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Effective Interventions for Secondary Students with Disabilities to be Successful in Mathematics

Garret Atteberry
atteberry.garret@gmail.com

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Garret L. Atteberry

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Bradley Kaffar, Chairperson
Hsueh-I Lo

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Chapter 1: Introduction

Many individuals have heard someone say, "I am not good at math?" You probably have listened to other adults, kids, or even personally with children saying this. Alternatively, maybe, you have said this about yourself before. Children in the United States have not made adequate gains on national assessments in mathematics (Strickland & Maccini, 2012). Based on the National Assessment of Education Progress (NAEP), 65% of 8^{th} graders and 76% of 12^{th}-graders scored below proficiency level, along with over 90% of secondary students with disabilities achieving below proficiency (Strickland & Maccini, 2012). The use of mathematics is not solely used in grade school. Math is global, whether in post-secondary schooling, jobs, and everyday life activities. The process of mathematics grows an individual's ability to compute thoughts and ideas. Our civilization has based itself on the information we obtain from computing (e.g., Why Math Is So Important, Roman).

It is no secret that many students struggle with mathematics in school. Students with disabilities have a much harder time with mathematics and are not performing at grade level related to their peers (Strickland & Maccini, 2012). Students with disabilities struggle with abstract thinking and following sequences, which is much of what mathematics is. Students with disabilities need specific interventions to help them be successful (Watt et al., 2016). These interventions include concrete examples, explicit teaching, concrete-representational-abstract model, graphic organizers, and cognitive thinking strategies.

Secondary students (students in grades 6-12) tend to take math classes to prepare for post-secondary education or prepare to enter the workforce following school. Students in Minnesota take the following routes in their mathematics education. During grades 6 through 8, students learn portions of numbers, operations, algebra, geometry/measurement, and data analysis/probability (Minnesota Department of Education, 2007). Students need to take three credits of mathematics during their high school careers in Minnesota. Generally, the courses are in grades 9 through 12 include: algebra, geometry, statistics, and probability. Since 2015, students need to complete an Algebra II credit or equivalent part of the 3-credit requirement (Minnesota Department of Education, 2020).

Students with math disabilities tend to have a hard time with basic math facts (Leach, 2016). Even though these students struggle with basic facts, they still move onto higher-level classes such as algebra and geometry. Algebra and geometry classes assume that students have the prerequisite knowledge of knowing basic math facts such as simple addition, subtraction, multiplication, and division. The struggle for students is noticed in regard to basic math facts based on personal experience teaching the content for several years. It takes them extended time to solve simple problems that other students at grade level may come to automatically. This is the reason students need to learn ways to heighten their knowledge of basic math facts so they can become successful in algebra, geometry, and higher-level classes. Once students have the basic math facts mastered, it can be appropriate for secondary students to start learning algebra and geometry. Students will be able to pick up skills and strategies quicker and focus on these skills' processes rather than get hung up on the basic facts that the problems require. A successful measurement for the students is demonstrating a level of mastery in the subject areas.

Other characteristics of students with disabilities related to mathematics include difficulty processing information, resulting in difficulty reading and problem-solving. They have trouble distinguishing relevant information in story problems and difficulty with motivation due to academic failure. Students also have a hard time self-regulation and monitoring during problem-solving (Gagnon & Maccini, 2016).

Research Question

One research question guided the review of this literature:

1. What are effective interventions for secondary students with disabilities in basic math facts, algebra, and geometry?

Theoretical Background

An educator’s goal in teaching mathematics to students who struggle is to help move cognitive functioning from necessary skills to higher levels, such as computation skills, in a variety of areas and real life. Basic math skills consist of adding, subtracting, multiplying, and dividing. Algebra consists of representing real-world and mathematical situations using equations and inequalities. The area of geometry includes working with angles, perimeter, area, and volume problems. By initially building student's ability and confidence in basic facts, teachers can start to integrate algebra and geometry interventions using explicit instruction, cognitive strategies, graphic organizers, and concrete-representational- abstract strategies.

Academic studies have evaluated each of these interventions and state the effects of students’ mathematical progress. There has been an improvement in special education research due to the Council for Exceptional Children’s (2014) standards. The standards provide many research requirements, including implementation, validity, outcome measures, context, and setting. Individuals are recommended to use these in studies to help find evidence-based research practices.

Focus of the Paper

This paper reviewed literature that examines the supports of effective interventions for students with disabilities in grades 6 through 12. Students with disabilities meet Minnesota State Criteria conducted by a comprehensive evaluation by a team from a public school for specialized instruction. Then, the students get placed on an Individualized Education Program (IEP) that outlines the specific student's needs to help make adequate educational progress (Minnesota Department of Education, Students with Disabilities, 2020). The focus of the review of the literature is on effective mathematic interventions for basic facts, algebra, and geometry.

Importance of the Topic

Individuals use math long after they leave school. Whether in the grocery store trying to budget and find sales, at a job trying to find correct dimensions of a building, in a kitchen baking, and cooking for a family. Individuals utilize math strategies even when they do not precisely know it. Students who have disabilities have to work extra hard because they process slower than a typical peer. These individuals may forget steps and need additional practice to master skills. Using effective math interventions will help students with disabilities succeed in their skills and help them have positive outcomes when they get older and need to use these skills in real-world situations.

