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Expropriation risk with social and political instability: a dynamic conservation modeling approach

Mémoire présenté
à la Faculté des études supérieures et postdoctorales de l'Université Laval
dans le cadre du programme de maîtrise en économique
pour l'obtention du grade de Maître ès arts (M.A.)

DÉPARTEMENT D'ÉCONOMIQUE
FACULTÉ DES SCIENCES SOCIALES
UNIVERSITÉ LAVAL
QUÉBEC

2011

Résumé

Ce travail de recherche propose un modèle dynamique des décisions de gestion du sol de la part d'un fermier lorsqu'il y a présence de conflits. Dans le contexte modélisé, la baisse de fertilité et la dégradation du sol entrave la sécurité alimentaire et le bien-être économique, comme par exemple en Afrique subsaharienne. La qualité du sol est modélisée comme une ressource naturelle renouvelable, alors que la présence de conflits est modélisé à l'aide d'un paramètre captant le risque de perde la terre (risque d'expropriation). Construire le problème économique de manière intertemporelle fait ressortir sous quelles conditions un fermier rationnel changera sa décision d'une stratégie de gestion durable à très long terme à une stratégie caractérisée par l'extinction defa la ressource. Nous caractérisons l'équilibre à long terme et comment celui-ci peut varier en fonction du risque d'expropriation.

Keywords: Risque d'expropriation, guerre, instabilité politique, sécurité des droits de propriété, dégradation du sol, conservation du sol

Abstract

This paper motivates the use of a dynamic model of on-farm soil management decisions, first to capture the intertemporal nature of the farmer's management problem and second to show how expropriation risk can affect these management decisions. The context is one where declining soil fertility and land degradation are impediments to food security and economic well-being in sub-Saharan Africa. This region is also plagued by a disproportionately high frequency of conflicts. Structuring the economic problem intertemporally highlights the stakes in terms of future agricultural production and public well-being. The results of the theoretical work define the conditions under which an economically rational farmer would switch from sustainable farm management to a resource-depletion pathway as a result of additional expropriation risk and the change in the long run steady state of each state and control variable in the model.

Keywords: Expropriation risk, war, political instability, property tenure, land degradation, soil conservation

Acknowledgements

Many thanks to Markus Herrman, my thesis director, who permitted me the mental space and time to explore economic aspects of challenges relating to long term management of renewable natural and agricultural resources. His patience, general knowledge and technical capacities were very helpful when challenges arose. Thanks also go to the professors, colleagues, family and friends who took the time to question me as I puzzled my way through some of the more complex aspects of such challenges. The project was financed in part by the *Fonds québécois de la recherche pour la société et la culture (FQRSC)*, a contribution that is much appreciated.

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1 Introduction

Agricultural productivity has a strong and direct impact on the well-being of the vast majority of the developing world. This document takes it as a given that the reader understands that low agricultural productivity and conflict are major challenges affecting the potential of many developing economies. The challenge of soil depletion in the developing world is one that has been recognized and continues to be addressed both in the literature and in the field (see, for example, Smith et al. (2010), Dreschel et al. (2001) and Bekunda et al. (2010), etc). It is also taken as a given that the reader has some appreciation of the fact that low agricultural productivity and the presence of conflict both limit individuals' capacity to reach their potential in many of these countries. The potential for a virtuous circle between improved agricultural productivity and improved political stability are further aspects of the context that do not warrant detailed discussion: it is hardly contentious to claim that equitable growth and improved well-being are good partners with stability. This study looks at the opposite case, where instability affects those with limited options to address the situation: smallholder famers.

This paper will not try to explain why conflicts start. What it does do is appropriate analytical tools from the field of resource economics to analyze the economic nature of smallholder farmers' rational management decisions when a conflict increases the perceived or real expropriation risk. At the aggregate, these decisions affect future agricultural wealth and public well-being in agricultural economies via management of agricultural resources (soil/land) used to produce agricultural goods.

The bodies of literature used in this paper include economic modeling of soil conservation, research on property tenure and investment in agriculture, a variety of sources to corroborate various claims about the nature of soil properties and plant growth, along with some generalities about conflict. These will be introduced as needed. This having been said, the basic chain of reasoning that drives the model in a context of expropriation risk is presented before outlining the remainder of the paper.

The economic rationale for the existence of a relationship between war and long term agricultural productivity is the following: changing economic conditions (including expropriation risk or changes in market conditions) can be expected to alter production choices that a rational smallholder farmer will consider as optimal, as described below. Stating that the farmer is rational is taken here to mean that the farmer maximizes their (or in an abstract sense, their household's) intertemporal utility given their preferences and endowments, with income flows resulting from production on a plot of land of a given initial soil quality. The farmer is not strictly considered to be a subsistence farmer, although they may or may not be extremely poor. This allows them to be affected by market disturbances and imperfections associated with war, instability, or perhaps even some forms of corruption. Current production choices affect future productivity in agriculture because some land uses tend to improve average soil productivity and other land uses tend to degrade it. For example, an intensely cultivated monocrop in rows is likely to decrease soil quality over time, while fallowing or combinations of productive trees, leguminous crops and limited grazing will improve soil quality under most conditions. A change in production choices can therefore affect long term soil quality. Technically speaking, since soil (land) is the most immovable aspect of agricultural production, it is highly appropriate to treat soil productivity as a state variable in a dynamic optimization problem. Introducing expropriation risk allows war and instability to affect economic variables. This then alters production decisions, in turn affecting levels of soil quality, carrying effects into the future. Future levels of agricultural production, economic growth and overall public well-being are thus at stake, particularly in agricultural economies.

That was in words. This paper will eventually communicate that logic in the form of a mathematical chain of reasoning, using economic theory to interpret the results along the way. Before we go there, the second section will review theoretical and empirical efforts by other researchers to operationalize the concepts and variables used in our model. This having been done, we will be in a good position to present the model along with the effects that expropriation risk has on current and future production decisions and outcomes. The fourth section produces and discusses results for the steady state, including the conditions where conflict causes a farmer to change their strategy to "mine" the agricultural resource rather than to manage it sustainably. The final section recaps the review, highlights the results with respect to long run economic welfare and presents interesting directions for further research.

2 Literature review

The main body of literature used to set up the economic problem in this study involves dynamic models of soil conservation decisions in agriculture. Dynamic models of soil conservation are a particular approach to structuring the economic problem faced by a farmer who is presumed to make current production decisions with full knowledge of the effects that these decisions have on future production conditions. Namely, current land use and production intensity decisions affect future soil productivity. Inquiring about the nature of the effects that trends of war and/or instability may have on these decisions will also involve some particular use of terms adopted in this document. Approaches to defining these terms are found in the relevant bodies of literature on dynamic soil conservation models, property tenure, soil sciences and, to a lesser extent, conflict. In each case, we will see that each concept can be understood in a variety of manners. Ideally, these precisions will allow conceptual ambiguities to remain in order to permit a highly general treatment of the issue. Discussion of these concepts, largely as presented in the literature, will form the bulk of this section and thus replaces some of the elaborations that may typically be expected to appear when presenting the model.

The following pages cover the most important variables that appear in the model. As such, it is useful to consider the subsections of the review as a presentation of the following: the state variable (soil quality), the first set of control variables (land use choices) and its relation to the state variable, the second set of control variables (input intensity and thereby output) and how market access may affect these decisions, property tenure (which affects the perceived value of future income flows) and expropriation risk, and war and instability (the novel variable added to the soil conservation model).

2.1 The state variable: Soil productivity

Soil quality can be defined in a large number of ways because different soil variables matter for different productive uses of soil. The collection of key soil variables can be expected to vary by region, according to current soil conditions, climatic and ecological variables, and according to cultural habits or traditions with respect to crop choice and land uses. An example of an indicator of soil quality is the percentage weight of soil organic carbon (SOC), a useful proxy of soil quality given the ease of measuring this variable in a laboratory setting. There are also strong explanations offered by the chemistry of soil science to explain how SOC supports plant growth, by acting as a substrate for nutrients, offering a structure for root growth, limiting erosion, holding water, etc. (Chapin et al., 2002). Yet it is not the SOC itself that the plant needs. Rather, it is bioavailability of nutrients in a healthy growing condition that allows plants to thrive. Evolutionary specialization, together with phenotypic plasticity, naturally imply that every type of plant responds differently to particular conditions. Fortunately for the empirical researcher or scientist, some generalities hold such as trends for the importance of bioavailability of various macro and micronutrients, water and light, along with the need for substrate of some sort for root growth.

In the field, plants behave quite differently across production environments, implying that a useful measure of soil quality in one context may not be so useful in another. McConnell (1983), for example, specifies the depth of soil in centimeters as a dynamic state variable that is directly subject to control. This could be an important consideration in areas where previous land degradation had already resulted in sufficiently extreme losses of soil cover for depth itself to be a limiting factor. The value of soil depth in terms of more reliable access to water and room for root development, allowing the uptake of nutrients, is unquestionable. As one would typically state in terms of agronomy or chemistry, however, one has to look hard to find specific cases where soil depth is the primary limiting factor. Typically, access to water, nutrients, along with appropriate soil conditions to confer bioavailability of these basic inputs, and appropriate levels of sunlight and temperature variation, are common limiting factors in plant growth. The matter of nutrients may appear obvious, but different plants require different levels of various macro and micronutrients and also have a varied range of abilities to uptake their required nutrients under various conditions. Relevant conditions for nutrient bioavailability may include soil acidity, alkalinity and salinity etc., which each affect the efficacy of cellular mechanisms that plants use to uptake nutrients. Optimal light and temperature conditions also vary across crops and vary for different stages of plant growth.

Soil depth hardly appears to be a good measure of soil productivity, in view of the various other important soil factors involved in plant growth. While the use of soil depth as a state variable by McConnell (1983) is somewhat simplistic, he does offer a clean theoretical structure for the soil conservation problem. The farmer has the "simple" objective of maximizing intertemporal benefits and decides soil levels to determine future production and flows of benefits. The choice to model soil quality in terms of its depth could be motivated by an apparent link between SOC and erosion, or perhaps due to the immediately clear (and easily visualized) signification of erosion as opposed to more elaborate measures of soil quality. It would also be unsurprising to find that the primary motivating factor for the use of soil depth as a proxy for quality is the existence of a policy debate relating to soil erosion in the USA. Expediency in this regard can be easily forgiven, however, given the level of clarity that his methodology gave to economic modelling of soil conservation. Erosion is certainly a fundamental part of the problem of land degradation and declining soil productivity on a non-negligible proportion of farms, yet it is but one of many manifestations of land degradation and declining soil productivity. An economist may be tempted to say that this distinction should not matter because the farmer can equate the intertemporal shadow prices of soil levels and marginal opportunity costs, and can also benefit from technological developments, but in reality a farmer could hardly expect to make optimal decisions if their sole measurement variable used to optimize production accounted for soil levels and erosion to the exclusion of other economically relevant soil variables. This justifies the need to take time to consider soil quality in greater detail.

Kiker and Lynne (1986) also look at an agricultural system where the state of soil quality is defined in relation to soil erosion. They do go so far as to mention that changes in product prices and input costs influence optimal soil erosion. This is just a hair short of moving towards an abstracted notion of soil productivity as the state variable of interest. But it still does not capture other elements of soil quality that could be important. For example, a plant could be particularly poor at dealing with the presence of a given mineral in the soil (such as aluminum), which would warrant monitoring soil quality for levels of aluminum. The level of this metal would then be considered as an indicator of soil quality for the particular context, even though this issue is largely irrelevant for the vast majority of farms. Alternatively, we could consider a crop which depletes the soil of a particular micronutrient. Monitoring levels of that micronutrient as a key indicator of soil quality would then be warranted. Ideally, this would lead to improved results via optimized nutrient replacement strategies or alternating land uses, although the range of economically feasible technical options actually considered may vary substantially depending on ecological conditions, cultural traditions and market factors.

