BRUNO LEMELIN

MINE PROJECT EVALUATION: A REAL OPTIONS APPROACH WITH LEAST-SQUARES MONTE CARLO SIMULATIONS

Thèse présentée
à la Faculté des études supérieures de l'Université Laval
dans le cadre du programme de doctorat en génie des mines
pour l'obtention du grade de Philosophiæ Doctor (Ph.D.)

DÉPARTEMENT DE GÉNIE DES MINES, DE LA MÉTALLURGIE ET DES MATÉRIAUX
FACULTÉ DES SCIENCES ET DE GÉNIE
UNIVERSITÉ LAVAL
QUÉBEC

2009

Résumé

Le but de cette thèse porte sur la compréhension et la résolution des problèmes décisionnels rencontrés durant la mise en place de la stratégie d'extraction minière tout en tenant compte des limitations des outils existants sur le marché pour l'optimisation de la valeur économique du site minier. Dans cette thèse, l'auteur amène une nouvelle approche pour évaluer correctement des zones minières avec des incertitudes relatives aux prix des métaux pour des cas réels et concrets. Ceci est destiné aux gestionnaires qui souhaitent utiliser un outil supplémentaire qui est à la fois pratique et compréhensif afin qu'ils établissent des plans d'affaires complets.

Abstract

The goal of this thesis is to understand and solve the decisional difficulties encountered when establishing mining extractive strategy while acknowledging the limitations of the existing tools on the market used for maximizing the economical value of the mining site. In this thesis, the author describes a new approach to properly evaluate mining zones with multiple price uncertainties using real situations. This is designed to give management a complementary tool that is both practical and understandable in order for them to establish sound business cases.

Preface

This thesis contains six chapters and includes one conference paper and two published papers. Although technical articles were inserted in the document, a thorough study on Real Options has been performed to develop the premise of using the new method with better comprehension. A full literature review in Chapter 2 presents the advantages and limitations of the existing evaluation models and sets the base to develop a new mining model based on recent mathematical advances in simulation. Chapter 3 explains the stochastic process of the underlying asset in the mine model, which is the metal price associated with its volatility.

Chapter 4 describes the original mine model that the author developed using a recent mathematical advance in Monte Carlo simulations that allows for consideration of as many uncertainties as the evaluator wants in the mine model. This corrects one of the main drawbacks of using a Real Options Approach with conventional closed form equations, stochastic differential equations or discrete time lattices. The paper inserted in this chapter was published in the *International Journal of Mining, Reclamation and Environment* in March 2006. The candidate, who is the primary author, developed the mine model. Sabry Abdel Sabour, post-doctoral student at Université Laval, programmed the simulator on MathLab, and Richard Poulin supervised the work. It was the author's original idea to break down the mine evaluation by zone, which is non-existent in the literature for an underground poly-metallic mine (more than one uncertainty).

Chapter 5 integrates the new Real Options Approach in a standard mining evaluation process for a marginal zone only. The author includes one technical paper to illustrate the feasibility of using the developed Real Options method in an industrial environment. Failure to transpose the option model in the industry has been one of the most important drawbacks of the Real Options Approach and one that many observers have compiled in the past. The candidate is the primary author of the published article; the Institute of Materials, Minerals and Mining approved the publication of the article in January 2008. The author has designed the mine option model to appraise a marginal ore body by subzones, which is original for underground operations. He has also explained the

Markovian property and illustrated the time connectivity of the stochastic process of the underlying asset. Sabry Abdel Sabour simulated the results, and Richard Poulin supervised the research.

Lastly, Chapter 6 relates on the new Real Options Approach in a standard mining evaluation process for a stand alone project. A conference paper is also included in the chapter. The article was presented at the 2007 APCOM conference in Chile. The candidate is the primary author of the paper that studied an entire underground mine with the Real Options Approach with the Least-Squares Monte Carlo simulations. The author has developed a technique to appraise separate ore zones with a new mine model. Sabry Abdel Sabour ran the simulations, and Richard Poulin directed the research.

Acknowledgements

I would first like to thank my research director, Dr. Richard Poulin, Ph.D., from Université Laval, for his collaboration, support and patience toward my doctoral studies while I was working for Xstrata. Dr. Poulin has been instrumental in helping me to get this far in my academic career.

I also want to thank Dr. Sabry Abdel Sabour, Ph.D., who not only gave me support but provided much technical savvy. Without him, I am sure I would not have completed this thesis at this date. Dr. Sabour was also very patient with my busy schedule and never gave up on me. Very rigorous, he helped me to generate sound and structured technical papers.

Sometimes timing and luck play into one's career. When I joined the Raglan staff in 2003, I could not have imagined my good fortune in meeting a great man who would change the destiny of one individual. Mr. Denis Lachance, former General Manager at Xstrata Raglan Mine, gave me his full support and guided me to make sure that I was doing the right thing at the right time. Thank you for giving me the time and the resources to complete my studies.

I have to thank Mr. Lee Weitzel, former Project Superintendent at Raglan. The numerous conversations we had about applying real options in a mining environment helped me to keep my bearings straight with the mining reality. If this thesis succeeds in showing the importance of applying the Real Options Approach on-site, Mr. Weitzel deserves part of the merit.

I would like to thank my family and my friends for supporting me during the long undertaking of my doctoral studies (2001–2008). I am amazed by and appreciative of how you could endure for so long my excitement, complaints, joys, regrets and expectations about this thesis.

Finally, I would like to thank my wife, Sophie, who always gave me the time to study, even when I was returning from a long trip abroad. Sophie always understood the importance I placed on this project and she gave me her total support to bring it to completion.

It is a simple task to make things complex, but a complex task to make them simple.

– Meyer's Law

Contents

List of tables

List of figures

List of appendices

List of abbreviations

NPV: Net present value

IRR: Internal rate of return

ROA: Real Options Approach

LSM: Least-Squares Monte Carlo simulations

Introduction

Mine management is characterized by the need to improve the existing mining processes in order to reduce costs and maximize the net present value (NPV) of the project. The ultimate goal is to create value and wealth for shareholders, who want to be rewarded for the risks taken by investing their money in the venture.

This thesis explains the necessity for the mining industry to study one of the most critical processes in mine management: the economic appraisal of a mining project. The actual evaluation process has not changed much in the last two decades. Interesting innovations now may modify the way the mine managers evaluate and prioritize projects. In this thesis, the author will show that, at the present time, mining firms possess all the tools needed to enhance the economical evaluation of any extractive resource project.

In Chapter 2, developments in the domain of economics-based approaches are reviewed. Conventional merit measures, like net present value (NPV) and the internal rate of return (IRR), show that these methods are limited for many aspects. The classic methods propose an economic appraisal under certainty, whereas most of the world projects exist under many uncertainties. For NPV calculations, the selection of a single discount weakly bears the risk associated with the nature of the project. The development of some alternative solutions has attempted to alleviate these shortcomings, but has been received with mixed opinions by the industry and by academia. The perpetual evolution of computation techniques has now revived the utilization of these new tools. Often seen as a black box, the Real Options Approach (ROA) is much easier to use for mine evaluation than ever before. In the second half of Chapter 2, the specific goals and aim of the research are described. Some shortcomings with the current mine planning process using actual merit measures, such as NPV criteria or IRR, are detailed. To evaluate mining projects using ROA, one has to understand the risk dimension the evaluator wants to include in his economic appraisal. Several uncertainties rule the financial performance of any organization. The literature review also includes a summary of the existing uncertainties affecting a mining project. The thesis assumptions are therefore disclosed showing the area of interest and limitations of the present document.

Chapter 3 of this thesis concentrates on assuming multiple uncertainties related to the metal price market. Two models are defined to indicate metal price behaviours. An example is also presented to illustrate the major econometric components for modelling the price of copper. With the help of a recent simulation advance, this puts the premise of using as many uncertainties in the mine model as the evaluator wants into his study. This research is unique, since academia has been studying mine models with no more than two uncertainties at one time.

The next three chapters are constructed from three papers written by the author to illustrate the use of ROA in a real-life environment on a continuous basis. In Chapter 4, the theory of ROA using the Least-Squares Monte Carlo (LSM) simulations is explained and described in the first half of the chapter. A case study shows the deployment of ROA using LSM. The results based on the first technical paper proved the importance of the new information that was gathered with the new approach.

In Chapter 5 and 6, the integration of ROA using LSM as a common tool for the mine planner is explained through two papers. The first article demonstrates the usefulness of ROA for marginal ore bodies. A case study depicts the economic evaluation for one ore lens that is considered non-economical under the conventional project appraisal procedure using NPV. Chapter 6 describes the integration of ROA in the mine planning process with a stand alone project. An existing mining project is taken as an example for valuing profitability using a classical merit measure (NPV) and ROA. This thesis is different and original when compared with what had been done in the past. The author attempts to use a new ROA with LSM simulation process, using up to seven uncertainties at the same time to evaluate real mining projects.

2 Literature review

In this section, existing conventional and modern evaluation methods are described. Advantages and shortfalls for each merit measure are portrayed and bring to light the necessity of developing a research project to enhance mining evaluation techniques in order to make sound financial decisions. Note that further detailed literature reviews are deferred to later chapters to assure continuity for the reader.

2.1 Classical methods of mine evaluation

Mining investments are carried out in an uncertain economic environment. In addition to technical and geological risks, the uncertainty of future mineral prices increases the complexity of evaluating mining investments, especially when the mine produces two or more minerals. In such a case, the investor should deal with the uncertainties of many mineral prices simultaneously when conducting a feasibility study.

Among the classical tools for evaluating mining investments are the net present value (NPV), payback period and internal rate of return (IRR) methods. The NPV method has been recognized for decades as one of the most efficient tools for analyzing mining investments. Gentry and O'Neil (1984) developed in great detail the application of the NPV as well as the IRR and the payback period for evaluating mining projects. Bhappu and Guzman (1995) conducted a survey on NPV criteria that is used extensively in the mining industry. More than 95% of the companies surveyed are using either the NPV or the internal rate of return. Payback period is another merit measure that is part of the decision-making process for more than 50% of the companies.

To analyze a project using the NPV method, all future cash inflows and outflows of the project are discounted back to time 0 at a predetermined discount rate and summed up to the capital investment to determine the NPV of the project. The decision rule is to go ahead with the investment if the NPV is positive. NPV measurement has two major advantages: it is universal and it is easy to understand the concept.

The internal rate of return consists of calculating the growth of wealth. Hence it gives an idea as to how fast an investor accumulates or loses money. Stermole (1971) extensively discussed the application of the rate of return in the mining industry. The payback period is also a helpful tool to describe how long it will take to recoup the initial investment. Hajdasinski (1989), who has long studied merit measures in the mining sector, describes the payback period as a conventional tool for miners.

These three merit measures have been used universally in the industry, but it is generally accepted that they might be suboptimal when it comes to dictating the true economic nature of some mining projects. For instance, the NPV methodology presents some drawbacks that bias cash flow calculations. It is believed that using the actual method of discounting cash flows through time with a single risk rate may not reflect the genuine value of the project in the end. Salahor (1998) discussed the failure of the NPV approach to appraise risk in the cash flow valuation model. A paper written by Samis et al. (2004) described many shortcomings with the NPV method, one of which being that the NPV method indiscriminately adjusts each cash flow with an all-use discount factor, regardless of the nature and the period of time when the monetary account is posted. If the evaluator is satisfied with the discount rate, he will continue to discount expenditures and revenues with one single rate, no matter the nature of the cash account. The corollary of this statement is that a risky project will require a high discount rate, leading to the depreciation of even more operating costs and resulting in a <u>relative</u> increase in the NPV of the project. In fact, the NPV method works best when future cash flows are certain or when significant financial attributes show few risks. The technique is better used for asset replacement strategy, cost-optimization projects and projects with a short lifespan.

Another problem with using a single discount factor lies in establishing it. In fact, it is up to the evaluator to assess the risk for the project. There are many ways to evaluate the discount rate. If the evaluator uses the capital asset pricing model (CAPM), the project will be risk adjusted against market expectations for investing in the mining sector, independently of the inherent risks of the project per se. If the evaluator uses risk premiums to calculate the discount rate, the estimate is still a gross representation of a biased perception. To illustrate, different evaluators might give distinct discount rates for the same project.

Others criticized the NPV approach, like Salahor (1998), who provided clear hindsight with respect to management choosing wrong projects using NPV criteria. He stated that an excessively high discount rate used in the NPV approach is believed to offer a conservative financial estimate, but, in fact, a higher discount rate may promote riskier projects for conservative companies. The non-linearity of some cash outflows is also another problem to take into account in the NPV model. Non-linear systems, such as taxation regimes, may be problematic. The project value may be underestimated if taxation levels change according to profitability. Bradley (1998) used a scenario tree for assessing royalty systems for petroleum projects. The modern asset pricing methodology (Real Options Approach) compared to the conventional discounted cash flows method showed greater results because taxation regime changes over time have a definite impact on profitability. The non-linearity of the project structure may be accounted for with modern techniques like ROA.

In addition to the problem of choosing an appropriate discount rate, the most critical shortcoming of the NPV method is that it cannot account for management flexibility to adjust the operating policy according to the status of future market conditions. Brennan and Schwartz (1985) considered the NPV approach as being a static method that ignores the possibility of future management decisions. The lack of flexibility may contribute to significantly affect the value of the project. Bhappu and Guzman (1995) supported the same idea when they explained in 1995 that NPV techniques could not capture the freedom of management to capitalize on metal price volatility or other external attributes that modify the value of a project. The NPV method ignores management operating flexibility. Many options are available to the mine manager throughout the existence of the mine. He may decide to temporarily close the mine during a low-price period or to reopen it if prices recover. Some scenarios may also suggest permanently abandoning the mine if prices continue to decline. Another option not captured in NPV calculations is the expansion of production at the mine, a scenario that arises during bold metal price periods.

An NPV value also excludes the flexibility for the project manager to defer its initial investment. It basically gives a go or no-go for the decision maker without the opportunity to appraise the value in delaying the project start-up date. An NPV model can be described as passive because it dictates two choices: the investment is either initiated now or it will never take place. There is no concept in the model to delay the investment decision to trigger future financial sustainability. As a result, some information remains lost in translation. The NPV is static in the sense that it does not calculate the what-if scenarios. For instance, the metal price may go higher or lower than forecasted. Management has the ability to temporarily wait for new information to come in before totally abandoning the project.

The NPV fails to capture the sum of these scenarios and ignores flexibility. Trigeorgis and Mason (1987) wrote in detail that "the conventional static NPV may undervalue projects by suppressing the 'option premium' component." The metal price market is volatile and fluctuates with time, and, as such, a conventional NPV will fail to measure the project flexibility and its value. The manager cannot know in advance the best course of action when market conditions change. In fact, the NPV method fails to explicitly calculate the flexibility value of a project that contains options to permanently or temporarily shut down the mine, to expand it or to keep the same pace. The lost opportunities caused by a lack of information from using classical discounting methodology have been dragging the mining industry into a concert of bad investments, oversupply and low returns.

The NPV method is best used when cash flows are certain and volatility is absent. When a large-scale mining project comprises many uncertainties and its time frame is relatively long, a new risk-management approach would better fit the investor. A sound model would adjust the risk on a quantitative and timely manner, as the mining production may not carry the same risk through time. On rare occasions, mining risks will remain constant through time. It is possible to discretely determine the risk for each annual cash flow component, although it might be long and suboptimal for multi-scenario analysis. When the evaluator has studied the uncertainty that has the largest impact on the project, he may model it with a continuous utility function that would facilitate the use of advanced evaluation techniques. The next section outlines the advances in the field of evaluating projects using modelled uncertainties.

2.2 Real Options Approach

Since the orthodox theory has failed to satisfy the industry, economists have been working to establish a new theory based on appraising real assets as options on non-financial assets. The new approach, called the Real Options Approach (ROA), has many advantages over the classical NPV. ROA has the ability to study and to include uncertainties in the evaluation model. The algorithm respectfully risk adjusts the cash account that is affected by the uncertainty, whereas NPV indiscriminately risk adjusts all accounts. It has been used in many industries, including the oil and gas sector, utilities services, manufacturing and in research and development.

Sometimes, the investment is not irreversible. While NPV cannot grab the essence of project flexibility when market conditions change, ROA provides optimal sequential decisions under uncertainty. The value of the information given by using ROA is vital for maintaining a satisfactory level of return on investment or for reducing operating losses when economic conditions shift.

What is an option?

> In finance, an option is a contract whereby one party (the holder or buyer) has the right but not the obligation to exercise a feature of the contract (the option) on or before a future date (the exercise date or expiry). The other party (the writer or seller) has the obligation to honour the specified feature of the contract. Since the option gives the buyer a right and the seller an obligation, the buyer has received something of value. The amount the buyer pays the seller for the option is called the option premium.
>
> Most often the term "option" refers to a type of derivative that gives the holder of the option the right but not the obligation to purchase (a "call option") or sell (a "put option") a specified amount of a security within a specified time span.

Source: Wikipedia.com

There are a series of different options. The European and American options constitute the majority of the options financially traded. An European option may be exercised at the expiry date only, whereas the American option may be exercised at any time before the expiry date. ROA differs from financial options because it is based on the right of the owner to change the input production (real assets), depending on the market conditions. In the natural resource sector, the underlying asset is the commodity price, which has an established derivative market.

The options approach in mining is important because there has always been a historical difference between the sum of all discounted cash flows from a mining company and its market price. Davis (1995) described that the difference can be explained by the managerial flexibility that the NPV methodology does not take into account. In fact, the miner has several management options while operating a plant. He may exercise his options or not, depending on certain economic conditions, such as the commodity price. This concept means that the production attributes are typically considered as being financial assets, hence financial options. It is assumed that a competent manager would change his extractive strategy when market conditions shift. He may decide to temporarily suspend operations or shut the mine down permanently. Many other options may be available to the mine manager, such as changing the block value grade, expanding production or stockpiling. Like a financial option (put or call), he might want to exercise his right to do so, therefore enabling the project to show a different spectrum of cash flows that will imply a new NPV. The commodity price volatility may not entirely impact NPV results if operating flexibility exists. It is possible for the mine manager to reduce losses during a harsh period of low price and to capture upsides when the metal price has recovered. It is important to note that the length of the project would normally favour managerial flexibility, but the NPV method sequesters distant future cash flows by heavily compound discounting them to insignificance.

2.3 Methods of valuing real options

The history of ROA started in 1973 with the seminal work of Black and Scholes (1973). The analytic approach drastically changed how financial markets previously operated. Then, the finite difference, a numerical approach, came in 1978 to enhance the practicality of ROA. Another numerical method, called the binomial methodology, was used in 1979 to measure option value. Monte Carlo simulations were developed by Boyle (1977) and were utilized to obtain a decent approximation of the results achieved from the other methods, but with more simplicity.

The Black and Scholes Model

The first ROA model was based on the works of Fisher Black and Myron Scholes, who made a breakthrough in evaluating financial assets at no risk. The model assumes that there is a constant risk-free rate and that the stock price evolves according to the geometric Brownian motion. It is then possible to capture, in an arbitrage-free environment, the value of the call option. The concept of justly pricing options was the first stepping stone to evaluating real assets using an analytic approach. The mathematical equations of the Black and Scholes formula (call option value) can be shown as:

$$C(S.T) = SN(d_1) - Ke^{-rT}N(d_2)$$

where

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)\cdot T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

S = Trading stock price

K = Exercise price

T = Exercise date period

r = Risk-free rate (which remains constant)

σ = Stock volatility
N = Standard normal cumulative distribution function

Although the model is quite simple to use in tracking financial option values, it becomes complex when this analytical tool is used to appraise real asset projects. It tends to become fairly complicated when the mining producer option changes operating policy, especially if the business model contains more than one uncertainty. The model is also based on European options, whereas the challenge for the manager is to continually track the value of his options and for the financial analyst to keep track of the value of his portfolio of American options.

Finite differences method

Many years later, there were other interesting models that generated greater interest in assessing mining properties with ROA. A numerical method based on finite differences was established by Brennan and Schwartz (1978). This method features approximating the partial differential equations that define the evolution of the option value by some approximate differential equations. In reality, the partial derivatives are replaced by finite differences equations. The first application of the approach for valuing mining investments was carried out by Brennan and Schwartz (1985), wherein a model for evaluating a copper mine with the management flexibility to stop production temporarily and close the mine prematurely was presented. The principles of using finite differences start with the concept that if the boundary and the terminal conditions are known, an option value matrix can be found for a definite subspace with the explicit finite differences formula:

$$V_{i,j} = \frac{a_i \cdot V_{i+1,j+1} + b_i \cdot V_{i,j+1} + c_i \cdot V_{i-1,j+1}}{1 + r\Delta t}$$

where

$$a_i = \frac{1}{2} \cdot \left(\sigma^2 \cdot i + r\right) \cdot i\Delta t$$

$$c_i = \frac{1}{2} \cdot \left(\sigma^2 \cdot i - r\right) \cdot i\Delta t$$

and

$$b_i = 1 - \sigma^2 \cdot i^2 \Delta t = 1 - (a_i + c_i)$$

V = Put value at node (i,j)

The boundary conditions are:

Lower boundary: F(0,T) = Exercise price

Upper boundary: F(S,T) = 0 (i.e., the share price is above the exercise price, hence no value)

The terminal condition is obviously the difference from the exercise price and the share price where option value equates:

Max [EX-S,0]

Under this environment, it is then possible to draw a matrix table that shows the initial conditions. A numerical example is shown in Table 1:

So=36$ E.P. = 40$ Sigma=0.2 T= 1 year rate r 0.06

		Time to maturity					
i	S	1	0.8	0.6	0.4	0.2	0
9	54	0	0	0	0	0	0
8	48						0
7	42						0
6	36						4
5	30						10
4	24						16
3	18						22
2	12						28
1	6						34
0	0	40	40	40	40	40	40

Table 1. Table using finite differences with initial conditions

where

S_0 = Spot price at time 0

E.P = Exercise price

T = Time to maturity

Sigma = σ = Volatility

r = Risk-free rate

The goal of the example is to define the put option value at time 1 if the spot price is $36. In the grey cells, the boundary conditions are filled. Once this table is set up, it is possible to revert back from the terminal condition to the present time. At the end, the option value can be found at the specific share price. As shown in bold in Table 2, the put option value can be established at $3.85.

So=36$ E.P. = 40$ Sigma=0.2 T= 1 year rate r 0.06

		Time to maturity					
i	S	1	0.8	0.6	0.4	0.2	0
9	54	0	0	0	0	0	0
8	48	0.457745	0.368608	0.256891	0.125107	0	0
7	42	1.491831	1.362296	1.190717	0.95635	0.608696	0
6	36	**3.85209**	3.855378	3.854204	3.855645	3.881423	4
5	30	8.120832	8.408077	8.732597	9.102704	9.525692	10
4	24	13.72103	14.15173	14.59785	15.05701	15.52569	16
3	18	19.68506	20.13646	20.59388	21.05701	21.52569	22
2	12	25.68401	26.13623	26.59388	27.05701	27.52569	28
1	6	31.67496	32.13074	32.5911	33.05607	33.52569	34
0	0	40	40	40	40	40	40

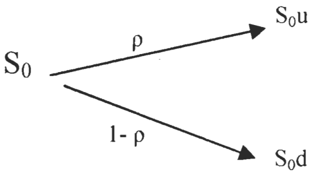
Table 2. Final table with put option value with finite differences

Frimpong and Whiting (1997) developed a mining evaluation model based on Brennan and Schwartz seminal works. Cortazar and Casassus (1999) also built a natural-resource evaluation model that helps to calculate operating flexibilities of the mine. Although the finite differences method is easy to implement, the definition of the option value for mining operations is limited by the number of uncertainties that can be simultaneously studied. Barraquand and Martineau (1995) proved mathematically that the method cannot be used if more than three variables are assumed. In a poly-metallic mine, using the finite differences will not capture the essence of the option value because many variables, including multiple metal prices, foreign exchange and fuel price, play a significant role in the profitability of the company.

Binomial method

The binomial methodology, first developed by Cox, Ross and Rubinstein (1979), consists of building a probabilistic tree wherein the option is calculated from the expected discrete state prices through time. It assumes that the option value evolves in a risk-neutral environment process.

The detailed mathematical formulas consist of:



$$\left(\rho \cdot S_0 u + (1-\rho) \cdot S_0 d\right) \cdot e^{-r\Delta t} = S_0$$

where

S_0 = Share price at time 0

S_0u = Share price at time Δt upside state

S_{0t} = Share price at time Δt downside state

ρ = Probability (risk-neutral environment)

hence

$$\rho = \frac{e^{r\Delta t} - d}{u - d} \text{ and } u = e^{\sigma\sqrt{\Delta t}}$$

When the share price at time Δt is calculated, the value of the option for the same time Δt is computed if the exercise price is known. With the "ρ" probability attribute, it is possible to revert back to time 0 to calculate the option value with the same discrete time model:

$$\left(\rho \cdot V_0 u + (1-\rho) \cdot V_0 d\right) \cdot e^{-r\Delta t} = V_0$$

where

V_0 = Option value at time 0

V_0u = Option value at time Δt upside state

V_{0t} = Option value at time Δt downside state

Hull and White (1993) depicted binomial and trinomial lattices to value American options. Kamrad (1995) developed a lattice model for mining applications that values projects that contain operating options. Although it becomes easier for calculating real assets projects, this approach can be difficult to handle if there are two or more sources of uncertainties. Grant et al. (1997) demonstrated that solving complex problems using a lattice method is mostly impracticable: computational requirements are too large for the methodology, forcing many authors to distort the algorithm so it reduces memory space needs. Kamrad compared the easiness of using simulations that handle multiple variable inputs to a decision tree methodology that is limited to one or two variables. There is a need in the industry to evaluate mining projects with multiple uncertainties, such as poly-metallic mining properties. Using a binomial approach would restrict the evaluator to determine a utility metal price function only on the most important commodity. In this case, the assessment of the Raglan properties would be based on the main metal price uncertainties, whereas other important uncertainties also have importance in the cash inflow composition. Ideally, the flexibility of a mining project composed of vital uncertainties, such as the foreign exchange rate and mining ore reserve, would be evaluated.

Simulation method

A third numerical technique, based on Monte Carlo simulation, slowly revolutionized the calculations of options values. First introduced by Boyle (1977), the model was simple, but was limited by the computational speed at the time. Tilley (1993) succeeded in valuing American put options with simulation and encountered the same problem. Broadie and Glasserman (1997) also developed a simulation technique to appraise a call option on a dividend-paying stock. Barraquand and Martineau (1995) studied simulated American put options value with restricted parameters. Limiting options values were found because heavy computational data was reducing speed and applications of the Monte Carlo simulation.

This approach has improved on two fronts. First, computational tools have improved since the last decade. The speed for solving complex problems has increased accordingly. Second, a promising numerical method developed by Longstaff and Schwartz (2001) enhances the simulation technique by introducing a least-squares method approach. It has the advantage of dealing with multiple sources of uncertainties, to reduce computational needs and is simple to use.

The methodology consists in using a simple regression for establishing the option price at time t. The technique is recursive, starting at the maturity of the option and going back to time 0, where it is now possible to get the option price. The simulated path is defined according a risk-neutral process. The numerical method by simulation is bound to make the quest for new information simpler. It tends to be as accurate and as precise as the other methods, but with the advantage of being user-friendly. With sound econometric parameters, the manager is expected to factor them into the model to properly assess its project portfolio.

In Table 3, a numerical example based on Longstaff and Schwartz is illustrated. To calculate the put option value if the current share price is \$36 and the exercise price is \$40. The risk free rate is 6%. As shown on the table below, eight simulated paths have been keyed in from the risk-neutral process. The simulated prices can be obtained from different distributions as well, like the geometric Brownian motion or a mean reverting process.

So=36\$ E.P. = 40\$ rate r 0.06

	Time			
Path	0	1	2	3
1	36	39.24	38.88	48.24
2	36	41.76	45.36	55.44
3	36	43.92	38.52	37.08
4	36	33.48	34.92	33.12
5	36	39.96	56.16	54.72
6	36	27.36	27.72	32.4
7	36	33.12	30.24	36.36
8	36	31.68	43.92	48.24

Table 3. Simulated price paths

The first step is to find the option value at the maturity date as shown in Table 4.

	Time			
Path	0	1	2	3
1	36			0
2	36			0
3	36			2.92
4	36			6.88
5	36			0
6	36			7.6
7	36			3.64
8	36			0

Table 4. Put value at maturity date

The second step is to determine if the evaluator wants to exercise or to keep the option at time 2. If the put value at time 3 is discounted at time 2, it will be possible to build a relationship between the put value and the share price both at time 2. A second order polynomial regression is simply used to determine this equation. For use of the regression, another matrix (Table 5) is built with discounted put value from time 3 to time 2 and the simulated value at time 2.

Time 2		
Path	Y	X
1	0	38.88
2	0	0
3	2.75	38.52
4	6.48	34.92
5	0	0
6	7.16	27.72
7	3.42	30.24
8	0	0

Table 5. Regression matrix at time 2

Note that x values taken into consideration in the regression are those that are in the money at time 2. By doing a second order polynomial regression, an equation is found that describes the put value at time 2 according to the simulated share price also at time 2 (Figure 1).

$$Y = -42.694 + 3.2749X - 0.055X^2$$

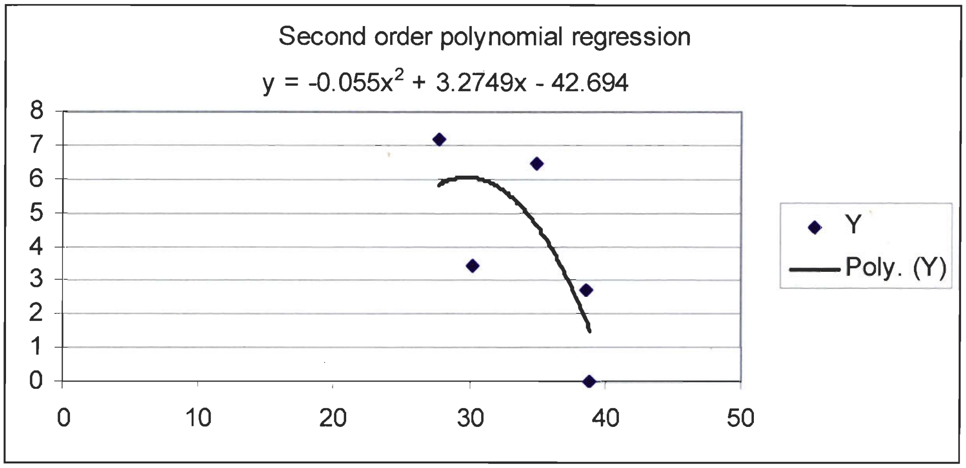


Figure 1. Second order polynomial regression at time 2

By substituting X with the share price, the evaluator obtains the put value if he desires to retain it until time 3. Therefore, a decision matrix (Table 6) can be established:

Decision matrix		
Path	Exercise	Continuation
1	1.12	1.49312
2	-	-
3	1.48	1.846676
4	**5.08**	4.598156
5	-	-
6	**12.28**	5.824316
7	**9.76**	6.043808
8	-	-

Table 6. Decision matrix at time 2

There are three paths at time 2 that the evaluator will exercise the put option, which are path numbers 4, 6 and 7. Then, a new profit matrix (Table 7) is done for time 2 and 3. Note the recursive nature of the methodology going from time 3 to time 2. At time 2, if the option is exercised, there is logically no value at time 3.

Profit matrix			
Path	Time 1	Time 2	Time 3
1	-	0	0
2	-	0	0
3	-	0	2.92
4	-	5.08	0
5	-	0	0
6	-	12.28	0
7	-	9.76	0
8	-	0	0

Table 7. Profit matrix for time 2 and 3

The following step is to repeat the algorithm until the profit matrix is completed. The final profit matrix can be calculated (Table 8).

	Profit matrix		
Path	1	2	3
1	0	0	0
2	0	0	0
3	0	0	2.92
4	6.52	0	0
5	0	0	0
6	12.64	0	0
7	6.88	0	0
8	8.32	0	0

Table 8. Profit matrix for time 1, 2 and 3

Hence, if all probable cash flows are discounted at time 0, the American put option value will be 4.35. It corresponds to the expected gain and takes into account all simulated paths. The evaluator needs to take the profit matrix found at Table 7 to calculate the European-style put value, which is 2.20.

The advantages of using the Least-Squares Monte Carlo (LSM) simulations approach reside in the fact that not only is computation easier with this method, but it is also possible to appraise the project value using multiple uncertainties that better reflect the mining reality. The LSM algorithm allows the framework to rule the Markovian property, which is vital to enhance the time connectivity of the simulations. The method is detailed further in Chapters 4 to 6 and in appendix C with specific numerical examples.

2.4 Nature of the problem and aim of the research

The research is to apply the LSM approach to an existing mining site. The evaluation model is original and is composed of multiple uncertainties. The development of an evaluation tool using ROA will allow mine management to carefully study the mine production entitlement and the long-term planning while trying to maximize the overall value of the site. The investigated applications will be tested at the Raglan Operations, where the actual mine planning techniques show a potential to deploy ROA. A short description of the Raglan property follows, as well as the current mine evaluation techniques in the next sections. By laying out the mine evaluation process map at Raglan, some actual technique deficiencies at Raglan state the true nature of the problem, which is apparent in other mining sites. The solution, which will overcome these deficiencies and which will be proven precise, applicable and accurate, can have a major impact on mining companies' financial results. More positive outcomes could be noted if the deficiencies in the mine evaluation process are resolved.

The ROA concept can include the study of many uncertainties that have the impact to modify production policies. We live in a world of uncertainties and it is multi-dimensional. In the mining arena, the principal factors that may alter financial results can be categorized into several entities: finance, operations and sustainable development (environmental, social, health and safety).

Financial uncertainties can be described as being economic parameters that fluctuate through time and impact cash flows. Most of these factors are external to the mining company. The metal prices follow a path that is far from being certain. The forward price curve is always in movement and many models try to mimic its inherent behaviour. The next chapter of the thesis explains metal price modelling. Some researchers studied the uncertainties related to metal prices and its effect on mining projects. Authors like Brennan and Schwartz (1985), Mardonnes (1993), Moyen & Al (1996), Frimpong (1997), Cortazar and Cassasus (1998), Samis and Poulin (1998), Cortazar and Reyes (2001), Samis & Al (2001), Samis (2001) and Sabour (2001) wrote diverse articles demonstrating the flexibility of management to react against market fluctuations. This is not an exhaustive list and more references can be found at the end of the document.

Other financial parameters impact cash flows generation by its respective variability. The forex exchange studied by Trigeorgis (1995), Jorion (1996), Davveta & Al (2002) and Samis, Poulin, Blais (2003) shows variability that can be modelled and inserted into a Real Options program. At some extents, the oil price which was studied by authors like Dias (1999,2001) and Lazo & Al (2003) should also be considered an uncertainty like the metal prices. The mining industry is an energy intensive sector that is directly affected by oil price variations. In this thesis, oil price is considered constant, signifying a limitation in this research and could be corrected in further studies. One original element of this future research would be to consider the oil price as an operating cost, a variable parameter modelled and incorporated into the Real Option model. There are other financial parameters that could influence cash flows generation such as royalties, taxation and inflation. Again, these aspects were considered constant in the thesis.

Operational uncertainties also exist in the valuation landscape. The most important one is geology. Ore tonnage and ore grade parameters are not constant at any stage of the exploration process and even during the mining of the zone; it rarely comes up exactly as predicted. Many works were written on the subject like Cortazar & al. (2001), Dimitrakopoulos & al. (2002), Godoy (2003), Ramazan and Dimitrakopoulos (2004), Dimitrakopoulos and Sabour (2007), Sabour & al. (2008). The metal output may greatly be affected by geological variations. This thesis is limited to deal with uncertainty related to metal prices only. Therefore, the ore grade and tonnage are assumed to be constant. Further research works including geological uncertainty will prove to be a good contribution to science especially when dealing with underground operations.

Sustainable development issues can also bring its share of uncertainty. Regulations are assumed constant although from time to time regulations changes, particularly in countries that pose great political risks. In North America, with a population who is more sensible with the environment, some legislation may change in the future such as the asset retirement obligations rules (ARO). The impact of any law changes with regards to sustainable development would impact directly each mining center. Although this aspect would be note of interest, this thesis has not taken into account specific changes in any existing environmental, social, health and safety regulations in Canada.

2.5 Describing the case at Raglan

Xstrata Nickel is one of the world's leading producers of nickel with investments in several countries. The Nickel Division includes mines such as Raglan, Montcalm, Falcondo and the Sudbury Operations. Xstrata Nickel also has processing facilities in Canada, Norway and the Dominican Republic. The Raglan property is situated at the northern tip of the Ungava Peninsula in the Nunavik region of the province of Quebec, Canada, north of the 55th parallel (Figure 2). The 100%-Xstrata-owned Raglan property encompasses 1,226 claims covering 48,149 hectares and nine 20-year mining leases covering 947 hectares. Raglan is a remote, fly-in working operation with the head office situated in Rouyn-Noranda, located 1,540 km southwest of Raglan. Workers access the project via a Xstrata-owned Boeing 737 jet (twice per week) from Montreal and Rouyn-Noranda, and an Air Inuit-operated Twin Otter from northern Nunavik communities (twice per week). Raglan's remote location has resulted in the development of a multicultural, trilingual (French, Inuktitut and English) workforce of roughly 400 people who live and work on rotation at the site. Approximately 15% of the workforce is from the local Inuit villages, such as Salluit (130 km to the northwest) and Kangiqsujuaq (60 km to the southeast). The operation runs 24 hours per day, 365 days per year.

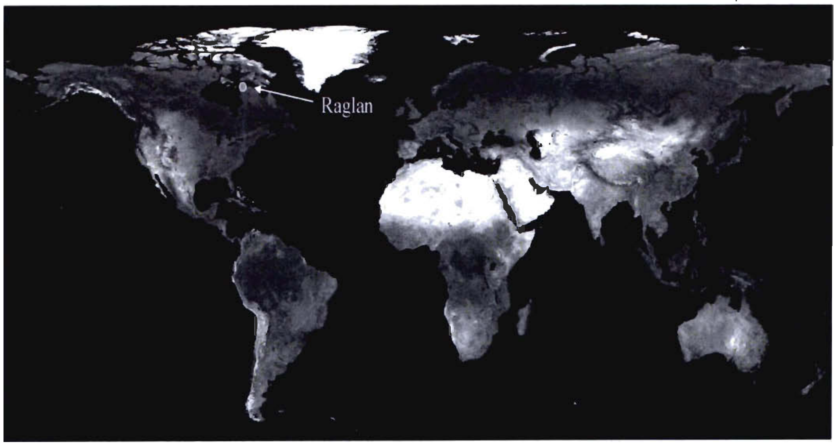


Figure 2. Raglan site in Nunavik, Northern Quebec, Canada

Xstrata's Raglan mine is a poly-metallic mining property that produces eight payable metals, the more important of which are nickel and copper. Raglan has been mining out an annual average of 906,500 tonnes of sulphide ore grading 2.7% nickel. There are three underground mines currently in production, and the Katinniq mine has produced 55% of the mill feed during the last eight years. Some small open pits contribute to the annual production, but their part has deeply decreased since 2002. The Raglan Operations represent an immense challenge for the mine evaluator, as the site is composed of more than 100 distinct mineralized bodies that range from 15,000 to 2,100,000 tonnes.

The komatiitic Ni-Cu-PGE sulphide deposits that define the Raglan Nickel Belt occur within the early Proterozoic volcano-sedimentary Cape Smith Belt (CSB) that extends 375 km across the Ungava Peninsula of Northern Quebec (Figure 3). The CSB represents the northernmost segment of the Circum-Superior Belt, a rifted continental margin and arc-continental collision zone juxtaposed against the Archean Superior Province (St-Onge et al. 1991). The CSB comprises two tectonostratigraphic domains (Figure 3). The Northern Domain consists of the Watts Group ophiolite, active volcanism within a continental subduction zone (Parent Group) and associated metasediments (Spartan Group). The Southern Domain hosts the Ni-Cu mineralization and comprises a basal volcano-sedimentary assemblage reflecting initial continental rifting (Povungnituk Group) and komatiitic basalt to mafic lavas representing the opening of an oceanic basin (Chukotat Group). The tectonostratigraphic domain containing the Chukotat and Povungnituk group rocks is preserved as remnant thrust imbricates that rest unconformably on Superior Province basement gneisses, having been translated southward as a result of convergence (St-Onge and Lucas 1991). Post thrusting regional folding has yielded a broad synclinorium of the Cape Smith stratigraphy.

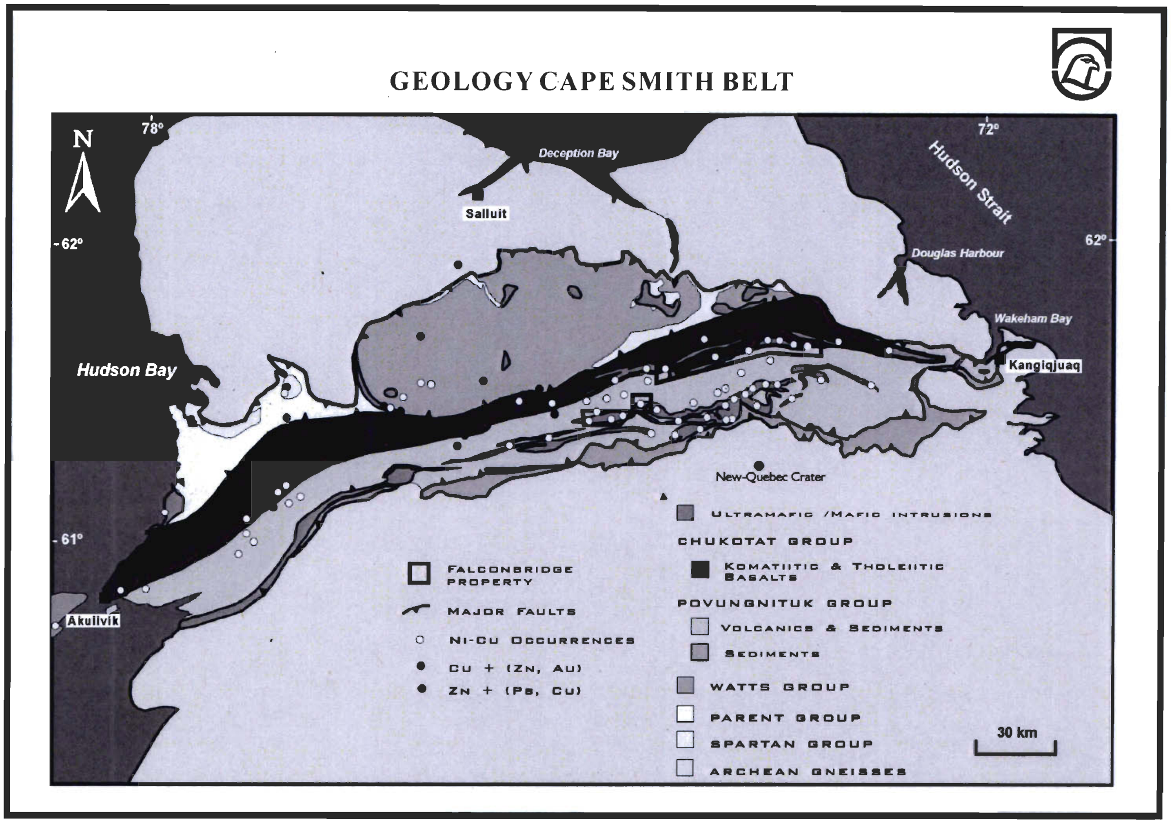


Figure 3. Cape Smith Belt geology (St-Onge and Lucas 1991)

The magmatic sulphide mineralization occurs within the Raglan Formation, a horizon of voluminous mafic-ultramafic volcanic and high-level intrusive rocks at the base of the Chukotat Group (Lesher 1998). The Raglan Formation is interpreted to have erupted in a relatively deep-water environment. Recent geologic and geophysical data support the suggestion of a large, east-west trending meandering lava channel system filling a broad embayment feature. The Raglan Formation dips north (roughly 45 degrees), extends to depths greater than 1 km and in general is between 200 and 400 m thick. The ultramafic rocks that make up the Raglan Formation in general change from peridotite to olivine pyroxenite to pyroxenite as one ascends through the volcanic pile from its base. The economic mineralization is associated with basal sub-channels in secondary embayment features and, as such, identification of the flow channel environment and determining its orientation is key, as multiple small, irregular, high-grade sulphide accumulations along the trend are the norm. The Ni-Cu-PGE mineral reserves and resources are focused in 10 major localities spanning the strike length of the property (Figure 4). The need for Raglan to periodically develop and sequence-in new mines is key to the mine plan, and it is this high number of potential zones of production that makes Raglan a challenge to optimize in the long-term planning.

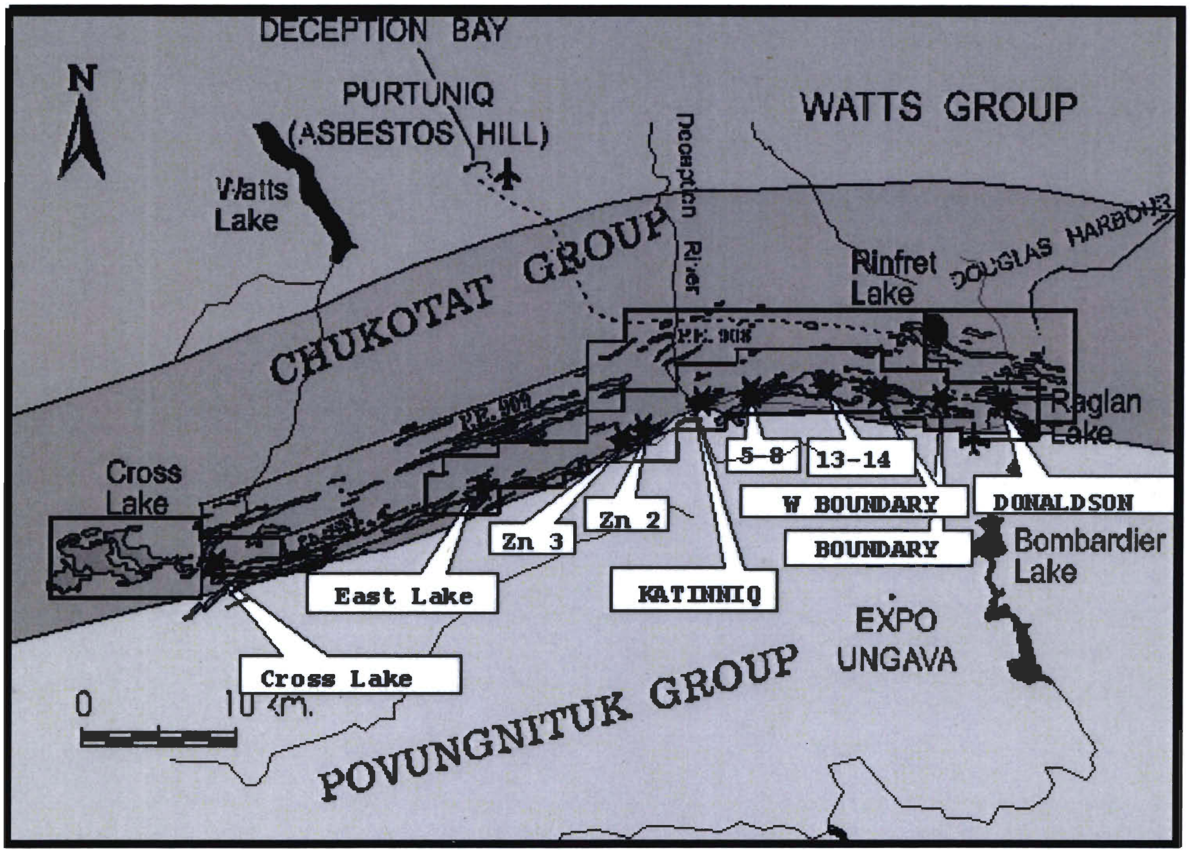


Figure 4. Location of the all-ore deposit zone for the Raglan project

2.6 The problem to be solved

One of the roles for the mine engineer is to find the optimum sequence that would trigger the highest net present value (NPV) possible with the current information he possesses. The rate of return and the payback period are also other merit measures that are taken into account in the feasibility studies. While these financial criteria can prove to be useful, such conventional techniques limit the evaluator in acknowledging the real economic nature of the mine for two main reasons: future cash flows are uncertain and the mine manager may have the flexibility to switch the production policy when market conditions shift. The NPV method is better used when inflows and outflows are relatively certain, as is the case with cost-reduction analysis or small scope projects.

For major capital projects that show relative longevity with reserves with multiple payable metals, the operating cash flows are uncertain over the scope of the mine life. Stochastic price models that reflect the historical degree of uncertainty of the metals benefit from using ROA. The problem that must be overcome here is to analyze and to process the behaviours and the interactions of up to seven payable metals at the same time into a financial model that would generate a new present value. Actually, the industry uses expected price list and Monte Carlo simulations to analyze the sensitivity of the project value. Using ROA might also be problematic: in this chapter, it has been shown that several types of ROAs failed to include numerous uncertainties due to its complexity to analytically treat the data. The Black and Scholes concept can be operated with one uncertainty. For the numeric models like the lattice or the finite difference approaches, multi-variable analysis can prove to be complex. This thesis will attempt to utilize a new approach using ROA to overcome the hurdle of multiple variables in modelling.