Definitions of Terms

Algebra: a mathematical process where a combination of real-world and mathematical situations using equations and inequalities involving linear quadratic, exponential, and nth root functions. Solve equations and inequalities symbolically and graphically. Interpret solutions in the original context (Minnesota K-12 Academic Standards in Mathematics, 2007).

Geometry: calculate measurements of a plane and reliable geometric figures. Know and apply geometric figures' properties to solve real-world and mathematical problems and to logically justify results in geometry (Minnesota K-12 Academic Standards in Mathematics, 2007).

Cognitive Strategies: the use of various tools to help students to organize and process information. Strategies often include mnemonic or heuristic to help students remember steps to solve a problem (Watt et al., 2016).

Concrete-Representational- Abstract Instructional Sequence: a gradual instructional method that moves students from concrete (Manipulatives) stage of learning to the representational stage (Pictures) and then to the use of abstract numbers and symbols (Watt et al., 2016).

Graphic Organizers: a pedagogical tool used to support vocabulary, organize work into steps, and to help students make connections between new and previously taught content (Watt et al., 2016).

Explicit Instruction: a type of teaching which incorporates validated teaching strategies such as cueing, modeling, rehearsal, and feedback. This instruction allows for teachers to adapt routine and instruction to accommodate strengths and weaknesses of students (Montague et al., 2011).

Chapter 2: Review of Literature

The purpose of this literature review was to examine the effectiveness of different interventions for secondary students with disabilities in basic math facts, algebra, and geometry. In the past few decades, there has been a large amount of research focused on mathematical interventions for students. This chapter is organized into three major sections: (1) studies that review the effectiveness of the Concrete-Representational-Abstract (CRA) teaching model, (2) studies that analyze the effectiveness of Explicit Teaching, and (3) studies that examine the effectiveness of Cognitive Strategies. Throughout the different learning models, the effectiveness of using them with algebra, geometry, and basic math facts will be provided. Studies within each group are presented in chronological order beginning with the oldest study.

Concrete-Representational-Abstract

Students with learning disabilities in mathematics need instruction that is broken apart and sequential in order for them to obtain the information adequately. The Concrete-Representational-Abstract (CRA) instructional sequence is a gradual method of instruction. Students proceed from the concrete stage using manipulatives to the representational stage using pictures, and then the final stage of using abstract numbers and symbols (Watt et al., 2016). The CRA method is meant for each stage to be interrelated, so there can be creative meaning between manipulatives and the abstract stages of learning. This section summarizes four articles that studied the CRA method.

Article One

Witzel et al. (2003) investigated the effects of the CRA instructional model. The study consisted of a combination of 34 matched pairs of 6^{th-} and 7^{th}-grade students. Participants were selected from 358 students based on assessments. These 34 matched pairs had Specific Learning

Disabilities (SLD) or were at risk for algebra. Students qualified for the intervention by having three points of criteria. Students needed the following: (1) performed below average in the classroom according to the teacher, (2) scored below the 50^{th} percentile in mathematics in recent state achievement test, and (3) were from a low socioeconomic background.

A pre-post follow-up approach was used by Witzel et al. (2003). Students were clustered by classroom and divided into two groups: (1) Students learning through CRA and (2) Students learning through repeated abstract instruction. The objective of the groups was to improve their pre-algebra skills. Materials used in this study included assessment instruments, daily learning sheets, and the treatment group used manipulative objects. The same math teacher taught both members of each matched pair of students throughout different classes. The classes were taught in the general education setting being inclusive with students with SLD.

The data were analyzed using the repeated measures of analysis of variance (ANOVA) because of the use of a pre- and post-testing phase along with the follow-up of the two levels of instruction, CRA and abstract. Results of this study indicated that students receiving algebra instruction through CRA outperformed peers receiving traditional instruction. Below are the statistics from the study.

- Pre-test Scores: $t(33) = 0.63$, $p = 0.27$. This supports that there were no significant differences between the treatment and condition group.
- Post-test Scores: $t(33) = 6.52$, $p < 0.01$. The group receiving CRA instruction ($M = 7.32$; $SD = 5.48$) outperformed the group receiving abstract instruction ($M = 3.03$; $SD = 4.39$).

- Post-test Scores: $t(33) = 3.28$, $p < 0.01$. The group that received CRA instruction ($M = 6.68$; $SD = 6.32$) also outperformed the abstract group ($M = 3.71$; $SD = 5.21$) on the 3-week follow-up test.

The study does have limitations that the assessment instrument used for pre-test, post-test, and follow-up was not thoroughly evaluated. The hands-on approach in other classrooms was also effective, and the lesson sequences were similar to a typical algebra textbook. Overall, both groups of students showed significant gains in learning from the pre-test to post-test. Though, on the post-test and follow-up assessments, the students who received CRA instruction significantly outperformed the students in the other groups and supported the idea of using the Concrete-Representational-Abstract teaching model is effective.

Article Two

A study was done by Browder et al. (2012) was implemented to research the effects of grade-aligned math instruction on math skill maintenance on four middle school students with moderate disabilities. These four students were from a large urban school system in the southeastern United States. The students were taught four mathematical units (geometry, algebra, data-analysis, and measurement) using a Concrete-Representational-Abstract (CRA) intervention. During the study, the CRA interventions included graphic organizers, hands-on and visual manipulatives, step-by-step training, and organizing key facts. Multiple probes across the unit design were used to measure data. The students were individually assessed for each of the four units of instruction during each baseline probe. After the baseline, the same procedures were followed to continue with each unit.