Erosion is also held as a major issue by Saliba (1985), but he views the soil's capacity to produce some level of yield as the variable of interest. Soil yield thus appears to be the variable describing the state of his agricultural system. Prices for produced goods enter the economic model in terms of goods' contribution to current and/or future profits. When connected to markets, however, the biological yield may not always be the variable of greatest interest to the farmer. Here, we could consider any cash crop associated with declining soil productivity, such as cotton. A cotton farmer who anticipates a future increase in cotton prices is more interested in the total value of profits resulting from their effort than the number of tonnes of cotton produced. An interest in yield rather than economic productivity could easily be motivated for the subsistence case however, where the farmer is interested in the level, rather than market value, of production because the market price signal does not reach this farmer. (Grepperud (1996) investigates the decisions of the subsistence farmer in much greater detail.) Despite the remaining question of an economically meaningful indicator of soil quality, Saliba's contribution to soil conservation modeling is notable, as will be seen in relation to land use choices.

The focus on erosion has carried through to empirical works, where the appeal of simplified proxy variables is immediately appealing: expensive lab equipment and testing is not required for the proxy of soil quality. For example, Barbier (1990) uses soil depth as a proxy for soil quality in his study of upland farming practices in Java. The particular context makes erosion the most obvious variable of interest, given the evident importance of soil erosion in areas with relatively steep slopes. In a few cases, Barbier even puts together some figures and compares the market prices of some different options and presents a profitability-to-erodability chart to illustrate the trade-offs involved in the farmer's decision. Again, in the empirical context, it is easy to understand why the abstract idea of soil productivity is less appealing than an easily measured proxy variable. (Of course we know the value of production, but production results from the combination of numerous inputs, one of which is a renewable natural resource, soil quality. Pinning down this variable for conceptual or empirical purposes is an entirely different question though.) Real world limitations are always a good reason to look for ways to simplify things, but from the theoretical perspective it is difficult to see a compelling reason to adopt these simplifications just yet. Indices of soil quality can easily be adjusted on an ad hoc basis to reflect specific contexts, much like any sensible farmer would make some effort to do, albeit perhaps on a more intuitive or less mathematical basis. On paper, however, there is no cost to defining abstract soil productivity as the state variable in the farmer's economic system. This specification is made by Larson and Bromley (1990) and by Wichelns and Burnes (2006).

Defining the state variable as soil productivity itself may appear as an expedient assumption because it does not specify which variables to include in a measurement index. It is argued here, however, that its simplicity is its beauty: this notion of soil productivity as soil quality is perfectly flexible for adjustments to specific contexts, to the extent that the researcher, farmer, or technical advisor are aware of which variables relate to soil productivity for the crops and land uses that are considered on a given plot of land. In our study, we will thus merely refer to soil productivity or quality in the abstract sense, as the collection of relevant variables which manifest themselves concretely in terms of economic productivity and contributions to household welfare.

2.2 Control variables: Soil productivity change and land uses

This section presents the land use control variable. We will first look at how the selection of appropriate indicators of soil productivity is context-specific for different kinds of land uses, then will present some approaches to economic modeling of land use decisions. This is briefly followed by a selection of land uses which are typically associated with either improving or declining soil productivity.

- **Changes in soil productivity**

Changes in soil conditions can be easily represented visually, in some cases to dramatic effect. Some examples include a dust storm in say Chad, Alberta or north China; landslides in mountainous agricultural areas in Indonesia, the Philippines or Nepal; the cracked soils of regions experiencing drought and/or long term problems with water tables (consider images of the recent drought in south China or in 1980s East Africa). Visualizing a change in soil productivity is typically much more difficult. Satellite photography could theoretically indicate general changes in green cover. While convenient for aggregate purposes, that proxy can hardly be argued to offer a complete story about changing production conditions for smallholder farmers in developing countries. At the more individual level, perhaps a before and after photo would work from the visual perspective: a positive change in soil productivity or overall profitability could be illustrated by a farmer with several cows, a new phone and children with textbooks in a school uniform; a decline in productivity or profitability could be illustrated by visibly undernourished children on a farm in a previously productive region, perhaps even one where conflict has flared up in the meantime or where instability and anarchy otherwise reign.

Changes in soil productivity are inextricably linked to the way that the state variable, soil productivity, is defined or represented. Its economic interpretation is obvious, but that does not help much in terms of looking at measurement variables for productivity stock, whether for theoretical or empirical applications. The most relevant measurement variables differ across circumstances, as discussed above, and as indicated throughout the literature. The exact time, place and environmental, cultural and market conditions will dictate which measurement variables are most relevant for changing soil productivity. This change can be described in terms of an increase or decrease in soil levels (as with McConnell (1983) or Barbier (1990)), as a change in yields (as with Saliba (1985) or as a change in productivity (Larson and Bromley (1990); Wichelns and Burnes (2006); also, the present study).

We will now illustrate why poor specification of this variable matters. Consider that we are taking soil depth as the state variable, and that soil depth is used to tell us how productive the farm is. If we are in a region where the level of soil is not a strong constraint on plant growth, but where aquifer problems or salinity is a problem, the measurement variable does not give us much useful information. A farmer could then claim success with respect to land degradation by reducing erosion, while ignoring the most economically relevant aspects of soil conditions for their particular context. Just how would they have achieved whatever improvements they nominally claimed to have achieved though? Generally, this is accomplished by allocating a share of their land among a number of possible land uses, some of which increase indicators of soil productivity and some of which reduce indicators of soil productivity.

- **Modeling land uses**

To start with, land use choices are not always considered as the control variable. Studies with just one land use may be looking to emphasize another aspect of the problem and instead provide their rational farmer with agency over the production intensity of the crop (McConnell (1983), or alternative technologies and input packages (Grepperud, 1996; Barbier, 1990). These three authors have respectively chosen a form that is appropriate to their contexts of monocropped grains in the USA, subsistence farming in the developing world and a mountainous region of Indonesia. In our case, we could consider a farmer's choice to allocate a plot between intensive cotton farming and a variety of pea (as seen later, leguminous plants improve soil nitrogen).

Any variety of other appropriate real-world land use choices could alternatively be considered. Goldstein and Udry (2008) estimate implicit discount rates in a region of Ghana by looking at fallowing decisions. Antle et al. (2006) show how farmers respond to support for terracing in Peru. Barbier (1990), Larson (1992), Besley (1995), Deacon (1999) and Goldstein and Udry (2008), each consider some level of investment in productivity. In most cases, this is measured as an investment in terms of money or opportunity costs, a measurement that is suitable for looking at decisions such as terracing where current investments are required for future productivity benefits.

In our case, however, the goal right here is to keep things as simple as possible. Thus, the choice in each period is between a land use that improves current profits and degrades soil productivity and a second land use that improves soil productivity but negatively affects current profits.

Much like McConnell (1983) serves as the founding author for the "modern" stream of dynamic modeling of soil conservation decisions, Saliba (1985) can be touted for bringing land use choice into the picture. He models the farmer's decision as a share of land allocated between two production options. One of these options has a higher contribution to profits in the present period but negatively impacts soil conditions, while the other land use has a lower contribution to current profits but has the ability to improve soil quality. This aspect of the model is also adopted in the following section of this paper. This type of decision does not necessarily have to reflect maximization of profits however, as illustrated by Lynne et al. (1988). They find that land use decisions depend on attitudes. Allowing attitudes and perceptions to drive economic decisions poses a certain challenge to the appropriateness of a dynamic economic optimization model for soil conservation issues. In principle, this could be addressed fairly easily: basic choice theory tells us that economic agents maximize utility by choosing from a strategy set. Attitudes which reduce the size of this strategy set will reduce the farmer's ability to respond to changing circumstances in a manner. A "suboptimal" outcome would not be a surprise in that case. In practice, the decision set or scope of strategies can be defined according to options that are most pertinent to the specific circumstance and can be as general or as specific as needed.

In the case of Pendelton and Howe (2002), the context of open-access cultivation at the edge of development in Bolivia leads them to pose this land use decision differently. The Tsimane hunter/collector/farmer in their study faces a decision to clear or not to clear a plot of land for short term cultivation. They also decide how long to cultivate the plot for. With open access and an abundance of land at the frontier, concerns about soil productivity are negligible, so they study the effects of market access on land use decisions made by the Tsimane people. Other authors (Grepperud, 1996; Antle et al., 2006) view the nature of this decision in yet another manner, with farmers allocating some land to uses which cost money and produce nothing, but which improve soil quality.

So there we have it: the use of land can be modeled in a variety of ways, and the previous paragraphs barely scratch the surface of the matter. It is nevertheless considered that studies which model the decision as a choice between land uses and as choices of input levels are intuitively preferable because the farmer actually has direct control over these variables. With an extension to a general case, as per Wichelns and Burnes (2006) who allow the farmer to choose between multiple land uses, there is also no reason that options ranging from monocropping versus intercropping, to the number of years of fallow (or equivalently, some notional average share of plots in fallow at each point in time), cannot be captured by representing the farmers' decision between land uses with either negative or positive effects on future soil productivity. In our case, a desire for simplicity again dictates the choice to model just two land use choices. Now, to move from the abstract to the concrete: what are some typical land uses options that an actual farmer may consider?

- **Soil productivity-enhancing and -degrading land uses**

We consider a context where agricultural land uses are generally the main cause of soil degradation (Sanchez, 2000).[1] While agricultural land uses tend to degrade soil quality, the distinction between soil productivity-enhancing and -degrading land uses implies that agricultural land uses may also be associated with improving soil productivity. The history of agrarian humanity, though, is necessarily riddled with cases of both successful and unsuccessful land management, whether choices were made in general ignorance or with awareness of the long term impacts of their choices.[2] In times when open access and land abundance were the norm, fallowing fields would be an obvious choice of land uses to replenish soil productivity, while slash and burn is another simple option. Fallowing remains a commonly used and easy-to-implement strategy to manage soil productivity, but more elaborate designs are often more appropriate and/or necessary with high population density or for particular crops. We will therefore categorize strategies which are used to counter cultivation-related soil degradation and provide some typical examples belonging to each of these categories.

We can group soil productivity-enhancing land uses in terms of their ability to improve the quantity or availability of nutrients, land uses which improve soil composition or texture and land uses or technologies which positively target water management. To head off a potentially contentious issue with respect to the first of these, let us consider inorganic fertilizer as an input that increases current

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[1] Demand for fuelwood is also an important factor driving land degradation (Larson and Bromley, 1990), as anyone who has traveled through various parts of the developing world can easily attest to. This factor will not be explicitly considered because specifications relating to common versus private management of fuelwood resources are not explicitly modelled in this paper.

[2] See (Diamond, 2004) for a highly readable and well-researched discussion of some more interesting examples of these impacts.

production but has ambiguous effects on soil quality. This assumption will be incorrect in some particular cases, but issues relating to fertilizer use are of more interest when considering off-farm effects or broader social benefits.[3]

Perhaps a relatively commonly known example of crop choices with the potential to improve soil productivity is the use of leguminous plants to increase soil nitrogen (Saliba (1985); Grepperud, (1997)). As evidenced by hundreds of studies published on journals listed with JSTOR, it is generally established that symbiotic relations form between root nodules in leguminous plants and mycorrhizal fungi to effectively fix atmospheric nitrogen, which yields nitrogen for both, say, a present pea crop and the cotton crop planted on that part of the plot in the following period. Decomposition of root and other plant matter from previous crops also releases nutrients to the soil.

In terms of soil composition or texture, land uses which tend to increase soil organic carbon, perhaps via deposition of plant detritus, are likely to have positive impacts. Investments such as terracing are also effective at reducing erosion, particularly when slope is an issue. With even steeper conditions (a 45-50% slope), terracing may have to be replaced by agroforestry to avoid excessive erosion (Barbier, 1995) if the farmer insists on cultivating such marginal lands. Production outputs from this type of strategy include cloves, citrus crops, fuelwood and fodder (Barbier, 1995).

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[3] McConnell (1983) and Wichelns and Burnes (2006) both touch on this issue. The first of these authors appears to disregard the general importance of externalities in most cases, whereas the second article focuses on the difference between on-farm and off-farm benefits as an important target of policy. Wichelns and Burnes (2006) go on to specify that "farm-level choices will not be consistent with those that would maximize the present value of social benefits. From the public's perspective, farmers will use too few inputs that generate off-farm benefits and too many inputs that generate off-farm damages" (p. 420). He then cites riparian buffers as a strategy with off-farm benefits that will be under-produced relative to the social optimum.