The second problem consists of including the flexibility dimension to the appraisal process of any mining project. In fact, many mining operations have the ability to switch production policy. For example, in 2006 at Raglan, the ore feed sources came from three underground mines and two open pits. The possibility to temporarily shut down an operation is feasible should it be deemed necessary and if it is economically reasonable. Switching production policy is applicable in other regions where the mining complex allows managerial flexibility.

For the Raglan example, the current mine planning process does not include the flexibility value of any project; rather it refers to operating the mine at its planned capacity until the ore reserve is exhausted.

In addition to that, many marginal ore bodies subsist in each mine. There is a need to know if it is worthwhile to mine out these secondary sources of metals. For instance, 3E zone at mine 3 consists of a half million tonnes of low-grade ore at the deepest part of the mine. What is the value of such an ore body? What is the impact of mining such a zone on the general mining plans? This information is crucial for the mine engineer to develop a sound strategic extractive schedule and sequence that will optimize the NPV of the operations. In Chapter 5, a case study is shown that illustrates the concept of analyzing marginal ore bodies.

2.7 Aim of the research

The goal of this thesis is to develop ROA for valuing mining properties based on LSM that takes into account the uncertainties associated with many state variables simultaneously. The research will be useful for mine managers, consultants and academics to evaluate more than one source of revenue at the same time. In order to achieve the objective, the author needs to choose some utility functions that admittedly represent the metal price behaviours. In Chapter 3, the metal price model for nickel, copper and cobalt (Ornstein-Uhlenbeck process) and the platinum group (PGE) (Wiener process) are presented.

Few studies have been carried out on dealing with multiple sources of uncertainties, and, as such, it may be difficult to get the financial parameters to work altogether. Some sources of uncertainties are correlated; some adjustments might be needed to create a stable and realistic model. This thesis is unique and original in using the LSM in the applications of the ROA in a real mining environment. The goal, if attained, is to apply the LSM framework for evaluating mining projects and to integrate the newly developed technique into a new mine evaluation process that would be used on a continuous basis. This objective is innovative since no mining operation in the world firmly uses the ROA with the LSM methodology integrated in its standard evaluating procedures. The outcome, if successful, will not only give more detailed information on the conventional NPV of the mining project, but will also provide critical information about the flexibility value of mining operations. The user can obtain a strategic advantage in optimizing the value of its natural ore reserves.

Chapters 4 to 6 treat specific applications of the ROA with integrated LSM. It aims to show the benefits for a mining company to complement its evaluation process with real options. The thesis aims to define a framework where flexibility can enhance the decision- making process for the mine engineer who schedules production operations. Although it is not the intent of this thesis to develop a new mathematical structure, some concepts are brought to light to illustrate the new mine planning configuration. The principal goal of the thesis is to show the viability of using the newly developed method on a real mining environment.

3 Modelling Uncertainties

The Real Options Approach seems more appropriate to evaluate projects with uncertain cash flows. As such, it holds great interest for an evaluator studying the main source of variation that could affect the profitability of the project. In fact, many parameters play an important role on the variability of the net present value (NPV) of a mining project. These components can include metal prices, exchange rate, capital expenditures, geology, etc. There are several means to conduct a search for those variables that have the most impact on the precision of the valuation. The tools that can be used are a sensitivity chart (commonly known as a spider diagram), tornado chart or Pareto chart. For the Raglan example, a sensitivity analysis was performed to determine which principal variables should be studied in the model according to historical performances. As shown on Figure 5, the impact of nickel price on the NPV is the greatest of all sources of uncertainties. Although the foreign exchange came second and the geological uncertainties also have a tremendous impact, the author will concentrate efforts on modelling the metal prices only. The problematic issue to deal with will be using the ROA with multiple metal price variables in the model: nickel, copper, cobalt, silver, gold, palladium and platinum.

The author feels that future research could integrate the foreign exchange and geological uncertainties as well as capital cost parameters along with other significant uncertainties to enhance the precision of the model.

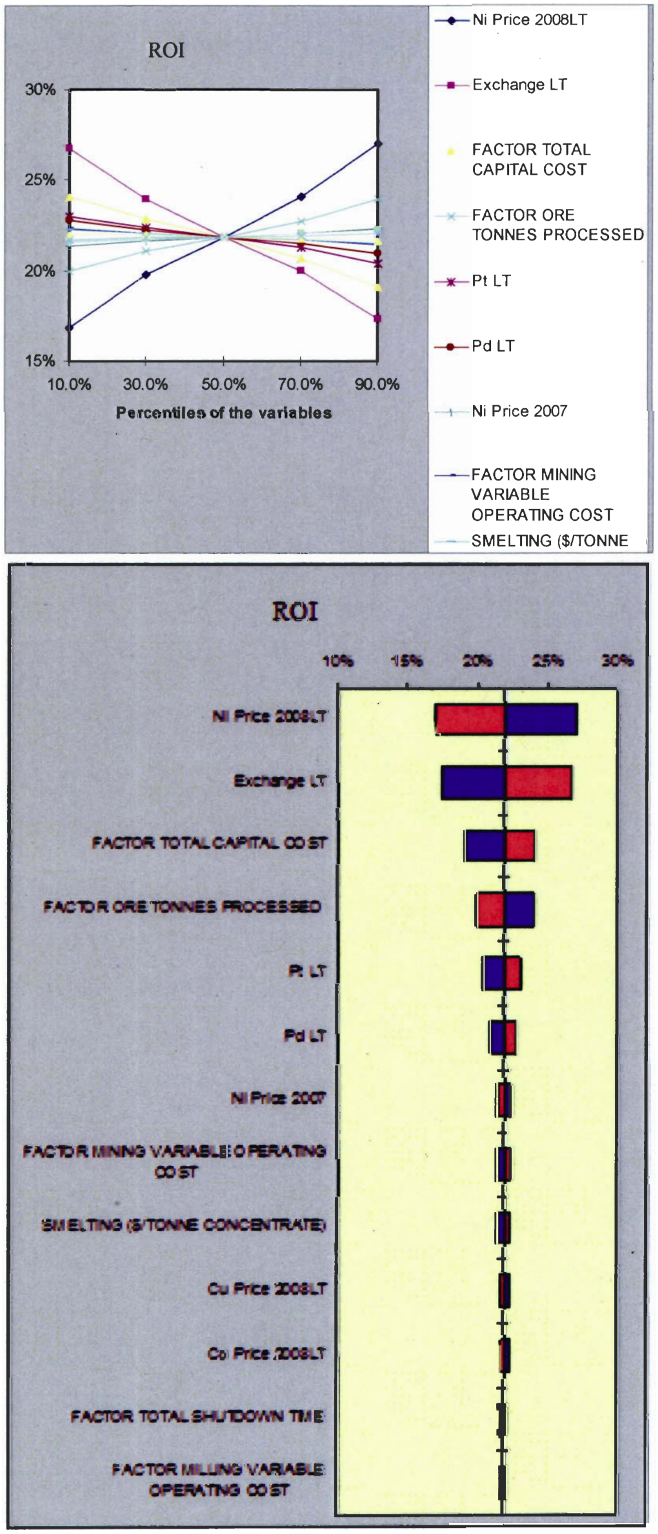


Figure 5. Spider diagram and tornado chart performed on Feb 23rd 2004 for project start-up in 2007

3.1 Metal price

Metal price is determined by various factors. For the most part, the long-term price reflects production costs. Price evolves over time as the cost to produce one unit of metal also evolves. If resources are becoming scarce, exploration costs and mining costs might drive the price up. On the other hand, technology can lower the cost to produce metal. Then, the long-term price should follow.

Supply and demand

In the short term, the price may fluctuate if the supply and demand is unbalanced. These discrepancies result from many factors, such as the lack of supply when demand suddenly shifts or an overcapacity in the system.

Supply

Many factors can influence the supply of the metal. The number of miners in the market plays a major role in establishing the metal price. The competition level may differ from one metal to another. For example, gold has so many producers that an individual would not be able to influence the market, whereas nickel can be influenced by a company like Russian Norilsk Nickel because the nickel miners are operating in conditions similar to an oligopoly. The price of a mineral can also be determined by one player, if that player has a monopoly like diamond producer De Beers. De Beers' Central Selling Organisation was a cartel that controlled the price of diamonds, until it was eventually dismantled in 2000. Other factors can have an impact on the price, like the geographic distribution of the miners, the geological condition of the metal and political and legal changes.

In general, the supply is driven by many variables:

- Available reserves
- Production capacity
- Quality of reserves
- Ore body nature and geometry (laterite vs. sulphite, long-hole vs. cut-and-fill)
- Nature of ore processing
- Exploration
- Inventory
- Strategic nature of the metal
- Recycling
- Etc.

Demand

The demand for a metal depends on its ability to bear value for a specific purpose. Gold is known as a protection against inflation for investors. In comparison, aluminum is worth less by weight, but it is a useful component in many industries, including automobile manufacturing, avionics and construction. The demand for a specific metal depends on:

- Economic conditions
- Potential markets (penetration)
- New applications
- Technological innovations
- Substitution (e.g., platinum vs. palladium)
- Consumption habits
- Regulations (e.g., asbestos)

With so many factors to consider, it is difficult to predict prices for the future. However, some useful clues can be discerned from studying the metal price behaviour over a long period of time. Some metal prices tend to return to their long-term trends after a price shock, while others do not. In the next section, a realistic price model is developed. This model takes into account the intrinsic properties of the metal price on the market.

3.2 Price models

Imagine comparable future cash flows through time of two different projects that give the same economical value. Such a concept can be expanded to develop similar assumptions for the mining industry as well. There should be no difference between mining a mineral to sell it on the market and buying some mineral contracts to sell again later. The forward contract can be seen as a way to simulate cash flows in the future, similar to cash flows generated by mineral production. At a certain cost for a specific delivery time, the miner can get a unit of the mineral by extraction or by buying a forward contract. With different delivery dates, a portfolio of forward contracts looks the same as extracting the ore and getting the metal out from the refinery.

There are two risks that we need to manage in our price model: time and price volatility. Both can be handled separately: the price volatility can be extracted at the source, and the time risk can be carried through discounting revenues each year at a certain rate (mostly risk-free rates like U.S. Treasury Bonds). An analysis of the revenue stream will show that if we extract the price volatility risk from the expected price in the future, we get the forward price contract.

$$E[S_t]\cdot RDF = K_{Mineral,t}$$

where

$E[S_t]$= Expected mineral price at time t

RDF = Risk discount factor (Price volatility risk discount factor)

$K_{\mathrm{Mineral,t}}$ = Forward price contract at time t

In the continuous model, the risk discount factor can be expressed by the natural logarithm factored by the price risk times time.

$\mathrm{RDF} = e^{-PRt}$

where

PR = Price volatility risk

There are multiple processes that can be used to develop a model to calculate the forward price contract. For base metals, historically, the price tends to be driven by a market force that pulls the price toward long-term equilibrium. Nickel, copper, cobalt and platinum prices can be modelled with the Ornstein-Uhlenbeck process, which is a lognormal price distribution reverting to the long-term equilibrium price. For gold, silver and palladium, this force has not been determined, probably because it does not exist. The geometric Brownian motion pattern, a generally accepted metal price model, can be used in the simulator for these three metals.

Modelling the uncertainty of metal prices using these two stochastic models, rather than subjective modelling in the conventional evaluation technique, has two major advantages. First, the suitable stochastic models are based on statistical parameters that take into account the historical behaviours of metal prices in the metal markets, which allows for modelling the probability distribution according to the specific nature of each metal price. Second, the simulated metal price paths can now be generated with a Markovian property that takes into account the metal price level in the immediate past period, which reduces absurd metal jumps and enhances time-connectivity.

3.2.1 Geometric Brownian motion price model

Gold, silver and palladium are modelled using an existing and well-accepted model called geometric Brownian motion. These metals do not seem to be affected by any force reverting them back to a long-term level. Its stochastic behaviour can be represented as follows according to Brennan and Schwartz (1985), Dixit and Pindyck (1994) and Glasserman (2004):

$$dP = \alpha P dt + \sigma P dz$$

where α is the constant trend, σ is the standard deviation and dz is the increment to a standard Wiener process.

3.2.2 Mean reverting price model

This stochastic model is based on the characteristic of a metal price to return back to its long-term equilibrium level after a shock in the market and is represented by the mean reverting process. Schwartz (1997) developed a form of mean reverting process that can be described as follows:

$$dP = k(\ln P^* - \ln P)Pdt + \sigma P dz$$

where k is the speed at which the price reverts to its long-term equilibrium level P^*, P is the spot price while σ is the associated standard deviation of the price, and dz is the increment to a standard Wiener process. Nickel, copper, cobalt and platinum are represented by this type of model with its respective econometric attributes.

For the sake of understanding the methodology behind the mean reverting process, the author hereby presents a description of the mean reverting process developed by Laughton and Jacoby (1993). It is derived from the above model and it shows in greater details the econometric parameters that influence the calculations of the metal price ruled by a mean-reverting force. This will constitute the base of this chapter. In this section, copper is used as the example to illustrate the live applications of the theory. The author created a database from the copper market integrating the daily prices, the market returns, the producer unit cash costs and other economic indicators that are in the example below. Although the exercise of building such database can prove to be extensive in nature, the results have permitted to develop a metal price model to be later used in the financial simulations for mining project evaluations. For commercial applications, it is suggested to go through a specialized firm that would identify the specific econometric parameters that are included in the model such as the reversion factor.

Future copper price model following a mean reverting process can be described as follows:

$$F(T) = \exp\left(e^{-\lambda T} \ln S + (1 - e^{-\gamma T}) \cdot (\ln S^{*} + \frac{\alpha^{*}}{\gamma} - PRisk_{Mkt} \cdot \rho_{Mkt,Min}) + \frac{\sigma^{2}}{4\gamma} \cdot (1 - e^{-2\gamma T}) \right)$$

where

F(T) = Future price
γ = Reversion factor
S = Spot price
T = Time period of the futures
S* = Long-term equilibrium price
α* = Growth rate of the equilibrium price
$\text{PRisk}_{\text{Mkt}}$ = Market price of risk
$\rho_{Mkt,Min}$ = Correlation of price and the market returns
σ = Short-term price volatility

It is possible to analyze each equation component, which will eventually lead to the expected price.

<u>Short-term price volatility (σ): Historic data method</u>

With the copper price database, it is possible to calculate the short-term copper price volatility by the given formula:

$$\sigma = \sqrt{\frac{\tau}{n-1} \cdot \sum_{i=1}^{n} (u_{i} - \bar{u})^{2}}$$

where

σ = Short-term price volatility

τ = Number of observation entries per year

n = Total number of observation entries

u_i = Natural logarithm relative price = $u_i = \ln\left(\frac{S_i}{S_{i-1}}\right), i = 1,\ldots,n$

$\bar{u}$ = Mean natural logarithm relative price

In the copper price example, the short-term standard deviation shows a value of 0.2455.

Risk-free interest rate (r):

As of February 12, 2005, the financial information on the bond markets was as is shown in Table 9:

	Canada	***U.S. Treasury***
2 years	2.89%	3.27%
5 years	3.53%	3.64%
10 years	4.15%	4.07%
Long-term	4.62%	4.46%

Table 9. Bond yield rates as of Feb 12th 2005

It is good practice to take the same term of the interest rate than the project in session. In the case study, it is relevant to take 4.15% as the risk-free rate on Canadian projects.

Current long-term equilibrium price (S*):

The current long-term equilibrium price (S*) can be seen as the metal price around which the market is evolving. In fact, it is meant to be equal to the production cost for the last marginal producer (swing producer) before it exits the market, or it equates to the long-term target price the industry is looking for, especially in a monopoly or oligopoly. The equilibrium price can be found by analyzing mining company annual reports. It consists of finding the cost to produce one unit of metal, one pound of copper for instance. At a certain unit cost, the mining company prefers to shut down the mine or to leave the mineral in the ground. The table below shows the cash cost to produce one pound of copper:

Company	Site	Unit cash cost
Noranda	Collahuasi	0.38
Noranda	Lomas Bayas	0.48
Noranda	Kidd Creek	0.87
Phelps Dodge	Phelps Dodge	0.52
Aur Resources	Aur Resources	0.5
Antofagasta	Los Pelambres	0.29
Antofagasta	El Tesoro	0.42
Antofagasta	Michilla	0.7

Table 10. Unit cash cost for several copper producers as of Feb 21st 2005

In the copper example, the value for the current long-term equilibrium price is $0.87/lb. This is basically where the price target should be in the copper market.

<u>Growth rate of the equilibrium price (α^*):</u>

The growth rate of the equilibrium price should be representative of the speed at which the metal price appreciates or depreciates through time. Given the non-renewable nature of mineral resources, it is believed that the growth rate should be positive according to Hotelling (1933). However, with technology breakthroughs, the unit cost has decreased facilitating the extraction at a lower cut-off grade as explained by Slade (1988). By analyzing the metal price time series, it is possible to capture the metal growth rate. In the copper example, the trend can be extracted from the daily copper price database. Three trend lines were calculated: the 1989–2005 period, the 1994–2005 period and, finally, the 1999–2005 period. As shown on the table and figures below, there are three distinct growth rates. During the last 15 years, the annual rate is around -2.5%. For the last 10 years, it improved to -2% per annum. Finally, the metal market has recovered at an appreciable 12.2% a year since 1999. For the copper metal price model, it would be wise to take into account that metal price is cyclical, thus it is best practice to keep a conservative number. In the copper example, the chosen growth rate of the equilibrium price is set to 0%, which reflects conservatism.

Period	Nb of years	Daily growth	Annual growth
89-05	15 years	-0.0090%	**-2.49%**
94-05	10 years	-0.008%	**-2.05%**
99-05	5 years	0.04%	**12.20%**

Table 11. Trend lines over certain periods (as of Feb 21st 2005)

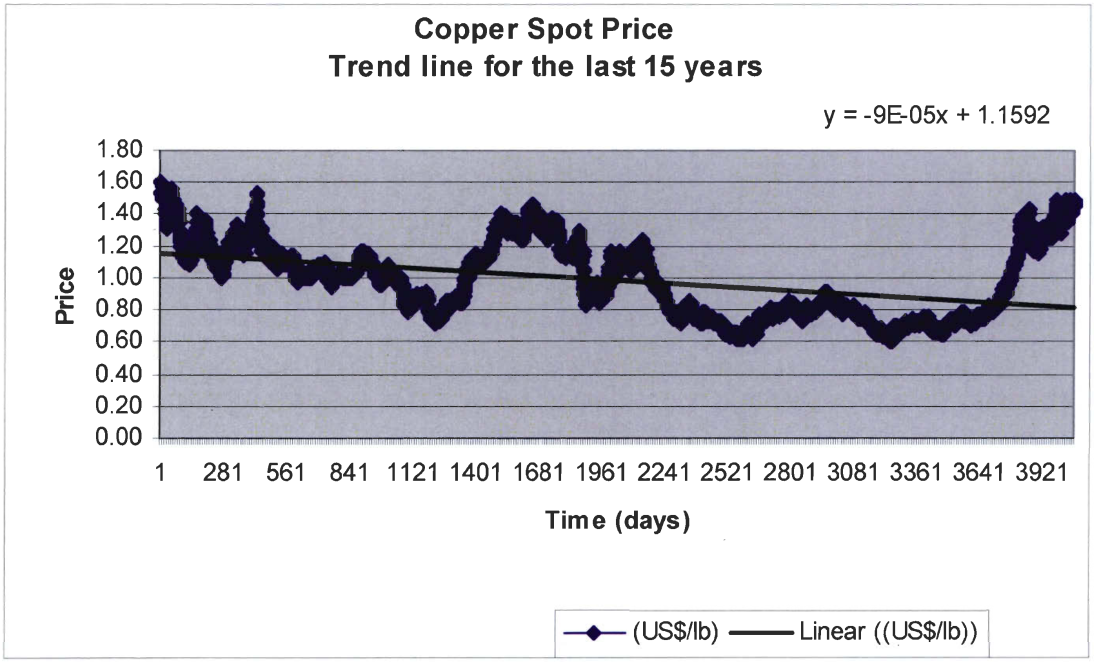


Figure 6. Copper spot price over the last 15 years

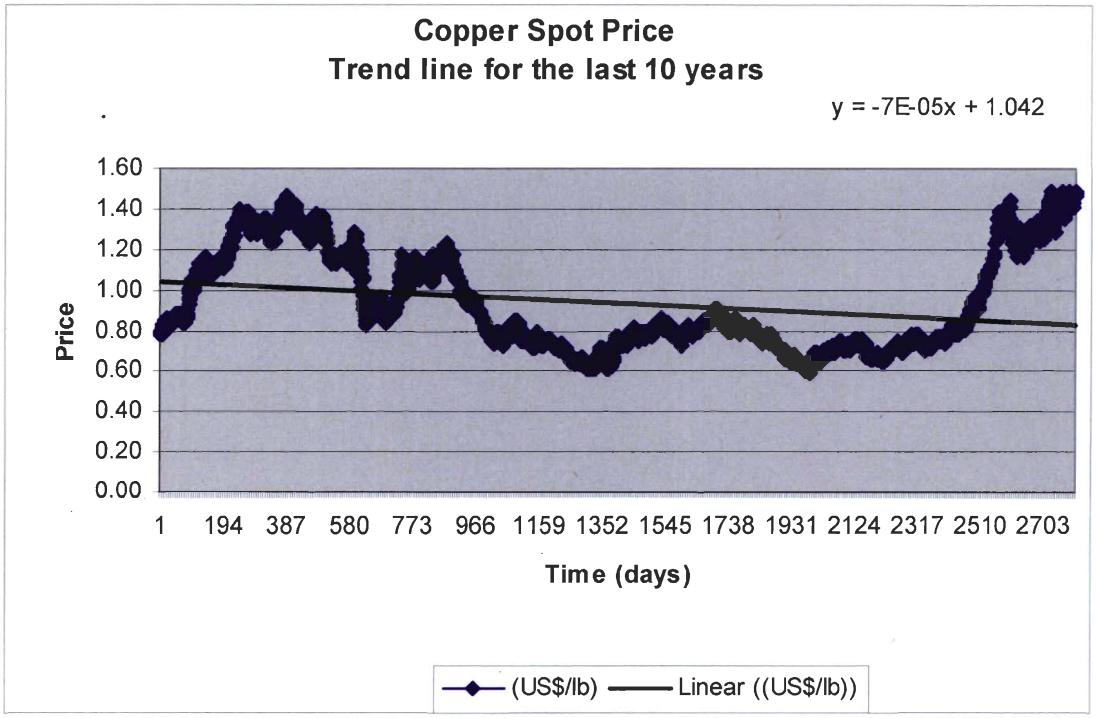


Figure 7. Copper spot price over the last 10 years

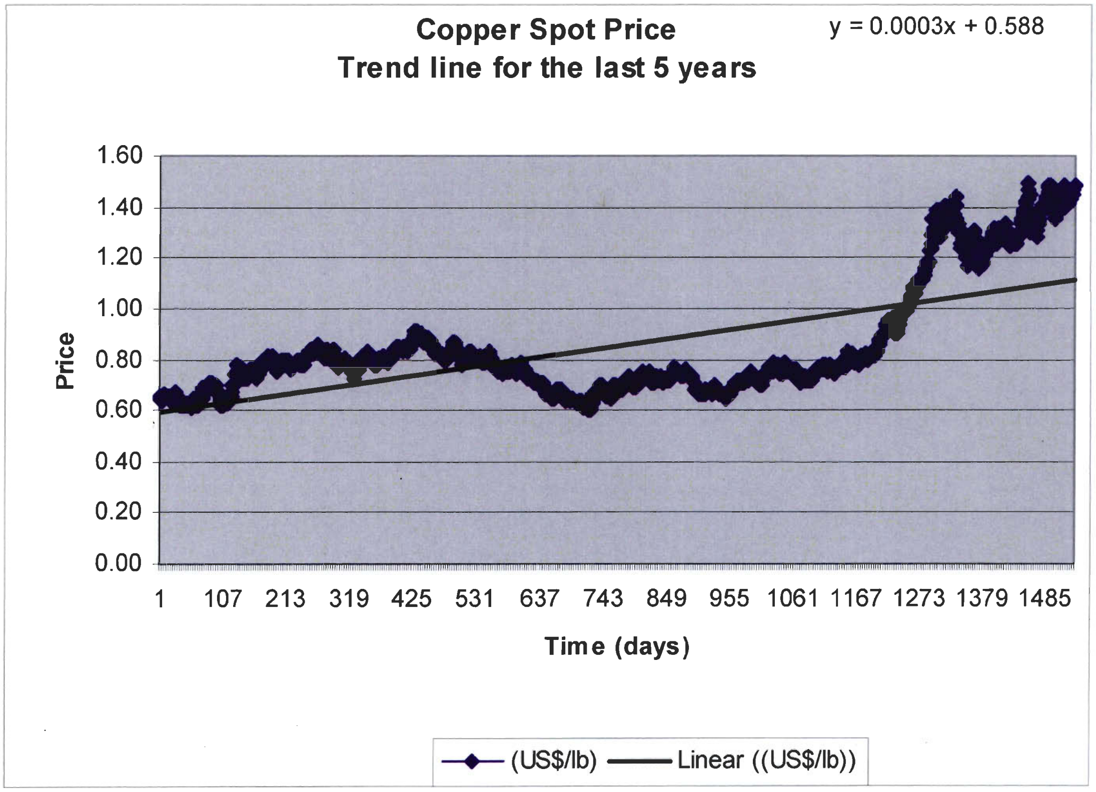


Figure 8. Copper spot price over the last five years

Correlation of price and market returns ($\rho_{Mkt,Min}$)

The correlation of price and market returns is another statistical tool that helps to figure the relation between two distinct parameters when they change. To represent the market, one can take the Dow Jones Industrial Average Index, the Standard and Poor's 500 Index (S&P 500) or any index that represent a global and significant market. In the copper model, the S&P 500 characterizes the market. The calculation of the correlation between the market and the copper spot price can be explained below:

$$\rho_{Mkt,Min} = \frac{1}{n \cdot \sigma_x \cdot \sigma_y} \sum_{i=1}^{n} (x_i - \bar{x}) \cdot (y_i - \bar{y})$$

where

$\rho_{Mkt,Min}$ = Correlation of price and market returns

n = Number of paired observations

x_i = Observed market data change from the last entry

$\bar{x}$ = Mean of the observed market data change in the population

y_i = Observed metal spot price change from the last entry

$\bar{y}$ = Mean of the metal spot price change in the population

σ_x = Standard deviation of series x (S&P 500)

σ_y = Standard deviation of series y (mineral price)

It is important to note that the correlation is calculated from the percentage change in the data and not directly from the data itself. The result is related to the change in percentage and not in the measurement unit. For instance, the observed market change from the last entry (x_i) of the S&P 500 is calculated by dividing the present value with the previous value minus one. The result gives a percentage of change from two data. It is not the difference between the actual value and the previous one, which would represent no useful reference. Results for the copper price are interesting. On a daily basis, for the last 15 years, the correlation between the market and the copper price is 0.021. If the monthly averages are taken into account instead of daily prices, the correlation climbs to 0.079. Averaging the indices on a monthly basis smoothes out variations in the raw data, and then the correlation coefficient grows larger. It shows that reliability can be argued depending on the source of the data and the subsets of the population.

Market price of risk ($PRisk_{Mkt}$)

The market price of risk is the return of the referred market minus the risk-free rate per unit of volatility in the market.

$$PRisk_{Mkt} = \frac{E(r_{Mkt}) - risk\ free\,rate}{\sigma_{Mkt}}$$

where

$PRisk_{Mkt}$ = Market price of risk

$E(r_{Mkt})$ = Expected annual return on market

σ_{Mkt} = Volatility (standard deviation) of the market

A review of the last 15 years of the S&P 500 shows that the expected return on the market is 9.18% and the associated volatility is 17.28%. Note that the volatility is calculated on a yearly basis. The risk-free rate of the 10-year Canadian bonds is 4.15%. The market price of risk is 0.2875, well below the historical 0.5 that the market has traditionally observed. It can partly be explained by the poor results of the market for the last five years (-0.54%).

Reversion factor (γ)

The reversion factor is the force at which the spot price moves in the direction of the market equilibrium price (S*). Some metals are not driven by a reversion factor. The price shocks or movements are permanently imprinted on the market. For instance, it is assumed that gold does not tend to come back to its long-term equilibrium after a change in pricing. For base metals like nickel and copper, it is possible to determine the reversion factor by iterations from the following formula:

$$F(T) = \exp\left(e^{-\lambda T} \ln S + (1 - e^{-\gamma T}) \cdot (\ln S^* + \frac{\alpha^*}{\gamma} - PRisk_{Mkt} \cdot \rho_{Mkt,Min}) + \frac{\sigma^2}{4\gamma} \cdot (1 - e^{-2\gamma T}) \right)$$

where

F(T) = Future price

γ = Reversion factor

S = Spot price

T = Time period of the futures

S* = Long-term equilibrium price

α^* = Growth rate of the equilibrium price

$PRisk_{Mkt}$ = Market price of risk

$\rho_{Mkt,Min}$ = Correlation of price and the market returns

σ = Short-term price volatility

In the copper price model, the calculated reversion model is 0.219, which is lower than what the economist community has estimated. It is generally accepted that the copper reversion factor plays around 0.4. The goal of this thesis is not to demonstrate a change in econometric conditions; it will be conservative and stay with numbers that have been validated by the experts. But it is interesting to see that it is possible to calculate a reversion factor from the "future price" formula that is simpler than the method currently used by economists. It also costs less than the conventional market survey that economists perform to come up with a precise number.

Median price ($Med_0(S,T)$)

Now that the intrinsic parameters of the model have been defined, the median price function can be determined. As part of a stochastic process, the median price evolves through time, which makes the model more dynamic than a deterministic one. The median price for each period of time is defined as the middle value of the price distribution at time T. It is important not to take the mean since the price follows a lognormal distribution. The median represents the value that separates the distribution into two equal parts. The median function can be expressed as:

$$Med_0\left[S(T)\right] = S^* \cdot \left[\frac{S}{S^*} \cdot e^{\frac{\alpha^*}{\gamma}\cdot(1-e^{-\gamma T})}\right]^{e^{-\gamma T}}$$

where

$Med_0\left[S(T)\right]$= Median price function of the metal

S^* = Long-term equilibrium price

S = Spot price

α^* = Growth rate of the equilibrium price

γ = Reversion factor

In the copper price model, the median table (Table 12) and curve (Figure 9) for the next 10 years are:

T	$Med_0\left[S(T)\right] = S^* \cdot \left[\frac{S}{S^*} \cdot e^{\frac{\alpha}{\gamma}(1-e^{-\gamma T})}\right]^{e^{-\gamma T}}$
0	1.47
0.5	1.391272133
1	1.324466623
1.5	1.267449069
2	1.218533479
2.5	1.176374677
3	1.139889397
3.5	1.108197721
4	1.080579155
4.5	1.056439331
5	1.035284519
5.5	1.016701945
6	1.00034446
6.5	0.985918487
7	0.973174488
7.5	0.961899359
8	0.951910315
8.5	0.94304995
9	0.935182219
9.5	0.928189138
10	0.92196807

Table 12. Copper median price over time

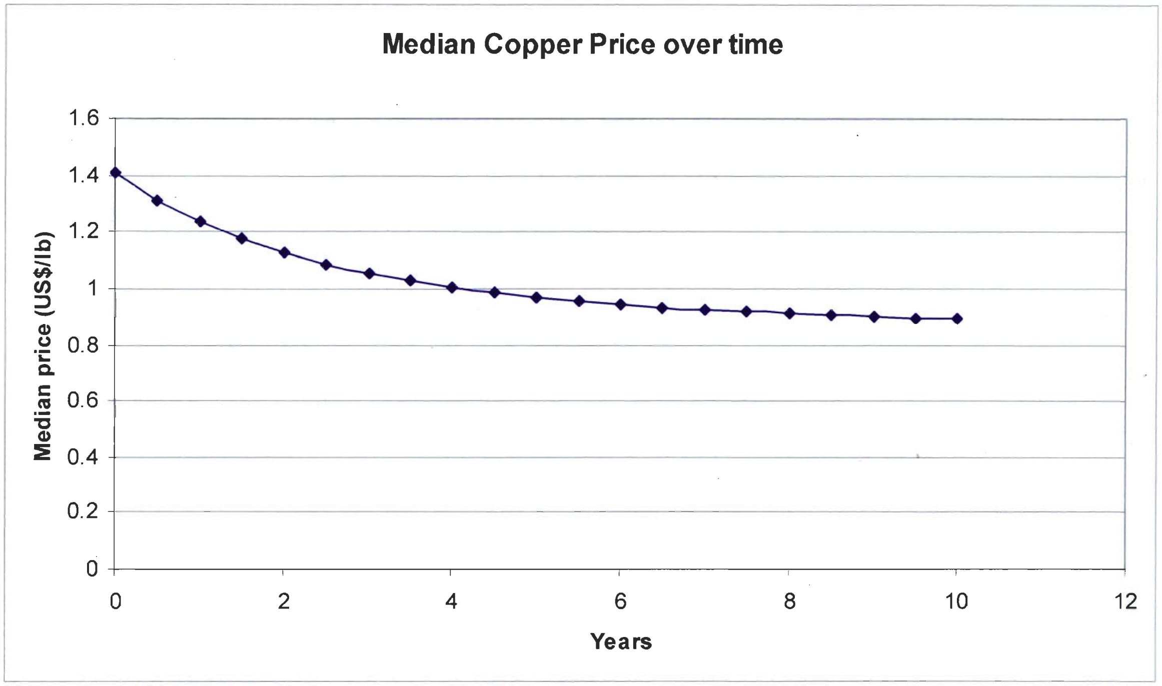


Figure 9. Median price over time

Variance ($Var_0(S,T)$)

In this stochastic process, the metal price follows a lognormal distribution. It has to be understood that for each period of time, the metal price distribution changes. It also means that the distribution parameters will change over time. The variance is likely to change (and to increase) as it moves away from the present time. The associated variance function in a reverting price process can be described and calculated as shown below:

$$Var_0(S,T) = \frac{\sigma^2}{2\gamma} \cdot (1 - e^{(-2\gamma T)})$$

where

$Var_0(S,T)$ = Variance of the price distribution at time T and price S

σ = Short-term price volatility

γ = Reversion factor

In the copper price model, the variance table (Table 13) and curve (Figure 10) for the next 10 years are:

T	$Var_0(S,T) = \frac{\sigma^2}{2\gamma} \cdot (1 - e^{(-2\gamma T)})$
0	**0**
0.5	**0.027081894**
1	**0.048837367**
1.5	**0.066314011**
2	**0.080353379**
2.5	**0.091631508**
3	**0.100691474**
3.5	**0.107969541**
4	**0.113816171**
4.5	**0.118512896**
5	**0.122285876**
5.5	**0.125316794**
6	**0.127751596**
6.5	**0.129707525**
7	**0.131278765**
7.5	**0.132540977**
8	**0.133554939**
8.5	**0.134369476**
9	**0.135023812**
9.5	**0.135549454**
10	**0.135971714**

Table 13. Copper distribution variance through time

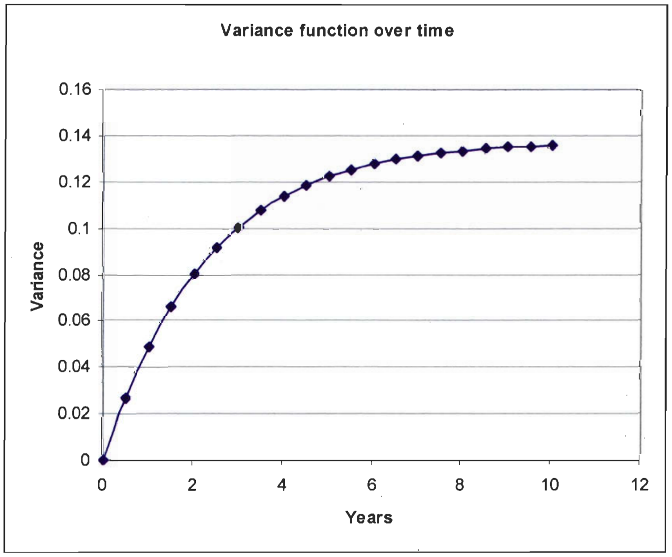


Figure 10. Variance function over time

Expected (mean) price ($E_0(S,T)$)

In a lognormal distribution, it is possible to calculate the mean if the median and the associated variance are known. The expected price is normally considered as the budget price for long-term forecast purposes when using classic NPV project evaluation. The expected mean price is calculated as follows:

$$E_0[S,T] = Med_0(S,T) \cdot e^{0.5 \cdot Var_0(S,T)}$$

where

$E_0[S,T]$= Expected mean price at time T

$Med_0(S,T)$ = Median price

$Var_0(S,T)$ = Associated variance

In the copper price model, the mean table (Table 14) and curve (Figure 11) for the next 10 years are:

T	$E_0[S,T] = Med_0(S,T) \cdot e^{0.5 Var(S,T)}$
0	1.47
0.5	1.410239402
1	1.357206459
1.5	1.310178358
2	1.268486882
2.5	1.231524898
3	1.198747155
3.5	1.169667817
4	1.14385626
4.5	1.12093211
5	1.100560092
5.5	1.082445029
6	1.066327176
6.5	1.05197796
7	1.039196169
7.5	1.027804566
8	1.017646912
8.5	1.008585355
9	1.000498148
9.5	0.993277669
10	0.986828681

Table 14. Copper expected mean price over time

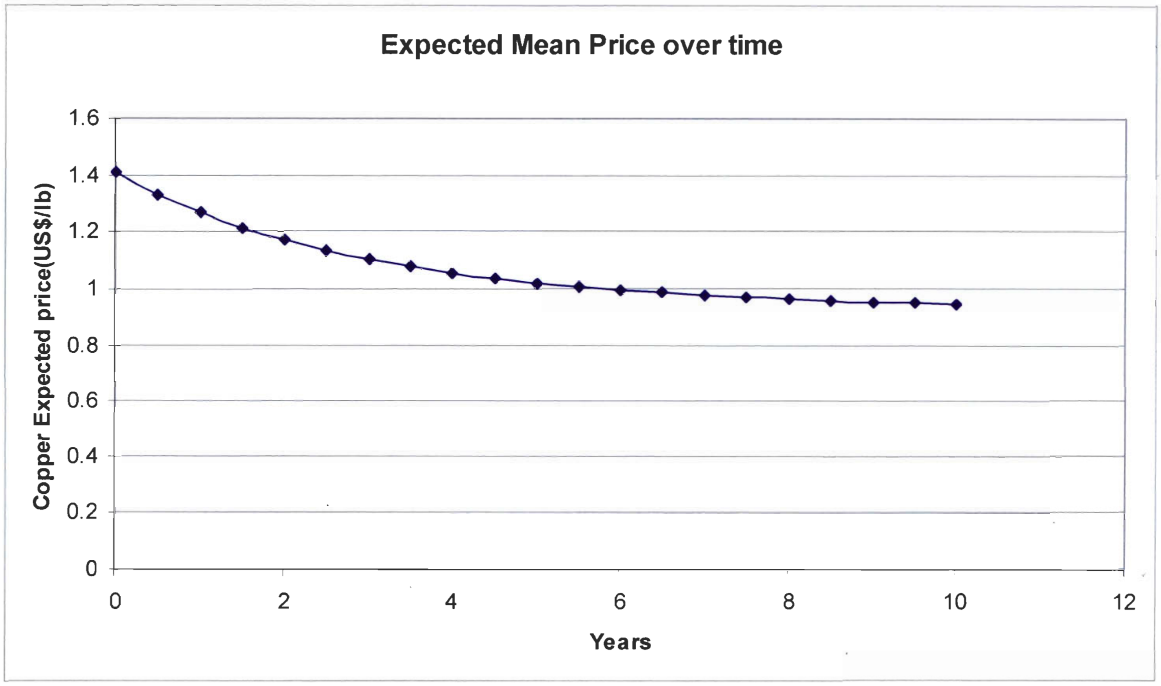


Figure 11. Expected mean price over time

Confidence intervals of the expected (mean) price ($S_{0,CI\%}(S,T)$)

From the median and the variance, it is possible to calculate the confidence intervals of the price model. The confidence intervals are the range through which the metal price has the probability to fall in at a specific degree of uncertainty. For instance, one can say with a predetermined spot price and with a confidence coefficient of 95%, the copper price range will be from $0.8/lb. to $1.2/lb. The confidence intervals for the metal price can be mathematically described using the following formula:

$$S_{0,CI\%} = Med_0(S,T)\cdot e^{(Z_{Normal,CI\%}\cdot Var_0(S,T)^{0.5})}$$

where

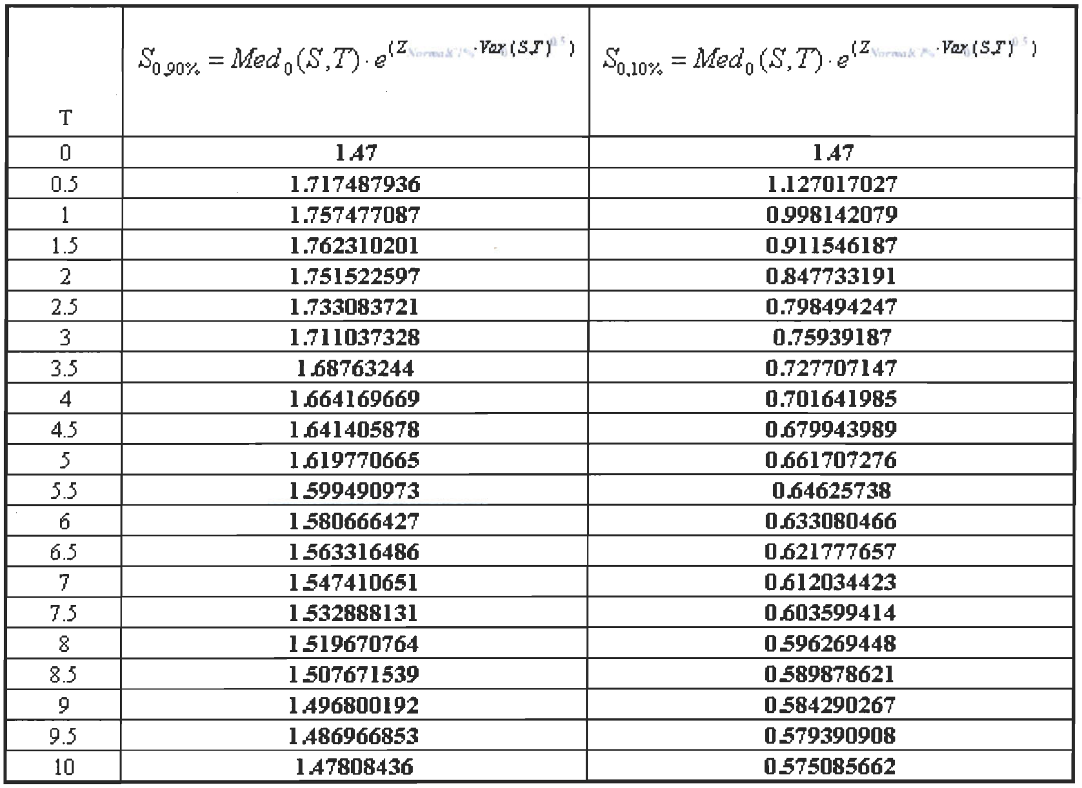
$S_{0,CI\%}$ = Confidence interval of the metal

$Med_0(S,T)$ = Median price

$Var_0(S,T)$ = Associated variance

$Z_{Normal,CI\%}$ = Z distribution at a specific confidence coefficient

In the copper price model, the confidence intervals table (Table 14) and curve (Figure 11) for the next 10 years are:



T	$S_{0,90\%} = Med_0(S,T)\cdot e^{(Z_{Normal,CI\%}\cdot Var_0(S,T)^{0.5})}$	$S_{0,10\%} = Med_0(S,T)\cdot e^{(Z_{Normal,CI\%}\cdot Var_0(S,T)^{0.5})}$
0	1.47	1.47
0.5	1.717487936	1.127017027
1	1.757477087	0.998142079
1.5	1.762310201	0.911546187
2	1.751522597	0.847733191
2.5	1.733083721	0.798494247
3	1.711037328	0.75939187
3.5	1.68763244	0.727707147
4	1.664169669	0.701641985
4.5	1.641405878	0.679943989
5	1.619770665	0.661707276
5.5	1.599490973	0.64625738
6	1.580666427	0.633080466
6.5	1.563316486	0.621777657
7	1.547410651	0.612034423
7.5	1.532888131	0.603599414
8	1.519670764	0.596269448
8.5	1.507671539	0.589878621
9	1.496800192	0.584290267
9.5	1.486966853	0.579390908
10	1.47808436	0.575085662

Table 14. Copper confidence intervals with 90% certainty

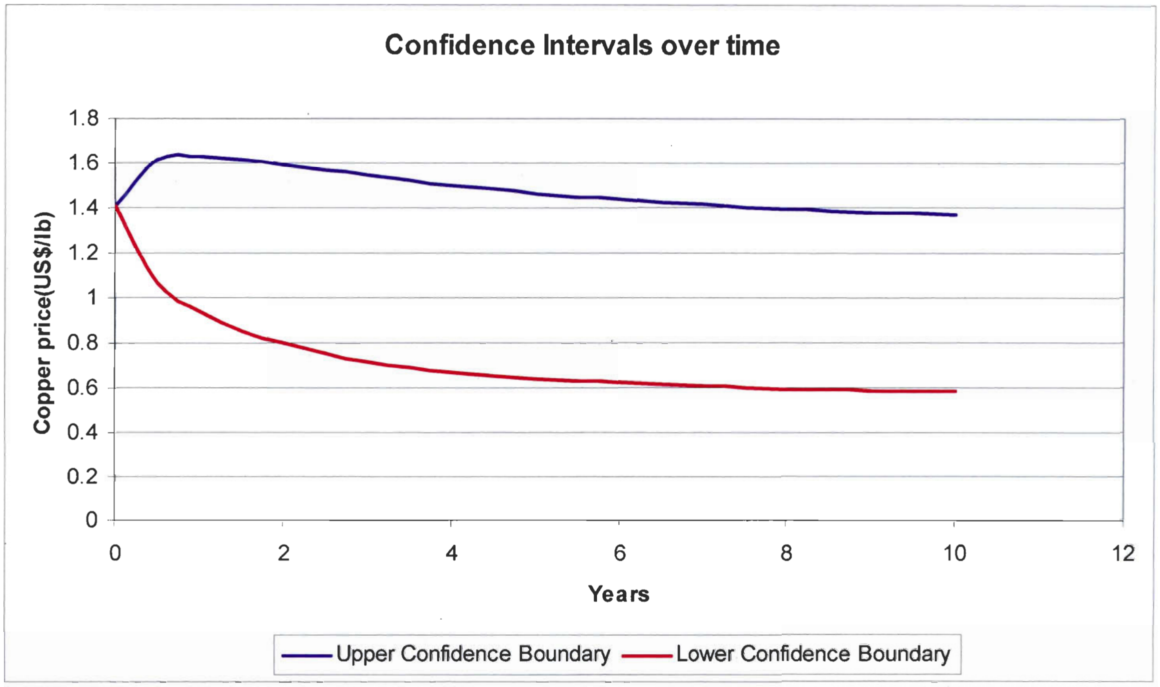


Figure 11. Copper confidence intervals with 90% of confidence

Price volatility risk discount factor (RDF)

As described in the introduction to this chapter, the risk associated with price volatility can be expressed by:

$$RDF = e^{-PRt}$$

where
PR = Price volatility risk

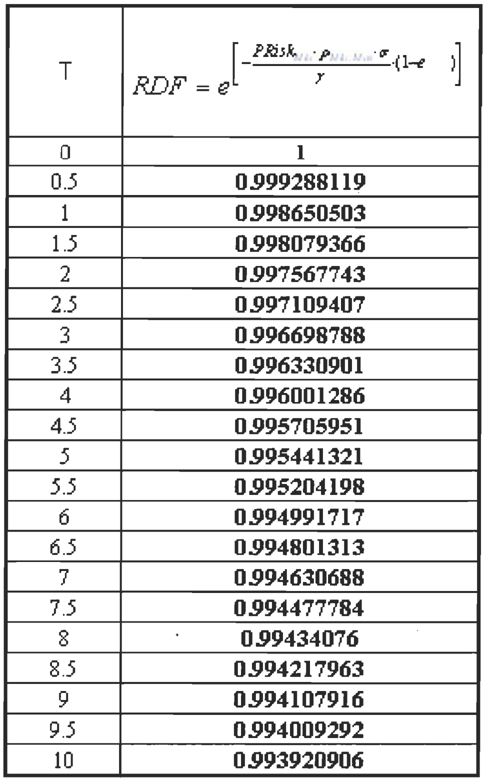
In the reverting model, the price volatility risk must include the reversion factor. Then, the risk is mathematically equal to:

$$PR = \left[-\frac{PRisk_{Mkt} \cdot \rho_{Mkt,Min} \cdot \sigma}{\gamma} \cdot (1 - e^{-\gamma T}) \right]$$

Thus, the risk discount factor for price volatility is:

$$RDF = e^{\left[-\frac{PRisk_{Mkt}\cdot\rho_{Mkt,Min}\cdot\sigma}{\gamma}\cdot(1-e^{-\gamma T})\right]}$$

In the copper price model, the risk discount factor table (Table 15) and curve (Figure 12) for the next 10 years are:



T	$RDF = e^{\left[-\frac{PRisk_{Mkt}\cdot\rho_{Mkt,Min}\cdot\sigma}{\gamma}\cdot(1-e\quad)\right]}$
0	**1**
0.5	**0.999288119**
1	**0.998650503**
1.5	**0.998079366**
2	**0.997567743**
2.5	**0.997109407**
3	**0.996698788**
3.5	**0.996330901**
4	**0.996001286**
4.5	**0.995705951**
5	**0.995441321**
5.5	**0.995204198**
6	**0.994991717**
6.5	**0.994801313**
7	**0.994630688**
7.5	**0.994477784**
8	**0.99434076**
8.5	**0.994217963**
9	**0.994107916**
9.5	**0.994009292**
10	**0.993920906**

Table 15. Copper price volatility risk discount factor

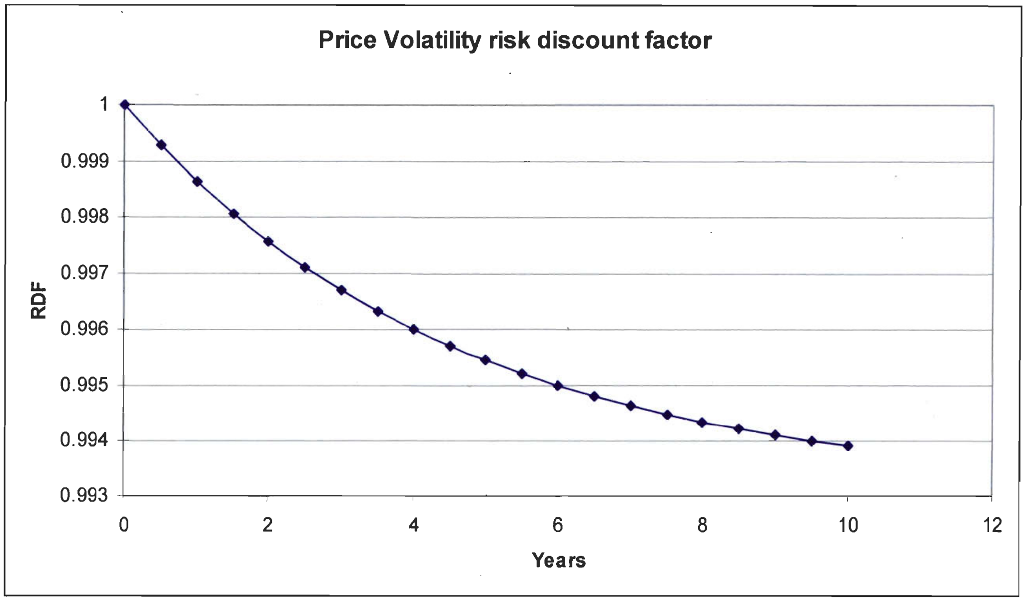


Figure 12. Price volatility risk discount factor

As shown in the table and in the graph, the risk discount factor is close to one for the next 10 years. This can be explained by the fact that the correlation of copper price and market returns is next to zero. The natural logarithm of zero is one, and as such, values close to one for the next 10 years come as no surprise, since the same correlation factor is carried forward for each calculation. This study is focused on the workings of the model mechanisms and not on using fully precise econometrics provided by experts. In reality, however, an evaluator would commission a study to determine the real correlation factor as well as other econometric values to develop the model. These studies can be costly, but easily affordable for a mining company. For this study, the author will use these calculated values in order to show how the model mechanisms work.