The results form Unit One, Geometry include Student 1 increased (from *M* = 1.3 to *M* = 5.1), Student 1 increased from (*M* = 1, to *M* = 6), Student 3 increased from (*M* = 0.33, to *M* = 1.7), and Student 4 increased from (*M* = 4, to *M* = 7.4).

The results from Unit 2, Algebra include Student 1 increased from (*M* = 3 to *M* = 3.9), Student 2 increased from (*M* = 1, to *M* = 4.7), Student 3 increased from (*M* = 1.7 to *M* = 6.3), and Student 4 increased from (*M* = 4 to *M* = 7.6).

The study also shares results from the student's ability to solve problems Unit Three: data analysis and Unit Four: measurement. Because this paper is meant to research the effectiveness of strategies on Algebra, Geometry, and Basic Math Facts, Units 3 and 4 results will not be shared. A few limitations identified in this study include that they did not teach students to identify the types of problems being solved, also that three of the four students made progress. It was minimal. A final limitation is that even though some problems had stories focused on real life, the teacher did not assess the subjects' generalizations.

Overall, the study was done by Browder et al. (2012) provided evidence that supports the idea that middle school students with disabilities can learn the mathematics standards (algebra, geometry, data analysis, and statistics) with a CRA learning model including reading aloud, task analytic practice, and the use of graphic organizers.

Article 3

Strickland and Maccini (2012) examined the effects of the CRA strategy on secondary students with learning disabilities to learn algebra strategies. Three boys participated in the study. There were two 9th-grade boys and one 8th-grade boy, and each were qualified SLD students. They were deemed eligible by meeting the following criteria: (1) have a history of difficulties in algebra stated by scores and teacher input, (2) currently enrolled in 8th grade or higher, (3) demonstrate a need for intervention by scoring below 60% on baseline test, (4) recommended by math teacher for the study, and (5) have signed parent permission. The study used three lessons related to multiplying linear expressions.

A multiple-probe design across three students was used to examine the effectiveness of the CRA strategy. This includes having a baseline and intervention phase along with three types of probes. The probes used were domain, transfer, and social validity measure probes. All directions and word problems were read aloud, and participates were given the option of dictating responses to open-ended essay questions.

The results that Strickland and Maccini (2012) found in this study indicated that secondary students with learning disabilities could learn algebra strategies when being taught through the CRA learning model. According to Strickland and Maccini, the baseline scores ranged from 0% to 17% accuracy, and that the scores ranged from 78% to 93% accuracy following the intervention. The three students showed a substantial increase in the mean domain probe, 77.5, 77.5, and 69.8 percentage points. In the study, they tested maintenance on the students 3 to 6 weeks following the intervention. Two of the three participants demonstrated a high retention rate (96%, 98%, and 52% accuracy). The mean score from the social validity measure equaled 4.6 (range = 3-5; mode = 5). All participants reported that they believed the intervention was beneficial, and using the manipulatives and box method helped. The study's limitations suggest that further research should be done due to the small number of participants and that the strategy should be used in group settings to see if it is effective. In summary, the study's results support that the CRA teaching model is an effective intervention in helping secondary students with learning disabilities.

Article Four

Watt et al. (2016) investigated effective interventions for teaching algebra to students with learning disabilities. This was an analysis of 15 studies that were all related to math interventions for algebra. The study used ten experimental and five single-subject designs for examination and comparison. Their study states that even though the United States is growing in math, there are large achievement gaps for students with disabilities. The studies that were incorporated met the following criteria for selection: (1) included students with learning disabilities (2) contain algebra content, (3) examine effective instructional interventions on student achievement, (4) use experimental, quasi-experimental, or single-subject designs, and (5) have been published from 1980 through 2014.

Throughout all the studies, there were 827 total participants: 398 males (53%) and 359 females (47%). Of all the studies included gender except for one. Students' grades ranged from 3rd to 12th grade, although 79% were secondary (grades 6-12). The interventions that were reviewed were a variety of different teaching methods. The most common intervention used in the studies was the CRA teaching model. Each study that used CRA, produced high effects (g= 0.53), tau-U = 1.00) on student achievement in algebra. From the overall analysis of the 15 studies, five interventions were identified as effective, and the CRA model was one of them. The study also notes that it would be effective to use instructional strategies such as using the CRA model and cognitive strategies.

This review from Watt et al. (2016) has three main limitations. Due to a large number of single-subject studies in the review, quantitative analysis was limited. Second, most of the researchers generated and reliability alphas were only reported in three studies. Lastly, the majority of the studies contained general education students and students with a variety of disabilities, so it was not an exact measure of learning disabilities. Overall, this review does support that the CRA teaching model has a positive effect on students with disabilities learning mathematics.

Summary

This section presented the findings of four studies that evaluated the Concrete-Representational-Abstract (CRA) teaching model's effectiveness**.** Table 1 provides a summary of these findings.

