, it is important to keep in mind that though this absolute world condition is absolute in the sense of not being relative to our ways of knowing, it is not absolute in the sense of being fixed and immobile. We are not drawing close to a static cosmos.

We said above that extrinsic measure differs from intrinsic measure in that whereas in the latter the relation between the measure and the object measured is only logical, in the former it is something real. Since all scientific measurement has to do with extrinsic measure it might be well before finishing the discussion of this point to try to determine as exactly as possible the nature of the relation that arises out of physical measurement.

Scholastics traditionally distinguish two types of relation: transcendental and predicamental. The former does not constitute a special category of being and hence is realized in several categories. It is found wherever an entity, though something absolute in itself, has in its very intrinsic nature a necessary orientation toward something else. The relation of act and potency is always a relation of this kind. Predicamental relation, on the other hand, is a special accident that is superadded to the absolute entity which it relates to something else. As Aristotle and St. Thomas point out, ⁽⁵⁶⁾ there are three species of predicamental relation: 1) those based on number and quantity; 2) those based on action and passion; 3) those based on measure. St. Thomas clarifies the meaning of the ⁽⁵⁷⁾ third species by explaining that measure here means something distinct from the measure of number or magnitude,

otherwise there would be no difference between the first and the third species. It has to do with the "measurement of being and truth." In this sense our knowledge of things is measured by the things known, that is to say, the truth of our speculative science is determined by objective reality.

These distinctions throw light upon the nature of our physical measurements. In the first place, there is a transcendental relation between the standards and the measuring instruments used and the reality that is measured, for neither standards nor measuring instruments have any intrinsic meaning except in relation to an object to be measured. In the second place, there is a real predicamental relation of the first species between our units of measurement and the quantity measured. Finally there is a predicamental relation of the third type between the knowledge that we gather from our measurements and the object measured. But here it is necessary to introduce a distinction. The knowledge that comes to us from physical measurement in science is at once both speculative and practical; from one point of view it reveals to us objective reality, while from another it reveals an article which we have manufactured. Hence there would seem to be a double predicamental relation of the third type involved. From one point of view objective reality is

the measure of our knowledge; from another point of view our mind is the measure of the object. But of the two the first relation is the most fundamental, for the second has only a functional character in relation to it. That is to say, the only reason why we become the measure of the object is to make it possible for the object to become the measure of our knowledge in a more perfect and adequate way. It is true that we choose the standard by which the quantity of reality is revealed, but it is also true that the object measured determines the measure. Some idealistic physicists tend to overlook this point.

3. The Limitations of Measurement

"If only the schoolmen had measured instead of classifying," writes Whitehead, "how much they would have learnt." (58) For historical reasons indicated in Chapter I it is doubtful perhaps just how much the medieval schoolmen would have actually learned if they had devoted themselves to science based on measurement. But there can be no doubt about how much has been learned in modern times through the systematic processes of measurement. The magnificent structure of modern physics is an eloquent

proof of the amazing fruitfulness of metrical method. Yet the epistemologist must not allow himself to become unduly impressed by this towering structure. He must strive to remain completely detached, and examine its foundations with as much objectivity as possible. His task is to assess its value, not from the point of view of practical success but from the point of view of pure knowledge.

This is the task we must now undertake. Having once recognized the amazing success and fruitfulness of the processes of measurement it is necessary to try to analyze their limitations. Many of these limitations have been more or less implicit in what we have been saying about the nature of measurement, but it is important to try to make them as explicit as possible. It is only in this way that we can come to see the true nature and value of the knowledge that is found in mathematical physics, since, as we have seen, all of this knowledge is in the last analysis derived from measurement.

In the first place, metric knowledge is able to come to grips only with the quantitative determinations of nature. As we explained in Chapter VII, it is utterly blind to all the determinant properties of things in their specific essences, to the very inner natures of things, to

all that seems to be of the highest significance for philosophy, for art, and for human life itself. The proper realm of metric knowledge is the homogeneous exteriority found in nature, and from the point of view of pure knowledge this is an extremely poverty stricken area, both because of the homogeneity and because of the exteriority.

Perhaps the following considerations may serve to make the outline of this important limitation more clear-cut. In the first place, it must be noted that measurement can reveal nature to us only in terms of its differences. This is in itself an extremely significant limitation, but it is only half of the story. Added to it is the further limitation that measurement can handle these differences only in terms of sameness. All this is but a corollary from the fact that the proper field of measurement is one that possesses exteriority and hence differences, and at the same time homogeneity and hence sameness. But perhaps we can make this point still clearer by rendering it more concrete and precise.

There are two types of variety in nature. Some objects differ in kind, as e.g. green differs from large and hot from hard. Other objects (or states of objects) though

of the same kind, differ by the fact that they possess their common character in various degrees. In face of the first type of difference measurement is wholly incompetent for the simple reason that it is a question of difference without sameness. (59) Measurement can come to grips with these differences only in an indirect way by introducing sameness through an artificial construction. That is to say, if changes in the one object are functions of changes in the other, or if certain occurrences in the one determine in some way corresponding occurrences in the other, then a correlation can be established between them. But it need hardly be remarked how limited is the type of knowledge that results from such correlations.

Measurement has far greater competence in relation to objects or states of objects which differ by degree. But even here an important distinction must be made -- the distinction between what have become known as "intensive quantities" and "extensive quantities." Examples of the former are density, hardness, temperature. The most important examples of the latter are length, time and mass, but there are many other examples of less importance, such as volume, electric resistance, momentum, etc. The measurements of both of these types of "quantities"

have this in common that their differences can be determined by a serial arrangement which will be both asymmetric and transitive. This is possible because there is a sameness uniting the differences. But they are distinguished from each other by the fact that in the case of intensive quantities the serial arrangement is not additive, whereas in the case of extensive quantities it is. It makes sense to say that eighty feet of length are twice as large as forty feet; but it is utterly devoid of sense to say that eighty degrees of temperature are twice as hot as forty degrees. This distinction arises from the fact that though in the case of "intensive quantities" there is sufficient sameness to allow the differences to be determined by a serial arrangement, this sameness is not true homogeneity, and consequently the series is not additive.

We have explained that all measurement consists in an attempt to assimilate in some way the object measured to the status of pure numbers. From this point of view there is a vast difference between intensive and extensive "quantities." In the first case there is an approach to ordinal number. It is only an approach because of the artificial and arbitrary elements entering into the arrangement of the order. In the second case, there is something

more: because of the additive quality there is an approach to cardinal number. But once again, it is only an approach, since, as we explained above, the measurement of magnitude can never escape the limitations of ratios.

Through processes of correlation similar to those mentioned a moment ago, the measurement of intensive "quantities" can to some extent be assimilated to that of extensive "quantities". This is done when the serial order of an intensive "quantity" is found to correspond to the serial order of an extensive "quantity". The most common examples of this are the correlation established between degrees of heat and degrees of length of a mercury column between the degree of color of a light and the degree of its refraction, between the degree of intensity of a sound and the length of a wave. Measurement obtained in this way is called derivative, whereas direct measurement of additive "quantities" is called fundamental. Now the indirect, artificial and arbitrary character of derivative measurement is so evident that it is hardly necessary to call attention to it. And obviously the knowledge which results from this measurement is extremely limited.

But even in the field most proper to it metric knowledge cannot get at the quantitative determinations of

the cosmos in the sense of being able to tell us what these determinations are. Precisely because it is "quantitative" knowledge it is not "quiditative" knowledge. It cannot answer the question "what", it can only answer the question "How much"? This is a profound limitation that must not be lost sight of. It makes little difference to what extremes of refinement we succeed in pushing our measurements, In the end the nature of the thing being measured is just as in-
(60)
scrutable as it was in the beginning.

But the metric knowledge that is found in physics cannot even tell us the "how much" of the quantitative determinations of nature in any absolute way. If mathematical physics were based upon the measurement of number, upon counting, it could tell us something absolute about nature. But as a matter of fact, it is based fundamentally upon the measurement of magnitude. And it is always a question of mere extrinsic measure, never of intrinsic measure. This means that it never tells us anything absolute of the object taken by itself independently of the standard. It only tells us how one object stands in comparison with another object under certain given circumstances. In other words metric knowledge in science gives us only ratios. This point is sometimes lost sight of. We tend to transform the ratios

into absolute properties of the objects measured. When, for example, we hear it said that the density of gold is 19. 32, it is easy enough to look upon this measure-number as designating something absolute that belongs to gold in se. As a matter of fact, it merely indicates the ratio between the weight of any piece of gold and that of a volume of water of equal size. Sir Arthur Eddington has brought out this point with his usual clarity:

So in any statement of physics we always have two objects in mind, the object we are primarily interested in and the object we are comparing it with. To simplify things we generally keep as far as possible to the same comparison object. Thus when we speak of size the comparison object is generally the standard metre or the yard. Since we habitually use the same standard we tend to forget about it and scarcely notice that a second object is involved. We talk about the properties of an electron when we really mean the properties of an electron and a yardstick - - properties which refer to experience in which the yardstick was concerned just as much as the electron. If we remember the second object at all we forget that it is a physical object; for us it is not a yard-stick, but just a yard. (61)

From what has been said thus far it should be fairly clear that strictly speaking metric knowledge does not reveal things to us. As Professor DeKoninck has remarked, "les entités fondamentales de la physique ne symbolisent que des coupures métriques dans les choses dont elles ne représentent qu'un aspect. Il est absurde

(62)
de considerer un atome comme une chose." One of the most common errors in science is to reify provisional metrical segmentations and to attribute to them the status of ontological entities. In this connection the following lines of Cassirer are extremely pertinent:

It seems almost the unavoidable fate of the scientific approach to the world that each new and fruitful concept of measurement, which it gains and establishes, should be transformed at once into a thing-concept. Ever does it believe that the truth and the meaning of the physical concepts of magnitude are assured only when it permits certain absolute realities to correspond to them. Each creative epoch of physics discovers and formulates new characteristic measures for the totality of being and natural process, but each stands in danger of taking these preliminary and relative measures, these temporarily ultimate intellectual instruments of measurement, as definitive expressions of the ontologically real. The history of the concept of matter, of the atom, of the concepts of the ether and of energy offer the typical proof and examples of this. All materialism -- and there is materialism not only of 'matter' but also of force, of energy, of the ether, etc., -- goes back from the standpoint of epistemology, to this one motive. The ultimate constants of physical calculation are not only taken as real, but they are ultimately raised to the rank of that which alone is real. (63)

The fact that metric knowledge in science gives us nothing more than the ratios between two objects brings to light further limitations that are intrinsic to it. If nature itself determined the standards, the resultant ratios would have a fixed and objective meaning. But as

Bergson has remarked, nature does not measure. And since the standards of measurement are not given in nature they must be established by convention. The intellect and will of man must enter into the process of measurement to determine the norm in relation to which the ratio must be established. Man becomes the legislator for nature. As Professor Beneze has remarked, "dire que le choix de l'unité est arbitraire, c'est dire que la volonté de l'opérateur va introduire dans la connaissance un élément sur lequel la sensibilité n'a plus aucune prise. Et cela ne signifie pas que le nombre qui va apparaître ne soit pas lié au sensible, mais il ne lui est lié que justement parce que la volonté de l'opérateur en a décidé ainsi."⁽⁶⁴⁾

All this evidently introduces an element of subjectivity and to a certain extent of arbitrariness into our metric knowledge. As a matter of fact, most of our systems of measurements derive originally from extremely arbitrary sources. In the English system of weights, for example, the weight of an average grain from the center of a head of wheat was originally selected as the standard, and the pound was consequently defined as the weight of seven thousand of these grains. The block of metal preserved in the United States Bureau of Standards now provides

a much more uniform standard, but the basic relativity and arbitrariness of the measuring system has not been changed. The same is true of the measurement of length, as Eddington has shown in his own whimsical way:

If report is to be trusted, King Henry I, about the year 1120, fixed the yard by stretching out his arm. King David of Scotland (c.1150) more democratically ordained that the inch should be the mean measure of the thumbs of three men, 'an merkle man' a man of measurable stature, and an 'lytell man', the thumbs being measured at the root of the nail. The meter less picturesquely embodies the mistakes of the early geodesists. Thus the result of all our careful measurement is to determine, for example, how many hydrogen atoms to the length of King Henry's arm or to the thumbs of three Scotchmen. That does not carry us very deeply into the mysteries of Nature. (65)

It is true that science does not rest content with the pure arbitrariness of the standards just mentioned. It has been possible to discover certain constants in the cosmos, such as Planck's constant, the velocity of light, the mass of a proton, etc. and these to some extent enable the scientist to measure nature with her own gauge, so to speak. But even these constants are determined in relation to the originally selected standards. And no matter to what extent science may go in its attempt to purify its processes of arbitrariness, in the last analysis the essential relativity intrinsic to the measurement of magnitude will remain untouched.

This essential relativity imposes an infinite limitation upon the metric knowledge that physics affords us. For no matter what extremes of refinement the progressive perfection of our processes of measurement may reach, the resultant measure-numbers are always an infinite distance from any absolute meaning. Sufficient attention is not always paid to this infinite limitation. The impression is often given that an absolute measure actually exists in nature, though profoundly hidden and extremely difficult to get at. This is, of course, an illusion.

Il pense volontiers que le nombre exact est là, caché dans le sensible, et il l'y poursuit comme on poursuit un gibier que l'on sait difficile à attraper. Métaphore trompeuse: l'impossibilité de l'atteindre ne tient pas au fait que la mesure exacte serait profondément cachée, mais au fait que le nombre est le résultat de cette tentative du Jugement d'imposer à la matière l'influence d'un élément, l'unité pure, qui lui est originairement étrangère. (66)

It should be clear why it is illegitimate to dismiss this question, as some authors do by merely stating that our measure-numbers are only approximative. For approximation implies a relation to a definite terminus and in this case no such terminus exists.

In order to make up in some way for this limitation science must seek to remain in a state of

tendency towards the dialectical limit of the minima mensura. The possibility of indefinite progress in this tendency, even though it would never succeed in triumphing over the limitation, would at least provide some compensation for it. But here we are brought up short before another restriction. For even though theoretically this indefinite progress is possible, practically it is not. There are, in fact, definite limits to the accuracy of our measurements in atomic physics. For no matter how highly refined our instruments of measurement become, they are in the last analysis made up of atoms themselves, and as Planck has remarked, "the accuracy of any measuring instrument is limited by its own sensitiveness." (67) Moreover, it is impossible for us to receive any message from nature of greater refinement than that brought to us by a complete photon. This is a very serious confinement, and at present at least there seems to be no way of evading it. As Sir James Jeans has said, "we have clumsy tools at best, and these can only make a blurred picture. It is like the picture a child might make by sticking indivisible wafers of colour on to a canvas." (68)

In relation to this question of the limitation of the accuracy of measurement in atomic physics, the much-discussed problem of indeterminism readily comes to mind.

So much has been written about this problem in recent years that it hardly seems necessary to go into detail in explaining its nature. It is well known that classical mechanics was rigorously deterministic. Its whole structure was built upon the assumption that every given state of universe was completely predetermined in its antecedent state, in such a way that if all the elements entering into this antecedent state had been known, it could have been mathematically deduced from it. And this applied not only to the universe as a whole but to every individual particle contained in it. The future state of each particle was already precontained in its present state. Past, present, and future were perfectly convertible. It is true that the existence of statistical laws was recognized, but this existence was attributed merely to subjective ignorance, and not to any objective indetermination in nature. That is why thermodynamics was for a long time considered to be the least scientific of all the branches of physics, and it was taken for granted that as science progressed the role played by statistical laws would inevitably decrease.

As a matter of fact, it is just the opposite that has taken place. Statistical laws now reign supreme in atomic physics, and classical physics' fond dream of determinism has been completely dissipated. Progress in science, in general,

and progress in the refinement of measurement in particular, has not provided us with greater power to predict future states of particles. On the contrary, it has demonstrated with increasing clarity our utter incapacity for making such predictions. It has now become generally recognized in physics that it is impossible to determine both the position and the velocity of a particle at the same time. It is possible to determine with great accuracy its position by prescind~~ing~~ from its velocity, or its velocity by prescind~~ing~~ from its position, but it is impossible to do both simultaneously. Not only that, but there is a constant proportion in our knowledge of these two facts; that is to say, in the precise measure in which our knowledge of the position increases in accuracy, our knowledge of the velocity decreases, and vice versa. And this proportion is equal to Planck's constant, h , the quantum of action.

All this has become known as Heisenberg's principle of indeterminacy, and a great deal has been written about how this principle should be interpreted. It would take us too far afield to attempt to analyze its philosophical significance here, but in so far as our present purpose is concerned, it is necessary to point out that there

are two fundamental issues involved in this question, and both of them reveal an intrinsic limitation of the process of measurement.

In the first place, the velocity and position of a particle cannot be simultaneously measured with a high degree of accuracy simply because such a thing is a contradiction in terms. A particle in motion is not in place; it is passing from one place to another. And the higher the velocity, the less is it connected with any one definite place. At any given instant one can speak of its position only by prescindng from its velocity. It is true that by being satisfied with rough and inexact measurements we can determine both the position and the velocity at the same time, especially if the velocity is low. But as soon as we try to determine both of them with a high degree of accuracy, we shall find that they are necessarily mutually exclusive, for a thing is moving to the extent in which it is not in any one position, and it is in a definite position to the extent in which it is not moving. It is not surprising, then, that science finds it impossible to measure both the position and the velocity simultaneously with any great degree of accuracy. And all this shows how the process of

measurement, by the very fact of its being perfected, leads us inevitably into an impasse from which there is no escape.

But we are far from pretending that this is an adequate solution to the problem of indeterminacy. There is in fact a good deal more involved in the question. And the principal issue is, of course, whether the indeterminacy which science has discovered in its processes is a revelation of an objective indeterminacy actually existing in nature itself. One must always be extremely diffident about attempting to determine the philosophical significance of the teachings of experimental science, and it would be foolhardy to arrive at hasty conclusions. But we feel that at least this much can be said: in the measure in which scientific indeterminacy is a revelation of ontological indeterminacy it is in perfect conformity with Thomism -- all the writings of contemporary Scholastics to the contrary notwithstanding. No one can read the works of Aristotle and St. Thomas without being impressed by the large measure of contingency and true objective indeterminism that they attribute to the material universe. It is something that is a pivotal point in the whole Thomistic system, since it is an immediate corollary of the doctrine of matter and form. To deny objective indeterminism to the material universe and to affirm at the same time that

one of the co-principles which constitutes the very essence of the things of the universe is a principle of pure indetermination -- prime matter, is a contradiction in terms.

An adequate discussion of this question cannot be given here. That has already been accomplished with admirable skill by Professor DeKoninck. (69) We have introduced the problem only because it reveals another important source of limitation of the measuring process. For, as we pointed out at the beginning of this Chapter, there is something at once both physical and mathematical about the process of measurement. The mathematical character is revealed in its attempt to arrive at exact determination. If measurement were being carried on in a mathematical world from which all contingency is excluded, the refinement of its exactitude could go on ad infinitum, but as a matter of fact, scientific measurement is carried on in a cosmos that is filled with chance, and that consequently is refractory to the exact determination which measurement seeks to realize.

This discussion of the progressive refinement in the exactitude of measurement raises a question which cannot be overlooked. We have said that the definitions which result from measurement can never be anything more than operational: physical properties are defined in terms of the concrete processes by

which they are determined. And at first sight this seems to involve us in an insolvable problem. For since physical properties are defined by the processes through which they are measured; since every measuring process involves the use of a physical instrument; and since an instrument cannot be known or defined except in terms of its properties, it is difficult to see how we can escape an immediate vicious circle except through another vicious circle, which would consist in falling back upon the senses from whose limitations the whole process of measurement is intended to deliver us.

It is true, as we pointed out in Chapter VII, that all physical experimentation involves an ultimate dependence upon sense. But this does not mean a going back to the limitations of the senses which physical science encounters at its point of departure. And we can escape this without getting involved in a vicious circle. It is not a question of a circle, but of an ascending spiral. In the beginning, science, by making use of ordinary sense data, arrives at an elementary physical theory. The substitution of measuring instruments makes it possible to correct the primary theory; the new theory helps to reveal the deficiencies of the instruments employed and makes it possible to perfect them; through the use of more perfect instruments science is able to arrive at a more perfect

theory, and so on ad infinitum.

There are two things that must be noted about this process. In the first place, it never arrives at perfect exactitude. And this is an important point to keep in mind. For it means that from this point of view mathematical physics does not have an absolutely certain point of departure. Its primary data, the measure-numbers, are not truly certain. And the fundamental reason why they are not certain is that they aim at a kind of certainty that cannot be attained in the realm in which it is being sought. From this point of view the primary data of the parts of the study of nature that are not mathematicized have greater certitude. This is true above all of the philosophy of nature. But lest this limitation appear greater than it actually is, attention must be paid to two points. First of all, even though the measure-numbers are not certain, they are certainly an approximation, and science is often able to determine with great exactitude the limits within which this approximation certainly falls. Secondly, because of its highly theoretical character, mathematical physics is not so essentially interested in the certainty of its point of departure as a purely inductive science must be. In a sense it is true to say that it is more interested in its point of arrival. It is satisfied

with any point of departure which will provide a sufficient basis for a theoretical structure which will eventually "save the phenomena."

The second thing to be noted about the process we have been discussing is that the more highly refined it gets, the more implicated it becomes in theory, and consequently the more deeply immersed in subjectivity. The use of the yard-stick does not depend upon very many theoretical assumptions. But the extremely elaborate and complicated instruments now employed by science are dependent upon a veritable maze of postulates and assumptions. As a matter of fact, does not our method of deciding that one process of measurement is more accurate than another consist in determining that it is more in accordance with our theories and with the laws which we have assumed to be true?

This brings us back to what we saw in Chapter IV about how the subjective logos is injected into nature through the processes of experimentation. Everything that was said in that connection applies with particular force to the processes of measurement. For measurement is an operation which we perform upon nature, and this operation has a double aspect. In the first place, it involves a mental procedure which gives the operation a meaning only by placing it in a highly com-

plicated pattern of interwoven assumptions. In the second place, it involves the actual physical procedure of measurement. Both of these aspects implicate measurement in a manifold of complex limitations. But for the moment we are interested only in the mental procedure by which hypothetical elements enter into the operation.

Measurement has been considered by some as a purely empirical procedure, dependent only upon perception and its means, and completely free of hypothetical assumptions. (70)

Nothing could be more false. Not even the simplest measuring operation has a purely empirical and immediately certain starting point. There is always a multiplicity of conceptual presuppositions lurking in the background, which, though subtly implicit, determine, nevertheless, the whole meaning of the procedure. If all the implicit assumptions upon which the ordinary process of measuring temperature by means of a column of mercury could be disengaged and laid bare the results would probably be startling. How much more is not the elaborate and complicated scientific processes of measurement dependent upon hypothesis. Innumerable theoretical assumptions go into the whole conceptual setting up of the experiment, into the construction of the instruments of measurement employed, into the precise way in which they are used, and, in

fact, into every operation that goes to make up the experimental procedure. (71) And every attempt to verify these assumptions only leads into a more complicated network of presuppositions.

Since a number of things have already been said about this general question in Chapter IV, we shall not attempt to develop it any further here. But we cannot refrain from quoting the following lines from Ernst Cassirer, who has laid considerable stress upon this point:

For any, even the simplest, measurement must rest on certain theoretical presuppositions, on certain 'principles', 'hypotheses,' or 'axioms,' which it does not take from the world of sense, but which it brings to this world as postulates of thought. In this sense, the reality of the physicist stands over against the reality of immediate perception as something through and through mediated; as a system not of existing properties, but of abstract intellectual symbols, which serve to express certain relations of magnitude and measure, certain functional coordinations and dependencies of phenomena. . . .

In this sense, each measurement contains a purely ideal element; it is not so much with the sensuous instruments of measurement that we measure natural processes as with our own thoughts. The instruments of measurement are, as it were, only the visible embodiments of these thoughts, for each of them involves its own theory and offers correct and useful results only in so far as this theory is assumed to be valid. It is not clocks and physical measuring-rods but principles and postulates that are the real instruments of measurement. For in the multiplicity and mutability of natural phenomena, the thought possesses

a relatively fixed standpoint only by taking it. In the choice of this standpoint, however, it is not absolutely determined by the phenomena, but the choice remains its own deed for which ultimately it alone is responsible. (72)

But not only do innumerable limitations result from the mental operations which construct the processes of measurement, they also result from the physical operations involved in the actual concrete processes. This is an extremely important point and too much attention cannot be paid to it. It immediately reminds us of all that was said in Chapter IV about the operational character of the definitions of experimental science. But a few special considerations must be introduced here which apply in a particular way to the process of measurement.

In the first place, it is important to keep in mind the proper reason why definitions of magnitudes are necessarily operational: the measurement of magnitude can never give us more than a proportion between the object measured and the standard employed. Consequently the whole meaning of the results depends upon the way in which the standard is chosen and the precise manner in which it is employed, and all this involves innumerable arbitrary elements, as we have already suggested. That is why the knowledge

which the measurement of magnitude gives us is always essentially relative, even when it is a question of the determination of the proper length of an object. By proper length in physics is understood the length which results from a measurement in which a standard is applied to an object that is at rest in relation to it. Later on we shall see that a second kind of relativity enters in when measurement is made of an object in motion.

Because number is something absolute, counting is an absolute operation. No matter how many different ways of counting a certain given plurality may be devised, their results must coincide exactly if they are to be true. As a matter of fact, counting is not essentially an experimental process, for it does not necessarily involve a manipulation of bodies. It is true that physical manipulation may be used as an aid, but in itself counting is a purely mental operation. Magnitude, on the other hand, is not something absolute, nor can the operation by which it is determined be considered absolute. It is possible for a number of individuals to measure the same extension by means of different operations and all arrive at different results. And it is possible to consider all of these results as equally true. To conceive the results of a certain measure

ment of magnitude as the revelation of something absolute in nature to which all other operations must conform is to misconstrue the whole nature of magnitude. That is why such measurement can never have any meaning independently of the concrete operations involved.

And all this means several things. In the first place, it means that if we wish to get at the exact significance of a definition of a length we must be able to specify completely and with perfect precision all of the operations which have entered into its determination. Because of the extreme complexity of even the simplest kind of measurement this seems to be an impossible task, not only because of the innumerable elements involved, but also because the operations interfere with each other, and there is no way of fixing upon the exact nature of the different interferences. But even if one could specify the operations completely and with perfect precision, the results would be very meager. For in the last analysis this specification would consist in merely pointing out certain processes and certain material instruments. One does not reveal very much about the nature of man by merely pointing out an individual man.

The operational character of the definitions of length means that when the operations change, the significance of the definition changes. As Professor Bridgman has pointed out, "In principle the operation by which length is measured should be uniquely specified. If we have more than one set of operations, we have more than one concept, and strictly there should be a separate name to correspond to each different set of operations." (73)

The primary meaning which measurement has in physics is that found in the determination of a length by the direct application or juxtaposition of a material standard to an object at rest in relation to it. But not all the measurements with which physics deals can be arrived at by the same operation, and when new types of operations are introduced, the meaning of the process changes. But lest confusion arise it might be well perhaps to point out that this does not mean that the results of the measurement depend solely upon the nature of the operations employed, for otherwise all objects measured in the same way would have the same length. We shall have a similar remark to make in connection with the second kind of relativity mentioned a moment ago: the results of the measurement of a body in motion do not depend solely upon the frame of

reference in relation to which it is measured, for otherwise every body measured in relation to the same frame would have the same length.

This relativity of measurement is often lost sight of. One type of operation is constantly being substituted for another on the presumption that they are equivalent and interchangeable. An operation proper to one field is projected into another field where determinant factors are different, and it is tacitly assumed that the operation preserves its original meaning. How is it possible to have any assurance that operations which give similar results under certain circumstances will necessarily give similar results under any other circumstances?

Perhaps a few concrete illustrations will serve to bring out more closely this important limitation of the measuring process. (74) In the first place, a very simple example is found in the difference between fundamental and derivative measurements. All too often these two types of measurement are considered to be practically equivalent; yet there is a vast difference in the operations by which they are determined. A more important case is that of the measurement of a body in motion. Such a process involves

operations that are quite different from those involved in the measurement of a body at rest, and the higher the velocities of the motion, the more complicated do these operations become. As a result the meaning of the process undergoes a profound change. We shall have more to say about this case later on because of its capital importance in modern physics.

Another way in which the concept of length is extended beyond its original meaning is found in the measurement of extremely large objects. Here the "tactual" operations which are employed in measurements that fall within the range of ordinary experience, and which consist in the successive direct application of the standard rod to the object, can no longer be employed, and optical operations are substituted. This is already found to some extent in terrestrial measurements, but it is particularly true of solar and stellar distances, where the character of space is entirely optical, and where no opportunity is given of making even a partial comparison between tactual and optical operations. And the complexity of the operations increases in proportion to the remoteness of the distance measured. As Bridgman has remarked:

At greater and greater distances not only does experimental accuracy become less, but the very nature of the operations by which length is to be determined becomes indefinite so that the distances of the most remote stellar objects as estimated by different observers or by different methods may be very divergent . . . We thus see that in the extension from terrestrial to great stellar distances the concept of length has changed completely in character. To say that a certain star is 10^5 light years distant is actually and conceptually an entirely different kind of thing from saying that a certain goal post is 100 meters distant. (75).

Something similar to this occurs when measurement is extended in the direction of the infinitely small. The operations involved change; they become more indirect and more highly complicated. Consequently, the results of microscopic measurements have a different meaning than those of molar physics. In this connection it is interesting to note that though in the determination of the number of molecules in a certain piece of matter we are forced to use indirect and complicated methods, and though different methods may give results that are systematically different, there can be no doubt but that the number of molecules is something absolutely determined in nature; consequently the results do not depend for their meaning upon the operations employed. In so far as these methods are theoretically good and accurate they must all arrive at the

same absolute result. But it does not seem to make any sense to say that in the determination of length, mass, force and other quantities of this kind involved in atomic physics, we must arrive at something absolutely given in nature independently of the operations which enter into the determination.

Of course in all of these cases of the extension of measurement beyond its original meaning, the changes which result do not occur in a fortuitous and uncontrollable way. That is to say, the new operations are not chosen in a purely arbitrary fashion; they are selected by design in such a way that within the realm in which both the original and the new operations may be applied, they both give the same numerical results within the limits of experimental error. Yet there is never any assurance that when the new operations are applied outside this realm where new circumstances are involved, the original coincidence will be preserved.

It is possible for several divergent definitions of length to be employed in circumstances in which direct measurement is impossible, such as, for example, in intense electric and magnetic fields. This is quite legitimate,

provided that, as the fields tend toward zero, they all converge towards the accepted definition. It is impossible to say that one of these definitions is right and the others wrong, For they will all be confirmed by observation, since the very observation will depend upon the theory that is originally accepted. But as Eddington has pointed out, it must be kept in mind that the distances thus measured will be pseudo-distances, "since they lack the most fundamental characteristic of the metrological conception of length, namely the correspondence between similarity of length and similarity of physical structure."⁽⁷⁶⁾

The second thing that must be noted in regard to this operational character of the measurement of magnitude is that the operations in question are concrete, physical, material operations. No matter how completely mathematicized or how highly theoretical physics may become, the definitions of the quantities involved in it are never independent of singular, concrete, material operations, nor do they ever have any meaning except in relation to them. The definition of length of a Relativity physicist is the same as that of an ordinary metrologist.

If, instead of length being defined observationally, its definition were left to the pure mathematician, all the other physical quantities would be infected

with the virus of pure mathematics . . .

In all orthodox physical theory, the metrological practice - - or more strictly the principle which it attempts to carry out - - supplies the theoretical definition. Thus it is secured that, when the experimenter checks the theorist, both are referring to the same thing.

Accordingly, by length in relativity theory we mean what the metrologist means, not what the pure geometer means. In accepting relativity principles, the physicist puts aside his paramour pure mathematics, dismisses their go-between metaphysics, and enters into honourable marriage with metrology. (??)

From the point of view of the logical structure of science, the limitations which all this implies are simply enormous. No definitions in physics are detached and universal; they are all tied down to particular material operations. They have no significance independently of the concrete instruments of measurement employed.

All too often measuring instruments are looked upon almost as if they were immaterial cognitive faculties which register events in a purely trans-subjective fashion. But a moment's reflexion will show how far this is from the truth. In the processes of measurement the instruments employed do not remain purely passive; they enter into the experiment in an active way. For obviously a physical instrument can reveal an event to us only if there is a

physical causal connection between the instrument and the event. And this causal connection inevitably involves an interference of the instrument in the event.

The seriousness of this interference depends upon several factors. In the first place, it is clear that the interference will ordinarily be greater in proportion to the greater imperfection of the instrument employed. And in this connection it is necessary to recall that perfect instruments exist only in the mind of the scientists; they do not exist in reality. Consequently, there is always something defective about every measurement made. Moreover, measuring instruments never remain the same; they are constantly in a state of flux. The very fact that instruments wear out is a sign that they are at all times subject to minute derangements. But even if measuring instruments were perfect there would still be considerable interference in the event that is measured. For purely material things cannot register objective events in a purely trans-subjective fashion.

Another important factor upon which the seriousness of the disturbance depends is the degree of refinement demanded by the experiment in question. In molar physics the interference is relatively light, though even here it cannot be over-

locked. But in the microscopic world the interference is of the same magnitude as the quantities measured, and consequently the limitations of measurement in this realm are simply enormous. The degree of intimacy in the causal connection between the measuring instrument and the quantity measured has also much to do in determining the seriousness of the disturbance. In the measurement of microscopic phenomena the causal nexus is extremely close, and as a result the interference is of great magnitude. This magnitude decreases in proportion to the increase of causal distance between instrument and event, but it can never be reduced to zero, since, as Planck has remarked "if the causal distance is assumed to be infinitely great, i.e. if we completely sever the object from the measuring instrument, we learn (78) nothing at all about the real event." Nor must the fact be overlooked that when experiments depend upon a multiplicity of pointer-readings, there is necessarily mutual interference between them.

Perhaps one might be tempted to think that this limitation of measurement is not so serious as it appears at first sight, since it is possible for scientists to take account of the interferences in question and to make compensations for them in their computations. It must be admitted

that certain possibilities of this kind lie open. But they are extremely meager in comparison with the problem in question -- if for no other reason than that every attempt to account for a disturbance involved in a measurement demands another measurement for its verification, and this obviously starts us out on an infinite series. (79)

In our discussion of this limitation of measurement arising from the causal influence of the instrument upon the quantity measured we have been using the terms "interference" and "disturbance" because they are the expressions which have become current in the modern scientific literature which has treated this problem, But perhaps they do not bring out the most profound aspect of the question as accurately as could be desired. For they tend to give the impression that the causal influence of the instrument is a purely accidental and extrinsic thing, or, in other words, that the measure-number emerging from a process of measurement is essentially a revelation of the object measured, but this revelation has been accidentally and extrinsically modified by the instrument used. To conceive the problem in this light is to miss the main issue. For measure-numbers are essentially the product of both the object measured and the instrument employed. And here we have in mind something more than the

point brought out above about measure-numbers being mere ratios resulting from a comparison of an object with a standard of measurement. We have in mind here something that has to do with physical causation. We mean that the measure-numbers are works of art produced by the co-causality of both the object measured and the measuring instrument.

Perhaps this point can be clarified to some extent by a simple distinction. The influences which an instrument has upon the results of measurement are of two kinds. Some of them are causal, and in a sense extrinsic, and these the scientist may labor to correct, or at least, to account for. But there are other influences which are essential, since they result from the very nature of the instrument and from the very purpose it was designed to achieve, and these it would be nonsensical for a scientist to attempt to eliminate.
(80)
nate.

Professor De Koninck has brought out with great exactness the fundamental issue involved in this question:

Entre ces nombres-mesures repérés sur l'échelle graduée d'un instrument et le sujet matériel, il y a la fabrication dont on ne peut faire abstraction sans tomber dans le subjectivisme. Ne confondons pas la donnée prescientifique avec le nombre-mesure, qui n'est pas une traduction immédiate et adéquate de cette donnée. Ce n'est pas l'objet sur le plateau de la balance qui sera le point de départ propre de l'élaboration scientifique, mais tel nombre sur l'échelle graduée auquel s'arrête l'aiguille. Une fois définie la propriété, je ne puis l'attribuer telle quelle à l'objet,

comme si la balance n'était qu'une espèce de rideau et que dans la pesée on épiait 'derrière' la balance pour surprendre l'objet tout nu. (Et c'est bien ce qu'on croyait faire avant la critique einsteinienne des mesures d'espace et de temps, oubliant que les circonstances mêmes de mensuration font partie d'une définition et que la différence de circonstances change qualitativement cette définition. Dire que des définitions de longueur qualitativement différentes doivent avoir la même valeur quantitative c'est tomber dans ce relativisme dont Einstein nous a libérés. (81)

One of the reasons why this point has often been lost sight of, at least to some extent, results from the innate and inevitable tendency of science to idealize the entities with which it deals. As we pointed out in Chapter IV, the physicist tends to substitute in his mind an ideal geometrical model for the physical apparatus with which he is working. He tends to de-materialize his instruments, in such a way that a concrete meter rod, for example is transformed into an immaterial meter. Speaking of this question Sir Arthur Eddington writes:

Primarily we say yard rather than yard-stick because a great many equivalent substitutes for the yard-stick are possible. But we do not generally think of a yard as a general name for one of a large variety of physical objects or systems; we do not think of it as an object at all. I grant that another physical object may be an equivalent substitute for a yard-stick, but I do not grant that a de-materialized yard is an equivalent substitute for a yard-stick.

When the quantum physicist employs a standard of length in his theory, he does not treat it as an object; if he did, he would according to the principles of his theory have to assign a wave function to it, as he does to the other objects concerned in the phenomena. In my view he is wrong. Either he is using the standard length as a substitute for the second body concerned in the observed relation of size, in which case he ought to attribute to it a wave function, so that he can bring it into his equations in the same way that the second body would have been brought in; or he is treating size as though it were not an observable relation between one physical object and another, and the lengths referred to in his formulae are not the lengths which we try to observe. We have to recognize then that what are called the properties of an electron are the combined properties or relations of an electron and some other physical system which constitutes a comparison object. For an electron by itself has no properties. If it were absolutely alone, there would be nothing whatever to be said about it -- not even that it was an electron. And we must not be misled by the fact that in current quantum theory the comparison is replaced by an abstraction, e.g. a metre, which does not enter into the equations in the way that an observable comparison object would do; for that is a point on which current quantum theory is clearly at fault. (82)

These considerations will serve to bring to light the position occupied by the instrument in the process of measurement. In some sense it is an ambiguous position, for the instrument belongs at the same time to the subject who is measuring, and to the object measured. For on the one hand, it is a kind of prolongation of the cognitive powers of the subject; it refines these powers and enables them to arrive at more exact and more sensitive discriminations. On the other

hand, it is one with the object both because it is one term of the comparison which every measurement implies, and because of the physical causality it exercises in the measuring process.

In connection with this limitation of measurement arising out of the part played by the instrument, another closely associated source of limitation must be touched upon. We are referring to the various cosmic influences that enter into every concrete measuring process. These influences are legion, and they have a very definite effect upon the results of the measurement. It is true that it is possible for scientists to cope with them to a certain extent. In every process of measurement there is an attempt to achieve an ideal state in which such influences as arise from electric and magnetic fields, unfavorable atmospheric conditions, strain, corrosion, flexure, etc. are either removed, or controlled, or accounted for Theoretically. And through the method of successive approximation employed (83) so extensively in physics science is able to achieve an ever increasing degree of perfection in the control of these influences. But no matter how much progress may be made in this direction, the goal will ever remain at an infinite distance, for it is a purely dialectical limit. In order to

be able to account for all of the cosmic influences which play a part in the measuring process, one would have to be perfectly acquainted with these influences, and that would demand an exhaustive knowledge of Nature. And perhaps it is not superfluous to note that this involves much more than a perfect knowledge of all the laws of nature. For chance plays such an important part in the cosmos that many of the influences that actually bear upon concrete experiments are pure chance events which have no determined cause, and which are therefore outside the pale of all law. It seems safe to conclude, then, that our actual knowledge of the influences entering into our experiments will ever remain infinitesimally small. And in this sense there is a great deal of wisdom in Planck's remark that "measurement gives no immediate results which have a meaning of their own."⁽⁸⁴⁾

What is it that we actually measure in our concrete processes? Perhaps it is not an exaggeration to say that even in such a trivial measurement as the weighing of a pound of meat, we are not merely measuring the weight of the meat - - we are actually measuring the whole cosmos. For the object measured and the instrument employed never constitute an isolated system. Nor can an isolated system

ever be achieved through successive approximation in the control of known cosmic influences. A perfectly closed system, other than the entire cosmos, is a pure idealization, It exists nowhere but in the mind of the scientist. The following lines of Louis De Broglie have considerable relevance here:

Le concept d'unité physique n'est donc vraiment clair et bien défini que si l'on envisage une unité complètement indépendante du reste du monde, mais, comme une pareille indépendance est évidemment irréalisable, le concept d'unité physique pris dans toute sa pureté apparaît à son tour comme une idéalisation, comme un cas qui jamais ne s'adapte rigoureusement à la réalité. Il en est de même, d'ailleurs, du concept de système. Le système, dans sa définition stricte, est un organisme entièrement fermé et sans relations avec l'extérieur; le concept n'est donc vraiment applicable qu'à l'univers entier. (85)

These general considerations lead us inevitably to a question which constitutes one of the most central problems in any discussion of the significance of measurement -- the question of the rigid scale. It is immediately evident that rigidity, or what Whitehead calls self-congruence, is the primary requirement for any standard of measurement. Elastic tapes are never used as standards, nor are easily expansible metals ever employed in measuring devices. And the fundamental reason for this has been brought out in our analysis of the nature of measurement.

But to what extent is self-congruence possible? Or, to put the question more pointedly, does the concept of self-congruence even have any meaning? If it is impossible to arrive at any definite determination of rigidity, and if the very notion of self-congruence is without meaning, then to say the least the validity and significance of the whole measuring process will be extremely questionable. And at first sight it might seem that we must be lead to this conclusion. For if the statement which we made a moment ago, that a length must be measured with a rigid scale, is to have any meaning for us, we must be able to define what we mean by a rigid scale. And the definition which naturally suggests itself to us is: a rigid scale is one that preserves the same length. But this immediately involves us in a vicious circle, for we have defined length in terms of a rigid scale, and a rigid scale in terms of length. (86) And as long as we cling to these two definitions we shall be confronted by an impasse. For, obviously, if length is a quantity obtained by means of measurement with a rigid scale, it will be necessary to have recourse to another rigid scale to decide whether or not the length of the first scale changes, and this sets us on an infinite series. The only possible way of surmounting this impasse is to revise one

of the two definitions. And a moment's reflection will show that the definition of length cannot be the one revised, since length can have no definite meaning except in terms of the self-congruence of a standard. We must then attempt a solution of the problem by seeking for a determination of rigidity independently of the notion of length. At first sight this may seem an impossibility, for it is difficult to see how one can decide whether an extension has increased, or decreased, or remained the same, except by means of measurement. And if measurement is employed, a vicious circle is inevitable.

Fortunately there is a way out of this impasse. And the way is suggested by a remark made earlier in this analysis: the standard of length has no length. Since we cannot speak of length in relation to a standard of length, it is illegitimate, and even nonsensical, to attempt to determine the rigidity of a standard in terms of length. Some might be tempted to object immediately that, far from leading us out of our impasse, this only complicates the problem all the more. For if the standard of length has no length, what sense is there in speaking of self-congruence or rigidity? No matter how much an elastic meter tape measure may be stretched, everything that is measured with it will

always be a meter in length. As a result the whole process of measurement loses its significance.

A moment's reflection will show that this objection arises from a confusion over the meaning of the term "length". As we have already pointed out, this term is susceptible of a multitude of meanings. But since we are dealing with physical science, we have been using it, and shall continue to use it, in the sense in which it is employed in physics: the measured magnitude of a sensible line. No standard has length in this sense. That is why we cannot employ measurement to determine the rigidity, for then the standard would be a measured magnitude. But obviously every standard has length in the sense that it is an object with a definite extension. And it is possible independently of any process of measurement and merely by having recourse to identity and non-identity ⁽⁸⁷⁾ to determine the constancy or inconstancy of this extension.

A number of bars of different material may be taken and their identical extension determined by noting the coincidence of extremities. These bars may then be subjected to a variety of influences such as pressure, temperature, atmospheric conditions, etc., and by comparison their coefficients of expansion or contraction observed.

The bar which comes closest to identity with the original extension is chosen as the standard. A special room is prepared in which conditions considered to be ideal are kept as constant as possible, and every effort is made to exclude disturbing influences. The chosen bar is then placed in this room, and at last a rigid scale has been achieved. This is, in substance, the way in which the international legal standard of length was arrived at -- the *Mètre des Archives*, which is a bar of platinum preserved in Paris at the temperature of melting ice and under atmospheric pressure.

This process of determining self-congruence may appear extremely dubious, and one might be tempted to ask, "Is this rigid scale actually rigid?" A question of this kind contains considerable ambiguity, and it is difficult to know how it should be answered. If its meaning is: "Can this meter rod ever be longer or shorter than a meter?", the answer must obviously be in the negative. Once a standard has been chosen, it is impossible for it to change qua standard. The question might also mean: does the scale remain absolutely rigid as far as science is concerned? and it is possible to answer such a question in the affirmative, in the sense that the whole structure of science is based upon the assumption that the scale is rigid.

Perhaps the word "assumption" will be immediately seized upon and the question pressed home: "But is it really rigid?" The answer to this question depends upon what is meant by "really". If it means that there is existing somewhere in the cosmos an ultimate and absolutely immobile ideal standard in relation to which the constancy or inconstancy of the chosen standard may be objectively determined, it is ~~extremely~~ doubtful just how much sense a question like that can have. It certainly has no sense from the point of view of physical science. (88) We do not see how it can even have sense from the point of view of philosophy. But if the question means: does the scale possess absolute objective immobility, then a definite answer can be given. And the answer is: certainly not, for the very notion of an absolutely immobile material object is a contradiction in terms.

And this brings us to the central point towards which most of this discussion has been directed: the whole significance of the measuring process depends upon the rigidity of the scale that is employed as a standard, and it is impossible to arrive at an absolutely rigid scale. The rigidity that is spoken of in science is one that is determined by fiat; it is a convention. And this obviously

introduces a profound limitation into the process of meaning. But it is impossible to have a clear notion of the nature of this limitation except by pointing out that, while it is meaningless to ask whether this convention is true or false, it is extremely important to determine to what extent it is arbitrary. It is obvious that like every convention, the determination of the rigid rod is in some measure arbitrary. But it is likewise obvious from what has been said that it is far from being purely arbitrary. In other words, it is something that is at once both subjective and objective. And though it will always remain impossible to determine the relative degrees of subjectivity and objectivity, it is important to note that purely objective rigidity is a dialectical limit to which science may draw constantly closer and closer, by means of its usual method of successive approximation through an ascending spiral similar to the one described above. When we stated that once a rigid scale has been chosen, it cannot change, we do not mean of course that science can never reject a chosen standard in favor of one that seems more perfect. In fact, it is of the very nature of physical science to be constantly in search for a more perfect standard. It is probable that the Paris meter will eventually be supplanted by another standard, such as, for example, the

grating space of a calcite crystal, whose lattice structure has the advantage of associating the standard with pure numbers. It is likewise probable that science will gradually achieve greater and greater rigidity in its standard. The only important point to keep in mind, as far as our present discussion is concerned, is that no matter what degree of rigidity may be attained, there will always be in the standard an indeterminable margin of subjectivity deriving from the free intervention of the human intellect and will.

This discussion of the rigidity of the measuring rod may perhaps bring to mind the question of the Fitzgerald contraction, first postulated to account for the absence of any indication of aether drag in the Michelson-Morley experiment and later confirmed by the electromagnetic researches of Larmor and Lorentz. According to the postulate of Fitzgerald, a material rod moving at high speed contracts in the direction of the line of motion. The consequences of this postulate for the problem of measurement are immediately apparent. What determined meaning can measurement have if the standard scale expands and contracts according to the velocity at which it is moving and according to the direction in which it is turned -- especially if

(as is the case) it is impossible to know in any absolute way the velocity of the scale. In ordinary circumstances this contraction is negligible; for example, the diameter of the earth contracts two and a half inches, or one part in two hundred million, in the velocity of nineteen miles a second of its movement around the sun. But at the speed of one hundred and sixty one thousand miles a second the contraction would be one half. And is there any way of knowing whether in relation to some point of reference in the cosmos, the whole solar system is not moving in a manner that approaches this velocity? What is worse, is there any way of knowing whether the whole frame of reference in relation to which we make our measurements is not moving in relation to other frames of reference in different directions and at different velocities, which perhaps do not remain constant?

It becomes immediately evident that all of our determinations of length (and of time also, as we shall see presently) are dependent upon the particular frame of reference within which they are made. And here we are touching upon the profound difference between Classical and Relativity physics. But the point is not that Classical physics failed to realize that different velocities and

different frames of reference have an influence upon the process of measurement. In fact, it provided formulae by which each observer could apply "corrections" to reduce his "fictitious" length to the "unique" Newtonian length. The whole crux of the matter lies in the meaning of the words "corrections", "fictitious", and "unique". In other words, Newtonian physics realized that measurements made by different observers will give different results. But it took it for granted that there was an absolute observer who occupied a privileged position -- a position that was Nature's own position. And from this supposition stemmed two implicit postulates: 1) that spatial relations determined by the measurement of length could be reduced to an absolute meaning; 2) that temporal relations had an absolute and independent character. Einstein was astute enough to see that both of these postulates were perfectly gratuitous, and he proposed to do without them. But in order to understand the significance of his doctrine for the question of measurement, it is necessary to return for a moment to the Fitzgerald contraction and try to fix upon its exact meaning.

At first sight, this contraction might seem to be in the same category with the changes in the standard scale,

discussed in connection with the problem of rigidity, but as a matter of fact, it constitutes an entirely different problem. Indeed, it is true to say that, paradoxical as it may seem, the Fitzgerald contraction has nothing to do with rigidity. The meaning of this statement will be fully explained in a few moments, and for the present it is sufficient to point out that the contraction is determined completely by the velocity of the motion and not by the specific nature of the rod in question. All rods moving at the same velocity undergo exactly the same contraction, no matter what degree of rigidity they may possess in relation to such influences as temperature, stress, etc. The contractions of a rod of platinum and a rod of rubber moving at the same speed are identical. Hence this contraction must not be looked upon as an imperfection of the rod. It must not be considered a deficiency in relation to an absolute rod. Such a rod does not exist, nor can it exist.

In order to come to understand how the problem of the Fitzgerald contraction differs radically from the problem of rigidity, it is important to note that the length of an object measured is in a sense completely independent of the difference between its temperature and that of the measuring rod. A cold scale may be brought into direct

contact with an extremely hot body and determine its length with precision. But the length of an object measured is not independent of the difference between its motion and that of the standard scale. In fact, it is, in a sense, completely dependent upon it.

When a scale and an object can be brought into immediate contact, or when their motions are correlated in such a way that they are moving with the same velocity and are thus at rest in relation to each other, the measurement gives us the proper length of the object. From one point of view, physics would be immensely simplified if it were always possible to arrive at the proper length of the objects measured. But, as Eddington has remarked, "it is not convenient to send your apparatus hurling through the laboratory - - after a pair of α particles, for example." (89)

Perhaps at first sight the difference between the determination of the proper length of an object and the determination of the length of an object in motion in relation to the scale may not seem to constitute any serious problem, since it appears to be a fairly easy matter to reduce the one to the other. Let us suppose, for example, that a straight rod is moving with uniform velocity with respect

to a certain frame of reference. It is possible to mark on the frame the simultaneous positions of the extremities of the rod, and then measure the distance between the two positions marked on the frame. Will the results correspond to the proper length of the object in motion? One might be tempted to answer in the affirmative, since the two positions were marked simultaneously, But then he will be obliged to tell us what he means by simultaneity. And therein lies the whole crux of the matter.

As we have already suggested, Classical physics attributed to the notion of simultaneity an absolute meaning. But Einstein pointed out that this attribution was based on an implicit assumption which was utterly incapable of being verified experimentally, since this verification would presuppose that signals announcing distant events could come to all observers instantaneously, that is, with an infinite velocity. Concepts have no meaning in physics unless they can be defined operationally, and Einstein made it very clear that every attempt to define simultaneity operationally inevitably results in making it something relative to the frame of reference in which the operation was carried on. In other words, the only kind of definition of simultaneity that has any meaning is such that if two

events verify it in one system they will not verify it in another system that is in motion with respect to the first. The measurement of time, then, becomes essentially relative to a given system. And since the determination of the length of a body in motion necessarily involves the notion of simultaneity, every determination of such a length is essentially relative to a certain frame of reference. Thus Einstein was able to arrive at the following statement: "If a body has the length l with respect to a system in which it is at rest, then with respect to a system in which it is moving with the velocity v it will have the length $l' = l\sqrt{1 - \frac{v^2}{c^2}}$, where c is the velocity of light. That is, length of a body has in each system a different value, depending on the velocity v of the body with respect to the system in question." This difference of value is equal to the Fitzgerald contraction. And since the determination of the other quantities which enter into physics is bound up with the reckoning of length, it follows that mass, periods of vibration, electric and magnetic fields, etc. become relative to a certain frame of space.