Other types of productivity-enhancing land uses may target water availability. For example, crops with deep roots may help maintain the amount of water in the soil near the surface which can then be used by other plants. Meanwhile, riparian buffers can be expected to improve water management (and also limit erosion) at the farm level and reduce sedimentation associated with runoff, while cropping systems that integrate drip irrigation both reduce erosion and conserve water. It should not go unmentioned that many of these approaches involve a significant input of labour.

For the sake of brevity, let it be stated that similar specifications can be made for land uses that negatively affect nutrient availability, soil composition and water availability. It is hoped that even this simple distinction between three determinants of soil productivity in the preceding paragraphs illustrates the difficulty of suitably defining variables for particular contexts. This problem of modelling the actual effects of land use choices makes it appealing to refer to an abstract notion of soil productivity as the state variable rather than a concrete soil quality index. Empirically, there are additional challenges because every additional specification in the measurement index costs more money for each complete data point.

A complicating caveat can be mentioned to close this section: some land uses have beneficial effects with respect to some variables and negative effects with respect to others. Consider the case of the lowly pea: it is good for soil nitrogen and is thus desirable in one respect, but cultivating this crop may still be associated with erosion and limited water retention, both of which are bad for soil productivity in many practical applications. Also, there is no particular reason that the farmer cannot use the land for non-agricultural purposes, but such land uses are not permitted in our model.[4]

2.3 Further control variables: Agricultural inputs and outputs

Every farmer, having decided what uses to make of their land, must then decide how intensely they wish to apply each of a selection of inputs. These inputs include basic ingredients like seeds and fertilizers, and a variety of equipment and labour inputs to plant, take care of, harvest and bring the product to market. These inputs and outputs are generally affected by market conditions and under all circumstances are determined in consideration of opportunity costs. Some of these assumptions may not be terribly unrealistic for some aspects of the American corn producer's decisions, but may also be unverified, unverifiable, or simply flat out wrong in the context of rural regions in some developing countries.[5]

What to do then? Well, one option is to look at the literature, a clear step above a purely ad hoc approach. There, we will find that, beyond the general form where the net present value is equal to $\sum_{t=0}^{T} e^{rt}\pi_t$, researchers in economic modeling of soil conservation tend to add just one or two, and at most three (Grepperud, 1996; Wichelns and Burnes, 2006) additional elements to the model, and typically do

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[4] This may smack of undue simplicity for the majority of us who live in or close to cities, and typically only see farms that could quite plausibly become suburban subdivisions in the near future. Our assumption is nevertheless realistic for most cultivated land, which is far from cities. Non-agricultural land uses in rural areas are especially unlikely to be considered in the context of a developing country with war or instability. This should be especially so where the quality of water, communications and energy supplies, for example, can easily preclude most alternative types of land development in the industrial or services sectors.

[5] For example, poor access to market information, undeveloped credit markets, limited availability of inputs, and undeveloped or unpredictable transportation infrastructure for outputs can be expected to feature heavily in much of the rural developing world. This makes it difficult to define the dynamics of production conditions on a farm operated by a rational farmer. No breakthroughs are required to mathematically model each of these. Taken together, however, the resulting model becomes much more unwieldy than necessary to capture the nature of the problem at hand.

not explicitly model market factors aside from some notional price for inputs and outputs. This does not need to prevent discussion of the effects of, for example, undeveloped credit markets, availability of inputs, even though these will not be explicitly modeled in the current paper.

In terms of market access, we can lean on Pendelton and Howe (2002).[6] In their model, however, market access boils down to implicit and explicit transportation costs. Given the empirical dimension of their study and the context of development at the frontier, this seems like a fairly reasonable measure of market access, which conveniently allows them to treat market access as a continuum.

From the theoretical perspective, it could be interesting to consider market access as a continuum ranging from 0 to 1. The upper bound only actually exists in a theoretical sense, when transactions costs and all imperfections are excluded from the problem. As for the lower bound, it would be fairly surprising to find a situation where the level of market access was effectively nil, as this would imply a purely subsistence situation where no trading was possible. Much like simplified representations of frictionless economies, this sort of Robinson Crusoe situation may be instructive for an introductory text in microeconomics. Here, access to at least basic implements can be presumed to exist in most practical situations.

While the extreme cases of perfect market access and zero market access may be rare in the real world, it may be instructive to consider some cases that appear to roughly approximate them. Realistically, most farmers in developing countries

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[6] The role of market access for their context of rural development may be summarized with the following quote: "Modern development strategies encourage market integration by reducing the transaction costs that separate rural and modern sectors. Road and other transportation improvements, technological transfers, and the provision of rural credit all contribute to the integration effort" (Pendelton and Howe (2002), p. 2). This is interesting to note because war and instability can easily be expected to negatively impact access to each of these types of market infrastructure.

are in an intermediate situation with respect to market access. Some seeds can be purchased and/or are affordable and standard fertilizer mixes and common pesticides are available at some price. The availability of more obscure micronutrients such as, say, molybdenum or equipment for easily-installed technologies such as a drip irrigation system, however, can easily be expected to be sparse in the rural developing world.

Concretely then, what is the logic behind the importance of market access for modeling farmers' soil conservation decisions? Consider the following chain of reasoning: war and instability likely affect market access, which affects actual prices paid for inputs and outputs as well as opportunity costs, affecting incentives for input and output intensity, and finally, input and output intensity affects future soil productivity. This is an area of study that almost certainly warrants further research, but is not fully explored in the theoretical developments of this paper.

2.4 Model parameter: Expropriation risk

This subsection touches on the historical and/or ideological nature of questions involving property tenure, presents common definitions to the term along with its use in economic literature and then discusses how expropriation risk can be modeled for the purpose of investigating the farmer's optimal long run management strategy in a context of expropriation risk.

• Property tenure

Differing views on individual and communal rights to land have played a non-negligible role in spawning some of the bloodiest conflicts in history. The surrounding debate comes with a whole trainload of philosophical, legal and ideological baggage, the discussions of which could easily fill up entire libraries. This paper, however, focuses on economic theory, not politics.

It should nevertheless be mentioned that the variety of approaches to managing land is all too often overlooked in economic analysis. The classical structure of soil conservation modeling often presumes perfect land tenure. This may be a fair assumption in many developed markets. A special report on risks associated with property ownership in *The Economist* (2011), for example, does not even mention the possibility that property tenure threats are of any relevance in this respect. We are often inclined to view this as the ideal state of things with respect to tenure. While I do not wish to comment on what state of affairs is ideal, it must absolutely be acknowledged that the reality on the ground in much of the world does not reflect this situation.

Some general references for this diversity can be pinpointed, however, along with more detailed study of individual cases. Ostrom (1990) provides an interesting collection of examples of how property rights regimes affect resource use decisions at the local level, while De Soto (2000) provides a detailed examination of how property rights and titling function (or do not function), with an almost amusing description of the absurd convolutions involved in obtaining formal title in 1990s Peru. The historian or philosopher may be interested in looking back to Locke's *Second Treatise on Government* or Hume's *An Enquiry Concerning Human Understanding*, standard texts in political philosophy, for evidence of the historical link between Western political thought and property ownership. These works, among others, are the product of an intellectual tradition that laid the foundations of the current dominant paradigm with respect to property, particularly in the West. Clearly there is no lack of options for modelling this problem, and the current document is far from the first to take up the issue of property tenure risk.

- **Views on property tenure in the literature**

Oxford dictionaries online[7] includes a definition of tenure as "the conditions under which land or buildings are held or occupied." For the purposes of economic modeling, this is most often considered in terms of an individual's ability to secure benefits flowing from the underlying value of a piece of land. A look through nearly any introductory- or intermediate-level microeconomics text will show that perfect tenure (i.e., perfect certainty of one's right to monopolize benefits flowing from land, including the right to sell) is an unstated yet defining paradigm underlying most economic analysis. This assumption may often provide a close approximation to reality in some jurisdictions, but may also be strongly contradicted by reality on the ground in other jurisdictions. (Hence, an existing literature on tenure risk.) Larson and Bromley (1990) describe this situation as a dichotomy between private and group land management regimes. In reality, most situations involve some combination of the two, and this variable could be considered as a continuum of sorts between the extremes of purely individual and communal tenure system. As per normal, the extreme cases are highly atypical or perhaps even only theoretical, but it is instructive to look at some examples that may nearly approximate these cases. In the West, for example, it is typically assumed that property rights have been purely instituted as an individual affair, except for when individuals contract together. This assumption would be the extreme scenario. In reality, local democracies at the municipal level engage in a fair degree of what could be loosely considered as group involvement in property decisions. Local development

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[7] Accessed March 3, 2011 from oxforddictionaries.com

and zoning requirements or property upkeep bylaws are fairly common examples of political engagement that determine the scope of land rights. At the other extreme, we could consider the case of 1970s to early 1980s communist China, where the property regime was communal, in some cases including features such as land redistribution to reflect demographic change at the household level (Deininger and Jin, 2003).

Changes in property tenure regimes are also found in other countries. (One could rhetorically ask just when are property regimes *not* in transition.) Besley (1995), for example, notes that property tenure systems in Ghana have been shifting from a traditional system where a chief regulates land transfers to a system with greater individual transfer rights and formalization via titling, etc. Goldstein and Udry (2008) also work in the Ghanaian context. Their study provides a unique lens through which property tenure can be viewed, namely, that social hierarchy and social relations at the local level matter for land rights. The strongest findings are that officeholders are less likely to lose a plot of land while it is in fallow, while women are more likely to lose a plot of land while it is fallow. Meanwhile, plot-level and household-level rights may differ, leading to divergent strategies on each of several plots. Goldstein and Udry (2008) address this by carrying out their theoretical developments and empirical work at the level of each plot owned by a household, then controlling for household-specific effects. This list of changing tenure systems could go on and on ...

When considering tenure or expropriation, we could also look at ownership structure. McConnell (1983), for example, points to family farms, rented family farms and corporate farms as the three types of tenure arrangements that should be considered for soil conservation decisions. The assumption of perfect tenure security is quite reasonable in the USA, making this type of distinction meaningful. His conclusion that incentives for soil conservation are lower on rented family farms is shared by Lynne et al. (1988) in their empirical work on farmers' conservation beliefs and practices, which found that owners make somewhat more of a conservation effort than renters. The rationale underlying these conclusions are similar to the logic behind the shift towards longer periods of tenure in rural China: the change from ongoing reallocations to 15, 30 and even 50 year land leases are specifically designed with a view to improving conservation incentives to protect future capacity to agricultural production and food security (Oi, 1999).

Another approach to classifying tenure is to focus on the distinction between open access and private property regimes. These property regimes are well-suited to analyzing situations where open access is the reality on the ground, as is the case in Pendelton and Howe's (2002) study on the decisions of the Tsimane people. The expected results in a classical context are fairly obvious: farmers have little incentive to conserve the resource. This probably reflects the actual story of much of history, when population was low enough for shifting cultivation to function effectively from the resource management perspective.

These and many other considerations are relevant to real-world applications and can be modelled individually. Since the focus of this paper is not to delve deeply into the nature of the variation of property tenure regimes across the localities and nations of the world, let it simply be noted that property tenure systems in much of the world are much different from what we may consider as normal in the West.

- **Modelling expropriation risk**

Modelling expropriation risk is of vital importance to this study. This is because it is a channel whereby war and instability affect farmers' production decisions. Other channels may include market disturbances. Much like changes in soil productivity can only be understood in the context of a particular definition of soil productivity itself, expropriation risk can only be understood in the context of a particular property regime. Loss of a plot of land may not always mean loss of income flows due to redistribution of resources within a community, such as by a chief or local official. A Burkinabe cocoa farmer in Côte d'Ivoire however, as an outsider, can be fairly certain that loss of a plot over the course of the present instability would be quite unlikely to result in compensation for their loss, whether financially or by access to some other plot to cultivate.[8] Property regimes are almost infinitely variable and are thus difficult to realistically capture in a model. The existence of at least some notional idea of "perfect" property tenure, however, may serve as a basis of comparison.