Table 1

Summary of CRA Studies

AUTHORS	STUDY DESIGN	PARTICIPANTS	PROCEDURE	FINDINGS
Witzel, Mercer, & Miller (2003)	Quantitative	358 6^{th-} and 7^{th}-grade students participated in instruction. Then 34 matched pairs of students with disabilities were selected to compare results.	Students had instruction of concrete-representational-abstract (CRA) in algebraic instruction. After completion of the course, the students took a 27-question assessment to reflect their knowledge.	Both groups of students showed significant learning from the pre-test to post-test. This shows that CRA is effective and that teachers should use concrete and pictorial representations to help teach students with disabilities.
Browder, Jimenez & Trela (2012)	Quantitative	Four secondary students with disabilities in a large urban school in the southeastern United States	Students received math intervention using task analysis processes using math problem stories and graphic organizers. The teachers implemented four math units (Algebra, geometry, measurement, and data analysis/ probability)	Results show a functional relationship between math instruction and student responses to questions. These interventions are effective.

Table 1 (continued)

AUTHORS	STUDY DESIGN	PARTICIPANTS	PROCEDURE	FINDINGS
Strickland & Maccini (2012)	Quantitative	Three total students with disabilities in grade 8 and grade 9.	Students were receiving Concrete-representation-abstract instruction in algebraic equations.	Using Concrete manipulatives, sketches, abstraction notation, and graphic organizers improved students understanding of algebraic equations
Watt, Watkins, & Abbitt (2016)	Quantitative	827 Students in grades 3-12 with disabilities across all studies	Reviewing 15 studies for effective interventions for teaching Algebra to students with Disabilities	Identified five effective interventions.

Explicit Teaching

Students with disabilities that struggle in math class benefit from individualized instruction at their level. These students need many examples and practice problems to help them generalize the skills. The Explicit Teaching Model (ETM) incorporates validated teaching strategies such as cueing, modeling, rehearsal, and feedback. This model uses structured, organized lessons, guided practice, and feedback. ETM is more flexible than other models because it is interactive, and instructors can modify it depending on their students' level (Montague et al., 2011).

Article One

Leach (2016) wrote an article about a detailed explanation of a tier three multiplication fact fluency intervention. This is a type of explicit teaching style. This intervention used high-probability instructional sequences, explicit, systematic, intensive instruction to increase motivation and fluency development. The intervention was used as a case study of one single student "Justin." The student, Justin, is in 4th grade and is being evaluated to determine if he is eligible for special education services due to his lack of mathematic skills. A key struggle for Justin is that he does not remember his multiplication facts. The school Justin is in provided intensive tier three interventions as part of the school's response-to-intervention.

This high-probability and explicit instruction involve presenting students with a series of facts that a student can quickly recall. This intervention is explicit because it directly teachers the approach that includes the procedures to solve the problems. It provides modeling, multiple times to practice, and frequent student responses/feedback. In this case study, Justin participated in the baseline assessment, the intervention, and the post-test assessment.

Leach (2016) began with a pre-test of 80 flashcards for numbers 2-9 multiplication facts. At baseline, Justin averaged 56 of 80 single-digit multiplication facts throughout the three assessments. The instruction was implemented one-on-one with the student and teacher. The intervention occurred 4 to 5 days a week for 10-minute sessions. At the end of each week, Justin would take the 80-flash card assessment. Table 2 has data of Justin's scores.

Table 2

Results of Leach (2016) Study

Week	Overall Facts
1	58/80
2	63/80
3	71/80
4	74/80
5	80/80

Justin maintained fluency in all the single-digit multiplication facts 3weeks following the end of the intervention. Justin's general education teacher reported that his success with the intervention made a positive impact on his overall mathematics performance in class, along with having an increased level of motivation and success. Leach's study did not note any limitations or implications to report. The only information to note is that this case study was only done with one student and was not on many students.

This method of explicit teaching turned to be very cost-effective due to only needing to purchase blank note cards. This intervention can be very flexible due to only needing 10 minutes of instruction daily. Another positive about this type of explicit teaching intervention is that it can teach fluency in addition, subtraction, division facts, and many other discrete skills. Overall, this case study supports the idea that explicit teaching can be an effective intervention to teach students basic math facts, which will overall help them with their ability to solve math problems.

Article Two

Ziegler et al. (2017) conducted a study to examine if explicit teaching would improve problem-solving knowledge and verbal explanation of algebraic addition and multiplication in the classroom. This article defines explicit learning and teaching as being intentional, deliberate learning that is striving at making students aware of rules and regularities. Explicit teaching can be incorporated by directing student attention to the structure of concepts and asking students to explain their thoughts verbally. The study looks at the introduction of algebra material with explicit teaching or implicit teaching style. For the explicit learning condition, they considered the former dataset with students who were trained via explicit verbally from Ziegler et al. (2017).

Participants in this study included 153 6^{th}-graders who had no prior formal instruction in algebra. A total of 155 students from eight classes participated. Students were randomly assigned to one of the two conditions. Students were instructed in mixed groups. The explicit learning condition included 79 students, and the implicit learning condition included 74 students. Two students did not finish the intervention; therefore, they were excluded. Ziegler et al. (2017) used a 2 x 3 mixed-factorial design with repeated measures. Conditions included explicit and implicit learning, and times were 1 day, 1 week, and 10 weeks after instruction.

The students participated in four intervention sessions and three follow-up sessions. The students had 90-minute sessions on four consecutive days. Each session included a self-study program with nine worksheets. After each worksheet, a learning test with three to eight problems were used to examine their progress. The intervention took place in groups of 10-15 students in rooms at the school.