Because of the way in which simultaneity is involved in our determination of length, it is clear that not only space, but time as well is implied in all our

measurements. In other words, to quote Eddington, "the fundamental measurement is not the interval between two points of space, but between two points of space associated with instants of time." (90) Events in nature are exterior to each other in four different ways, of which three are spatial and one temporal, and the order of these events constitutes one indissoluble four-dimensional space-time order. It is the purpose of the laws of physics to express this order in the form of numerical relations, and this can be done without ambiguity only by having recourse to a system of reference of four coordinates. That is why non-Euclidian geometry has become the instrument of Relativity physics.

Science has been led to reconstruct the world in this four-dimensional order, not by any arbitrary choice, but by the very nature of extrinsic measurement upon which its whole method is founded. Because the bodies which constitute the cosmos are in motion with respect to each other length can be measured only in relation to time, and time only in relation to length. Consequently, observers with different motions will have different reckonings of space and time, and each observer by merely changing his motion will make a different division of the four-dimensional

order into space and time. In other words, each observer, according to the different operational definition he gives of simultaneity, will cut up the space-time continuum into space and time in different ways. But while the determinations of length and time are relative, the space-time continuum which they constitute has an absolute character. And it was Einstein's chief aim to attempt, by a comparison of measures made with respect to different systems, to arrive at elements which would be independent of particular observers.

But, to get back to the main purpose of this analysis, it follows from what has been said that the same body may have any number of different lengths, depending upon the frame of reference in relation to which the length is determined. It makes no sense to say that one of these lengths is true and all the others are false. They are simply different. Nor is it legitimate to give to one of them a special meaning by attributing to its frame of reference a privileged position in the cosmos. Nature has not revealed any privileged frame. And the profound significance of the theory of Relativity is not that it discovered that the frame of reference used by Classical physics was wrong or that it involved experimental incon-

sistencies, for such a discovery would not have produced any great revolution; but rather that it brought to light the fact that neither this frame nor any other frame that might be chosen can be considered unique.

It is clear, then, that there is a double relativity involved in every determination of length. In the first place, every length is essentially relative to the chosen standard, and this standard is arbitrary. Secondly, it is essentially relative to the particular frame of reference by which it is determined, and this frame is also arbitrary. That is why length has no absolute meaning. All this refers, of course, to extrinsic measure which alone has significance for physical science.

But what about intrinsic measure? Is that also essentially relative? This question has already been solved, at least in a general way, earlier in our analysis. But perhaps it will be worth while to bring it back into focus again in relation to what we have been saying about Fitzgerald contraction. Scientists are often asked: does the Fitzgerald contraction actually take place? Such a question is extremely ambiguous, and no definite answer can be given until several important distinctions are made. In the first

place, the question might be taken to mean: do measurements of a body in motion with respect to a given frame of reference give results which differ from those obtained by the measurement of a body at rest, and if so, is this difference equal to the Fitzgerald contraction? Taken in this sense, the question will receive an affirmative answer from scientists. And this seems to be the only sense in which the question can have any significance for them. For in physics the phrase "actually takes place" can only refer to what actually takes place in measuring instruments.

But perhaps one might be tempted to push the question further and ask: But does velocity make the length of a rod contract in the same way that a change in temperature does? This question is still ambiguous, since it attempts to establish a comparison between "lengths" which have entirely different meanings. But perhaps the issue can be clarified by putting the problem in these terms: does the motion of a body decrease its intrinsic measure? And then the answer is: first, the Fitzgerald contraction certainly does not imply such a change, since it has nothing at all to do with intrinsic measure; secondly, there is no way of knowing what actually happens to the intrinsic measure of a body in motion, for in order

to determine the dimensions of such a body we are forced to have recourse to extrinsic measurement made in relation to a particular frame of reference.

In this discussion of the limitations of measurement it has been necessary to restrict ourselves to rather general and superficial considerations. A more refined analysis of particular processes of measurement, such as those which have to do with time, for example, would throw fuller and more definite light upon the extremely limited character of the knowledge which measurement affords us. But perhaps enough has been said to show how highly artificial and subjective this knowledge is. There is, indeed great wisdom in Bergson's remark that nature does not measure. It is man that measures. And he cannot measure without projecting his own logos into nature. At every step in the measuring process there is a projection of the human intellect and will. And the more perfect this process becomes, the greater becomes the part played by the subjective elements. In a very true sense, measure-numbers are not found in nature. They are imposed upon nature by man.

But lest all this seem to give too much aid and comfort to idealism it is worth while pointing out, as we

bring this question to a close, that measurement is after all a real physical operation which comes to grips with the real world. And the relations which arise out of it are basically real relations, in spite of the large margin of subjectivism. Moreover, the subjective element is^{*} purely functional; it exists only to enable us to come into more intimate relation with the objective world.

CHAPTER NINE

THE MATHEMATICAL TRANSFORMATION OF NATURE

1. The Transformation of Natural Science.

"The mathematician," Goethe once remarked, is like a Frenchman: if you speak to him, he translates it into his own language, and at once it becomes something altogether different." In this Chapter we must endeavor to see at least in a summary and schematic way, how the mathematician who is called in to assist the physicist in the study of nature translates the world of the physicist into his own language and makes of it something altogether different. And we shall consider this transformation from two points of view. First we shall try to see the way in which the introduction of mathematics into physics effects the very structure of physical science itself; and secondly, we shall attempt to bring out the change that this produces in the reflection of nature that is found in physical science.

In the last Chapter we considered the preliminary step in the mathematical transformation of physical science. In order for science to be mathematicized, all of its processes of experiment must be transformed into processes of measurement; all of the phenomena with which it deals must be translated into pointer readings. This preliminary step provides the scientist with a collection of measure-numbers, by which are determined various properties of bodies such as mass, volume, temperature, pressure, viscosity, valence, molecular weight, various optical, electrical and magnetic properties, etc. But just as physics is not a collection of phenomena, so mathematical physics is not a collection of measure-numbers. In order for science to emerge, the unifying process described in Chapter IV must undertake, by using measure-numbers as materials, to construct out of them an integrated and coordinated system. And the first step in this process is the establishment of law.

Since the only materials of construction available are numbers, laws in mathematical physics can be nothing but the expression of relations between numbers. Since a law must be universal, that is to say formulate a constant relation, a physico-mathematical law will express a relation between variable magnitudes, and consequently will (not) be algebraic and

not arithmetical (in the restricted sense of the term "arithmetical"). The uniformity of association which constitutes the essence of experimental law finds its best expression in the language of numbers because it is at once both exact and universal. This expression usually takes the form of differential equations.

"A physical law", writes Planck, "is any proposition enunciating a fixed and absolutely valid connection between measurable physical quantities -- a connection which permits us to calculate one of these quantities if the others have been discovered by measurement." ⁽¹⁾ In other words, a physical law is a constant relation between variable quantities; it takes the form of an algebraic equation which expresses a functional relationship indicating the precise value of any one of the measures that corresponds to any given value of the other measures. Once the concrete measure-numbers are absorbed into mathematical equations they become susceptible of all the pliancy of mathematical manipulation. The mathematician is free to have recourse to all of the resources at his disposal: powers, roots, divisors, dividends, sines, cosines, vectors, etc. There is nothing to prevent him from squaring the symbol for time, for example. These manipulations, obviously, do not effect the concrete properties from which the original measure-numbers have arisen,

but they may lead to the discovery of new properties.

It is extremely important to grasp the true nature of the functional relationship of physico-mathematical law. As is evident from our analysis of the nature of mathematical abstraction, mathematics prescind from all causality except a type of formal causality that is found in formal relationships. For example, the geometric "law" $B = S/H$: the base of a rectangle is equal to the surface divided by the height does not mean that a surface can actually be divided by a length. And if B varies it is not because (in the sense of true causality) S varies, or vice versa. The law merely states that if the base is changed, the nature of a rectangle is such that the surface will undergo a proportional change. The "if" makes all efficient causality extrinsic to the law, and the phrase "the nature of a rectangle is such that" shows that the law deals with formal causality, since it is the form of a thing which determines its nature. (2) Consequently, in the measure in which physical laws are expressed in mathematical equations they are stripped of all true causality. Genuine causal statements are irreversible, that is to say they always involve ontological symmetry and usually temporal asymmetry. The effect depends upon the cause for its being and not vice versa. Formulae of covariation and purely functional statements, on the other

hand, are essentially symmetrical. Any one of the variables may be arbitrarily considered as independent or dependent.

When a mathematical physicist states that the movement of the planets is in accordance with the following law: the force of attraction between bodies is directly proportional to the product of their masses and inversely proportional to the square of the distance between them, he is not expressing the cause of planetary movement. He cannot treat force as a true cause since for him it is reduced to a measure-number which is a product of the multiplication of the numbers derived from the measurement of mass and acceleration. He is merely expressing a formal interrelatedness emerging from a comparison of the mass, distance, and acceleration of planets. Force and movement, then, are not related as cause and effect. They are simply two data which are mutually dependent in somewhat the same way as the diameter and circumference of a circle. (3)

Poincaré has insisted upon this point in La Science et l'Hypothèse:

Qu'est-ce que la masse? C'est, répond Newton, le produit du volume par la densité. Il vaudrait mieux dire, répond Thompson, que la densité est le quotient de la masse par le volume. Qu'est-ce que la force? C'est, répond Lagrange, une cause qui produit le mouvement d'un corps ou tend à le reproduire. C'est, dira Kirchoff, le produit de la masse par l'accélération. Mais, alors, pourquoi ne pas dire que la masse est le quotient de la force par l'accélération? Ces difficultés sont inextricables. Quand on dit que la force est la cause d'un mouvement, on fait

de la métaphysique, et cette définition, si on devait s'en contenter, serait absolument stérile. Pour qu'une définition puisse servir à quelque chose, il faut qu'elle nous apprenne à mesurer la force, cela suffit d'ailleurs, il n'est nullement nécessaire qu'elle nous apprenne ce que c'est que la force 'en soi', ni si elle est la cause ou l'effet du mouvement. (4)

Ohm's law merely signifies that the numbers obtained by the measurement of the intensity of an electric current, the electromotive force, and the resistances are so related that they always verify the equation: $I = E/R$, whatever be the numerical values of the symbols in individual cases. The law of Mariotte is likewise stripped of causality when it is transformed into a mathematical equation. It does not mean that the pressure is the cause of the increased volume; in so far as the mathematical physicist is concerned. Both the pressure and the volume may be considered either as the independent ("cause") or the dependent ("effect") variable. The law merely states that when all other measures are equal, if the measure of temperature increases, there is a definite corresponding increase in the measure of the volume. Or to put it in other words which will bring out the assimilation of a physical law to a geometrical law, and show what type of causality is in question; the law states that if a cause should increase the temperature of a gas, the nature of the gas is such that there will be a

proportional increase in its volume.

The same is true of all the laws of mathematical physics: they do not declare that A is the cause of B; they merely state that one set of events B is a function of another set of events A. If the mathematical formulation of the law expressed causality, the causality would have to be reversible. Perhaps one might be tempted to think that the intervention of a time measure into a law might introduce causality since this measure will indicate which of the variables is the antecedent and which the consequent. But a moment's reflection will show that this is not true. This intervention of a time measure merely expresses the fact that the other measures vary in relation to the time measure. An expression of antecedence does not involve causality.

It is clear, then, that the mathematical formulation of physical laws empties them of all true efficient causality. And the same must be said of final causality. Just how profound this change is becomes evident when one stops to consider that all law essentially involves finality. By its very nature law means an inclination, an ordination to an end. We shall return to this question later on.

From all that was said in the last Chapter on the

nature of measurement it follows that, in spite of the exact mathematical formulation by which they are expressed, and in a certain sense precisely because of it, the laws of physics do not have exact and absolute validity. (5) In fact any mathematical expression of physical constancy is only one of an infinite number of slightly different expressions which might possibly be employed to formulate the same phenomenon. All physical laws are essentially provisional. And they are provisional for two reasons in particular; first because they are merely approximative, and in this sense neither true nor false; secondly because they are schematic. They are approximative because the measures whose relations they express are never made with absolute exactitude. That is why they must ever remain open to successive corrections, for progress in the refinement of measurement will continually introduce slight changes in the numerical coefficients, and there is no limit to this process of refinement.

Laws are schematic because they include only a small fraction of the possible measures that could have been made; that is to say, they express a relation between certain chosen properties, independently of all the other properties which may be connected with the ones chosen. Consequently,

as science progresses its laws must be constantly modified in such a way as to take into consideration attributes previously omitted. Physical properties are defined by the description of their process of measurement, and as we noted in the last Chapter all of the circumstances entering into this process can never be enumerated. Progress in experimentation reveals an increasing multiplicity of circumstances which have a definite influence upon the results of the measuring processes, but which were neglected in the original formulation of a general law. That is why all laws must remain forever open to a progressive modification by which these newly discovered influences are integrated into its structure. This modification does not change the form of the law or its numerical coefficients, as does the modification occasioned by its approximative character. The newly discovered circumstances can be introduced only by the introduction of new measures and consequently new properties. Thus progress in experimentation with gases revealed the fact that in order to determine with precision the relation between pressure and volume attention must be paid to the mutual attraction of the molecules and their proper volume. A determination of these additional circumstances results in the transformation of the law of

Mariotte into that of van der Waals.

Thus, as science progresses its laws become increasingly complicated by the integration of newly discovered influences. This complication results in investing general laws with greater precision and accuracy. But as we saw in the last Chapter, even the simplest measuring process involves the whole universe. That is why a perfectly exact law would require an exhaustive descriptive of the entire cosmos.

But while this process of complication is taking place there is a concomitant process of simplification going on, which consists in the reduction of the ever increasing multiplicity of measures to a few fundamental measures. This is done in two ways. In the first place it is discovered that a number of different instruments give the same results. Since physical properties are defined by their processes of measurement it remains theoretically true that two different processes define two different properties. Nevertheless it sometimes becomes evident that the results of two or more different processes coincide, as for example when heat is measured by the expansion of a metal spring and by the expansion of a column of mercury. But even more

important than this is the simplification resulting from the discovery that the results of certain processes of measurement coincide with mathematical combinations of other processes. Laws reveal constant relations between the measures of different properties. These constant relations make it evident that certain measures can be replaced by a combination of other measures. In this way it is possible to reduce a vast multiplicity of measures to a few fundamental measures. In fact science has been able to push this process of simplification to the extent of reducing all physical measures to combination of the fundamental measures of length, mass and time, in such a way that the former may be considered as functions of the latter. It thus becomes possible to define the multiplicity of physical properties in terms of combinations of a few irreducible properties, This does not mean that bodies have no other properties but the three that are measured by a rule, a balance and a clock. It merely means that when the variety of physical properties are measured by different measuring processes the results are numerically the same as certain mathematical combinations of the measure numbers provided by a rule, a balance and a clock. By this simplification the scientist is able to synthesize his knowledge into a small number of propo-

sitions into which only a few basic measures enter, and from the relations existing between the fundamental measures it becomes possible to deduce the multiplicity of relations existing between the particular measures.

All this shows how this process of simplification opens the way for the scientist to take the next step in the unification of his knowledge -- to ascend from laws to theories. But before passing on to an analysis of the nature of physical theory it is necessary to remark that because of the approximative and schematic character which we have been discussing, physical laws are always a simplification of the mind and in this sense a product of the mind. And their provisional nature cannot be lost sight of without undermining their objective significance. Casting physical reality in mathematical form has the advantage of providing it with great openness, that is to say, of opening it up to the unlimited reaches of mathematical speculation which affords such abundant sources of explanation. But at the same time, it has the disadvantage of imposing upon reality a frame which because of its exact determination is too closed. And in this connection it is worth while recalling the well-known remark of Einstein that in so far as the theses of mathematics are certain they do not refer to

physical reality, and in so far as they are made to refer
to physical reality they are not certain. (7)

But perhaps one might be tempted to object to the statement that all of the laws of physics are provisional on the grounds that there are certain fundamental laws known as principles which are not subjected to the successive change about which we have been speaking and which consequently seem to have an absolute and not a provisional character. The conservation laws, the law of inertia etc. are all laws of this kind. The answer to this objection is that the absolute character of these principles is a pure gift of the mind. The principles of experimental science are laws which have been merely suggested by nature, but which the mind has arbitrarily erected into fixed and absolute principles. The reason why progressive experimentation does not modify them is simple: the mind has accepted them as conventional definitions of the very objects to which they apply. Consequently it is impossible for these objects not to be in accord with them. And now, having examined the nature of physical laws we must take up the problem of physical theories.

For reasons explained earlier in this study, the

the mind cannot rest satisfied with an a posteriori possession of physical laws. It will never feel that it has assimilated them perfectly until it is able to possess them in an a priori fashion. Just as the formulation of laws makes it possible for the mind to arrive at the results which a certain measuring process would give without actually effecting the process, so the scientist instinctively seeks for a point of departure which will enable him to arrive at a certain law in a way that does not depend upon experience. In other words, having arrived at physical laws by induction, the scientist is led to attempt to arrive at them by deduction; having posited their existence, he must attempt to explain them; having arrived at universal functional relationships, he must try to show that these relationships are necessary. This is done by making the laws appear as logically necessary conclusions. (8) Since the laws themselves are numerical relations, the point of departure from which they are to be deduced must be general numerical relations. These general numerical relations constitute what is known as a mathematical theory. A theory has been defined by Duhem in the following terms: "un système de propositions mathématiques, déduites d'un petit nombre de principes, qui ont pour but de représenter aussi simplement, aussi complètement, et aussi exactement que possible un ensemble de lois expérimentales." (9)

Not only does a physical theory synthesize the laws which experience has suggested, but it tends to fill in the gaps which observation has left open by substituting what Cassirer has called "a continuous connection of intellectual consequences."⁽¹⁰⁾ In this way science becomes a conordinated system. And this system is perfected by a continual simplification and reduction of the principles which form its point of departure and a continual increase of the experimental propositions which constitute its terminus. As Whyte has remarked, "the highest possible aim for science is the formulation of a self-consistent closed chain of concepts and principles permitting deductive argument in one direction at every point of the chain."⁽¹¹⁾ The dialectical limit of this movement would be a science in which the whole universe could be deduced from one mathematical formula.⁽¹²⁾

On more than one occasion in this study we have insisted upon the fact that the fundamental reason why physical science reaches out to mathematics is to discover an explanation which it finds itself unable to provide for physical phenomena, in other words, to discover a reason or propter quid for its experimental propositions. But perhaps what has been said thus far in the present Chapter about the

mathematical transformation of physical science may give rise to doubts as to whether this goal is actually achieved. As a matter of fact, a number of authors explicitly deny that a physico-mathematical theory is an explanation. Duhem, for example, writes: "Une théorie physique n'est pas une (13) explication." We believe that the difficulty here arises from the ambiguity of the word "explanation". As a matter of fact, it is a term that is susceptible of a variety of meanings. In its most fundamental sense it means to give the proper reason for a thing by presenting one or several of the four causes by which reality is constituted. This is the type of explanation that is employed in the philosophical sciences.

There is another sense in which the term explanation is used and which has long been associated with experimental science. It consists in presenting a model whose structure and functions reproduce the structure and function of the phenomena to be explained. We understand the term "model" here in the sense of a mechanical construct or at least of a pictorial image, and not in the sense in which it is now sometimes used and which includes mathematical "patterns" such as "tensors and matrices, manifolds and their curvature, differential forms and their

(14)
invariants." It is well known that mechanical models constructed out of pulleys, wires, rubber tubes, etc. were the favorite form of explanation employed by the classical physicists, particularly those of the English School, such as Lord Kelvin, Oliver Lodge, Faraday, Maxwell, etc. We have already quoted Kelvin's well-known remark that for him to understand reality meant to be able to construct a mechanical model of it and apart from such a model no explanation of reality could have any meaning for him.

But even when less emphasis was put upon concrete mechanical models and more upon abstract mathematical conceptualization, there was, until recently, always lurking in the background of mathematical theories physical models of some kind. For example in the background of the mathematical kinetic theory of gases there has always been a fairly definite physical model constructed of molecules which are so idealized and so simplified that they are susceptible of accurate mathematical treatment, even though spectrum analysis has given abundant evidence of a considerable gap between the idealized and simplified molecules and the actual molecules. These idealized and simplified physical models have served as a kind of bridge between actual physical reality and mathematical theory.

Because of their physical character they have been considered to be in contact with reality; at the same time their simplified and idealized state makes them directly amenable to mathematical manipulation. Recent physics has discovered however that it can get along without this bridge, that independently of any physical model it can set up a correspondence between the results of its mathematical constructions and the physical system. This has been particularly true of the quantum mechanics of Dirac. (15) Speaking of this significant change Professor Bridgman writes:

What we now have is in effect mathematical models rather than physical models. This emancipation I feel to be a very important step forward toward greater theoretical power, because there is an enormously greater wealth of possibility among the structures of mathematics than in the physical models which we can visualize and which have a simple enough mathematical theory. It cannot be denied, however, that a mathematical model cannot be visualized in the same sense that a physical model can be. Although we may recognize with our intellect that the mathematical model is just as good as the physical model if it only enables us to answer any question that we may propose about the behaviour of the physical system, nevertheless we have an uncomfortable feeling that we have lost something. (16)

Professor Bridgman is correct in maintaining that this recent change in physical theory represents significant progress. As a matter of fact, the identification of scientific explanation with the construction of mechanical

models, such as is found in the writings of Lord Kelvin, and the classical physicist's insistence upon physical models as the criterion of the value of theories, make the intellect the slave of the imagination. Moreover, they destroy the true notion of science, since they seem to make the sensible as such the formal object of science. In a word, they amount to a confusion of the material and the formal object of science.

It is true that this tendency to explain reality in terms of physical and mechanical models reveals a trait that is native to the mind in the sense that it is natural to man to want to reduce the unfamiliar to the level of the common and the familiar. But to tie science down to this type of explanation can only result in creating insurmountable obstacles in the path of progress. For reality is infinitely richer than any fixed frames that derive from ordinary experience. Moreover it is presuming a great deal to expect to find in familiar molar experience counterparts of microscopic reality. Scientists are coming to realize this more clearly every day, especially in the field of wave mechanics, and the work of Dirac, Schrodinger, etc. has put particular emphasis upon this point. But the most important aspect of this question is that true progress in

science, as we saw in Chapter IV, does not consist in transforming things into what is most knowable for us, but in approaching closer and closer to what is most knowable in se, though least knowable for us. In other words, it does not consist in imposing our measure upon reality, but in allowing reality impose its measure upon us. And if it becomes necessary to have recourse to art, the only reason is, as we have seen, to open up reality more and more as an object.

But in this question it is not necessary to be a purist. The remark made by Dirac to Schrodinger "Beware of forming models or pictures at all," must not be taken too literally. ⁽¹⁷⁾ Even though physics has recently taken a very definite step in asserting its emancipation from physical models, it is doubtful that this emancipation will ever be complete, or even that such a complete emancipation would be desirable. Imaginative construction inescapably accompanies intellectual activity. Moreover, this imaginative construction may often prove useful for the physicist, as Professor Bridgman has pointed out:

I think that the ordinary physicist will want to keep his physical models as long as he can.. Unless one has supreme power as a mathematician, one may well find it useful to have at his command methods of reasoning by analogy that

will give him an insight into the nature of the solution of special problems, and one may cheer from the sidelines any attempt to invent combinations of the elements of the mathematical analysis which may be handled somewhat like the elements of ordinary experience, and of which we may hope ultimately to acquire a more intuitive command. I suspect that Bohr's attempt to find a dualistic aspect of nature is an attempt of this sort.(18)

Even mathematical conceptualization is necessarily tied up with the imagination, as we saw in Chapter VI. The imaginative construction which accompanies this conceptualization, while on the one hand less free than that found in metaphysical knowledge, is freer and less determined than that found in physical knowledge. In mathematical theory it is of little importance what the nature of the imaginative construction is, provided that it prove useful and that it remain in continuity with the measure numbers out of which the theory is evolved.

Consequently, the physicist is free to employ any physical models that may prove helpful to him, provided he remain critically conscious of their true significance. He is free to conceive of light in terms of "waves" or "corpuscles" or both, provided he does not allow himself to slip into the delusion of thinking that the ontological nature of light is actually like waves of water or like

tiny pellets. The most important function that these models can play is to provide suggestive sources of mathematical manipulations and an imaginative support which will aid the mind in coordinating experimentally observed relations.

The fruitfulness of Bohr's theory of the structure of the atom did not consist so much in the planet-like circulation of electrons around a nucleus as in the fact that this structure provided a basis for mathematical speculation. By considering seven electrons circulating in one atom and eight electrons in another, one is enabled in some way to seize upon the difference between nitrogen and oxygen. (19)

In La Science et l'Hypothèse Poincaré has brought out the true function played by models in physical theory and showed that they are essentially transitory while the mathematical relations which they suggest constitute the essential and permanent part of physical theory:

... ces équations expriment des rapports et, si les équations restent vraies, c'est que ces rapports conservent leur réalité. Elles nous apprennent, après comme avant, qu'il y a tel rapport entre quelque chose et quelque autre chose; seulement, ce quelque chose nous l'appelions autrefois mouvement, nous l'appelons maintenant courant électrique. Mais ces appellations n'étaient que des images substituées aux objets réels que la nature nous cachera éternellement. Les rapports véritables entre ces objets réels sont la seule réalité que nous puissions atteindre, et la seule condition,

c'est qu'il y ait les mêmes rapports entre ces objets qu'entre les images que nous sommes forcés de mettre à leur place. Si ces rapports nous sont connus qu'importe si nous jugeons commode de remplacer une image par une autre. (20)

And he goes on to explain that the scientist may employ models that are mutually contradictory:

Il peut se faire qu'elles expriment l'une et l'autre des rapports vrais et qu'il n'y ait de contradiction que dans les images dont nous avons habillé la réalité. Les hypothèses de ce genre n'ont donc qu'un sens métaphorique. Le savant ne doit pas plus se les interdire que le poète ne s'interdit les métaphores; mais il doit savoir ce qu'elles valent. (21)

We believe that this view of the meaning of scientific models is correct and that it fits in perfectly with the Thomistic doctrine of the nature of mathematical physics. For in a science which is formally mathematical and terminative physical, the explanatory constructions will be essentially mathematical. It will not be necessary that in these constructions there be physical re-embodiments of nature. All this is required is that the mathematical constructions be in the end verifiable in physical experiment.

But, to return to our original question: is a mathematical theory an explanation? Professor Bridgman,

after noting that the emancipation from physical models gives us an uncomfortable feeling that we have lost something, goes on to say: "I think that we discover on analysis that it is the explanation which we feel we have lost". It is certain that a mathematical theory is not an explanation in the sense of a reduction to familiar experience, nor does it provide an explanation of the type that philosophy affords. That is to say, the purpose of physical theory is not to give us the real foundation of the laws, but a logical foundation. For theories are mental constructs, and it must be kept in mind that mathematical physics is dialectics. Nevertheless, we feel that a physical theory may be called an explanation in a true sense of the term.

Qu'est-ce donc qu'expliquer? C'est tout uniquement faire rentrer un fait dans une forme. Le fait est expliqué lorsqu'il apparaît identique à l'un des phénomènes qu'engendre un de ces sorites indéfinis que nous appelons théorie ou forme.(22)

Physical theory provides an explanation of reality in the sense of making it deducible and thus rational. It is an explanation in the line of formal causality, even though it is not a question of the proper ontological formal cause that is found in nature. It is a mere substitute formal causality - - and never more than provisional. Nevertheless by means of it mathematical physics

truly achieves the aim of subalternation.

And it must be pointed out that the emancipation from physical models of which we have been speaking does not in any sense dissolve the intimate union between mathematics and physics that subalternation implies. To the question: what is the theory of Maxwell, Hertz is supposed to have replied: "The Theory of Maxwell is Maxwell's system of equations." (23) And Poincaré writes: "Une loi pour nous ... en un mot, c'est une équation différentielle." (24) There is obviously a sense in which these expressions are correct. And yet it would be false to suppose that Maxwell's Theory or any other theory in physics consisted merely of mathematical equations and nothing more. In so far as science remains materially physical, there must be a link binding these mathematical equations with physical reality - even when the bridge constituted by a physical model has been removed. This link is provided by what is known as a text or a dictionary. This text reveals the physical significance of the mathematical equations and shows how these equations are to be used in order for that significance to be maintained. For example, the formula $s = \frac{1}{2}gt^2 + v_0t$ has no physical significance unless it be accompanied by a dictionary which explains that it is the formula for falling

bodies and that the symbols s, g, t, v refer to distance, gravitational attraction, time and original velocity, or, to be more accurate, to sets of concrete measuring operations whose resultant measure-numbers represent the properties of distance, attraction, etc.. To say that this is the equation for falling bodies means that the numbers obtained by the concrete processes of measurement determined by the text satisfy the equation when they are substituted into it. The text determines not only the nature of the measurements involved, but also the precise connection between the various symbols used in the equation. If for example the time and the distance must be obtained by simultaneous measurements, this must be specified by the text. It is clear that in the dictionary we shall find the way in which the multiplicity of individual measures are reduced to the fundamental measures and how particular measures, such as that of temperature, for example, become absorbed by the theory and lose themselves, so to speak, in combinations of the basic measures of length, time and mass.

It is easy to lose sight of the importance of the dictionary in physical theory. And yet its function is essential, for it maintains the intimate union between the mathematics and the physics. It is precisely by means

of the text that the mathematical physicist is able to keep in mind that what he is dealing with directly is a physical element, and that the mathematical element enters into his object only by way of connotation. To quote Bridgman once again:

It appears, therefore, that a complete mathematical formulation requires equations plus text, and the text may perform a variety of functions. The necessity for a text is almost always overlooked, but I think it must be recognized to be essential, and a study of what it must contain is as necessary for an adequate conception of the nature of the mathematical theory as is the study of the equations themselves. One of the functions of the text, we have seen, is to tell us how to set up the correspondence between the numbers given by the equation and the numbers obtained by manipulations of the physical system. The text cannot tell us what it is that the correspondence is to be set up with without going outside the system of the mathematical theory and assuming an intuitive knowledge of the language of ordinary experience. In classical mechanics, the geometrical variables in the equations of motion are the coordinates of massive particles, but unless we know intuitively what a massive particle is, we simply cannot make connection with equation or theory. Not only is the theory powerless to describe, either in text or equations, what the elements are to which correspondences are to be made, but all the more is it powerless to explain why the elements have the properties that they do. (25)

The truly great physicist never allows the symbolism of mathematics to make him lose intimate contact with physical reality. Of Einstein Langevin could write:

Pour lui jamais le voile du symbole ne masque la réalité. Nombreux sont les esprits pour lesquels le signe cache souvent la chose signifiée; Einstein se meut à l'aise dans le monde des symboles, mais jamais ceux-ci ne lui dissimulent l'aspect physique des choses. (26)

But this union between mathematical construction and physical reality must be correctly understood. In the simple example cited above of the formula for falling bodies there is a one to one correspondence between the mathematical symbols and operationally defined physical properties. Must we expect this same correspondence to be found in all mathematical formulations and throughout the whole of physical theory? Such an expectation would misconstrue the proper function of mathematics in physics and would impose sterilizing restrictions upon the theoretical power of mathematical construction. (27)

There is no reason why each symbol in the mathematical equations, nor even each step in the structure of mathematical theory, should have a definite counterpart in the physical system. Nor is it necessary that all of the operations performed by the mathematician in his interpretation of nature should have a physical meaning, or that all of the quantities manipulated be accessible to experience.

It is true that all physico-mathematical theories must originate in measure-numbers produced by physical processes and must ultimately terminate in formulas which have direct physical relevance and which correspond to concrete measure-numbers. But between this point of departure and this terminus the theoretical physicist is free to create any auxiliary mathematical quantities which will help him to carry forward his task, even those whose realization in nature would involve a contradiction. Nor is there any contradiction in maintaining that fictitious entities can make a positive contribution to the explanation of reality. It is well known how the fictitious constructs of the Theory of Relativity both provided an explanation for phenomena previously inexplicable, such as the anomaly of Mercury, and led to the new discovery of the deviation of luminous rays in the neighborhood of the sun. And if pure logical entities and fictitious constructs can be efficaciously used to solve practical problems, as in the rather well-known case of Steinmetz's use of the mathematical surd, $\sqrt{-1}$, to solve the problem of getting electrical locomotives over the Continental Divide, they can a fortiori serve as efficacious explanatory devices to solve theoretical problems.

(28)

Modern physics has exercised wide freedom in this regard. It has felt free to push the theoretical power and the creativity of mathematics to the limit, provided only that in the end there result formulae that can be given a physical meaning. Weyl has claimed for physics the right to make use of every possible resource no matter how strange the results may appear. (29) In this connection Eddington writes:

The pure mathematician, at first called in as a servant, presently likes to assert himself as master; the connexus of mathematical propositions becomes for him the main subject, and he does not ask permission from Nature when he wishes to vary or generalise the original premises. Thus he can arrive at a geometry unhampered by any restriction from actual space measures; a potential theory unhampered by any question as to how gravitational and electrical potentials really behave; a hydrodynamics of perfect fluids doing things which it would be contrary to the nature of any material fluid to do. (30)

We see in this exercise of freedom a confirmation of the Aristotelian and Thomistic interpretation of mathematical physics as opposed to that of the ancient and neo-Pythagoreans. As Bridgman has remarked, the feeling that all the steps in the structure of mathematical theory must have their counterpart in physical reality derives from the Pythagorean belief that the mathematical interpretation of nature means a discovery of mathematics in nature, which (31)

is in the last analysis a mathematical construction. In the doctrine of Aristotle and St. Thomas the mathematical world is extrinsic to the physical world (in the sense already explained) and consequently the use of mathematics in the study of the physical world is not a discovery; it is an application. As a result the theorist in making this extrinsic application is granted all of the freedom that is native to the world of mathematics. It took the genius of Einstein to fully realize that geometrical conceptions must be manipulated with the utmost freedom in order to provide an explanation of physical phenomena.

The following lines of Cassirer are relevant here:

For it is precisely the complex mathematical concepts, such as possess no possibility of direct sensuous realization that are continually used in the construction of mechanics and physics. Conceptions, which are completely alien to intuition in their origin and logical properties, and transcend it in principle, lead to fruitful applications within intuition. This relation finds its most pregnant expression in the analysis of the infinite, yet is not limited to the latter.(32)

This brings us to the mooted question of the geometrical structure of "real" space. It is a question that has been rendered obscure by the ambiguity of the terms employed. As a matter of fact, the word "real" can have

more than one meaning. For the physicist, if he so desires, is entitled to consider a space as "real" when the geometry to which it corresponds provides the greatest theoretical power in explaining (in the sense determined above) the concrete measure-numbers derived by actual experimentation with the physical world, and has the greatest success in synthesizing in an exact, simple, coherent and complete fashion all of the experimental laws. The only meaning that reality can have for the mathematical physicist is the numbers that are the results of concrete measuring processes. That is why the geometry that best "explains" these results is for him the geometry of reality.

When the question is understood in this sense, it is clear that no particular type of space, and no particular system of geometry is privileged. Any geometry which at a given stage in the development of physics provides the greatest explanatory power for all of the discoveries that have been made up to that point may be considered to be the geometry of real space. And just as soon as any other system of geometry provides greater explanatory power or is better able to meet the problems arising from newly discovered phenomena, it must supplant its predecessor and become the geometry of "real" space. In this sense it is perfectly

legitimate to say that "real" space is spherical or elliptical, or that the geometry of nature is not Euclidian but Reizmanian.

But for the philosopher, the geometry of "real" space is not merely the geometry which best "saves" a collection of measure-numbers. It is the geometry which is realized (though not of course in the abstract state proper to the mathematical world) in the quantity of the objective world condition. There is a vast difference between this objective realization and an explanatory saving of a collection measure-numbers, and that is something that more than one modern scientist and philosopher of science have overlooked. Theoretical continuity between a geometrical system and a collection of measure-numbers does not constitute an experimental proof of the objective character of that system. In fact, it seems necessary to insist that as long as it remains true to its proper method mathematical physics can neither prove nor disprove that the absolute world condition is either Euclidian or non-Euclidian. Nor does the theoretical continuity just mentioned prove as some contemporary scientists have claimed, that the distinction between geometry and physics has been wiped out, in such a way that

the former must be considered an experimental science.

If the ability of a mathematical system to provide numerical values which coincide with those derived from physical measurement were a sufficient proof of the ontological character of the space proper to that system, then any and all fictitious constructs which could be put into continuity with measure numbers would have to have objective existence. Some modern scientists seem to have recognized this fact and have consequently felt the necessity of attempting to establish the possibility of some connection between non-Euclidian space and sensory perception. The results of these attempts have only served to show their (33) utter futility. Sir James Jeans, for example, while admitting the obvious difficulty encountered in trying to imagine "spherical space" believes that this difficulty derives merely from its unfamiliarity. He holds that our intuitive belief that space is Euclidian is similar to the "common sense" belief that the earth is flat, and compares the difficulty of imagining non-Euclidian space with the difficulty that a child has in imagining people existing (34) on the other side of the earth without falling off. A moment's reflection will show that there is no parity between these two cases. It is possible for the imagination

to cope with the sphericity of the earth, but it is utterly impossible for it to cope with the concepts of non-Euclidian space.

Consequently, when "real space" is understood in the philosophical sense of the term it becomes necessary to say that the geometry proper to it can be nothing but Euclidian. The modern non-Euclidian geometries are purely dialectical structures and that they cannot be applied to real quantity without a contradiction. Only the entities of Euclidian geometry are capable of construction in the imaginative intuition, and this capability is necessary for realization in the objective world, since this realization means to exist with sensible existence. The entities of non-Euclidian geometry require Euclidian geometry as a foundation of their conceptual existence; consequently, their objective existence would involve a contradiction since it would deprive them of this foundation. For this reason the non-Euclidian constructions which have proved so fruitful in modern physical theory cannot be considered to have actual physical counterparts in nature; they must be looked upon not as something which directly reveal the quantitative nature of the objective world, but as pure geometrical symbols of this objective world. And the same

must be said, mutatis mutandis, of the mathematical constructions of Quantum physics: they are pure mathematical symbols, which, without possessing any direct physical counterparts in the objective world, provide the best theoretical schema to explain and synthesize the results of our measuring processes.

It must be pointed out however that there is a sense in which the mathematical constructions which constitute modern physical theory have objective significance. Though without direct physical counterpart, they do, nevertheless, succeed in a certain fashion in seizing upon the structure of the objective world. By providing an intelligible scheme of relationship which establishes continuous connection between the members of the manifold which constitutes nature, they succeed in reflecting the interrelatedness of cosmic reality and the harmonious order that prevails in it. Were this not so, the value of modern science would be extremely dubious. For it would have gained very little for having condemned decadent Scholasticism for transforming facts into mere names, if in the end it resulted in nothing more than the transformation of facts into symbols. As a matter of fact, however, the sacrifice which mathematization has imposed upon it of renouncing the inner natures of things

is repaid by a reflection of the all-inclusive structure of nature.

All that the 'thing' of the popular view of the world loses in properties it gains in relations; for it no longer remains isolated and dependent on itself alone, but is connected inseparably by logical threads with the totality of experience. Each particular concept is, as it were, one of these threads, on which we string real experiences and connect them with future possible experiences. (35)

By reducing nature's manifold to a rational unity through relatedness in a number system, mathematical physics provides a quasi solution for the problem of the one and the many. By creating an order of pure homogeneous relatedness it affords a quasi sapiential view of the universe which enables the mind to derive the manifold from the one, even though the one be a pure substitute and the manifold be reached only in its purely material and numerical diversity and not in its proper specific nature. This explains why it is so easy for the mind to mistake mathematical physics for true wisdom.

All this is a great achievement. But it is paid for with a great price. Perhaps nowhere does the adage traddutore - - traditore obtain with greater force than in the mathematical "translation" of physical science. And that is what we must now try to see by considering

the transformation that this translation produces in the reflection of nature that is found in physical science.

2. The Transformation of Nature.

It would be virtually an endless task to attempt to bring out in full detail the profound metamorphosis that the mathematization of physics produces in the scientific view of nature, and we must limit ourselves to touching briefly upon a few of the most characteristic and significant points. And a moment's reflection will suggest that the pivotal point of this whole transformation is found in the concept of motion, which, as St. Thomas says, is, so to speak, the very "life" of the world. In Chapter II we went to considerable lengths to show that physical science is essentially a study of mobility. For nature is necessarily defined in terms of motion (37) and that is why Aristotle could say that he who is ignorant of motion is ignorant of nature. (38) On the other hand, we explained in Chapter VI that mathematics essentially excludes motion. The mathematical world is a world of immobility. It is true that mathematicians speak of a kind of motion, but as we pointed out, this motion is only a dialectical, imaginary, intramental thing, which does

not involve true becoming. As we mentioned in Chapter I, this opposition between the mobility of nature and the immobility of mathematics was traditionally one of the stumbling blocks for those who wished to mathematicize the cosmos, and provided one of the bases for Aristotle's criticisms of the Platonists and the Pythagoreans. We must now try to see in what sense mathematics may be applied to motion and what is the effect of this application.

Aristotle and St. Thomas explain that motion may be considered under two different aspects. In the first place, it may be considered in its proper and specific essence, and in this sense it signifies a coming into being. Considered in this way, it is something profoundly obscure, since it lacks the determination and actuality of being. Consequently, it can be correctly defined only in a way which will bring out this profound obscurity. Aristotle has given us the essential definition of motion in the third book of the Physics: the act of a thing in potency in so far as it is in potency. This coming into being is realized both in the substantial and in the accidental order, and in the latter case (which is the strictest meaning of the Thomistic term "motus") it is found in the three categories of quantity (growth in living beings), quality and place

(local motion). All substantial change involves accidental changes, and motion in the predicaments of quantity and quality always involve local motion of some sort. ⁽³⁹⁾ This, in a sense all motion may be reduced to local motion. This kind of motion is the most superficial and the one which realizes the least the concept of becoming. It involves essentially an extrinsic denomination. To say, therefore, that all the motion in the universe may be reduced to local motion, is to say that it may be reduced to a system of extrinsic relations.

The second aspect under which motion may be considered is brought out by St. Thomas in his Commentary on the Fifth Book of the Metaphysics. ⁽⁴⁰⁾ In analyzing the notion of quantity, he tells us that there are various kinds of quantitative modes. Some things are quantitative per se, such as "line", others are quantitative per accidens. Among those which are quantitative per accidens some are such by the fact that they are accidents inhering in a quantified subject; others however are quantitative by the fact that they are divisible according to quantity. In this category are found motion and time. St. Thomas writes:

Alio modo dicuntur aliqua quanta per accidens non ratione subjecti, in quo sunt, sed eo quod dividuntur secundum quantitatem ad divisionem alicuius quantitatis; sicut motus et tempus, quae

dicuntur quaedam quanta et continua, propterea quod ea, quorum sunt, sunt divisibilia et ipsa dividuntur ad divisionem eorum. Tempus enim est divisibile et continuum propter motum; motus autem propter magnitudinem; non quidem propter magnitudinem eius quod movetur, sed propter magnitudinem eius in quo aliquid movetur. Et eo enim quod illa magnitudo est quanta, et motus est quantus. Et propter hoc quod motus est quantus, sequitur tempus esse quantum. Unde haec non solum per accidens quantitates dici possunt, sed magis per posterius, in quantum quantitatis divisionem ab aliquo priori sortiuntur. (41)

In his Commentary on the De Trinitate he shows how this quantitative aspect makes it possible for mathematics to enter into the study of motion:

Ad quintum dicendum, quod motus secundum naturam suam non pertinet ad genus quantitatis, sed participat aliquid de natura quantitatis aliunde, secundum quod divisio motus sumitur ex divisione spatii vel ex divisione mobilis; et ideo considerare motum non pertinet ad mathematicum, sed tamen principia mathematica ad motum applicari possunt: et ideo secundum hoc quod principia quantitatis ad motum applicantur, naturalis considerare debet de divisione et continui, et motus, ut patet in VI Physicorum. Et in scientiis mediis inter mathematicam et naturalem tractatur de mensuris motuum, sicut in scientiis de sphaera mota, et in astrologia. (42)

These distinctions make it clear how it becomes possible for mathematics to be applied to the motion in the universe. By reducing all motion to local motion or movement in space, by considering this local motion not as a

coming into being but as a pure extrinsic relation, and by considering this extrinsic relation purely in terms of its quantitative aspect, mathematics is able in some way to seize upon motion. But in doing so it transforms it into the only sense in which it can have meaning for a mathematician -- the simple variation of the relations of a point with coordinated axes. And thus in mathematical physics movement becomes nothing more than a variation of spatial relationship between two or more bodies which remain intrinsically unchanged. Lenzen, for example defines it as a "change of position in space with time."⁽⁴³⁾ A continual series of spatial points are united with a continuous series of temporal points and the four dimensional curve which results becomes the model of motion.

It should be immediately evident that such a notion of motion empties it of its proper physical essence. It is no longer a true change, but a mere displacement of a point, no longer a process but a relation, no longer a becoming, but a state which has a certain determined value⁽⁴⁴⁾ that can be measured.

Things do not come into existence at a certain place and at a certain instant of time -- they simply exist at a certain point in a continuum.

That physico-mathematical motion is emptied of all becoming is clearly brought out by Sir Arthur Eddington:

Events do not happen; they are just there, and we come across them. "The formality of taking place" is merely the indication that the observer has on his voyage of exploration passed into the absolute future of the event in question; and it has no important significance.(45)

It is clear, then, that there is no true becoming in the physico-mathematical world, and consequently no rest. (46) A good analogy of the difference between the physico-mathematical world and the real world may be found in the difference between a piece of music played by a symphony orchestra and a record made of the piece. There is something on the static record to correspond to all the movements and nuances of the piece, but the movements and nuances themselves have been lost. They have all been spatialized.

Because mathematatized motion is not a coming into being but a pure relation, it is perfectly reciprocal. That is why Descartes who had identified real motion with mathematicized motion could say that it is perfectly indifferent whether we say that we are moving towards a goal or that the goal is moving towards us, since in both cases the variation of the relations of distance remains exactly the same. (47)

It is easy to see what has happened in this mathematization of motion. Nothing is so irrational, so refractory to the intellect as potentiality. That is why the mind in its attempt to rationalize the universe as completely as possible is inevitably led to the attempt to wipe out potentiality and to reduce everything to the plane of actuality. From this point of view Bergson is correct in maintaining that experimental science deals only with the "tout fait."⁽⁴⁸⁾ But in wiping out potentiality it destroys all true mobility. It thus succeeds in explaining nature only at the expense of destroying it. It reduces motion to something that is perfectly clear and intelligible, but in so doing it sacrifices its very essence, for motion, as we said, is something essentially obscure. That is why mechanism taken as a philosophy of nature involves an intrinsic contradiction. For in attempting to give an adequate account of reality by means of motion and extension it empties motion itself of its reality. It was because Descartes failed to realize that his mathematicized motion was not true motion that he heaped such supercilious scorn upon Aristotle's definition:

*At vero nonne videntur illi verba magica proferre,
quae vim habeant occultam supra captum humani
ingenii, qui dicunt motum, rem unicuique notissiman,
esse actum entis in potentia, prout est in potentia?*

quis enim intelligit haec verba? quis ignorat quid sit motus? et quis non fateatur illos nodum in scirpo quaesivisse? Dicendum est igitur, nullis unquam definitionibus eiusmodi res esse explicandas, ne loco simplicium compositas apprehendamus; sed illas tantum, ab aliis omnibus secretas, attente ab unoquoque et pro lumine ingenii sui esse intuendas. (49)

It is to be noted that Descartes did not say: "quis ignorat quod sit motus," but "quid sit motus?" For Aristotle the existence of motion was perfectly clear; it had all the clarity of a direct intuition. Descartes thought that this perfect clarity of direct intuition could be extended to the very essence of motion. That is why to his question: "quis ignorat quid sit motus?" one is justified in answering: "Descartes." Pasteur's dictum: "Je plains les gens qui n'ont que des idées claires", is especially applicable to the realm of Nature where things are essentially obscure. (50)

All this helps us to understand the solution to the antinomy mentioned in Chapter I between the ancient and modern concepts of motion. For Aristotle, as we saw, it was evident that the continuance of a body in motion demanded a cause and without this cause the body would come to rest. For Descartes, on the other hand, the principle of inertia was perfectly evident, and according (51)

to this principle the cessation of the motion of a body demands a cause, and without this cause the motion will continue ad indefinitum. The enigma of this striking paradox immediately vanishes when we call to mind that Aristotle and Descartes are talking about two different things. For Aristotle motion means a coming into being, and since nothing can bring itself into being, there must be a cause to explain the process of becoming: quidquid movetur ab alio movetur. For Descartes motion is a state, that is to say a kind of entity which will retain its existence until robbed of it by some cause. The principle of inertia has to do with mathematicized motion, that is to say with a motion that is infinitely uniform and rectilinear. This principle does not in any way involve the falsity of Aristotle's notion of motion. They belong to two different orders. Aristotle made no attempt to treat the mathematical aspect of local motion. It is extremely important to keep in mind that this mathematization is not a substitute for Aristotle's definition; it is a passing to an entirely different order. All too many historians make the mistake of treating Aristotle's Physics as though he were attempting to write a treatise on mathematical physics.

This question has an important corollary in the problem of prima via in St. Thomas' demonstration of the existence of God. In this demonstration motion is considered as a becoming and not as a state. And that is why it makes no sense to say that the argument is disproved by the principle of inertia. Obviously, if motion is conceived as a state there is no need to have recourse to an actio to explain it. This shows that the mathematization of the cosmos has a profound effect upon the problem of causality in the universe. But before turning to this question we must consider in a summary way how this mathematization effects a notion that is intimately connected with that of motion, namely time: "tempus habet fundamentum in motu."⁽⁵²⁾

Contemporary Scholastics have insisted upon the difference between Aristotelian time and Einsteinian time to the extent of denying that they have anything more in common than the name.⁽⁵³⁾ They have furthermore claimed that what Einstein has to say about the impossibility of simultaneity at a distance has nothing to do with the time of which Aristotle speaks. We feel that this is extremely ambiguous. For the term "time" does not always have exactly the same meaning in Thomistic terminology.

In the first place, it signifies the duration of mobile beings, that is to say, the persistence in existence of beings whose existence is successive. But the "time" which Aristotle defines in the fourth book of the Physics does not exactly coincide with this primary notion, although it is essentially connected with it. For by defining time as the measure of motion according to a relation of priority and posteriority he makes it clear that he is speaking of an extrinsic determination of this duration in relation to a chosen standard, that is to say, of a measurement of this duration. Consequently, in so far as both Aristotelian time thus defined and Einsteinian time have to do with measurement they coincide. And we believe that what Einstein has to say about the impossibility of simultaneity at a distance applies to the time defined by Aristotle in the Physics. For we know of no way in which the measure of motion according to a relation of before and after can be determined so that distant events can be fixed as simultaneous. Of course, in so far as time is successive duration there is such a thing objectively as distant simultaneity even though that simultaneity cannot be determined by us.

But it would be illegitimate to conclude from

this that the time defined by Aristotle in the Physics is same as the time of which Einstein speaks. For in the time of the Physics the notion of true physical motion is involved. Consequently, this time can truly be said to "flow" from past to future. In Relativity physics, on the contrary, the notion of motion has been emptied of its proper physical meaning. There is no true process, no becoming. Consequently, Einsteinian time does not really flow; it is a mere dimension. (54)
It is studied in terms of geometry.

But even before the advent of the Theory of Relativity the notion of time had already undergone a profound transformation by the mathematization of nature. (55) We have already spoken of the symmetry of mathematical equations. The processes of classical dynamics are reversible, that is to say, if the velocities of the particles of a system should at any given moment be reversed the motion would proceed in accordance with the same equations in the reverse direction. In so far as the notion of time is concerned, this means that the equations of classical dynamics make no distinction between the positive and negative directions along the time axes. Professor Cunningham does not hesitate to say that in so far as time is determined mechanically, past and future are interchangeable. (56) And Lindsay and Morgenau

write: "If equations predict future events they predict past ones as well. (57) Of course the physicist in order to discover "time's" arrow may have recourse to entropy-gradient, but even then the irreversibility is only highly improbable (58) and never absolutely impossible.

In Relativity physics the mathematical transformation of the notion of time becomes complete. It is assimilated to the notion of space - united with space as a dimension in the four dimensional continuum called space-time which, as we saw in the last Chapter, may be cut up in different ways according to the position and velocity of the individual observer. Time then becomes "the totality (59) of possibilities of relative temporal position of events."