Modelling tenure in our context involves some positive possibility that the farmer will lose the farm. This expropriation may be due to exogenous or endogenous factors. The first case would suggest that the farmer was completely unable to mitigate threats to their tenure security, while the second would imply that the farmer's level of tenure security is influenced by decisions that they can control. Examples of this second case involve farmers' effort to improve their tenure. Besley (1995) refers to Atwood, who views the planting of trees on a plot as an African approach to Lockean property rights. Expenditures on fencing or formal titling (Hotte, 2001), are also among approaches adopted by farmers to improve

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[8] Carol Off (2006) discusses the generally unfavourable view towards those who have migrated to the country over the last few decades, many of whom have no access to formal land title.

their tenure.

For the time being, let us consider property tenure as something that is generally exogenous to the farmer's problem. That is to say, that there is some positive probability that the farmer will lose the farm in each period. Besley (1995), in the context of squatters in Latin America, views tenure insecurity as similar to a random tax on land. He eventually discounts the total present value of profits by a proportion equal to the probability of losing the plot. Hotte (2001) uses an exponential distribution for the risk of expropriation in his model, which he takes as a justification for considering eviction risk as the same as an increase in the effective discount rate. The higher effective discount rate thus encourages a settler to "mine" the natural resource rather than manage it. Mendelsohn (1994), in his investigation of the role of property threats in deforestation incentives, similarly shows that the size of the addition to the discount rate is equivalent to the probability that the plot will be lost in any given period.

Levhari and Mirman (1977) show that lifetime uncertainty makes it less likely that an individual will make permanent investments. This can be taken as analogous to the number of periods a farmer exploits a plot for, and is not intended to reflect any physical risk to the farmer themselves, although this may also be relevant. Our appropriation of this approach is justified and explained briefly in the modelling section and in more detail in appendix A.

So, what is "expropriation risk" supposed to mean on the ground, if not frontier competition as modeled by Hotte (2001), or squatter eviction as considered by Besley (1995)? One possible way to investigate this question would be to go to refugee camps and ask farming households why they left their farms. Unfortunately, the likely response would be the proximate cause, the proverbial straw that broke the camels back. Ad hoc reasoning can hardly be presumed a better alternative, but for lack of an alternative let us consider the following stylized situation. A highly local view of tenure is required here.

Consider a Burkinabe farming household in Côte d'Ivoire. They may have excellent relations with their neighbours, but when economic or security problems arise, public officials point to non Ivoirians as the root of the problem. Relations deteriorate with some neighbours, and as conflict in other parts of the country intensifies and/or spreads, the family is increasingly the target of unfavourable attention. This may involve intimidation, threats, or even attacks on property and person. At some point, the family may finally decide that enough is enough and go somewhere else. Or, local leaders will refuse to protect their interests when native Ivorian farmers either threaten to or actually forcibly take the plot. This last scenario would actually be fairly unexpected in most cases, while the first appears more likely. Not an encouraging example, as this does not exactly corroborate the idea of a fixed probability of losing land in each period. But, perhaps the farming household perceives some probability that an ultimate calamity will occur, affecting both the conservation decisions and the decision to leave.

The white Zimbabwean family of the 1990s may be more amenable to viewing property threats as a random probability of losing a plot. Anyone who has discussed the matter with these farmers knows that the types of threats and aggression described above were not unheard of at that place and time: there was always some background knowledge that tomorrow, or the day after, or ten thousand days after, some mob could come along and ultimately remove the farming household from the property. The eventual results of these farmers' "choice" to leave the land were declining agricultural production followed by a complete collapse in food production and a humanitarian crisis. With so much land taken out of production, soil conditions are presumably improving, but at what cost of human suffering? As for the appropriateness of the choice to model property threats as a probability of losing land in each period? It is somewhat more appealing for the Zimbabwean case than the Ivorian one.

In general, we could also consider the more systemic risks of nationalization, land redistribution, or corrupt land expropriations from officials.

Whether the plot is formally appropriated or not, an absence of effective land markets is all that is required for the first case to function in a mathematically convenient manner. Just picture the sad state of the farmer in Côte d'Ivoire walking from neighbour to neighbour offering to sell rights to the land. The odds of that transaction being respected are slim to none in a situation of political and military instability. They may have chosen to leave, but any previously expected revenues flowing from the land are utterly worthless. This can therefore be viewed as functionally identical to having lost the plot outright if we assume that the straw that breaks the proverbial camel's back will come at an unknown time.

2.5 Root cause of parameter shocks: War and instability

The onset of war and instability is the most important independent variable in this study. Its treatment as an independent variable can easily be questioned, but the ambiguous results with respect to the literature on the causes of conflict, as reported by Ross (2004a), suggests that it may be preferable to suppose that the cause of the conflict is essentially unknown. As per normal, there are a variety of approaches to operationalizing concepts and experiences of war and instability for the purpose of theoretical and empirical investigation.

On the qualitative side of things, we could look to the University of Peace's recent glossary for peace and conflict studies (Miller, 2005). It defines conflict as "either manifest, recognizable through actions or behaviours, or latent, in which case it remains dormant for some time, as incompatibilities are unarticulated or are built into systems of such institutional arrangements as governments, corporations, or even civil society." This rings well with the definition of war at oxforddictionaries.com: war is "a state of armed conflict between different countries or different groups within a country." The sociologist or institutionalist may be content with such definitions, but "numbers people" like most economists are likely to find such definitions problematic: how will mathematical or empirical work progress with such 'soft' definitions?

For those with more empirical inclinations, and who wish to investigate the more violent side of conflict, the Uppsala Conflict Data Program[9] produce a variety of conflict-related annual data. War for them is defined as "a contested incompatibility that concerns government and/or territory where the use of armed force between two parties, of which at least one is the government of a state, results in at least 25 battle-related deaths." This definition, along with the cumulative 1000-death threshold used by the Collier and Hoeffler team (Collier and Hoeffler, 1998, 2002, etc.) among others seem to be the most commonly used measures of conflict for empirical applications.[10] With varying datasets and variable definitions, divergent results are not exactly shocking. This literature, however, is primary interested in explaining the causes of war, whereas the current paper

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[9] See pcr.uu.se/research/UDCP, current as of March 24, 2011, for details on their program and data availability.

[10] Ross (2004b) points to Sambanis (2001) and Gleditsch et al. (2002) as good resources for discussion of the differences between these datasets and definitions.

presumes that the initial cause of conflict is essentially unknown.

What about instability? Researchers may also want to capture other aspects of conflict that are not reflected by the annual number of deaths. Other commonly used indices that reflect instability rather than violent conflict include the Rule of Law Index, hosted by the World Justice Project, and the Corruptions Perceptions index carried out by Transparency International. The first of these is more an evaluation of governance than stability, but the index includes some variables that could potentially be of interest when looking for intermediary factors to explain how property tenure may be affected differently across contexts. For example, access to civil justice and informal justice are both included in this index for each country (Agrast et al., 2010). Transparency International's corruptions perceptions index is similarly difficult to tie directly to instability, particularly when looking at the effects of conflict rather than its determinants. This index is mentioned because it was inferred earlier that corruption, poor market access and conflict may all have similar effects (increased input prices and deflated output prices due to additional market friction) on farmer's market decisions. The Economist's Political Instability Index would be another interesting source of information in this regard, but is not freely available to the public.

We can see here that empirical applications have options in terms of the data they can use to investigate the causes and effects of conflict. We could continue on, as do Stigns (2006) and Wicke and Bulte (2006), to pick apart the numerous ways that resources are defined. These articles show that the resources-to-conflict connection depends on how the variables are defined and thus that the definitions are interesting in their own right.

A short note on instability in our model: this is essentially the part of "war and instability" that negatively affects production conditions for all producers, including farming households. This occurs via higher input prices and lower output prices in most cases. Initially poor transportation infrastructure, combined with an excessive number of checkpoints (and by checkpoints here, we mean bribes), closed trade routes and generally poor transmission of market information and goods all result in less complete markets, i.e., more market friction. At the social or local level, instability may also be reflected in terms of tensions that affect the household's perception of tenure, personal endangerment and their production decisions. "War and instability", as conceived in this study, is said to negatively affect perceptions of property tenure. It could alternatively affect prices of market inputs and outputs. This is convenient from the theoretical or modeling perspective, but any empirical application would have a strong interest in motivating a definition or alternative terminology to reflect the nature of data that is actually used.

If it sounds like the description of the effects of "war and instability" bear strong resemblance to common problems seen in rural development, it is because it is true. In fact, the modeled effects on expropriation risk can probably be considered as a fairly realistic representation of the situation before the onset of conflict. While the problem posed in this study involves the effects of war and instability, via expropriation risk, it is safe to say that any factors which affect these two variables should carry similar effects regardless of whether war and/or instability were the root of the risk. For the sake of clarity and brevity, the present document will nevertheless assume frictionless markets in order to focus on the effects that instability can have on agricultural systems via expropriation risk.

3 The general model

This section will first introduce the components of the model, including the farmer's control variables z_{1t} and z_{2t} (input packages for soil productivity-enhancing and -degrading land uses, respectively), u_t (the share of land allocated to land uses that bring greater profits in the current period and are associated with poorer future soil quality), the law of movement for the state variable X_t (soil productivity), its implicit value, the utility function, the intertemporal objective and the process to linearize what is an essentially stochastic expropriation risk. The general form of the five-variable model, including X_t and the series of implicit values of on-farm soil stock and production intensity, is then introduced. Expropriation risk is then inserted into model before deriving and interpreting the general conditions for the model in a situation of expropriation risk. Developments relating to the steady-state dynamics and bioeconomic sustainability are undertaken in the following section.

3.1 Components of the model

The context we are investigating is one where a farmer makes production decisions in each period with full knowledge of the effects that these decisions have on expected future productivity. This implies that, given the system's conditions in the initial period, a growth function and economic parameters, the farmer's optimal strategy over an infinite time horizon can be determined. These production decisions involve production intensity and the choice of land uses on a plot of land. These land uses can be entirely different crops or two production techniques for the same crop. The potential movement toward some steady-state level of soil productivity thus depends on production decisions. We then model the farmer's decisions with the presence of expropriation risk.

In order to allow this change to affect the components of the farmer's economic decision, the probability of losing the farm must be allowed to enter the problem. This economic problem can then be modeled with control variables including crop choice and production intensity, along with a law of movement involving changes in soil productivity, the state variable. This will allow us to derive the general nature of the optimality requirements for the five-variable case (which has five variables when including the state variable and its implicit values over time) and to describe the impact that expropriation risk can have on the steady state for the farmer's optimal management strategy. Parameters will be introduced as necessary throughout this section.

- **The farmer's objective**

Here, we describe an inherently dynamic problem with a utility function, two production functions, a current profit function and an intertemporal objective with constraints. We then add the risk of expropriation to the problem. The problem is presented using the Lagrangian approach for discrete time in the following subsection.

The first control variable is u_t, the share of land allocated in period t to the soil-degrading land use 1. The share of land allocated to land use 2, the production technology associated with relatively better effects on future soil productivity, is $(1-u_t)$. The first of these is thus linked to production intensity z_{1t} and $(1-u_t)$ is linked to intensity z_{2t}. This control variable can be easily extended to the general case with j land uses if desired, as done by Wichelns and Burnes (2006). As discussed in the review, however, the actual number of choices that realistically belong to a farmer's decision set may actually be fairly limited in any given context. Together with a desire to keep the model as simple as possible, this justifies the decision to include just two production options for land uses in the model, such that $0 \leq u_t \leq 1$. As also discussed in the review section, the actual land uses considered by the farmer will differ depending on their climatic, ecological and cultural conditions.

The second and third control variables are the levels of effort that the farmer puts into producing each of these two goods. This is indicated by the levels of each variable in the z_{1t} and z_{2t} vectors. We will later consider that the elements of each of these inputs are used in fixed proportions, a simplifying assumption that reduces the dimensionality of the problem. These two vectors can include inputs such as labour, fertilizers, seed, etc., that are needed to produce the two agricultural goods.