Results of this study showed a main effect of condition was observed in favor of the explicit learning condition, $F(1149) = 7.42$, $p = .004$, $n^2\ p = 0.5$. The post hoc *t-tests* revealed differences at all measure points with no weak to moderate effects. On three follow-up tests over 10 weeks, the students who received explicit learning demonstrated better problem-solving knowledge. Overall, explicit learning outperformed the implicit problem-solving on the main measures assessing algebraic term transformation. This supports the idea that explicit teaching strategies and interventions can help secondary students learn algebra facts and be successful in mathematics.

Article Three

Satsangi et al. (2019) compared the use of video modeling to explicit face-to-face instruction for teaching students with learning disabilities geometry problems. The reason behind this study was to compare the effectiveness of using new technology as an intervention or face-to face explicit teaching. Students with learning disabilities have weaknesses in problem-solving and short-term and long-term memory, and poor organizational skills. There have been lots of special education research that states for mathematics instruction to be structured and explicit.

This study defines explicit instruction as reviewing prerequisite knowledge skills, modeling the explicit actions needed to use, and then using guided and independent practice.

In this study, the participants were three female 9th-grade students who had a Specific Learning Disability in mathematics. The students needed to meet the following qualifications to participate: (1) currently, in grades 9-12 and enrolled in an Algebra 1 course, (2) identified with a learning disability in mathematics, (3) recommended for participation by their mathematics teacher based on performance, (4) scored below 50% on research pre-assessment on the ability to solve geometry problems. The intervention was taught one-on-one during five 20-minute sessions of each of the two treatment conditions: video modeling or explicit teaching. The interventions taught students to solve geometry word problems in which they were focused on area and perimeter. This study's experimental design was a single-subject alternating-treatments design across three students was used to compare the effects of video modeling and explicit face-to-face instruction.

Results from this study indicate that all three students scored above their baseline levels with both treatments. The students earned 80% or higher scores with each across all intervention sessions. When comparing the scores, the explicit instruction intervention earned higher average accuracy scores than the video modeling for two out of three students. The third student showed identical performance with both. By looking at the effect size of both treatments the Tau-U scores was 1.0, 95% confidence interval (CI; [0.5666, 1.0]), for video modeling and 1.0, 95% confidence interval (CI; [0.5666, 1.0]), for explicit instruction. This study's limitations state that the authors assessed only three variations of the area and perimeter word problems. The difficulty level of the problems should have been noted, and the authors did not incorporate a continuous baseline condition. Even though both interventions produced higher accuracy scores, overall, the explicit instruction condition generated slightly greater accuracy scores for two of the three students. This supports the idea that interventions involving explicit teaching can improve students' math skills with learning disabilities in geometry.

Summary

This section presented the findings of three studies that evaluated the explicit teaching model's effectiveness**.** Table 3 provides a summary of these findings.

Table 3

Summary of Explicit Teaching Studies

AUTHORS	STUDY DESIGN	PARTICIPANTS	PROCEDURE	FINDINGS
Leach (2016)	Case Study	One student with math difficulties.	Students are given high-probability instruction sequences (RTI) to help build basic multiplication facts.	Using these RTI strategies are feasible and effective. They are easy to implement and effective with students.
Ziegler, Edelsbrunner, & Stern (2017)	Quantitative	153 6th-graders	Study researched if explicit teaching would be able to improve problem-solving knowledge and verbal explanation of algebraic addition and multiplication.	Explicit learning outperformed the implicit problem solving on the main measures assessing algebraic term transformation.
Satsangi, Hammer, & Hogan (2018)	Quantitative	Three 9th grade students who have Specific Learning Disabilities in Mathematics	The study compared the effects of video modeling and explicit teaching in the area of geometry.	Both interventions produced positive scores, but Explicit teaching had higher percentage of correct answers with two out of three students.

Cognitive Strategy

The final method of intervention, Cognitive Strategies Instruction (CSI), is used in the world of mathematics. CSI focuses on educating students with a range of cognitive and metacognitive processes, strategies, and mental activities (Montague et al., 2011). The processes used may be simple to complex and can be used in many different areas. The cognitive strategy instruction has been useful for increasing the performance of students with learning disabilities.

Article One

Montague et al. (2011) performed a study on improving mathematical problem-solving for middle school students with learning disabilities. The intervention they implemented was the “Solve It” Cognitive Strategy. The study identifies mathematical problem solving as a complex, cognitive activity involving multiple cognitive processes. The “Solve It” cognitive strategy was implemented for 7 months, and there was consistent progress monitoring.

Montague et al.’s (2011) study included 40 middle schools (grades 6-8) in a large urban district. Due to attrition at the outset, 24 schools completed the study. Eight schools were in the intervention groups and 16 were in the comparison groups.

In these schools, 319 students received the “Solve It” intervention, and 460 received typical classroom instruction. The study grouped students by math levels: learning disabilities = 78, low achieving = 344, and average achieving = 258. The “Solve It” Cognitive Strategy began in October of the school year and consisted of 3 days of intense instruction followed by weekly problem-solving practice sessions. Students in the comparison group received only typical classroom instruction that followed the class’s pacing guide. Curriculum-Based-Measurements (CBMs) were given to each intervention teacher’s class six times, baseline and monthly for the remainder of the school year. The comparison groups had four CBMs; baseline and then three more times throughout the year.