To quote Eddington once again:

In the four-dimensional world ... the events past and future lie spread out before us as in a map. The events are there in their proper spatial and temporal relation; but there is no indication that they undergo what has been described as 'the formality' of taking place,' and the question of their doing or undoing does not arise. We see in the map the path from past to future or from future to past; but there is no sign-board to indicate that it is a one-way street. Something must be added to the geometrical conceptions comprised in Minkowski's world before it becomes a complete picture of the world as we know it. We may appeal to consciousness to suffuse the whole - - to turn existence into happening, being into becoming. But first

let us note that the picture as it stands is entirely adequate to represent those primary laws of Nature which, as we have seen, are indifferent to a direction of time. (60)

In this spatialization of time, mathematical physics has achieved the goal at which it has aimed from the beginning -- the transformation of all sensuous and intuitive heterogeneity into pure homogeneity. The first step in this transformation was the homogenization which gradually emptied external experience of its proper and specific content. But even when this had been accomplished there still remained untouched the "form of the inner sense" -- the process of duration which is so intimately connected with internal experience. Through the spatialization of time this last barrier of specific experiential content was broken down. Speaking of this transformation Cassirer writes:

This transformation of the time-value into an imaginary numerical value seems to annihilate all the 'reality' and qualitative determinateness, which time possesses as the 'form of the inner sense', as the form of immediate experience. The stream of process, which, psychologically, constitutes consciousness and distinguishes it as such, stands still; it has passed into the absolute rigidity of a mathematical cosmic formula. There remains in this formula nothing of that form of time, which belongs to all our experience as such and enters as an inseparable and necessary factor into all its content. But, paradoxical as this result seems from the

standpoint of this experience, it expresses only the course of mathematical and physical objectification, for, to estimate it correctly from the epistemological standpoint, we must understand it not in its mere result, but as a process, a method. In the resolution of subjectively experienced qualities into pure objective numerical determinations, mathematical physics is bound to no fixed limit. It must go its way to the end; it can stop before no form of consciousness no matter how original and fundamental; for it is precisely its specific cognitive task to translate everything enumerable into pure number, all quality into quantity, all particular forms into a universal order and it only 'conceives' them scientifically by virtue of this transformation. Philosophy would seek in vain to bid this tendency halt at any point and to declare ne plus ultra. The task of philosophy must rather be limited to recognizing fully the logical meaning of the mathematical and physical concept of objectivity and thereby conceiving this meaning in its logical limitedness. (61)

Once again it is important to recognize this spatialization of time as an attempt of the mind to triumph over its greatest enemy: potentiality. Designated points in space are all actual, and when time is homogenized with space, t_1 , t_2 , t_3 , etc. become but a series of actual "nows". Perhaps it is legitimate to see in this spatialization of time a striving of the human intellect towards the duration of perfect actuality that is proper to pure
(62)
Intellect. But this attempt only results in the destruction of time:

... si le devenir doit se transformer en être (selon M. Einstein), au point que l'acte de se produire, pour un événement, devient une simple formalité dénuée d'importance (selon M. Eddington), si la succession n'est qu'une illusion (selon M.H. Marais) et si tout système physique constitue une entité privée de changement (selon M. Cunningham), cela ne peut signifier qu'une chose: l'abolition et la disparition du temps. Aussi M. Cunningham n'hésite-t-il point à parler de l'univers non-temporel de Minkowski. (63)

The destruction of mobility in the universe has many far-reaching consequences, but perhaps the most significant from the point of view of science, which is a knowledge of things in their causes, is its effect upon causality. In the second book of the Physics Aristotle and St. Thomas place considerable emphasis upon the fact that the science of nature must study its object from the point of view of all of the four fundamental types of causality: efficient, final, formal and material.

Dicit ergo primo quod cum quatuor sint causae, sicut supra dictum est, ad naturalem pertinet et omnes cognoscere et per omnes naturaliter demonstrare, reducendo quaestionem propter quid in quamlibet dictarum quatuor causarum, scilicet formam, moventem, finem et materiam. (64)

The reason for this is fairly obvious: there is an analytical connection between mobility, the formal object of the science of nature, and quadruple causality.

Necesse est autem quatuor esse causas. quia cum causa sit ad quam sequitur esse alterius, esse eius quod habet causam, potest considerari dupliciter: uno modo absolute, et sic causa essendi est forma per quam aliquid est in actu; alio modo secundum quod de potentia ente fit actu ens. Et quia omne quod est in potentia, reducitur ad actum per id quod est actu ens; ex hoc necesse est esse duas alias causas, scilicet materiam, et agentem qui reducit materiam de potentia in actum. Actio autem agentis ad aliquid determinatum tendit, sicut ab aliquo determinato principio procedit: nam omne agens agit quod est sibi conveniens: id autem ad quod tendit actio agentis, dicitur causa finalis. Sic igitur necesse est esse causas quatuor. Sed quia forma est causa essendi absolute, aliae vero tres sunt causae essendi secundum quod aliquid accipit; inde est quod in immobilibus non considerantur aliae tres causae, sed solum causa formalis. (65)

The last lines of this passage throw great light upon the effect that the mathematical transformation of physics has upon causality. The student of nature as long as he stays within his own field is bound to reduce natural phenomena to all of their four causes: "In naturalibus redendum est propter quid penitus" (66) But unable to discover any universal and necessary propter quid for experimental propositions he is forced to have recourse to mathematics. Since mathematics, however, is a world of immobility, the only type of propter quid he can borrow from it is the unique type that is proper to it: propter quid in the line of formal causality.

In a passage immediately preceding the one just quoted St. Thomas gives an example of what he means by formal causality:

Quandoque enim propter quid reducitur ultimo in quod quid est, idest in definitionem, ut patet in omnibus immobilibus, sicut sunt mathematica; in quibus propter quid reducitur ad definitionem recti vel commensurati vel alicuius alterius quod demonstratur in mathematicis. Cum enim definitio recti anguli sit, quod constituatur ex linea super aliam cadente, quae ex utraque parte faciat duos angulos aequales; si quaeratur propter quid iste angulus sit rectus, respondetur quia constituitur ex linea faciente duos angulos aequales ex utraque parte; et ita est in aliis. (67)

It is clear that the only type of causality that can be found in mathematical physics is a kind of formal causality consisting in an expression of the metric coherence of phenomena. This metric coherence constitutes what is known as the causal structure of world occurrences. It is true that physicists may speak in terms which seem to indicate other types of causality. They may for example, use the expression "efficient causality", but in doing so they merely refer to a relation between the states of physical systems at different points of time, which are connected in such a way that, given the determination of the state of the system at any one point of time, its state at any designated future point of time can be logically deduced. (68) St. Thomas brings out the incompetence of

mathematics in the field of efficient causality:

Mathematica accipiuntur ut abstracta secundum rationem, cum tamen non sint abstracta secundum esse. Unicuique autem competit habere causam agentem, secundum quod habet esse. Licet igitur ea, quae sunt mathematica, habeant causam agentem; non tamen secundum habitudinem, quam habent ad causam agentem, cadunt sub consideratione mathematici. Et ideo in scientiis mathematicis non demonstratur aliquid per causam agentem. (69)

In pre-Relativity physics the mathematization of the cosmos had already resulted in the disappearance of true efficiency from the concept of efficient causality, but in Relativity physics this effacement is made even more complete. For now, the concept of force, for example, is completely absorbed into a system of determinations bound together by mathematical relations implemented by the differential and tensorial calculus, etc.

In somewhat the same way, physicists often speak of matter, but their matter is far from being the material cause of which Aristotle speaks. It is something that is completely actual and not a potential principle of becoming. In fact, in Relativity physics matter becomes so formalized that it is absorbed into isotropic space. On the other hand it must be noted that if matter is formalized, it is also true to say that the formal cause is materialized.

That is to say, the formal cause that is treated of in mathematical physics is not the proper specific formal cause which reveals the nature of things in their heterogeneous interiority, but a homogeneized formal cause of spatial relations. (70)

In insisting upon the necessity of studying nature in terms of all four causes, Aristotle and St. Thomas place special emphasis upon the importance of final cause. "Et haec species causae potissima est inter alias causas: est enim causa finalis aliarum causarum causa." (71) In fact, after explaining in a general way how nature involves all four types of causality, they single out only final causality for particular attention. The whole last part of the second book of the Physics is devoted to a study of it, and to an insistence of its prime importance in the study of nature. Yet of all the causes that disappear in the mathematization of the cosmos, this is perhaps the type that is most efficaciously and most completely effaced. One looks in vain for anything that even remotely corresponds to finality in mathematical physics. And the fundamental reason for this has already been pointed out in Chapter VI: since there is no good in mathematics, there can be no final causality:

Ex hoc enim quod finis non potest esse in rebus immobilibus, videtur procedere quod in scientiis mathematicis quae abstrahunt a materia et motu, nihil probatur per hanc causam, sicut probatur in scientia naturali, quae est de rebus mobilibus, aliquid per rationem boni. Sicut cum assignamus causam quare homo habet manus, quia per eas melius potest exequi conceptiones rationis. In mathematicis autem nulla demonstratio fit hoc modo, quod hoc modo sit quia melius est sic esse, aut deterius si ita non esset. Puta si diceretur quod angulus in semicirculo est rectus, quia melius est quod sic sit quam quod sit acutus vel obtusus. Et quia posset forte aliquis esse alius modus demonstrandi per causam finalem, puta si diceretur, si finis erit, necesse est id quod est ad finem praecedere: ideo subjungit, quod nullus omnino in mathematicis facit mentionem alicuius talium pertinentium ad bonum vel ad causam finalem. Propter quod quidam sophistae, ut Aristippus, qui fuit de secta Epicureorum, omnino neglexit demonstrationes quae sunt per causas finales, reputans eas viles ex hoc quod in artibus illiberalibus sive mechanicis, ut in arte 'tectonica,' idest aedificatoria, et 'coriaria,' omnium rationes assignantur ex hoc quod est aliquid melius vel deterius. In mathematicis vero, quae sunt nobilissimae et certissimae scientiae, nulla fit mentio de bonis et malis.(72).

From all this it follows that it is entirely illegitimate for critics to reproach scientists as some modern Scholastics have done, for failing to take all types of causality into consideration. The very nature of his science makes it impossible for the mathematical physicist to consider anything but formal causality. And it is important for the scientist to be aware of his own

limitations, so that he will not, for example, confuse his substitute for efficient causality with true efficient causality. There is particular danger of this happening with this type of causality since it is the best known and the most manifest to the mind.

At first sight it might appear that this banishment of causality from the cosmos might make the physico-mathematical world like Malebranche's world of occasionalism. As a matter of fact, there is only a surface likeness between the two. In a deeper sense they are opposed. For in the world of Malebranche it is necessary to have constant recourse to God, since every event is the occasion of His action. In the physico-mathematical world, on the other hand, God is completely dispensed with; there is no need to go to Him at all; nor is it even possible to go to Him. Because of its rationality, its ever increasing unity and its immutability, the physico-mathematical world is more like the Parmenidian sphere.

This analysis of the effects of the mathematical transformation of the cosmos might go on interminably. We might for example show that it destroys not only the becoming of the universe, but in a certain sense even its

being. For as we saw in Chapter VI, mathematics prescind
from existence, and the only meaning that being has in the
physico-mathematical world is the occupation of a "place"
in a certain order, in a space-time schema. In this sense,
Bergson is correct in saying that in modern science "
l'existence concrète des phénomènes de la nature tend à
(73)
s'évanouir . . . en fumée algébrique." We might also show
how the concept of substance is transformed into the notion
(74)
of persistent system. But we feel that enough has al-
ready been said to show that the nature of which the mathe-
matical physicist speaks is not the nature that is defined
by Aristotle and St. Thomas in the second book of the Physics
as a principle of motion and of rest and as a "ratio" or
rational principle put into things which directs them in
(75)
their striving for ends. The nature of the mathematical
physicist is, as Eddington has remarked, "only an empty
(76)
shell." In other words, as we have already remarked,
in order to explain nature the physicist has found it
necessary to destroy it.

Obéissant aux deux tendances, nous avons, de
théorie en théorie, et d'identification en
identification, fait complètement disparaître
le monde réel. Nous avons d'abord expliqué,
c'est-à-dire nié le changement, identifiant
l'antécédent et le conséquent, et la marche
du monde s'est arrêtée. Il nous restait un
espace rempli de corps. Nous avons constitué

les corps avec de l'espace, ramené les corps à l'espace, et les corps se sont évanouis à leur tour. C'est le vide, 'rien du tout', comme dit Maxwell, le néant. Car le temps et l'espace se sont dissous. Le temps, dont le cours n'implique plus de changement, est indiscernable, inexistant; et l'espace, vide de corps, n'étant plus marqué par rien, disparaît aussi. (77)

It need hardly be pointed out, of course, that the great loss resulting from this destruction of nature has rich compensations that are daily becoming more apparent. For even though in destroying nature we destroy the intelligence that Aristotle saw in it and rob it of its seeking for ends, at the same time we make nature more intelligible than it is by injecting our own intelligence into it. The mathematical representation of nature is an improvement of it, in the sense in which a mathematical line is an improvement of a physical line. We construct a model for nature, and this construction forces nature to yield up its secrets. (78)

From all that has been said about the nature of this rationally constructed physico-mathematical world it is clear why it should inevitably appear to Sir James Jeans as a world consisting of pure thought, the thought of a mathematical thinker. (79) But it should also be clear why it is illegitimate for him to conclude that the objective

universe, that is to say, the absolute world condition, is nothing but pure thought and the product of a pure mathematician acting as a pure mathematician. For even though a physico-mathematical world may tend towards the absolute world condition as though towards its asymptote, a pure mathematician acting purely as such, neither would nor could create a physical universe. As Bridgman has remarked, "What Jeans might have said is that Man is a mathematician, and reflected that it is no accident that he forms nature in his own image."⁽⁸⁰⁾

CHAPTER TEN

A SHADOW WORLD OF SYMBOLS

1. The Nature of Symbolism.

Having seen how the mathematician transforms the physical universe into a new world of his own making, we must now try to analyze briefly the nature of this new world. All the best philosophers of science are now unanimous in characterizing the physico-mathematical world as a symbolic universe. Sir Arthur Eddington, for example, has, as is well-known, repeatedly described it as "a shadow world of symbols." ⁽¹⁾ We believe that if this phrase be rightly understood, it brings out with great accuracy the true nature of the universe constructed by mathematical physics. Let us try to determine what precise meaning must be given to it.

In the first place, it is necessary to fix upon the meaning of the word "symbol". And here we come upon a great lack of unanimity. All will agree that in its primitive meaning the term "Symbolon" signifies a mark or emblem or index employed to designate something, and that consequently every symbol is a sign. But is every sign a symbol? Not a few authors seem to think so. Thus R. B. Perry writes: "Any datum may be a symbol if it means something or operates

(2)
as a sign." And he goes on to explain that such data may include:

... conspicuous features of nature, monuments, written or spoken words, small images or familiar objects easily duplicated or distributed. Any of these is a symbol provided it directs expectation or interest to something other than itself. Symbolism is, then, the study of the part played in human affairs by all these signs and symbols, especially their influence on thought. Symbols direct and organize, record and communicate. For words, arrangements of words, images, gestures, and such representations as drawings or kinetic sounds we use the term symbols.

To make the sign and the symbol coterminous in this way is to rob symbolism of all precise meaning. (3) And the ordinary usage of the term seems to insist upon a precise meaning. Clouds are considered to be signs of rain, and smoke a sign of fire, but they are never referred to as symbols. It is necessary, therefore, to try to press the meaning of the term a bit closer.

In the first place, the examples just referred to make it clear that purely natural signs (i.e. those which have a natural and real connection with the thing signified, prior to any connection established by the mind) must be excluded from the notion of symbolism. To apply the term "symbol" to (4) a natural sign is actually a distortion of language. In other words, symbols are necessarily arbitrary or conventional signs, i.e. signs in which the connection with the thing signified is not found in nature as such, but created by the

mind. This does not, however, exclude the possibility of there being in nature a foundation for the connection established by the mind.

Having made this important distinction we are faced with this problem: are all conventional signs necessarily symbols? Once again, a good many authors seem to think so -- at least if it be question of the most important type of conventional signs, namely those which make up language. Miss Stebbing, for example, tells us that "a word is a special kind of sign called a symbol."⁽⁵⁾ And again she writes: "A sign consciously designed to stand for something will be called a symbol." This opinion seems to be shared by Professor Whitehead: "The word symbolizes the thing. Language almost exclusively refers to presentational immediacy as interpreted by symbolic reference."⁽⁶⁾ This tendency to make all language and even all thought symbolic⁽⁷⁾ makes it difficult to attach any precise and proper meaning to the term.

Since the word is currently employed in such a loose way it is necessary for us to try to fix upon the particular meaning it is to have for us in this discussion of the symbolism of science. Its etymology provides us with a helpful suggestion. The Greek words $\sigma\upsilon\nu$ and $\beta\acute{\alpha}\lambda\lambda\epsilon\iota\nu$ mean "to throw together". Now, whatever may have been the original

historical usage of these words which gave rise to the term we are analyzing, it is clear that they suggest a collection of things among which there is no strict natural unity -- an aggregate whose principle of unification is purely extrinsic. If we keep this in mind we shall be able to see why Saint Thomas, in his Commentary on the Sentences ⁽⁸⁾ gives this description of the symbol: "...nomen symboli similitudinem et collectionem importat." It would seem that a symbol must be defined as an artificial sign established to signify a determined object that is one only according to the mind. In order to bring out the meaning of this definition, it is necessary to see the difference between a symbol and a name. ⁽⁹⁾

In his Commentary on the Perihermeneias, St. Thomas explains the important distinction between the name and the infinite name:

Deinde cum dicit (Aristoteles) "non homo vero non est nomen" etc., ~~excludit~~ quaedam a nominis ratione. Et primo, nomen infinitum; secundo casus nominum; ibi: "Catonis autem vel Catoni" etc. Dicit ergo primo quod "non homo" non est nomen. Omne enim nomen significat aliquam naturam determinatam, ut "homo"; aut personam determinatam, ut pronomen; aut utrumque determinatum, ut Sortes. Sed hoc quod dico "non homo", neque determinatam naturam neque determinatam personam significat. Imponitur enim a negatione hominis, quae aequaliter dicitur de "ente" et "non ente". Unde "non homo" potest dici indifferenter, et de eo quod non est in rerum natura; ut si dicamus, "chimaera est non homo", et de eo quod est in rerum natura; sicut cum dicitur, "equus est non homo". Si autem imponeretur a privatione, requireret subiectum ad minus existens; sed quid imponitur a negatione, potest dici de ente et de non ente, ut Boethius et Ammonius dicunt. Quia tamen significat per modum nominis,

quod potest subici et praedicari requiritur ad minus suppositum in apprehensione. Non autem erat nomen positum tempore Aristotelis sub quo huiusmodi dictiones concluderentur. Non enim est oratio, quia pars eius non significat aliquid separata, sicut nec in nominibus compositis; similiter autem non est negatio, id est oratio negativa, quia huiusmodi oratio superaddit negationem affirmationi, quod non contingit hic. Et ideo novum nomen imponit huiusmodi dictioni, vocans eam nomen "in-finitum" propter indeterminationem significationis, ut dictum est. (10)

It is clear from this passage that the name must signify something that is one by nature. Because of its indetermination the infinite name does not signify something that is one by nature. Because it is a pure negation, it does not even have the determination of privation which must always be in the same genus as the thing of which it is the negation.

Nevertheless, in spite of the indetermination of the infinite name, it has a significance; in some way it signifies something that is one. St. Thomas explains this in his Commentary on the Second Book of the Perihermeneias:

...nomen infinitum quodam modo significat unum. Non enim significat simpliciter unum, sicut nomen finitum, quod significat unam formam generis vel speciei aut etiam individui, sed in quantum significat negationem formae alicuius, in qua negatione multa conveniunt, sicut in quodam uno secundum rationem. "Unum" enim eodem modo dicitur aliquid, sicut et "ens"; unde sicut ipsum "non ens" dicitur "ens", non quidem simpliciter, sed secundum quid, id est secundum rationem, ut patet in IV Metaphysicae, ita etiam negatio est unum secundum quid, scilicet secundum rationem, Introducit autem hoc, ne aliis dicat quod affirmatio,

in qua subicitur nomen infinitum, non significat unum de uno, quasi nomen infinitum non significet unum. (11)

There is, then, a unity in the infinite name -- a unity that is founded upon the unity of the thing negated. It is possible to predicate the infinite name of anything except the thing negated. But it is important to note that even though the infinite name can be applied to any one of the things that fall within the class which includes everything except the thing negated, it does not properly signify any one of them. Nor does it signify the class of all those things, as a genus signifies everything that falls within it. The infinite name is not a collective noun; there is a class of things to which it may be applied, but it does not express any of them.

Now all this has a very important bearing upon the nature of the symbol. For we believe that the symbol falls somewhere between the name and the infinite name. The name may signify a collection, but it never signifies a collection qua collection, i.e. as a mere accidental union. The infinite name on the other hand, though it may be applied to a collection, does not formally signify a collection, because of its indetermination. The symbol alone signifies a collection formally as a collection. Unlike the universal name, the symbol

does not abstract from multiplicity; in fact, it is precisely the multiplicity that it signifies. Like the name and unlike the infinite name, the symbol signifies a determined object; but unlike the name and like the infinite name it does not signify anything that is one by nature.

A simple example will serve to clarify the issue. Is the sign "3" a symbol? That depends upon what it is taken to signify. If it represents the three which is a numbering number, a pure aggregate, a collection of $1+1+1$, it is a symbol in the strict sense of the term. If however, it is employed to signify numbered number, or predicamental number, which is not three ones, but one three, because the three have a common physical genus and constitute an unum per se, it is not a symbol in the strict sense, but merely a convenient substitute for the name "three". In other words, in order for a sign to be a symbol it must signify something that possesses only logical unity; it must signify a collection in its pure collectivity. If Russell's definition of number as "the class of all classes that are similar to it" were correct, all numbers would be nothing but symbols.

(12)

The transcendent terms of logic used so extensively in the Priors Analytica are illustrations of the symbol, for they signify at the same time everything and nothing. Of them St.

Albert the Great writes: "Ideo terminis utimur transcendentibus, nihil et omnia significantibus. Nihil dico, quia nullam determinant materiam. Omnia vero dico significantibus: quia omnibus materiis sunt applicabiles, sicut sunt
(13)
a, b, c."

It is clear that a symbol is something quite different from a mere abbreviation. An abbreviation has only the outward appearance of a symbol, and is in reality nothing but a convenient substitute for a name. Vossler's remark that the language of mathematics is pronominal, must be rightly understood. If it means that the language of mathematics consists in signs that substitute for names, it is true of traditional mathematics. If it means that mathematical signs stand in the place of names in the sense of signifying collections which names cannot signify, it is true only of the dialectical part of modern mathematics.

Nominalism is at bottom nothing but a denial of the important distinction we have just drawn between name and symbol. By a strange paradox, it is a rejection of the name in the true sense of the term, for if all names signify nothing but a collection of singulars, if "being" for example, means
(14)
nothing but the whole collection of beings, all names can be nothing but symbols.

If names in the last analysis were only symbols, and if reality were such that it could be represented and expressed only by means of symbols, then there would be no true natures in existence and all things would constitute nothing more than an accidental collection without any intrinsic or essential unity. Universal nominalism which denies all determined natures must necessarily conceive all language in terms of pure symbolism. That is why Whitehead, for whom reality is a process, is logical in holding that all names are symbols. And in this connection it is interesting to note that Cratylus, who pushed universal nominalism to its absolute extreme, held that words should not be employed at all, and had recourse to the movement of a finger in order to express himself. (14a)

And now, having fixed upon the precise meaning to be attached to the term "symbol" let us try to see in what sense the physico-mathematical world can be truly called a world of symbols.

2. Symbolism and Mathematical Physics.

It has long been customary for scientists with a penchant towards scientism to ridicule the philosophical sciences for their "verbalism". This attitude has been based upon the assumption that philosophy deals essentially with vague

and shadowy concepts which have no definite counterparts in reality, and that only in experimental science are things laid hold of in their true objective natures. The new self-revelation that has occurred in the realm of experimental science has done much to mitigate this naive view. It has become increasingly evident that experimental science, in so far as it attempts to employ names, is the most verbalistic of all the sciences. The philosopher can define with precision the fundamental concepts which he employs such as substance, accident, motion, time, etc., he can set forth the nature, the quod quid est of things. The physicist, on the other hand, is hard put to it to define what he means by even the simplest and most basic notions that enter into his science, such as body, energy, matter, mass. As we shall see presently, every attempt to define these notions involves him in an endless circle from which there is no exit.

The fact of the matter is that experimental science is essentially nominalistic in the sense defined above. By its very nature it is committed to the use of symbols rather than of names. And nothing could be more striking than the contrast between the vagueness of scientific language when interpreted in terms of names, and its precision when interpreted in terms of symbols.

It has taken science a long time to realize this. Because experimental science necessarily tends towards the condition of science in the strict sense of the term, it was only natural that in its origin and development it should aspire towards a state in which its language could consist of names in the proper sense of the word. The great mistake of scientists has been to believe that this state was already a fait accompli. This was characteristic of classical physics. It was particularly characteristic of a view that was current in the nineteenth century, especially among such men as T.H. Huxley, which held that science is nothing but organized and refined common sense, and that its language is only the ordinary language of common sense rendered more precise and accurate.

This view is no longer popular. The cleavage between science and common sense has become so profound that it has caused dismay not only in the minds of laymen who are interested in trying to find out what science is about, but even in the minds of the scientists themselves who desire to comprehend the meaning of their science. How for example, can Schrodinger's oscillations signs operating in multi-dimensional space be expressed in the ordinary language of common sense? We believe that this state of affairs can be understood only by

becoming conscious of the fact that experimental science is essentially symbolic, that is language is not a language of names, but a language of symbols. Let us try to see why this is so.

As Saint Thomas points out in the lines cited above from the Commentary on the Perihermeneias, a name in the strict sense of the term always stands for a definite nature (or person); it indicates something that is an unum per se -- a quod quid est. Now we have seen that though experimental science tends towards laying hold of natures, it necessarily falls short of its goal. Pure induction by enumeration can never of itself disclose a nature that is strictly one. That is why from the very start, experimental science is doomed to deal with collections, no matter how it may strive to rise above their multiplicity and arrive at the unity of a strict nature. What the nominalists taught about knowledge is perfectly correct when applied to experimental science. "Science, writes Weyl, "concedes to idealism that this its objective world is not given but only propounded (like a problem to be solved) and that it can be constructed (15) only by symbols." But that is not all.

In this striving to rise above multiplicity, it is forced to operate upon nature. This operation, as we saw in

Chapter IV, never reveals the objective nature of things; its results depend essentially upon the whole collection of concrete elements which entered into it. Since, then, the definitions of physics can be nothing but operational, none of its notions can stand for a strictly unified objective nature. They can mean nothing more than the whole collection of elements entering into the operations from which they derive; they can signify only a collection qua collection, that is to say an accidental aggregate of nature plus a multiplicity of operational elements, all of which have a unity that comes from the mind alone. Symbols alone, and not names can stand for collections of this kind. That is why all of the language which physics uses, whether it consist of words or any other type of signs, is necessarily symbolic. As a consequence, when the physical world is identified with the world in se it is impossible to escape transcendental symbolism. Likewise to look upon these signs as names is to confuse art with nature, subjective construction with objective reality what is one only in the mind with what is one by nature; it is to fall into a very pernicious type of idealism, as we shall point out in a later context.

It should be evident from the foregoing that science is symbolic not merely in its more theoretical superstructures

(16)

but in the very results of its primary contact with nature.

Lindsay and Margenau bring this out in the following passage:

It thus appears that the symbol here is but a shorthand expression for the results of a given operation leading to the assignment of a number value to the symbol. Instead of describing in words the entire series of acts involved in the setting of the tubes and the reading of the scale, the whole matter is summed up in the one phrase: measurement of P. Is this then all that there is to the meaning of symbolism? If it were necessary to associate a symbol with the results of every single physical operation the description of these operations might indeed be simplified but it would not constitute what we now consider theoretical physics. The real power of symbolism in physics first becomes clear when we envisage the possibility of letting a symbol stand for a concept which is, so to speak, the synthesis of the results of a whole set of operations which may appear to be superficially dissimilar, but are assumed by the physicist to have a common element. (17)

It should also be evident from the foregoing that the symbolic character of science does not consist in its abstractness, as some seem inclined to believe. The language of the philosophical sciences is abstract, but it is not essentially symbolic. There is, as we observed earlier in this Chapter, a profound difference between symbols and names which stand for abstract natures. Duhem has endeavored to clarify this distinction in La Théorie Physique:

Prenons une loi de sens commun, une des plus simples

comme une des plus certaines: Tout homme est mortel. Cette loi, assurément relie entre eux des termes abstraits, l'idée abstraite d'homme en général, et non l'idée concrète de tel ou tel homme en particulier; l'idée abstraite de la mort et non l'idée concrète de telle ou telle forme de la mort; c'est en effet à cette seule condition de relier des termes abstraits qu'elle peut être générale. Mais ces abstractions ne sont nullement des symboles théoriques; elles extraient simplement ce qu'il y a d'universel dans chacun des cas particuliers auxquels la loi s'applique; aussi, dans chacun des cas particuliers où nous appliquons la loi, trouverons-nous des objets concrets où seront réalisées ces idées abstraites; chaque fois que nous aurons à constater que tout homme est mortel, nous nous trouverons en présence d'un certain homme particulier incarnant l'idée générale d'homme, d'une certaine mort particulière impliquant l'idée générale de mort. . .

Il n'en est plus de même pour les lois de la Physique. Prenons une de ces lois, la loi de Mariotte, et examinons-en l'énoncé, sans nous soucier, pour le moment, de l'exactitude de cette loi. A une même température, les volumes occupés par une même masse de gaz sont en raison inverse des pressions qu'elle supporte; tel est l'énoncé de la loi de Mariotte. Les termes qu'elle fait intervenir, les idées de masse, de température, de pression, sont encore des idées abstraites; mais ces idées ne sont pas seulement abstraites, elles sont, de plus, symboliques, et les symboles qu'elles constituent ne prennent un sens que grâce aux théories physiques. Plaçons-nous en face d'un cas réel, concret, auquel nous voulons appliquer la loi de Mariotte; nous n'aurons pas affaire à une certaine température concrète réalisant l'idée générale de température, mais du gaz plus ou moins chaud; nous n'aurons pas devant nous une certaine pression, mais une certaine pompe sur laquelle on a pesé d'une certaine manière. Sans doute, à ce gaz plus ou moins chaud correspond une certaine température, à cet effort exercé sur la pompe correspond une certaine pression; mais cette correspondance est celle d'une chose signifiée au signe qui la remplace, d'une réalité au symbole qui la représente. Cette correspondance n'est nullement immédiate; elle s'établit au moyen d'instruments, par l'intermédiaire souvent très long

et très compliqué des mesures; pour attribuer une température déterminée à ce gaz plus ou moins chaud, il faut recourir au thermomètre; pour évaluer sous forme de pression l'effort exercé par la pompe il faut se servir du manomètre et l'usage du thermomètre, l'usage du manomètre, impliquent, nous l'avons vu au chapitre précédent, l'usage des théories physiques.(18)

The symbolism of experimental science may take on various forms. In the first place, it may take the form of words. But words serve the purpose of symbolism very inadequately. For they are primarily designed to signify natures. That is why their use as symbols presents the constant danger of their being mistaken for names, and it is well known how many scientists and philosophers have fallen prey to this danger. It is a sign of extreme naiveté on the part of philosophers to rejoice over the fact that certain terms, such as "substance", "matter", "body", etc. are shared in common by both philosophy and science, and to believe that it is legitimate for them to incorporate into their philosophical system these notions as they are understood in science. Moreover, there is an isolation about words which makes them incompetent to express the interconnectedness that science tries to achieve. Because therefore experimental science must necessarily speak in symbols and because words serve this purpose so inadequately, there is a natural tendency, especially in mathematical physics, to draw away as completely as possible from words, to have recourse to other signs, and to construct a language of its own which defies all translation into the

ordinary language of common sense -- much to the discomfiture of the popularizers of science.

A second form which scientific symbolism may take is that of models. These serve the purpose of symbolism somewhat more effectively than mere words. The danger of their being mistaken for natures in the strict sense of the word is to some extent diminished. Besides this they have the advantage of giving a direct and immediate expression of interconnectedness. But they are still extremely inadequate. For one thing, because of their direct connection with intuition they all too easily give the impression that they represent nature in its pure objectivity, independently of the manufacturing processes of the scientist who works upon nature. This easily leads to the delusion that they are direct and immediate copies, or pictures, or at least schemas of objective natures. That the classical physicists labored under this delusion constantly is a matter of history, and it is now generally recognized how great an obstacle this delusion placed in the path of scientific progress. Models are not well adapted to symbolize the true collections that are involved in the notions of experimental science. Moreover, their immediate connection with intuitive schemas makes their capacity for expres-

sing interconnectedness extremely limited. For these reasons science has in recent years tended to free itself more and more from the restrictions of these models. As we intimated in the last Chapter, however, since experimental science deals with the realm of the physical, it is doubtful if it will ever be able to dispense entirely with the sensible support that such sensible constructs provide. But it is extremely important to remain conscious of the fact that they are mere constructs, mere symbols, and to be aware of what they actually signify.

The next step in science's search for adequate symbolic forms has been the use of what have sometimes been called pseudo-sensible constructs. ⁽¹⁹⁾ These constructs include such entities as atoms, electrons, etc. Though some of these constructs may be said to be closer to nature than others, none of them has any immediate correspondence with anything in reality. As Professor Margenau points out, their value has no relation to their mode of existence. There is less resemblance between them and objective entities than there is between clues and criminals. As Thompson has remarked: "We may well say of them what Hobbes said of words: 'They are wise men's counters, they do but reckon by them, but they are the money of fools.'" ⁽²⁰⁾ Constructs of this kind may

be generated by science ad libitum, for since they are merely counters by which to reckon, their nature and validity is essentially functional. And their function is to construct and shape a body of doctrine which will explain natural phenomena. Though they do not correspond to anything encountered in experience, they serve to give systematic form to the data of experience. As Cassirer has observed: "thought only separates itself from intuition in order to turn to it with new instruments, thereby to enrich it in itself... They render insight into relations possible, and guarantee it, although they themselves can never be perceived after the fashion of isolated objects." (21) They differ from the data of experience by their essential interconnectedness. Because of this interconnectedness they can serve to erect a coherent organism which can substitute for the disconnected mass of experiential data and thus rationalize it. In other words, by mapping the elements of nature which by themselves appear as incoherent, contingent and unpredictable upon constructs, science is able to create a symbolic system which is more coherent, more necessary, more rational than nature. More or less arbitrary rules of combinations may be employed in relation to these constructs which gives great freedom for the mind to reason about them and which gives great pliancy to the constructional system.

The results of this rational transformation are ultimately mapped back upon nature in such a way as to predict phenomena.
(22)

In this way science succeeds in building up a world of its own . . . a world that is rationally organized, and intrinsically coherent, and all the elements of which mutually imply each other. The validity and significance of the individual constructs which go to make up this symbolic system cannot be established by themselves alone by appealing to experience. In so far as the notion of verification can be applied to them, it cannot mean the establishment of any direct referends in reality. Their validity and significance is derived from the role that they play as members of a theoretical complex.

It is evident that these pseudo-sensible constructs go far beyond the strictly physical models in their capacity to serve as symbols. But in so far as they resemble in some respects these physical models they share to some extent in the limitations attached to the latter. Both types of constructs provide the sensible support that physical speculation needs. But though they may for a while stand the weight of speculation placed upon them, they tend eventually, as Jeans has remarked, "to break in our hands."
(23)

That is why physics must reach beyond the limitations of
(24)
these constructs to a more perfect type of symbolism.

This more perfect type of symbolism is found
(25)
in mathematics. As is well known, mathematics, especially
in its modern dialectical form, is admirably suited to
play the role of symbolism. Its abstraction from existence,
from nature, and from all specific substances, and its empty
forms make it an apt instrument to signify collections and
the relations among manifolds without signifying the nature
of the relata. Through mathematical symbolism alone can
the diverse phenomena of nature be reduced to a high degree
of interconnectedness. That is why physics is learning to
express itself more and more fully in the abstract forms of
mathematics. One has only to recall Heisenberg's, Dirac's
and Schrodinger's recent developments in quantum physics
to realize how far this tendency has gone. As we have
already remarked, sensible and pseudo-sensible constructs
will never be completely dispensed with, but as Jeans has
put it, they will remain mere parables -- mere clothing
(26)
which we drape over our mathematical symbols.

3. A World of Shadows.

"The frank realisation that physical science is

concerned with a world of shadows," writes Eddington, "is one of the most significant of recent advances. I do not mean that physicists are to any extent preoccupied with the philosophical implications of this. From their point of view it is not so much a withdrawal of untenable claims as an assertion of freedom for autonomous development."⁽²⁷⁾

Nothing could be more striking than the paradoxical fact that by attempting to introduce the brilliance of cartesian clarity everywhere in the physical world, science has made of it a world of shadows. We must now try to see why the world of physics has necessarily become a world of shadows and what some of the philosophical implications of this fact are.

The shadowy character of the physical world derives principally from its symbolic nature. But even independently of the use of symbols there are a number of reasons why the world with which physics deals can be truthfully called a world of shadows. To begin with, all human knowledge is by its very nature shadowy. For the human intellect is the lowest intellect that could possibly exist; it is essentially united with matter, and dependent upon it (at least extrinsically) for its functioning. As a consequence its realm of knowledge is at best a mere shadowland.

That is why Aristotle tells us that it is like the eyes of an owl which can see well only in the deep twilight and in the dark. And the more it attempts to penetrate into the realm of the sensible, the more does its knowledge become shadowy. Sense knowledge is truly an obscure knowledge. For it is at the utmost extreme of knowledge, where immateriality peters out into materiality, where the light of the intentional world is mingled with the darkness of the purely physical world. It is a very late twilight when darkness has almost entirely taken over, and when only obscure shadows can be seen. Now physics deals with everything in terms of sensible matter. Not only that, but it is the part of natural doctrine that is the farthest advanced in the direction of concretion, that is the most profoundly immersed in the obscurity of matter. That is why its object is essentially a shadowland.

The dialectical character of physics gives us another reason why it necessarily deals with shadows. For since it is a scientia quia and not a scientia propter quid, it can get at phenomena alone; it is restricted to mere appearances. The nature behind the appearances remains in the dark. In attempting to get at this nature, physics throws up a scaffolding against reality -- a scaffolding

which is like a shadow of reality, roughly, and sometimes grotesquely reflecting its outline. Though there is always some relation between the proportions of a shadow and the reality, this relation is not definite, particularly with regard to specific details. The relation between the world constructed by the physicist and the world of reality, is of this kind.

By the fact of its being subalternated to mathematics, the world of physics takes on an even stronger resemblance to a shadowland. For a shadow is something that reduces the heterogeneity of the object it represents to pure homogeneous exteriority. The qualitative is swallowed up in the quantitative. To be more specific, the mathematical line is a shadow of the physical line, and when the physicist studies the physical line in terms of the mathematical line, he is getting at reality only by means of its shadow.

But it is principally because of its symbolic character that the world of physics is a world of shadows. And the reason for this should be fairly evident. We have seen that symbols differ from names in that they do not stand for natures in the strict sense of the term. That is why when they are used as signs, the precise nature of the

things signified remains blurred and hidden in the background. And no manipulations of symbols can make them emerge from this background.

As science perfects its symbolic forms, the physical world takes on more and more the character of a self-authenticating formal system in which the inter-relatedness of nature's manifold is seized upon and reflected. The principal criterion for the use of these symbolic forms is not that they should individually have a direct correspondence with something intuitively given, but that they be able to fit coherently into the self-authenticating system. From one point of view the increasing perfection of the symbolic reflection of nature's inter-relatedness throws greater light upon the relata, but from another point of view it makes them more like shadows.

Sir Arthur Eddington has laid great emphasis upon this point. In the introduction to The Nature of the Physical World he writes:

Science aims at constructing a world which shall be symbolic of the world of commonplace experience. It is not at all necessary that every individual symbol that is used should represent something in common experience or even something explicable in terms of common experience. The man in the street is always making this demand for concrete explanation of the things referred to in science, but of necessity he must be

disappointed. It is like our experience in learning to read. That which is written in a book is symbolic of a story in real life. The whole intention of the book is that ultimately a reader will identify some symbol, say BREAD, with one of the conceptions of familiar life. But it is mischievous to attempt such identifications prematurely, before the letters are strung into words and the words into sentences. The symbol A is not the counterpart of anything in familiar life. To the child the letter A would seem horribly abstract; so we give him a familiar conception along with it.

"A was an Archer who shot at a frog." This tides over his immediate difficulty; but he cannot make serious progress with word-building so long as Archers, Butchers, Captains, dance round the letters. The letters are abstract and sooner or later he has to realize it. In physics we have outgrown archer and apple-pie definitions of the fundamental symbols. To a request to explain what an electron really is supposed to be we can only answer. "It is a part of the A B C of physics. The external world of physics has thus become a world of shadows...

It is difficult to school ourselves to treat the physical world as purely symbolic. We are always relapsing and mixing with the symbols incongruous conceptions taken from the world of consciousness. Untaught by long experience we stretch a hand to grasp the shadow, instead of accepting its shadowy nature. Indeed, unless we confine ourselves altogether to mathematical symbolism it is hard to avoid dressing our symbols in deceitful clothing. When I think of an electron there rises to my mind a hard, red, tiny ball; the proton similarly is neutral grey. Of course the colour is absurd - - perhaps no more absurd than the rest of the conception - - but I am incorrigible. I can well understand that the younger minds are finding these pictures too concrete and are striving to construct the world out of hamiltonian functions and symbols so far removed from human preconception that they

do not even obey the laws of orthodox arithmetic. For myself I find some difficulty in rising to that plane of thought; but I am convinced that it has got to come.

Later in the same work he brings out this point more specifically in connection with his explanation of the cyclic method employed in physics. (28) All of the constructs out of which the structure of physics is formed, such as point-events, potentials, matter, etc. are definable and translatable only in terms of each other, not in terms of anything else, and in particular not in terms of any underlying reality that is independent of the mind of the scientist or the physical objects of the perceptual world. These constructs form a closed circle. By beginning at any point on this circle we may define any one of the members which form it in terms of the others, and from it deduce the others. But as we travel around the circle at no point do we make fresh contact with reality. At a certain point, e.g. "matter" we may think that we are talking about something which has a direct embodiment in the world of reality, but in point of fact, the "matter" that is dealt with in physics has no direct counterpart in nature. It is by working around this circle that we derive the physical laws.

In this way physics remains within its own domain; it constitutes a closed world of its own, and this world is but a shadowland reflecting the underlying reality which can never be made to emerge from its obscurity:

And you can see how by the ingenious device of the cycle physics secures for itself a self-contained domain for study with no loose ends projecting into the unknown. All other physical definitions have the same kind of interlocking. Electric force is defined as something which causes motion of an electric charge; an electric charge is something that exerts something that produces motion of something that exerts something that produces . . . ad infinitum. The supposed approach through the physical world leads only into the cycle of physics, where we run round and round like a kitten chasing its tail and never reach the world-stuff at all . . . However much the ramifications of the cycles may be extended by further scientific discovery, they cannot from their nature trench on the background in which they have their being -- their actuality. (29)

It is particularly in its use of the theory of groups that the physical world takes on the character of a world of shadows. As we saw in the last Chapter, it is possible to give an exact mathematical description of patterns, while the nature of the entities involved in them remain in the dark. "It (mathematics) dismisses the individual elements by assigning to them symbols, leaving it to non-mathematical thought to express the knowledge, if any, that we may have of what the symbols stand for . . .

Every path to knowledge of what lies beneath the structure
is then blocked by an impenetrable mathematical symbol." (30)

All this discussion about the shadow world of physics calls to mind the famous shadows of the Platonic cave. In fact, the well-known passage from the Republic is so relevant here that we cannot refrain from quoting it:

And now, I said, let me show in a figure how far our nature is enlightened or unenlightened:- Behold! human beings living in an underground cave, which has a mouth open towards the light and reaching all along the cave; here they have been from their childhood, and have their legs and necks chained so that they cannot move, and can only see before them, being prevented by the chains from turning around their heads. Above and behind them a fire is blazing at a distance, and between the fire and the prisoners there is a raised way; and you will see, if you look, a low wall built along the way, like the screen which marionette players have in front of them, over which they show the puppets.

I see

And do you see, I said, men passing along the wall carrying all sorts of vessels, and statues and figures of animals made of wood and stone and various materials, which appear over the wall? . . .

You have shown me a strange image, and they are strange prisoners.

Like ourselves, I replied; and they see only their own shadows, or the other shadows which the fire throws on the opposite wall of the cave.

True, he said; how could they see anything but the shadows if they were never allowed to move their heads?

And of the objects which are being carried in like manner they would only see the shadows?

Yes, he said.

To them, I said, the truth would be literally nothing but the shadows of images. (31)

All that has been said in the course of this study about the nature of experimental science makes it evident how much the scientist is like a prisoner in a dark cave. The very method to which he is committed are the chains which bind him and prevent him from turning his head and seeing reality in its objectivity. As Plato's observer saw both other shadows and his own thrown against the wall of the cave, so in the shadow world of physics the scientist sees both the shadows of objective reality and his own, but in this case the two are inextricably blended together.

The following parable brings out still further the similarity between the physicist and the cavedweller of Plato:

An aged college Bursar once dwelt secluded in his rooms devoting himself entirely to accounts. He realised the intellectual and other activities of the college only as they presented themselves in the bills. He vaguely conjectured an objective reality at the back of it all - - some sort of parallel to the real college - - though he could only picture it in terms of the pounds, shillings and pence which made up what he would call "the common sense college of everyday experience." The method of account-keeping had become inveterate habit handed down from generations of hermit-like bursars; he accepted the form of accounts as being part of the nature of things. But he was of a scientific turn and he wanted to learn more about the college. One day in looking over his books he discovered a remarkable law. For every item on the credit side an equal item appeared somewhere else on

the debit side. "Ha" said the Bursar, "I have discovered one of the great laws controlling the college. It is a perfect and exact law of the real world. Credit must be called plus and debit minus; and so we have the law of conservation of L s. d. This is the true way to find out things, and there is no limit to what may ultimately be discovered by this scientific method. I will pay no more heed to the superstitions held by some of the Fellows as to a beneficent spirit called the King or evil spirits called the university Commissioners. I have only to go on in this way and I shall succeed in understanding why prices are always going up."

I have no quarrel with the Bursar for believing that scientific investigation of the accounts is a road to exact (though necessarily partial) knowledge of the reality behind them. Things may be discovered by this method which go deeper than the mere truism revealed by his first effort. In any case his life is especially concerned with accounts and it is proper that he should discover the laws of accounts whatever their nature. But I would point out to him that discovery of the overlapping of the different aspects in which the realities of the college present themselves in the world of accounts, is not a discovery of the laws controlling the college; that he has not even begun to find the controlling laws. The college may totter but the Bursar's accounts still balance.(32)

However much symbols and shadows may cut off the scientific observer from reality their essential purpose is to unite him to it. For the nature of symbols is to signify something and the nature of shadow is to be a
(33)
reflection of reality. That is why, after having seen the nature of the physico-mathematical world, we must now

try to analyze its relation to the objective world. The nature of this relation has been more or less implicit in much that has been said thus far, and has, we feel, already begun to take on fairly definite outline. But it is of supreme importance for a right understanding of the validity of scientific knowledge to endeavor to make it as explicit as possible. That is not an easy thing to do, for it should be evident from all that has been said up to now that this relation is far from being the simple thing that the classical physicists and the majority of modern Scholastics have imagined it to be. We can only hope to treat the problem in its general aspects without descending to details.

CHAPTER ELEVEN

THE ABSOLUTE WORLD CONDITION

1. Isomorphism.

By the absolute condition of the universe is meant the objective world as it is itself -- the world as it is contemplated by supramundane intelligences which do not have to depend upon the manifold subjective and relative conditions that necessarily accompany all knowledge derived through the senses, which are free of the barriers that result from the limitations of the human intellect, which do not have to probe the world with appliances that are within it, and a part of it, and subject to its laws, and which do not have to reconstruct the world, and thus remodel and change it, in order to know it. (1)

That this absolute world condition is not identified with the physico-mathematical world is only too evident. We must beware of the ambiguity of the term "physical world." (2) Originally it was employed to designate the objective cosmos. Physical science was born of a desire

to lay hold of this cosmos in its objectivity. But as science grew, it gradually evolved, for reasons already set forth, a world quite distinct from the objective cosmos — a world of its own making. It is to this latter world that the term "physical world" now usually refers when it is employed by physicists.

Progress in science has resulted, from one point of view at least, in an ever widening gap between these two worlds. The scientific universe has become more and more independent of the objective universe, more and more closed in upon itself, more self-sufficient. This has come first of all from the steadily increasing use of hypothetical elements logically interwoven into a coherent structure, but it has been carried to great lengths by the subalternation of physics to mathematics, which, as we have seen, is independent of existence and of any necessary order to existence, and which constitutes a closed and autonomous universe determined only by its own intrinsic logic. In this way, physical science has tended to become more and more a formal, self-authenticating system, even the raw materials of which are no longer taken directly from the objective world, but are subjectively created constructs.

From this point of view, then, the scientific world

is a self-contained world, distinct from and independent of the absolute world condition. Science has become like a platonic demiurge, fabricating a universe out of its own subjective constructs and rationalizing it by means of mathematics. And in this perspective there is a great deal of truth in Maritain's remark: "ce n'est pas la réalité qui demandera à la science d'être vraie, c'est la science qui demandera à la réalité d'être 'scientifique', et de lui présenter ses papiers." (3) In order to know that there is a vast difference between the scientific world and the absolute world condition it is not necessary, as some might be tempted to suppose, that we have direct knowledge of the world in itself and thus be able to compare the two. For in the first place we know that there is a negative distance between the two universes by our experience with the kind of knowledge we have, which must go from the more general to the more concrete without ever being able to exhaust the concrete. The history of science brings out this point and underscores our great ignorance. In a positive way we know that there is a vast difference between the two universes because we know that in order to carry on scientific endeavor we must construct and must inject mathematics. (4)

But this independence of the scientific world

from the absolute world condition is only one side of the picture, and to exaggerate it to the extent of obscuring the other side would be to vitiate the whole meaning of scientific knowledge. (5) The objective world is not merely a malleable matter which allows the scientist to make any constructions he may wish. In erecting his scientific world he enjoys a great measure of freedom, but he is not completely free. (6) Though mathematical physics is formally mathematical and from this point of view independent of the real world, it is terminative naturalis; its whole purpose is to get to know objective nature. The scientific world remains bound down to the objective world at both ends; that is to say, the scientist must both begin and terminate his work in contact with nature. (7) While it is true to say that in one sense, the theory of Relativity, for example, as it pursues its constructive elaborations never returns to the world of experience but seems to draw farther and farther away from it, in another sense it does return. Einstein knew before he started that all of his mathematical calculations and constructive elaborations had, in the end, to lead back into the black - bands of the Michelson interferometer. The scientist must solve problems that are initially given in the objective world; his solutions must explain facts as found in

experience. While the experimental operation measures the world condition, there is a sense in which it is true to say that the absolute world condition measures the experimental operation. As Eddington has observed, "The study of physical quantities, although they are the results of our own operations (actual or potential), gives us some kind of knowledge of the world-conditions, since the same operations will give different results in different world-conditions."⁽⁸⁾ Moreover, there is a sense in which it is true to say that the scientist deals with familiar objects of the objective world. A sign of this is found in the fact that commercial companies concerned with these objects always have recourse to the help of scientists.⁽⁹⁾

All this helps us to understand the problem that the meaning of real existence presents to the mind of the modern scientist.⁽¹⁰⁾ If the question is raised: "Does the scientific world really exist?" or "Does an electron really exist?" it is impossible to answer either yes or no, for we are dealing with constructs composed of both reality and mind. Taken from the point of view of the subjective elements they contain, they do not really exist. But taken from the point of view in which they are a reflection of reality, they do really exist. In fact, in the latter perspective we may

say that they exist in a more real sense than the sense world or the world of philosophy of nature, for science, in coming closer to concrete objectivity becomes more like the knowledge that God and the separated substances have of the absolute world condition than any other type of knowledge we have. (11) That is why Eddington, writing of Rossetti's Blessed Damosel who contemplates the world from heaven, can say: "If the Blessed Damosel sees the earth in the Einsteinian way she will be seeing truly -- I can feel little doubt as to that -- but she will be missing the point. It is as though we took her to an art gallery, and she (with that painful truthfulness which cannot recognize anything that is not really there) saw ten square yards of yellow paint, five of crimson, and so on." (12) The scientific world is made up of yards of paint taken from the objective world; but these yards of paint have been caught up into a composition that is not found in nature.

In the light of these remarks a number of passages in the writings of modern scientists which at first sight might appear baffling are rendered perfectly intelligible. A good example is the following passage of Eddington:

However, so far as I can judge the meaning of the question, the answer appears to be in the affirmative -- the external world described in physics {E. & O. E.} really exists.

One thing can perhaps usefully be added. I do not think that with any legitimate usage of the word it can be said that the external world of physics is the only world that really exists. (13)

There are in fact an infinite number of "physical universes". There was for example the original universe of Einstein which was full of matter and static. That has now been abandoned. There was likewise the universe of De Sitter which was empty. There is now the universe of Abbé Lemaître, which contains matter in constant expansion. These "physical universes" may be multiplied endlessly. All of them can be said to really exist in the sense just determined, but none of them can be considered the only one that really exists.