The farmer has two production functions: one for each land use considered. A subsistence farmer consumes all of their income from production each period and, in the extreme case, is unable to purchase production inputs or sell production outputs. The only form of savings in this case is productive capital, with soil productivity acting as productive capital. Alternatively, a farmer with full access to markets may purchase inputs and sell outputs at their market value, whether or not there are financial markets. We assume that the household consumes the entire value of their production, either through direct consumption or trade in each period.[11] Let us also assume in this paper that markets are complete for both

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[11] Grepperud (1996) and Larson and Bromley (1990) are among other soil conservation studies making this assumption.

production inputs and production outputs. This allows the farmer to consume their income by trading on the market. This specification could be relaxed to allow changing market friction to influence production conditions. When three control variables are considered, the production functions can be described as:

$$f_{1t} = f_{1t}(X_t, z_{1t}) \quad \text{and} \quad f_{2t} = f_{2t}(X_t, z_{2t}) \tag{1}$$

where f_{1t} is the quantity of good 1 produced in period t, X_t represents soil quality in period t and z_{1t} refers to inputs for good 1. Production of good 1 is assumed to bring greater immediate profits per unit land but has negative effects on soil productivity. The quantity of good 2 produced in period t is f_{2t} and is a function of X_t and z_{2t}, inputs used to produce this second type of good. Production of good 2 contributes less to current profits but has relatively better (but still possibly negative) impacts on future soil productivity.

Since the agricultural system of interest is the farm,[12] the state variable X_t, soil productivity, appears in the production function. Soil productivity represents the state variable for the system, which has the following dynamic over time:

$$X_{t+1} - X_t = F(X_t) + y(X_t, u_t, z_{1t}, z_{2t}) \tag{2}$$

Together, the elements of (2), the dynamic state equation, describe the change in soil quality from one period t to period $t+1$ and will be described in what follows.

Specification of a growth function for soil productivity is important because it amounts to treating soil productivity as a renewable resource. This describes the trajectory of potential productivity if the farm remains unused. The dynamics

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[12] Many rural households produce on a number of small plots that are in different locations. The difference between plot-level and household-level optimization is thus an important one. Plot-level considerations are taken up in greater detail by Goldstein & Udry (2008).

of this bioeconomic system are, however, also a function of agriculture-related soil productivity losses. These changes in production conditions are given by $y(X_t, u_t, z_{1t}, z_{2t})$, which is generally a negative number. It is notionally comprised of partial effects of cultivation-related factors which affect the average rate of soil regeneration due to agriculture across the farm and other factors which lead to cultivation-related soil productivity loss on the farm.

In a general sense, the farmer derives utility in each period from production:

$$V_t = V(f_{1t}, f_{2t}) \tag{3}$$

where V_t is utility in period t, f_{1t} is the level of production of good 1 in period t and f_{2t} is the level of production of good 2 in period t. f_{1t} is associated with declining soil quality and higher current profits, while f_{2t} is associated with improved future soil quality but lower current profits. We can define profits as

$$\pi_t = u_t[p_{1t}f_1(X_t, z_{1t}) - c_1(z_{1t})] + (1 - u_t)[p_2 f_2(X_t, z_{2t}) - c_2(z_{2t})] \tag{4}$$

where c_1 and c_2 are the costs of producing goods 1 and 2 in period t.

Assuming complete markets, we can now incorporate econonomically relevant variables such as input levels and costs to consider that $V_t = V(\pi_t)$ where π_t is equal to the profits in period t, a strong but common simplifying assumption. The farmer discounts future utility by ρ per period. The absence or paucity of financial markets in rural areas of many developing countries implies that ρ does not necessarily depend on the market interest rate. ρ itself is considered constant over the time horizon.[13] Note that while input costs and product prices affect

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[13] This assumption is also a strong one. Becker and Mulligan (1997) and Weitzman (2001) offer convincing accounts of the difficulties associated with this assumption. Intertemporal consistency is nevertheless a common assumption, and one that is followed in this paper.

optimal production levels, the farmer's decisions relating to that level of production are essentially limited to land use and input decisions.

The most typical elements of the model are now in place, largely as seen in the literature. We have yet to develop an approach to including expropriation risk in the model that is both justifiable and convenient. Modelling these common features of war and instability are covered next.

Adding expropriation risk to the objective function is a very important methodological consideration in this paper. The process used to achieve this goal is sufficiently important that a more detailed illustration, found in appendix A, is warranted.

A positive probability ϕ that the farmer will lose the farm in a given period is introduced. The assumption that ϕ is equal in each period after the conflict starts is a strong one. Let us look to another time-oriented statistic reported by Collier (2009). He shows that the risk of the conflict itself is relatively steady over the entire first decade after a ceasefire has been agreed upon. This is different from the actual risk *during* conflict, but it is the closest thing that we found to justify *any* assumption with respect to the temporal distribution of risk. The assumption of a constant expropriation probability can thus be questioned, but it is not purely ad hoc and could even be superior to other more elaborate forms. It also allows us to arrive at a convenient mathematical property, as we will see shortly. The following is motivated by Levhari and Mirman (1977).

There is a probability ϕ in each period that the farmer will lose their farm and the cumulative probability of expropriation trends to 1 over an infinite time horizon. Recall that, in the most general sense, the goal of the farmer is to maximize their welfare over time, typically stated as maximizing intertemporal utility, as given by:

$$\max E_T \sum_{t=0}^{T} \rho^t V(C_t) \tag{5}$$

where C_t refers to wellbeing associated with consumption in period t. This consumption arises from profits generated by production of goods 1 and 2, a figure that is given in (4). The expected value operator is denoted as E_T and reflects the possibility that expropriation will put an end to profit flows from the farm. Most of the developments are left for the appendix, but let us look at one key step to explain the logic of what is happening here. The following series shows the possible set of consumption and production plans that the farmer can engage in, given the possibility of losing the plot after the first period, or after the second period, etc. To keep things manageable, we ignore the possibility that the farm is expropriated or otherwise lost between planting and the harvest.

$$\begin{aligned}
\max \sum_{t=0}^{T} \phi_t \sum_{i=0}^{t} \rho^i V(C_i) &= \phi_0 \rho^0 V(C_0) \\
&\quad + \phi_1[\rho^0 V(C_0) + \rho^1 V(C_1)] \\
&\quad + \phi_2[\rho^0 V(C_0) + \rho^1 V(C_1) + \rho^2 v(C_2)] \\
&\quad \vdots \\
&\quad + \phi_T[\rho^0 V(C_0) + \ldots + \rho^T V(C_T)] \\
&= \sum_{t=0}^{T} \rho^t P_t V(C_t)
\end{aligned}$$

where P_t is the probability of having lost the farm by the end of period t, i.e., $P_t = \sum_{t=0}^{t} \phi_t$.[14] This representation of the objective as a hypergeometric series essentially allows us to linearize the inherently stochastic expropriation risk, a nontrivial result. The mathematical legwork that follows is largely produced in

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[14] The risks of appropriation in $\phi_t, \phi_{t+1}, \phi_{t+2}$, etc., are all independent. This implies that the farmer evaluates the probability of expropriation as identical in each period.

the appendix, but the result that it leads to is key: the effect of expropriation risk in this context is to effectively transform the discount factor from ρ to $(1-\phi)\rho$. This is the form that will be used in the model once the necessary conditions for an optimal strategy are developed. As for those conditions, we will see them after brief discussions of discount factors and discount rates in a context of expropriation risk.

The discount rate is typically referred to as δ and it is often analogous to a periodic interest rate. It could also be considered as a point of reference for a risk-free investment, although we do not consider that the smallholder farmer has access to financial markets. The discount factor is typically referred to as ρ. The relationship between these two parameters is given as $\rho = \frac{1}{1+\delta}$, which can be manipulated to yield $\delta = \frac{1}{\rho} - 1$.

If we now consider the case where there is a risk of expropriation of ϕ in each period, the discount factor effectively goes from ρ to $\rho(1-\phi)$. The equivalent to δ in this case is given as

$$\delta(\rho, \phi) = \frac{1}{\rho(1-\phi)} - 1 \tag{6}$$

The relationship expressed in (6) will be useful later on when discussing the system dynamics.

A quick example will make the effects of ϕ on the discount rate and the discount factor obvious, if this was not already the case. Consider a discount factor of $\rho = 0.9$ and an expropriation risk of $\phi = 0.1$. Equation (6) tells us that we can calculate the change in the discount factor as going from 0.9 to 0.81 and a change in the discount rate as going from $1.\bar{1}$ to 1.234. This example suggests an interpretation of expropriation risk as a factor that decreases the discount factor and increases the discount rate, both of which imply that less importance will be attached to production conditions and profits in future periods.

3.2 Introducing expropriation risk to the five-variable model

This subsection presents the Lagrangian form of the farmer's economic objective and then derives and discusses the necessary conditions for an economically optimal strategy.

A student of resource economics or dynamic optimization should be able to easily recognize the general form of the following objective, presented in its Lagrangian form. The following presentation is largely as per Conrad (1999), for discrete time optimization of a renewable natural resource. The structure of the farmer's decision is largely as per Saliba (1985). A probability ϕ of the farm being expropriated or otherwise lost without compensation is introduced for each period. As detailed in the appendix, the end result is that the discount factor ρ is transformed into $\rho(1-\phi)$. The farmer's objective in this context is

$$\max_{u_t, z_{1t}, z_{2t}} \quad \sum_{t=0}^{\infty} \rho^t (1-\phi)^t \pi_t(u_t, z_{1t}, z_{2t}, X_t) \tag{7}$$

subject to:

$$\begin{aligned} X_{t+1} - X_t &= F(X_t) + y(X_t, u_t, z_{11}, z_{2t}), \qquad X_{t=0} \quad \text{given}, \\ 0 \leq\ &u \leq 1 \end{aligned} \tag{8}$$

and the terminal condtion

$$\lim_{t \to \infty} X_t \geq 0 \tag{9}$$

Together, (7), (8) and (9) describe the bioeconomic system faced by the farmer.

Forming this problem as a Lagrangian function, we have

$$\begin{aligned} L &= \sum_{t=0}^{\infty} \rho^t (1-\phi)^t \{\pi_t(\cdot) \\ &+ \rho(1-\phi)\lambda_{t+1}[-X_{t+1} + X_t + F(X_t) + y(X_t, u_t, z_{1t}, z_{2t})]\} \end{aligned} \tag{10}$$

where λ_{t+1} is the adjoint variable linked to the change in the state variable at time t. We also have the transversality conditions

$$\lim_{t\to\infty} X_t \geq 0, \ \lim_{t\to\infty} \rho^t(1-\phi)^t\lambda_t \geq 0 \text{ and } \lim_{t\to\infty} \rho^t(1-\phi)^t\lambda_t \cdot X_t = 0 \tag{11}$$

Together, the auxiliary function (10) and transversality conditions (11) can be used to describe the optimal trajectory of a system over time. Here, L is the function to be maximized over a theoretically infinite number of periods. The number of unknown variables to this problem, X_t, u_t, λ_t (and λ_{t+1}, etc.), z_{1t} and z_{2t}, is thus infinite. It is, however, possible for the system's dynamics and economic conditions to result in an endogenous and finite value for T. Current profits are represented by π_t, as defined in equation (4). ρ is the discount factor, $\rho\lambda_{t+1}$ is the marginal benefit of a marginal increase in soil productivity in the following period and the inner square brackets enclose the system's law of motion.

The farmer's objective as expressed in equation (10) is to maximize net benefits by choosing input intensities (which affect both π_t and $y_t(X_t, u_t, z_{1t}, z_{2t})$ and the shares of land associated with production of each of two goods, accounting for the effects that current production decisions have on the future level of the resource. The decisions of u_t, z_{1t} and z_{2t} are made in each period in consideration of the effect that it has on the following period.[15] This is captured by the insertion of

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[15] Technically speaking however, the farmer's optimal management policy can be traced out in advance, given the initial conditions and parameters. This property relates to the time consistent formulation of our problem.

$\rho\lambda_{t+1}$ in front of the implicit form of the state equation (the constraint). The additional ρ in front of the λ_{t+1} brings this marginal benefit from $(t+1)$ to period t.

The benefit from each period is brought into the current period by a discount factor ρ. It could be said that $(1-\phi)^t$ acts as an expected value operator for the benefits that may flow in each period t to reflect the risk of expropriation. Alternatively, we can consider $(1-\phi)$ as suggested above, as an additional discount factor, such that the overall discount factor is equivalent to $\rho(1-\phi)$.