This study used a three-level model using repeated measures within students while the students were within the schools. They did use multilevel modeling (MLM) techniques for all analyses. The analysis indicated that individual differences between students accounted for

28.7% of the variability, and mean differences between schools accounted for 19.8% of the problem-solving variation. The 51.5% that was left was the variability of within-person score differences. Overall, the study found that the students who received the cognitive strategy intervention showed significantly more significant growth in math problem-solving throughout the school year than the comparison students who received typical instruction. The intervention effects did not differ for students with learning disabilities, low-achievers, and average-achieving students.

A few implications that Montague et al. (2011) addressed were that the intervention teachers were reluctant to use the intervention to give up class time of their curriculum. Another concern was the pace of the instruction of the cognitive strategy. Students with learning disabilities typically do have instruction slowed down (Montague et al., 2011). However, as the students got used to the instruction, they could work faster and still be successful. Overall, this study supports that cognitive strategies can be effective interventions for students with disabilities in mathematical problem-solving.

Article Two

Burns et al. (2016) performed a study on a cognitive strategy intervention called Incremental Rehearsal (IR). The IR method was used to teach students single-digit multiplication facts. Students would benefit from having the prerequisite knowledge of knowing basic math facts, including multiplication, from succeeding in higher-level math classes such as algebra and geometry. Students may struggle with complex math problems because they have not mastered the basic early math skills (Burns et al., 2016).

This intervention began by presenting the unknown fact on a flashcard to the student while reading it out loud to them and stating the correct answer. Then, the student was asked to restate the same thing that the teacher said. Lastly, the student was asked a final time to complete the card's problem and answer it. Once a new unknown fact was presented, the previously unknown fact was treated as the first known facts. The previous eight known facts were removed from the deck of cards, and the process began over again.

The participants of this study included 55 students in 3rd or 4th grade. Students receiving special education services were not included in this study. Even though students with disabilities were omitted, that this study supported the cognitive strategy of IR as an intervention. IR should be used with students who may or may not have disabilities but do struggle with basic math facts. The instructional and assessment sessions were conducted at a small table in a quiet place such as a media center, hallway, or breakout room. The student would work individually with the data collector while being timed to measure intervention efficiency. The next school day, an interventionist returned to the school, and tested the retention of the students. Therefore, each student participated in one intervention session and one assessment session.

The instructional set size effect was done by comparing the percent of facts retained for each condition. A one-way ANCOVA analyzed the data. The condition was the independent variable, the percentage of the facts retained was the dependent variable, and the OLPA math score was the covariate. The data showed that the students retained 76% of the math facts taught with the IR cognitive strategy but remembered less than 50% when taught two facts and remembered 33% when taught eight facts. The ANCOVA was significant, $f\,(2,52) = 8.29$, $p < 0.17$, and the effect was large ($n^2 = .25$). This supports that there was a significant difference between the three conditions on the number of facts retained.

On average, the students who received the cognitive strategy IR were taught 4.05 facts ($SD = 0.71$) and retained 3.16 facts ($SD = 1.21$). Students who were taught two facts retained an average of 0.94 facts (*SD* = 0.87), and those who were taught eight facts retained an average of 3.11 (*SD* = 1.64). The *post hoc* analysis determined that the condition led to significantly higher retention rates than students who were taught 2 facts, $t(35) = 2.55$, $p < .017$, or 8 facts, $t(35) = 5.22$, $p < .017$.

Limitations stated by the researchers, including student ages of 3rd and 4th grades, were only conducted with students who knew all eight known facts (Burns et al., 2016). The instructional stimuli were unknown facts taken from the most challenging facts to learn. Overall, this study supports that the cognitive strategy of IR is an effective intervention for students to learn and retain basic multiplication facts.

Article Three

Hraste et al. (2018) examined the efficiency of a new integrated mathematics/geometry physical activity (PA) and cognitive strategy program (CSP). The study indicated that there is a theory of a close relationship between motor and cognitive development. There are studies that discuss how motor development and cognitive development affect each other in childhood. When children use mathematics, a focus on shifting attention between dimensions and objects or other problems' characteristics exists.

This study used 36 students in 4th grade assigned to an experimental ($n = 19$) or control group ($n = 17$). The investigation was done over 4 weeks and performed in the mornings at school. The students participated in a pretest and post-test. The study of using CSP and PA involved four integrated lessons of mathematics/geometry and PA. Each lesson lasted 45 minutes. The children were learning topics of rectangles and squares with perimeters. During intervention lessons, the students would run, walk, and use other game assignments. The assignment was used to get the students moving, combining walking and running at a low to medium intensity, along the edges of rectangles and squares. Other parts of the lesson included the students acquiring geometrical knowledge of the shapes and angles with various games.

A factorial ANOVA was used to record the results of this study. The results after 4 weeks indicated that the group of students who used the intervention of PA and CSP were significantly more successful ($p < 0.05$) than the control group. Other statistics include the factor *Group* F (1, 36 = 5.051; $p = 0.031$; $p2 = 0.123$) and the factor *Treatment F*(1, 36 = 7.760; $p = 0.008$; $p2 = 0.177$). The significant impact of the factor group supports the differences between the two groups. Limitations to the study include: (a) short in duration of length (b) student's first experience of this type of teaching, and (c) a different teacher was teaching the lessons to the children in the experimental and control groups.

Overall, the study conducted supports the idea that an intervention using physical activity and cognitive processes can be useful in teaching children new mathematical and geometrical skills (Hraste et al., 2018), Even though the study was not conducted with students with disabilities, this intervention should be considered with students with disabilities who are at the skill level of mathematical and geometrical problem-solving.