Perhaps the central problem with which we are concerned in this Chapter can be made clearer by casting it in the following form: is the scientific world true? Is it the truth about objective nature? What exact sense can be attached to Eddington's statement that if the Blessed Damosel sees the objective world in the Einsteinian manner she sees it truly? (14)

As is well known, truth may be defined either in terms of intrinsic coherence or in terms of extrinsic conformity. Every science in so far as it constitutes a body of doctrine and takes on systematic form must possess truth in the former sense. There are some sciences in which this kind of truth is of pri-

mary concern. These are particularly the mathematical sciences which deal with abstracta ut abstracta, and which prescind from any actual order to existence. But in those disciplines which deal with reality and which are sciences in the strict meaning of the term it is truth in the sense of extrinsic conformity that is of primary concern.

Now from what was said above about the scientific world constituting a closed and intrinsically coordinated system and about the criteria for the choice and elaboration of constructs being not correspondence with objective entities but their capacity to serve as principles of internal coherence, it would seem to follow that it is truth in the first sense of the term that is characteristic of experimental science. This would seem to derive both from the vast use of hypothesis and especially from the introduction of mathematics. It is true that there is some connection between scientific constructs and objective reality, but it would seem that this connection must be viewed not so much in terms of truth as in terms of goodness, since the validity of these constructs is judged by their functional role, by their explanatory efficacy. The whole question comes down to this, then: can the conformity definition of truth be applied to the relation between the scientific world and the absolute world condition?

It should be immediately evident that if the conformity definition be taken in its full and absolute meaning, the answer must be no. Truth in this sense has the implication of uniqueness and to apply it to the ever changing scientific world would make of it an extremely protean thing. On the other hand, it is equally evident that there is some correspondence and some kind of conformity between the scientific world and the absolute condition of the universe, that some relation similar to truth obtains between them, if for no other reason than that verisimilitude is, as we saw in Chapter V, of the very nature of experimental science. This conformity is found even with regard to the most theoretical parts of science, for since theory is the source from which the phenomena of nature logically flow, and the objective essences of things are the source from which they really flow, it is obvious that there must be some kind of correspondence between the two, even though theory may not give an explanation of reality that is true in the strict sense of the word. And as theory is perfected this correspondence becomes more and more exact.

Moreover, the scientific world is made up of reality as well as of mind, and it must not be forgotten that even the subjective elements derive their whole meaning from their orientation towards the real world.

In other words, scientific symbols like all symbols are a mixture of truth and fiction, as Urban has observed:

It is, as we have seen, of the very nature of the symbol that it contains both truth and fiction, both the real and the unreal. This principle follows, in a sense, from the two preceding. We have already seen that a symbol must stand for something, otherwise it would not be a symbol. We have also seen that it cannot stand for anything in a wholly unambiguous way. If it did it would not be a symbol. A fictional element in every symbol is made necessary by the principle of dual reference. It is of the nature of the symbol that if either reference is taken exclusively it becomes unreal or else a mere substitutional sign.

A relation of two domains is involved in every symbolic function. If the symbol is taken literally, as we say, if, in other words, the reference to the primary domain is taken exclusively the symbol is a fiction and mis - represents. If it is taken wholly as a sign without any reference to the intuitive domain out of which it springs, it is again a fiction, in this case a merely conventional sign. The symbolic function, as distinguished from literal representation or description and from the merely conventional, is not only this dual reference but the combination of truth and fiction which arises out of it. This is as true in the region of scientific symbolism as in any other. It is, in fact, one of the main issues in modern, scientific concepts is truth and how much fiction. (15)

It is clear, then, that in spite of its self-authenticating character, mathematical physics has a definite relation of correspondence with the real world. By the very fact that it is terminative naturalis, it must in

some way realize the conformity definition of truth that
is characteristic of all sciences which deal with reality. (16)

And if mathematical physics appears as something that is from one point of view essentially dominated by the coherence definition of truth and at the same time from another point of view principally dominated by the conformity definition, it is chiefly because it is a scientia media.

Let us try to fix upon the nature of the correspondence between the two worlds.

This obviously depends upon one's theory as to the nature of scientific knowledge. For those who press operationalism to the limit of maintaining that science reveals nothing but a set of operations carried on by the scientific worker, this correspondence is extremely tenuous when it exists at all. At least this is true if the notion of correspondence be considered from the point of view of speculative truth, as it is being considered in this context. For many operationalists, scientific symbols do not represent the objective universe at all; they merely reveal how one has operated upon nature and how one must operate upon nature in order to control it. (17) These authors fail to realize that the art that is involved in experimental science is purely functional and that its whole purpose is to serve science by

helping to disclose the objective logos. In other words, scientific symbols are like poetic symbols in this that they turn aside from a direct expression of reality only that in some sense they may express it more profoundly. (18)

The majority of modern scientists and philosophers of science hold that the physico-mathematical world must be considered at least a partial representation of reality. This opinion is held by Einstein and Planck, among others, and according to Cassirer it constitutes the essential modern scientific standpoint. It adopts a mediate position between the copy theory of the classical physicists and extreme operationalism. For most of those who hold this view, the scientific representation of reality consists in a reflection of nature's order, structure and inter-relatedness, rather than in a direct representation of intuitively given natural phenomena.

We believe that this opinion is essentially correct. But it is necessary to try to give greater philosophical determination to the correspondence between the scientific world and the absolute world. Some have sought to solve this problem by saying that the scientific universe is analogically true. Hoenen has been particularly favorable to this solution. (19) He holds that physical theories express an ana-

logical relation to reality and that if all the superfluous elements in them are eliminated by means of experiment and reasoning, it is possible for the relation to become univocal. We shall not linger over the latter part of this opinion, for all that has been said in preceding Chapters makes it abundantly evident how utterly untenable such a view is. In so far as analogy is concerned, we believe that this opinion is extremely ambiguous. It is clear that if the term analogy be taken in a broad and loose sense it may be applied to the knowledge that science gives of the objective world, in that the scientific world is partly like and partly different from the absolute world condition. But it is extremely important to keep this use of the term distinct from the proper use that is found in metaphysics. In true analogy we find a totum acutale that is the analogum in which the parts are known. In the case in hand, on the contrary, the parts are not known well enough. The objective and subjective elements in the scientific world are so intimately interpenetrated and fused, that it is impossible to distinguish between them; it is impossible to say what is in conformity with objective reality and what is not; it is impossible to determine which particular part is due to nature and which is due to mind.

We believe that the correspondence between the scientific world and the absolute world condition can best be explained in the following terms. In the first place, the scientific universe is a sign of the objective universe. Every sign represents an object distinct from itself to a cognitive power. But because there are two essentially different ways in which this representation can be effected, there are, as is well known, two essentially different kinds of signs: formal and instrumental. Since every sign is a means by which a cognitive power gets to know an object, even a formal sign is a kind of instrument. But it differs from an instrumental sign in this that it delivers the object it represents so directly and immediately to the mind that in this deliverance it does not itself constitute an object of knowledge. Thus the concept which the mind forms of an objective entity is a formal sign of that entity because it does not interpose itself as an object between the mind and the entity. An instrumental sign, on the other hand, is one that is first known in itself as an object in its own right, and only by being known in this way does it represent another object distinct from, but virtually im-
(20) plied in itself. In other words, as Cajetan has remarked, (21)
there are two kinds of beings; some are primarily designed to be and only secondarily do they represent; others are pri-

marily designed to represent other things. The former are instrumental signs and the latter are formal signs. In Thomistic terminology, a formal sign is id in quo aliquid cognoscitur, an instrumental sign is id per quod aliquid cognoscitur, the first is a forma intra potentiam informans, the second is an objectum extra potentiam movens.

Now the great error of many of the classical physicists and of the majority of modern scholastics is that they have looked upon the scientific world as a kind of formal sign directly and immediately revealing the absolute world condition. To view the scientific world in this light means to fall a prey to a great illusion. It means to destroy the scientific world's character as a sign, for it wipes out the true revelation it gives of the objective universe.

The physico-mathematical world is not a formal sign, but an instrumental sign of the absolute world condition. It constitutes an object in its own right, and must be known as such before it can reveal the objective universe. Like all instrumental signs, it hides the object it represents at the same time that it reveals it. And it is only by viewing the scientific world in this light that through it we can in some fashion come to know the objective world as it is in itself.

No other notion brings out so accurately the true character of the relation between the two worlds than this notion into which enters both instrumentality and signification. It explains how the physico-mathematical world can be at the same time completely closed in upon itself and completely opened to the objective world. It reveals why the criteria of the validity of the scientific structure can be both goodness and truth, with the goodness entirely subservient to the truth, why the scientific universe is at once practical and speculative, with the practical completely orientated towards the speculative, at once art and science, with the art entirely ordered to the science, (both in the sense in which fine art reveals an original, and in the sense in which useful art serves a purpose -- the practical purpose in this case being found in the speculative order). Neither pure instrumentality alone, nor pure formal signification can bring out all of these paradoxical elements and serve to establish them in their proper relations.

The physico-mathematical world is in many ways a particularly perfect type of instrumental sign and it tends towards the perfection of a formal sign. Even those elements in it which are not taken directly from the objective universe

and which consequently from one point of view serve to hide it, are introduced into it only to reveal the absolute world condition all the more. In this it is similar to a work of art into which the artist's own logos has been injected only for the purpose of revealing the original with greater clarity.

But the scientific world is an even more perfect sign than a work of art in that the fabrication found in it, while interposing an object between the mind and the real world, can never constitute an end in itself. The scientific world is art, but not simpliciter. It is essentially speculative knowledge, and as such its whole raison d'être consists in its orientation towards the real world. In this it is similar to a formal sign.

John of St. Thomas assigns five conditions which must be present if one thing is to be the sign of another. (22) First, the sign must be something distinct from both the object signified and the cognitive potency. This condition is fairly obvious and needs no comment. Secondly, it must have the nature of a representation. This establishes a transcendental relation between the sign and the thing signified. Thirdly, the sign must be more knowable than the

thing signified. By reconstructing the objective universe, by injecting his own logos into it, by introducing the rationality of mathematics, the scientist succeeds in rendering it more intelligible. Fourthly, the sign must be less perfect than the thing signified and inferior. This recalls to mind what we said in an earlier Chapter about the physico-mathematical world being worse than the real world precisely because it is better. It ever remains a mere substitute for the real world. Its role is purely functional. This means that over and above the transcendental relation mentioned a moment ago, there is a predicamental relation between the scientific world as sign and the absolute world condition as thing signified. This relation belongs to the species of relation that exists between a measure and a thing measured, (in the sense explained in Chapter VIII in connection with the various types of relation]. The absolute world condition is the measure of the physico-mathematical world. It is this predicamental relation and not the transcendental relation that constitutes the latter as the sign of the former. (23) The fifth condition laid down by John of St. Thomas is that the sign and the thing signified must be dissimilar. The vast difference between the scientific universe of discourse and the objective universe has already been sufficiently stressed.

The foregoing makes it clear that in experimental science the mind does not assimilate the objective world directly, but rather reflects it by constructing a schema of its own that is founded upon reality. But it is important to try to determine the nature of this schema and thus bring out as accurately as possible the exact character of the instrumental sign. We believe that this can be done by having recourse to the notion of isomorphism. Isomorphism, as the word implies, signifies identity of structure or form, and it is commonly defined in the following terms: Given two classes: S, composed of elements a, b, c, ..., and S', composed of elements a', b', c', ...; if the elements of S can be placed in one-one correspondence with those of S', in such a way that a corresponds to a', b corresponds to b', etc.; and if for every relation R between the elements of S (e.g. a R b) there exists a relation R' between the corresponding elements of S' (e.g. a' R b'), the two classes are said to be isomorphic. A familiar example of isomorphism is found in an ordinary map. There is identity of structure between the relations between the points on the map and the corresponding points on the countryside to which the map is related. It is important to insist upon the fact that isomorphism is not founded upon

a material correspondence between the elements involved, but the identity of structural form. It prescind from the proper nature of the matter to which the forms are applied. But this prescinding is not a negation. In fact if the heterogeneity of the matter of the different systems were destroyed, the isomorphism would also be destroyed.

Now this notion of isomorphism brings out the nature of the relation between the physico-mathematical world and the absolute world condition. For mathematical physics is a search for system and order. As we have seen, it constructs its own organized system, but in so doing it is determined in its every move, either directly or indirectly, by measurements made upon the real world. In spite of the arbitrary elements in measurement, the absolute world condition remains the measure of the measuring process, in such a way that although different codes of measurement employed in relation to the same world condition will render different results, as long as the same code is employed in relation to the same world condition, the results will be identical. (24) That is why, after the physicist has constructed his schema he is able to map it back upon nature and predict (25) natural phenomena.

As Duhem has observed, the relation between the

scientific world and the objective world may be compared to the relation between the form of a suit of armour and the form of the body of the knight who wears it. (26) There is always a similarity of structure in this relation no matter how imperfect the suit of armour may be. This similarity grows as the suit becomes more perfect, as the number of pieces of metal which compose it increases, and as its structure becomes more complex. At the limit the form of the suit would be identified with the natural form of the body. This limit can never be actually reached, obviously; but it can be indefinitely approached. And as the artificial form of the suit gets closer to the natural form of the body, it is at the same time drawing farther away from it in the sense that it is constantly becoming more artificial.

2. Logical Identity.

The gap that exists between the absolute world condition and the structures manufactured by the scientist is something that the mind must seek to bridge. It must seek to go beyond the relation of isomorphism of which we have been speaking and arrive at some kind of identity. In order to see how this may be accomplished, how what is at

once both reality and artifice can be erected into a unified object, it is necessary to have recourse to the notion of predication of identity.

Aristotle and St. Thomas speak of this notion in several places, notably in the fifth book of the Meta-
(27) (28)
physics and the fourth book of the Physics. In the latter text we read:

Genus potest cum additione unitatis vel identitatis praedicari de pluribus individuis existentibus in una specie, et similiter genus remotum de pluribus speciebus existentibus sub uno genere propinquo; neque tamen species de individuis, neque genus propinquum de speciebus diversis potest praedicari cum additione unitatis vel identitatis... Et huius assignat (Aristoteles) rationem: quia cum idem et diversum seu differens opponantur, ibi possumus identitatem dicere, ubi differentia non invenitur, sed non possumus dicere identitatem ubi invenitur differentia.

In order to make a predication of identity of things that are different it is necessary to ascend to a genus that is not divided by their proper differences. Aristotle and St. Thomas explain this by having recourse to examples taken from mathematics. Thus it is possible to say that a scalene triangle and an equilateral triangle are the same figure. But it would be incorrect to say that they are the same triangle. The reason is fairly obvious. For the one condition for identity is absence of difference. Now the

scalene triangle and the equilateral triangle divide the genus triangle by a difference that is proper to the triangle, since they are different species of triangle. That is why we cannot say that they are the same triangle without falling into a contradiction. But they do not differ by a difference of figure, since they both fall under the same difference which divides the genus figure, namely triangle. And that is why we can say that they are the same figure.

Aristotle and St. Thomas give another example taken from the realm of number. Even though it is impossible to say that ten cows and ten dogs are the same ten, it is possible to say that they are the same number. In other words, there are two different species of ten, but the same number. The same number is neither the ten cows (for then either the dogs would not be ten or they would be identified with the ten cows), nor the ten dogs (for similar reasons). It is neither the one nor the other determinately, but different from both. It is not different, however, in the sense of being non-ten, as three or twelve. It is ten, but indifferent to the particular species of ten.

From this example it is clear that the relation

of identity is something created by the mind. For to be the same number does not mean to be identical, otherwise the ten cows and the ten dogs would be the same. Hence the identity in question in this whole context is constituted by a relation of reason added to that which is predicable as genus of individuals or as remote genus of species.

What has been said of figures and numbers may be applied to the ratio entis. Both real being and logical being may be said to be the same being, provided that the ratio entis in question be not identified with either the one or the other. In other words, the ratio entis can be said to be the "same" only on condition that it be "other", that is to say, it can be the "same" only if it is not identified with any of the terms in relation to which it is said to be the same. It must be like the ration "ad" of relation, which is indifferent to "in esse", or "Non- in esse", or like mathematical quantity which is indifferent to real or logical being.

It is to be noted that this predication of identity is not tautological. When we say that a scalene triangle and an equilateral triangle are both the same figure, we

do not merely wish to say that figure is predicable of both of ~~them~~ in so far as both of them are figures. For the same could be said of triangle since both of them are triangles. Predication of identity does not merely have to do with what is the "same" in the species, namely the genus, or with what is the "same" in the individuals, namely the species. It has to do with the differences in their very difference - - not in an absolute way, of course, for that would make them absolutely identical, but in their relation to the genus that is predicated of them by identity. Thus, this predication is not made after the terms in question have been stripped of their difference, for any genus may be predicated of its inferiors in this way. On the contrary it presupposes the differences. It is this, in fact, that gives it its special significance.

That in relation to which the differences are said to be the "same" is something purely logical, namely the logical genus in so far as it takes on a potentiality that derives from our mode of conception. The indetermination in question is not found either in the terms themselves to which identity is attributed or in that which is attributed to them, for both a scalene and an equilateral triangle on the one hand, and figure on the other, are in themselves

definitely determined things. The indetermination is found in the figure in so far as it is considered as a predicable genus. In other words, predication of identity can exist only because it involves logical intentions.

It is clear, then, that by withdrawing into the potentiality of the logical order where differences can be blended it is possible to predicate the "same" of things that are essentially diverse, to unite into one things that are divided secundum rem. And this is of extreme importance for the question of the relation between scientific constructions and the absolute world.

In order to see why this is so, let us take a simple example. When after an ordinary process of measurement we declare that the proper length of a certain body is two meters, this statement can be taken in two ways. It may, in the first place, mean simply that a meter measure has been placed twice end to end along the body; in other words that the length of the body is equal to the length of two meters. As a matter of fact, however, when we say that a certain body has a length of two meters, we are not speaking formally of the relation of equality between the body and the meter placed twice end to end along its surface. We are not speaking

formally either of the absolute length of the body, nor the absolute length of the meter placed twice along its surface, nor of the relation of equality between the two, though all this is presupposed. In order to be able to say that a certain body has a length of two meters it is necessary to go beyond a mere relation of equality and arrive at identity. If the length of the body is equal to the length of two meters they are the same length, but they are not the same length of two meters, just as ten cows and ten dogs are the same number but not the same number ten.

In other words, we have seen that operational definitions do not allow absolute attributions, since the practical operation involved separates us from the terminus to which it is ordered. Now when we say that a body has a length of two meters we have in a certain sense surmounted the gap created by this separation, for merely to describe the measuring operation and to say that the body has a length of two meters are not the same thing. This has been done by ascending to a logical genus to which we have added the relation of identity. In this way it has become possible to predicate the "same length" of the body in question. But, we repeat, the same length is not the same length of two meters. In other words, we have attributed to the body a

logical genus which cannot be identified with it. We are not in the real order, but merely turned towards it. If in this predication we actually reached the real order there would be contradiction, for the length that is said to be the same for the body of two meters and for the meter placed twice along its surface would be identified with both of them and one would be two.

It is clear that this identity adds something to the unum secundum quid constituted by the operational experiment and the absolute condition of the world. By arriving at identity even though it be merely logical, we have in some way surmounted the diversity involved in the unum secundum quid, and have achieved a kind of counterfeit unum per se.

What has just been said about the simple process of measurement can be applied in a general way to all of the constructions manufactured by the scientist. Mathematical physics deals neither with the world of its own constructions as such, nor with the absolute world as such; it deals formally with a world that is a logical identity of the two.

But this logical identity is not an end in itself; it is only a means. And its purpose is to draw the scientist

closer to the absolute state of the universe. In so far as it keeps the scientist in the logical order, and in so far as the goal sought for is the world in se, experimental science must ever strive to escape from this purely logical identity and to draw ever closer to the real world. In other words, logical identity is not sufficient. Science must seek to surpass it by tending towards real identity. We have seen that mathematical physics is dialectics and that "omnis dialectica est ~~tentativa~~". From the construct which is the physico-mathematical world it must ever strive to reach the real world. To this dialectical movement we must now turn our attention.

3. Movement towards Real Identity.

That the scientific world is constantly in movement is a fact of history. But there are two things to be noted about this movement. First, it is something that is essential to the scientific world. Without it science would lose its meaning. In this experimental science differs radically from all the sciences in the strict sense, which, though caught in the flux of history and in some measure subject to it, are intrinsically independent of all movement. The reason for this character-

istic property of experimental science has already been emphasized: the scientific universe is essentially a dialectical construct which must ever seek to go beyond itself; it is a vehicle of progress and not a mansion of residence.

The second thing to be noted about this movement is that it has a very definite direction. "It is plain, writes Planck, "that when regarded as a whole, all the changes in the different views of the world of Physics do not constitute a rhythmical swing of the pendulum. On the contrary, we find a clear course of evolution making more or less steady progress in a definite direction." (29) From this point of view it is interesting and instructive to contrast the history of experimental science with the history of philosophy. Though philosophy in its inner essence is independent of movement, as we pointed out a moment ago, it appears to be much more a prey of the irrational flux of history. When viewed in its entirety, the history of philosophy presents no definite direction; it is constantly repeating and refuting itself.

As Poincaré has observed, to those who are unacquainted with the true meaning of experimental science

the ephemeral character of scientific views and the constant succession of new theories may seem to have the same aimlessness. As a matter of fact, however, these views and theories are continually tracing out a definite pattern.

"Sans doute, au premier abord, les théories nous semblent fragiles, et l'histoire de la science nous prouve qu'elles sont éphémères; elles ne meurent pas tout entières pourtant, et de chacune d'elles il reste quelque chose." (30)

The following comparison of Duhem brings out with great exactness the existence of a definite direction in the movement of science underneath a superficial appearance of aimlessness:

Celui qui jette un regard de courte durée sur les flots qui assaillent une grève ne voit pas la marée monter; il voit une lame se dresser, courir, déferler, couvrir une étroite bande de sable, puis se retirer en laissant à sec le terrain qui avait paru conquis; une nouvelle lame la suit, qui parfois va un peu plus loin que la précédente, parfois aussi n'atteint même pas le caillou que celle-ci avait mouillé. Mais sous ce mouvement superficiel de va-et-vient, un autre mouvement se produit, plus profond, plus lent, imperceptible à l'observateur d'un instant, mouvement progressif qui se poursuit toujours dans le même sens, et par lequel la mer monte sans cesse. Le va-et-vient des lames est l'image fidèle de ces tentatives d'explication qui ne s'élèvent que pour s'écrouler, qui ne s'avancent que pour reculer; au-dessous, se poursuit le progrès lent et constant de la classification naturelle dont le flux conquiert sans cesse de nouveaux territoires, et qui assure aux doctrines physiques la continuité d'une tradition." (31)

As science advances it often happens that the new theories which supplant each other appear, in their external form at least, to become increasingly divergent. This is in itself a significant fact, for it is a sign that the scientific world is becoming more and more a subjective construct. But no matter how divergent new theories may be, they are never born in a vacuum; there is always a continuity with the past. "It happens", says Weyl, "that broadened or more precise experiences and new discoveries do not overthrow old theories but simply correct them. One looks for the least possible change in the historically developed theory that will account for the new facts."⁽³²⁾ The Bohr atom did not destroy the Rutherford atom, but merely corrected and developed it. And the same is true of other changes through which physical science has passed. This does not refer merely to the gradual changes that take place in physics. Even in the so-called revolutions there is always continuity with the past. The formulation of the Quantum Theory, as Planck himself admits, was prepared by Lummer's, Pringsheim's, Ruben's and Kurlbaum's measurements of the spectral distribution of energy, by Lenard's experiments on the photoelectric effect, and by Franck and Hertz's experiments on the impact of electrons. In the same way, the Theory of

Relativity was prepared by Michelson's experiments on optical interference. But more than that, it is a mistake to believe, as many do, that the theory of Relativity and the theory of Quanta mean a complete destruction of classical physics. For it is necessary to assume the classical theory in order to define the experimental conditions in which the theory of Relativity obtains to a higher approximation. And that is why Einstein begins his first paper on the special theory of Relativity with the statement: "Let us have given a system of coordinates, in which the equations of Newtonian mechanics hold to the first approximation."

Like the system of Euclid, or Ptolemy, or Newton, which have served their turn, so the systems of Einstein and Heisenberg may give way to some fuller realization of the world. But in each evolution of scientific thought new words are set to old music, and that which has gone before is not destroyed but refocussed. Amid all our faulty attempts at expression the kernel of scientific truth steadily grows; and of this truth it may be said — The more it changes, the more it remains the same thing. (33)

It is clear, then, that the development of the scientific world does not take place in a haphazard fashion but follows a very definite direction. At the end of chapter V we noted that there is a similarity between experimental science and the type of knowledge described by Russell in Mysticism and Logic in which deductions are drawn from

freely chosen hypotheses. Now it is necessary to see that there is also a vast difference between them. For in the type of knowledge considered by Russell, there is no direction; we may, as he says, take any hypothesis which seems amusing. Experimental science, on the contrary, is knowledge that is essentially ordered towards a definite goal.

Now the relation between the scientific world and the absolute world condition cannot be properly grasped unless it be viewed in terms of a movement that is essential to the former and essentially orientated towards the latter. And we know of no way of bringing out accurately this dynamic relation except by having recourse to a notion which plays its most familiar role in mathematics and especially in the calculus, but which can be fruitfully applied to other fields as well. We have in mind the notion of a variable ordered towards a limit. A brief analysis of this notion will throw great light upon the orientation of the scientific universe towards the absolute world condition. (34)

This notion, in its most simple and generic form, is usually expressed in terms similar to the following: A variable quantity x is said to tend towards a determined

limit if the successive values of x approach a certain fixed number a in such a way that the difference $x-a$ becomes less than any given number ϵ , no matter how small it may be. Thus, for example, the number 2 may be defined as the limit towards which the following series tends:

$$1, 1 + \frac{1}{2}, 1 + \frac{1}{2} + \frac{1}{4}, \dots$$

In the same way, a circle may be defined as the limit towards which tends a regular inscribed polygon whose sides increase indefinitely. Applying this now to the question in hand, we hold that the scientific world may be considered as a variable quantity which by passing through the successive stages of its evolution approaches the absolute world condition as its limit.

An analysis of this notion reveals that it involves both a heterogeneity and a homogeneity, both an otherness and a likeness. The heterogeneity, the otherness, consists in the fact that there are necessarily two terms which belong to different orders or to different species: e.g. discontinuous-continuous; point-line; line-surface; polygon-circle; curved-straight, etc. Heterogeneity is essential to the notion of limit, even under the aspect in which the limit is considered as a value of the variable term: it is precisely in its heterogeneity that it is the limit value of the variable. It is not a polygon (no matter of how many sides) that is the limit of the polygon

whose sides increase indefinitely, but a circle. On the other hand, even though the polygon becomes more and more like a circle, it is not changing in its species (in which it remains essentially a polygon), but merely in its values. Now this heterogeneity is found in the relation between the scientific universe and the absolute universe. A great deal of emphasis has already been laid upon their essential otherness. It is not an advanced stage in its own development that the scientific world is attempting to reach in the movement that is essential to it, but something beyond itself and essentially other than itself, namely the absolute state of the universe. On the other hand, even though the scientific world in its development comes ever closer to this absolute state, it does not in any degree lose the otherness which derives from the fact that it is essentially a construct. On the contrary, this otherness increases, just as the polygon becomes, in a sense, more of a polygon, i.e. a many-sided figure, the more its sides are increased.

But along with this heterogeneity there is an essential homogeneity involved. This is evident by the very fact that one term is said to be the limit of the other. When we say that x has a as its limit ($\lim x = a$), the fixed term a is considered as the limit value of x , in such a way that

$\lim (x - a) = 0$. From this point of view the heterogeneous terms are considered as belonging to the same order, that is to say, one is considered as a value or a case of the other. A polygon with a hundred sides is considered as a case of the polygon; a circle is considered (in a hypothetical way) as another case — the limit case; if the limit could be reached the case of the polygon which is the circle would differ from all the other polygons in that it would have the greatest number of sides possible. From this point of view there is an order of continuity between the variable and the limit. And it must be noted that the "more" or "less" of the formal order of the variable quantity is not merely quantitative. That is to say, a certain given value of the variable is not merely greater than any preceding value; it is at the same time more like the limit. In other words, by running through its values the variable is related to the formal structure of the limit. The increasing structural similarity tends towards structural identity.

A homogeneity of this kind is found in the relation between the scientific world and the absolute world. The former tends to issue into the latter. If the limit of scientific development could be reached there would be

identity of structure between the two. And as the limit is approached the likeness of structure which we explained above by the notion of isomorphism, increases. While at any given stage of the development there is a certain likeness of structure between the two worlds, it is inadequate and often extremely misleading to consider this static relation independently of the dynamic relation that the movement which is essential to the scientific world involves. This is suggested in the following passage of Sir Arthur Eddington:

Scientific discovery is like the fitting together of the pieces of a great jig-saw puzzle; a revolution of science does not mean that the pieces already arranged and interlocked have to be dispersed; it means that in fitting on fresh pieces we have had to revise our impression of what the puzzle-picture is going to be like. One day you ask the scientist how he is getting on; he replies, "Finely. I have very nearly finished this piece of blue sky." Another day you ask how the sky is progressing and are told, "I have added a lot more, but it was sea, not sky; there's a boat floating on the top of it." Perhaps next time it will have turned out to be a parasol upside down; but our friend is still enthusiastically delighted with the progress he is making. The scientist has his guesses as to how the finished picture will work out; he depends largely on these in his search for other pieces to fit, but his guesses are modified from time to time by unexpected developments as the fitting proceeds. These revolutions of thought as to the final picture do not cause the scientist to lose faith in his handiwork, for he is aware that the completed portion is growing steadily. Those who look over his shoulder and use the present partially developed picture for the purposes outside science, do so at their own risk.(35)

There is, then, in the notion of limit the paradox of heterogeneity and homogeneity. And the key to this paradox, as has just been intimated, is found in movement. For one term is ordered towards another as its limit, not in its proper specific character, but only in so far as it is considered as a variable whose successive values approach the term which is the limit. These successive values must be indefinite; between any given value and the limit there must always be an infinity of other possible values in potency. But this potential infinity is not sufficient. It is merely the foundation of something more, namely a progression, a movement, a becoming. Because of this movement the difference between the two terms decreases indefinitely. In this way the variable tends to enclose the limit as its own final value. Heterogeneity tends towards homogeneity. The variable tends to bound itself by going beyond itself, that is to say by going beyond any value actually given within itself; it tends to break through its own form and thus destroy itself by taking on the form of the limit. ⁽³⁶⁾ In other words, both the variable and the limit have a double state: an absolute state which consists in their irreducible otherness, and a state of becoming by which they tend to reduce this otherness to sameness. The variable is always essentially other than

the limit, but at the same time it is always becoming the limit. In the same way, the limit has an absolute state by which it is essentially different from the variable, but at the same time it has a state of becoming -- a state of "coming from" the variable. "The limit must be coming from the otherness that is the variable, as if it were pre-contained in that otherness. The variable -- whose proper values are being more and more actualized, so that the variable itself is becoming more and more the self that it ever more can be -- must at the same time be moving away from itself and becoming identical with what is otherness to it, viz. the limit." (37) In so far as the limit is considered as coming from the variable it may be said to be generated by the progression of the variable. Thus this progression triumphs over the givenness of the limit and in this sense rationalizes the irrationality of this mere givenness.

This movement of which we have been speaking is sui generis, for by its very nature it is a movement that can never arrive. Whereas the terminus of every other movement, such as the becoming of a house, is defined by the possibility of its actually being reached (whether it actually will be reached or not) the limit of this movement

is defined by the impossibility of its being reached. Whereas the terminus of movement in the ordinary sense can still be considered the terminus even though the movement towards it has actually ceased, the limit of this movement ceases to be a limit once the getting closer ceases. In other words, the notion of limit supposes an actual and indefinitely prolonged movement. Just as all relations consist in an "esse ad", so all movements are towards something other. But just as some relations are of such a nature that they cannot "be in" that "toward" which they are, so this movement cannot actually reach the limit towards which it tends. From one point of view this movement seems to be an end in itself, since it can never arrive at anything beyond itself. But from another point of view, it is not an end in itself, since it must ever tend towards the limit which is beyond itself.

Now all this has an application to the relation between the scientific world and the objective world. Both of them have an absolute state by which they are essentially heterogeneous. But they also have a state of becoming which tends to reduce this irreducible heterogeneity to homogeneity. In so far as the scientific world is concerned this state of becoming consists in a continuous development by which it

draws ever closer to the objective world. In so far as the objective world is concerned this state of becoming does not, obviously, mean a real change; it merely means that as the scientific world draws closer to the absolute world, the latter may be considered as coming from the former. In this way, the absolute world condition may be viewed as being generated by the construction of the scientist; thus its pure givenness is triumphed over and the irrationality of this givenness rationalized. As we remarked in Chapter IV, if the scientist could reach his goal, man would be God. But there is one difference to be noted here between the movement of a variable towards a limit and the movement of the scientific world towards the objective world. In the former case the limit is already known before the movement towards it begins. In the latter case, this is not true: the absolute world is an unknown quantity that gradually reveals itself as the movement towards it progresses. In this way the state of becoming of the objective world has more of the nature of a generation.

It is clear that the objective world as a limit cannot be reached by the progress of science. The aim of science, writes Planck, "is an incessant struggle towards a goal which can never be reached. Because the goal is of

its very nature unattainable. It is something that is essentially metaphysical and as such is always again and again beyond each achievement." (38) The very method to which experimental science and especially mathematical physics is committed makes it impossible for it to ever reach the objective universe as it is in se. And yet by a strange paradox, it is only by remaining faithful to this method that it can be carried closer and closer to this goal. (39)

All this brings us back to what was said earlier in this study: experimental science is essentially a vehicle of progress and can never become a mansion of residence. And to consider it as a mansion of residence is the most effective way of destroying its true relation to the absolute world condition. From this point of view, the movement of the scientific world may be considered as an end in itself, and in this sense we may accept the dictum of Gotthold Lessing to which frequent reference is found in the writings of modern scientists: "Not the possession of truth but the effort in struggling to attain it brings joy to the researcher." (40) But from another point of view it is obvious that the movement of the scientific world is not an end in itself. The end must ever remain the absolute world condition. The scientist who loses himself in the develop-

ment of his own subjective constructions is not true to his science. It must be noted, moreover, that while it is better to be able to move towards truth than not to be able to approach it at all, it is absolutely speaking far better to be in the full possession of truth than merely to be approaching it.

It is obvious that the reason why the variable cannot arrive at the limit is that this arrival would involve a contradiction. The limit of a polygon would be both a circle and a polygon, that is to say, both a circle and a non-circle, both a one-sided and a many-sided figure, both an unbroken and a broken line. This contradiction is an essential condition for terms to be related as variable and limit. When it is stated that a polygon and a circle meet at infinity, this merely means that they would meet if per impossible "at infinity" could be. The variable tends towards its ultimate value and at the same time at something that is essentially other than any of its values. In other words, the tendency to realize itself is a tendency to destroy itself. But this does not mean that the dialectical movement towards the limit is in itself contradictory and meaningless. The contradiction that would be is only at the limit, which cannot be attained. The movement itself

cannot be considered contradictory simply because it cannot attain a contradiction. The possibility of this movement does not depend upon the possibility of attaining the limit but upon the possibility of considering the term toward which the movement tends as the limit of this movement. The movement in itself is meaningful precisely because it never goes beyond the stage of "being towards".

Now the movement of the scientific world is a movement towards contradiction. This has already been alluded to on several occasions throughout our study. We have seen in a general way that the scientific universe in seeking to posit itself more fully tends to negate itself and to vanish into emptiness. Several particular forms of this tendency towards contradiction have already been indicated. But it is of extreme importance to examine this question more closely here, for nothing could bring out more clearly and fully the noetic structure of the scientific world. And this can best be done by showing that the most fundamental and most proper principles of experimental science are such that they could not be really true without contradiction, that is, they could not be true without being false. These fundamental principles are the methodological principles such as the principles of definition, of

identity, of unity, of order, of induction, of simplicity, etc. Let us consider a few examples in detail.

The first example to be examined is the principle of definition. We have seen that in mathematical physics all definitions are in terms of operations of measurements. Now both from the point of view of measurement and from the point of view of operation this principle of definition involves mathematical physics in a movement towards a limit, the attainment of which would imply a contradiction. In so far as measurement is concerned this is evident from all that was said in Chapter VIII about the search for a minima mensura. Progress in measurement must consist in a movement towards greater precision and certitude. The limit of this movement would be an absolutely minimum measure. But such a measure is a contradiction since it implies a quantity that is at once continuous and non-continuous.

A similar movement towards contradiction is discovered when the nature of operational definitions is analyzed. We saw in Chapter IV that these definitions express a mixture of nature and art, of a quod and a quo, of subject and object. The thing defined is neither a pure operation, nor a pure objective quantity, but an inextricable mixture of the two. In other words, the definitum is only

an unum per accidens and not an unum per se. The unity is conferred upon it by the mind. If it were a per se unity, the world would be at the same time nature and a human work of art. This is the position of the Marxists.

It is clear, then, that while operational definitions are destined to help us to know the real in se (for operations are not carried on for their own sake, and physics does not consist in mere descriptions of what physicists do), a reality which could be known in se by means of operational definitions is an impossibility. By means of operational definitions we tend towards a limit which cannot be attained by means of operational definitions. The practical operation involved separates us from the terminus towards which it leads us. Arrival at the limit would involve a complete arrival at the limit and a complete separation from it at the same time.

Another good example is found in the principle of induction. Poincaré's statement that all generalization is an hypothesis is true of the type of induction that is characteristic of experimental science -- induction by enumeration. When a general proposition "Every A is B" is founded merely upon the enumeration: "A is B", "A is B", "A is B", etc., it cannot be true. For if "Every A is B" is true, "Some A is non-B" is false. But in so far as

"Every A is B" is founded merely upon a collection of particular cases, it cannot be said that "Some A is non-B" is false. Hence, "Every A is B" is a logical proposition that tends towards reality without being able to attain it. It is, so to speak, a relation "ad" without "inesse". If "Every A is B" were true in so far as founded upon a collection, all A's would not only be alike; they would be identified — they would be the same A. For if induction by enumeration could give a universal in the strict sense, this universal would be the particular cases, and the particular cases would be the same particular case. Hence there would be contradiction.

The principle of causality as employed in physics offers a third example for our analysis. Events are knowable by us only in so far as they are determined. Hence the future can be adequately known by us only to the extent in which it is already determined in the present. The future is, of course, of great importance in the fluid universe that constitutes the object of physics. The future is a part of our world, for without before and after there could be no time. Now it is evident that there must be a certain amount of determination in the relation between present and future, since the universe is not run by pure

chance. The question is, however, is this determination absolute?

It is the purpose of science to get at the determination in the cosmos. In order that no real determination may escape it, it must consider all the indetermination that appears as merely provisional. It is necessary for science to act in practice as though the determination in the world were absolute and without limitation. It must, as Laplace said, consider the present state of the universe as the effect of its anterior state and as the cause of the state to follow." In this sense, then, science must take determinism as a methodological principle.

But this methodological principle cannot be made a real principle without a contradiction being involved. For in order for this principle to be real, it would have to be verified in experience. Such a verification, However, is impossible. More than that, even if it were possible the principle would be absurd. For this principle has to do with the future, and with the future in its entirety. Hence the verification of it would have to mean verification for the whole future. The known present cannot

serve to confirm it. Now if the future in its entirety were present to us, this principle would be useless, for it would be a pure tautology. In order for it to have any meaning at all, it is necessary that the present truth of the future be future in relation to us. If the truth of the future were present, the principle would not need to be confirmed in the present experience.

In other words: on the one hand, the validity of the principle which affirms the present truth of the future depends upon the future as non-present. But the truth considered by this principle is not known as certain except in so far as the future is effectively present. On the other hand, once this future which must confirm the principle is present, it no longer fulfills the conditions required for a confirmation that would establish the validity of the principle for the whole future. It is precisely in so far as it is future and not in so far as it is present that the future is not certain. The verification of the principle of determinism would have to consist in rendering the future evident in the present by means of the future which is non-evident, in other words, to make the future certain by means of the uncertain future.

It is clear, then, that the principle of causality in physics has a meaning, and can be said to be true, only in so far as it is not really true. It is merely a methodological principle; it tells us how to proceed and not what things are objectively. In so far as it tells us how to proceed it is true. In so far as it attempts to tell us how things are in themselves, it involves a contradiction.

These examples suffice to show that the movement of the scientific world tends towards a contradiction. The meaning of this tendency will be made clearer if we return to the notion of predication of identity discussed earlier in this Chapter. We saw that in this predication we consider the terms which are either specifically or individually different not merely in what they have in common absolutely, but in their very formal differences. It is this, in fact, which characterizes predication of identity. Polygon and circle, for example, have an identity in their very differences (considered of course in relation to their remote genus). Now in our discussion of the notion of limit we saw that it supposes two terms which are at once the same and different. That is to say, the limit must be comprised in the variable; since it is the limit of the variable it must be considered as comprised in the order of the variable. Now this identity

which the limit supposes is accomplished by passing to the genus that is predicable by identity. Consequently, the notion of limit is founded on a predication of identity of the differences.

Now the dialectical movement consists precisely in the tendency of one difference towards another difference within their abstract identity. This identity in the difference is a principle of dialectical movement, but it is not the terminus. The tendency of one difference towards another difference within one abstract identity, is a tendency towards an identity of another order, namely real identity. It is the realization of this real identity that is impossible.

All this makes it evident once again how much truth there is in Meyerson's central theme that if science could arrive at the goal which it is constantly seeking the result would be a vast tautology, and how correct DeBroglie is in quoting in connection with his description of Meyerson's doctrine the remark of Vallery to the effect that what science seeks to achieve is an absurdity. It is clear, however, that this absurdity is not merely that of a vast tautology, but that of an intrinsic contradiction.

If the identities which we posit in science were real identities, the logical and the real order would be identified.

In spite of the contradiction at the limit, science tends to emerge from mere logical identity to real identity. We find this tendency on every level of the scientific structure. In the definitions we tend to pass from logical identity to the absolute world condition, even though the arrival would be contradictory. The same is true of scientific laws: generalization tends towards a universal nature, even though if such a nature were achieved it would be contradictory. The case of hypotheses is very much the same: they are destined to make the truth known, but they cannot provide truth ex propriis. The truth which they help to reveal does not depend in any way on them. If hypotheses could be identified with their terminus (which is known by experience) they would destroy themselves as hypotheses. Finally, scientific deduction is orientated towards a true conclusion. But it cannot provide this true conclusion, that is to say, the conclusion cannot be true qua conclusion. Between the conclusion taken as such and the truth that it permits us to discover there is only an accidental connection, since another deduction could serve

to reveal the same truth.

Since therefore, the initial definitions cannot give us the real as it exists in itself; since physical laws are only generalizations which are never really founded in any absolute sense; and since deductions cannot be true as such, it is evident that the physical world cannot be identified with the absolute world condition. It is, consequently, merely a construction of the mind -- a construction which imitates more or less the absolute world. It is turned towards the absolute world, and can approach it indefinitely without ever being able to reach it.

The Marxists have sought for a proof of their dialectical materialism in this characteristic nature of science. Their line of argument may be reduced to this: The methodological principles are true. But if they are true, the world is contradictory; it is at the same time affirmation and negation of itself, at the same time true and false; there is no absolute truth. Consequently, since this state of things cannot satisfy speculative thought, man is not made for thought, but for action. (41) The error of this argument consists in an exploitation of the ambiguity of the term "true" in the proposition "the methodological principles are true". The foregoing analysis has

made it clear that they are true only as logical principles and not in the sense in which they would signify the truth of the world in se. In other words, there is a confusion here between the logical and the real order. Logical possibilities have greater freedom than real possibilities. In the logical order it is reasonable to build structures with elements that are not capable of realization. Nor does the lack of this capability prevent the possibility of drawing closer and closer to the real. It is possible for logical constructions to comprehend being and non-being at the same time. "Non homo", for example, is an indetermination which comprises at the same time both being and non-being.

Because the scientific world is a logical construction, because it is dialectics, there is deep within it an essential conflict from which it ever seeks to deliver itself. In the first place, there is the conflict between being and non-being. Experimental science tends towards being by means of the impossible. It tends towards the real by means of the purely logical. There is, moreover, a conflict between the one and the many; it tends towards the one by means of the many. There is a conflict between the speculative and the practical, between science and art.

Because of its operationalism, mathematical physics tends in its experimentation towards the res in its physical, entitative status; at the same time it tends towards pure science in the intellect. For this reason it tends to issue into two contrary directions: on the one hand pure science, independent of physical operations of things in their entitative status; on the other hand, pure operation by which things are mastered through action. That is why there will always be two fundamental tendencies in mathematical physics: one towards a kind of Platonic mathematicism, and the other towards a kind of dialectical materialism whose ultimate aim is to master things through and for practical action.

Perhaps the general drift of this whole Chapter can be summed up by saying that the scientific world is a structure composed of both the subjective and the objective and that if the goal towards which it strives could be reached it would be at the same time completely subjective and completely objective. For this reason it is necessary before bringing this study to a close, to turn our attention to the question of the subjective and the objective in mathematical physics. It has been customary for scientists

to claim that philosophical and theological knowledge are essentially subjective and that only experimental science is capable of giving true objective knowledge. We must try to see why just the opposite is the case.

CHAPTER TWELVE

OBJECTIVE SUBJECTIVITY

1. Subjectivity and Objectivity.

As we explained earlier in this essay, all knowledge is by its very nature objective, since to know is to become another thing in its very otherness. But not all knowledge is equally objective, for there is a direct proportion between objectivity and the perfection of the knower. In God alone is perfect objectivity found.

Now the word subject can be taken in two ways. In the first place, it can be understood to mean simply a knower. In this sense, all knowledge, in so far as it implies that a known thing is in a knower (cognitum est in cognoscente) involves both a subject and an object. In its proper meaning, however, the term subject implies a state of subjection. This involves passivity, and conse-

quently limitation and imperfection.

When the term is understood in the first way there is no opposition between it and objectivity. In this sense it may be applied even to God, in Whom knowledge is so perfect and therefore so objective that there is no real distinction between the knower, the knowledge and the object known. In its proper meaning, however, there is an opposition between it and objectivity. In fact, a pure subject in this sense is an object which does not know at all.

Now in the knowledge of all creatures, the knower is in some measure a subject in the proper sense of the word. For all creatures receive their knowledge from without and their state of being recipients involves passivity and subjection. This is true even of the angels, for their intelligible species are impressed upon them by God. An object, in its full formality as object, is above every created intellect, for in so far as an intellect is a subject in the proper sense of the term it is measured by the object, and a measure, from the point of view in which it is a measure, is always more perfect than the thing measured. Creatures cannot be the measure of objects because

their being is not the source of these objects. Their cognitive powers cannot reach the very root of these objects because they are not the root.

This subjectivity (in the sense of the term in which it is opposed to objectivity), already found in the highest angel, increases as we descend the hierarchy of created beings. It is found in the fullest measure in which it can be found in sense knowledge, for here a material organ, which in itself is a pure subject and hence absolutely opposed to objectivity, enters into the very intrinsic structure of the cognitive power. But already in the human intellect (which is the lowest type of intellect that could possibly exist) a large measure of subjectivity is found. For the human intellect has this in common with the senses that it receives its species from things. This involves a greater measure of subjection and passivity than is found in angelic knowledge in which the species though coming from the outside, do not come from things (they are, in fact, prior to things) but from God. Now the obscurity arising from this passive subjectivity forces the human intellect to have recourse to a kind of active subjectivity. That is to say, it can know only by constructing logical beings, by composing and dividing in its

judgments, by fabricating formal discourses in its processes of reasoning. This active subjectivity is also an obstacle to pure objectivity. For all of these reasons it is necessary to agree with Eddington that "it is the inexorable law of our acquaintance with the external world that that which is presented for knowing becomes transformed in the process of knowing."⁽¹⁾

But this subjectivity of the human intellect must not be exaggerated. For there is a sense in which it is true to say that the mind is capable of a kind of pure objectivity. In its ordinary processes and in the way in which it functions in the philosophical sciences it is able to disengage the quod quid est of things — their objective essences. There is always a certain amount of subjectivity involved, to be sure, but it is a kind of subjectivity that attaches not so much to that which is known as to the way in which it is known or the state in which it is known. To use Scholastic terminology, it is a subjectivity that affects rather the modus quo cognoscitur than id quod cognoscitur. There is, of course, a kind of subjective element entering into the object known, but it is more of a negative than a positive thing. That is to say, in comparison with the object in se the object as known is always

imperfect and inadequate. But this does not transform the object in the sense of making it a new object. Definitions of the mind can apply with perfect truth to things as they are in se. In other words, the mind does not project a new positive element into the essence it knows in such a way that this essence is reconstructed into something different. In this the intellect differs essentially from the senses which in knowing their object necessarily transform it into something different because of the physical interaction which takes place between object and organ.

Now, as we have seen, physics deals with sensible things under the aspect in which they are the most profoundly immersed in sensible matter. That is why the obscurity of sensible matter and the subjectivity and anthropomorphism attached to sensibility are of major concern for it. We have seen what means it has devised to triumph over these obstacles and how great has been their success. We have noted that Planck was correct in writing "that as the view of the physical world is perfected, it simultaneously recedes from the world of sense; and this process is tantamount to an approach to the world of reality." (2) But we have also insisted upon the fact that

this movement away from the world of sense and towards the world of reality is at the same time a movement away from the world of reality towards a subjective world in such a way that if it be asked which of the two famous tables of Eddington, ⁽³⁾ (the familiar table and the scientific table) is the more objective and which the more subjective, it is necessary to make a very important distinction: the scientific table is at once more subjective because of the essential subjectivity of scientific method, and more objective, i.e. more like a table as it is known by a superior intellect.

The profound subjectivity of the physico-mathematical world is now generally admitted by all the better scientists. ⁽⁴⁾ But it is important to try to determine the nature of this subjectivity. By a strange paradox, the movement of science away from the sense world towards the world of reality is at the same time a movement away from the world of reality to a world that is, from one point of view, subjective in essentially the same way as the sense world. What we mean here is that, just as the sense world is subjective in a way that puts a positive subjective element into the object and reconstructs it to the extent of transforming it into something different, so mathematical

physics projects a positive subjective element into its object and reconstructs it into something essentially different. There is therefore, a sharp distinction to be drawn between the type of subjectivity that is characteristic of experimental science and the type mentioned a few moments ago that accompanies other kinds of intellectual knowledge. In the latter case, art merely surrounds the object, whereas in the case of experimental science art enters intrinsically into the object and constructs it. And just as in the case of sense knowledge the objective and the subjective are so interpenetrated that it is impossible for the knower to draw a line between them and thus set forth the object in its pure objectivity, so in mathematical physics the subjective and the objective are so fused that it is impossible for the scientist to disentangle them. ⁽⁵⁾ In order to do this he would have to have direct intellectual intuition of the real world.

In the course of this study we have endeavored to indicate the most important ways in which subjectivity enters into scientific knowledge. ⁽⁶⁾ All of them, as has already been suggested, may be traced back to two sources. First there is a physical intrusion of the subject in the experimental operation in which the object known becomes

irretrievably confused with the way by which it is known. Secondly, there is an intellectual intrusion consisting in a a priori hypothetical construction. Mathematical physics has no other means of getting to know reality except by refashioning it in these two ways. It cannot assimilate reality directly; it can only reconstruct it. It is, as Einstein and Infeld have suggested, ⁽⁷⁾ in a position something like that of a man trying to understand the mechanism of a closed watch. Since he has no way of opening the case, he cannot know the inside of the watch as it is in itself. All he can do is construct something that will account for the moving of the hands and the ticking. As Meyerson has remarked, "nous voulons le réel conforme à la raison, mais nous comprenons en même temps que s'il était, ⁽⁸⁾ la raison devrait pouvoir le recréer."

Since, then, the scientific world is formally a subjective construction, it follows that its constitution is predetermined by the methodological principles employed in constructing it. "Operabilia sunt quorum principia sunt in nobis." It also follows that to the extent in which it is so predetermined it can be known a priori by a close analysis of these principles and their implications. This, it seems, is the gist of Eddington's The Philosophy

of Physical Science, the substance of which he has expressed in the following passages:

Let us suppose that an ichthyologist is exploring the life of the ocean. He casts a net into the water and brings up a fishy assortment. Surveying his catch, he proceeds in the usual manner of a scientist to systematise what it reveals. He arrives at two generalisations:

- (1) No sea-creature is less than two inches long.
 - (2) All sea-creatures have gills.
- These are both true of his catch, and he assumes tentatively that they will remain true however often he repeats it.

In applying this analogy, the catch stands for the body of knowledge which constitutes physical science, and the net for the sensory and intellectual equipment which we use in obtaining it. The casting of the net corresponds to observation; for knowledge which has not been or could not be obtained by observation is not admitted into physical science.