Forward price ($K_0(\sigma_s,t)$)

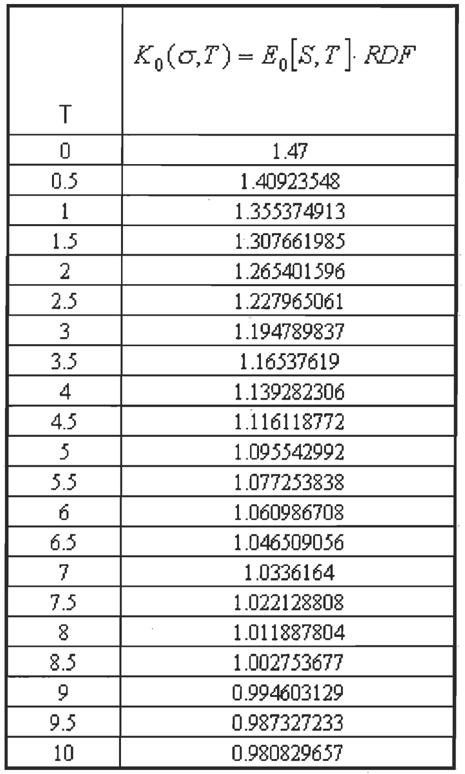
Finally, the forward price can be calculated from the expected price discounted by the price risk. Note that the expected price is discounted at the source and not discounted over the years as it is with the conventional net present value (NPV) methodology using a 15% include-all-risk-rate, for instance. The forward price can easily be obtained by:

$$K_0(\sigma,T) = E_0[S,T] \cdot RDF$$

$E_0[S,T]$ = Expected mean price at time T

RDF = Price volatility risk discount factor

In the copper price model, the forward price table (Table 16) and curve (Figure 13) for the next 10 years are:



T	$K_0(\sigma,T) = E_0[S,T] \cdot RDF$
0	1.47
0.5	1.40923548
1	1.355374913
1.5	1.307661985
2	1.265401596
2.5	1.227965061
3	1.194789837
3.5	1.16537619
4	1.139282306
4.5	1.116118772
5	1.095542992
5.5	1.077253838
6	1.060986708
6.5	1.046509056
7	1.0336164
7.5	1.022128808
8	1.011887804
8.5	1.002753677
9	0.994603129
9.5	0.987327233
10	0.980829657

Table 16. Forward copper price over time

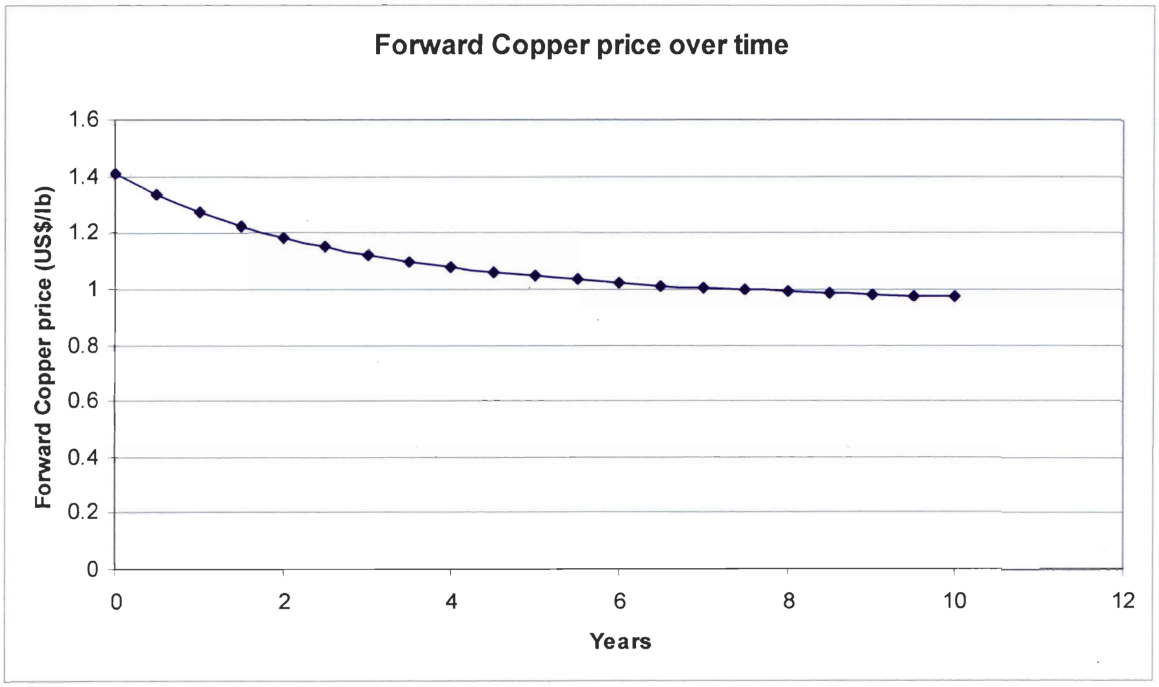


Figure 13. Forward copper price over time

As a result, the forward copper price for the next 10 years can be figured as below. When the forward components are put altogether on a graph; it gives an illustration like the following graph (Figure 14):

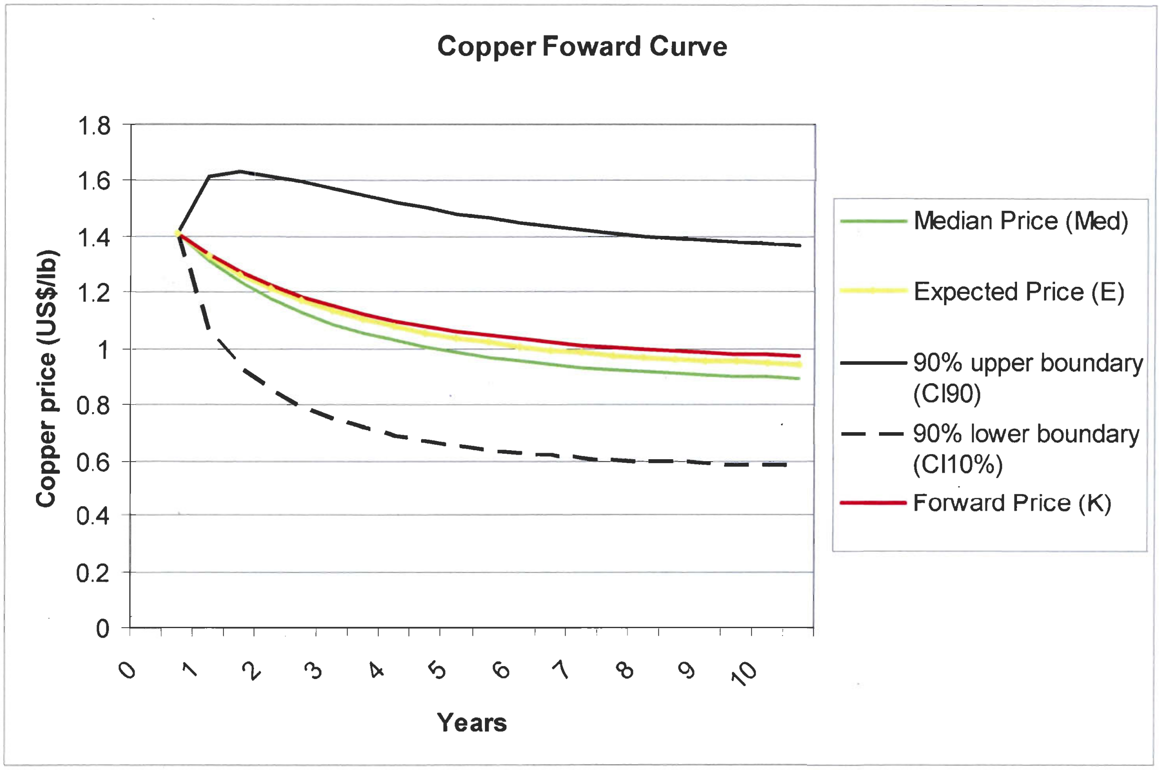


Figure 14. Forward, expected and median copper price over time

3.2.3 Probability distribution

With the forward price, it is possible to calculate the NPV of the project, the base case scenario for revenue generation. To determine a range for the sensitivity of the project, one needs to establish the price variation process through time. In order to build the model, one needs to find the lognormal probability density function (PDF) by using two parameters u' and σ'_S. The distribution can be described as follows:

$$f(S_t') = \frac{1}{\sigma_{S_t'}\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{S_t' - u'}{\sigma_{S'}}\right)^2}$$

where

S_t' = ln(S_t), natural logarithm of the price u' = Mean of the natural logarithm of the price

$\sigma_{St'}$= Associated standard deviation of the natural logarithms of the price

A variable price is lognormally distributed if Y = LN(S) is normally distributed with the natural logarithm. Assuming that the incremental areas under the normal and the lognormal probability density functions are equal, then:

$$f(S_t)dS_t = f(S_t')dS_t'$$

and

$$dS_t' = \frac{dS_t}{S_t}$$

substituting dS_t' by $\frac{dS_t}{S_t}$,

$$f(S_t) = \frac{f(S_t')}{S_t}$$

thus

$$f(S_t) = \frac{1}{S_t \cdot \sigma_{S_t'} \sqrt{2\pi}} \cdot e^{-\frac{1}{2} \cdot \left(\frac{\ln(S_t) - \ln(Med_t)}{\sigma_{S_t'}} \right)^2}$$

Note that ln(u) = u'= ln($e^{u'}$) and $e^{u'}$ = Median of the lognormal distribution.

Then, u' = ln(Med_t).

By plotting the density functions for all periods, graphs like those below (Figures 15 and 16) are generated to show the probabilities shifting toward its long-term equilibrium.

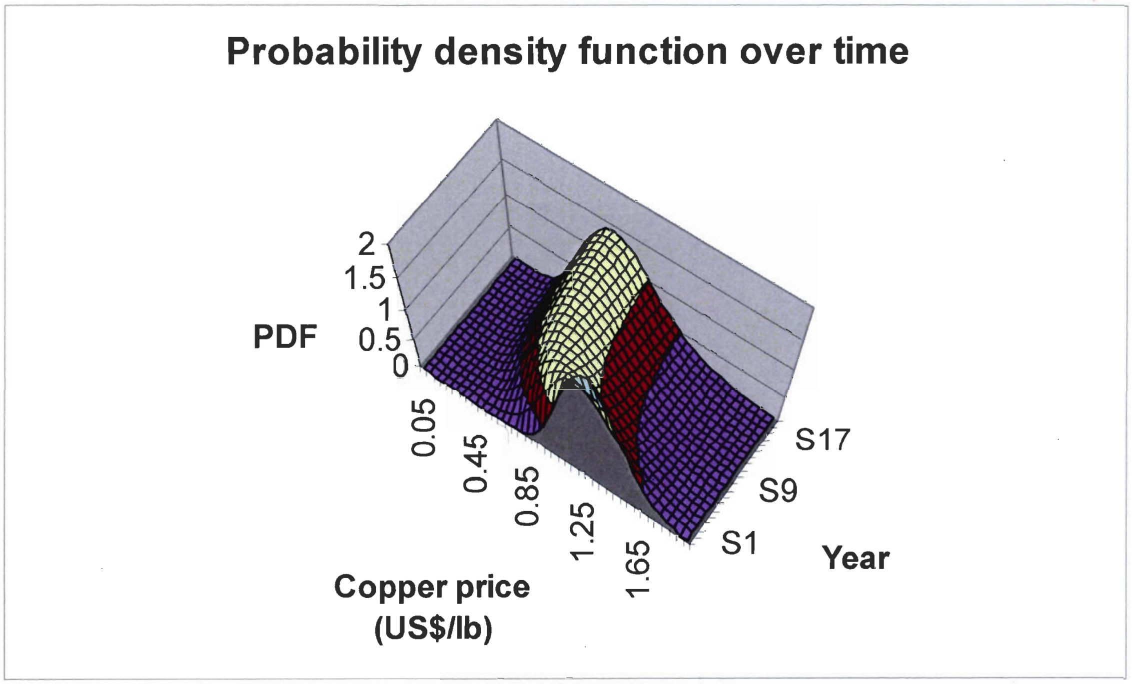


Figure 15. Probability density function over time (3-D view)

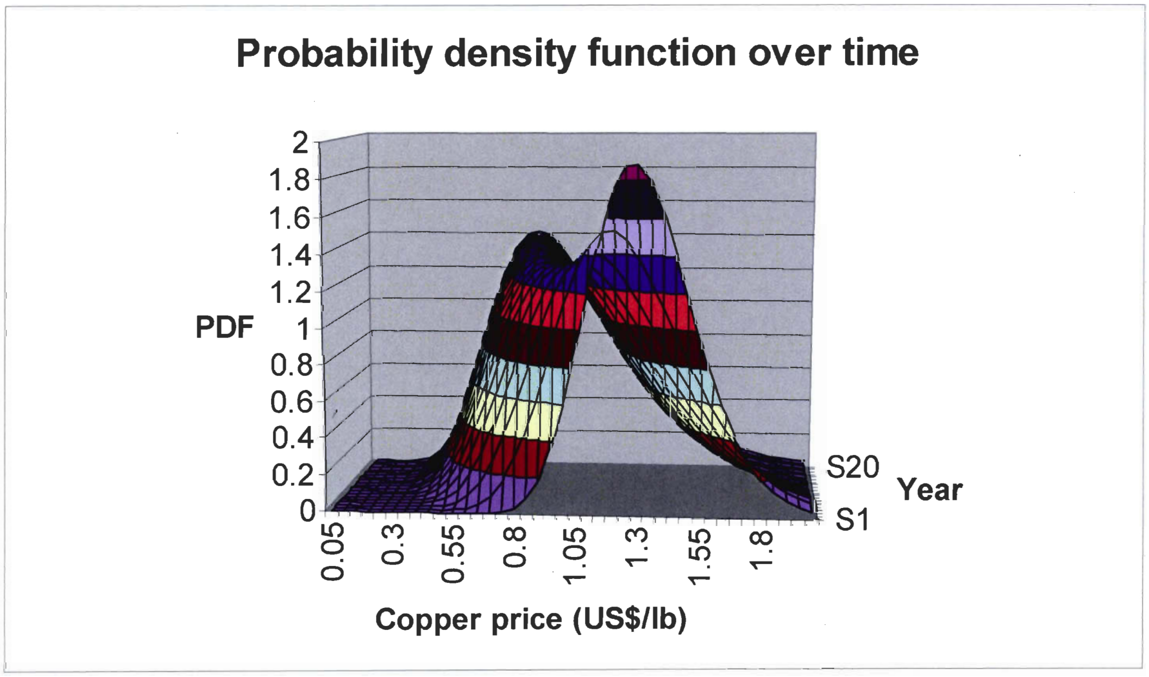


Figure 16. Probability density function over time (alternative 3-D view)

These functions can be implemented into any simulator that will run Monte Carlo simulations on price. However, it is possible to program a code that will calculate price probabilities when one inputs the model price parameters.

Assuming the natural log of the commodity price is normally distributed, one can obtain the price at time t + Δ by utilizing the median at time t and its associated variance. This methodology has the strong advantage of being dynamic, while the simulation price profile will be more realistic. The price follows a pattern that takes into account the previous price data, which generates a realistic price profile, whereas a typical Monte Carlo simulation in the mining industry forces the price pattern to come back to its mean for each iteration. In a conventional Monte Carlo simulation, there is usually no link to the previous data, basically because the simulation is run from a price list with particular variation properties that take into account the predicted price at time t. Moreover, it is tricky to set up linked assumptions in commonly used software like Crystal Ball because formulas are not accepted in the assumed cells (defined assumptions).

3.3 Dynamic mean reverting simulated price profile

In the copper price model, the price at time t + Δ is linked to the median at time t along with its associated variance. Note that the defined assumption draw (Crystal Ball draw) can now be defined and utilized since the simulated item is not the price per se. Then, the formula to calculate the price outcome comes from applying the natural log function on the first assumption where the natural log of the commodity is normally distributed:

$$S_{t+\Delta} = Med_t(S_{t+\Delta}) \cdot e^{(Defined\ Assumption\ Draw \cdot \sqrt{Var_t(S_{t+\Delta})})}$$

where

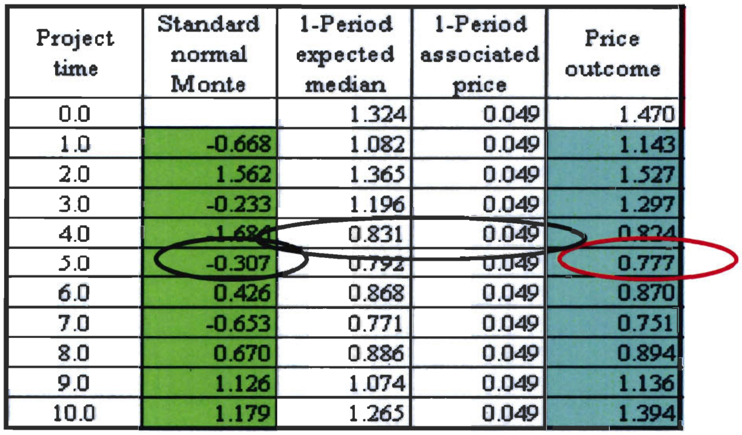
$S_{t+\Delta}$ = Price outcome at time t + Δ

$Med_t(S_{t+\Delta})$ = Median price at time t

$Var_t(S_{t+\Delta})$ = Associated variance at time t

Defined assumption draw = Normal distribution draw where the mean is 0 and standard deviation = 1

Using an Excel sheet, it is possible to build a table that gathers the parameters stated above to obtain the price outcome at different periods of time. Table 17 shows an example of a copper price profile over time using a Monte Carlo simulation.



Project time	Standard normal Monte	1-Period expected median	1-Period associated price	Price outcome
0.0		1.324	0.049	1.470
1.0	-0.668	1.082	0.049	1.143
2.0	1.562	1.365	0.049	1.527
3.0	-0.233	1.196	0.049	1.297
4.0	1.684	0.831	0.049	0.824
5.0	-0.307	0.792	0.049	0.777
6.0	0.426	0.868	0.049	0.870
7.0	-0.653	0.771	0.049	0.751
8.0	0.670	0.886	0.049	0.894
9.0	1.126	1.074	0.049	1.136
10.0	1.179	1.265	0.049	1.394

Table 17. Copper price profile for one Monte Carlo simulation

In the latest example, the price outcome (red circle) at time 5 is determined by the median at time 5 along with its associated variance (large black circle). The Monte Carlo draw is defined by the probabilities of a normal curve. With 20,000 iterations, the author should have enough points to project the curve and eliminate subset systematic errors. In Figure 17 below, the results from the simulations shows the expected price outcome at time 5.

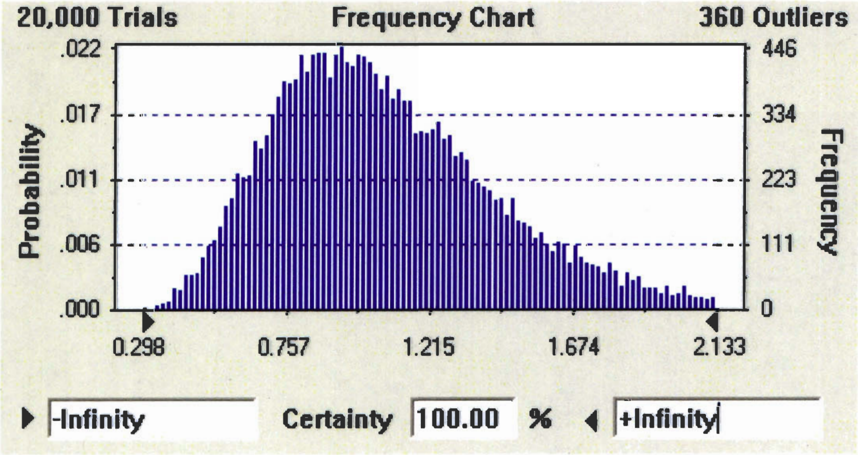


Figure 17. Crystal Ball Monte Carlo simulation for the expected price at time 5

If the Real Options Approach is applied, then the forward price is taken in the cash flow calculations. The forward price is the expected price minus the metal risk on the market.

$$E[K_{t+\Delta}] = E[S_{t+\Delta}] \cdot RDF$$

where

$E[K_{t+\Delta}]$ = Forward price at time t plus delta

$E[S_{t+\Delta}]$ = Expected price at time t plus delta

RDF = Risk discount factor

Table 18 below shows the forward price when the price outcome is multiplied by the risk discount factor.

Project time	Standard normal Monte	1-Period expected median	1-Period associated price	Price outcome	Risk discount factor	Risk-adjusted median
0.0		1.324	0.049	1.470	1.000	1.470
1.0	0.000	1.218	0.049	1.324	0.999	1.323
2.0	0.000	1.138	0.049	1.218	0.998	1.215
3.0	0.000	1.077	0.049	1.138	0.997	1.134
4.0	0.000	1.031	0.049	1.077	0.996	1.073
5.0	0.000	0.994	0.049	1.031	0.995	1.026
6.0	0.000	0.966	0.049	0.994	0.995	0.989
7.0	0.000	0.943	0.049	0.966	0.995	0.960
8.0	0.000	0.925	0.049	0.943	0.994	0.938
9.0	0.000	0.910	0.049	0.925	0.994	0.920
10.0	0.000	0.899	0.049	0.910	0.994	0.905

Table 18. Risk-adjusted median (forward price)

It is now easy to obtain simulated price profiles. By linking the forward price from time 0 to time 10, it is possible to graph the results. Figures 18 and 19 below show how the simulation mechanisms take place at each run. For each year, the price continues its path from its last position. This way, the volatility from year 0 to year 10 reflects the copper price market. The mechanism is called the Markovian property.

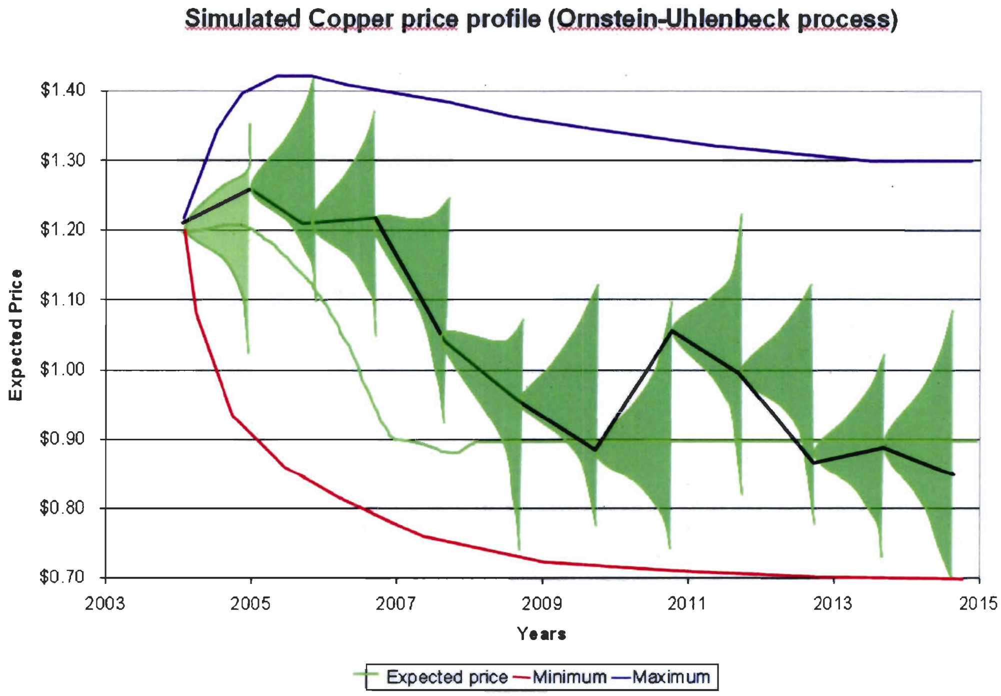


Figure 18. Simulated copper price profile using the Ornstein-Uhlenbeck process

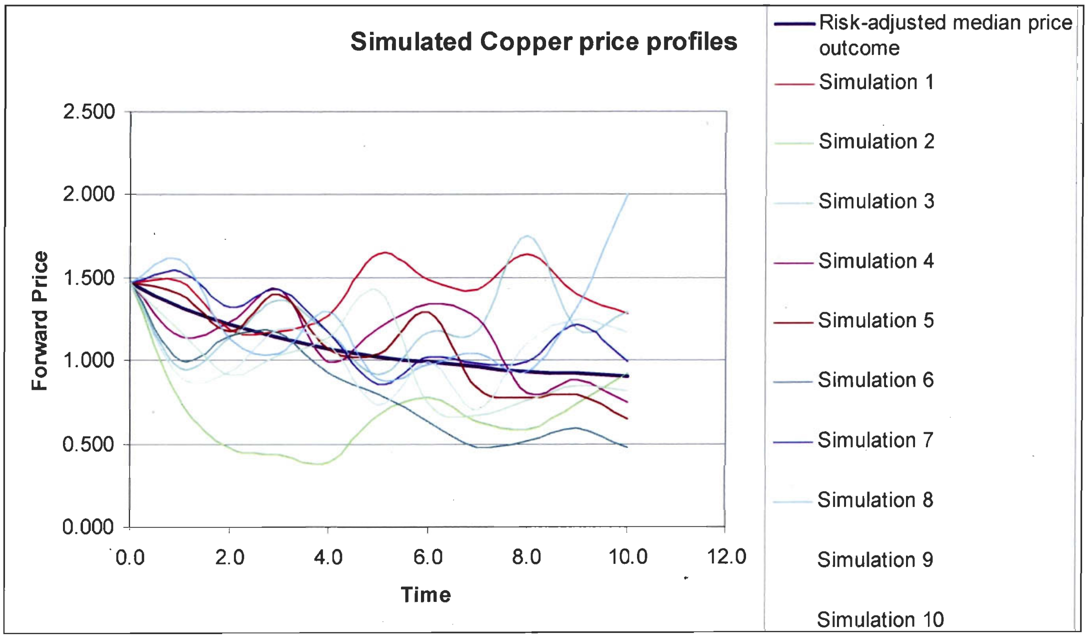


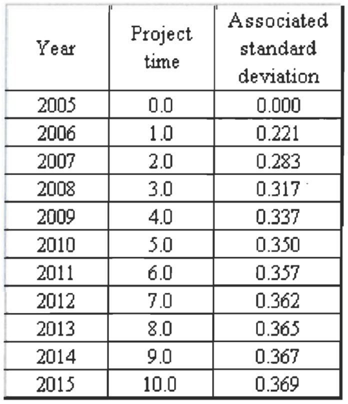
Figure 19. Forward price curve with 10 simulated copper price profiles

For the volatility of the simulated copper prices in the example, an averaged standard deviation of 0.2405 is found, whereas the standard deviation of copper is 0.2456. The results are excellent and the copper price seems to be well built, reflecting the intrinsic properties of copper variation on the market. With 20,000 trials, the mining project is assessed with a range value instead of a point estimate. At a certain interval of confidence, the mine manager can state that the project will be reporting a NPV ranging from X to Y with a mean of Z. It is very important then that the simulated price profiles are realistic.

3.4 Other simulation techniques in the mining industry

If one sets a dynamic mean reverting model as a reference for comparing other simulation techniques, there are two conditions that should be met to mimic copper price behaviour on the market.

1) The standard deviation of the simulated profiles should be close to 0.2456 (copper price volatility)
2) The simulated annual price should have an associated standard deviation next to the associated standard deviation of the mean reverted copper price (see Table 19):



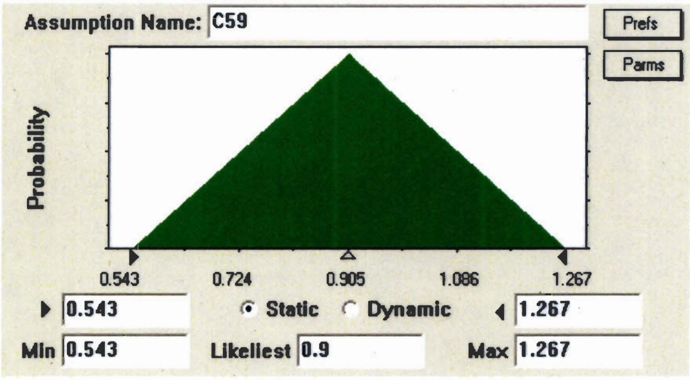
Year	Project time	Associated standard deviation
2005	0.0	0.000
2006	1.0	0.221
2007	2.0	0.283
2008	3.0	0.317
2009	4.0	0.337
2010	5.0	0.350
2011	6.0	0.357
2012	7.0	0.362
2013	8.0	0.365
2014	9.0	0.367
2015	10.0	0.369

Table 19. Associated standard deviation that reflects copper price behaviour

There are two common simulations techniques that are used in the industry for running Monte Carlo simulations on Crystal Ball.

<u>Triangular distribution with a percentage range technique</u>

The first one is to define a range in percentage that the metal price floats over time. For instance, it is common in the mining industry to set the copper price at $0.9/lb. plus or minus 10%. As shown in Figure 29, it is a plain triangular distribution. In this example, the range is set to ±40% to get the same volatility as the copper price on the market at 0.2456.



Assumption Name: C59

▶ 0.543 ● Static ○ Dynamic ◀ 1.267

Min 0.543 Likeliest 0.9 Max 1.267

Figure 20. Triangular assumption of the copper price with a ±40% range at year 2015

In Figure 21 below, the simulation mechanisms show that the volatility is less than the dynamic mean reverting model, since the price is forced back to its long-term price at each run.

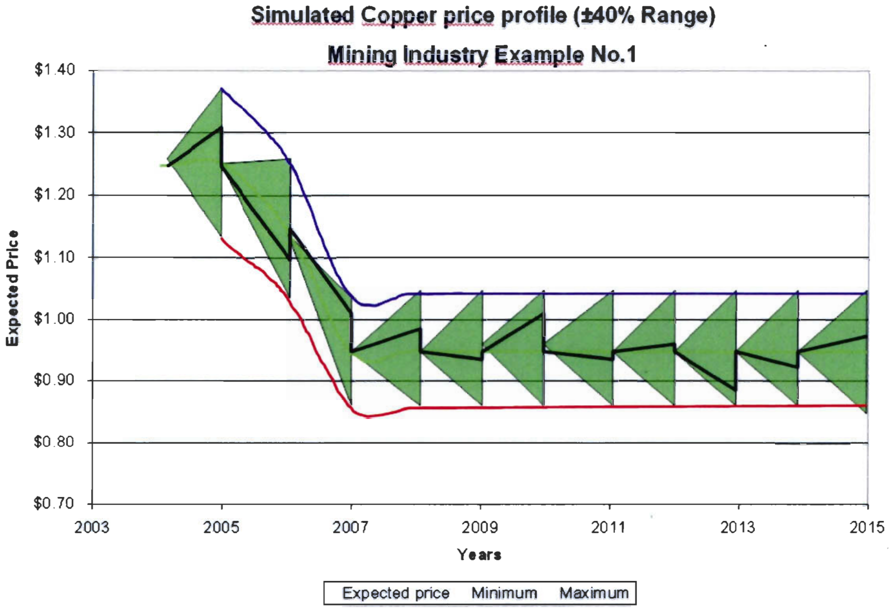


Figure 21. Simulated copper price profile with a ±40% range

Combined with a triangular distribution, it greatly limits the price from drifting over time (see Figure 22). It is as though the simulated price gives results the evaluator wants to see. This technique underestimates the risk associated to the metal price. When evaluating a mining project, the NPV could be deceptive.

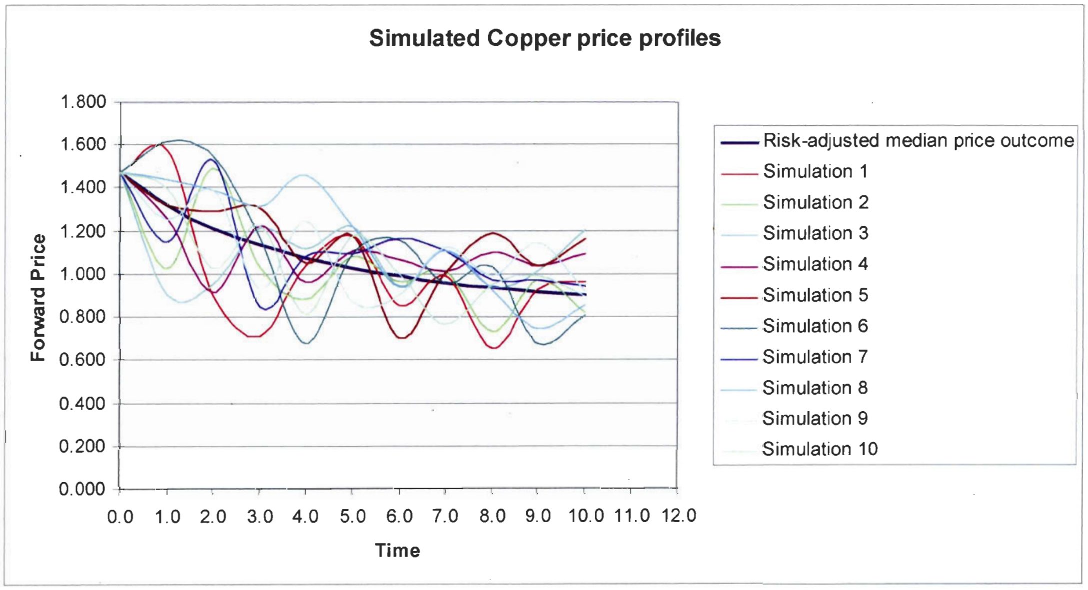


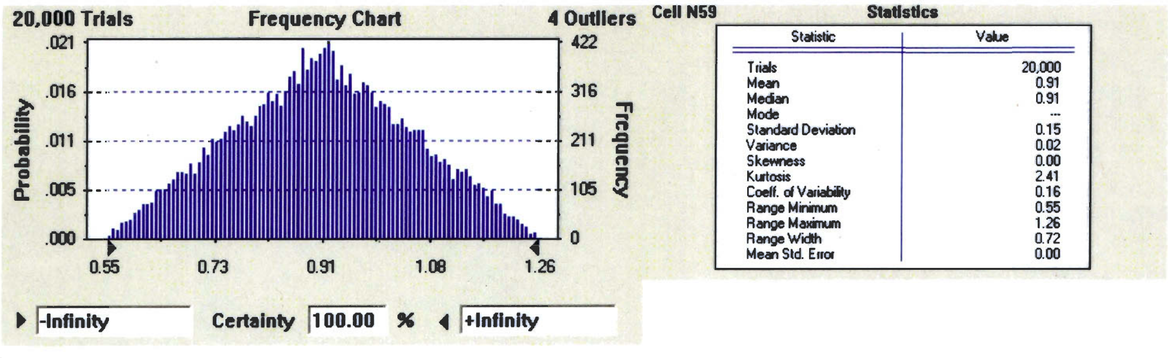
Figure 22. Crystal ball simulation with a ±40% range

The results are interesting since they appear to be satisfying for the modest evaluator. Table 20 below shows the simulated copper price profile with its respective standard deviation.

Simulated price profiles with +/- 40% range (Crystal Ball) Industry example no.1										
Risk-adjusted median	Simulation 1	Simulation 2	Simulation 3	Simulation 4	Simulation 5	Simulation 6	Simulation 7	Simulation 8	Simulation 9	Simulation 10
1.470	1.470	1.470	1.470	1.470	1.470	1.470	1.470	1.470	1.470	1.470
1.323	1.577	1.028	0.906	1.262	1.323	1.618	1.150	1.435	1.251	1.394
1.215	0.906	1.491	0.950	0.917	1.292	1.546	1.524	1.391	1.391	1.027
1.134	0.714	1.032	1.213	1.225	1.304	1.168	0.851	1.316	0.930	1.225
1.073	1.041	0.880	1.116	0.966	1.051	0.676	1.084	1.459	1.245	0.815
1.026	1.174	1.083	1.220	1.106	1.174	1.094	1.094	1.220	0.866	1.174
0.989	0.857	0.967	0.945	1.066	0.703	1.154	1.165	0.934	0.901	1.000
0.960	0.993	1.014	1.110	1.014	1.003	0.960	1.110	1.110	1.121	0.768
0.938	0.656	0.729	0.938	1.104	1.188	1.032	0.969	0.927	0.990	0.948
0.920	0.930	0.971	1.012	1.042	1.042	0.674	0.971	0.746	1.144	0.991
0.905	0.965	0.825	1.207	1.096	1.166	0.804	0.945	0.855	0.915	0.895
Avg	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7	σ_8	σ_9	σ_{10}
0.2435	0.31989151	0.27260155	0.22708411	0.18038475	0.2384275	0.30517183	0.24597201	0.15531821	0.24237001	0.24781143

Table 20. Simulated copper price profiles with a ±40% range and its respective volatility

The averaged standard deviation of the simulations is close to what it is observed on the copper price market at 0.2435. If the analysis of the simulations stops right here, however, some important information about apparent project risks could be missed. When we analyze the standard deviation of copper price distribution, we observe that it is far from correctly assessing the volatility of the metal price. In fact, with this technique, the risk associated to the metal price drops as time goes by. In the year 2015, the volatility of the simulated copper price is 0.15 (see Figure 23).



Statistic	Value
Trials	20,000
Mean	0.91
Median	0.91
Mode	---
Standard Deviation	0.15
Variance	0.02
Skewness	0.00
Kurtosis	2.41
Coeff. of Variability	0.16
Range Minimum	0.55
Range Maximum	1.26
Range Width	0.72
Mean Std. Error	0.00

Figure 23. Frequency chart for simulated copper price in the year 2015

This raises doubt because the value is quite low for a price that is predicted to happen in 10 years. Then, the evaluator needs to trace the volatility for 2005 to 2015 in order to see if it makes sense.

Year	Simulated standard deviation	Mean reverting s.deviation
2005	0	0
2006	0.22	0.221
2007	0.2	0.283
2008	0.19	0.317
2009	0.18	0.337
2010	0.17	0.350
2011	0.16	0.357
2012	0.16	0.362
2013	0.15	0.365
2014	0.15	0.367
2015	0.15	0.369

Table 21. Simulated standard deviation compared to mean reverting standard deviation

In Table 21 above, volatility is seen to decrease with time. It should be the contrary. The mean reverting standard deviation seems to give better results. With a 40% range that was originally needed to mimic the standard deviation of the copper price, the annual standard deviation is compromised. The volatility decreases with time because it is related to the price estimates that themselves decrease for the next 10 years. Volatility is related to price. When a percentage range is applied, low price calls for lower volatility.

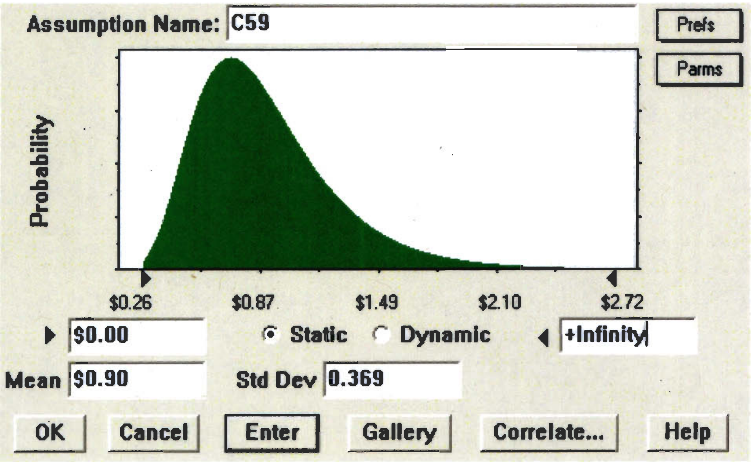
Lognormal distribution with associated standard deviation technique

There is another common way of simulating metal price in the mining industry that tries to eliminate the problem of having unrealistic annual standard deviation.

This method uses the same long-term price profile as the percentage range method. It also uses the associated standard deviation profile of the mean reverting process found in section 3.2. The simulated price profiles are then produced by associating the price profile to its associated standard deviation. Hence, in the simulator, the "defined" assumptions can be easily integrated (see Table 22 and Figure 24).

Industry example no.2		
Year	Risk-adjusted median price outcome	Mean reverting s.deviation
2005	1.470	0
2006	1.323	0.221
2007	1.215	0.283
2008	1.134	0.317
2009	1.073	0.337
2010	1.026	0.350
2011	0.989	0.357
2012	0.960	0.362
2013	0.938	0.365
2014	0.920	0.367
2015	0.905	0.369

Table 22. Simulation assumptions (long-term price profile and its associated standard deviation)



Assumption Name: C59

▸ $0.00 ● Static ○ Dynamic ◂ +Infinity

Mean $0.90 Std Dev 0.369

Figure 24. Defined assumption on copper price in the year 2015

In Figure 25 below, the simulation mechanisms show that volatility is again less than the dynamic mean reverting model because the price is also forced back to its long-term price at each run. It is mainly attributed to the fact that this technique is not derived from the Markovian property.

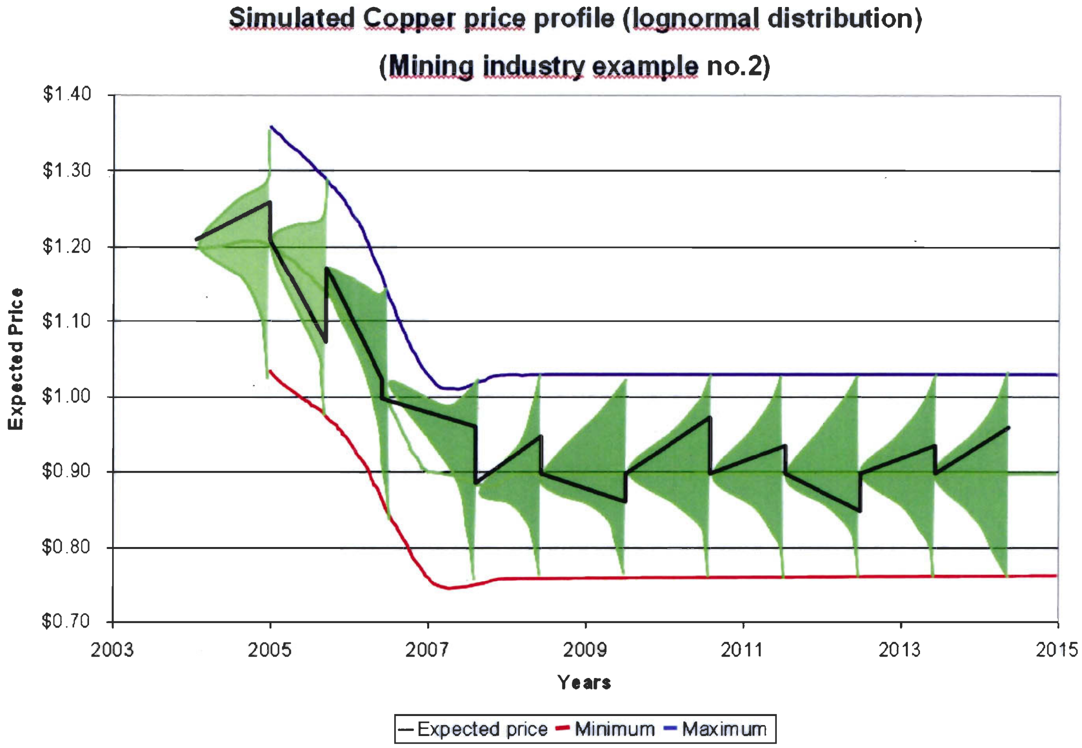
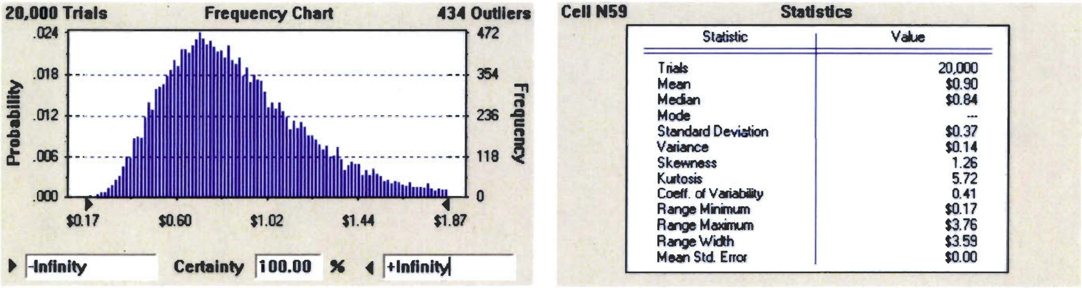


Figure 25. Simulated copper price profile with a lognormal distribution

This model is not restrained to a triangular distribution and, as such, the volatility should be greater. However, compared to the results from the dynamic mean reverting process, the volatility of the price profile is still lower because the price movements are limited by forcing each run to start over from the mean.