Article Four

A study conducted by Watt et al. (2016) was used in an earlier section of Chapter 2 under the Concrete-Representational-Abstract (CRA) teaching method. Their study also examined the use of CRA to teach algebra to students with learning disabilities. The study was an analysis of 15 studies that were all related to math interventions for algebra. The study used 10 experimental and five single-subject designs for examination and comparison. The studies that were incorporated met the following criteria for selection: (1) included students with learning disabilities, (2) contain algebra content, (3) examine effective instructional interventions on student achievement, (4) use experimental, quasi-experimental, or single-subject designs, and (5) have been published from 1980 through 2014 (Watt et al., 2016).

Throughout all the studies, there were 827 total participants: 398 males (53%) and 359 females (47%). Gender was not reported. Students' grades ranged from 3rd to 12th grade although, 79% were secondary students (grades 6-12). The interventions that were reviewed were a variety of different teaching methods. Using the cognitive strategy instruction is using various tools to help students organize and process information. These strategies are commonly mnemonics to help students remember the steps to solve specific problems. CRA that supported positive growth in students include the "STAR" learning strategy done by Maccini (1998), the "DOTS" strategy done by Xin et al. (2008) and the "COMPS" and "SOLVE" strategy done by Xin et al. (2011), as cited in Watt et al. (2016).

Results from this study indicate that the use of cognitive strategy instruction is highly effective as an intervention (g = 0.83, *tau*-U = 1.00). The use of CRA was one of the top five interventions found within the overall study. As stated earlier, this review has three main limitations (Watt et al., 2016). Due to a large number of single-subject studies in the review, quantitative analysis was limited. Second, most of the researchers generated and reliability alphas were only reported in three studies. Lastly, majority of the studies contained general education students and students with a variety of disabilities, so it was not an exact measure of learning disabilities. Overall, the use of Cognitive Strategy Instruction is an effective intervention to teach algebra to students with learning disabilities.

Summary, this section presented the findings of four studies that evaluated the effectiveness of the cognitive strategy instruction teaching model**.** Table 4 provides a summary of these findings.

Table 4

Summary of Cognitive Strategy Instruction Studies

AUTHORS	STUDY DESIGN	PARTICIPANTS	PROCEDURE	FINDINGS
Montague, Enders, & Dietz (2011)	Quantitative	319 middle school students receiving intervention and 460 who received typical instruction	Schools using the “Solve It!” cognitive strategy math curriculum during a 7-month school year.	Students receiving the “Solve It!” intervention showed more significant math problem-solving ability.
Burns, Zaslofsky, Maki, & Kwong, (2016).	Quantitative	55 third and fourth-grade general education students.	Students received Incremental rehearsal (IR) instruction on single-digit multiplication facts. Each student was randomly assigned to be taught two multiplication facts, 8 multiplication facts, or a set size determined by their acquisition rate.	Incremental rehearsal (IR) supported to be an effective strategy to teach students multiplication facts.
Hraste, De Giorgio, Jelaska, Padulo, & Granic (2018)	Quantitative	36 fourth-grade general education students.	Students were taught cognitive learning strategies along with a physical activity to learn mathematical and geometrical facts, including perimeter, angel, of shapes.	The results after four weeks indicated that the group of students who used the intervention of PA and cognitive strategy were significantly more successful ($P < 0.05$)
Watt, Watkins, & Abbitt (2016)	Quantitative	827 Students in grades 3-12 with disabilities across all studies	Reviewing 15 studies for effective interventions for teaching algebra to students with Disabilities	Identified five effective interventions.

Chapter 2 Summary

A total of 11 studies were reviewed in for this chapter to examine the effectiveness of different interventions for secondary students with disabilities in basic math facts, algebra, and geometry. Conclusions and recommendations are discussed in Chapter 3.

Chapter 3: Conclusions and Recommendations

The purpose of this research paper was to evaluate effective interventions for secondary students with disabilities in basic math facts, algebra, and geometry. Chapter 1 provided background information on the topic and Chapter 2 presented a review of the research literature. Chapter 3 discusses findings, recommendations, and implications from research findings.

Conclusions

I reviewed 11 studies that evaluated interventions for secondary students with disabilities in basic math facts, algebra, and geometry. Three of the studies reviewed Concrete-Representational-Abstract (CRA) teaching model (Browder et al., 2012; Strickland & Maccini, 2012; Witzel et al., 2003). Three studies examined Explicit Teaching Strategies (Leach, 2016; Satsangi et al., 2018; Ziegler et al., 2017). Three other studies reviewed Cognitive Strategy Instruction (Burns et al., 2016; Hraste et al., 2018; Montague et al., 2011). The study by Watt et al. (2016) examined Cognitive Strategy Instruction and CRA teaching model.

Of the 11 studies reviewed, two reviewed interventions related to basic math facts (Burns et al., 2016; Leach, 2016). Four of the studies reviewed interventions based on algebra (Strickland, & Maccini, 2012; Watt et al., 2016; Witzel et al., 2003; Ziegler et al., 2017). Two articles investigated interventions in the area of geometry (Hraste et al., 2018; Satsangi et al., 2018). The study completed by Browder et al. (2012) reviewed the effects of interventions in the area of both algebra and geometry. The study conducted by Montague et al. (2011) investigated interventions in each area: basic math skills, algebra, and geometry.