An onlooker may object that the first generalisation is wrong. "There are plenty of sea-creatures under two inches long, only your net is not adapted to catch them." The ichthyologist dismisses this objection contemptuously. "Anything uncatchable by my net is ipso facto outside the scope of ichthyological knowledge, and is not part of the kingdom of fishes which has been defined as the theme of ichthyological knowledge. In short, what my net can't catch isn't fish." Or -- to translate this analogy -- "If you are not simply guessing, you are claiming a knowledge of the physical universe discovered in some other way than by the methods of physical science, and admittedly unverifiable by such methods. You are a metaphysician. Bah,"

The dispute arises, as many disputes do, because the protagonists are talking about different things. The onlooker has in mind an objective kingdom of fishes. The ichthyologist is not concerned as to whether the fishes he is talking about form an objective or subjective class; the property that matters is that they are catchable. His generalisation is perfectly true of the class

of creatures he is talking about -- a selected class perhaps, but he would not be interested in making generalizations about any other class. Dropping analogy, if we take observation as the basis of physical science, and insist that its assertions must be verifiable by observation we impose a selective test on the knowledge which is admitted as physical. The selection is subjective, because it depends on the sensory and intellectual equipment which is our means of acquiring observational knowledge. It is to such subjectively-selected knowledge, and to the universe which it is formulated to describe, that the generalizations of physics -- the so-called laws of nature -- apply.

It is only with the recent development of epistemological methods in physics that we have come to realize the far-reaching effects of this subjective selection of its subject matter. We may at first, like the onlooker, be inclined to think that physics has missed its way, and has not reached the purely objective world which, we take it for granted, it was trying to describe. Its generalizations, if they refer to an objective world, are or may be rendered fallacious through the selection. But that amounts to condemning observationally grounded science as a failure because a purely objective world is not to be reached by observation...

Suppose that a more tactful onlooker makes a rather different suggestion: "I realize that you are right in refusing your friend's hypothesis of uncatchable fish, which cannot be verified by any tests you and I would consider valid. By keeping to your own method of study, you have reached a generalisation of the highest importance to fishmongers, who would not be interested in generalizations about uncatchable fish. Since these generalisations are so important, I would like to help you. You arrived at your generalisation in the traditional way by examining the fish. May I point out that you could have arrived more easily at the same generalisation by examining the net and the method of using it?"

The first onlooker is a metaphysician who despises physics on account of its limitations;

the second onlooker is an epistemologist who can help physics because of its limitations. It just because of the limited - - some might say, perverted - - aim of physics that such help is possible . . .

Generalisations that can be reached epistemologically have a security which is denied to those that can be reached empirically... some laws of nature may have an epistemological origin. These are compulsory; and when their epistemological origin is established, we have a right to our expectation that they will be obeyed invariably and universally. The process of observing, of which they are a consequence, is independent of time or place.(9)

It would take us too far afield to analyze and assess the validity of the development and applications which Eddington subsequently makes of the principles laid down in these passages. But after all that has been said about the subjective construction of the scientific world we do not see how the principles themselves can be called into question. Moreover, we feel the implications of these principles are so far reaching that all of the laws of physics without exception must be recognized as subjective. (10)

Later in the same work Eddington lays great stress upon a point that is vital for the question which forms the subject of this chapter: the scientific world is not simply discovered, it is manufactured by the scientist:

The question I am going to raise is - - how much do we discover and how much do we manufacture by our experiments? When the late

Lord Rutherford showed us the atomic nucleus did he find it or did he make it? It will not affect our admiration of his achievement either way -- only we should rather like to know which he did. The question is one that scarcely admits of a definite answer. It turns on a matter of expression, like the question whether the spectroscope finds or whether it makes the green colour which it shows us. But since most people are probably under the impression that Rutherford found the atomic nucleus, I will make myself advocate of the view that he made it. The tendency of writers on quantum theory has been perhaps to go farther than I do in emphasising the physical interference of our experiments with the objects which we study. It is said that the experiment puts the atoms or the radiation into the state whose characteristics we measure. I shall call this Procrustean treatment. Procrustes, you will remember, stretched or chopped down his guests to fit the bed he had constructed. But perhaps you have not heard the rest of the story. He measured them up before they left next morning, and wrote a learned paper "On the Uniformity of Stature of Travellers" for the Anthropological Society of Attica . . .

Suppose an artist puts forward the fantastic theory that the form of a human head exists in a rough-shaped block of marble. All our rational instinct is roused against such an anthropomorphic speculation. It is inconceivable that Nature should have placed such a form inside the block. But the artist proceeds to verify his theory experimentally— with quite rudimentary apparatus too. Merely using a chisel to separate the form for our inspection, he triumphantly proves his theory. Was it in this way that Rutherford rendered concrete the nucleus which is scientific imagination had created? . . .

It is difficult to see where, if at all, a line can be drawn. The question does not merely concern light waves, since in modern physics form, particularly wave form, is at the root of everything. If no line can be drawn, we have the alarming thought that the physical analyst is

an artist in disguise, weaving his imagination into everything - - and unfortunately not wholly devoid of the technical skill to realise his imagination in concrete form . . .

The question is raised whether the experimenter really provides such an effective control on the imagination of the theorist as is usually supposed. Certainly he is an incorruptible watch-dog who will not allow anything to pass which is not observationally true. But there are two ways of doing that - - as Procrustes realised. One is to expose the falsity of an assertion. The other is to alter things a bit so as to make the assertion true. And it is admitted that our experiments do alter things. (11)

All this undoubtedly conjures up the dreadful spectre of idealism in the minds of many and particularly the neo-Scholastics for whom the stigmatizing phrase "ducit ad subjectivism" is sufficient to demolish every doctrine which does not propose the univocal type of realism which they consider inseparable from all knowledge. As a matter of fact however, it is only by recognizing the essential subjectivity of scientific knowledge that one can be a true realist. It is for this reason that we have entitled this Chapter "Objective Subjectivity". Most of the critics who have belabored with the redoubtable club of accusation of idealism Eddington and other modern scientists who have tried to bring to light this subjectivity are far more idealists than their victims. For they project into the objective world something that is essentially the product

of the mind. They are in many respects worse than the Platonists of whom St. Thomas writes: "Ex hoc in sua positione erravit (Plato) quia credidit quod modus rei intellectae in suo esse sit sicut modus intelligendi rem ipsam." (12) From what was said in the last Chapter about logical identity it is evident that they identify the logical and the real in reality, and that is essentially idealism. Nor can the subjectivity of scientific knowledge be considered a falsification of reality, as Professor De Koninck has pointed out:

Ne disons pas que les concepts de la science reposent en définitive sur une distorsion du monde et que dès lors les documents du physicien sont par avance forgés et trahissent la réalité. Mais justement il ne faut pas se laisser abuser par cette distorsion. Les documents sont fidèles à leur façon et ne nous trompent que lorsque nous leur prêtons une signification à laquelle ils ne prétendent pas. Est-ce que la lumière est un malin génie qui se joue de nous lorsqu'un bâton plongé dans l'eau paraît brisé? Pas plus que mon poste de T.S.F. n'est responsable de ce que mes enfants croient qu'il y a un monsieur caché dans la boîte. (13)

It is futile to try to rule out the subjectivity of mathematical physics as some modern Scholastics have done by appealing to the Thomistic doctrine that ideas are not id quod sed id quo cognoscitur. (14) For while it is true that in non-reflexive knowledge an idea is a mere quo which

which carries the mind to a quod and not just to itself known as an idea, the quod to which the mind is thus carried may be either objective reality or a construction of the mind. We hold that in mathematical physics the quod to which the mind is carried is formally something that is manufactured by the mind -- though not without dependence upon objective reality.

If the subjectivity which we have attributed to the scientific world be rightly understood there is no reason to fear idealism. While insisting upon this subjectivity Eddington likewise insists upon the fact that it can never be more than partial -- that objectivity is also
(15)
essential to physical science. Meyerson has shown how great and how constant is the concern on the part of all the greatest scientists to remain in as close a contact as possible with an objective universe. This is true even of physicists like Einstein and Schrodinger whose theories
(16)
seem to have the greatest likeness to idealism. Whereas idealism begins with a denial of the objective universe,
(17)
physical science begins by postulating its existence.

All through its development the contact with this objective universe remains unbroken. And even though science constructs its own subjective world as something distinct from the

objective world, the latter is reflected in the former and is grasped in some way through it. Whereas idealism seeks to arrive at a maximum of ideas with a minimum of experience, physical science tends towards a maximum of experience in order to arrive at a minimum of ideas. Meyerson has shrewdly pointed out that having started with sensible reality, it is the sensible rather than the reality that physical science tends to dissolve and that this dissolution of the sensible actually results in a reinforcement of the reality. Idealism does just the opposite -- the sensible remains but the reality becomes nothing apart from the ego. (18)

It may readily be admitted that as physics advances in its theoretical elaborations it seems to take on more and more the character of idealism. (19) But the likeness is only superficial. For in idealism subjectivity is an end in itself. In physics, on the contrary, it is only a means; its character is purely functional. Because the whole purpose of the subjectivity of physics is to carry the mind to a greater measure of objectivity, it is essentially different from the subjectivity of idealism. There can be no doubt that Relativity physics for example is much more subjective than Classical physics was. But at the same time it is far more objective, for it has purged physics of innumerable subjective elements that were lurking unsuspected in the

Classical system. It delivered physics from the subjectivism of individual observers and made all systems of coordinates equivalent for the expression of the general laws of nature.

There is another side to the theory of relativity. We have pointed out in the beginning how the development of science is in the direction to make it less subjective, to separate more and more in the observed facts that which belongs to the reality behind the phenomena, the absolute, from the subjective element, which is introduced by the observer, the relative. Einstein's theory is a great step in that direction. We can say that the theory of relativity is intended to remove entirely the relative and exhibit the pure absolute.(20)

2. Mathematical Physics and Kantianism.

Eddington sums up the substance of his Philosophy of Physical Science in the following terms:

The subjective laws are a consequence of the conceptual frame of thought into which our observational knowledge is forced by our method of formulating it, and can be discovered a priori by scrutinising the frame of thought as well as a posteriori by examining the actual knowledge which has been forced into it.(21)

It is impossible to read these lines without finding them reminiscent of Kantianism. And as a matter of fact, as we noted in Chapter I, Eddington himself draws explicit

attention to the remarkable affinity between Kantian epistemology and the modern developments of physics. Let us recall his words once again:

If it were necessary to choose a leader from among the older philosophers, there can be no doubt that our choice would be Kant. We do not accept the Kantian label; but, as a matter of acknowledgement, it is right to say that Kant anticipated to a remarkable extent the ideas to which we are now being impelled by the modern developments of physics. (22)

Nor is Eddington the only one who has drawn attention to this affinity. From the start the Theory of Relativity has seemed to have profound philosophical implications and it has been a natural tendency to attempt to associate it with some philosophical system. And, as Meyerson has remarked, ⁽²³⁾ the philosopher whose name has been mentioned the most frequently by the relativists themselves (Einstein seems to be an exception) has been Kant.

As is well-known, Kant was perfectly conversant with Newtonian physics, and had a vast admiration for it. This admiration led him into two serious errors. First, he considered Newtonian physics to be definitive. For him it was not merely dialectical; on the contrary it had the supreme certitudè of science in the strict sense of the word. Secondly, not only was it a perfect science,

but it was the perfect science. In other words the properties of physics became for him the criteria for all speculative science. And that is why the Critique of Pure Reason is in the last analysis nothing but a critique of physical science, or more exactly, a critique of speculative knowledge in terms of physical science. These two fundamental errors necessarily compromised the validity of the whole epistemological structure of Kant, but they did not prevent him from seizing upon the proper nature of physical science -- at least in an obscure way. That is what we must now try to see. And our brief analysis will consider two points: first we shall try to see how Kant seized upon the general nature of physical science; secondly we shall consider the relevance of his doctrine for mathematical physics in particular, and especially with regard to its object. It is this second point that is of greater interest for us.

It is well-known that Kant erected his philosophical system as a reaction to the empiricism of Hume in which he recognized the utter destruction of all true science. But this reaction did not blind him to the essential role that experience plays in science. In his introduction to the Critique of Pure Reason he makes it clear that all

speculative knowledge is reducible to objects of ex-
(24)
perience alone. At the same time, however, he insists upon the fact that experience alone is not sufficient to explain scientific knowledge, that the mind cannot simply be measured by external reality but must in some way become its measure; in other words, that true scientific knowledge must be a priori knowledge. His intimate acquaintance with the physics of his time made it evident to him that the universality and necessity of scientific concepts could not be derived from the singularity and contingency of experience and consequently had to be a contribution of the mind.

We have already intimated, particularly in Chapter IV, to what extent Kant was justified in arriving at this conclusion. We have seen that experimental science by its very nature demands that the mind by means of hypothetical constructions of its own making supplies for the universality and necessity which experience cannot provide, and even ~~predetermines~~ experience. We have seen that he was correct in maintaining that in experimental science the mind cannot know reality as it is in itself; it can only approach it provisionally. And in getting to know reality, it necessarily fashions and forms it according to

its own preconceived ideas. Kant's great mistake as we said a moment ago consisted in ~~making~~ experimental science the pattern and norm of all speculative knowledge. This mistake did not derive from the fact that he conceived all speculative science as necessarily composed of an a priori element as well as an element drawn from experience, for that is perfectly true, but rather in the fact that he failed to recognize that there are two essentially different kinds of a priori elements. For in so far as philosophy of nature, for example, is universal and necessary it contains an a priori element in the sense that this universality and necessity rises above, and hence is independent of singular contingent experience. This a priori element, however, does not consist in something posited by the subject, but in something revealed by the object, namely an analytical and hence necessary truth concretized in the singular contingent experience.

As we saw in Chapter IV, it is precisely because the mind is unable to discover truths of this kind in experimental science that it is forced to have recourse to another kind of a priori element which is conferred by the mind. And in so far as this type of knowledge is concerned, Kant was justified in making synthetic a priori judgments the

pivotal point of science. It should be recalled that for Kant synthetic a priori judgments were those in which there is added to a subject a predicate that is essentially extrinsic to it. As a result such judgments were a purely artificial synthesis consisting in an accidental composition whose unity derived from the mind. Their truth was not founded upon the principle of contradiction as was that of analytical judgments, but on the possibility of experimental verification.

Now all this is a fairly accurate description of the type of judgments that are characteristic of experimental science. We have seen that experimental science is based essentially upon induction by enumeration. If it were to limit itself to the individual cases of the enumeration ("This A is B") its judgments would be purely synthetic, and it would be completely deprived of the character of science. On the other hand, induction by enumeration can never give true universal natures and hence analytical judgments with the a priori knowledge that is characteristic of such judgments. That is why experimental science must necessarily have recourse to synthetic a priori judgments in which the a priori element is something conferred by the mind. When, therefore, experimental science declares: "Every A is B",

this judgment is at once synthetic, because based on purely synthetic judgments ("This A is B", "That A is B", etc.) and a priori, because the form of universality is conferred by the mind without adequate foundation in nature. However, because of the regularity found in the multiplicity of cases, it must be noted that such a judgment is neither purely synthetic nor purely a priori.

Because judgments of this kind are not founded upon the principle of contradiction but upon the possibility of experimental verification they can never be anything more than hypothetical. Because of his belief in the definitive character of Newtonian physics Kant failed to recognize their hypothetical nature and attributed to them perfect necessity that derived from absolutely fixed forms of thought which were his categories. The dissolution of the Classical system has shown how unwarranted his assumptions were in this regard. Nevertheless it must be noted that, in spite of the essentially transitory character of the hypothetical constructions of experimental science, Kant was not wholly wrong in attributing a fixed and necessary character to the a priori element found in it. For earlier in this Chapter we saw that the construction of the scientific world is predetermined and shaped by the

methodological principles which constitute the very essence of the scientist's approach to reality, and that as a consequence a close examination of these principles makes it possible to know a priori the fundamental lines of this construction, just as the examination of the fisherman's net makes it possible to know a priori a great deal about the nature of his catch. Because these methodological principles do not change, because they are fixed forms which are essential to the very nature of experimental science, the laws which are known in this a priori way have a necessity that those deriving from experience do not have. And in all this there is certainly a striking affinity with the Kantian categories.

But of greater importance in this study of the relation between Kantianism and mathematical physics is the consideration of the similarity between Thomistic doctrine with regard to the object of mathematical physics and Kant's doctrine of sensible intuition. Let us recall the substance of what Kant has to say about sensible intuition. Early in his Critique of Pure Reason he explains what he means by intuition in general. He defines it as the necessary means by which all knowledge is related to objects and which all thought uses in order to attain them.

Kant agreed with Aristotle that all our knowledge begins in the senses and he held that all intuition as found in man is necessarily sensible — it has to do with an object furnished by sensation. Nevertheless, he felt that sensible intuition could not consist merely in the reception of physical data coming from external reality. For his whole purpose, as is well known, was to save science from the devastation it had received at the hands of both the extreme rationalists who had followed in the wake of Descartes and of the extreme empiricists such as Hume. And he thought that this could be accomplished only by considering the whole structure of science as determined by a kind of noetic hylemorphism in which the matter would be a posteriori and furnished by physical reality and the form would be a priori and provided by the subject. That is why in setting out to disclose and analyze the a priori forms of cognition he felt that such forms should be found even in our sensible intuition of the external world, in such a way that even our direct experience with nature would consist in a fashioning of physical reality by the subject.

And in order to explain how this is possible he distinguished between two aspects of intuition: pure intui-

tion and empirical intuition. The former is sensible intuition considered from the point of view of pure sensibility, that is to say, from the point of view of the capacity of the knower to receive objects coming from the sensible world, prescindng from actual sensation and from any particular objects that such sensation might furnish. The latter is sensible intuition considered from the point of view of actual sensation of physical objects. In pure sensibility he discovered certain forms or determinations which were a priori in the sense that they were prior to all actual sensation and hence completely independent of it. These a priori forms of sensibility which constituted pure intuition were space and time.

Now it is extremely significant that for Kant space and time were the object of mathematics. He defined mathematics as the science which considered these two a priori forms of sensibility in abstraction from all concrete sensible data. Space constituted the object of geometry which deals with lines and figures; time constituted the object of arithmetic because it deals with numbers which are a succession of units.

It is evident from what has just been said that

for Kant sensible intuition involves something more than just sensibility in the ordinary sense of the word. It is in fact not merely sensible knowledge, but intellectual knowledge. It is called sensible because of its dependence upon sensation which provides it with the matter to which the a priori forms are applied.

Now the two a priori forms of space and time which when taken by themselves in abstraction constitute the object of mathematics, when applied to actual sensation caused by physical reality constitute something that Kant calls a phenomenon. This phenomenon is a composite made up of two elements: a material element which is a posteriori and derived from nature through actual sensation, and a formal element which is a priori and consists in the forms of pure sensibility. Only by the application of the latter to the former can the raw materials of knowledge coming from nature be unified, ordered, rationalized, made significant, and rendered capable of entering into the structure of science.

Ce qui, dans le phénomène, correspond à la sensation, je l'appelle matière de ce phénomène; mais ce qui fait que le divers qu'il y a en lui est ordonné suivant certains rapports, je le nomme la forme du phénomène. Comme ce en quoi seul les sensations peuvent s'ordonner, ou ce qui seul leur permet de les ramener à une certaine forme, ne saurait être

lui-même sensation, il suit que, si la matière de tout phénomène ne nous est donnée qu' a posteriori, la forme en doit être a priori dans l'esprit, toute prête à s'appliquer à tous, et que par conséquent, on doit pouvoir la considérer indépendamment de toute sensation. (27)

It should be fairly evident that pure intuition and the form of the phenomenon are merely two aspects of the same thing. Pure intuition is the a priori form in so far as it is considered as a determination of pure sensibility. The form of the phenomenon is the same a priori form considered in relation to the manifold of sensations to which it is applied and to which it gives order and unity.

It is to be noted that in the passage just cited, Kant, in speaking of the union of the a priori form with the matter of sensation, uses the word "application". This is significant. For it brings out the fact that in this union the form is essentially extrinsic to the matter. If the very being of the phenomenon arises from the extrinsic application of one of its composing elements to the other, it follows that it can be nothing but an artificial composite whose unity is purely accidental.

Now the close affinity between this object of sensible intuition and the object of mathematical physics

as analyzed in this study should be immediately apparent. This affinity is found both in the fact that the two objects are accidental composites, and in the very nature of the elements which enter into the composition. In so far as the composition itself is concerned, it is clear that in both cases there is a union of two elements one of which plays the part of matter and the other that of form. In both cases the form is something essentially extrinsic to the matter, and as a result the union consists merely in an application of one to the other effected by the knowing subject. Consequently, the union is in both cases something purely accidental, something due to the mind rather than to nature, and hence the resulting composite is an artefactum.

A similar affinity is found in the very elements which go to make up the composite. For in both cases the material element is a sensible datum, something deriving from physical nature, and the formal element is something drawn from mathematics. In both cases the mathematical form orders and rationalizes the physical datum and gives it scientific significance.

It is easy to see why for Kant the application of mathematics to nature is not only possible but even

necessary. Without this application no true knowledge of physical reality is conceivable. That is why physics for Kant is necessarily mathematical physics. But more than that, since the whole Kantian structure of speculative science is based upon sensible intuition, the speculative reason is, in the last analysis, capable of nothing but physico-mathematical knowledge. Kantianism is the most radical form of scientism.

But in spite of this profound epistemological aberration there is much to be said for Kant if his Critique be limited to the realm of mathematical physics. For in mathematical physics the mind does form and fashion reality in an a priori way; it does become the lawgiver of nature. And that is why we can find no better way of summing up the general theme of this Chapter than by quoting the following lines of Eddington:

... We have found that where science has progressed the farthest, the mind has but regained from nature that which the mind has put into nature. We have found a strange foot-print on the shores of the unknown. We have derived profound theories, one after another, to account for its origin. At last we have succeeded in reconstructing the creature that made the foot-print. And lo! it is our own. (28)

CHAPTER THIRTEEN

THE NATURE OF MATHEMATICAL PHYSICS

1. The Essence of Mathematical Physics.

By way of conclusion it will be well perhaps to give a brief resumé of some of the more important points in this study, and thus try to fix upon the specific nature of mathematical physics in the light of the foregoing analyses. And we know of no better way of going about doing this than by returning to something we saw in Chapter I. After presenting the various opinions proposed by philosophers of science with regard to the fundamental meaning of the mathematization of the cosmos, we pointed out that in a general way all of them may be reduced to two extreme positions. In the first place, there is the opinion of those who, like Pythagoras, bring the mathematical world and the physical world into so close a union as to arrive, in one way or another, at an identification between them. In this position the object of mathematical physics is simply and perfectly one. At the other

extreme there is the position of those who remove the mathematical world so far from the physical world that in mathematical physics the former remains a pure instrument, a pure logical or linguistic tool, in relation to the latter. In this position the object of mathematical physics is also simply and perfectly one; that is to say, it is a pure physical object to which mathematics remains completely extrinsic.

There is something highly significant in the wide divergence of these two opinions. For it brings out the fact that the mathematical world is at once extremely close to and extremely distant from the physical world. When this is grasped, it becomes easy to understand why modern authors such as Einstein have divided geometry into two branches of which one is very distant from the physical world, and the other identified with it. The first branch consists in purely formal knowledge based on free creations of the mind and schematic concepts devoid of all content, and the second in a natural science known as practical geometry. As we noted in Chapter VI this is actually a denial of the true nature of geometry, since the first branch seems to be nothing but dialectics, and the second nothing but a physical science. The distance between

the physical world and the mathematical world and the closeness of them was also a problem for Plato, as we saw in Chapter I. On the one hand he drew them into a union that was extremely intimate in the sense that he made the physical world indefinitely amenable to mathematization and conceived of this mathematization as a revelation of a logos that is proper to nature. On the other hand, he created an immeasurably wide gulf between them by conferring upon the mathematical world an ontological existence that was independent of the physical world. There is this to be noted immediately about the distance created by Einstein between the two worlds and that created by Plato: in the first case the gulf can be bridged in the sense that the dialectics can be successfully and fruitfully applied to the physical universe as an instrument, even though it must ever remain essentially extrinsic to the object of physics, whereas in the case of Plato, as we intimated in Chapter I, in the measure in which the mathematical world is conceived to have an ontological existence of its own, not only must it remain extrinsic to the object of physics, but it cannot even be used as an instrument in relation to the physical world.

We believe that it is possible to hit the very heart of the problem of mathematical physics by saying that both Plato and the moderns have erred by making the mathematical world at once both too close to the physical world, and too distant from it. In the Thomistic solution of the problem they are brought together without identification and separated without the creation of a gulf between them. And once this has been understood it becomes possible to see how mathematics can enter intrinsically into the object of physics and at the same time remain extrinsic to it and serve as an instrument. It also becomes possible to see that the object of mathematical physics is not something simply and perfectly one, but rather something that is under one aspect one, and under another dual. Because it is one, Aristotle and St. Thomas could conceive of mathematical physics as a science. But because it is at the same time dual, they found it necessary to conceive of it as a scientia media. Let us try to analyze these points and see how they fit together.

In the first place, Aristotle and St. Thomas make a definite and clear-cut distinction between the physical world and the mathematical world by means of their doctrine of the different degrees of formal abstraction.

The physical world must be studied in the light of the first degree of formal abstraction. It is a world of mobility and everything in it must be defined in terms of sensible matter. The mathematical world is the result of the second degree of formal abstraction. It is a world of immobility and everything in it must be defined without sensible matter. Once we have made this initial distinction and turn to examine the nature of the abstraction by which the mathematical world is set off from the physical world something very significant immediately strikes us. For there is a peculiar quality about mathematical abstraction that is not found in either physical or metaphysical abstraction. In both of these latter cases there is a correspondence between the way the object concerned exists outside the mind and the way it exists inside the mind. The object of physics depends upon sensible matter both for its being and for its "being known". The object of metaphysics is independent of sensible matter both for its being and for its "being known". But the object of mathematics is on the one hand dependent on sensible matter for its being, that is to say, for any existence it can be said to have outside the mind, and on the other independent of sensible matter for its "being known". In this dichotomy between the way mathematical objects are conceived and the way they exist lies the secret

of the distance between the mathematical world and the physical world and their closeness. But before attempting to see why this is so, it is significant to note that both Plato and the moderns conceive of the distance between the two worlds in a way that puts mathematics in a state which can in some sense be said to correspond to the third degree of abstraction. We explained in Chapter II that both metaphysics and logic fall within the general category of those sciences whose object is free of all matter. Metaphysics arrives at this state by means of positive abstraction, logic by means of negative abstraction. Now in so far as Plato attributes an ontological existence to abstract mathematical forms he conceives of them as though they were separated substances. And that is why, as we noted in Chapter I, his metaphysics is a kind of mathematical metaphysics. On the other hand, in so far as the moderns identify mathematics with dialectics they make of it a kind of logic. To put mathematics into the third degree of abstraction is to separate it too far from the physical world and at the same time not far enough. It is only by analyzing the proper nature of the second degree that we can understand the true nature of its separation. But before insisting upon this separation, let us try to see why mathematics ever remains in close contact with physical reality.

The mathematical world is intrinsically and essentially linked to the physical world. As we remarked in Chapter VI, if the material world were impossible, the mathematical world would also be impossible. Since prime matter is the principle of homogeneity, and since homogeneity is the fundamental postulate of all mathematics, there is no possibility of mathematics without an intrinsic reference to prime matter. In other words, it is only in a world of composed essences, in which formal oppositions are incomplete because of the common matrix of prime matter that the mathematical world can originate. All mathematical notions are drawn from the physical universe, and even after the separation of abstraction has taken place, they still retain a necessary connection with the world of matter. For unlike the case of metaphysical abstraction, the separation effected by the mind in simple apprehension cannot in the case of mathematics be transposed to the second operation of the mind. The essence of the judgment is the copula, and this expresses existence, and if mathematical entities are to exist at all they must exist in the physical world. In the universe of matter there are lines and circles and triangles which may be considered the physical counterparts of mathematical lines and circles and triangles, even though the realization of the latter in the former is not perfect

since they lose what is proper to them as abstract entities through their realization in the material universe.

The fact of this loss suggests how far the mathematical world is from the physical world in spite of the nearness upon which we have just been insisting. In a sense the mathematical world is farther removed from the physical world than is the world of metaphysics. For while mathematical being has a necessary relation with the real physical world, it never retains the ontological essence of the thing with which it is connected. Metaphysical abstraction does. And that is why the communia entis can be said to be realized directly in the physical world as well as in the world of separated substances. Mathematical entities are not realized directly in the physical world. In other words, by the very fact that metaphysics deals with sensible beings in so far as they are beings, its notions can be predicated of the physical universe. Mathematical entities on the other hand can be predicated directly of nothing existing in physical reality, precisely because they are defined in a way in which they cannot exist, that is, as separated from sensible matter.

While all sciences deal with the abstract, the mathematical sciences are the only sciences which deal with the abstract precisely as abstract. Their world is an

autonomous world, set apart from reality, and governed by its own intrinsic laws. In it the mind is eminently free. It deals with notions originally drawn from the physical world, but notions which have been transformed into a condition that is especially congenial to its own nature. Though dealing with things originally connected with sense matter, it is not bound down to the necessity of having its processes terminate in the external senses. Though its notions always retain some kind of physical reference, they acquire a pliancy and a capacity for manipulation that are utterly foreign to the physical world.

All this is at the basis of the doctrine of John of St. Thomas that the mathematical world prescind not only from the actual exercise of existence, but also from any intrinsic order to existence, and that as a consequence mathematical being is indifferent to either real or logical being, just as the essence of relation consisting in the esse ad is indifferent to either real existence or purely logical existence. And this explains why it has been possible for modern mathematicians to build elaborate dialectical superstructures upon mathematical foundations -- dialectical superstructures which, while essentially distinct from mathematical structures, are nevertheless based upon them.

and in some way patterned after them. These dialectical superstructures have immeasurably increased the pliancy and instrumentality of mathematics.

The foregoing makes it clear that the mathematical world is an intermediary world between the purely material and the purely immaterial worlds. And this explains why mathematics can at the same time enter intrinsically into the object of mathematical physics and at the same time remain extrinsic and serve as an instrument. And while being a medium between the material and the immaterial, mathematics is at the same time a medium between the objective and the subjective, as is evident from the last paragraph. This immeasurably increases its effectiveness as a scientific instrument, because it gives freedom to the mind to elaborate its own rational schemas, and at the same time provides the possibility of these schemas being applied to cosmic reality.

Having in this way solved the problem of the distance and the closeness between the mathematical world and the physical world and explained in a general way how it is possible for mathematics in mathematical physics to enter intrinsically into the object and at the same time remain extrinsic as an instrument, it remained for Aristotle, St. Thomas, Cajetan and John of St. Thomas to work

out this possibility in fuller and more specific detail. This they did in their doctrine of subalternation and scientia media.

In mathematical physics, physics is subalternated to mathematics in the fullest sense of the word; that is to say, there is subalternation by reason of the object. This means that object of the subalternated science contracts the object of the subalternating science by adding something to it. The addition, however, can be only an accidental difference, for otherwise there would be no formal distinction of sciences. This is an important point because it means that the matter of the subalternated science remains extrinsic to that of the subalternating science even though the two enter into composition.

As soon as we examine the nature of the elements entering into mathematical physics another reason for this extrinsic character presents itself. For mathematical entities are united with physical elements in the state of idealization that is proper to mathematical abstraction. This union is, therefore, not a direct concretion of mathematical entities in sensible matter. It does not consist in something that would be merely the reverse of mathematical abstraction -- the mere putting back of mathematical entities

into the sensible matter from which they were drawn. This means that the composition of the two can never be anything more than the application of the former to the latter. In other words, it is a composition that is not discovered, but created by the mind; it is a logical composition. It is something remarkably similar to the Kantian "phenomenon", and from this point of view as well as from the point of view of the innumerable predetermining a priori elements that the mind contributes to reality in all experimental science, many concessions must be made to Kantianism by a realistic philosophy of mathematical physics.

Now the union between the two worlds is effected by the mind principally through a process of measurement which lays hold of the quantitative determinations in nature directly, and indirectly of the other determinations in so far as the former can serve as surrogates of the latter. But our processes of measurement can never be anything more than approximative, and herein we find a third reason why the mathematical world remains essentially extrinsic to the physical world. If it were merely a question of the first two reasons, mathematical physics could still be a science in the strict sense of the word. The third reason, however, prevents it from being a true science and makes it dialectics.

In fact, at this level it has already become doubly dialectical. For by the very fact that it is experimental science, physics is without a true propter quid and has to have recourse to mere probable reasoning; and the attempt to find a propter quid in mathematics only results in an approach to nature which is so extrinsic that it provides nothing better than a substitutional and approximative propter quid.

In so far as the mathematical element which enters into composition with the physical element always remains extrinsic to it, the object of mathematical physics is dual. But from another point of view it is one. For in the first place, even though the composition in question is logical, it is not completely logical. The elements involved are brought together by the mind - - but for an objective reason. Even though the mathematical entities applied to nature retain their abstract and idealized state, the fact remains that they do have physical counterparts in nature. And the union between the two elements is so intimate that mathematical physics employs a unique type of abstraction, an intermediary abstraction which participates in the nature of both mathematical and physical abstraction at the same time.

But the most important point in connection with the unity of the object of mathematical physics is that a scientia media does not have as its object simply and directly the composite of the two elements considered as an accidental being. In mathematical physics only the physical element is considered directly. The mathematical element is considered obliquely, in so far as it is connoted by the physical element and in so far as it informs and modifies it and thus makes it scientifically fruitful by providing a source of new properties. In this way, even though there is no res media, there can be a scientia Media.

In this notion of connotation we touch the very heart of the Thomistic philosophy of mathematical physics. For it explains how the object of the science can be at the same time one and dual, how mathematics can be brought into intimate contact with physics and yet retain its distance, its autonomy and freedom, and how it can enter intrinsically into the object which specifies mathematical physics and at the same time remain an instrument. The very fact that it is the physical element that is considered directly and per se, whereas the mathematical element is brought into the consideration obliquely and connotatively makes the role of the latter essentially functional. Moreover, while this gives wide

scope to the exercise of the functional role by leaving mathematics the autonomy that is native to it and by thus making it possible for it to exploit all of the conceptual richness and virtuosity that is intrinsic to its nature, it keeps the mathematical elaborations completely subordinated to, and always essentially orientated towards, the physical element.

One gets an idea of how wide is the scope granted to mathematics in Thomistic philosophy of mathematical physics when one recalls that in the structure of a mixed science an accidental element taken from the lower science is added to the object of the higher science. This means that from the point of view we have in mind here the physical element is merely an accidental addition to the mathematical element. Moreover, the latter plays the role of form in relation to the former. This means that in mathematical physics the illumination and conceptual determination comes from mathematics. As a result, even the things that are most proper to the study of nature lose their purely physical status and are mathematicized: motion is transformed from a becoming into a state; the flow of time becomes a dimension; the four causes are reduced to the formal cause; etc.

In taking advantage of the freedom that all this gives to mathematics, the mathematical physicist is not obliged to have a direct and immediate physical counterpart for every mathematical element he incorporates into his conceptual structure. The notion of connotation keeps the mathematical elaborations essentially orientated towards physical reality, but this orientation must not be understood in too narrow a sense. It is possible to maintain the essential contact that connotation implies even though mathematical elements which have no direct physical counterparts are introduced in order to enhance the theoretical power of mathematics in so far as it is employed as an instrument. In elaborate physico-mathematical theories the essential connotation is maintained by means of the text or dictionary.

The mathematical physicist, therefore, is free to push the pliancy and instrumentality of mathematics to the limit. In doing so, he may, if he wishes, go out beyond the limits of mathematics in the strict sense of the word and construct dialectical superstructures which will give greater scope to this theoretical explanation of physical reality. Even though the application of these dialectical constructions to physical reality does not

constitute mathematical physics in the strict Thomistic sense of the word, it is governed by the same general principles and follows the same general pattern as the latter. Through the use of these dialectical constructions mathematical physics, which is already doubly dialectical, becomes triply dialectical.

The objectum formale quod of mathematical physics is the physical considered as connoting the mathematical, and hence from this point of view it is more physical than mathematical ("magis naturalis quam mathematica"): its whole aim is to get to know the physical world and not the mathematical world. Its objectum formale quo is the special type of abstraction that is proper to it, which, while it participates in the nature of both mathematical and physical abstraction, is more mathematical than physical, since mathematics gives the propter quid and plays the part of form; hence from this point of view, mathematical physics is more mathematical than physical ("magis affinis mathematicis"). Though formally mathematical, it is not specifically mathematical. For in it mathematics is applied to a physical object in order to constitute a new subject and new principles proper to a science concerned with physical reality. Consequently

it is specifically distinct from both pure physics and pure mathematics. Since it is not a science in the strict sense of the word, but dialectics, it has no habitus that is proper to it. The habitus that rectifies the intellect in it is the habitus of logic. However, mathematical physics is not pure dialectics. It proceeds per modum scientiae.

2. The Existence of Mathematical Physics.

Having seen how in relation to the problem of the essence of mathematical physics Thomism steers a middle course between the two extreme positions indicated at the beginning of this Chapter it will be helpful in order to round out this summary to explain how it likewise steers a middle course in relation to a problem which in a general way can be called the problem of the existence of mathematical physics. We have intimated that for some Scholastics the grounding of physics upon mathematics is an error which should never have been committed or at best a mere ~~historical~~ accident. At the other extreme is the opinion of those who hold that this grounding of physics upon mathematics is so necessary that no other valid way of studying reality is possible. True Thomism accepts neither of these opinions.

Against the first opinion it holds that the subalternation of physics to mathematics is not only legitimate, but necessary and inevitable. In the course of our analyses we have indicated a number of reasons why this is so. Perhaps it would be well to recall the more important reasons. The very definition of science itself *cognitio certa per causas*, gives us the central reason. For experimental science is neither certain knowledge, nor is it knowledge of things in their proper causes. Hence physics has a double reason for reaching out to a scientia propter quid, i.e. mathematics, in order to obtain for itself at least a substitute certitude and a substitute propter quid. Doxa naturally aspires to the status of episteme; the "infirmus modus demonstrandi" that is characteristic of study of material nature, particularly in its concretion, seeks support in the more sure type of demonstration that is found in mathematics.

Moreover, physics is inevitably lead to abandon the attempt to treat nature in terms of the proper sensibles and to substitute the common sensibles for them. For sense cognition is to some extent necessarily subjective, and at the same time extremely limited, and as a consequence knowledge of nature in its concretion that depends upon the

proper sensibles is necessarily anthropomorphic. Hence it lacks the objectivity and intersubjectivity that all science seeks to attain. Moreover, the proper sensibles are in many respects irrational: they cannot be defined; they are incapable of analysis; they are deficient in communicability; they can neither be demonstrated nor be the principles of demonstration; they are isolated. For all these reasons physics is led to treat nature in terms of the common sensibles, and since these are all reducible to quantity, this inevitably results in the subalternation of physics to mathematics. For only the consideration of quantity in the light of mathematical abstraction has sufficient rationality to carry physics forward towards its goal.

Physics becomes subalternated to mathematics because through this subalternation the mind is able to realize its natural desire to triumph over the heterogeneity of reality through homogeneity. The mathematization of the cosmos provides a homogeneity which while it breaks down the barriers isolating the specific properties of nature and thus triumphs over their pure givenness, at the same time makes it possible to maintain contact with these specific properties through their quantitative surrogates.

In other words it affords at the same time both a unity to provide for what is lost by the emergence of physics from generalities, and a distinctness to enable the mind to follow its natural movement towards concreteness. The mathematization of nature makes it possible for the intellect to realize its instinctive desire to know reality in terms of what is most knowable for it (and thus make up for what is lost by drawing away from generalities) and at the same time in terms of what is most knowable in se (and thus make up for the deficiencies of purely generic knowledge.) In pure physics there is always an opposition between what is most knowable for the mind and what is most knowable in se. Hence the inevitable tendency to ground physics upon the one science in which what is most knowable for the mind is at the same time most knowable in se. And this grounding enables the intellect to realize its natural desire for deduction. Since the universals found in pure natural doctrine are merely universals in praedicando, natural science if left to itself cannot become a purely deductive system. Hence the inevitable turning to mathematics which is the deductive science par excellence because its universals are similar to universals in causando.

As natural doctrine moves towards concretion it is getting farther and farther away from the knowledge of nature that is most in conformity with the human intellect. In this can be found another reason for its turning to that science which is of all the sciences the most in conformity with the human mind. The least rational of the speculative sciences reaches out to the most rational to supply for its deficiencies. In this way the mind is able to study its most natural object (the essence of material things) through the science that has the greatest connaturality for it. The mathematization of the cosmos enables the mind to fulfill its natural tendency to dominate its object, to impose its laws upon it, to become prior to it, to triumph over its givenness, to construct it, and to get at its most profound aspect: the order of the whole.

A final reason for the subalternation of physics to mathematics must be added here. We have seen that by its very nature experimental science is led to express itself through symbols rather than through names. Mathematics provides the most perfect symbolic system for this expression.

Because of these reasons and many others that might be added, it is manifestly erroneous to consider the grounding of physics on mathematics an accident or a mistake. On the other hand it is equally erroneous to make this grounding so necessary that no other valid approach to reality remains possible. Thomism avoids this opposite extreme by situating mathematical physics accurately in the whole epistemological scheme. When this is done it becomes evident that not only is mathematical physics not the only approach to reality in general, since metaphysics is a valid science and the most important of all the purely human sciences, but it is not even the only approach to physical reality, since it is only the part of natural doctrine that is advanced towards concretion that requires subalternation to mathematics. Philosophy of nature remains a valid approach to the cosmos, and one which in many respects is of greater importance than the approach of mathematical physics, since it deals with the most fundamental problems of the universe and since it provides knowledge of the most noble natural form -- the human soul.

Thomism recognizes the worth and importance of mathematical physics. It believes that the most profound knowledge one can have of reality is knowledge of it in its

proper causes, and from one point of view at least mathematical physics comes closer to this type of knowledge than philosophy of nature. Thomism even goes so far as to hold that in mathematical physics the mind possesses a knowledge of the cosmos which in many respects is like the knowledge that God has of nature, since it carries the mind far along the road ~~towards~~ knowing reality in its specific concretion. At the same time Thomism insists upon the many profound limitations that are inherent to the type of knowledge that mathematical physics provides. In the first place, it is not science in the strict sense of the word, but merely dialectics. It is not a mansion of residence, but a vehicle of progress -- a ~~vehicle~~ of progress that must travel over a road that has no end. Thomism believes that even though it is better to make progress than to stand still, per se a mansion of residence is more perfect than a vehicle of progress. By the very fact that it is experimental science mathematical physics can never arrive at universal and necessary propositions, and must remain in probable reasoning. Its definitions are operational and cannot give the quod quid est of things. It can get at the objective logos only by projecting a subjective logos into nature, in such a way that the two become inextricably intermingled. Because it is subalternated to mathematics, the only type of knowledge

it can give of nature is that provided by measurement. The data out of which its whole structure is built is, in the last analysis nothing but pointer readings. Now metric knowledge is at best an extremely meager kind of knowledge. For it comes to grips only with the quantitative determinations of nature; it is utterly blind to all the determinant properties of things in their specific essences, to the very inner nature of things, to all that is of greatest significance for philosophy, for art, and for human life itself. But it cannot even get at the quantitative determinations of reality in the sense of being able to tell us what these determinations are. By the very fact that it is "quantitative" knowledge it is not "quidditative" knowledge. It cannot answer the question "what", but only the question "how much?" And it cannot answer this question in any absolute way, since a minima mensura in continuous quantity is a contradiction in terms. It can give us only knowledge of ratios determined by arbitrary standards. Nor is it possible to progress indefinitely in the direction of a minima mensura. And besides all this, other innumerable limitations of metric knowledge result from the maze of hypotheses in which all measuring processes are involved, from the physical interactions

between the measuring instrument and reality, from all the cosmic influences that enter into every measurement, etc.

For all these reasons the physico-mathematical world can be nothing more than a shadow world, in spite of (or rather precisely because of) all the Cartesian clarity with which it becomes suffused in the light of mathematical intelligibility. The true natures of things remain in the background. As a matter of fact, mathematical physics does not get to know the objective world in its absolute state directly; it knows it indirectly by constructing an imitation of it - - an imitation which is better than the objective world because more rational, but at the same time worse, because its whole purpose is to lead to the objective world in its absolute condition. The physico-mathematical world is not a formal sign, but an instrumental sign of the absolute world condition. Between the two there is a relation of isomorphism. The mind must ever try to bridge the gap between the two worlds by bringing the scientific world ever closer to the absolute world. But in coming continually closer, the two continually get farther apart. The reason is that the scientific world is at once essentially subjective and essentially objective, and the more objective it gets, the more subjective does it become.

This subjectivism of the scientific world does not favor idealism, since its whole purpose is to orientate the mind toward the absolute world condition. As a matter of fact it is only by admitting this subjectivity that it is possible to escape idealism, for otherwise one inevitably mistakes one's own mental constructions for objective reality.

While rejecting the exaggerations of scientism which have tended to make physico-mathematical method the only valid approach to reality, Thomism recognizes the truths which scientism has exploited for its own ends, and the source from which has come the spell that mathematical physics exercises over the mind. In mathematical physics the intellect is allowed to indulge in unlimited speculation in the realm that is most connatural with it -- that of mathematics, and this speculation is inseparable from construction in which the intellect posits its own object. At the same time this speculation brings it closer to the object that is most proper to it -- the essence of material things. And this intimate knowledge of material things reveals the plasticity and malleability that is native to them and thus gives the mind the power to re-fashion nature to its own image and likeness. Because man

is composed of matter and spirit there are two fundamental tendencies in him: to draw everything from matter, and to draw everything from spirit. The quantitative homogenization of the cosmos and the study of it in the light of the abstract rationality of mathematics makes it possible for him to realize both of these tendencies simultaneously. Or to put the thing in a slightly different way: the combination of the first and second degrees of formal abstraction enables a man to be at once an idealist and a realist. The induction of experimental physics satisfies his desire to know cosmic reality; the deduction of mathematics satisfies his desire for perfect rationality. The first without the second leads him into impenetrable obscurity; the second without the first cuts him off from reality. The combination of the two provides a way out of obscurity and a way back to reality. More than that, it provides man with a kind of wisdom -- not the divine wisdom of metaphysics which is so far above him, which is only loaned to him in a very inadequate way and never really given to him, and in which he must make his way with continual strain and effort, but a human wisdom -- one to which his mind is particularly attuned, and in which he can move with comparative ease and security. It is a

wisdom whose ideal is to see the whole of cosmic reality in the light of a few fundamental mathematical formulae. Already the Einsteinian system has brought us far along towards this ideal. And if, as it is only natural to hope, Relativity and Quantum physics can eventually be integrated into a unified system, man will have come near to realizing his ideal. This is the wisdom to which Descartes dedicated himself -- a wisdom that is not restricted to an elite, but one in which all men can share on equal footing, a wisdom so connatural to man that as he tells us in his Regulae, if a student only follows the right rules "there is nothing, generally speaking, that any other man is able to know that he himself will not be capable of knowing." And this wisdom not only satisfies the mind's desire to dominate its object in the speculative order, it also satisfies its desire to dominate it in the practical order, for, as is well known, technological fruitfulness has inevitably followed in the wake of every advance in theoretical physics. Small wonder then that this type of knowledge has been transformed into a philosophy of life, that it has become the light of the world.

The great error of scientism has been to believe that the knowledge most connatural to man is also the knowledge most essential for him.

APPENDIX

PART I

NOTES

Chapter I

- (1) Mayne I. Logsdon: A Mathematician Explains, University of Chicago Press, Chicago, Ill., 1935, p. 158.
- (2) University of Chicago Press, 1944, pp. 3 - 4.
- (3) La Théorie Physique, (deuxième ed. Paris, Marcel Rivière & Cie., 1914, p. 158. Cf. also p. 166.
- (4) The New Background of Science, Cambridge University Press, 1933, p. 296.
- (5) Cf. Pierre Duhem: Le Système du Monde, Paris, Librairie Scientifique A. Herman et Fils, 1916, I, pp. 128 - 129.
- (6) Met. I, ch. 5; 985 b 23 - 986 a 10. Trans. by W.D. Ross.
- (7) Cf. Roy Hack; God In Greek Philosophy To The Time of Socrates, Princeton University Press, 1931, pp. 47 ff.
- (8) The Mysterious Universe, (second ed.) Cambridge University Press, 1937. p. 117. As we shall see later, there is a sense in which it is true to say that mathematics enters into physics from above: mathematics is the limit towards which matter is drawn. But Jeans has something quite different in mind here.
- (9) Ibid, p. 122. Cf. also New Background of Science, pp. 296 - 297. It is interesting to note in passing that this view is vigorously contested by Eddington. Cf. The Philosophy of Physical Science, Cambridge U. Press, 1939 p. 137, etc. We shall consider Eddington's opinion later.
- (10) Science and the Modern World, pp. 36, 47 - 48 Cf. Sir William Dampier: History of Science: Cambridge University Press, 1943, "In our own day, Aston with his integral atomic weights, Moseley with his atomic numbers, Planck with his quantum theory, and Einstein with his claim that physical facts such as gravitation are exhibitions of local space-time properties, are reviving ideas that, in older, cruder forms, appear in Pythagorean philosophy." - - p. 20.

- (11) An Essay On Man, Yale University Press, 1944, pp. 210, 211, 214.
- (12) Whitehead: Adventures of Ideas, Cambridge, University Press, 1933, pp. 194 - 195.
- (13) De L'Explication dans les sciences, Paris, Payot, 1927, p. 133.
- (14) Op. cit. p. 101.
- (15) Cf. St. Thom. In I Met. Lect. 10, no. 143: "Unde et Plato tamquam eius auditor, recipiens Socratem, idest sequens, suscepit hoc ad inquirendum in rebus naturalibus, quasi in eis hoc posset evenire, ut universale in eis acciperetur de quo definitio traderetur, ita quod definitio non daretur de aliquo sensibilibus, quia cum sensibilia sint semper 'transmutantium,' idest transmutata, non potest alicuius eorum communis ratio assignari. Nam omnis ratio oportet quod et omni et semper conveniat, et ita aliquam immutabilitatem requirit."
- (16) Cf. Aristotle: I Met., ch. 6, 387 a 30. After discussing the position of the Pythagoreans he goes on to say: "After the systems we have named came the philosophy of Plato, which in most respects followed these thinkers, but had peculiarities that distinguished it from the philosophy of the Italians. For, having in his youth first become familiar with Cratylus and with Heraclitean doctrines (that all sensible things are ever in a state of flux and there is no knowledge about them), these views he held even in later years."
- (17) I Met. ch. 6, 987 b 23 - 33.
- (18) Philebus, 55, 56. Cf. Field: "Plato and Natural Science", in Philosophy, Vol. VIII, No. 30 p. 139.
- (19) Substance and Function and Einstein's Theory of Relativity, Chicago, The Open Court Co., 1923, p. 134.
- (20) A. E. Taylor: "Forms and Numbers", in Philosophical Studies, London, Macmillan and Co., 1934, pp. 149 - 150.
- (21) I. Met. 992 a 30: Cf. St. Thom. lect. 17, no. 259: "Sed Platonicis praetermittentibus huiusmodi causis facta sunt naturalia, ac si essent mathematica sine motu, dum principium et finem motus praetermittebant. Unde et dicebant

quod mathematica debent tractari non solum propter seipsa, sed aliorum gratia, idest naturalium, inquantum passiones mathematicorum sensibilibus attribuebant".

- (22) "Number and spatial magnitudes cannot exist apart from things." Met. XIII, 1085 b 35.
- (23) Procedures and Metaphysics, University of California Press, 1936, pp. 24 - 25; 27 - 28.
- (24) Substance and Function and Einstein's Theory of Relativity, p. 155.
- (25) Cf. pp. 40 ff.
- (26) Pp. 184 - 186, 214 - 215, 217.
- (27) Cf. F.S.C. Northrop: Science and First Principles, New York, The Macmillan Co., 1931. p. 16: "The third consequence of the mathematical theory is methodological in character. Since mathematical forms are not observed in nature, and, as Plato said, are suggested by, not contained in the world of observation, it follows that one cannot proceed, as did the physical theory of nature, from the facts of observation to one's scientific principles by the necessary relation of formal implication. The facts merely suggest the mathematical forms; they do not imply or contain them. Hence, as Plato maintained, the fundamental scientific method in this theory is the method of hypothesis. Since this method always commits the logical fallacy of affirming the consequent, Plato tried to introduce the dialectic, which is not the vicious thing its modern connotation suggests to certain minds, but the simple sound idea that all hypotheses must be traced to their common presuppositions and unified into a consistent deductive system. When we attempt to do this for psychology and epistemology as well as mathematics and astronomy, we find ourselves face to face with all the problems with which Plato wrestled in his Dialogues. To see the second movement of Greek science in the light of its bearing on first principles is to understand the philosophy of Plato. It was the mathematical and rational character of the inorganic universe that made man an idealist in ancient times."
- (28) Opus Majus, ed. Bridges, II, 173. Cited by Randall: The Making of the Modern Mind, Boston, Houghton Mifflin Co., 1940, p. 211.