To assume accuracy in the results as described in Sabour and Poulin (2005), the result of running 20,000 simulations on copper prices from the year 2005 to the year 2015 shows, for each year, a lognormal price distribution pattern as had been assumed in the first place. Figure 26 shows the frequency chart for copper at year 15, while Figure 27 illustrates only 10 simulation price paths that closely follow the long-term trend line.



Cell N59 **Statistics**

Statistic	Value
Trials	20,000
Mean	$0.90
Median	$0.84
Mode	---
Standard Deviation	$0.37
Variance	$0.14
Skewness	1.26
Kurtosis	5.72
Coeff. of Variability	0.41
Range Minimum	$0.17
Range Maximum	$3.76
Range Width	$3.59
Mean Std. Error	$0.00

Figure 26. Frequency chart for simulated copper price (associated volatility) in the year 2015

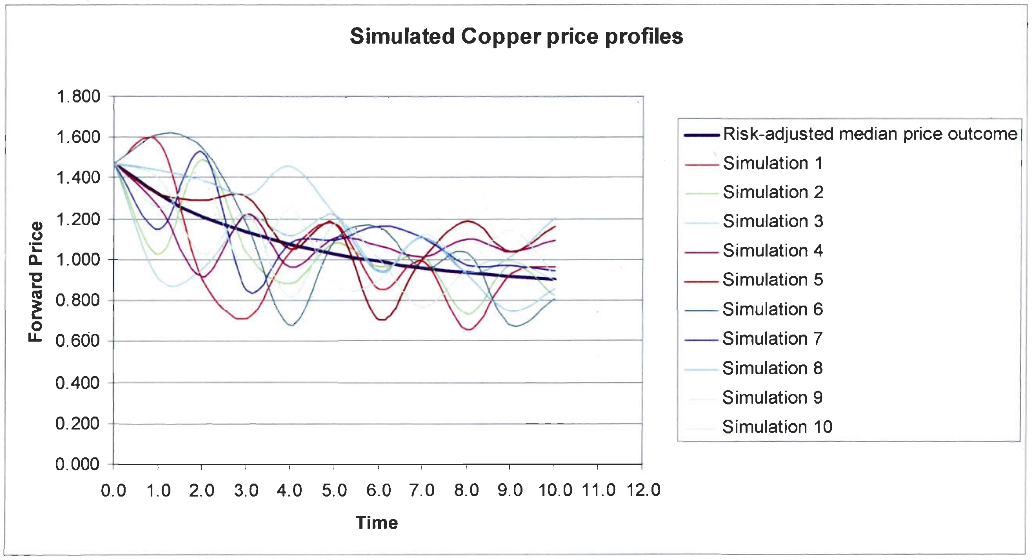


Figure 27. Crystal ball simulation with associated standard deviation

Now copper price volatility is locked to the long-term price. Since it starts at the mean for each run, the deviation cannot go far since the next year the price is set back to the long-term price. It is an artificial volatility that mimics copper price behaviour but does not generate natural price profiles.

If the evaluator does not pay attention to that fact, he could assess a high-risk mining project with a low-risk price profile. It might result in accepting bad projects, due to the narrow NPV range that complies with the merit criteria of the company. In the next section, the effect of the three price simulating techniques on a typical copper mine project is presented.

3.5 Effect of using different simulation techniques on the net present value (NPV)

Given a 10-year mine project that will be extracting 100M pounds of copper a year, the capital cost to start up the mine is $300M and the unit cash cost is estimated to be $0.50/lb., at a 15% rate of return and with the price models set on the previous section, what is the NPV for the three simulation techniques?

The figures shown below reveal what was explained in the previous section. With the triangular distribution, the NPV is $45.6M. The percentage of profitability is established at 90.22%, which is considered high. At a 95% confidence interval, the project ranges from -$24M to $113M. It is the narrowest range within the group, mainly due to the low volatility of annual copper prices (remember the standard deviation of 0.15 at year 15 when it should be close to 0.38).

The second technique with the lognormal distribution with associated deviation (not dynamic) has a similar NPV at $45.1M. More volatility is observed due to the fact that it has a realistic annual standard deviation, but the NPV range is still narrow. The project ranges from -$36M to $139M, which is due to the lack of volatility in the price profiling. The simulated prices are forced to come back to the base case price.

Finally, the results for the dynamic mean reverting model (as shown in Table 23) are interesting. There is a higher NPV at $70.6M, which is more than the two previous methods, but the risk is greater, with a profitability index of 70.89%. The NPV range is wide, from -$115M to $328M. The project, even if it seems profitable at first glance, has a risk profile. The evaluator can get a clear picture of the challenges posed by the project. If the company is not financially solid, they might want to reduce the risk by hedging future production, by having a joint venture with a major or by delaying production until market conditions are more stable.

	Scenario		
Measure	40% Range	Associated SD	Mean Reverting
Mean	45.6M$	45.1M$	70.6M$
Standard Deviation	35M$	45M$	113M$
% Profitability Certainty	90.22%	84.36%	70.89%
95% Confidence Level	-24M$ to 113M$	-36M$ to 139M$	-115M$ to 328M$

Table 23. Economic measures according to three price simulation technique

This chapter reviewed the driving forces behind metal prices. Primarily, supply and demand constitute the leading factors that regulate price. On a long-term basis, price should reflect the unit cost to the last marginal producer in the market. A study of the market price of the metal will reveal parameters inherent to the metal price behaviour that is suitable for evaluation. These parameters help the evaluator to build a realistic price model that can be used for evaluating future projects.

In section 3.2.2, a dynamic lognormal mean reverting price model with copper was used as an example to show the mechanisms of the methodology. It is important to note that many of the model's parameters are difficult to calculate without a thorough econometric study. However, it is always possible to commission economists to obtain missing data in order for the evaluator to perform a project assessment with a credible price model. As part of the thesis, the mean reverting technique based on Schwartz will be used in the next chapters for the simulations to ensure a relatively good data set to illustrate the differences between the classical evaluation approach (NPV) and the Real Options Approach (ROA).

4 Valuing Mining Properties Using ROA

In this chapter, the Least-Squares Monte Carlo (LSM) simulation is discussed in further detail. The LSM is a numerical simulation technique that forms the basis of the thesis for mine evaluation using ROA. In Section 4.2, a case study was performed on a mining project at Xstrata Raglan Mine. The deployment of ROA shows the measurement of operating flexibility that was not previously taken into account in the mine model at the existing operations.

4.1 The LSM method

After selecting the appropriate stochastic model as seen in Chapter 3 for each metal and estimating its parameters, the next step is to generate correlated sample paths according to the Markovian property. These correlated sample paths are generated using the stochastic model determined in the previous step and the correlation matrix that determines the correlation of each metal price with the prices of other metals produced simultaneously from the mine. In this respect, the continuous time is divided into small time steps, each with a length of Δt. If the price of a metal follows, for example, the stochastic model presented in Section 3.2.2, future sample paths of risky metal prices can be generated as follows Glasserman (2004):

$$P_{(i+1,j)} = P_{(i,j)}e^{-k\Delta t} + P^{*}(1-e^{-k\Delta t}) + \sigma\sqrt{\frac{1}{2k}(1-e^{-2k\Delta t})}W_{(i+1,j)}$$

As indicated from the above equation, the price at period i+1 along the sample path j depends on the price at the previous period i along the same path j. The change in price between two successive periods is directly related to the expected long-term equilibrium level p^{*}, the speed of reversion k, the standard deviation σ, the length of each time step Δt and the correlated random number W. This simulation process, based on the Markovian property, ensures that the evolution of future prices is realistically represented.

After generating simulated metal price paths as described above, the cash flows at each discrete date along the simulated metal price paths are defined based on the metal price, the unit production cost and the tax and royalty rates. Then, the optimal switching policy among the different production alternatives is determined, conditional on the metal market condition using an extended version of the LSM method. As described by Abdel Sabour and Poulin (2005), to evaluate mining projects under the flexible production model, it is assumed that the mine status can be changed at some discrete dates per year with a time step of Δt apart. Then, the optimum production policy at each date is determined by comparing the expected values of the mine in the three alternative modes: open, closed and abandoned. Let $H_0(i,j,R)$ denotes the value of the open mine at time i along the sample path j where the remaining reserve was R and $H_c(i,j,R)$ denotes the value of the closed mine. The condition for closing the active mine at time i along the path j is:

$$\hat{H}_o(i,j,R) \leq \hat{H}_c(i,j,R) - K_c$$

and the condition for reopening the closed mine is:

$$\hat{H}_c(i,j,R) \leq \hat{H}_o(i,j,R) - K_o$$

where K_c and K_o are the switching costs to close and reopen the mine respectively.
Otherwise, the open mine should be permanently abandoned if:

$$\hat{H}_o(i,j,R) \leq K_a \geq \hat{H}_c(i,j,R) - K_c$$

While the closed mine should be abandoned if:

$$\hat{H}_c(i,j,R) \leq K_a \geq \hat{H}_o(i,j,R) - K_o$$

where K_a is the abandonment cost.

The expected values of the mine in the open and the closed modes, H_o and H_c, are estimated conditional on the metal prices at each date using the conditional expectation functions for the open and the closed mine. The coefficients of these basis functions are determined using the lease-squares regression. As explained in Longstaff and Schwartz (2001), the unknown functional form of H_o and H_c can be approximated by a linear combination of M basis functions $L_n(P)$, including Laguerre polynomials, trigonometric series or simple powers of the metal price P, such as

$$H_o(i,j,R) = \sum_{n=0}^{M} a_n L_n(P)$$

and

$$H_c(i,j,R) = \sum_{n=0}^{M} b_n L_n(P)$$

The coefficients a_n and b_n for the values of the open and the closed mine functions are estimated at each date using the least-squares regression. Then, at time i along the path j the values of the open and the closed mine can be obtained by substituting for the metal price P(i,j) in the basis function, such as

$$\hat{H}_o(i,j,R) = \sum_{n=0}^{M} \hat{a}_n L_n(P(i,j))$$

and

$$\hat{H}_c(i,j,R) = \sum_{n=0}^{M} \hat{b}_n L_n(P(j,i))$$

After revising the cash flows according to the determined optimum operating policy of the mine at the discrete exercise points throughout all the simulated metal price paths, the mine value at time 0 is determined by discounting the cash flows and averaging over the number of simulated metal price paths.

4.2 Article abstract no. 1: Mine 2 evaluation method at Raglan[2]

Since the premises were set to evaluate mining projects using ROA with LSM, a case study was performed on Mine 2 at Raglan. Mine 2 underground operations were first analyzed and appraised using conventional merit measures like NPV and IRR criteria. Following the current mine planning process, the engineer obtained economical values for the project that dictated the decision-making process, whether it would be a go or a no-go. Using ROA with LSM as described in the previous sections, the mining project was evaluated on a continuous horizon where the decision making process can take place in multiple moments in the future. Operating flexibility values were found and a new mine planning approach took place.

Résumé de l'article no. 1: Évaluation de mine 2 à Raglan.

Les prémisses d'évaluation avec la technique des options réelles avec LSM ayant été démontré, une étude de cas a été fait sur le projet de mine 2 à Raglan. Les opérations minières souterraines de mine 2 sont premièrement analysées avec les critères de mesure conventionnels tels que le VPN et le TRI. En suivant le processus actuellement en place, l'ingénieur obtient un résultat lui permettant de justifier ou non le départ du projet. En utilisant la technique des options réelles avec LSM comme décrit dans les sections précédentes, le projet minier sera plutôt évalué sur un horizon continuel où la décision de départ peut survenir à des moments multiples dans le futur. La valeur de cette flexibilité d'opération a été trouvée et une nouvelle approche de planification minière a pris place.

---

[2] Lemelin, B., S. A. Abdel Sabour and R. Poulin. 2006. Valuing Mine 2 at Raglan using real options. *International Journal of Mining, Reclamation and Environment* 20:46–56.

Valuing Mine 2 at Raglan Using Real Options: a Least-Squares Monte Carlo Approach

Bruno Lemelin[*], Sabry A. Abdel Sabour[†] and Richard Poulin[‡]

[*]Falconbridge Ltd., Canada

[†]Assiut University, Egypt

[‡]Université Laval, Canada

Valuation of a mining project is always challenging. For complex projects, such as the case of Raglan, when the mine consists of numerous mineralized zones and produces many payable metals, the valuation process becomes very complicated. In this case, while performing typical discounted cash flow analysis (DCF) may give a gross outlook of the project value, it is well understood that such conventional methods cannot capture the value of management flexibility to change decisions over time in response to the new market conditions.

In contrast to the conventional DCF methods, the Real Options Approach (ROA) is more efficient for dealing with the management responses to the uncertain future outcomes and consequently can determine the more precise and accurate value of a project through its production stages. For a project with only one source of uncertainty, real options valuation can be carried out using either finite difference or binomial trees methods. In the case of Raglan, where there are eight payable metals, neither of the methods can be applied since they suffer the curse of dimensionality as the number of state variables is greater than one. In such cases, where there are two or more uncertain state variables, the newly developed Least-Squares Monte Carlo (LSM) approach is the most practical tool for valuing real options.

Mine 2 is the next underground mine to come into production at Raglan. This paper presents the Real Options Approach and its application for valuing Mine 2. The LSM approach is applied for valuing the mineralized zones of Mine 2 taking into account the uncertainty associated with the prices of the all payable metals simultaneously as well as the management flexibility to switch among the different operating alternatives. The paper also demonstrates the flexibility options the manager gets with ROA by analyzing multiple production scenarios.

Keyword: Mine valuation; Real options; Raglan mine

Introduction

Falconbridge's Raglan Mine has begun its commercial production in 1998. The Raglan operations have started with Katinniq mine, which at the time of the commissioning represented an area with the best prospects. Along with Katinniq, the company exploited two open pit mines: Mine 2 and Mine 3. Throughout the years, the ore at surface depleted and underground operations started at Mine 3. The actual plan is to pursue mining of Mine 2 mineralization from underground with cut and fill and long-hole stopes. At the beginning of 2005, an economic study was deposited addressing positively the long-term viability of Mine 2. It is scheduled to be in development in 2005 starting first ore in 2006.

A conventional financial analysis was performed to economically appraise Mine 2. Like Katinniq and Mine 3, Mine 2 is composed of a multitude of mineralized zones which makes it hard to establish an optimized mine planning on the first pass. The zones are formed apart from each other at different depths with different grades and tonnages. Indeed, several scenarios have been studied with the long-term mine engineering team. At some point, there is a certainty that with the conventional financial techniques, the value of Mine 2 is not maximized, mainly due to the linear process of evaluating the project with merit measures like NPV and IRR.

A classic NPV study fails to address the issues of numerous uncertainties like the metal prices. When market conditions are shifting, the change in the value of each zone is important. The production strategy might be different, especially at Mine 2, which is considered a marginal satellite source of ore. If the nickel price plummets enough to affect the profitability, Falconbridge must have the flexibility of temporarily shutting down high cost zones. On the other hand, when the metal prices soar, the operations may consider expanding its production. This flexibility has a value that cannot be accounted for in a basic financial model. The economic evaluation of Mine 2 can improve in precision and accuracy using the Real Options Analysis (ROA) methodology. The business model includes the flexibility of choosing different production options throughout Mine 2 lifetime depending on variables such as metal prices, foreign exchange and fuel. It is a dynamic process that forecasts the value of the project should an expansion or abandonment is decided.

Aside from giving detailed information about flexibility, the ROA handles better the financial risks of the project. It segregates the risk at the source of each cash flow since each of them has a different risk profile. The ROA framework is not limited by one fits-all discount rate. It risk-adjusts every cash flow at root. Once they are adjusted for risk (risk-free cash flows), they are discounted from the future to the present time at a risk free rate like the US treasury bonds. On the other hand, a conventional NPV analysis does not separate the risk from the different source of inflow and outflows (Lemelin et al, 2004). For example, the nickel and gold prices do not act the same on the market. Why should the associated revenues be discounted at the same rate? Many good projects will be dismissed whereas conservative low return projects will be capitalized.

As explained by Miller and Park (2002), although the classic NPV methodology exists since the 50's, its worldwide application begun seriously in the 80's. The same phenomenon applies for the ROA. In 1973, it was an innovation in the financial world when Black and Scholes (1973) initiated a method for evaluating financial assets. Later, Myers (1977) developed the first framework that would generate an evaluation tool for real assets that takes into account future opportunities inherent to a project like expanding or abandoning production. Academicians have worked hard since then to develop real options models. Application of ROA in the mining industry has started in 1985 with the seminal paper of Brennan and Schwartz (1985). Since then, many studies have been carried out in this subject include, for example, Trigeorgis (1990), Mardones (1993), Moyen et al. (1996), Samis and Poulin (1998), Cortazar and Casassus (1998), Abdel Sabour (1999; 2001), and Kamrad and Ernst (2001).

Generally, the value of an option can be estimated using either analytical or numerical methods. Analytical methods like the one developed by Black and Scholes (1973) can be valuable as the options pricing results can be computed quite easily once the assumptions defining the process are in place. However, the assumptions can be too limiting. It has also the disadvantages to become knotty if the model integrates more than one source of uncertainty.

Numerical methods started to generate greater interest in assessing mining properties with real options. The numerical methods include lattice methods, finite difference, and Monte Carlo simulations. The lattice valuation methodology proposed by

Cox et al. (1979) consists in building a probabilistic tree where the option is calculated from the expected state prices through time. Although it is visually attractive, it is too difficult to apply this approach if there are two or more sources of uncertainty. The same fact can be said for the finite difference method proposed by Brennan and Schwartz (1977). This method resides in approximating partial derivative equations that define the evolution of the option value by finite differential equations. Monte Carlo simulation method has been proposed by Boyle (1977) for valuing European options. Many modifications have been introduced to the method; include the work of Tilley (1993), Carriere (1996) and Longstaff and Schwartz (2001), in order to make it suitable for valuing American options.

Thanks to the least-squares method, recently developed by Longstaff and Schwartz (2001), this promising numerical method by simulation is bound to set the quest for new information simpler. It tends to be as accurate and precise as the other methods but with the advantage of being user-friendly. With sound econometric parameters, the manager is expected to factor them in into the model to properly assess the project portfolio. This is what intended in this paper. The authors will extend the least-squares Monte Carlo method (LSM) developed by Longstaff and Schwartz (2001) to evaluate Mine 2 at Raglan. The LSM method has the advantage of being able to deal with multiple sources of uncertainty. The methodology consists in determining the optimal stopping policy using the least-squares regression. The technique is recursive, starting at the maturity of the option and going back to time 0. For the sake of the study, the production capacity, the unit operating cost per zone, and the foreign exchange rate are assumed to be constant. The ROA takes into account seven payable metals at Raglan, and then it deals with seven sources of uncertainty. The aim of the paper is to perform a ROA with multiple uncertainties for each zone at Mine 2. The observation of the results will show us at what point the mine manager gains in additional information. The next section of the paper describes the stochastic processes of metal prices. Then, the valuation results of the mining zones at Mine 2 are presented in Section 3 with the analysis and discussions.

Modelling Metal Prices

The common stochastic processes describing the behaviour of commodity prices are: the Brownian motion models, the mean-reverting models based on Ornstein-Uhlenbeck process, and jump processes (see Chapter 3 in Dixit and Pindyck, 1994, for description of these models). Based on these basic models, Schwartz (1997) has presented a two-factor and a three-factor model that can handle the uncertainty of two and three variables. The most important reason limiting the applicability of the two and three-factor models in practice lies in the difficulty of estimating some of their parameters. In contrast, the parameters of the one-factor models such as the geometric Brownian motion (GBM) and the mean-reverting process (MRP) can be easily estimated from the historical dada of commodity prices. Therefore, these two models are the most widely applied for modelling the behaviour of commodity prices. As explained in Dixit and Pindyck (1994), with the GBM the commodity price S evolves according to the following stochastic model:

$$dS = \alpha S dt + \sigma S dz \tag{1}$$

where α is the drift, σ is the standard deviation and dz is the increment of a Wiener process which equals $\varepsilon_t dt$. The random variable ε_t has zero mean and unit standard deviation. The GBM represented by Equation (1) implies that the commodity price has an expected growth rate of α per year, so that the expected commodity price after time t equals $S_0 e^{\alpha t}$, where S_0 is the current spot price. While some commodities like gold have a constant drift, some other commodities such as base metals do not have a constant drift. Therefore, the suitable stochastic process for base-metal prices is the mean-reverting process (MRP). Schwartz (1997) presented a MRP in which the spot price evolves such as:

$$dS = \eta(\mu - \ln S)Sdt + \sigma S dz \tag{2}$$

where η is the reversion speed and μ is the logarithm of the equilibrium price. According to this process, the expected instantaneous drift is not constant, but depends on the difference between the logarithms of the spot price S and the equilibrium price. If S is lower than the equilibrium price, then the expected drift for the next period is positive. If S is higher than the equilibrium price, the expected drift will be negative. Therefore, the stochastic process of Equation (2) is well reflecting the forces of supply and demand that control the commodity markets.

In valuing Mine 2 at Raglan, the nickel, copper, cobalt and platinum prices are modelled using the MRP described in Equation (2), while the prices of gold, silver, and palladium are modelled according to the GBM of Equation (1). Both Equations (1) and (2) describe the risky commodity prices. In order to simulate the risk-adjusted future prices, the drift in each model should be adjusted for the risk associated with commodity prices. In this respect, the market risk premium of each commodity is estimated using the capital asset pricing model and subtracted from the drift to get the risk-neutral stochastic process. Using the risk-adjusted process for each commodity, sample paths of correlated prices can be simulated at each time step taking into account the correlation coefficients between the metal prices. An illustrative example for the simulated gold and nickel prices using 200 sample paths are presented in Figures 1 and 2.

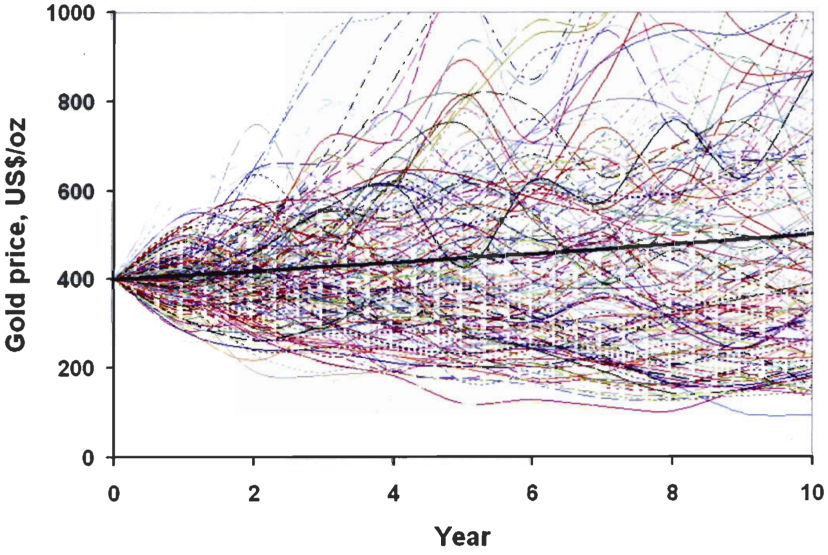


Figure 1. Sample paths of gold price

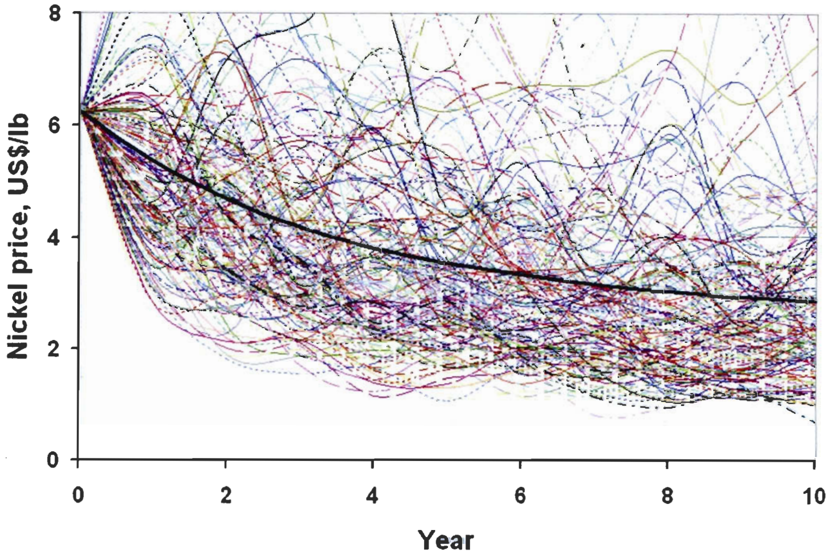


Figure 2. Sample paths of nickel price

Application of the LSM to the case of Raglan properties (Mine 2)

In this section, the results of valuing the 4 zones of Mine 2 using both the classical NPV method and the modern real options technique are presented. First, the mine data are presented, then the valuation methods are briefly described, and finally, the valuation results of both techniques are discussed.

Mine data:

The data of the mining zones at Mine 2 are listed in Table 1. The switching cost to close the zone is estimated at $CAD 50,000 and that to reopen the zone is $CAD 100,000. If the zone is temporarily closed a maintenance cost of $CAD 100,000 per year is incurred to keep the zone ready for production.

Table 1. Data of the 4 mining zones at Mine 2

	Zone			
	2A	**2CAN**	**2ANW**	**2D**
Tonnage, tonne	855214	165105	130530	119532
Full production rate tonne/year	142536	82553	65265	59766
Starting Production	Mid 2006	Mid 2006	Mid 2007	Mid 2008
Average unit operating cost $CAD/tonne	214	255	289	262
Ni grade, %	2.03	3.08	4.1	3.03
Cu grade, %	0.61	0.58	1.04	0.88
Co grade, %	0.04	0.05	0.06	0.07
Au grade, g/tonne	0.3	0.14	0.24	0.14
Ag grade, g/tonne	2	2	2	2
Pt grade, g/tonne	0.5	0.56	1.08	0.79
Pd grade, g/tonne	1.2	2.25	2.5	2.19
Rh grade, g/tonne	0.25	0.48	0.53	0.46

Valuation methods: NPV vs ROA

The NPV valuations are carried out using Monte Carlo simulations while the real options valuations are carried out using the least-squares Monte Carlo method. In order to be able to compare easily the two sets of valuation results, the time step in Monte Carlo simulations is set to be 0.5 year. However, it is well understood that in order to well approximate the early-exercise feature of American-like real options and have accurate valuation results, the time step should be set to be as small as possible. Based on the risk-adjusted stochastic process for each metal, 20,000 simulated paths for the prices of nickel, copper, cobalt, gold, silver, platinum, and palladium are generated using a correlated-random-numbers generator. The rhodium prices are assumed to be constant throughout the mine life due to the lack of the historical date to define its stochastic process.

The NPV valuations are performed by defining the cash flows at each time step throughout all the simulated paths based on the capital spending and production schedules. With the NPV method there is no operating flexibility, then, the NPV of the mine is estimated directly by discounting all cash flows throughout all the simulated paths and averaging over the number of paths. Since the metal prices have been already adjusted for risk, the resultant cash flows are risk-free and are discounted back to time 0 using the riskless discount rate.

In contrast to the NPV method, the Real Options Approach (ROA) takes into account the management flexibility to change the mine operating mode according to the status of metal prices. In reality, a mine manager is not obligated to keep the mine producing when the unit price falls below the unit operating cost. In such cases, the manager should evaluate the other alternatives and select the one that maximizes the mine value. The other alternatives include shutting the mine temporarily and reopen it when the metal prices improve, or permanently abandon the mine. The sound decision cannot be taken without knowing the value of the mine when it is open, temporarily closed, and abandoned. If the mine values corresponding to these three operating modes are estimated, the manager can easily compare the three values and choose the operating mode with the highest value. Then, the bottleneck in maximizing the mine value is to estimate the value corresponding to each operating mode. If the mine produces one metal and has simple capital and production plans, the problem can be easily solved using finite difference or binomial lattice method. In the case of Mine 2, there are 8 payable metals produced from the mine. In addition, the capital spending and production schedules are very complex to be handled by finite difference or binomial lattice. Having 8 uncertain prices and complex capital and production schedules, the suitable method for valuing Mine 2 with the operating flexibility model is the least-squares Monte Carlo method (LSM).

For valuing Mine 2 with the flexibility model using the LSM, it is assumed that every six months the management will optimally choose to: (1) keep the mine status unchanged for the next six months, (2) change the mine status (temporarily shutdown if the mine is open or re-open the mine if it is temporarily closed), or (3) permanently abandon the mine. After defining the cash flows at each time step, the valuation process is carried out recursively. The mine value corresponding to each operating mode is estimated conditional on the current state using the basis function whose parameters are estimated from the least-squares regression. Switching from one operating mode to the other is optimal if the value of the current mode is less than the value of the other mode minus the switching cost. After determining the optimum operating modes and the corresponding cash flows throughout all the paths, the mine value is estimated by discounting the cash flows at the risk-free rate and averaging over the number of simulated paths.

Results and discussion

To save space, the detailed cash flow calculations of zone 2A only will be presented. For the other zones, the estimated value of each one using both techniques along with the total mine value will be reported. Table 2 lists the cash flows of zone 2A throughout its expected life. Production from the zone will start by the mid of 2006 (time 1), the first revenue will be generated by the end of 2006 (time 2) and the last one will be in the mid of 2012 (time 13). Since the NPV method does not account for the operating flexibility, zone reserves are extracted over the estimated 6 full production years in all the simulated paths regardless the metal price status. Therefore, as shown in Table 2, the last cash flow will be at time 13. In real options valuations, the management flexibility to stop production temporarily and reopen the mine later according to the metal price status is taken into account. Accordingly, there is a probability that the zone reserves will not be completely extracted by the end of the 6 years estimated life. This is clearly demonstrated in Figure 3. The expected operating income with ROA extends beyond time 13, which indicates that the reserves have not been depleted throughout some simulated paths.

Table 2. Cash flows of zone 2A using NPV and ROA

NPV valuation (all cash flows are in $CAD millions)																	
Calendar	End 2005	Mid 2006	End 2006	Mid 2007	End 2007	Mid 2008	End 2008	Mid 2009	End 2009	Mid 2010	End 2010	Mid 2011	End 2011	Mid 2012	End 2012	Mid 2013	......
Time	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	......
Operating income	0	0	9.30	7.81	6.55	5.46	4.53	3.73	3.04	2.45	1.93	1.48	1.09	0.75			
Depreciation	0	0	0.32	1.25	1.25	2.32	2.32	2.81	2.81	2.88	2.88	3.13	3.13	3.13			
Taxes	0	0	3.59	2.63	2.12	1.25	0.88	0.37	0.09	-0.17	-0.38	-0.66	-0.82	-0.95			
Capital cost	3.87	0	10.15	0.00	9.71	0.00	3.42	0.00	0.33	0.00	0.76	0.00	0.00	0.00			
Net cash flow	-3.87	0	-4.44	5.19	-5.29	4.21	0.23	3.36	2.62	2.62	1.55	2.14	1.91	1.70			
Discounted net cash flow	-3.87	0	-4.26	4.87	-4.87	3.79	0.20	2.91	2.22	2.17	1.26	1.70	1.49	1.30			
NPV of zone 2A	8.92																
ROA valuation (all cash flows are in $CAD millions)																	
Operating income	0.00	0.00	9.30	7.86	6.70	5.77	5.02	4.41	3.90	3.50	3.16	2.90	2.67	2.47	0.70	0.61	.....
Depreciation	0.00	0.00	0.32	1.20	1.12	1.90	1.72	1.89	1.73	1.62	1.51	1.53	1.41	1.32	0.51	0.45	.....
Taxes	0.00	0.00	3.59	2.66	2.24	1.55	1.32	1.01	0.87	0.76	0.67	0.56	0.51	0.47	0.09	0.07	.....
Capital cost	3.87	0.00	10.15	0.00	9.71	0.00	3.42	0.00	0.33	0.00	0.76	0.00	0.00	0.00	0.00	0.00	.....
Net cash flow	-3.87	0.00	-4.44	5.19	-5.25	4.22	0.28	3.40	2.69	2.74	1.73	2.34	2.15	2.00	0.61	0.53	.....
Discounted net cash flow	-3.87	0.00	-4.26	4.88	-4.83	3.80	0.25	2.94	2.28	2.27	1.41	1.86	1.68	1.53	0.46	0.39	.....
Value of zone 2A	14.56																

As shown in Figure 3, from time 2 to 13, the operating income estimated with both techniques decreases smoothly. This decrease of the operating income is due to the combined effect of mean-reversion in nickel, copper, cobalt and platinum prices, and the risk adjustment. After time 13, the curve representing the operating income of the NPV method is disappeared since the reserves have been completely extracted. For the operating income of the ROA, there is a sharp decline in the curve at time 14, which indicates that the reserves have been completely depleted throughout the majority of the simulated paths. For the remaining paths the zone is either permanently abandoned or the reserves have not depleted due to the temporarily shutting down of the mine in some previous periods. So that, beyond time 13, the operating income of ROA decreases due to the above mentioned reason in addition to the decrease of the number of paths in which the reserves have not been depleted. The small maintenance cost to be incurred during the temporarily closure of zone 2A causes the zone life to be very long throughout some paths. This is because, with the small maintenance cost, it will not be optimal to abandon the zone even when the prices fall substantially below the unit operating cost. Therefore, the expected cash flows extend beyond time 13 until time 40. Also, as shown in Figure 3, the operating income of both techniques are almost the same during the earlier periods, then the one obtained with ROA becomes greater than that of the NPV and the difference between them increases steadily until time 13. This is because with the NPV the mine is kept producing regardless the metal price level so that the reported operating income is the average of the operating incomes for the 20,000 paths. Differently from this, with the operating flexibility analyzed by ROA, there is a possibility that the mine will close temporarily if the prices fall below the unit operating cost. Based on this, the management will avoid the losses during low-price periods and save the reserves until the prices recover back. To illustrate this point, assume that there are only two simulation paths. At some time step, the operating income at the first path is $10 and at the second path is -$5. Also assume that the maintenance cost during the temporarily closure period is $1. Having these data, the average operating income with the NPV will be $2.50. With ROA, there is a flexibility to close the mine if the price goes down (like in the second path), then the cash flow in the second path will be -$1 (the maintenance cost) instead of -$5. Then the average operating income of the two paths will be $4.50, much higher than that of $2.50 estimated with the NPV method.

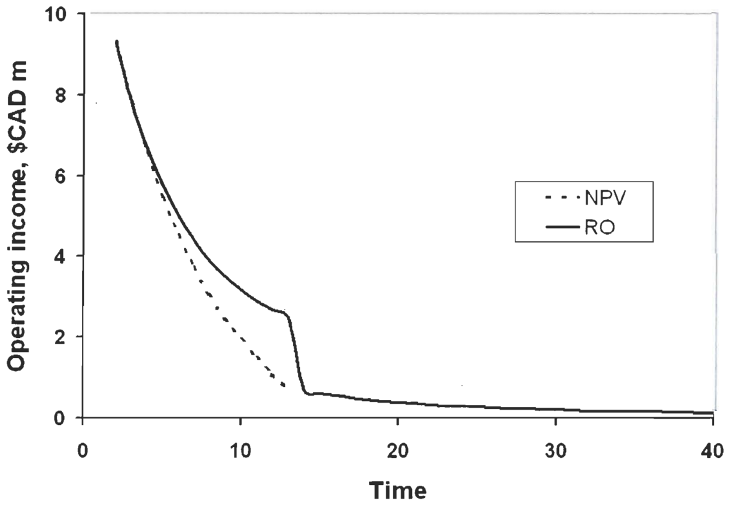


Figure 3. Operating incomes versus time

Table 3 lists the valuation results of both the NPV method and ROA for the 4 mining zones as well as the total mine value. The difference between the value estimated with ROA and that of the NPV represents the value of the operating flexibility to shutdown the zone temporarily and to abandon it. The last column of the table reports the value of the operating flexibility as percentage of the zone value estimated by the NPV method. As indicated in Table 3, the value of the operating flexibility ranges from about 0.5% to 63% of the NPV estimates and that of the mine is 14% (these results agree with the findings of Moyen et al. 1996). Since all other factors are the same for the 4 zones, the difference in the value of the operating flexibility among the zones is due the difference in their lifetimes and the ratio of the unit revenue to the unit operating cost (R/C ratio) for each zone. Looking at Table 3, since both zones 2CAN and 2ANW have the same lifetime and approximately the same R/C ratio, they have very close operating flexibility values. Zone 2D has the same lifetime as zones 2CAN and 2ANW but has a lower R/C ratio, so that its operating flexibility value is much higher than those of zones 2CAN and 2ANW. To investigate how the value of the operating flexibility is related to the zone lifetime, compare the results of zones 2D and 2A. Both zones have the same R/C ratio but the lifetime of zone 2A is three times that of zone 2D. Due to the difference in lifetime between the two zones, the value of the flexibility at zone 2A is much higher than that of zone 2D.

In summary, other things held constant, the additional value that would be gained by shutting down the mine temporarily and abandon it in response to the low metal prices increases with the mine lifetime and decreases with R/C ratio. For short lifetime mines, if the R/C ratio is high, the probability that the prices will fall below the unit operating cost during the short time is very low. Accordingly, the probability to shutdown or abandon the mine is very low. As the lifetime of the mine increases, the probability that the prices will fall below the unit operating cost increases, the probability that the mine will be closed or abandoned increases, and consequently the value of the operating flexibility increases.

Table 3. Comparison between the valuation results of NPV and ROA

Zone	Full-production life, years	Unit revenue/unit operating cost during the first production period	Value, $CAD m		Value of flexibility	
			NPV	ROA	$CAD m	% of the NPV
2A	6	1.61	8.92	14.56	5.64	63.23
2CAN	2	2.00	12.92	12.99	0.07	0.54
2ANW	2	2.10	16.56	16.66	0.10	0.60
2D	2	1.60	6.19	6.59	0.40	6.46
Total value of Mine 2			**44.59**	**50.80**	**6.21**	**13.93**

Finally, the question to be answered is; how to validate the technique used in this article for valuing the zones of Mine 2 using ROA, or, how to be sure that the ROA valuation results are correct. Generally speaking, in order to validate and benchmark a new approach or valuation technique, a valuation example should be worked out using both the new technique and another well-known approved technique. If the new technique produced the same, or very close, valuation results, this means that it is a valid and accurate technique and can be relied upon in more advanced and complex valuation problems. For the case at hand, the validity and accuracy of least-squares Monte Carlo method (LSM) have been examined in Abdel Sabour and Poulin (2005) by applying it to value a gold mine and a copper mine that have been worked out in previous literature using finite difference method. Since the LSM method generated almost the same valuation results as the finite difference technique for both mines, then, the LSM is a valid technique and can be applied to more complex valuation problems. Differently from the simple cases of the gold and the copper mines where the mine produces only one metal and consequently there is only one source of uncertainty to deal with, in the case of Mine 2, there are 7 sources of uncertainty and a complex capital spending schedule. With these complexities, it is very difficult, if it is not impossible, to apply any other technique to validate the valuation results of the LSM. However, the theory of option pricing can be used to give a general validation for the valuation results obtained by the LSM. For example, the theory of option pricing tells us that when the price is much higher than the unit production cost, the ROA and the NPV produce very close valuation results. We can check this point by comparing the valuation results of both techniques for the 4 mining zones. For zones 2A and 2D where the price to cost ratio is relatively low, there is a large difference between the value estimated by ROA and that of the NPV, especially for zone 2A. In contrast, with higher price to cost ratios at zones 2CAN and 2ANW, the value estimated by ROA is very close to that of the NPV method.

Conclusion

In this article, the mining zones of Mine 2, at Raglan mine, have been valued using both the classical NPV method and the ROA. The ROA valuations have been conducted by extending the recently developed least-squares Monte Carlo method. From the results of this article it could be concluded that the difference between the values estimated by the NPV and the ROA depends on the mine lifetime and the selling price to unit operating cost ratio. This difference is significant when the lifetime is long and when the price to cost ratio is low. For higher price to cost ratios, the difference between the value estimates of both techniques is not important, especially when the lifetime is relatively short.

In the real asset valuation world, the business environment is paved with uncertainties and complexities, which delayed the application of ROA at the corporate level. The implementation of the financial model can be tricky for many users. Without a doubt, the construction of an engine that rolls the financial simulations of a mining project is more complex than what the evaluator used to. However, the price of complexity is far less than the reward to gain additional information on a multi-million dollar investment. The LSM method applied in this paper can provide a practical valuation tool that can enhance the application of ROA at the corporate level of mining companies. The method can deal easily with multiple sources of uncertainty and with the complexity of mining investments.

References

Abdel Sabour, S.A., Decision making with option pricing and dynamic programming: development and application. Resources Policy, 1999, 25 (4), 257-264.

Abdel Sabour, S.A., Dynamics of threshold prices for optimal switches: the case of mining. Resources Policy, 2001, 27 (3), 209-214.

Abdel Sabour, S.A., and Poulin, R., Accuracy of least-squares Monte Carlo in valuing mineral properties. In Proceedings of the Fifth Canadian Conference on Computer Applications in the Mineral Industry (CAMI 2005), October 31 - November 3, 2005, Banff, Alberta, Canada.

Black, F., and Scholes, M., The pricing of options and corporate liabilities. Journal of Political Economy, 1973, 81(3), 637- 654.

Boyle, P., Options: a Monte Carlo approach. Journal of Financial Economics, 1977, 4(3), 323-338.

Brennan, M. J., and Schwartz, E. S., The valuation of American put options. Journal of Finance, 1977, 32(2), 449-462.

Brennan, M.J., and Schwartz, E.S., Evaluating natural resource investments. Journal of Business, 1985, 58(2), 135-157, republished in Real Options and Investment under Uncertainty: Classical Readings and Recent Contributions, edited by E.S. Schwartz and L. Trigeorgis , Chapter 16, pp. 335-358, 2001 (The MIT Press: Cambridge, Massachusetts).

Carriere, J., Valuation of the early-exercise price for options using simulations and nonparametric regression. Insurance: Mathematics and Economics, 1996, 19(1), 19-30.

Cortazar, G., and Casassus, J, Optimal timing of a mine expansion: implementing a real options model. The quarterly Review of Economics and Finance, 1998, 38(3), 755-769.

Cox, J.C., Ross, S.A., and Rubinstein, M., Option pricing: a simplified approach. Journal of Financial Economics, 1979, 7(3), 229-263.

Dixit, A.K., and Pindyck, R.S., Investment under Uncertainty. 1994 (Princeton University Press: New Jersey).

Kamrad, B., and Ernst, R., An economic model for evaluating mining and manufacturing ventures with output yield uncertainty. Operations Research, 2001, 49(5), 690-699.

Lemelin, B., Clouet, J-M., Poulin, R., Evaluating multi zone ore bodies of the Raglan nickel deposit using current planning techniques complemented by real option pricing. 2004 CIM Proceedings, Edmonton.

Longstaff, F.A., and Schwartz, E.S., Valuing American options by simulation: a simple least-squares approach. The Review of Financial Studies, 2001, 14(1), 113-147.

Mardones, J.L., Option valuation of real assets: application to a copper mine with operating flexibility. Resources Policy, 1993, 19(1), 51-65.

Miller, L.T., and Park, C.S., Decision making under uncertainty-real options to the rescue? The Engineering Economist, 2002, 47(2), 105-150.

Moyen, N., Slade, M., and Uppal, R., Valuing risk and flexibility: a comparison of methods. Resources Policy, 1996, 22(1/2), 63-74.

Myers S., Determinants of corporate borrowing. Journal of Financial Economics, 1977, 5, 147-175.

Samis, M., and Poulin, R., Valuing management flexibility: a basis to compare the standard DCF and MAP frameworks. CIM Bulletin, 1998, 91(1019), 69-74.

Schwartz, E. S., The stochastic behaviour of commodity prices: implications for valuation and hedging. Journal of Finance, 1997, 52(3), 923-973.

Tilley, J.A., Valuing American options in a path simulation model. Transaction of the Society of Actuaries, 1993, 45, 83-104.

Trigeorgis, L., A real options application in natural resource investments. Advances in Futures and Options Research, 1990, 4, 153-164.

5 Integration of ROA as a Common Tool in Mining: Marginal Zone Analysis

In this chapter, the Real Options Approach (ROA) is integrated into the mine evaluation and planning process in a real mining environment. The case study describes the evaluation of one zone, called 441 lens. The goal of this paper is to show the usability of ROA using the Least-Squares Monte Carlo (LSM) simulation in a real mining context, specifically on one marginal zone.

5.1 Article abstract no. 2: Marginal zone study[3]

An original interest of the author was to apply the Real Options concept to a marginal underground ore zone. Academia has mostly been studying the application of Real Options with open pit with one rich ore body and one low grade ore body or mineralization coutour. This section will study the feasibility and the usefulness of applying ROA to a marginal underground ore zone.

There is a possibility to excavate ore tonnage from 441 lens at East Lake Mine, Raglan. The nature of the mineralization is interesting since it is the underground continuation of the existing open pit. While literature has been studying with interest the calculation of an open-pit expansion with pushback, this 441 lens case is more complex because the mineralized ore zone is divided into three subzones: Kiruna, Classic and Salvage. The article not only treats the econometric measure of zone flexibility, including its value, but also delineates the important mechanism principles that rule the simulations. The evaluation approach follows a stochastic path with a Markovian price profile and, as such, a better value is found. The results show the applicability of the Real Options with new

---

[3] Bruno Lemelin, Sabry Abdel Sabour and Richard Poulin, "A flexible mine production model based on stochastic simulations " paper accepted by the *Institute of Mining and Metallurgy* 23p.

intelligence of the project and give a better understanding of the viability of certain subzones, sometimes going against gut feelings.

Résumé de l'article no.2 : étude de zone marginale

Une idée originale de l'auteur a été d'appliquer le concept des options réelles à une zone minéralisée marginale souterraine. Les académiciens ont pour la plupart étudié l'application des options réelles sur des exploitations de mines à ciel ouvert avec une zone riche et une zone pauvre. Cette section étudiera la faisabilité et l'utilité d'appliquer la technique des options réelles à une zone souterraine marginale. Il existe une possibilité d'excaver du minerai de la lentille 441 à la mine East Lake au site de Raglan. La nature de la minéralisation est intéressante puisqu'elle est la continuité minérale de la mine à ciel ouvert. Alors que la littérature scientifique s'est concentrée sur les opérations à ciel ouvert avec possibilité d'expansion vers les zones en teneur plus faible, le cas de la lentille 441 est plus complexe parce que la zone minéralisée se divise en trois chantiers : « Kiruna », « Classic » et « Salvage ». L'article ne discute non seulement des mesures de flexibilité économique de la zone mais souligne aussi les principes importants qui régissent les règles de simulation. L'approche de l'évaluation suit un chemin stochastique avec un profil de prix Markovien. Une valeur plus précise y est retrouvée. Les résultats démontrent l'application des options réelles avec des informations nouvelles donnant une meilleure compréhension de la viabilité de certains chantiers, parfois allant au contraire de l'instinct.

New planning process

Figure 28 below outlines a mine planning process that includes ROA with one step further than the conventional mine planning process seen in the paper. After the zone is evaluated with the classical merit measures, such as the net present value (NPV) and the internal rate of return (IRR), if the viability of the studied zone is questionable, ROA is launched. Instead of abandoning the zone, new mining strategies can be deployed in order to capture value if market conditions shift. As a result, the flexibility gained offers the miner options to delay further capital investments elsewhere on the site, which could enhance cash flow generation.