Let us now draw attention to the marginal implicit benefit of on-farm soil quality in period $t+1$, given as $\rho(1-\phi)\lambda_{t+1}$ in equation (10). This future implicit value in period $t+1$ is brought into the current period by $\rho(1-\phi)$, the expropriation risk-adjusted discount factor derived in appendix A. When $\phi = 0$, we have the simplified case where the risk of expropriation is zero (perfect property tenure).

Before we move on to the first order conditions, let us take a few words to look at the transversality condition. This appears in (11) and indicates a non-negativity constraint for the state and adjoint variable. The weakness of the inequality with 0 reflects the possibility that the farm could be sustainably managed over a theoretically infinite time horizon in some conditions, while the soil resource could also be depleted under other conditions. The second set of the transversality condition states that the expected, present value of the shadow price of the soil quality must be non-negative. The third set of transversality conditions given in (11) relates to convergence for the long run steady state, and has two components: The first is $\rho^t(1-\phi)^t\lambda_t$, the discounted shadow price of a unit of soil productivity, and the second is X_t, the stock of soil productivity in period t. The transversality condition essentially requires that at least one of these terms trends to zero over a relatively long period of time. In some cases, this could reflect complete exhaustion of the resource: i.e., depletion of the soil stock to the point that it would be economically unprofitable to operate a farm. A farm that can be sustainably managed over an infinite time horizon, however, has a positive value of X_t for all t. Assuming that $\rho < 1$ and given that $(1-\phi) \leq 1$, it holds that $\lim_{t\to\infty} \rho^t(1-\phi)^t \cdot \lambda_t = 0$, as long as λ_t does not "explode," i.e. does not trend to infinity. Thus, when X_t is finite, its discounted expected shadow value must equal zero.

3.3 The necessary conditions for optimality

The necessary conditions for an optimal, *interior* solution to this problem are given by

$$\frac{\partial L}{\partial u_t} = \frac{\partial L}{\partial z_{1t}} = \frac{\partial L}{\partial z_{2t}} = \frac{\partial L}{\partial X_t} = \frac{\partial L}{\partial[\rho(1-\phi)\lambda_{t+1}]} \equiv 0.$$

After multiplying each of the conditions by $\frac{1}{\rho^t(1-\phi)^t}$ and rearranging the results, we have:

$$\begin{aligned}
\frac{\partial \pi_t(\cdot)}{\partial u_t} &= \rho(1-\phi)\lambda_{t+1}\left(\frac{\partial y_t}{\partial u_t}\right) && (12)\\
\frac{\partial \pi_t(\cdot)}{\partial z_{1t}} &= \rho(1-\phi)\lambda_{t+1}\left(\frac{\partial y_t}{\partial z_{1t}}\right) && (13)\\
\frac{\partial \pi_t(\cdot)}{\partial z_{2t}} &= \rho(1-\phi)\lambda_{t+1}\left(\frac{\partial y_t}{\partial z_{2t}}\right) && (14)\\
\lambda_t &= \frac{\partial \pi_t}{\partial X_t} + \rho(1-\phi)\lambda_{t+1}\left(1 + \frac{\partial F(X)}{\partial X_t} + \frac{\partial y_t}{\partial X_t}\right) && (15)\\
X_{t+1} &= X_t + F(X_t) + y(X_t, u_t, z_{1t}, z_{2t}) && (16)
\end{aligned}$$

The development of the conditions presented in equation (15) with respect to X_t is not entirely typical and thus deserves a few extra words. First, recall that equation (10) is a summation series and that the value of the farmer's objective in period

$(t-1)$ is also a function of the value of productivity (X_t) in period t. A λ_t thus appears in the farmer's objective in period $(t-1)$ and reappears in (15).

The first optimality condition, expressed in (12), requires the farmer to optimally select u_t, the share of land allocated to production of good 1. In so doing, $(1-u_t)$ is implicitly chosen as well. If already situated at the optimum u_t^* (not to be confused with u^*, the steady-state equilibrium), any increase in u_t would lead to lower future soil productivity, and the gains in terms of higher current profits would not justify an increase in u_t if the farmer were already planning to operate at u_t^*. Generally speaking, (12) states that the farmer should choose u_t such that the positive impact of a marginal change in u_t on profits is perfectly countered by its negative effects on the marginal benefits of future soil productivity stock.

Optimality with respect to z_{1t}, as presented in equation (15), requires z_{1t} to be chosen in consideration of two kinds of costs. As per the simplified case presented in Conrad (1999), both the standard current costs associated with producing more of good 1, $\frac{\partial c_1(z_{1t})}{\partial z_{1t}}$, and the costs in terms of lost future productivity, $\rho(1-\phi)\lambda_{t+1}\cdot\frac{\partial y}{\partial z_{1t}}$, must be accounted for. When situated at z_{1t}^*, if the farmer were to decrease their use of z_{1t}, future soil productivity levels would be "too high": the farmer could increase the value of the objective function by a small increase in the amount of the z_{1t} input factors, which would increase current profits by more than it would decrease the discounted expected value of future soil productivity. The opposite case holds for a farmer who is situated at z_{1t}^* and is considering an increase in application of z_{1t}. Since $\frac{\partial y_t}{\partial z_{1t}} \leq 0$ and $\frac{\partial \pi}{\partial z_{1t}} \geq 0$ by definition, we can state that optimally choosing the level of inputs for land use 1 requires marginal effects on current and future benefits to cancel each other out. The second condition, in equation (14), is essentially similar to the first condition, except that $\frac{\partial y_t}{\partial z_{2t}} \geq 0$.

Here, z_{2t} can contribute positively to both current profits and future productivity.

The fourth condition, in (15), may have the most interesting interpretation. Each of the two components of λ_t, the marginal contribution of a unit of soil productivity to the farmer's objective, will be considered in turn. The first component is $\frac{\partial \pi_t(\cdot)}{\partial X_t}$, the marginal contribution of soil productivity to current profits. If it were certain that the farm would be expropriated in the following period (i.e. if $\phi = 1$) then the right hand term would be eliminated, leaving a value of $\lambda_t = \frac{\partial \pi_t(\cdot)}{\partial X_t}$ because future soil productivity would be of no value to the farmer. (This could also be said to hold true for cultivation towards the end of a lease, for example, although the case of leases is not explored in detail in this paper).

For any value of $\phi < 1$, the farmer will also account for the contribution of future soil productivity to future profit flows. This is captured by $\rho(1-\phi)\lambda_{t+1}[1 + \frac{\partial F(X_t)}{\partial X_t} + \frac{\partial y_t}{\partial X_t}]$. So long as $\frac{\partial F(X_t)}{\partial X_t} + \frac{\partial y_t}{\partial X_t} > 0$, the positive contribution of X_t is not only in relation to current profits via improved current productivity, but is also in relation to future benefits via its effects on the future resource growth rate. The total marginal benefit, evaluated in period t, of a relatively small unit of soil productivity in period $(t+1)$ is $\rho(1-\phi)\lambda_{t+1}$. As seen above, $(1-\phi)$ discounts the value of λ_{t+1} due to the risk of expropriation between periods and ρ brings this expected value into the period that the management decision is made.

The fifth of the first order conditions, in (16), simply restates the law of motion that describes the dynamics of the state variable, soil productivity.

4 The reduced-form model

This section simplifies the problem into three essential variables in order to more easily represent the problem graphically. The steady-state conditions are then used to describe the long term dynamics of the problem. This allows us to develop a modified version of the fundamental equation of renewable resources, which is then used to derive the conditions under which an increase in risk would lead an "economically rational" producer to switch from a sustainable management strategy to a renewable resource depletion trajectory.

4.1 Three-variable case for graphical representation of the farmer's intertemporal problem

It has been instructive to investigate the nature of the farmer's problem when three control variables are considered: u_t, the share of land allocated to the land use associated with higher current profits and lower future soil productivity, z_{1t}, inputs for this type of land use and z_{2t}, inputs for land uses associated with lower current profits and relatively better future production conditions on the farm. This has allowed us to consider the costs and benefits associated with these input decisions in some detail. It would also be interesting to visualize the changes in the state and control variables over time. A basic question of dimensionality suggests that the problem should be simplified to graph the problem in order to more easily view the long term stakes of the concepts involved. We simplify the problem by considering that the farmer essentially selects u_t in each period and we normalize costs with respect to land use u_t. An additional cost c (not to be confused with $c_1(z_{1t})$ and $c_2(z_{1t})$ from above) is associated with the second land use. The sole control variable is now u_t.

All other notation will follow as per above. We have a state equation with soil quality as the state variable. It is given as

$$X_{t+1} - X_t = F(X_t) + y(u_t, X_t) \tag{17}$$

The farmer's objective can then be expressed for the three-variable case (X_t, u_t, λ_t) in the following Lagrangian form:

$$L = \sum_{t=0}^{\infty} \rho^t (1-\phi)^t \left[\pi(X_t, u_t) + \rho(1-\phi)\lambda_{t+1}\left(-X_{t+1} + X_t + F(X_t) + y(u_t, X_t)\right)\right] \tag{18}$$

The first order conditions from the Lagrangian, again multiplied by $\frac{1}{\rho^t(1-\phi)^t}$ and rearranged, give us

$$\begin{aligned} \frac{\partial \pi_t}{\partial u_t} &= -\rho(1-\phi)\lambda_{t+1} \cdot \frac{\partial y_t}{\partial u_t} && (19) \\ \lambda_t &= \rho(1-\phi)\lambda_{t+1}(1 + \frac{\partial F(X_t)}{\partial X_t} + \frac{\partial y_t}{\partial X_t}) && (20) \\ X_{t+1} &= X_t + F(X_t) + y(u_t, X_t) && (21) \end{aligned}$$

and current profits can be defined as

$$\pi_t(X_t, u_t) = up_1 f_1(X_t) + (1-u)[p_2 f_2(X_t) - c] \tag{22}$$

The farmer decides u_t, the share of land allocated to a land use that is more profitable in the current period and has more negative effects on future soil productivity (i.e. we require $\partial \pi_t / \partial u_t > 0$). The remaining $(1-u_t)$ share is allocated to a land use that has positive effects on future soil productivity, such that $0 \leq u_t \leq 1$. In particular, we assume that $\frac{\partial y_t}{\partial u_t} < 0$ to reflect deteriorated soil productivity in the next production period, while the second land use is said to be related to future soil conditions such that $\frac{\partial y_t}{\partial (1-u_t)} > 0$ to reflect improved soil productivity for the next production period. This assumption could be easily relaxed if necessary, but it will make interpretation of our version of the fundamental equation of natural resources more straightforward. As it stands, this leads us to the following condition that parameters and functional forms have to satisfy:

$$p_1 f_1(X_t) > p_2 f_2(X_t) - c, \qquad \forall X_t.$$

We also know that $\rho > 0$, $(1-\phi) > 0$ and $\lambda_{t+1} > 0$, so the equality in (19) requires that $\frac{\partial y_t}{\partial u_t} < 0$. When viewing y_t as the amount of soil stock "harvested" in relation to production, this may seem a little odd. Recall, however, that y_t is the change in soil quality directly resulting from production in period t, and this is typically expected to be negative. A negative value for $\frac{\partial y_t}{\partial u_t}$ means that an increase in the share of land allocated to land use 1 increases soil loss: the change in soil quality (y_t) is thus negative. As a whole, (19) states that the marginal instantaneous benefit of an increase in u_t is equal to the marginal loss relating poorer soil conditions in the following period.

The optimality condition with respect to X_t, soil quality, is given in (20) as $\lambda_t = \rho(1-\phi)\lambda_{t+1}(1 + \frac{\partial F(X_t)}{\partial X_t} + \frac{\partial y_t}{\partial X_t})$. This condition expresses that the current implicit value of a marginal unit of soil quality, i.e., the opportunity cost of using the resource, is equal to the marginal benefit of preseving soil quality for the following period, evaluated at the shadow price of soil quality, properly discounted. The second of these, the marginal benefit in the future incorporates two effects: $\frac{\partial F(X_t)}{\partial X_t}$ is the growth in soil stock as a function of current stock (its internal growth) and the change in soil quality resulting from production.