- (29) Cited by Weyl: Mind and Nature, Philadelphia, University of Pennsylvania Press, 1934. p. 50.
- (30) Docta Ignorantia, I, 1.
- (31) De Mente, Ch. VI.
- (32) De Possesit (Op. S., 259).
- (33) Ernst Cassirer: "Giovanni Pico della Mirandola. A Study in the History of the Renaissance", Journal of the History of Ideas, Vol. III, No. 3, p. 321. Cf. Cassirer's Individuum Und Kosmos in der Philosophie der Renaissance, Leipzig Teubner, 1927, pp. II, 15, etc.
- (34) Cf. Frammenti, 85 - 88, 113. etc. Cited by Randall, op. cit. 221, 236.
- (35) Cf. Dampier, op. cit. p. 115.
- (36) Epitome Astronomiae Copernicae, p. 1
- (37) Harm. Mundi, Lib IV, cap. 1.
- (38) Cf. Strong, op. cit. Chapter VII: "Method and Metaphysics in Kepler".
- (39) Ed. Naz. VIII, 613. Cited By Randall, op. cit. p. 237.
- (40) Opere Complete di Galileo Galilei, Firenze, 1842, Vol. IV, p. 171. Cited by Burtt, op. cit. p. 64.
- (41) Opere, IV, 333, Cited by Burtt, op. cit. p. 75.
- (42) De L'Explication Dans Les Sciences, p. 134.
- (43) Le Songe de Descartes, Paris, Editions R. A. Correa, 1932, p. 40.
- (44) Descartes, Haldane and Ross ed. I, 13, 11.
- (45) Lettre à Mersenne, ed. Cousin, T. VII, p. 121.
- (46) Principia, II, 64.
- (47) Principia, II, 4, 9.

- (48) De Potentia, III, I ad 17.
- (49) Cf. Regulae ad Directionem Ingenii, ed. Adam et Tannery, pp. 426 - 427.
- (50) L'Expérience Humaine, Paris, 1922, p. 185. Cited by Meyerson: La Déduction Relativiste, Paris, Payot, 1925, p. 254.
- (51) La Théorie Physique, pp. 169 - 170.
- (52) Principia, IV, 199.
- (53) Whitehead: Science and the Modern World, p. 69. It would be difficult to exaggerate the importance of the discovery of the calculus for the mathematical interpretation of the physical universe. Cf. Randall, *op. cit.* p. 258: "Such a method of measuring movement and continuous growth Newton discovered; he had arrived at the most potent instrument yet found for bringing the world into subjection to man. Since any regular motion, be it of a falling body, an electric current, or the cooling of a molten mass, can be represented by a curve, he had forged the tool by which to attack, not only the figures, but the processes of nature -- the last link in the mathematical interpretation of the world. By its means a Lagrange in the eighteenth or a Clerk-Maxwell in the nineteenth century could bring all measurable phenomena into the unified world of mathematics, and calculate, predict, and control light, heat, magnetism and electricity."
- (54) Op. cit. 255.
- (55) Cited by Burtt, op. cit., pp. 204 - 205.
- (56) The Limitations of Science, New York, The Viking Press 1933, p. 6.
- (57) Science and Human Experience, London, Williams and Norgate, Ltd., 1931, p. 38.
- (58) The Philosophy of Physical Science, pp. 188 - 189.
- (59) Prolégomènes à toute Métaphysique Future, trad. J. Gibelin, dans "Bibliothèque des Textes Philosophiques", 1ère partie, pr. 10, pp. 44 - 45.

- (60) Cited by Meyerson De L'Explication dans Les Sciences, p. 458.
- (61) Throughout this study the phrase "classical physics" refers to Newtonian physics and not to the physics of the Greeks.
- (62) The Mysterious Universe, pp. 111 ff.
- (63) Some authors have attempted to press this continuity to the extent of seeing in the distinctive achievements of recent physics the realization of the main trends in the history of mathematical physics. Cf. Juvet: La Structure des Nouvelles Théories Physiques, Paris, Alcan, 1933, p. 177: "Les Nombres de Pythagore, les idées de Platon, la mathématique universelle de Descartes, la caractéristique de Leibniz sont de belles anticipations métaphysiques que la nouvelle philosophie naturelle fondée sur les travaux d'Einstein, de De Broglie, d'Heisenberg, la mathématique moderne créée par Galois, Lie, Cartan, Weyl confirment et précisent avec l'aide de la méthode axiomatique d'Hilbert. Les Nombres et les Idées sont les groupes, le symbolisme des axiomes, c'est la Caractéristique Leibnizienne, et les succès sans fin de la mathématique, qui ne se justifient que par la cohérence créée grâce à l'emploi des groupes, font du rêve de Descartes la réalité d'aujourd'hui." We believe that the true nature of the continuity between contemporary mathematical physics and the past is something quite different from what is indicated here by Juvet.
- (64) Introduction à l'Etude de la Médecine Expérimentale, pp. 94 - 95.
- (65) The Logic of Modern Physics, New York, The Macmillan Co., 1932, p. 61. Cf. Woodbridge: An Essay on Nature, New York, Columbia University Press, 1940, p. 124.
- (66) The Method of Theoretical Physics, Oxford, Clarendon Press, 1933, pp. 11 - 13. Cf. The World as I See it, pp. 32 - 34.
- (67) The Mysterious Universe, p. 113.
- (68) Op. cit. p. 121. Cf. "The Mathematical Aspect of the Universe," by the same author in Philosophy, Vol. VII, No. 25, pp. 3 - 14. It is not immediately evident how this doctrine of Jeans fits in with what he says in Physics

and Philosophy: "The same is true of all the discoveries of the pure mathematician: they are universal in the sense that they would be true in any world, and so cannot tell us anything about the special properties of this particular world." -- Cambridge University Press, 1943, p. 49.

- (69) Op. cit. p. 62.
- (70) Op. cit. p. 176.
- (71) "Mathematics For The Doctor in The Million," in Philosophy of Science, Jan. 1944, Vol. II, No. 1. pp. 48 - 49. Cf. also Lenard: Great Men of Science, New York, The Macmillan Co., 1933, pp. 220 - 222.
- (72) The Principles of Quantum Mechanics, Oxford Press, 1935, Preface, p. vii.
- (73) p. 137.
- (74) Cf. also New Pathways in Science, Cambridge University Press, 1935, Ch. XII.
- (75) Cf. Dantzig: Aspects of Science, New York, The Macmillan Co., 1937, p. 74.
- (76) An Essay on Man, New Haven, Yale University Press, 1944, pp. 211 - 212. This position is developed at great length in his Philosophie der symbolischen Formen, and in Substance and Function.
- (77) Since a number of historians have seen fit to consider the doctrine of St. Thomas as a perversion of the philosophy of Aristotle, it is worth while noting perhaps that we consider Thomism to be in the strictest Peripatetic tradition. It would take us too far afield, however, to attempt to establish this point here.
- (78) Our Knowledge of the External World, p. 240
- (79) Op. cit. p. 40. Cf. Randall, op. cit. p. 236.
- (80) Cf. Science and the Modern World, pp. 15 - 16: "But for science something more is wanted than a general sense of the order of things. It needs but a sentence to point out how the habit of definite exact thought was implanted in the European mind by the long dominance of scholasticism."

tic logic and scholastic divinity. The habit remained after the philosophy had been repudiated, the priceless habit of looking for an exact point and sticking to it when found. Galileo owes more to Aristotle than appears on the surface of his Dialogues; he owes him his clear head and his analytic mind. I do not think, however, that I have even yet brought out the greatest contribution of medievalism to the formation of the scientific movement. I mean the inexpugnable belief that every detailed occurrence can be correlated with its antecedents in a perfectly definite manner, exemplifying general principles. . . My explanation is that the faith in the possibility of science, generated antecedently to the development of modern scientific theory, is an unconscious derivation from medieval theology.

- (81) Paris, Librairie Félix Alcan, (4^e ed.) 1932, p. 40.
- (82) Paris, Librairie Félix Alcan, 1931, p. 149. Cf. ibid. 695; De l'Explication dans les Sciences. pp. 489, 528, etc., etc.
- (83) "Physique Ancienne et Physique Moderne: Leurs Conceptions de l'Intelligible", in Travaux du IXe Congrès International de Philosophie, V, Partie II., Paris, Hermann et Cie, 1937, pp. 197 - 198.
- (84) Science and the Modern World, pp. 36 - 37. Cf. Adventures of Ideas: Aristotle's Logic "entirely leaves out of account the interconnections between real things. . . (It) renders an interconnected world of real things unintelligible. The universe is shivered into a multitude of disconnected substantial things. . . But substantial thing cannot call unto substantial thing. . . But the Platonic doctrine of the interweaving of harmony with mathematical relations has been triumphantly vindicated. The Aristotelian classifications based upon qualitative predicates have a very restricted application apart from the introduction of mathematical formulae. Indeed, Aristotelian Logic, by its neglect of mathematical notions, has done almost as much harm as good for the advancement of science. We can never get away from the questions: How much -- In what proportions -- and, In what pattern of arrangement with other things." -- pp. 169 - 170, 196.
- (85) Philosophy and History, Oxford, Clarendon Press, 1936, pp. 61 ff.
- (86) Pp. 71 - 74.
- (87) P. 73.

- (88) In Philosophia et Sciences, (Journées d'Etudes de la Société Thomiste, Louvain, 1935), Les Editions du Cerf, Juvisy.
- (89) In Philosophie et Sciences, pp. 26 - 28.
- (90) "La Conception Scolastique de la Physique," in Philosophie et Sciences, pp. 48 - 49.
- (90a) Ibid. pp. 55 - 56.
- (91) Cited by Riezier: Physics and Reality, New Haven, Yale University Press, 1940, p. 119. Cf. Strong: op. cit. pp. 161 - 162: "Galileo treats Aristotle in The Two New Sciences as a predecessor in the inquiry into problems of vacuum, infinity, and continuity, and mechanics. He introduces principles advanced by Aristotle in order to agree as well as disagree with them. In the earlier work, The Two Great Systems of the World, Galileo presents a more impatient Salviati and a Simplicio who is less of a student and more of a controversialist than in the later treatise. The "Peripatetics" receive the brunt of an attack launched against those who will not receive the evidence of the telescope and the demonstrations of mathematics; but this is not done to convict Aristotle, since Galileo believes that he should have changed his opinion in the light of the new evidence. In neither of his two major works does Galileo present an opposition between Platonic and Aristotelian metaphysics. Aristotle is rather presented in The Two New Sciences as more closely affiliated with mechanical questions than is Plato." In the preface to his Etudes Sur Leonardo da Vinci, Duhem shows that it is a mistake to believe that the science of Galileo is a victory of modern science over medieval philosophy. It is rather a victory of a mechanics that was born in Paris in the fourteenth century and based upon the critical treatment of doctrines derived from Aristotle and Averroes.
- (92) "The Birth of Science among the Greeks," in Encyclopedia of Modern Knowledge, Edited by Sir John Hammerton, part 15, p. 1425.
- (93) Cf. De Cosmo, II, lect. 17, no. 2; I, 32, 1, ad 2.
- (94) Op. cit., p. 74 (footnote).
- (95) Ch. VIII, 1073 b : 7 - 18.

- (96) Loc. cit.
- (97) Cf. St. Thom. Lect. 9, No. 2566: "Quot autem sint motus planetarum, nos nunc dicemus ea quae circa haec mathematici dicunt, ut circa haec reddamur attentis, ut aliqua pluralitas determinata mente concipiatur a nobis."
- (98) Philosophical Essays for Alfred North Whitehead, Chapter I: The Mathematical Background and Content of Greek Philosophy, p. 39. Cf. A.E. Taylor: "Numbers and Forms" in Philosophical Studies.
- (99) Op. cit. pp. 25 - 27.
- (100) De l'Explication Dans Les Sciences, p. 174.
- (101) 78 b 33 - 79 a 15.
- (102) 193 b 23 - 194 a 10.
- (103) That is to say, scientific in the Aristotelian sense of the term.
- (104) Cf. Met. III and XIII.
- (105) The Nature of Physical Theory, New York, John Wiley and Sons, 1931, pp. 39 - 45.
- (106) Cf. In II Phys, lect. 11, no. 3.
- (107) This notion of the intrinsic incorruptibility of the heavenly bodies has, of course, been rejected by modern Thomists. But the question of the possible existence of such bodies still remains open.
- (108) In this connection it is interesting to note that in some respects the physics of the medieval Scholastics was more mathematized than modern physics: "Les mêmes conceptions mathématiques se trahissent d'ailleurs dans le détail des théories particulières. C'est ainsi que la doctrine arithmétique classique des cordes vibrantes se complétait au moyen âge par une esthétique musicale entièrement fondée sur les nombres qui définissent les sons. Fait plus remarquable, ces mêmes doctrines pythagoriciennes étaient mises en oeuvre dans les théories parallèles de la vue, du goût et de l'odorat. Le blanc, par exemple, était réputé contenir le maximum possible

de lumière, le noir par contre le minimum (De Sensu, VII, 97 - 102, 104, 115. Cf. Ibid. 95, la doctrine qui fait du noir le minimum et non la privation). Toutes les autres couleurs résultaient ensuite d'un mélange en proportion définie de ces deux composantes simples (Ibid., 102) Pour les belles couleurs, dont on ajoutait mélancoliquement, qu'elles étaient aussi rares que les beaux accords musicaux, le rapport des composantes sera une fraction particulièrement simple, pour les vilaines un incommensurable (Ibid. 101, 104, avec la théorie mathématique aux 98 - 100) Les qualités perçues par le goût résultent pareillement d'un mélange en proportion définie de deux composantes simples le doux et l'amer. (De Sensu XI, 148). Et l'esthétique se faisant, cette fois gastronomie, on ajoute que les goûts agréables correspondent aux fractions simples, les mauvais goûts aux incommensurables. (Ibid. 149) Les odeurs, enfin, qui sont définies par les goûts auxquels elles sont conjuguées, suivent la même loi. (De Sensu, XII, 174 - 176; XIII, 177 - 178) Et sur tous ces points la physique ancienne se montre manifestement beaucoup plus mathématisée que celle des modernes." R.P. Salnan: "La Conception Scolastique de la Physique" in Philosophie et Sciences, pp. 46 - 47.

(109) We believe that M. Maritain has thrown some confusion upon this point: "Cet usage des principes mathématiques dans la connaissance de la nature peut ou bien rester accidentel et représenter un emprunt fait aux mathématiques par le naturalis, ou bien être essentiel à la science considérée, qui est alors proprement une scientia media; et il est clair que divers degrés de 'mathématisation' accidentelle doivent conduire progressivement de la science purement physique à la scientia media. La physico-mathématique des modernes réalise le type de la scientia media d'un façon parfaite. Au contraire nous pensons que l'usage des mathématiques en biologie par exemple ou en psychologie, n'arrivera jamais à subordonner typiquement ces disciplines aux règles d'explication mathématiques." Degrés du Savoir, p. 284 footnote.

(110) Cf. W.R. Thompson: Science and Common Sense, London, Longmans Green and Co., 1937, p. 29: "Eminent men of Science, indeed, maintain that the perfection of scientific synthesis depends on the extent to which it has resolved itself into mathematical concepts. There are, however, vast fields

of Science in which the data seem extremely refractory to mathematical interpretation; where we cannot conceive that they could by any process of distillation or boiling down, be reduced entirely to measurable elements. Science, in its present form, thus presents itself, not as a deliquescent and temporary construction melting to a dead level of mathematical material, but as a solid and ordered hierarchy of different kinds of knowledge: all true, all scientific, but diverse."

- (111) "Réflexions sur le problème de l'indéterminisme" in Revue Thomiste, nov. - dec. 1937, p. 394.
- (112) Le Cosmos (revised edition) p. 7.
- (113) "Et c'est aussi en obéissant à cette même tendance éternelle de l'esprit humain que la chimie s'étonne de la diversité des substances et que cet étonnement, selon le témoignage autorisé de M. Job, constitue le point de départ de cette science tout entière." Meyerson: De L'Explication dans les Sciences, p. 181 - 182. Cf. also Essais, Paris, Librairie J. Vrin, 1936, p. 18 - 19.
- (114) Cf. Cours de Philosophie Positive, Vol. II, 281; III, 29. Comte taught that the extensive use of mathematics in physics was simply an inexcusable prejudice. Cf. Cours de Politique Positive, 3e ed. IV, Appendice Général, pp. 1 - 3. It is also interesting to note that J.S. Mill considered the hope of applying mathematics to chemistry and physiology as something chimerical -- Cf. Meyerson: Du Cheminement de la Pensée, p. 190.
- (115) Critique of Physics, London, Kegan Paul, Trench, Trubner and Co., 1931, p. 142.
- (116) For a good summary discussion of this point see W.R. Thompson: Science and Common Sense, chapter VI.
- (117) One of the simplest examples of how mathematical formulation may serve to throw light upon biological phenomena is indicated by W.R. Thompson in the following lines: "In the simplest cases, such as the cylindrical or spherical form, which we find in a multitude of organic formations, the mathematical formation is instantly made. It might seem, at first sight, that the

recognition of a raindrop or an Echinoderm egg as a sphere, or a plant stem as a cylinder, add nothing to our knowledge of these forms. But the recognition of a sphere, for example, implies the knowledge of its mathematical law, as expressed in the equations indicating the order between its quantities, such as $S = 4\pi r^2$ and $V = 4/3\pi r^3$. By the knowledge of the law of its quantities, these are reduced from multiplicity to unity so that the form of the sphere becomes intelligible and we have, in the formal order, a cognitio per causas. Furthermore, the mathematical definition of a sphere is the basis of a true deductive argument. Knowing that a cell is spherical, we know that it is something whose surface is smaller for the volume it encloses than any other possible figure: and we see also, looking at the matter from another angle, that it exemplifies the law according to which 'any system of bodies arranges itself in such a manner that the potential energy of the system is a minimum' (E. Edser, General Physics, 1913, p. 290)" -- Science and Common Sense, pp. 76 - 77.

- (118) Cf. Thurston: Vectors of Mind, Chicago 1935; Spearman: The Abilities of Man, New York, 1928; Edgeworth: Mathematical Psychics, London 1881, etc.
- (119) It is important to keep in mind that whenever intelligence is studied by any method of this kind, what is understood by intelligence can be known only operationally, that is to say, it can be defined only by a description of the concrete procedures of measurement by which the results have been arrived at.
- (120) Introduction to Mathematics, The Home University Library, 1931, p. 223.
- (121) The Physical Principle of Quantum Theory, Chicago University Press, 1930, p. 62.
- (122) Pp. 4 - 7.
- (123) L'Idée de Loi Naturelle dans la Science et la Philosophie, Paris, Librairie Philosophique J. Vrin, 1925, p. 136.
- (124) Philosophical Aspects of Modern Science, New York, The Macmillan Co., 1932, p. 63.

- (125) The Mysterious Universe, pp. 122 - 123.
- (126) New Background of Science p. 298. Cf. also pp. 296 - 297. Cf. "The Mathematical Aspect of the Universe," in Philosophy, Vol. VII, p. 14.

Chapter II

- (1) Cf. Thompson: An Introduction to Science, New York, Henry Holt and Co., 1911, Chapter IV.
- (2) The Philosophy of Physics, London, George Allen and Unwin, 1936, p.81.
- (3) The Three Reformers, London, Sheed and Ward, 1936, p. 64.
- (4) Cf. Gilson: "L'Idée de l'unité du corps des sciences.. est inséparable, chronologiquement et logiquement, de l'extension de la méthode mathématique à la totalité du domaine de la connaissance." Discours de la Méthode, Texte et Commentaire, Paris, 1925, p. 214.
- (5) Cursus Phil. I, Q. XXVII, a. 1.
- (6) Cf. Charles DeKoninck: "La dialectique des limites comme critique de la raison", Laval Théologique et Philosophique, Vol. I, No.1, pp. 177 ff.
- (7) Cf. III, 11, 6; Cf. also John of St. Thomas, Cur.Phil., I, Part II, Q. XXVII, a. 1. (Reiser ed. pp. 819 b and 824 b).
- (8) Cf. Contra Gentes, III, Chap. 100.
- (9) Cf. Henri Pichette: "Considération sur quelques principes fondamentaux de la doctrine du spéculatif et du pratique", Laval Théologique et Philosophique, Vol. I, No. 1, pp. 52 ff.
- (10) Cf. III De Anima, c. 10 433 a 10.
- (11) II Met. I, 993 b 20; In Boet. de Trin., V, I, c.

- (12) I - II, 57, 1, ad 1.
- (13) I - II, 3, 5, ad 2.
- (14) Cf. I, 14, 16.
- (15) De Ver. III, 3, ad 4.
- (16) De Veritate, I, 2, c.
- (17) Cursus. Theol., I, disp. 2, a. 10, no. 18.
- (18) Curs. Theol. I, disp. 2, a. 10, no. 5.
- (19) In II Eth. lectio 12, no. 256. Cf. In III Sent. d. 23, q. 2, a. 3, sol. 2.
- (20) Cf. S. Thomas, Comment. Polit., Prol.; De Ver., III, 3, c.
- (21) Lect. 14, no. 8.
- (22) Discours de la Méthode, Edition Hatier, Paris, p. 76.
- (23) "Hypothesis" in Studies in the History and Method of Science, edited by Charles Singer, Oxford, 1921, Vol. II, pp. 429 - 431.
- (24) London, N.C.L.C. Publishing Society, pp. 7 and 9.
- (25) Language and Reality, London, George Allen & Unwin, 1939, p. 505.
- (26) I, 1, 7.
- (27) I - II, 46, 2; Cf. In VI Eth., lect. 3, no. 1145; St. Albert: Liber I Phys. I, cap. 5.
- (28) Cf. Post. Anal. I, c. 2, 71 b 10.
- (29) Cf. De Anima II; I 77, 3, etc.
- (30) In Post Anal. I, lect. 41; De Trin. VI, etc.
- (31) In De Generatione et Corruptione, Proem.; Cf. III De Anima.

- (32) In I, 1, 3, no. 3, 4, 5; Cf. also John of St. Thomas loc. cit. p. 819. and 260.
- (33) This objective light must, of course, be distinguished from the interior light by which the cognitive potency is actualized.
- (34) I, 1, 3.
- (35) Cf. St. Thomas in Boeth. De Trin. V, 1.
- (36) Cf. St. Thomas I, 14, 1; In De Anima II, lect. 24; De Veritate, II, 2, etc.
- (37) Cf. In VI Eth. lect. 3, no. 1145 - 1149, etc.
- (38) De Trin. V, 1; Cf. Post. Anal. I, lect. 41; In VI Met. lect. 1; I, 85, 1, ad 1 and 2; In De Sensu et Sensato, lect. 1, etc.
- (39) In De Ente et Essentia, Proem.
- (40) In Boeth. De Trinitate, V, 3.
- (41) "... propriam et determinatam rationem praedicati inferioris (non) accipiat." John of St. Thomas: Ars Logica, p. 31 a 22.
- (42) Cf. John of St. Thomas, Curs. Phil. p. 265 a 30 - 35.
- (43) V, 1.
- (44) Cf. St. Thomas: I, 85, 1 ad 1; In VII Met.; De Trin. V, 2, etc. It is hardly necessary to point out that the student of nature uses individual observations and experiments, but only as a point of departure and as a means to arrive at the common sensible matter.
- (45) Lest confusion arise, it must be pointed out that modern authors use the word "metaphysic" in a much broader sense than the traditional Thomistic acceptance of the term. It is now generally employed in such a way as to include philosophy of nature as well as metaphysics. We shall use the word in its strict Thomistic meaning.
- (46) Cf. In I Phys. lect. 1; In VI Met. lect. 1; In XI Met. lect. 4, etc.

- (47) E.g. Maritain: Les Degrés du Savoir, Paris, Désclée de Brouwer et Cie., 1932, pp. 77, 79, etc.
- (48) De Subjecto Naturalis Philosophiae.
- (49) Curs. Phil. II, q. I, a. 1.
- (50) I, lect. 1.
- (51) In Phys. I, lect. 1.
- (52) Curs. Phil. I, 1. This point may give rise to a difficulty: a metaphysical definition (actus entis in potentia) seems to be employed in philosophy of nature. The answer is that the word potency has a different meaning when the definition is used in metaphysics and in philosophy of nature. In the latter case, it means the physical potency of matter. In the former it is considered in its general meaning as a principle of being. Every act of a being in potency is necessarily the act of a material thing, but while in reality there is identification, the aspect under which this reality is considered is different.
- (53) "...ita quod ens mobile, licet complexionem nominum contineat, incomplexum tamen et per se unum quod quid est significat, sicut ens per se." De Subjecto Naturalis Philosophiae, Laval edition, 1939, pp. 9 and 10.
- (54) "Ens mobile non sumitur complexe pro aggregato ex ente et mobilitate ut duobus, sed incomplexo pro quidditate, cui convenit mobilitas." Cursus Phil. II, q. I, a. 1.
- (55) In IV Met. lect. 13, no. 683. Cf. In III Met. lect. 3.
- (56) Cf. In Boeth De Trin. V, 2; In VI Eth. lect. 3, etc.
- (57) In VI Eth. lect. 1, no. 1123.
- (58) In Anal. Post. I, lect. 16. Cf. I, 86, 3.
- (59) In De Trin. V, 2.
- (60) In VI Met. lect. 1.
- (61) Cf. St. Thomas: In II Phys., lect. 3; In De Trinitate, V, 3.
- (62) Cf. St. Thomas: In De Trinitate, V, 3.

- (63) In Boeth. De Trin., VI, 2 Cf. De Coelo et Mundo, III.
- (64) Cf. De Anima I.
- (65) De Veritate XII, 3 ad 2.
- (66) Cura. Phil. I, Pars I, q. 27, a. 1.
- (67) E.g. II-II, 1, 1; I-II, 54, 2, ad 2; etc.
- (68) E.g. In VI Met. lect. 1; In I Poster. Anal. lect. 41, etc.
- (69) VI, lect. 1.
- (70) Cf. e.g. L.M. Regis: "La Philosophie de la Nature: Quelques apories", in Philosophie, Cahier I, Etudes et Recherches, Collège Dominicain, Ottawa, 1936.
- (71) "La Physique Aristotélicienne et la Philosophie," in Philosophie et Sciences, pp. 25 - 26.
- (72) Ibid. p. 24.
- (73) It may be true that Aristotle himself never brought out this hierarchical structure as explicitly as St. Thomas. Yet the latter merely clarified what was already implicit in Aristotle's doctrine. That is why we see no point in Mansion's argument when he writes: "On ne voit donc pas ce qui justifie, en bonne doctrine aristotélicienne, l'abstraction mathématique entendue comme un degré spécial d'abstraction. Il faudrait pour cela que les notes quantitatives possédassent vis-à-vis des autres notes auxquelles elles sont unies dans la réalité physique, une antériorité logique ou métaphysique, qu'Aristote n'a point cherché à établir. Les efforts faits dans ce sens par les scolastiques -- Saint Thomas ou d'autres -- ne peuvent pas entrer en ligne de compte pour formuler une appréciation concernant la position doctrinale du Stagiritique lui-même." Ibid. p. 25.
- (74) Ibid. 25.
- (75) Ibid. p. 25.
- (76) Ibid. p. 23 - 24.

- (77) Ibid. p. 24.
- (78) In De Trin., V, 3, ad 5.
- (79) Ibid. p. 27 - 28.
- (80) Loc. cit.
- (81) Chap. 27, Trans. by G.R.G. Mure; McKeon edition: The Basic Works of Aristotle, New York, Random House, 1941, p. 153.
- (82) Lectio 41, no. 5.
- (83) La Théorie Physique, p. 167.
- (84) Curs. Phil. II, Q. I, a. 2.
- (85) Loc. cit.
- (86) Meyerson makes it clear that the superior immateriality of arithmetic has been quite generally recognized. Cf. Du Cheminement de la Pensée, p. 302: "Telle était déjà l'idée de Gauss. 'Nous devons admettre humblement, écrivait-il à l'astronome Bessel, que, le nombre est uniquement le produit de notre esprit, l'espace, même au point de vue de notre esprit, constitue une réalité à laquelle nous ne pouvons a priori dicter complètement ses lois'. Dedekind, dans la préface de son fameux opuscule sur la nature du nombre a vivement insisté sur cette idée de l'autonomie de l'arithmétique à l'égard du réel. Le nombre est 'une émanation immédiate des lois pures de la pensée' et 'entièrement indépendant des concepts de temps et d'espace'; les nombres sont 'des créations libres de l'esprit humain, ils servent de moyen pour saisir plus aisément et avec plus de précision la diversité des choses' (Was sind und was sollen die Zahlen? 52 ed., Brunswick 1923, p. III... "Mais Locke, déjà jugeait que 'le nombre est la plus simple et la plus universelle de toutes nos idées" (Essai Philosophique, II, Ch. XVI, no. 1), et Hume considérait la géométrie comme moins assurée que l'arithmétique et l'algèbre au point de vue de la valeur apodictique de ses affirmations. (Psychologie, tr. Renouvier et Fillon, Paris 1878, p. 98)".
- (87) Cf. R. P. Salnan: "La Conception Scolastique de la Physique", in Philosophie et Sciences p. 37: "Ceux-ci (les

anciens) connaissaient bien des distinctions: les sciences et les arts, les arts libéraux et les arts serviles, les sciences pratiques et spéculatives, ces dernières diversifiées selon leur degré d'abstraction; mais jamais dans aucun domaine, ils n'ont opposé une "science" à une "philosophie."

- (88) Cf. Maritain: La Philosophie de la Nature; "Toutefois cette vérité capitale était payée chez les anciens, chez Aristote lui-même et chez les anciens scolastiques également, au prix d'une grave faute de précipitation intellectuelle. . . Pour l'optimisme des anciens, qui se portait très rapidement à des raisons d'être quelquefois très hypothétiques quand il s'agissait du détail des phénomènes, philosophie et sciences expérimentales étaient un seul et même savoir, et toutes les sciences du monde matériel étaient des subdivisions d'une seule et unique science spécifique qui s'appelait 'philosophia naturalis' . . . --p. 31.
- (89) Cf. Réflexions Sur l'Intelligence, Les Degrés du Savoir, La Philosophie de la Nature, Science et Sagesse, etc.
Cf. also Yves Simon: "Maritain's Philosophy of the Sciences," in The Thomist, Vol. V. pp. 85 - 102.
- (90) Art. cit. p. 95. (Italics ours)
- (91) Curs. Phil. II, q. I, a. 2.
- (92) Cf. In I Phys. lect. 1, nos. 6 - 8; De Sensu et Sensato, lect. 1, no. 2; De Generatione et Corruptione, Proem, In De Anima, lect. 1 no. 1; In Meteorolog. lect. 1; De Coelo et Mundo, lect. 1; etc.
- (93) The full significance of this statement will be brought out in the next chapter which the question of subalternation will be studied in detail.
- (94) In VI Met. lect. 1, no. 1147.
- (95) Ibid. no. 1165.
- (96) Ibid. no. 1149. Even on this point M. Maritain seems to be anti-Thomistic, for he writes: "Je note, entre parenthèses, que l'étude des premiers fondements ontologiques des mathématiques, la philosophie du nombre et du continu,

rentre dans la sphère de la philosophie de la nature, car l'abstraction mathématique, ne portant pas de soi sur l'être réel, ne comporte pas de sagesse dans son ordre propre." — Degrés du Savoir, p. 92-93. Cf. p. 345. Cf. also La Philosophie de la Nature, p. 91. It is difficult to see how the philosophy of mathematics, the problems of number and continuity fall within the sphere of philosophy of nature, which is the study of things in terms of mobility. Philosophy of nature is, indeed, a kind of wisdom within its own realm, in the sense that the general principles of mobile being which it studies give order to the entire study of natural things, but it is a wisdom only in terms of mobility. May not the source of Maritain's confusion be, at least in part, his substitution of sensible being for mobile being? The Philosophy of mathematics pertains to metaphysics not only for the reason given above, but also because, being wisdom, metaphysics has as one of its functions not only the critique of its own nature, but also of that of all the other sciences.

- (97) Degrés du Savoir, p. 352, footnote 1.
- (98) Ibid. Cf. La Philosophie de la Nature, pp. 88.
- (99) V, 1, ad 5.
- (100) Art. cit., Cf. St. Thomas: In De Sensu et Sensato, lect. 1 no. 16.
- (101) Cf. Les Degrés du Savoir, pp. 77 ff., 94 - 95, 352, etc.
- (102) After consistently assigning sensible being as the formal object of the study of nature throughout Les Degrés du Savoir, M. Maritain notes in La Philosophie de la Nature, (p. 113) that as Cajetan explains in his opusculum, De Subjecto Naturalis Philosophiae, the expression ens sensible is less apt than ens mobile. He still insists, however, that it is legitimate to assign ens sensible as the formal object. For reasons indicated earlier in this chapter we feel that there is much more involved here than a question of aptness.
- (103) "Maritain's Philosophy of the Sciences" in The Thomist, Vol. V, pp. 90 - 91.

- (104) We prescind here from the important difference that the first definition in so far as it has to do with a living being, is based upon both internal and external experience, whereas experimental scientists with the exception of the experimental psychologists in some cases, have adopted the method of drawing only from external experience even when dealing with living beings. We shall return to this point in chapter VIII. For the moment it is sufficient to note that a difference in the sources of experience employed cannot, obviously, constitute a specific difference between sciences.
- (105) In I Post. Anal. lect 16, no. 5.
- (106) Art. cit., p. 94.
- (107) Ibid. p., 92.
- (108) Degrés du Savoir pp. 74 - 75. Having made this admission he illogically holds that the experimental sciences come before the philosophy of nature in pedagogical order.
- (109) Cf. Degrés du Savoir, pp. 344 ff.
- (110) La Philosophie de la Nature, pp. 91 - 92.
- (111) The Sophist 219 a ff. Cf. Aristotle's criticism of this doctrine: Prior Anal. I, 31; Post. Anal. II, 5 and 13.
- (112) Morceaux Choisis, pp. 44 - 47.
- (113) In I Phys. lect. 1, no. 8.
- (114) Cf. La Philosophie de la Nature, p. 23.
- (115) Cf. Nos. 1147, 1151, 1165.
- (116) Degrés du Savoir, p. 77.
- (117) Ibid. p. 351.
- (118) Cf. Fulton J. Sheen: The Philosophy of Science, Milwaukee, The Bruce Publishing Co., 1934, pp. 164 - 189; Father Whittaker, O.P.: "The Position of Mathematics

in the Hierarchy of Speculative Science," in The Thomist, Vol. III, no. 3, p. 503.

- (119) Cf. Cajetan: De Subjecto Naturalis Philosophiae, ed. Laval, pp. 19 - 20.
- (120) Ibid. p. 91.
- (121) 639 b 13 - 640 a 9.
- (122) St. Thomas, lect. 15.
- (123) Lect. 15, no. 5.
- (124) De Part. An., loc. cit.
- (125) Cf. Phys. II.
- (126) In VI Met. lect. 1, no. 1149.
- (127) Ibid. no. 1146. Cf. I, 46, 1 ad 3.
- (128) We prescind here from the special case of man, whose future existence could have been demonstrated with apodictic necessity once given the existence of a material cosmos, for here the basis of demonstrability was something extrinsic.
- (129) Cf. St. Th. In De Trinitate, VI, 1: Ex hoc autem quod consideratio naturalis est circa materiam a pluribus dependet, scilicet a consideratione materiae et formae, et dispositionum materialium et proprietatum quae consequuntur formam in materia, Ubi cumque autem ad aliquid cognoscendum oportet considerare plura, est difficilior cognitio: unde in I Posteriorum dicitur, quod minus certa scientia est quae est ex additione, ut geometria ad arithmetica. Ex hoc vero quod eius consideratio est circa res mobiles, et quae non uniformiter se habent, eius cognitio est minus ferma, quia eius demonstrationes, ut in majori parte sunt ex hoc, quod contingit aliquando aliter se habere: et ideo quando aliqua scientia magis appropinquat ad singularia, sicut operativae, ut medicina, alchimia, et moralis, minus possunt habere de certitudine propter multitudinem eorum quae consideranda sunt in talibus scientiis, quo quodlibet si omittatur, frequenter erratur, et propter eorum variabilitatem.

- (130) Cf. St. Thom. In I Eth. Nic. lect. 1.
- ~~(131) Cf. infra Chapter III, note~~
- (132) The uniqueness of the method proper to each science does not, of course, exclude the possibility of a general treatise on scientific method, for logic, writes St. Thomas, "trahit communem modum procedendi in omnibus aliis scientiis. Modus autem proprius singularum scientiarum, in scientiis singulis circa principium tradi debet." (In II Met. lect. 5, no. 335) In view of this distinction of St. Thomas, the following assertion of Gilson is at best extremely ambiguous: "An Aristotelian discourse on method is. . . an impossibility; it is possible to speak only of a discourse on methods." (Op. cit. p. 71) Far from being an impossibility, a discourse on scientific method was actually written by Aristotle, namely the Posterior Analytics.
- (133) VI, 2.
- (134) La Philosophie de la Nature p. 24.
- (135) Lect. 21.
- (136) Ibid. no. 2.
- (137) Ibid. no. 6.
- (138) Lectio 22.
- (139) Lectio 15.
- (140) Ibid. no. 4.
- (141) Du Cheminement de la Pensée, p. 481.
- (142) No. 4.
- (143) No. 4.
- (144) Ibid. nos. 5 - 7.

Chapter III

- (1) In maintaining that philosophy of mathematics is an intermediary science between the second and third degrees of abstraction, Father Whittaker has confused the kind of application just mentioned with true subalternation. Cf. "The Position of Mathematics in the Hierarchy of Speculative Science," in the Thomist, Vol. III, No.3, p.496.
- (2) Cf. John of St. Thomas: Curs.Theol., I, q.1, d.2, a.6.
- (3) Cf. Gilson: L'Esprit de la Philosophie Médiévale, Paris Librairie, J.Vrin, 1932, P.4: "Alors que le rationaliste pur place la philosophie au sommet et l'identifie à la sagesse, le néo-scolastique la subalterne à la théologie, qui reste seule à mériter pleinement le nom de sagesse; mais pourquoi certains néo-scolastiques pensent-ils que même subalternée à la théologie, leur philosophie demeure identique en nature à celle qui ne reconnaît aucune sagesse au-dessus d'elle."
- (4) Cf. Ars. Logica: P.I, Q. XXVI, a.2.
- (5) In De Trinitate, VI, 1.
- (6) The ancient Thomists sometimes called this type of dependence subalternation secundum quid, but denied that it was subalternation simpliciter. Cf. John of St. Thomas, Curs. Phil. Ars. Logica P. II, q. XXVI, a.2, pp. 798.
- (7) Loc. cit. p. 796.
- (8) V, 1, aq.5.
- (9) Ibid.
- (10) Loco. cit. p 796 b 43. Philosophy for John of St.Thomas means the science of nature.
- (11) Cf. Fulton Sheen: "Furthermore, the more developed the empirical sciences the better is the raw material upon which metaphysics may speculate to build a scientia media

or the Philosophy of Nature." The Philosophy of Science, p. 189. "Just as the science of mathematical physics is formed by the application of mathematics to physics, so too, the science of the philosophy of nature is formed by the application of the fundamental principles of metaphysics to the natural sciences." Ibid. 184. Cf. Whitaker: "The philosophy of nature is the intermediary science between the physical and metaphysical orders." Op. cit. p. 503.

- (12) Cf. Annibaldus: In I Sent. dist. 1, q. 1, a. 1: "Primarum scientiarum proximum principium est intellectus, earum vero scientiarum, quae sua principia ab aliis supponunt, proximum principium est credulitas principiorum ab aliis suppositorum; primum vero earum principium est intellectus. Perficitur tamen certitudo istarum scientiarum cum per viam resolutionis in ipsum intellectum primorum principiorum perveniunt."
- (13) John of St. Thomas, Curs. Theol. I, q. 1, d. 2, a. 6 p. 369 b.
- (14) Curs. Theol. I, q. 1, d. 2, a. 5, p. 364 a.
- (15) In I, 1, 2.
- (16) Curs. Theol. I, q. 1, d. 2, a. 5, p. 364 a.
- (17) Curs. Phil. Ars Logica, II, q. XXVI, a. 3. pp. 799 ff.
- (18) De Veritate XIV, 9.
- (19) De Veritate, XIV, 9, ad 3.
- (20) Curs. Theol. loc. cit. p. 369.
- (21) In De Trinitate, V, 3, ad 6.
- (22) Cf. Vassily Pavlov: "Mathematics for the Doctor in the Million," in Philosophy of Science, Vol. II, no. 1, p. 48: "...an effort has been made...to a pretense of applying the concrete sciences to the abstract ones. It has gone to the extent of naming new hybrids in inverse order as physical mathematics, (compare Einsteins' physical geometry)...biological mathematics, and the like.. ...To this writer it still looks like the application of mathematics to biology rather than the reverse."

- (23) I, lect. 5, no. 7.
- (24) In De Trinitate, V, 3, ad 7.
- (25) In I Post. Anal., lect. 25, no. 2.
- (26) Ars Logica, Pars II, Q. XXVI, a. 2, p. 797.
- (27) Ars Logica, Pars II, q. XXVII, a. 1, p. 827.
- (28) "La Conception Scolastique de la Physique" in Philosophie et Sciences, pp. 48 - 49.
- (29) In I, 1, 2.
- (30) Ars Logica, Pars II, Q. XXVI, a 2, pp. 798 - 799.
- (31) Lest this definition seem to exclude a posteriori knowledge by which we know things through their effects, it is necessary to note that the term "cause" in the definition refers to the cause of science.
- (32) Ars Logica, loc. cit. p. 798 b.
- (33) Cf. James A. McWilliams: "Idealism in Science," in The Modern Schoolman, Vol 14, p. 7: "These scientists are, in their turn, victims of the initial error of grounding their partial science on another partial science, on mathematics instead of metaphysics."
- (34) Loc. cit. p. 799.
- (35) In I Post. chap. 12.
- (36) I Post. Anal. lect. 25. no. 4.
- (37) I Post. Anal. ch. 13, no. 6.
- (38) In II Phys. lect. 3, no. 8.
- (39) Substance and Function, p. 117.
- (40) Cf. "Physique et Philosophie", in Philosophie et Sciences, p. 86: "Dès que nous entrons en physique, nous ne traitons que de rapports entre grandeurs. Mais la physique ne devient pas pour cela de la mathématique, et il n'y a ici aucune attraction d'une science moins noble par une science plus noble. C'est le donné initial qui impose cette forme

mathématique à la physique. Les relations entre grandeurs variables sont données implicitement quand ces grandeurs sont données; mais il faut les expliciter et les synthétiser." Cf. also p. 81: "Puis tout se traduit en nombres concrets. Les nombres fournis par les instruments ne sont pas des nombres abstraits ni des êtres de raison mathématiques; ce sont des nombres 'qualifiés par l'instrument qui les a fournis'. 7 volts et 7 degrés ne sont pas la même chose parce que le premier s'obtient avec un voltmètre et le second avec un thermomètre".

- (41) In I Post. Anal. lect. 25, no. 4.
- (42) Phys. II, 2
- (43) In De Trin., V, 3, ad 6.
- (44) Curs. Phil., Vol. II, Q. I, a. 1.
- (45) In II Phys., lect. 3, no. 8. Many modern authors hold that in this passage St. Thomas is guilty of misreading Aristotle. Maritain, for example, has this to say: "Ici j'ouvre une parenthèse d'ordre historique, Aristote, en réalité, n'a pas dit cela expressément, c'est saint Thomas qui l'a dit en s'appuyant sur un texte d'Aristote pour notre plus grand profit mal compris. Aristote, au livre II de la Physique, chap. 2, 194 a, 7, parle de la connaissance mathématique, et il parle des parties des mathématiques qui sont plus physiques que les autres, qui concernent d'avantage les choses physiques, c'est ce qu'il appelle τὰ φυσικώτερα τῶν μαθηματικῶν, les traducteurs modernes traduisent à bon droit: "les parties les plus physiques des mathématiques". Saint Thomas, au contraire, dans sa troisième leçon sur le Livre II de la Physique, entend qu'il s'agit non pas des parties les plus physiques des mathématiques, mais de sciences plus physiques que mathématiques, magis naturales quam mathematicae." — La Philosophie de la Nature, p. 36. Cf. also Mansion, op. cit. p. 27. In spite of the fact that a superficial reading of the Greek phrase cited by Maritain might seem to favor his interpretation, we prefer to believe that St. Thomas' reading of Aristotle is correct. There are three reasons. First, the rendition of St. Thomas is not incompatible with the construction of the Greek phrase. Secondly, the Latin translation which Saint Thomas followed was that made by William of Moerbeke, and it is considered

by the most competent of modern critics as extremely accurate. Thirdly, the exactness of the version of St. Thomas is clear from the whole paragraph in which the disputed phrase appears, for in it Aristotle shows precisely that optics, for example, is more physical than mathematical. In order to bring out this last point we give here the whole paragraph in Greek, together with William of Moerbeke's translation:

Δηλοῖ δὲ καὶ τὰ φυσικώτερα
τῶν μαθημάτων, οἷον ὀπτική
καὶ ἁρμονική καὶ ἀστρολογία
ἐν ἀπείρῳ γὰρ τρόπον τινὶ
ἔχουσι τὴν γεωμετρίαν. Ἀλλ' ἡ
μὲν γεωμετρία περὶ γραμμῆς
φυσικῆς σχοπεῖ, ἀλλ' οὐχ ἡ
φυσική· ἡ δ' ὀπτική μαθηματική
μὲν γραμμῆν, ἀλλ' οὐχ ἡ
μαθηματική, ἀλλ' ἡ φυσική.

"Demonstrant autem et quae magis physica quam mathematica, ut perspectiva et harmonica et astrologia: e contrario enim quodammodo se habent ad geometriam. Geometria quidem enim physicam intendit lineam, sed non in quantum est physica: sed perspectiva quidem mathematicam lineam, sed non in quantum mathematica, sed in quantum est physica.

- (46) II - II, 9, 2, ad 3.
- (47) Curs. Phil. II, Q. I, a. 1.
- (48) Cf. Fulton Sheen: "Every science is constituted of a material and formal object. The material object is what is studied; the formal object is the aspect or the how it is studied. The new mathematical physics is, from the material point of view, a science of the real world, but it soon leaves that concrete, real world to manipulate it in terms of mathematical symbols." -- The Philosophy of Science, p. 83.
- (49) Op. cit. p. 28.
- (50) On the Method of Theoretical Physics, p. 12. Cf. The World As I See It, by the same author, pp. 32 - 34.
- (51) Ibid. p. 7.
- (52) Op. cit. p. 173 footnote.
- (53) Du Cheminement de la Pensée, p. 482. In spite of the great name that Meyerson has won for himself in the philosophy of science, and especially in the historical background of science, we find it necessary to remark that throughout his many writings he has consistently misinterpreted Aristotelianism and Thomism.

Chapter IV

- (1) Lect. 1, no. 1.
- (2) Cf. I Met. c. 2, 982 a 17.
- (3) Cf. De Coelo et Mundo, Prooemium; De Generatione et Corruptione, Prooemium; De Meteorologicis, I; De Anima, I, lect. 1; De Sensu et Sensato, lect. 1; De Partibus Animalium, II, c. 1; De Generatione Animalium, I, c. 1, etc.
- (4) Lectio 1, no. 2.
- (5) It is worth while noting in this connection that the scholastic manuals which make the study of nature a part of metaphysics are perfectly logical in placing the study of general metaphysics before that of cosmology and psychology.
- (6) Cf. Harold R. Smart: The Logic of Science, New York, D. Appelton and Co., 1931, p. 80.
- (7) The Analysis of Matter, New York, Harcourt, Brace and Co., 1927, p. 130.
- (8) Cf. Whitehead: Science and the Modern World, p. 41:
"Nothing is more impressive than the fact that as mathematics withdrew increasingly into the upper regions of ever greater extremes of abstract thought, it returned back to earth with a corresponding growth of importance for the analysis of concrete fact. . .
The paradox is now fully established that the utmost abstractions are the true weapons with which to control our thought of concrete fact."
- (9) Translation by R.P. Hardie and R.K. Gaye.
- (10) Lect. 1, no. 6.
- (11) Cf. St. Thom: In I Post. Anal. lect. 4, no. 16: "In omni enim generatione, quod est in potentia est prius tempore et posterius natura, quod autem est completum in actu est prius natura et posterius tempore. Cognitio autem generis est quasi potentialis in comparatione ad cognitionem speciei, in qua actu sciuntur omnia essentialia rei. Unde in generatione scientiae nostrae prius est cognoscere magis commune quam minus commune." Cf. John of St. Thom., Logica II, q. I, a. 1.

- (12) I, lect. 1, no. 1.
- (13) De Pot. III, 2, ad 2.
- (14) pp. 128 - 129.
- (15) "Que si, cependant, on considère les limites de ce qu'on entendait ainsi déduire, on s'aperçoit que Hegel est resté bien en deçà de son modèle. Aristote et ses sectateurs au moyen âge, nous l'avons vu à propos de Gersonide, limitaient sans doute la déduction à l'universel, mais ces universaux comprenaient tout ce qui constitue la science, puisque celle-ci ne peut traiter que du genre. Hegel ne déclare déductibles que certains aspects très généraux de la science, tout le reste étant issu de l'arbitraire, de la nature et justiciable seulement de savoir empirique. C'est qu'en dépit de toute son 'arrogance logique', la philosophie hégélienne est obligée de tenir compte de ce fait qu'un énorme acquis scientifique s'interpose entre elle et les derniers sectateurs de la physique péripatéticque, et que cette évolution lui interdit de pousser sa régression au delà de certaines limites." pp. 476 - 477.
- (16) No. 8.
- (17) In II Phys., lect. 6, no. 3; De Trin. V, 4, etc.
- (18) Methodologie Scientifique, Laval, p. 26.
- (19) Cf. for example Rey: "La Physique scolastique avait la prétention d'atteindre directement les propositions générales dont se déduisait le système complet de la nature. Contre cette prétention s'éleva la physique de la Renaissance." -- La Théorie Physique, p. 344.
- (20) De Partibus Animalium, Ch. 5. Transl. by Ogle.
- (21) I, c. 2, 316 a 5 - 15.
- (22) Cf. C. Singer: The Story of Living Things, pp. 9 - 44; "The Birth of Science Among the Greeks" in Enc. of Modern Knowledge, pp. 1415 - 1427.

- (23) De Trin. VI, 2.
- (24) The Philosophy of Physical Science, p. 10.
- (25) Lect. 14, no. 8. Cf. In XII Met. lect. 12.
- (26) Cf. Phys. II, lect. 2.
- (27) Loc. cit. Cf. also page 133: "Ainsi le panlogisme péripatéticien et le panmathématisme et panmécanisme platonicien et moderne se rencontrent dans cette foi en la rationalité complète, et, partant, en la déductibilité de la nature."
- (28) The Mysterious Universe, pp. 123 - 124.
- (29) Cf. Spinoza: Ethics, Part I, Prop. XXIX: "Things could not have been produced by God in any other way or in any other order than the way and the order in which they have been produced."
- (30) "None of even the relatively gross structures that the microscope has revealed was suspected to exist before it was seen." -- Yves Delage. Cited by W.R. Thompson, Science and Common Sense, p. 45.
- (31) Paul Valéry, Cited by Louis De Broglie: Matière et Lumière, p. 318.
- (32) Cf. Meyerson, Identité et Réalité, p. 368: "On ne peut mieux caractériser les traits distinctifs de cette méthode que ne l'a fait Paul Tannery: 'D'une part, tendance à s'attacher aux phénomènes tels que les sens les révèlent à l'observation superficielle et grossière, on peut même dire respect marqué pour les croyances vulgaires, du moment où elles ne sont pas visiblement erronées; d'autre part, tendance à remonter le plus haut possible et le plus tôt possible dans la série des causes, mais cela par simple analyse du concept et sans aucun retour nouveau à l'expérience.' Les Principes de la science de la nature chez Aristote" in Congrès de Philosophie, 1900, Vol. IV, p. 214.
- (33) Principia, III Pars, ch. 1. Cf. Lettre à Mersenne (1632) Ed. Adam et Tannery, t. I, p. 250: "Je suis devenu si

hardi que j'ose maintenant chercher la cause de la situation de chaque étoile fixe."