CRA Teaching Model

All four of the articles that examined the CRA teaching model were found to improve math skills in secondary students. Witzel et al. (2003), Strickland and Maccini (2012) and Watt et al. (2016) studied the effects on algebra content. Browder et al. (2012) studied the CRA teaching model in four unit including geometry, algebra, data-analysis and measurement. These studies differed in the ways they used the CRA teaching model within each step. Each student increased from pretest to post-test in the geometry and algebra sections. Witzel et al. (2003) used daily learning sheets, manipulatives and assessment instruments. The group receiving CRA instruction ($M = 7.32$; $SD = 5.48$) outperformed the group receiving abstract instruction ($M = 3.03$; $SD = 4.39$). The Browder et al. (2012) study, used graphic organizers, hands on manipulatives, step-by step training and organizing key facts. The study done by Strickland and Maccini (2012) used word problems and did reading out loud, and were able to use open ended essay questions to assess. Then, Watt et al. (2016) did an analysis of 15 studies that were related to math interventions and discovered that the CRA teaching model had high effects of achievement in the area of algebra.

Explicit Teaching

The three articles that reviewed the math intervention of explicit teaching strategy to be an effective strategy. Leach (2016) studied the effects of explicit teaching in the area of basic math fact fluency. This intervention used high-probability instructional sequences, explicit, systematic, intensive instruction to increase motivation and fluency development. Leach's student greatly improved during the 5-week study. The student was able to maintain fluency 3e weeks following the end of the intervention. Ziegler et al. (2017) studied the effects of explicit teaching in the area of problem-solving knowledge of algebra addition and subtraction. For the explicit learning condition, they considered the former dataset with students who were trained via explicit verbally from Ziegler et al. (2017). The results they found were that the students used intervention showed the main effect $F(1149) = 7.42$, $p = .004$, $n2\ p = 0.5$. Satsangi et al. (2018)

studied explicit teaching instruction with geometry content. Their study used explicit face-to-face instruction as their intervention. Their students all scored above their baseline levels with the treatment. The students earned 80% or higher with the explicit teaching.

Cognitive Strategies

Each of the four studies that studied Cognitive Strategy Instruction (CSI) were found to be highly effective interventions. Montague et al. (2011) examined the effects of the "Solve It" strategy which used weekly problem-solving practice sessions which followed 3 days of intense instruction. Overall, the study found that the students who received the cognitive strategy intervention showed significantly more significant growth in math problem-solving throughout the school year than the comparison students who received typical instruction. Burns et al. (2016) used a cognitive strategy known as Incremental Rehearsal (IR), to teach basic math facts. the students who received the cognitive strategy IR were taught 4.05 facts (SD =0.71) and retained 3.16 facts (SD = 1.21). Hraste et al. (2018) studied a mathematics/geometry physical activity and cognitive strategy program. The study examined the strategies when used with geometry topics. The results after four weeks indicated that the group of students who used the intervention of PA and CSP were significantly more successful ($p < 0.05$) than the control group.

Overall, the studies that were reviewed in this paper indicate that the CRA teaching model, explicit teaching, and cognitive strategy instruction are effective interventions for secondary students with learning disabilities in the area of basic math facts, algebra, and geometry. These interventions should be used to grow students learning and understanding to help them be successful in math and lead to having success in the future once their academic lives are completed.

Recommendations for Future Research

Through the studies being examined, there were limitations throughout the process that should be considered. The studies varied when it came to their limitations. Two of the studies listed that they had a small number of participants and that it would be effective to test larger numbers. Another limitation included that the interventions were used in small groups and should be assessed in large group instruction to see if they are as effective. Furthermore, a couple of studies tested the students on their progress, but they did not assess the subject's generalization.

The review of studies from Watt et al. (2016) determined a few limitations that came across multiple studies. These limitations included that the quantitative analysis was limited. Second, most of the researchers generated and reliability alphas were only reported in three studies. Lastly, the majority of the studies contained general education students and students with a variety of disabilities, so it was not an exact measure of learning disabilities. The review by Witzel et al. (2003) specifically stated that the assessment instruction used for the pretest, post-test, and follow-up were not thoroughly evaluated. If the assessment instruments were formally evaluated, this would support the study as being more creditable.

Implications for Current Practice

Working with students who having learning disabilities has its ups and downs. Students need individualized instruction that has a step by step process with much practice. This is very true especially in the area of mathematics which is very formula and step by step based. These studies helped support the idea that CRA, Explicit Teaching, and Cognitive Strategy Instruction are effective interventions to help secondary students with learning disabilities learn math in the area of basic facts, algebra and geometry.

I will work to use these strategies in my current high school, special education math classes. It will take some time to master each intervention and to use it to its full potential. I will also recommend these interventions to colleagues and other teachers who teach math in the world of special education. It is important for students to build these math skills so they can gain confidence in the classroom and to be able to use basic math skills in their everyday life. A goal of being an educator is to support and teach students so they can become successful, and functional members of society.

Summary

The findings of these studies support that there are effective interventions to teach students in different areas of math. If teachers use the CRA teaching model, Explicit Teaching, and Cognitive Strategy Instruction, they should see growth in student performance and improvement in students' knowledge in the area of mathematics, specifically in basic facts, algebra, and geometry. Helping students improve their classroom performance will increase their self-esteem in many ways and they will begin to feel the success they deserve.

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