This paper has largely been concerned with interior solutions to the problem, i.e., solutions where u_t is neither 0 nor 1. Both of these extreme cases are possible, and would be reflected by cultivation via just one land use. We can therefore state the Kuhn-Tucker conditions for corner solutions as follows:

$$u_t \geq 0; \quad \frac{\partial L}{\partial u_t} \leq 0 \quad \text{and} \quad \frac{\partial L}{\partial u_t} \cdot u_t = 0 \tag{23}$$

$$u_t \leq 1; \quad \frac{\partial L}{\partial u_t} \geq 0 \quad \text{and} \quad \frac{\partial L}{\partial u_t} \cdot (1 - u_t) = 0 \tag{24}$$

These conditions imply that extreme cases are not covered by the model, whereas any intermediate case is.[16] The condition in (23) expresses that the optimal solution to u_t can only be equal to zero if the marginal benefit of an increase in u_t, given by $\frac{\partial L}{\partial u_t}$, is negative at the optimum. The third statement in (23) can be referred to as complementary slackness, which means that either $\frac{\partial L}{\partial u_t} = 0$ or $u_t = 0$. The second Kuhn-Tucker condition in (24) essentially mirrors the above for the case $u_t = 1$.

4.2 The stationary state

The bio-economic system is in a stationary state when the following holds:

$$X_{t+1} = X_t = X^*; \qquad \lambda_{t+1} = \lambda_t = \lambda^*; \qquad u_{t+1} = u_t = u* \tag{25}$$

These equations jointly comprise the steady-state equilibrium of (X^*, λ^*, u^*).

The first of these conditions, $X_{t+1} = X_t = X^*$, implies that

$$F(X^*) = -y(X^*, u^*) \tag{26}$$

This tells us that, at the steady state, the system's natural regeneration as a function of soil quality itself is equal to agriculture-related soil quality change

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[16] It should be noted that any additional benefits of multicropping through plant or ecosystem interactions are not captured by this precise formulation of the problem.

(degradation), y^*. This last variable is negatively related to u to reflect lower future soil quality associated with land use 1.

Using the steady-state condition that $\lambda_{t+1} = \lambda_t = \lambda^*$ together with (19) allows us to state that:

$$\lambda^* = \frac{\frac{\partial \pi}{\partial u}}{\rho(1-\phi)(-1)\frac{\partial y}{\partial u}} \tag{27}$$

i.e. the shadow value of marginal soil stock is equal to the ratio between the marginal contribution of u to current profits and its marginal contribution to agriculture-related future – and hence properly discounted – soil loss.

We can replace the expression for λ^* from (27) into (20), which can now be modified to state that

$$\frac{\partial \pi}{\partial X} - \frac{\frac{\partial \pi}{\partial u}}{\frac{\partial y}{\partial u}}\left[1 + \frac{dF(X)}{dX} + \frac{\partial y}{\partial X}\right] = -\frac{\frac{\partial \pi}{\partial u}}{\frac{\partial y}{\partial u}} \cdot \frac{1}{\rho(1-\phi)} \tag{28}$$

Making use of (6), which defines the relation between the risk-adjusted discount factor and risk-adjusted discount rate $\delta(\rho, \phi)$, we can further manipulate the former equation to yield

$$\delta(\rho, \phi) \;\; = \;\; \frac{\partial F(X)}{\partial X} + \frac{\partial y}{\partial X} - \frac{\frac{\partial \pi}{\partial X}}{\frac{\partial \pi}{\partial u}}\frac{\partial y}{\partial u} \tag{29}$$

which is our analogue to the *fundamental equation of renewable resources.*

The first two terms on the right hand side of (29) can be referred to jointly as the marginal net growth rate at the steady state. The block of partial derivatives on the far right is the marginal stock effect, itself linked to u because y is defined in terms of u here.

In the particular case where $y_t = y(u_t)$, such as $y_t = -u_t$ for example, then $\frac{\partial y}{\partial X} = 0$ and the fundamental renewable resources equation can be written as

$$\delta(\rho, \phi)\frac{\partial \pi}{\partial u} = \frac{\partial F(X)}{\partial X}\frac{\partial \pi}{\partial u} + \frac{\partial \pi}{\partial X}$$

The left hand side represents the (risk-adjusted) benefit of a marginal increase in u. These marginal benefits must, at the equilibrium, equal the opportunity cost of preserving soil quality. This total opportunity cost includes the effect that a change in u has on soil productivity growth ($\frac{\partial F(X)}{\partial X}$) and the effect that the change in soil stock X has on profits in each period.

In principle, any number of functional forms could be used to find the system's steady-state triple optimum at (X^*, u^* and λ^*) depending on the dynamics of the bioeconomic system, along with values for all the related ecological and economic parameters for the actual context. Specifying these functions and parameters in more detail, however, is an empirical question which lies outside the scope of the current research.

4.3 Sustainability of the bioeconomic system

We will now work towards describing the system at the steady state to find: a) the effects of expropriation risk on long term soil stocks and economic welfare and b) the conditions under which an additional expropriation risk will tilt the bioeconomic system from one with an optimal management policy of a sustainable harvest to one where resource depletion is optimal. The sustainability of the bioeconomic system depends on biological factors specified in the resource growth function, economic parameters specified in relation to discounting, prices and production functions, and economic variables such as the state variable and control variables.

For a simple representation of the biological parameters in a resource growth function, we can adopt the logistic growth function $F(X) = rX(1 - \frac{X}{K})$, where r is the intrinsic growth rate with respect to soil quality and K can be interpreted as the long-run level of soil quality in the absence of any agricultural activity.[17]

The marginal change in soil regeneration as a function of the current soil quality is given by $\frac{\partial F(X)}{\partial X} = r(1 - \frac{1}{2}\frac{X}{K})$. Setting $\frac{\partial F(X)}{\partial X} = 0$ and solving for X allows us to characterize the steady-state level of soil quality related to the maximum sustainable "yield." We find that $X_{MSY} = \frac{K}{2}$ and that the amount of soil quality that is used in relation to agricultural land uses equals $r\frac{K}{4}$.

Having specified a growth function and the soil loss function (in this case a logistic growth function and a linear soil degradation function), we only need to specify a production function in order to use equation (29) to solve for X^*. This result is the steady-state level of soil productivity satisfying the fundamental equation of natural resources. Intuitively, we can expect $X^*_{\delta(\rho,0)}$ to sit to the left of X_{MSY} and $X^*_{\delta(\rho,\phi)}$ to be even further to the left. This reflects the fact that, in the face of risk and uncertainty, the farmer values the future resource stock less, so there is less incentive to maintain a high level of soil stock over the long run.

Generally speaking, we expect to find a certain type of relationship between the optimal level of soil quality and the economic and ecological parameters ρ, ϕ, r and K. In particular, the optimal steady-state level of soil quality varies positively with ρ, the discount factor, varies negatively with ϕ, the periodic expropriation risk, varies positively with r, the resource's intrinsic growth and varies positively with K, the resource's carrying capacity. The second of these is perhaps the most interesting: this result specifically tells us that the steady-state value of X^* will be lower when there is a higher risk of expropriation.

Figure 1, below, represents soil growth as a logistic growth function. The

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[17] In the natural resources literature, when the logistic growth function is used to model biological or ecological stock dynamics, K refers to a given environment's carrying capacity for that resource.

maximum sustainable yield is $r\frac{K}{4}$, found by determining the maximum point on the growth function. An intermediate case is seen with normal economic discounting, where the optimal steady-state soil stock is denoted as $X_{\delta(\rho,0)}$ in the figure. Finally, we have the case of expropriation risk, with $X^*_{\delta(\rho,\phi)}$ as the economic optimum. This illustrates that we can expect expropriation risk to be associated with lower steady-state levels of soil productivity.

Here, $\frac{\partial F(X)}{\partial X}$ must be equal to the resource's total economic return of $\delta(\rho,\phi) - \frac{\frac{\partial \pi}{\partial X}}{\frac{\partial \pi}{\partial u}}$.

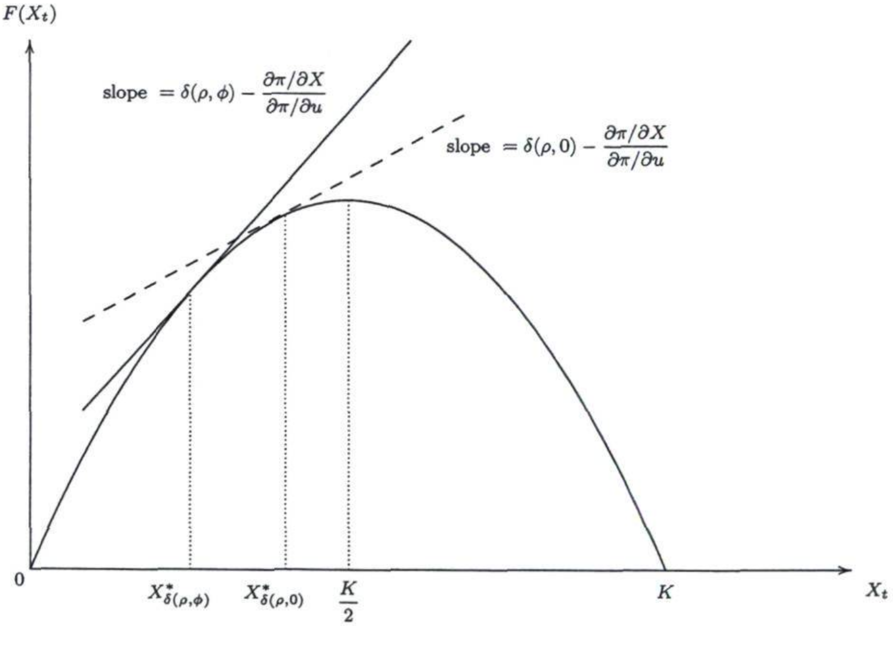

Figure 1: Graphical representation of the steady state

Generally speaking the total economic returns of the resource must be equal – in the steady state – to the effective discount rate, accounting for expropriation risk. When total returns to the resource are greater than the risk-adjusted discount rate, the producer will have an interest in managing the resource such that the system will converge to a strictly positive level of soil productivity stock in the long run (Dockner et al., 2000). In our case, this situation is more likely to hold if there is no expropration risk because $\delta(\rho, \phi) < \delta(\phi, 0)$

If there was an initial perceived level of risk, ϕ_A, followed by a deterioration in expected conditions such that the farmer evaluated the new ongoing risk of expropriation as ϕ_B, with $0 \leq \phi_A < \phi_B \leq 1$, then we would be interested in knowing whether the farmer would rationally wish to change from a sustainable management policy to a resource-depletion strategy as a result of the underlying changes that led to greater perceived expropriation risk.

The shift in the optimal management policy from sustainable management to resource depletion can be described in relation to the fundamental renewable resources equation expressed in (29) for the three-variable model used for the steady-state analysis, by the following inequality

$$\frac{1}{\rho(1-\phi_A)} \leq \frac{\partial F(X)}{\partial X} + \frac{\partial y}{\partial X} + \frac{\frac{\partial \pi}{\partial X}}{\frac{\partial \pi}{\partial y}} + 1 < \frac{1}{\rho(1-\phi_B)} \tag{30}$$

where $\frac{\partial y}{\partial X}$ will be zero if y is only a function of u and is not a function of X.

A producer who maximizes the present value of profit flows, as the one in our model does, will add to the stock of the resource so long as the marginal cost of increasing the level of the resource is less than the marginal benefit in terms of possible current uses. Conversely, there is a cost to holding a stock of soil: it's potential value if it is used for production in the current period. When the total effective discount factor is affected by expropriation risk, this reduces both future costs and benefits but does not affect benefits in the current period.

Assuming that $\frac{\partial y}{\partial X} = 0$, the inequality on the left side of (30), equivalent to $\frac{1}{\rho(1-\phi_A)} - 1 \leq \frac{\partial F(X)}{\partial X}$, reflects a situation where the farmer's optimal resource management strategy is to maintain the resource for sustainable production in the long run. Here, the farmer's risk-adjusted discount rate is lower than the resource's total internal economic return. In this situation, the farmer has an interest in depleting the resource until the resource's internal economic return is equal to the farmer's risk-adjusted discount rate, which explains the weakness of the inequality. The situation described by (30), with an increase in the expropriation risk from ϕ_A to ϕ_B, is one where the farmer will choose to move to a resource depletion strategy because the resource's internal economic return is not sufficient to warrant maintaining the resource in the long run under the new level of risk.