- (34) Science et Hypothèse, p. 168.
- (35) De Coelo, III, c. 7, 306 a 5 ff. Trans. by J.L. Stocks.
- (36) De Coelo, I, c. 3, 270 b 10.
- (37) Lect. 7, no. 6. Cf. lect. 3: "Unde hoc non est demonstratum sed suppositio quaedam."
- (38) Ch. 12.
- (39) Lect. 17, nos. 1 and 2.
- (40) I q. 32, a. 1, ad.2. Cf. In I Meteor. lect. 11, no. 1: "Postquam Philosophus reprobant opiniones aliorum, hic incipit ponere opinionem propriam de comitis. Et primo ostendit modum certitudinis qui est in hac materia exquirendus. Et dicit quod de talibus quae sunt immanifesta sensui, non est exquirenda certa demonstratio et necessaria, sicut in mathematicis et in his quae subiacent sensui; sed sufficit per rationem demonstrare et ostendere causam ita quod quaestionem solvamus per aliquam solutionem possibilem, ex qua non sequatur aliquod inconveniens, per ea quae hic apparent secundum sensum. Unde hoc modo in proposito ad habendam causam est procedendum." Cf. also In XII Met. lect. 10, no. 2586., etc., etc.
- (41) Cf. La Théorie Physique, pp. 54 ff.
- (42) Prodromus Dissertationum Cosmographicarum, Continens Mysterium Cosmographicum, in Joannis Kepleri Astronomi Opera Omnia, t. K, p. 112 - 153 -- Cited by Duhem, op. cit. p. 58.
- (43) Op. cit. p. 59.
- (44) Cf. Meyerson: De l'Explication dans les Sciences, p. 100: "Pour Comte, en effet, les lois découvertes, si elles atteignent un certain degré de généralité (comme, par exemple, la loi de Mariotte), doivent demeurer à tout jamais. Toute recherche ultérieure tendant à les ébranler, ou seulement à en modifier ou à en préciser le contenu, est jugée parfaitement oiseuse et doit être rigoureusement proscrite. C'est là un thème sur lequel Comte est revenu maintes reprises et au sujet duquel il s'est exprimé avec

l'énergie la plus grande. Accumulant les termes de réprobation, il a déclaré 'incohérents ou stériles', procédant d'une 'curiosité toujours vaine et gravement perturbatrice', d'une 'puerile curiosité stimulée par une vaine ambition', les travaux où l'on se sert d'instruments de mesure trop précis; il a protesté hautement contre 'l'abus des recherches microscopiques et le crédit exagéré qu'on accorde trop souvent encore à un moyen d'investigation aussi ambiguë.'"¹¹¹

- (45) Posterior Analytics, I, c. 27, 87 a 30 ff. -- Trans. by G.R.G. Mure.
- (46) Cf. Meyerson: "L'irrationnel scientifique ressemble donc, à certains égards, à celui que, selon Renouvier, constituerait un acte de libre arbitre; il représente aussi dans un ordre de considérations tout différent il est vrai, un 'commencement absolu.'" De L'Explication dans les Sciences, p. 545.
- (47) Cf. infra Ch. VI.
- (48) Lect. 41, no. 3.
- (49) Met. II, C. 1, 993 b 3. Trans. by W.D. Ross.
- (50) Cf. St. Thomas, In De Trin. V, 1.
- (51) Curs. Phil., I, p. 767 b 28 - 41.
- (52) "Le Problème de l'indéterminisme" in L'Académie Canadienne Saint Thomas d'Aquin, Sixième Session, Québec, 1937, Typ. l'Action Catholique, p. 67.
- (53) Curs. Phil., I, p. 200. Cf. p. 790.
- (54) Cf. infra Chapter VI.
- (55) Cf. John of St. Thomas: Logica p. 60.
- (56) Cf. e.g. Bertrand Russell: "The general principles of science, such as the belief in the reign of law, and the belief that every event must have a cause, are as completely dependent upon the inductive principle as are the beliefs of daily life. All such general principles are believed

because mankind have found innumerable instances of their truth, and no instances of their falsehood. But this affords no evidence for their truth in the future unless the inductive principle is assumed." Problems of Philosophy, London, Thornton Butterworth, Ltd., 1912, p. 107.

- (57) Cf. e.g. Mansion: "De ces données -- encore incomplètes -- il faudrait conclure: pour Aristote, la science -- prise au sens moderne du mot ou dans un sens analogue -- n'existe pas." Op. cit. p. 19.
- (58) Cf. e.g. texts of Maritain considered in ch. II.
- (59) Ch. I, 642 a 5.
- (60) Ch. 5, 645 a 5.
- (61) Essay Concerning Human Understanding, IV, ch. 12 sect. 10.
- (62) Cf. St. Thomas: In I Eth. lect. 4, no. 52: . . . Et quia nos ratiocinando notitiam aequirimus, oportet quod procedamus ab his quae sunt magis nota nobis; et si quidem eadem sunt magis nota nobis et simpliciter, tunc ratio procedet a principiis, sicut in mathematicis. Si autem alia magis nota sint simpliciter, et alia quoad nos, tunc oportet e converso procedere sicut in naturalibus et moralibus." Cf. also In I Poster. Anal. lect 4, no. 16. This is just one of the several points in which modern scholastics have made philosophy a kind of mathematics.
- (63) Lect. 6, no. 10.
- (64) Philosophy of Science p. 167.
- (65) Cited by Thompson: Introduction to Science, pp. 124 - 125. Cf. also Cohen: and Nagel: An Introduction to Logic and Scientific Method, New York, Harcourt, Brace, and Co., 1934, p. 149.
- (66) Cf. Rey: La Théorie Physique; "La relativité des connaissances physico-chimiques leur permet de croire à côté de ces connaissances à des affirmations au sujet desquelles

la physique est incompetente. Elle ne leur permet pas d'en connaître l'objet." p. 367.

- (67) Physics of the Twentieth Century, New York, The Philosophical Library, 1944, p. 142. Cf. Dingle: Through Science to Philosophy: "If, without violating the principles on which physics and biology have developed science can extend its correlations over the whole of experience it will become philosophy." p. 34. Cf. also Whitehead: Process and Reality, Cambridge University Press, 1929, pp. 12 - 13.
- (68) La Philosophie de la Nature, p. 141. We believe that Maritain's error is at least partially due to the fact that he looks upon the whole of philosophy as pedagogically posterior to the experimental sciences. He writes: "Il y a, c'est bien sûr, une forte dépendance matérielle de la philosophie à l'égard des sciences. Tout d'abord, dans la hiérarchie des connaissances la philosophie est comme le terme culminant, et qui par suite vient pédagogiquement en dernier lieu. -- Les Degrés Du Savoir, p. 101. In our opinion the correct pedagogical order of the speculative sciences is as follows: mathematics, philosophy of nature, the experimental sciences, metaphysics. We shall try to explain in chapter VI why mathematics is put in the first place: of all the speculative sciences it has the greatest harmony with the human mind. That is why there are child prodigies in mathematics and not in the other speculative sciences.
- (69) The simplicity and commonness of the experience that is sufficient for philosophy of nature has led some authors to make of philosophy a kind of logic. Thus Professor Watson writes: "The student of philosophy already knows how to speak in the manner that is understood by his fellow in every-day affairs. When he begins philosophy, questions are asked which he can answer without learning new facts about the world...It must be the nature of philosophical problems that their solution does not have to await the becoming of facts. It must be irrelevant to philosophy what actually happens in the world. If this were not so how would philosophy differ from the sciences whose business is with facts. " -- On Understanding Physics, Cambridge University Press, 1938, p. 6. cf. also p. 21.

- (70) Introduction to What Man Has Made of Man, pp. xi - xii. Cf. Dotterer: Philosophy By Way of the Sciences: "If then, while including the descriptions arrived at by common sense within the class of genuine descriptions of reality we should deny this status to the results attained by scientific research, our attitude toward science would seem to imply the principle that the more pains we take in trying to discover the nature and structure of the actual world, the less likely we are to succeed in the attempt." New York, the Macmillan Co., 1929, p. 297.
- (71) "Les Sciences expérimentales sont-elles distinctes de la philosophie de la nature?" in Culture, 1941, IV, p. 473.
- (72) The Philosophy of Physical Science, p. 8.
- (73) In Met. I, lect. 1. Cf. In I Sent. d. 38, q. 1, a. 5, c.
- (74) Cf. Duhem: La Théorie Physique, p. 248: "Si donc l'interprétation théorique enlève aux résultats de l'expérience de Physique la certitude immédiate que possèdent les données de l'observation vulgaire, en revanche, c'est l'interprétation théorique qui permet à l'expérience scientifique de pénétrer bien plus avant que le sens commun dans l'analyse détaillée des phénomènes, d'en donner une description dont la précision dépasse de beaucoup l'exactitude du langage courant." Cf. also Pp. 246 - 247.
- (75) An exception to this last statement is found in divine knowledge.
- (76) De L'Explication dans les Sciences p. 214. Cf. Cheminement de La Pensée, p. 52.
- (77) La Théorie Physique, p. 29. Cf. p. 497.
- (78) Cf. Meyerson: Identité et Réalité, p. 20: "La loi qui régit l'action du levier n'envisage que le 'levier mathématique'; or, nous savons fort bien que nous ne rencontrerons jamais rien de pareil dans la nature. De même nous n'y rencontrerons jamais les 'gaz idéaux' de la physique ni les cristaux tels que nous les montrent les modèles cristallographiques. . . On connaît l'ensemble formidable de travaux auxquels Stas a dû se livrer pour obtenir de l'argent à peu près chimiquement pur; on sait

d'ailleurs qu'il avait choisi ce corps comme point de départ de ses déterminations parce qu'il lui paraissait offrir le plus de facilités, et l'on sait aussi que l'argent obtenu par lui n'était pas réellement pur, de sorte qu'il a fallu depuis rectifier les résultats auxquels il était parvenu. On peut voir, par cet exemple topique, combien le substrat même de la loi, le concept généralisé, est chose de notre pensée." Cf. also De L'Explication Dans les Sciences, pp 25, 26, etc.

- (79) Substance and Function and The Theory of Relativity.
p. 130.
- (80) Preface to the second edition.
- (81) Theoretical Biology, Introduction.
- (82) "La seule science qui mérite proprement ce nom est celle dont la certitude est apodictique; la connaissance qui ne peut contenir d'une certitude empirique est ce qu'on n'appelle qu'improprement un savoir." un savoir théorique "ne mérite le nom de science de la nature que dans le cas où les lois de la nature qui en sont le fondement sont connues a priori et ne sont pas de simples lois de l'expérience." Kant: Premiers Principes Métaphysiques de la Science de la Nature, French transl. by Andler et Chavannes, Paris, 1892, p. 4. Cited by Meyerson; De l'Explication Dans les Sciences, p. 458.
- (83) Science et Hypothèse, p. 170. Cf. Whitehead: Process and Reality, p. 22: "Every scientific memoir in its record of the 'facts' is shot through and through with interpretation." And Schrodinger writes: "We cannot close the door to the entry of subjective factors in determining our scientific policy and in giving a definite direction to our line of further advance."--Science and the Human Temperament, New York, W. W. Norton and Co., 1935, p. 87.
- (84) Pp. 39, 44, 53.
- (85) Pierre Duhem is particularly eloquent on this point. He has shown with great penetration that what Bernard says is true of biological and medical science, applies with infinitely greater force to physics, in which the part

that theory plays is so great. Cf. La Théorie Physique, pp. 276 - 278, et passim. "L'enseignement de la Physique par la méthode purement inductive, telle que l'a définie Newton, est une chimère. Celui qui prétend saisir cette chimère se leurre et leurre ses élèves. Il leur donne, pour faits vus des faits simplement prévus; pour procédés réalisables, des expériences purement idéales; pour lois expérimentales, des propositions dont les termes ne peuvent sans contradiction, être pris comme exprimant des réalités. La Physique qu'il expose est une Physique fautive et falsifiée." p. 309.

- (86) Du Cheminement de la Pensée, p. 463.
- (87) Cf. Meyerson: "De plus, exposés sans hypothèses, les résultats expérimentaux nous apparaissent comme quelque chose de définitif, d'achevé, sans que nous apercevions la voie qui y a mené, ni celle qui pourra nous conduire plus loin; car la science n'est pas baconienne, et l'expérience seule, sans le secours de l'hypothèse, ne saurait y mener bien loin. C'est ce qui fait que l'image de la science ou d'une partie de la science que l'on nous offre ainsi sera en quelque sorte statique, alors que la science se trouve en réalité dans un flux perpétuel et dynamique." -- Identité et Réalité, p. 468.
- (88) Where is Science Going? p. 97 Cf. Jeans: The New Background of Science, pp. 46 - 47.
- (89) Du Cheminement de la Pensée, p. 45.
- (90) Cf. Lindsay and Margenau: Foundations of Physics, New York, John Wiley and Sons, Inc., 1936, pp. 4 - 5.
- (91) Cf. Cassirer: Nevertheless it would be a mistake to assume that exact science, owing to this characteristic feature of its concepts, withdraws more and more from the tasks offered by concrete empirical existence. Precisely in this apparent turning away from the reality of things, science is directed upon them in a new way." "They only go beyond the given in order to grasp the more sharply the systematic structural relations of the given." -- Substance And Function and Einstein's Theory of Relativity, pp. 228 - 229, 128.

- (92) Cf. Jules Tannery: *Science et Philosophie*, pp. 322 - 333: "Je pourrais bien ne pas parler de la masse puisque c'est une unité fondamentale et non une unité dérivée, mais je ne vois aucun inconvénient à ce qu'on définisse la masse d'un corps en décrivant la façon dont on fait une pesée (une double pesée, si l'on veut) au moyen d'une balance. Si l'on équilibre avec des grammes on dira que la masse en grammes est exprimée, ou encore, qu'on a pris le gramme pour unité de masse; c'est la même chose. Quant au 'gramme' il ne me gênerait nullement que les élèves pensassent au petit cylindre de cuivre que l'on sait, mais il est bien entendu que pour satisfaire tout le monde on parlera du morceau de platine irradié, etc... en n'oubliant pas, si l'on veut être dans le train, de dire que ce morceau de platine est déposé au pavillon de Bellevue, et non aux Archives. Je sais bien qu'on criera au cercle vicieux. . . "
- (93) Cf. Eddington: *The Philosophy of Physical Science*, p. 70.
- (94) Cf. Eddington: *The Philosophy of Physical Science*, p. 189: "Reference may also be made to another general philosophical system, namely logical positivism. Our insistence that physical quantities are to be defined in such a way that the assertions of physics admit of observational verification, may suggest an affinity with logical positivism. The meaning of a scientific statement is to be ascertained by reference to the steps which would be taken to verify it. This will be recognized as a tenet of logical positivism -- only it is there extended to all statements. When it is limited as here to items of physical knowledge, it is in no sense a philosophical tenet; it is only a bringing into line of the language of theoretical and of experimental physics, so that we may not claim the support of observation for assertions which have no observational foundation. If it were a general characteristic of knowledge, it would not be so useful to us in discriminating physical knowledge from other kinds of knowledge. We are therefore not particularly predisposed to favour the more general assertion of logical positivism that the meaning of all non-tautological statements is to be ascertained in the same way, namely by reference to the procedure of verifying them."

- (95) Pp. 1 - 4. Cf. The Nature of the Physical World, pp. 154 - 155; "Our knowledge of the external world cannot be divorced from the nature of the appliances with which we have obtained the knowledge. The truth of the law of gravitation cannot be regarded as subsisting apart from the experimental procedure by which we have ascertained its truth. The conception of frames of space and time, and of the non-emptiness of the world described as energy, momentum, etc., is bound up with the survey by gross appliances. When they can no longer be supported by such a survey, the conceptions melt away into meaninglessness." Et passim. For a detailed analysis of the meaning of operationalism and its implications cf. Percy Bridgman: The Logic of Modern Physics, New York, The Macmillan Co., 1932., and The Nature of Physical Theory. Princeton University Press, 1936.
- (96) Methodologie Scientifique, p. 16.
- (97) Principles of the Quantum Theory, p. 3.
- (98) Loc. cit.
- (99) Cf. St. Thomas: "Sunt enim quidam, qui veritatem intelligibilem capere non possunt, nisi eis particulatim per singula explicetur. Et hoc quidem ex debilitate intellectus eorum contingit." I, 55, 3.
- (100) Cf. Einstein: Introduction to Where is Science Going? p. 13; "In every important advance the physicist finds that the fundamental laws are simplified more and more as experimental research advances. He is astonished to notice how sublime order emerges from what appeared to be chaos." Cf. also Hermann Weyl: The Open World, p. 41: "The astonishing thing is not that there exist natural laws, but that the further the analysis proceeds, the finer the details, the finer the elements to which the phenomena are reduced, the simpler -- and not the more complicated, as one would originally expect -- the fundamental relations become and the more exactly do they describe the actual occurrences."
- (101) Physics, Bk. II
- (102) Cf. Boutroux: De L'Idée de la Loi Naturelle dans la Science et dans la Philosophie, p. 42.: Les lois mé-

caniques ne peuvent donc être considérées comme réalisées telles quelles dans la nature des choses. Les concepts dont elles se composent deviennent intelligibles quand on en fait des êtres."

- (103) Les Principes de la Physique, (adaptation of Physics, The Elements), Paris, Librairie Felix Alcan, 1929, p. 22.
- (104) La Théorie Physique, p. 252.
- (105) The Philosophy of Physical Science, pp. 66 - 67.
- (106) Cf. Einstein and Infeld: Evolution of Physics, pp. 11 - 12 Eddington: The Nature of the Physical World, pp. 126 ff.
- (107) For other examples cf. Duhem, op. cit. pp. 325 - 327.
- (108) Cited by Duhem, op. cit., p. 318.
- (109) The Nature of the Physical World, p. 234.
- (110) L'Evolution des Idées en Physique, Paris, Flammarion, 1938, p. 286. Cf. Meyerson, De L'Explication, etc. p. 60.
- (111) Cf. Louis De Broglie: Matière et Lumière, Paris, Editions Albin Michel, 1937, pp. 319 - 320.
- (112) Cf. Einstein: The World As I See It, pp. 35 - 36:
 "The natural philosophers of those days were, on the contrary, most of them possessed with the idea that the fundamental concepts and postulates of physics were not in the logical sense free inventions of the human mind but could be deduced from experience by 'abstraction' -- that is to say by logical means. A clear recognition of the erroneousness of this notion really only came with the general theory of relativity, which showed that one could take account of a wider range of empirical facts, and that too in a more satisfactory and complete manner, on a foundation quite different from the Newtonian. But quite apart from the question of the superiority of one or the other, the fictitious character of the fundamental principles is perfectly evident from the fact that we can point to two essentially different principles, both of which correspond with experience to a large extent; this proves at the same time that every attempt at a logical deduction

of the basic concepts and postulates of mechanics from elementary experiences is doomed to failure."

- (113) Cf. Arist: I Anal. Post., 22: "Nam ab effectu qui a pluribus causis procedere potest, non potest una illarum concludi."
- (114) Cf. Meyerson: "Une science privée de théorie apparaîtrait en quelque sorte comme entièrement achevée, statique, alors que la vraie science, nous le sentons, doit être en flux, évoluer, progresser. -- De L'Explication, etc. p. 55.
- (115) For an excellent example of how science advances by successive theoretical syntheses, see Louis De Broglie, Matière et Lumière, pp. 157 - 177.
- (116) Cf. Duhem, op. cit. pp. 268 - 269; Rey, op. cit. p. 194, etc.
- (117) Cf. Cassirer: "It is only owing to the fact that science abandons the attempt to give a direct, sensuous copy of reality, that science is able to represent this reality as a necessary connection of grounds and consequents. It is only through going beyond the circle of the given, that science creates the intellectual means of representing the given according to laws."--op. cit. pp. 164 - 165. Cf. p. 280.
- (118) Cf. Petit, C.S.C.: Methodologie Scientifique, p. 10: "Au-dessus du sujet, au delà de l'objet immédiat, la science se fonde sur le projet. Dans la pensée scientifique, la méditation de l'objet par le sujet prend toujours la forme du projet."
- (119) L'Intellectualisme de St. Thomas d'Aquin, Paris, Beauchesne, 1924, p. 146.
- (120) Cf. Curs. Phil. I, ed. Reiser, p. 86 a 18; 87 a 49 - b 23.
- (121) Cf. St. Thomas: "A forma quae est in anima nostra procedit forma quae est in materia in artificialibus, in naturalibus autem e contrario." In Met. VII.

- (122) V, 1, ad 3. In the context of this passage St. Thomas mentions explicitly only logic and mathematics, but from the examples he cites it is evident that he includes mathematical physics under mathematics.
- (123) I, 93, 1.
- (124) Cf. Goethe: "Hypotheses are the scaffolds which are erected in front of a building..." in Maxims and Reflections. Cited by Frank: Between Physics and Philosophy, p.30, Cf. also von Uexkull, loc. cit. p.x.
- (125) Cf. e.g. Sir James Jeans: The New Background of Science: "The history of physical science in the twentieth century is one of a progressive emancipation from the purely human angle of vision." p.5. Cf.pp. 227 - 228.
- (126) Op. Cit. p. 445.
- (127) Cf. Niels Bohr: "The present day situation in physics brings forcefully home to us the old adage that we are actors as well as spectators of the grand drama of existence." Cited by Tobias Dantzig: Aspects of Science, p. 135.
- (128) Evolution Créatrice, pp.131, 101, 356. Cf. Matière et Mémoire, 3e éd. Paris 1903, p.201.
- (129) Les Principes de la Physique, p.166
- (130) "L'Outillage Mental," in Encyclopedie Francaise, Paris 1937. Cited by Supple: Dialectics and Experimental Biology, p.20
- (131) Cf. C. de Koninck: De La Primauté Du Bien Commun Contre Les Personalistes, Editions de l'Université Laval, Québec, 1943, pp. 159 ff.
- (132) Experience and Nature, pp. 357 - 358. Cf. also The Quest For Certainty, p. 202; "The doctrine that nature is inherently rational was a costly one. It entailed the idea that reason in man is an outside spectator of a rationality already complete in itself. It deprived reason in man of an active and creative office; its business was simply to copy, to re-present symbolically, to view a given rational structure. Ability to make a transcript

of this structure in mathematical formulae gives great delight to those who have the required ability. But it does nothing; it makes no difference in nature. In effect, it limits thought in man to retraversing in cognition a pattern fixed and complete in itself. The doctrine was both an effect of the traditional separation between knowledge and action and a factor in perpetuating it. It relegated practical making and doing to a secondary and relatively irrational realm."

- (133) Onze Theses sur Feuerbach, pp. 87, 95. Cf. Jean Langevin: "Sciences et Industrie" in A la Lumière du Marxisme, p. 114: "La méthode expérimentale, est véritablement active, puisqu'elle consiste précisément à maîtriser ou à modifier les circonstances naturelles. C'est à elle, en premier lieu que s'applique la phrase de Goethe: 'Au commencement de tout, il y a l'action.'"
- (134) Freedom Versus Organization, New York, 1934, p. 192. Nor was Hitler unaware of the essential meaning of Marxism, as the following passage clearly indicates: "Ce qui reste du marxisme, c'est la volonté de construction révolutionnaire, qui n'a plus besoin de s'appuyer sur des béquilles idéologiques et qui se forge un instrument de puissance implacable pour s'imposer aux masses populaires et au monde tout entier. D'une téléologie à base scientifique, il sort ainsi un vrai mouvement révolutionnaire, pourvu de tous les moyens nécessaires à la conquête du pouvoir. Et quel est le but de cette volonté révolutionnaire? Il n'y a pas de but précis. Rien qui soit fixé une fois pour toutes. Avez-vous tant de peine à comprendre cela? Nous sommes en mouvement. Voilà le mot qui dit tout...mais nous savons, nous qu'il n'y a pas d'état définitif, qu'il n'y a rien de durable, qu'il y a une évolution perpétuelle. Ce qui ne se transforme pas, c'est ce qui est mort. Le présent est déjà passé. Mais l'avenir est le fleuve inépuisable des possibilités infinies d'une création toujours nouvelles." in Hermann Rauchning: Hitler M'A Dit, pp. 211 - 214 -- Cited by C. De Koninck. Méthodologie Scientifique, 1939, p. 7.
- (135) Morceaux Choisis, p. 222.
- (136) Cf. Poincaré: "C'est la connaissance qui est le but et l'action qui est le moyen." -- Science et Hypothèse, p. 258.

Chapter V

- (1) Physics and Philosophy, pp. 179, 181.
- (2) The Mysterious Universe, pp. 138, 111. cf. The New Background of Science, p. 60.
- (3) Cf. The Scientific Outlook, p. 88.
- (4) Cf. Reg. II and III.
- (5) New Pathways in Science, p. 24.
- (6) Cf. John of St. Thomas, Arts. Logica, p. 839.
- (7) I, lect. 20.
- (8) Lect. 20, no. 574.
- (9) Cf. St. Th. In Met VII, lect. 2, no. 1280: "Et quia posset alicui videri, quod ex quo philosophus ponit omnes modos, quibus dicitur substantia, quod hoc sufficeret ad sciendum quid est substantia; ideo subjungit dicens, quod nunc dictum est quid sit substantia 'solum type', idest dictum est solum in universali, quod substantia est illud quod non dicitur de subjecto, sed de quo dicuntur alia; sed oportet non solum ita cognoscere substantiam et alias res, scilicet per definitionem universalem et logicam: hoc enim non est sufficiens ad cognoscendum naturam rei, quia hoc ipsum quod assignatur pro definitione tali, est manifestum; Non enim huiusmodi definitione tanguntur principia rei, ex quibus cognitio rei dependet; sed tangitur aliqua communis conditio rei per quam talis notificatio datur."
- (10) For other examples cf. Post. Anal., lect. 27, no. 7; lect. 33, nos. 1 - 2; lect. 38, no. 6.
- (11) Cf. III Phys. lect. 8, nos. 1 - 4. When in this passage St. Thomas points out that the arguments of Aristotle for finiteness of bodies are purely dialectical because they proceed ex communibus, he does not mean common principles such as are found in the second type of dialectical reasoning, but principles that are commonly accepted and hence probable.

- (12) Cf. St. Thomas lect. 2, nos. 24 - 28; cf. also St. Albert the Great: De Anima, I, tract. 1, cap. 7: "Physicus autem et dialecticus diffiniunt differenter unumquodque istorum quae diximus esse animae opera et passiones. Si enim quaerimus quid est ira, intendentes de diffinitione quarere, dicet dialecticus quod est appetitus contrarii doloris, aut aliquid huiusmodi diffiens per intentiones communes formales, quae non sunt vera causa rei propria, sed intentiones communes inventae in pluribus et nulli propriae: et ideo diffinit per formam quae forma de se communis est, et non appropiatur ad esse rei nisi per propriam materiam uniuscuiusque rei. Physicus autem dicet quod ira ascensus vel ascensus et calefactio sanguinis circa cor, tangens propriam causam efficientem quae est ascensus et calefactio sanguinis, et propriam materiam quae est sanguis cordis bulliens, et subjectum quod est cor. Et horum quidem alius reddit materiam propriam scilicet physicus: alius autem reddit speciem et intentionem formae simplicem et communem quae est rei ratio communis. Hic enim considerat rationem sive intentionem communem rei, eo quod non descendit ad propria: ille autem considerat principia realia quae dant esse rei. Necessarium autem est quod ista realia principia sint in materia huiusmodi quae determinata et propria est, si erunt et habent esse in natura."
- (13) Lect. 5, no. 9.
- (14) Cf. Cajetan, in I, 17, 3, nos. 7 - 8.
- (15) Cf. St. Thomas In I Sent. d. 38, q. 1, a. 5, c: "In istis causis effectus futuri non habent certitudinem absolutam, sed quandam, in quantum sunt magis determinatae causae ad unum quam ad aliud; et ideo per istas causas potest accipi scientia conjecturalis de futuris, quae tanto magis erit certa quanto causae sunt magis determinatae ad unum; sicut est cognitio medici de sanitate et morte futura, et iudicium astrologi de pluviis et ventis futuris."
- (16) Topics, I, 1, 100 b 21 - 23.
- (17) 121 b 2 - 3.

- (18) I, 1, 1355 a 14.
- (19) Ibid.
- (20) In I Top. c. XII, no. 4.
- (21) In his Commentary on the Topics, St. Albert the Great brings out the meaning of probability and its connection with dialectics: "Probabilia autem sunt verisimilia. Dupliciter autem verisimilia: aut enim in se sunt verisimilia, eo quod ipsa habitudo praedicati ad subjectum verisimilis est, eo quod nec praedicatum est in subjecto per se, nec subjectum in praedicatione per se, nec utrumque in utroque, nec praedicatum necessarium et essentialem inhaerentiam habet cum subjecto, sed verisimile est in signis non in causis necessariis acceptum. Aut quia necessarium habet inhaerentiam, sed non accipitur nisi per signum: et hoc est probabile secundum modum acceptionis, quamvis in se sit necessarium: sicut solem esse majorem terra (eo quod ubique unius quantitatis apparet) probabiliter acceptum est. Solem autem esse majorem terra per quantitatem diametri acceptum est necessarium et non probabile, secundum quod probabile et necessarium opponuntur. Probabile autem sicut dictum verisimile est quod per sui ipsius veritatis figuram videtur omnibus aut pluribus aut sapientibus, et his sapientibus videtur omnibus aut pluribus aut maxime notis et probabilibus: ita quod sapientibus et his vel omnibus sapientibus vel pluribus vel maxime notis vel probabilibus, totum pro uno membro ponatur. "Signa vero versimilitudinis, aut occurrunt statim in superficie et in exterioribus rei quae accipit sensitiva potentia comparans sensata ad invicem: et si talia sunt signa, probabile est quod videtur omnibus, sicut nivem esse albam per hoc quod nix est parvae partes perspicui in parva conjuncti, in diuis partibus undique lux diffunditur: hoc enim signum sensui est medium. Si autem signa indicium facientia de verisimilitudine sunt non in superficie, sed aliquantulum profunderata, non ad necessaria, sed nec in superficie extrinsecus manentia: tunc est id quod videtur pluribus: quia sensui aliquid miscent rationis, sicut quod stella in cauda minoris urseae sit polus, eo quod non deprehenditur eius singularis motus: hoc enim rationis iudicium sensui est permixtum. Si autem signum

verisimilitudinis profundatur in essentialium et convertibilium causas quae sunt convertibilia sicut causae: tunc est quod videtur sapientibus, sicut est, quod luna moveatur in epicyclo; quia profundius et altius transit per umbram terrae; hoc enim non est causa sed signum.

"Ideo illud quod videtur sapientibus gradus habet, quia aut videtur omnibus, aut pluribus, aut maxime notis vel probabilibus. Quia signum convertibile cum causa, vel apparet mixtum sensui, et tunc videtur omnibus; vel in ipsis substantialibus profundatur, et tunc non videtur nisi probatis et probabilibus sapientibus vel medio est acceptum, et hoc dupliciter. Si enim plus est inclinatum ad sensum: tunc est quod videtur pluribus sapientibus. Si autem plus est profundatum ad necessaria essentialia et intellectualia: tunc est quod videtur maxime notis, qui ex potestate scientiae et artis hoc deprehendere noverunt. Hoc igitur est probabile, ex quo fit syllogismus dialecticus, quod tali et taliter diversificato deprehenditur signo. Haec est sententia commentarii Arabici: et sic scientia demonstrativi et etiam dialectici syllogismorum determinata est." -- Lib. I, tract. I, Ch. 2.

- (22) Indetermination in things may, of course, be a cause of the indetermination in the mind that is proper to opinion, as St. Thomas points out on numerous occasions, but this latter indetermination may also be had when things are objectively determined. Cf. De Veritate, XV, 2, ad 3: "Sunt autem quaedam in quibus non est possibile talem resolutionem facere ut perveniatur usque ad quod quid est, et hoc propter incertitudinem sui esse; sicut est in contingentibus in quantum contingentia sunt; unde talia non cognoscuntur per quod quid est, quod erat proprium objectum intellectus, sed per alium modum scilicet per quandam conjecturam de rebus illis de quibus plena certitudo haberi non potest. Unde ad hoc alia potentia requiritur. Et quia haec potentia non potest reducere rationis inquisitionem usque ad suum terminum quasi ad quietem, sed consistit in ipsa inquisitione quasi in motu, opinionem solummodo inducens de his quae inquirunt; ideo quasi a termino suae operationis haec potentia ratiocinativum vel opinativum nominatur." Cf. also De Anima III, lect. 16.

- (23) Cf. St. Albert the Great, In Perihermeneias, I, tract. II, c. 5, p. 493 b, Vives T. I.: "Sed attende quod licet nomen infinitum nihil ponat et nihil significet, non tamen est vox non-significativa ut britabaf: quia vox non-significativa non excitat intellectum ad aliquid de aliquo intelligendum, sed non-homo excitat ad intelligentiam de hoc quod est privatae qualitatis, quod tamen in rerum natura nihil est, quamvis sit in apprehensione: et hoc modo opinio est circa id quod est in apprehensione tantum. Et est simile sicut quando dicimus innominabile, hoc nihil est, et tamen prout cadit in apprehensione per suum oppositum quod est nominabile, ad aliquid excitat intellectum."
- (24) In I Post. Anal. lect. 1, no. 6. Cf. St. Albert, In I Physicorum, Tract. 1, cap. 5: "Dico autem quod omnis scientia quae habet principia sic procedit, et illa sola est vere scientia: quia est demonstrativa, et effectus solius demonstrationis est scire. Si autem ipsa non habeat verum nomen scientiae, tunc ipsa erit scientia topica dialecticae vel rethoricae, et effectus eius non erit scientia, sed opinio. . . "
- (25) Ars Logica., II, Q. I, art. 5, pp. 278, 280.
- (26) IV, lect. 4, nos. 576 - 577.
- (27) The Philosophy of Physical Science, p. 1.
- (28) Ars Logica, Pars I, p. 5.
- (29) Ibid, Pars II, p. 250.
- (30) I, c. 12, 105 b 10.
- (31) St. Thomas In I Poster, lect. 9, no. 4.
- (32) Cf. I Post. Anal., lect. 5, nos. 7 - 8; lect. 19 nos. 4 - 5.
- (33) H. Poincaré: La Science et l'Hypothese, p. 245.
- (34) In I Post. Anal. lect. 5, no. 4.

- (35) I, lect. 21, no. 3.
- (36) Cf. Top. Ch. 14.
- (37) Matière et Lumière, p. 177. M. Jean Perrin writes:
"Tout concept finit par perdre son utilité, sa signification même, quand on s'écarte de plus en plus des conditions expérimentales où il a été formulé."
Cited by Petit: Méthodologie Scientifique, p. 18.
- (38) Philosophy by Way of the Sciences, p. 124.
- (39) Op. cit. p. 66.
- (40) When experimental science is made the only valid type of knowledge, and when it is applied to social and economic problems, it is easy to see that radical and revolutionary social doctrines are bound to be the result. Marxism is a proof of it.
- (41) Top. I, tract. III, c. 1.
- (42) The New Background of Science, pp. 46 - 47.
- (43) Op. cit., preface p. x.
- (44) Cf. I. Post. Anal. lect. 1; I Top., c. 2.
- (45) Loc. cit. It must be noted in passing that the statement: "In nature everything is certain" is at best ambiguous. In relation to subjective probability it is true, but in relation to objective probability as defined above it is false.
- (46) Op. cit. preface p. ix.
- (47) Cf. for example, I De Generatione et Corruptione, c. 2, 316 a 5 - 15.
- (48) Topics, I, c. 1.
- (49) An Outline of Philosophy, p. 163.
- (50) Mysticism and Logic, London, Allen & Unwin, 1927, p. 75.

Chapter VI

- (1) Bertrand Russell: Mysticism and Logic, p. 91.
- (2) It is worth noting that the Thomists are not the only ones who insist upon the essential relation between mathematics and quantity. A number of modern thinkers are beginning to realize that the only adequate solution for many of the problems concerning the nature of mathematics is a return to this traditional notion. Cf. Harold R. Smart: The Logic of Science, Chapter III.
- (3) Cf. Burtt, op. cit. p. 43: "...the orthodox Aristotelian school minimized the importance of mathematics. Quantity was only one of the ten predicaments and not the most important."
- (4) In Met V, lect. 15, no. 983.
- (5) Some modern Thomists erroneously make quantity a common sensible. Thus Maritain, who, after asserting that quantity precedes the whole sensible order says: "Elle (la quantité) est un 'sensible commun'." --Les Degrés du Savoir, p. 281.
- (6) Cf. also I, 40, 3, c.
- (7) De Trinitate, loc. cit.
- (8) In I, V, 3, ad 4, no. 4.
- (9) Op. cit.
- (10) Loc. cit.
- (11) Curs. Theol., Ia, Q.V and VI, disp. 6, art. 2.
- (12) John of St. Thomas, Loc. cit. no. 17.
- (13) John of St. Thomas, loc. cit. no. 20.
- (14) Science and the Modern World, p. 44. Earlier in the same work he writes: "Mathematics is thought moving in the sphere of complete abstraction from any particular instance of what it is talking about." This is perfectly true, but it does not bring out the par-

ticular character of mathematical abstraction, for the same statement could be made of other types of abstraction.

- (15) The Nature of Physical Theory, p. 67.
- (16) In Phys. II, lect. 3, no. 6.
- (17) Substance and Function and Einstein's Theory of Relativity, pp. 19 - 20. Later in the same work (pp. 229 - 230, footnote) he writes: "The 'concrete universality' of the mathematical concepts has also incidentally been recognized and emphasized from the standpoint of Richert. 'The gap for conceptual knowledge between the universal and the particular,' says Lask in his work, Fichtes Idealismus und die Geschichte, and the consequent irrationality is bridged in the mathematical view through the possibility of construction. The individual cases realizing the mathematical concept can be generated by the concept itself. From the concept of the circle, we can attain by construction the mathematical individuality of the particular circle, and thus go from the universal to the individual in its individuality... In mathematics, also, the intuitive object is an individual concrete and given object; but it is given a priori, not a posteriori like the material of sensation it is a logical unique, something individual, but at the same time capable of being construed a priori! We see here also that Richert's criticism would have taken another form if he had conceived the concepts of natural science decisively and from the beginning as products of constructive mathematical, rather than as results of 'abstractive' procedure. The insight once gained for mathematics would have had to be transferred to physics; for precisely here lies the real problem -- that mathematics is no 'logical unique,' but that it progressively provides the 'special' natural sciences with its own characteristic form of concept. The form of mathematical 'deduction' is already contained in the form of physical 'induction', by which we grasp the empirically real, and thus the same method of mastery of the particular by the universal is achieved.
- (18) I a, Q. V and VI, disp. 6, art. 2 (T. I, pp. 532 - 536).

- (19) Du Cheminement de la Pensée, p. 694.
- (20) Loc. cit., no. 20.
- (21) Physics and Philosophy, p. 16.
- (22) III Met. Ch. 4.
- (23) I, 5, 3, ad 4.
- (24) Loc. cit. no. 29.
- (25) Cf. In I Post. Anal. lect. 19, no. 6.
- (26) I, 5, 4, ad 1.
- (27) Met. XIII, ch. 3, 1078 b, 1 - 5.
- (28) Pp. 120 - 121.
- (29) P. 121.
- (30) I, 85, 1, ad 2.
- (31) In De Trin. V, 3, c.
- (32) In De Anima III, lect. 8, no. 708. "Quaedam ergo sunt formae quae materiam requirunt sub determinata dispositione sensibilibus qualitatum; et huiusmodi sunt omnes formae naturales; et idcirco naturalia concernunt materiam sensibilem. Quaedam vero sunt formae, quae non exigunt materiam sub determinata dispositione sensibilibus qualitatum, tamen requirunt materiam sub quantitate existentem: sicut triangulus, et quadratum, et huiusmodi: et haec dicuntur mathematica; et abstrahunt a materia sensibili, sed non a materia intelligibili, in quantum in intellectu remanet continua quantitas, abstracta a sensibili qualitate."
- (33) In De Anima III, lect. 8, no. 714 - 715.
- (34) Cf. In VIII Met. lect. 5, no. 1761.
- (35) I Met. Ch. 6, 987 b 15.

- (36) I, lect. 41, no. 5., Cf. De Veritate, II, 6, ad 1.
- (37) Opera I, 31. Cited by Burtt: The Metaphysical Foundations of Physics, p. 57.
- (38) Cf. II Met. lect. 5, no. 336; VI Eth., lect. 7, no. 1209; De Trin. VI, 1, etc.
- (39) II Met., Ch. 1; St. Th. lect. 1, no. 281.
- (40) In II Met., lect. 5, no. 336.
- (41) Lect. 7, nos. 1209 - 1210.
- (42) "Movet circa hoc quaestionem scilicet quare puer potest fieri mathematicus non autem potest fieri sapiens idest metaphysicus vel physicus, idest naturalis. Ad hoc respondet Philosophus, quia haec quidem, scilicet mathematicalia cognoscuntur per abstractionem a sensibilibus quorum est experientia; et ideo ad cognoscendum talia non requiritur temporis multitudo. Sed principia naturalia quae non sunt abstracta a sensibilibus, per experientiam considerantur, ad quam requiritur temporis multitudo. Quantum autem ad sapientiam, subiungit quod iuvenes sapientialia quidem scilicet metaphysicalia non credunt, idest non attingunt mente, licet dicant ore; sed circa mathematica non est immanifestum eis quod quid est, quia rationes mathematicorum sunt rerum imaginabilium, sapientialia autem sunt pure intelligibilia. Iuvenes autem facile capere possunt ea quae sub imaginatione cadunt. Sed ad illa quae excedunt sensum et imaginationem non attingunt mente, quia nondum habent intellectum exercitatum ad tales considerationes, tum propter paucitatem temporis, tum propter plurimas mutationes naturae." Ibid.
- (43) In II Met., lect. 5, no. 334.
- (44) Ibid no. 336.
- (45) Cf. Gérard Petit, C.S.G.: Methodologie Scientifique, pp. 72, 78 etc.
- (46) Cf. Timaeus 35 a.

- (47) Cf. Meyerson: La Dédution Relativiste, p. 320:
"Nous dirons donc que c'est parce que dans le spatial,
l'esprit semble s'accorder parfaitement avec le réel
que nous sommes embarrassés pour déterminer ce que
nous devons attribuer à l'une ou l'autre source, et
que nous pouvons, en fin de compte, selon des rai-
sonnements qui s'appuient plus ou moins sur des ex-
périences, modifier cette attribution."
- (48) Du Cheminement de la Pensée, pp. 658 - 659.

Chapter VII

- (1) La Théorie Physique, pp. 158 - 159.
- (2) Ibid.
- (3) The Nature of the Physical World, pp. 251, 253.
- (4) Saunderson, author of a treatise on optics, was blind from the first year of his life.
- (5) The Philosophy of Physics, p. 16. Cf. also: Theoretical Physics, Columbia Univ., 1915, pp. 4 - 5.
- (6) Cf. Opticks, Bk. I, Pt. II, 1931 ed., pp. 124 - 125.
- (7) Leviathan, p. 3.
- (8) Essay Concerning Human Understanding, Bk II, ch. 8 par. 9 ff.
- (9) The Universe in the Light of Modern Physics, p. 14.
- (10) Cf. Lindsay and Margenau: Foundations of Physics, p. 20; Norman R. Campbell: Physics - The Elements; Stebbing: Philosophy and the Physicists, p. 80; Bertrand Russell: The Scientific Outlook, p. 67, etc.
- (11) Cf. Med. VI; Principia, IV, 198, 199. etc.
- (12) Cf. I Phys., lect. 2, no. 7; II Phys., lect. 1, no. 8; VIII Phys., lect. 1, no. 3. etc.
- (13) Dominique Salmon: "La Conception Scolastique de la Physique," in Philosophie et Sciences, p. 54.
- (14) Méthodologie Scientifique. Laval Univ. 1939, pp. 18 - 19.
- (15) I, 27, 5, c.
- (16) In De Anima III, lect. 2, nos. 592 - 593.
- (17) In IV Met. lect. 12, no. 673.
- (18) In De Anima II, lect. 10, no. 350.

- (19) Science and the Modern World, p. 113.
- (20) New Pathways in Science, p. 4.
- (21) Met. IV, 5, 1010 b 33; Cf. Coment. of St. Th. lect. 14, no. 706; De Anima, III, lect. 2.
- (22) The ambiguity of the word "physical" may give rise to some confusion on this point. We understand it here in its primitive meaning in which it signifies something pertaining to objective material nature. In the passage which we are about to quote from Eddington it has an entirely different meaning; it designates the world constructed by science. That is why there is no contradiction between Eddington's position and ours. "Writing this chapter on an autumn day, I feel myself in a familiar world whose most prominent characteristic is colour. There is no colour in the physical world. I think that that is the right way to put it. It is true that each colour is represented in the physical world by a number supposed to indicate the length of a wave of some kind. Similarly I am represented at the telephone exchange by a number indicating a hole in a switch-board; but it would not be correct to say that I inhabit the telephone exchange. To put it another way, there is nothing in the accepted description of the physical world which owes its acceptance to the fact that we have a sense of colour. Everything that we assert can be verified by a colour-blind person; and indeed most of our accurate knowledge has been ascertained through the medium of a colour-blind photograph plate." -- New Pathways in Science, pp. 11 - 12.
- (23) P. 152. Cf. Whitehead: The Concept of Nature, p. 29: "For natural philosophy everything perceived is in nature. We may not pick and choose. For us the red glow of the sunset should be as much part of nature as are the molecules and electric waves by which men of science would explain the phenomenon. It is for natural philosophy to analyze how these various elements of nature are connected".
- (24) Dissertation of 1770.
- (25) In IV Met., lect. 14, nos. 705 - 706. Cf. De Anima III, lect. 2, no. 596.

- (26) Cf. I, 78, 3, ad 2.
- (27) In V Met., lect. 15, no. 985.
- (28) In I Met., lect. 2, nos. 5 - 8.
- (29) Méthodologie Scientifique, pp. 19 - 20.
- (30) Lect. 1, no. 6.
- (31) Lect. 1, no. 6.
- (32) Cf. I, 69, 2, ad 1.
- (33) Cf. IV Phys., lect. 20, 22.
- (34) In De Anima I, lect. 2.
- (35) Cf. Planck: The Philosophy of Physics, p. 17: "Once the specific perceptions of the senses as fundamental concepts of physics had been eliminated from that science it was a logical step to substitute suitable measuring instruments for the organs of sense. The eye gave way to the photographic film, the ear to the vibrating membrane, and the skin to the thermometer. The introduction of self-registering apparatus further eliminated subjective sources of error. The essential characteristic of this development, however, did not consist in the introduction of new measuring instruments of steadily growing sensitiveness and exactitude: the essential point was that the assumption that measurement gave immediate information about the nature of a physical event -- whence it followed that the events were independent of the instruments used for measuring them -- now became the foundation of the theory of physics."
- (36) Les Principes de la Physique, p. 16 - 18
- (37) La Déduction Relativiste, p. 11. Cf. De L'Explication dans les Sciences, p. 184: "Il est manifeste, en effet, qu'aucune propriété physique ne saurait nous apparaître comme réellement motivée par la raison suffisante, qu'au contraire toute qualité dont nous essaierons de doter la matière nous apparaîtra forcément comme une qualité occulte, seules les propriétés spatiales se révélant comme conformes aux exigences de notre esprit, comme réellement nécessaires. C'est donc que la matière véritablement rationnelle ne peut être au fond que de l'espace." Cf. also Rey: La Théorie Physique, p. 214.

- (38) Sir James Jeans: The New Background of Science, 29 - 31. Cf. Dingle: Science and Human Experience, pp. 81 - 82: "Thus a colour-blind person may not be able to appreciate the full subtlety of Swinburne's observation: "Those eyes, the greenest of things blue, the bluest of things grey," but give him a spectroscope and he will discriminate colours by wave-lengths a million times as finely as the eye of the keenest artist. A deaf person cannot distinguish the horrors of modern dance music from the sonatas of Beethoven, but by the use of Lisajou's figures he can detect differences of pitch of which the ear of the most sensitive musician would be unconscious."
- (39) Mind and Nature, p. 15.
- (40) Identité et Réalité, p. 392.
- (41) Cf. Rey: "Pour Poincaré, comme pour le mécaniste, la matière du physicien implique une certaine homogénéité. Ce n'est pas l'homogénéité simple et absolue que la mathématique réclame de son objet, mais elle s'en rapproche indéfiniment comme vers sa limite naturelle. Cette marche vers l'homogénéité explique la possibilité pour la physique de prendre la forme mathématique." La Théorie Physique, p. 186. Cf. also pp. 263 - 265.
- (42) This does not mean that quantity is strictly the subject or the root of the other accidents, but the medium by which they are rooted in the substance. "Accidens non potest per se esse subiectum accidentis, sed unum accidens per prius recipitur in substantia quam aliud, sicut quantitas quam qualitas." I, 77, 7, ad 2. "... subiectum recipit unum accidens alio mediante, sicut corpus recipit calorem mediante superficie, et sic unum accidens dicitur alteri inesse." I - II, 7, 1, ad 3.
- (43) Cf. St. Thomas: "Ex velocitate et tarditate motuum contingit gravitas et acuitas in sonis." In X Met., lect. 2, no. 1948.
- (44) I, 42, 1, ad 1; cf. I C. G. 43, etc.
- (45) A. 11, ad 10.

- (46) Cf. Meyerson: "C'est, encore une fois, l'accord entre la réalité et la mathématique, plus particulièrement, la géométrie, dont nous avons traité au chapitre précédent, en tant que fondement du panmathématisme. Mais ce que nous devons constater ici, où il s'agit de concepts du sens commun, c'est qu'il n'y a pas seulement accord, mais union, union immédiate et, au fond, indissoluble. Tout ce que notre perception nous présente comme réellement existant assume aussitôt la forme spatiale, et cette forme, nous ne pouvons l'en dépouiller sans atteindre par là l'existence elle-même. . . Existence et spatialité sont donc ici synonymes ou, du moins, inséparables, et c'est là encore un aspect de cette supériorité du panmathématisme en tant que conception métaphysique applicable à la science, que nous avons constatée." De L'Explication dans les Sciences, pp. 576 - 577.
- (47) The Philosophy of Physical Science, p. 122.
- (48) Eddington, op. cit. p. 124.
- (49) Op. cit. pp. 123 - 125.
- (50) P. 464. Cf. also Rey, Op. cit., pp. 214 - 215. Professor Dewey in the Quest For Certainty explains the significance and fruitfulness of this homogenization of nature from the point of view of instrumentalism: "Physical science disregards the qualitative heterogeneity of experienced objects so as to make them all members in one comprehensive homogeneous scheme, and hence capable of translation or conversion one into another. This homogeneity of subject-matter over a broad range of things which are as disparate from each other in direct experience as sound and colour, heat and light, friction and electricity, is the source of the wide and free control of events found in modern technology. Common-sense knowledge can connect things as sign and thing indicated here and there by isolated couples. But it cannot possibly join them all up together so that we can pass from any one to any other. The homogeneity of scientific objects through formulation in terms of relations of space, time and motion, is precisely the device which makes this indefinitely broad and flexible scheme of

transitions possible. The meaning which one event has is translatable into the meanings which others possess. Ideas of objects, formulated in terms of the relations and changes bear to one another, having common measures, institute broad, smooth highways by means of which we can travel from the thought of one part of nature to that of another. In idea at least, we can travel from any meaning --or relation --found anywhere in nature to the meaning to be expected anywhere else." In John Dewey's Philosophy by Ratner, p. 337.

- (51) De L'Explication dans les Sciences, pp. 25 - 27.
- (52) De L'Explication dans les Sciences, p. 14. Cf. La Déduction Relativiste, p. 258, etc.
- (53) Matière et Lumière, p. 316.
- (54) Cf. Boutroux: La Contingence des Lois de la Nature, p. 71: "Pour que la loi mécanique puisse être considérée comme la traduction de la loi physique proprement dite, il faut que l'équivalent existe, non seulement entre les deux ordres de faits, mais entre les deux ordres de rapports, entre l'enchaînement des faits physiques et l'enchaînement de leurs conditions mécaniques. Or cette seconde équivalence semble inintelligible parce que, tandis que la variable est homogène, l'élément qui doit en être la fonction est hétérogène."
- (55) "C'est la qualité même, non seulement l'étendue où elle apparaît, qu'on réussit à mesurer."--La Pensée et la Quantité, Paris, Librairie Félix Alcan 1927. p. 34.
- (56) "La quantité n'est rien d'original, pas plus matière, étendue ou durée, que grandeur purement logique; elle est une construction conceptuelle, fondée à la fois sur la diversité et l'homogénéité qualitative des contenus de pensée." Ibid. p. 379. "C'est la mesure qui crée la quantité."--p. 273.
- (57) Cf. Sir James Jeans: Physics and Philosophy, p. 197.
- (58) Cf. Benjamin: The Logical Structure of Science, p. 317:

"Now as a methodological postulate this can hardly be criticized. But as is often the case in science, a methodological postulate is given the status of a meta-physical judgment. The quantitative aspects of the world are soon looked upon as representative of its essential nature. To explain qualities is to explain them away. To understand them is to be convinced that they are mere appearances. To rationalize them is to construct a system in which they do not function at all as explicit elements. To talk about them is to talk about them vicariously. To grasp them is to realize that they cannot be grasped."

- (59) Cf. Tannery, Science et Philosophie, p. 38.
- (60) W. R. Thompson: Science and Common Sense, p. 69.
- (61) "And so we have our schedule of pointer readings ready to make the descent. And if you still think that this substitution has taken away all reality from the problem, I am not sorry that you should have a foretaste of the difficulty in store for those who hold that exact science is all-sufficient for the description of the universe and that there is nothing in our experience which cannot be brought within its scope." Eddington: The Nature of the Physical World, p. 254.
- (62) Curs. Phil. T. II, p. 764.
- (63) Rempserber, Philosophies of Science, New York, F. S. Crofts, 1942. p. 233.
- (64) The Quest For Certainty, in John Dewey's Philosophy, by Ratner, pp. 338 - 341.
- (65) Cf. Max Planck: The Philosophy of Physics: "The essential characteristic of this development, however, did not consist in the introduction of new measuring instruments of steadily growing sensitiveness and exactitude: the essential point was that the assumption that measurement gave immediate information about the nature of a physical event -- whence it followed that the events were independent of the instruments used for measuring them -- now became the foundation of the theory of physics. On this assumption a distinction must be made, whenever a

physical measurement takes place, between the objective and actual event, which takes place completely independently, and the process of measuring, which is occasioned by the event and renders it perceptible. Physics deals with the actual events, and its object is to discover the laws which these events obey." pp. 17 - 18.