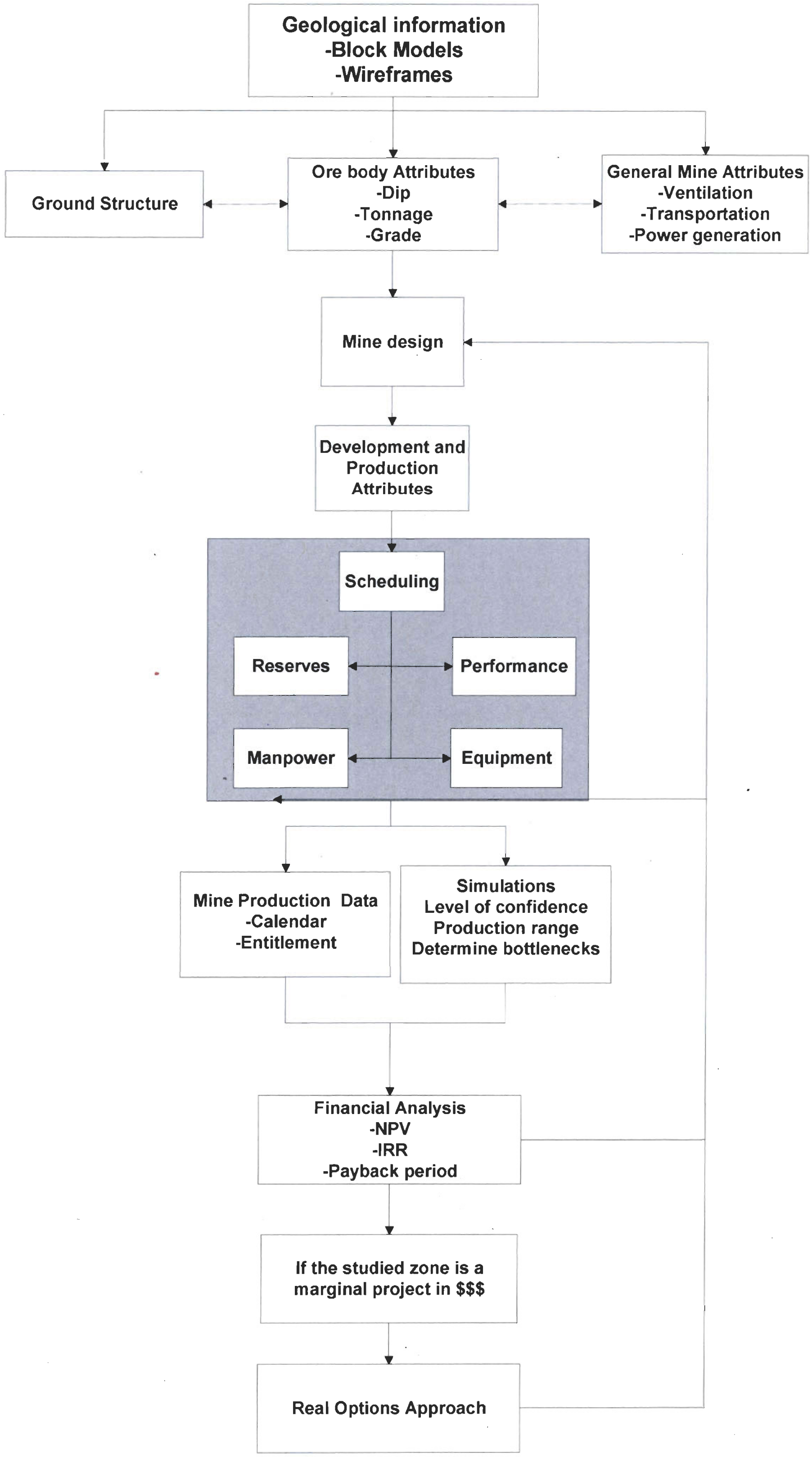


Figure 28. Mine planning process with the Real Options Approach (ROA)

A flexible mine production model based on stochastic price simulations: Application at Raglan mine, Canada

Bruno Lemelin[4], Sabry A. Abdel Sabour[5] and Richard Poulin[*6]

---

[4] Xstrata Nickel, Raglan Mine, Tel : (819)762-7800 ext.5163 E-mail: BLemelin@katinniq.xstratanickel.ca

[5] Mining & Metallurgical Engineering Department, Assiut University, Assiut 71516, Egypt.
E-mail : sabrysabour@yahoo.com ; sabry.abdel-hafez.1@ulaval.ca

[6] Département du génie des mines, des matériaux et de la métallurgie, Université Laval, Québec (Québec) G1K 7P4, Canada. Tel : (418)656-5273 Fax : (418)656-5131 E-mail : Richard.Poulin@gmn.ulaval.ca

* Corresponding author

A flexible mine production model based on stochastic price simulations: Application at Raglan mine, Canada

Abstract:

The conventional economic evaluation technique that is currently used to evaluate the economic viability of mining operations has three important pitfalls. First, the probability distributions of key variables fed into the simulation process are in most part subjective and not based in a solid scientific ground. Second, the simulation method applied to generate metal prices paths results in unrealistic jumps and falls between the consecutive discrete time steps throughout the same simulated path. Third, the conventional technique implements a static production model in which the flexibility to alter the production policy is not applicable. These three pitfalls can impact the accuracy of evaluation results and consequently can lead to suboptimal production decisions.

This paper presents an economic evaluation technique for mining projects based on the real options theory. This technique is based on generating future simulated metal price paths using the appropriate stochastic process for each metal. More important, the technique implements a flexible production model in which the production policy can be revised according to the new market information.

To illustrate the difference the proposed improvements can make in the evaluations results, both the conventional and the new technique are applied to investigate the economic viability of some marginal mining zones based on data from Xstrata's Raglan mine. It has been found that the differences in the evaluation results between the two techniques range between $CAD0.82 million to as high as $CAD3.25 million depending on the size of the subzone, and for the total zone value, the difference is $CAD6.55 million.

Keywords: Mining economics; Stochastic metal price modelling; Flexible production

1. Introduction

Conventional financial evaluation methods such as the net present value (NPV), the internal rate of return (IRR) and the payback period are usually deployed in feasibility studies of mining projects. Monte Carlo simulation method is also applied to obtain a general idea about the probability distributions for the profitability measures such as the NPV or IRR instead of the single value estimates. However, the usefulness of this technique is limited since the flexibility of mining operations is not included in the valuation model. The conventional discounted cash flow (DCF) methods assume that the mine will continue producing at the pre-determined rate until the ore reserve is exhausted regardless of the metal market condition. Therefore, the DCF methods always undervalue mining properties since they ignore the management flexibility to stop production temporarily during low price periods and restart operations subsequently if the prices increased or abandon the mine if the prices continued decreasing. As outlined by Moyen et al.[1], the value of the management flexibility ranges between 0 %, for low cost mines, and 100 %, for high cost mines, of the mine value estimated by the NPV method. Also, Keswani and Shackleton[2] explained that the value of a project can increase substantially when taking into account the management flexibility to revise the original decisions.

In order to build a better extraction strategy, the economic value of mining operations should be estimated taking into account all operating possibilities. To accomplish this, a more efficient technique that allows the evaluator to identify the value of flexibility, obtain an accurate merit measure, and develop an active mining strategy should be implemented. In this respect, the Real Options Approach (ROA) can provide a great help in developing a flexible mine plan that interacts with the unforeseen market conditions. Unlike the DCF methods, the ROA can deal efficiently with the issue of management flexibility to alter or revise decisions when the uncertainty about some key variables is resolved. The first model for pricing financial options was developed by Black and Scholes[3]. Application of the ROA for valuing natural resource projects has started by the mid 1980s when Brennan and Schwartz[4] have valued a copper mine with the management flexibility to switch among different operating modes in response to the copper price movements. Since then, the research related to the ROA has been extended in many directions and applications. For example, McDonald and Siegel[5] analyzed the optimum investment timing when both the revenues and the costs are uncertain. Olsen and Stensland[6] presented a model for the optimal timing of the shutdown decision when both the output prices and the production quantities are uncertain. Mardones[7] applied the ROA to value the management flexibility to modify the cutoff grade. Frimpong and Whiting[8] evaluated a copper project using the derivative mine valuation method based on the dynamic arbitrage theory. Cortazar and Casassus[9] extended the ROA for valuing the management flexibility to expand production capacity. Samis and Poulin[10] and Kamrad and Ernst[11] incorporated the heterogeneity of mineral deposits into the real options valuation model. Also, the ROA has been applied for valuing oil projects with many types of operating flexibility such as the flexibility to defer, start or accelerate project development and the flexibility to change the production rate (see Armstrong et al.[12] for a review of the ROA applications in the oil industry).

Empirical testing of the ROA against actual industry data has indicated its efficiency as a valuation as well as a decision making technique. Kelly[13] examined the accuracy of the ROA in valuing mining properties by comparing the valuation result of the ROA to the final offer price of a gold property. Moel and Tufano[14] tested the real options model of the operating flexibility to open and shutdown a mine that was developed by Brennan and Schwartz[4] against the actual opening and closing decisions of 285 gold mines in the period 1988-1997 and found that the opening and shutting decisions of the ROA are consistent with the actual industry practice.

The typical practice in real options valuations was to assume a single source of uncertainty and apply numerical methods such as the finite difference and lattice methods. Until recently application of the ROA for valuing real capital investments having multiple uncertainties such as the case of poly-metallic mines was not possible. It was just the beginning of the new millennium when Monte Carlo simulation method has really taken advantage over the other numerical methods since the information systems have demonstrated a rapid growth in the speed to treat massive data volumes. Hence, valuing financial options with multiples variables using Monte Carlo simulation method as established by Longstaff and Schwartz[15] has been made possible. The least-square Monte Carlo simulation originally developed for valuing American financial options has permitted the implementation of a new method for evaluating real capital investments with multiple uncertain variables. Abdel Sabour and Poulin[16] have presented an extended version of the least-squares Monte Carlo method for valuing mining investments. The accuracy of that extended version has been examined using valuation data of a copper and a gold mine[16]. Lemelin et al.[17] have applied the extended version for valuing some mineralized zones at Raglan mine.

In this work, the classical mine evaluation technique at Raglan mine, Canada is revisited to allow for an effective interaction with the ROA. In the next section, the main features of the proposed evaluation technique based on a flexible production model are outlined. Then, an explanation of the pitfalls of the current classical evaluation technique at Raglan is provided in the application section with a comparison between the evaluation results of the conventional and the proposed techniques in valuing a mine zone.

2. The Method

This work deals with the uncertainty related to metal prices only. The geological uncertainty is not considered in this work. Therefore, throughout this work both the ore grade and the ore reserves are assumed to be known with certainty. Details on dealing with geological uncertainty can be found elsewhere[18-22]. Also, throughout this work, the term "Monte Carlo simulation" is used in the context of metal price simulation and mine project valuation and finance. The term "flexible production" will be used to indicate the management flexibility to stop production temporarily, reopen and abandon the mine.

This section outlines the three main features of the new economic evaluation technique at Raglan. The first feature of the new technique is related to the modelling of metal prices evolution. Choosing the appropriate model describing the evolution of a metal price is very important in forecasting the future trends of that metal in the market. Basically, prices of most commodities can have either a constant trend or an average equilibrium level. When the price of a commodity, P, shows a constant trend, its stochastic behaviour can be represented by the geometric Brownian motion such as:[4,23,24]

$$dP = \alpha P dt + \sigma P dz \qquad (1)$$

where α is the constant trend, σ is the standard deviation, and dz is the increment of a Wiener process. When the price of a metal follows the geometric Brownian motion in Eq. (1), the expected value of that price after time, t, given the current price, P_0, is:

$$E(P_t) = P_0 e^{\alpha t} \qquad (2)$$

Based on the properties of the lognormal distribution, the variance, V, after time, t, is:

$$V = P_0^2 e^{2\alpha t}(e^{\sigma^2 t} - 1) \tag{3}$$

On the other hand, when the price of a commodity has a long-term equilibrium level, its evolution could be represented by a mean-reverting process. The simplest form of mean-reverting processes is:[23,24]

$$dP = k(P^* - P)dt + \sigma dz \tag{4}$$

where k is the speed at which the price reverts to its long-term equilibrium level P^*. The expected value of the price and the associated variance after time, t, are:

$$E(P_t) = P_0 e^{-kt} + P^*(1 - e^{-kt}) \tag{5}$$

$$V = \frac{\sigma^2}{2k}(1 - e^{-2kt}) \tag{6}$$

A more complex form of the mean-reverting processes is the one presented in Schwartz[25] in which:

$$dP = k(\ln P^* - \ln P)Pdt + \sigma P dz \tag{7}$$

Selection among the geometric Brownian motion and the mean-reverting processes to model each metal price should be based on the actual behaviour of each metal in the market. Analysing the historical market prices of a metal can provide evidence whether the evolution of that metal should be modelled by a geometric Brownian motion or a mean-reverting process. Also, using simple statistical analysis, it is straightforward to estimate the key parameters of the stochastic model such as the expected trend, variance, reversion speed, and long-term equilibrium level (more details on this are found in Dixit and Pindyck, Chapter 3[23]).

After selecting the appropriate stochastic model for each metal and estimating its parameters, it is important in the simulation process to generate sample paths according to the Markov property. This is the second feature of the proposed new evaluation technique. Sampling metal price paths using the Markov property enhances time-connectivity, reduces the unrealistic shifts throughout the single path and in the same time guarantees that the confidence levels are naturally respected according to the probability function at each time. According to the Markov property, the probability distribution of a state X at time t+1 depends only on the current state at time t such as:[26-28]

$$Q(X_{t+1} = a_{t+1} | X_t = a_t, X_{t-1} = a_{t-1}, X_{t-2} = a_{t-2}, ...) = Q(X_{t+1} = a_{t+1} | X_t = a_t) \quad (8)$$

This means that the probability, Q, that the state value at the next time, t+1, will be a_{t+1} depends only on the state value at the present time, t, and independent of the past states at times t-1, t-2, ….. In other words, the Markov property suggests that only the current information is sufficient to forecast the future value of the process. For example, if the time step is set to be one month when forecasting future copper prices, the expected price of copper one month from now depends on the current spot price not on the past prices. Therefore, in order to forecast the expected copper price after one month, only the current spot price is sufficient to provide a good forecast. If the copper price evolves according to the mean-reverting process of Eq.(4), future sample paths of risky prices can be generated based on the Markov property as follows:[24]

$$P_{(i+1,j)} = P_{(i,j)} e^{-k\Delta t} + P^*(1 - e^{-k\Delta t}) + \sigma \sqrt{\frac{1}{2k}(1 - e^{-2k\Delta t})} W_{(i+1,j)} \quad (9)$$

As indicated from Eq.(9), the copper price at period i+1 along the sample path j depends on the price at the previous period i along the same path j. The change in price between two successive periods is directly related to the expected long-term equilibrium level p^*, the speed of reversion k, the standard deviation σ, the length of each time step Δt, and the random number W.

The third feature of the new technique is the integration of a flexible production model based on the Real Options Approach. This model allows for revising the production policy according to the metal price level. If the metal price market goes bust for instance, it is likely that some ore bodies will not be suitable for a foreseeable economical gain. Hence, if the operator has the flexibility to temporarily shutdown some marginal zones, it should be noted that some losses will be prevented or limited while additional costs will be incurred to maintain the closed zone. The same logic is applied as well during booming metal price cycles where a miner will likely be tempted to expand production to profit from the lucrative period of high prices. This temporary closure flexibility has been studied empirically by Moel and Tufano[14] using a data set of 285 North American gold mines in the period 1988-1977. In that study, it has been found that 86 mines have been closed during the specified period due to low gold prices and 10 mines have been re-opened as a result of high gold prices. Also, it has been found that the probability that the temporary closure option will be exercised depends, among other factors, on the operating costs structure, the shutting down costs, the maintenance costs during the temporary closure period and the re-opening costs. In order to have a sound value estimate, it is important that all those costs are accurately quantified and fed into the valuation model. This enhances the decision making process that carried out internally in the valuation process since the optimum operating mode of the mine will be based on all quantifiable costs. However, in some cases it is difficult to include some social costs since these costs cannot be quantified. Dealing with such unquantifiable social costs and impacts is beyond the scope of the proposed economic evaluation model.

The economic analysis provided in this study is well suited for applications related to optimal management of mine production when the mine consists of multiple mining zones differ from each other in metal grade, mineable tonnage and production costs. In such cases, the mine manager will have the flexibility to revise long-term production plans and production rates of one or more mining zones so as to maximize the overall value of mining operations without significant influence in the production target of the company. Having multiple production alternatives with different economic performances, it is possible to stop production temporarily from high-cost low-grade mining zones during low metal price periods while continue producing from other mining zones with better economic performance. Obviously, this contraction or downsizing option aims to avoid negative cash flows and to conserve ore reserves for future production when metal price increases sufficiently. This flexibility comes at a cost since capital investments have to be incurred to shutdown and reopen mining zones and maintenance costs are also required during the shutting periods. Therefore, in addition to the temporary closure option, the manager may elect to permanently stop operations and abandon a mining zone if the metal prices reached a very low level. These scenarios are included in the new financial model.

In the proposed model, the optimal switching policy among the different production alternatives is determined conditional on the metal market condition using an extended version of the least-squares Monte Carlo method. As described by Abdel Sabour and Poulin,[16] to evaluate mining projects under the flexible production model, it is assumed that the mining zone status can be changed at some discrete dates per year with a time step of Δt apart. After generating simulated metal price paths according to the Markov property described above, the cash flows at each discrete date along the simulated metal price paths are defined based on the metal price, the unit production cost and the tax and royalty rates. Then, the optimum production policy at each date is determined by comparing the expected values of the mining zone in the three alternative operating modes: “open mining zone”, “closed mining zone”, and “abandoned mining zone”. Let $H_o(i,j,R)$ denotes the value of the “open mining zone” at time i along the sample path j where the remaining reserve was R and $H_c(i,j,R)$ denotes the value of the “closed mining zone”. The condition for closing the active mining zone at time i along the path j is:

$$\hat{H}_o(i,j,R) \leq \hat{H}_c(i,j,R) - K_c \tag{10}$$

and the condition for re-opening the closed mining zone is:

$$\hat{H}_c(i,j,R) \leq \hat{H}_o(i,j,R) - K_o \tag{11}$$

where K_c and K_o are the switching costs to close and reopen the mining zone respectively. Otherwise, the open mining zone should be permanently abandoned if:

$$\hat{H}_o(i,j,R) \leq K_a \geq \hat{H}_c(i,j,R) - K_c \tag{12}$$

While the closed mining zone should be abandoned if:

$$\hat{H}_c(i,j,R) \leq K_a \geq \hat{H}_o(i,j,R) - K_o \tag{13}$$

where K_a is the abandonment cost.

The expected values of the mining zone in the open and the closed modes, H_o and H_c, are estimated conditional on the metal price at each date using the conditional expectation functions for the open and the closed mining zone. The coefficients of these basis functions are determined using the lease-squares regression. As explained in Longstaff and Schwartz,[15] the unknown functional form of H_o and H_c can be approximated by a linear combination of M basis functions $L_n(P)$ such as Laguerre polynomials, trigonometric series or simple powers of the metal price P such as

$$H_o(i,j,R) = \sum_{n=0}^{M} a_n L_n(P) \tag{14}$$

and

$$H_c(i,j,R) = \sum_{n=0}^{M} b_n L_n(P) \tag{15}$$

The coefficients a_n and b_n for the values of open and the closed mining zone functions are estimated at each date using the least-squares regression. Then, at time i along the path j the values of the open and the closed mining zone can be obtained by substituting for the metal price P(i,j) in the basis function such as

$$\hat{H}_o(i,j,R) = \sum_{n=0}^{M} \hat{a}_n L_n(P(i,j)) \tag{16}$$

and

$$\hat{H}_c(i,j,R) = \sum_{n=0}^{M} \hat{b}_n L_n(P(j,i)) \tag{17}$$

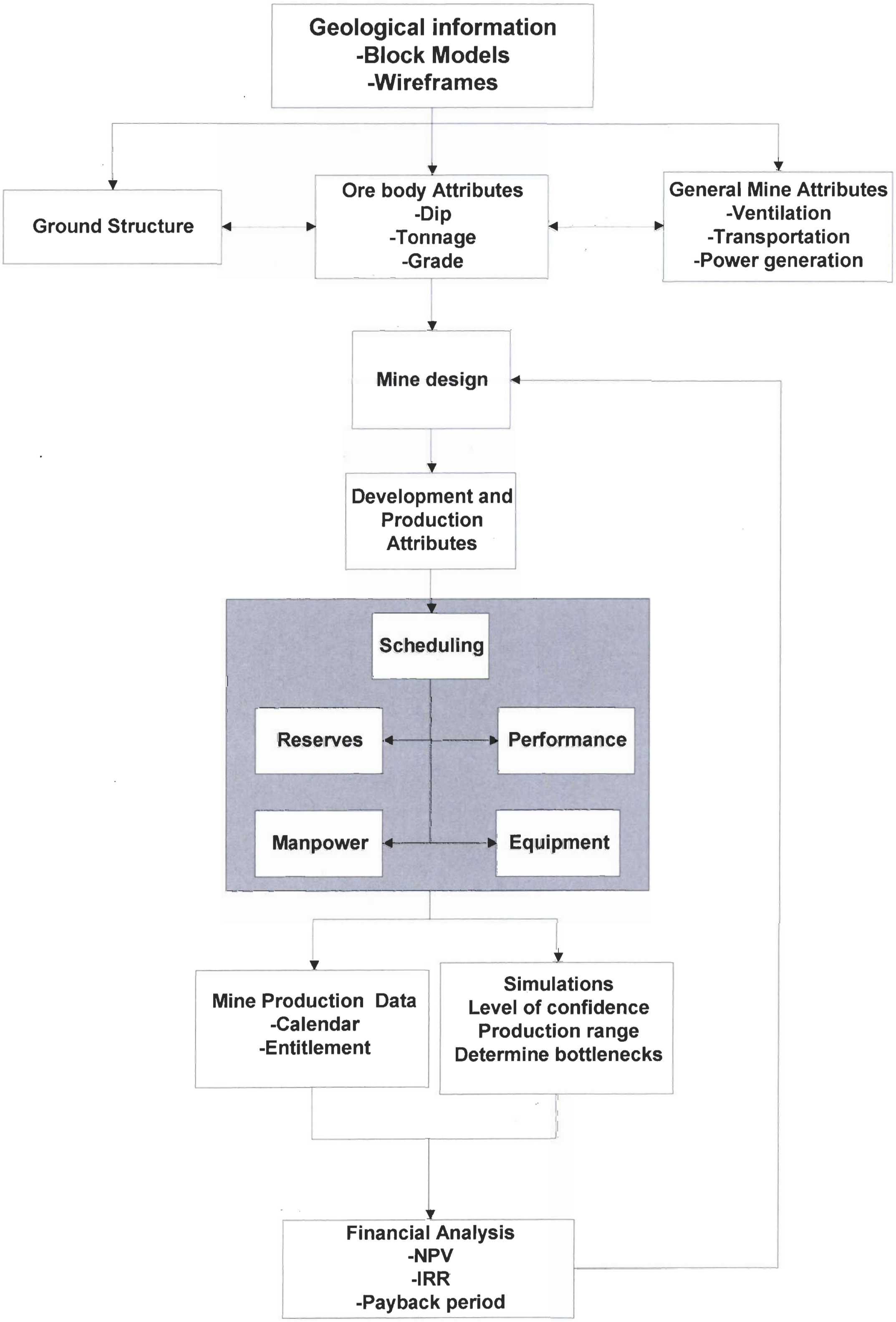
After revising the cash flows according to the determined optimum operating policy of the mining zone at the discrete exercise points throughout all the simulated metal price paths, the mining zone value at time 0 is determined by discounting the cash flows and averaging over the number of simulated metal price paths.

3. Application

This section provides an application of the proposed economic evaluation model to a case study at Raglan mine. The mine is located in Nunavik, Northern Quebec, Canada and operated by Xstrata Nickel. Raglan mine is a poly-metallic mining property producing 8 payable metals, principally nickel and copper. It consists of multiple mineralized deposits alongside a 65 km strike creating a very complex nickel mining camp. Raglan operations have historically mined out 900 000 tonnes annually grading 2.7% nickel in average. The company is contemplating options to increase the production in the coming years. There are two underground mines currently in production (Katinniq and Mine 3) along with one underground mine in development (Mine 2). The rest of production comes from small open pits. The contributions of these pits are decreasing year after year.

In this section, the current conventional economic evaluation process along with its shortcomings is first described. Then, both the conventional and the proposed model that has been outlined in the previous section will be applied to evaluate a mining zone at Raglan to emphasize the differences in the valuation results.

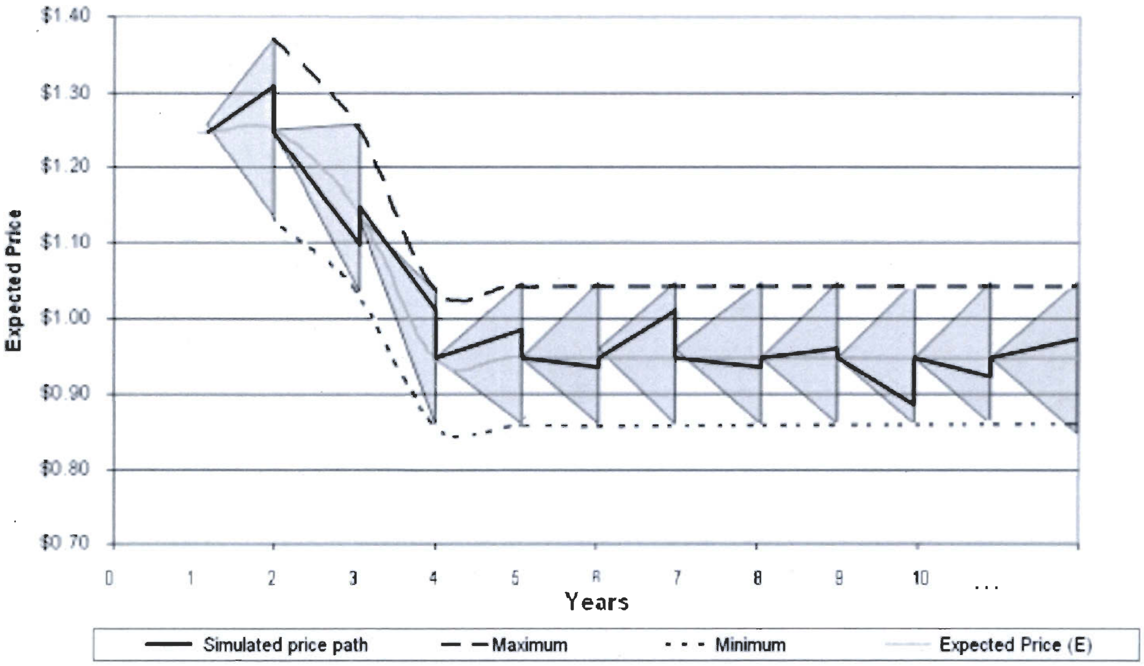
The current process of evaluating an ore body at Raglan mine is outlined in Fig. 1. The process starts with the block model of the zone grading as much as 8 payables metals in 2.5m x 2.5m cells. Depending on the cut-off grade required, the geological department provides wire frames of the interpreted mineralization. As demonstrated Fig. 1, during the scheduling process, valuable information such as the manpower and equipment as well as their respective performance is needed to establish the other parameters that affect the mine production. Many factors are considered such as manpower and equipment availability, distance between workplaces and amount of time to perform a task. Most of the inputs come from time studies and all these data are incorporated in the model. A commercial simulator has been utilized to treat all the numbers of the model and export the results into a report giving the engineer the expected range of production with a certain level of confidence. As shown in Fig. 1, a financial model has been appended to produce directly the financial merit measures like the NPV and the IRR. This financial model makes the process of evaluating a zone quite easy by permitting the engineer to see the interactions between mine design changes and the pecuniary bottom line. Also Monte Carlo simulation is included in the financial model, which enables the mine engineer to get a range for the project value depending on the probability distributions of the key variables fed to the model by the evaluator.



1 The conventional mine evaluation process at Raglan

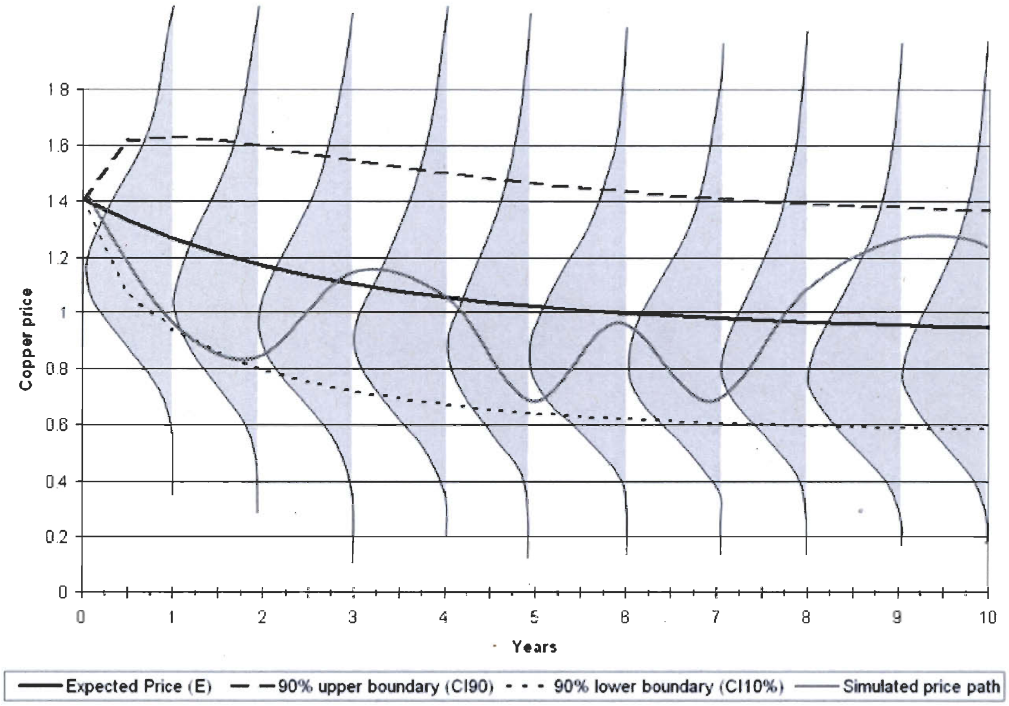
The results generated by the financial model are sensitive to the inputted probability distributions of the key variables. For example, the metal prices can be represented with a normal, Poisson or even a Weibull distribution. Two problems are noted at this point of the process. First, does a metal price follow a pre-defined simple distribution function? Likely it does not. In fact, it has been well accepted by the economists' community that the metal prices can behave distinctly in two ways depending on the nature of the metal. Gold and precious metal prices, for instance, are often modelled using the geometric Brownian motion while the prices of base metals are modelled using the mean-reverting process (for more details about the two models see for example Dixit and Pindyck[23]; Schwartz[25]). Choosing the suitable stochastic model according to the historical behaviour of the metal price prevents absurd errors in pre-defining probability distributions and setting confidence levels. This makes the simulation results more realistic and improves its accuracy in estimating the true economic value of the property.

The second problem with the current evaluation technique is related to the simulation process applied to generate simulated future price paths. In the current process, the simulated future metal prices are not generated in a path-wise following the Markov property, and consequently, they are not linked through time. The future prices in the current model are simulated assuming that the future average metal price will follow a certain path with a certain confidence level. The simulated metal prices are then generated from the pre-defined distribution without a link to the simulated value at the immediate previous year in the same path. This leads to unusual jumps and falls in the simulated metal price paths. To illustrate this, Fig. 2 shows an example for simulating future copper prices using the triangular distribution. As shown in Fig. 2, the simulated copper price along the path represented by the solid line shows instantaneous sudden shifts at each period. These sharp shifts are unlikely to occur instantaneously in the real metal market, which shows a smooth transition in the spot price over the short time intervals. The main reason for the unrealistic transition in the simulated prices outlined in Fig. 2 is that, in the current technique, the reference price at all future times is the pre-defined expected price based on the information available at the current time, not the price at the immediate previous year. Ignoring the price level at the immediate past when generating a simulated price path reduces the time-connectivity between the successive price simulations and results in a poor representation for the possible future price movements even if the time interval between the successive periods is set to be very short.



2 A simulated copper price path with a triangular distribution forced at the expected price

To demonstrate the difference in the simulated prices when considering the Markov property, Fig. 2 has been re-constructed respecting the Markov property, and the results are depicted in Fig. 3. Differently from the price path of Fig. 2, the path generated under the Markov property in Fig. 3 shows a smooth transition from one period to the other with the absence of any instantaneous sharp jumps and falls. This indicates that the simulation process based on the Markov property ensures that the evolution of future prices is realistically represented. Another important point to be explained for the example of the simulated copper price path based on the mean-reverting process of Fig. 3 is that the variance of the price becomes constant after a period of time depends on the reversion speed, among others. It is a stylized fact about the mean-reverting models that the uncertainty saturates with time (see Laughton and Jacoby[29]; Salahor[30]).

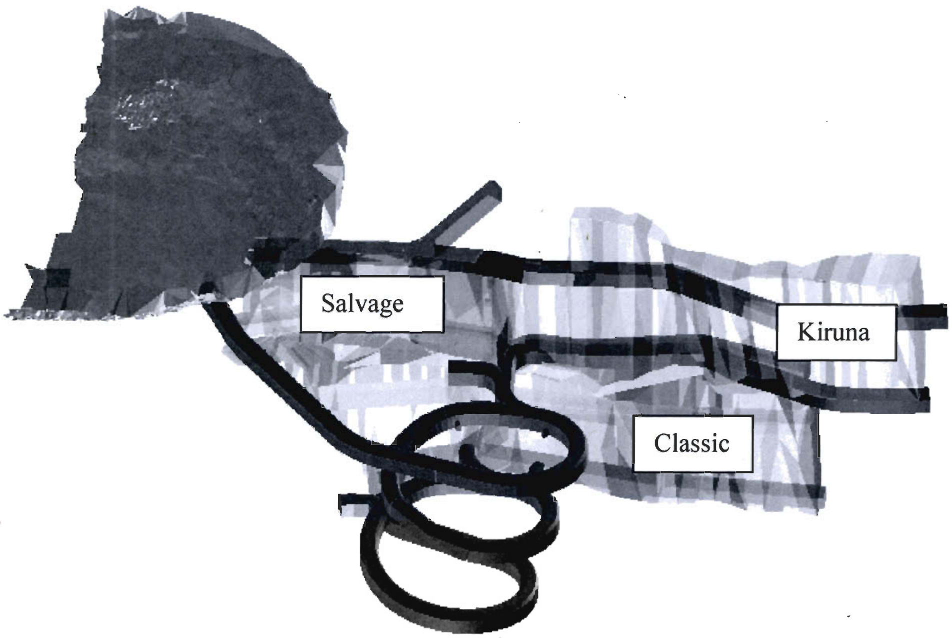


3 Stochastic price path with the mean-reverting process based on the Markov property

Once the simulations are performed on the model, two merit measures are extracted from the analysis: the NPV and the IRR. The payback period can always be calculated from the net cash flows in the financial model. Then, the company's current evaluation techniques indicate another point where improvement could be achieved. The conventional method of financial evaluation is quite discrete. In fact, by its nature, the mine manager possesses only information about the current market conditions and price expectations in the future. Starting from this point, the professional looks for an expected NPV within a range of confidence. The decision will then be made to go or not to go into production. The decision to start production is based on a feasibility study founded only on one scenario, a determined production. It does not include any conditional options for mining the ore zone. This is the third problem of a static evaluation technique that rejects any flexibility of the operator to temporarily shutdown the mine if the price is getting just too low or to expand production if the metal price has soared to some record levels. In reality, the mine manager may have some flexibility to change the rate of extraction depending on the markets conditions. At a certain point, the owner will prefer to abandon the mine than to continuously lose money. In the simulated metal price paths, some scenarios show that the metal price can go below the unit cost of production for a long period of time. These simulated scenarios might end up giving negative NPV results, which will lower the estimated average NPV of the whole paths when added to the paths with positive NPV. In fact, the problem of the conventional DCF methods with simulated prices is that they take every scenario from the beginning and calculate the net cash flows until the end of the mine life assuming that the production will continue regardless the level of metal prices in the simulated paths. In reality, a tool that reports expected values including temporarily shutting down, abandonment or expansion would give the evaluator a better assessment of the mineralized zone value.

4. Comparison of evaluation results

In this section, the evaluation results of both the current and the new techniques are presented. A case study for this issue will assess the economic value of the East Lake 441 zone. As shown in Fig. 4, the East lake 441 zone consists of three subzones: Kiruna, Classic and Salvage. According to the company's production plan, the reserves of the three subzones will be mined out by the long-holes panel method. The development workings consist of 2 portals with one of them headed along the strike of the mineralized zone. The other portal is utilized to gain access to the lower levels of the zone. An interesting feature about this zone is that it contains half a million tonnes and as the ramp is driven down it will provide access to additional mineralized zones contained in the East Lake area. In fact, the East Lake 441 zone provides Xstrata with the flexibility to operate on and off in this zone while developing the mine.



4 The East Lake 441 zone

The data of the subzones at the East Lake zone are presented in Table 1. Each subzone produces eight metals with different tonnages, production rates, production costs and grades. This implies that there are eight sources of uncertainty, corresponding to the eight produced metals, affecting the economic value of each subzone. Having these eight sources of uncertainty makes the accuracy of the simulation process a key factor for obtaining accurate evaluation results. With Monte Carlo simulation, it is possible to deal with these eight sources of uncertainty simultaneously by generating correlated future realizations of the eight produced metals. The operating flexibility is also an important issue to be considered in the evaluation process. With the availability of many potential mining zones at the site, it is always possible to shutdown the unprofitable zones temporarily when the metal prices are low and start production at some other richer zones. This helps the company to maintain its targeted production rate while maximizing the economic value of its operations. Also, the good roof conditions at the different mining zones of Raglan property allows for closing the zones temporarily at very low maintenance costs which makes the flexibility of shutting down and reopening a viable solution in response to the uncertain market conditions. The shutting down and re-opening costs are

$CAD50,000 and $CAD100,000 respectively, while the maintenance costs during the temporary closure periods is $CAD100,000/year.

Table 1. Data of the East Lake zone

	Subzone			**Total zone**
	Kiruna	**Classic**	**Salvage**	
Reserve , tonne	211465	260010	68609	540084
Full production rate, tonne/year	105732	86670	68609	261011
Average operating cost, $CAD/tonne	179.00	184.00	181.00	181.66
Average grade:				
Nickel, %	1.08	1.27	1.21	1.19
Copper, %	0.30	0.36	0.35	0.34
Cobalt, %	0.03	0.04	0.04	0.04
PGM	Not Disclosed			

The evaluation results of the three subzones of the East Lake using both the current and the new evaluation techniques are listed in Table 2. While conducting the economic evaluation using both techniques, the metal grades, the production rate and the unit production cost are all assumed to be constant throughout the subzone as indicated in Table 1. Column 2 in Table 2 lists the evaluation results using the current evaluation system described above and outlined in Fig. 4. Under this system the subzone will continue in production until its reserve is exhausted without any operating flexibility. Column 3 reports the evaluation results of the new technique where the behaviour of metal prices is represented by stochastic models based on the actual behaviour of each metal in the market and the simulations are carried out in accordance with the Markov property. In the new technique, the operating flexibilities to shutdown the subzone temporarily and abandon it permanently in response to the metal market are considered. Therefore, the lifetime of each subzone throughout some simulated metal price paths can be greater or lower than that estimated under the no-flexibility system.

Table 2. Evaluation results of the two techniques

Subzone	**Evaluation results, $CAD**		**Difference, $CAD**
	The current technique	The new technique	
Kiruna	-1 910 000	570 000	2 480 000
Classic	-340 000	2 910 000	3 250 000
Salvage	-270 000	550 000	820 000
Total zone value	-2 520 000	4 030 000	6 550 000

As shown in Table 2, using the current evaluation technique all the subzones have negative net values. This indicates that it is not economical to extract each of these subzones. In contrary, the net values of the subzones as well as the total zone value are positive under the new technique with the flexible production system. The last column of Table 2 lists the difference between the net value estimates of both techniques. This difference ranges from as low as $CAD 820 000 for the Salvage subzone to as high as $CAD 3 250 000 for the Classic subzone, while the difference in the total value of the East Lake zone between the two techniques is $CAD 6 550 000. This difference is significant when considering the small tonnage of each subzone. For bigger zones the difference in the evaluation results between the two techniques can be much higher. This gives a clear idea about how can the evaluation results, and consequently the decision making process, affected by the quality of the evaluation technique.

Conclusions

In this paper, it has been shown that the mine evaluator can gain in precision and accuracy if the current evaluation process is revised to overcome three main pitfalls. First, the stochastic models for the precious and the base metals need to be more realistic based on the actual historical behaviour in metal markets rather than depending on pre-set probability distributions. For instance, the Nickel price can be better modelled with the mean-reverting process since it tends, in the real metal market, to return back to its long-term equilibrium level after a shift in the spot price. The next elaborated point was the importance of generating realistic price paths. According to the Markov property, the probability distribution of a state depends on its immediate predecessor. Hence, it ensures a better time-connectivity and decreases sharp instantaneous jumps in the simulated metal price paths. Finally, the most important point is the ability to calculate the value of flexibility to revise the operating mode of a mining zone depending on the market conditions. The ability to shutdown operations even temporarily improves the appraisal value of the mining zone since it is unlikely that an operator will continue to mine out minerals through time at a loss.

A case study was carried out at Raglan property to illustrate the significant difference in the evaluation results when applying these three improvements. In this respect, economic evaluations were performed on the East Lake 441 zone with the conventional method as well as the proposed new technique. With half a million tonnes of mineralized ore, the studied zone has different extraction rates through the years. Some assumptions were done to perform the real options valuations such as a constant rate of extraction for each sub zone with the half rate rhythm for the first and the last years of production, which is very close to reality. However, it would be interesting to compare the results with a model that accepts different annual rates of extraction for each sub zone. Kiruna, Classic and Salvage subzones were first assessed as being non-economical with the current evaluation process. In contrary, these subzones became economically viable with the new technique, basically due to the concept of being able to shutdown the zone when the metal market conditions turn unfavourable. In reality, the East Lake 441 zone has this attribute to be temporarily shutdown and re-open as long as the other East Lake zones are not mined out. When a mine shows signs of economic marginality and/or can be operated with flexibility, it is then diligent to be able to calculate the true economic value of the mining property using the proposed new evaluation technique based on flexible operations.

References

1. N. Moyen, M. Slade and R. Uppal: 'Valuing risk and flexibility: a comparison of methods', Resources Policy, 1996, **22**, 63-74.
2. A. Keswani and M.B. Shackleton: 'How real option disinvestment flexibility augments project NPV', European Journal of Operational Research, 2006, **168**, 240-252.
3. F. Black and M. Scholes: 'The pricing of options and corporate liabilities', Journal of Political Economy, 1973, **81**, 637- 654.
4. M.J. Brennan and E.S. Schwartz: 'Evaluating natural resource investments', Journal of Business, 1985, **58**, 135-157.
5. R. McDonald and D. Siegal: 'The value of waiting to invest', The Quarterly Journal of Economics, 1986, **101**, 707-727.
6. T.E. Olsen and G. Stensland: 'Optimal shutdown decisions in resource extraction', Economic Letters, 1988, **26**, 215-218.
7. J.L. Mardones: 'Option valuation of real assets: application to a copper mine with operating flexibility', Resources Policy, 1993, **19**, 51-65.
8. S. Frimpong and J.M. Whiting: 'Derivative mine valuation: strategic investment decisions in competitive markets', Resources Policy, 1997, **23**, 163-171.
9. G. Cortazar and J. Casassus: 'Optimal timing of a mine expansion: implementing a real options model', The quarterly Review of Economics and Finance, 1998, **38**, 755-769.
10. M. Samis and R. Poulin: 'Valuing management flexibility: a basis to compare the standard DCF and MAP frameworks', CIM Bulletin, 1998, **91**, 69-74.
11. B. Kamrad and R. Ernst: 'An economic model for evaluating mining and manufacturing ventures with output yield uncertainty', Operations Research, 2001, **49**, 690-699.
12. M. Armstrong, A. Galli, W. Bailey and B. Couët: 'Incorporating technical uncertainty in real option valuation of oil projects', Journal of Petroleum Science and Engineering, 2004, **44**, 67-82.
13. S. Kelly: 'A binomial lattice approach for valuing a mining property IPO', The Quarterly Review of Economics and Finance, 1998, **38**, 693-709.
14. A. Moel and P. Tufano: 'When are real options exercised? An empirical study of mine closings' The Review of Financial Studies, 2002, **15**, 35-64.

15. F.A. Longstaff and E.S. Schwartz: ‘Valuing American options by simulation: a simple least-squares approach’, The Review of Financial Studies, 2001, **14**, 113-147.
16. S.A. Abdel Sabour and R. Poulin: ‘Accuracy of least-squares Monte Carlo in valuing mineral properties’, in Proceedings of the Fifth International Conference on Computer Applications in the Mineral Industries, Banff, Alberta, Canada, Oct-Nov 2005, 1099 – 1124.
17. B. Lemelin, S.A. Abdel Sabour and R. Poulin: ‘Valuing Mine 2 at Raglan using real options’, International Journal of Mining, Reclamation and Environment, 2006, **20**, 46-56.
18. P. A. Dowd, ‘Risk in minerals projects: analysis, perception and management’ Transactions of the Institutions of Mining and Metallurgy, Section A: Mining Technology, 1997, **106**, A9-A18
19. R. Dimitrakopoulos, C.T. Farrelly and M. Godoy: ‘Moving forward from traditional optimization: grade uncertainty and risk effects in open-pit design’ Mining Technology : IMM Transactions section A, 2002, **111**, A82-A88.
20. N. Grieco, and R. Dimitrakopoulos: ’Managing grade risk in stope design optimisation: probabilistic mathematical programming model and application in sublevel stoping’ Mining Technology : IMM Transactions section A, 2007, **116**, 49-57.
21. A. Leite, and R. Dimitrakopoulos: ‘A stochastic optimization model for open pit mine planning: Application and risk analysis at a copper deposit’ IMM Transactions, Mining Technology, 2007 (in press).
22. R. Dimitrakopoulos, and S.A. Abdel-Sabour: ‘Evaluating mine plans under uncertainty: Can the real options make a difference?’ Resources Policy, 2007, **32**, 116-125.
23. A.K. Dixit and R.S. Pindyck: ‘Investment under Uncertainty’, 59:92, 1994, Princeton University Press, New Jersey.
24. P. Glasserman: ‘Monte Carlo Methods in Financial Engineering’, 93:111, 2004, Springer-Verlag New York, Inc, New York.
25. E.S. Schwartz: ‘The stochastic behaviour of commodity prices: implications for valuation and hedging’, Journal of Finance, 1997, **52**, 923-973.
26. E. Seneta: ‘Markov and the birth of chain dependence theory’, International Statistical Review, 1996, **64**, 255-263.

27. Y. Ait-Sahalia: 'Telling from discrete data whether the underlying continuous-time model is a diffusion', The Journal of Finance, 2002, **57**, 2075-2112.
28. D. Filipovic and J. Zabczyk: 'Markovian term structure models in discrete time', The Annals of Applied Probability, 2002, **12**, 710-729.
29. D.G. Laughton and H.D. Jacoby: 'Reversion, timing options, and long-term decision-making" Financial Management, 1993, **22**, 225-240.
30. G. Salahor: 'Implications of output price risk and operating leverage for the evaluation of petroleum development projects' The Energy Journal, 1998, **19**, 13-46

6 Integration of ROA as a Common Tool in Mining: Stand Alone Mine Analysis

The previous chapter supports the application of the ROA to mineralized zones where as this chapter consists in applying the ROA to an entire mine project. Although the concept of using a Real Options model that takes into account several metal price uncertainties is original for mine evaluation purposes, another original concept is the application of the method to an underground mine development. No serious paper in the literature has been close to come to an evaluation as such it is presented in this chapter.

6.1 Article Abstract no. 3: Mine evaluation study[7]

In this section, ROA is fully integrated in the mine evaluation system. Not only are the NPV and the IRR calculated, but the flexibility value for each mining zone is evaluated to complement the information file on the project. A base case study was performed on the East Lake Underground Mine Project at Raglan Mine with four evaluated ore zones: 441 lens, 1040, C and Lake. An article was written to explain the process of using ROA in a global feasibility study. The research has permitted the author to further expose the sensitivity of each mining zone when cash flows are uncertain due to metal price fluctuations. With seven modelled payable metal prices, the article describes ROA integration into the mine planning process with the other mining and geological systems. This represents an advance in the applied economic science, since most of the mine managers agreed that such tools did not exist for a practical solution or were too complex, as exposed by Busby and Pitts (1997).

---

[7] Bruno Lemelin, Sabry Abdel Sabour and Richard Poulin, "An integrated evaluation system for mine planning under uncertainty " paper submitted for the *2007 APCOM conference*, Chile 12p.

Résumé de l'article no.4: Étude économique d'une mine

Dans cette section, la technique des options réelles est pleinement intégrée dans le système d'évaluation minière. Non seulement les standards classiques d'évaluation y sont calculés, mais la valeur de la flexibilité pour chaque zone minéralisée est prise en compte complémentant ainsi l'analyse économique de la mine. Une étude de cas permet de mettre en relief le projet East Lake de la mine Raglan qui contient quatre zones : la lentille 441 et les zones 1040, C et Lake. Un article a été écrit afin d'expliquer le procédé d'utilisation des options réelles dans une étude globale de faisabilité. La recherche a permis à l'auteur de mettre davantage en valeur la sensibilité de chaque zone minéralisée dans un contexte de prix des métaux incertain. Avec sept prix des métaux modélisés, l'article décrit l'intégration dans le processus de la planification minière des options réelles en interaction avec d'autres systèmes miniers et géologiques. Ceci représente une avancée dans les sciences économiques appliquées puisque la majorité des gestionnaires miniers s'entendent qu'aucunes techniques d'options réelles pratique n'existaient ou bien qu'elles étaient trop complexes telles qu'exposé par Busby et Pitts (1997)..