Thus, as per the logic of Dockner et al. (2000) mentioned above, a change in ϕ results in a different optimal trajectory on the farm. The lower value of future soil stock will induce the farmer to increase current production, with the result of lower future soil quality stock and lower future production. In the case where the change in expropriation risk is sufficiently high for (30) to hold, a change in the political and social environment that is associated with higher perceived expropriation risk would cause an economically rational farmer to shift from sustainable management to resource depletion over time. If soil growth dynamics and economic conditions verify the inequality in (30) on any significant share of farms across an economy, then the amount of viable farmland may be expected to decline over time as a result of elevated expropriation risk, for any given set of technological options. Fortunately, the opposite also holds true for an improvement in relation to expropriation risk, regardless of whether the improvement is linked to political and/or civil instability.

5 Concluding remarks

The context of this paper is one where agricultural development is called upon to be an engine for growth in developing countries. However, it appears that some regions are more prone to conflict than others. This promotes an interest in knowing how war, conflict or political instability may affect management of these agricultural resources. This paper motivated the use of a dynamic model of on-farm soil management decisions, first to capture the intertemporal nature of the farmer's management problem and second to show how expropriation risk can affect these management decisions. As shown in the review, the best approach to measuring soil quality is far from straighforward to identify, whether for particular or general cases.

The review section presented some stylized facts about the main variables of interest in the soil conservation model. We saw that specifying the determinants of the state variable, soil productivity, is a less than straightforward task because of ecological, economic and cultural contexts. We saw that the relationship between land uses and soil productivity change is difficult to operationalize for a general case because a given land use may have different directions of effects on different indicators of soil quality. This is often overcome in soil conservation models by specifying some abstract variable of soil quality. Additional intertemporal effects in soil quality are also discussed in relation to the input packages that may be used for different land uses. Further research possibilities with respect to market completeness are also explored.

The issue of property tenure was also introduced in order to open the door to the complexity of how tenure manifests itself on the ground and what that means for expropriation risk may manifest itself on the ground. This study investigated the effects that civil and political instability have on a farmer's resource management decisions through an effect on the risk of the farm being expropriated or otherwise lost. As such, the review closed with a brief review of the challenges involved in defining and operationalizing variables linked to war or civil and political instability.

We initially set up the problem as a five-variable one then derive and interpret the conditions for an optimal trajectory of all the variables. In order to keep the problem analytically tractable when developing, discussing and graphing the long run steady-state situation, a number of simplifications and assumptions were made to model this situation. The results show that a higher risk of expropriation will result in a lower long run level of soil stocks and will cause farmers in some circumstances to shift from from sustainable resource management to resource depletion. This result is particularly relevant for countries which struggle with instability and conflict and are also largely agricultural economies.

A number of possible avenues remain for further exploration. It would be interesting to model market incompleteness, with instability decreasing output prices at the gate and increasing prices for inputs, perhaps even to the extent that some markets are eliminated altogether due to scale economies in supply chains. Another interesting area for exploration would be to specify a range of growth functions across an economy to produce a "land desertion function": some farming households would move following the increase in risk and others would shift to resource depletion over time and desert the farm at some later point in time. Incorporating labour markets to reflect opportunity costs linked to the land-desertion/migration decision would further extend that approach. Ex post empirical work on agricultural productivity and production before and after conflict could also be carried out for countries with available household-level, regional or nationally aggregated data, so long as the level of aggregation of data on agricultural household choices and conflict were comparable.

For the time being however, the conclusion is clear: expropriation risk has negative effects at the individual and societal level, particularly in the long run. Improving farmer's certainty of tenure, whether through formal or informal means, as well facilitating the supply of a broader range of options to respond to the ecological, economic and social situations they operate in, can be expected to improve outcomes at the individual and societal level.

6 References

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7 Appendix: Bringing expropriation risk into the objective function

Let us begin by declaring T as the expected date of expropriation, with a probability of expropriation in period t defined as ϕ_t. The risks of expropriation $\phi_t, \phi_{t+1}, \phi_{t+2}$, etc., are all independent and necessarily satisfy

$$\sum_{t=0}^{T} \phi_t = 1 \tag{31}$$

which states that the cumulative probability that the plot is expected to be expropriated by the time that it is expropriated is equal 1. As discussed above, we can then insert the claim of an even temporal distribution of risk such that $\phi_t = \overline{\phi}\ \forall t$.

Recall that the farmer's goal is to maximize their welfare over time, typically stated as maximizing their intertemporal utility. This initially appears as an objective in relation to the following series of Von Neumann-Morgenstern utility functions:

$$\max E_T \sum_{t=0}^{T} \rho^t V(C_t) \tag{32}$$

which, by the expected utility theorem, can be restated as equal to

$$\sum_{t=0}^{T} \phi_t \sum_{i=0}^{t} \rho^i V(C_i) \tag{33}$$

The right-hand summation series is the flow of utilities if the plot is expropriated in each respective period t. The following illustration presents the problem as one where the economic actor considers a consumption plan that could be taken for each possible outcome, weighted for the possibility of expropriation in each year.

We then have a problem where

$$P_t = \sum_{i=t}^{T} \phi_i \tag{34}$$

which effectively states that the probability of the plot having been expropriated by period t is equivalent to the sum of the probabilities that it would be expropriated in each of the preceding periods.

Let us now return to the farmer's objective function before expropriation risk is accounted for. This can be stated as:

$$\sum_{i=0}^{t} \rho^t V(C_i) = \rho^0 V(C_0) + \rho^1 V(C_1) + \rho^2 V(C_2) + \ldots + \rho^t V(C_t) \tag{35}$$

However, the risk of expropriation still needs to be considered. The following series represents the role of expropriation risk in the objective function.

$$\begin{aligned} \max \sum_{t=0}^{T} \phi_t \sum_{i=0}^{t} \rho^i V(C_i) &= \phi_0 \rho^0 V(C_0) \\ &\quad + \phi_1 [\rho^0 V(C_0) + \rho^1 V(C_1)] \\ &\quad + \phi_2 [\rho^0 V(C_0) + \rho^1 V(C_1) + \rho^2 V(C_2)] \\ &\quad \vdots \\ &\quad + \phi_T [\rho^0 V(C_0) + \ldots + \rho^T V(C_T)] \\ &= \sum_{t=0}^{T} \rho^t P_t V(C_t) \end{aligned}$$

The reader may wish to view this series as the set of possible consumption and production plans that the farmer can engage in, given the possibility of losing the plot after the first period, or after the second period, etc. To keep things manageable, we ignore the possibility that the plot is expropriated or otherwise lost between planting and harvest.

Under this interpretation, the first figure on the right hand side of the question is the probability-weighted value of utility flows associated with the case that the plot is lost after the first period, the next line is the probabiltiy-weighted value of utility flows for the case that the plot is lost after the second period, and so on. The farmer then maximizes his expected utility given all possible outcomes. This hypergeometric series thus describes the farmer's problem. However, such notation could easily become unwieldy for further developments, so it will be convenient to develop this objective such that the risk of expropriation can be more easily included in the dynamics of the problem. A couple points are worth mentioning before those final steps though. First, that the hypergeometric series implies that $\phi_t = \phi(1-\phi)^t$. In this case, the objective becomes:

$$\sum_{t=0}^{\infty} T(1-\phi)^t \rho^t V(C_t) = \sum_{t=0}^{\infty} [(1-\phi)\rho] \tag{36}$$

It should be recalled here that the farmer is said to maximize intertemporal utility based on the probability of a variety of outcomes, but that in each case, the farmer maximizes benefits flowing from market income and autoconsumed production, where we can consider that $u(C_t) = u(wi_t)$ because the benefits from production are assumed to be completely consumed in each period. This specification emphasizes the assumptions with respect to risk aversion, which are not formally modeled but are rather informally discussed along with the results. For now, we will suppose that $u(w) = w$ where $w = p_1 f_1 + p_2 f_2 + \cdots + p_n f_n$ for n types of goods produced. This can be justified in a weak sense by supposing that the economic agent of interest in this study, the farmer, is not remotely near any point of income saturation, and that well-being almost certainly increases with income across the relevant range of likely income values.

Let us start by returning to the construction that the probability of losing the plot by the end of period t is equal to

$$\begin{aligned} P_T &= \sum_{i=t}^{T} \phi_i, \qquad \text{where} \quad \phi_i = \phi(1-\phi)^i \\ &= \sum_{i=t}^{T} \phi(1-\phi)^i && (37) \\ &= \phi[(1-\phi)^t + (1-\phi)^{t+1} + \cdots + (1-\phi)^{T-t}] && (38) \end{aligned}$$

which allows us to state that

$$\begin{aligned} P_t &= \phi(1-\phi)^t \cdot Y && (39) \\ &= \phi(1-\phi)^t \cdot \frac{1-(1-\phi)^{T-t+1}}{1-(1-\phi)} && (40) \\ &= (1-\phi)^t(1-(1-\phi)^{T-t+1} && (41) \\ &= (1-\phi)^t - (1-\phi)^{T+1} && (42) \end{aligned}$$

This last term is problematic, and some more mathematical games will be required to complete the proof relating to the form that expropriation risk can take in the model. The first line of this proof may be contentious, since it effectively states the assumption that the plot will eventually be expropriated. For the infinite timeframe, this quite reasonably stands as true. This presumably does not hold true for shorter time frames.

$$\begin{aligned} \sum_{t=0}^{T} \phi_t &= 1 && (43) \\ &= \sum_{t=0}^{T} \phi(1-\phi)^t = 1 && (44) \\ &= \phi \sum_{t=0}^{T} (1-\phi)^t = 1 && (45) \end{aligned}$$

Here, we are presented with a series that takes the following form:

$$\begin{aligned}\Sigma &= (1-\phi)^0 + (1-\phi)^1 + (1-\phi)^2 + \cdots + (1-\phi)^T \qquad (46)\\ &= 1 + (1-\phi)^+(1-\phi)^2 + \cdots + (1-\phi)^T \qquad (47)\\ &= 1 + (1-\phi)[1 + (1-\phi)^1 + \cdots + (1-\phi)^{T-1} + (1-\phi)^T - (1-\phi)^T](48)\end{aligned}$$

Where defining $T = (n-1)$ (and thus also that $n = (T+1)$), then it follows that

$$\begin{aligned}\Sigma &= \frac{1-a^n}{1-a} \qquad (49)\\ &= \frac{1-(1-\phi)^{T+1}}{\phi} \qquad (50)\end{aligned}$$

when $(1-\phi)$ is defined as a. Reinserting $a = 1-\phi$ into the previous equality and solving for $(1-\phi)^{T+1}$ we find that this last term is equal to zero. This allows us to eliminate $(1-\phi)_{T+1}$ from the conclusion that $P_t = (1-\phi)^t - (1-\phi)T + 1$, with the result that

$$P_t = (1-\phi)^t \qquad (51)$$

Thus, as per Levhari and Mirman (1977) we can state that the effect of a probability ϕ that the plot will be expropriated or otherwise lost is mathematically equivalent to multiplying the discount rate by $(1-\phi)$. Since $\rho = \frac{1}{1+\delta}$, this gives us the final result of $\rho = \frac{1}{1+(1-\phi)\delta}$. That was a lot of legwork, but it will make it possible to follow a more conventional approach for the typical optimality conditions that will appear soon enough.

This exact procedure could also be proposed for the risk of life and limb itself too by allowing q_t to take on larger values after war and instability commence . For example, if the risk of death was *abc*, we could then state the overall discounting rate as $(\delta + abc)$. It should probably be fairly clear that this does not have any additional effects of interest on the dynamics of the problem, since the direction of effect will be the same in all cases. Thus, again, the desire to maintain a relatively straightforward model suggests that it should be sufficient to stick with the $(1-\phi)$ factor, with the knowledge that the effects of this additional discount factor will be underestimated because the physical risk to the farmer and his household's life and limb is not explicitly accounted for.