- (66) Planck: The Philosophy of Physics, p. 95.
- (67) Ibid. p. 104.
- (68) Eddington: Space, Time, and Gravitation, pp. 15, 31.
- (69) Cf. Eddington: New Pathways in Science, pp. 12 - 13: "When we have eliminated all superfluous senses, what have we left? We can do without taste, smell, hearing, and even touch. We must keep our eyes -- or rather one eye, for there is no need to use our faculty of stereoscopic vision. The eye need not have the power of measuring or graduating light and shade; I think it is sufficient if it can just discriminate two shades so as to detect whether an opaque object is in a certain position or not. . . .
In 1915 Einstein made another raid on their sensory equipment. He removed all the retina of the eye except one small patch. The observer could no longer recognize form or extension in the external world, but he could tell whether two things were in apparent coincidence or not."
- (70) Eddington: New Pathways in Science, pp. 2 - 3. Cf. The Philosophy of Physical Science, p. 77, etc. Cf. Bertrand Russell: The Analysis of Matter, p. 6: "All empirical evidence consists in the last analysis of perceptions; thus the world of physics must be, in some sense, continuous with the world of our perceptions, since it is the latter which supplies the evidence for the laws of physics."
- (71) Pp. 69-70. Cf. Lenzen: "The qualitative phase of physics is an essential constituent of even the most highly developed system of physical theory, for the ultimate significance of physical concepts lies in their connection with immediate qualitative experience." -- The Nature of Physical

(67)

Theory, p. 46.

(72) Introduction, viii.

Chapter VIII

- (1) Curs. Phil. T. II, Q. I, a. 1.
- (2) L'Evolution Créatrice, p. 360.
- (3) Cf. Planck: The Universe in the Light of Modern Physics: "The occasion of this development was that extreme refinement in measurement which is an essential condition of the progress of science." pp. 27 - 28. Cf. also pp. 58 - 60; 73 - 74.
- (4) "Je crois que la prédominance de la physique est due principalement à sa méthode. Elle a l'avantage sur les autres sciences d'introduire la mesure le plus loin possible dans ses raisonnements. Tout le secret de sa valeur et de son influence est dans le fait qu'elle est la science de la mesure." -- Les Principes de la Physique, p. 19.
- (5) Cf. Eddington: Space, Time, and Gravitation, Prologue, p. 2: "Physicist: 'I really cannot tell you anything about it, if you will not let me make measurements of any kind. Measurement is my only means of finding out about nature. I am not a metaphysicist.'"
- (6) In spite of the numerous criticisms that certain aspects of Eddington's position have evoked, we believe that his fundamental ideas on the relation between physics and measurement are quite correct, and that they represent an opinion that has become generally accepted in recent years, at least among those most competent to assess the true meaning of physical science. He himself writes: "I should like to make it clear that the limitation of the scope of physics to pointer readings and the like is not a philosophical craze of my own but is essentially the current scientific doctrine. It is the outcome of a tendency discernible far back in the last century but only formulated comprehensively with the advent of the relativity theory." -- The Nature of the Physical World, p. 254.
- (7) The Nature of the Physical World, pp. 252 - 258.
- (8) Cf. Renoirte: "La Théorie Physique" in Revue Néoscholastique, Nov. 1923, p. 353: "Il convient, en effet, de ne

pas s'arrêter aux mots: les noms que l'on donne aux attributs étudiés en physique ont un rapport évident et immédiat avec des hypothèses sur la nature de ces attributs. L'expression 'longueur d'onde d'une lumière' a un sens obvié dans la théorie de l'ondulation et elle ne répondrait à rien dans la théorie de l'émission. Mais elle correspondra toujours à un procédé opératoire par lequel on trouve un nombre-mesure. Quoi qu'on imagine sur la nature de la matière, le procédé fera trouver le même nombre; on continuera sans doute à le représenter par λ , mais on préférera l'appeler autrement que 'ci-devant longueur d'onde.'

- (9) La Science et L'Hypothèse, p. 193.
- (10) In X Met. lect. 2, no. 1938.
- (11) Curs. Theol. Prima Pars, Q. X, disp. 9, a. 1 (Ed. Solesmes, p. 50 b).
- (12) Cf. "Quarta Via", I, 2, 3, c.
- (13) "Réflexions sur le Problème de l'Indéterminisme," in Revue Thomiste, novembre-décembre, 1937, pp. 393 - 394.
- (14) Met. X, Ch. 1.
- (15) Ibid. no. 1938.
- (16) Cf. John of St. Thomas. Cursus. Theol., loc. cit., p. 53 a.
- (17) Ibid no. 1938. Cf. In V Met. lect. 8, no. 872: "Ratio unius est in hoc, quod sit principium alicuius numeri. Quod ex hoc patet, quia unum est prima mensura numeri, quo omnis numerus mensuratur: mensura autem habet rationem principii, quia per mensuram res mensuratae cognoscuntur, res autem cognoscuntur per sua propria principia. Et ex hoc patet, quod unum est principium noti vel cognoscibilis circa quodlibet, et est in omnibus principium cognoscendi."
- (18) Cf. Cajetan: Ens autem minime unum est, unum quidem est pro quanto non est in se divisum proportionaliter, minime vero est pro quanto tantam diversitatem formalem cum sua unitate

compatitur." De Ente et Essentia, cap VI, edit. Laurent, p. 92, no. 58. Cf. St. Thomas In V Met. lect. 8, no. 875: "Sciendum est autem quod esse mensuram est propria ratio unius secundum quod est principium numeri. Hoc autem non est idem cum uno quod convertitur cum ente, ut in quarto dictum est. Ratio enim illius unius in sola indivisione consistit: Huiusmodi autem unius in mensuratione."

- (19) In X Met. lect. 2, no. 1945. Cf. John of St. Thomas, Curs. Theol., In Primam Partem, Q. X, disp. 9, a. 1, (ed. Solesmes, p. 49): "Perfectissimum seu magis indivisibile in uno genere, est mensura ceterorum: eo quod quanto est magis indivisibile, est magis certum: si quidem minus illi additur, vel aufertur; et sic quod est simplicius in aliquo genere deservit pro mensura, quia ad mensuram pertinet certificare de suo mensurato."
- (20) Ibid., no. 1953.
- (21) I - II, 91, 3, ad 3.
- (22) In V Met., lect. 8, no. 875.
- (23) "Et hoc maxime dicitur in quantitate, et inde derivatur ad alia genera ratio mensurae." Ibid., no. 1938.
- (24) Ibid., nos. 1939 - 1940.
- (25) Ibid., no. 1944.
- (26) Ibid., no. 1945 - 1946.
- (27) It is interesting to compare this doctrine of St. Thomas on the difference between the measurement of number and of magnitude with what has been written on the question by modern philosophers of science, particularly Sir Arthur Eddington: Cf. e.g. The Nature of the Physical World, p. 23: "Number (of discrete individuals) is absolute. It is the result of counting, and counting is an absolute operation. If two men count the number of people in this room and reach different results, one of them must be wrong. "The measurement of distance is not an

absolute operation. It is possible for two men to measure the same distance and reach different results, and yet neither of them be wrong."

- (28) In Met. V., lect. 17, no. 1007.
- (29) Ibid., 1935.
- (30) In Post. Anal. I., lect. 36, no. 11.
- (31) In De Coelo II., lect. 6, no. 4.
- (32) Ibid.
- (33) Phys. IV., lect. 19.
- (34) Cf. Sir James Jeans: Physics and Philosophy, pp. 7 - 9
- (35) Cf. St. Thomas, loc. cit., no. 1950: "Nam sensus non percipit differentiam valde parvorum, sed eorum differentia percipitur 'in rationibus', idest secundum diversas rationes proportionum quae ex diversis proportionibus numeralibus causantur.
- (36) Cf. St. Thomas, ibid. no. 1953: "...sicut mensura pedalis, quae quidem indivisibilis est proportione, sed non natura."
- (37) Cf. also Bridgman: The Logic of Modern Physics, Chap. I.
- (38) Cf. St. Thomas In V Met. lect. 15, no. 978: "Si esset longitudo infinita, non esset linea. Linea enim est longitudo mensurabilis. Et propter hoc in ratione lineae ponitur, quod eius extremitates sunt duo puncta."
- (39) John of St. Thomas: Curs. Theol., loc. cit. p. 52 a. Cf. ibid. p. 92 a: "De ratione mensurae est quod sit notificativa quantitatis mensurae ratione majoris uniformitatis suae: quia scilicet quantitas mensurati confusior vel inaequalior est, et reducta seu comparata ad uniformiorem et simpliciolem quantitatem notificatur et explicatur eius confusio."
- (40) Curs. Theol., loc. cit., p. 49 b.

- (41) Cf. DeKoninck, Méthodologie Scientifique, Laval, p. 61: "Alors que l'hypothèse du physicien classique définit l'étalon de mesure et sa fonction par rapport à une limite réelle il faut définir l'étalon par sa fonction. Alors que le physicien classique croyait s'assimiler l'univers en l'abordant de face, supposant tout droit devant lui la limite qu'il voulait atteindre, le physicien moderne avance à reculons, les yeux tournés vers l'ombre du monde, laquelle se précise à mesure qu'il recule."
- (42) I - II, 97, l. ad 2.
- (43) John of St. Thomas, Curs. Theol. loc. cit., p. 50.
- (44) In X Met., no. 1954. Cf. John of St. Thomas: Curs. Theol., loc. cit. p. 49. "Nihil enim mensuratur mensura propria et adaequata, nisi per aliquid quod est sui generis. Unde distinguuntur diversae mensurae secundum diversa genera; et perfectissimum seu magis indivisibile in uno genere, est mensura ceterorum: eo quod quanto est magis indivisibile, est magis certum."
- (45) No. 1955.
- (46) I, - II, 19, 4, ad 2.
- (47) Cf. St. Thom., I, 3, 5, ad 2: "Objectio illa procedit de mensura proportionata, hanc enim oportet esse homogeam mensurato. Deus autem non est mensura proportionata alicui; dicitur tamen mensura omnium ex eo quod unumquodque tantum habet de esse, quantum si appropinquat." Cf. Comm. of Cajetan, nos. 9 ff.
- (48) Lect. 15, no. 978.
- (49) Cf. De Ver. I, 1, 5, c.; II Sent., d. 2, q. 1, a. 2, ad 1.
- (50) Curs. Theol., loc. cit. p. 50 a. Cf. Curs. Phil. T. II, P. I, q. 18, a. 3, ed. Reiser, pp. 381 - 382.
- (51) Curs. Theol., loc. cit., p. 53 a.
- (52) Curs. Theol. loc. cit. p. 57.
- (53) John of St. Thomas: Curs. Phil. T. I, P. I, Q. XVIII, a 3,

ed. Reiser p. 382.

- (54) Curs. Theol. loc. cit. p. 92.
- (55) Cf. e.g. R. Dalbiez: "Dimensions Absolues et Mesures Absolues" in Revue Thomiste, 1925, pp. 147 ff.
- (56) V Met. Chap. 5, lect. 17.
- (57) Ibid. no. 1003.
- (58) Science and the Modern World, p. 37.
- (59) Cf. Benjamin, The Logical Structure of Science, p. 326: "The fifth objection is the general incapacity of a quantitative system to represent dichotomous divisions, i.e. to handle two-value systems. Where qualities manifest themselves not by degrees, but by complete presence or complete absence, there can be no quantitative representation. Thus it is impossible to show quantitatively how two qualities may be at the same time similar because species of the same genus and yet contradictory because implying contradictory differentia."
- (60) Cf. Eddington: "Distances are linkages whose intrinsic nature is inscrutable; we do not deny the inscrutability when we apply measure numbers to them -- 2 yards, 5 miles etc. -- as a kind of code of distinction." -- The Nature of the Physical World, p. 81.
- (61) New Pathways in Science, p. 224.
- (62) "Le problème de l'Indéterminisme" in L'Académie Canadienne Saint-Thomas D'Aquin, 1935, p. 100.
- (63) Substance and Function and the Theory of Relativity, p. 358.
- (64) Critique de la Mesure, (Actualités Scientifiques et Industrielles Paris, Hermann and Cie, 1937, p. 10.
- (65) New Pathways in Science, p. 229.
- (66) Beneze, op. cit. p. 14. Cf. Frank: Between Physics and Philosophy, pp. 94 - 97.
- (67) The Philosophy of Physics, p. 26.

- (68) Physics and Philosophy, p. 142.
- (69) Cf. "Réflexions sur le problème de l'indéterminisme," in Revue Thomiste, juillet-septembre, novembre-décembre, 1937; also: "Le Problème de l'indéterminisme." in Académie Canadienne Saint-Thomas D'Aquin, Sixième Session, 1935, Québec.
- (70) This seems to be the opinion of Ostwald, for example: "When every magnitude appearing in the formula is itself measurable, then we are concerned with a lasting formula or with a law of nature; . . . if, on the contrary, magnitudes, which are not measurable, appear in the formula, then we are concerned with an hypothesis in mathematical form, and the worm is in the fruit." -- Vorlesungen über Naturphilosophie, Leipzig, 1902, p. 213. Cited by Cassirer, Substance and Function, p. 141.
- (71) Cf. Planck: Where is Science Going? pp. 92 - 93: "Every measurement first acquires its meaning for physical science through the significance which a theory gives it. Anybody who is familiar with a precision laboratory will agree that even the finest and most direct measurements -- such as those of weight and current -- have to be corrected (93) again and again before they can be employed for any practical purpose. It is obvious that these corrections cannot be suggested by the measurement process itself. They must first be discovered through the light which some theory or other throws upon the situation; that is to say, they must arise from an hypothesis." Cf. also p. 95.
- (72) Substance and Function, pp. 357, 365.
- (73) The Logic of Modern Physics, p. 10.
- (74) Cf. Bridgman, The Logic of Modern Physics, pp. 9 - 25.
- (75) Ibid., pp. 17 - 18.
- (76) The Philosophy of Physical Science, p. 81.
- (77) Eddington, The Philosophy of Physical Science, pp. 73 - 74.
- (78) The Philosophy of Physics, p. 22.

- (79) Cf. Lindsay, "Where is Physics Going?" in Scientific Monthly, Vol. 38, p. 248.
- (80) Cf. Eddington, The Nature of the Physical World, p. 142, footnote.
- (81) "Réflexions sur le Problème de l'indéterminisme." in Revue Thomiste, novembre-décembre, 1937. pp. 395 - 396.
- (82) New Pathways in Science, pp. 224 - 225. Cf. The Mathematical Theory of Relativity, pp. 5 - 6. Cf. Max Planck: The Philosophy of Physics, pp. 95, 104.
- (83) Cf. Lenzen, Procedures of Empirical Science, pp. 15 - 19, etc.
- (84) Where is Science Going, p. 95.
- (85) Matière et Lumière, p. 312.
- (86) Cf. Eddington, Space, Time and Gravitation, p. 3 sqq.
- (87) Cf. Rencirte, "La Critique Einsteinienne ~~des~~ Mesures D'Espace et ~~de~~ Temps," in Revue Néoscholastique, 1924.
- (88) Cf. Eddington: "It is perhaps not superfluous to add that no question arises as to whether the standard of length defined in this way is really constant at all times and places. The question implies that we have in mind some more ultimate standard (invested with 'reality') by which to define the delinquencies of the physical standard. The conception of physical quantities having to conform to some particular role allotted in advance in a vaguely imagined realm of reality, is not recognized in physical science; quantities such as length and time-extension are introduced solely for the purpose of succinct description of observational measurements actual or hypothetical." The Philosophy of Physical Science, p. 76.
- (89) Space, Time, and Gravitation, p. 11.
- (90) Space, Time and Gravitation, p. 12.

Chapter IX

- (1) The Universe in the Light of Modern Physics, p. 58.
- (2) Cf. Renouirte: Éléments de Critique des Sciences et de Cosmologie, p. 135.
- (3) Cf. Boutroux: L'Idée de Loi Naturelle p. 33.
- (4) p. 120.
- (5) Cf. Duhem: La Théorie Physique, pp. 249 - 269; Renouirte: "La Théorie Physique" in Revue Neo-scholastique, nov. 1923.
- (6) Cf. Renouirte: Critique des Sciences et Cosmologie, p. 148.
- (7) Cf. Geometrie und Erfahrung, Berlin, 1921.
- (8) "La science ne se contente pas de formuler les lois d'expérience elle cherche bien plutôt à construire un système logique, reposant sur un minimum de prémisses et comprenant dans ses conséquences toutes les lois de la nature." -- Einstein: La Théorie de la Relativité éd. Rouvière, Paris, 1921, p. 109.
- (9) Op. cit. p. 26. In this connection it is interesting to note that in Le Système du Monde, Duhem claims that the Aristotelian doctrine of homocentric spheres was the first physical theory in the modern sense of the word: "Pour la première fois, en effet, dans la constitution de cette théorie, on vit le géomètre partir d'un certain nombre de principes simples qui lui étaient donnés d'ailleurs et, conformément à ces principes, construire un système mathématique hypothétique, retoucher, compliquer ce système jusqu'à ce qu'il sauvât avec une exactitude suffisante les apparences décrites par les observateurs.
"Lorsque l'observation eut fait connaître des phénomènes que tout système de sphères homocentriques était, à tout jamais, impuissant à sauver, les astronomes géomètres acceptèrent d'autres principes et, à l'aide de ces nouveaux principes, combinèrent de nouvelles hypothèses; mais la méthode qu'ils suivirent pour construire de nouveaux systèmes astronomiques ne différa pas de celle qui avait servi à édifier le système des sphères homocentriques."-- I, p. 128.
- (10) Substance and Function, etc. p. 135.

- (11) Critique of Physics, p. 159.
- (12) Cf. Eddington: "An ideal shines in front of us, far ahead perhaps but irresistible, that the whole of our knowledge of the physical world may be unified into a single science which will perhaps be expressed in terms of geometrical or quasi-geometrical conceptions. Why not? All the knowledge is derived from measurements made with various instruments. The instruments used in the different fields of inquiry are not fundamentally unlike. There is no reason to regard the partitions of the sciences made in the early states of human thought as irremovable."--The Nature of the Physical World, p. 137. Cf. also Einstein: The World As I See It, pp. 33 - 34.
- (13) Op. cit., p. 24. Cf. also Garrigou-Lagrange: Le Sens Commun, La Philosophie de l'être, p. 70: "Les sciences positives ne peuvent jamais que classer des faits généraux par des hypothèses provisoires (hypothèses représentatives et non explicatives)..."
- (14) Cf. Dantzig: Aspects of Science, p. 231. "The continual use of such terms and phrases has finished by converting them into so many new patterns, and to the extent that they conjure up in the minds of the experts definite physical situations, these weird patterns fulfill their purpose as fully as did the classical mechanical models." Cf. Dirac: Quantum Mechanics, p. 10: "One may extend the meaning of the word 'picture' to include any way of looking at the fundamental laws which makes their self-consistency obvious. With this extension, one may acquire a picture of atomic phenomena by becoming familiar with the laws of the quantum theory."
- (15) Cf. The Principles of Quantum Mechanics, by Dirac.
- (16) The Nature of Physical Theory, p. 62.
- (17) Cf. Shrodinger Science and the Human Temperament, p. 160. Cf. Dirac: The Principles of Quantum Mechanics pp. vi and 10.
- (18) Op. cit., pp. 63 - 64.
- (19) Cf. Eddington, The Nature of the Physical World, pp. 198

- 199: "In Bohr's semi-classical model of the hydrogen atom, there is an electron describing a circular or elliptic orbit. This is only a model; the real atom contains nothing of the sort. The real atom contains something which it has not entered into the mind of man to conceive, which has, however been described symbolically by Schrodinger." Cf. Urbain: Les Notions Fondamentales d'Éléments Chimique et d'atome. Paris, 1925, p. 114: "Cette hypothèse (de Bohr) est exceptionnellement grave. Elle est, en effet, en contradiction formelle avec les lois de l'électrodynamique. De ce chef, elle peut être qualifiée d'absurde. Si donc les théories étaient faites pour expliquer les phénomènes et donner ainsi à l'esprit la satisfaction de les comprendre, on aboutirait à ce résultat singulier qu'il faut recourir à l'absurde pour faire une théorie cohérente du monde. Mais, comme une pareille manière de voir est tout à fait inacceptable, on doit conclure que les théories mécaniques n'ont d'autre fin que de créer des modèles commodes. Toutes les hypothèses relatives à ces modèles sont acceptables, car, d'une part, le monde idéologique auquel ils appartiennent ne saurait être que conventionnellement astreint aux lois du monde sensible; d'autre part on ne saurait exiger des modèles que de schématiser des faits et de permettre à l'esprit de prévoir d'autres faits par les raisonnements relativement simples qu'ils peuvent suggérer." -- Cited by Rencirte, Op. cit., p. 158.

(20) Pp. 189 ff.

(21) Ibid. Louis De Broglie has brought out the true relation between models and mathematics in physical theory: "Une autre conclusion s'impose à nous. Les représentations concrètes ont souvent aidé et aideront encore souvent les théoriciens dans leurs recherches, mais elle constituent en réalité la partie fragile et périssable des théories; ce qui subsiste ce sont les formes abstraites auxquelles ces représentations ont conduit. Fresnel était parvenu à l'équation des Ondes en imaginant un éther élastique vibrant. Maxwell et ses continuateurs remplacent cet éther élastique par un éther électromagnétique déjà beaucoup moins concret. Einstein et les Relativistes abandonnent complètement l'idée d'éther et réduisent la vibration électromagnétique à n'être qu'une pure grandeur géométrique dirigée. La nouvelle Mécanique, enfin, ne peut pas encore attribuer une nature physique précise aux ondes qu'elle

envisage, et cela ne l'empêche nullement de se développer: "Le véritable but de la physique théorique paraît donc être de découvrir et d'étudier les formes mathématiques dans lesquelles les phénomènes physiques peuvent venir se loger. Assigner ce rôle à la physique théorique, c'est sans doute faire participer cette science à la rigueur des Mathématiques, mais c'est aussi lui marquer ses limites: derrière l'harmonie que nous révèle la possibilité de couler les faits dans des moules analytiques se cache une Réalité dont l'essence nous demeure prodigieusement inconnue." Recueil d'exposés sur les Ondes et Corpuscules, Paris, 1930 pp. 24 - 25. Cited by Rencirte, op. cit. pp. 162 - 163.

- (22) Bouasse: De La Méthode dans les Sciences, Paris, Alcan 42 éd., 1915, p. 124.
- (23) Cited by Duhem, op. cit. p. 116.
- (24) Science et Hypothèse, p. 174.
- (25) Op. cit. pp. 60 - 61.
- (26) "L'Oeuvre d'Einstein et l'Astronomie" in L'Astronomie, juillet, 1931. Cited by Maritain: Les Degrés du Savoir, p. 305 footnote.
- (27) Cf. Duhem: "En exigeant que les opérations mathématiques par lesquelles les postulats produisent leurs conséquences aient toujours un sens physique, on impose au géomètre d'insupportables entraves qui paralysent toutes ses démarches; avec B. Robin, il en vient jusqu'à redouter l'emploi du calcul différentiel; en fait, s'il se piquait de satisfaire sans cesse et scrupuleusement à cette exigence, il ne pourrait presque plus développer aucun calcul; dès ses premiers pas, la deduction théorique se trouverait arrêtée. Une idée plus exacte de la méthode physique, une plus juste démarcation entre les propositions qui ont à se soumettre au contrôle des faits de celles qui en sont dispensées, rendront au géomètre toute sa liberté et lui permettront d'user, pour le plus grand développement des théories physiques, de toutes les ressources de l'Algèbre." op. cit. p. 316.
- (28) Cf. Milhaud: Le Rationnel, p. 105: "Comment nier que la science tire le plus grand profit de notions fictives, invérifiables, échappant, par leur nature, aux conditions

ordinaires de détermination des choses..., comme en mathématiques, chaque symbole nouveau introduit par généralisation précisément dans les cas où en vertu des conditions premières, il cessait de rien représenter? Et ce ne sont pas seulement des notions fictives qui peuvent réussir, ce sont parfois des vues manifestement absurdes.' Cited by Meyerson: Du Cheminement de la Pensée, p. 388, Cf. W. R. Thompson Science and Common Sense, p. 87. Obviously fictitious entities can serve as an explanation only if the explanation be conceived as not being definitive, nor ontological nor proper. Cf. Meyerson: "Il nous semble aller en quelque sorte de soi que la véritable explication soit en même temps une explication réelle, par ce qu'il y a au-dessous de phénomène, par ce qui est. Seuls, les habitants d'un asile d'aliénés, dit avec raison Hartmann, pourraient tenir des explications physiques à l'aide de concepts sciemment irréels."--De L'Explication dans les Sciences, p. 61. Cf. Whitehead: The Concept of Nature, pp. 44 - 46.

- (29) Cf. Space, Time and Matter, [French ed.] p. 280.
- (30) A.S. Eddington: The Nature of the Physical World, pp. 161-162 Cf. Cassirer: Substance and Function, p. 116.
- (31) "The feeling that all the steps in a mathematical theory must have their counterpart in the physical system is the outgrowth, I think, of a certain mystical feeling about the mathematical construction of the physical world. Some sort of an idea like this has been flitting about in the background of the paraphernalia of the thinking of civilization at least since the days of Pythagoras, and every now and then, perhaps after some particularly striking mathematical success, it bursts forth again like a crop of mushrooms after a rain, as in the recent fervid exclamation of Jeans that 'God is a mathematician.'" -- Op. cit. p. 67.
- (32) Substance and Function, p. 116. (Italics ours)
- (33) Cf. Prof. G. Castelnuovo: Scientia, Vol. XXXIII, pp. 169 - 180.
- (34) The New Background of Science, pp. 60, 114, 136.
- (35) Cassirer: Substance and Function, p. 156. Cf. Eddington:

"We must seek a knowledge which is neither of actors nor of actions, but of which the actors and actions are a vehicle. The knowledge we can acquire is knowledge of structure or pattern contained in the actions." -- New Pathways in Science, p. 256.

- (36) I, 18, 1, ad 1.
- (37) Physics II.
- (38) Physics, III, 1.
- (39) Cf. Phys, IV, lect. 1.
- (40) Lect. 15.
- (41) No. 985.
- (42) V. 3.
- (43) The Nature of Physical Theory, p. 73.
- (44) Cf. Meyerson: "Nous avons vu qu'à l'origine le concept de la vitesse n'est qu'un rapport entre deux termes limités et que le mouvement apparaît comme un changement analogue au changement de couleur. Il n'en est plus ainsi pour nous: le mouvement nous apparaît comme un état, analogue par conséquent non au changement de couleur, mais à la couleur elle-même."--Identité et Réalité, p. 159.
- (45) Space, Time, and Gravitation, p. 51.
- (46) Cf. Phys. IV, lect. 20: "Non omne quod non movetur, quiescit; sed quiescens est privatum motu, quod tamen aptum natum est moveri."
- (47) Cf. Eddington: The Nature of the Physical World, pp. 132 - 135.
- (48) Cf. Riezler: Physics and Reality, p. 91: "The world of your physics is finished before it begins -- an accomplished work extended in time, a realm of established laws, natura naturata. . . Your world is the plane of actuality. Your laws relate actualities to one another. They are verified

by experience in a stratum detached by the anonymous observer from the totality of phenomena. Its content is the behavior of classes and aggregates. So far as certain conditions prevail -- and they do prevail in your large scale inorganic world--the plane of actuality is governed by your sort of physical laws. Hence your straight-line causality. But the plane of actuality is not the entire body of reality. Reality embraces both actuality and potentiality, the surface and the depth. . ."

- (49) Regulae ad Directionem Ingenii, edit. Adam et Tannery, pp. 426 - 427.
- (50) Cf. Bertrand Russell: Mysticism and Logic, pp. 80 - 84: "Weierstrass, by directly banishing from mathematics the use of infinitesimals, has at last shown that we live in an unchanging world, and that the arrow in its flight is truly at rest. . . As regards motion and change, we get similarly curious results. People used to think that when a thing changes, it must be in a state of change, and that when a thing moves, it is in a state of motion. This is now known to be a mistake...we may now indulge the comfortable belief that a body in motion is just as truly where it is as a body at rest. Motion consists merely in the fact that bodies are sometimes in one place and sometimes in another, and that they are at intermediate places at intermediate times. Only those who have waded through the quagmire of philosophic speculation on this subject can realise what a liberation from antique prejudices is involved in this simple and straightforward commonplace."
- (51) Cf. Principia, c. 37 and 39.
- (52) I Sent. d. 19, q. 10, a. 4.
- (53) Cf. e.g. Maritain: Théonas, p. 78: "Car le sens commun et la philosophie parlent du temps réel, d'une réalité physique qui est la durée de ce qui change; tandis qu'Einstein -- de fait, si non d'intention -- parle de tout autre chose, d'une entité mathématique qui est une variable dans une équation, et qui n'a que le nom de commun avec le temps."
- (54) "You define motion. But this motion when measured by your own acting and being acted on does not move. To define motion you use time. But time as a dimension of your space of num-

bers is extended and stands still. This time is not the living time you are familiar with, marching without rest and respite, turning future into past and ever generating and devouring yourselves and your present." -- Riezler: Physics and Reality, pp. 54 - 55.

- (55) Cf. Eddington: "Objection has sometimes been felt to the relativity theory because its four-dimensional picture of the world seems to overlook the directed character of time. The objection is scarcely logical, for the theory is in this respect no better and no worse than its predecessors. The classical physicist has been using without misgiving a system of laws which do not recognize a directed time; he is shocked that the new picture should expose this so glaringly." -- The Nature of the Physical World, pp. 68 - 69.
- (56) The Principle of Relativity, p. 213.
- (57) The Foundations of Physics, p. 76. Cf. Eddington: "So if the laws of Nature are indifferent as to the doing and undoing of an event, they must be indifferent as to a direction of time from past to future. That is their common feature, and it is seen at once when (as usual) the laws are formulated mathematically. There is no more distinction between past and future than between right and left. In algebraic symbolism, left is $-x$ and right is x ; past is $-t$ and future is t ." -- The Nature of the Physical World, p. 66.
- (58) Cf. Eddington, op. cit. Chapter IV and V.
- (59) Lenzen: The Nature of Physical Theory, p. 75.
- (60) Op. cit. p. 68.
- (61) Substance and Function, pp. 449 - 450.
- (62) "Cette vue de l'univers est ... la vue d'une intelligence qui serait capable d'embrasser d'un seul coup d'oeil le tout de l'espace et du temps. Mais les limitations de l'intellect humain résolvent ce tout privé de changement en ses aspects temporel et spatial, et le passé et l'avenir du monde physique est le passé et l'avenir de l'intelligence qui le perçoit." -- Cunningham: The Principle of Relativity,

Cambridge, 1914, p. 213 -- Cited by Meyerson: La Déduction Relativiste, p. 101.

- (63) Meyerson: La Déduction Relativiste, p. 102. As is well-known, Bergson has treated the problem of the spatialization of time at great length. We do not agree with many of his views on the problem, but at least he has effectively demonstrated that modern science has destroyed the true notion of time. Cf. Les Données Immédiates de la Conscience, Durée et Simultanéité, La Pensée et le Mouvant, Matière et Mémoire, etc.
- (64) Lect. 11, no. 1. Cf. In I Phys. lect. 1, no. 5.
- (65) Ibid. lect. 10. no. 15.
- (66) Ibid. lect. 11, no. 7.
- (67) No. 14.
- (68) Cf. Northrop: Whitehead's Philosophy of Science, p. 187, Cf. also Riezler, op. cit. p. 42: "If I am not mistaken there is some confusion about causality. Many of you, it seems to me, mix up the principium rationis with the law of causality. Each ought to be kept distinct. The principium rationis binds reason and consequence. When you draw your conclusions in the realm of your mathematics you are inclined to call reason the thesis to start from -- say, a given triangle having a right angle. From this you proceed to the Pythagorean proportion of the squares, speaking of consequence. In doing so you refer merely to the process of your thinking. You may also start from the squares and conclude that the angle is a right angle. Then reason and consequence exchange places."
- (69) I, 44, 1, ad 3.
- (70) "Ainsi, il est impossible d'en douter, la diversité dans l'espace constitué pour nous une énigme, un sujet d'étonnement d'essence sinon identique, du moins très semblable à celui que nous découvrons dans la diversité dans le temps, et dès lors nous ne pouvons échapper à cette conclusion que si nos raisonnements sont exacts, le but vers lequel tendent explication et théories consiste réellement à remplacer ce monde infiniment divers qui nous entoure par de l'iden-

tique dans le temps et l'espace, lequel, évidemment, ne peut être que l'espace lui-même."--Meyerson, De L'Explication dans les Sciences, p. 186.

- (71) In II Phys. lect. 5. no. 11. Cf. In I Phys. lect. 1, no. 5: "Nam materia est propter formam forma autem est ab agente propter finem."
- (72) In III Met. lect. 4, no. 375.
- (73) Essai sur les données immédiates de la conscience, Paris, 1906, 5e ed. p. 157.
- (74) Cf. Lenzen: "Bodies and processes are represented by numbers or by symbols which may be represented by matrices. Hence the search for substance becomes the search for constants and invariants. There are functional relations between numbers. Thus the permanence, objectivity, and self-determination of substance are replaced by the constancy, invariance and functional relationship of numerical measures."--The Nature of Physical Theory, p. 277. Cf. Eddington: The Philosophy of Physical Science, pp. 129 - 130.
- (75) Lect. 1, no. 5; lect. 14, no. 8.
- (76) Space, Time, and Gravitation, p. 200.
- (77) Meyerson Identité et Réalité, p. 285. Cf. La Dédution Rélativiste, p. 258.
- (78) Cf. Meyerson: Du Cheminement de la Pensée, p. 707: "N'y-a-t-il pas là, véritablement, sujet à l'émerveillement le plus profond? Comment, en s'écartant ainsi du réel concret, en le foulant aux pieds intentionnellement (l'expression ne semble certes pas trop forte dans ce cas particulier), le mathématicien a-t-il pu néanmoins rester aussi intimement en accord avec son rythme profond?"
- (79) Cf. The Mysterious Universe, pp. 113 - 124.
- (80) The Nature of Physical Theory, p. 67. Cf. Stebbing: Philosophy and the Physicists; p. 28 - 29: "Given this exclusion, then the sounds of a Beethoven sonata could be replaced by a series of curves or a set of mathematical formulae. By studying these formulae we might discover that Beethoven was a mathe-

matician. We should not be able to discover that he was a musician because we have replaced the sounds by the mathematical expressions by means of which they could be mathematically but not musically described. To discover that musician we need further what Jeans would no doubt call musical concepts. But it then becomes impossible to maintain that the universe is God's mathematical thoughts or God thinking mathematically. "Perhaps the source of the confusions into which Jeans falls lies in the fact that he believes both that a mathematical description of a phenomenon can give complete knowledge of the phenomenon and also that the phenomenon is indeed an appearance of an unknown reality."

Chapter X

- (1) Cf. e.g. The Nature of the Physical World, Introduction, et passim.
- (2) A General Theory of Value, p. 408.
- (3) Cf. Urban: Language and Reality, London, George Allen and Unwin, 1939, pp. 405.
- (4) Cf. Urban, op. cit., p. 405: "From recent psychological literature I gather the following 'gem': 'My behaviour symbol relative to steaming foods may be a reacting of the salivary glands.' To say that the reaction of my salivary glands is a sign of the presence of food is entirely appropriate, but to call it a symbol is a linguistic distortion which is not only in itself inexcusable, but bars the way to any proper use of the concept of symbol."
- (5) A Modern Introduction to Logic, p. 13.
- (6) Process and Reality, p. 263. Ogden and Richards make symbolism coterminous with all uses of language with the exception of the emotive and the evocative.-- The Meaning of Meaning, Chapter X. "A symbol," they tell us, "symbolizes an act of reference, that is, among its causes in the speaker, together no doubt with desires to record and communicate, and with attitudes towards hearers, are acts of referring. Thus a symbol
- (7) Cf. Delacroix: Le Langage et La Pensée, Paris, 1924, p. 580: "Toute pensée est symbolique." Cf. also Lambert: Organon, part II.
- (8) III, d. 25, q. 1, a. 1.
- (9) In this connection the term "name" includes the verb.
- (10) In I Perih. lect. 4, no. 13.
- (11) Lect. 1, no. 3.
- (12) Cf. Introduction to Mathematical Philosophy, Macmillan, 1938, p. 18.

- (13) Priora Analytica, I, tract. I, c. 9 (Vives-Borgnet, p. 472 b)
- (14) Cf. John of St. Thomas: Curs. Phil. T. I, Pars II, Q. III, a 2, pp. 315 ff.
- (14a) Cf. In IV Met., lect. 12, no. 684.
- (15) Mind and Nature, p. 38.
- (16) We believe that M. Maritain has misconstrued Eddington's doctrine on this point: "M. Eddington paraît oublier ici que non seulement les mesures recueillies dans la nature par nos appareils nous livrent quelque chose de réel (qui peut sembler une 'ombre' au regard de notre univers familier, le philosophe cependant sait que ce sont autant de points d'émergence par où un aspect des choses existant en soi nous apparaît), mais encore que le premier degré ou le premier temps de conceptualization, parfois très élaborée, où nous dégageons de ces mesures une description du comportement observable des choses nous met aussi en présence de réalités -- je dis observables et mesurables, et prises précisément comme telles, -- nous introduit dans un monde de faits, de causations observables et de structures observables que le physicien théoricien a tendance à tenir pour une simple matière offerte à son génie constructeur, mais dont le physicien de laboratoire n'est pas disposé à laisser méconnaître qu'ils font déjà authentiquement partie de la science physique elle-même. Ces faits peuvent être établis d'une manière plus ou moins certaine ou plus ou moins hypothétique, ils peuvent impliquer à un degré ou à un autre un achèvement idéal du réel par la raison, ils n'en ressortent pas moins à l'ordre de l'être réel. Des notions comme celle de la constitution des gaz par des molécules individuelles en agitation sans fin, ou de la structure réticulaire des cristaux, et une foule de notions semblables, doivent être tenues pour autre chose que des symboles, -- je dis en tant même que traductions du mesurable et de l'observable, et avant que l'effort théorique, en s'appliquant à approfondir leur signification et à découvrir, dans une explication complète, de quoi elles nous parlent, nous permette de comprendre qu'en dernière analyse nous ne savons

que symboliquement de quoi elles nous parlent. Mais c'est précisément ce second degré ou ce second temps de conceptualisation scientifique que M. Eddington a en vue; et là il serait téméraire de récuser son témoignage." -- Les Degrés du Savoir, pp. 314 - 316.

- (17) The Foundations of Physics, pp. 12 - 13.
- (18) Pp. 250 - 251.
- (19) Cf. H. Margenau: "Methodology of Modern Physics", in Philosophy of Science, Vol. II, Nos. 1 and 2 (Jan. and April, 1935.)
- (20) Introduction to Science, p. 137.
- (21) Substance and Function, p. 229.
- (22) Cf. H. Margenau: "Metaphysical Elements in Physics", in Reviews of Modern Physics, Vol. 13, no. 3.
- (23) The Universe Around Us, pp. 133 - 134.
- (24) Cf. Eddington: "In short, the physicist draws up an elaborate plan of the atom and then proceeds critically to erase each detail in turn. What is left is the atom of modern physics. I want to explain that if the erasure is carefully carried out, our conception of the atom need not become entirely blank. There is not enough left to form a picture; but something is left for the mathematician to work on."-- New Pathways in Science, p. 259.
- (25) Some modern authors reserve the term "symbolism" to this perfect type that is provided by mathematics, and they describe the evolution of physics from the use of sensible and pseudo-sensible constructs to the use of pure mathematical signs as a progress from schematism to symbolism. See in this connection the writings of Ernst Cassirer.
- (26) The New World Picture of Modern Physics, British Association for the Advancement of Science, Aberdeen, 1934.

- (27) The Nature of the Physical World, Introduction.
- (28) Cf. also Science, Religion, and Reality.
- (29) The Nature of the Physical World, pp. 264, 280, 282.
- (30) Eddington: The Philosophy of Physical Science, pp. 141 - 142.
- (31) Book VII, Cf. Jeans: The Mysterious Universe, pp. 111 - 113.
- (32) Eddington: The Nature of the Physical World, pp. 237 - 238 Cf. Riezler, Physics and Reality, p. 34.
- (33) Cf. Eddington: Science and the Unseen World, p. 37.

Chapter XI

- (1) Cf. Eddington: New Pathways in Science, p. 315: "In Rossetti's poem the Blessed Damozel looked down from the golden balcony of Heaven across 'The void, as low as where this earth spins like a fretful midge.' Looking from the abode of truth, perfect truth alone can enter her mind. She must see the world as it really is..." Cf. The Mathematical Theory of Relativity, p. 1; The Nature of the Physical World, p. 225. Cf. also H. Brunschwig, Le Progrès de la Conscience, 702: "Jamais n'est apparu aussi chimérique l'espoir que l'homme réussisse à forcer la barrière de l'expérience humaine et qu'une fois de l'autre côté il aperçoive les choses à la manière dont on suppose que Dieu les contemple dans son éternité". — Cited by Meyerson, Du Chéminement de la Pensée, p. 689.
- (2) Cf. Eddington: The Philosophy of Physical Science, pp. 157 ff.
- (3) Le Songe de Descartes, p. 63.
- (4) Cf. Eddington: New Pathways in Science, p. 45: "We must concede therefore that 'the universe as it is conceived in modern physics' is not identical with what a philosopher would call 'the objective physical universe.' When we cease to think of it there is no reason why it should be. The task of physical science is to disclose the scheme of the recurrences in the combined experience of conscious beings. We have seen that the universe which constitutes the solution of this problem must necessarily have the characteristics of regularity and externality; we said nothing about objectivity. And so it happens that the aim of science and the search for an objective universe follow the same road up to a certain point and then part company."
- (5) Cf. Planck: The Universe in the Light of Modern Physics, Pp. 11 - 12: "As soon as contact with reality has been lost, physical law ceases to be felt as the relation between a number of magnitudes which have been ascertained independently of one another, and becomes a mere definition by which one of these magnitudes is derived from the others. In this method there is a particular attraction, due to the fact that a physical magnitude can

be defined far more exactly by means of an equation than by means of measurement. But at the same time, this method amounts to a renunciation of the true meaning of magnitude; while it must also be remembered that confusion and misunderstanding result when the same name is retained in order to denote a changed meaning."

- (6) Cf. Cohen: Reason and Nature, p. 277: "But this fails to explain why phenomena seem to occur as if the law of gravitation with its inverse squares were true, or why the properties of circular functions have proved most potent instruments for the discovery of important facts in almost all branches of physics. Doubtless equations are not vibrating strings; but is it not straining the dualistic dogma to assert that they have nothing in common with each other? Do not let us be misled by the terms 'expedient' or 'invention'. A map or a chart is an expedient or invention. Yet if it fairly represents its objects, is it not because certain relations between its parts are precisely those between corresponding parts of the objects represented?" Cf. also "The Logic of Fictions," Journal of Philosophy, 1923, p. 447.
- (7) Cf. De Sitter; Kosmos, p. 6.
- (8) The Mathematical Theory of Relativity, p. 3.
- (9) Eddington rightly objects to Professor Stebbing's contention that physicists are not concerned with chairs; "Physicists are not concerned with chairs! Are we really expected to take this sitting down? . . . Why is it that a Transport Company, wishing to improve its arrangements for seating, consults a physicist who is not concerned with the chairs we sit upon, instead of a philosopher who is?" The Philosophy of Physical Science, pp. 159 - 160.
- (10) Cf. e.g. Eddington: The Nature of the Physical World, and The Philosophy of Physical Science.
- (11) Cf. Jeans: The Mysterious Universe, pp. 70 - 71: "It may be well to state our conclusion in advance. It

is, in brief, that the ethers and their undulations the waves which form the universe, are in all probability fictitious. This is not to say that they have no existence at all: they exist in our minds, or we should not be discussing them; and something must exist outside our minds to put this or any other concept into our minds. To this something we may temporarily assign the name 'reality', and it is this reality which it is the object of science to study. But we shall find that this reality is something very different from what the scientist of fifty years ago meant by ether, undulations and waves, so much so that, judged by his standards and speaking his language for a moment, the ethers and their waves are not realities at all. And yet they are the most real things of which we have any knowledge or experience, and so are as real as anything possibly can be for us."

- (12) New Pathways in Science, p. 315.
- (13) New Pathways in Science, p. 26. (E. & O.E. means: errors and omissions excluded).
- (14) Elsewhere Eddington writes: "We asked why the story teller should be believed when he talks about galvanometers, although he is untrustworthy when he talks of familiar objects. I think the answer is that the truth of the story is not the point in question; the physicist is concerned only with the scraps of cipher contained in it. The galvanometer is a device for leading the story into situations in which the underlying cipher becomes less baffling to interpret; it is not a bridle on the story teller's imagination."--New Pathways in Science, pp. 10 - 11.
- (15) Op. cit. pp. 424 - 425.
- (16) Cf. Duhem; La Théorie Physique. p. 509: "En un mot le physicien est forcé de reconnaître qu'il serait déraisonnable de travailler au progrès de la théorie physique si cette théorie n'était le reflet de plus en plus net et de plus en plus précis, d'une Métaphysique; la croyance en un ordre, transcendant à la Physique, est la seule raison d'être de la théorie physique."

- (17) Cf. John Dewey: The Quest for Certainty, Chapter V.
- (18) Cf. Urban, op. cit. p. 514: "There is, as we have seen no possible type of symbol which does not contain some element 'of fiction' (of the factitious, to use Descartes' terms), and which does not in some way and to some degree 'distort' reality. In the case of the aesthetic symbol the artist seeks to achieve deviations from reality in order, paradoxically, to represent reality better or to penetrate more deeply into it. In the case of the scientific symbol the scientist also deviates from the intuitive, phenomenal reality -- in this case, however, to explain and ultimately to control reality and predict happenings."
- (19) Cf. "De Valore Theoriarum Physicarum" in Acti Primi Congr. Thom. Roma, 1925, pp. 61 - 74; 269 - 275; "Inquisitiones criticae in Theoriam Atomicam Physico-chimicam Eiusque Valore Pro Philosophia Naturali" in Gregorianum, 1925; pp. 248 - 265; 1927; pp. 229 - 242; 1928; pp. 417 - 460.
- (20) Cf. John of St. Thomas: Ars Logica, p. 681 b: "...non aliter signatum representat quam prius ac ut obiectum representando, ulterius extendendo representationem sui ad aliud in se virtualiter implicitum et contentum."
- (21) In I, 55, 3, no. 13.
- (22) Curs. Phil., Ars Logica, p. 692 b.
- (23) Cf. John of St. Thomas, loc. cit., p. 647 a: "Ratio signi formaliter loquendo non consistit in relatione secundum dici, sed secundum esse."
- (24) Cf. Eddington: The Mathematical Theory of Relativity, Introduction.
- (25) Cf. Duhem: La Théorie Physique, p. 452: "Pourquoi donc le physicien peut-il, sans prêter à rire, affirmer que l'expérience découvrira une certaine loi parce que sa théorie réclame la réalité de cette loi, tandis que le

conchyliologiste serait ridicule si la seule présence d'une case vide dans ses tiroirs, consacrés aux diverses couleurs du spectre, le menait à conclure qu'il y a des coquilles bleues dans l'Océan? C'est que visiblement, la classification de ce collectionneur est un système purement arbitraire, qui ne tient aucun compte des affinités réelles entre les divers groupes de mollusques; tandis qu'en la théorie du physicien, transparait comme le reflet d'un ordre ontologique."

- (26) Op. cit. pp. 265 - 266.
- (27) Chapter 6, 1016 a 25, lect. 7, no. 863.
- (28) Chapter 14, 224 a 2, lect. 23, no. 13. Cf. St. Albert the Great. Ibid., Tract. III.
- (29) The Universe in the Light of Modern Physics, p. 13.
- (30) Science et Hypothèse, p. 6 Cf. Jeans; New Background of Science, p. 51: "The layman sees Science, as it seems to him, forever changing her mind, hesitating, turning back on her tracks, and repudiating her earlier opinions. The scientist sees her ever progressing through a succession of theories, each of which covers more phenomena than the predecessor it displaced, towards the goal of a single theory which shall embrace all the phenomena of nature."
- (31) La Théorie Physique, p. 53.
- (32) Mind and Nature, p. 46.
- (33) Edington: The Nature of the Physical World, p. 353. Cf. Planck: The Philosophy of Physics, p. 31.
- (34) For a study of this notion and its philosophical implications see Juvenal Lalor, O.F.M. "Notes on the Limit of a Variable" in Laval Théologique et Philosophique, Vol. I, Nos. 1 and 3.
- (35) The Nature of the Physical World, pp. 352 - 353.

- (36) Cf. Lalor, op. cit. No. 1. p. 143.
- (37) Lalor, op. cit. p. 146.
- (38) Where is Science Going?, p. 82. Cf. The Universe in the Light of Modern Physics, p. 15., The Philosophy of Physics, p. 31. Cf. also De Broglie, op. cit. p. 319.
- (39) Cf. Duhem: La Théorie Physique, p. 450.
- (40) Cf. Planck: Where is Science Going?, p. 200, The Universe in the Light of Modern Physics, p. 57 - 58. Cf. also Eddington: Science and the Unseen World, p. 23: "We seek the truth; but if some voice told us that a few years would see the end of our journey, that the clouds of uncertainty would be dispersed, and that we should perceive the whole truth about the physical universe, the tidings would be by no means joyful. In science as in religion the truth shines ahead as a beacon showing us the path; we do not ask to attain it; it is better far that we be permitted to seek."
- (41) Cf. Morceaux Choisis Sur le Marxisme, pp. 51, 52.

Chapter XII

- (1) New Pathways in Science, p. 7.
- (2) The Universe in the Light of Modern Physics, p. 15.
- (3) The Nature of the Physical World, Introduction.
- (4) Cf. "Interviews with Eminent Scientists" in The Observer, April 13, 1930 by J.W.N. Sullivan: "I found that not only Einstein, but also Planck and Schrodinger fully recognized the subjective element in science. Planck in particular...regards science as a constructed work of art, expressing a certain side of man's nature."
- (5) Cf. Rey: La Théorie Physique, p. 350.
- (6) Cf. Jeans: The New Background of Science, pp. 2 - 3; 67, etc.; Physics and Philosophy, pp. 143 - 144.
- (7) The Evolution of Physics, p. 33.
- (8) Du Cheminement de la Pensée, p. 654.
- (9) Pp. 16 - 20.
- (10) Cf. Ibid. p. 57.
- (11) Pp. 108 - 112.
- (12) In I Met. lect. 10. no. 158.
- (13) "Réflexions sur le problème de l'indéterminisme" in Revue Thomiste, 1937, p. 396.
- (14) Cf. Fulton Sheen: Philosophy of Science, p. 76: "The problem whether science has a real value is much like a modernization of the Scholastic dispute of whether an idea is an id quo or an id quod. In modern language this means, do mathematics have a relation to reality or are they only a mathematical symbol? The modern idealist would hold that scientific knowledge is "That which is known" instead of that "by which" reality is known. St. Thomas' criticism of the subjective theory of knowledge is therefore quite to the point."

- (15) "We must therefore remember that not all our knowledge of the physical universe is comprised in knowledge of the laws of nature. The warning is not so superfluous as it seems. I have often found an impression that to explain away the laws of nature as wholly subjective is the same thing as to explain away the physical universe as wholly subjective. Such a view is altogether unfounded." Op. cit. p. 15. Cf. also pp. 104, 178, 217, etc.
- (16) Cf. e.g. Du Cheminement de la Pensée.
- (17) Cf. Meyerson: La Déduction Relativiste, pp. 134, 143.
- (18) De L'Explication dans Les Sciences, pp. 526 - 528.
- (19) "La science est réaliste; mais nous savons cependant que d'explication en explication, elle ne peut aboutir qu'à l'acosmisme, à la destruction de la réalité. Or, dans le relativisme, précisément parce qu'il constitue, une forme très avancée, très parfaite, de l'explication théorique, ces deux extrêmes de l'existence et de la non-existence se trouvent très rapprochés l'un de l'autre. D'où une sorte de conflit douloureux dans la conscience du physicien." -- Meyerson: La Déduction Relativiste, p. 205.
- (20) De Sitter: Kosmos, p. 108.
- (21) P. 104.
- (22) Ibid. pp. 188 - 189.
- (23) La Déduction Relativiste, pp. 209 - 210.
- (24) Preface to the second edition (French version) p. 26.
- (25) Cf. Eugene Babin, Op. cit.
- (26) Cf. Tome I, p. 61 (French edition).
- (27) Op. cit. pp. 61 - 62.
- (28) Space, Time, and Gravitation, p. 201.

PART II

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