An Integrated Evaluation System for Mine Planning Under Uncertainty

Bruno Lemelin
Falconbridge Ltd., Raglan Mine, Tel.: (819) 762-7800, ext. 5163
Email: blemelin@falconbridge.com

Sabry A. Abdel Sabour
Mining and Metallurgical Engineering Department, Assiut University, Assiut 71516, Egypt
Email: sabrysabour@yahoo.com, sabry.abdel-hafez.1@ulaval.ca

Richard Poulin
Département du génie des mines, des matériaux et de la métallurgie, Université Laval,
Québec (Québec) G1K 7P4, Canada. Tel.: (418) 656-5273 Fax.: (418) 656-5131
Email: Richard.Poulin@gmn.ulaval.ca (Corresponding author)

Mine designer usually evaluates different technically-feasible alternatives in a search for the optimum plan with the highest net economic value. This requires dealing with complex issues such as how to model and manage metal prices volatility. The classical evaluation methods that implement a static production model are not well-suited to face these challenges. In order to obtain reliable evaluations, a more advanced technique that depends on simulating metal price scenarios and implementing a dynamic optimization process is necessary. This process is lengthy and requires handling and processing massive amount of data, especially in case of poly-metallic multi-zone mines. To facilitate this, some kind of automation is needed to enable evaluators to examine different alternatives in shorter times.

The recent progress in computer manufacturing has created a revolution in all businesses and industries. Mining industry is not an exception. It became possible now to depart from the classical evaluation methods and adopt advanced numerical techniques that require large storage capacities and fast data transfer rates. This article proposes a techno-economic system for automating mine evaluations. In this system, an economic model is integrated so as to give monetary reflections for changes in mine design. The model is based on simulating a sufficiently large number of price paths from the stochastic process of each metal and implementing a dynamic optimization using the option pricing technique. The results of applying the proposed system at Raglan mine, Falconbridge Ltd., are presented.

Keywords: Mine evaluation; Uncertainty; Simulation; Flexibility.

INTRODUCTION

Falconbridge Limited is one of the world's leading producers of copper and nickel, with other investments in zinc and aluminum. Most of the nickel production comes from Raglan, Montcalm and the Sudbury operations. Falconbridge also has processing facilities in Canada, Norway and Dominican Republic. The Raglan property, shown in Figure 1, is situated at the northern tip of the Ungava (Nunavik) Peninsula in the Province of Quebec, Canada (Nouveau Québec), north of the 55th parallel. The 100% Falconbridge owned Raglan property encompasses 1,226 claims covering 48,149 hectares and nine 20-year mining leases covering 947 hectares. Raglan is a remote, fly-in working operation with the head office situated in Rouyn-Noranda, located 1540 kilometres southwest of Raglan. Workers access the project via a Falconbridge owned Boeing 737 jet (twice per week) from Montreal and Rouyn Quebec, and an Air Inuit operated Twin Otter from northern Nunavik communities (twice per week). Raglan's remote location has resulted in the development of a multicultural, trilingual (French, Inuktitut and English) workforce of approximately 400 people live and work on rotation at the site. Approximately 15% of the workforce is from the local Inuit villages such as Salluit (130 kilometres to the northwest) and Kangiqsujuaq (60 kilometres to the southeast). The operation runs 24 hours per day, 365 days per year.

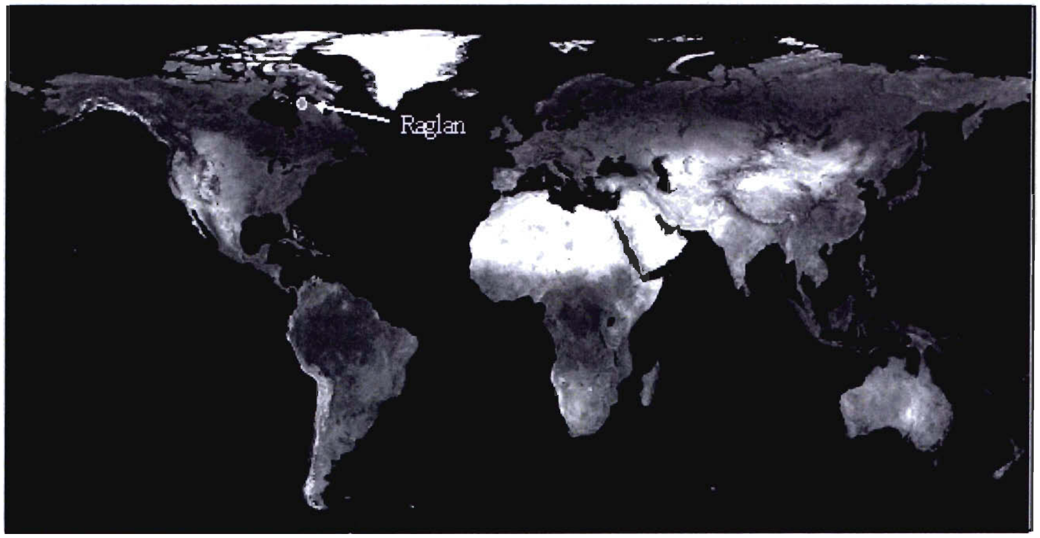


Figure 1. Raglan site in Nunavik, Northern Quebec, Canada

Falconbridge's Raglan mine is a poly-metallic mining property producing 8 payable metals, mainly nickel and copper. Raglan has been mining out an average of 906,500 tonnes of sulphide ore annually grading 2.7% nickel in average. There are three underground mines currently in production. Approximately, 55% of the mill feed during the last 8 years has been produced from Katinniq mine. Also there are some other small open pits contributed to the annual production but their output has deeply decreased since 2002.

The Raglan operations represent a deep challenge for the mine evaluator since the site is composed of over 100 distinct mineralized bodies ranging from 15000 to 2,100,000 tonnes. The role of the mine engineer is to find the optimum production sequence that maximizes the net present value (NPV) based on the information available at the planning time. The internal rate of return (IRR) and the payback period are also other merit measures that are taken into account in the conventional feasibility studies performed at Raglan. While these financial criteria can be useful for giving a gross estimate, they fail to estimate accurately the real economic value of the mine since the flexibility of mining operations is not included in the valuation model. The conventional discounted cash flow (DCF) methods suppose that the miner will continue extracting the ore at a pre-conceived capacity regardless of the metal market condition. This does not account for the fact that the miner can exercise several production options when the metal price noticeably changes in one way or the other. For instance, there is often a possibility to stop production in some mining areas temporarily during low price periods and restart operations subsequently if the prices increased or abandon the mine if the prices continued decreasing. Also the miner can choose to expand production during peak price periods. As reported by Moyen et al. (1996), the value of the management flexibility ranges between 0 % for low cost mines and 100 % of the mine value estimated by the NPV method for high cost mines. Also, Keswani and Shackleton (2006) explained that the value of a project can increase substantially when taking into account the management flexibility to revise the original decisions.

Evaluating mining operations under uncertainty may turn out to be complex when this uncertainty comes from multiple sources. In such cases, the Monte Carlo method is generally used to run simulations using the probability distributions of the uncertain variables such as metal prices, fuel costs and foreign exchange rates. The advantage of applying Monte Carlo simulations method is that it gives the miner a range of value for the project instead of the single value estimate generated by the classical evaluation methods. However, the usefulness of this technique is limited since the flexibility of mining operations is not included in the valuation model. In order to increase the usefulness of the Monte Carlo simulation method, the value of the management flexibility to change or modify decisions over time in response to the uncertain exogenous conditions should be precisely estimated. This can be carried out using the Real Options Approach (ROA).

The ROA is well suited for valuing capital investments under uncertainty, where the management flexibility to revise scenarios with time can represent a significant part of the total project value. In addition to its ability to capture the value of flexibility, as explained by Samis et al. (2006), the ROA has the advantage over the classical DCF methods in allowing for discounting the different future cash flow components according to their unique risk characteristics. The foundation of the ROA is the model developed by Black and Scholes (1973) for pricing financial options. The first application of the approach for valuing mining investments has been carried out by Brennan and Schwartz (1985), in which a model for evaluating a copper mine with the management flexibility to stop production temporarily and close the mine prematurely has been presented. Following Brennan and Schwartz (1985), McDonald and Siegel (1986) discussed the effect of the uncertainties associated with both the revenue and the costs on the optimum investment timing. Olsen and Stensland (1988) analyzed the relation between the uncertainties of both the output prices and the production quantities and the optimal timing of the shutdown decision. Mardones (1993) valued the management flexibility to modify the cutoff grade using the ROA. Frimpong and Whiting (1997) applied the derivative mine valuation method based on the dynamic arbitrage theory to evaluate a copper project. Cortazar and Casassus (1998) presented a real options model for valuing the management flexibility to expand production capacity. In a step toward increasing the practical applicability of the Real Options Approach in dealing with the specific characteristics of ore deposits, Samis and Poulin (1998) and Kamrad and Ernst (2001) introduced the heterogeneity of mineral deposits into the real options valuation model.

The most widely known numerical methods for valuing the real options are the finite difference method, the lattice methods and Monte Carlo simulation method. Both the finite difference and lattice methods are practically useful for valuing the real options if the uncertainty comes from a single source, or at most two sources. For applications that require handling multiple sources of uncertainty simultaneously both methods become impractical since their level of complexity grows exponentially with the number of the uncertain sources. In contrary, Monte Carlo simulation method can handle easily the cases of multiple sources of uncertainty. However, the original version of Monte Carlo simulation developed by Boyle (1977) was restricted only to valuing European options. Since most real options of capital investments are American-style, until recently application of the Monte Carlo simulation method for valuing real capital investments was not possible. This restriction has been completely relaxed in the new version called the least squares Monte Carlo (LSM) developed by Longstaff and Schwartz (2001) that enabled applying the Monte Carlo method for valuing American-style securities. An extended version of the LSM method for valuing capital investments having multiple uncertainties, such as the case of poly-metallic mines, has been presented by Abdel Sabour and Poulin (2005). Lemelin et al. (2006) have applied that extended version of the LSM method for valuing some mineralized zones at Raglan mine.

In this article, the improvements carried out to the economic evaluation technique at Falconbridge Ltd. will be presented. The aim of these proposed improvements is to build an effective mine evaluation tool that takes into account all sources of uncertainty affecting the economic value of the mine as well as the operating flexibility to readjust the production policy with time. This new evaluation tool is based on valuing the operating flexibility available to mine managers using the Monte Carlo simulation method. First, the current evaluation technique will be presented to illustrate the critical points and problems that need to be revisited. Second, the proposed improvements carried out to the current model in order to allow for an accurate evaluation under uncertainty will be explained. Third, a case study will be evaluated using both the conventional and the improved evaluation techniques to visualize the impact of integrating the uncertainty and the operating flexibility on the economic evaluation results.

THE CONVENTIONAL MINE EVALUATION PROCESS

The conventional mine evaluation process currently in use at Raglan consists of three main steps. First, the evaluation process starts with gathering the geological information from drill core logging and electronic data storage using the Century system, DHLogger and BHManager systems. Prior 1999, the mineral reserves were manually calculated using the sectional polygonal method. Since then, the Geology department uses Datamine to create wireframe models such as that illustrated in Figure 2 and all existing mineral reserves were converted to Datamine block models.

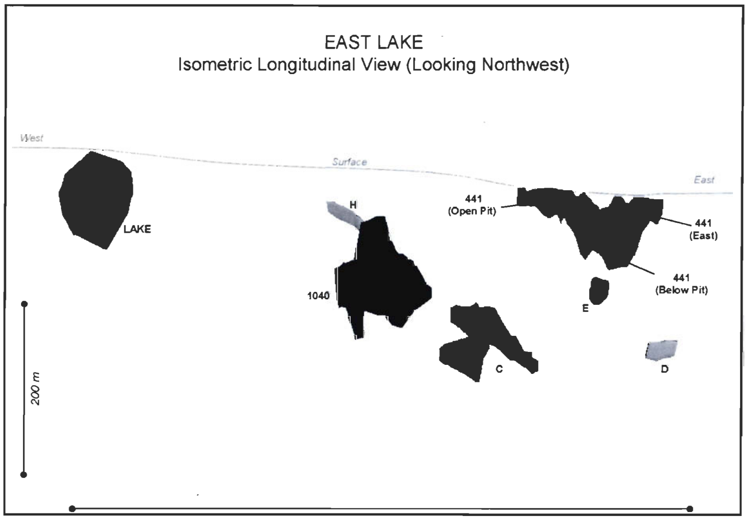


Figure 2. Datamine wireframe models of East Lake

Falconbridge geologists create block models of the mineralized zone grading as much as 7 payables metals in 2.5m x 2.5m cells. They also carry important geotechnical information such as the Rock Quality Designation (RQD) data, rock type and ground structures that helps the mine engineer in designing mining plans according to the best practices to allow safe and secure extraction. While Datamine can offer geological sections, the print layouts are done in CAMP, an in-house mining program using solid modelling (Figure 3 illustrates an example of this process).

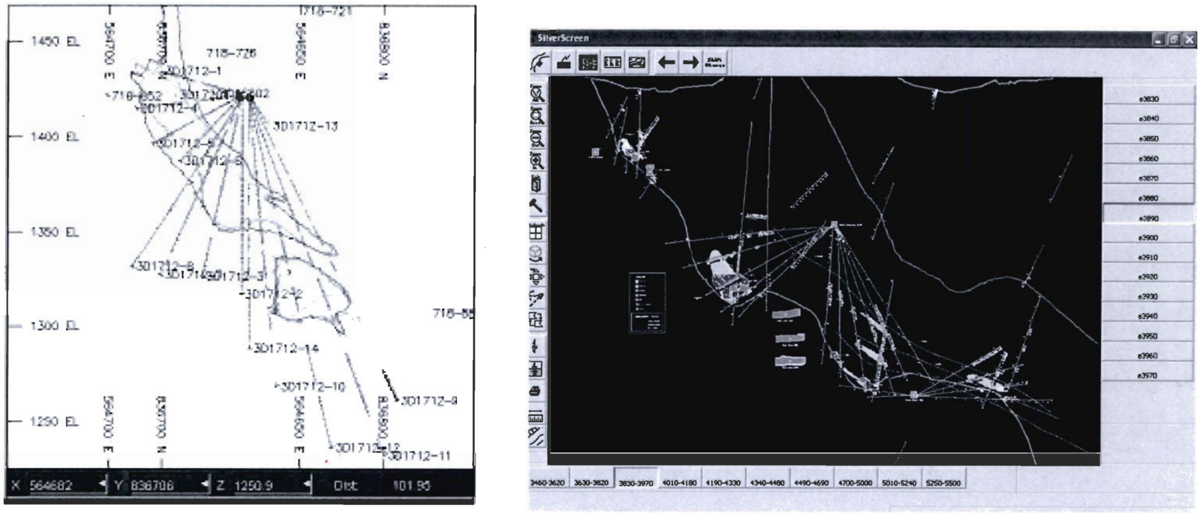


Figure 3. Datamine and CAMP geological sections

The second step includes the mine design and production planning. The long range planning is carried out using AST-Gijima Mining Suite with Mine2-4D. The latter serves as a platform to create rough design of the entire future mine. The software permits the mine engineer to quickly lay out a ramp or a vertical hauling system and to map different level drifts and ore access points. The easiness of the system to change designs according to additional geology information enables the user to make several iterations before retaining the final draft. Many scenarios can also be performed to optimize the ventilations system, power and water management systems as well as ensuring that the rules of art have been respected. Once the mining parameters are well set up in Mine 2-4D, the software deploys another feature by coordinating the scheduled workplaces linked to each others, predecessors and successors. It takes into account every activity the mine planner wants to schedule from driving the ramp to backfilling the ore stope. This exercise serves multiple purposes. It generates a visual animation, as shown in Figure 4, which shows the user the interaction between different workplaces and activities illustrating critical paths that can be optimized as well. The data are also utilized for the scheduler program. The Earthworks Productions Scheduler (EPS), another program from the AST-Gijima mining suite, activates mining schedule from productivity rate resulting in monthly tonnages and grades. It also generates Gantt chart showing scheduled tasks against a specific calendar. EPS gives a first glance on production capacity of the project. It gives a point estimate of the production rate the project will achieve with the productivity rate that has been given for each activity. Thereafter, the workplaces data such as tonnage, grades, meter of development are exported into an excel spreadsheet (Figure 5) that gather productivity information that will be useful to run Monte Carlo simulations onto a commercial simulator, AutoMod. With its module AutoStat, it generates confidence intervals of the project productivity. It calculates simulated performance from time studies and probability distributions like failures on equipment, absenteeism, etc and take into account the travel distances from a workplace to another for the crew and the equipment.

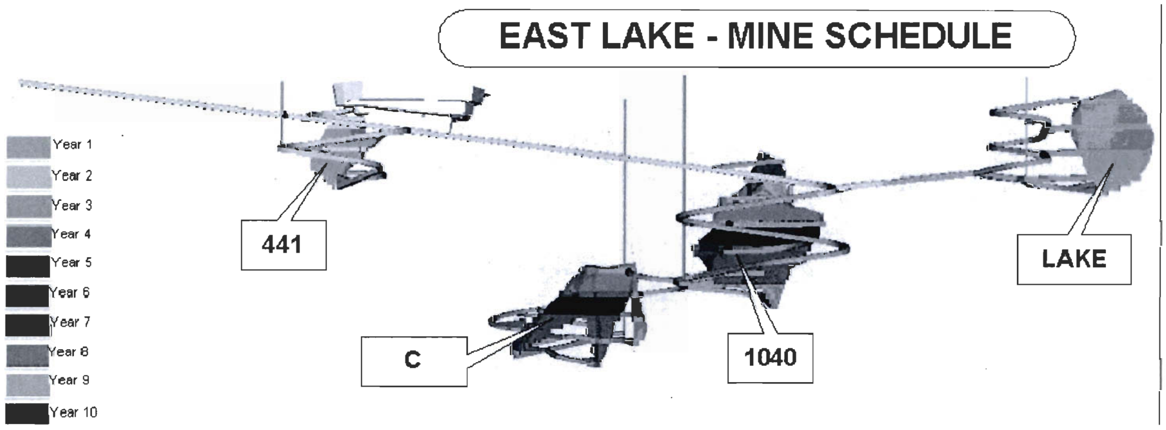
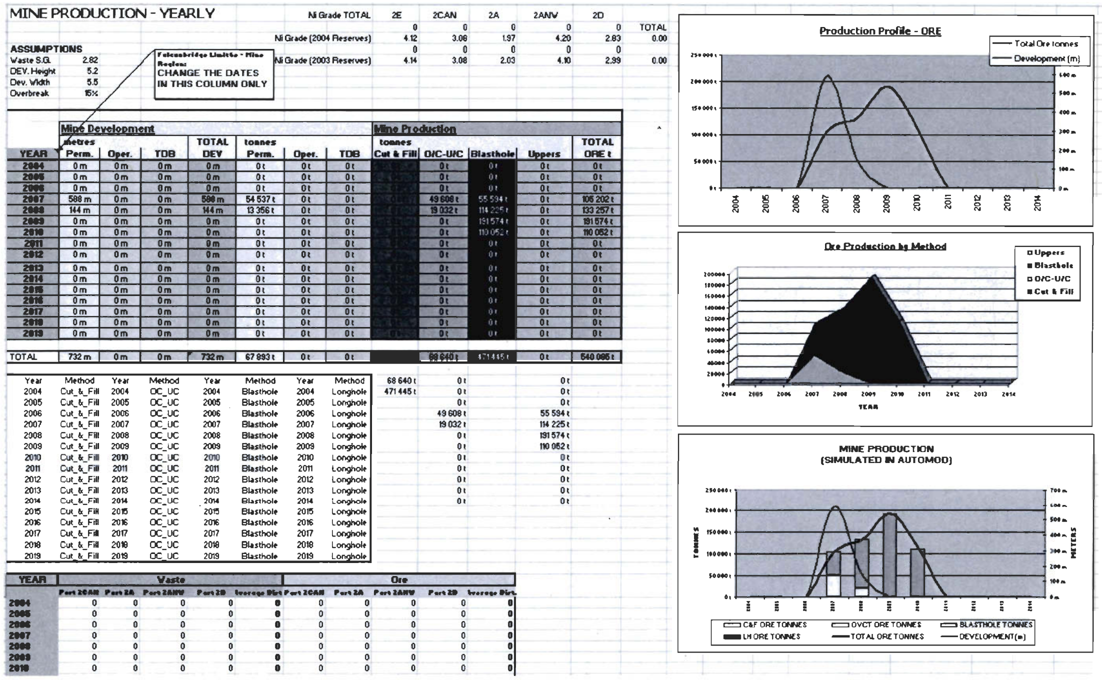


Figure 4. Mine2-4D long range design



MINE PRODUCTION - YEARLY

Ni Grade TOTAL	2E	2CAN	2A	2ANV	2D	
	0	0	0	0	0	TOTAL
Ni Grade (2004 Reserves)	4.12	3.08	1.97	4.20	2.83	0.00
	0	0	0	0	0	
Ni Grade (2003 Reserves)	4.14	3.08	2.03	4.10	2.99	0.00

ASSUMPTIONS

Waste S.G.	2.82
DEV. Height	5.2
Dev. Width	5.5
Overbreak	15%

Falconbridge Limited - Mine Region:
CHANGE THE DATES IN THIS COLUMN ONLY

	Mine Development							Mine Production				
	metres			TOTAL	tonnes			tonnes				TOTAL
YEAR	Perm.	Oper.	TDB	DEV	Perm.	Oper.	TDB	Cut & Fill	O/C-U/C	Blasthole	Uppers	ORE t
2004	0 m	0 m	0 m	0 m	0 t	0 t	0 t	[illegible]	0 t	0 t	0 t	0 t
2005	0 m	0 m	0 m	0 m	0 t	0 t	0 t	[illegible]	0 t	0 t	0 t	0 t
2006	0 m	0 m	0 m	0 m	0 t	0 t	0 t	[illegible]	0 t	0 t	0 t	0 t
2007	588 m	0 m	0 m	588 m	54 537 t	0 t	0 t	[illegible]	49 608 t	55 594 t	0 t	105 202 t
2008	144 m	0 m	0 m	144 m	13 356 t	0 t	0 t	[illegible]	19 032 t	114 225 t	0 t	133 257 t
2009	0 m	0 m	0 m	0 m	0 t	0 t	0 t	[illegible]	0 t	191 574 t	0 t	191 574 t
2010	0 m	0 m	0 m	0 m	0 t	0 t	0 t	[illegible]	0 t	110 052 t	0 t	110 052 t
2011	0 m	0 m	0 m	0 m	0 t	0 t	0 t	[illegible]	0 t	0 t	0 t	0 t
2012	0 m	0 m	0 m	0 m	0 t	0 t	0 t	[illegible]	0 t	0 t	0 t	0 t
2013	0 m	0 m	0 m	0 m	0 t	0 t	0 t	[illegible]	0 t	0 t	0 t	0 t
2014	0 m	0 m	0 m	0 m	0 t	0 t	0 t	[illegible]	0 t	0 t	0 t	0 t
2015	0 m	0 m	0 m	0 m	0 t	0 t	0 t	[illegible]	0 t	0 t	0 t	0 t
2016	0 m	0 m	0 m	0 m	0 t	0 t	0 t	[illegible]	0 t	0 t	0 t	0 t
2017	0 m	0 m	0 m	0 m	0 t	0 t	0 t	[illegible]	0 t	0 t	0 t	0 t
2018	0 m	0 m	0 m	0 m	0 t	0 t	0 t	[illegible]	0 t	0 t	0 t	0 t
2019	0 m	0 m	0 m	0 m	0 t	0 t	0 t	[illegible]	0 t	0 t	0 t	0 t
TOTAL	732 m	0 m	0 m	732 m	67 893 t	0 t	0 t		68 640 t	471 445 t	0 t	540 085 t

Year	Method	Year	Method	Year	Method	Year	Method			
								68 640 t	0 t	0 t
2004	Cut_&_Fill	2004	OC_UC	2004	Blasthole	2004	Longhole	471 445 t	0 t	0 t
2005	Cut_&_Fill	2005	OC_UC	2005	Blasthole	2005	Longhole		0 t	0 t
2006	Cut_&_Fill	2006	OC_UC	2006	Blasthole	2006	Longhole		49 608 t	55 594 t
2007	Cut_&_Fill	2007	OC_UC	2007	Blasthole	2007	Longhole		19 032 t	114 225 t
2008	Cut_&_Fill	2008	OC_UC	2008	Blasthole	2008	Longhole		0 t	191 574 t
2009	Cut_&_Fill	2009	OC_UC	2009	Blasthole	2009	Longhole		0 t	110 052 t
2010	Cut_&_Fill	2010	OC_UC	2010	Blasthole	2010	Longhole		0 t	0 t
2011	Cut_&_Fill	2011	OC_UC	2011	Blasthole	2011	Longhole		0 t	0 t
2012	Cut_&_Fill	2012	OC_UC	2012	Blasthole	2012	Longhole		0 t	0 t
2013	Cut_&_Fill	2013	OC_UC	2013	Blasthole	2013	Longhole		0 t	0 t
2014	Cut_&_Fill	2014	OC_UC	2014	Blasthole	2014	Longhole		0 t	0 t
2015	Cut_&_Fill	2015	OC_UC	2015	Blasthole	2015	Longhole			
2016	Cut_&_Fill	2016	OC_UC	2016	Blasthole	2016	Longhole			
2017	Cut_&_Fill	2017	OC_UC	2017	Blasthole	2017	Longhole			
2018	Cut_&_Fill	2018	OC_UC	2018	Blasthole	2018	Longhole			
2019	Cut_&_Fill	2019	OC_UC	2019	Blasthole	2019	Longhole			

YEAR	Waste					Ore				
	Port 2CAN	Port 2A	Port 2ANV	Port 2D	Average Dirt	Port 2CAN	Port 2A	Port 2ANV	Port 2D	Average Dirt
2004	0	0	0	0	0	0	0	0	0	0
2005	0	0	0	0	0	0	0	0	0	0
2006	0	0	0	0	0	0	0	0	0	0
2007	0	0	0	0	0	0	0	0	0	0
2008	0	0	0	0	0	0	0	0	0	0
2009	0	0	0	0	0	0	0	0	0	0
2010	0	0	0	0	0	0	0	0	0	0

Figure 5. Automod simulation results

The third step is to perform the financial analysis based on the mine layout and the forecasted production schedule. In this respect, the simulated production information from the previous step is fed into a financial model. The mine engineer inputs the project capital cost, variable and fixed operating costs into the model. With a specific metal price model, financial Monte Carlo simulations are run to establish not only the net present value of the project but also the variability of its economic value. Crystal ball software is utilized to run the simulations and also produce the tornado chart that indicates the sensitivity of the estimated project value to the key parameters. Samples of the financial model output are presented in Figure 6.

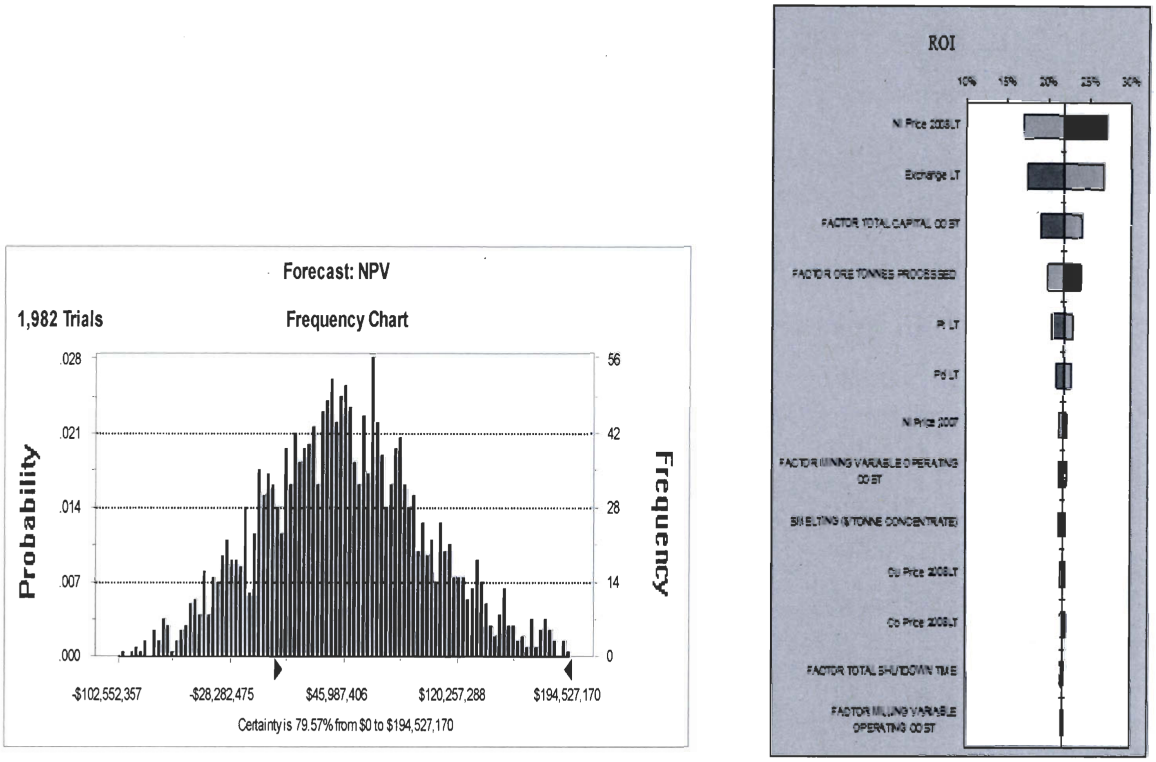


Figure 6. Estimated NPV with confidence interval with Crystal Ball Simulation and tornado chart

Although the above described evaluation technique provides the evaluator with a range of value for the mining project as well as the sensitivity of the project to its key parameters, its usefulness for handling the uncertainty is limited for two main reasons. First, the uncertainty of the key parameters is modelled in a subjective manner in which the evaluator inputs the minimum, the maximum and the most likely value for each parameter. Then, the simulator uses these subjectively inputted limits for generating simulated values at each year without any link to the simulated values at the immediate past year along the same simulation paths. This results in simulated behaviours that do not represent the actual behaviour of metal prices in the real metal markets.

The second reason for the limited usefulness of the current evaluation process is the ignorance of the value that would be generated by revising the production scenarios according to the arrival of new information from the metal markets. The current evaluation process implements a static future production scenario that is independent of the future metal price levels. In this respect, the mining operations are assumed to continue at the predetermined capacity until the ore reserves are exhausted. The operating flexibility to stop production temporarily or permanently during low price periods is not integrated in the evaluation process. The value of such operating flexibility can be significant especially in case of marginal (low profit) mining zones. Therefore, ignoring that value will underestimate the value of a project and consequently can lead to wrong investment decisions.

INTEGRATING THE UNCERTAINTY AND THE OPERATING FLEXIBILITY IN THE EVALUATION PROCESS

In order to obtain reliable evaluation results, the evaluation process is improved in two directions. First, the uncertainty of future metal prices is modelled and the simulation paths are generated so as ensure a close representative of the actual behaviour of metal prices in the real metal markets. Generally, there are two well accepted stochastic price models that correspond to the metals extracted at Raglan. The first stochastic model is based on the characteristic of a metal price to return back to its long-term equilibrium level after a shock in the market. This model is represented by the mean reverting process. The following form of mean-reverting processes was developed by Schwartz (1997):

$$dP = k(\ln P^* - \ln P)Pdt + \sigma Pdz \tag{1}$$

Where k is the speed at which the price reverts to its long-term equilibrium level P^*, P is the spot price while σ is the associated standard deviation of the price, and dz is the increment to a standard Wiener process. Nickel, copper, cobalt, and platinum are represented by this type of model with its respective econometric attributes. Gold, silver and palladium are modelled using another existing and well-accepted model called the geometric Brownian motion. These metals do not seem to be affected by any force reverting it back to a long-term level. Its stochastic behaviour can be represented such as (Brennan and Schwartz (1985); Dixit and Pindyck (1994); Glasserman (2004)):

$$dP = \alpha P dt + \sigma P dz \qquad (2)$$

Where α is the constant trend, σ is the standard deviation, and dz is the increment to a standard Wiener process.

Modelling the uncertainty of metal prices using these two stochastic models, rather than the subjective modelling in the conventional evaluation technique, has two major advantages. First, the suitable stochastic models are based on statistical parameters that take into account the historical behaviours of metal prices in the metal markets which allows for modelling the probability distribution according to the specific nature of each metal price. Secondly, the simulated metal price paths can now be generated with a Markovian property that takes into account the metal price level at the immediate past period which reduces absurd metal jumps and enhances time-connectivity.

The second proposed improvement to the conventional economic evaluation technique is the integration of the operating flexibility to revise production scenarios in the future based on the metal price levels using the Real Options Approach. Based on the operating flexibility model, the Real Options Approach evaluates at regular discrete time points the value of the different production options and policies available to the miner. The policies that are regarded if the zone is active are: to keep the mining operations as planned, to shut down the zone temporarily, or to abandon it completely. If the zone is in the temporarily closed mode, the production polices that will be evaluated are: to keep it closed until the next period, re-open the zone, or permanently abandon it. Based on the metal price at each discrete decision time, the flexibility-based model will mimic reality by analysing the current situation as if it is unfolded. Typically, if there is a zone where negative cash flows are deemed to stay for a long period of time, in the real mining world, the manager will direct the production effort where it pays better and will shut down the zone. The conventional evaluation technique does not account for such operating flexibility in its calculations structure, and then it can bring sub-optimal project value and, consequently, may result in rejecting good investment opportunities. In the flexibility-based evaluation technique, the expected values of the different production alternatives are estimated conditional on the simulated metal prices using the least-squares regressions. More details about this process are found in Longstaff and Schwartz (2001) and Abdel Sabour and Poulin (2005; 2006).

CASE STUDY: THE EAST LAKE MINE

In this section, a case study of the East Lake project at Raglan mine is presented. The East Lake project is a future mine target with a reserve of 2,000,000 tonne grading 2.97% Ni in average. The mining operations are scheduled to start at the beginning of 2008. As shown in Figure 7 four mineralized zones that are qualitatively and quantitatively different are included in the evaluation study. The first zone is named 441, which will be the extension of the existing pit. It contains a low grade ore body with a tonnage of 443,000 tonne at 1.29% Ni. In the mining plans, this zone is not scheduled to start production until 2014. However, right at the beginning of the operations in 2008, there is flexibility to commence the development of the overcut and later on, the development of the undercut which would give the company a good opportunity to marginally mine the zone at any time. 1040 zone is definitely the main zone of the mine, which pays for all the surface infrastructures such as the generators, buildings, main ventilation system, etc. This zone contains a tonnage of 1,000,000 tonne at 3.76% Ni. The Mining operations in this zone are expected to last five years. C zone is a relatively low grade ore body of 430,000 tonne grading 2.12% Ni that will begin production in 2011 for three years. The zone remains open at depth and offers a geological upside potential to get higher grade ore. Lake zone is a relatively small nickel ore body containing 75,000 tonne at a very high grade ore of 6.5%. Two years only will be sufficient to mine out this zone.

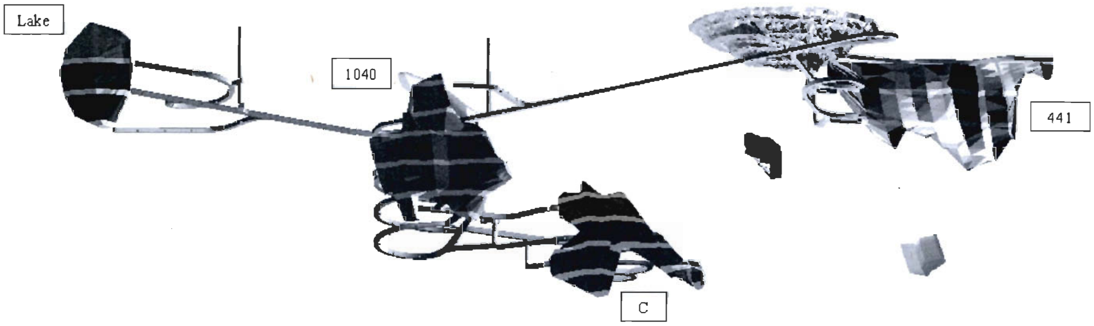


Figure 7. East Lake Mine design on CAMP

The evaluation results of the different mining zones using both the conventional technique without accounting for the operating flexibility (NPV-based) and the improved one based on the ROA are presented in Table 1. The closing cost represents all the costs required to close a zone temporarily while the opening cost is the total costs incurred to reopen a temporarily closed zone. The maintenance cost is the cost incurred to maintain the temporarily closed zone during the closure period. Some assumptions were made in order to simplify the evaluation process using the ROA. In this respect, for each mining zone, the extracted semi-annual tonnages, the ore grade, and the unit extraction cost are assumed to be constant throughout the zone lifetime. On a zone analysis basis, as outlined in Table 1, 1040 and Lake zones are clearly positive while C zone is marginally positive and 441 zone is not economical. The results indicate that since 1040 and Lake zones are highly profitable, there is no significant difference between the evaluation results of NPV-based and the ROA-based techniques. This is because under the high grade condition of both zones and the current high market prices of metals, the probability that the zones will be temporarily closed or abandoned due to economic reasons is negligible. Consequently, the value of the operating flexibility to shutdown the zone temporarily or permanently is close to zero.

Table 1. Evaluation results of the mining zones at the East Lake project

Zone	**Closing cost, CAD M**	**Opening cost, CAD M**	**Maintenance cost, CAD M**	**Value, CAD M**	
				Conventional NPV-based technique	**Improved ROA-based technique**
1040	1.0	1.000	0.50	99.39	100.11
Lake	0.1	0.100	0.05	18.59	18.61
C	0.1	0.125	0.15	5.17	8.02
441	0.1	0.075	0.14	-2.90	2.98

In contrary, for both C and 441 zones, there is a significant difference between the results of the NPV-based and the ROA-based evaluation techniques. As presented in Table 1, the conventional NPV-based technique suggests that the C zone would generate a small net present value of CAD 5.17 million while the ROA-based technique shows a higher value of CAD 8.02 million. This means that the additional value that could be generated by the operating flexibility is CAD 2.85 million, which is approximately 55 % of the value estimated with the static NPV-based evaluation technique. This difference can be understood by the fact that, since the C zone is low in nickel grade, there is a high probability that the ore extraction operations will become uneconomical due to unexpected shifts in metal prices. The static NPV-based evaluation process calculates the net value for each simulated metal price path during the time C zone will be in production regardless of the net losses generated throughout low price paths. In reality, if this situation is encountered, the miner might choose to temporarily shut down the zone and wait for better times with high metal prices, or abandon the zone permanently. The conventional evaluation technique based on the rigid NPV method does not account for such operating flexibility. In contrary, the ROA-based process permits to capture the value of such operating flexibility, which consequently leads to improving the precision of estimating the real economic value of the ore body. Since C zone remains open at depth at the present time, it would be important to incorporate a geological dimension into the economic evaluation model. In this case, the geological information can be regarded as an uncertainty attribute such as the metal prices. Modelling of such geological uncertainty and integrating it into the economic evaluation model is out of the scope of this article.

As indicated in Table 1, completely different evaluation results have been generated for the 441 zone using the conventional evaluation technique and the improved one based on the ROA. The conventional NPV-based technique gives a negative value of CAD 2.9 million for this zone whereas the ROA-based technique gives a positive value of CAD 2.98 million. This indicates that the 441 zone is not economically viable from the point of view of the conventional economic evaluation technique while it represents a potentially viable zone from the ROA-based evaluation technique. This result is not surprising given the difference between the specific mechanisms of the static NPV method and the more dynamic ROA. Differently from the assumption of the NPV method that the zone will be mined out during a prescribed period, in the ROA model the miner has the option to mine out the zone at anytime during the total lifetime of the project. Therefore, when the metal price in the world market will be bullish, the miner will have the flexibility to extract the ore and to generate positive cash flows out of the 441 zone. However, The 441 should be regarded as an incremental ore tonnage that adds up to the existing planned capacity rather than a main zone that will displace the existing production schedule in the future.

CONCLUSIONS

This article proposed a techno-economic evaluation tool for mining operations based on the ROA that allows for taking into account the market uncertainty and the management flexibility to revise production plans. Taking advantage of the recent progress in computer manufacturing, the proposed technique applies the Monte Carlo simulation method for valuing the real options associated with mining operations. The main advantage of the Monte Carlo simulation is its ability to process multiple sources of uncertainty simultaneously. Although it requires handling massive data, this is not a problem with the current high-speed processors that can deal with huge data and perform the computations in reasonably short times. The ROA-based evaluation tool presented in this article has been developed to support the decision making process related to the investment and production strategies and to help in implementing a sound long-term mining strategy that would maximize the overall value of the mining projects. From data logging in geology to designing a mine, the mining engineer can use the proposed evaluation tool to directly observe the changes in the net economic value of the project when implementing different planning and scheduling actions.

A case study has been presented in this article for the East Lake project at Raglan mine, Falconbridge Ltd. This project, which is a poly-metallic mining operation, is a good example for the cases in which the project value is affected by multiple sources of uncertainty. The economic viability of the 4 mining zones of the East Lake project has been evaluated using the conventional NPV-based technique and the improved one based on the ROA that allows for incorporating the operating flexibility. From the case study presented in this paper, it has been found that using the ROA-based evaluation technique can bring completely different economic evaluation results, especially for low grade or marginal zones, and consequently can modify the exploitation decisions and strategies.

REFERENCES

Abdel Sabour, S.A. and Poulin, R. (2005). Accuracy of least-squares Monte Carlo in valuing mineral properties, in Proceedings of the Fifth International Conference on Computer Applications in the Mineral Industries (CAMI 2005), 1099 – 1124.

Abdel Sabour, S.A. and Poulin, R. (2006). Valuing real capital investments using the least-squares Monte Carlo method. The Engineering Economist, 51(2), 141-160.

Black, F., and Scholes, M. (1973). The pricing of options and corporate liabilities. Journal of Political Economy, 81(3), 637- 654.

Boyle, P. (1977). Options: a Monte Carlo approach. Journal of Financial Economics, 4(3), 323-338.

Brennan, M.J., and Schwartz, E.S. (1985). Evaluating natural resource investments. Journal of Business, 58(2), 135-157.

Cortazar, G., and Casassus, J. (1998). Optimal timing of a mine expansion: implementing a real options model. The quarterly Review of Economics and Finance, 38, 755-769.

Dixit, A.K., and Pindyck, R.S., Investment under Uncertainty. 1994 (Princeton University Press: New Jersey).

Frimpong, S., and Whiting, J.M. (1997). Derivative mine valuation: strategic investment decisions in competitive markets. Resources Policy, 23(4), 163-171.

Glasserman, P., Monte Carlo Methods in Financial Engineering. 2004 (Springer-Verlag New York, Inc: New York).

Kamrad, B., and Ernst, R. (2001). An economic model for evaluating mining and manufacturing ventures with output yield uncertainty. Operations Research, 49, 690-699.

Keswani, A., and Shackleton, M.B. (2006). How real option disinvestment flexibility augments project NPV. European Journal of Operational Research, 168(1), 240-252.

Lemelin, B., Abdel Sabour, S.A., and Poulin, R. (2006). Valuing Mine 2 at Raglan using real options. International Journal of Mining, Reclamation and Environment, 20(1), 46-56.

Longstaff, F.A., and Schwartz, E.S. (2001). Valuing American options by simulation: a simple least-squares approach. The Review of Financial Studies, 14, 113-147.

Mardones, J.L. (1993). Option valuation of real assets: application to a copper mine with operating flexibility. Resources Policy, 19, 51-65.

McDonald, R., and Siegal, D. (1986). The value of waiting to invest. The Quarterly Journal of Economics, 101(4), 707-727.

Moyen, N., Slade, M., and Uppal, R. (1996). Valuing risk and flexibility: a comparison of methods. Resources Policy, 22(1/2), 63-74.

Olsen, T.E., and Stensland, G. (1988). Optimal shutdown decisions in resource extraction. Economic Letters, 26, 215-218.

Samis, M., and Poulin, R. (1998). Valuing management flexibility: a basis to compare the standard DCF and MAP frameworks. CIM Bulletin, 91(1019), 69-74.

Samis, M., Davis, G.A., Laughton, D., and Poulin, R. (2006). Valuing uncertain asset cash flows when there are no options: A Real Options Approach. Resources Policy, 30(4), 285-298.

Schwartz, E. S. (1997). The stochastic behaviour of commodity prices: implications for valuation and hedging. Journal of Finance 52, 923-973.

Conclusion and Further Research

During the last 20 years, the mining industry has struggled with low returns. Only recently have miners been able to gain a foothold in profitability. Yet, failing to obtain accurate information, miners might not be in a position to assess whether the market still shows great business opportunities. Miners might end up investing only when market conditions seem promising, without knowing that they might already be at the peak of the cycle. Worse, mineral resources may be wasted in years to come because managerial flexibility has not been taken into account. Millions of dollars of mineral might be written off the resource book. This thesis explains and proves the necessity for the mining industry to adopt a new mine planning process that includes the Real Options Approach (ROA) in order to enhance the decision-making process and to plan more strategically to capture opportunities from existing uncertainties like metal price fluctuations.

The two major goals of this thesis were to create the very first mine model capable of dealing with multiples sources of uncertainties and to implement it into an existing standard mine planning process. The thesis is successful because the author proved that using the Real Options Approach with the least-squares Monte Carlo Simulation could deal with multiple sources of uncertainties for mine modeling. This was never performed before and its originality was published in two academic papers. Two other conference papers were written to show the applicability of the method inside a typical mine planning process. With great enthusiasm and excitement, the author succeeded in bringing a performing tool for the mine planner to properly evaluate the financial flexibility of a mining project with regards to the uncertainties in place.

The thesis is important because it advances the science of mine engineering It was showed that conventional evaluation models, like net present value (NPV), have some limitations when it comes to capturing the flexibility of mining projects. The NPV algorithm calculates all operating cash flows, even when it does not make sense economically and ignores managerial flexibility. It also denies investment initiation policy. Rather, it considers a project a go or no-go without the ability to revise the decision should market conditions change over time. The NPV discounting mechanism was also proven to fail to reflect risk according to monetary cash inflows and outflows. Finally, the NPV method fails to capture the project volatility and to turn this data into valuable information. The classical approach is rather static and would be efficient for asset replacement strategy, cost-optimization projects and projects with a short lifespan, hence the need to create a better tool for mine appraisal with multiple risks.

This thesis has proposed an original mine model to better compute the volatility of the underlying asset. It has gathered up to seven uncertainties within the same model, something that has not been performed in mining academia. In the past, the development of some alternative solutions attempted to ease the problem. Often seen as a black box and restricted to dealing with only two uncertainties, the Real Options Approach (ROA) has been limited in fulfilling its promises. With a real mining environment with multiple uncertainties, it is quite problematic for the manager to calculate an accurate project value that captures managerial flexibility. The model created by the author comes off in dealing with multiple sources of uncertainties and incorporates the underlying asset by the use of a stochastic process. The main uncertainties that have been studied in this thesis and that affect a mining project are the commodity prices. Base metals, such as nickel and copper, follow a mean reverting process where the metal price is under specific market influences to revert back to a long-term equilibrium. The nickel example is further studied in Appendix A. Some other commodities follow a geometric Brownian motion stochastic process where there is no apparent drift in the price movement. Gold follows this pattern and is further elaborated in Appendix B.

By developing the model using the recent Least-Squares Monte Carlo simulation technique (LSM), the author has improved the mine evaluation process by solving two drawbacks of the option framework. First, multiple uncertainty sources can be modelled following a stochastic process and can be included in the project evaluation, enabling the decision maker to gather new information on the project volatility. The second problem that has been solved is that the newly developed model can be used in a real-world environment. The author has proved that the technique can be applied for existing underground operations. Three case studies showed the applicability of the method for a real mining project. The results proved the importance of the new information that can be gathered with the new approach. It is possible to calculate the flexibility value for each mining zone and be broken down by subzones. The additional information helps the mine engineer to schedule mining activities so business opportunities can be grabbed if specific market conditions occur. The conditions apply well when the mining project has a long-term horizon and the profit ratio is barely positive. The mine engineer has then the flexibility to add new mineralized zones in the mining plan. Later, these zones may economically reward the operator. The decision maker also will be rewarded by getting a fair share of the new planned production: the life of the mine can extended, which is beneficial for the worker community, and taxes are levied on the new fortune. All in all, a company who uses an option framework to evaluate mining projects may create more wealth for its shareholders. The thesis has definitely achieved its goals to improve the mine evaluation process. The managerial flexibility when multiple sources of uncertainties exist must be known by the mine owner. The author's model is a proven tool that now exists to help every mine manager to carefully evaluate the true economic nature of the mining project.

This technique can be used where mining operations have a capital project under uncertainty. By understanding that uncertainties can taint a profitable project, the mine manager takes into account that the economic environment of the underlying asset may change over time and that it must dynamically be accounted for in the feasibility study. With the option of paying a premium to get more information as the project goes, it is more than necessary for the manager to use the Real Options techniques in a high capital cost environment. Multibillion-dollar projects are now more than ever present in the actual mining outlook, and decision makers need more than an NPV statement to go ahead or not with massive capital outlays.

In this thesis, the author succeeds in integrating the ROA using LSM as a common tool for the mine planner, and it was depicted through two published articles. The first article showed the usefulness of the ROA at an existing mining operation in Northern Quebec. ROA using LSM was conducted on a new project called Mine 2, owned by Xstrata Raglan Mine. Three conclusions were drawn. First, the ROA using LSM is practicable in a real-life environment. Second, the longer a mineralized zone production life, the better flexibility an operator has to switch its production policy, hence increasing its value. Finally, the unit revenue per unit cost ratio helps to focus on marginal ore bodies where ROA results show more gains to apply the new technique. In a context where the producer has the ability to temporarily shut down a mining zone, marginal ore bodies prove to be the ones with greater flexibility value as opposed to rich zones, where metal price outlook has a weak effect on its production. The second published paper also described a Real Options study at the Raglan Mine, but for a different project. ROA was performed on a marginal ore zone called 441 lens at East Lake mine. It was considered non-economical under the conventional project appraisal procedure using NPV. In this paper, it was demonstrated that using a stochastic metal price process to model the underlying asset proves time connectivity and avoids absurd price jumps in the simulations. The article concludes by analyzing three subzones of 441 lens, enabling the manager to decide whether or not to write off the mineralized zone from the reserve book. The use of ROA showed that the mineralized zone gathered economic value depending on specific timing in the mine schedule.

This thesis is original when compared with what has been done in the past, and has succeeded in using a new ROA model with LSM for up to seven uncertainties at the same time to evaluate real and practical mining projects. The integration of ROA in an entire mine planning process is realizable if not essential. Three case studies were performed and showed that results may differ from the classic NPV method to the ROA using LSM, leading the author to propose ROA as a tool in the mine planning strategy that can be utilized in most every mining operation.

While performing simulations on different mining projects, the author has identified another source of uncertainty that would need consideration for further studies. Geological attributes may prove to be strategic for the mining engineers as new information coming into play may change the mine planning. Wireframe envelopes are designed by the geologist to illustrate ore bodies. Typically, ore body wireframes are created from diamond drill holes data spaced according to a specific pattern. It is up to the geostatistician to graphically link the data together to form a mineralized shape.

When the miners excavate toward the zone, the production might end up different from what was first established in tonnage and grade. A statistical study on mined-out reserves could illustrate enough volatility that impacts on mining. For instance, a zone might be wiped out from the mining plan as the expected ore zone disappeared once the mining drift got through the expected pay zone. It is also true that a new mineralized zone can be observed where no diamond drilling was performed. The newly interpreted zone might have an influence on the mining method. If the expected mineralized zone were to be mined by cut and fill stoping, new geological information may contribute to change the engineer's decision to extract the ore by long hole panels, which is less manpower-intensive and less expensive. These changes have tremendous effects on mining costs, and consequently on profitability. Sometimes, mineralized targets are written off from the reserve book because they do not show a financial gain, or other zones are assumed to be economical until further information is collected.

Another research area that could be enhanced with regards to using ROA is the implementation of a global model that takes into account the impact of the production changes of singular project on a mining company that owns several mines in its portfolio. This aspect was not considered in this thesis and may prove to be of great importance should a mining company get additional penalties from ceasing production. These variations may hurt existing vertical profit centers such as the smelter and the refinery and may even penalize the marketing department.

With recent computation techniques and new hardware, managers can extend the economical study to the next level. It is important for them to do so, since success in mining is tied to the bottom line. Not only has the mining company had the responsibility to minimize the impact of its footprint on its properties, it is imperative that it creates wealth and optimizes the value of them. It is only reasonably diligent to think that with all the monetary resources in hand, a mining company has the instruments to study the economical value of the mining zones according to the ROA or at least to contract an engineering firm to do so. The classic NPV evaluation method has been the preferred tool for the financial community for the last two decades. Remember, however, that the NPV methodology was first developed in the 1950s and encountered the same kind of resistance that the ROA faces today. But in the end, the companies that adapt to this new method will have a competitive edge over their rivals and will bring better returns to their shareholders. This thesis will decrease this resistance because it reveals the importance of using the LSM mine model when appraising the economic value of a mining project. It also demonstrates the applicability of the method inside a standard mine planning process.

With new mining projects around the world, the mining community has the required tools to plan and to schedule ore production that secures operations and captures mining opportunities. Mining companies have the responsibility to mine out respectfully these non-renewable resources to create wealth that society will enjoy while minimizing the impact on the environment.

References

Abdel Sabour, S. A. 1999. Decision making with option pricing and dynamic programming: Development and application. *Resources Policy* 25 (4): 257–64.

Abdel Sabour, S. A. 2001. Dynamics of threshold prices for optimal switches: The case of mining. *Resources Policy* 27 (3): 209–14.

Abdel Sabour, S. A., and R. Poulin. 2005. Accuracy of least-squares Monte Carlo in valuing mineral properties. *Proceedings of the Fifth International Conference on Computer Applications in the Mineral Industries* (CAMI 2005): 1099–124.

Abdel Sabour, S. A., and R. Poulin. 2006. Valuing real capital investments using the least-squares Monte Carlo method. *Engineering Economist* 51 (2): 141–60.

Abdel-Sabour, S.A., R.G. Dimitrakopoulos, M. Kumral. 2008. Mine design selection under uncertainty. *IMM transactions, Mining Technology.* Institute of Mining and Metallurgy 2008.

Ait-Sahalia, Y. 2002. Telling from discrete data whether the underlying continuous-time model is a diffusion. *Journal of Finance* 57: 2075–112.

Armstrong, M., A. Galli, W. Bailey and B. Couët. 2004. Incorporating technical uncertainty in real option valuation of oil projects. *Journal of Petroleum Science and Engineering* 44:67–82.

Barraquand, J., and D. Martineau. 1995. Numerical valuation of high dimensional multivariate American securities. *Journal of Financial and Quantitative Analysis* 30: 383–405.

Bhappu, R. R., and J. Guzman. 1995. Mineral investment decision making: A study of mining company practices. *Engineering and Mining Journal* (July): 36–8.

Black, F., and M. Scholes. 1973. The pricing of options and corporate liabilities. *Journal of Political Economy* 81: 637–54.

Boyle, P. 1977. Options: a Monte Carlo approach. *Journal of Financial Economics* 4 (3): 323–38.

Bradley, P. 1998. On the use of MAP for comparing alternative royalty systems for petroleum development projects. *Energy Journal* 19 (1): 47–82.

Brennan, M. J., and E. S. Schwartz. 1977. The valuation of American put options. *Journal of Finance* 32 (2): 449–62.

Brennan, M. J., and E. S. Schwartz. 1978. Finite difference methods and jump processes arising in the pricing of contingent claims: A synthesis. *Journal of Financial and Quantitative Analysis* 13: 461–74.

Brennan, M. J., and E. S. Schwartz. 1985. Evaluating natural resource investments. *Journal of Business* 58: 135–57.

Broadie, M., and P. Glasserman. 1997. Pricing American-style securities using simulation. *Journal of Economic Dynamics and Control* 21: 1323–52.

Busby, J., and C. Pitts. 1997. Real Options in Practice: an Exploratory Survey of How Decision Makers in Industry Think About Flexibility. *Management Accounting Research* 8: 169–86.

Carriere, J. 1996. Valuation of the early-exercise price for options using simulations and nonparametric regression. *Insurance: Mathematics and Economics* 19 (1): 19–30.

Cortazar, G., and J. Casassus. 1998. Optimal timing of a mine expansion: Implementing a real options model. *Quarterly Review of Economics and Finance* 38: 755–69.

Cortazar, G., and J. Casassus. 1999. A compound option model for evaluating multi-stage natural resource investments. *In Project Flexibility, Agency, and Competition.* New York: Oxford University Press.

Cortazar, G, E.S. Schwartz, J, Cassasus. 2001. Optimal exploration investments under price and geological-technical uncertainty: a real options model. *R&D Management*, 31, 2, pp.181-189

Cox, J. C., S. A Ross and M. Rubinstein. 1979. Option pricing: A simplified approach. *Journal of Financial Economics* 7: 229–63.

Davveta, A., G.Felice, J. Hakala, and U. Wystup. 2002. A model for long term foreign exchange options. *Foreign exchange risk: Models, instruments and strategies.* Risk Books, London. England. Pp.317-325.

Davis, G. A. 1995. An investigation of the underpricing inherent in DCF valuation techniques. Society of Mining Engineers of AIME, Society for Mining, Metallurgy, and Exploration.

Dias, MAG. 2000. A step-by-step approach to improving investment decisions and asset valuation, Real Options in exploration & production of Petroleum. *Proceedings ROV Conference* March 2000.

Dias, MAG. 2001. Selection of investment in information for oilfield development using evolutionary real options approach. *5th Annual International Conference on Real Options – Theory Meets Practice*, UCLA-Los Angeles, USA. July 2001

Dimitrakopoulos, R.G, C.T. Farrelly, M. Godoy. 2002. Moving forward from traditional optimization: grade uncertainty and risk effects in open-pit design. *IMM transactions, Mining Technology.* January-April 2002. Institute of Mining and Metallurgy 2002

Dimitrakopoulos, R.G, S.A. Abdel Sabour. 2007. Evaluating mine plans under uncertainty : Can the real options make a difference? *Resource Policy* (2007).

Dixit, A. K., and R. S. Pindyck. 1994. *Investment under uncertainty.* New Jersey: Princeton University Press.

Dupras, N., and A. H. Green. 1999. Exploration and development history of the Raglan Ni-Cu-(PGE) deposits. Mineral Exploration Research Centre Guidebook Series, Vol. 2: 1–16.

Filipovic, D., and J. Zabczyk. 2002. Markovian term structure models in discrete time. *Annals of Applied Probability* 12: 710–29.

Frimpong, S., and J. M. Whiting. 1997. Derivative mine valuation: strategic investment decisions in competitive markets. *Resources Policy* 23 (4): 163–71.

Glasserman, P. 2004. *Monte Carlo methods in financial engineering.* New York: Springer-Verlag.

Godoy, M.C 2003, The efficient management of geological risk in long-term production scheduling of open pit mines, PhD thesis, University of Queensland, Brisbane, 256p.

Grant, D., V. Gautam and D. Weeks. 1997. Path-dependent options: Extending the Monte Carlo simulation method. *Management Science* 43 (11): 1589–602.

Hajdasinski, Miroslaw. 1989. Complete definition and computer aided examination of the payback period project evaluation criterion. *Proceedings of the 21st International*

Symposium on Application of Computers and Operations Research in the Mineral Industry: 765–80.

Harvey, C. R. 1999. Identifying real options. Lecture given at Fuqua School of Business, Duke University, Durham NC.

Hotelling, H. 1931. The economics of exhaustible resources. *Journal of Political Economy* 30: 137–75.

Hull, J., and A. White. 1993. Efficient procedures for valuing European and American path-dependent options. *Journal of Derivatives* 1: 21–31.

Hull, J. 1993. *Options, futures, and other derivative securities*. 2nd ed. New Jersey: Prentice Hall.

Hull, J. 2003. *Options, futures, and other derivatives*. 5th ed. New Jersey: Prentice Hall.

Jorion, P. R. Sweeney. 1996. Mean Reversion in Real Exchange Rates: Evidence and Implications for Forecasting. *Journal of International Money and Finance*, Vol. 15, Issue 4, 535-550

Kamrad, B. 1995. A lattice claims model for capital budgeting. *IEEE Transactions on Engineering Management* 42 (2): 140–9.

Kamrad, B., and R. Ernst. 2001. An economic model for evaluating mining and manufacturing ventures with output yield uncertainty. *Operations Research* 49: 690–99.

Kelly, S. 1998. A binomial lattice approach for valuing a mining property IPO. *Quarterly Review of Economics and Finance* 38: 693–709.

Keswani, A., and M. B. Shackleton. 2006. How real option disinvestment flexibility augments project NPV. *European Journal of Operational Research* 168: 240–52.

Laughton D. G., and H. D. Jacoby. 1993. Reversion, timing options, and long-term decision making. *Financial Management Association* 22 (3): 225–40.

Laughton, D., J. Sagi, and M. Samis. 2000. Modern asset pricing and project evaluation in the energy industry. *Journal of Energy Literature* 7 (1): 3–46.

Laviéville, M. 1990. *Statistiques et probabilités*. Paris: Dunod Éditeur.

Lazo JGL, MAC Pacheco, M Vellasco, MAG Dias. 2003. Real option decision rules for oil field development under market, Pontifical Catholic University of Rio de Janeiro, realoptions.org

Lemelin, B., S. A. Abdel Sabour and R. Poulin. 2006. Valuing Mine 2 at Raglan using real options. *International Journal of Mining, Reclamation and Environment* 20: 46–56.

Lemelin, B., J.-M. Clouet and R. Poulin. 2004. Evaluating multi zone ore bodies of the Raglan nickel deposit using current planning techniques complemented by real option pricing. *Proceedings of the 2004 CIM Mining Industry Conference*.

Lesher, C. M., 1998. Raglan/Katinniq Field Guide. Falconbridge Limited Internal Report.

Longstaff, F. A., and E. S. Schwartz. 2001. Valuing American options by simulation: A simple least-squares approach. *Review of Financial Studies* 14 (1): 113–47.

Mardones, J. L. 1993. Option valuation of real assets: Application to a copper mine with operating flexibility. *Resources Policy* 19: 51–65.

McCarthy, J., and P. Monkhouse. 2002. To open or not to open, or what to do with a closed copper mine. *Journal of Applied Corporate Finance* 15 (2): 63–73.

McDonald, R., and D. Siegal. 1986. The value of waiting to invest. *Quarterly Journal of Economics* 101: 707–27.

Miller, L. T., and C. S. Park. 2002. Decision making under uncertainty: Real options to the rescue? *Engineering Economist* 47 (2): 105–50.

Modigliani, F., and M. M. Miller. 1958. The cost of capital, corporation finance and the theory of investment. *American Economic Review* 48: 261–97.

Moel, A., and P. Tufano. 2002. When are real options exercised? An empirical study of mine closings. *Review of Financial Studies* 15: 35–64.

Moyen, N., M. Slade and R. Uppal. 1996. Valuing risk and flexibility: A comparison of methods. *Resources Policy* 22: 63–74.

Myers, S. 1977. Determinants of corporate borrowing. *Journal of Financial Economics* 5: 147–75.

Nemcsok, G. January 24, 2004. Datamine Software reference. Xstrata internal memorandum.

Olsen, T. E., and G. Stensland. 1988. Optimal shutdown decisions in resource extraction. *Economic Letters* 26: 215–8.

Ramazan, S, R.G. Dimitrakopoulos. 2004. Traditional and new MIP models for production scheduling with in-situ grade variability. *International Journal of Surface Mining, Reclamation and Environment*, 18(2), pp 85-98

Salahor, G. 1998. Implications of output price risk and operating leverage for the evaluation of petroleum development projects. *Energy Journal* 19 (1): 13–46.

Samis, M., and R. Poulin. 1998. Valuing management flexibility: A basis to compare the standard DCF and MAP valuation frameworks. *CIM Bulletin* 91 (1019): 69–74.

Samis, M. 2001. Multi-zone mine valuation using modern asset pricing (real options) techniques. National Library of Canada, ISBN 0612566145.

Samis, M., R. Poulin and V. Blais. 2003. Using real options to analyze the effect of contractual investment restrictions: The case of construction guarantees. CIM Annual General Meeting, Montreal.

Samis, M., G. Davies, D. Laughton and R. Poulin. 2005. Discounting risky project cash flows when there are no options: A Real Options Approach. *Resources Policy* 30: 285–298.

Samis, M., and G. Davies. 2004. Actively recognizing cash flow uncertainty within the net present value calculation. Mineral Economics and Management Society *MEMS Newsletter* 31: 13–19.

Samis, M., D. Laughton and R. Poulin. 2001. Valuing a multi-zone mine as a real asset portfolio: A modern asset pricing (real options) approach. SSRN.

Schwartz, E. S. 1997. The stochastic behavior of commodity prices: Implications for valuation and hedging. *Journal of Finance* 52: 923–73.

Seneta, E. 1996. Markov and the birth of chain dependence theory. *International Statistical Review* 64: 255–63.

Slade, M. E. 1988. Grade selection under uncertainty: Least-cost last and other anomalies. *Journal of Environmental Economics and Management* 15 (2): 189–205.

Smith, L. 2000. Discounted cash flow analysis: Methodology and discount rates. *Mineral Property Valuation Proceedings*. Canadian Institute of Mining, Metallurgy and Petroleum, Montreal: 85–100.

Stermole, F. Colorado School of Mines Research Foundation Inc. 1971. Economic evaluation of mineral investment alternatives. *Mineral Industries Bulletin* 15 (3).

St-Onge, M. R., S. B. Lucas and R. R. Parish. 1991. Terrane accretion in the internal zone of the Ungava Orogen, northern Quebec. Part 1: Tectonostratigraphic assemblages and their tectonic implications. *Canadian Journal of Earth Sciences* 29.

Tamhane, A. C., and D. D. Dunlop. 2000. Statistics and data analysis from elementary to intermediate. New Jersey: Prentice Hall.

Tilley, J. A. 1993. Valuing American options in a path simulation model. *Transactions of the Society of Actuaries* 45: 83–104.

Trigeorgis, L., and S. P. Mason. 1987. Valuing managerial flexibility. *Midland Corporate Finance Journal* (Spring): 14–21.

Trigeorgis, L. 1990. A real options application in natural resource investments. *Advances in Futures and Options Research* 4: 153–64.

Trigeorgis L., 1995. Real Options in Capital Investment, *Harvard Business Review*, May-June.

Appendix A. Nickel Price Modelling Using a Mean Reverting Price Model

This stochastic model is based on the characteristic, called the mean reverting process, of a metal price to return to its long-term equilibrium level after a shock in the market. Schwartz (1997) developed a form of mean reverting process that is described in detail in Chapter 3. It is generally well accepted in academia that the analytic solution of the mean reverting stochastic process can be expressed as shown below:

$$P_{t+g} = P^* \cdot \left[\frac{P_t}{P^*} \cdot e^{\frac{\alpha^*}{\gamma} \cdot \left(1 - e^{-\gamma g}\right)} \right]^{e^{-\gamma g}} \cdot e^{\left(RD \cdot \sqrt{\frac{\sigma^2}{2\gamma} \cdot \left(1 - e^{-2\gamma g}\right)}\right)}$$

where

P_{t+g} = Median price function of the metal

P^* = Long-term equilibrium price = \$5/lb.

P_t = Spot nickel price = \$23.74/lb.

α^* = Growth rate of the equilibrium price = 0.056

γ = Reversion factor = 0.22

g = Time period

σ = Short-term price volatility = 0.455

RD = Random draw from a normal distribution with a mean of 0 and a standard deviation of 1

This equation corresponds to the next price after period "g" and establishes a continuous time process for metal price that behaves according a mean reverting process. Since nickel tends to revert to a long-term equilibrium price, simulations with the mean reverting process are performed to simulate a series of price paths later used for the LSM method. The example provided in this section illustrates 10 nickel price simulations.

A.1 Important data for modelling the nickel price

For the sake of illustrating how to model the nickel price, some database arrays were shortened to allow a quicker and easier validation. The database that was built and used for the extensive work in the thesis was based on a daily nickel price in the London Metal Exchange (LME). Here in the example in Table A.1, only the annual nickel price is shown to illustrate the methodology of the calculations that will reveal some important parameters of the nickel price model.

Annual Nickel Spot Price in LME	
LME Copper	(US$/lb)
03-janv-89	8.53
02-janv-90	3.57
02-janv-91	3.78
02-janv-92	3.25
04-janv-93	2.75
04-janv-94	2.36
03-janv-95	4.03
02-janv-96	3.49
02-janv-97	2.89
05-janv-98	2.71
04-janv-99	1.81
04-janv-00	3.74
02-janv-01	3.17
02-janv-02	2.65
02-janv-03	3.27
02-janv-04	7.57
04-janv-05	6.40
03-janv-06	6.13
02-janv-07	15.22

Table A.1. Historical annual nickel spot price in the LME

Current long-term equilibrium price (P*):

The actual nickel price outlook raises some questions with regard to the long-term nickel price consensus. It had long been established that the nickel price would evolve around US$3.25/lb. Current mining projects with lower ore grades and the high cost of capital suggest that the long-term nickel price might have found a new base around US$5/lb.

Spot price (P_t):

As of April 10, 2007, the nickel spot price was at US$23.74/lb. The reference was taken from the Xstrata corporate office. Table A.2 below shows the last five days of nickel trading with inventory data.

London Metals Exchange Prices			
	NICKEL		
	CASH	3 MONTH	STOCK
DATE	**US$/lb**	**US$/lb**	MT
4/1/2007	NA	NA	NA
4/2/2007	21.6137	20.7292	5232
4/3/2007	22.5685	21.5933	5.66
4/4/2007	22.9881	22.0264	5124
4/5/2007	23.7569	22.4551	4812
4/8/2007	NA	NA	NA
4/9/2007	NA	NA	NA
4/10/2007	23.7456	22.4778	4632

Table A.2. Nickel spot price on April 10, 2007

Growth rate of the equilibrium price (α^*):

The growth rate for the price of nickel has been on an upward trend for the last 18 years. From January 1989 to December 2007, the price of nickel has gained 5.6% annually. Over the last 10 years, nickel has increased with an impressive 18.8% annual rate. Since January 1999, the silvery white metal has soared with a spectacular 41.8% rate a year. Table A.3 below demonstrates the strong acceleration of nickel appreciation over the last five years. Also, Figures A.1, A.2 and A.3 express the ascendant trends of nickel over different periods of time.

Period	Nb of years	Daily growth	Annual growth
89-07	18	0.0540%	**5.60%**
97-07	10	0.062%	**18.07%**
92-07	5	0.13%	**41.85%**

Table A.3. Annual growth rates for nickel

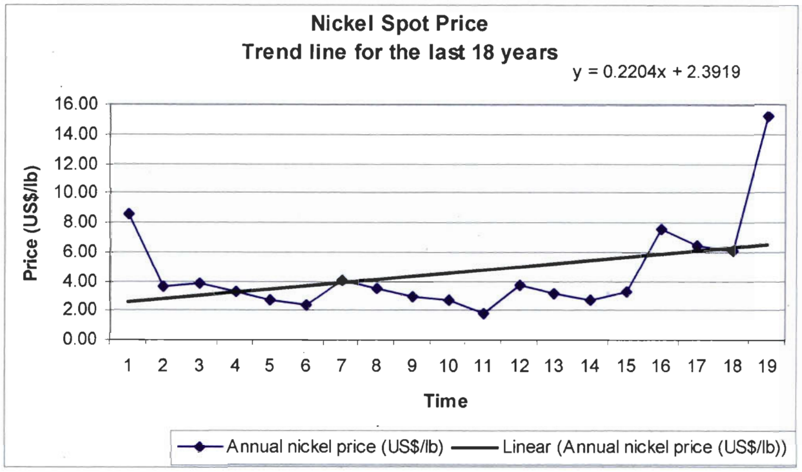


Figure A.1. Annual price of nickel over the last 18 years

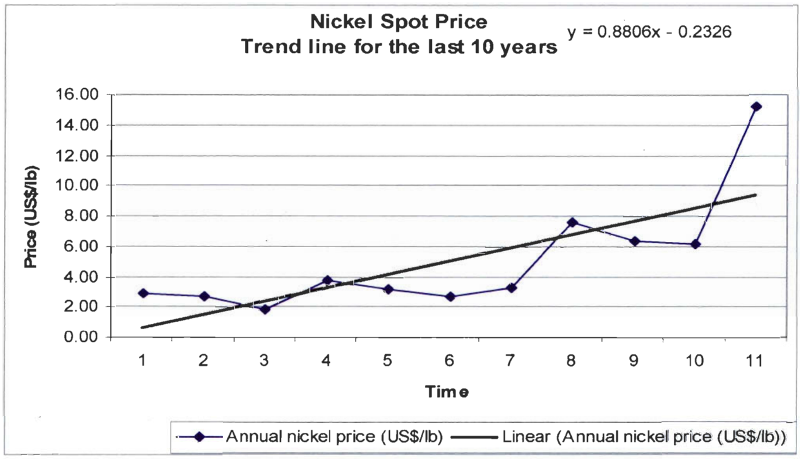


Figure A.2. Annual price of nickel over the last 10 years

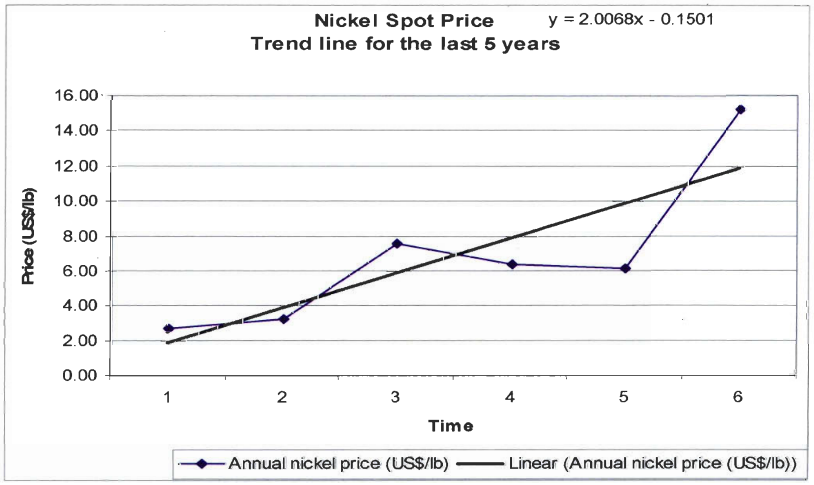


Figure A.3. Annual price of nickel over the last five years

A.2 Simulation results for nickel

With all parameters in hands, it is now possible to perform simulation on the nickel price according to the mean reverting process as shown at the beginning of this chapter. Note that the simulated paths respect the Markovian property that avoids absurd price jumps through time. Table A.4 shows 10 simulated price paths that have been computed with the mean reverting process, and Figure A.4 illustrates their associated profile.

Nickel price simulation paths										
Time	Path 1	Path 2	Path 3	Path 4	Path 5	Path 6	Path 7	Path 8	Path 9	Path 10
0	23.74	23.74	23.74	23.74	23.74	23.74	23.74	23.74	23.74	23.74
0.5	34.30	17.28	26.05	21.77	27.68	35.70	19.19	19.87	21.70	21.70
1	22.91	13.56	34.67	22.37	24.41	31.03	22.50	17.05	17.48	17.48
1.5	33.93	13.88	36.93	22.36	24.17	20.45	11.97	16.40	30.31	30.31
2	52.61	10.44	40.88	30.00	24.99	21.95	11.73	15.72	28.13	28.13
2.5	41.20	5.45	43.88	25.00	33.79	13.31	12.16	31.08	23.10	23.10
3	29.20	4.68	27.25	19.18	37.63	7.32	10.83	38.63	12.98	12.98
3.5	18.56	5.15	26.81	25.14	39.24	6.59	11.45	38.02	11.43	11.43
4	23.81	3.13	31.84	15.10	40.02	7.48	9.81	36.45	10.97	10.97
4.5	15.77	2.57	15.74	14.47	39.54	8.39	15.91	30.60	8.64	8.64
5	9.91	1.85	20.40	14.32	33.27	4.75	14.23	33.74	9.02	9.02
5.5	11.31	1.11	24.43	13.57	31.07	3.18	15.30	42.12	8.91	8.91
6	6.73	1.64	17.79	10.58	28.58	2.97	9.37	53.61	6.90	6.90
6.5	4.30	1.20	15.30	19.97	20.31	2.53	7.84	34.17	9.76	9.76
7	6.44	1.11	28.58	21.98	16.21	2.01	4.70	19.21	9.57	9.57
7.5	8.06	0.85	19.87	25.56	24.88	1.75	6.13	18.46	6.71	6.71
8	7.04	0.82	21.11	27.40	16.26	2.08	6.61	17.32	5.12	5.12
8.5	10.70	0.46	26.38	31.68	14.55	1.30	7.40	33.17	6.36	6.36
9	14.33	0.31	29.78	23.66	8.59	0.98	8.55	29.08	7.80	7.80
9.5	11.41	0.40	29.56	15.80	7.66	1.12	18.52	50.20	9.37	9.37
10	9.88	0.25	25.55	14.81	5.05	1.28	16.62	45.88	11.58	11.58

Table A.4. Nickel price simulation paths

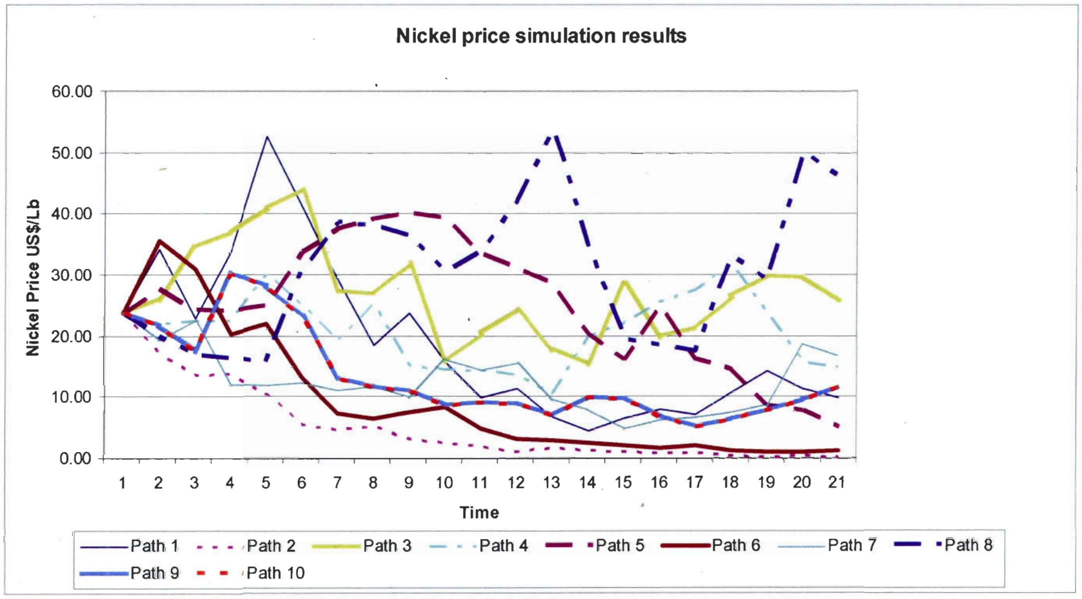


Figure A.4. Simulated nickel prices

Appendix B. Gold Price Modelling Using a Geometric Brownian Motion (GBM)

The analytic solution of the geometric Brownian motion stochastic process can be found in the literature, such as in Dixit and Pindyck (1994), Glasserman (2004) and Jorion (2003). It is expressed as shown below:

$$P_{t+g} = P_t \cdot e^{\left[(\alpha - 0.5\sigma^2)\cdot g + (\sigma \cdot RD \cdot \sqrt{g})\right]}$$

Where

P_{t+g} = Gold price at time t+g

P_t = Spot gold price = \$678.70/oz.

α = Growth rate = 0.0276

g = Time period

σ = Gold price volatility = 0.128

RD = Random draw from a normal distribution with a mean of 0 and a standard deviation of 1

This equation allows tracing multiple simulated price paths for metal that does not revert toward a long-term equilibrium price. For instance, gold price follows a geometric Brownian process with a reversion factor close to nil. The following example illustrates the mechanisms behind gold price simulations that are later required in the Least-Squares Monte Carlo (LSM) simulation, the last step to establish the real options value of a mining project.

B.1 Important data for modelling the gold price

Again, some database arrays were shortened to illustrate gold price evolution over time. The data was provided by the Xstrata corporate office in Toronto. Annual gold prices from 1992 to 2007 are represented in Table B.1.

Gold	
London Metals Exchange Prices	AMFIX
DATE	US$ /oz
January 2, 1992	351
January 4, 1993	332.3
January 4, 1994	386.5
January 3, 1995	383.6
January 2, 1996	387.4
January 2, 1997	369
January 5, 1998	292.5
January 7, 1999	289
January 4, 2000	281
January 30, 2001	265.9
January 7, 2002	279
January 3, 2003	345.3
January 5, 2004	422.5
January 5, 2005	425.7
January 3, 2006	533.5
January 3, 2007	628.5

Table B.1. Annual gold price since 1992

Gold price volatility (σ):

The gold price volatility can be expressed by simple standard deviation calculations. It is important to refer to relative volatility rather than absolute volatility since the gold price appreciation or depreciation in the simulation path follows a lognormal stochastic process. In the equation below, gold price volatility calculations are demonstrated at full lengths. Table B.2 illustrates measurement data for gold price volatility.

Where

$$\sigma = \sqrt{\frac{\tau}{n-1} \cdot \sum_{i=1}^{n} (u_i - \bar{u})^2}$$

σ = Price volatility

τ = Number of observation entries per year

n = Total number of observation entries

u_i = Natural logarithm relative price = $u_i = \ln\left(\frac{P_i}{P_{i-1}}\right), i = 1,\ldots,n$

$\bar{u}$ = Mean natural logarithm relative price

P_i = Price at time i

Annual Gold Spot Price in London		Gold price volatility (σ): Historic data method						
LME Copper	Pi (US$/lb)	u_i	τ	μ_{avg}	n	$(u_i-u_{iavg})^2$	$\Sigma((u_i-u_{iavg})^2$	σ
January 2, 1992	351		1	0.038836654	15		0.230031401	**0.1281827**
January 4, 1993	332.3	-0.054748				0.008758096		
January 4, 1994	386.5	0.1510937				0.012601643		
January 3, 1995	383.6	-0.007532				0.002150008		
January 2, 1996	387.4	0.0098574				0.000839797		
January 2, 1997	369	-0.048661				0.007655858		
January 5, 1998	292.5	-0.232332				0.073532426		
January 7, 1999	289	-0.012038				0.002588228		
January 4, 2000	281	-0.028072				0.00447677		
January 30, 2001	265.9	-0.055234				0.008849358		
January 7, 2002	279	0.0480915				8.56519E-05		
January 3, 2003	345.3	0.2132018				0.030403212		
January 5, 2004	422.5	0.2017758				0.026549179		
January 5, 2005	425.7	0.0075454				0.000979141		
January 3, 2006	533.5	0.2257242				0.034926954		
January 3, 2007	628.5	0.163877				0.015635078		

Table B.2. Gold price volatility

Spot price (P_t):

As of April 10, 2007, the gold spot price was established at US$678.70/oz. The reference was again taken from the Xstrata corporate office. Table B.3 below shows the last eight days of gold trading with inventory data.

London Metals Exchange Prices	**Gold**
	AMFIX
DATE	US$/oz
4/1/2007	664.30
4/2/2007	663.60
4/3/2007	664.20
4/4/2007	673.40
4/5/2007	674.40
4/8/2007	671.10
4/9/2007	675.00
4/10/2007	678.70

Table B.3. Gold spot price for April 1–10, 2007

Growth rate of gold price (α*):

The gold price growth rate for the three studied periods shows a positive trend. From January 1992 to December 2007, the price of gold has gained in value with an annual increase of 2.76%. Gold has accelerated its climb in the last 10 years with an impressive 8.74% growth rate. Only during the last five years has gold been spectacular with an annual price expansion of 21%. Table B.4 and Figures B.1, B.2 and B.3 express the ascendant trends of gold over different periods of time.

Period	Nb of years	Daily growth	Annual growth
92-07	15	0.0100%	**2.76%**
97-07	10	0.031%	**8.74%**
02-07	5	0.071%	**20.99%**

Table B.4. Annual growth rates for gold

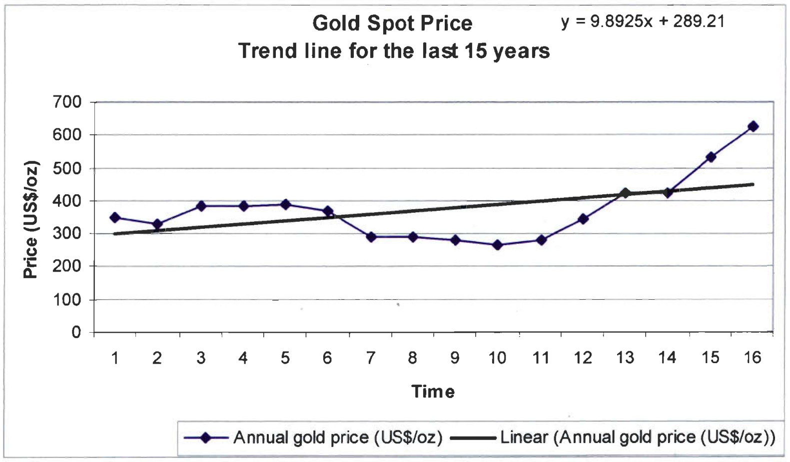


Figure B.1. Annual price of gold over the last 15 years

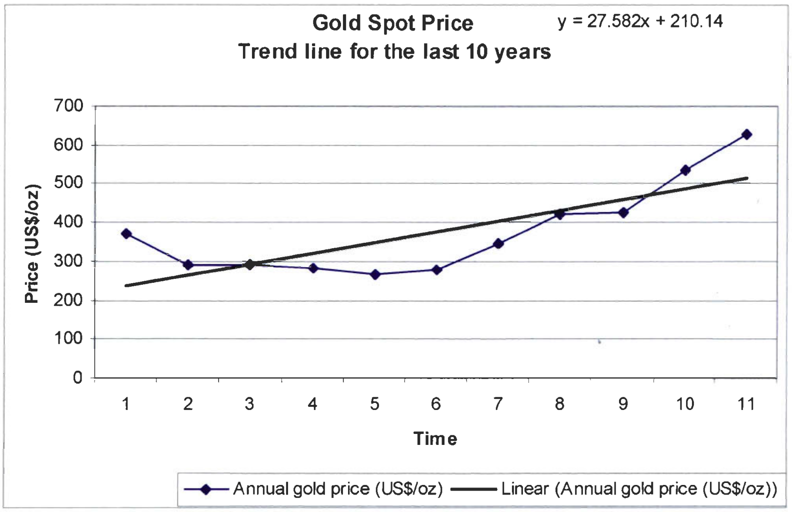


Figure B.2. Annual price of gold over the last 10 years

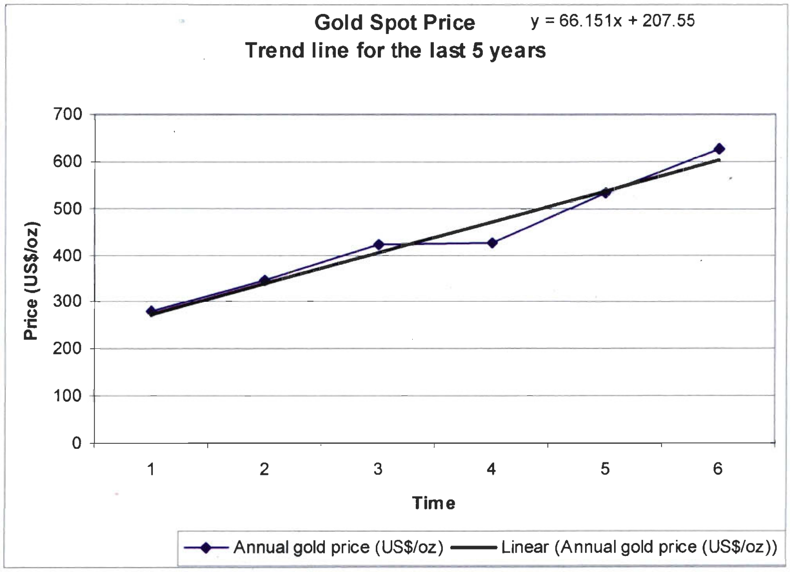


Figure B.3. Annual price of gold over the last five years

B.2 Simulation results for gold

Based on the parameters provided above, the simulation of the gold price according to the geometric Brownian motion can be carried out. To illustrate gold price behaviours, 10 simulated paths were performed (see Table B.5 and Figure B.4). In general, a minimum of 20,000 price simulations are required to obtain valid results according to Sabour et al (2005).

Gold price simulation paths										
Time	Path 1	Path 2	Path 3	Path 4	Path 5	Path 6	Path 7	Path 8	Path 9	Path 10
0	678.70	678.70	678.70	678.70	678.70	678.70	678.70	678.70	678.70	678.70
0.5	679.33	625.56	634.25	649.31	773.30	652.67	627.06	786.76	653.68	595.11
1	765.85	716.36	691.93	610.65	859.45	581.84	549.66	849.76	690.91	564.47
1.5	740.80	698.51	774.25	548.65	833.21	582.54	494.36	885.06	820.63	565.81
2	803.74	738.64	642.63	522.64	773.63	681.41	491.97	741.86	803.04	592.68
2.5	897.64	782.62	644.33	521.20	800.26	685.01	507.20	778.08	810.83	577.48
3	834.16	694.04	603.23	556.22	661.19	685.61	493.23	845.63	902.79	594.73
3.5	916.66	733.40	546.75	595.26	652.43	675.75	502.80	725.28	1115.06	663.02
4	796.13	792.06	651.32	714.81	632.71	674.46	481.06	680.30	942.97	613.23
4.5	801.16	896.41	677.32	684.84	561.80	728.07	541.79	676.78	1013.98	594.90
5	952.25	857.83	737.70	634.81	522.25	740.83	530.07	739.54	1014.56	547.67
5.5	964.07	1004.35	695.90	593.54	459.17	720.66	550.22	716.55	1012.65	708.92
6	1030.64	1007.64	651.47	554.59	425.36	699.70	561.27	615.48	777.85	727.19
6.5	1150.13	925.09	682.31	584.96	386.00	726.21	624.85	531.82	829.25	719.22
7	1218.75	1001.48	730.75	622.27	434.34	695.01	634.98	538.70	724.92	777.93
7.5	1195.71	1026.05	820.78	540.60	486.54	733.82	646.57	569.91	738.81	747.21
8	1205.26	1070.69	756.07	512.73	472.52	838.07	672.31	571.92	809.27	713.26
8.5	1140.74	921.60	712.27	533.31	481.94	892.45	670.93	513.74	815.70	741.34
9	1160.60	955.69	757.10	532.81	471.21	827.53	648.70	493.33	783.22	701.98
9.5	1287.42	1008.94	798.79	527.42	507.32	858.41	667.31	481.71	851.34	759.96
10	1101.60	851.11	761.37	520.67	513.29	841.25	629.12	441.26	749.11	791.31

Table B.5. Gold price simulation paths

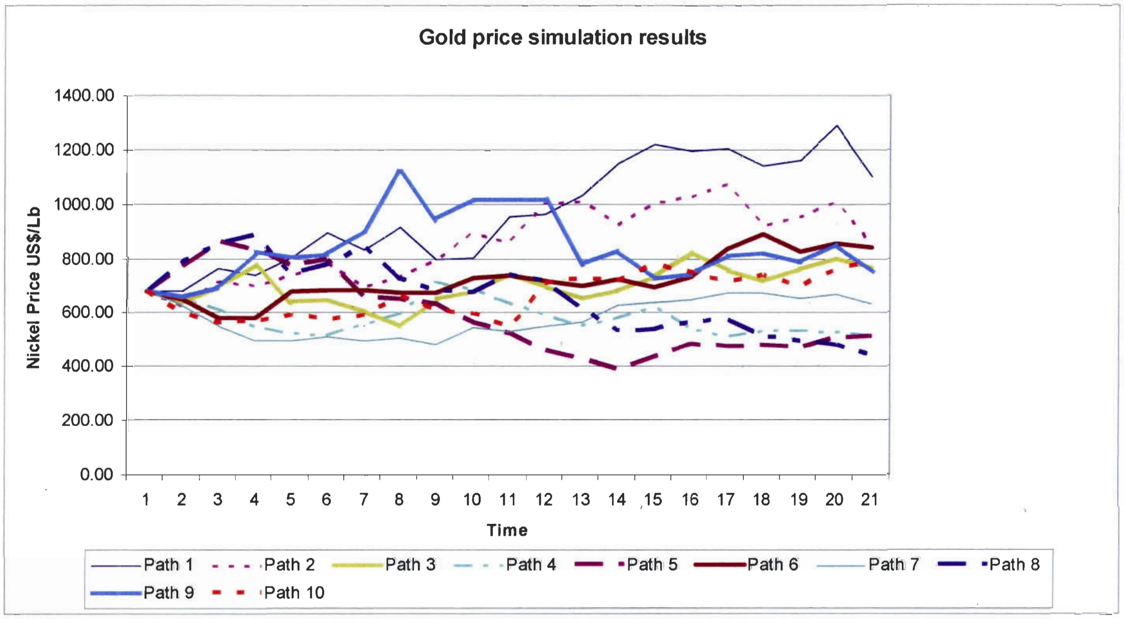


Figure B.4. Simulated gold prices

Appendix C. Numerical Examples for Applying the LSM Method in Mining Investments

In this section, some numerical examples illustrate the application of the Least-Squares Monte Carlo (LSM) simulation method as described in the thesis. It enables the evaluator to assess the managerial flexibility to abandon or to shut down a mine at any time desired. For the example below, a hypothetical gold mine has its production and cost data listed in Table C.1. In this table, the maintenance cost represents the annual cost required to maintain the mine when it is temporarily closed. This cost can be regarded as the fixed-cost portion of the annual operating cost. The shutting and the reopening costs represent the expenses the owner must pay to shut down temporarily or to reopen the mine. The extracted value from the operations does not include capital outlay or tax payments. Note that all figures are in US dollars.

Item	Description
Production rate	500,000 oz/year
Production cost	$450/oz
Risk-free interest rate	3%/year
Cost inflation	3%/year
Maintenance cost	$10M/year
Shutting cost	$5M
Reopening cost	$5M
Abandonment cost	0

Table C.1. Data for a hypothetical gold mine

The first step in assessing the mining operations is to simulate a series of metal price paths according to the appropriate stochastic methodology as shown in Sections A and B. To simplify the example, only 10 simulated gold price paths evolving over three periods of time are used. In reality, simulations must be numerous enough to ensure replicating the future volatility of the underlying metal price. Table C.2 shows the 10 simulated gold price paths in years t, t+1 and t+2.

Simulated prices ($/oz)			
Path	**Year t**	**Year t+1**	**Year t+2**
1	563.33	541.81	550.83
2	336.63	469.55	512.54
3	549.68	537.65	487.7
4	390.65	350.84	483.28
5	643.49	728.94	632.73
6	350.03	420.12	430.79
7	634.19	804.16	748.41
8	424.45	519.34	595.31
9	597.71	700.17	615.59
10	440.83	540.34	674.47

Table C.2. Simulated gold prices over a three-year period

The data in Tables C.1 and C.2 will be used to illustrate how to value the operating flexibility to abandon the mine as well as the flexibility to shut down the mine temporarily.

C.1 Valuing the flexibility to abandon

At time t, the operator has the option to produce 500,000 ounces of gold with an exercised price t+1. The time step used in this simulation is one year, but it should be set to a smaller period of time to reflect the American-style options (which can be exercised at any time, contrary to European options that can be exercised only at the end of the contract). It should reflect the ability for management to change the operating policy during an operating year. The second step is to calculate the cash flows of year t+1 using the simulated gold prices in year t+1, as shown in Table C.3.

Path	Gold price at time t+1 ($/oz)	Production cost ($/oz)	Production rate (oz/yr)	Cash flows ($M)
1	541.81	450	500,000	45.91
2	469.55	450	500,000	9.77
3	537.65	450	500,000	43.82
4	350.84	450	500,000	-49.58
5	728.94	450	500,000	139.47
6	420.12	450	500,000	-14.94
7	804.16	450	500,000	177.08
8	519.34	450	500,000	34.67
9	700.17	450	500,000	125.09
10	540.34	450	500,000	45.17

Table C.3. Cash flow matrix in year t+1

The third step is to develop a continuous function that will estimate the expected present value of the cash flows from time t+1 with predicted price of time t. Using the least-squares regression method, it is possible to define a simple function with its associated coefficients and constant. In this section, a second order function will be used to describe the expected present value of the cash flows at time t+1. The form of the function is shown below:

$$y = a_2x^2 + a_1x + a_0$$

where y represents the expected present value of cash flows, x is the simulated gold price at time t and a_2, a_1 and a_0 are the coefficients to be determined from the least-square regression. Table C.4 shows the data used in determining the coefficients of the basis function.

Path	Simulated cash flows at time t+1 ($M)	Simulated discounted cash flows at time t+1 ($M)	Gold price at time t ($/oz)
1	45.91	44.57	563.33
2	9.77	9.49	336.63
3	43.82	42.55	549.68
4	-49.58	-48.14	390.65
5	139.47	135.41	643.49
6	-14.94	-14.5	350.03
7	177.08	171.92	634.19
8	34.67	33.66	424.45
9	125.09	121.44	597.71
10	45.17	43.86	440.83
Average	55.65	54.03	493.10

Table C.4. Data used in the regression

From Figure C.1, coefficients of the function are found and can be mathematically expressed as:

$$y = 0.00204x^2 - 1.49596x + 269.40891$$

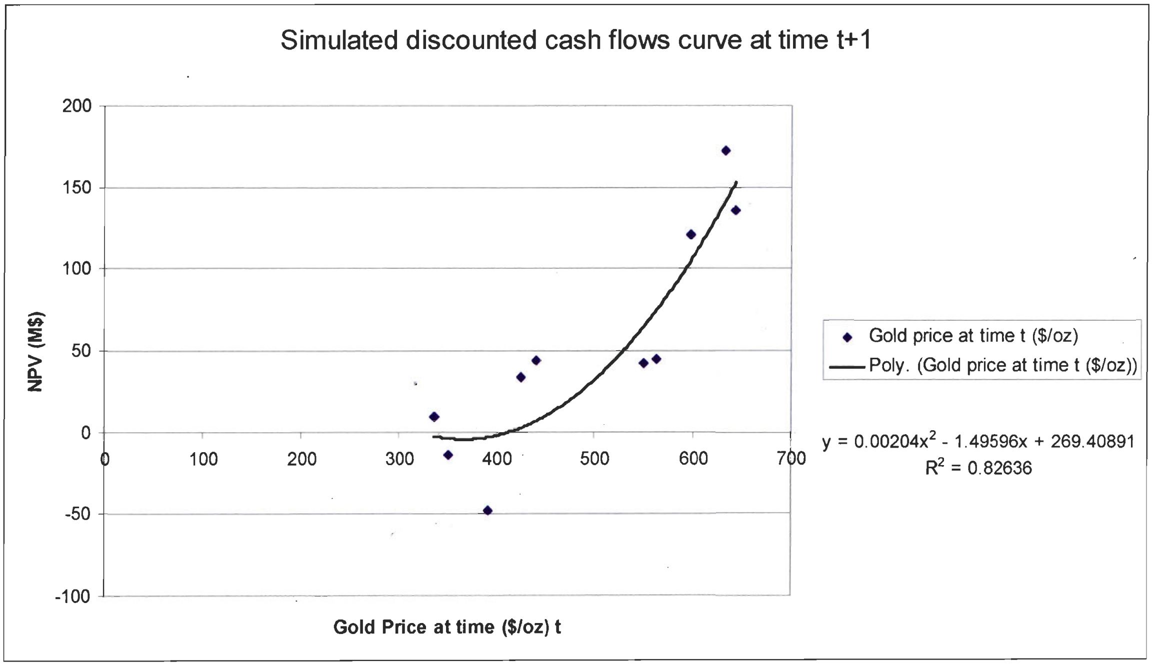


Figure C.1. Estimating the coefficients of the basis function for the abandonment option

The fourth step is to build another data array that will be named the expected present value at time t+1. The array will be constructed from the 10 simulated paths of the gold price at time t. Then, the manager has the option to produce the 500,000 gold ounces depending on the results he gets. If the NPV is negative, then he will never start production. This action will prevent the operation from losing money during that specific year. With 10 simulated prices, the manager approves or disproves production on the basis of profitability. As a result, only simulated discounted cash flows associated with positive expected cash flows will be taken in the average value of the operations. The calculations of the new value from the decision whether to approve or not the production show a different value of the project for the first year of the operations. Whereas the first average of the simulated discounted cash flows showed a value of $54.03M, the flexibility of not starting the production according to some price levels and prices expectations indicates a value of $59.34M, more than $5M above the first estimate. Table C.5 shows results data and the decision table for calculating the new discounted cash flow value of the project for the first year of operations.

Path	Gold price at time t ($/oz)	Expected cash flows at time t+1 ($M)	Decision	Simulated cash flows at time t+1 ($M)	Simulated discounted cash flows at time t+1 ($M)
1	563.33	74.06477	Produce	45.91	44.57
2	336.63	-3.003801	Abandon	0	0
3	549.68	63.49175	Produce	43.82	42.55
4	390.65	-3.668722	Abandon	0	0
5	643.49	151.4955	Produce	139.47	135.41
6	350.03	-4.279127	Abandon	0	0
7	634.19	141.1678	Produce	177.08	171.92
8	424.45	1.970605	Abandon	34.67	33.66
9	597.71	104.0634	Produce	125.09	121.44
10	440.83	6.380285	Abandon	45.17	43.86
Average	493.10	53.17		61.12	59.34

Table C.5. Decision matrix for the abandonment option

C.2 Valuing the flexibility to temporarily shut down the mine

Now that the value to open the mine or not has been determined, the next option for the producer (should he decide to start production) is to continually challenge his decision to remain open. This section will establish the flexibility value to postpone production by one year instead of pursuing the production efforts. Note that postponing production means only temporarily shutting down the mine instead to permanently abandoning it. Two parameters have a direct impact on the degree of flexibility of the producer in the option to shut down the mine for a year: maintenance costs for the mine while it is closed and switching costs to close and reopen the mine. In this example, maintaining the closed mine costs $10M a year while switching operating policy costs the producer $5M.

To calculate the flexibility value of the shutdown option, the evaluator needs to assess the cash flows from year t+1 and year t+2. As shown on Table C.6, the average results are the simulated undiscounted cash flows for years t+1 and t+2, which can be calculated by subtracting the production cost from the revenues.

Path	Gold price at time t ($/oz)	Gold price at time t+1 ($/oz)	Gold price at time t+2 ($/oz)	Production cost ($/oz)	Production rate (oz/yr)	Cash flows when producing in year t+1	Cash flows when producing in year t+2
1	563.33	541.81	550.83	450	500,000	45.91	50.42
2	336.63	469.55	512.54	450	500,000	9.77	31.27
3	549.68	537.65	487.7	450	500,000	43.82	18.85
4	390.65	350.84	483.28	450	500,000	-49.58	16.64
5	643.49	728.94	632.73	450	500,000	139.47	91.36
6	350.03	420.12	430.79	450	500,000	-14.94	-9.61
7	634.19	804.16	748.41	450	500,000	177.08	149.21
8	424.45	519.34	595.31	450	500,000	34.67	72.66
9	597.71	700.17	615.59	450	500,000	125.09	82.79
10	440.83	540.34	674.47	450	500,000	45.17	112.24
Average						**55.65**	**61.58**

Table C.6. Cash flows calculations for the temporary closure option

The cash flows when producing at year t+1 are then discounted at the risk-free rate of 3% as illustrated in Table C.7. The cash flows of year t+2 are discounted twice since two years have passed and the maintenance cost and the switching cost are subtracted from the discounted value to reflect the present value of the operation at time t+2 if the operations were closed in year t+1.

Path	Gold price at time t ($/oz)	Cash flows when producing in year t+1	Cash flows when producing in year t+2	Discounted cash flows when producing in year t+1	Discounted cash flows when producing in year t+2 minus $15M (maintenance + reopening costs)
1	563.33	45.91	50.42	44.57	32.53
2	336.63	9.77	31.27	9.49	14.47
3	549.68	43.82	18.85	42.55	2.77
4	390.65	-49.58	16.64	-48.14	0.68
5	643.49	139.47	91.36	135.41	71.12
6	350.03	-14.94	-9.61	-14.5	-24.06
7	634.19	177.08	149.21	171.92	125.64
8	424.45	34.67	72.66	33.66	53.49
9	597.71	125.09	82.79	121.44	63.04
10	440.83	45.17	112.24	43.86	90.80
Average				**54.03**	**43.05**

Table C.7. Data used in the regression

With spot prices at time t, it is now possible to calculate the value of the next production according to the state of production at time t+1. With the help of Table C.7, the regression analysis is performed on the discounted cash flows at time t+2 for the scenario in which the manager reopens the mine after year t+1. The function can be expressed as follows:

$y = 0.00012x^2 + 0.12441x - 48.91913$

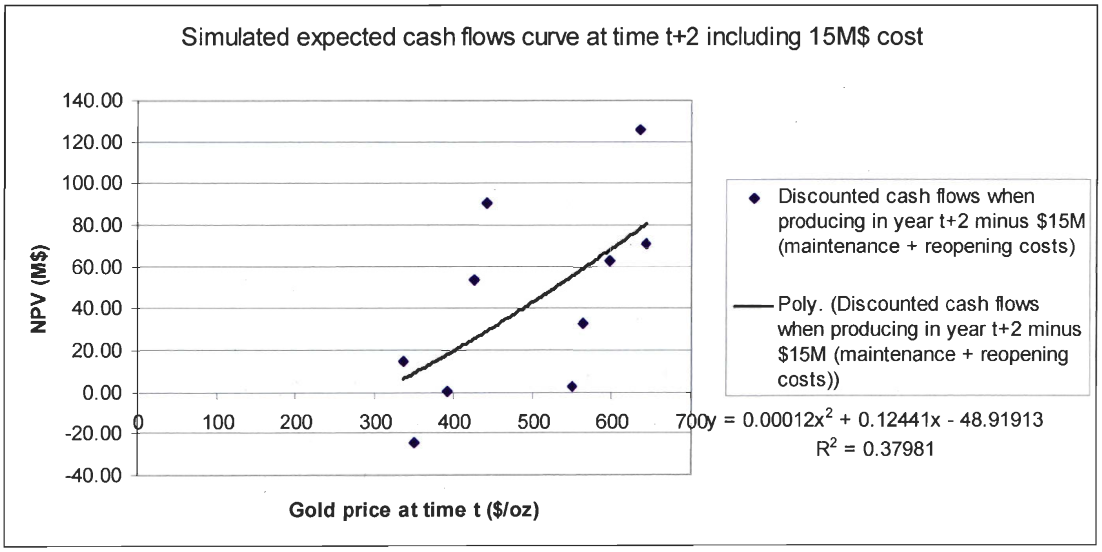


Figure C.2. Estimating the coefficients of the basis functions for the temporary closure option

For the scenario in which the mine stays open, the equation remains the same as in Section C.1, which represents the expected cash flows at time t+1.

Is it more profitable to delay the production by one year or to keep the production on-line according to the gold price at time t? With the two regression equations, both expected values at the mine (whether it is open or closed) can be inputted according to their respective expected functions. The expected value of the mine at time t+1 if the mine is open remains the same as in Section C.1 since the regression equation is the same. The expected cash flows from the reopening of the mine at time t+2 can be calculated from the regression formula found in Figure C.2. Of course, the decision to keep the mine open at time t+1 must be greater than the expected value of shutting down the mine, which includes a $5M fee. On Table C.8, the flexibility value is shown in the last column, where the expected present value of the production is compiled ordering an average of $65.13M, which is about $9.5M above the average value without the flexibility to temporarily shut down operations for a year.

Path	Gold price at time t ($/oz)	Expected cash flows at time t+1 ($M)	Expected cash flows at time t+2 ($M)	Expected cash flows at time t+2 minus the shutting cost ($M)	Decision	Present cash flows
1	563.33	74.06	59.25	54.25	Keep open	44.57
2	336.63	-3.00	6.56	1.56	Shut down	14.47
3	549.68	63.49	55.72	50.72	Keep open	42.55
4	390.65	-3.67	17.99	12.99	Shut down	0.68
5	643.49	151.50	80.83	75.83	Keep open	135.41
6	350.03	-4.28	9.33	4.33	Shut down	-24.06
7	634.19	141.17	78.24	73.24	Keep open	171.92
8	424.45	1.97	25.51	20.51	Shut down	53.49
9	597.71	104.06	68.31	63.31	Keep open	121.44
10	440.83	6.38	29.24	24.24	Shut down	90.80
Average		53.17	43.10	38.10		65.13

Table C.8. Decision matrix for the temporary closure option

Appendix D Cash Flow Discounting Mechanisms: NPV vs. ROA

In this section, a paper that was presented at the 2004 CIM Mining Industry Conference in Edmonton describes the current mine planning process at Raglan as well as its limitations. Once the deficiencies are exposed, a comparison between the conventional net present value methodology (NPV) and the Real Options Approach (ROA) is made in order to underline the difference in the cash flow discounting mechanisms. The paper also describes to a great extent the mine planning process that is currently in place at Raglan.

D.1 Article Abstract: the current evaluation method at Raglan[8]

Before ROA methodology can be applied on the Raglan mining projects, it is important to illustrate the current evaluation planning process. When studying any mine appraisal technique, it is vital to divide the analysis into four sections. The first element is to know what type of merit measures are used to select profitable projects at the site. It can be surprising to see different tools among different mine sites. The second aspect is to identify shortcomings of the actual process or to evaluate where improvements can be made to enhance the precision and the accuracy of the economic results. The third feature is to identify the uncertainties that have the most impact on the business plan of the company. If the merit measures do not capture the volatility of such uncertainties, the results may be biased. Then, it is important to be familiar with the most significant risks of a mining project. The fourth section is to set a plan as to how the technique would resolve the problem.

---

[8] Lemelin, B., J-M. Clouet and R. Poulin. 2004. Evaluating multi zone ore bodies of the Raglan nickel deposit using current planning techniques complemented by real option pricing. *Proceedings of the 2004 CIM Mining Industry Conference.*

These four parameters were described in a article written in 2004. The article took into account the current planning techniques used by the engineer at Raglan. It could be seen that the site used conventional merit measures, including the discounted cash flows net present value (NPV), the internal rate of return (IRR) and the payback period. The article showed the disadvantages of using such merit measures by explaining the multiple biases that may occur from a single discount factor managing the project risk profile. In fact, it showed that several shortcomings could be found when cash flows are discounted uniformly with a simple discount rate. The article also confirmed that it is possible to expose the most important uncertainties in the evaluation model by using certain analytical tools, such as a sensitivity chart. It demonstrated the importance of the price of nickel as well as foreign exchange rates in the economic model at Raglan. A spider diagram and a tornado chart were presented to illustrate the significance of these parameters. The author explained that it is possible to enhance the evaluation analysis by using a different form of discounting called “differential discounting,” which set the base for using ROA. In truth, the new discounting mechanism improved the evaluation techniques by taking the risk out of the cash flows at the specific time it would occur. It also explained the theory of how the cash flows are then time discounted to the present time.

Résumé de l’article: la méthode d’évaluation actuelle à Raglan

Avant que la méthodologie des options réelles puisse être appliquée à la mine Raglan, il est important d’illustrer le processus actuel d’évaluation des projets miniers. Est-ce que la théorie s’applique aux opérations de la mine? Pour y répondre, il est vital de diviser l’analyse en quatre parties. Le premier élément consiste à identifier les outils de mesure économique qui sélectionnent les projets rentables au site. Il peut être surprenant de voir différents types d’outils utilisés à travers les sites miniers. Le deuxième aspect est d’identifier les faiblesses du processus actuel et les points de développement pour savoir si la technique des options réelles pourrait améliorer la précision et l’exactitude des résultats économiques. Le troisième point est d’identifier les incertitudes qui ont le plus d’impact sur le plan d’affaires de la compagnie. Si les critères de mesures économiques n’incluent pas la volatilité des incertitudes identifiées, les résultats peuvent être biaisés. C’est pourquoi il est nécessaire d'être familier avec les risques les plus significatifs ayant un impact sur un projet minier. Le quatrième volet de l'analyse est d'établir un plan à savoir comment la technique des options réelles pourraient résoudre les problèmes d'évaluation. Ces quatre paramètres sont décrits dans l'article écrit par l'auteur en 2004. L'article met au jour le processus actuel de l'évaluation de projet minier utilisé par l'ingénieur à Raglan.

Il est démontré que les critères de mesure en vigueur à la mine sont les méthodes conventionnelles de la valeur présente nette (VPN), du taux de rendement interne (TRI) et de la période de recouvrement. Le texte démontre les désavantages d'utiliser ces critères en expliquant les biais multiples que peuvent occasionner par l'utilisation d'un taux d'actualisation simple comme simple outil de gestion du risque.

De plus, l'article démontre les défauts qui peuvent être retrouvés dans l'évaluation économique d'un projet lorsque les flux monétaires sont actualisés uniformément en suivant un seul taux d'escompte.

L'auteur confirme également qu'il est possible de mettre en relief les plus importants risques dans le modèle d'évaluation en utilisant certains outils analytiques comme la charte de sensibilité. Il est démontré que le prix du nickel et que le taux de change sont les dimensions d'incertitudes ayant le plus d'impact sur la valeur d'un projet à Raglan. Un diagramme araignée et une charte de forme "tornade" illustrent l'importance de ces paramètres.

L'auteur explique qu'il est possible d'améliorer l'analyse économique du projet en utilisant l'actualisation différentielle qui est la prémisse à l'utilisation des options réelles. En réalité, le nouveau processus d'actualisation permet le découplage du risque de chaque poste monétaire à chaque moment du projet. Il est ensuite expliqué comment les flux monétaires y sont par la suite actualisés en valeur d'aujourd'hui.

EVALUATING MULTI ZONE ORE BODIES OF THE RAGLAN NICKEL DEPOSIT USING CURRENT PLANNING TECHNIQUES COMPLEMENTED BY REAL OPTION PRICING

Bruno Lemelin, P.Eng, Falconbridge Ltd, Raglan Mine
Jean-Marie Clouet, P.Eng, Falconbridge Ltd, Raglan Mine
Dr. Richard Poulin, Université Laval

Abstract: Falconbridge Ltd Raglan Mine has been in production since 1998 and has produced annually some 900 000T of ore grading 3.1% nickel, 0.9% copper and 0.06% cobalt. The mine is located in Nunavik, Northern Quebec. The current mine life is estimated at approximately 20 years. Reserves are composed of numerous small ore bodies situated across a 65 km strike length. Planning the extraction of the ten small mineralized zones represents a great challenge to optimize.

The overall objective is to maximize the company's return on net assets. Actual planning uses computerized technology tools like Automod and Mine2-4D, and a conventional discounted cash flow method (DCF) of calculating net present value (NPV). The complex setting of Raglan could benefit from advance valuation techniques such as the real options method (RO). This paper will present the characteristics if the mines that make it a good candidate to use RO to gain further insight.

Geology

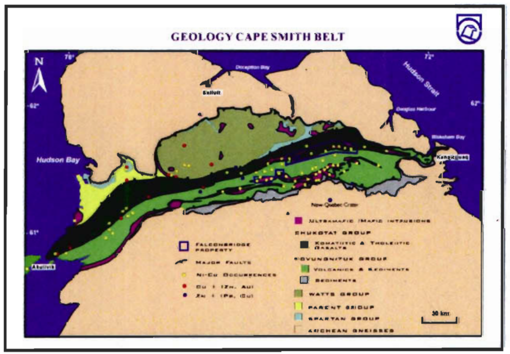


Figure 1. CSB Geology (St. Onge and Lucas, 1991).

The komatiitic Ni-Cu-PGE sulphide deposits that define the Raglan Nickel Belt occur within the early Proterozoic volcano-sedimentary Cape Smith Belt (CSB) extending 375km across the Ungava Peninsula of Northern Quebec. The CSB represents the northernmost segment of the Circum-Superior Belt, a rifted continental margin and arc-continental collision zone juxtaposed against the Archean Superior Province (St-Onge et al., 1991). The CSB is comprised of two tectonostratigraphic domains (Figure 1). The Northern Domain consists of the Watts Group ophiolite, active volcanism within a continental subduction zone (Parent Group) and associated metasediments (Spartan Group). The Southern Domain hosts the Ni-Cu mineralization and is comprised of a basal volcano-sedimentary assemblage reflecting initial continental rifting (Povungnituk Group) and komatiitic basalt to mafic lavas representing the opening of an oceanic basin (Chukotat Group).

The tectonostratigraphic domain containing the Chukotat and Povungnituk group rocks is preserved as remnant thrust imbricates that rest unconformably on Superior Province basement gneisses, having been translated southward as a result of convergence (St. Onge and Lucas 1991). Post thrusting regional folding has yielded a broad synclinorium of the Cape Smith stratigraphy. The magmatic sulphide mineralization occurs within the Raglan Formation, a horizon of voluminous mafic-ultramafic volcanic and high-level intrusive rocks at the base of the Chukotat Group (Lesher, 1998). The Raglan Formation is interpreted to have erupted in a relatively deep water environment and recent geologic and geophysical data supports the suggestion of a large, east-west trending meandering lava channel system filling a broad embayment feature. The Raglan Formation dips north (roughly 45 degrees), extends to depths greater than 1km and in general is 200-400m thick. The ultramafic rocks that make up the Raglan Formation in general change from peridotite to olivine pyroxenite to pyroxenite as one ascends through the volcanic pile from its base. The economic mineralization is associated with basal sub channels in secondary embayment features and as such, identification of the flow channel environment and determining its orientation is key as multiple small, irregular, high-grade sulphide accumulations along the trend are the "norm".

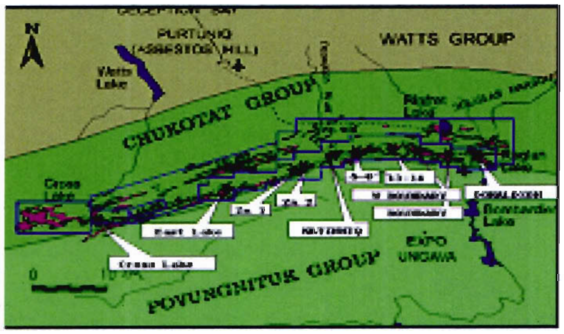


Figure 2. Location of the all ore deposit zone for the Raglan project

The Ni-Cu-PGE mineral reserves and resources are focused in ten major localities spanning the strike length of the property (Figure 2). To date, roughly 100 distinct mineralized bodies make-up the mineral reserve and resource and range in size from 15000 to 1 000 000 tonnes. The need for Raglan to periodically develop and sequence-in new mines is key to the mine plan and it is this high number of potential zones of production that makes Raglan a challenge to optimize the long-term planning.

Problem statement

Uncertainty of metal price, mineral reserves and diesel cost create a high risk environment. A conventional financial analysis such as the Discounted Cash Flow (DCF) cannot capture a multivariable risk concept. The introduction of Real Option (RO) represent an avenue that could integrate risk parameter in a meaningful way inherent in a complex project such as Raglan. This advanced financial technique is successfully used in other industries and is getting more and more attention from the mining sector.

Conventional DCF project evaluation will use standard NPV, IRR and payback period criteria. The introduction of a more complex advance procedure such as RO will meet some resistance. Even though users of DCF are instinctively aware that the method is not sensitive to project risk (modifying the discount rate with each project would be seen as fudging), the adoption of an alternative will take time and will require the demonstration that the method is robust, reproducible and correct in a wide range of mining applications. A body of example in peer-review literature is building (Brennan and Schwartz, 1985; Mardones, 1993; Frimpong and Whiting, 1997; Cortazzar and Casassus, 1998; Samis et Poulin, 1998; Sabour, 1999, 2001; McCarthy and Monkhouse, 2002). With time, we are seeing that the process is becoming more and more transparent.

The objective of this paper is not to promote RO but to evaluate how it could be applicable and practical for the Raglan case. We will describe the actual procedure and see how RO can apply by describing the logic of the method. We would like to highlight some of the strength and the shortcomings of what we are doing actually and see if with the help of RO we can improve.

Actual Process

Long-term planning at Raglan uses numerous tools for an in depth approach. Datamine Studio software is employed to create wireframe shells at 0.65%, 1.0%, 1.5% and >5% Ni grades from the available diamond drill hole information across all Zones at Raglan. Ordinary kriging is employed within the confines of these orebody wireframes and block models are created within their limits. In one pass, block model cell grades for six payable metals, Ni, Cu, Co, Au, Pt and Pd are determined along with FE, S and density. The block models may be individually assessed, by element, as Ni equivalent, dollar equivalent or any combination(s) thereof in real 3D space (see Figure 3).

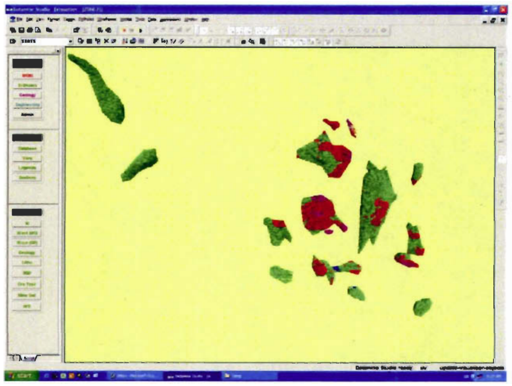


Figure 3. Block model made with Datamine

Using reserve calculations and the wire frame shells, the long-term planning engineer design the production plans and the mining schedule with the Mine2-4D module (see Figure 4). Unless the new zone shows abnormal characteristics the planning will be based on parameters determined from previous experience of similar zones being mined or mined out. Ventilation and backfill systems are conceptualized following general standards and later optimized as the mine design is firmed up.

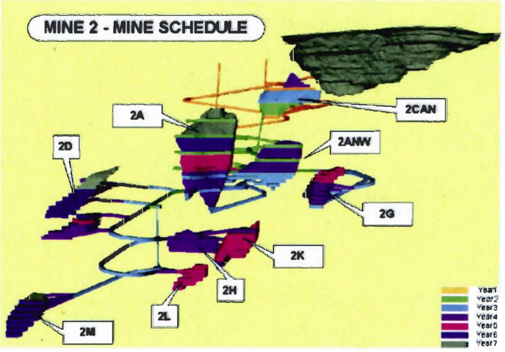


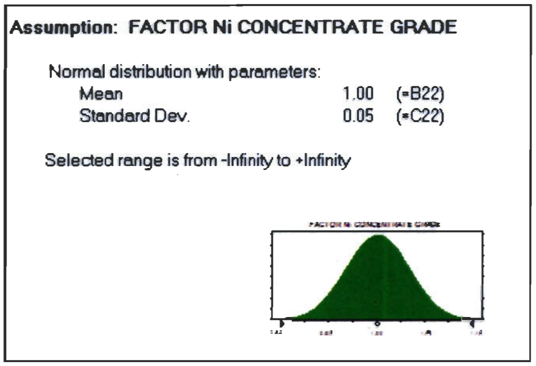
Figure 4. Mine design and scheduling with Mine 2-4D

The geotechnical analysis of the mineralized zone is used to optimize, following an iterative procedure, the mine design in relation to ground support. Simultaneously, another iterative procedure is initiated using VNETPC to create a ventilation simulation model based on the characteristics of the mine design. Some touch up to the model normally will validate the needs in primary ventilation for the mining of the up coming zone.

The following step is to extract of Mine2-4D the mine schedule and to generate a production calendar with Earthworks Production Scheduler (EPS) software. Based on time studies and on historic data concerning production rate coming from mined out zones, a mine production capacity is establish. The EPS software shows what is expected from the specific production teams, each one having a set performance rate, as well as grade and tonnage per period. However, EPS does not take into account the interactions of production activities and their variability, i.e. the resulting schedule is just a point estimate. To confirm the production capacity of the mine and establish a confidence interval, simulations are made with the Automod software. Finally, a ground condition analysis is performed to assess the level of risk of each stope to optimize the mining sequence and to avoid long periods in high risk areas.

Combining each mine production plan and driven by mill operational limits, Raglan long-term plan is created as an excel model. The long-term equipment strategy and the electric consumption model are linked to the overall production plan. The main document includes production, revenues, operating cost, recovery, development, Capex cost and merit measures. This is visible on the engineering and financial interactions flow chart given at the end of the paper (Figure 9). This result is based on two additional set of data. A first one comprises metal prices, foreign exchange rates (Canadian, American and Norwegian currencies) and diesel fuel cost. The list of price originates from the Toronto head office research team. The second one is a recovery model for the mill.

The results are given to give a clear picture of mine design and economic implications (NPV, IRR and payback period). A sensitivity analysis is applied on the parameters displaying uncertainty such as metal prices, foreign exchange rate, production rate, reserves and grade (Figure 5). The distribution of the uncertain variable is base on historic data or performance measures. Calculations are made using the user friendly Crystal Ball simulation software. The Monte Carlo simulation will show the estimated value as well as its variance inside a confidence interval (Figure 6). A spider diagram and tornado chart (Figures 7 and 8) also show graphically the influence of parameters on the total variability of the project and their impact on the net present value, the internal rate of return and the pay back period.



Assumption: FACTOR Ni CONCENTRATE GRADE

Normal distribution with parameters:

Mean	1.00	(=B22)
Standard Dev.	0.05	(=C22)

Selected range is from -Infinity to +Infinity

Figure 5. Parameter variance with Crystal Ball

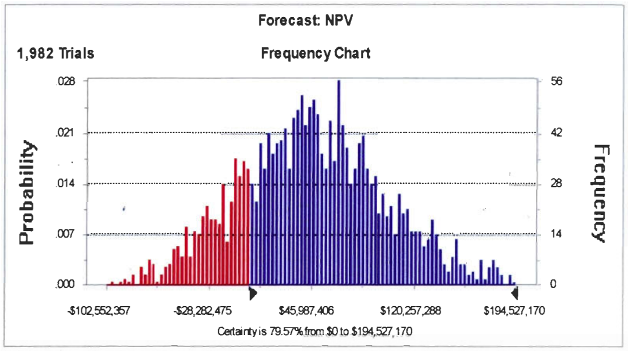


Figure 6. Estimated value with confidence interval with Crystal Ball Simulation

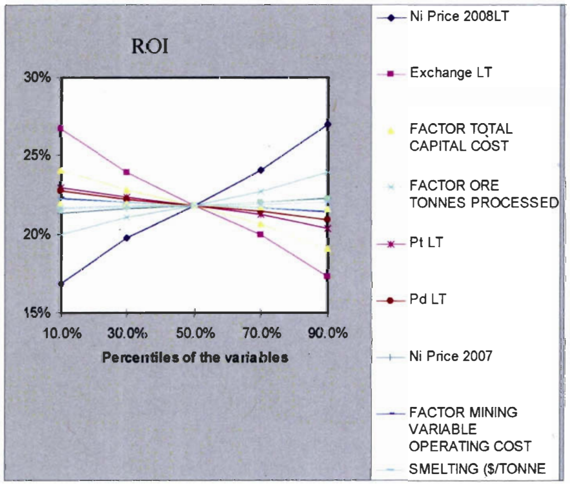


Figure 7. Spider diagram

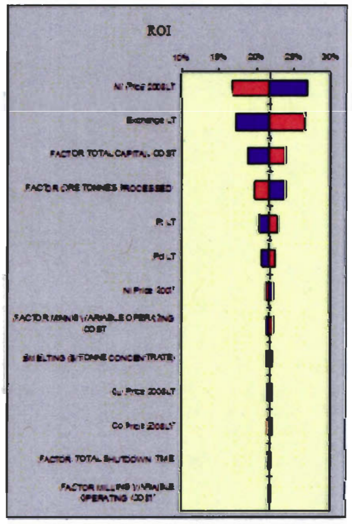


Figure 8. Tornado chart

Note: Some geological or economical parameters or values were changed or omitted for corporate confidentiality.

Real Option introduction

The DCF method, widely used in the mining industry including Raglan, has its weakness. First, a discount rate needs to be chosen. A popular way is to use associated risk premiums (Smith, 2000). For example, starting with a riskless discount rate to which is added a risk premium for each category of uncertainty. An uncertain production could bring 2% to be added to the initial rate. This premium is not based on a model, but rather on the experience of the evaluator. The probability that for each source of uncertainty be estimated the same by different evaluators is remote.

Another way to choose a discount rate is using the weighted average cost of capital (WACC) method. It is believed that the cost of financing is the market evaluation of the level of risk associated with the firm. The use of the WACC as a proxy for a risk adjusted discount rate as merit when looking at the global portfolio of a firm. It does not apply the individual project being analyzed unless they are "average".

Secondly, the DCF method will discount cash flows independently of their nature and their evolution over time. Often, a feasibility study will comprise a wealth of information (production, processing, scheduling) that will be boiled down to a simple discount rate, questionably adjusted (Samis et al., 2004). Looking at Falconbridge operations at Raglan, different mineralized bodies have production parameters that are specific and have different operating lives. Uncertainty and accordingly risk varies between them. By using a single discount rate, each is equalized to the same level.

Finally, DCF does not differentiate the real economic nature of the project by amalgamating the risk with the return on investment. The RO should show a more realistic value of the project. It will then be to the manager to decide if the project is a good fit for the investment portfolio.

How can RO overcome some of the deficiency of the traditional DCFC method of project evaluation?

The RO technique adjusts each cash flow with is associated risk. After, the cash flows are discounted with riskless discount rate. For example, Falconbridge has to evaluate the merit in investing into the East Lake deposit in the Raglan basin. This deposit shares the same geological and geotechnical characteristics with "Mine 3" already in production. If we take only the price of metal as an economic variable, RO will discount each year's cash flow adjusted for the variability of metal price. The adjustment for other variables, such as operating costs, is not lumped with the revenue but based on its historic production data. Doing this, the adjustment for well known production costs will not be done with the same factor than metal price for example. This would mean that with RO cash flow components are adjusted for their risk before discounting.

The mathematical logic of NPV and RO can be put in parallel to better visualize the differences. The formula for conventional discrete DCF follows, with "t" for time and "r" the discount rate. The cash flows here are discounted to time 0. The discount rate can often vary around 15% depending on company policy.

$$NPV = \sum_{t=o}^{T} \frac{(Revenues_n - Operating\,costs_n) - Capital\ Expenditures_n}{(1+r)^n}$$

The NPV is a criterion often used. A positive answer will mean that the investment is worthwhile. However, we can see that in fact the Revenue, the OPEX and the CAPEX are discounted with the same risk-adjusted discount rate whatever their uncertainty. If RO is used, the discounting procedure is maintained. What is different is that each component of

$$NPV_{project} = \sum_{t=o}^{T} \frac{\left[(Revenues_n \times RDF(R)_n) - Operatingcosts_n \times RDF(O)_n) - Capital\,Expenditues_n \times RDF(C)_n\right]}{(1+rsklr)^n}$$

the cash flow is adjusted for each period n of time. Here, $RDF(R)_n$ is the factor that adjusts the revenue for time n. $RDF(O)_n$ and $RDF(C)_n$ are respectively the factors that adjust for the OPEX and the CAPEX risks. Now that risk is properly taken into account, time discounting of cash flows can be performed with a discount rate without risk (rsklr) such as the rate of American Treasury Bonds.

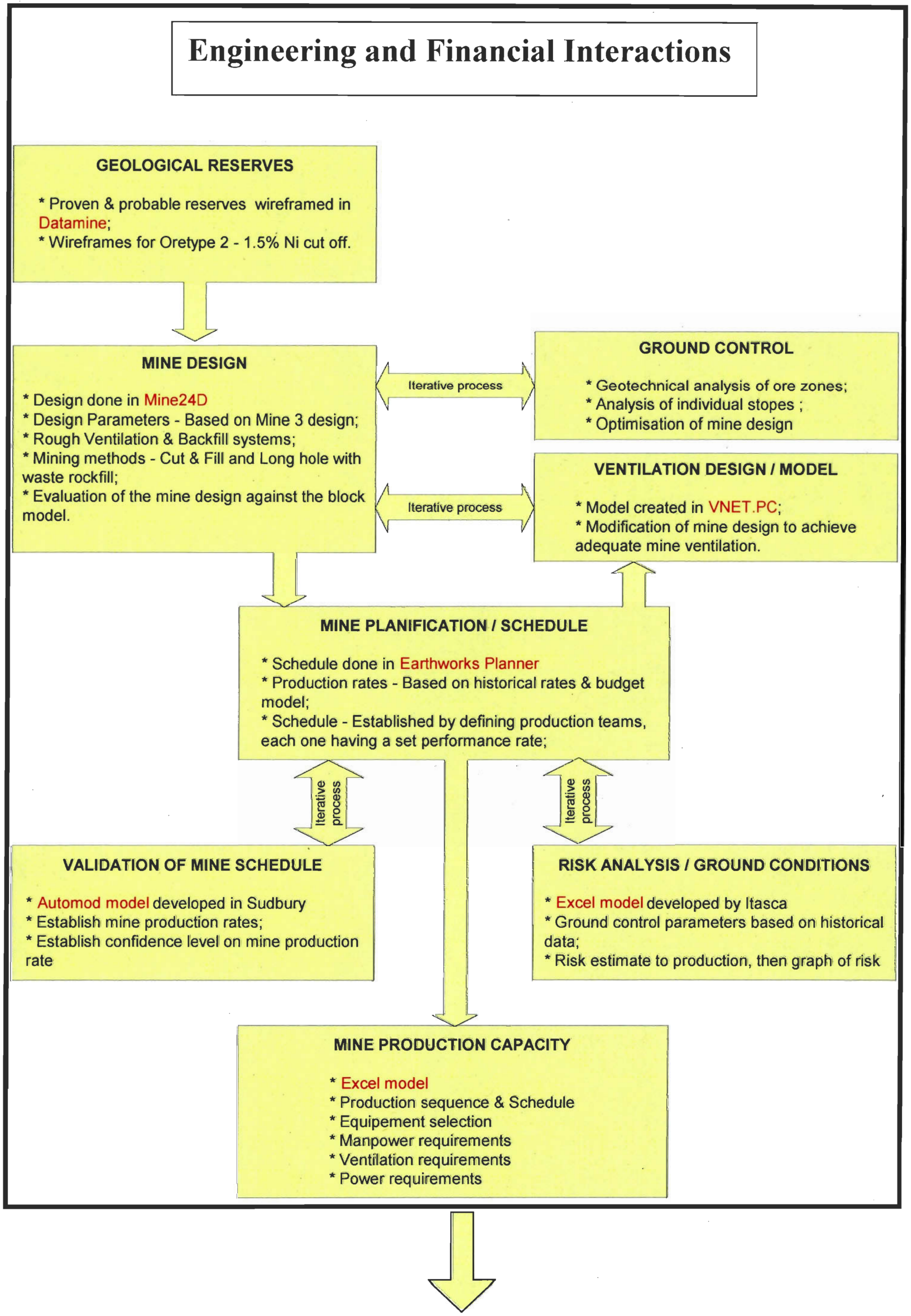
The result should be closer to the true value of the project. Utilizing the real options theory has a greater benefit in that it has the capacity to add a temporal discount model based on advanced predictive patterns. The use of a more sophisticated tool to analyze project investment requires an elaborate mathematical procedure which can be too complex for the average manager to utilize properly. The manager is required to do the interpretation. The model is submitted by an economist familiar with the uncertainty components of the parameters associated with the mining world. For example, metal price model can be used with different dispersion techniques or with a mean reverting function or with out provision for shocks (see Schwartz, 1997, for example).

We have seen in the Raglan example that the financial results are highly sensitive to input components such as nickel price and foreign exchange rate. These values are where the larger uncertainty resides. It would be of interest to see the East Lake project under a RO scenario with a mean reverting price model and a foreign exchange model. The influence on foreign exchange rate is not negligible for a mineral project when metal price is given in American dollars and cost incurred in local currency (Samis et al, 2003).

With RO, it is possible to put forward a profitability criterion in accordance with the objectives of the firm. In fact, the internal rate of return criteria can also be applied because the IRR methodology calculates a yield from expected cash flows. Then, the internal rate of return of the project should not change with the real options method.

Conclusion

The objective of this paper is to better understand the RO approach and to evaluate the potential benefits if used in a decision model by a mining company such as Falconbridge Ltd. The mine planning of the ore zones at the Raglan property follows an extensive procedure. Especially complex is the determination of its mining sequence. Optimization of return are a priority after that the safety of worker and environmental protection of the area. DCF technique is simple and well accepted to evaluate the economic merit of a project. However, this simplicity is a disadvantage if we were to choose to adjust for the risk associated with each of the input variables. The discount rate used in a DCF analysis is a trade off between one-size fits all for calculation and expressing the risk involved. The RO technique appears to have potential that we would like to put to the test in a practical case at Raglan. The application of the technique is favor by the existence of future market for commodities and exchange rate. It is possible that RO permit a better comprehension of the economics of the many mineralized zones present at Raglan and could eventually influence the sequencing of their mining. We have the tools to perform advance calculations making possible the use of sophisticated models to run elaborate simulations. We are not required to accept over simplification for calculating purposes. We are interested to see if in the end the use of RO will bring meaningful difference or academic nuances.



Engineering and Financial Interactions

GEOLOGICAL RESERVES

* Proven & probable reserves wireframed in Datamine;
* Wireframes for Oretype 2 - 1.5% Ni cut off.

MINE DESIGN

* Design done in Mine24D
* Design Parameters - Based on Mine 3 design;
* Rough Ventilation & Backfill systems;
* Mining methods - Cut & Fill and Long hole with waste rockfill;
* Evaluation of the mine design against the block model.

GROUND CONTROL

* Geotechnical analysis of ore zones;
* Analysis of individual stopes ;
* Optimisation of mine design

VENTILATION DESIGN / MODEL

* Model created in VNET.PC;
* Modification of mine design to achieve adequate mine ventilation.

MINE PLANIFICATION / SCHEDULE

* Schedule done in Earthworks Planner
* Production rates - Based on historical rates & budget model;
* Schedule - Established by defining production teams, each one having a set performance rate;

VALIDATION OF MINE SCHEDULE

* Automod model developed in Sudbury
* Establish mine production rates;
* Establish confidence level on mine production rate

RISK ANALYSIS / GROUND CONDITIONS

* Excel model developed by Itasca
* Ground control parameters based on historical data;
* Risk estimate to production, then graph of risk

MINE PRODUCTION CAPACITY

* Excel model
* Production sequence & Schedule
* Equipement selection
* Manpower requirements
* Ventilation requirements
* Power requirements

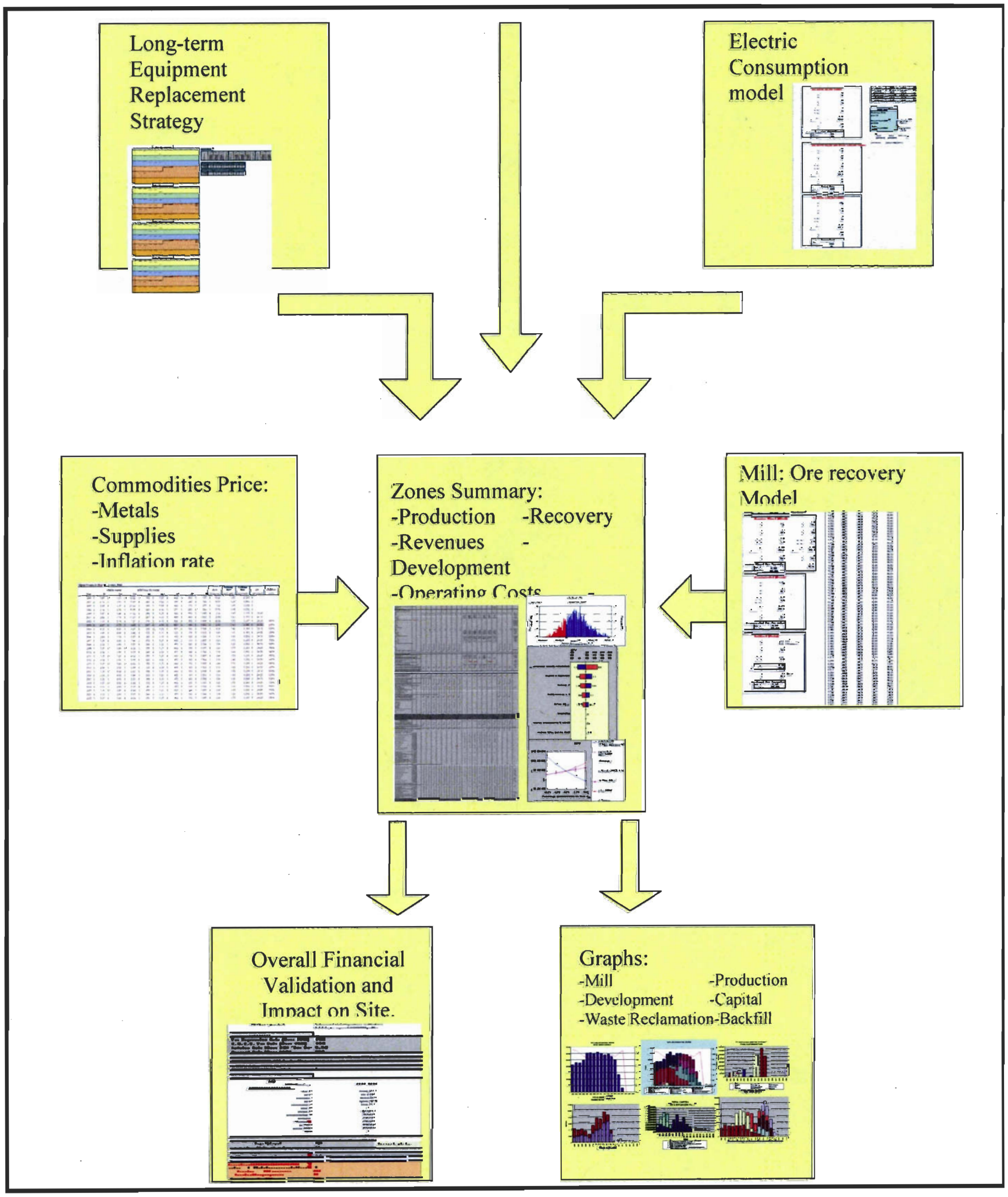


Figure 9. Engineering and Financial Interactions Flow Chart

References

Abdel Sabour, S.A., 1999. Decision making with option pricing and dynamic programming: development and application. *Resources Policy*, 25(3)257-264.

Abdel Sabour, S.A., 2001. Dynamics of threshold prices for optimal swiches: the case of mining. *Resources Policy*, 27(4)209-214.

Brennan, M.J.and Schwartz, E. (1985): Evaluating Natural Resource Investments, *Journal of Business*, 58(2) 135-157.

Cortazzar, G. and Casassus, J., 1998. Optimal timing of a mine expansion: Implementing a real options model. *The Quarterly Review of Economics and Finance*, 38 755-769.

Dupras, N and Green, A.H. Exploration and Development History of the Raglan Ni-Cu-PGE Deposits, Merc Guidebook Series Volume 2 1-16

Frimpong, S., and J. M. Whiting, 1997. Derivative mine valuation: Strategic investment decisions in competitive markets. *Resources Policy* 23 (4) 163-71.

Lesher, C.M., 1998. Raglan/Katinniq Field Guide, Falconbridge Limited, Internal Report.

Mardones, J. L., 1993. Option valuation of real assets: application to a copper mine with operating flexibility. *Resources Policy* March: 51-65.

McCarthy, J. and P. Monkhouse, 2002. To open or not to open, or what to do with a closed copper mine. *Journal of Applied Corporate Finance* 15 (2) 63-73.

Nemcsok, G., January 24, 2004Datamine Software reference, Internal memorandum.

Samis, M. and Poulin, R., 1998. Valuing management flexibility: A basis to compare the standard DCF and MAP frameworks. *CIM Bulletin*, 91(1019)69-74.

Samis, M., Poulin, R. and Blais, V., 2003. Using Real Options to Analyze the Effect of Contractual Investment Restrictions : The Case of Construction Guarantees. CIM Annual General Meeting, Montréal.

Samis, M., Davies, G., Laughton, D. and Poulin, R., upcomming. Discounting Risky Project Cash Flows When There are No Options: A Real Options Approach. *Resources Policy.*

Samis, M. and Davies, G. Actively recognizing cash flow uncertainty within the net present value calculation, Draft article for 2003 Winter Newsletter of the Mineral Economics and Management Society. 8p

Schwartz, E. S., 1997. The stochastic behavior of commodity prices: implications for valuation and hedging. Journal of Finance 52 (3): 923-973.

Smith, L, 2000 Discounted cash flow analysis: methodology and discount rates. Mineral Property Valuation Proceedings: Papers Presented at Mining Millennium 2000. Canadian Institute of Mining, Metallurgy and Petroleum, Montreal, Quebec: 85-100.

St. Onge, M.R., Lucas, S.B., and Parish, R.R., 1991, Terrane accretion in the internal zone of the Ungava Orogen , northern Quebec. Part 1: Tectonostratigraphic assemblages and their tectonic implications. Canadian Journal of Earth Sciences Vol. 29.

St. Onge, M.R. and Lucas, S.B., 1991, Terrane accretion in the internal zone of the Ungava Orogen , northern Quebec. Part 2: Structural and metamorphic history. Canadian Journal of Earth Sciences Vol. 29.

Wabete, I. Rapport sur la zone 3D, Mine 3, Raglan Mine, Falconbridge Ltd, September 2003 1-73

Acceptance Letters

De :em.mnt.0.8abd4.3f972884@editorialmanager.com
[mailto:em.mnt.0.8abd4.3f972884@editorialmanager.com]De la part de Mining Technology
Envoyé : 31 janvier 2008 00:47
À : Richard Poulin
Objet : Your submission MNT19R2

Ref.: MNT19R2
A flexible mine production model based on stochastic price simulations: Application at Raglan mine, Canada
Mining Technology (TIMM A)

Dear Professor Poulin

Thank you for submitting a revised version of the above submission and your response to the comments made by the reviewers. I am pleased to confirm that the paper is accepted for publication in Mining Technology (TIMM A). It was accepted on 31/01/2008

Please ensure that you have supplied a text file prepared according to the specification in the instructions to authors (see http://www.maney.co.uk/journals/notes/mining) and a high resolution image file of each figure. Failure to do this may result in a delay in the typesetting of your paper.

Please contact us immediately if you wish to make any changes to the final submitted version of your paper. We will contact you again only if problems arise with the text or figure files. Otherwise the paper will be passed to the Production Department and you will receive PDF proofs by email in the near future. If you require further information on the progress of your paper, please contact the Production Editor at mnt.ed@materials.org.uk.

Thank you for submitting your work to Mining Technology (TIMM A).

With kind regards

Peter Alan Dowd, PhD, MSc, BSc
Editor
Mining Technology (TIMM A)